

# Lecture 4: Transformations and Scene Organization

DHBW, Computer Graphics

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1.2.2023.

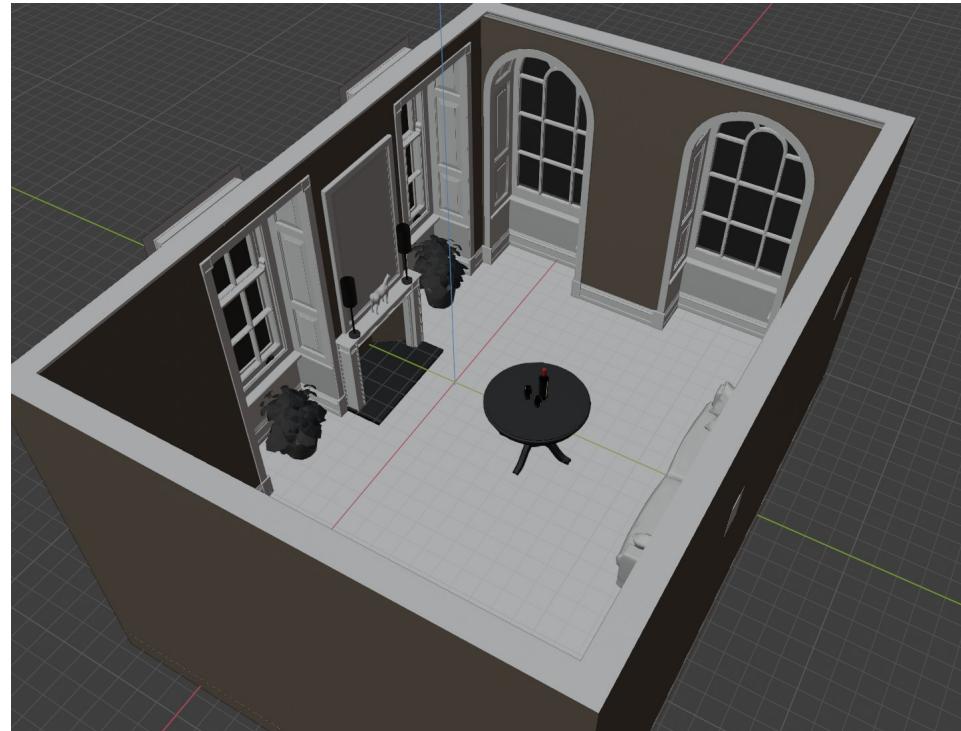
# Syllabus

- 3D scene
    - Objects
    - Lights
    - Cameras
  - Rendering
  - Image and display
- 
- The diagram illustrates the progression of topics in the syllabus. It starts with a list of basic 3D scene components: objects, lights, and cameras. A blue bracket groups these three items. An arrow points from this group to a second, larger blue bracket containing a list of more advanced concepts: 3D space, transforms, and scene organization.
- 3D scene
    - Objects
    - Lights
    - Cameras
  - 3D scene
    - 3D space
    - Transforms
    - Scene organization

3D space

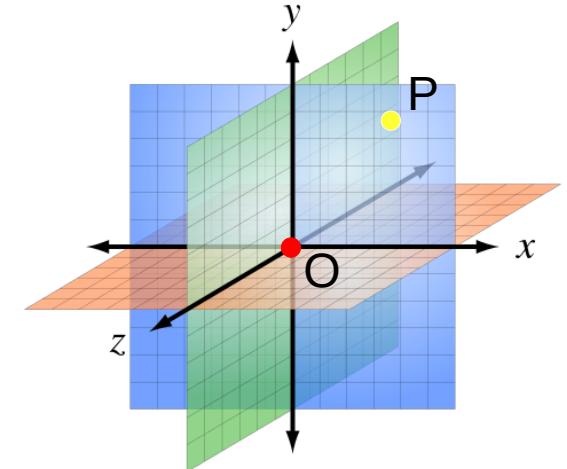
# Introduction

- Shape, positions and orientations of objects, lights and cameras in 3D scene are defined with **points**, **vectors** and **coordinate systems**.



