

Lecture 11: Rendering overview

DHBW, Computer Graphics

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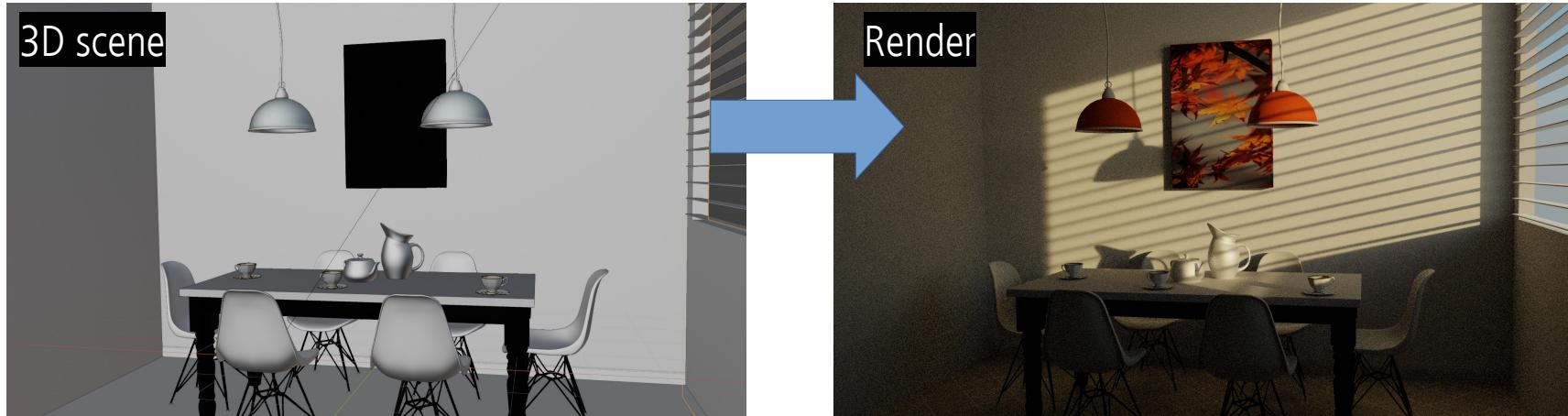
8.3.2023.

Syllabus

- 3D scene
 - Object
 - Light
 - Camera
 - Rendering
 - Image and display
-
- The diagram illustrates the flow of the syllabus. It starts with a list of components for a 3D scene: Object, Light, and Camera. An arrow points from this list to a rounded rectangle containing a list for 'Rendering'. Another arrow points from the 'Rendering' section to the final topic, 'Image and display'.

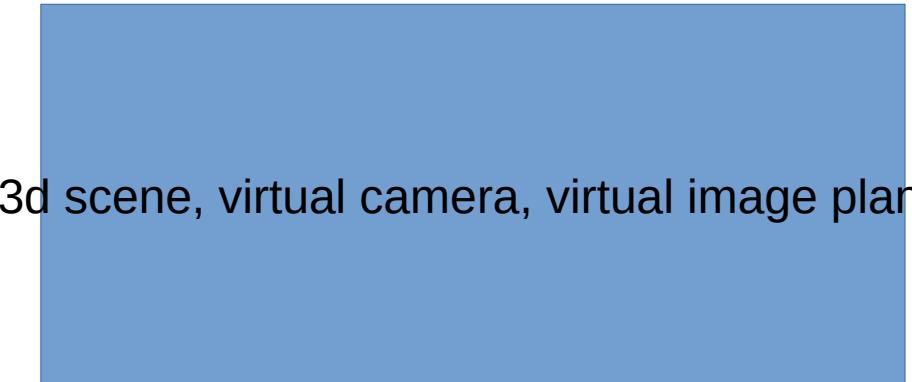
3D scene and rendering

- We have discussed how to model elements of 3D scene:
 - Objects: shape and material
 - Lights
 - Cameras
- Rendering creates an image from 3D scene
 - Information on 3D scene is used to simulate interaction of light with objects



Rendering

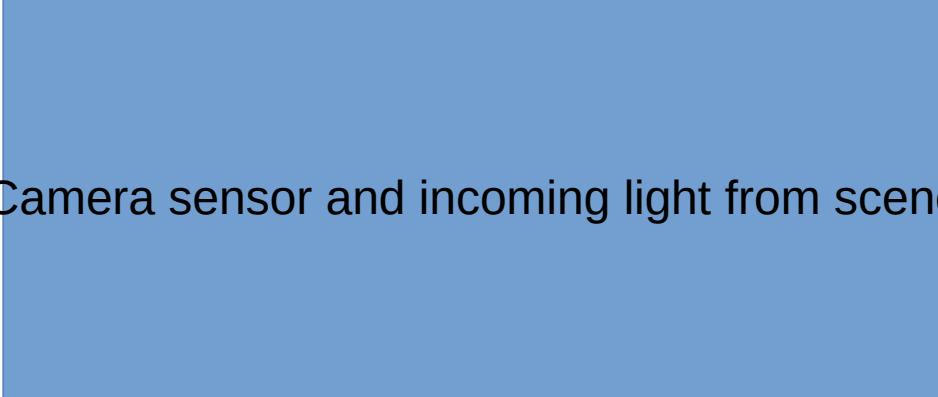
- The goal of rendering is to create viewable 2D image from 3D scene which can be displayed on raster display devices
 - Images are 2D array of pixels
- The task of rendering is to compute color for each pixel of virtual image plane placed in 3D scene
 - Find which objects are visible from pixels or what is covered by pixels
 - Calculate color of visible objects



3d scene, virtual camera, virtual image plane

Rendering: intuition

- Idea of computing pixel colors of virtual image plane is similar to how real camera works
 - Virtual image plane simulates digital camera sensor
- Digital camera sensor is made of array of photo-sensitive cells which convert incoming light into colors
- Light falling on camera is reflected from objects and emitted from light sources
- Therefore, we are only interested into light falling on camera sensor



Camera sensor and incoming light from scene

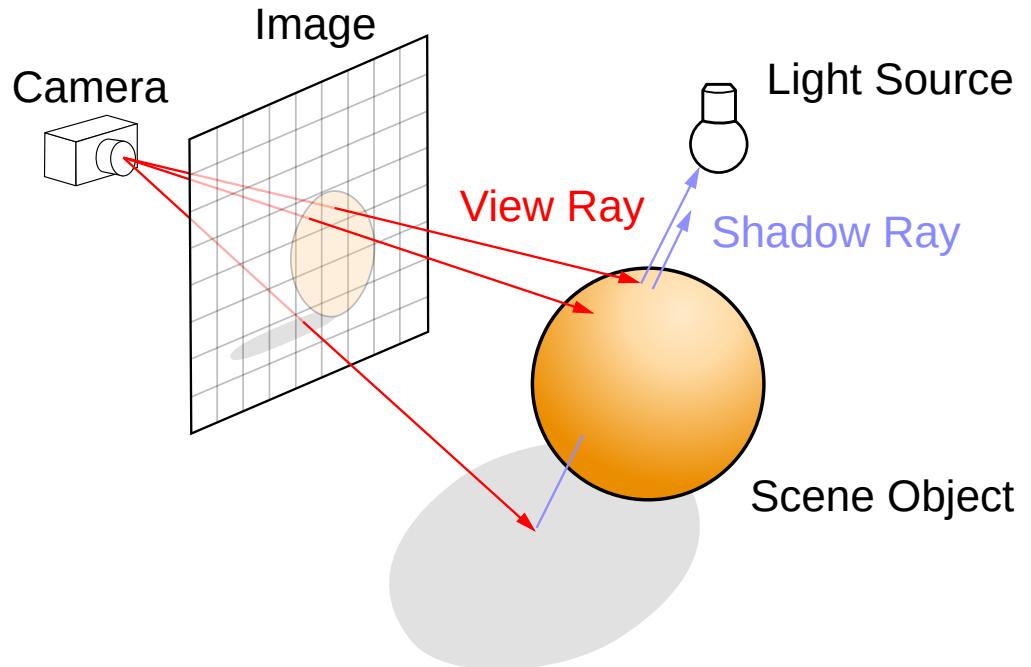
Rendering: physical and perceptual foundations

- Light emission, travel and interaction with objects and camera sensor is well described in physics (wave and geometrical optics)
- Light and image formation can not be only simulated on physical level; we need to **take in account human visual system**
 - Size and shape of objects gets smaller if are more further → **foreshortening effect**
 - Cameras produce images on this principle
 - In rendering we project objects on flat plane (image plane) using **perspective projection**
 - Which objects can we see → **visibility problem**
 - How objects appear → **color** visible objects

Rendering: main steps

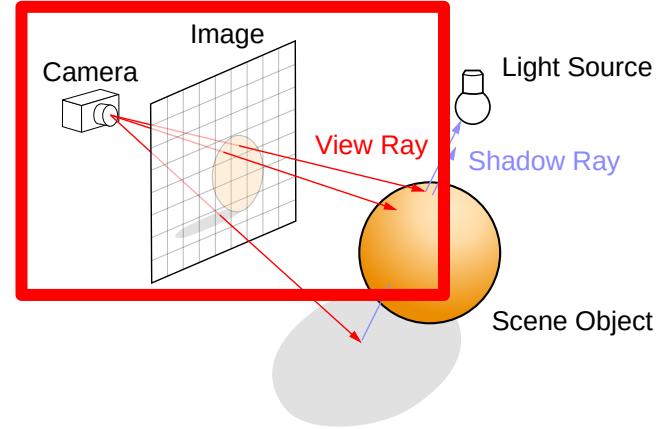
Rendering is solving:

- **Visibility** problem – which objects and surfaces are visible to each other or from camera
 - Rasterization
 - Ray-tracing
- **Shading** problem – how does visible objects and surfaces look like → color



