Lecture 12: Ray-tracing based rendering

DHBW, Computer Graphics

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Syllabus

- 3D scene
  - Object
  - Camera
  - Light
- Rendering
  - Ray-tracing based rendering
  - Rasterization-based rendering
- Image and display

- Ray-tracing based rendering
  - Overview
  - Camera rays
  - Intersection testing (visibility)
  - Shading
  - Light transport

Inter-reflections

Glossy reflections

Soft shadows

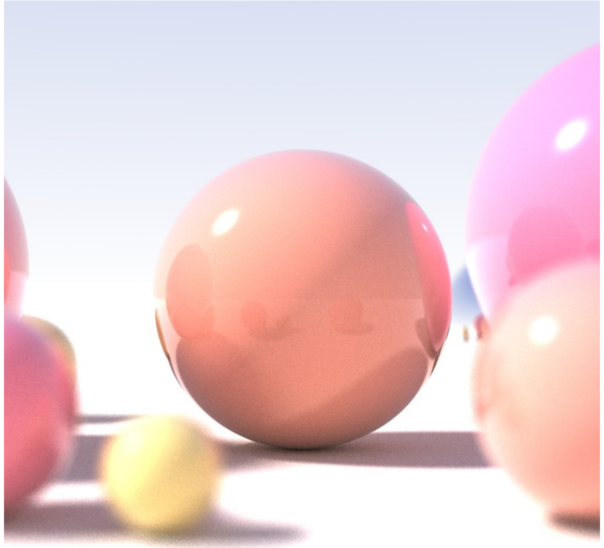
Transmission

Ray-tracing-based rendering:
https://www.realtimerendering.com/Real-Time_Rendering_4th-Real-Time_Ray_Tracing.pdf

Ray-tracing based rendering overview

Introduction

- Ray-tracing is a method **inspired by physics of light** → realistic image synthesis
- Ray-tracing is considered one of the **most elegant techniques** in computer graphics
- Many phenomena such as **shadows, reflections and refracted light are intuitive and straightforward** to implement



Robert L. Cook Thomas Porter Loren Carpenter. "Distributed Ray-Tracing" (1984).

https://en.wikipedia.org/wiki/Ray_tracing_(graphics)

Rendering recap

- **Rendering: generating 2D image from 3D scene**
  - **Calculating a color of each pixel of the virtual image plane**
- **Rendering process: visibility and shading**
  - **Visibility**: find objects that are visible from camera
    - Find which surfaces are visible, ignore participating media
  - **Shading**: what is the color of visible objects
    - Use incoming light, view, object shape and material information

Ray-tracing for visibility

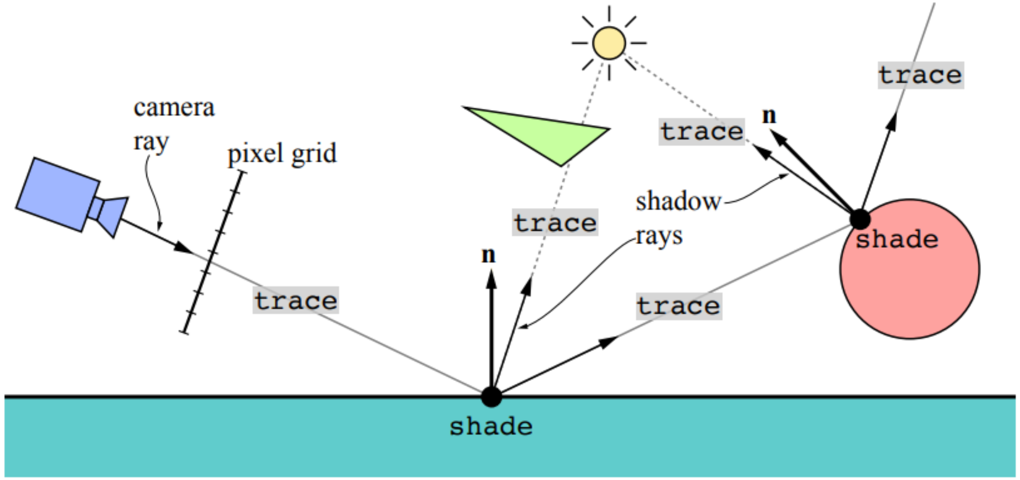
- **Visibility problem**: two points are visible to each other if line segment that joins them does not intersect any obstacle.
- Ray-tracing fundamentally **solves visibility problem using ray-casting**
  - **Finding objects visible from camera**: generate ray from camera and test intersections with objects in 3D scene
  - **Shading, light transport:** generate additional rays from intersected objects into 3D scene to find amount and direction of incoming light

Ray-tracing based rendering

- **Generate ray** for each pixel of virtual image plane → **camera ray**
  - **Perspective camera**: generate rays from aperture to each pixel
  - **Orthographic camera**: generate parallel rays for each pixel
- **Ray-object intersection**: testing intersection of generated ray and 3D scene objects to find closest intersection → objects visible from camera
- **Shading**: calculating the color and intensity of intersected points
  - **Light-matter calculation**: light absorption and reflection
  - **Light transport for computing incoming light**: tracing rays to potential sources of light

Ray-tracing-based rendering: `trace()` & `shade()`

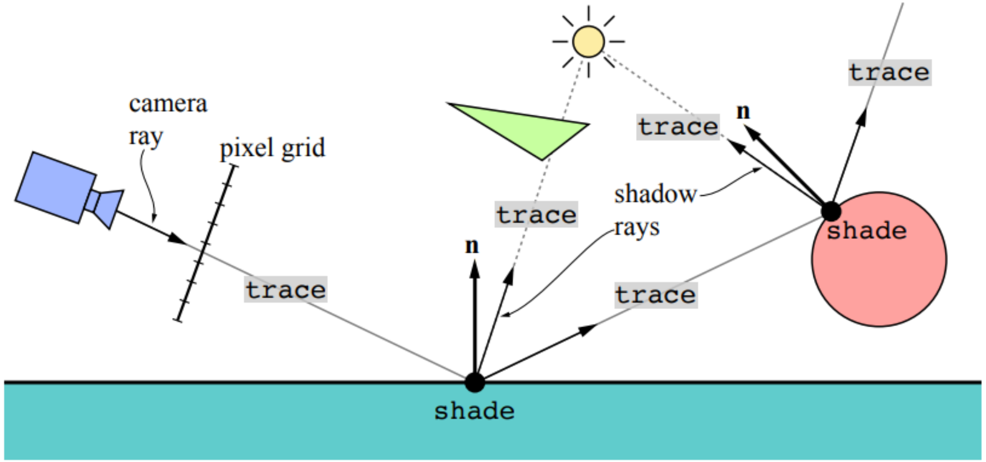
- Raytracing can be described with two functions **`trace()`** and **`shade()`**
- **`trace()`** is geometrical part of algorithm responsible for finding closest intersection between ray and the objects in 3D scene → solving visibility problem
- **`Shade()`** returns color of the ray intersecting object found by **`trace()`**



Ray-tracing-based rendering:
https://www.realtimerendering.com/Real-Time_Rendering_4th-Real-Time_Ray_Tracing.pdf

`trace()` for camera rays

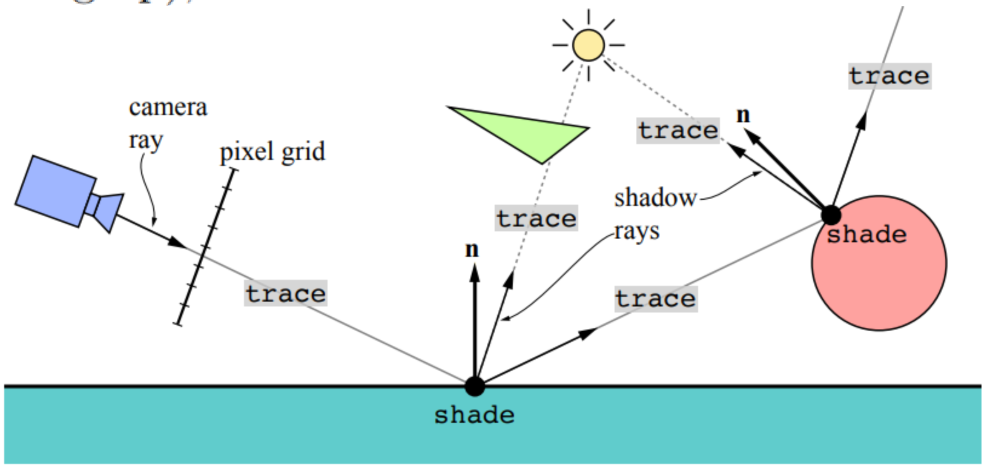
- **Rendering starts by generating camera rays for each pixel in the image**
- **`trace()` function is used on generated camera rays**
  - Find closest intersection of camera ray with 3D scene objects
  - Naive `trace()` function: loop through all n objects in the scene and returns closest intersection
    - `O(n)` performance. Spatial acceleration structure (BVH or k-d tree) → `O(log(n))` performance



`trace()` for camera rays

```
function RAYTRACEIMAGE
    for p do in pixels
        color of p = TRACE(camera ray through p);
    end for
end function
```

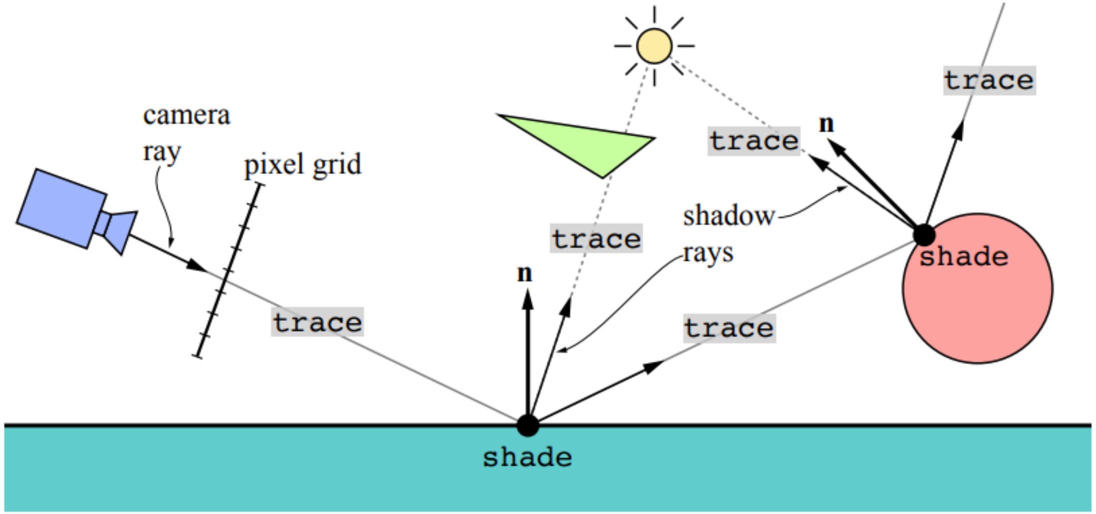
```
function TRACE(ray)
    pt = find closest intersection;
    return SHADE(pt);
end function
```



Ray-casting is often used in `trace()` function for determining visibility between two points.

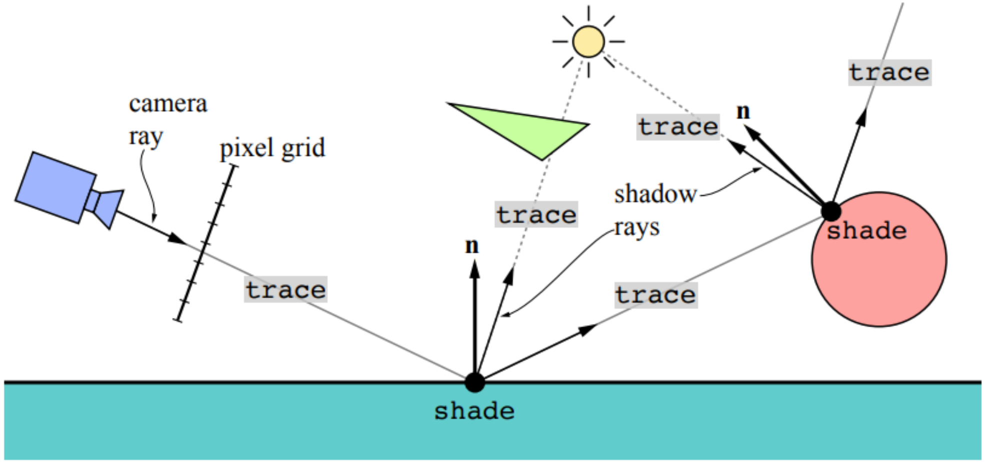
`shade()` for intersections

- `shade()` calculates color in intersection.
- `shade()` can be arbitrarily complex:
  - It can just return the color of object
  - It can use material information in intersected point with incoming light information
    - Use `trace()` for closest light sources
    - Use `trace()` to gather incoming light from all directions (e.g., other surfaces)



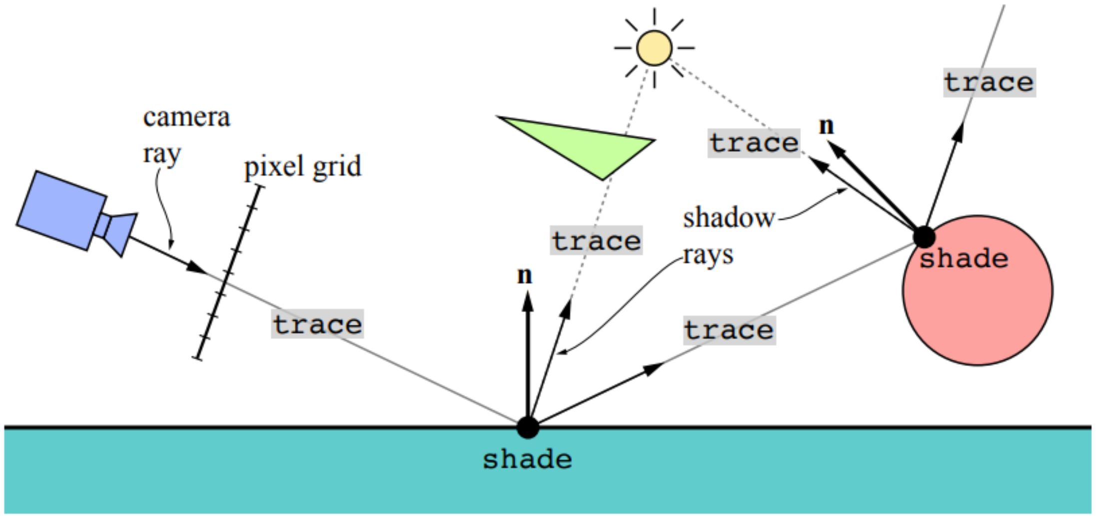
`trace()` for light transport

- **Light transport: obtaining incident light on shading point using** `trace()`
  - Calculate **visibility between shaded point and light source (direct illumination)**
  - Compute light from **specular surfaces (Whitted ray-tracing)**
  - Compute **indirect light reflected from other surfaces (e.g., path-tracing)**



`shade()` for intersections

```
function SHADE(point)
    color = 0;
    for L do in light sources
        TRACE(shadow ray to L);
        color += evaluate material;
    end for
    color += TRACE(reflection ray);
    color += TRACE(refraction ray);
    return color;
end function
```

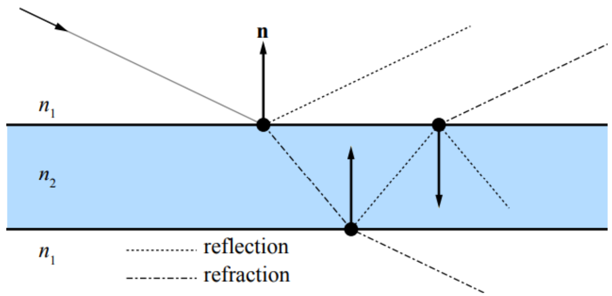



```
function TRACE(ray)
    pt = find closest intersection;
    return SHADE(pt);
end function
```

- Each `shade()` can call `trace()` and each `trace()` can call `shade()`
- **Ray depth** is term with indicates number of rays that have been shot recursively along a ray path

`shade()` and `trace()`

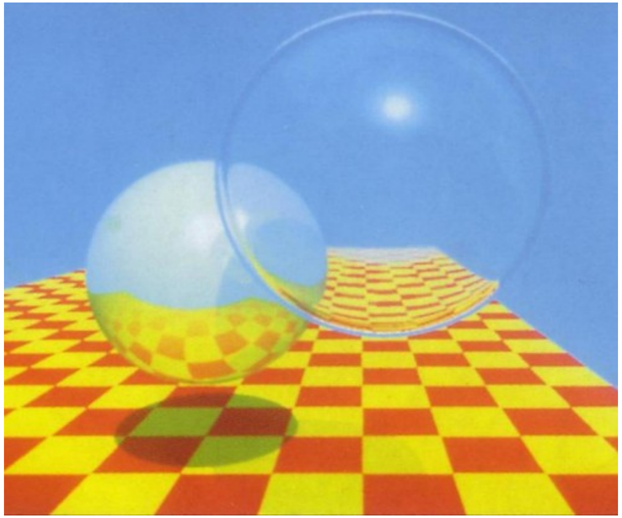
- `shade()` function is implemented by user as a **shader** program
  - Material (scattering functions and textures) are used to define light-surface interaction – amount of absorption and reflection, that is, color
- Traversal and intersection testing which takes place in `trace()` function is often implemented on CPU
  - GPU rendering uses compute or ray-tracing shaders in Vulkan or DXR



`shade()` defines interaction of light with surface, its color.

Ray-tracing structure

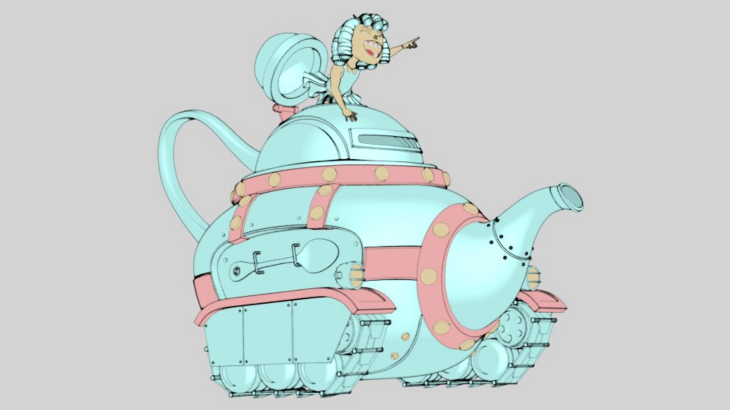
- Ray-tracing structure consisting of `shade()` and `trace()` is basis for **Whitted ray-tracer.**
  - Assumptions: surface are perfectly sharp (specular) reflections and refractions or diffuse.
  - Computes indirect light coming from specular surface
- Whitted ray-tracing is foundation of many other rendering variants such as path-tracing which are solving rendering equation and global illumination.



