

# Lecture 5: Mesh Shape Representation

DHBW, Computer Graphics

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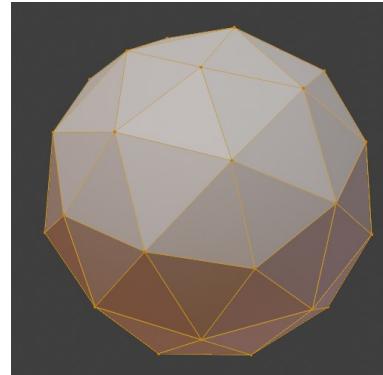
1.2.2023.

# Syllabus

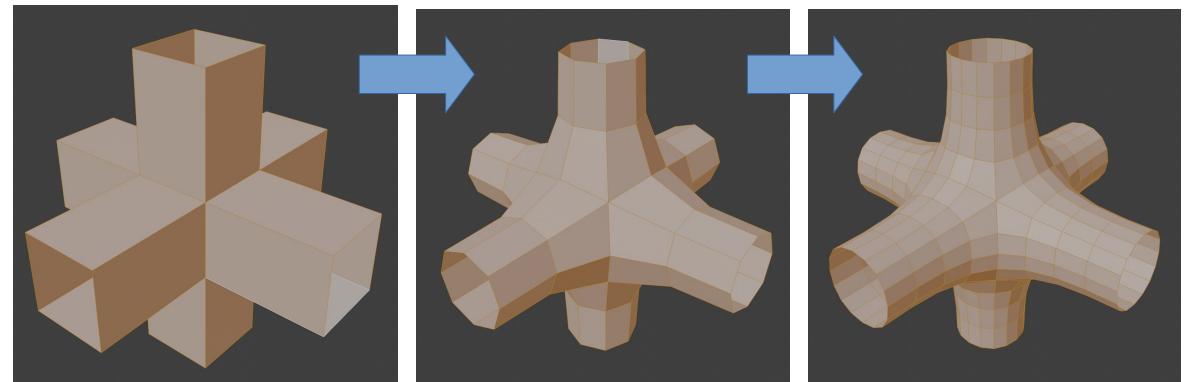
- 3D scene
    - Objects
      - Shape
    - Material
    - Lights
    - Cameras
  - Rendering
  - Image
- Object shape
    - Polygon mesh
    - Subdivision surfaces
    - Mesh and rendering
    - Mesh data structures
    - Mesh modeling and acquisition
- 
- ```
graph LR; A[Shape] --> B[Object shape]
```
- The diagram illustrates a flow from the 'Shape' item in the 3D scene section to the 'Object shape' section. An arrow originates from the 'Shape' bullet point under 'Objects' and points to the 'Object shape' bullet point within the callout box.

# Recap: shape representations

- Points
  - Point clouds
  - Particle systems
- Surfaces:
  - Polygonal mesh
  - Subdivision surfaces
  - Parametric surfaces
  - Implicit surfaces
- Volumetric objects/solids
  - Voxels
  - Space partitioning data-structures



Examples of polygonal mesh



Subdivision surfaces

# Polygon meshes

- Polygon mesh (shortly mesh) representation is one of the oldest, most popular and widespread geometry representation used in computer graphics
  - All DCC tools or game engines offer mesh representation that is used either for modeling or for rendering

Blender: <https://docs.blender.org/manual/en/latest/modeling/meshes/index.html>

Maya: <https://help.autodesk.com/view/MAYAUL/2023/ENU/>

Houdini: <https://www.sidefx.com/docs/houdini/nodes/lop/mesh.html>

Unity: <https://docs.unity3d.com/Manual/class-Mesh.html>

Unreal: <https://docs.unrealengine.com/4.26/en-US/WorkingWithContent/Types/StaticMeshes/>

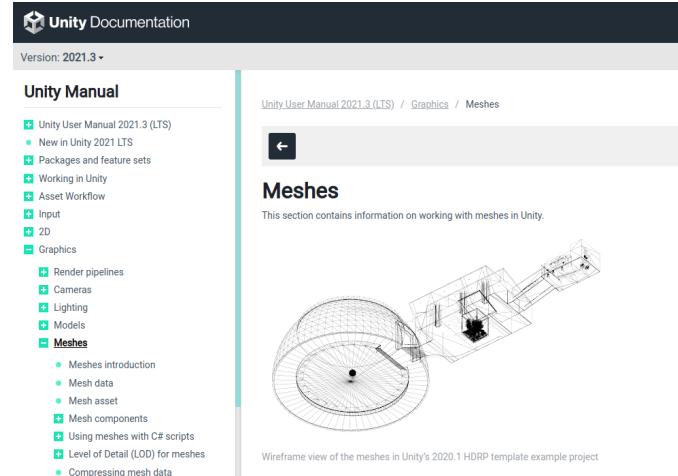
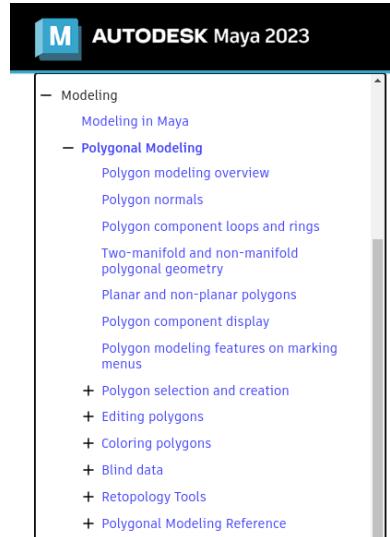
Houdini™

- Material Linker
- Material Variation
- Merge LOP
- Mesh
- Modify Paths
- Modify Point Instances

Houdini 19.5 > Nodes > LOP nodes >

Mesh

Creates or edits a mesh shape primitive.



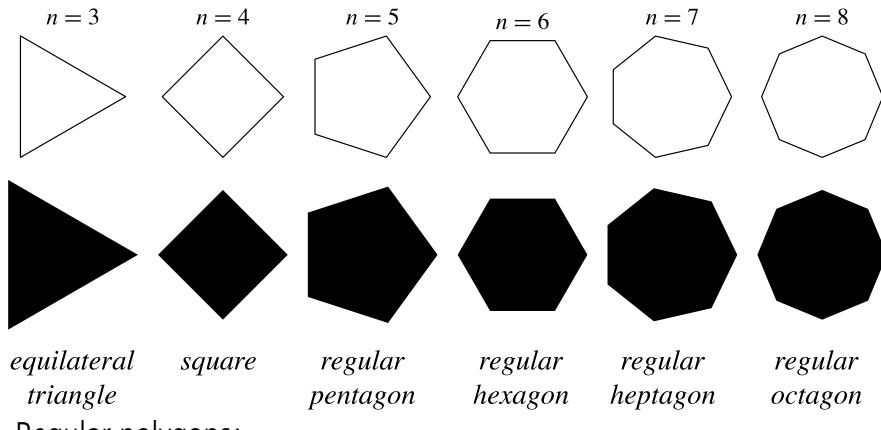
The screenshot shows the Blender 3.4 Manual. The title bar says "Blender 3.4 Manual". The left sidebar has a tree view. Under "Modeling", "Meshes" is selected, and its sub-sections are listed: "Introduction", "Structure", "Primitives", "Tools", "Selecting", "Editing", "Properties", and "UVs". The main content area shows a 3D render of a character model with a wireframe overlay, illustrating mesh editing.

\* Very often, mesh is commonly used for transporting models and scenes from DCC tools to game engines. DCC tools enable modeling using different shape representations, but in a lot of cases, all shapes are transformed to mesh representation and exported to other programs.

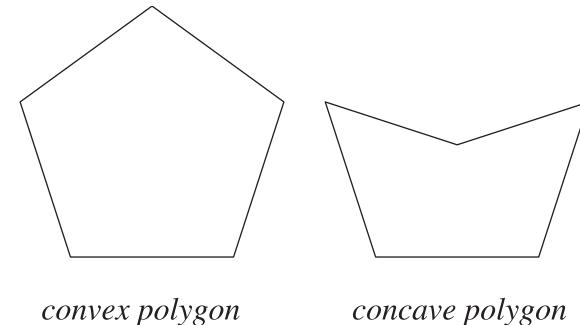
# Polygon mesh

# Polygon

- Polygon is planar shape defined by connecting array of points.
- **Vertices** (vertex, singular) - individual points
  - In 2D they are defined using two coordinates, e.g., (x,y)
  - In 3D they are defined using three coordinates, e.g., (x,y,z)
- **Edges** - lines connecting two vertices
- **Polygon (face)** – inner part of a connected and closed line of edges
  - Order of connecting vertices (**winding direction**): **clockwise** or **counterclockwise**
  - Face orientation, defined by **normal**, depends on winding direction



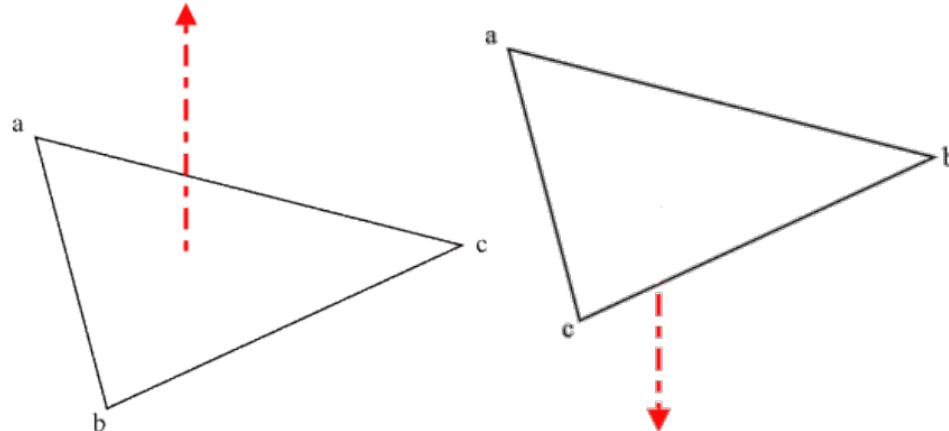
Regular polygons:  
<https://mathworld.wolfram.com/RegularPolygon.html>



Convex polygon: connection of any two points completely lies inside polygon:  
<https://mathworld.wolfram.com/ConvexPolygon.html>

# Polygon

- Winding order: order of connecting vertices **clockwise** or **cOUNTERCLOCKWISE**
- Face orientation, defined by **normal**, depends on winding direction

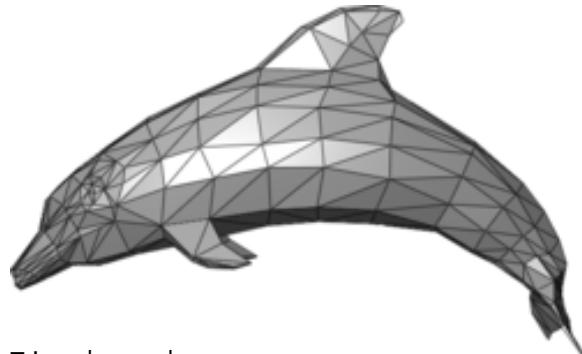


Left: anticlockwise. Right: clockwise

<https://research.ncl.ac.uk/game/mastersdegree/graphicsforgames/indexbuffers/Tutorial%208%20-%20Index%20Buffers.pdf>

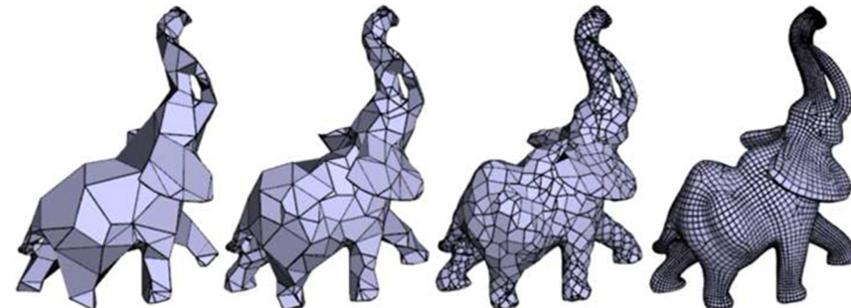
# Polygon mesh

- **Polygonal (surface) mesh:** boundary representation of an object
  - 2D surface embedded in 3D space
- Assumption: objects are hollow and can be represented by an **approximation** of their surface using polygons
  - Note that this representation is not good for example smoke and volumetric effects.
- Typical polygons: **triangles** and **quads**



Triangle mesh:

[https://en.wikipedia.org/wiki/Polygon\\_mesh](https://en.wikipedia.org/wiki/Polygon_mesh)

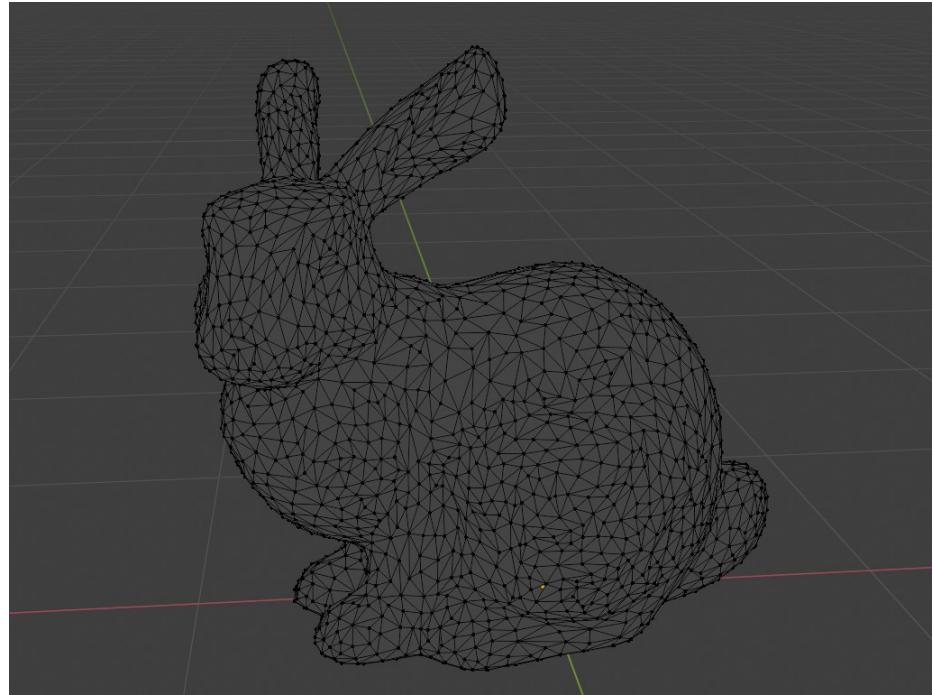
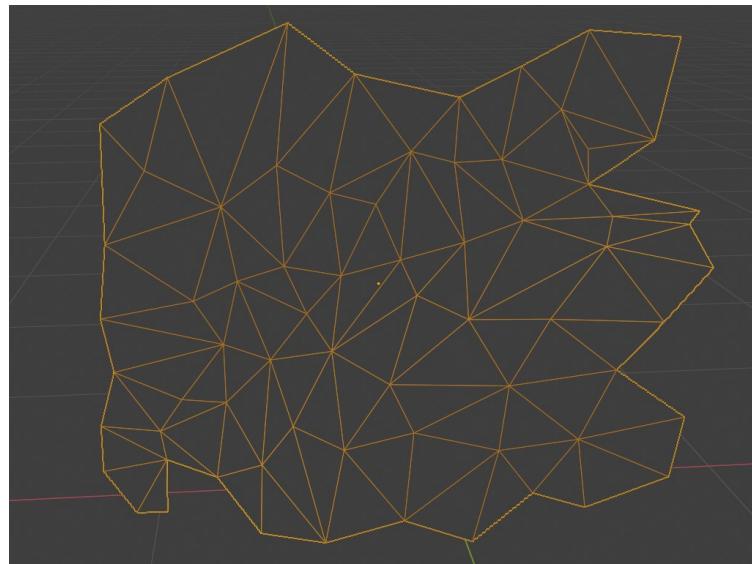
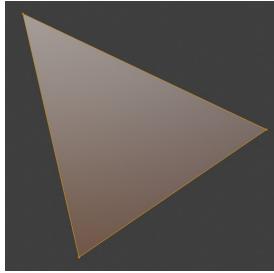


Quad mesh:

<https://www.sciencedirect.com/science/article/abs/pii/S0097849312000623>

# Triangle mesh

- Polygon with three vertices → **triangle polygon**.
- **Triangle mesh** consists of many triangles joined along their edges to form a surface

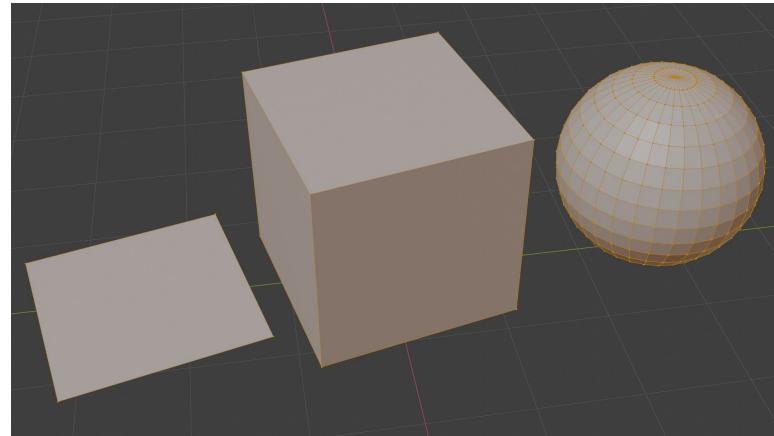


# Triangle mesh

- Triangle mesh is foundational and most widely used data-structure for representation of a shape in graphics
  - All vertices lie in the same plane – **always coplanar**
  - GPU graphics rendering pipeline is optimized for working with triangles
  - Easy to define ray-triangle intersections needed for ray-tracing-based rendering
  - Easy to subdivide in smaller triangles – triangle is replace with several smaller triangles (e.g., smoothing)
  - Texture coordinates and other data are easily interpolated across triangle
- Different shape representations used in modeling and acquisition can be transformed to triangle mesh

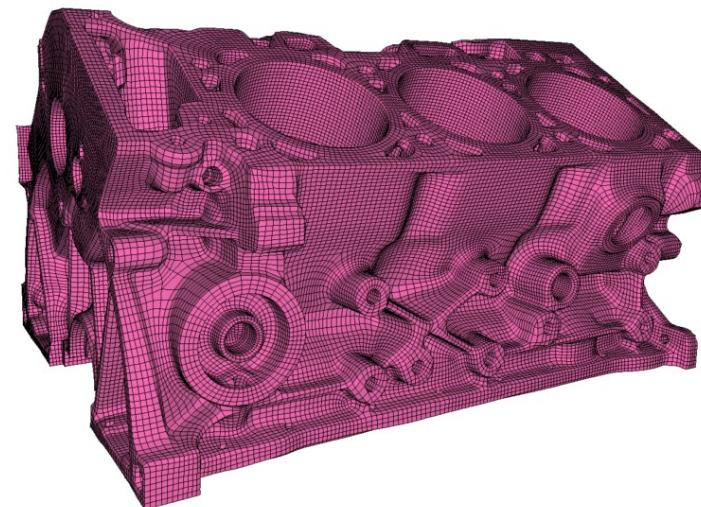
# Quad mesh

- Polygon with four vertices → quadrilateral polygon, shortly quad
- Often used as a modeling primitive
- Problem: not all quad vertices lie on a plane (not necessarily coplanar)
  - More complex ray intersection testing
- For rasterization-based rendering converted to triangle mesh
- Ray-tracing rendering can define ray-plane intersection



Blender mesh:

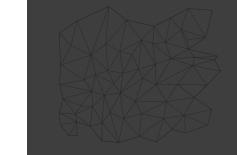
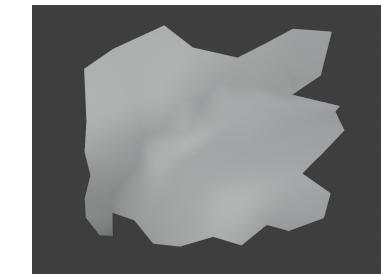
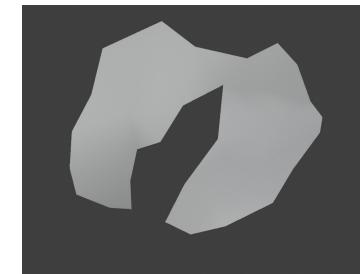
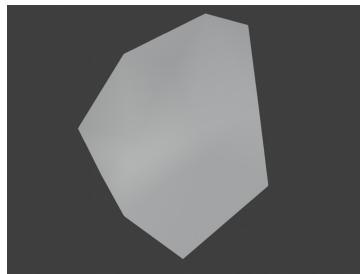
<https://docs.blender.org/manual/en/latest/modeling/meshes/index.html>



<https://geometryfactory.com/products/igm-quad-meshing/>

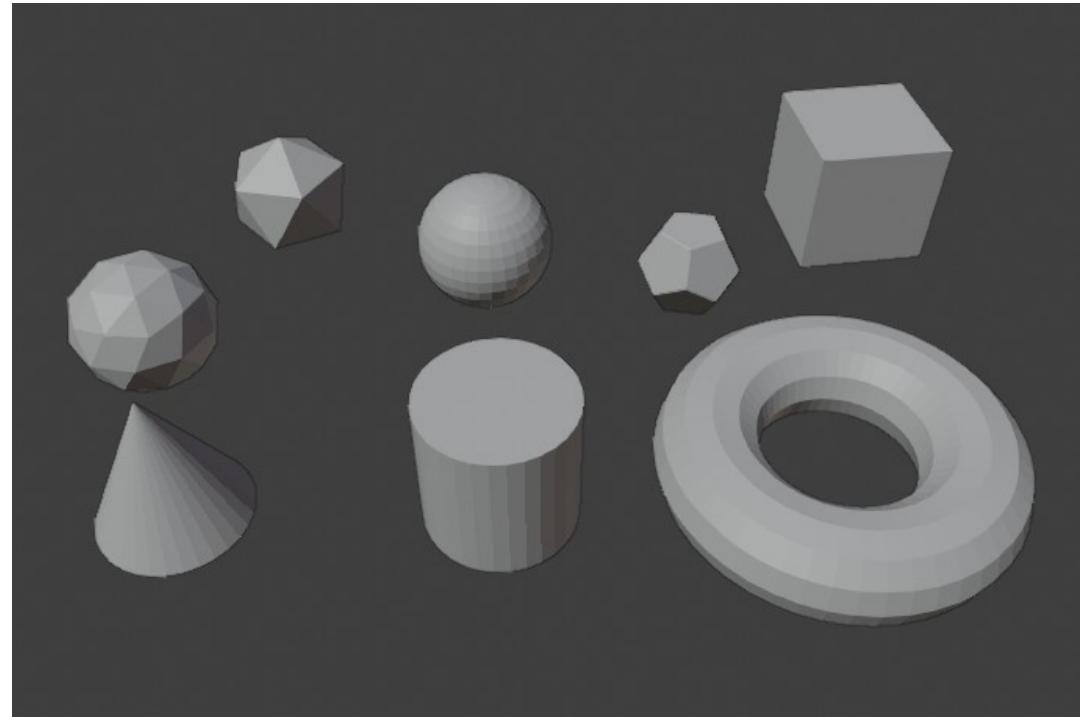
# General polygon

- **General polygon:** polygon with more than four vertices
  - Can be quite complex, e.g., holes.
- **Face (atomic element)** of mesh can be any polygon.
  - Good practice: keep atomic elements as simple as possible (so that computation is easier) and combine those atomic elements into more complex shapes, e.g., convex or concave meshes or meshes with holes.