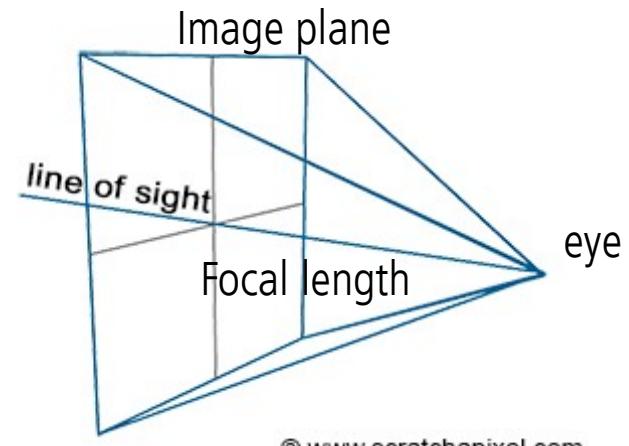
Visibility

- Visibility determines if two points are visible one to another
- Often, used to determine which objects are visible to camera
 - Human visual system is simulated → perspective projection



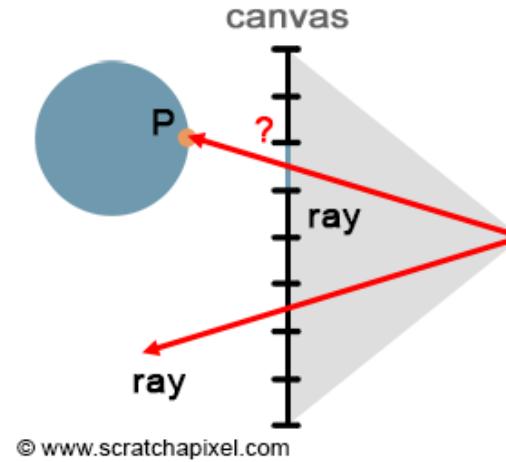
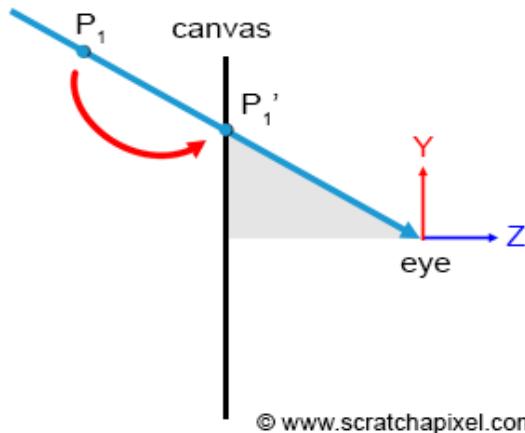
Perspective projection

- Image is representation of 3D scene on a flat surface, e.g., image plane
- Perspective projection simulates how human visual system forms images
 - Objects are projected on image plane
 - Foreshortening effect: more distant objects appear smaller
- Projection defines view frustum
 - Objects outside of frustum are not visible
 - Shape of frustum depends on image planes size and focal length
- Different projections are possible, e.g., orthographic projection



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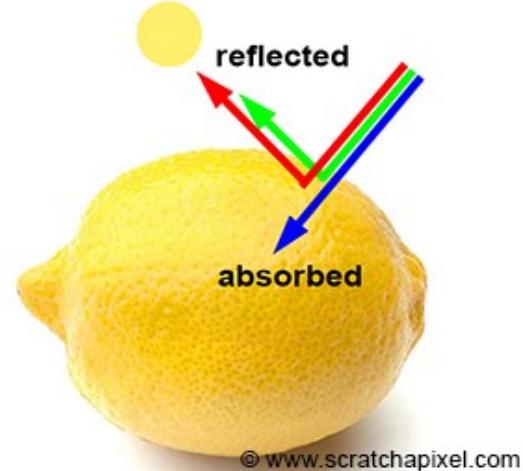
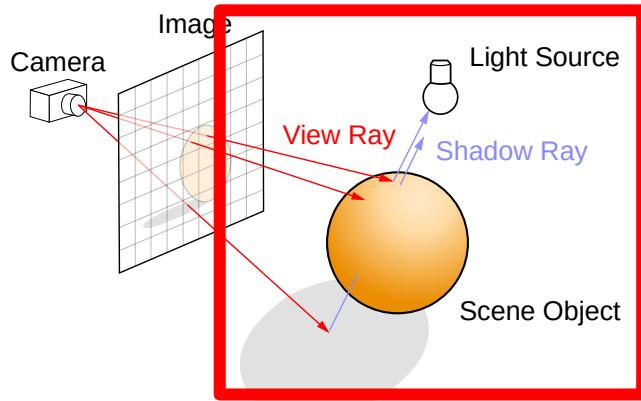
Visibility and projection



- Rasterization-based rendering projects object vertices on screen to solve visibility
 - Perspective projection matrix is used
- Ray-tracing based rendering generates rays from camera to solve visibility
 - Perspective projection will be inherently present

Shading

- Shading calculates appearance (color) of objects visible to camera
- Color of object is determined by:
 - Amount of light falling on it
 - Material which defines how light reflects and absorbs



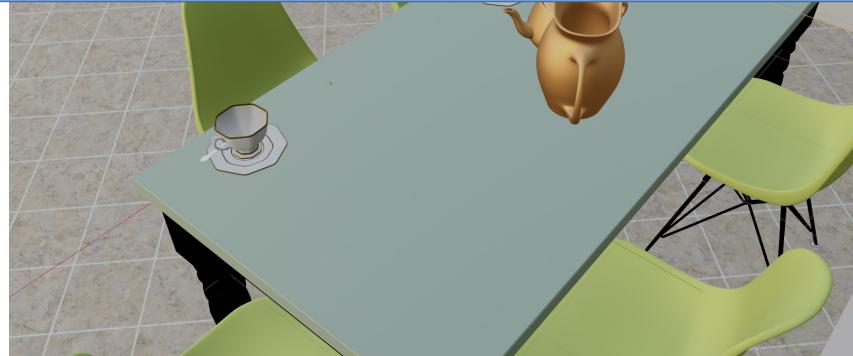
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Shading



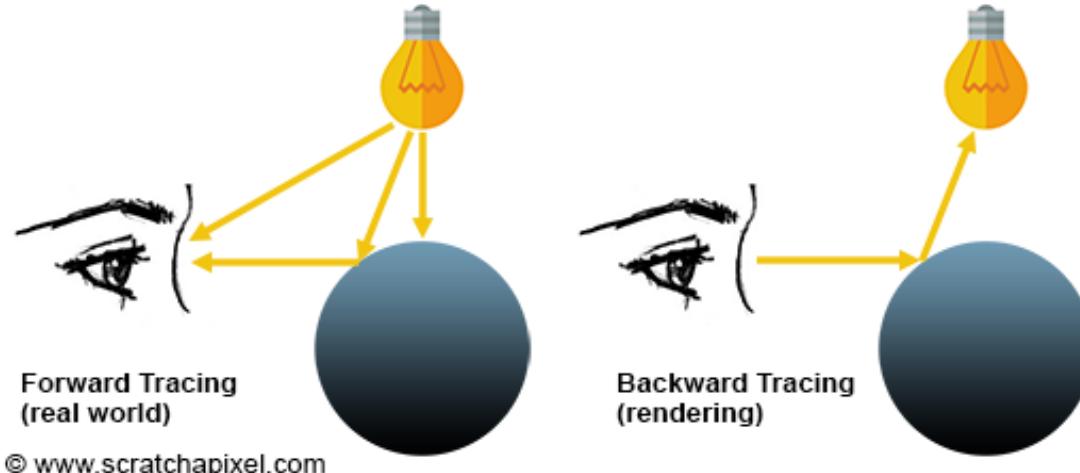
add: <https://www.scratchapixel.com/lessons/3d-basic-rendering/rendering-3d-scene-overview/introduction-to-shading>

- Form of visibility problem: which light sources and surfaces reflecting light are visible from shaded surface
- This computation is called **light transport**



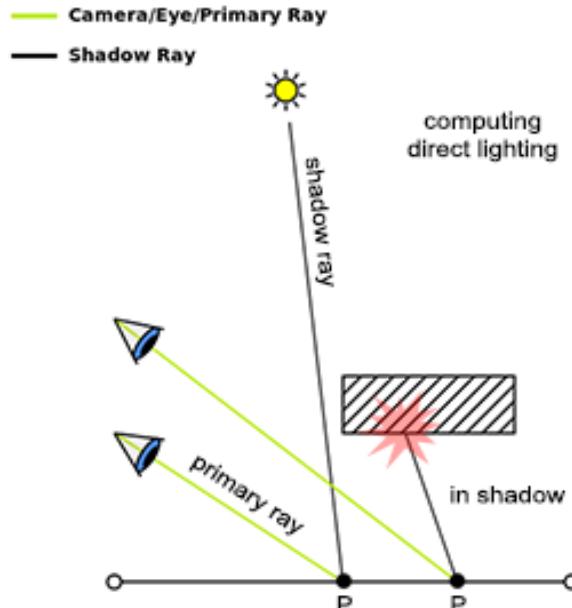
Light transport

- Information on light is crucial for shading
- Forward tracing starts from light, bounces around the scene and enters camera
- Backward tracing starts from camera, finds a way to light source
- Light transport is used to find amount of light falling on objects visible from camera

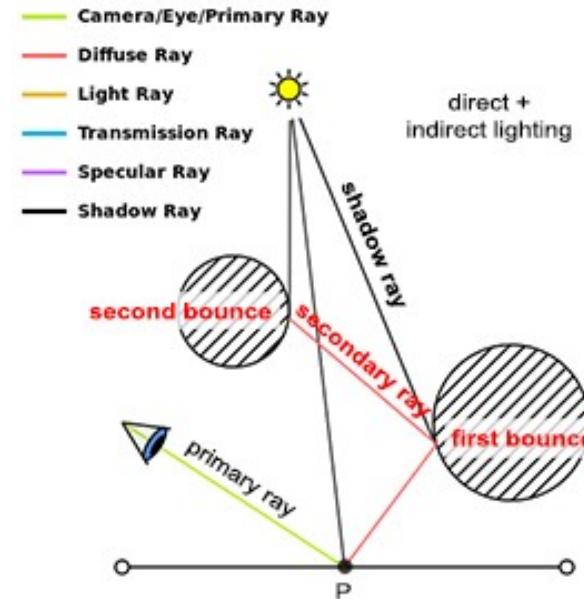


Light transport

- Backward tracing: direct illumination
 - Only takes in account direct contribution from light source (shadow ray)