(Non-)Photo-realistic rendering

- **Level of photo-realism depends on reproducing the color and intensity of objects - appearance**
- Shading plays crucial role for object appearance:
  - **Photo-realistic rendering**
  - **Expressive or artistic rendering ➞ non-photo-realistic rendering**
- Photo-realistic rendering is useful for understanding physically-based shading from which non-photo realistic rendering can be derived with means of exaggeration

https://www.artstation.com/artwork/rANRe5



"Rolling Teapot" - Model by Brice Laville, concept by Tom Robinson, render by Esteban Tovagliari - RenderMan 'Rolling Teapot' Art Challenge: https://appleseedhq.net/gallery.html#https%3A%2F%2Fappleseedhq.net%2Fimg%2Frenders%2Frolling-teapot.jpg

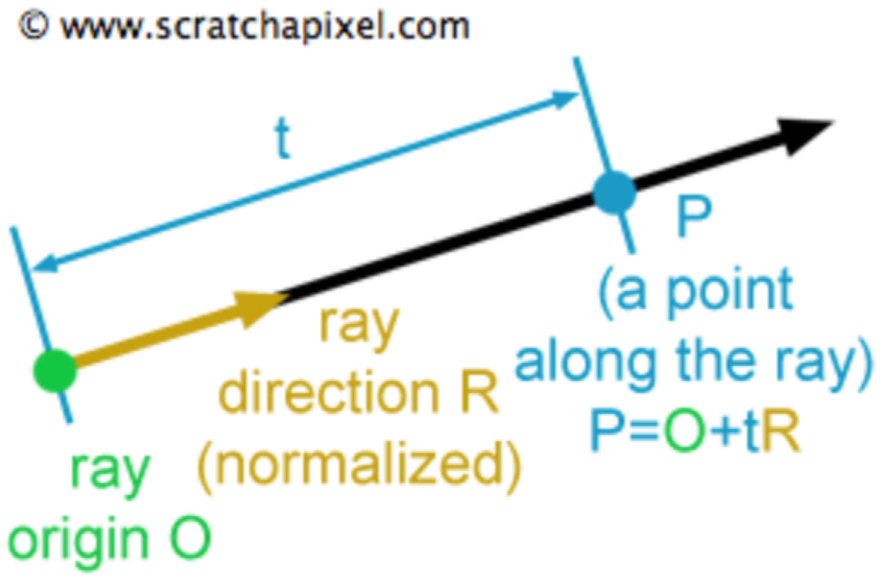
Generating camera rays

Ray

- **Ray is fundamental element of ray-tracing used for solving visibility**
- Ray is defined as:
  - `vector3f origin;`
  - `vector3f direction;`
- Any point on ray is is defined with parametric equation:

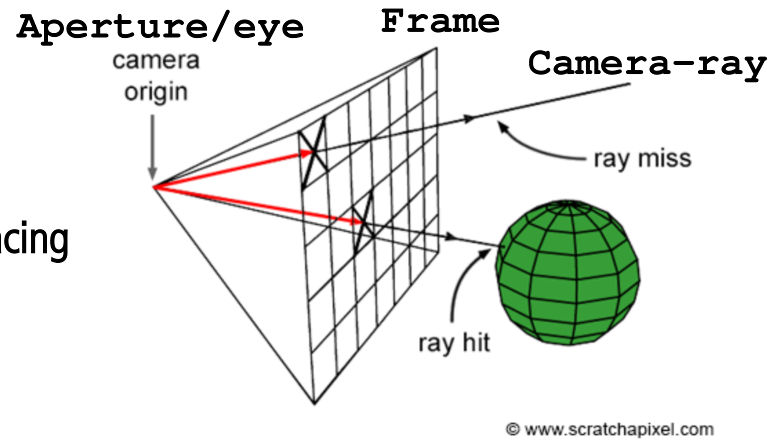
```
P(t) = origin + t * direction;
```

  - `t` – distance from `origin` to `P(t)`
  - **`t > 0`** → `P(t) is` in front of ray's `origin`
  - `t < 0` → `P(t) is` behind the ray's `origin`



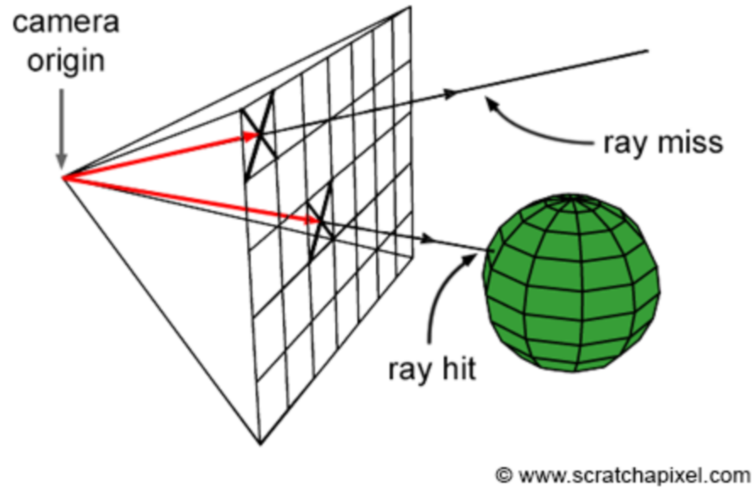
Generating camera rays

- Camera:
  - **Aperture** (eye) position
  - **Orientation** of camera line of sight
  - **Field of view**: how much of the scene we see
  - **Frame (image)**: array of pixels where image is formed
- **Camera rays** are generated starting from camera aperture and passing through each pixel in the image plane into 3D scene → **backward/eye-tracing**
  - Used to compute the visible objects → **ray-casting**
- Camera rays are also known as: **primary ray, view ray**



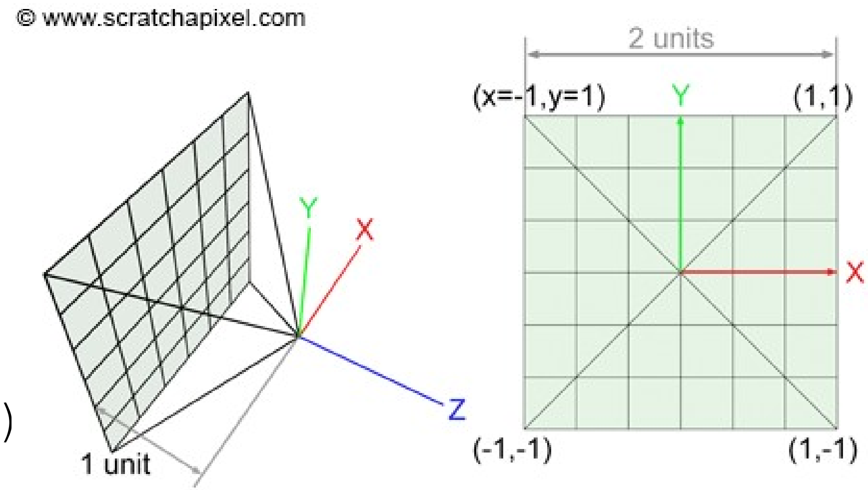
Generating camera rays

```
Vector3f ImageBuffer[imageWidth, imageHeight];
For (int j = 0; j < imageHeight; ++j)
{
    For (int i = 0; i < imageWidth; ++i)
    {
        Ray ray = buildCameraRay(i, j);
        For  (int k = 0; k < nObjectsInScene; ++k)
        {
            if (intersect(ray, object[k], intersectionContext)
            {
                // Object hit. Compute shading using intersectionContext.
                ImageBuffer[j * imageWidth + i] = shadingResult;
            }
            else
            {
                // Background hit. Compute background color...
                ImageBuffer[j * imageWidth + i] = backgroundColor;
            }
        }
    }
}
```



Generating camera rays: basic setup

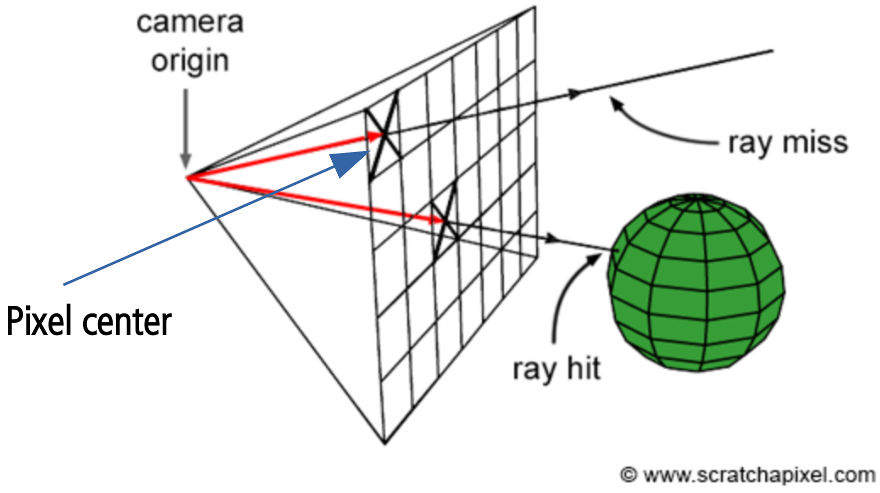
- Basic setup:
  - Camera origin (aperture, eye) is in `(0,0,0)`
  - Camera is looking in negative `Z` axis
  - Film (image) plane is placed 1 unit from from camera's origin
  - Film dimensions are `2x2` units
  - Film is centered around `Z` axis
  - Image is square (`image_width == image_height`)



https://www.scratchapixel.com/lessons/3d-basic-rendering/ray-tracing-generating-camera-rays/generating-camera-rays.html

Generating camera rays: basic setup

- World-space ray is created by connecting world-space points:
  - Camera origin – already aligned with world coordinate system origin `(0,0,0)`
  - Pixel center – requires transformation from raster space to world space
- World-space ray is needed for intersection since all objects in scene are also defined in world space



Pixel center coordinates

- Pixel coordinates are originally defined in **raster space `[image_height, image_width]`**
  - Integer coordinates $(\texttt{Pixel}_x, \texttt{Pixel}_y)$ where left-top corner of frame is `(0,0)`
- Pixel position must be first normalized using frame dimensions giving **normalized device coordinates (NDC) [0,1]**

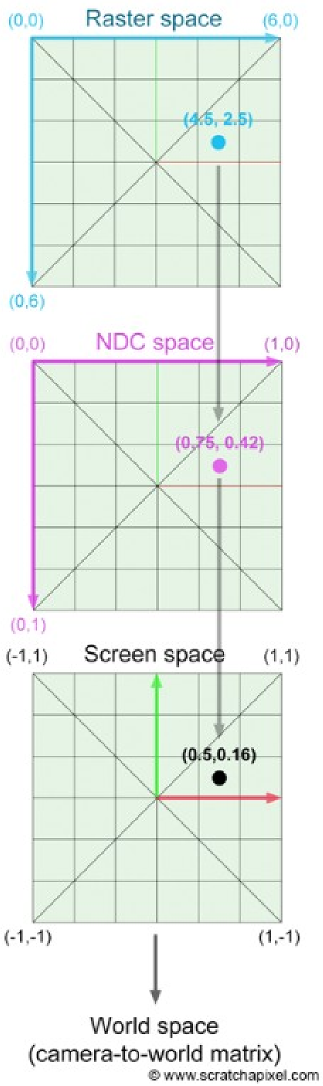
$$PixelNDC_x = \frac{(Pixel_x + 0.5)}{ImageWidth},$$

$$PixelNDC_y = \frac{(Pixel_y + 0.5)}{ImageHeight}.$$

- Finally, pixels are transformed from NDC to **screen space**

$$PixelScreen_x = 2 * PixelNDC_x - 1,$$

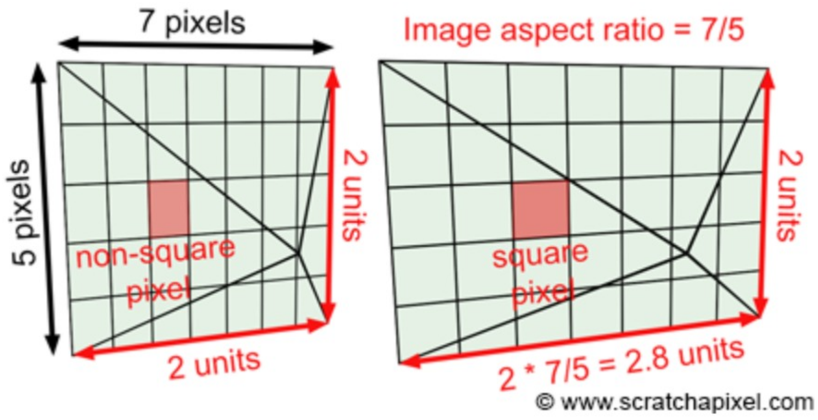
$$PixelScreen_y = 1 - 2 * PixelNDC_y.$$



Pixel center coordinates: arbitrary image aspect ratio

- To ensure square pixels for image arbitrary aspect ratio, use image aspect ratio to scale frame size.

$$ImageAspectRatio = \frac{ImageWidth}{ImageHeight},$$
$$PixelCamera_x = (\quad PixelScreen_x \quad) * ImageAspectRatio,$$
$$PixelCamera_y = (\quad PixelScreen_y \quad).$$

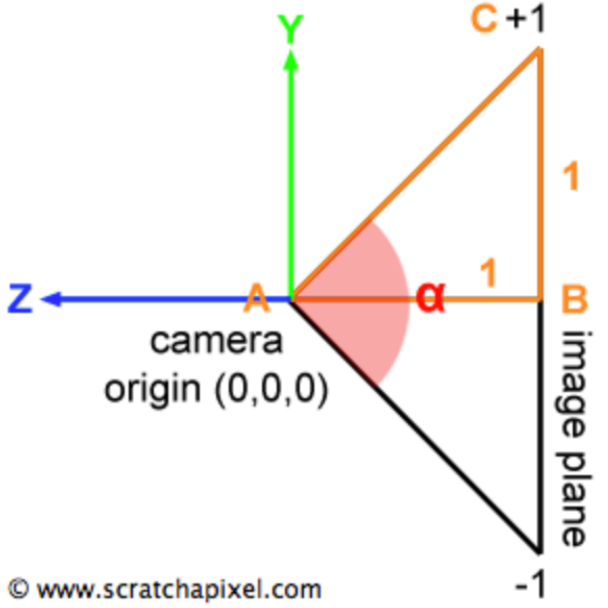


Pixel center coordinates: arbitrary field of view

- Field of view α defines how much we see or zoom level.
- To incorporate arbitrary field of view:

$$PixelCamera_x = (\quad PixelScreen_x \quad) * ImageAspectRatio * tan(\frac{\alpha}{2}),$$

$$PixelCamera_y = (\quad PixelScreen_y \quad) * tan(\frac{\alpha}{2}).$$

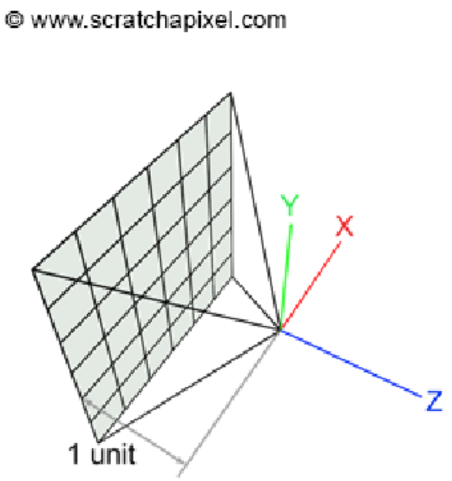


$$|||BC|| = tan(\frac{\alpha}{2}).$$

Pixel center coordinates: camera space

- **Now, pixel coordinates are expressed in camera coordinate system space**
  - Pixel coordinates are defined with regards to camera's image plane → missing 3rd coordinate
- Currently, **camera is in default position** (camera coordinate system is aligned with world coordinate system)
  - Camera origin O (aperture): `(0,0,0)`
  - Orientation: negative Z axis
  - Image plane: 1 unit away from camera's origin
- 3D Pixel coordinate position on the image plane in camera space

$$P_{cameraSpace} = (PixelCamera_x, PixelCamera_y, -1)$$



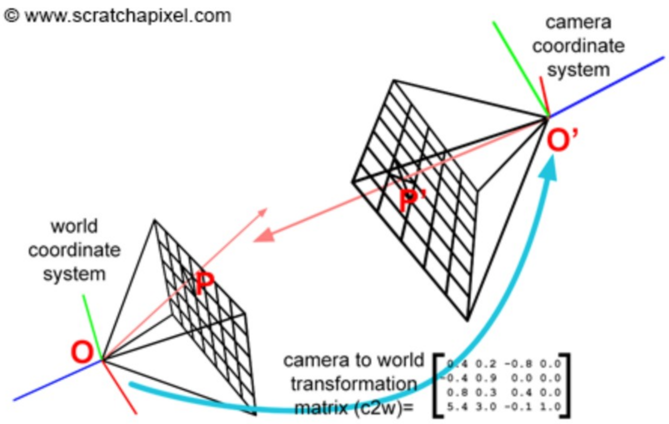
Generating camera rays: world space

- **Camera space ray** can be constructed using camera origin and pixel position in camera space

```
vector3 RayOrigin = cameraOrigin;
vector3 rayDirection =
    normalize(pixelPositionCamera - cameraOrigin);
```

- **World space ray** can be constructed using camera origin and pixel position in world space
  - `Camera-to-world matrix` is first applied applied on pixel position and camera origin
  - `Camera-to-world` can be constructed using `look-at` notation.

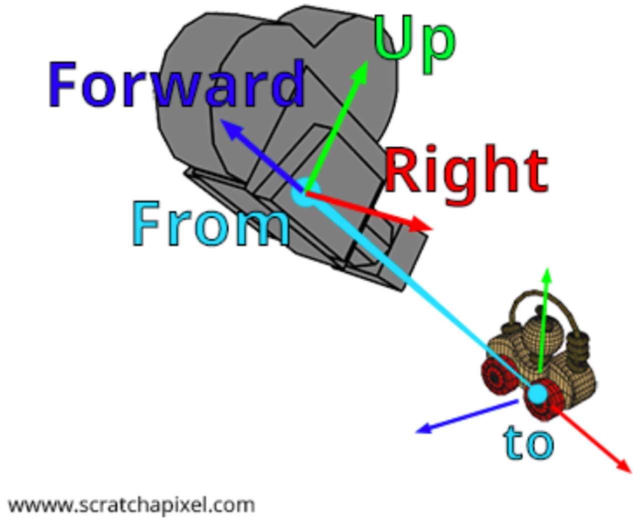
```
vector3 RayOrigin = cameraOriginWorld;
vector3 rayDirection =
    normalize(pixelPositionWorld - cameraOriginWorld);
```



Camera is originally set in its default position. Camera-to-world matrix is used to move camera origin and pixel position for generating world space rays.

Camera-to-world matrix

- **4x4 transformation matrix** → transformation of camera from its local (camera) space to world space in 3D scene: **camera-to-world** and its inverse **world-to-camera** matrix
- Can be constructed from **look-at notation**



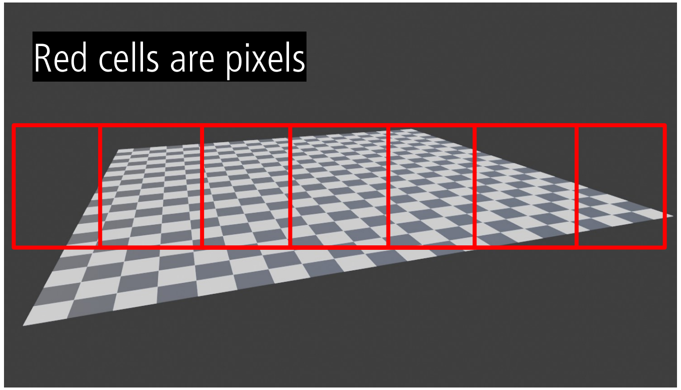

```
Forward = normalize(From - To)
Right = crossProduct(randomVec, Forward)
Up = crossProduct(forward, right)
```

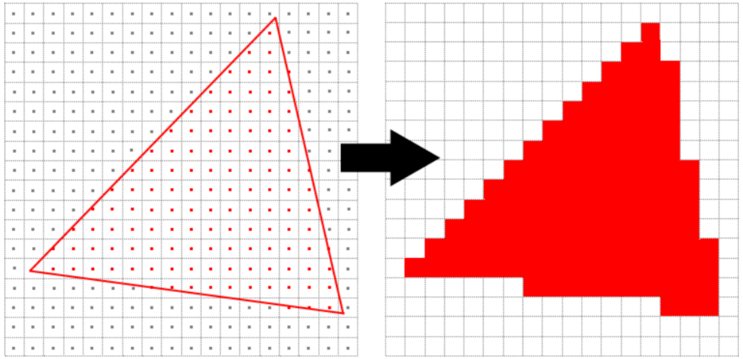
$$\begin{matrix} Right_x & Right_y & Right_z & 0 \\ Up_x & Up_y & Up_z & 0 \\ Forward_x & Forward_y & Forward_z & 0 \\ \mathtt{From}_x & \mathtt{From}_y & \mathtt{From}_z & 1 \end{matrix}$$

`RandomVec = (0,1,0)` or other if `Forward` is close to `(0,1,0) or (0,-1,0)`

Generating multiple camera rays per pixel

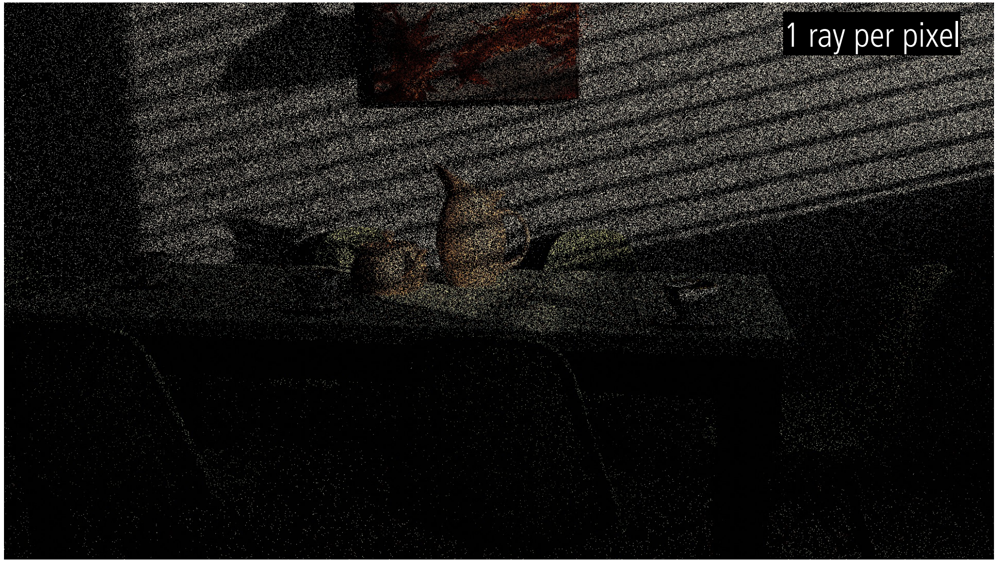
- **Pixel footprint in 3D scene can cover large area with different textures and objects**
  - **Problems: noise and aliasing**
- Since **pixel can represent only one color, it is important to use multiple rays** to obtain the color which is the most representative for the part of the scene covered by that pixel
  - Instead of pixel center, random points on pixel area are taken for building rays (samples per pixel SPP)





Generating multiple camera rays per pixel

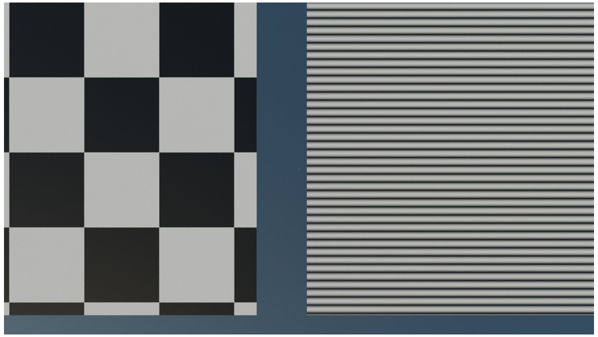
- Using multiple rays per pixels, **reduces noise**