# Polygon mesh representation

- Basic information needed for representing and storing any polygonal mesh:
  - **Vertex positions:** represented by coordinates  $(x,y,z) \rightarrow$  geometry
  - **Vertex connectivity:** how are vertices connected to form polygon mesh  $\rightarrow$  topology



# Properties of meshes

- Manifold mesh

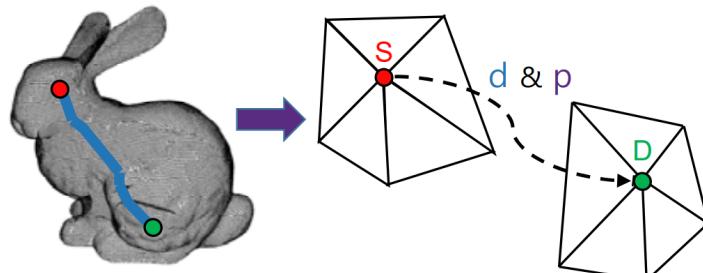
- Mesh is manifold if the intersection of two polygons is empty, a common vertex or a common line
- Desired property of many algorithms and general in graphics

- Mesh genus

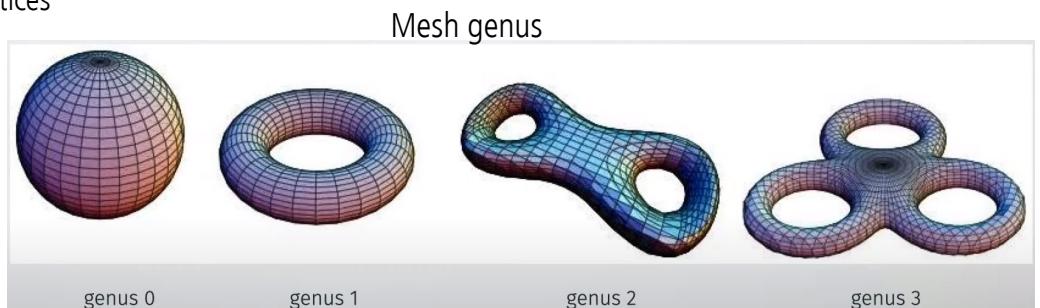
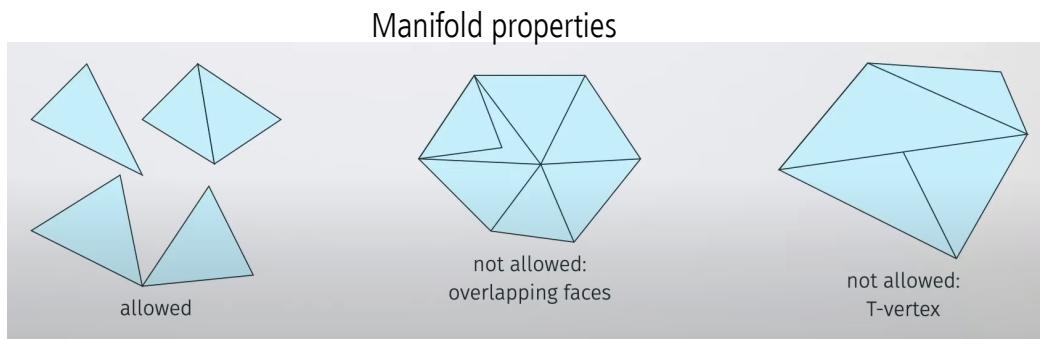
- Intuition: "number of holes"
- Important for computational topology

- Meshes as graphs

- Vertices, edges and faces
- Application of graph algorithms, e.g., shortest edge path between two vertices



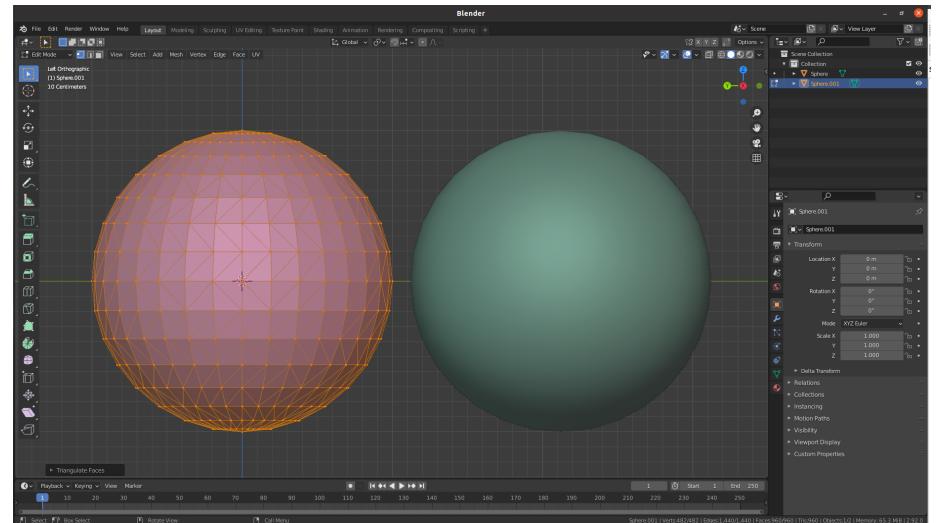
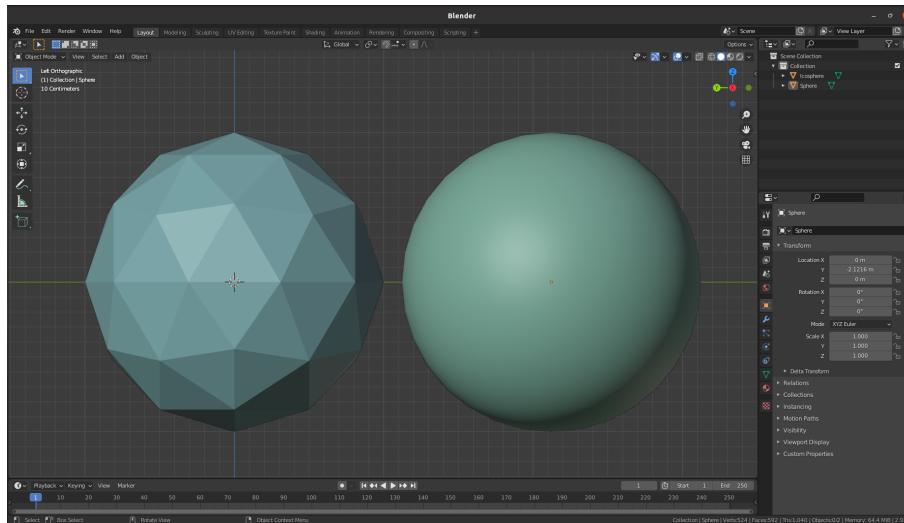
Mesh as graph: <http://www.jatit.org/volumes/Vol95No18/12Vol95No18.pdf>



[https://www.youtube.com/watch?v=V3Npa0uZYIE&list=PL4TpdkuzgxxUVZ-\\_DiO33kp4\\_rkoAy1BC&index=6&ab\\_channel=ChristophGarth](https://www.youtube.com/watch?v=V3Npa0uZYIE&list=PL4TpdkuzgxxUVZ-_DiO33kp4_rkoAy1BC&index=6&ab_channel=ChristophGarth)

# Polygonal approximation

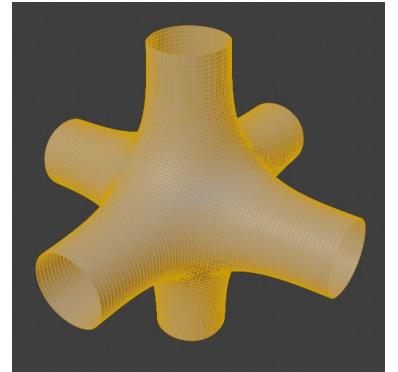
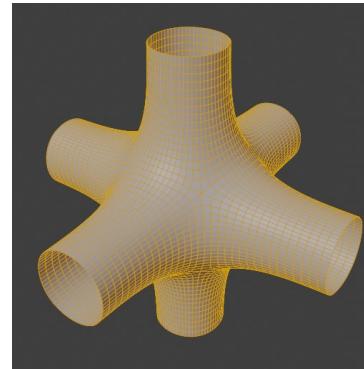
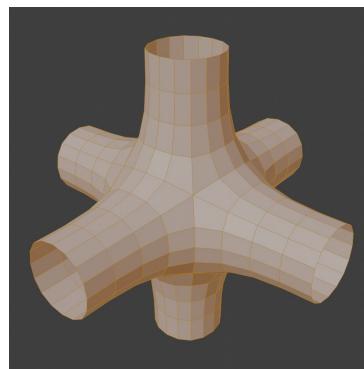
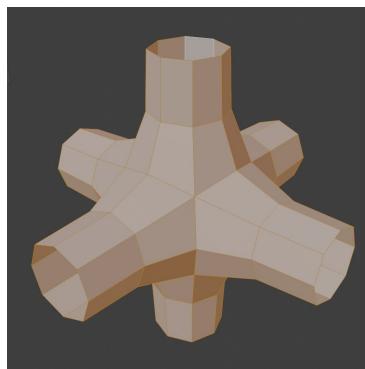
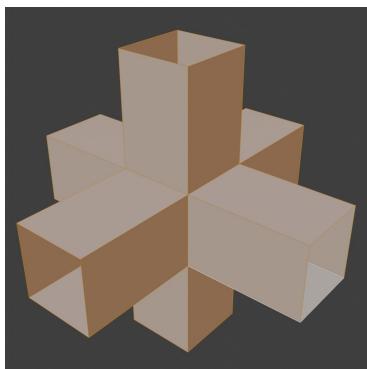
- Example: ideal sphere vs polygon sphere
- **Polygonal approximation:** piece-wise linear (flat) approximation of continuous surface with polygonal mesh



# Subdivision surfaces

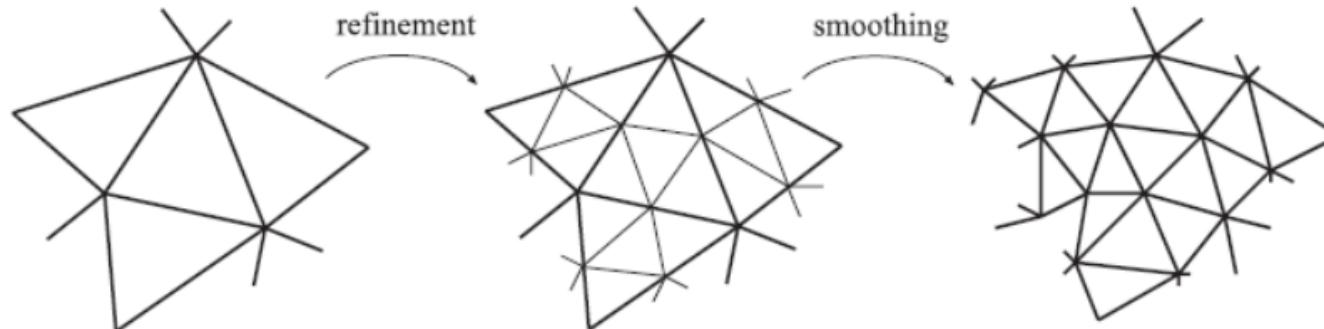
# Representing smooth surfaces

- **Subdivision surfaces:** methods for defining smooth and continuous surfaces from meshes with arbitrary topology
  - Infinite level of detail: as many as needed triangles can be generated



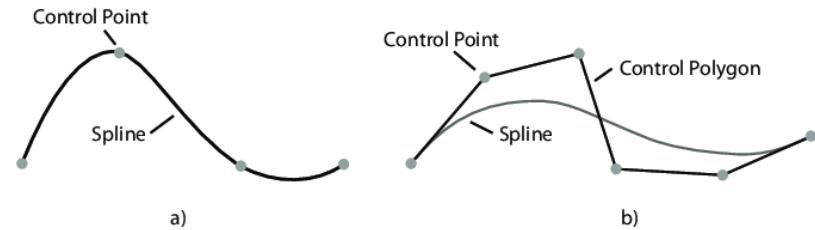
# Subdivision surfaces

- Initial input is polygonal mesh: **control mesh (cage)**
- Two-phase process:
  - First, **refinement phase**, creates new vertices and reconnects them to create new smaller polygons
  - Second, **smoothing phase**, computes new positions for some or all vertices in the mesh



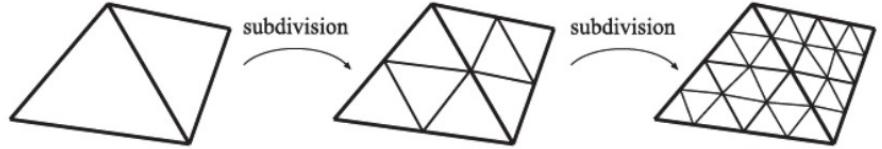
# Subdivision surfaces

- Different **subdivision schemes** are determined by **Refinement** and **Smoothing**:
  - Loop subdivision
  - Catmull-Clark subdivision
  - Doo-Sabin subdivision
- Choice of rules gives different surfaces:
  - Level of **continuity**
  - **Approximative** or **interpolating** surface
- Subdivision schemes characterization:
  - **Stationary vs non-stationary**: always use same rules at every subdivision step or change rules
  - **Uniform vs non-uniform**: use same rules for all vertices and edges or change rules
  - **Triangle vs polygon**: only triangles or arbitrary polygon

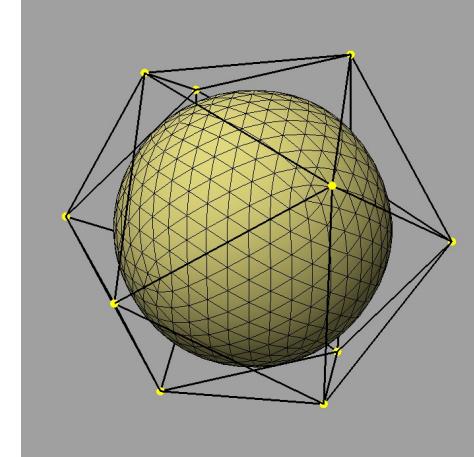
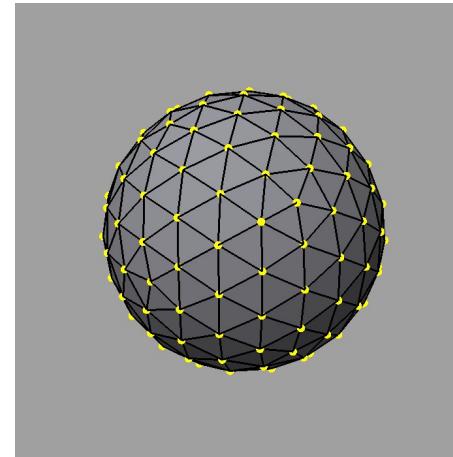
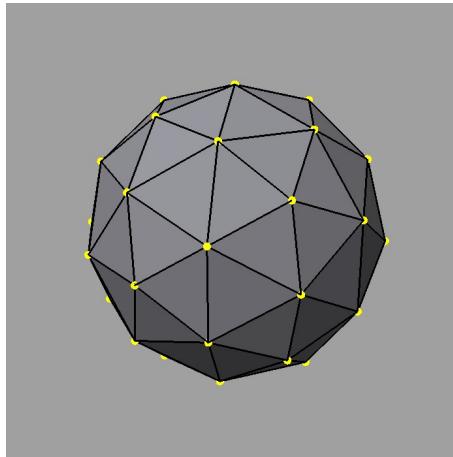
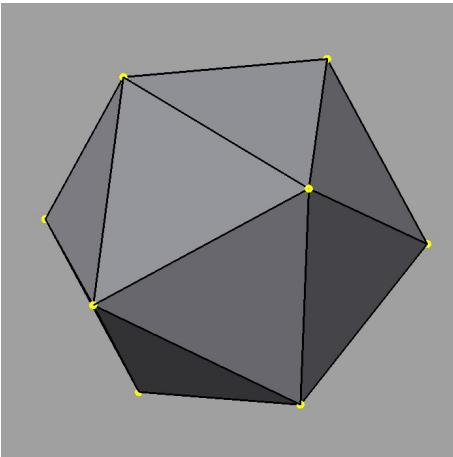


Interpolation – passing through control points (a)  
Approximation – doesn't lie on control points (b)

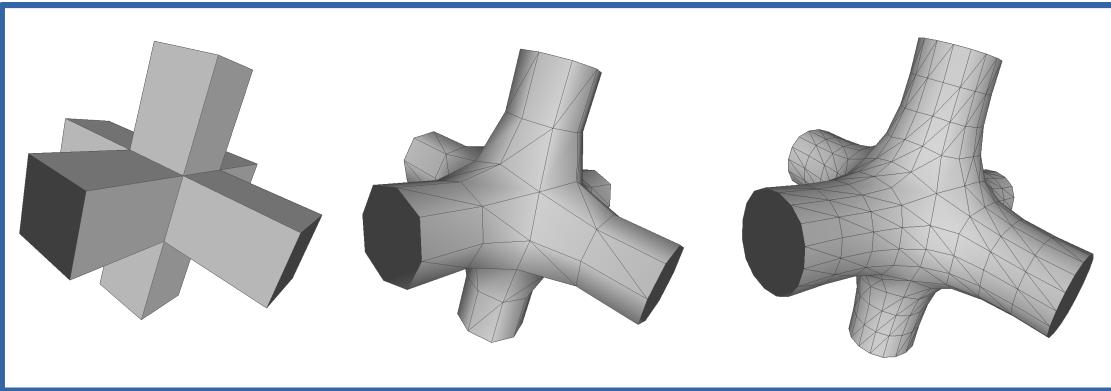
# Loop subdivision



- Stationary, triangle based
- Starts with control mesh, updates each existing vertex and creates new vertex for each edge
  - At each step, triangle is subdivide into four new triangles. After  $n$  steps  $\rightarrow 4^n$  triangles
- Resulting surface is approximative  $\rightarrow$  smooth but shrinked
  - To reduce shrinking, more vertices must be defined in control mesh
  - Final surface is contained in convex hull of the original control points



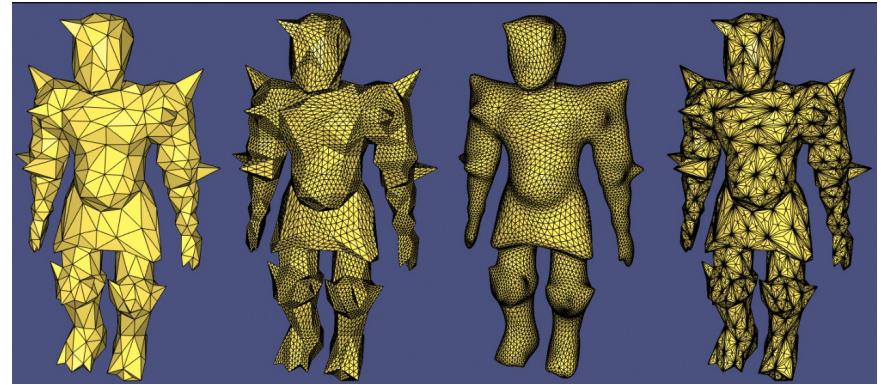
# Loop subdivision



Meshlab loop subdivision:

[https://pymeshlab.readthedocs.io/en/latest/filter\\_list.html#meshing\\_surface\\_subdivision\\_loop](https://pymeshlab.readthedocs.io/en/latest/filter_list.html#meshing_surface_subdivision_loop)

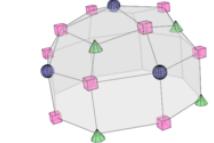
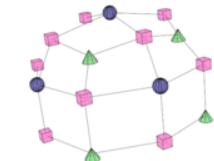
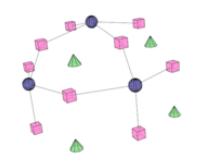
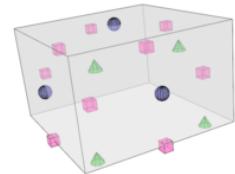
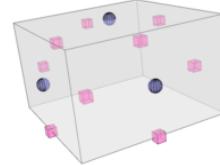
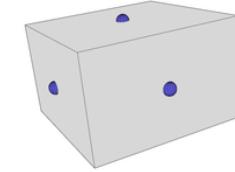
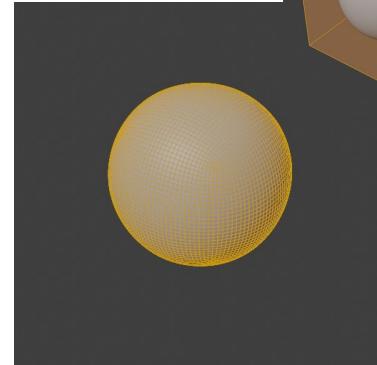
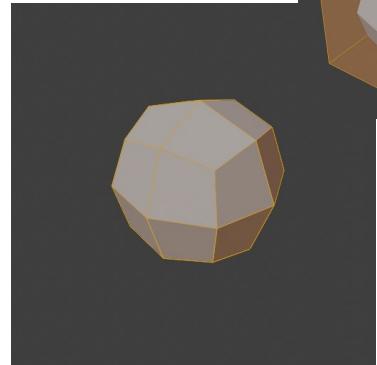
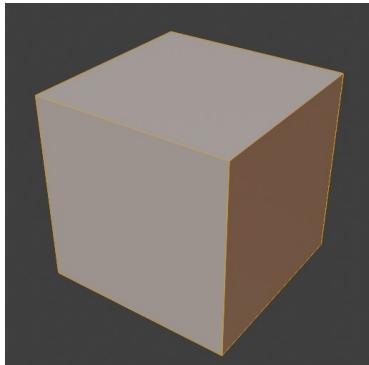
<https://github.com/cnr-isti-vclab/meshlab>



libigl subdivision: <https://libigl.github.io/tutorial/> (original, upsample, **loop** and barycentric).