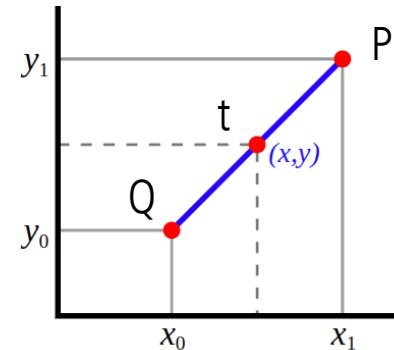
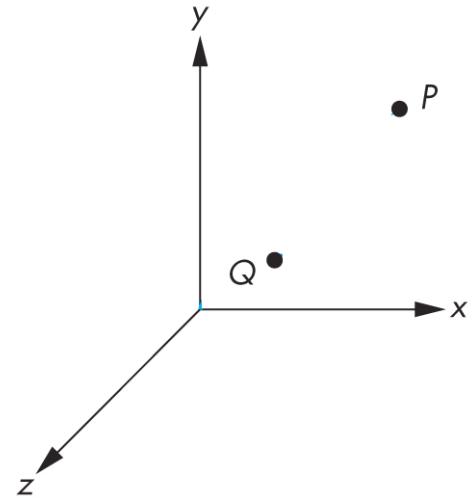
# Coordinate system

- Points and vectors are represented with **three coordinates**:  $(x,y,z)$
- These values are meaningless without a **coordinate system**:
  - Origin (**point**) of the space
  - **Basis**: three linearly independent vectors that define **X, Y and Z axis** of a space.
- Origin and three vectors define a **frame** which defines coordinate system.
  - **Cartesian coordinate system**: perpendicular axes
  - **Euclidean space**



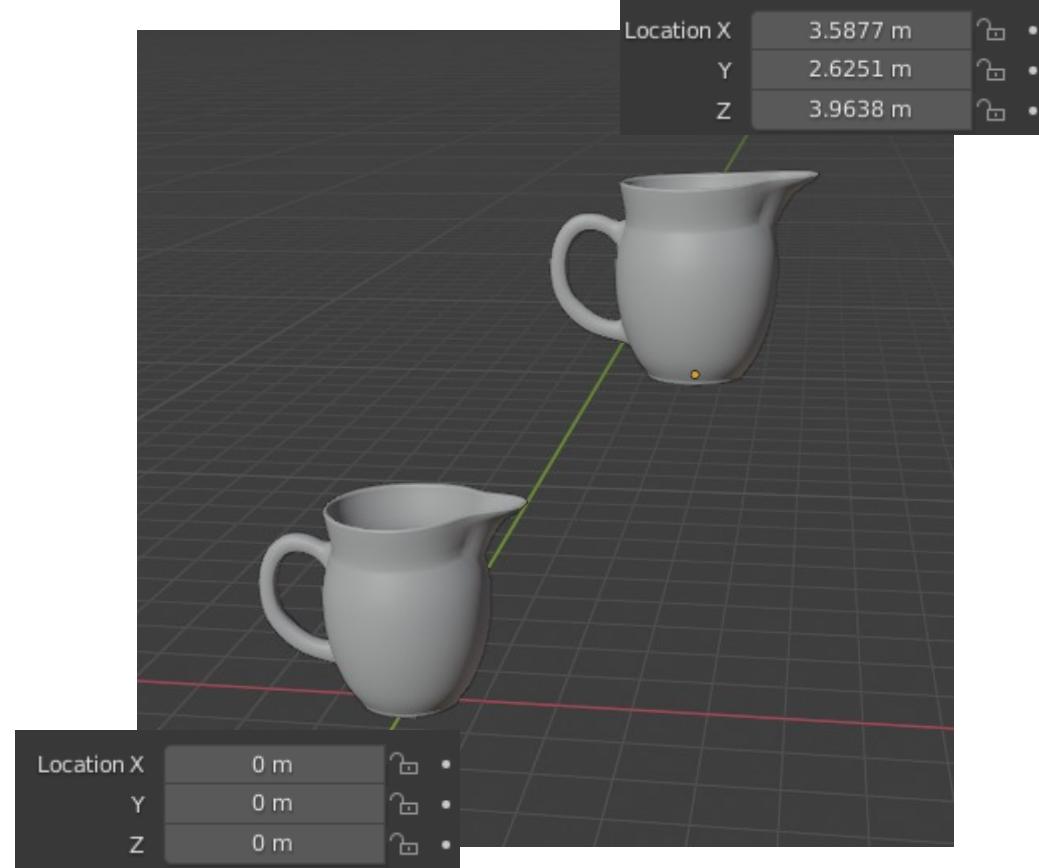
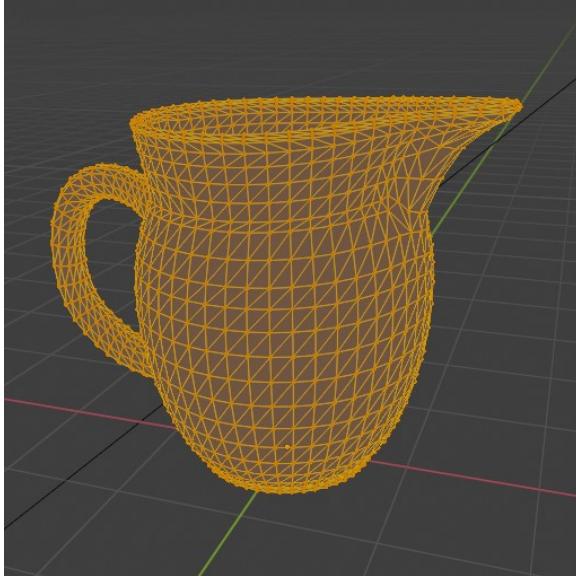
# Points

