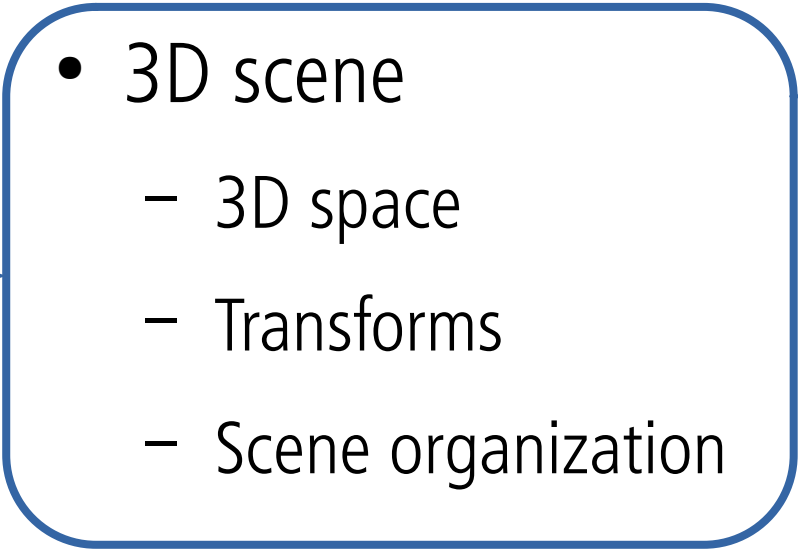
3D space, transformations and scene organization

Syllabus

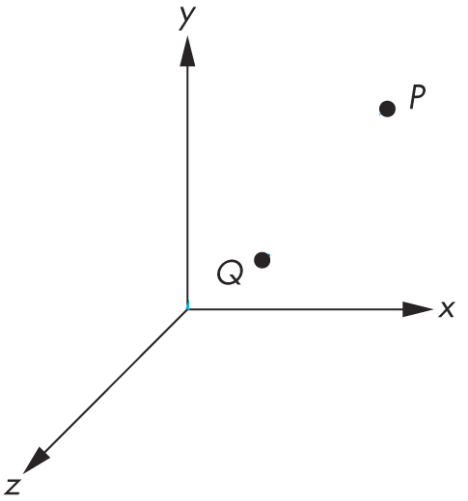
- 3D scene
  - Objects
  - Lights
  - Cameras
- Rendering
- Image

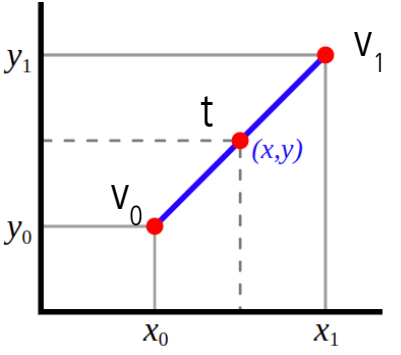


3D space

Points

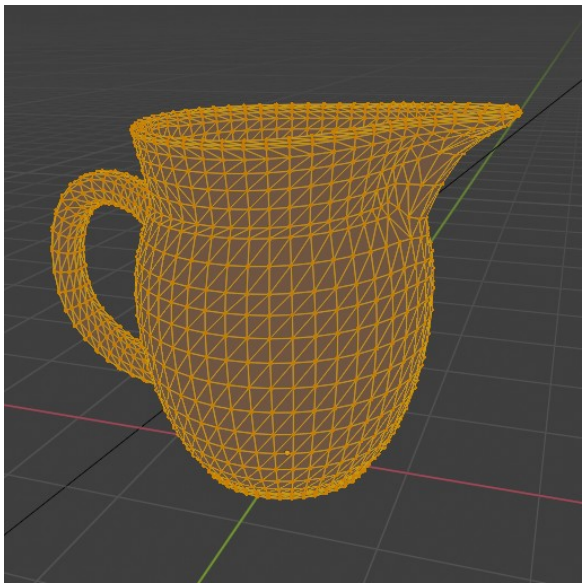
- Zero-dimensional **location** with respect to coordinate system:
  - 2D space `(x,y)`
  - 3D space: `(x,y,z)`
- **Homogeneous points**
  - Adding 4th element to point: `(x,y,z,w), w = 1`
  - Used when multiplying with matrices
- Interpolation between points: **linear interpolation**
  - `lerp(v1, v2, t) = (1 - t) * v0 + t * v1;`

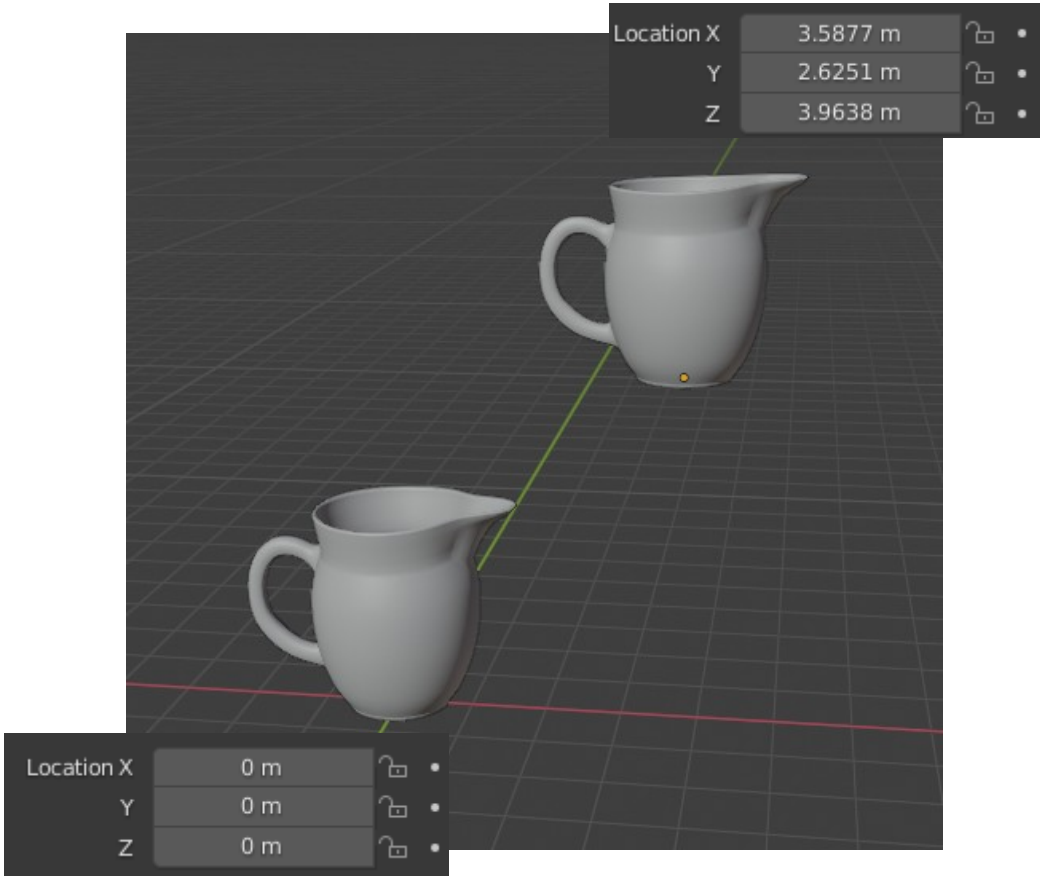




Points

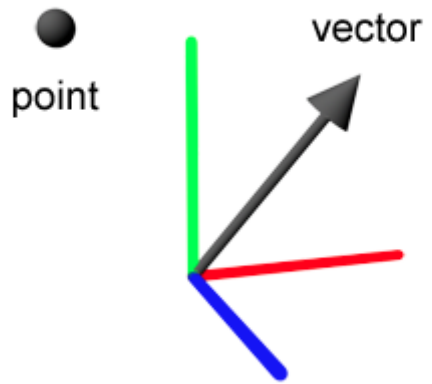
- Points are used to:
  - Describe shape
  - Define position of object in space





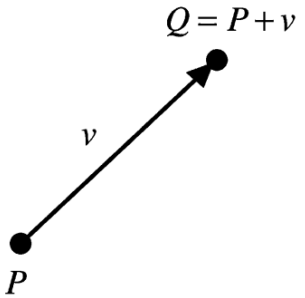
Vectors

- **Magnitude (norm/length)** and **direction** in 2D or 3D space
  - Two coordinates `(x,y)` – usually for texture space
  - Three coordinates `(x,y,z)` – for any 3D elements

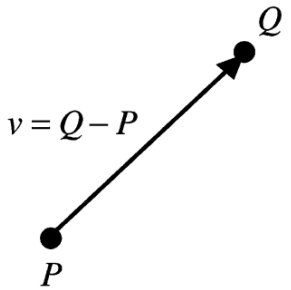


Points and vectors

- Subtracting or adding point with vector results in new point
- Distance between two points results in vector which contains length and direction.
- Points and vectors are transformed – moved in space - using **linear transformations** – multiplication with matrix



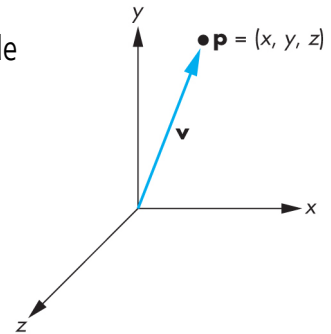
Adding point to vector



Subtracting point from a point

$$\text{2D: } |\mathbf{v}| = \sqrt{x^2 + y^2}$$
$$\text{3D: } |\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$$

Magnitude (length)



$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

Direction obtained by normalization

https://www.researchgate.net/publication/220184055_On_the_algebraic_and_geometric_foundations_of_computer_graphics

http://www.c-jump.com/bcc/common/Talk3/Math/Vectors/W01_0090_dot_product.htm

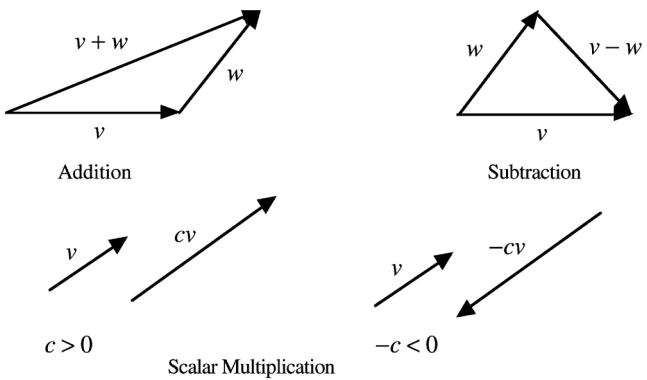
Row major and column major

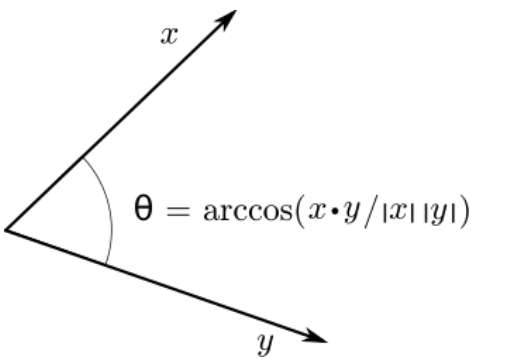
- Points and vectors can be written as:
  - [1x3] matrix → **row major order** (Direct X, Maya)
  - [3x1] matrix → **column major order** (OpenGL, PBRT, Blender)

$$V = \begin{bmatrix} x & y & z \end{bmatrix} \qquad V = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Common vector operations

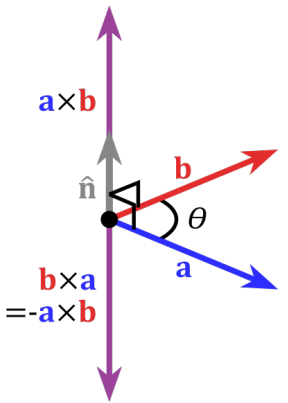
- Addition, substracton, multiplication with scalar, etc.
- Dot (`dotProduct(a,b)`) and cross product (`crossProduct(a,b)`) operators
- Normalization → length of vector = 1 **(unit vector)**