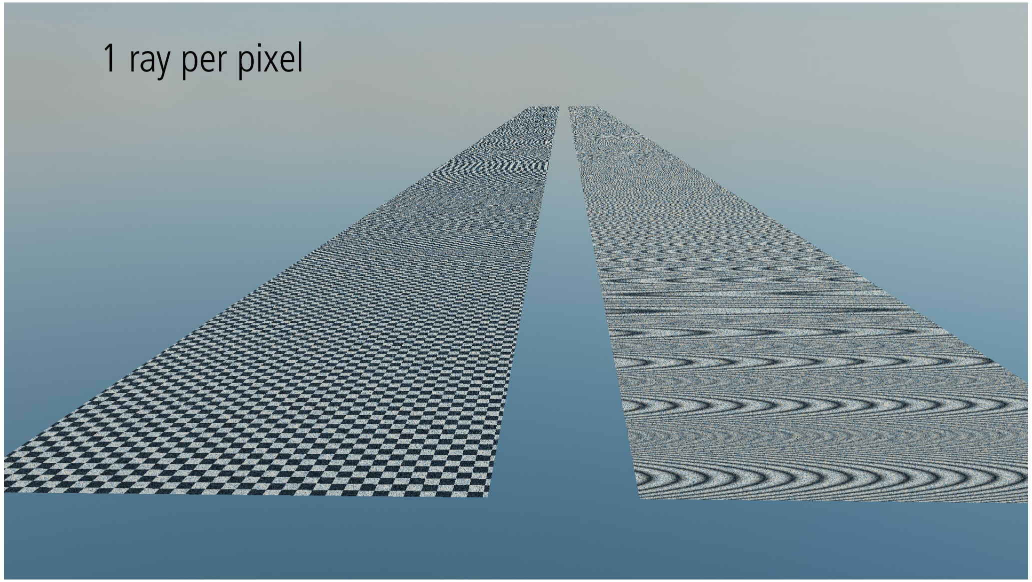


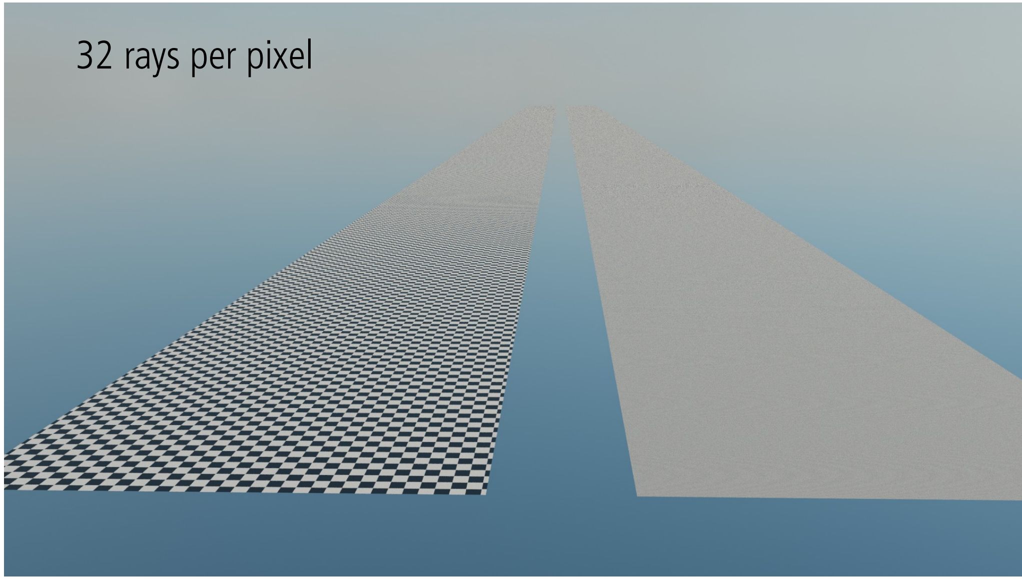
- Using multiple rays per pixel is especially important for methods building on raytracing and using stochastic sampling (e.g., Monte Carlo) such as path tracing.

Generating multiple camera rays per pixel

- Using multiple rays per pixels, **reduces aliasing**







Testing ray-object intersections

Ray-tracing: image centric method

- Since rendering starts from virtual image plane, ray-tracing is called **image centric approach**
- **Once camera rays are generated, they are traced into scene and tested for intersections with objects**
  - Looping over all objects in 3D scene is performed and each is tested for intersection with current ray → **visibility test**
  - Testing intersections with objects **depends on object shape, i.e., triangulated mesh**

for P **do** in pixels
    for T **do** in triangles
        determine if ray through P hits T
    end for
end for

Camera ray-objects intersection testing

```
…
Ray ray = buildCameraRay(i, j);
for  (int k = 0; k < nObjectsInScene; ++k)
{
    if (intersect(ray, objects[k], intersectionContext)
    {
        // Object hit. Compute shading using intersectionContext.
        framebuffer[j * imageWidth + i] = shadingResult;
    }
    else
    {
        // Background hit. Compute background color...
        framebuffer[j * imageWidth + i] = backgroundColor;
    }
}
...
```

- `intersect()` method depends on object shape
- `ShadingResult` depends on object material.
- That is why decoupling material and shape of 3D object is useful.

Testing ray-object intersections

- Objects in 3D scene can be represented with different **shape (geometry) representations**
  - Parametric representations
    - Spheres, disks, planes, etc.
    - Surfaces and curves (e.g., Bezier curves, NURBS, etc.)
  - Implicit surfaces SDFs: spheres, cubes, etc.
  - Polygonal meshes (e.g., triangles, quads) and subdivision surfaces
  - Voxels
  - Etc.
- We will discuss:
  - **Ray-sphere intersection**
  - **Ray-triangle intersection and its extension to triangulated meshes**

Ray-sphere intersection

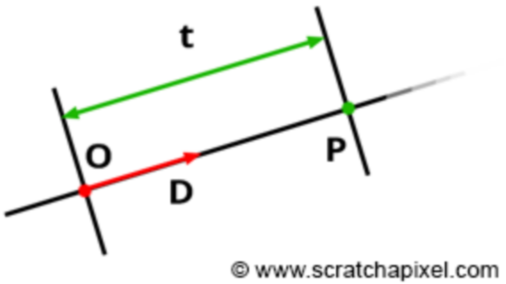
- Ray-sphere intersection is simplest ray-geometry intersection
- Parametric ray description: `P(t) = O + t * D`
- Implicit (algebraic) sphere form at world origin and radius R:

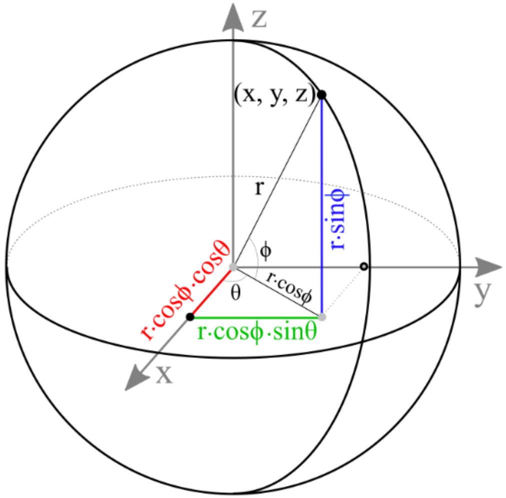
$$x^2 + y^2 + z^2 = R^2$$

  - `x,y,z` are coordinates of a point on a sphere

$$P^2 - R^2 = 0$$

  - Implicit function which defines implicit sphere shape
  - Equation is equal to zero if point `P` is on sphere





http://www.songho.ca/opengl/gl_sphere.html

Ray-sphere intersection

- Start with $P^2 - R^2 = 0$ and $P(t) = O + t * D$
- **Substitute P with ray equation**: $|O + t * D|^2 - R^2 = 0$
  - Develop: $D^2t^2 + 2ODt + O^2 - R^2 = 0$
- Quadratic equation: $f(x) = ax^2 + bx + c$:
  - $a = D^2$, $b = 2OD$, $c = O^2 - R^2$. Solution:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

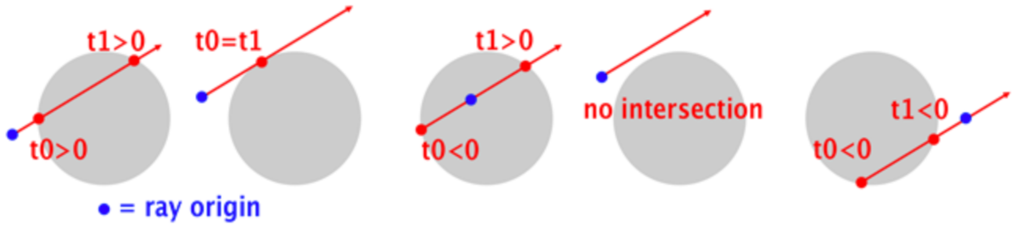
  - $\Delta > 0$: ray intersects sphere in two points (t_0 and t_1):

$$\frac{-b + \sqrt{\Delta}}{2a} \quad and \quad \frac{-b - \sqrt{\Delta}}{2a}$$

  - $\Delta = 0$: ray intersects sphere in one point ($t_0 = t_1$):

$$-\frac{b}{2a}$$

  - $\Delta < 0$: ray doesn't intersect the sphere



Ray-sphere intersection: arbitrary sphere position

- If sphere is translated from origin to point C, then:
  - $|P - C|^2 - R^2 = 0$
- Substituting the ray equation:
  - $|O + t * D - C|^2 - R^2 = 0$
- Solving quadratic equation gives t_0 and t_1:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = D^2 = 1$ (ray direction D is normalized)

$b = 2D(O - C)$

$c = |O - C|^2 - R^2$

Ray-sphere intersection: intersection point and intersection context

- t_0, when inserted into ray equation `P(t) = O + t * D` gives closest **intersection point** of ray with a sphere

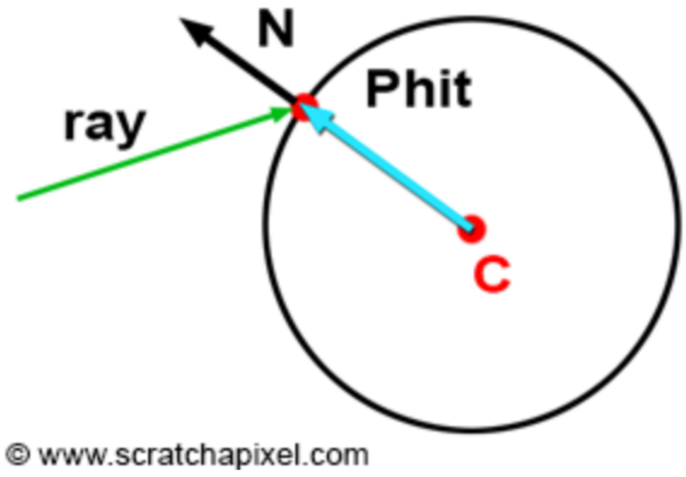
  `vector3 Phit = O + D * t0`

- Next to intersection point `Phit`, renderer often computes additional intersection information – **intersection context:**
  - **Normal** in intersection point
  - **Texture coordinate** in intersection point
  - Object/triangle index → material assigned to this object
  - Etc.
- **Intersection context computation depends on object shape**. Example: texture coordinates
  - For parametric surfaces (u,v) parameters can be used as texture coordinates
  - For polygonal (e.g., triangle) mesh, texture coordinates must be precomputed and stored per vertex. Intersection point is used to interpolate texture coordinates per triangle face (e.g., barycentric interpolation)
- **Intersection context contains information which is used for shading of intersection point**

Ray-sphere intersection: normal

- Normal calculation in intersection point depends on shape representation
- For implicit sphere with center `C(x,y,z)`

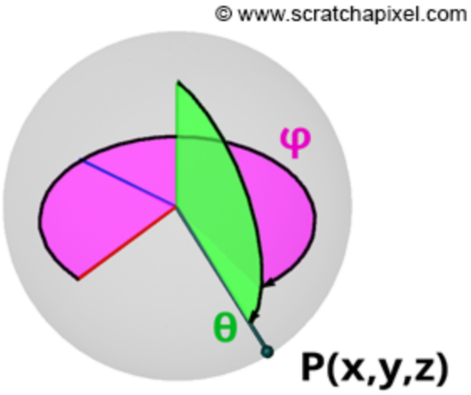
```
vector3 N = normalize(Phit - C)
```



Ray-sphere intersection: texture coordinates

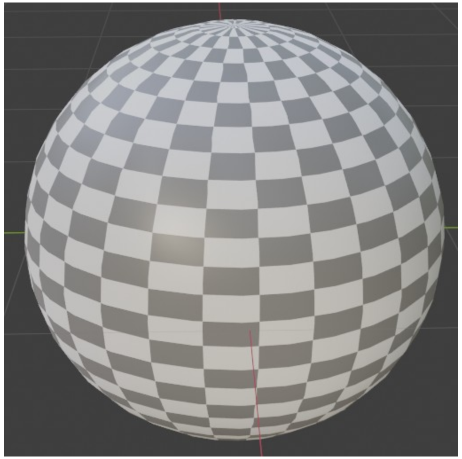
- Sphere can be written in parametric form:

$$
\begin{aligned}
P.x &= \cos(\theta)\sin(\phi),\\
P.y &= \cos(\theta),\\
P.z &= \sin(\theta)\sin(\phi).
\end{aligned}
$$



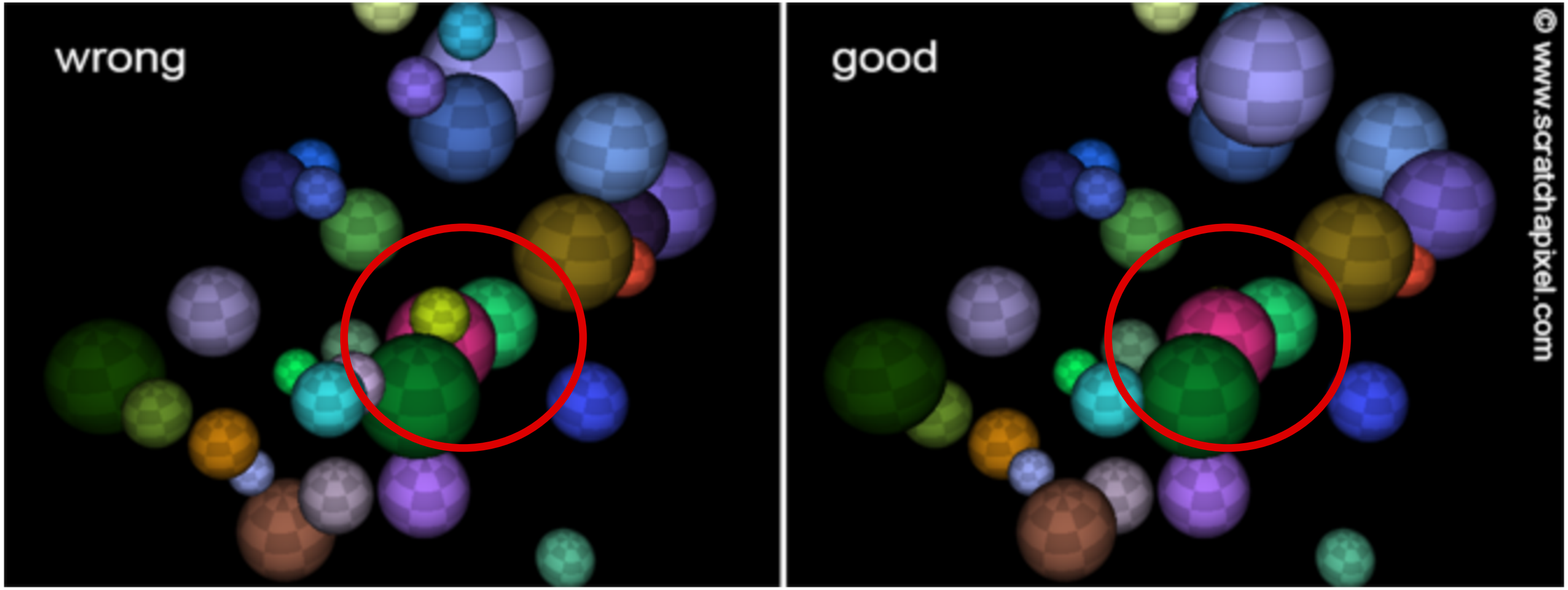
- Texture coordinates for sphere are simply spherical coordinates:

$$
\begin{aligned}
\phi &= atan(z, x),\\
\theta &= acos(\frac{P.y}{R}).
\end{aligned}
$$



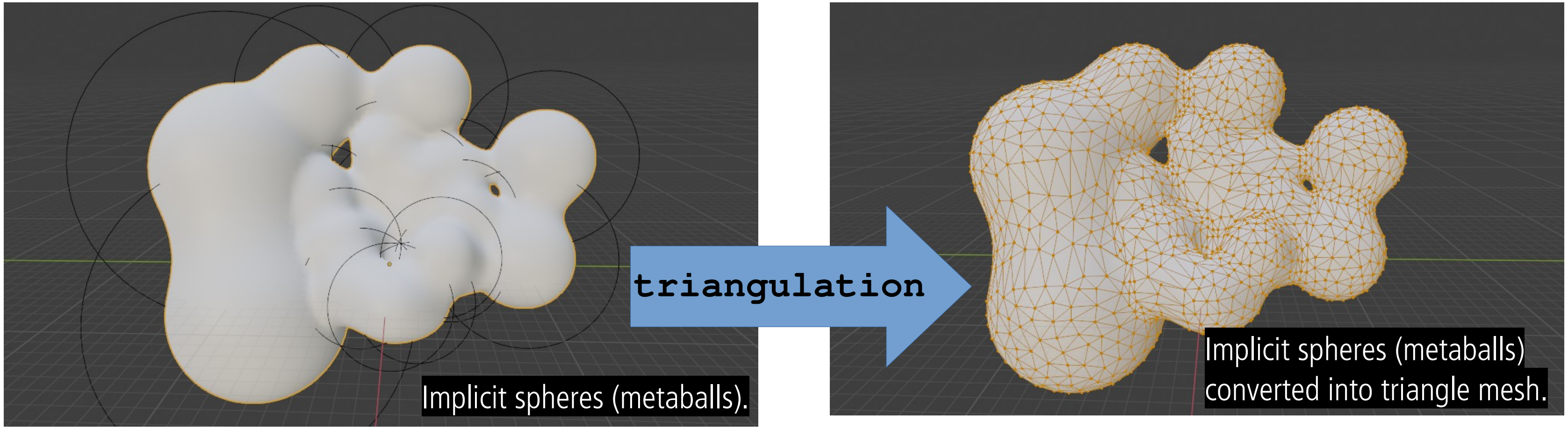
Ray-sphere intersection: intersection order

- If scene contains multiple objects then certain **depth sorting** is needed
  - Keep track of closest intersection distance, closest `t`, and use this value for comparison when testing intersection with new object



Intersecting other shape representations

- Define ray-shape intersection for each representation separately (e.g., triangulated mesh and parametric curves)
- Alternative solution: **convert each shape representation to same internal representation which is used for rendering**
  - Professional rendering software often work on **triangulated mesh** which is obtained using **triangulation**



Implicit spheres (metaballs).

Implicit spheres (metaballs) converted into triangle mesh.

Tessellation and triangulated mesh

- **Two sides of the computer graphics/image generation: authoring of 3D scene and rendering**
  - Some representations are much more easier and efficient to handle on authoring level
  - Some are much more efficient to handle for rendering purposes
  - Therefore, efficient mapping of those representations quite important and researched.
- **Renderer working with single primitive is much more efficient than supporting various primitives**
- **Why triangles?**
  - Can approximate any surface and shape well
  - Conversion of almost any type of surface to a triangulated mesh (tessellation) is well researched and feasible
  - Triangulated mesh is also basic rendering primitive for rasterization-based renderers
  - Graphics hardware is adapted and optimized to efficiently process triangles
  - Triangles are necessary co-planar which makes various computations, such as ray-triangle, much easier
  - Lot of research was devoted to efficient computation of ray-triangle intersection: https://www.realtimerendering.com/intersections.html
  - For triangles we can easily compute barycentric coordinates which are essential to shading
- **Tessellation process can be done after modeling of shape is done and when exporting takes place or it can be done during rendering (render time)**

Ray-triangle intersection test

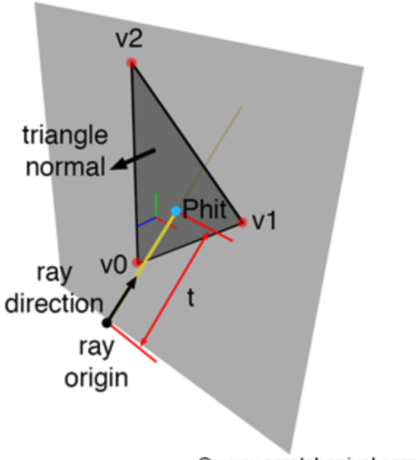
- **Basic ray-triangle intersection is straight forward**
- **Complexity comes due to multitude of different cases** which must be taken in account
  - Various ray-triangle intersection tests exist: https://www.realtimerendering.com/intersections.html
- **Ray-triangle intersection testing steps**:
  1. Does ray **intersect a plane** defined by a triangle?
  2. Does ray **intersect point inside the triangle**?