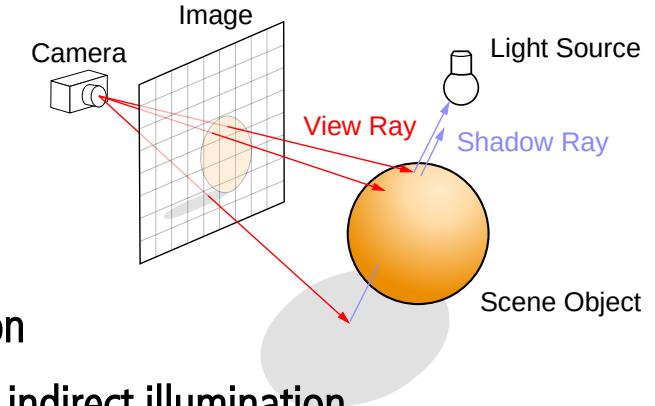


- Backward tracing: direct and indirect illumination
 - Takes in account both direct and indirect light contribution (shadow and secondary rays)



Light transport

- Relies on concept of visibility
 - Which light sources are visible to shaded surface → direct illumination
 - Which surfaces are visible to shaded surface that may reflect light → indirect illumination
- Light transport significantly determines the resulting image realism



Rasterization (EEVEE, Blender)

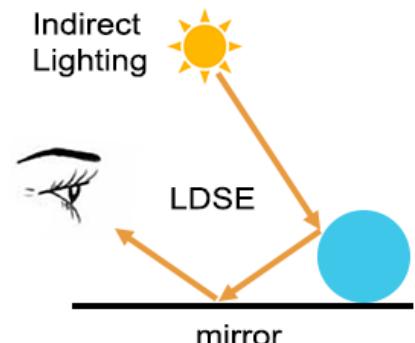
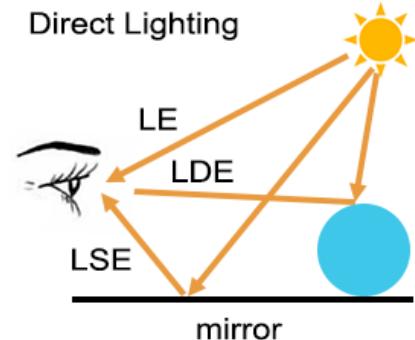
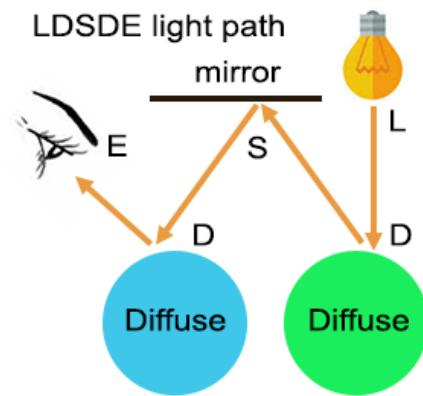


Ray-tracing (Cycles, Blender)



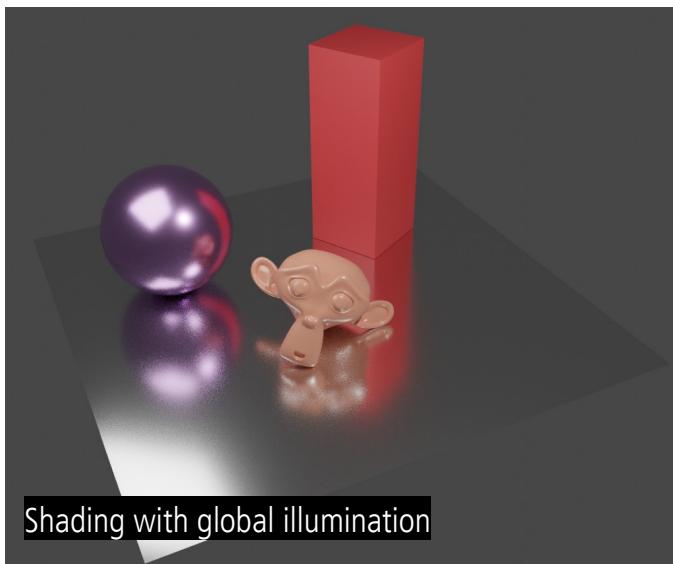
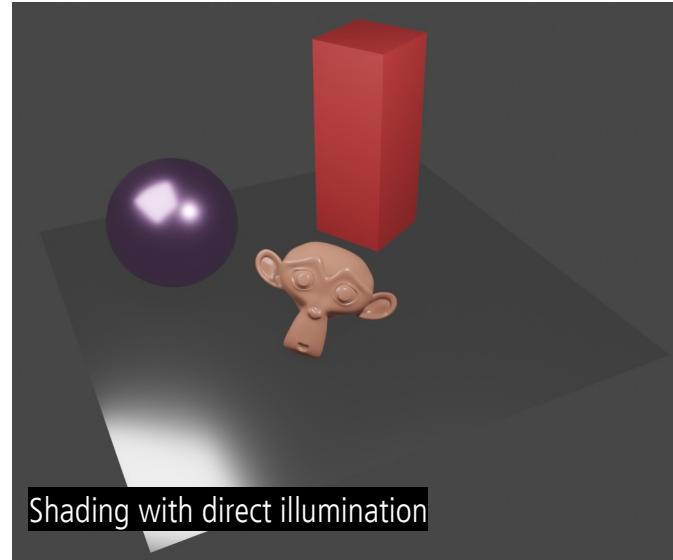
Light transport

- Light is emitted from light source (**L**), reflects on objects, small amount enters the eye (**E**)
- Direction of light reflection depends on object material:
 - Diffuse surface (**D**)
 - Specular surface (**S**)
- Light paths solved by light transport can be characterized using **path labeling**: **L, E, D, S**
 - Paul Heckbert: "Adaptive Radiosity Textures for Bidirectional Ray Tracing"



Object appearance

- Object appearance (color) depends on:
 - How light interacts with objects
 - How light travels between objects
- **Shading:** light-object interaction
 - Reflection, refraction, transparency, diffuse, specular, glossy, etc.
- **Light transport:** how much light falls on surface
 - **Direct illumination:** light from light sources
 - **Indirect illumination:** inter-reflections (indirect diffuse, indirect specular), soft shadows, transmission
 - **Global illumination:** direct + indirect illumination



Decoupling rendering steps

- Visibility & projection
 - Perspective projection
 - Rays and camera
 - Ray-triangle intersection
 - Ray-mesh intersection
 - Ray-shape intersection
 - Object transformations
 - Rasterization
 - Ray-tracing
 - Etc.
- Shading & light transport
 - Rendering equation
 - Light transport algorithms: path-tracing
 - Non-physical lights (e.g., point lights)
 - Physical (area) lights
 - Material
 - Texture
 - Scattering functions; BRDF
 - Diffuse, glossy, specular
 - Etc.

Rendering equation

- Shading and light transport are described by rendering equation:

$$L_o(p, \omega_o) = L_e(p, \omega_o) + L_s(p, \omega_o)$$

Emission Scattering → reflection

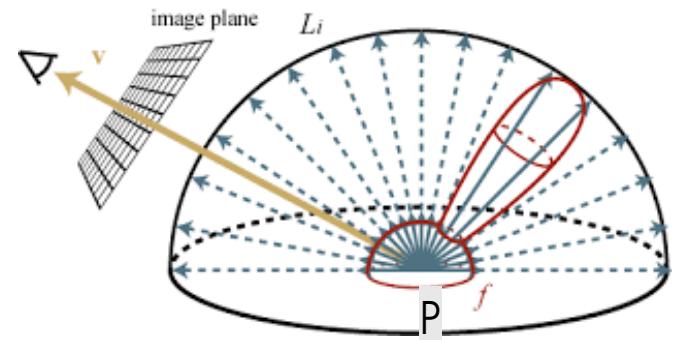
$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega} f(p, \omega_o, \omega_i) L_i(p, \omega_i) (\omega_i \cdot n) d\omega_i$$

Emission BRDF Incoming light Attenuation due to surface orientation

- Rendering equation is foundation of physically-based rendering
- All rendering approaches are solves rendering equation to some extent or just some of its parts.

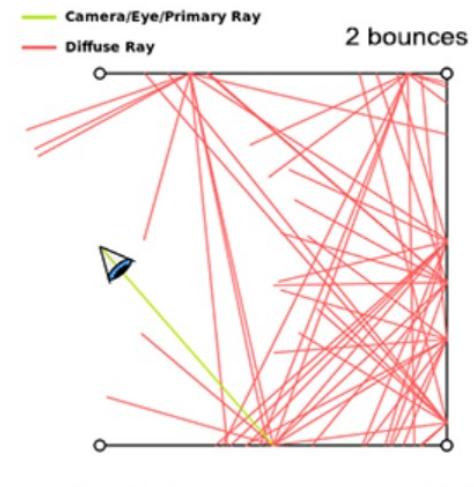
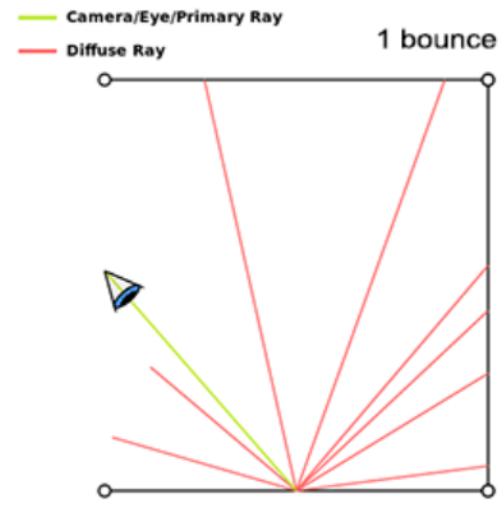
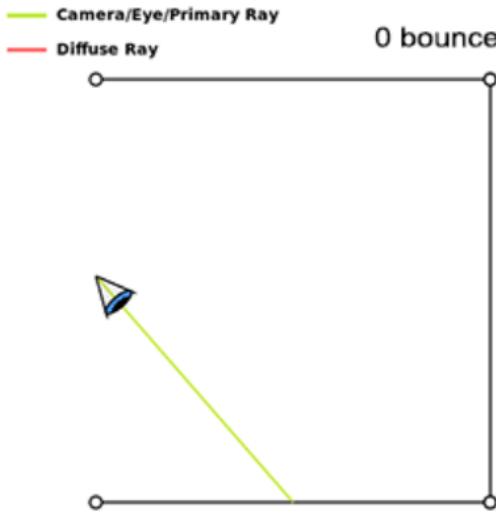
Rendering equation: incoming light directions