- Zero-dimensional **location** with respect to coordinate system:
  - 2D space  $(x, y)$
  - 3D space:  $(x, y, z)$
- Interpolation between points: **linear interpolation**
  - $\text{lerp}(Q, P, t) = (1 - t) * Q + t * P;$



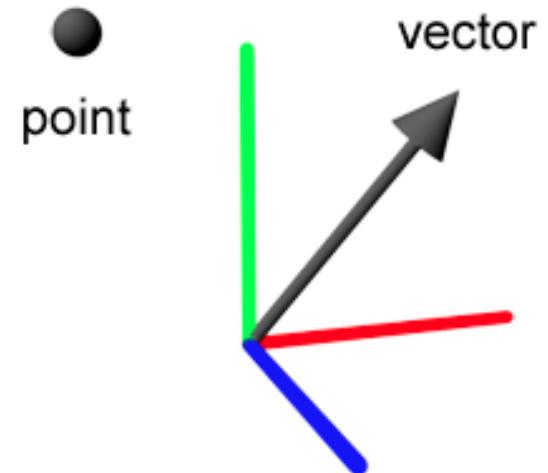
# Points

- Points are used to:
  - Describe shape
  - Define position of object in space



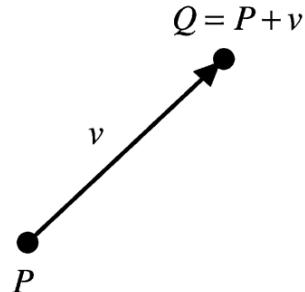
# Vectors

- **Magnitude (norm/length)** and **direction** in 2D or 3D space
  - Two coordinates ( $x, y$ ) – usually for texture space
  - Three coordinates ( $x, y, z$ ) – for any 3D elements

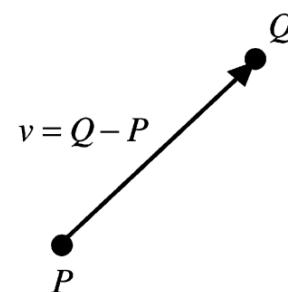


# Points and vectors

- Subtracting or adding point with vector results in new point
- Distance between two points results in **vector** which contains **length** and **direction**.



Adding point to  
vector

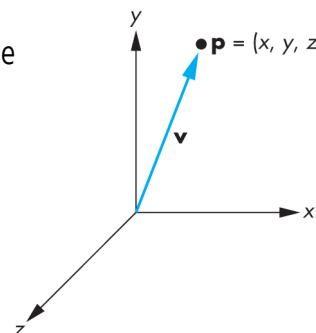


Subtracting point  
from a point

$$2D: \|v\| = \sqrt{x^2 + y^2}$$

$$3D: \|v\| = \sqrt{x^2 + y^2 + z^2}$$

Magnitude  
(length)



$$\hat{v} = \frac{v}{\|v\|}$$

Direction  
obtained by  
normalization

# Row major and column major - notation

- Points and vectors can be written as:
  - [1x3] matrix → **row major order** (Direct X, Maya)

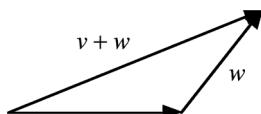
$$V = [x \quad y \quad z]$$

- [3x1] matrix → **column major order** (OpenGL, PBRT, Blender)

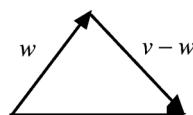
$$V = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

# Common vector operations

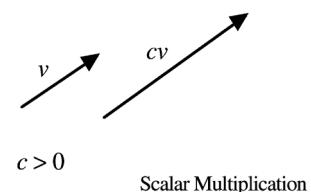
- Addition and subtraction
- Addition, subtraction, multiplication with scalar, etc.
- Dot (dotProduct ( $a, b$ ) ) and cross product (crossProduct ( $a, b$ ) ) operators
- Normalization → length of vector = 1 (unit vector)



