Machine Learning

Contents

1 Regression

1.1 Linear regression

1.1.1 Squared error cost function

Measures how well line fits training data

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})^2$$

m = num of training examples
$y^{(i)}$ = training example
$\hat{y}^{(i)} = wx^{(i)} + b$
$\frac{1}{m}$ finds average error for larger data sets, $\frac{1}{2m}$ makes later calculations neater

1.1.2 Gradient descent

Find w, b for minimum of cost function $J(w,b)$

1. Start with some w, b (commonly $0, 0$)
2. Look around starting point and find direction that will move the point furthest downwards for a small step size

α = learning rate
Must simultaneously update w and b

$$w_1 = w_0 - \alpha\frac{\partial}{\partial w}J(w_0, b_0)$$
$$b_1 = b_0 - \alpha\frac{\partial}{\partial b}J(w_0, b_0)$$
$$\frac{\partial}{\partial w}J(w,b) = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})x^{(i)}$$
$$\frac{\partial}{\partial b}J(w,b) = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})$$

1.2 Multiple linear regression

n_f = number of features
m = number of data points
$\vec{w}$ = vector of length n_f
X is a matrix of size $m \times n_f$
Sum of predictions of all features is the prediction of multiple linear reg

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$

Gradient descent

$$\vec{w}_j = \vec{w}_j - \alpha\frac{\partial}{\partial \vec{w}_j}J(\vec{w}, b)$$
$$b = b - \alpha\frac{\partial}{\partial b}J(\vec{w}, b)$$

Cost function and its partial derivatives

$$J(\vec{w},b) = \frac{1}{2m}\sum_{i=0}^{m-1}(f_{\vec{w},b}(X^{(i)}) - y^{(i)})^2$$
$$\frac{\partial}{\partial \vec{w}_j}J(\vec{w},b) = \frac{1}{m}\sum_{i=0}^{m-1}(f_{\vec{w},b}(X^{(i)}) - y^{(i)})x_j^{(i)}$$
$$\frac{\partial}{\partial b}J(\vec{w},b) = \frac{1}{m}\sum_{i=0}^{m-1}(f_{\vec{w},b}(X^{(i)}) - y^{(i)})$$

1.3 Logistic regression

Sigmoid function

$$g(z) = \frac{1}{1+e^{-z}}$$
$$0 < g(z) < 1$$

From sigmoid function to logistic regression formula

$$z = \vec{w} \cdot \vec{x} + b$$
$$f_{\vec{w},b}(\vec{x}) = g(z)$$

The output of f can be interpreted as the "probability" that class is 1.
ex. $f_{\vec{w},b}(\vec{x}) = 0.7$ means there is a 70% chance y is true
Logistic regression requires a new cost function because $f_{\vec{w},b}(\vec{x})$ for logistic regression is non-convex, trapping gradient descend in local minima.
Cost function

$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m} L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)})$$
$$L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}) = \begin{cases} -\log(f_{\vec{w},b}(\vec{x}^{(i)}) & \text{if } y^{(i)} = 1 \\ -\log(1 - f_{\vec{w},b}(\vec{x}^{(i)})) & \text{if } y^{(i)} = 0 \end{cases}$$

Simplified form

$$L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}) = -y^{(i)}\log(f_{\vec{w},b}(\vec{x}^{(i)})) - (1-y^{(i)})\log(1 - f_{\vec{w},b}(\vec{x}^{(i)}))$$

The loss function will decrease as f approaches $y^{(i)}$ on a graph of L vs f.
$\frac{\partial J(\vec{w},b)}{\partial w_j}$ and $\frac{\partial J(\vec{w},b)}{\partial b}$ are the same as in linear regression, just the definition of f has changed.

1.4 Softmax regression

Generalization of logistic regression, y can have more than two possible values.
The most probable value of y is the value that when given to L yields the largest loss.
Calculate z_i with $\vec{x}$ only consisting of data points that have label i?
n_f = num features
n_y = number of possible y outputs
W is now a matrix of dimensions $n \times n_f$.
$\vec{b}$ is now a vector of length n_y.

$$1 \le i \le n_y$$
$$z_i = \vec{W}_i \cdot \vec{x} + b_i$$
$$a_i = \frac{e^{z_i}}{\sum_{k=1}^{n_y} e^{z_k}}$$
$$L(\vec{a}, y) = \begin{cases} -\log a_1 \text{ if } y = 1 \\ -\log a_2 \text{ if } y = 2 \\ \vdots \\ -\log a_n \text{ if } y = n \end{cases} \tag{1}$$

1.5 Feature scaling: z-score normalization

After z-score normalization, all features will have a mean of 0 and a standard deviation of 1
μ_j = mean of all values for feature j
σ_j = standard deviation of feature j

$$x_j^{(i)} = \frac{x_j^{(i)} - \mu_j}{\sigma_j}$$
$$\mu_j = \frac{1}{m}\sum_{i=0}^{m-1} x_j^{(i)}$$
$$\sigma_j^2 = \frac{1}{m}\sum_{i=0}^{m-1}(x_j^{(i)} - \mu_j)^2$$