- Light can come from any direction on point P
- There are infinite number of possible directions (thus integral sign)
- This can not be solved analytically, solutions:
 - Simplify possible incoming directions: direct illumination – only look for light sources
 - Approximate by sampling smaller number of directions: indirect illumination – other surfaces may contribute



Rendering equation: recursive nature

- For each light direction incoming from another surface, the rendering equation must be evaluated again



Rendering equation solvers

- Rendering equation is very general: describes wide range of light transport phenomena
 - Not possible to solve analytically
- Different methods compute incoming light with simplifications or approximations:
 - Direct illumination: usually rasterization-based rendering
 - Consider only light sources, and light from specular reflection and transmission: Whitted ray-tracing
 - Direct and indirect illumination solvers:
 - Stochastic approximation based on Monte-Carlo ray-tracing → path tracing, bidirectional path tracing, metropolis light transport, etc.
 - Approximation using finite element method → radiosity

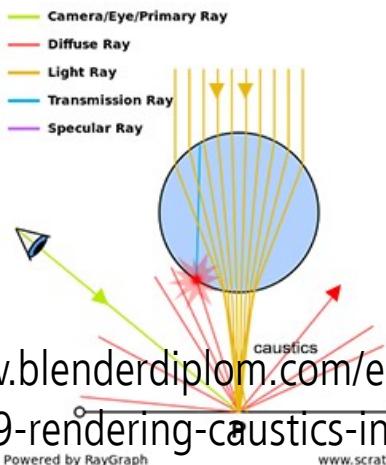
Comparisons of different approaches:

1. direct illumination
2. ray tracing
3. global illumination: path tracing, radiosity

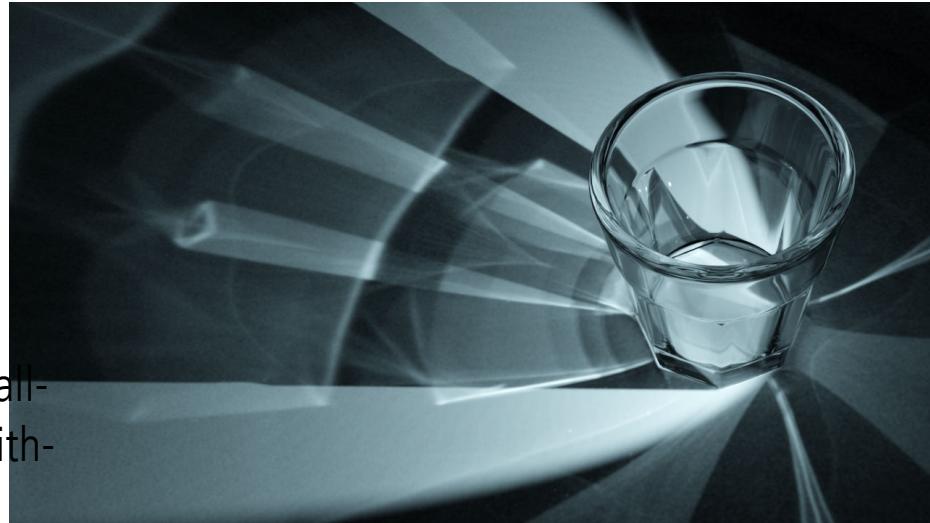
Rendering equation solvers

re-read

- Different light transport methods were developed for rendering different materials
- Light paths labeling can be used to categorize different approaches
- Example: hard problem are specular surfaces (S) illuminating diffuse surfaces (D): **LSDE**
 - Light is emitted, refracted through glass object and due to refraction, light is concentrated towards few points → **caustics**
 - In rendering, we start from eye and diffuse surface and try to find incoming direction



<https://www.blenderdiplom.com/en/tutorials/all-tutorials/649-rendering-caustics-in-blender-with-appleseed.html>
Powered by RayGraph www.scratchapixel.com



Rendering equation extensions

- Rendering equation describes light emission and surface reflection in vacuum.
- Limitations:
 - Transmission
 - Sub-surface scattering:
<https://www.pbr-book.org/3ed-2018/Vol>umes
 - Volumetric scattering:
<https://www.pbr-book.org/3ed-2018/Vol>processes
 - Wave optics effects: polarization, diffraction, interference, etc.

Practical rendering

Practical rendering approaches

Two main rendering approaches are based on:

- Ray-tracing
- Rasterization

Ray-tracing vs Rasterization

- Compared to rasterization, ray-tracing is more directly inspired by the physics of light
 - As such it can generate substantially more realistic images: shadows, multiple reflections, indirect illumination, etc.

Rasterization (EEVEE, Blender)



Ray-tracing (Cycles, Blender)



Visibility solvers

- Rasterization and ray-tracing are methods for solving visibility
- Both methods solve **visibility from camera**
- Ray-tracing can further be used to solve visibility between any points in 3D scene
 - particularly useful for shading and light transport

Practical tip: Ray-tracing vs Rasterization

- Important difference is that **ray-tracing** can shoot rays in any direction, not only from eye or light source
 - As we will see, this makes it possible to render **reflections**, **refractions** and **shadows**
 - On other words, images just look better
- Content creation is simpler for ray-tracing than rasterization since
 - Additional artist work is required when rasterization-based rendering is used

Merge with prev slide



Practical tip: Ray-tracing vs Rasterization

- In production where final images are generated using ray-tracing, rasterization is used for preview and modeling, e.g., animation films
- In production where final images are generated using rasterization. Sometimes ray-tracing can be used to precalculate some effects.
- <image: animation film: rasterization for preview and modeling
- <image: games: rasterization as final image with hacks and hacks

Merge with prev slide

Comparison with rasterization

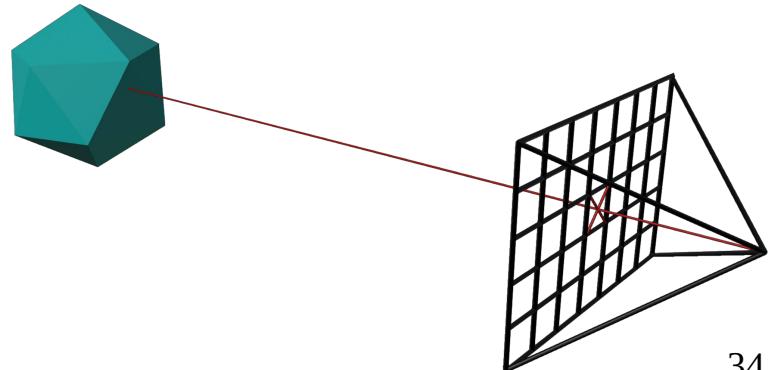
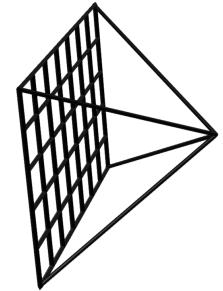
- Recursive ray-casting used in ray-tracing is something that rasterization-based rendering must use different techniques to achieve just what can be obtained with ray-tracing.
- <image: example of approximation and ray-traced effect>

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Visibility: ray-tracing

```
for P do in pixels
    for T do in triangles
        determine if ray through P hits T
    end for
end for
```

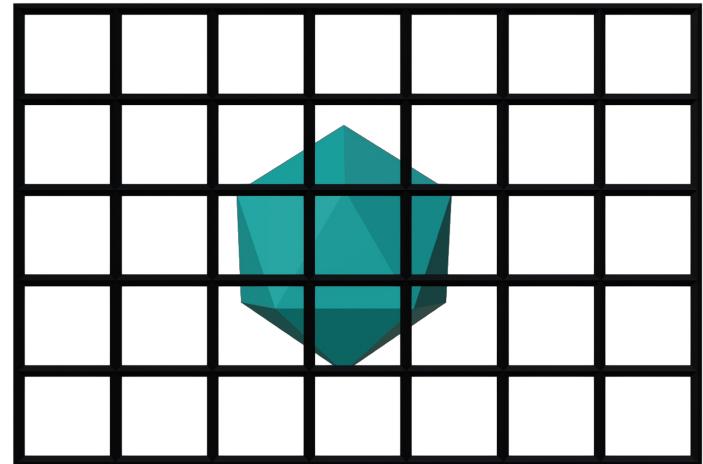
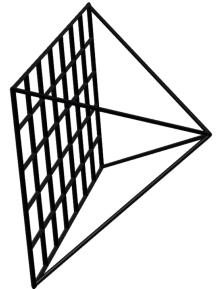
- Ray-tracing generates rays for each pixel of virtual image plane
- Rays are constructed using camera and are tested for intersection with objects in 3D scene
- Closest intersection determines objects visible from camera



Visibility: rasterization

```
for T do in triangles  
    for P do in pixels  
        determine if P inside T  
    end for  
end for
```

- Rasterization projects objects on virtual image plane
- Camera information is used to construct (perspective) projection matrix
- For each pixel of virtual image plane find which objects are covered by pixel and take closest

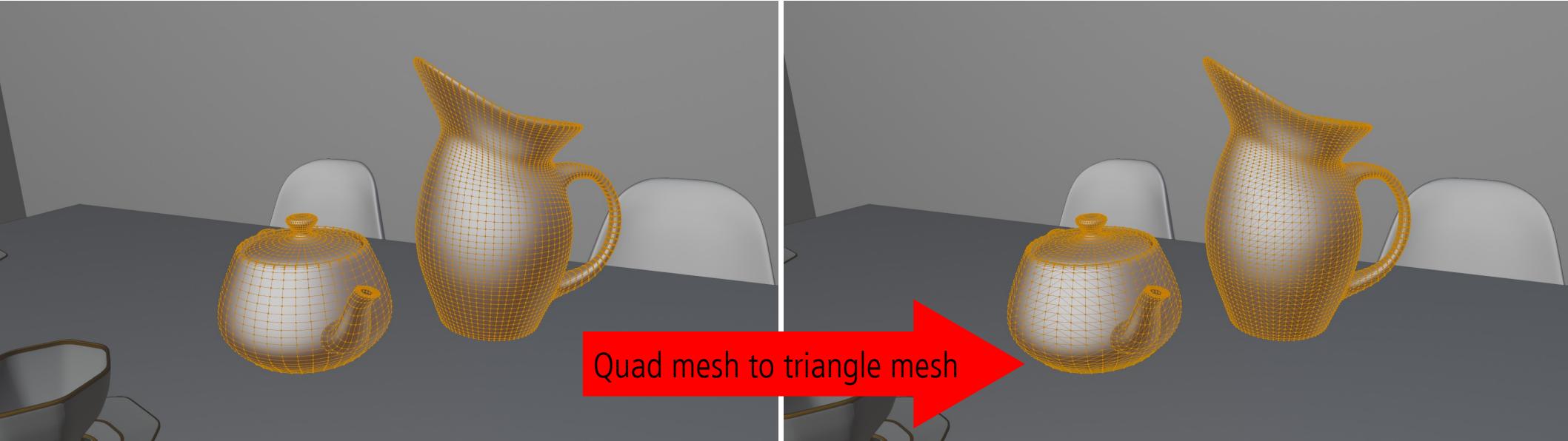


Raytracing vs rasterization

- **Raytracing** iterates through all pixels of image plane, generates rays and then iterates over all objects (triangles) in the scene.
 - This approach is called **image centric**.
- **Rasterization** loops over all geometric primitives in the scene (triangles), projects these primitives on image plane and then loops over all pixels of the image plane.
 - This approach is called **object centric**.