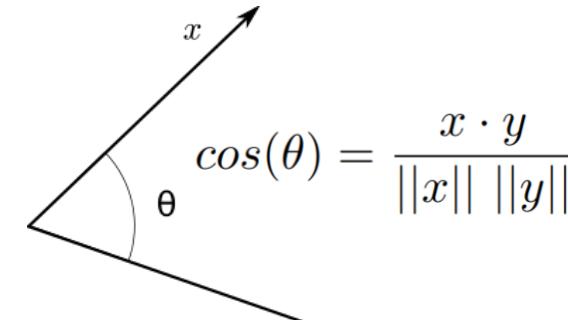
Addition



Subtraction



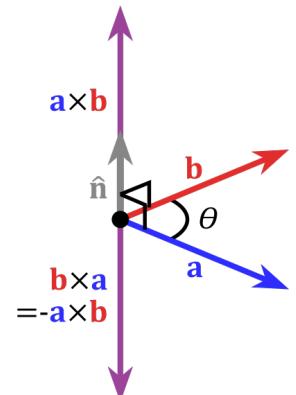
Scalar Multiplication



$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^n \mathbf{a}_i \mathbf{b}_i = \mathbf{a}_1 \mathbf{b}_1 + \mathbf{a}_2 \mathbf{b}_2 + \dots + \mathbf{a}_n \mathbf{b}_n$$

Dot product

[https://en.wikipedia.org/wiki/Dot\\_product](https://en.wikipedia.org/wiki/Dot_product)



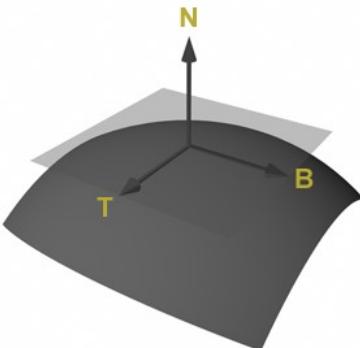
$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Cross product

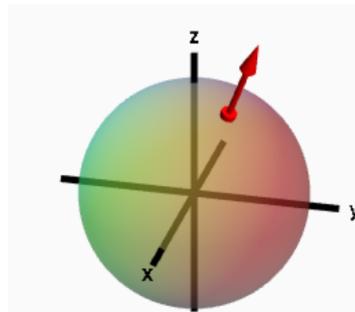
[https://en.wikipedia.org/wiki/Cross\\_product](https://en.wikipedia.org/wiki/Cross_product)

# Normals

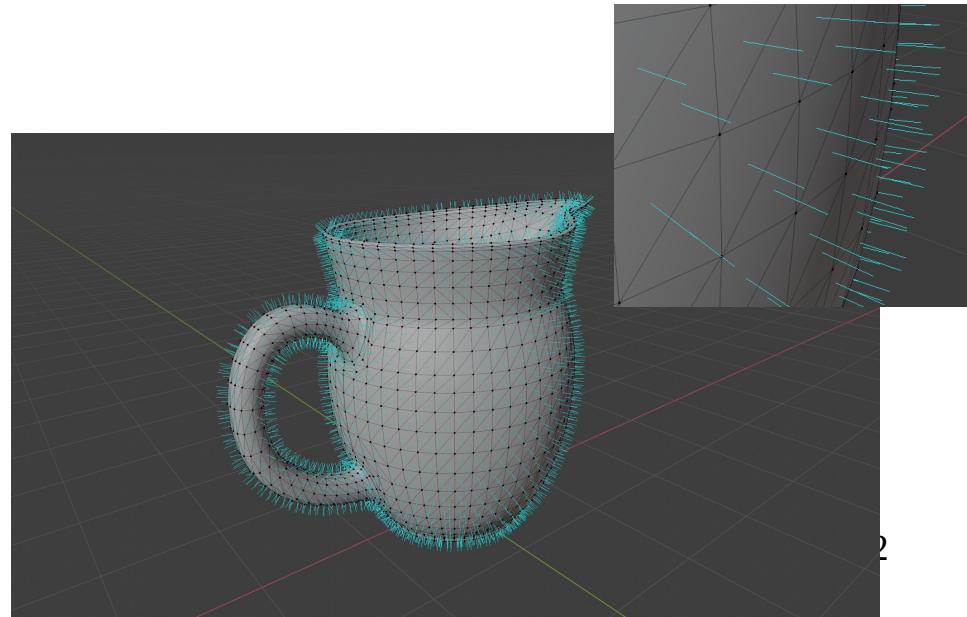
- Normal  $(x, y, z)$  describes **orientation of surface** of a geometric object at a point
  - Perpendicular to surface at a point
  - Similar to vectors but they are defined in relationship to a particular surface: special care needed when **applying transformations**
- It can be defined as cross product of any two non-parallel tangent vectors at that point.