Ray-triangle intersection tools

- **Triangles are coplanar**: vertices are lying on a plane and plane can be defined with those vertices
- **Using triangle vertices, normal can be computed**
  - Plane on which triangle lies has the same normal

```
vector3 a = v1 - v0
vector3 b = v2 - v0
vector3 c = cross(a, b)
vector3 normal = normalize(c)
```

  - Winding order of vertices defines normal and thus **surface orientation – important for shading!**



© www.scratchapixel.com

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$
$$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

https://en.wikipedia.org/wiki/Cross_product

Ray-triangle intersection test: intersecting plane

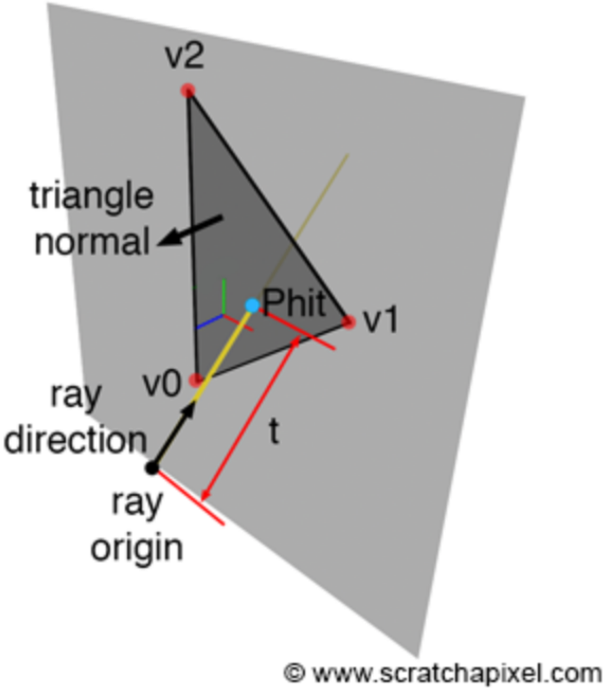
- Intersected point is somewhere on ray:
  - `Phit = P(t) = O + t * R`
- Plane equation:

  `Ax + By + Cz + D = 0`

  `D = -(Ax + By + Cz)`

  - `A, B, C` are coordinates of plane normal `N = (A,B,C)`
  - `D` is distance from origin `(0,0,0)` to the plane
  - `x, y, z` are coordinates of point on a plane
- Normal `N` can be calculated using triangle vertices
- D can be calculated using any triangle vertex `v` and normal `N`:

  `D = dotProduct(N, v0) = -(N.x * v0.x + N.y * v0.y + N.z * v0.z);`



Ray-triangle intersection: intersecting plane

- Substitute ray equation `P(t) = O + t * R` into plane equation `Ax + By + Cz + D = 0`:

  `A * P.x + B * P.y + C * Pz + D = 0`

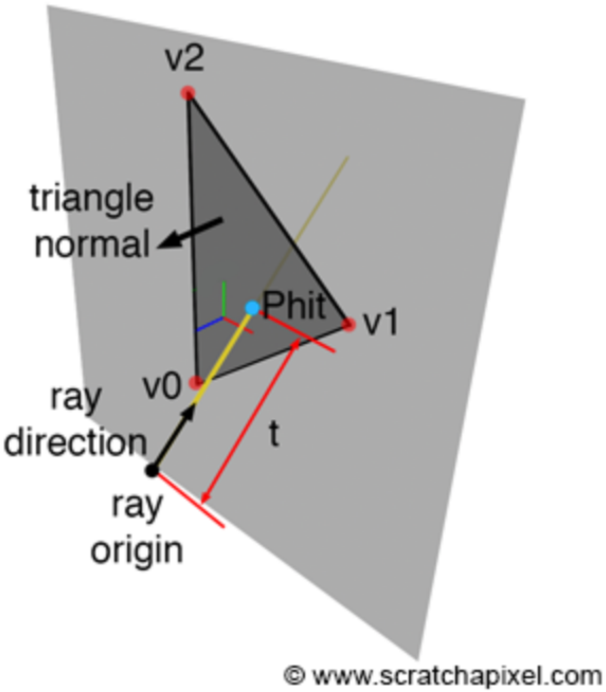
  `A * (O.x + t * R.x) + B * (O.y + t * R.y) + C * (O.z + t * R.z) + D = 0`

- Solving by `t`:

$$t = -\frac{N(A,B,C) \cdot O + D}{N(A,B,C) \cdot R}$$

- Finally, intersection point of ray and plane:

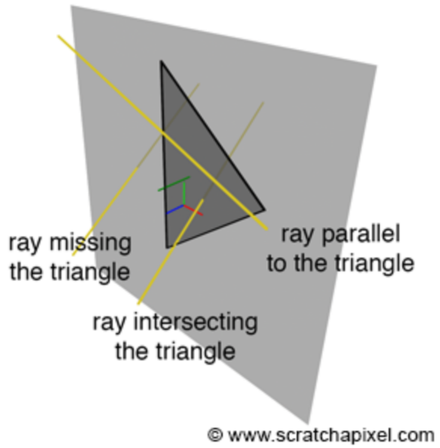
```
float t = - (dot(N, O) + D) / dot(N, R)
Vector3 Phit = O + t * R
```

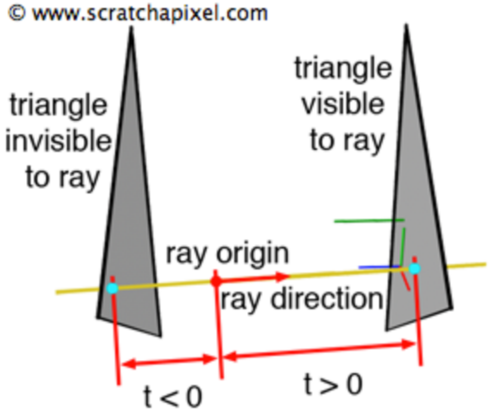


Ray-triangle intersection: intersecting plane

- Special cases of non-intersection:
  - **Ray and triangle (plane) are parallel**
    - Triangle's normal and ray direction are perpendicular

      `dot(N, R) = 0`
  - **Triangle is behind ray origin**
    - If `t < 0` → triangle behind ray origin. Else, triangle is visible

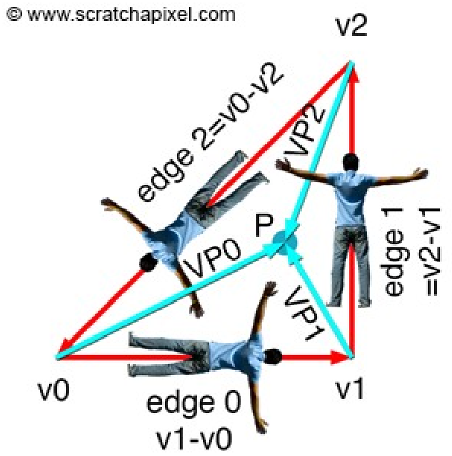




Ray-triangle intersection: point inside triangle?

- We have found intersection point `P` on plane. Is it inside triangle?
- **Inside-out test**

```
vector3 edge0 = v1 - v0;
vector3 edge1 = v2 - v1;
vector3 edge2 = v0 - v2;
vector3 C0 = P - v0;
vector3 C1 = P - v1;
vector3 C2 = P - v2;
bool q1 = dotProduct(N, crossProduct(edge0, C0)) > 0;
bool q2 = dotProduct(N, crossProduct(edge1, C1)) > 0;
bool q3 = dotProduct(N, crossProduct(edge2, C2)) > 0;
If (q1 and q2 and q3) then inside;
```



Inside-outside test:

- Using information of **vector along triangle edge** and **vector defined with first vertex of edge and intersection point** P test if intersection point P is on the left side of the edge.
- If P is on the left side of all three edges, then P is inside triangle

Ray intersection with triangle mesh

- We have a **routine to compute ray-triangle intersection**
- **To test ray intersection with object which is represented as triangulated mesh:**
  - **Loop over all triangles of triangulated mesh**
    - Test if ray intersects triangles of triangulated mesh
    - Respect depth sorting by keeping track of nearest object
- **For generated camera ray, we can write `intersect()` function which:**
  - Takes triangulated meshes and ray
  - Returns information on intersection
  - Return information on intersection context

https://www.scratchapixel.com/lessons/3d-basic-rendering/ray-tracing-polygon-mesh/ray-tracing-polygon-mesh-part-1.html

Testing ray intersections: `intersect()` function

```
bool intersect (Ray ray, Object objects, &intersectionContext)
{
    bool intersected = false;
    for (int k = 0; k < objects.nObjects; ++k)
    {
        for (int n = 0; n < objects[k].nTriangles; ++n)
        {
            if(rayTriangleIntersect(ray, objects[k].triangle[n])
            {
                intersected = true;
                IntersectionContext.objIdx = k;
                IntersectionContext.triIdx = n;
                IntersectionContext.N = N;
                ...
            }
        }
    }
    return intersected;
}
```

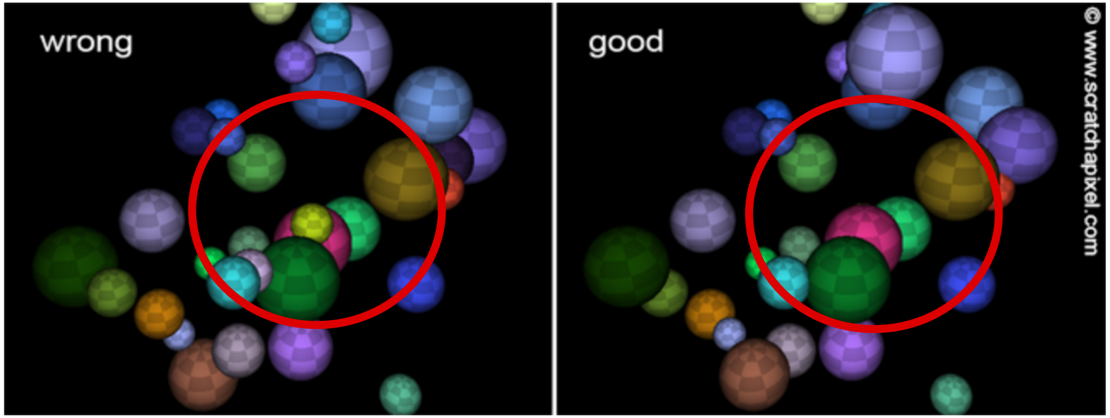
Intersect function can be used for:

- finding intersections between camera rays and objects → **camera visibility**
- Finding intersections between secondary rays and objects → **light transport**

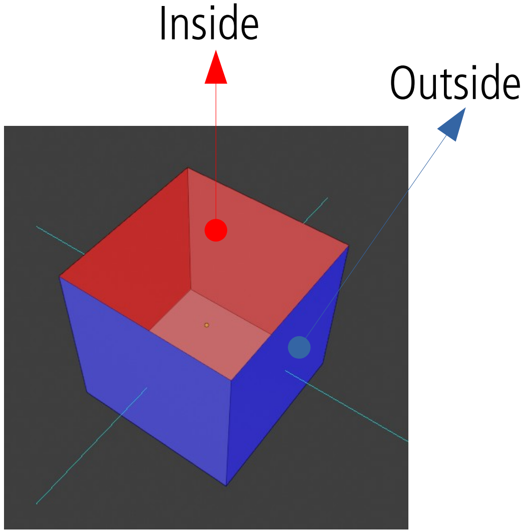
`intersect()`: depth sorting

```
bool intersect (Ray ray, Object objects, &intersectionContext, &tNearest)
{
    bool intersected = false;
    tnearest = INFINITY;
    for (int k = 0; k < objects.nObjects; ++k)
    {
        for (int n = 0; n < objects[k].nTriangles; ++n)
        {
            if(rayTriangleIntersect(ray, objects[k].triangle[n], t) and t < tNearest)
            {
                intersected = true;
                IntersectionContext.objIdx = k;
                IntersectionContext.triIdx = n;
                tNearest = t;
            }
        }
    }
    Return intersected;
}
```

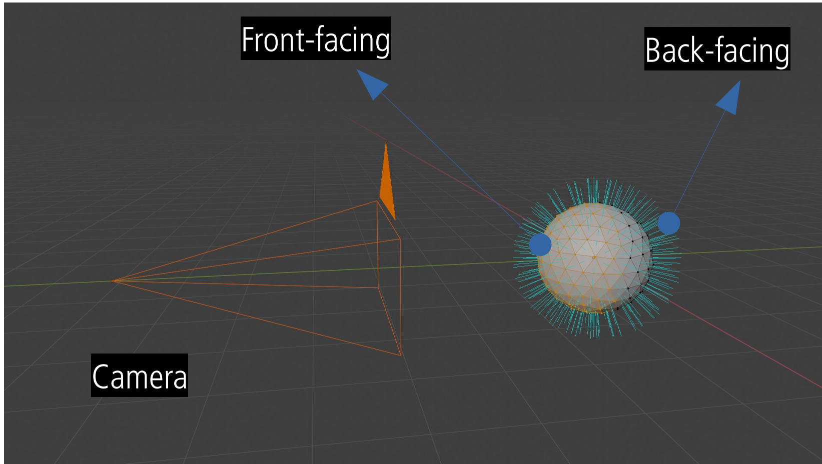
- Ray may intersect several triangles.
- Keep track of closest intersection and update it with each intersection



Triangle (surface) orientation

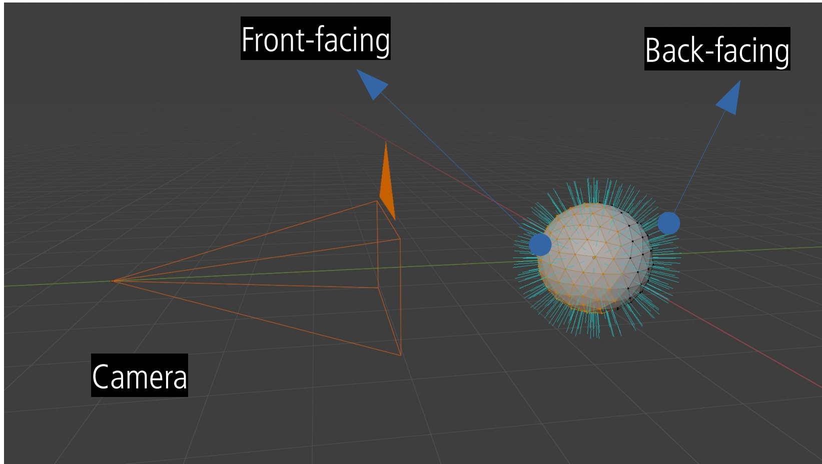


- **Winding order of triangle vertices and handedness of coordinate system defines normal orientation** → surface orientation
  - Surface normal pointing outward → **outside surface**
  - Surface normal pointing inward → **inside surface**
- Based on camera view, triangle can be:
  - **Front-facing**: if outside surface is facing camera
  - **Back-facing**: if outside surface is not facing camera



Single and double sided triangle

- **During rendering, that is visibility computation**, it is possible to specify:
  - **Single-sided triangles**: only visible if are front-facing
  - **Double-sided triangles**: visible for both front-facing and back-facing
- **Back-face culling**: discarding back-facing triangles during rendering
  - For casting shadows, back-face culling can not be used.
  - Test: `dotProduct(ray.direction, N) > 0`



`intersect()` function: intersection context

- `intersect()` function must return information **if ray intersects object** (triangle) as well as **intersection context**:
  - **Object index of intersected triangle**: `objIdx` → `object material information`
  - **Triangle index of intersected triangle**: `triIdx` → `triangle geometry and/or material information`
  - **Ray parameter `t` for nearest intersection**: `tNearest`
  - **Intersection point coordinates**
  - **Intersection point normal**
  - **Intersection point texture coordinates**
  - etc.
  - Information of `objIdx`, `triIdx` and `tNearest` can be used to compute the rest of intersection context information
- **Often, triangulated meshes contain data stored per vertices → vertex attributes**
  - **Barycentric interpolation is used to obtain value for any intersection point from vertex attributes**
- **Intersection context information will be used for shading the intersected point**

Recap: barycentric interpolation

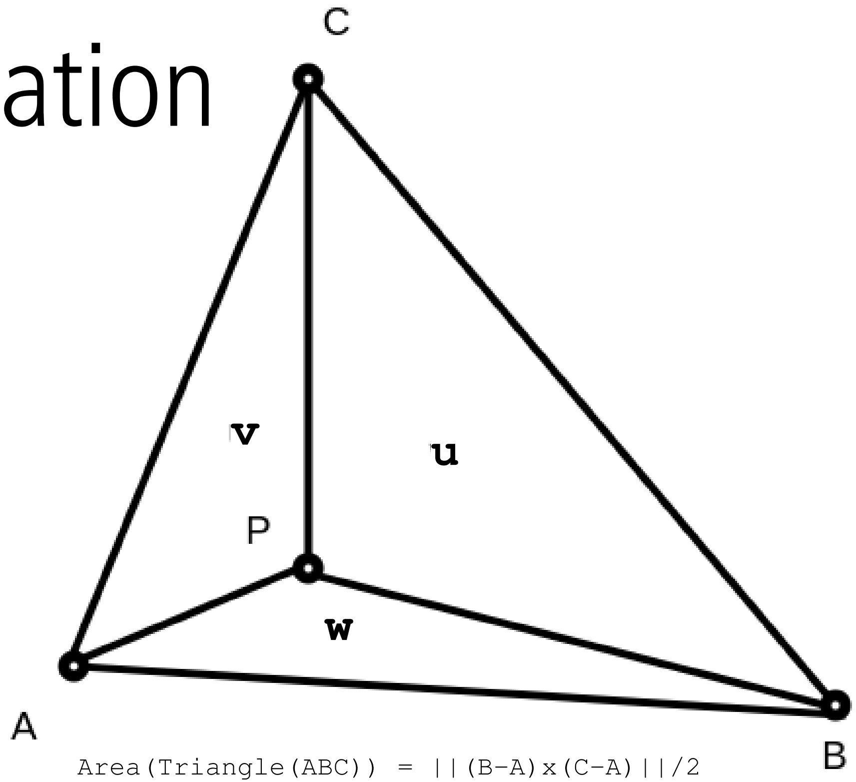
- **Any point `P` on triangle can be described with barycentric coordinates `u,v,w`:**

  `P = uA + vB + wC`, where `A,B,C` are triangle vertices

  `u + v + w = 1`

  If `0 <= u,v,w <=1` then `P` inside or on triangle edge. Otherwise `P` is outside of triangle.

- **Barycentric coordinates: `u,v,w` are proportional to area of sub-triangles defined by P**
- **To interpolate vertex attributes:**
  - Using `P, A,B,C` calculate `u,v,w`
  - Interpolate vertex arbitrary attribute in point `P`
    - $P_{attrib} = A_{attrib} * u + B_{attrib} * v + C_{attrib} * w$



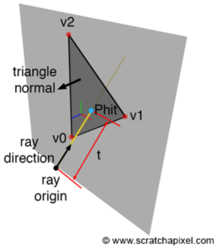

```
Area(Triangle(ABC)) = ||(B-A)x(C-A)||/2

u = Area(Triangle(CAP)) / Area(Triangle(ABC))
v = Area(Triangle(ABP)) / Area(Triangle(ABC))
w = Area(Triangle(BCP)) / Area(Triangle(ABC))
```

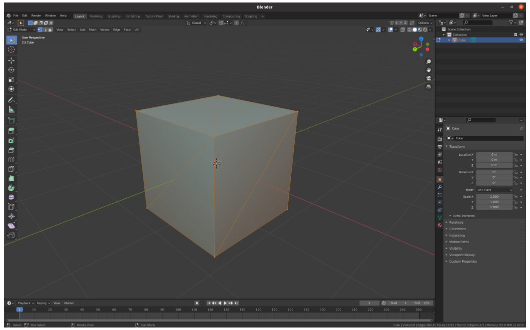
Intersection context: normal

- **Flat normal in intersection point can be computed using triangle vertices:**

```
vector3 a = v1 – v0
vector3 b = v2 – v0
vector3 c = cross(a, b)
vector3 normal = normalize(c)
```



- **Smooth normals can be created when modeling triangulated mesh:**
  - Those are stored per vertex
  - To obtain smooth normal in intersection point, barycentric interpolation can be performed using intersection point and triangle vertices

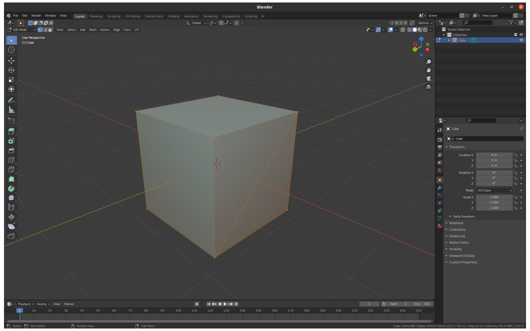


```
# Blender v2.92.0 OBJ File: ''
# www.blender.org
o Cube_Cube.002
v -1.000000 -1.000000 1.000000
v -1.000000 1.000000 1.000000
v -1.000000 -1.000000 -1.000000
v -1.000000 1.000000 -1.000000
v 1.000000 -1.000000 1.000000
v 1.000000 1.000000 1.000000
v 1.000000 -1.000000 -1.000000
v 1.000000 1.000000 -1.000000
vt 0.625000 0.000000
vt 0.375000 0.250000
vt 0.375000 0.000000
vt 0.625000 0.250000
vt 0.375000 0.500000
vt 0.625000 0.500000
vt 0.375000 0.750000
vt 0.625000 0.750000
vt 0.375000 1.000000
vt 0.125000 0.750000
vt 0.125000 0.500000
vt 0.875000 0.500000
vt 0.625000 1.000000
vt 0.875000 0.750000
vn -1.0000 0.0000 0.0000
vn 0.0000 0.0000 -1.0000
vn 1.0000 0.0000 0.0000
vn 0.0000 0.0000 1.0000
vn 0.0000 -1.0000 0.0000
vn 0.0000 1.0000 0.0000
s off
f 2/1/1 3/2/1 1/3/1
f 4/4/2 7/5/2 3/2/2
f 8/6/3 5/7/3 7/5/3
f 6/8/4 1/9/4 5/7/4
f 7/5/5 1/10/5 3/11/5
f 4/12/6 6/8/6 8/6/6
f 2/1/1 4/4/1 3/2/1
f 4/4/2 8/6/2 7/5/2
f 8/6/3 6/8/3 5/7/3
f 6/8/4 2/13/4 1/9/4
f 7/5/5 5/7/5 1/10/5
f 4/12/6 2/14/6 6/8/6
```