# Catmull-Clark

- Method of arbitrary polygons
- Starts with control mesh, recursive process defining rules for\*:
  - Computing new face point
  - Computing new edge point
  - Updating vertices, creating new edges and faces



# Catmull-Clark subdivision

- Most commonly used subdivision surface method
  - Used in all Pixar feature films from Toy Story 2 onward
  - Used from making models for games
- Tends to generate more symmetrical surfaces



Catmull-Clark Subdivision in Blender (OpenSubdiv backend):

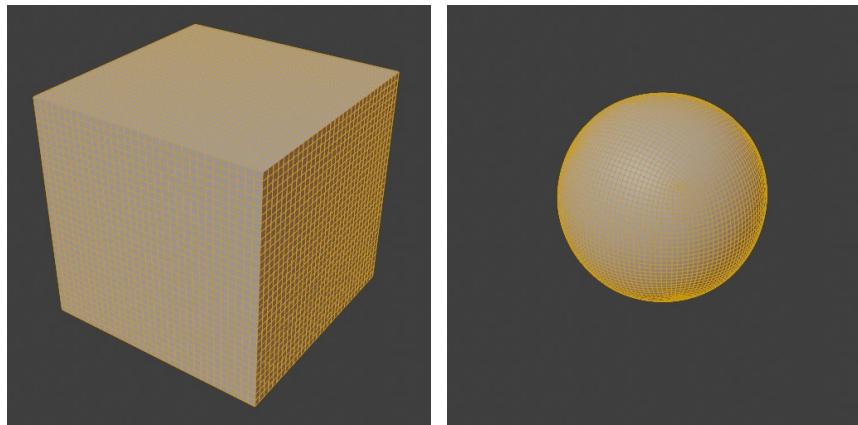
[https://docs.blender.org/manual/en/latest/modeling/modifiers/genenerate/subdivision\\_surface.html](https://docs.blender.org/manual/en/latest/modeling/modifiers/genenerate/subdivision_surface.html)



OpenSubdiv: State of the art subdivision library for production:  
<https://graphics.pixar.com/opensubdiv/docs/intro.html>

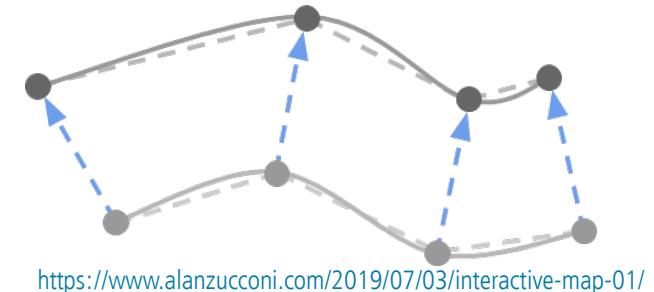
# Surface details

- Flat and curved surfaces are too smooth and thus not realistic while lacking detail
- Detail on surface can be added with:
  - Geometrical enhancements: displacement
  - Material modeling:
    - Scattering function parameters variation: e.g., color patterns
    - small scale surface variations: surface normal perturbation

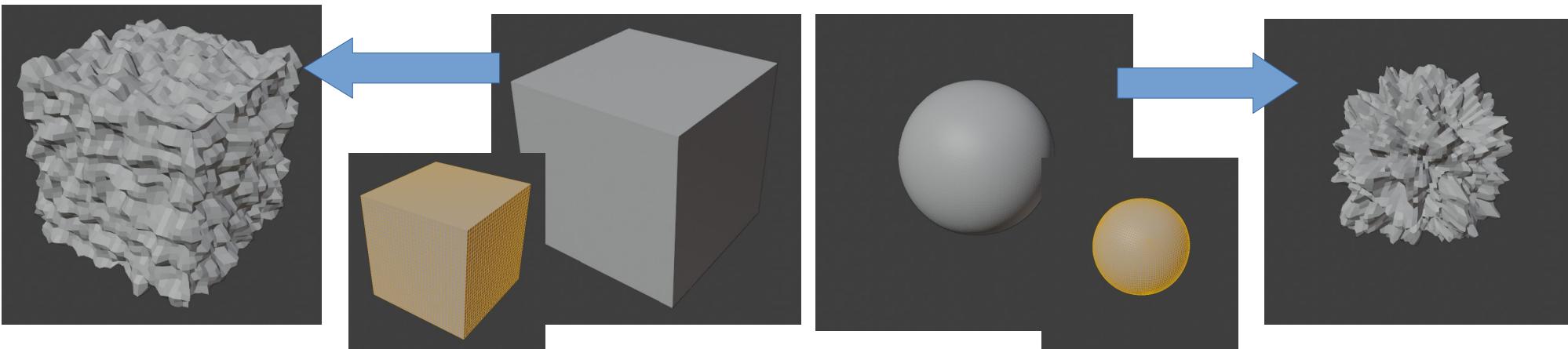


# Displaced subdivision

- Detail can be added on subdivided surface using **displacement mapping**
- Often displacement mapping moves vertices ( $v$ ) in direction of surface normal vector ( $N$ ) for distance ( $d$ ):  
$$v' = v + d * N$$
- Generally, displacement can be done using arbitrary vector ( $M$ )  
$$v' = v + M$$
- Scalar distance values ( $d$ ) or vector values ( $M$ ) used for displacement are described with image or procedural texture



<https://www.alanzucconi.com/2019/07/03/interactive-map-01/>



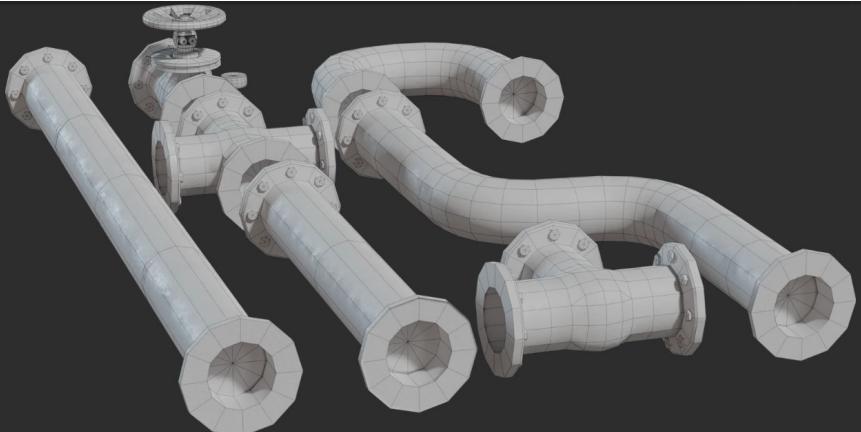
# Examples of polygonal meshes

<https://polyhaven.com/>



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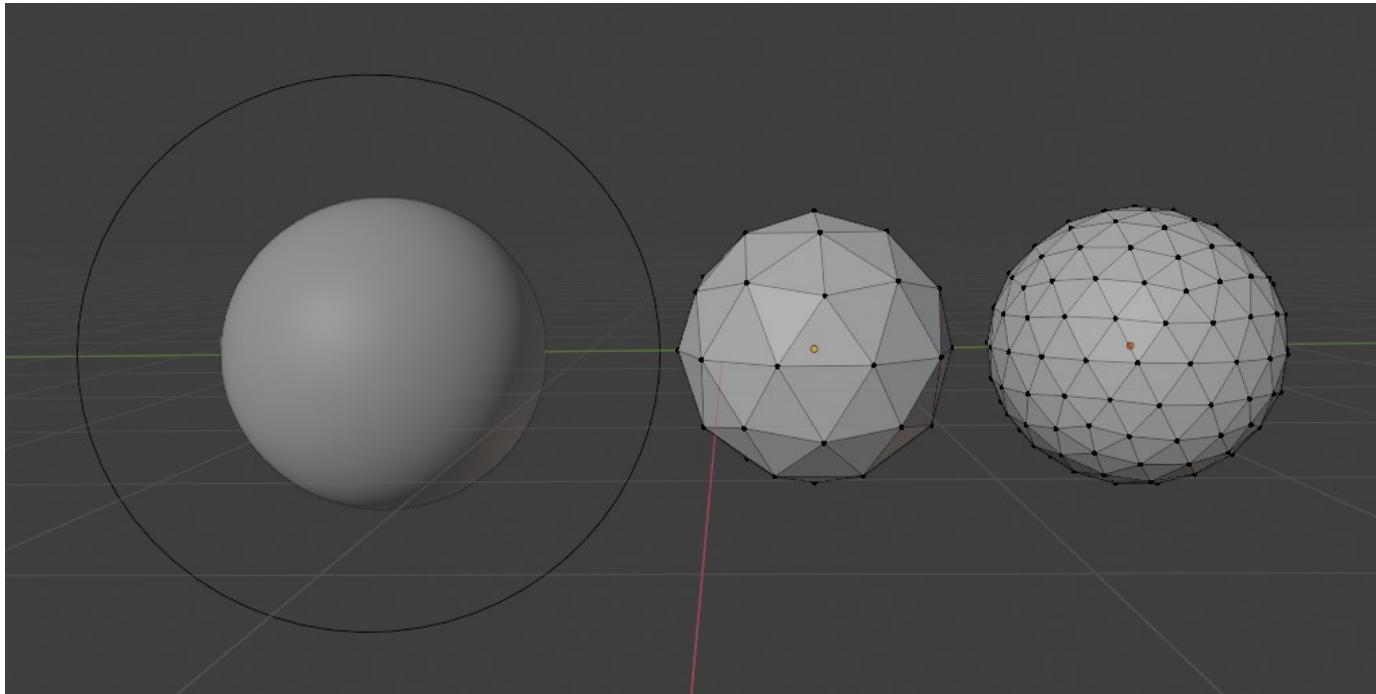
# Mesh and rendering

# Mesh and rendering

- Rendering uses object shape to solve **visibility problem**
  - What is visible from camera
  - Which surfaces are visible to each other → which might reflect light onto each other (light transport, shading)
- **Rasterization and ray-tracing based rendering are optimized for working with triangulated mesh**
- Any shape representation can be transformed into triangulated mesh using **tessellation**, specifically, **triangulation**:
  - Uniform
  - Adaptive

# Uniform tessellation

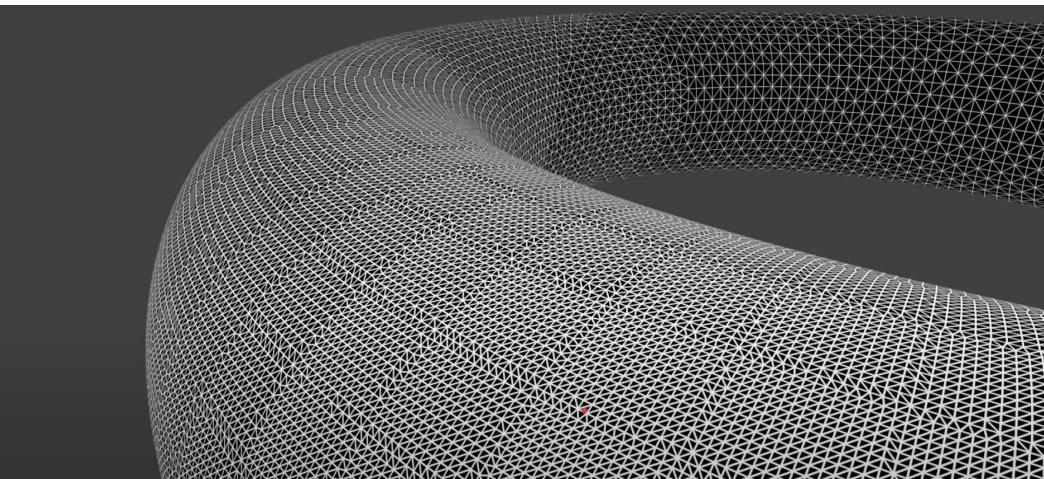
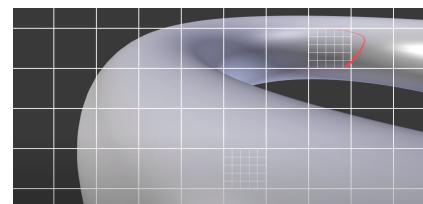
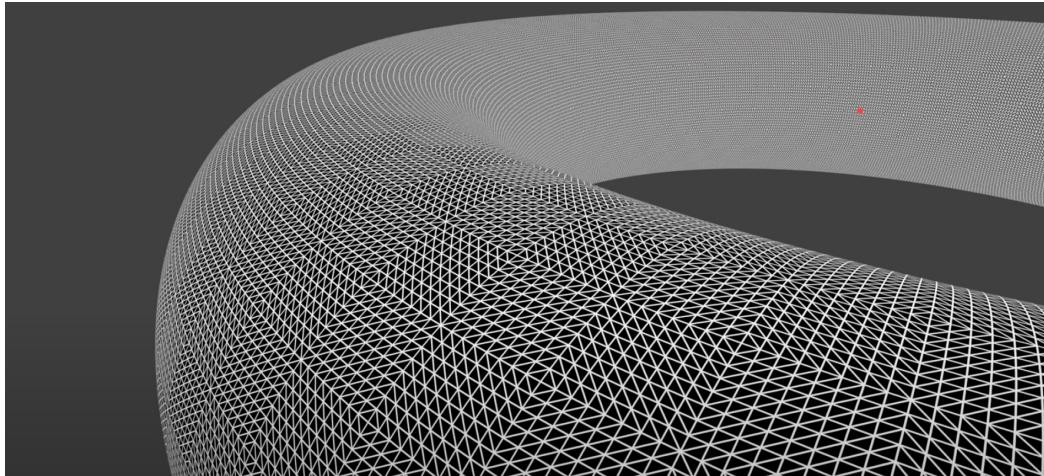
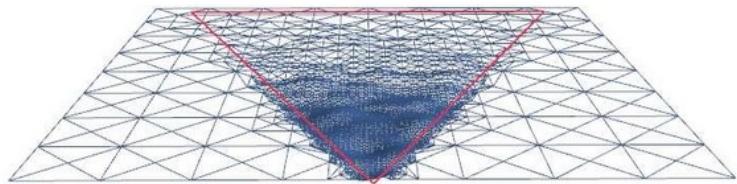
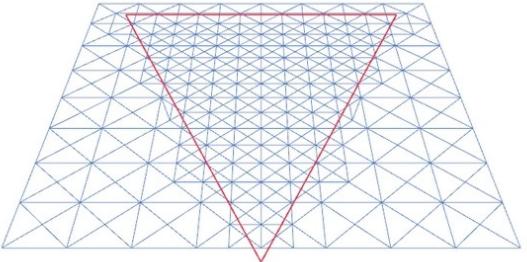
- uniformly sized triangles are created for whole surface.



Uniform tessellation of implicit representation of a sphere.

# Non-uniform (adaptive) tessellation

- Triangles of varying sizes are created for surface, based on:
  - Distance to surface

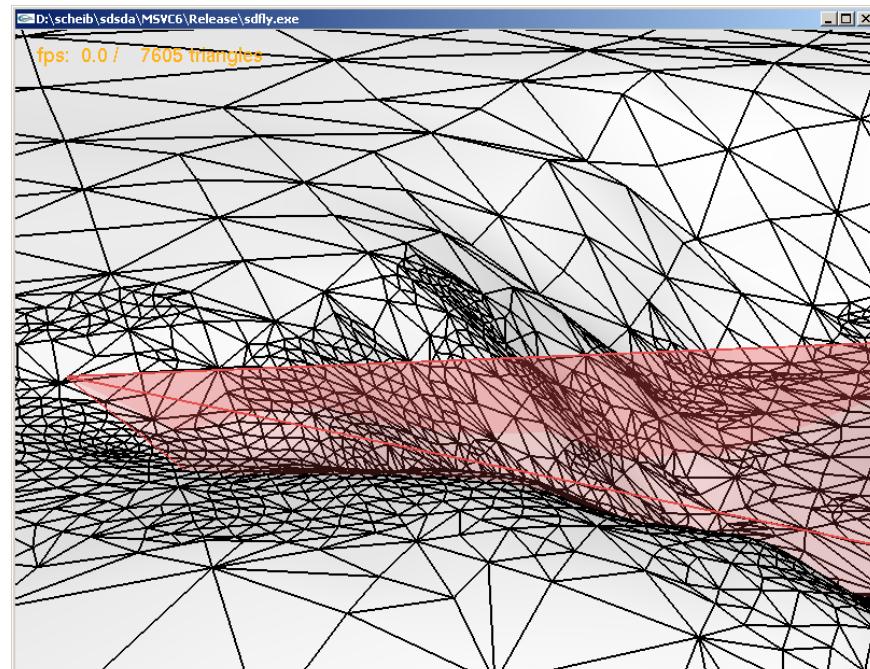
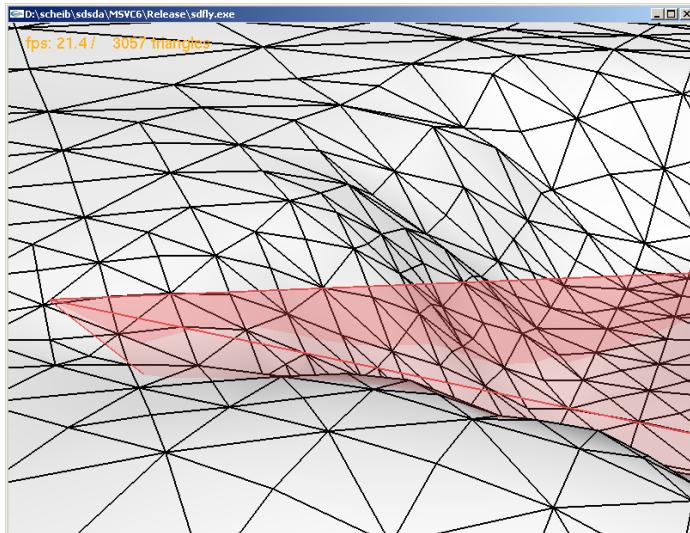


<https://www.hindawi.com/journals/tswj/2014/979418/>

<https://www.blenderguru.com/tutorials/introduction-microdisplacements>

# Non-uniform (adaptive) tessellation

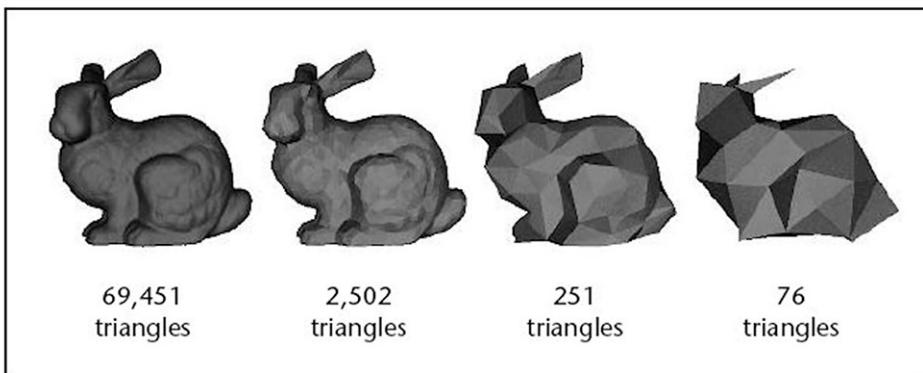
- triangles of varying sizes are created for surface, based on:
  - Distance to surface
  - Amount of surface details (surface curvature)



Curved parts of surface are represented with more triangles while flat parts can be represented with less triangles:  
<https://gamma.cs.unc.edu/SDSDA/>

# Level of detail

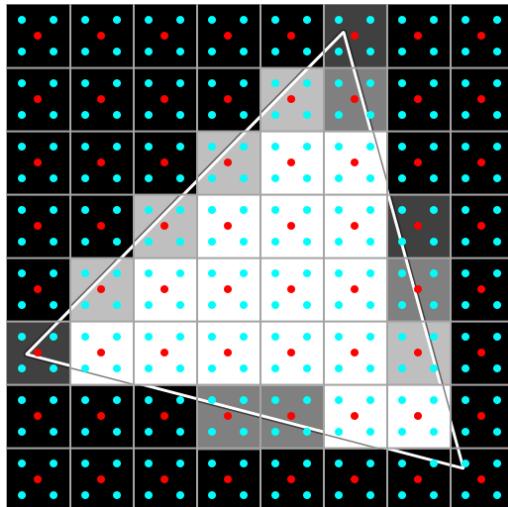
- Depending on camera distance, mesh can contain more or less details
- **Static:** multiple object shapes with different amount of polygons (polygonal resolutions) are stored and used depending on camera distance.
- **Dynamic:** amount of polygons in object shape is computed and used on the fly
  - <https://www.gamedeveloper.com/programming/real-time-dynamic-level-of-detail-terrain-rendering-with-roam>



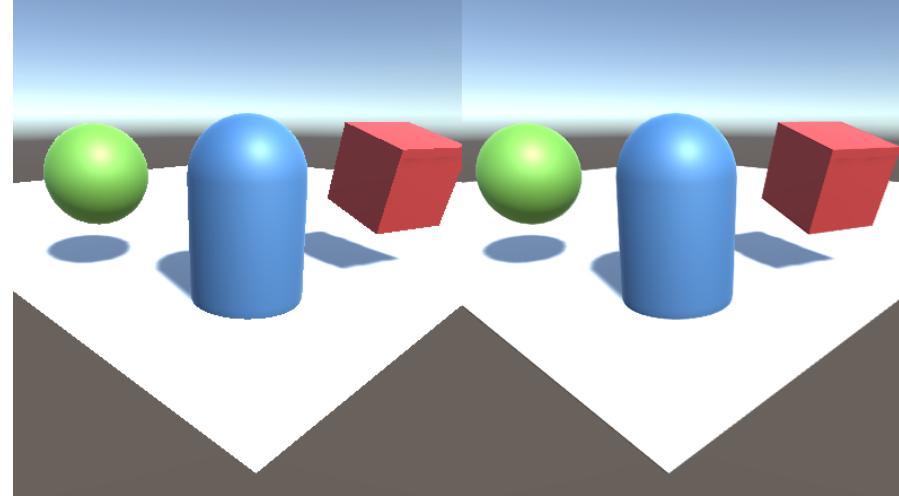
Static level of detail: <https://3dstudio.co/3d-lod-level-of-detail/>

# Triangulated mesh and rendering

- Triangle surface is continuous, to be displayed on raster image discretization must be performed
  - Due to discretization, aliasing problems might appear
  - Solution: multiple rays/samples per pixel



<https://www.beyond3d.com/content/articles/122/4>



<https://docs.unity3d.com/560/Documentation/Manual/PostProcessing-Antialiasing.html>

# Vertex attributes

- Besides vertex positions (geometry) and connectivity information (topology), additional information is used for rendering.
- This information is stored per vertex - **vertex attributes\***:
  - Normal
  - Texture coordinate
  - Color
  - Any kind of information that can be encoded and used for rendering: weights, temperature, etc.