$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

Dot product
https://en.wikipedia.org/wiki/Dot_product

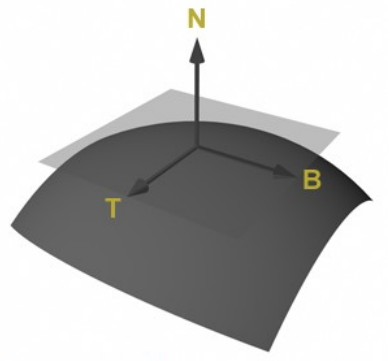


$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

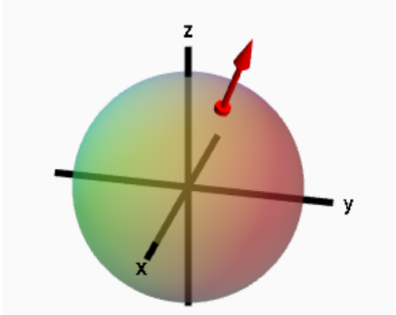
Cross product
https://en.wikipedia.org/wiki/Cross_product

Normals

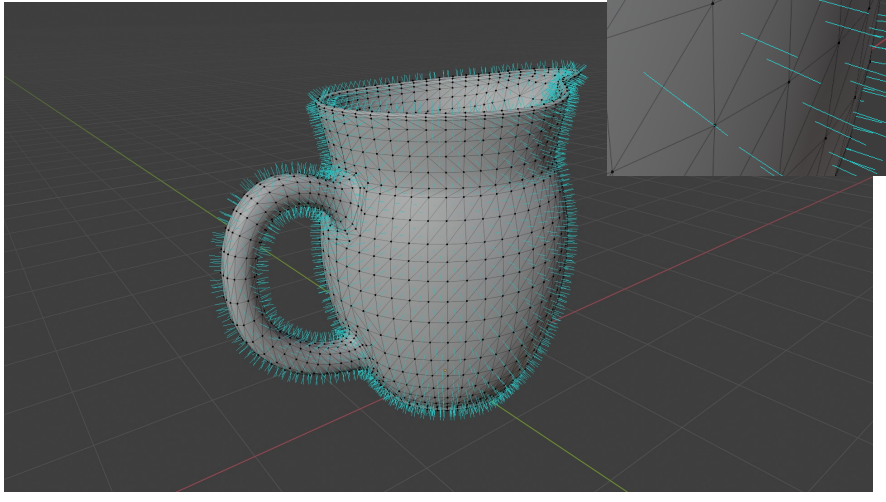
- Normal `(x,y,z)` describes **orientation of surface** of a geometric object at a point
  - Perpendicular to surface at a point
  - Similar to vectors but they are defined in relationship to a particular surface: they behave differently in some situations, particularly when **applying transformations**
- It can be defined as cross product of any two non-parallel tangent vectors at that point.



https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/geometry/points-vectors-and-normals.html

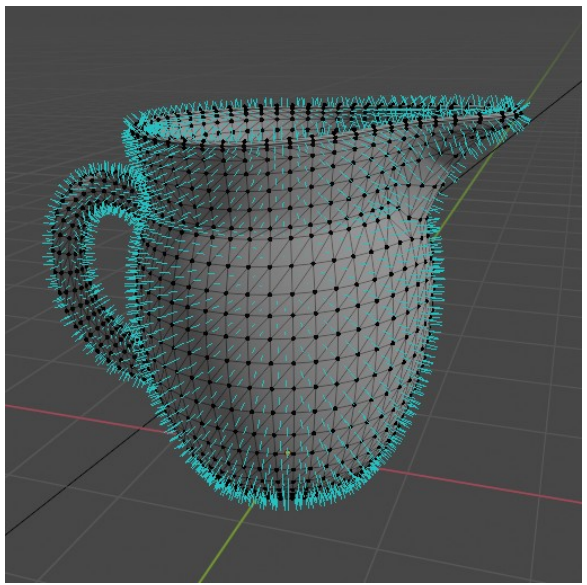


https://mathinsight.org/parametrized_surface_orient



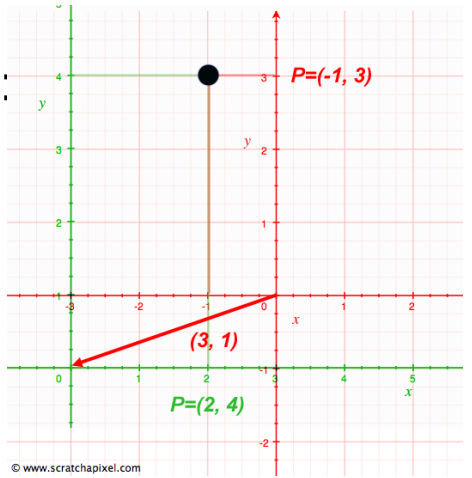
Normals

- Crucial information for rendering and modeling
  - Example: brightness of object



Coordinate system

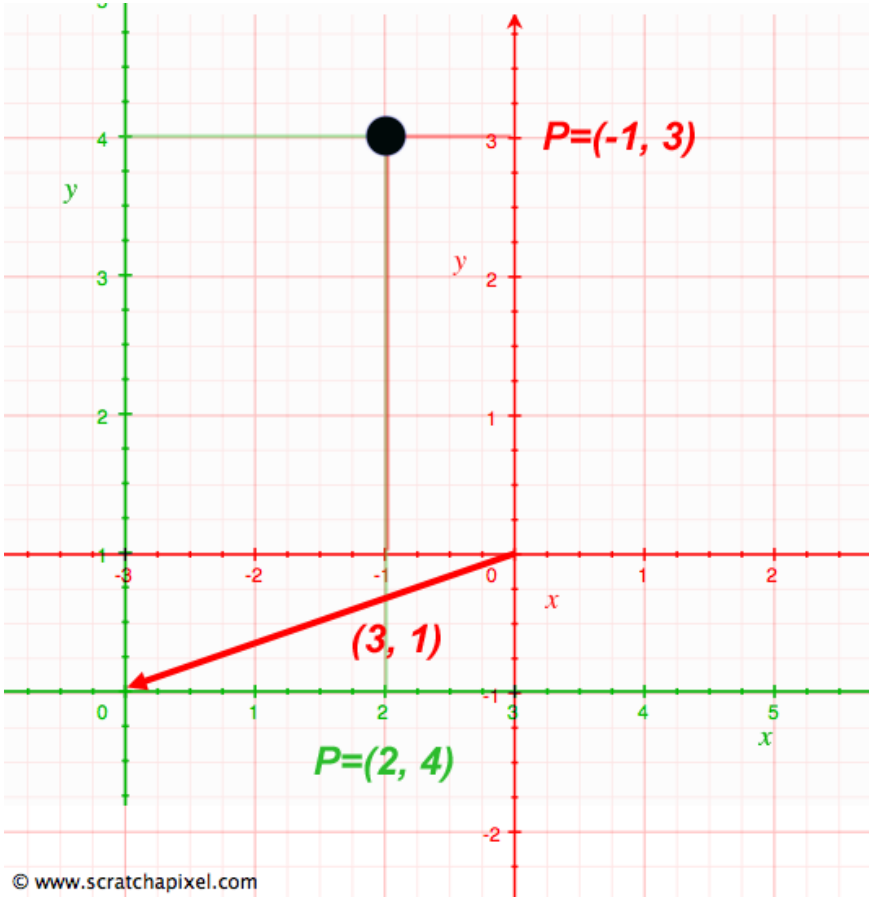
- Defining shape, location, orientation and other properties of 3D scene elements relies on **points and vectors.**
- Points and vectors are represented with **three coordinates: (x,y,z)**
- These values are meaningless without a **coordinate system** which defines:
  - **Origin** of the space: a point
  - **Basis**: three linearly independent vectors that define X, Y and Z **axis** of a space.
- Origin and three vectors define a **frame** which defines coordinate system.
  - **Cartesian coordinate system:** perpendicular axes
  - **Euclidean space**



Point or vector in 3D space depend on its relationship to the frame – point can have same absolute position in space but its coordinates depend on frame.

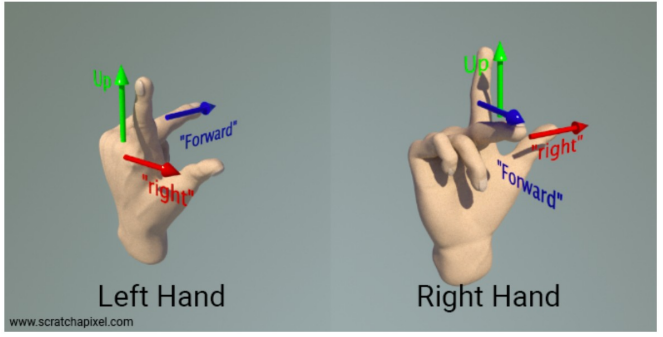
Coordinate system

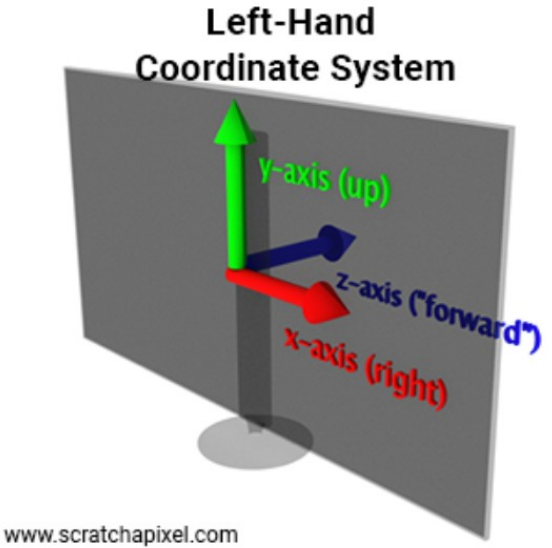
- Infinite number of coordinate systems can be defined
  - Coordinates of point depend on referent coordinate system
- Transformation of point from one coordinate system to another is done by matching their origins and basis - **transformation**

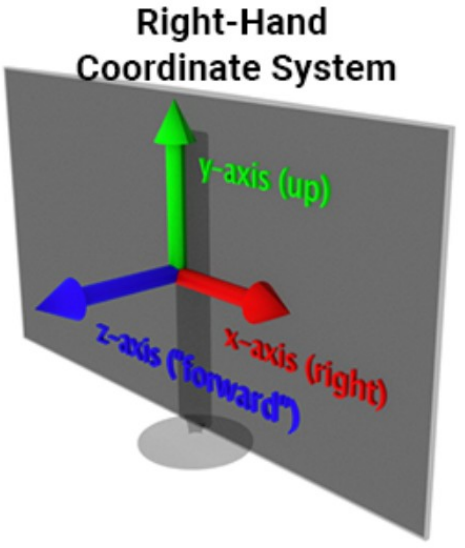


Coordinate system handedness

- Axis of X, Y and Z vectors defining coordinate system can face in one of two directions:
  - Left-handed coordinate system
  - Right-handed coordinate system





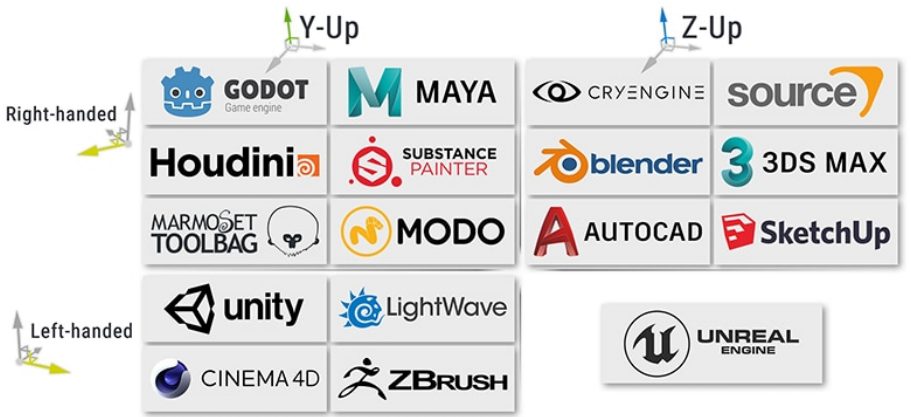


https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/geometry/coordinate-systems.html