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<https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/geometry/points-vectors-and-normals.html>

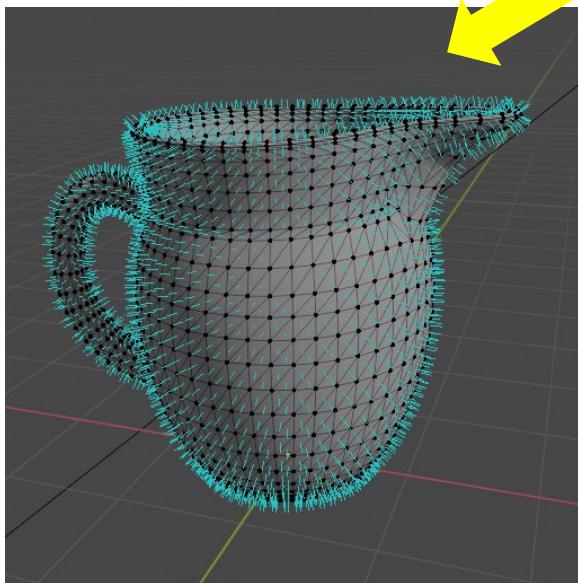


[https://mathinsight.org/parametrized\\_surface\\_orient](https://mathinsight.org/parametrized_surface_orient)



# Normals

- Crucial information for rendering and modeling
  - Example: **brightness of object** – (clamped\*) dot product of surface normal and light direction

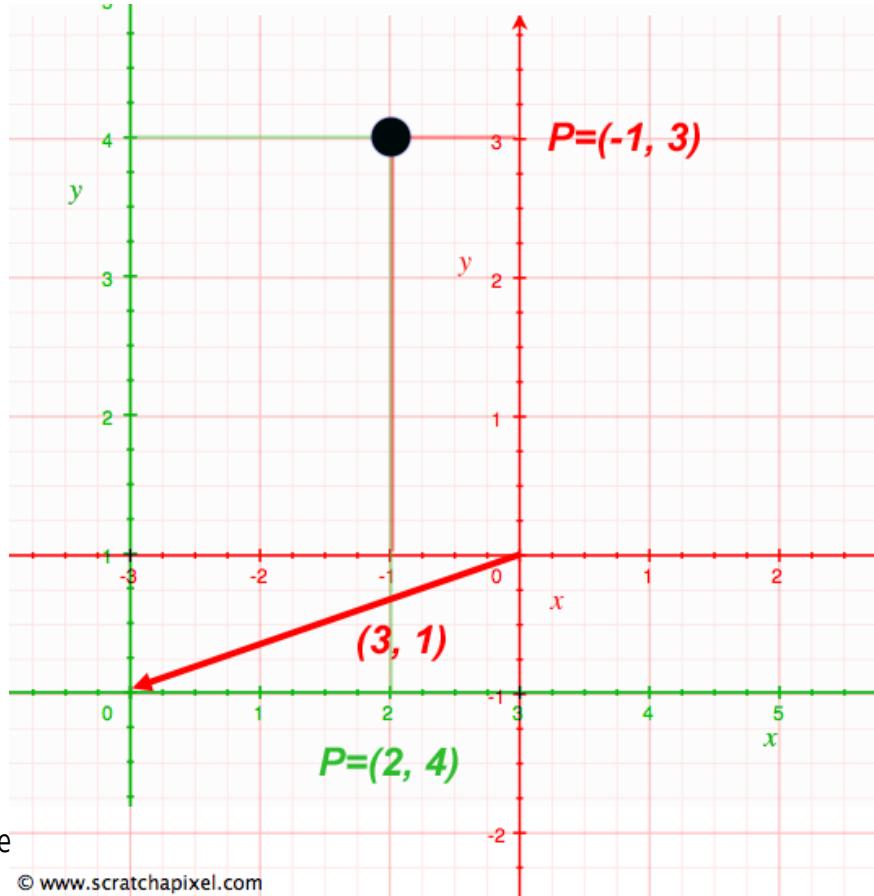


\* Convention is to define light and view vectors facing away from surface. If dot product is negative, clamping operation sets result to 0