Reminder: triangulated mesh

- Although various shape representations exists for modeling purposes, when it comes to rendering, often all object shapes are transformed to triangles using tessellation process.



Rendering speed: Ray-tracing and Rasterization

3D scenes are complex and often large

24 million unique triangles

Object reuse via instancing results in 3.1 billion triangles



Rendering speed

- Number of rendered images for display per second is called **frames per second (FPS)**:
 - Real-time graphics > 60 FPS
 - Interactive graphics $1 < x < 20$ FPS
 - Offline graphics < 1 FPS

Speed: Ray-tracing vs Rasterization

- Both algorithms are conceptually simple
- To make them fast and usable in practice for n objects:
 - Ray-tracing is utilizing acceleration spatial data structures for achieving $O(\log(n))$
 - Acceleration data structures: bounding volume hierarchy (BVH) or Kd tree
 - Rasterization relies on culling and hardware support ensuring better running time than $O(n)$
 - Culling methods for avoiding full scene processing: occlusion culling, frustum culling, backface culling
 - Deferred shading
 - GPUs are adapted for rasterization-based rendering

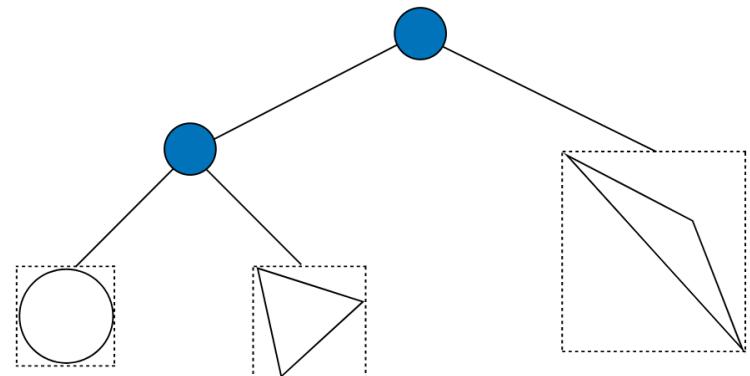
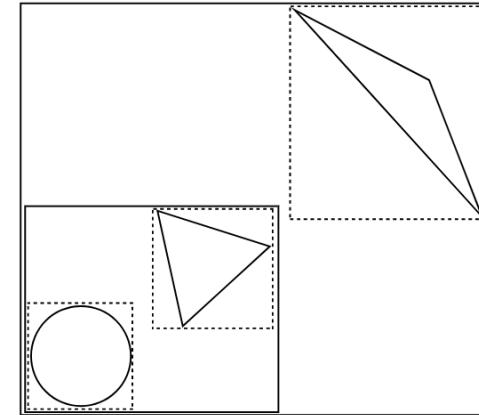
Re-read, see what can be removed
And is mentioned in next slides

Ray-tracing acceleration

- Tracing single ray through the scene naively would require testing intersection with all objects in 3D scene → linear in the number of primitives in the scene → slow!
- This is not optimal since rays might pass nowhere near to vast majority of primitives
- To reduce number of ray-object intersection tests, acceleration structures are used
- Two main categories of acceleration structures:
 - Object subdivisions
 - Spatial subdivisions

Object subdivisions: BVH

- Progressively breaking down objects in the scene into smaller parts of objects
 - Example: room can be separated into walls, floor, ceiling, table, chairs, etc.
- Resulting parts are in a hierarchical, tree like structure
 - For each ray, instead of looping through objects, traversal through the tree is performed
- If ray is not intersecting the parent object then its parts, children objects, are not tested for intersection
 - Example: if room is not intersected, then table can not be intersected as well



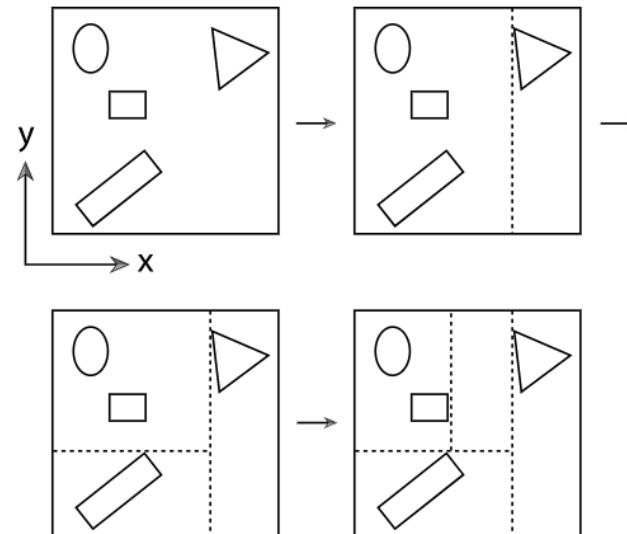
- Root node holds the bounds of the entire scene
- Objects are stored in the leaves, and each node stores a bounding box of the objects in the nodes beneath it

Spatial subdivisions: BSP trees

- Binary space partitioning trees adaptively subdivide scene into regions using planes and record which primitives are overlapping regions
 - Process starts by bounding box encompassing whole scene
 - If the number of objects is larger than some defined threshold, box is split
 - Repeated until maximum depth or tree contains small enough regions
- Two BSP variants:
 - Kd trees
 - Octree
- When tracing rays, only objects in regions through which ray passes are tested for intersection

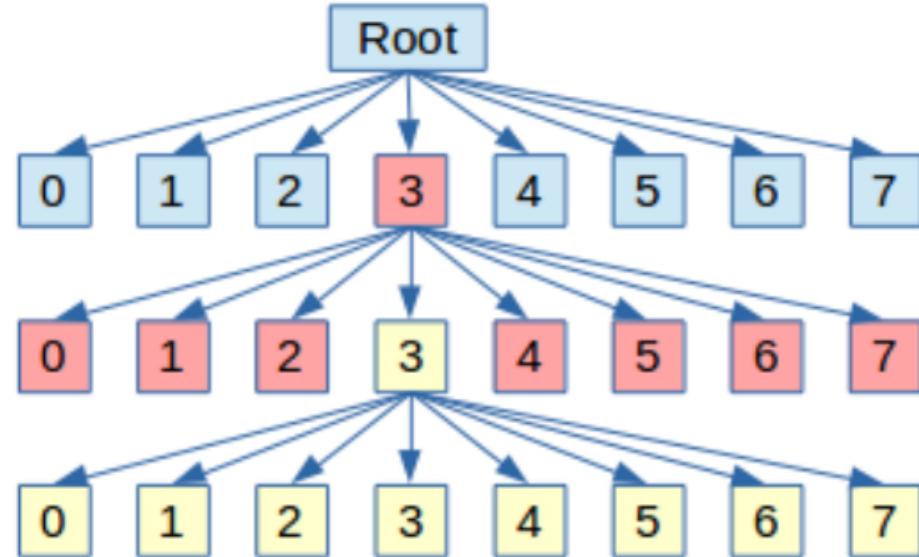
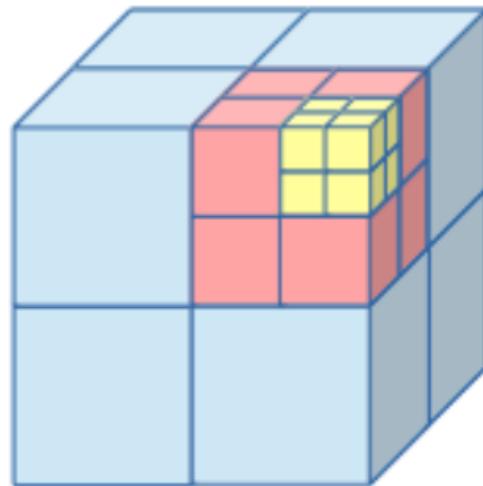
Spatial subdivisions: kd-trees