Coordinate system handedness and naming

- **Handedness** of coordinate system is defined by orientation of left/right vector relative to up and forward vectors
- **Naming convention** – how axis are labeled (e.g., X, Y or Z) has nothing to do with handedness, e.g. up is not necessary Z axis, this depends on renderer/3D application.
  - Industry standard: right-handed, x – right, y – up and z – outward
  - Maya and OpenGL use right-handed
  - DirectX, PBRT and RenderMan use left-handed coordinate system

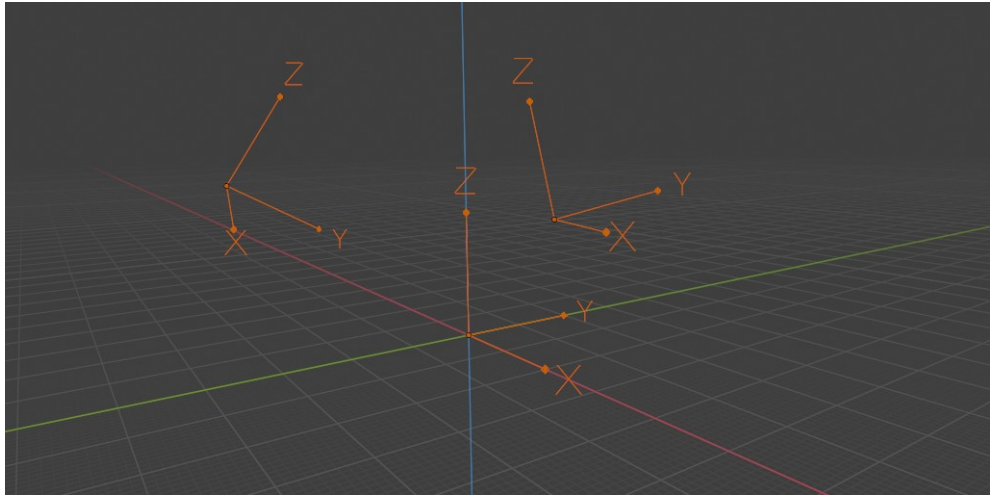
Exporting from one application to another requires special care of coordinate systems!

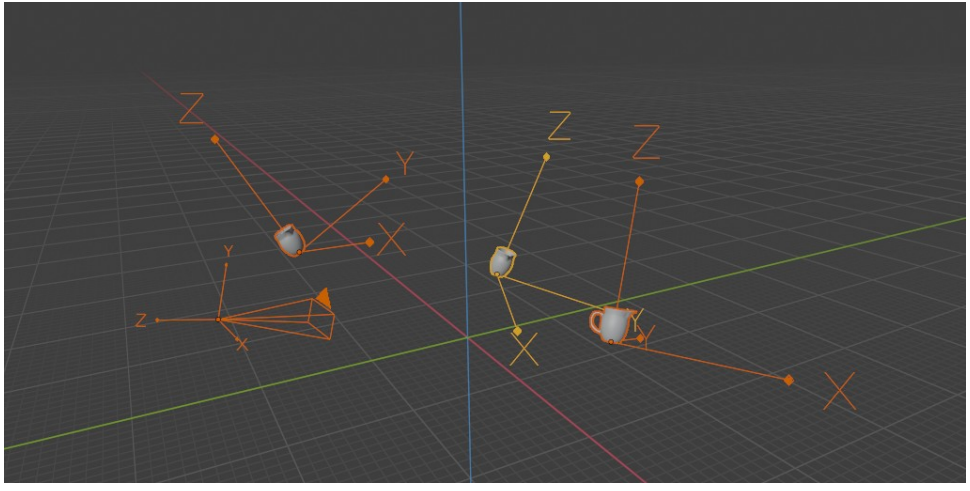


https://www.techarthub.com/a-guide-to-unitys-coordinate-system-with-practical-examples/

World coordinate system

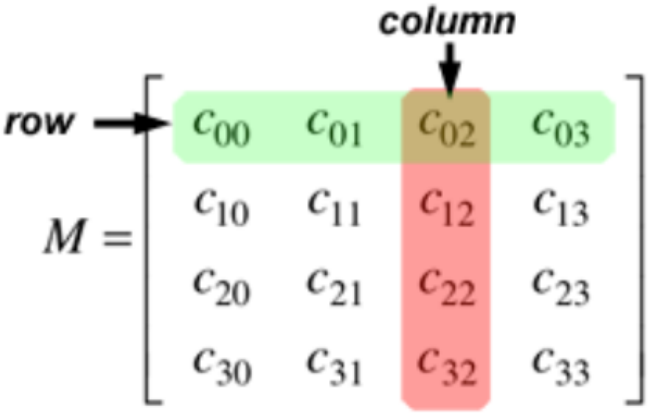
- Coordinate system is defined with origin and basis.
- Point and vectors depend on coordinate system!
- **World coordinate system** – a **standard frame**:
  - Origin
  - Basis
- All other frames – **local coordinate systems** - will be defined with respect to this world coordinate system.





Matrices

- Matrices are essential for **moving elements** within 3D scene
  - Scaling, rotation and translation transformations are described with matrices
  - Multiplying point or vector with matrix returns transformed point or vector
- Matrix (M): 2D array of numbers: `m x n` – number of **rows** (m) and **columns** (n)
  - M_{ij} – matrix element at `(i,j)` position
- For computer graphics **square matrices** 3x3 and 4x4 are most important



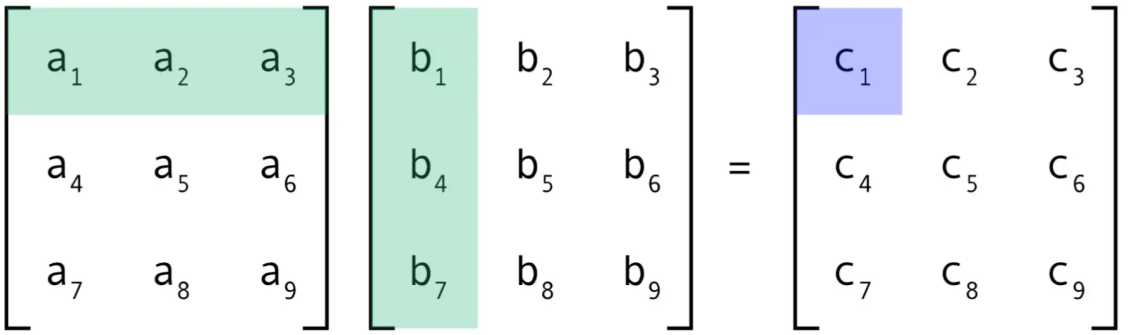
Row major and column major

- Matrices can be written as:
  - **Row major order** (Direct X, Maya, PBRT)
  - **Column major order** (OpenGL)

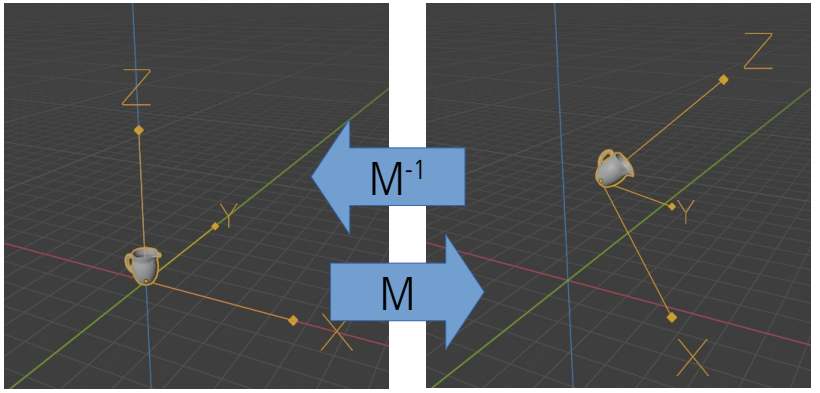
$$\begin{bmatrix} c_{00} & c_{01} & c_{02} & 0 \\ c_{10} & c_{11} & c_{12} & 0 \\ c_{20} & c_{21} & c_{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} c_{00} & c_{01} & c_{02} & 0 \\ c_{10} & c_{11} & c_{12} & 0 \\ c_{20} & c_{21} & c_{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix operations

- **Matrix-matrix multiplication** gives matrix
  - Useful for representing multiple transforms with one matrix
  - Not commutative: order of multiplications, thus transforms is important!



Matrix operations



- Operators:
  - **Inverse**: multiplying point A with matrix M given point B. Multiplying point B with inverse of matrix M gives point A.
    - $MM^{-1} = I$ (identity matrix)
    - Gauss-Jordan method: https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/matrix-inverse/matrix-inverse.html
  - **Transpose**: switch row and column indices of a matrix. Row-mayor to column-mayor and vice versa
  - **Determinant:**

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - ceg - bdi - afh.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity matrix (I)

Row-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Column-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Matrix operations

- **Matrix-point/vector multiplication** gives new point/vector → **transform**
  - **Row-major/column-major** vector order dictates matrix operation order and matrix order
  - Row-major points and vectors are written as [1x3] or [1x4] matrices and then multiplied with matrix which is [3x3] or [4x3]

$$[x \quad y \quad z] * \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$x' = x * a + y * d + z * g$$
$$y' = x * b + y * e + z * h$$
$$z' = x * c + y * f + z * i$$

$$P' = P * T * R_z * R_y$$

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$P' = R_y * R_z * T * P$$

Both conventions are correct and give the same result but operations must be consistent.

Row major order: point is on left side
Column major: point is on right side
https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/geometry/row-major-vs-column-major-vector.html

Matrix operations on points and vectors

- Point `(x,y,z)` can be written as `[1x3]` matrix
  - Often, transformation matrices are `[4x4]`
  - To multiply point with such matrices we need to present point as **homogeneous point/coordinate**: `(x,y,z,w)`, `[1x4]`.
  - If the resulting w coordinate is not 1, then `x,y,z` must be divided by `w` to obtain usable Cartesian point.

$$(x, y, z, w) \rightarrow \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right).$$

- As vectors represent only direction, translation transformation is meaningless.
- https://www.pbr-book.org/3ed-2018/Geometry_and_Transformations/Applying_Transformations

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \\ 1 \end{pmatrix} = \begin{pmatrix} v_x + t_x \\ v_y + t_y \\ v_z + t_z \\ 1 \end{pmatrix}$$

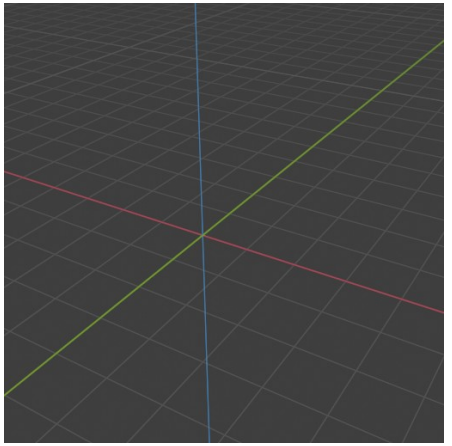
Column-major notation

Matrix and coordinate system

- Matrices can represent basis of a coordinate system
  - Each row of matrix represents an axis of coordinate system – orthogonal vectors → **orthogonal matrix**
  - Such matrix is called **orientation matrix – no translation**
- **Inverse of orthogonal matrix is equal to its transpose**

$$\begin{bmatrix} c_{00} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{matrix} x-axis \\ y-axis \\ z-axis \end{matrix}$$