# Coordinate system

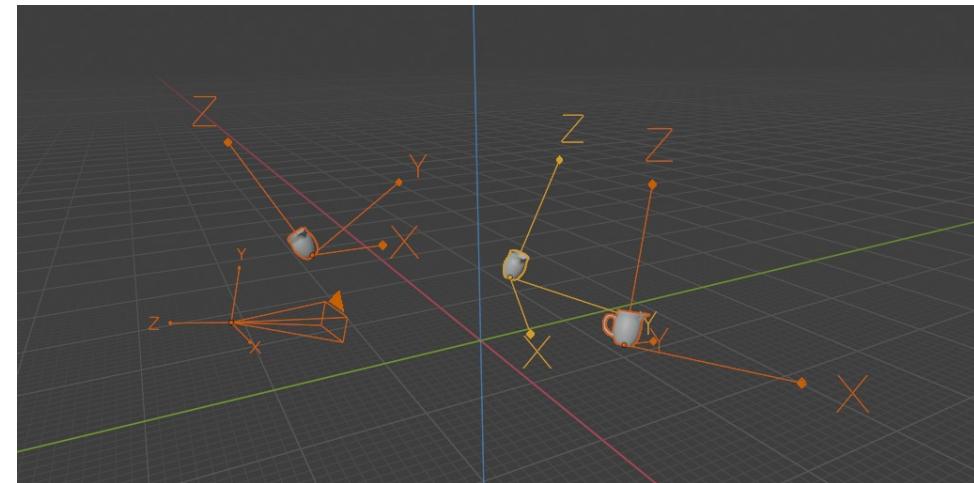
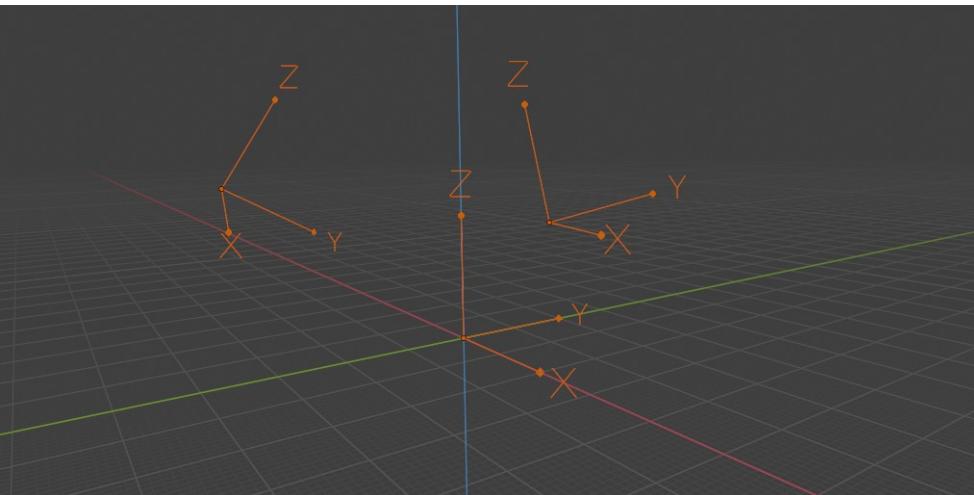
- Coordinate system is defined with **origin** (point) and **basis** (three unit vectors).
  - But points and vectors depend on coordinate system!?
- Infinite number of coordinate systems can be defined
  - Referent coordinate system must be defined!

Point or vector in 3D space depend on its relationship to the frame – point can have same absolute position in space but its coordinates depend on frame.



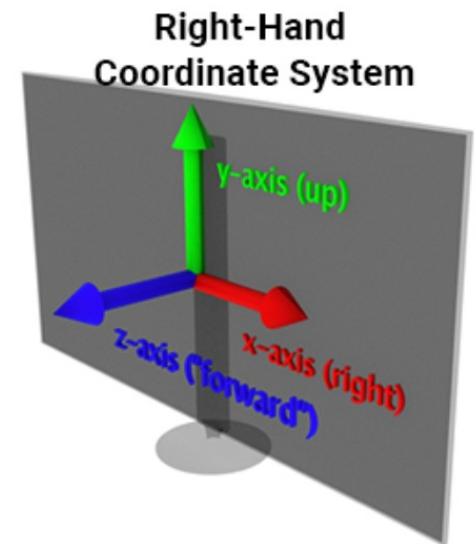
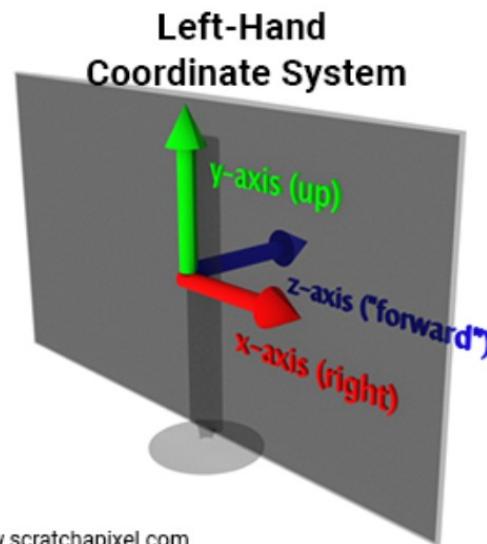
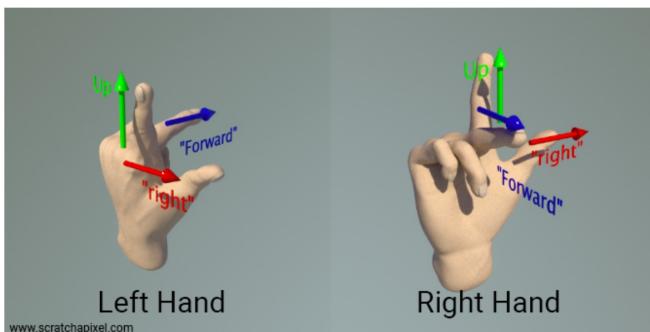
# World coordinate system