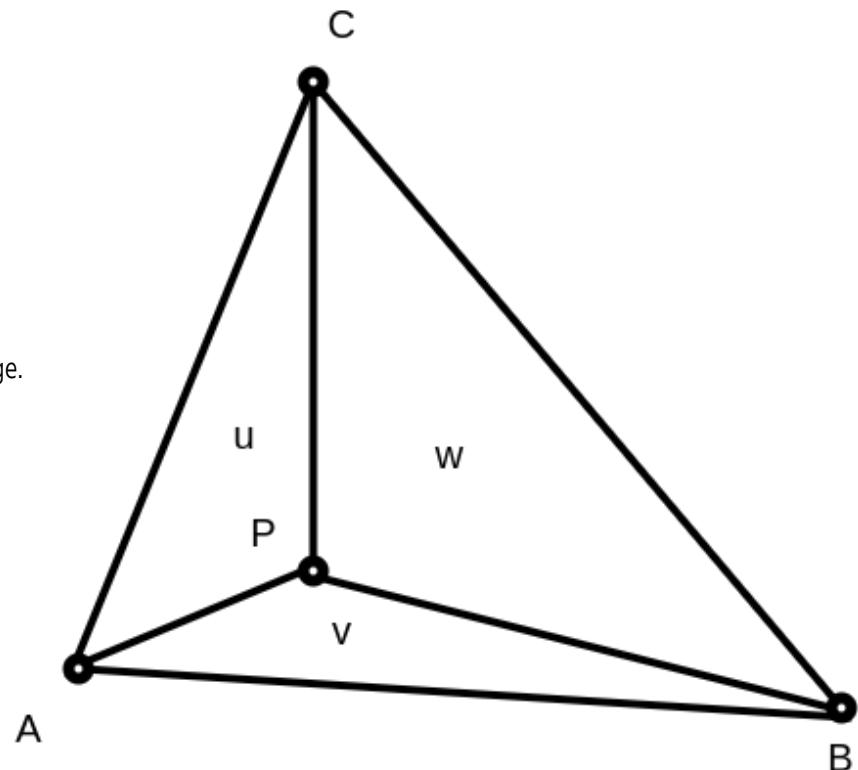
\* Those are in general called primitive variables. Primitive is generic term in computer graphics which describes object which is understandable by program.

# Vertex attributes

- Vertex attributes are, during rendering (shading), interpolated across triangle or quad
  - Example ray-tracing: ray hitting inside of a polygon
  - Example rasterization: color of pixel inside polygon
- Interpolation depends on polygon types:
  - Triangles: **barycentric interpolation**
  - Quads: **bilinear interpolation**

# Barycentric interpolation

- Express position of any point located on triangle with three scalars.
  - $P = uA + vB + wC$ , where  $A, B, C$  are triangle vertices
  - $u, v, w$  are barycentric coordinates
    - $u + v + w = 1$  and
    - $0 \leq u, v, w \leq 1$ , otherwise  $P$  is outside of triangle. If equal then  $P$  is on triangle edge.
- Areal coordinates:  $u, v, w$  are proportional to area of sub-triangles defined by  $P$ 
  - $u = \text{Area}(\text{Triangle}(CAP)) / \text{Area}(\text{Triangle}(ABC))$
  - $v = \text{Area}(\text{Triangle}(ABP)) / \text{Area}(\text{Triangle}(ABC))$
  - $w = \text{Area}(\text{Triangle}(BCP)) / \text{Area}(\text{Triangle}(ABC))$
  - $\text{Area}(\text{Triangle}(ABC)) = ||(B-A) \times (C-A)||/2$
- Interpolating vertex data
  - We know  $P$  and  $A, B, C$ . Calculate  $u, v$  and  $w$  and use these factors for interpolating

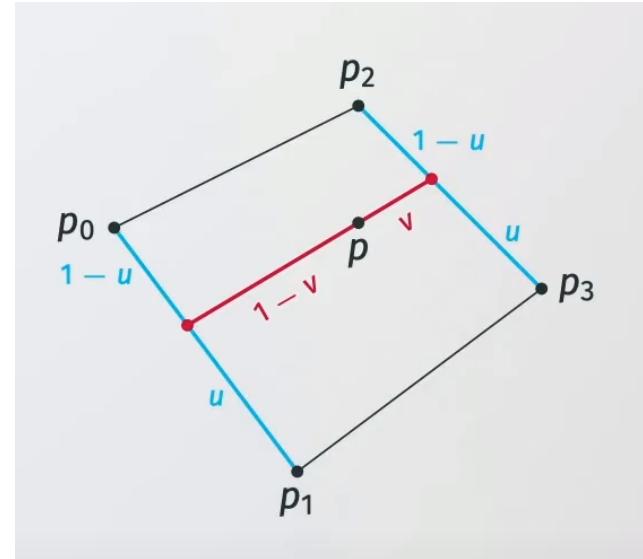


# Bilinear interpolation

- Any point on quad can be found with bilinear interpolation
  - Thus, and value stored per vertex of quad can be interpolated in the same way

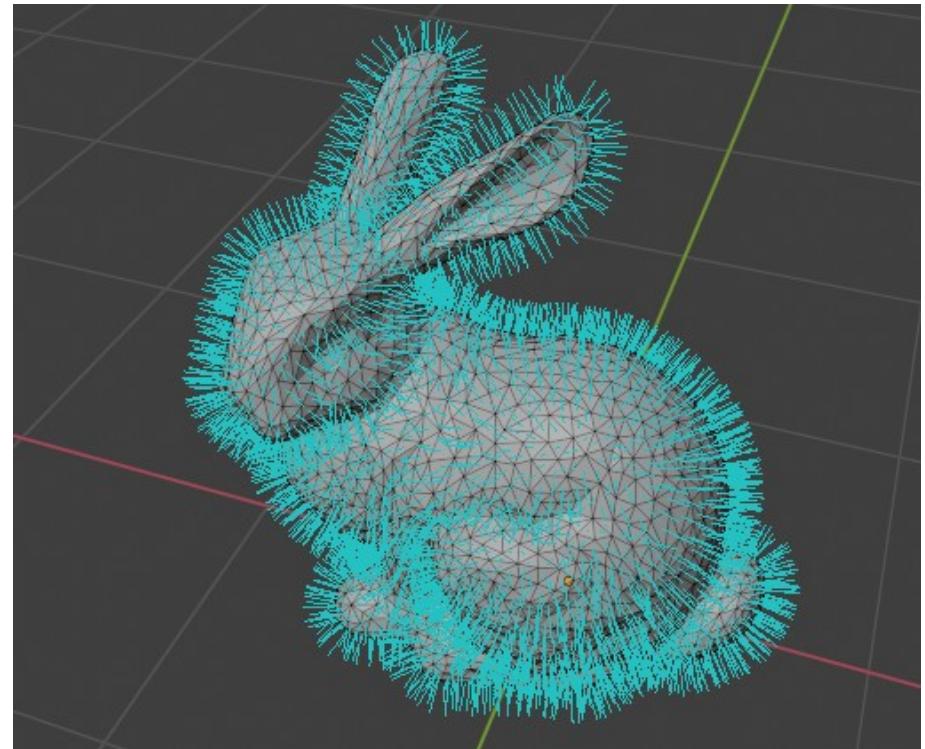
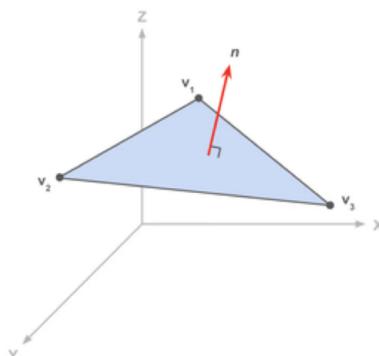
$$f(p) = (1 - v) ((1 - u)f(p_0) + uf(p_1)) + \\ v ((1 - u)f(p_2) + uf(p_3))$$

$$u, v \in [0, 1]$$



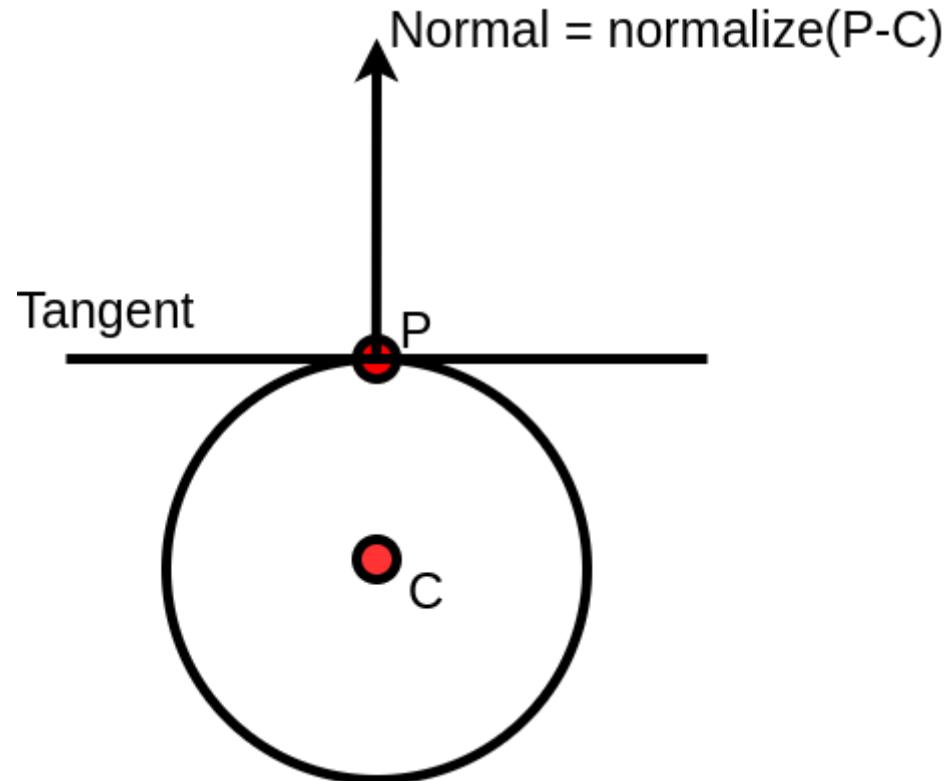
# Normals

- Orientation of surface in each point is determined by normal vector.
- Normal vector is **core information for rendering (shading) and modeling.**
- Mesh normal can be defined per:
  - face
  - per vertex



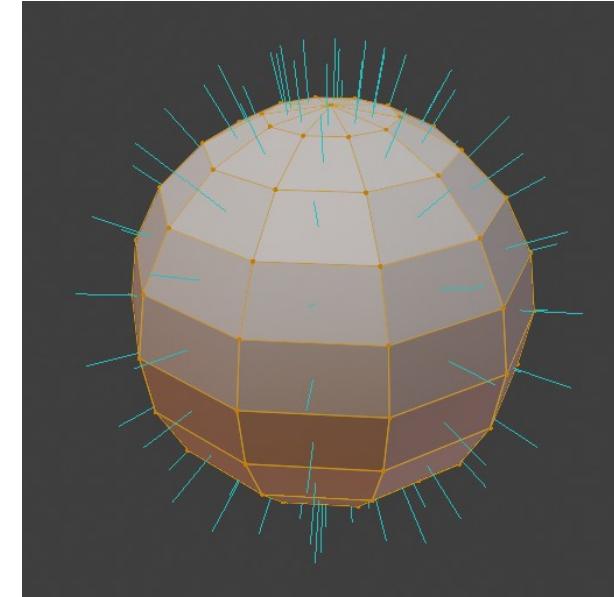
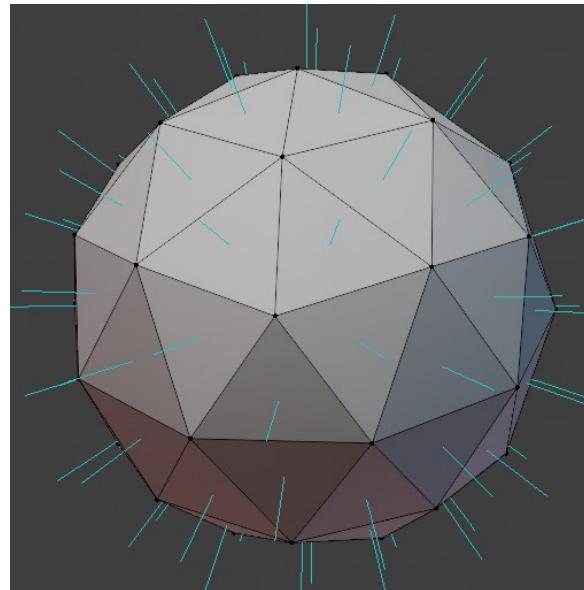
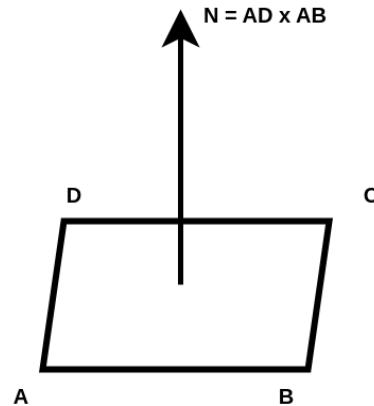
# Computing normals

- Normal in surface point is **vector perpendicular to tangent** in that surface point
- Computation of normals depends on shape representation
  - Sphere normal
  - Mesh face normals can be calculated using triangle or quad polygon edges.

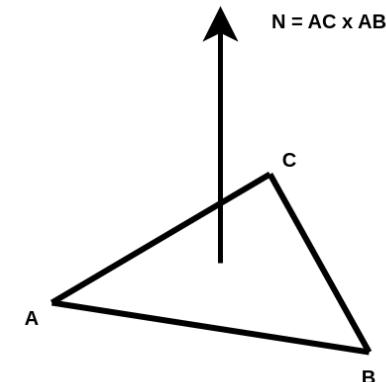


# Normals: per face

- Cross product between triangle or quad edges
- Winding order of vertices defines orientation of normal
- For right-handed coordinate system, polygons with counter clockwise winding will be front facing

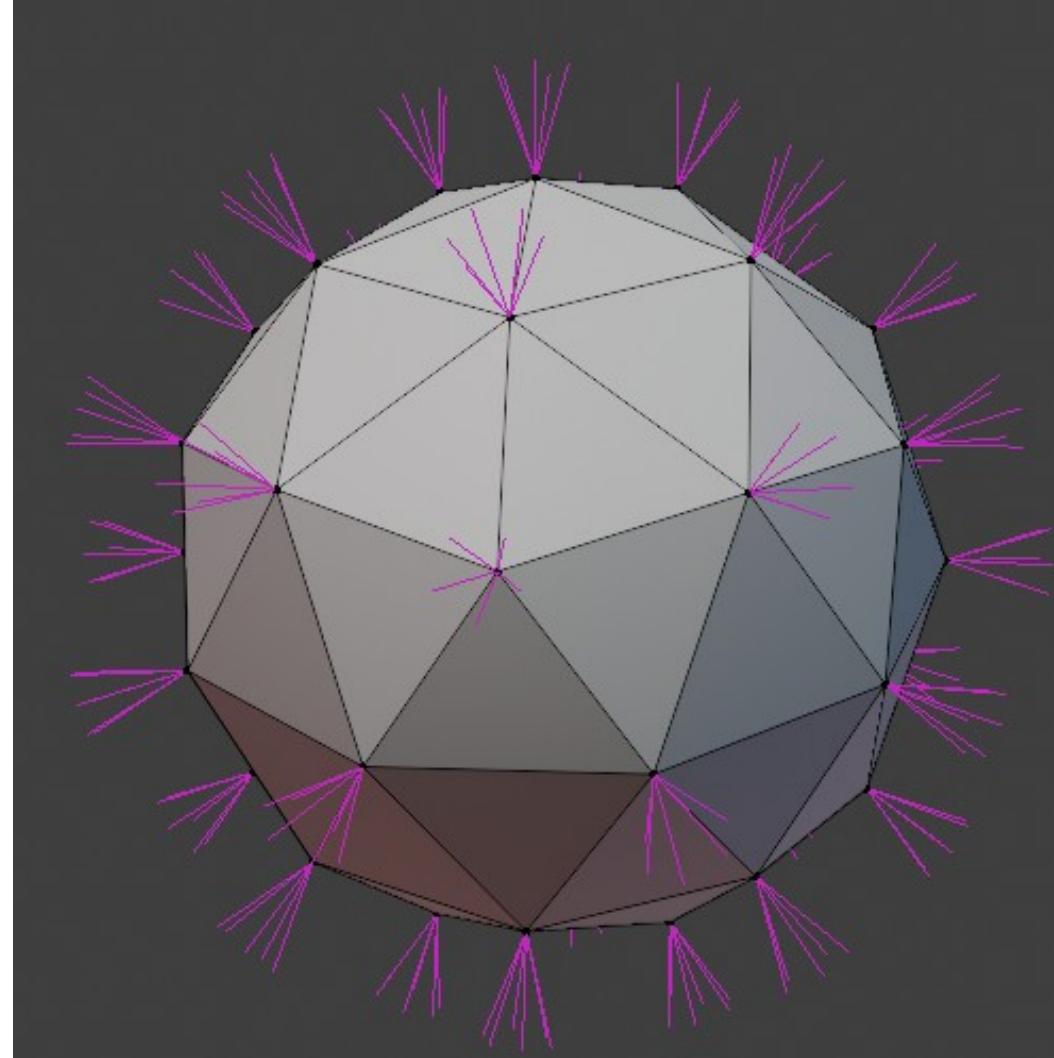


$$N = (B-A) . \text{crossProduct}(C-A);$$



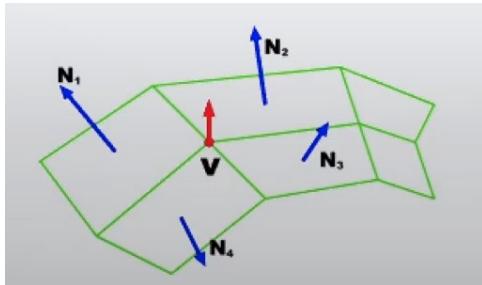
# Normals: per vertex

- Multiple normals, one for each face, can be stored per vertex.