Row-major notation



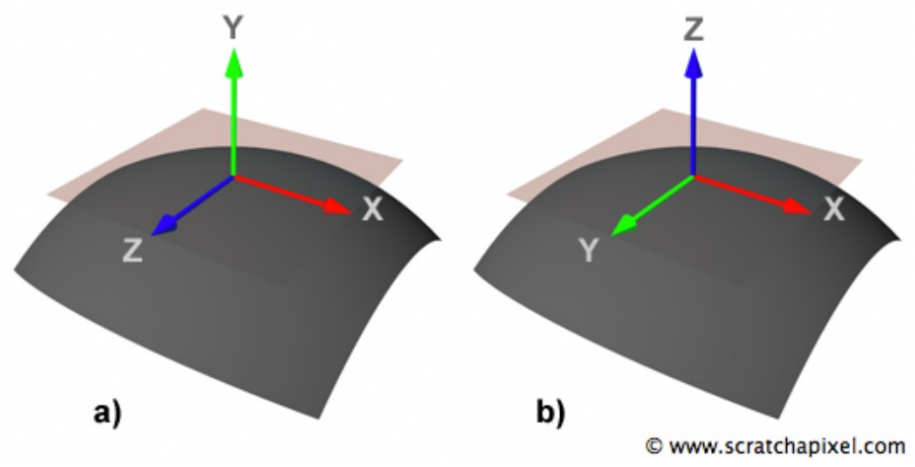
Local coordinate system

- Local coordinate system can be constructed using single vector using cross product.
- Often, local coordinate system using normal is constructed
  - Normal is one axis of local coordinate system
  - Tangent and bi-tangent are other to axes
  - Example: Normal corresponds to up vector, tangent to right vector and bi-tangent to forward vector

Up is Y

$$\begin{bmatrix} T_x & T_y & T_z & 0 \\ N_x & N_y & N_z & 0 \\ B_x & B_y & B_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row-major notation



Up is Z

$$\begin{bmatrix} T_x & T_y & T_z & 0 \\ B_x & B_y & B_z & 0 \\ N_x & N_y & N_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row-major notation

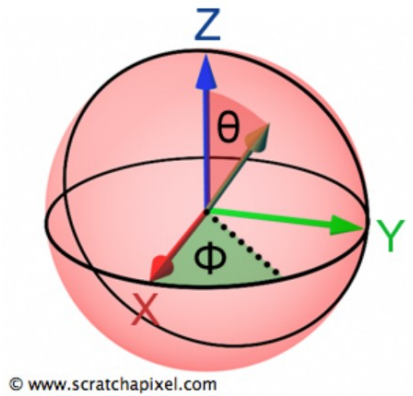
Matrix, coordinate system and transform

- Transforming points from one coordinate system to another is done by transformation matrix
  - Orientation (rotation), size (scale), and position (translation) of coordinate system represents the transformation that will be applied to the points when they are multiplied by this matrix
  - TODO: https://www.pbr-book.org/3ed-2018/Geometry_and_Transformations/Transformations- discussion on frame transforms
  - Matrix M transforms points from one coordinate system to another
    - Basis vector and origin of current coordinate system are transformed by matrix M
    - Points and vectors are expressed in terms of current coordinate system frame. Applying transformation matrix to points and vectors is equivalent to applying transformation matrix to current coordinate system frame

$$\mathbf{M} = \begin{pmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{pmatrix}.$$

Spherical coordinate system

- Spherical coordinates simplify computation needed for rendering
- Representing vectors in spherical coordinate system
  - Two angles: polar angle `[0, PI]` and azimuth angle `[0, 2PI]`
- Converting from spherical to Cartesian coordinate system



Vector representation in spherical coordinate system using polar and azimuth angle.

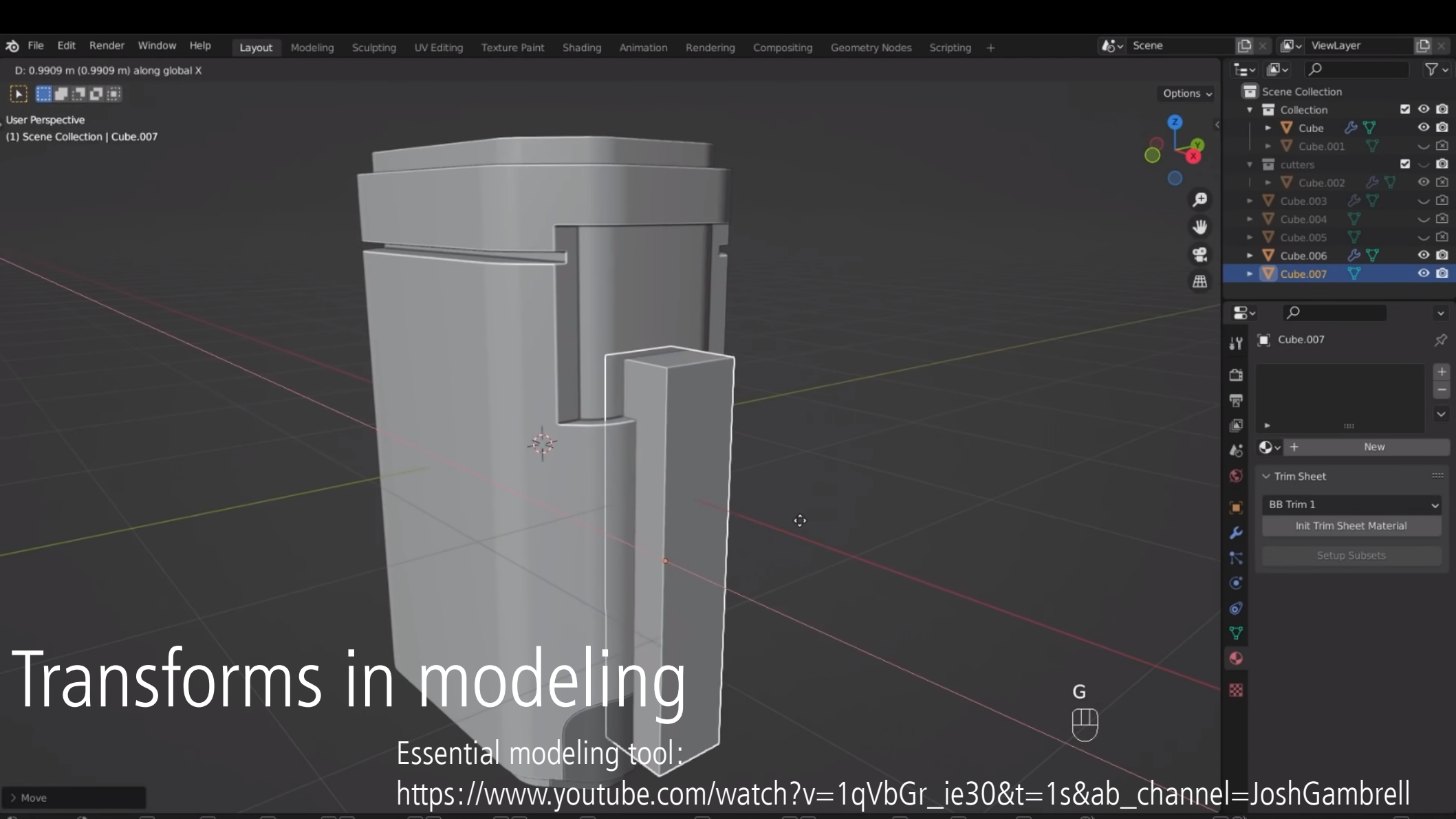
$$x = \cos(\phi)\sin(\theta)$$
$$y = \sin(\phi)\sin(\theta)$$
$$z = \cos(\theta)$$

Converting from spherical to Cartesian coordinate system

Transforms

Transforms

- Basic tool for manipulating 3D scene elements; points and vectors:
  - Position, orientate, reshape and animate objects, lights and cameras (3D scene modeling)
  - Essential for rendering computations
- **Linear transforms**: scaling, rotation
- **Affine transforms**: translation, rotation, scaling, reflection, shearing
  - Preserve parallelism of lines but not necessary lengths and angles
  - Require homogeneous point notation



Transforms in modeling

Essential modeling tool:
https://www.youtube.com/watch?v=1qVbGr_ie30&t=1s&ab_channel=JoshGambrell

Transforms in animation

Crucial tool for animation: https://www.youtube.com/watch?v=CBJp82tlR3M

Transforms in rendering

- All calculations must be preformed in the same coordinate system
  - Example: light-surface interaction calculation
  - Example: camera space – easier calculations
- Another example is projecting objects onto plane which is used for rasterization-based rendering.

Transforms in professional software

- Godot: https://docs.godotengine.org/en/stable/classes/class_transform.html
- Unity: https://docs.unity3d.com/ScriptReference/Transform.html
- Unreal: https://docs.unrealengine.com/4.27/en-US/BlueprintAPI/Math/Transform/
- Blender: https://docs.blender.org/manual/en/latest/scene_layout/object/properties/transforms.html
- GLM: https://github.com/g-truc/glm

Basic transformations

- Modeling elements, 3D scene and animations relies on transformations
  - Translation
  - Rotation
  - Scale
  - Shear
  - Look-at notation

Identity transform

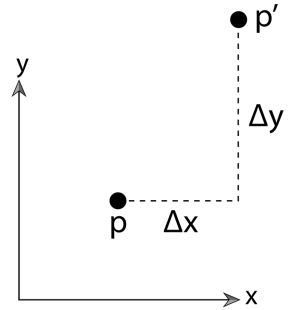
- Identity transformation is default transformation
- Represented by **identity matrix (I)**
  - `A * I = A`, A – vector or point

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Transforming points and vectors

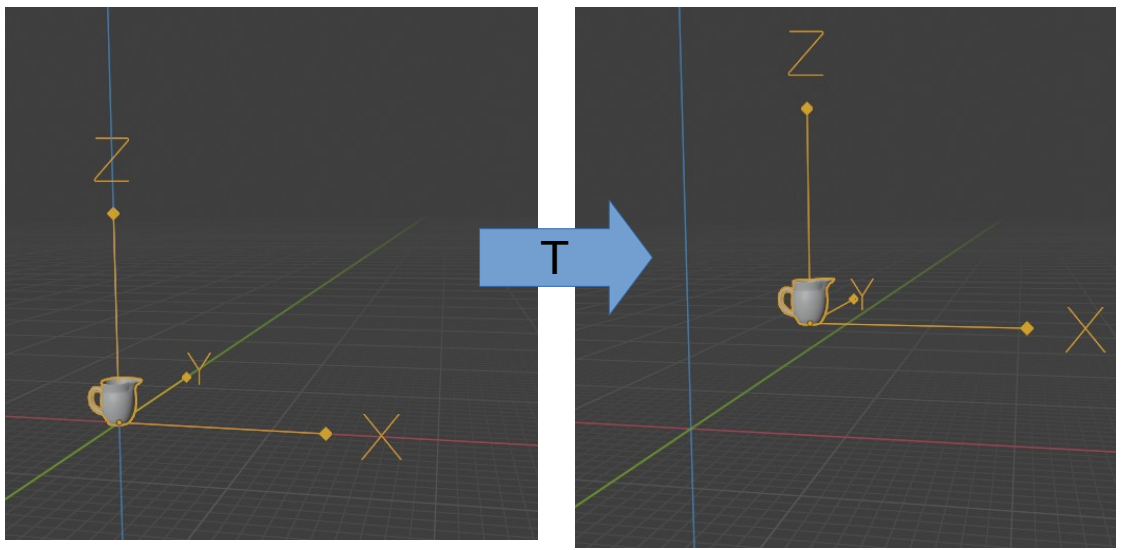
- Transformation matrices are `[4x4]` matrices.
- Homogeneous notation is needed:
  - Points: `P(x,y,z,1)`
  - Vectors: `V(x,y,z,0)`

Translation transform



- Changes point from one location to another
  - Translation transform translates coordinates of point `P(x,y,z)` by `(Δx, Δy, Δz)`
  - Translation only affects points, leaving vectors unchanged!
- Represented by **translation matrix T**

$$\mathbf{T}(\Delta x, \Delta y, \Delta z) = \begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

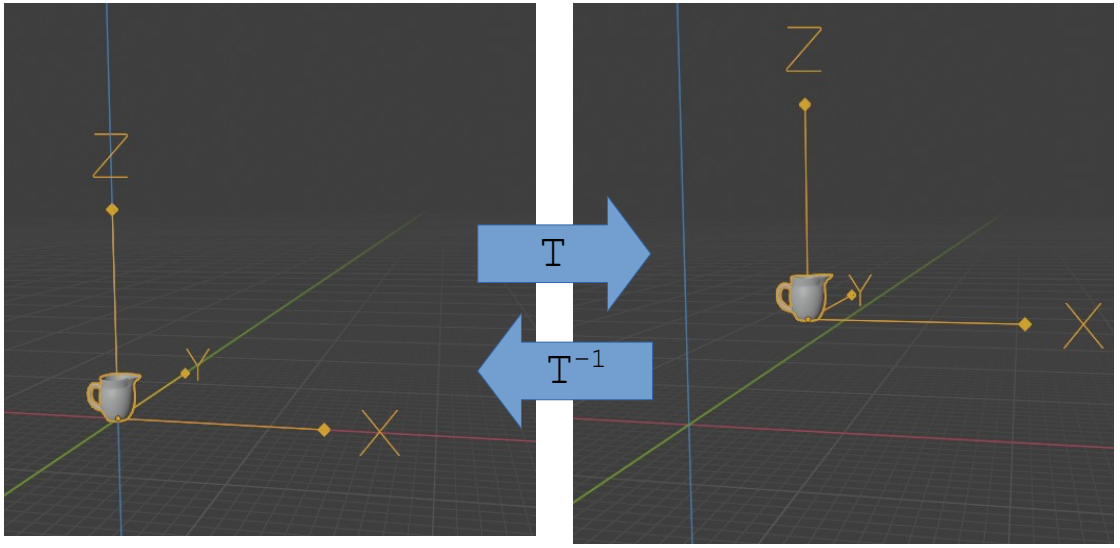


$$\begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{pmatrix}.$$

Translation matrix on point.