Intersection context: texture coordinates

- Texture coordinates for each vertex of triangulated meshes are created during mesh modeling using:
  - **Mesh unwrapping**
  - **Texture projections**, e.g., spherical, cylindrical, triplanar, etc.
- To compute texture coordinate in intersection point, barycentric interpolation is used
- Note: texture projections can be also used to create texture coordinates on the fly
  - **Texture projections**, e.g., spherical, cylindrical, triplanar, etc.

```
# Blender v2.92.0 OBJ File: ''
# www.blender.org
o Cube_Cube.002
v -1.000000 -1.000000 1.000000
v -1.000000 1.000000 1.000000
v -1.000000 -1.000000 -1.000000
v -1.000000 1.000000 -1.000000
v 1.000000 -1.000000 1.000000
v 1.000000 1.000000 1.000000
v 1.000000 -1.000000 -1.000000
v 1.000000 1.000000 -1.000000
vt 0.625000 0.000000
vt 0.375000 0.250000
vt 0.375000 0.000000
vt 0.625000 0.250000
vt 0.375000 0.500000
vt 0.625000 0.500000
vt 0.375000 0.750000
vt 0.625000 0.750000
vt 0.375000 1.000000
vt 0.125000 0.750000
vt 0.125000 0.500000
vt 0.875000 0.500000
vt 0.625000 1.000000
vt 0.875000 0.750000
vn -1.0000 0.0000 0.0000
vn 0.0000 0.0000 -1.0000
vn 1.0000 0.0000 0.0000
vn 0.0000 0.0000 1.0000
vn 0.0000 -1.0000 0.0000
vn 0.0000 1.0000 0.0000
s off
f 2/1/1 3/2/1 1/3/1
f 4/4/2 7/5/2 3/2/2
f 8/6/3 5/7/3 7/5/3
f 6/8/4 1/9/4 5/7/4
f 7/5/5 1/10/5 3/11/5
f 4/12/6 6/8/6 8/6/6
f 2/1/1 4/4/1 3/2/1
f 4/4/2 8/6/2 7/5/2
f 8/6/3 6/8/3 5/7/3
f 6/8/4 2/13/4 1/9/4
f 7/5/5 5/7/5 1/10/5
f 4/12/6 2/14/6 6/8/6
```

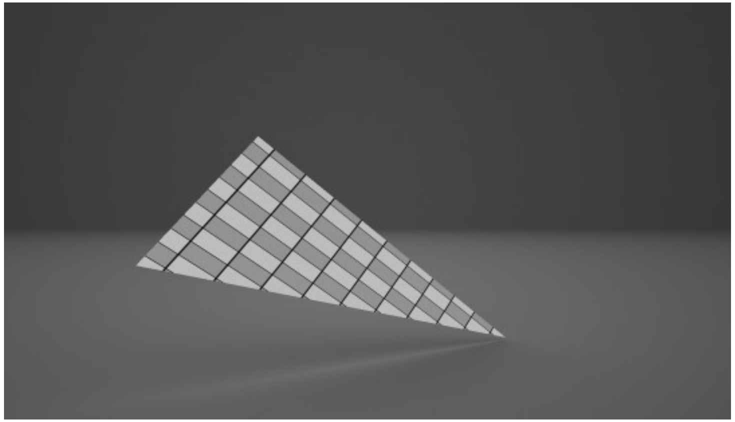


Testing intersections: complexity

- **Complexity of ray-tracing algorithm depends on:**
  - **Number of pixels of virtual image plane**, e.g., 1024x768 → number of camera rays
    - Note that often multiple rays per pixel are used, e.g., 256
  - **Number of objects in the scene**
- Each **camera ray** must be tested for intersection with every object
- Every **secondary and shadow rays** must be tested for intersection with every object
  - Additional rays are generated per secondary ray intersections → recursion

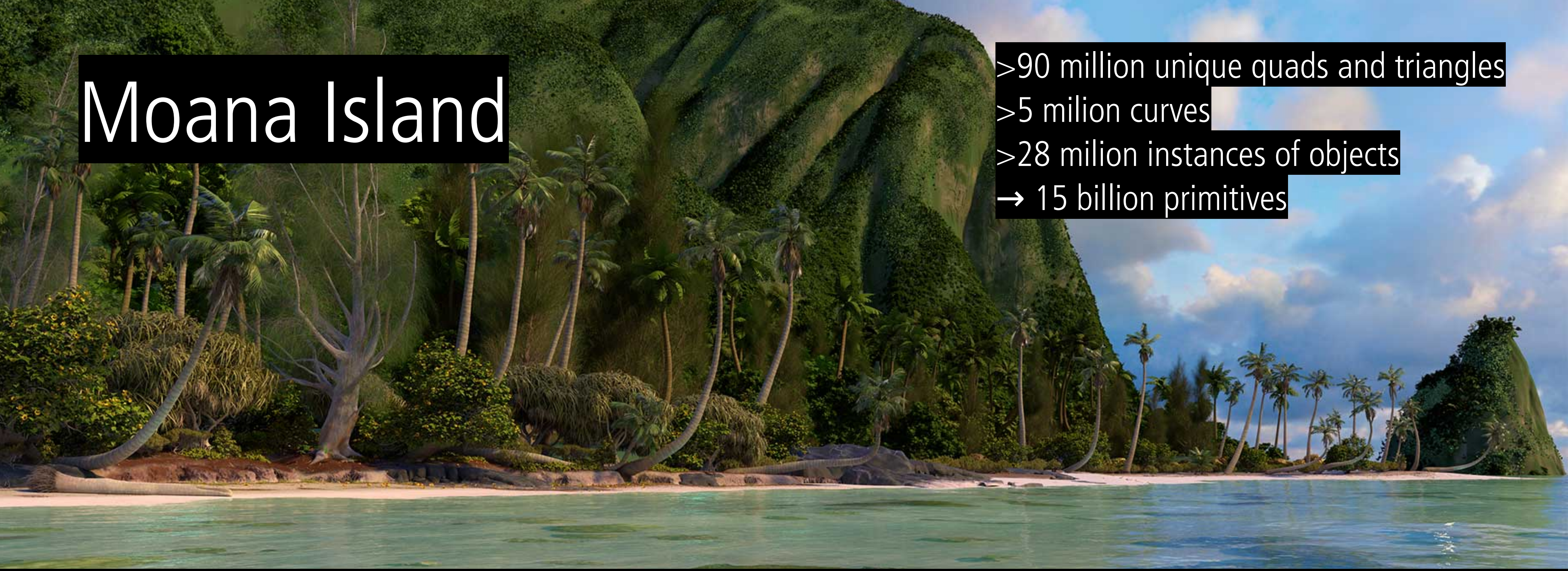
Testing intersections: efficiency

- Time to render a scene is (directly) proportional to the number of triangles in the scene
  - For every camera ray, looping over all objects, that is triangles is needed
- For shading purposes, all triangles must be stored into memory and each must be tested for intersection for ray
- Therefore, various efficient ray-triangle and other ray-object intersection tests exist:
  - Möller-Trumbore method: https://www.tandfonline.com/doi/abs/10.1080/10867651.1997.10487468
  - Philip Dutré and Ares Lagae: https://www.tandfonline.com/doi/abs/10.1080/2151237X.2005.10129208
  - Marta Löfstedt and Tomas Akenine-Möller: https://www.tandfonline.com/doi/abs/10.1080/2151237X.2005.10129195
  - Inigo Quilez: https://www.shadertoy.com/view/MlGcDz
  - More: https://www.realtimerendering.com/intersections.html



Moana Island

>90 million unique quads and triangles
>5 milion curves
>28 milion instances of objects
→ 15 billion primitives

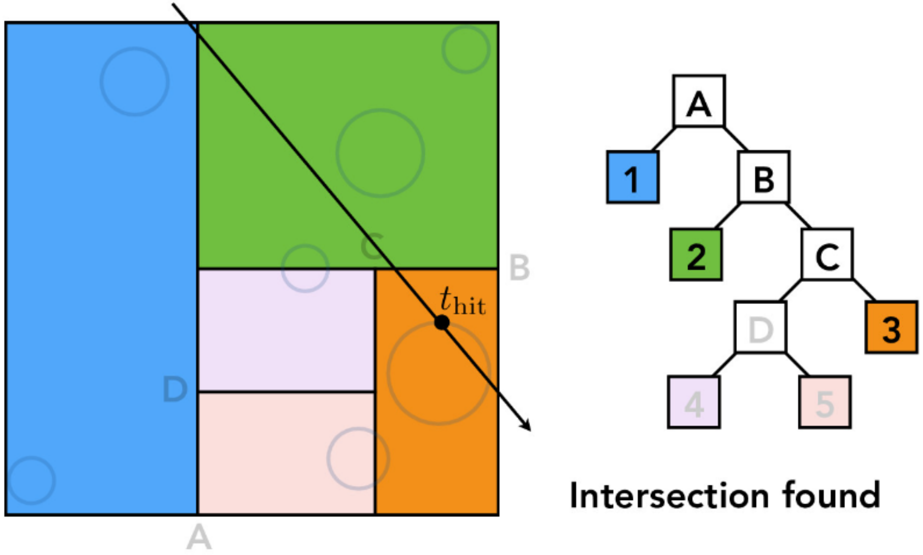


https://disneyanimation.com/resources/moana-island-scene/



Accelerating intersection testing

- If certain parts of the scene are not relevant for intersecting with current ray, they can be skipped
- **Spatial acceleration data structures**
  - 3D scene and scene is spatially subdivided
  - Now larger volumes of the scene can be tested for intersection
  - If ray is not intersecting volume, all objects inside do not have to be tested for intersections
- Acceleration structures require **pre-computation overhead**:
  - **Static scenes**: only once before rendering
  - **Animated scenes**: each frame, each time objects move

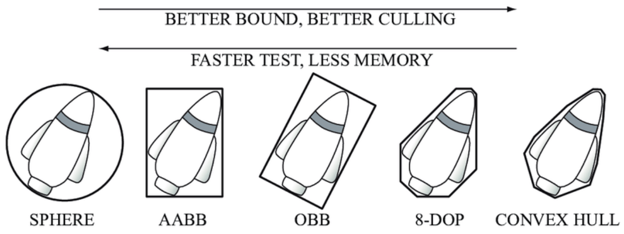


https://cs184.eecs.berkeley.edu/sp22/lecture/10/ray-tracing-acceleration

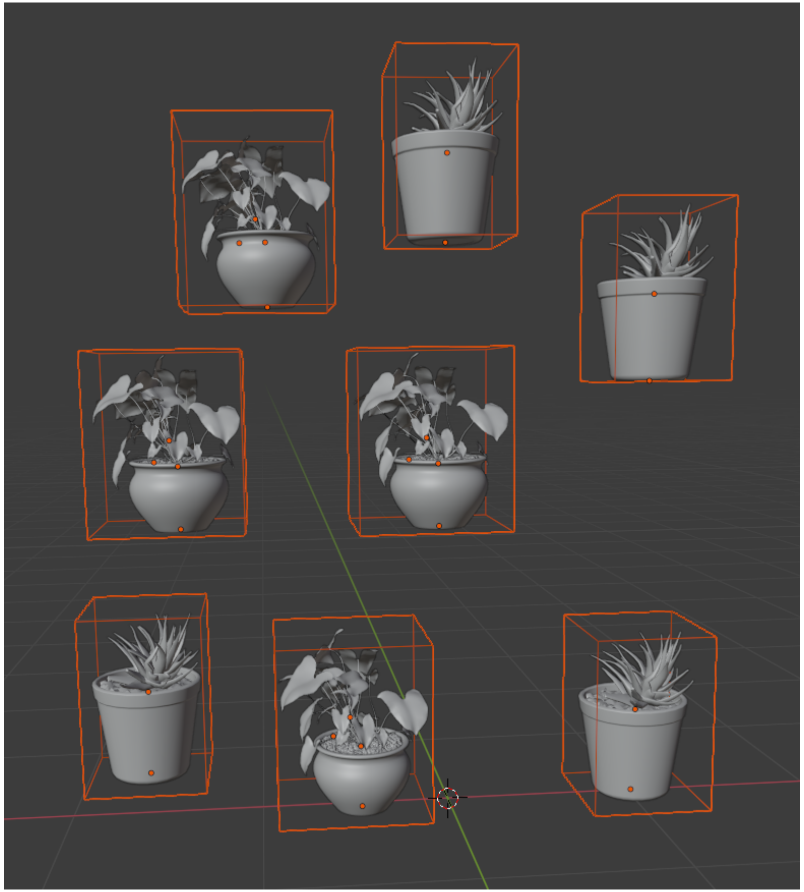
Spatial acceleration data structures

- **Bounding volumes**
  - **Bounding box**, sphere, etc.
- **Hierarchical spatial data structures**
  - **Bounding volume hierarchy (BVH)**
  - **Binary space partitioning (BSP) tree**

Bounding volume

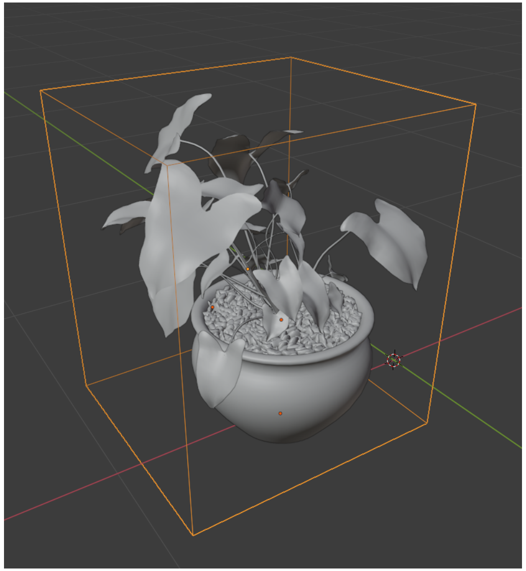


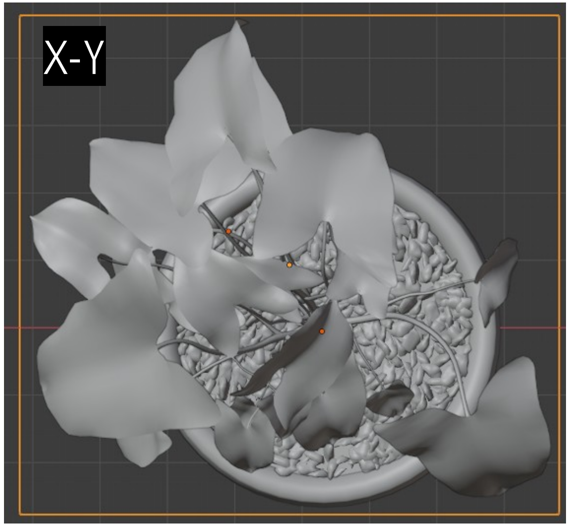
- Bounding volume: tightest possible volume (e.g., box or sphere) surrounding object
- Idea behind bounding volume:
  - For all objects in 3D scene, first test bounding volume intersection with ray
    - If ray doesn't intersects volume, then skip complete object inside
    - If ray intersects volume, loop over object triangles and test for intersection
- Testing intersection between ray and bounding volume is simple:
  - Sphere bounding volume: ray-sphere intersection
  - Box bounding volume: box is triangulated mesh → ray-triangle intersection



Bounding box

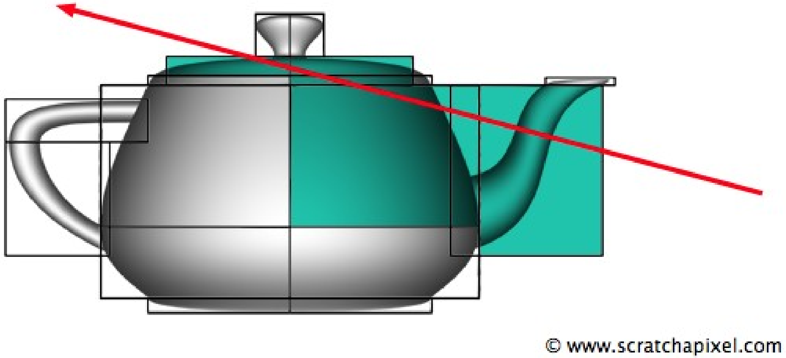
- **Creating bounding box for any object is simple:**
  - Loop over all vertices of object mesh and find minimum and maximum value for each vertex `(x,y,z)`
  - Found minimum and maximum coordinates are bounding box corners
- Building bounding box can be **constructed before rendering and and reused**

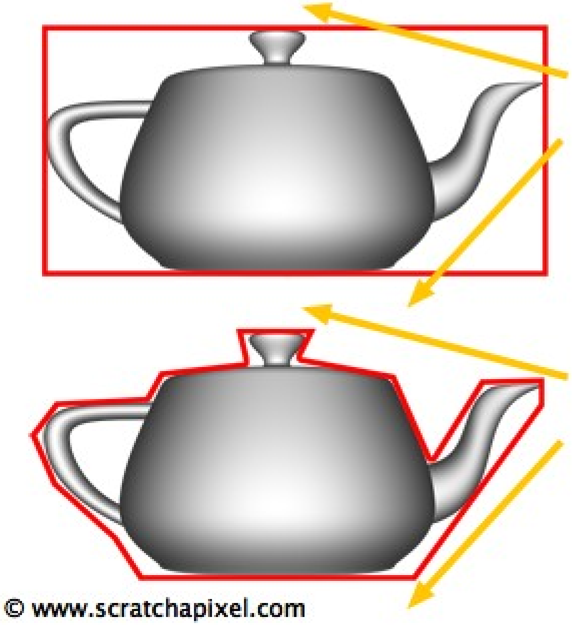




Bounding volume optimization

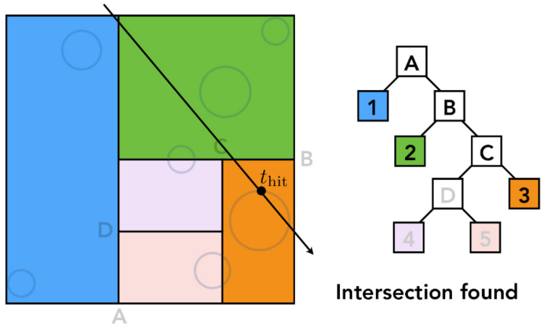
- **Single bounding volume (e.g., box) may fit object too loosely**
  - Ray that intersect bounding box will often miss object
- **Bounding extent can be created out of multiple bounding volumes to fit objects more closely**
  - Gives more correct intersection results but it is more costly to ray-trace



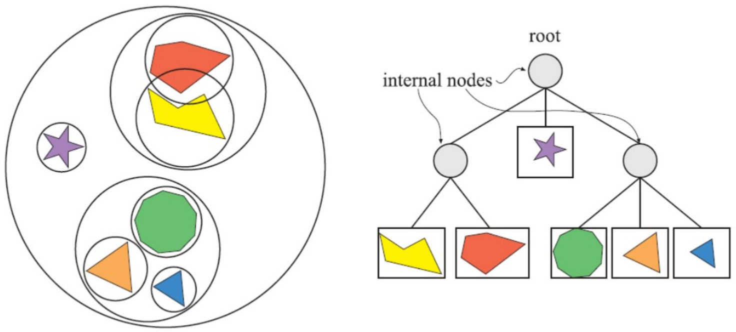


Spatial data structures

- **Organizes objects in 3D scene using hierarchical, tree-like structure**
  - Root defines whole scene
  - Children nodes define its own volume of space which in turn contains its own children
- **Structure is nested and recursive**
- For n objects it gives **improvement from `O(n)` to `O(log(n))`** for testing intersections
- **Main types:**
  - **Bounding volume hierarchies** → object subdivisions
  - **Binary space partitioning trees** → space subdivision



Bounding volume hierarchies (BVH)

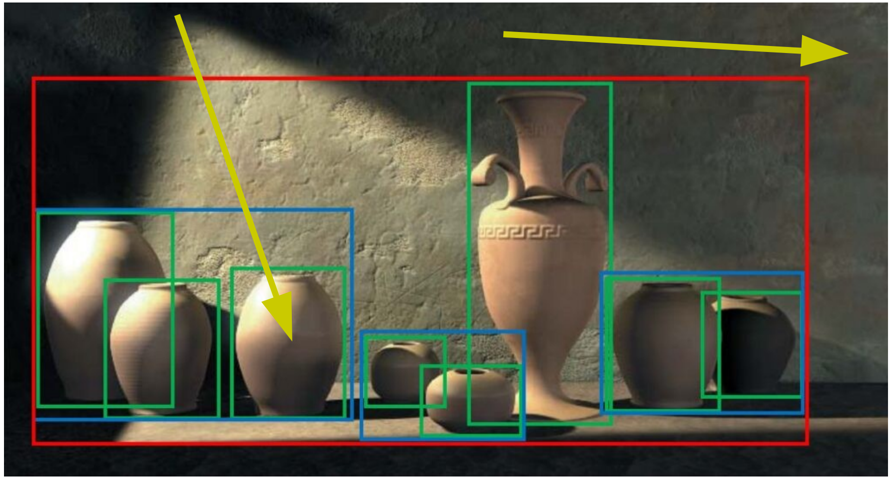


- Encloses the regions of space surrounding objects
- **Bounding volumes are much simpler to intersect than the contained objects**
- **Examples of bounding volumes:**
  - Spheres
  - Axis-aligned bounding boxes (AABB)
  - Oriented bounding boxes (OBB)
- **Scene is organized into a hierarchical tree structure consisting of a set of connected nodes**
  - Root node contains whole scene
  - Internal nodes contain intermediate volumes and point to other internal nodes with smaller volumes
  - Leaf nodes are volumes containing actual object geometries

https://courses.engr.illinois.edu/cs419/sp2017/Lecture-19-BVH%20and%20Octrees.pdf