- Subdivision rule: planes must be perpendicular to one of coordinate axes



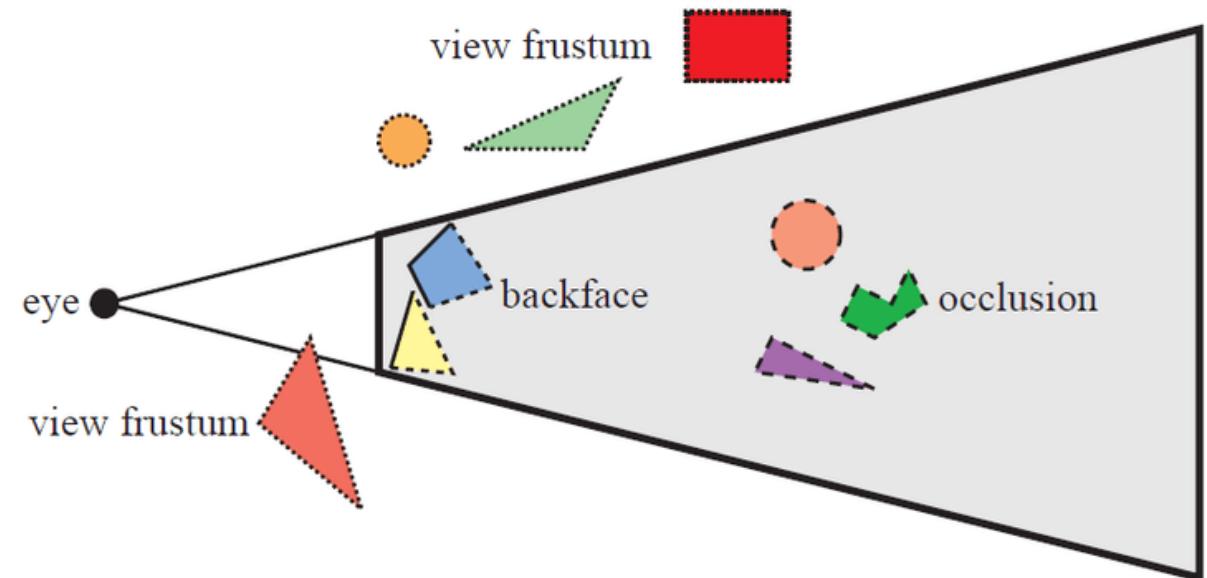
Spatial subdivisions: octrees

- Subdivision is done by three axis-perpendicular planes which splits box into eight smaller boxes



Rasterization acceleration

- Whole scene processing is avoided using:
 - Frustum culling
 - Occlusion culling
 - Backface culling



Frustum culling

- <https://learnopengl.com/Guest-Articles/2021/Scene/Frustum-Culling>
- Horizon example

Backface culling

- <https://learnopengl.com/Advanced-OpenGL/Face-culling>

Occlusion culling

- <https://www.gamedeveloper.com/programming/occlusion-culling-algorithms>

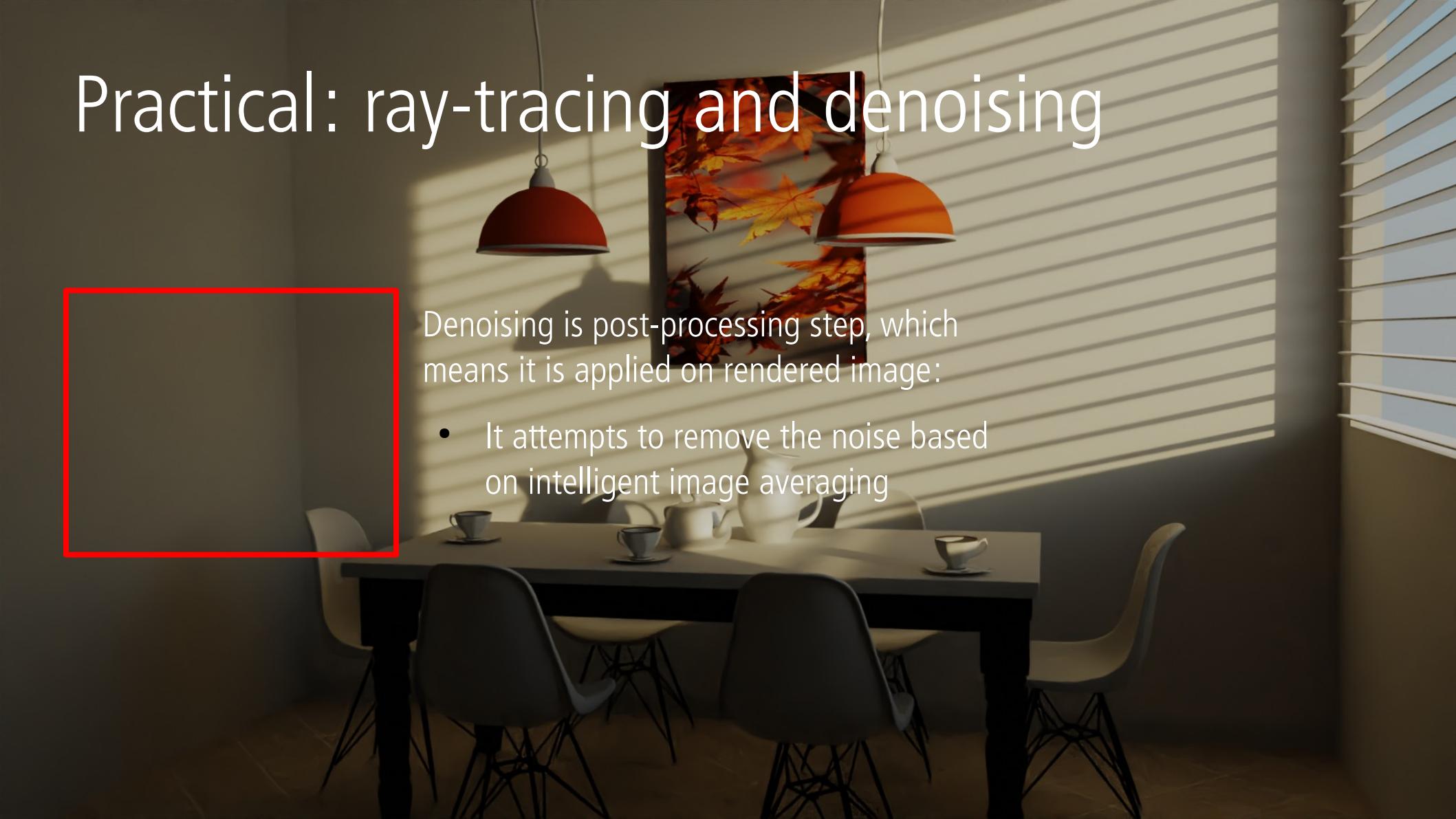
Practical: ray-tracing and noise

Often, in ray-tracing, noise is present* :

- More samples per pixel → expensive!
- Denoising algorithms?

* Noise comes when area lights, glossy surfaces, environment map or path-tracing is used.

Practical: ray-tracing and denoising



Denoising is post-processing step, which means it is applied on rendered image:

- It attempts to remove the noise based on intelligent image averaging

Towards real-time ray-tracing

- Additional techniques and hardware
- Denoising is promising solution.
- For short-term, clever combinations are expected.
 - <https://docs.unrealengine.com/4.26/en-US/RenderingAndGraphics/RayTracing/>
 - <https://unity.com/ray-tracing>
 - Rasterization seems to be not going anywhere any time soon!

re-read

] are

Note: complexity of scenes and rendering

- Although HW and methods a rendering the scenes stayed complexity of the 3D scenes algorithms increased.
 - Blinn law

re-read

Raytracing and rasterization

- Raytracing is historically early method for image creation.
- Raytracing has always been recognized as best method for image creation.
However, up until 1990-2000s it hasn't been used much.
- Due to hardware advancements, rasterization-based rendering has become dominant.
- Raytracing is intuitive and straightforward method of creating images. It is easy to understand how image is created.
 - Based upon raytracing, more advanced rendering methods are build.
 - Furthermore, understanding ray-tracing gives good foundations for understanding rasterization-based rendering systems.

Practical ray-tracing and rasterization

- Raytracing method can be separated in:
 - **Determining visibility from camera:** determine which point in 3D scene projection is inherent due to camera which is used for generating image
 - **Shading:** calculating the color of visible point – this operation besides all light which influences the color and intensity of that point.
- Both steps require extremely time-consuming intersections between rays and objects
 - Efficient ray-object intersection method is needed
 - 3D scene must be represented for fast and efficient spatial search of objects which should be tested for intersection
- On the other hand, rasterization can solve the visibility problem (camera-object) very fast. That is why real-time graphics is predominantly using GPU rasterization approach. But when it comes to shading, due to the available information on 3D scene, it is not as good as raytracing.
- Raytracing is slow for solving visibility problem, but it enables high-quality shading. For high-quality, photo-realistic production rendering, ray-tracing is almost always used*.

* However, real-time ray-tracing is now possible to certain degree with certain hardware and it is hot research topic!

Rasterization and raytracing

- Both **raytracing** and **rasterization** are used to solve the **visibility problem** (which also contains **perspective/orhographic projection solving**).
- Solving the visibility problem is determining the color of each pixel. This step is called **shading** and it uses the information from the rasterization step to determine the color of each pixel based on the visible surfaces and calculates the lighting effect.

Extend based on prev chapter

Other part
This step is
on visible

Importance of visibility calculation for rendering

- Based on previous discussions, we can highlight the importance of visibility calculation and how it is related to all concepts in rendering.
- Visibility problem is concerned with determining visibility between points
- First, when determining visible objects from camera, we solve visibility problem and utilize perspective/orthographic projection
- Then, when calculating shading we require light transport information which is again based on visibility between surface and lights in 3D scene
 - This is required to calculate shading
- Rasterization is good for solving visibility problem in scene which is important for shading
- Raytracing is good for solving visibility problem in scene which is important for GI

<IMAGE: visibility in rasterization

Discuss how visibility is important for shading, GI, and shadows in 3D
Then say that ray-tracing is for GI
But GI effects can be simulated on rasterization

A0: approximation of GI effects: shadows

- RTR 11.3

Rasterization and Raytracing in practice

- Examples of rasterization-based renderers in production
 - Godot, Unity, Unreal
- Examples of raytracing-based renderers in production
 - Appleseed, Arnold, etc.

<IMAGE: differences and comparison between rasterizers

- Important note that we can learn here, that occurs in all renderers: **Trade-off between speed and quality.**
 - Depending on application, best option for speed of rendering and quality of rendered images must be found. For games, it is important that frames are rendered in real-time (>30fps) and certain quality must be sacrificed. For animated films, rendering time can take up to several hours for only one frame therefore, focus can be put on quality.