1.6 Over / underfitting

Underfit / high bias: does not fit training set well ($wx + b$ fit onto data points with $x + x^2$ shape)

Overfit / high variance: fits training set extremely well but does not generalize well ($w_1x + w_2x^2 + w_3x^3 + w_4x^4 + b$ fit onto training set of shape $x + x^2$ can have zero cost but predicts values outside the training set inaccurately)

Addressing overfitting

- Collect more data
- Select features ("Feature selection")
- Reduce size of parameters ("Regularization")

1.6.1 Regularization

Small values of $w_1, w_2, \cdots, w_n, b$ for simpler model, less likely to overfit

Given n features, there is no way to tell which features are important and which features should be penalized, so all features are penalized.

$$J_r(\vec{w}, b) = J(\vec{w}, b) + \frac{\lambda}{2m} \sum_{j=1}^{n} w_j^2$$

Can include b by adding $\frac{\lambda}{2m} b^2$ to J_r but typically doesn't make a large difference.

The extra term in J_r is called the regularization term.

Effectively, $\lambda \propto \frac{1}{w}$. When trying to minimize cost, either the error term or the regularization term must decrease. The larger the lambda, the more the regularization term should decrease to minimize cost, decreasing w parameters.

Regularized linear regression

$$J_r(\vec{w}, b) = \frac{1}{2m} \sum_{i=1}^{m} \left[(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)})^2 \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} w_j^2$$

For gradient descent, only $\frac{\partial J_r}{\partial w_j}$ changes (b is not regularized):

$$\frac{\partial J_r}{\partial w_j} = \frac{1}{m} \sum_{i=1}^{m} \left[(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] + \frac{\lambda}{m} w_j$$

Regularized logistic regression

$$J_r(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^{m} L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}) + \frac{\lambda}{2m} \sum_{j=1}^{n} w_j^2$$

For gradient descent, only $\frac{\partial J_r}{\partial w_j}$ changes (b is not regularized):

$$\frac{\partial J_r}{\partial w_j} = \frac{1}{m} \sum_{i=1}^{m} \left[(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] + \frac{\lambda}{m} w_j$$

2 Neural networks

n_ℓ = num layers excluding input
$n_n^{[\ell]}$ = n neurons in layer ℓ
n_f = num features
$\vec{W}$ is a vector (length n_ℓ) of matrices of size $n_n^{[\ell]} \times n_f$
$\vec{x}$ is a vector of outputs from each neuron in previous layer
$\vec{b}$ is a vector (length n_ℓ) of vectors (length $n_n^{[\ell]}$)
$\vec{z}$ and $\vec{a}$: same dimensions as $\vec{b}$
g: activation function
$1 \leq i \leq n_\ell$

$$\vec{z}_i^{[\ell]} = \vec{W}_i^{[\ell]}\vec{x} + \vec{b}_i^{[\ell]}$$
$$\vec{a}^{[\ell]} = g(\vec{z}^{[\ell]})$$

a (activation) = scalar output of a single neuron
Superscript $[i]$ is used to notate information relating to the ith layer in a neural network.
Activation functions are functions a neuron uses to output a value, more in section 2.2.
ReLU activation function: $g(z) = \max(0, z)$

2.1 Choosing an activation function

sigmoid: $a = \frac{1}{1+e^{-z}}$
tanh: $a = \frac{e^z - e^{-z}}{e^z + e^{-z}}$
ReLU: $a = \max(0, z)$
Leaky ReLU: $a = \max(\epsilon z, z)$ where ϵ is a small nonzero positive value < 1

For output layer

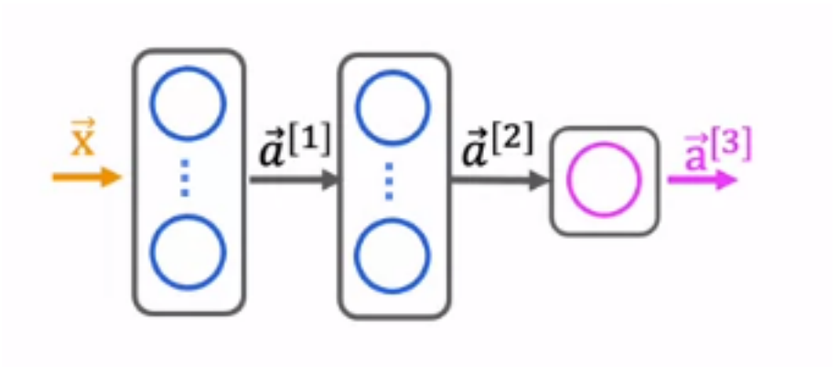
Binary classification, y = 0 or 1: use sigmoid
Regression, y $\in \mathbb{R}$: use linear activation function
Regression, y ≥ 0: use ReLU

For hidden layer

ReLU is most common

2.2 Layer types

Dense layer



Activation value of unit (neuron) j in layer ℓ

$$a_j^{[\ell]} = g(\vec{w}_j^{[\ell]} \cdot \vec{a}^{[\ell-1]} + b_j^{[\ell]})$$

Input layer is $\ell = 0$.

Convolutional layer

Each neuron only looks at a part of the previous layer's output.
May have faster computation, and needs less training data (less prone to overfitting)