- World coordinate system – a standard frame (**origin and basis**)
- All other – frames, **local coordinate systems**, are defined with respect to world coordinate system



# Coordinate system handedness - notation

- Axis of X, Y and Z vectors defining coordinate system can face in one of two directions:
  - Left-handed coordinate system
  - Right-handed coordinate system



<https://www.scratchapixel.com/lessons/mathematics-physics-for-computer-graphics/geometry/coordinate-systems.html>

# Coordinate system handedness and naming - notation

- Handedness of coordinate system: orientation of **left/right**, **up** and **forward** vectors
- Labeling axis (e.g., X, Y or Z) is matter of convention:
  - Industry standard: right-handed coordinate system: x – right, y – up and z – outward

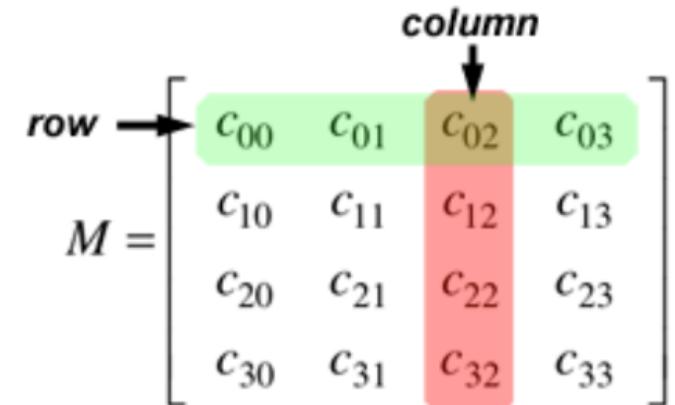


Exporting from one application to another requires special care of coordinate systems!

<https://www.techarthub.com/a-guide-to-unitys-coordinate-system-with-practical-examples/>

# Matrices

- Points and vectors are moved in space using **linear transformations**
  - multiplication with matrix
    - Transformations: scaling, rotation and translation
    - Multiplying point or vector with matrix returns transformed point or vector
- Matrix ( $M$ ): 2D array of numbers:  $m \times n$  – number of **rows** ( $m$ ) and **columns** ( $n$ )
  - $M_{ij}$  – matrix element at  $(i, j)$  position
- For computer graphics **square matrices**  $3 \times 3$  and  $4 \times 4$  are most important since all transformations can be described with them



# Row major and column major - notation

- Matrices can be written as:
  - Row major order (Direct X, Maya, PBRT)
  - Column major order (OpenGL)

$$\begin{bmatrix} \color{red}{c_{00}} & \color{red}{c_{01}} & \color{red}{c_{02}} & 0 \\ \color{green}{c_{10}} & \color{green}{c_{11}} & \color{green}{c_{12}} & 0 \\ \color{blue}{c_{20}} & \color{blue}{c_{21}} & \color{blue}{c_{22}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \color{red}{c_{00}} & \color{green}{c_{01}} & \color{blue}{c_{02}} & 0 \\ \color{red}{c_{10}} & \color{green}{c_{11}} & \color{blue}{c_{12}} & 0 \\ \color{red}{c_{20}} & \color{green}{c_{21}} & \color{blue}{c_{22}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