Translation transform

- Inverse: `T⁻¹(t) = T(−t)`
- **Rigid-body transform**: preserves distances between points and headedness



$$\begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{pmatrix}.$$

Translation matrix on point.

$$\begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}.$$

Translation matrix on vector.

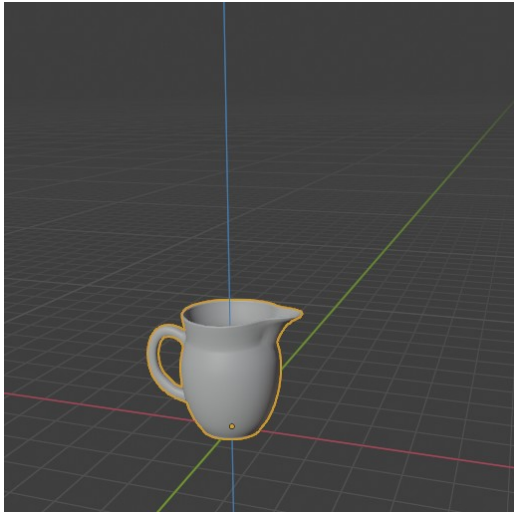
Translation transform

- Instancing example

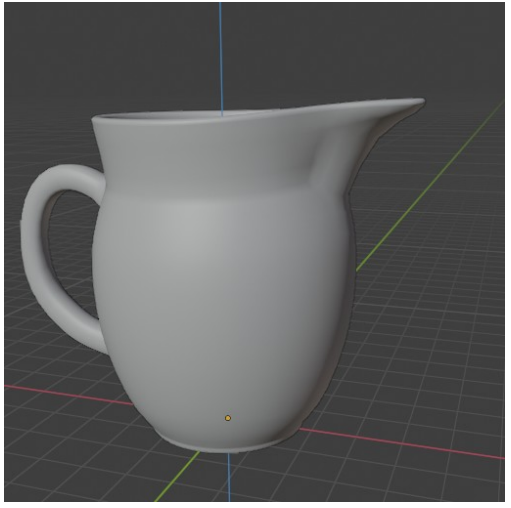
Scale transform

$$\mathbf{S}(x, y, z) = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

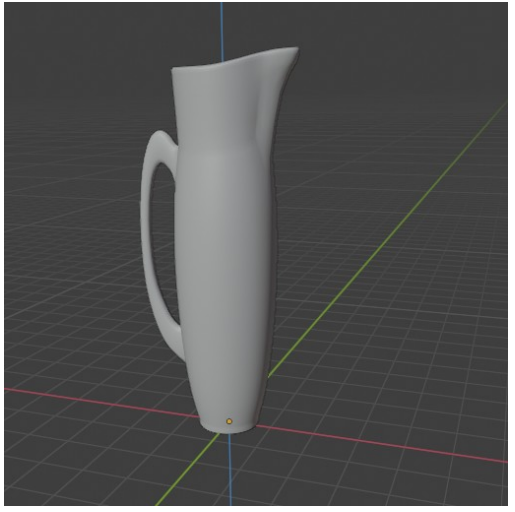
- Enlarging or diminishing objects
  - Multiply points `P` or vectors `V` components `(x,y,z)` by scale (s_x, s_y, s_z)
  - Represented by **scaling matrix S**
- If all scaling factors are same: **uniform** (isotropic) scaling, else **non-uniform** (anisotropic).



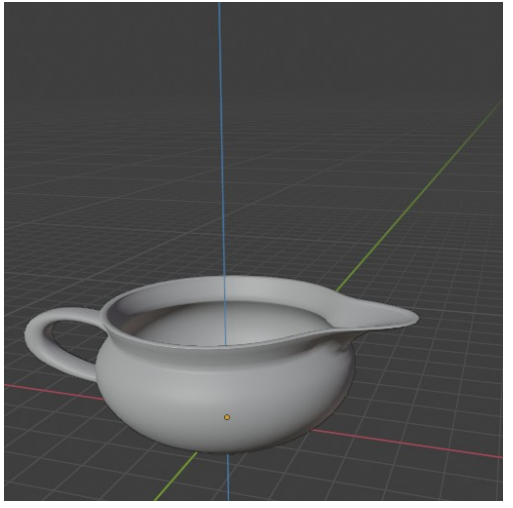
Original



Uniform



Anisotropic (z)

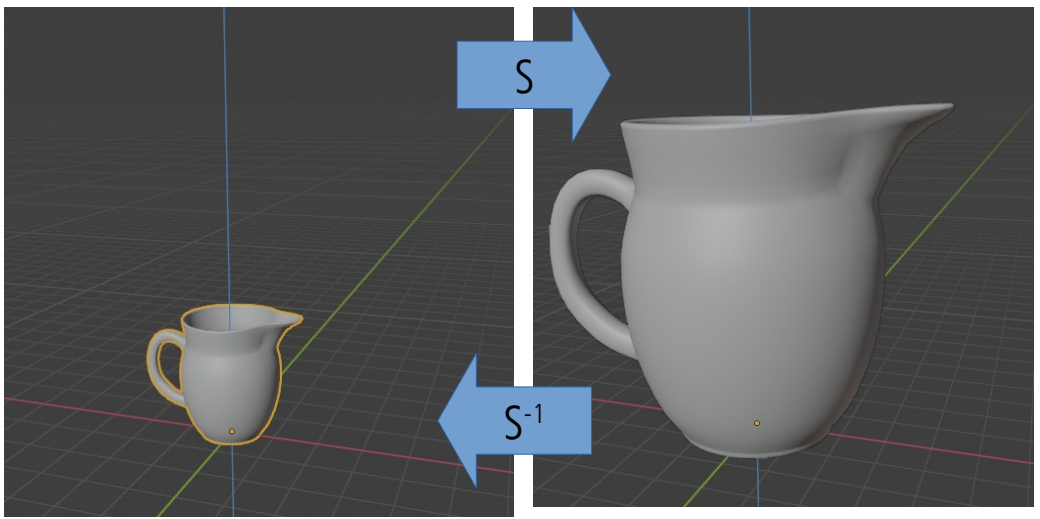


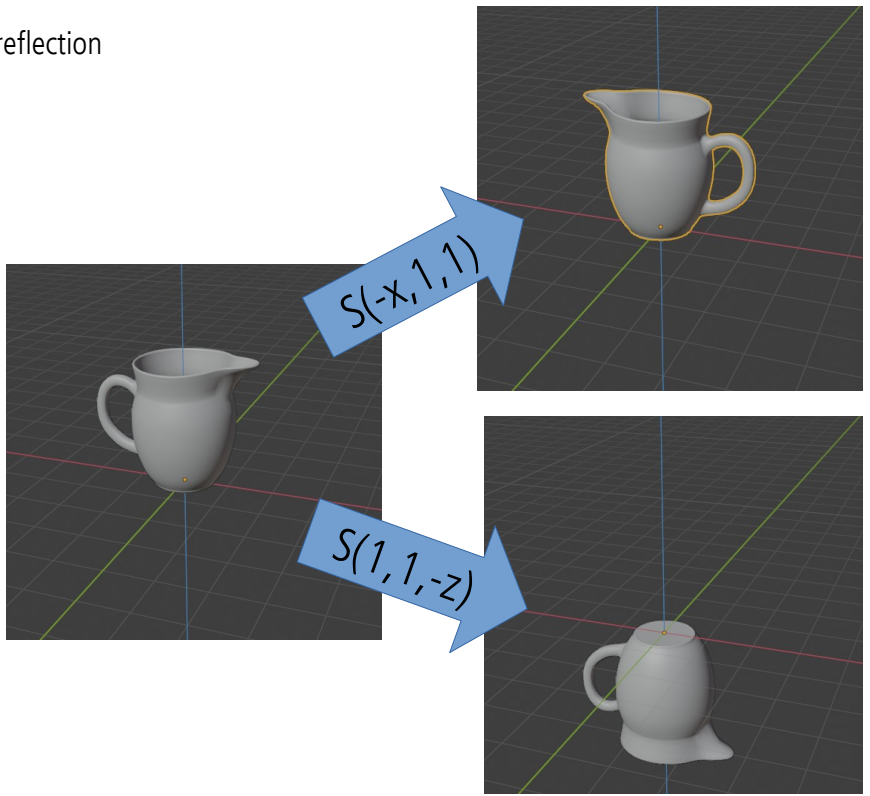
Anisotropic (x,y)

Scale transform

- **Inverse**
- Negative value of scaling factor gives **reflection (mirror) matrix**
  - Triangle with clockwise orientation will have counter-clockwise orientation after reflection

$$\mathbf{S}^{-1}(x, y, z) = \mathbf{S}\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right).$$