Bounding volume hierarchies

- **Once BVH is constructed it can be used for intersection testing given ray**
  - Testing starts at the root
  - If ray misses volume defined by root then skip testing for the rest of the BVH
  - Otherwise, testing continues recursively by testing the children (internal) nodes.
    - Any time when ray misses volume defined by child node the whole sub-tree can be discarded
  - When ray hits leaf node, ray is tested for intersection with actual object geometry
- **For dynamic (animated) scenes, BVH must be recomputed at each frame when objects move**
  - Example: temporal bounding volume: https://ieeexplore.ieee.org/document/1274066



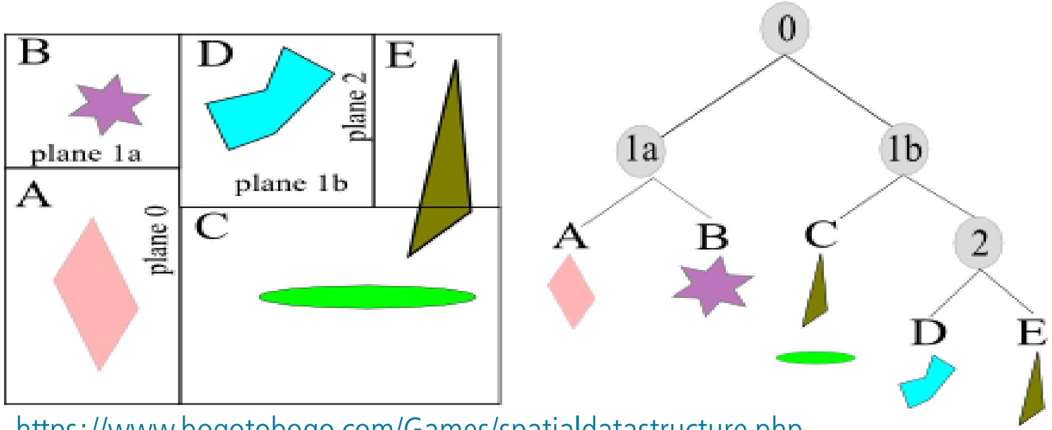
Ray intersection testing with BVH: https://jacco.ompf2.com/2022/04/13/how-to-build-a-bvh-part-1-basics/

Binary space partitioning (BSP) trees

- Entire space is subdivided and encoded into tree data structure
- **Tree is created by plane subdividing space in two, sorting geometry into these two spaces. Further division is done recursively.**
- Main types:
  - **Axis aligned BSP AKA kd trees**
  - **Octrees**
  - Polygon aligned BSP

Axis-aligned BSP construction

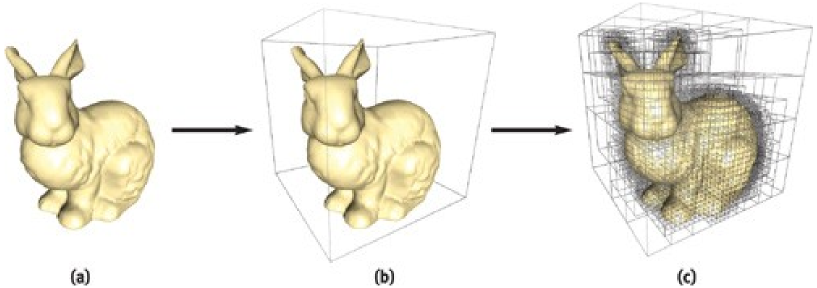
- Fist, whole scene is enclosed in axis-aligned bounding box (AABB)
- Then, initial AABB is recursively subdivided into smaller boxes
- At any level of recursion: one axis of box is choosen and perpendicular plane is used to divide space into two boxes
  - Uniform: plane is positioned exactly at half of the box
  - Non-uniform: plane is positioned adaptively inside box → more balanced tree
- If Objects in 3D scene intersect the planes:
  - Truly split object using plane into two separate objects
    - Only once copy of object, deletion is easier
    - Inefficient for smaller objects
  - Create duplicate and place at both nodes
    - Can give tighter bounds to larger objects
    - Problem of duplicating object

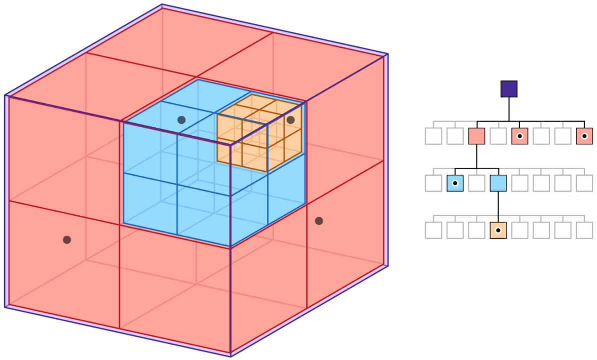


https://www.bogotobogo.com/Games/spatialdatastructure.php

Octrees

- **First, scene is enclosed in a minimal axis-aligned (AA) box**
- Similarly, to AA-BSP the **box is then split simultaneously along three axes recursively where split point is the center of the box**
  - Result is eight new boxes, which can be described with tree-like structure: octree
  - Recursive splitting stops when maximum depth is achieved or when there are certain number of objects (triangles) in a box



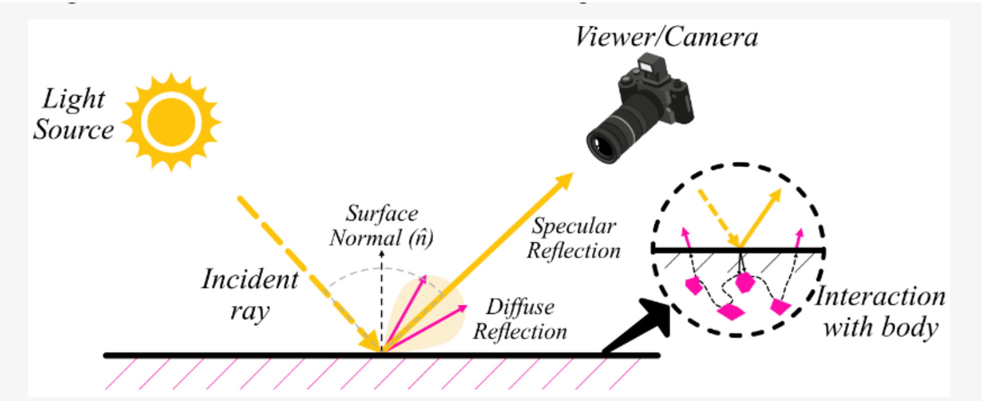


https://developer.nvidia.com/gpugems/gpugems2/part-v-image-oriented-computing/chapter-37-octree-textures-gpu
https://developer.apple.com/documentation/gameplaykit/gkoctree

Shading

Shading

- Once **intersection of camera ray with object** is found (visibility is solved), we need to calculate color and intensity of that point → **shading**
  - Intersection point → **shading point**
- **Shading and thus appearance of object (color) depends on:**
  - **Material of the object surface (scattering model, texture)**
  - **Shape of the object surface (normal)**
  - **Amount of Incoming light on surface (direction, color, intensity)**
  - **Camera viewpoint (position and orientation)**



https://www.mdpi.com/1424-8220/22/17/6552

Rendering equation

- Light (color) in shading point towards view direction is described with **rendering equation**
  - **Light** L_o in **intersected (shading) point** `p` towards **view direction** ω_o is results of summing all **incoming light** L_i from hemisphere of directions Ω, multiplying it with **BRDF** `f(p, ω_o, ω_i)` and **attenuation factor** `max(0, dotProduct(n, ω_i))`

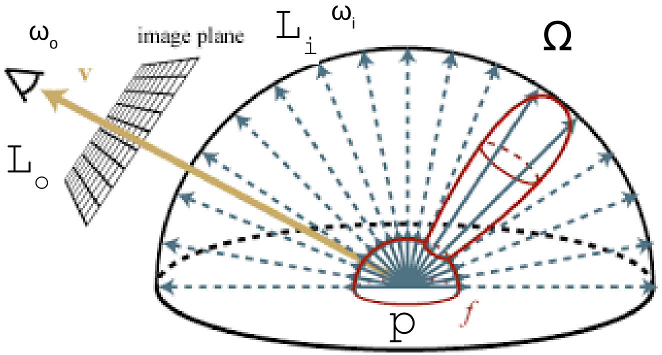
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega f(p,\omega_o,\omega_i) L_i(p,\omega_i)(\omega_i \cdot n) d\omega_i$$

Emission
- Take in account contribution of light if surface is emissive

BRDF
- Defines surface material
- Uses texture information

Incoming light over hemisphere of directions

Attenuation due to orientation of surface towards light.
- Depends on surface shape (normal)



https://www.mdpi.com/1424-8220/22/17/6552

Rendering equation decomposition

- BRDF **describes light reflection on surface**, e.g., specular and diffuse surfaces
- Attenuation factor: alignment of surface normal and incoming light direction
- Incoming light: compute all possible incoming light on shading point → not possible to solve analytically → **different light transport methods**
  - Solve using assumptions:
    - Only take in account light coming from light sources: **direct illumination**, e.g., point and directional lights
    - Only take in account light from point light source s and specular surfaces: **Whitted ray-tracing**
  - Solve using estimation of rendering equation → **global illumination**
    - Monte-Carlo ray-tracing: path-tracing
    - Finite elements methods: radiosity

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega \boxed{f(p,\omega_o,\omega_i)}\boxed{L_i(p,\omega_i)}\boxed{(\omega_i \cdot n)}d\omega_i$$

Rendering equation: assumptions

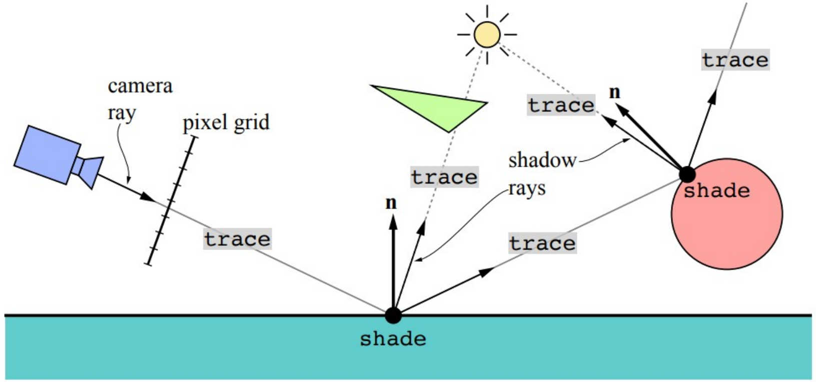
- **Assume: between objects is vacuum**
  - **No participating media: fog, smoke, clouds, etc. →** volumetic rendering equation is needed for describing light transport
    - Light can not get attenuated while traveling between objects
- **Only surface interactions:** e.g., specular or diffuse reflections
  - **No sub-surface scattering and transmission** → further extensions on rendering equation
    - Light will not scatter inside objects

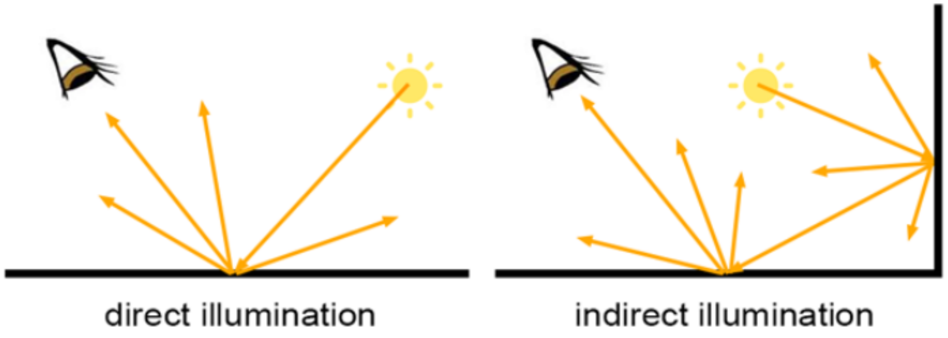
https://www.pbr-book.org/3ed-2018/Light_Transport_I_Surface_Reflection

Incoming light

$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega f(p,\omega_o,\omega_i)\boxed{L_i(p,\omega_i)}(\omega_i \cdot n)d\omega_i$$

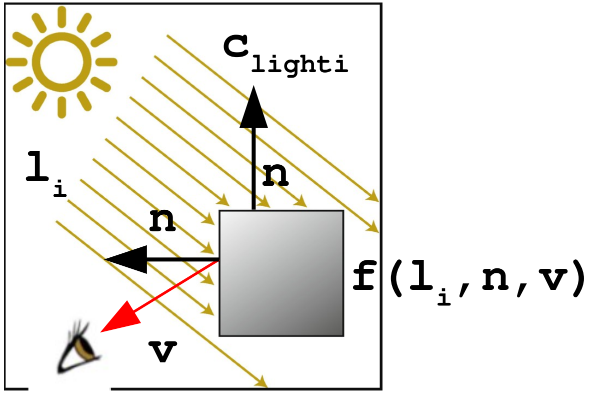
- Computing incoming light on surface is essential to shading
  - **Direct illumination**: compute contribution directly and only from light sources
  - **Indirect illumination:** light reflected from another object surface





https://www.realtimerendering.com/Real-Time_Rendering_4th-Real-Time_Ray_Tracing.pdf

Direct illumination: directional lights



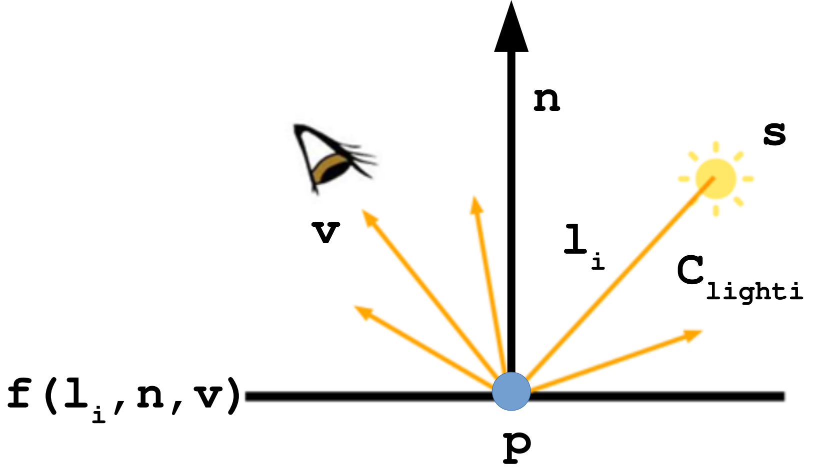
- **Loop over all directional light sources and evaluate surface color using:**
  - **Light direction (`l`$_i$)**: direction of directional lights can be used directly
  - **Light color and intensity (`c`$_{lighti}$)** is given are parameter of directional light and is constant over scene

$$c_{shaded(p,v)} = \sum_{i=1}^{n} f(l_i, n, v) c_{light_i} (n \cdot l_i)^+$$

Light source color

BRDF

Direct illumination: point lights



- Loop over all point (or spot) light sources and evaluate surface color:
  - Ray is generated from shading point `P(x,y,z)` to light source `S(x,y,z)` (visibility solving):
    - Light direction: l_i `= normalize(S - P);`
    - Light color and intensity: c_{lighti}
      - Given as parameter of point light
      - **Inverse square law**: intensity falls with squared distance to light source

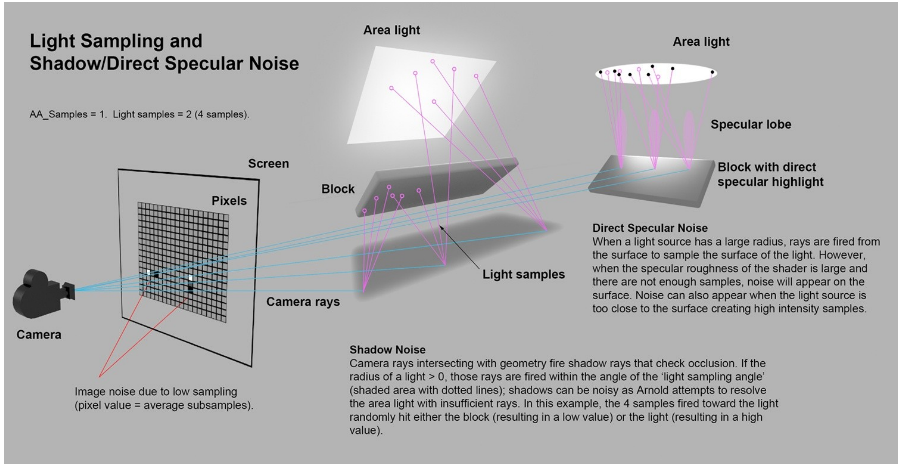
$$c_{shaded(p,v)} = \sum_{i=1}^{n} f(l_i, n, v) c_{light_i} (n \cdot l_i)^+$$

BRDF

Light source color

Direct illumination: area lights

- **Loop over all area light sources and evaluate surface color**
  - Sample points on light source geometry (Monte-Carlo methods)
  - For each sampled point on light source geometry:
    - **Generate ray from shading point to sampled point**
    - Evaluate contribution of light
    - Evaluate surface color using amount and direction of incoming light

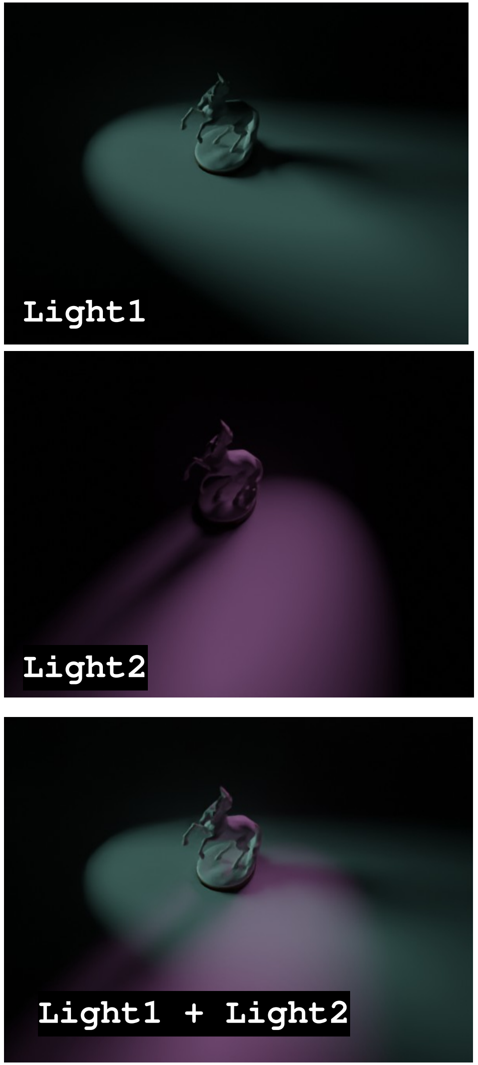


https://www.pbr-book.org/3ed-2018/Light_Transport_I_Surface_Reflection/Direct_Lighting
https://www.pbr-book.org/3ed-2018/Light_Transport_I_Surface_Reflection/Sampling_Light_Sources

Shading: Multiple lights

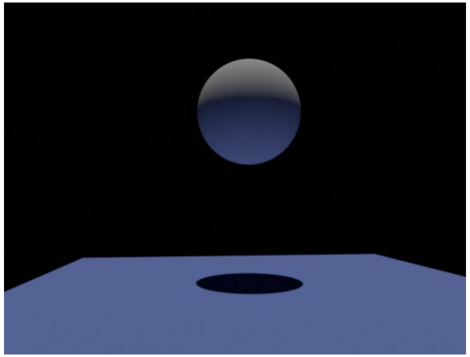
- 3D scenes often contain multiple light sources
- Contribution of each light source adds up linearly

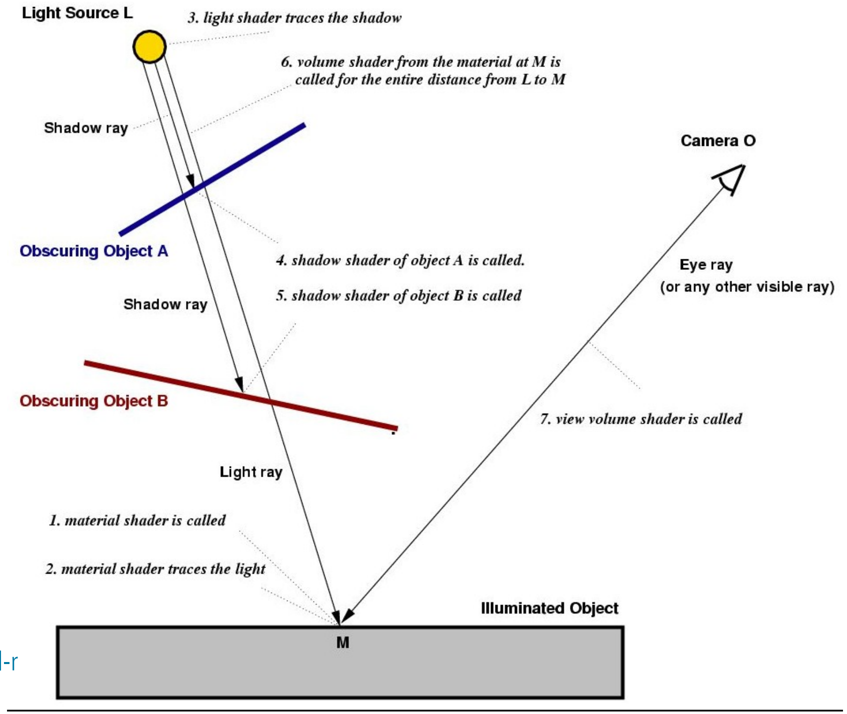
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \boxed{\int_\Omega} f(p,\omega_o,\omega_i) \boxed{L_i(p,\omega_i)} (\omega_i \cdot n) \boxed{d\omega_i}$$