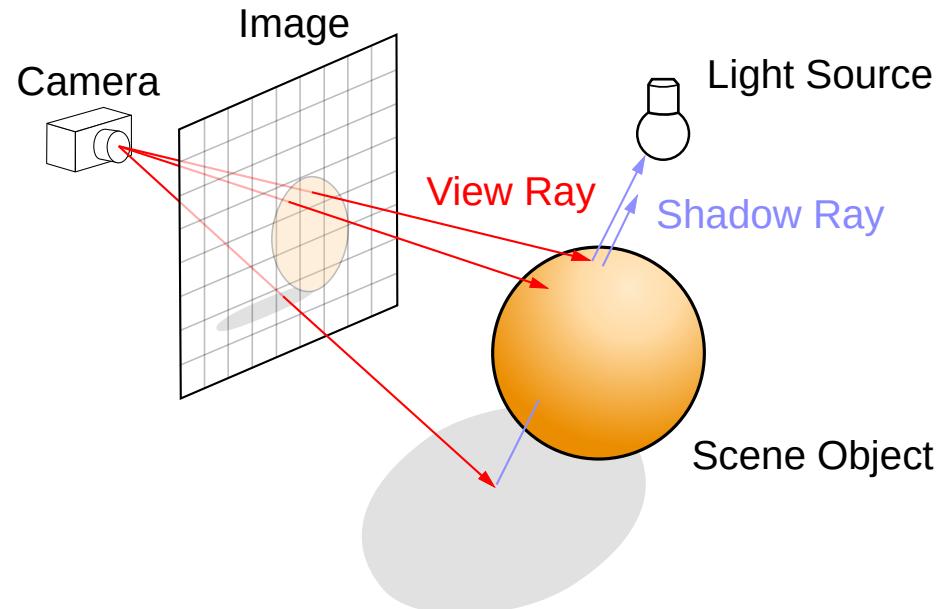
re-read

off

Rendering: intuition via ray-tracing

Ray-tracing based rendering

- Rendering: process of creating 2D image from 3D scene
- Task: calculate pixel colors of virtual image plane of virtual camera in 3D scene
 - This implies computing light falling into camera



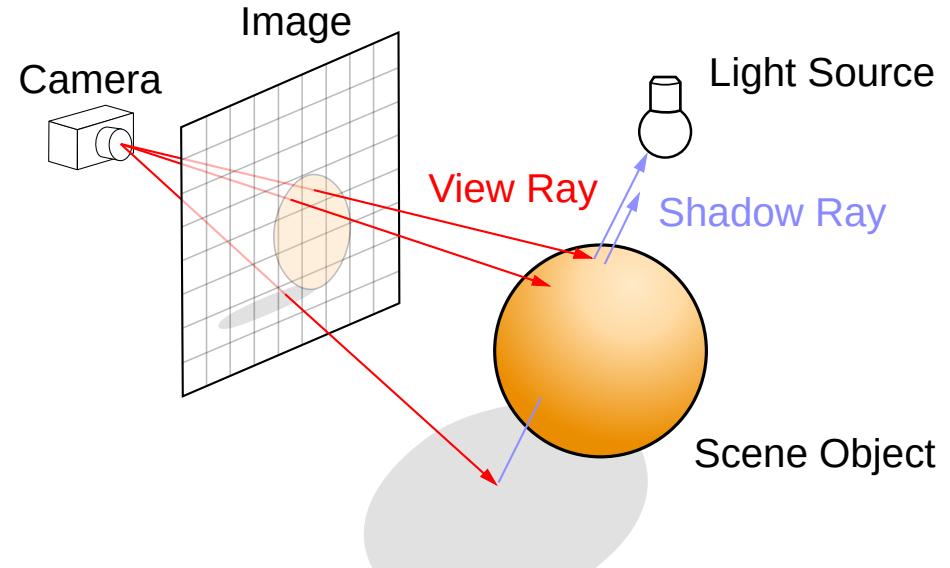
Simulating light

- Simulating physical (real-world) light-object interaction means simulating all light rays paths from light and their interactions with objects. Furthermore, only small amount of that light actually falls on camera forming an image. This kind of simulation is called **forward ray-tracing or light tracing**. Simulating this is not tractable!
- We reverse the process: we start with rays that fall into camera and build from there.
- Based on previous discussion, we trace rays from sensor to the objects. This simulation is called **backward ray-tracing or eye-tracing***.
- In rest of the lecture, by ray-tracing we mean backward ray-tracing.

* Introduced by Turner Whitted in paper "An Improved Illumination Model for Shaded Display". Method in computer graphics using the concept of shooting and following rays from light or eye is called path-tracing.

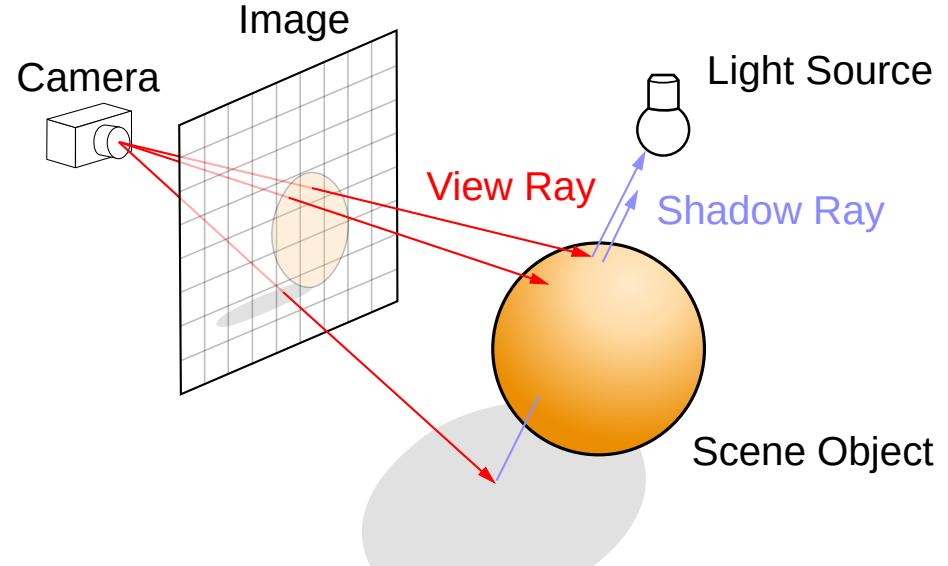
Pixel colors → camera rays

- Camera defines from where we look at 3D scene and a 2D surface on which 3D scene will be projected as an image
- Compute only light rays which are contributing to virtual image plane
- **Backward tracing:** start from virtual image plane, that is camera and generate camera rays for each pixel into the scene
 - Camera rays are generated from aperture position through each pixel of film plane
- Computed color (light) of each ray will correspond to pixel color from which this ray was generated



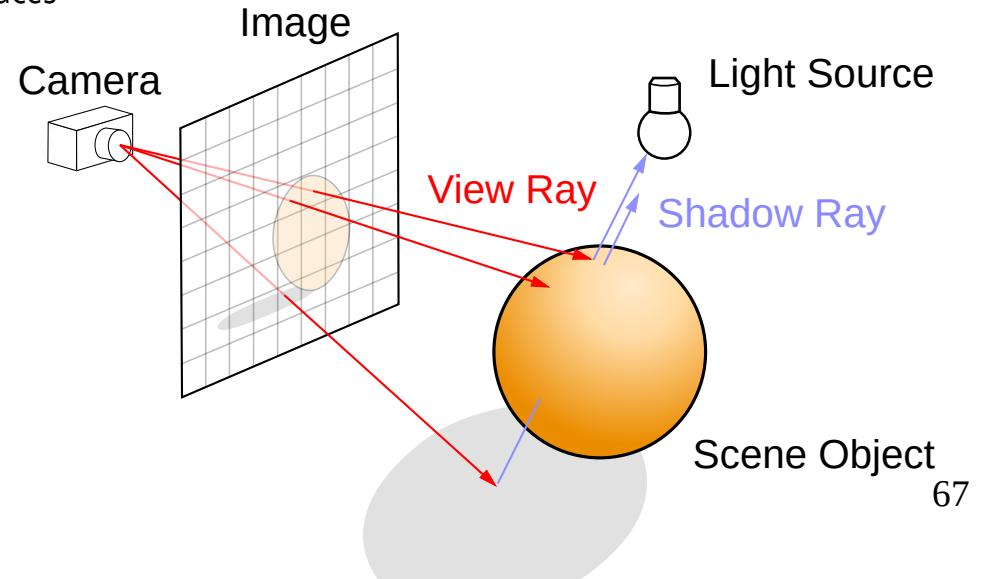
Camera ray intersections

- To compute color (light) for each camera ray, first find intersection of this ray and objects in 3D scene
- Ray-object intersection is a way of finding closest objects to camera, that is, objects visible to camera which will reflect light into camera
- Ray-object intersection is geometrical problem and it depends on object shape representation
- Although magnitude of different shapes exist, we can assume that in rendering time we will have triangulated mesh



Intersection point color → shading

- Once intersection which object is found we need to find amount of light reflected from this point into camera
- This is equivalent to object color and its computation is called shading
- For shading we need:
 - Object material and shape in point
 - Incoming light on point → light sources and/or other surfaces
 - View direction on point



Shading

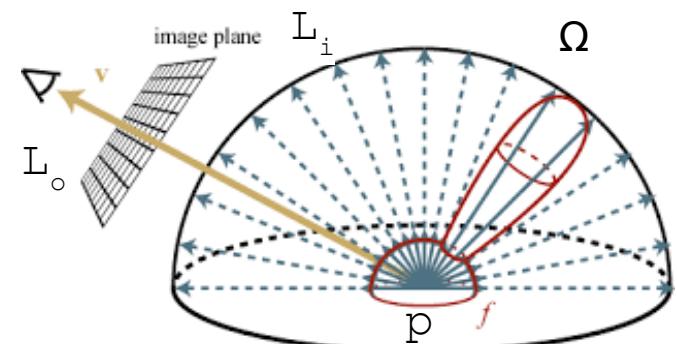
- Amount of light reflected in view direction, that is color of visible surface is defined with **rendering equation**
- Computing incoming light on shading point is called **light transport**

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega} f(p, \omega_o, \omega_i) L_i(p, \omega_i) (\omega_i \cdot n) d\omega_i$$

BRDF
Defines surface material
Uses texture information

Incoming light

Attenuation due
to orientation of
surface towards
light.
Depends on
surface shape
(normal)