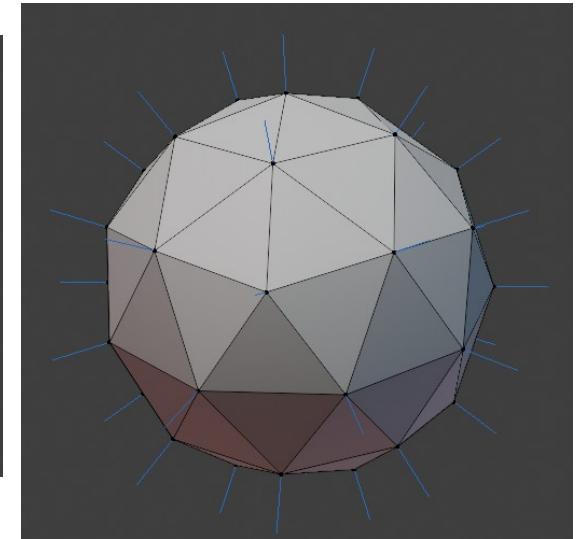
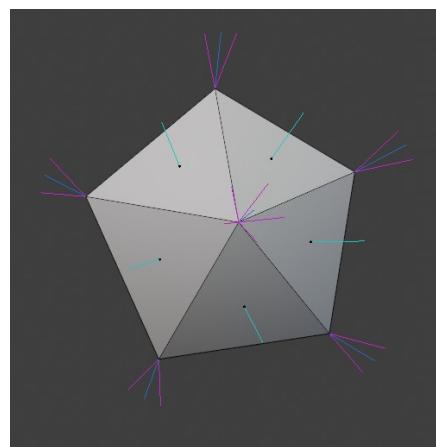
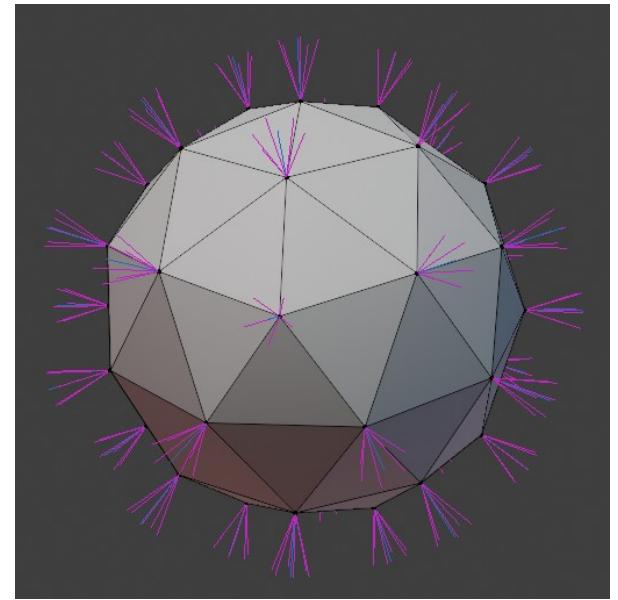


# Normals: per vertex

- One normal per vertex is calculated as:
  - Calculate normal for each face connected to vertex
  - Average calculated face normals

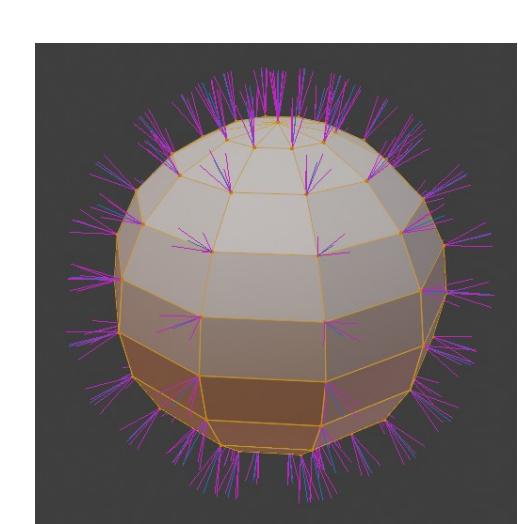
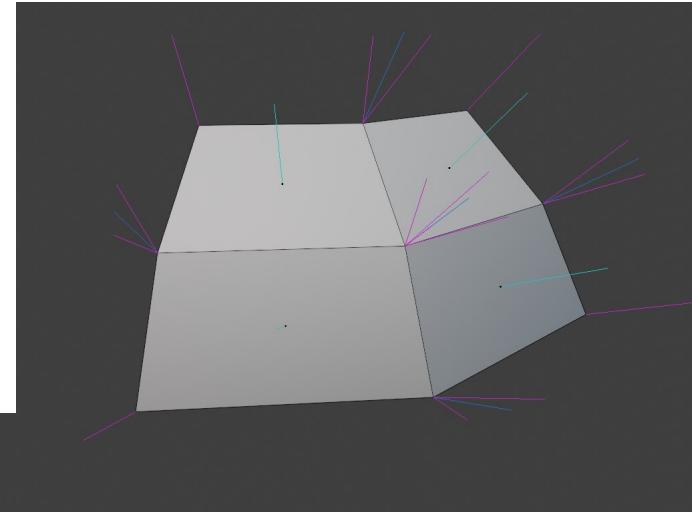
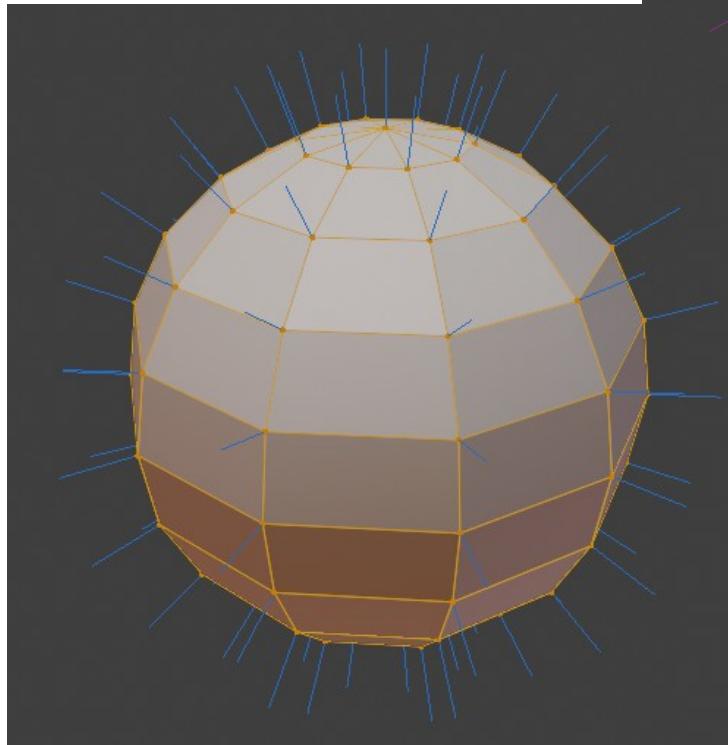
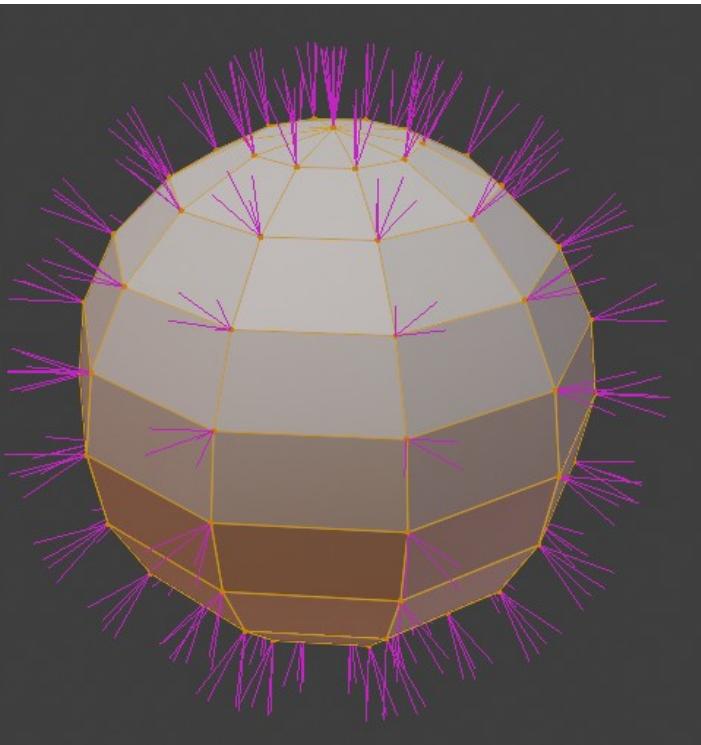


$$\vec{N}_v = \frac{\sum_{i=1}^k \vec{N}_i}{\left\| \sum_{i=1}^k \vec{N}_i \right\|}$$



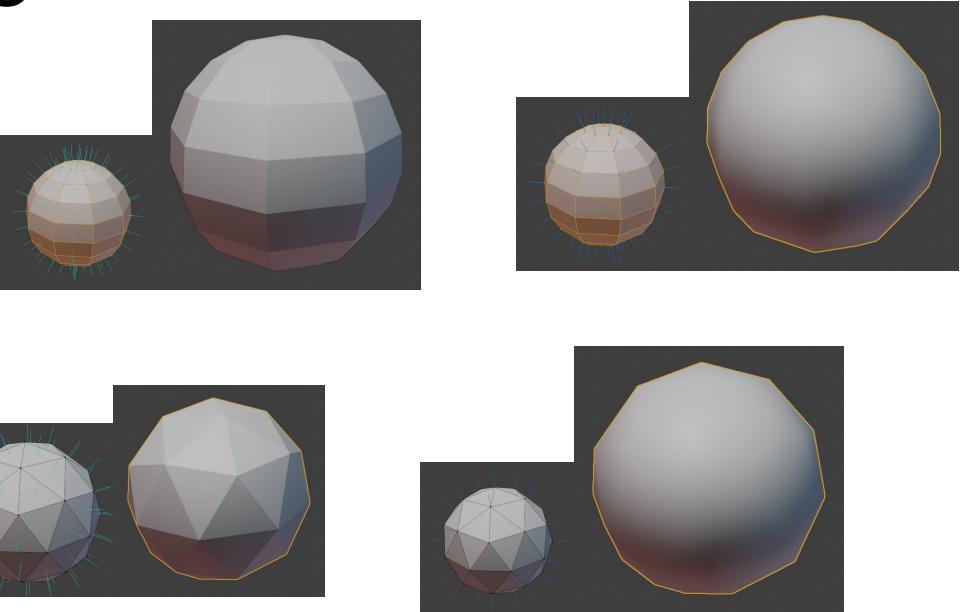
# Normals: per vertex

- One normal for each face per vertex vs one normal per vertex



# Using normals: shading

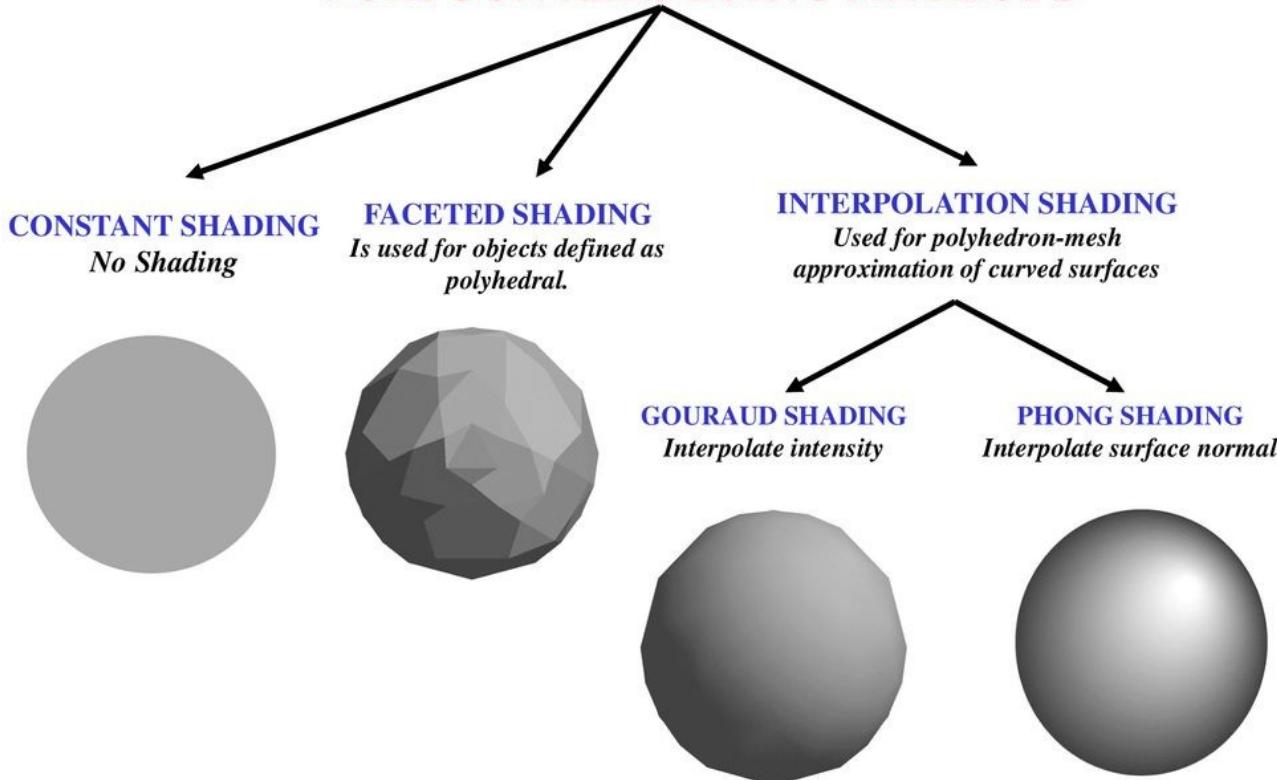
- Appearance of object depends both on shape and material.
  - **Normal is main shape information used in shading:** evaluation of scattering function depends on normal
- **Flat shading:** evaluation of scattering model using per-face normals (no interpolation)
- **Gouraud (smooth) shading:** evaluation of scattering model at vertices and interpolating resulting color (e.g., barycentric interpolation)
  - Problem: unnatural highlights for larger polygons since largest highlights appear at vertices of mesh and less on polygon during interpolation
  - “Vertex” light calculation
- **Phong (smooth) shading**
  - Normals per vertex are linearly interpolated over triangles (using barycentric coordinates) and then scattering model is evaluated.
  - “Per-pixel” light calculation
  - Note: phong shading vs phong lighting model



Smooth shading:

- Geometry is not changed: note faceted silhouette
- Illusion of smooth surface is created during shading

## SHADING METHODS POLYGON RENDERING METHODS



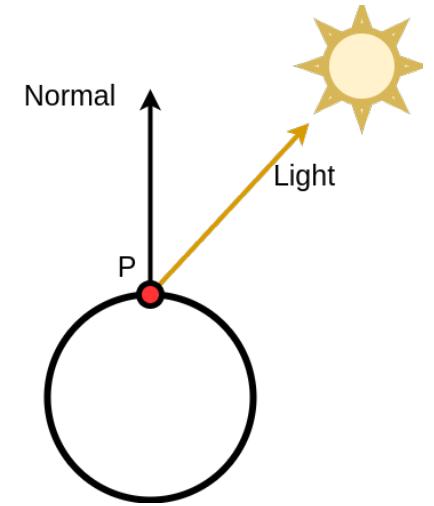
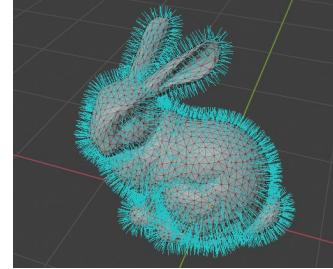
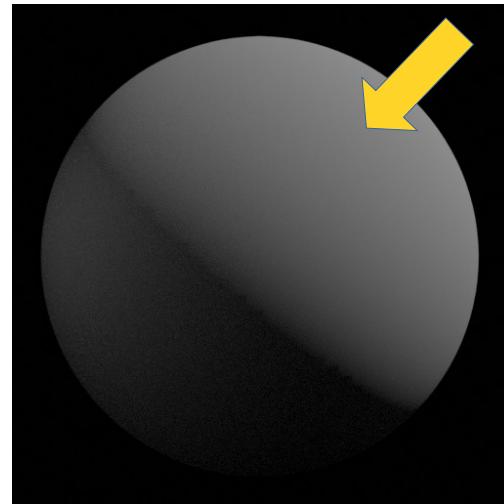
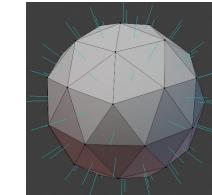
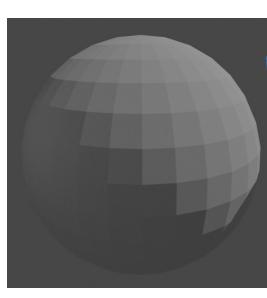
<https://slideplayer.com/slide/16683696/>

# Flat vs smooth shading



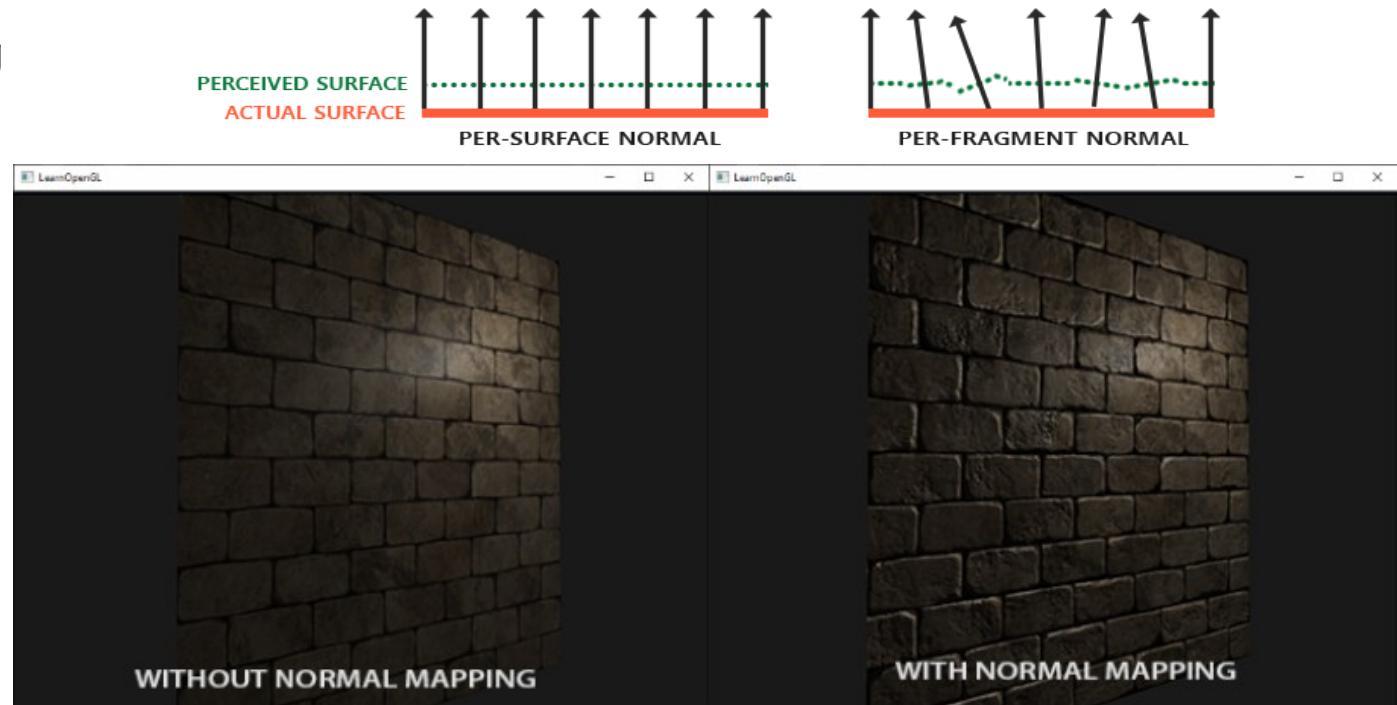
# Using normals: shading

- Amount of light incident on surface depends on cosine of the angle between light direction and surface normal
  - Lambert's Cosine law
  - $\cos(\theta) = \text{dot}(N, L)$



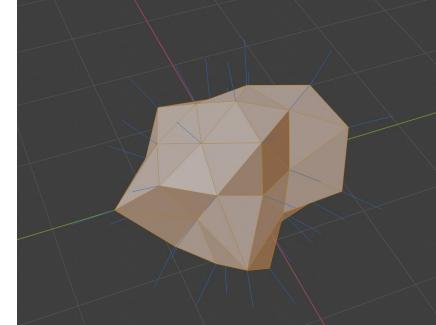
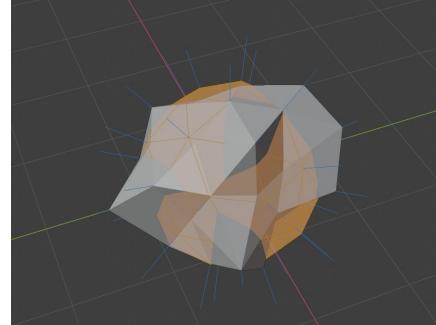
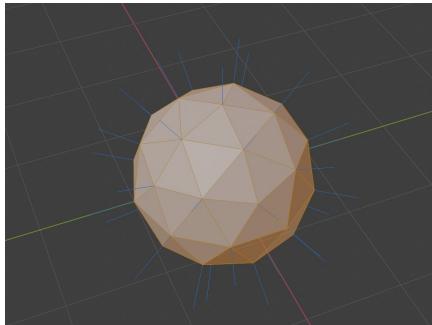
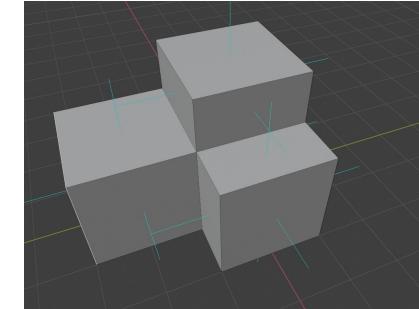
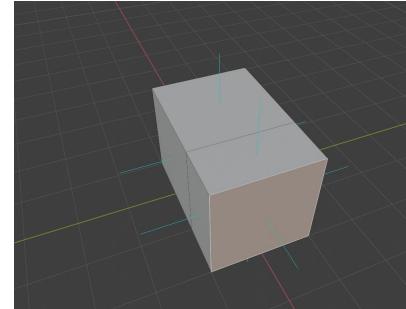
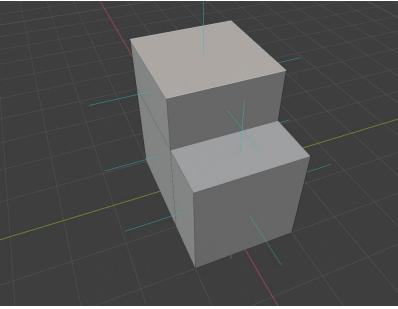
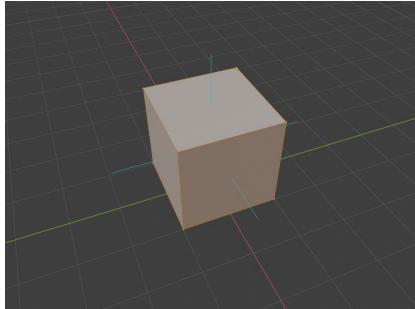
# Using normals: surface details