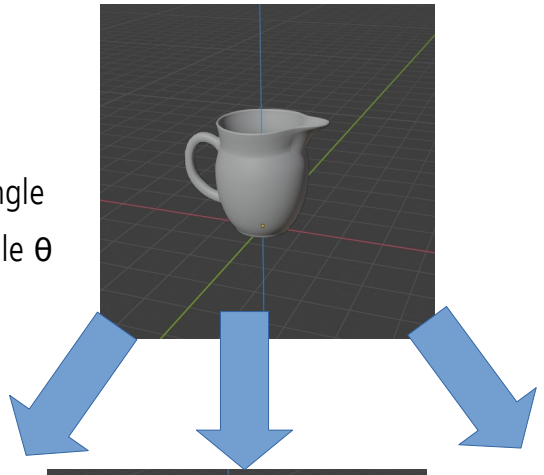


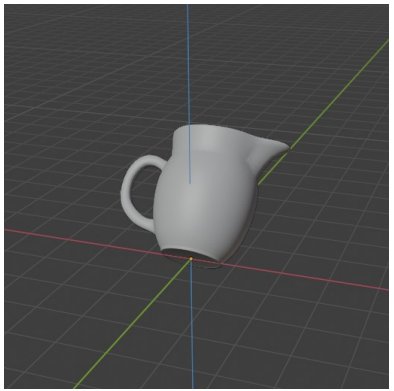
Scale transform

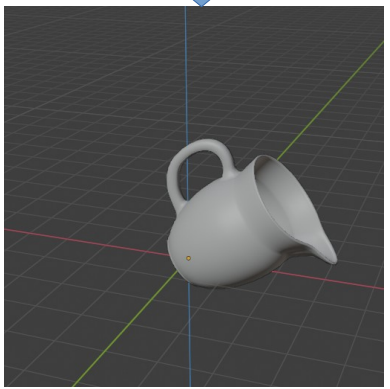
- Instancing example

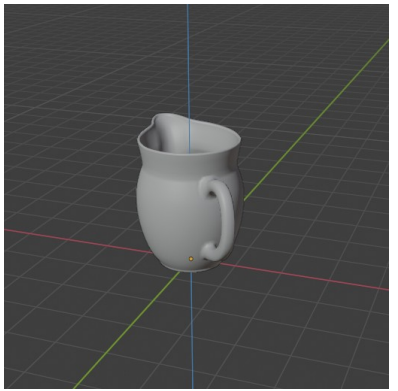
Rotation transformation

- Rotates objects around arbitrary axis from origin to any direction for a given angle
  - Most common rotations: rotation around X, Y and Z coordinate axes by angle θ
  - Represented by **rotation matrices** $R_x(\theta)$, $R_y(\theta)$, $R_z(\theta)$
  - Clockwise vs counter-clockwise rotation by angle θ









left-handed coordinate system, the matrix for clockwise rotation

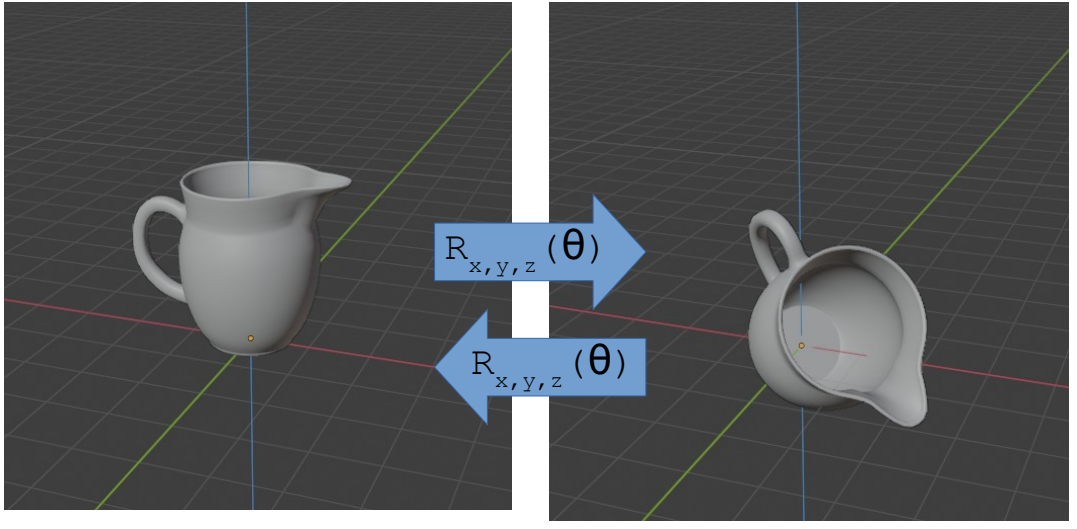
$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Rotation transformation

- **Rigid-body transform:** preserve distance between points and headedness
- **Inverse** $\mathbf{R}_a^{-1}(\theta) = \mathbf{R}_a(-\theta) = \mathbf{R}_a^T(\theta),$
- **Rotation around arbitrary axis**
  - Rotation around arbitrary axis (x, y, z) is represented by rotation matrix $R_{x,y,z}(\theta)$ – a combination of $R_x(\theta)$, $R_y(\theta)$, $R_z(\theta)$
  - Matrix consist of unit vectors → **orthogonal matrix**

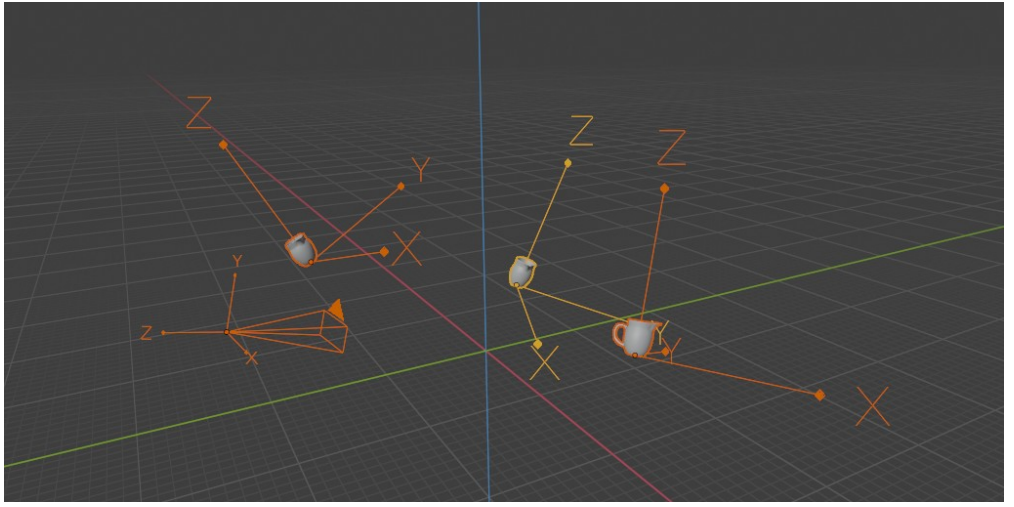


Rotate transform

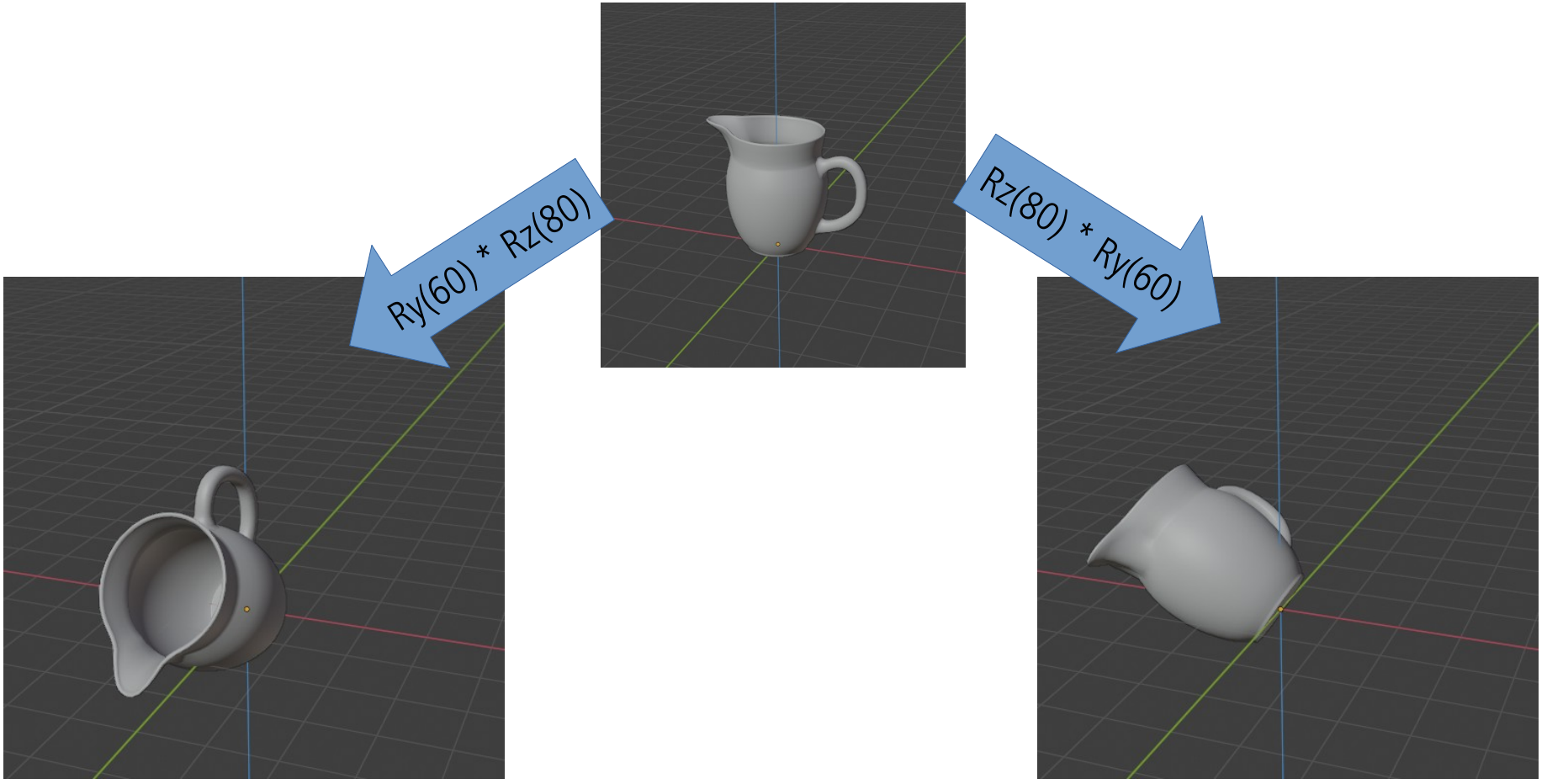
- Instancing example

Rotation transformation

- **Orientation matrix**
  - Rotation matrix associated with camera view or object → local coordinate system
  - Defines its orientation in space: directions for up and forward are needed to define this matrix