Light and shadows

- While evaluating incoming light, **rays are traced from shading point to light sources** (or other surfaces) → **shadow/secondary/light rays**
- Next to light direction, intensity and color, the purpose of shadow ray is to **determine if it is obstructed by objects**
- **If shadow ray is obstructed, it has zero light contribution** → black color





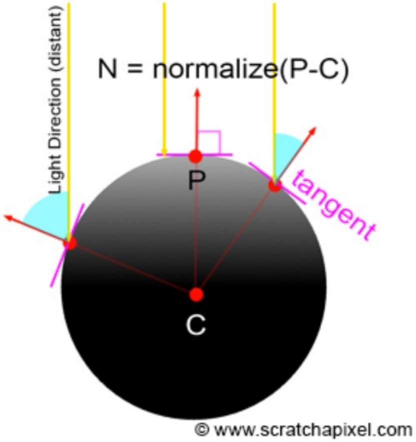
https://help.autodesk.com/cloudhelp/2016/ENU/mental-ray-docs/mr_docs/manual/node102.html

Light attenuation

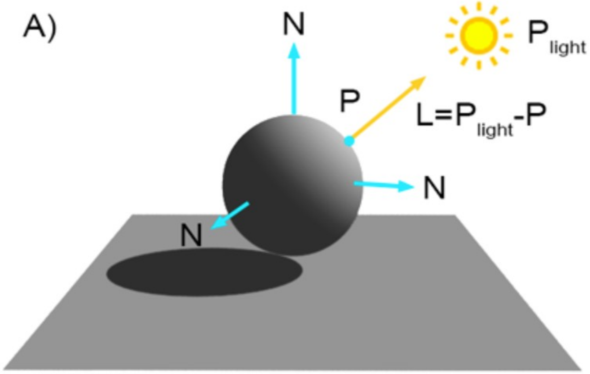
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega f(p,\omega_o,\omega_i)L_i(p,\omega_i)\boxed{(\omega_i \cdot n)}d\omega_i$$

- **Surface normal** `N` in shading point defines surface orientation
- **Alignment of surface normal `(N)` and incoming light direction `(L)` determines surface brightness**:

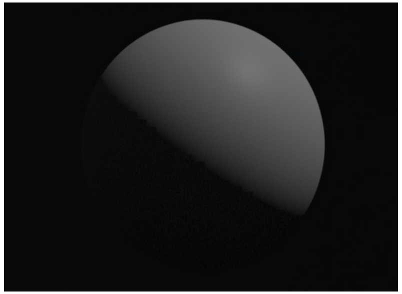
```
max(0, dotProduct(N, L))
```



Light attenuation in case of directional lights



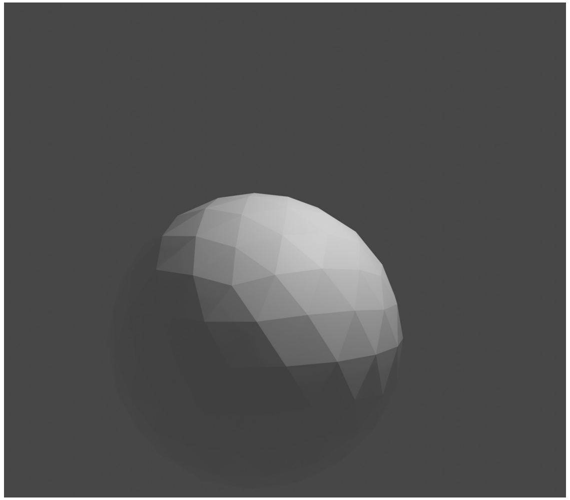
Light attenuation in case of point lights

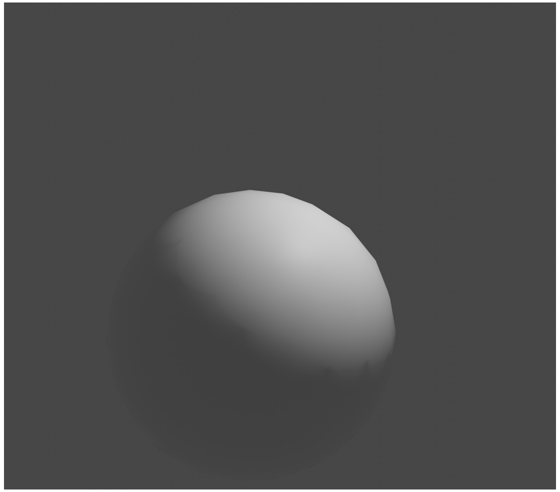


Incoming light (from any direction or lights source) can get attenuated due to surface orientation

Surface normal and attenuation

- Triangulated mesh can’t represent perfectly smooth surface
- Since shading depends on surface normal, triangulated meshes result in faceted look
  - **Flat shading**
- Solutions:
  - Gourad or Phong shading → **Smooth shading**
  - **Subdivision surfaces** → more smoother surface geometry

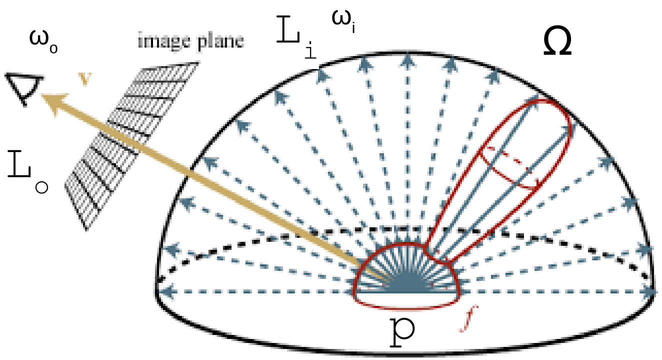




BRDF

- **Given incoming light, color of object is result of how light is reflected in view direction**
- **Incoming light is attenuated** using the information on normal alignment with incoming light direction
- **The amount of light reflected in viewing direction is computed using BRDF** `f(L,N,V)` which depends on:
  - Incoming light direction `L` → ω_i
  - Surface normal `N` in shading point `p`
  - View direction `V` → ω_o

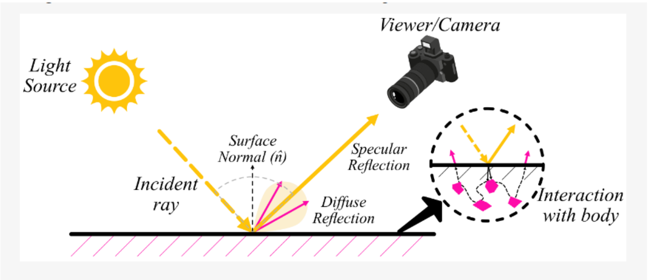
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega \boxed{f(p,\omega_o,\omega_i)} L_i(p,\omega_i)(\omega_i \cdot n)d\omega_i$$

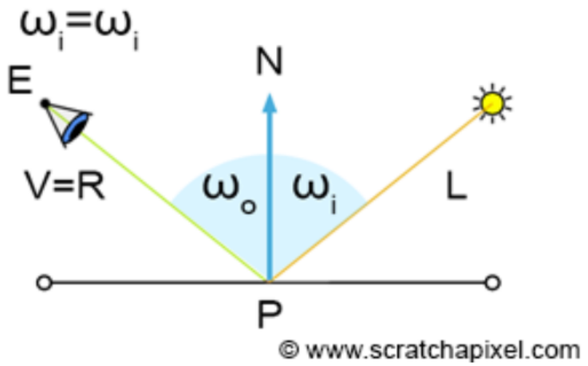


BRDF

- Mathematical model **approximating interaction of light and microscopic structure of object material**
  - Diffuse/Lambertian, specular, Phong, Blinn-Phong, Lafortune, Torrance-Sparrow, Cook-Torrance, Ward, Oren-Nayar, etc.
- BRDF contains number of parameters which are varied over surface using **texture**
  - **Complete material description with BRDF and texture is defined in shader**
- Shading using direct illumination with one point light:

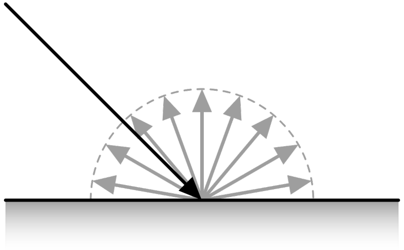
```
shaded_color = f(L, V, N) * light_color *
light_intensity * max(0, dotProduct(N, L))
```

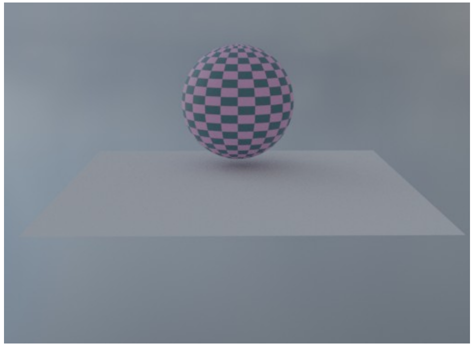


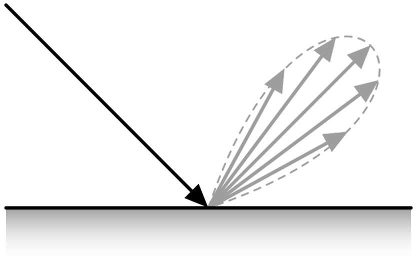


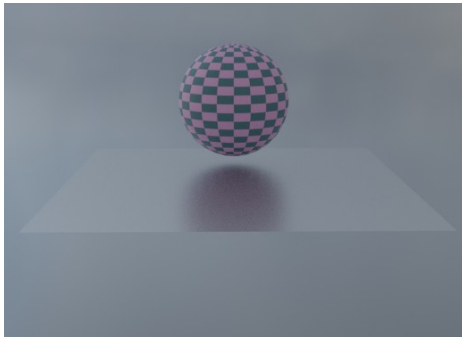
Common BRDFs

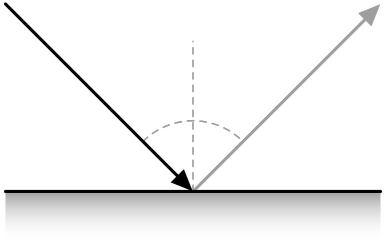
- Most real world surface can can be described with following BRDFs:
  - Diffuse
  - Glossy
  - Specular

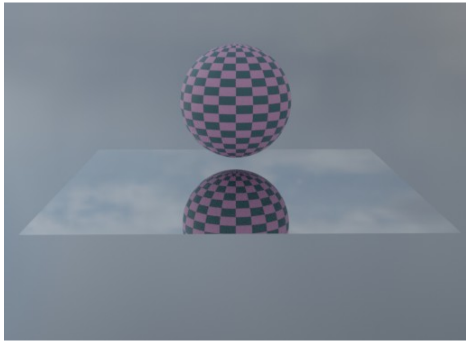






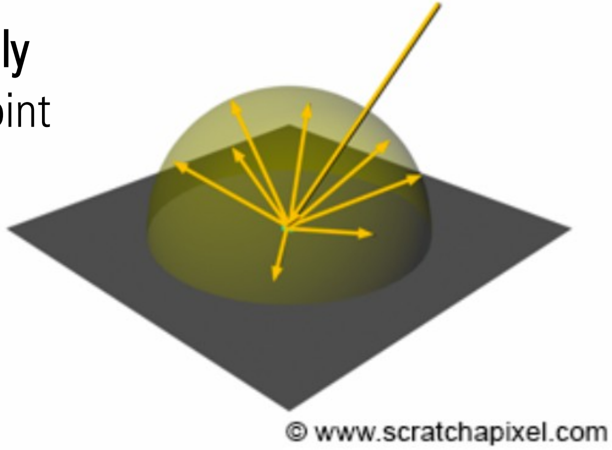






Diffuse BRDF (reflection)

- **Light incoming from any direction on diffuse surface will be equally reflected in all directions** – over whole hemisphere at shading point `P` around normal `N`.
  - **Independent of view ($\boldsymbol{\omega}_o$) and light ($\boldsymbol{\omega}_i$) direction**
- Diffuse BRDF `f(p,`$\omega_{0,}\omega_o$`)` parameters:
  - **`albedo`** `= reflected_light / incident_light`
  - Albedo is given as RGB in `[0,1]`



$$f(p, \omega_o, \omega_i) = \frac{albedo}{\pi}$$

Diffuse BRDF (reflection)

- Color of diffuse object intersected by camera ray is result of how light reflects from diffuse surface

$$L_o(p, \omega_o) = \int_\Omega \boxed{\frac{albedo}{\pi}} L_i(p, \omega_i)(\omega_i \cdot n) d\omega_i$$

Specular BRDF (reflection)

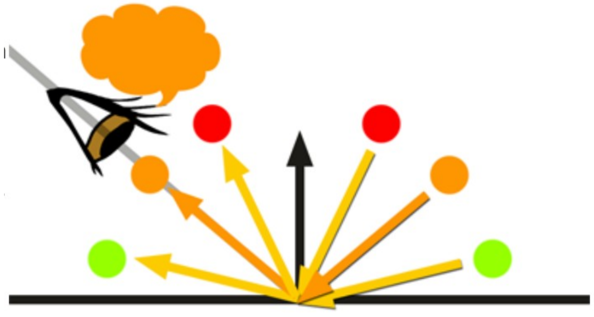
- **Light reflects in direction symmetrical to incident direction around normal** at shading point

```
incident_angle = reflected_angle
```

- **Reflected direction** (R) is calculated using light direction (L) and normal (N) → **law of reflection**

```
R = L - 2 (N * L) * N
```

- **Reflection depends on view and light directions**
- Specular BRDF parameters:
  - **Reflectivity** given as RGB in [0,1]



$$f(p,\omega_o,\omega_i) = \begin{cases} reflectivity \text{ if } & \omega_o = \omega_i - 2(\omega_i \cdot n)n \\ 0 & otherwise \end{cases}$$

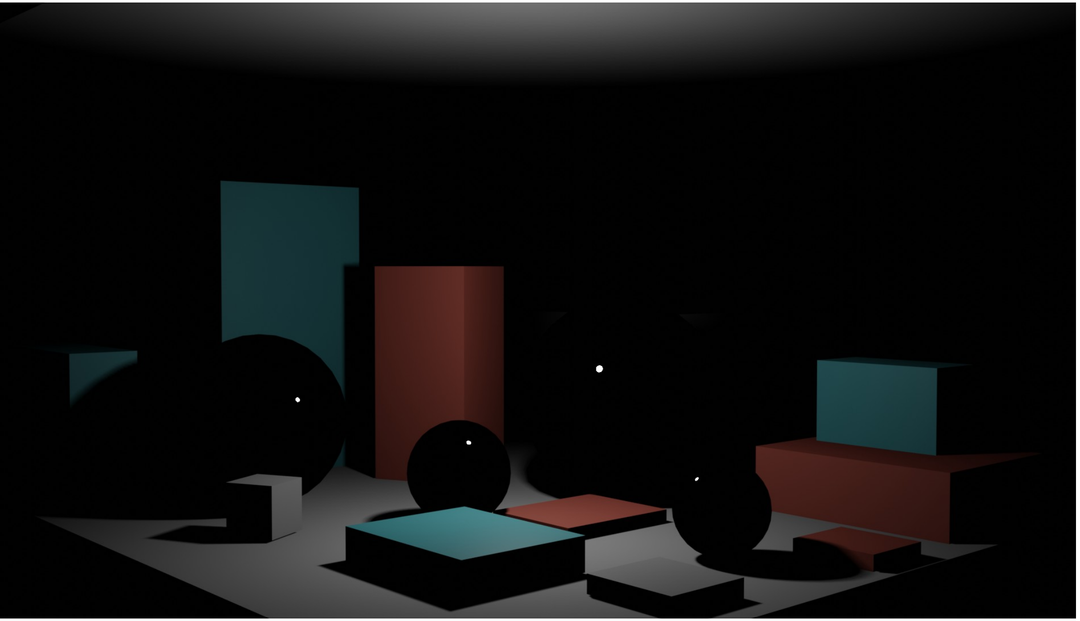
$$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_\Omega \boxed{f(p,\omega_o,\omega_i)} L_i(p,\omega_i)(\omega_i \cdot n)d\omega_i$$

Specular BRDF (reflection)

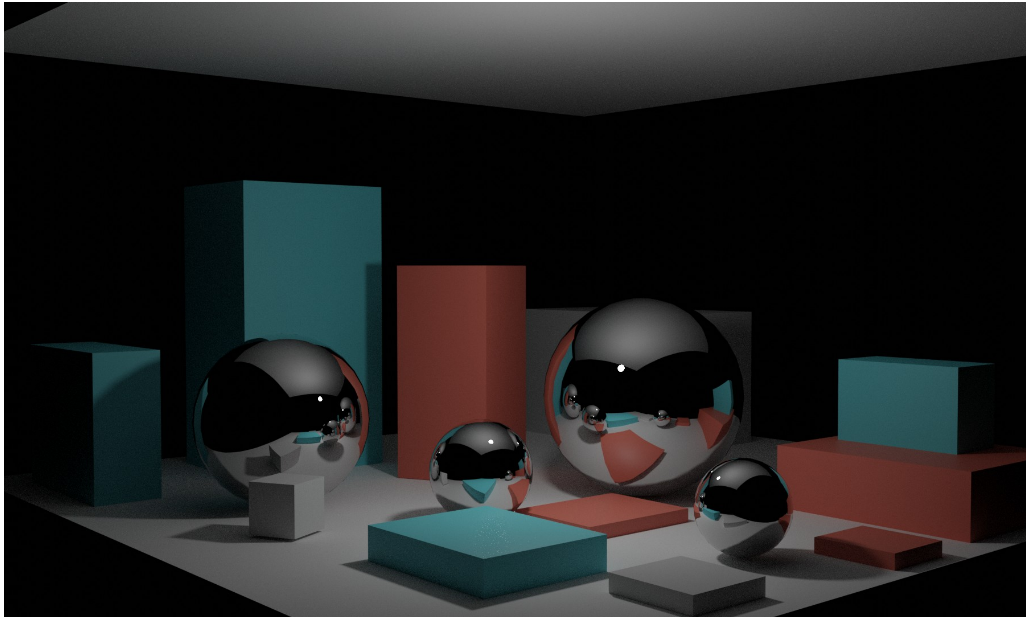
Specular surfaces are only reflecting light from other surfaces or light sources

Without objects in 3D scene, specular sphere is not visible since there is not objects to reflect

Specular BRDF (reflection)



Using only **direct illumination** as method for computing incoming light will result in specular surface **reflecting light from light source** and not other surfaces!