- Face or vertex normals result in smooth and too perfect surface
- These normals, during rendering, can be perturbed for each point on surface which give appearance of surface details
  - Bump/normal mapping



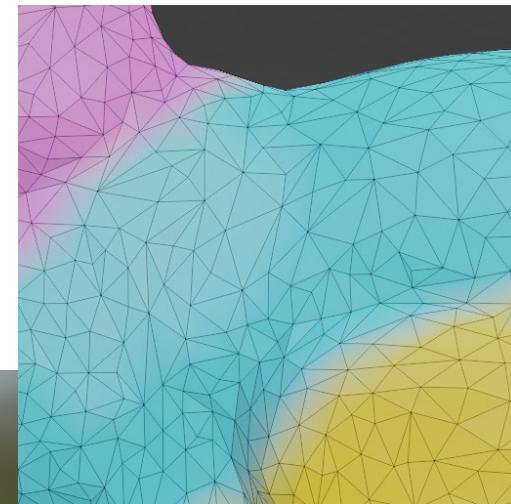
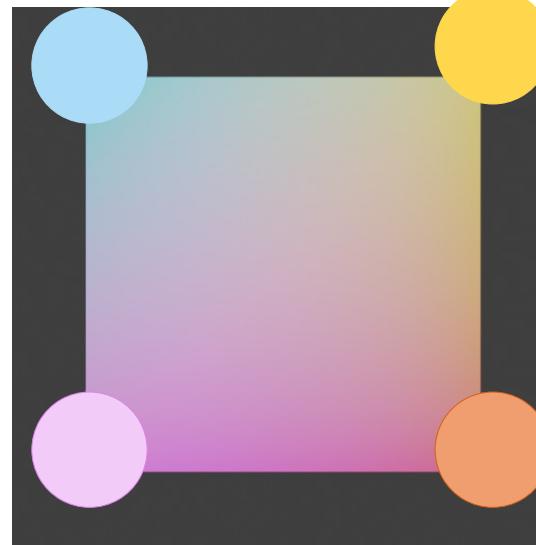
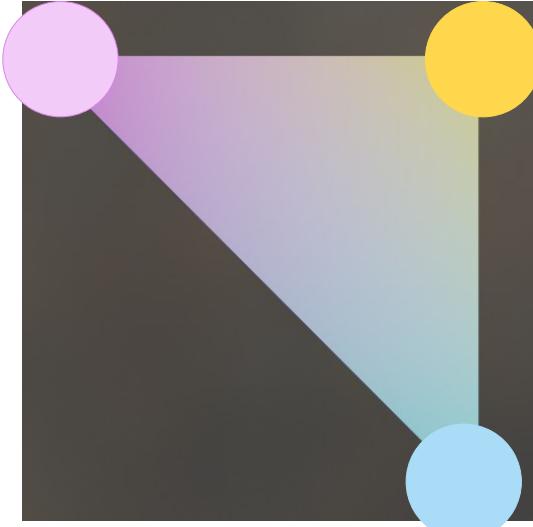
# Using normals: geometric manipulation

- Example: extrusion
  - Basic modeling operations which moves mesh face in direction of normal
- Example: procedural modeling
  - Using normal direction for **displacing mesh vertices**



# Additional mesh information: vertex colors

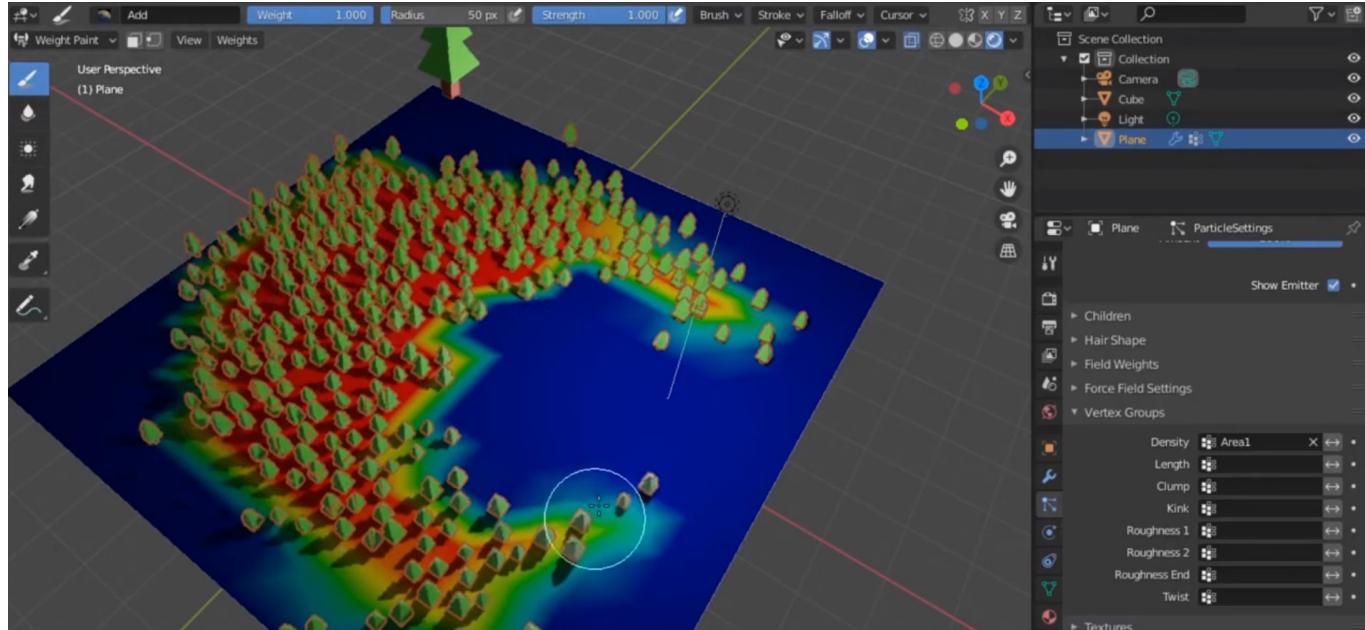
- Similarly as normals are defined per vertex and interpolated over triangle for shading purposes, the same can be done with color
- Simplest way to introduce variation over object surface – a form of **texture**
- This method works fine for meshes with finer structure → more colors can be assigned to more vertices.
  - When it is not possible to have fine mesh, then texture is used.



[https://docs.blender.org/manual/en/latest/sculpt\\_paint/vertex\\_paint/index.html](https://docs.blender.org/manual/en/latest/sculpt_paint/vertex_paint/index.html)

# Additional mesh information: vertex weights

- Vertex attributes can be any information that can be used for rendering
- Example: vertex weight designating density for instancing objects (e.g., particles)



[https://www.youtube.com/watch?v=oPenYcM6Usw&ab\\_channel=HelperGround](https://www.youtube.com/watch?v=oPenYcM6Usw&ab_channel=HelperGround)

# Additional mesh information: texture coordinates

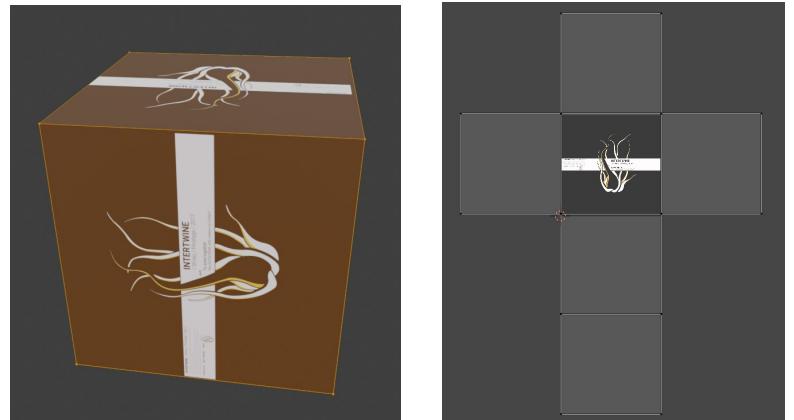
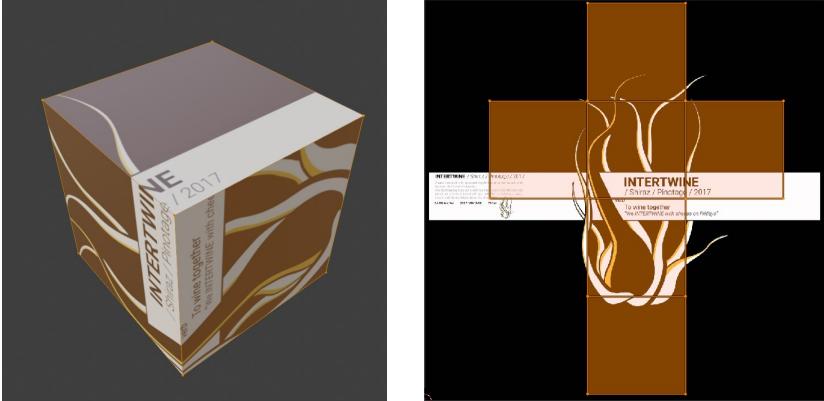
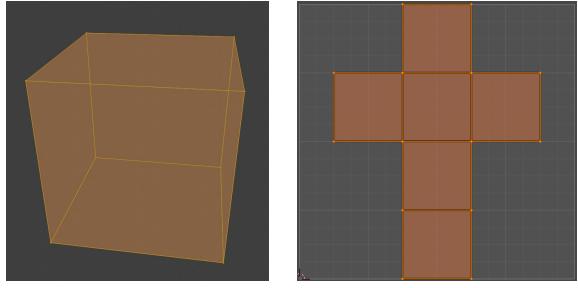
- To add more details on object, often images and procedural patterns are used → **textures**.
- The problem of applying texture image on 3D object is often quite complex than on flat plane.
  - A way of mapping a 2D image/pattern to 3D shape can be done by “unwrapping” mesh onto 2D plane.



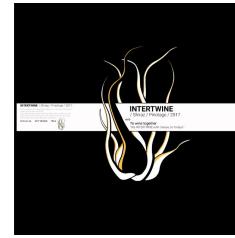
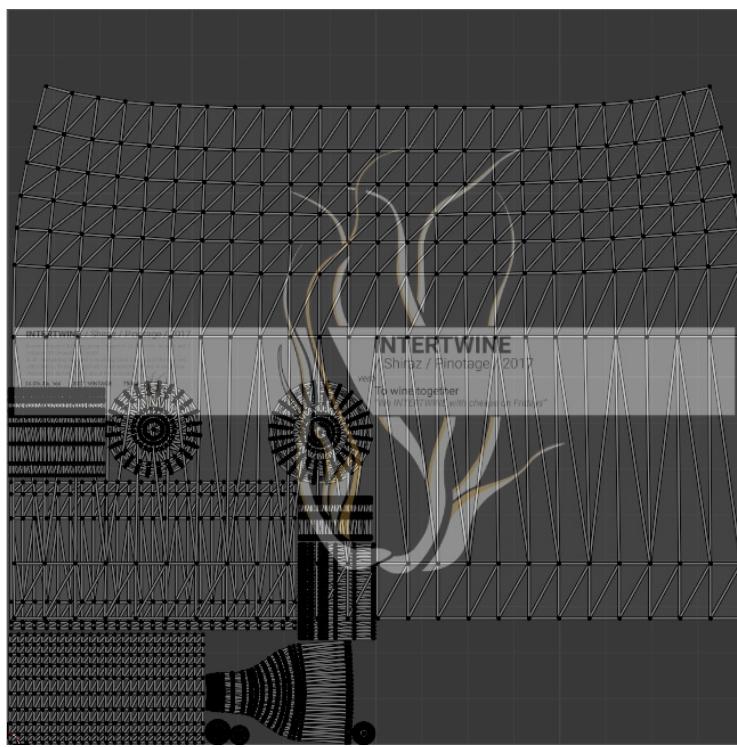
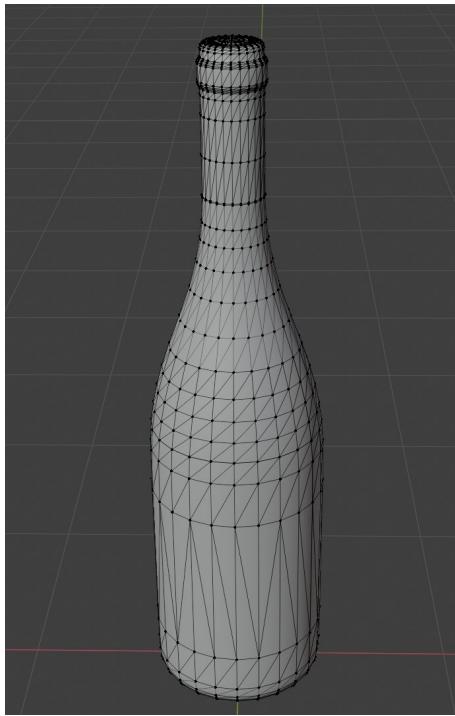
[https://polyhaven.com/a/wine\\_bottles\\_01](https://polyhaven.com/a/wine_bottles_01)

# Mesh unwrapping: intuition

- Mesh is unwrapped into 2D plane we can imagine it lies in 2D coordinate system.
- Vertices of unwrapped mesh now have coordinates in this 2D coordinate system ( $u,v$ ) → **texture coordinates\***
  - $(u,v)$  are in  $[0, 1]$  range
  - Unwrapped faces can be manipulated, but then texture might get deformed
  - Whole unwrapped mesh can be translated or rotated or scaled to achieve different texture positioning
  - Several faces can overlap



# Texture coordinates: mesh unwrap



\* Note: this is one way of calculating texture coordinates. Texture coordinates can also be calculated on the fly during rendering or other methods that we will discuss later.

# Polygon Mesh: datastructures, storing and transfer

# Datastructures and representation

- There are multiple data structures for polygonal meshes that offer trade-off between run-time complexity and memory requirements
  - Efficient topology traversal
  - Efficient mesh updates
- Different data structures are better for rendering, storage or modeling
- Often algorithms on polygonal meshes perform:
  - Computation of normal vectors
  - Mesh smoothing, refining, simplification, compression, etc.

# Mesh data structure

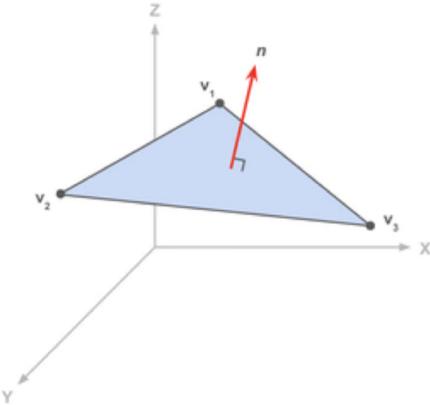
- Mesh data structure must at least describe:
  - **Geometry**: position of vertices specified using coordinates
  - **Connectivity/topology** information for vertices → how are edges and faces formed from the vertices.  
Options:
    - For each polygon, list all vertices that define it
    - Polygon topology: specify edges of mesh in terms of pairs of vertices (connection between vertices and edges)
    - Mesh topology: for each edge, state which polygons are incident connection between edges and polygons (connection between edges and polygons)
  - Other information not explicitly given can be calculated from given information

# Explicit face representation

- **Triangle soup:** each triangle is represented using vertex coordinates (topology is not represented explicitly)
  - Triangle: 3 vertices each having 3 coordinates ( $x, y, z$ ) .
  - $n$  triangles:  $n * 3 * 3 = 9 * n$  floats  $\rightarrow$  high redundancy!
- Data format used in production:
  - **STL** (STereoLitography)

| $t$ | $v_i$              | $v_j$              | $v_k$              |
|-----|--------------------|--------------------|--------------------|
| 0   | ( 1.0, 1.0, 1.0)   | ( -1.0, 1.0, -1.0) | ( 1.0, 1.0, 1.0)   |
| 1   | ( 1.0, 1.0, 1.0)   | ( -1.0, -1.0, 1.0) | ( 1.0, -1.0, -1.0) |
| 2   | ( 1.0, 1.0, 1.0)   | ( 1.0, -1.0, 1.0)  | ( -1.0, 1.0, -1.0) |
| 3   | ( 1.0, -1.0, -1.0) | ( -1.0, -1.0, 1.0) | ( -1.0, 1.0, -1.0) |

# Example: STL file format



```
facet normal  $n_i$   $n_j$   $n_k$ 
  outer loop
    vertex  $v1_x$   $v1_y$   $v1_z$ 
    vertex  $v2_x$   $v2_y$   $v2_z$ 
    vertex  $v3_x$   $v3_y$   $v3_z$ 
  endloop
endfacet
```

```
foreach triangle
  REAL32[3] – Normal vector
  REAL32[3] – Vertex 1
  REAL32[3] – Vertex 2
  REAL32[3] – Vertex 3
  UINT16   – Attribute byte count
end
```

STL file format (ASCII and binary type) stores triangles by explicitly listing vertices and normals: <https://all3dp.com/1/stl-file-format-3d-printing/>

# Shared vertex representation

- AKA: indexed face set
  - Reduce redundancy
- Store vertex coordinates in a separate array: **vertex array** ( $3 \times n$  floats)
- Specification of triangles refers to vertex array table by storing indices of triangles: **triangle/face array** ( $3 \times n$  integers)
- Data formats used in production:
  - OBJ
  - OFF

| v | x    | y    | z    |
|---|------|------|------|
| 0 | 1.0  | 1.0  | 1.0  |
| 1 | -1.0 | 1.0  | -1.0 |
| 2 | -1.0 | -1.0 | 1.0  |
| 3 | 1.0  | -1.0 | 1.0  |

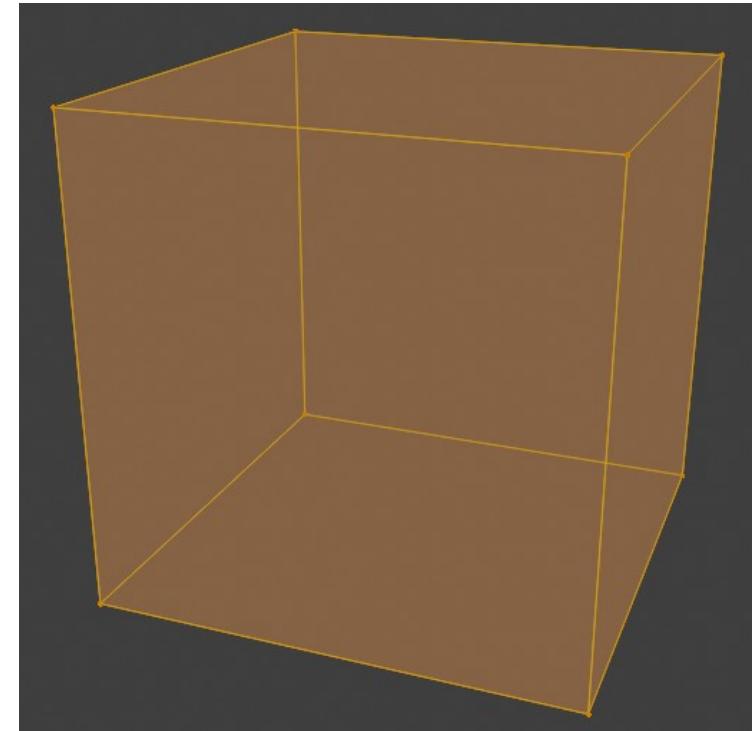
vertex array

| t | i | j | k |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 0 | 2 | 3 |
| 2 | 0 | 3 | 1 |
| 3 | 3 | 2 | 1 |

triangle array

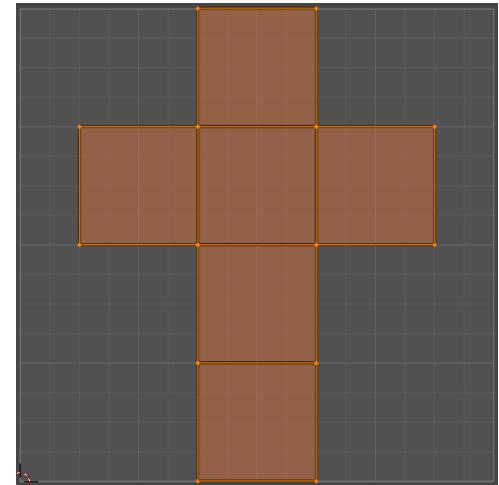
# Example: representing a cube in a computer

- 8 vertices: **vertex array – geometry information**
  - `vert_array = {{-1,1,1},{1,1,1},{1,1,-1}, {-1,1,-1}, {-1,-1,1}, {1,-1,1}, {1,-1,-1}, {-1,-1,-1}}`
- 6 quads for representing faces. Each quad requires 4 vertices: **face index array – topology information.**
  - `face_index_array = {4, 4, 4, 4, 4, 4}`
  - `size(face_index_array)` = number of polygons used for representing a shape
- For each face, which indices of vertex array are used: **vertex index array – topology information.**
  - `vertex_index_array = {0, 1, 2, 3, 0, 4, 5, 1, 1, 5, 6, 2, 0, 3, 7, 4, 5, 4, 7, 6, 2, 6, 7, 3}`
  - `size(vertex_index_array)` = sum of all values in face index array
  - Note: each face of the cube shares some vertices with other faces



# Texture coordinates

- Representing texture coordinates:
  - Texture coordinate per vertex: list of (u,v) coordinates equal to the number of vertices.  
Example: **texture\_coordinates** = {{0,0.5}, {1,0.5}, {0.5,1}, {0,0.5}, {0.5,0}, {1,0.5}}
  - In UV space it is possible that multiple vertices have same texture coordinates thus connectivity information can be used to reduce number of texture coordinates to be written down