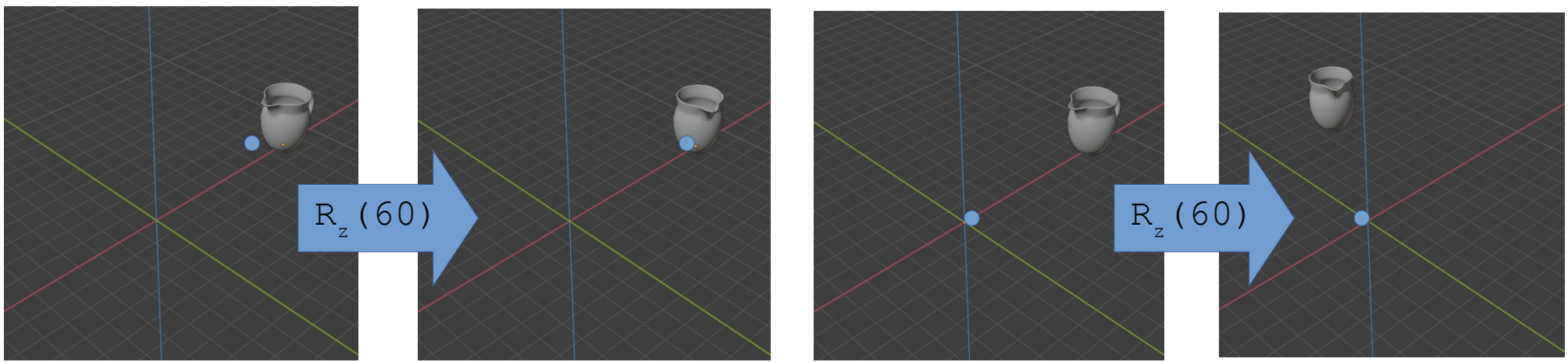


Combining rotation matrices



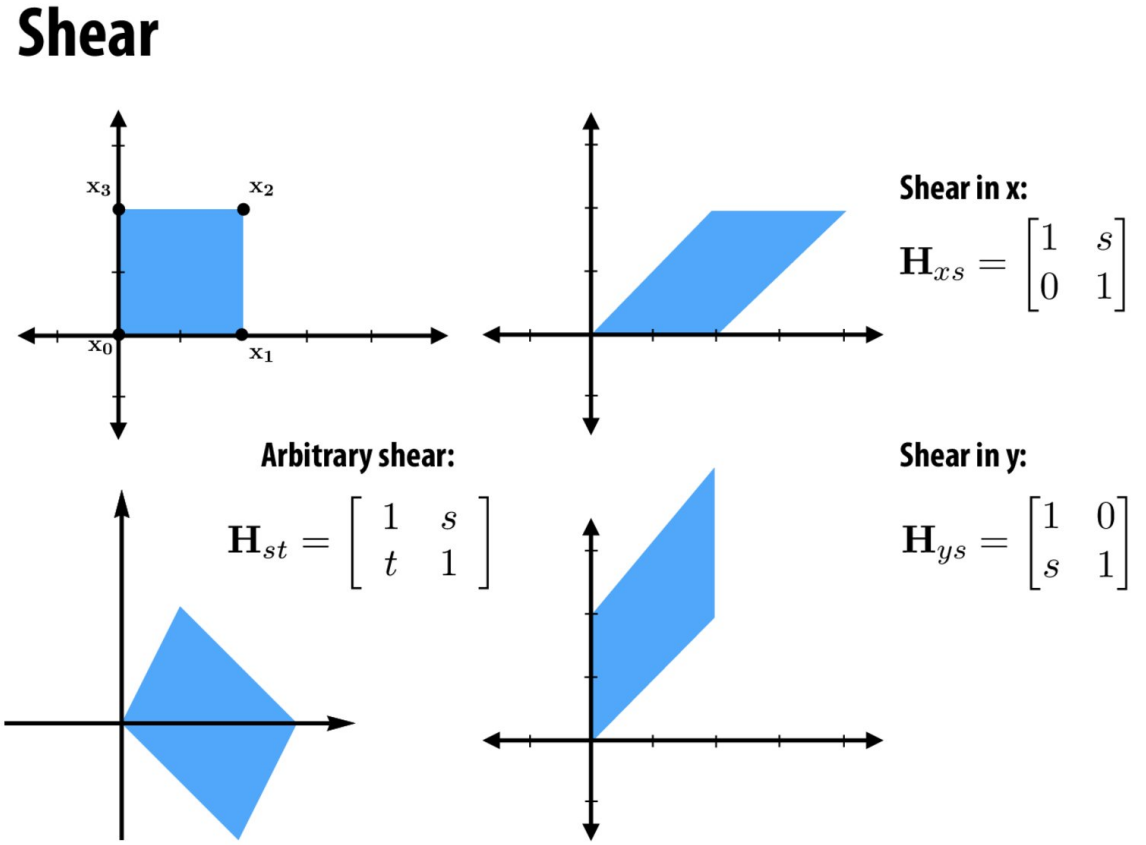
Rotation and translation

- If object is not in world origin, rotation origin can be local or world coordinate system
  - **Pivot point**
- For orienting object which is not in world origin
  - object is first translated so that pivot point in the center and then rotation is performed after which object is again transformed so that pivot point is in the same position as before.



Shear transform

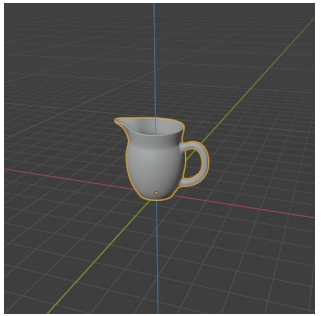
- 6 basic shearing matrices for 3D case: xy, xz, yx, yz, zx, zy

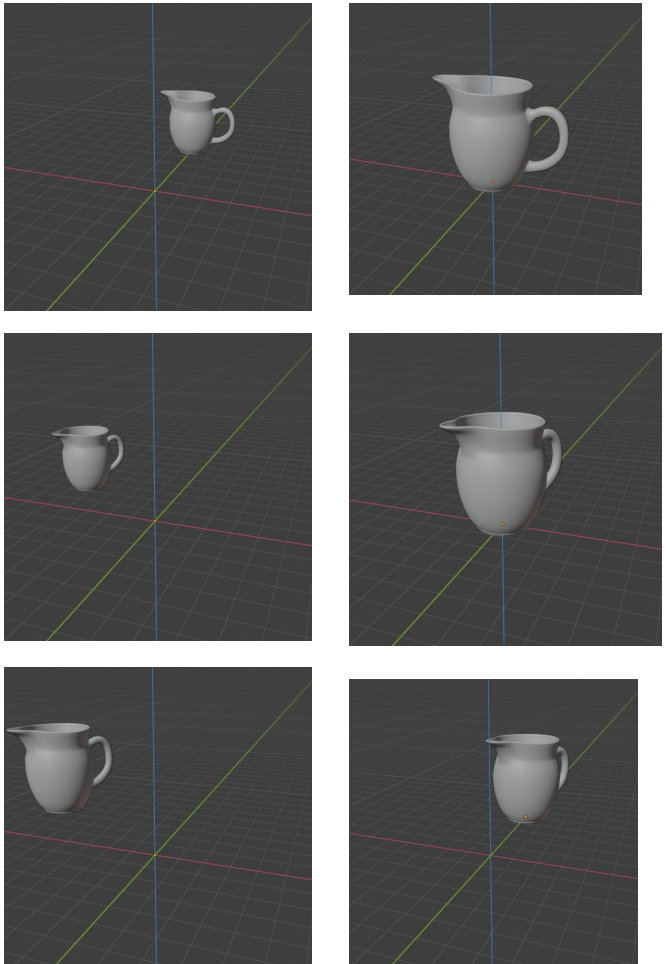


http://15462.courses.cs.cmu.edu/fall2019/lecture/transformations/slide_028

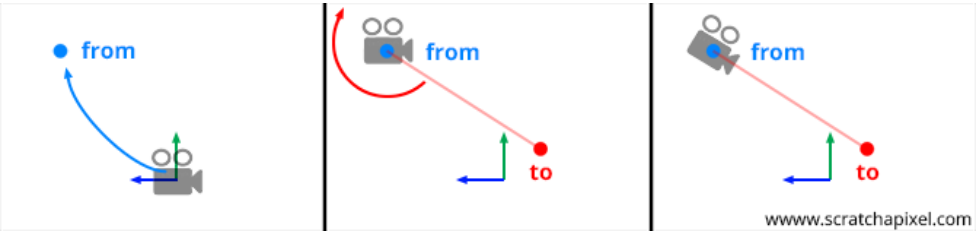
Concatenation of transforms

- Matrix multiplication is non-commutative: **order of multiplication matters**
- Example: objects in 3D scene must be scaled, rotated and translated
  - Matrix is concatenated into one matrix which is used for multiplication of points P
  - Such matrix must be composited as C = TRS, C * P = TRS * P
    - Scaling is applied first, then rotation and finally translation
- Concatenation of only translation and rotation matrices results in **rigid-body transform**.



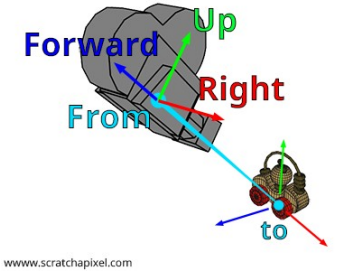


Look-at transform



- Often used for transforming camera in 3D scene
- **Look-at methods gives look-at transform which defines transformation matrix**

$$\begin{matrix} Right_x & Right_y & Right_z & 0 \\ Up_x & Up_y & Up_z & 0 \\ Forward_x & Forward_y & Forward_z & 0 \\ T_x & T_y & T_z & 1 \end{matrix}$$

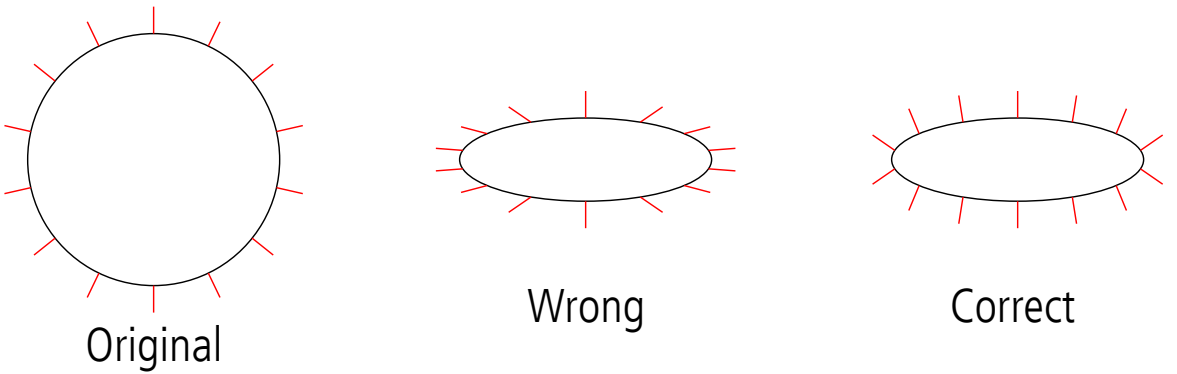


Row-major, right-handed coordinate system.

```
Forward = normalize(from - to)
Right = crossProduct(tmp, forward), tmp = (0,1,0)
Up = crossProduct(forward, right)
```

Special care: transformations of normals

- Normal can not be transformed with the same matrix used for transforming points and vectors.
  - It has to be multiplied with transpose of the inverse of that matrix.



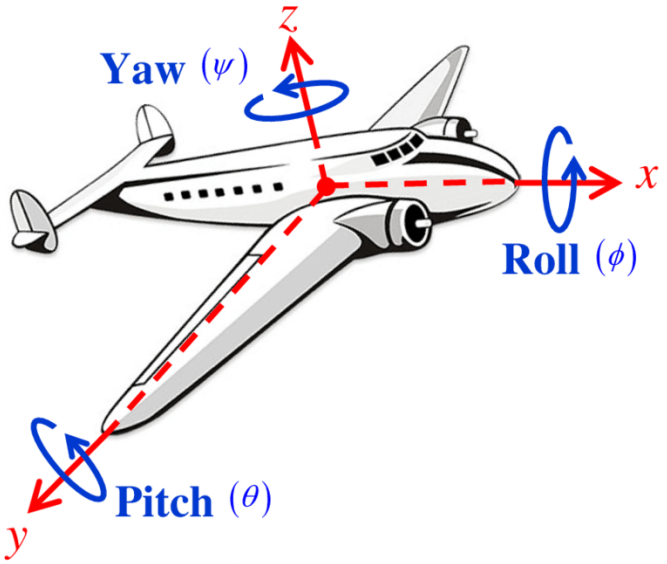
https://paroj.github.io/gltut/Illumination/Tut09%20Normal%20Transformation.html

Special transforms

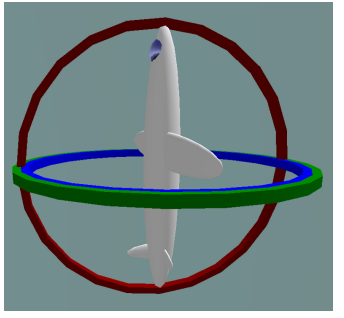
- Euler transform
- Rotation about an arbitrary axis
- Quaternions
- Othographic projection
- Perspective projection

Euler transform

- Represented by Euler matrix, a concatenation of $R_x(\theta)$, $R_y(\theta)$, $R_z(\theta)$ – rotation matrices around X, Y and Z axes
- Establish default view, e.g., facing positive X with Z up.
- Angles of rotation: **yaw**, **pitch**, **roll**
- Problems:
  - Gimbal lock: e.g., when pitch and roll become aligned, changes to roll and yaw result in same rotation
  - Two different sets of Euler angles can give same orientation
  - Interpolation between two Euler angle sets is not as simple as interpolating each angle
- Good: angles can be easily extracted from Euler matrix → matrix decomposition