To compute realistic light reflection from specular surfaces, **indirect illumination must be used**, e.g., **Whitted ray-tracing**

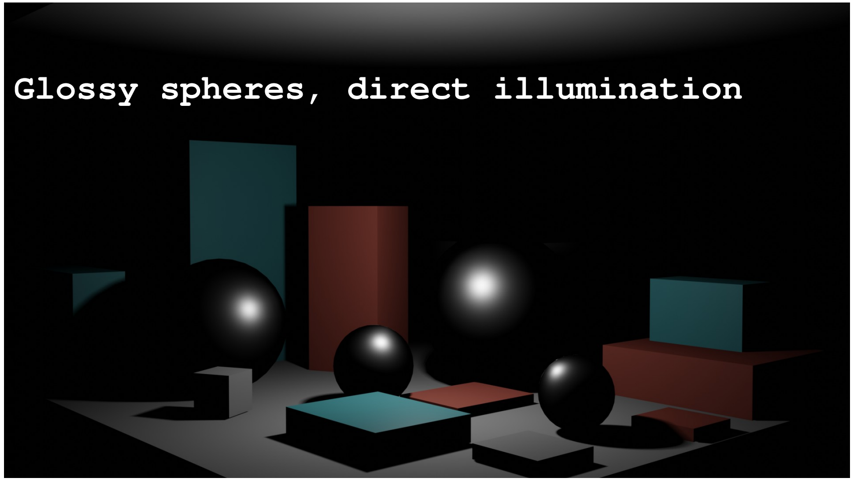
Glossy BRDF

$$R_{Cook-Torrance} = R_s + R_d$$

$$R_s = \frac{D \cdot G \cdot F}{dot(N,V) \cdot dot(N,L)}$$

- **R_s – Specular component**
  - D – distribution of microfacet normals
  - G – geometry term
  - F – Fresnel term
- **R_d – Diffuse (Lamberitan) reflection model**

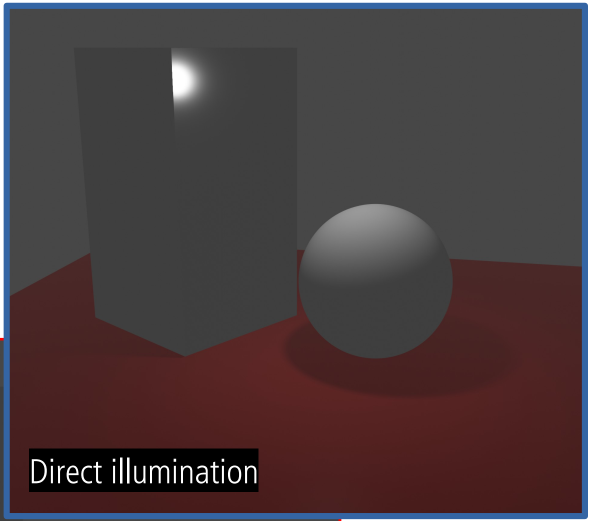
- **Microfacet-based BRDFs**
  - **Cook-Torrance**, Oren-Nayar
- **Glossy reflection, similarly as specular, reflects other surfaces and light sources but reflections are blurred**
  - Amount of blur depends on **surface roughness**

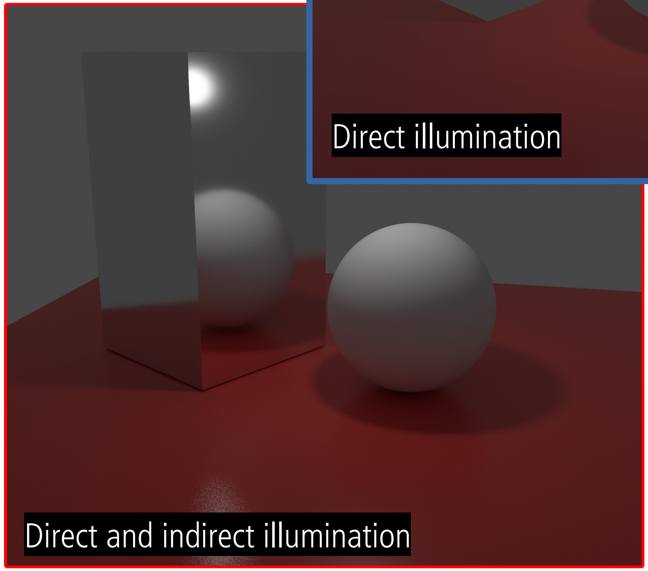


Light transport

Indirect illumination

- **Next to light from light sources, large contribution of light is also indirect**
- Light travels from light source, reflecting from objects and finally fall on shaded surface → **indirect illumination**
  - **Indirect diffuse**
  - **Indirect specular**
  - **Soft shadows**
- Gathering indirect light on shading point is called **light transport**
  - **Depends on visibility → geometrical problem**
  - **Whitted ray-tracing**





Indirect diffuse – diffuse objects reflect light which illuminates other objects in the scene
Indirect specular - specular objects reflect light which illuminates other objects in the scene

Shading and light transport

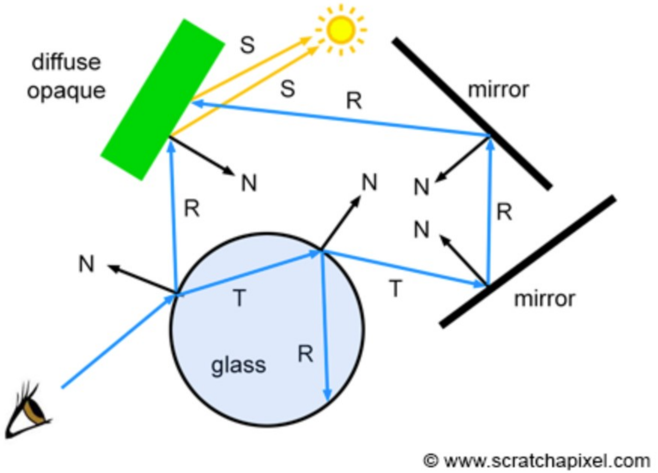
- Appearance of object and can be divided into:
  - **Shading**: how light absorbs and reflects; light-surface interaction
    - How light interacts with matter: what happens to light which reaches the surface and how light leaves the surface
    - Uses scattering model (e.g., BRDF) and texture to compute color
    - Effects: reflection, transparency, specular reflection, diffuse reflection, sub-surface scattering.
  - **Light transport**: how much light object receives
    - How light bounces from surface to surface
    - Which paths light take and how does it depends on material
    - Is light blocked by another surface?
    - Uses scattering model (e.g., BRDF) for computing light reflection
    - Effects: Indirect diffuse, indirect specular, soft shadows
- Distinction between shading and light transport is very thin

Global illumination and light transport

- Rendering equation describes **global illumination**: **direct and indirect light**
- Various light transport strategies based on ray-tracing for solving full rendering equation, most popular is **path-tracing**
- Classical example of light transport on which other advanced methods are built is **Whitted ray-tracing**
  - Not solving full global illumination: only light from light sources and specular surfaces
  - Based on backward-tracing: tracing starts from camera
  - Utilizing ray-tracing solving visibility and for "gathering" indirect illumination
- Note: ray-tracing is method for solving **visibility** while path-tracing or Whitted ray-tracing are methods for solving **light transport**

Whitted ray-tracing

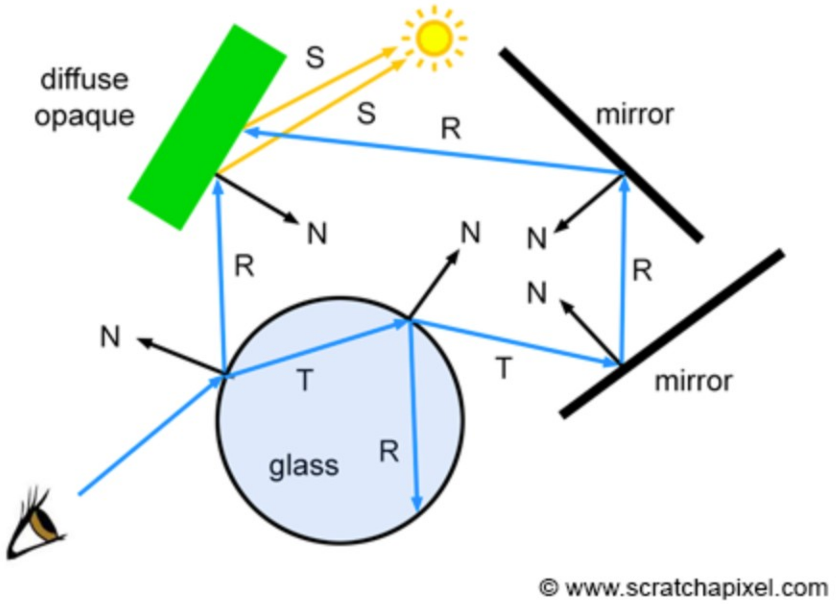
- **Whitted ray-tracing is solving light transport** needed for shading points found by **camera ray** intersection
- **Assumptions**
  - Scene contains only point lights
  - Surfaces are only **specular** (reflection, refraction) or **diffuse**
- Light transport
  - **Generate secondary rays** from intersection, found by camera ray, into the scene
  - **At every intersection color and direction of the ray depends on surface**:
    - Diffuse surface
    - Specularly reflective surface
    - Specularly transmissive surface



Whitted ray-tracing: case 1 – diffuse surface

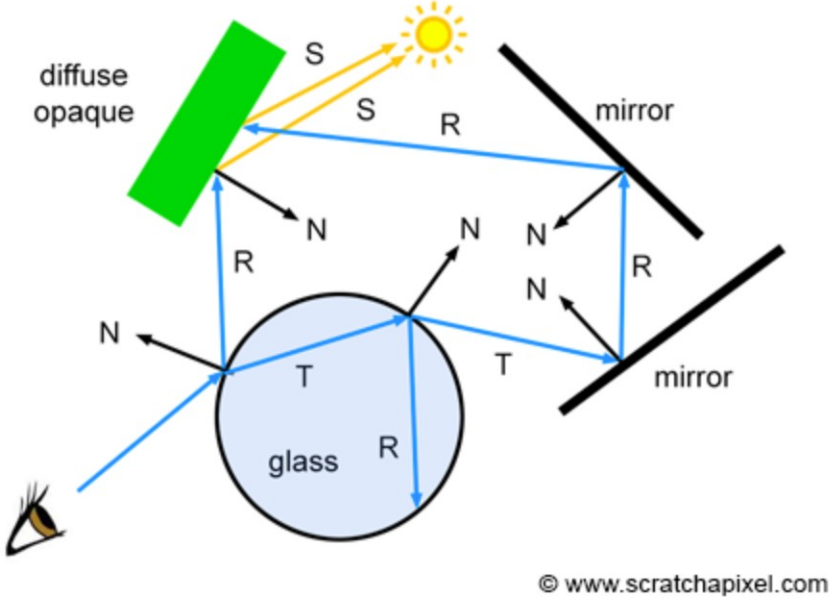
- Surface at intersection is point is **diffuse**:
  - Cast a **shadow ray** in direction of light to find light direction, intensity, color or if it is shadowed
  - If not in shadow use **scattering model (BRDF) to compute color** of the object: e.g., Lamberitan BRDF
  - Path is here terminated, computed color is added to camera ray intersection color

Note: Whitted-ray tracing is terminating path when reaching diffuse surfaces and only directional illumination is used to compute the color.



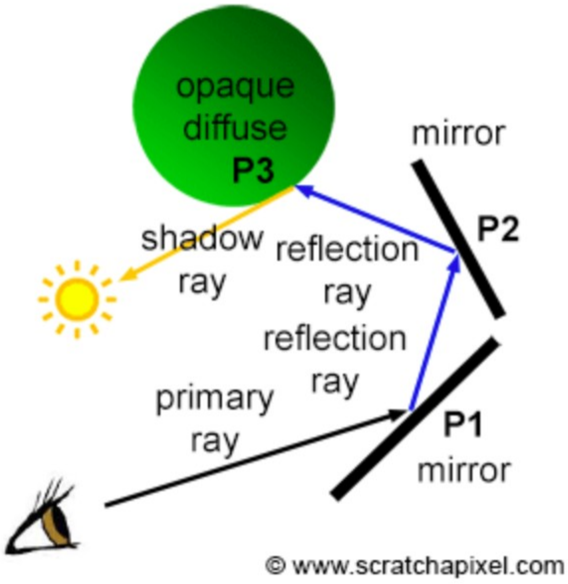
Whitted ray-tracing: case 2 – specular reflection

- Surface at intersection is point is **specular**:
  - Trace secondary ray – **reflection ray** from intersection point in direction of specular reflection
    - Reflected ray is calculated using incoming ray and surface normal at intersection → **law of reflection**
  - This process is **recursive** until it hits background or diffuse surface
    - Terminate with **ray depth –** trade off between render time and quality



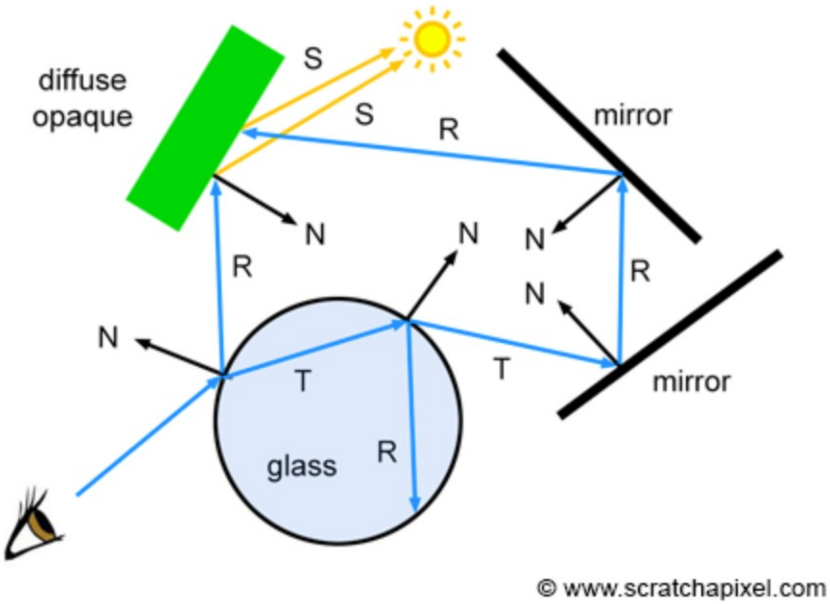
Whitted ray-tracing: case 2 – specular reflection

- Backward ray-tracing:
  - Primary ray is generated and intersects specular surface at P1
  - At intersected point P1 reflection ray is generated and intersects mirror surface at P2
  - At intersected point P2 reflection ray is generated and intersects diffuse surface at P3
  - At P3 color of the surface is calculated
- Return color on created path
  - Color at P3 becomes color at P2
  - Color at P2 becomes color at P1
  - Color at P1 is color of primary ray
  - Color of primary ray is color of pixel from which primary ray was generated



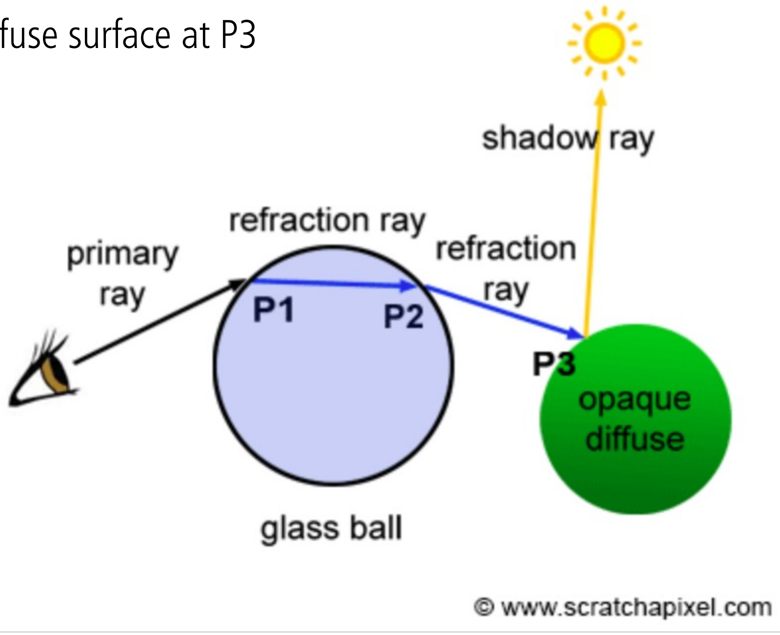
Whitted ray-tracing: case 3 – specular transmission

- Surface at intersection is point is **transparent (specular transmission)**:
  - Trace secondary ray – **refraction ray** from intersection point in direction of specular refraction
    - Refraction ray direction is computed using **Snell's law**
  - Trace secondary – **reflection ray** in direction of specular reflection
- Similarly as for specular reflection, specular transmission is recursive process and terminates once diffuse surface or background is intersected



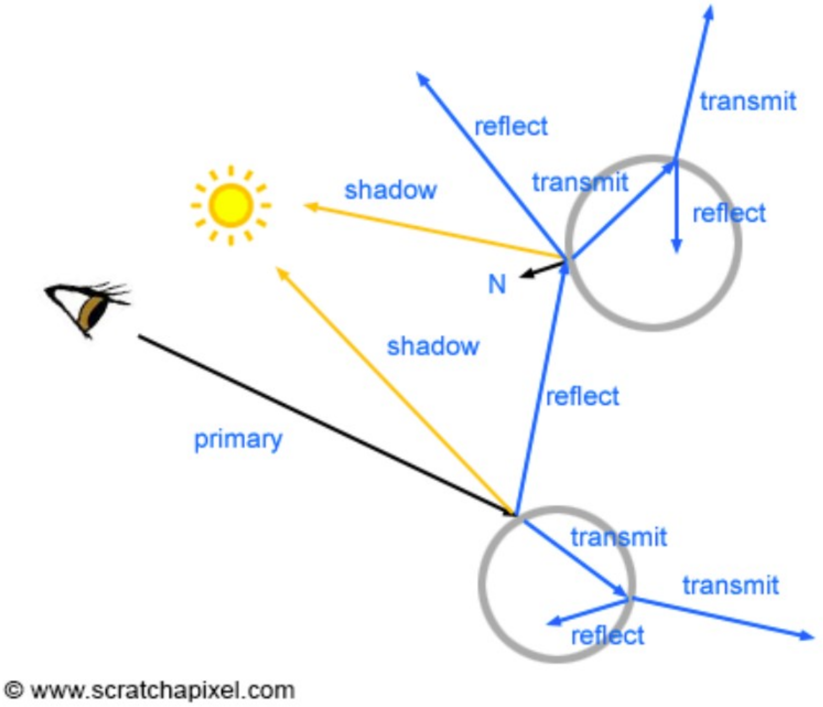
Whitted ray-tracing: case 3 – specular transmission

- Backward ray-tracing:
  - Primary ray is generated and intersects transparent surface at P1
  - At intersected point P1 refraction ray is generated and intersects transparent surface at P2
  - At intersected point P2 refraction ray is generated and intersects diffuse surface at P3
  - At P3 color of the surface is calculated
- Return color on created path
  - Color at P3 becomes color at P2
  - Color at P2 becomes color at P1
  - Color at P1 is color of primary ray
  - Color of primary ray is color of pixel from which primary ray was generated



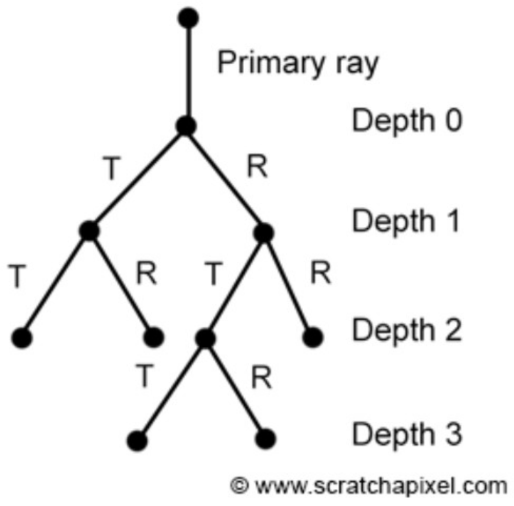
Whitted light-transport properties: recursive nature

- **Intersecting reflective surface** causes generation of reflection ray
- **Intersection of transparent surface** causes generation of two new rays: reflected and refracted
- These rays can further intersect reflective or transparent surfaces → **recursion**
  - To evade explosion, max recursion depth is exposed as parameter



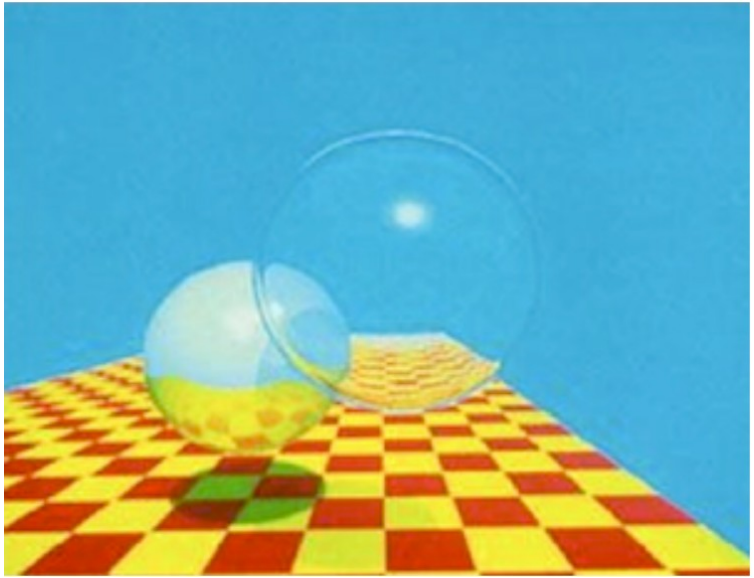
Whitted light-transport properties: tree of rays

- All secondary rays (reflection or refraction) spawned by primary or other secondary rays can be represented tree-like structure
- Each intersection marks new depth/level of the tree and thus one level of recursion



Whitted ray-tracing and beyond

- **Whitted ray-tracing - classic light transport** algorithm which computes indirect light from specular surfaces
- **Advanced light transport strategies** simulate indirect light transport in scenes containing more complex surfaces
  - Monte Carlo ray-tracing: path-tracing, bidirectional path-tracing, etc.
  - Finite elements methods: radiosity



An Improved Illumination Model for Shaded Display. Turner Whitted, 1980

Path-tracing in production: Renderman

https://renderman.pixar.com/tech-specs

Ray-tracing based rendering: verdict

- Good:
  - Ray-tracing based rendering is **clear and straightforward method to implement**
  - It represent **unified visibility solving framework**
    - Computing what is visible from camera (primary ray intersections), with inherent perspective/orthographics projection
    - Computing light transport – visibility between surfaces and light sources in 3D scene (secondary ray intersections) required for the shading process.
  - **Powerful shading capabilities due to visibility computation**

Ray-tracing based rendering: verdict

- Bad:
  - **Requires efficient methods for testing ray-shape intersections**:
    - Sometimes hard to develop (good understanding of mathematics, particularly geometry and linear algebra for geometrical or analytical solutions)
    - Core element during rendering and they are often very expensive
  - For advanced light transport needed in shading stage, **ray-tracing-based rendering must store all scene objects** must be readily available for intersection testing and material evaluation.
    - An example where is this needed is to compute indirect light or shadows casted by different objects.
  - **Additional acceleration datastructures are often desired to accelerate ray-scene intersections**.
    - Acceleration data structures require efficient development, cause additional pre-computations and require a lot of memory

More into topic

- Generally about ray-tracing:
  - https://www.scratchapixel.com
  - https://raytracing.github.io/books/RayTracingInOneWeekend.html#overview
- Acceleration datastructures:
  - https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-acceleration-structure/bounding-volume-hierarchy-BVH-part2.html
  - https://www.pbr-book.org/3ed-2018/Primitives_and_Intersection_Acceleration/Bounding_Volume_Hierarchies
  - https://www.realtimerendering.com/

Summary questions

- https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/12_rendering_raytracing

Literature

- https://github.com/lorentzo/IntroductionToComputerGraphics