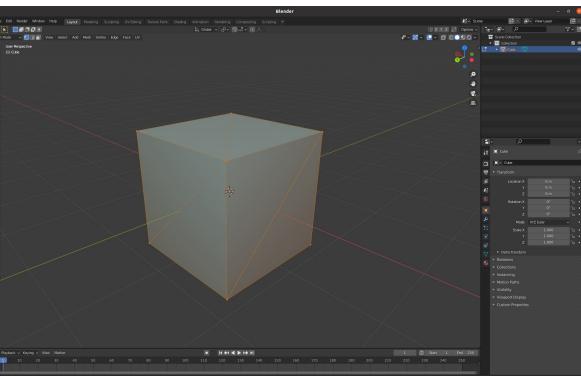


# Example: OBJ file format

- Less redundancy than explicit representation (e.g., STL)
- Separates geometry (vertices) and topology (connectivity)
- This representation is costly for computation
- Each line starts with letter representing type of data:
  - v – vertices
  - vt – texture coordinates
  - vn – normals
  - f – faces (index of vertex, vertex texture coordinate, vertex normal)

OBJ file format is further extensible:

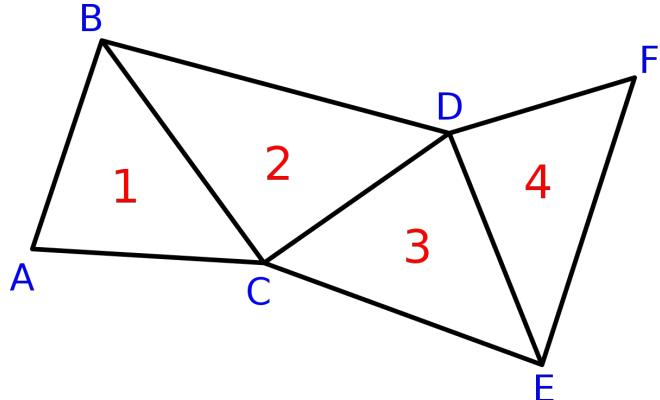
[https://en.wikipedia.org/wiki/Wavefront\\_.obj\\_file](https://en.wikipedia.org/wiki/Wavefront_.obj_file)



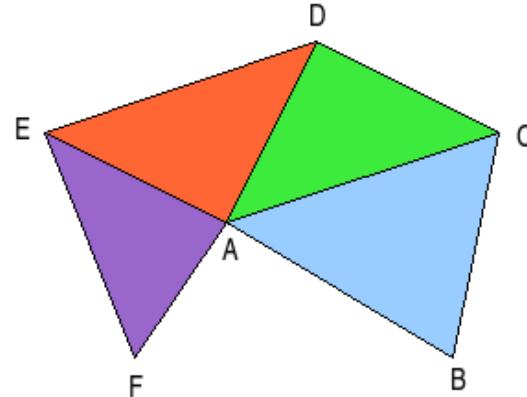
```
1 # Blender v2.92.0 OBJ File: ''
2 # www.blender.org
3 o Cube_Cube.002
4 v -1.000000 -1.000000 1.000000
5 v -1.000000 1.000000 1.000000
6 v -1.000000 -1.000000 -1.000000
7 v -1.000000 1.000000 -1.000000
8 v 1.000000 -1.000000 1.000000
9 v 1.000000 1.000000 1.000000
10 v 1.000000 -1.000000 -1.000000
11 v 1.000000 1.000000 -1.000000
12 vt 0.625000 0.000000
13 vt 0.375000 0.250000
14 vt 0.375000 0.000000
15 vt 0.625000 0.250000
16 vt 0.375000 0.500000
17 vt 0.625000 0.500000
18 vt 0.375000 0.750000
19 vt 0.625000 0.750000
20 vt 0.375000 1.000000
21 vt 0.125000 0.750000
22 vt 0.125000 0.500000
23 vt 0.875000 0.500000
24 vt 0.625000 1.000000
25 vt 0.875000 0.750000
26 vn -1.0000 0.0000 0.0000
27 vn 0.0000 0.0000 -1.0000
28 vn 1.0000 0.0000 0.0000
29 vn 0.0000 0.0000 1.0000
30 vn 0.0000 -1.0000 0.0000
31 vn 0.0000 1.0000 0.0000
32 s off
33 f 2/1/1 3/2/1 1/3/1
34 f 4/4/2 7/5/2 3/2/2
35 f 8/6/3 5/7/3 7/5/3
36 f 6/8/4 1/9/4 5/7/4
37 f 7/5/5 1/10/5 3/11/5
38 f 4/12/6 6/8/6 8/6/6
39 f 2/1/1 4/4/1 3/2/1
40 f 4/4/2 8/6/2 7/5/2
41 f 8/6/3 6/8/3 5/7/3
42 f 6/8/4 2/13/4 1/9/4
43 f 7/5/5 5/7/5 1/10/5
44 f 4/12/6 2/14/6 6/8/6
```

# Example: specific data structures

- Data structures for minimal redundancy and memory efficient representation by implicit topology:
  - Triangle/quad strip
  - Triangle fan
- Complex meshes are represented as set of strips and fans
  - Used for hardware rendering (OpenGL - <https://www.khronos.org/opengl/wiki/Primitive>)



Triangle strip: implicit representation of strip-like sequence of triangles. [https://en.wikipedia.org/wiki/Triangle\\_strip](https://en.wikipedia.org/wiki/Triangle_strip)



Triangle fan: set of triangles sharing one vertex  
<https://www.khronos.org/opengl/wiki/Primitive>

# Recap: representing and storing mesh

- To describe a mesh we need:
  - Vertex array (vertex positions)
  - Face index array (how many vertices each face is made of)
  - Vertex index array (connectivity)
  - Primitive variables:
    - Vertex color
    - Normals
    - Texture coordinates
    - Other optional application-specific values

# Storing polygons: practical note

- Even single polygon mesh in a 3D scene can be quite large ( $10^5$ - $10^6$  vertices is not unusual)
- Storing vertices and connectivity information must be performed efficiently
  - All vertices must be stored
  - Different techniques exist which try to minimize the amount of data needed for representing connectivity → yielding different standards, formats and API specifications for storing and transferring mesh data, e.g., OBJ or FBX\*.

\* Those are popular and widely used standards. We will discuss them more when we will be talking about triangle meshes. RenderMan, on the other hand, defines API specification for representing mesh data.

# Storing and transferring mesh objects

- Mesh data is often transferred between different modeling, rendering and interactive tools.
- Interface between different tools and specification of how mesh should be stored is defined by standards\*: [https://renderman.pixar.com/resources/RenderMan\\_20/ribBinding.html](https://renderman.pixar.com/resources/RenderMan_20/ribBinding.html)
- Different implementations of mesh storage formats exists, which are:
  - Are more or less compact
  - Are more or less human-readable
  - Can contain additional object data which is described with the mesh (textures, materials, etc.)
  - Can contain various metadata (e.g., physical behavior of object described with mesh)
  - Store only mesh information
  - Store whole scene and mesh is only one of elements
- Popular formats:
  - <https://all3dp.com/2/most-common-3d-file-formats-model/>
  - [https://www.sidefx.com/docs/houdini/io/formats/geometry\\_formats.html](https://www.sidefx.com/docs/houdini/io/formats/geometry_formats.html)
- 3D scene is not necessarily created, rendered and used in same software. Usually, whole pipeline of software is used, at least:
  - DCC → game engines
- Formats: OBJ, GLTF, USD
- Interesting: <https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-polygon-mesh/polygon-mesh-file-formats>

\* note that tendency is towards standardization of whole scene description. Which, besides mesh polygons, include materials, lights, cameras, different shape representations, etc.

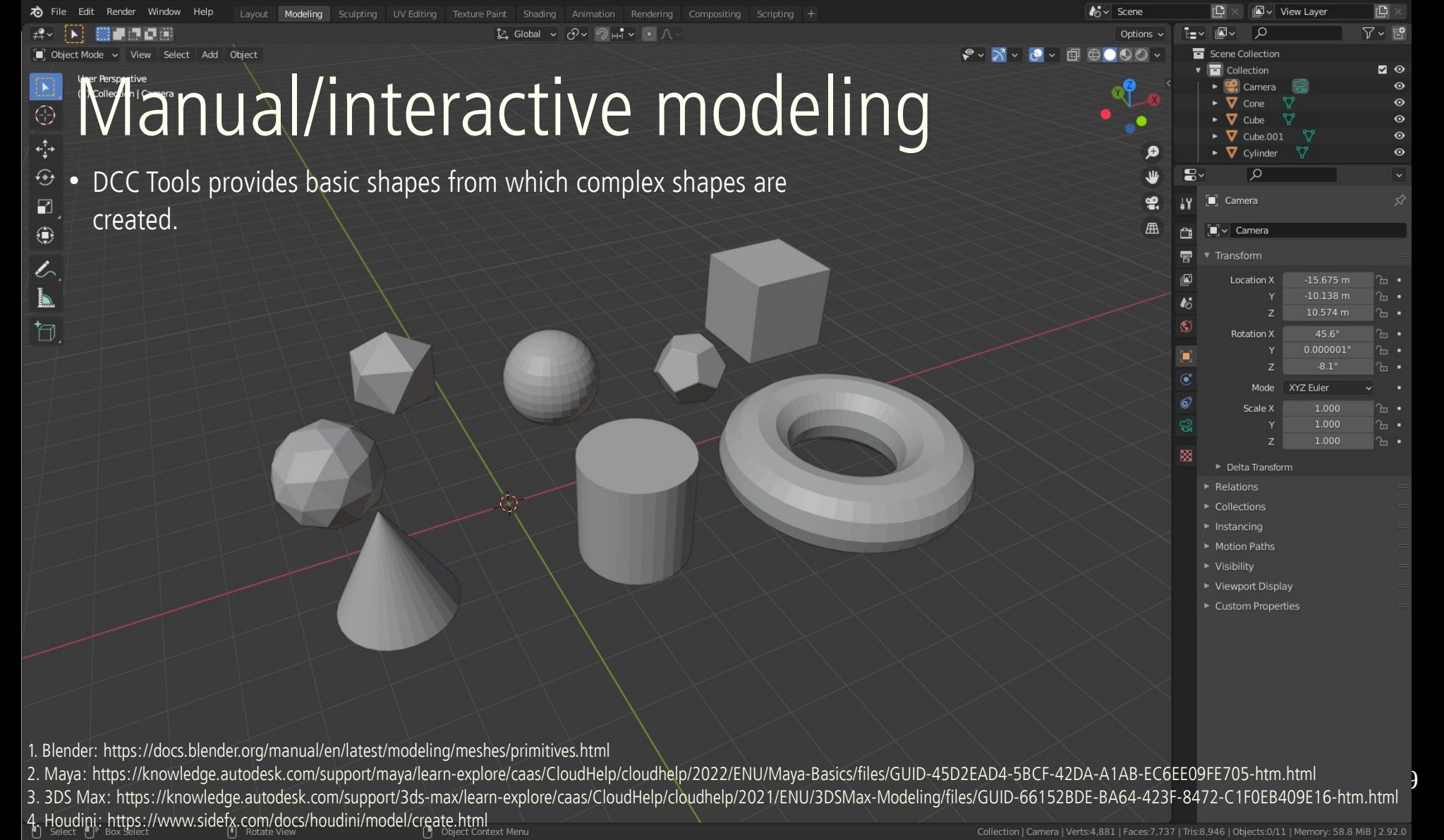
# Practical note: using different formats

- Professional file formats and representations are commonly used and are very efficient
- Using modeling/interactive/rendering tools, user is often provided with “importer/exporter”
  - feature which enables importing and exporting different file formats
    - Example: <https://www.sidefx.com/docs/houdini/io/formats/index.html>
- The problem comes if one writes its own rendering program and parsing file formats for import can be not that easy. Luckily, different libraries can be used for this purposes:
  - Example: <https://github.com/assimp/assimp>

# Practical note: units

- Note that until now we haven't discussed units in which shape is stored.
- Units are important for preserving consistent scale among different modeling/rendering tools in which object might be transferred.
- Let's discuss creation of two meshes in Blender. Mesh, that is, vertices are defined in a coordinate system. In this coordinate system we can have two same objects where one is larger and another is smaller. By relation/proportion we can distinguish the scale and we might not care about units.
  - Since positions of vertices are relative to coordinate system, the axis of coordinate system must specify units.
- But what happens if one of this objects is exported into another tool, for example Unity? Unity might use different coordinate system unit scale and object might be too big or too small in this coordinate system!
  - Example: different scales with same coordinates.
- Unit is defined by user who models the object and it must be taken care of during transfer.

# Mesh polygons: modeling and acquisition



# Manual/interactive modeling

- DCC Tools provides basic shapes from which complex shapes are created.

1. Blender: <https://docs.blender.org/manual/en/latest/modeling/meshes/primitives.html>

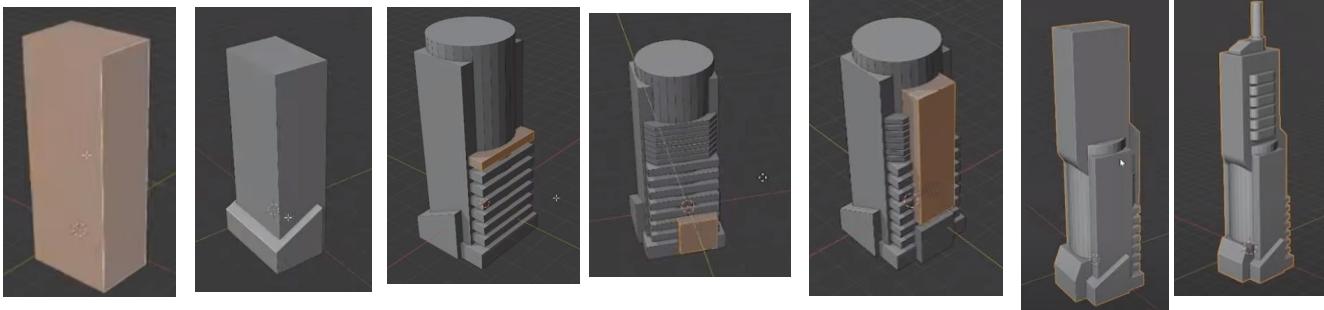
2. Maya: <https://knowledge.autodesk.com/support/maya/learn-explore/caas/CloudHelp/cloudhelp/2022/ENU/Maya-Basics/files/GUID-45D2EAD4-5BCF-42DA-A1AB-EC6EE09FE705.htm.html>

3. 3DS Max: <https://knowledge.autodesk.com/support/3ds-max/learn-explore/caas/CloudHelp/cloudhelp/2021/ENU/3DSMax-Modeling/files/GUID-66152BDE-BA64-423F-8472-C1F0EB409E16.htm.html>

4. Houdini: <https://www.sidefx.com/docs/houdini/model/create.html>

# Practical tip: complex shapes from base shapes

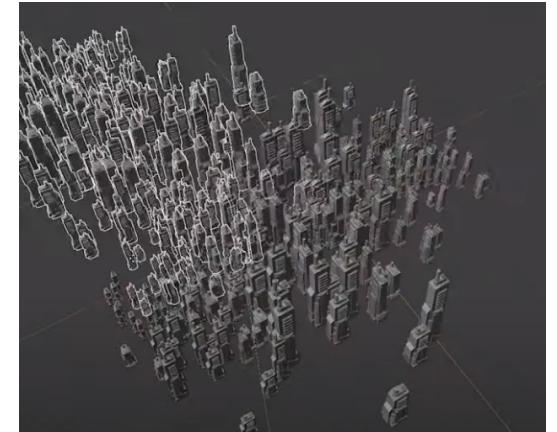
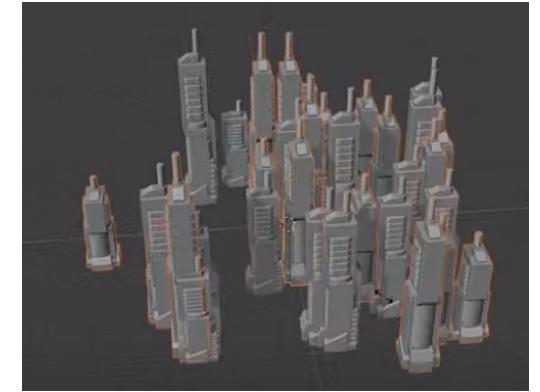
- Anything can be decomposed in simple forms<sup>1,2</sup>: box, sphere, cylinder, torus, cones, etc.



[https://www.youtube.com/watch?v=Q0qKO2JYR3Y&ab\\_channel=BlenderSecrets](https://www.youtube.com/watch?v=Q0qKO2JYR3Y&ab_channel=BlenderSecrets)

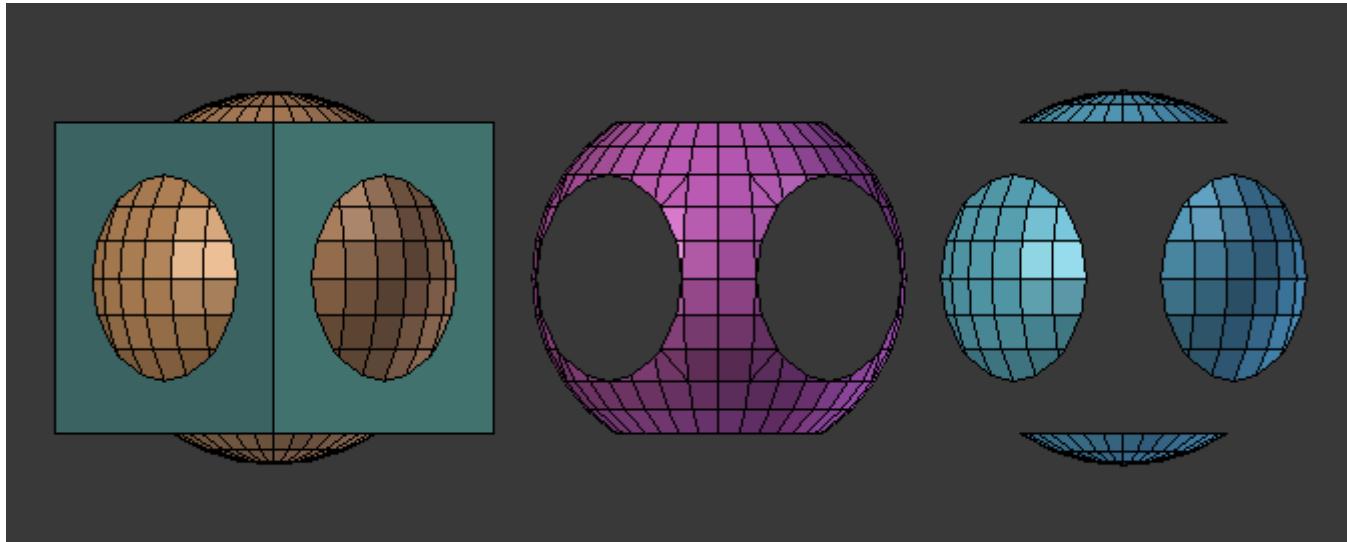
1. <http://www.thedrawingwebsite.com/2015/02/18/practicing-your-draw-fu-forms-forms-are-like-sentences/>

2. [https://www.youtube.com/watch?v=6T\\_-DiAzYBc&list=RDCMUCIM2LuQ1q5WEc23462tQzBg&start\\_radio=1&rv=6T\\_-DiAzYBc&t=1343&ab\\_channel=Proko](https://www.youtube.com/watch?v=6T_-DiAzYBc&list=RDCMUCIM2LuQ1q5WEc23462tQzBg&start_radio=1&rv=6T_-DiAzYBc&t=1343&ab_channel=Proko)



# Constructive solid geometry

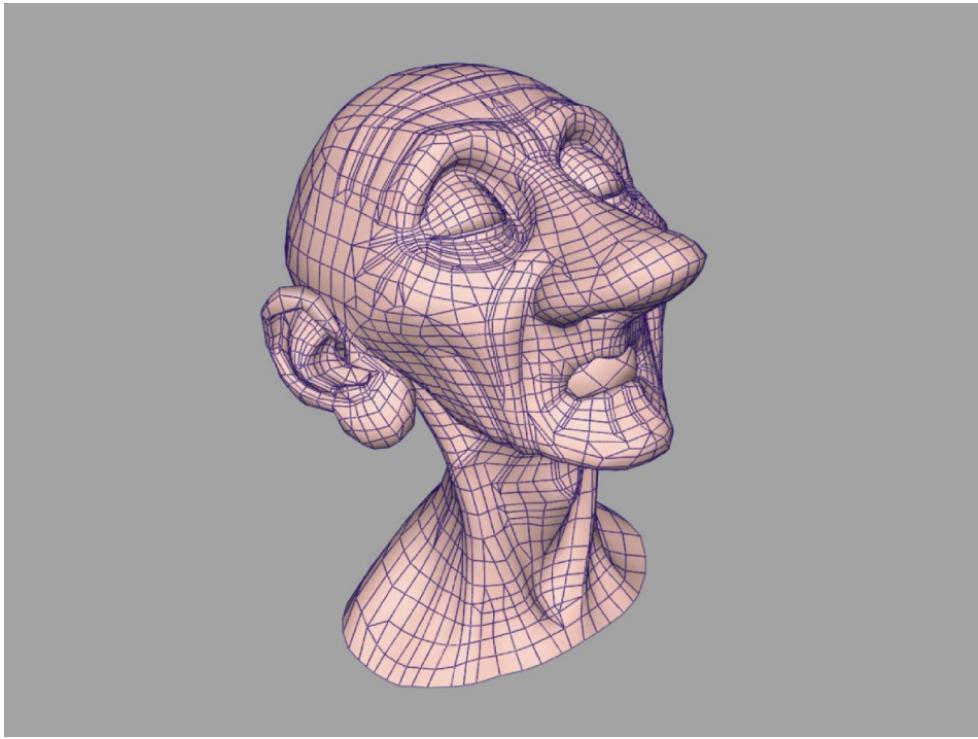
- Boolean operations applied on meshes:
  - Union
  - Intersection
  - Difference