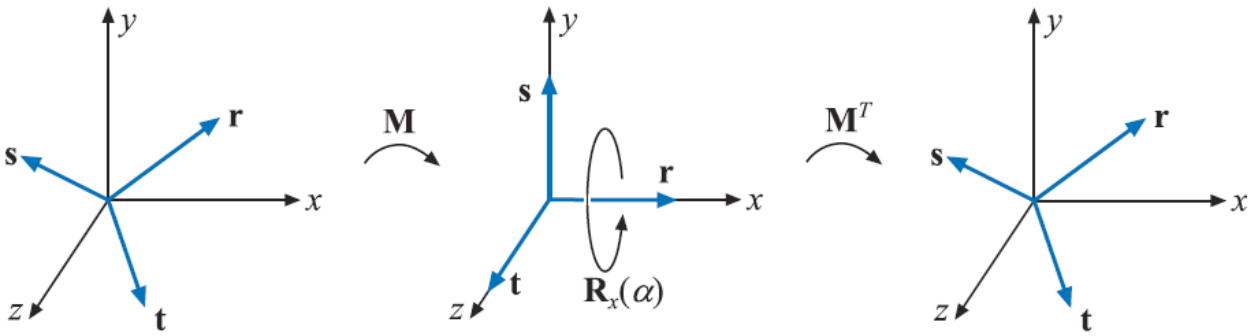
https://www.researchgate.net/publication/335854843_Special_Orthogonal_Group_SO3_Euler_Angles_Angle-axis_Rodriguez_Vector_and_Unit-Quaternion_Overview_Mapping_and_Challenges



Gimbal lock: one degree of freedom (rotation angle) is lost.
https://en.wikipedia.org/wiki/Gimbal_lock

Rotation about arbitrary axis

- Rotate by some angle θ around arbitrary axis `r`
  - Transform to space where axis we want to rotate around is X
    - Local coordinate system is created using axis `r` via cross product → basis
    - Obtained basis is aligned with world coordinate system basis and `r` is aligned with `X`
  - Perform rotation
    - Rotation around X axis
  - Return to original space

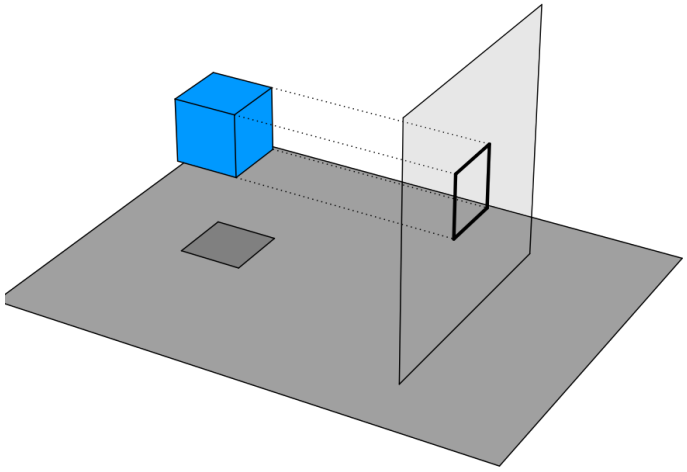


Quaternions

- Orientation is represented by a single rotation around particular axis
  - Quaternion: $q = s + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
    - s – scalar
    - i, j, z – three spatial axes
    - (x,y,z) represent axis
- Unit quaternion can represents any 3D rotation
- Advantage: Interpolating between two quaternions is stable and constant
- Supported in various modeling tools and APIs:
  - https://github.com/g-truc/glm/blob/master/manual.md#-311-quaternion-functions
  - https://docs.blender.org/api/current/mathutils.html#mathutils.Quaternion

Orthographic projection

- Projects points P(x,y,z) on plane z = 0



$$Pv = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ 0 \\ 1 \end{bmatrix}$$

Orthographic projection

- Projects points P(x,y,z) on plane z = 1

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective projection

- Projects point P (x,y,z) on plane $z = 1$

Scene organization

3D scene as scene-graph

- 3D scene modeling goes hand in hand with object oriented design.
- 3D scene representation has inherent tree-like structure thus often represented with so called **scene-graph**
  - Arrangement between user who builds 3D scene and renderer
  - https://learnopengl.com/Guest-Articles/2021/Scene/Scene-Graph
  - Scene modeling tools: DCC examples

<IMAGE: COMPONENTS OF 3D SCENE AND SCENE GRAPH>

Scene graph

- Hierarhical datastructure for organizing and structuring storing whole 3D scene, with all its elements
- User oriented datastructure for modeling and organizing elements and their relationships in hierarchy for easier manipulation and construction of scene
  - It is edited and created by user: artists and designers
  - Enables easier modeling and animation (e.g., whole subtrees can be translated)
- It serves as support to rendering algorithms. Pass through scene graphs pass for rendering
  - Depending on rules, rendering algorithm might use different materials, geometries, lights, animations, etc.

Scene graph

- Example of simple scene graph

Scene graph

- Elements:
  - Nodes: root, internal and leaf. Each node contains data, at least its position in graph which gives consistent structure
  - Edges
- Graph scene:
  - Root – starting point for whole scene
    - Data: type of coordinate system, scene units, etc.
  - Internal nodes – organize scene into hierarchy. Often those are transformation information (where and how are objects positioned)
  - Leaf nodes – contain elements of the scene: objects, camera, lights. As objects in the scene can repeat, these nodes can be duplicated
  - image

Leaf nodes

- Contain elements of the scene:
  - Object meshes
  - Object materials
  - Lights
  - Cameras
  - EXAMPLES

Internal nodes

- Often those are transformation nodes defining positions of sub-trees: translations, rotations, scaling, etc.
- Other types:
  - Grouping – contain nodes without any other function
  - Conditional – type of grouping node which enables activation only the particular children node
  - Level of detail – contains children nodes where each has a copy of objects with varying level of detail and only one is active depending on camera distance
  - Billboard – groupping node which orientates all children node towards camera
  - EXAMPLES
- As materials and textures are often shared between different objects, they can be stored as internal node which is references by children nodes

Scene graph traversal

- Recursive operation starting from root going through hierarchy
- Scene graph traversal can be used during rendering
- Therefore, it serves as **standardized scene description** that can be shared between different rendering, modeling and interaction tools

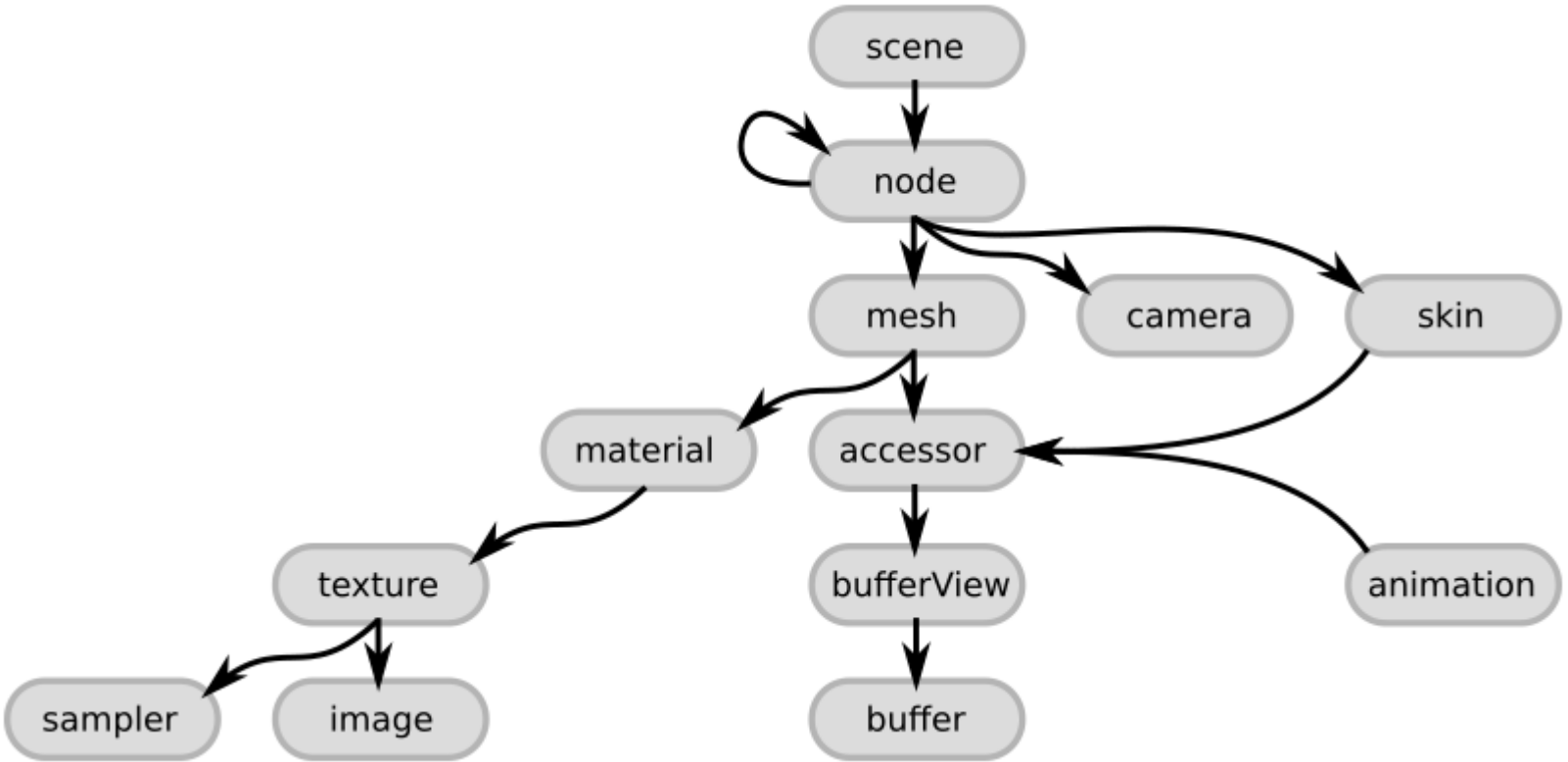
Scene graph and scene transfer

- Different modeling, rendering and interaction software has different formats for scene description
  - Transfer between them requires **standardized formats**
  - For different formats, different **importer/exporter** functions are needed
  - Different formats exists which differ by **scene elements support**
- Storing and transferring scene requires data described by scene graph

Scene graph and scene transfer

- Tendency towards standardized formats is required
- Popular scene description formats:
  - glTF
  - USD: https://graphics.pixar.com/usd/release/index.html

Scene graph example: glTF



https://github.com/KhronosGroup/glTF-Tutorials/blob/master/gltfTutorial/gltfTutorial_002_BasicGltfStructure.md

Scene graph example: glTF

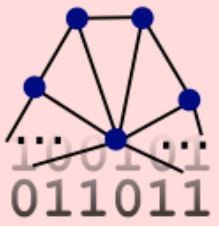
.gltf (JSON) file

```
"scenes": [ ... ],
"nodes": [ ... ],
"cameras": [ ... ],
"animations": [ ... ],
...



"buffers": [
  {
    "uri": "buffer01.bin",
    "byteLength": 102040
  }
],



"images": [
  {
    "uri": "image01.png"
  }
],
```

The JSON part describes the general scene structure, and elements like cameras and animations.
Additionally, it contains links to files with binary data and images:

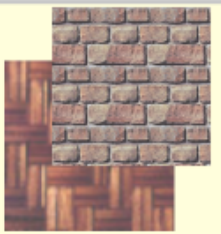
.bin files

Raw data for geometry, animations and skins



.jpg or .png files

Images for the textures of the models



Elements of 3D scene in production

- https://github.com/appleseedhq/appleseed/wiki/Project-File-Format
- Scene graph can be stored in various formats: e.g., XML

- `<project>` !
  - `<scene>` !
    - `<assembly>` *
      - `<assembly>` *
      - `<assembly_instance>` *
        - `<transform>` *
      - `<bsdf>` *
      - `<color>` *
      - `<edf>` *
      - `<light>` *
        - `<transform>` *
      - `<material>` *
      - `<object>` *
      - `<object_instance>` *
        - `<assign_material>` *
        - `<transform>` *
      - `<surface_shader>` *
      - `<texture>` *
      - `<texture_instance>` *
    - `<assembly_instance>` *
      - `<transform>` *
    - `<camera>` *
    - `<color>` *
    - `<environment>` ?
    - `<environment_edf>` ?
    - `<environment_shader>` ?
    - `<texture>` *
    - `<texture_instance>` *
  - `<rules>` +
    - `<render_layer_assignment>` ?
  - `<output>` !
    - `<frame>` ?
  - `<configurations>` !
    - `<configuration>` *

Literature

- https://github.com/lorentzo/IntroductionToComputerGraphics/wiki/Foundations-of-3D-scene-modeling