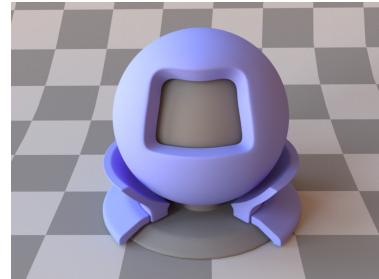
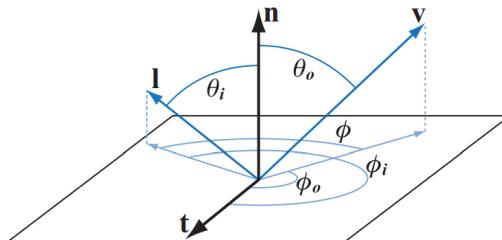


Light transport: incoming light

- **Direct illumination:** calculating visibility between shading point and light source
- **Indirect illumination:** calculating visibility between shading point and all possible surfaces above shading point which may contribute with light
 - Additional rays are generated from shading point into hemisphere of directions above surface
 - For any found intersection, rendering equation must be re-evaluated
 - Material determines where light scatters (direction) and how much it scatters in particular direction (intensity)
 - Light transport algorithms vary based on light paths they can simulated: diffuse, specular, etc.
- **Global illumination:** direct and indirect illumination
 - Path tracing represents method solving global illumination extending ray-tracing

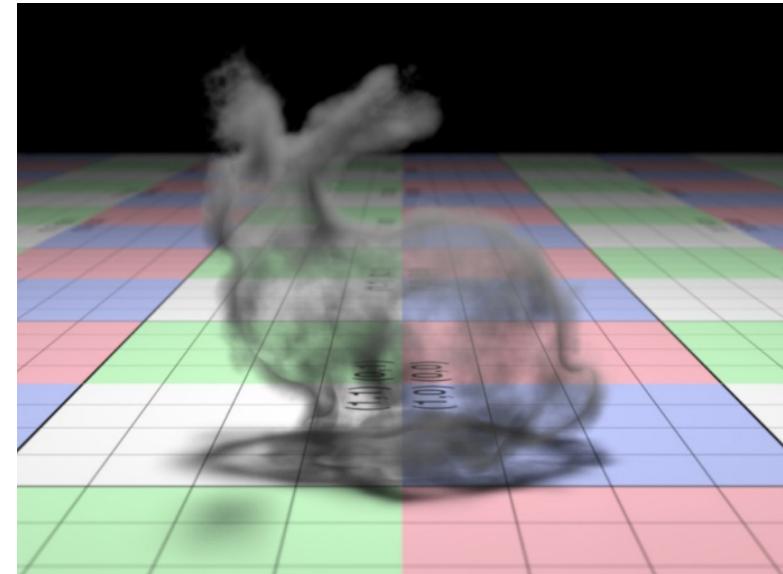
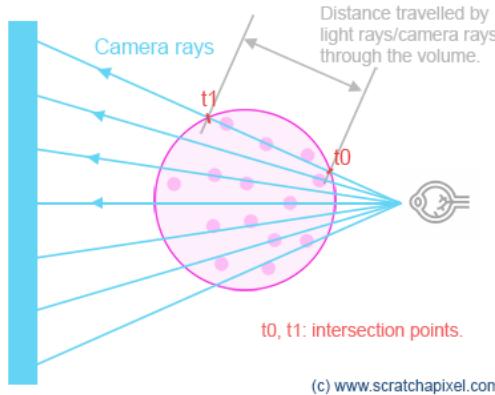
Shading: light-matter interaction

- Once incoming light on shading point is computed, shading must use material information to determine amount of light being absorbed and reflected in view direction
- For this purpose, BRDF is used
 - Given light and view directions, BRDF determines ratio of reflected light in view direction
 - BRDF further uses texture parameters which varies with shading point position



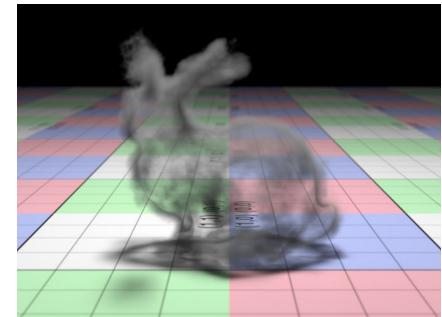
Between surfaces*

- We assume that media between object surfaces is vacuum
 - In this case, light which is reflected from object or incoming on object will not be attenuated
- If other medium is present then we call it participating media
 - In this case, light will be attenuated by scattering and absorption while traveling
 - To solve this, volumetric rendering equation is used



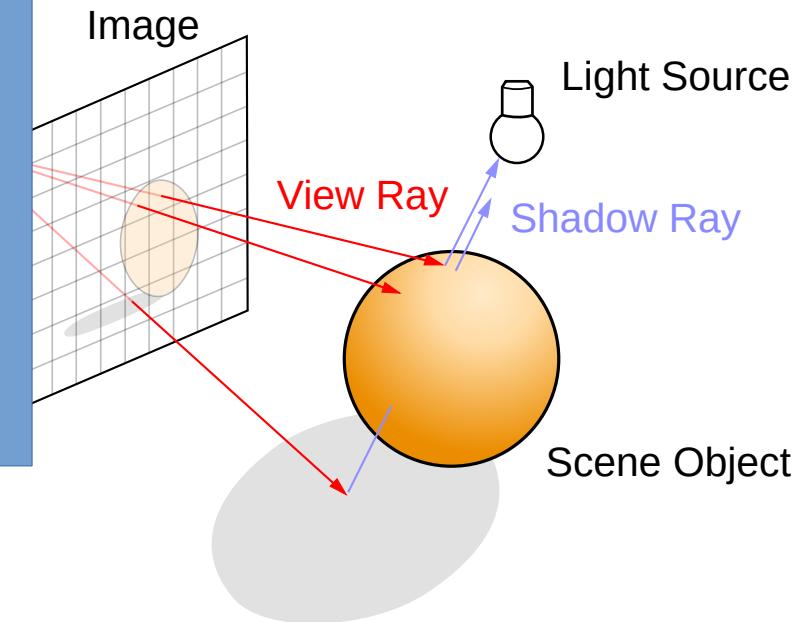
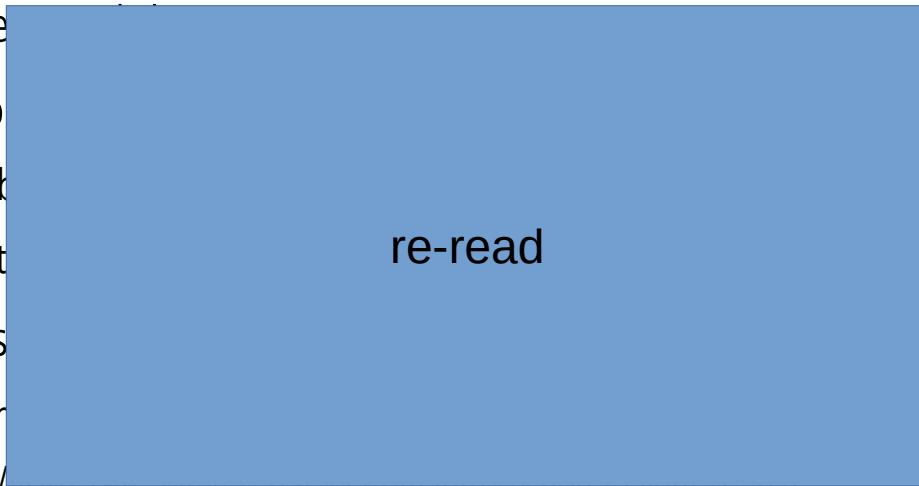
Surface vs volume rendering

- Surfaces:
 - Shape representation: meshes, parametric surfaces, implicit surfaces, etc
 - Material: transmissive, reflective, glossy, diffuse, specular, etc.
- Volumes (participating media):
 - Shape representation: voxels, meshes (consider interior density), etc.
 - Material: sub-surface scattering, participating media density scattering



Ray-tracing-based rendering: complete

- Generate ray using camera information → primary/camera ray
- Trace ray into scene
 - Intersects object
 - No intersection
- Surface intersection
 - Calculate area shading
 - Shadow/light ray: directly traced from intersection to light source → direct illumination
 - Advanced: sample various directions from this point in 3D scene to obtain reflections of other objects → global illumination
- Calculate amount of light reflected in primary ray direction using incoming light and material specification → shading



Algorithm*:

- For each pixel in image:
 - Generate primary ray using camera
 - Shoot primary ray
 - For each object
 - Check if intersected by multiple objects
 - For intersections, record intersection point to all lights in the scene
 - If shadow ray is not intersecting anything, lit the pixel

re-read

```
for j do in imageHeight:  
    for i do in imageWidth:  
        ray cameraRay = ComputeCameraRay(i, j);  
        pHit, nHit;  
        minDist = INFINITY;  
        Object object = NULL;  
        for o do in objects:  
            if Intesects(o, cameraRay, &pHit, &nHit) then  
                distance = Distance(cameraPosition, pHit);  
                if distance < minDist then  
                    object = o;  
                    minDist = distance;  
                end if  
            end if  
        end for  
        if o != NULL then  
            Ray shadowRay;  
            shadowRay.direction = lightPosition - pHit;  
            isInShadow = false;  
            for o do in objects:  
                if Intesects(o, shadowRay) then  
                    isInShadow = true;  
                    break;  
                end if  
            end for  
        end if  
        if not isInShadow then  
            pixels[i][j] = object.color * light.brightness  
        else  
            pixels[i][j] = 0;  
        end if  
    end for
```

* Arthur Appel in 1969 - "Some Techniques for Shading Machine Renderings of Solids"

More into topic

- Surface rendering
 - https://www.pbr-book.org/3ed-2018/Light_Transport_I_Surface_Reflection
 - <https://www.scratchapixel.com/lessons/3d-basic-rendering/rendering-3d-scene-overview/computer-discrete-raster.html>
- Volumetric rendering
 - <https://www.scratchapixel.com/lessons/3d-basic-rendering/volume-rendering-for-developers/volume-rendering-summary-equations.html>
 - https://www.pbr-book.org/3ed-2018/Light_Transport_II_Volume_Rendering
 - <https://graphics.pixar.com/library/ProductionVolumeRendering/paper.pdf>

Summary questions

- https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/11_rendering_overview

Reading Materials

- <https://github.com/lorentzo/IntroductionToComputerGraphics>