Union, intersection and difference Boolean operators applied on meshes:

<https://docs.blender.org/manual/en/latest/modeling/modifiers/generateBOOLEANS.html>

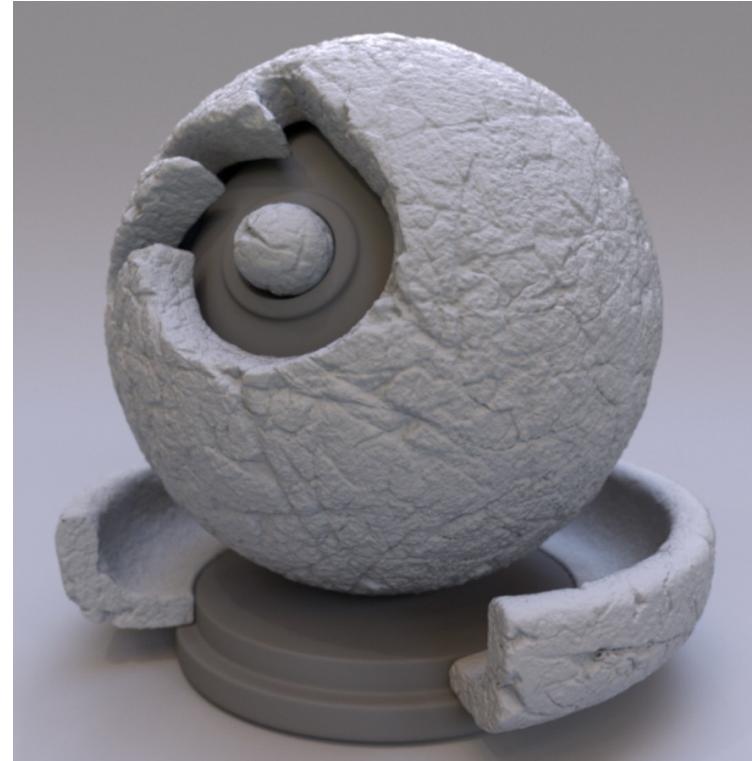
# Subdivision surfaces and displacement



Subdivision surfaces for smooth surfaces in Pixar:

<https://graphics.pixar.com/library/Geri/paper.pdf>

<https://www.fxguide.com/fxfeatured/pixars-opensubdiv-v2-a-detailed-look/>

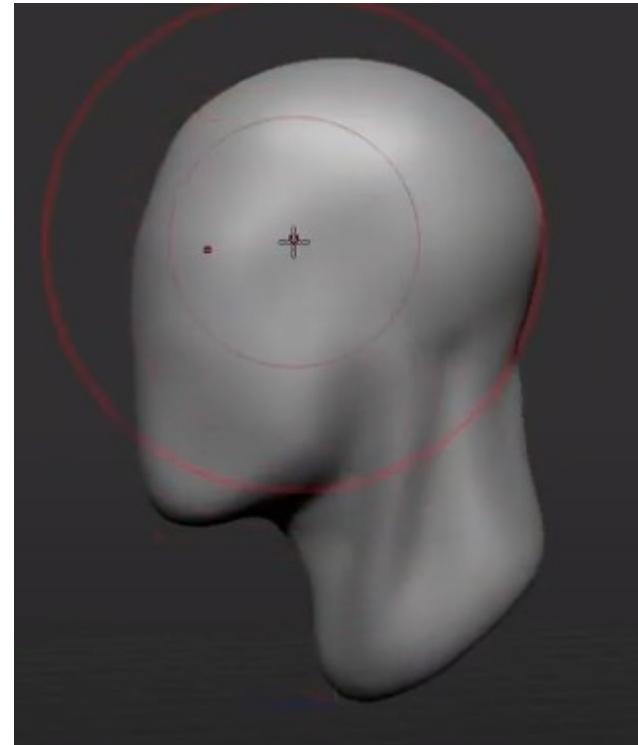
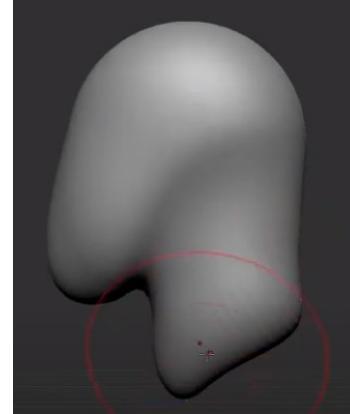
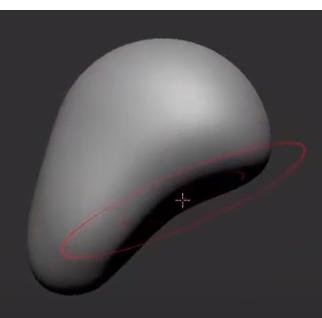
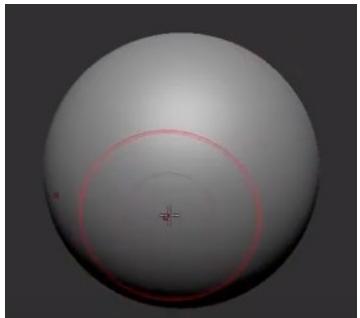


Blender displacement of subdivided surface:

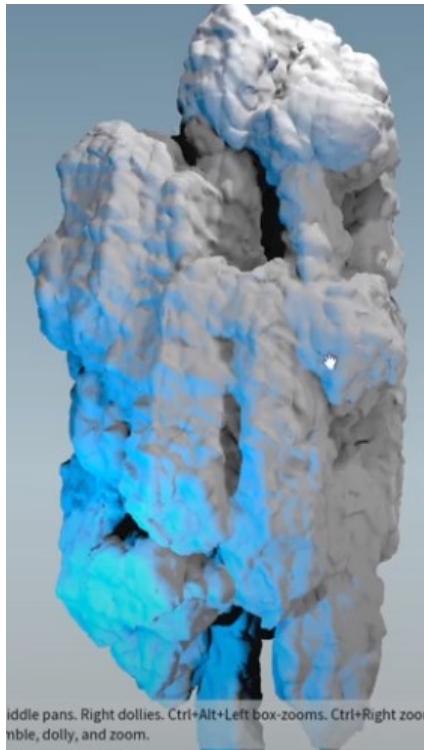
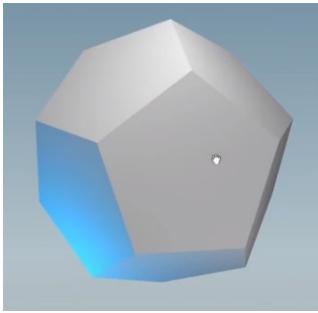
<https://docs.blender.org/manual/en/latest/render/materials/components/displacement.html>

# Practical tip: complex shapes from base shapes

- Sculpting base shapes.



# Practical tip: complex shapes from base shapes

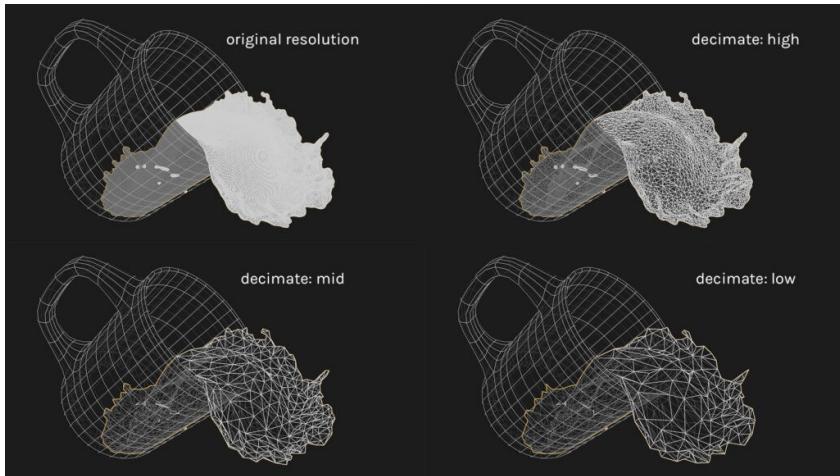


- Procedural modeling



# Simulations

- Polygonal model can be generated using simulations (e.g., fluid simulation using voxel representation) which is then tessellated into polygons for rendering.



Fluid simulation in Blender:  
<https://andi-siess.de/designing-a-book-cover-as-3d-vector/>



Houdini fluid simulation: <https://www.sidefx.com/tutorials/crashing-wave/>

# Procedural modeling

- Similarly to simulations, polygonal mesh can be created by writing programs that create such data – procedural modeling



Blender procedural modeling using geometry nodes:

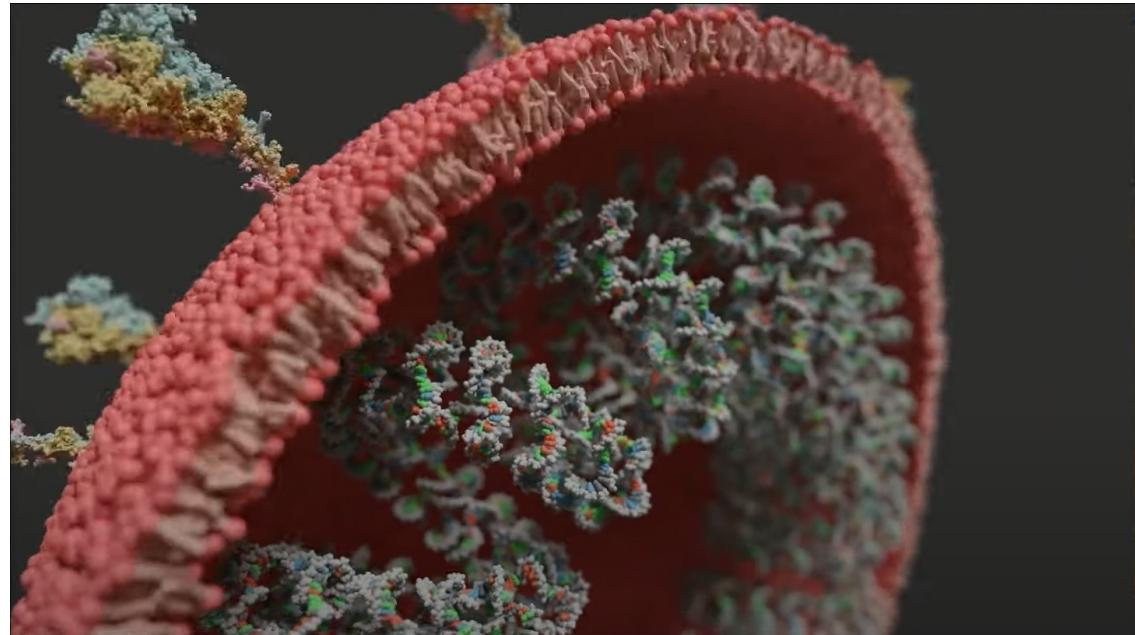
<https://blenderartists.org/t/procedural-abandoned-house-with-geometry-nodes/1363024>



Procedural modeling in Houdini:  
<https://entagma.com/category/tutorials/>

# Transforming data into visualizations

- Data found in different forms can be converted to polygon data. Example: visualizing protein data using spheres.

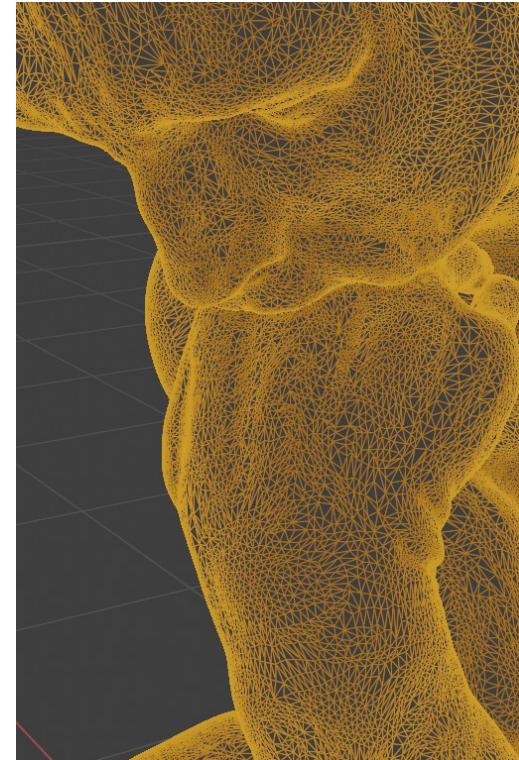
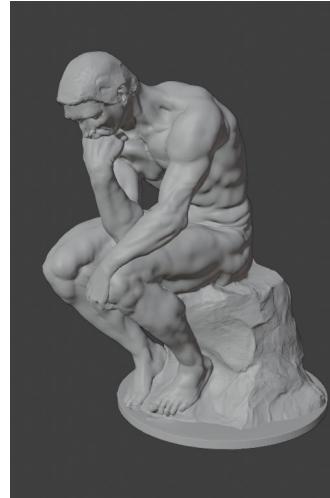
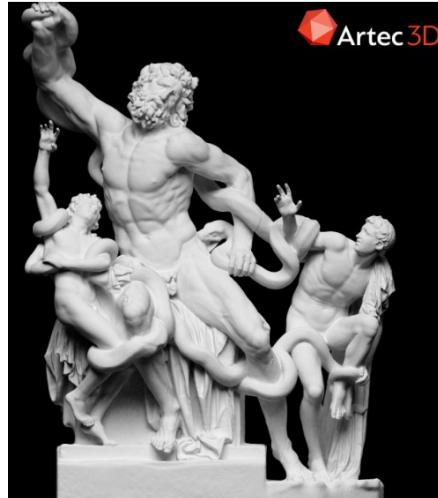


Visualizing viruses in Blender:

[https://www.youtube.com/watch?v=adhTmwYwOjA&ab\\_channel=Blender](https://www.youtube.com/watch?v=adhTmwYwOjA&ab_channel=Blender)

# Scanning and photogrammetry

- Polygonal model can be generated using scanning and photogrammetry which results in point clouds and then is transformed into polygons for rendering.



Scan the world: <https://www.myminifactory.com/scantheworld/>

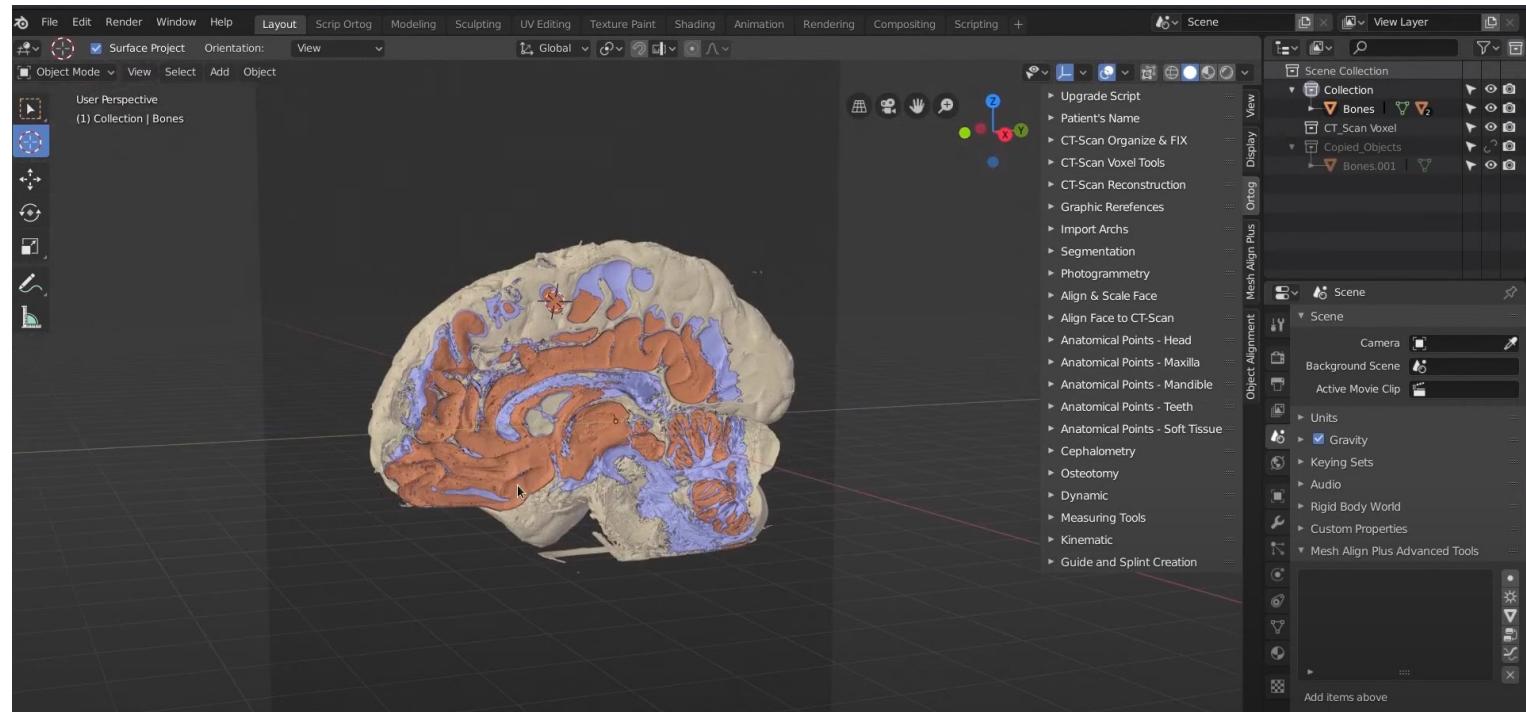
Sans Factory: <https://www.scansfactory.com/>

Mega Scans: <https://quixel.com/megascans/home/>

Art Station 3D scanning and photogrammetry: [https://www.artstation.com/channels/photogrammetry\\_3d\\_scanning?sort\\_by=popular](https://www.artstation.com/channels/photogrammetry_3d_scanning?sort_by=popular)

# Data from volumetric scans

- Generating surface from volume data scanned with CT or MRI.

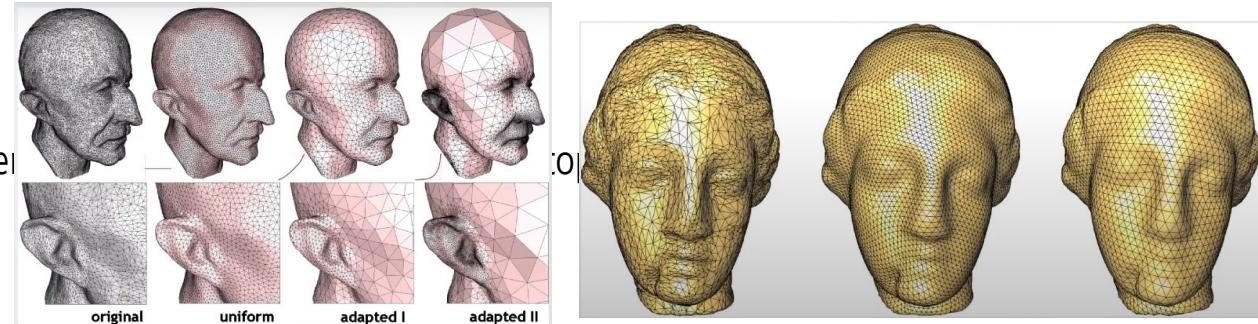


MRI to mesh in Blender: <https://www.blendernation.com/2019/07/15/converting-a-video-into-a-dicom-and-a-3d-mesh/>

# Mesh polygons: outlook

# Deeper into topic

- Polygonal mesh is highly used and researched method in computer graphics. We have covered foundations. There are many other topics. Some of those will be discussed later, some are out of scope for this course:
  - Remeshing
  - Simplification, level of detail
    - replacing the mesh with the simpler one
  - Mesh compression
  - Consolidation, mesh reparation
  - Animation, skinning and deformation



# Exploring meshes

- **Meshlab** is open-source tool for mesh manipulation and processing
  - Mesh smoothing and sharpening
  - Re-meshing: subdivide, re-sample, simplify,
  - Topological operations: fill holes, fix self-intersections
  - Boolean operations
- **libigl** and **CGAL** libraries for mesh processing
- **Blender**
  - Modeling with meshes
  - Procedural meshes with Python
- **Sources of mesh data:**
  - <https://casual-effects.com/data/index.html>
  - <https://polyhaven.com/models>
  - <https://sketchfab.com/>
  - <http://graphics.stanford.edu/data/3Dscanrep/>
- **More informations:**
  - <https://www.realtimerendering.com/#polytech>

# Mesh shape representation: verdict

- Pros:
  - Most common surface representation
  - Simple for representing and intuitive for modeling and acquisition
  - A lot of effort has been made to represent various shapes with meshes
  - Lot of research has been done to convert other shape representations to mesh representation
  - Graphics hardware is adapted and optimized to work with (triangle) meshes → fast rendering
- Cons:
  - Not Guaranteeing smoothness
  - Triangle meshes are not efficient or intuitive for manual/interactive modeling
  - Not every object is well suited to mesh representation:
    - Shapes that have geometrical detail at every level (e.g., fractured marble)
    - Some objects have structure which is unsuitable for mesh representation, e.g., hair which has more compact representations

# Reading material

- [https://www.youtube.com/watch?v=V3Npa0uZYIE&list=PL4TptkuzgxxUVZ-\\_Di033kp4\\_rkoAy1BC&index=6&ab\\_channel=ChristophGarth](https://www.youtube.com/watch?v=V3Npa0uZYIE&list=PL4TptkuzgxxUVZ-_Di033kp4_rkoAy1BC&index=6&ab_channel=ChristophGarth)
- <https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-polygon-mesh/introduction.html>
- <https://www.pbr-book.org/3ed-2018/Shapes>
- Real-time rendering book
  - Chapters: 16 and 17
- Computer graphics practices and principles book
  - Chapters 8, 9, 23 and 25