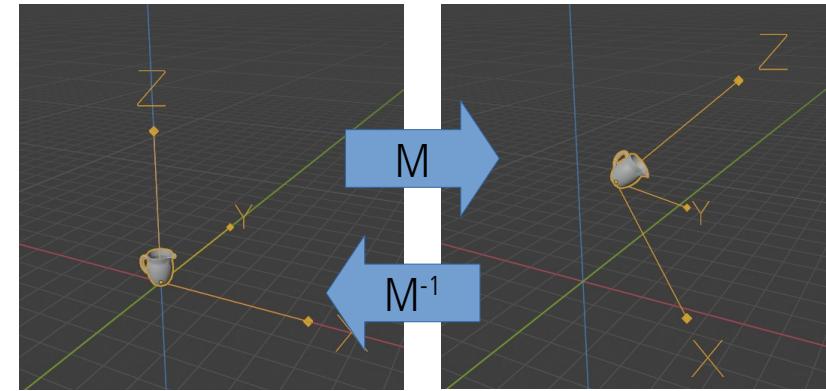
# Matrix operations

- **Matrix-matrix multiplication** gives matrix
  - Useful for representing multiple transforms with one matrix
  - Not commutative: order of multiplications, thus transforms is important!

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{bmatrix}$$

# Matrix operations

- Operators:
  - **Inverse**: multiplying point A with matrix M given point B. Multiplying point B with inverse of matrix M gives point A.
    - $MM^{-1} = I$  (identity matrix)
    - Gauss-Jordan method\*
  - **Transpose**: switch row and column indices of a matrix. Row-major to column-major and vice versa



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity  
matrix ( $I$ )

Row-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Column-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

# Matrix-point multiplication

- Matrix-point/vector multiplication gives new point/vector → transformation
  - Row-major or column-major convention dictates matrix multiplication order

Row major order: point is on left side

$$[x \ y \ z] * \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad x' = x * a + y * d + z * g \\ y' = x * b + y * e + z * h \\ z' = x * c + y * f + z * i$$

$$P' = P * T * R_z * R_y$$

Column major: point is on right side

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$P' = R_y * R_z * T * P$$

Row-major points and vectors are written as [1x3] or [1x4] matrices and then multiplied with matrix which is [3x3] or [4x3]

# Homogeneous coordinate

- Point  $(x, y, z)$  can be written as  $[1 \times 3]$  matrix
  - Often, transformation matrices are  $[4 \times 4]$
- To multiply point with transformation matrices, write points as **homogeneous point/coordinate**:  $(x, y, z, w)$ ,  $[1 \times 4]$ .
  - If the resulting  $w$  coordinate is not 1, then  $x, y, z$  must be divided by  $w$  to obtain usable Cartesian point.

$$(x, y, z, w) \rightarrow \left( \frac{x}{w}, \frac{y}{w}, \frac{z}{w} \right).$$

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \\ 1 \end{pmatrix} = \begin{pmatrix} v_x + t_x \\ v_y + t_y \\ v_z + t_z \\ 1 \end{pmatrix}$$

Column-major notation: matrix-point multiplication

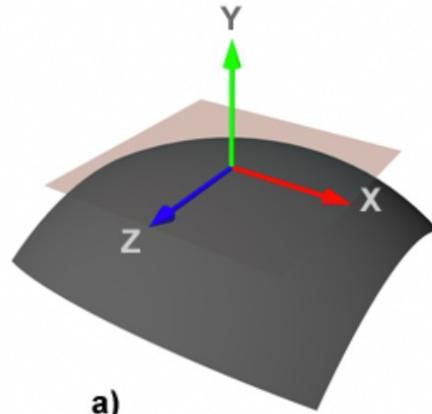
# Local coordinate system

- Local coordinate system can be constructed using single vector using cross product.
- Often, local coordinate system using normal is constructed
  - Normal is one axis of local coordinate system
  - Tangent and bi-tangent are other to axes
  - Example: Normal corresponds to up vector, tangent to right vector and bi-tangent to forward vector

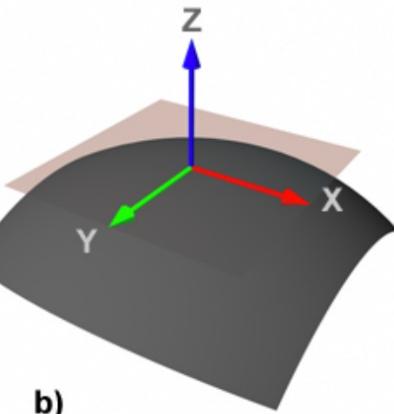
Up is Y

$$\begin{bmatrix} T_x & T_y & T_z & 0 \\ N_x & N_y & N_z & 0 \\ B_x & B_y & B_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row-major notation



a)



b)

Up is Z

$$\begin{bmatrix} T_x & T_y & T_z & 0 \\ B_x & B_y & B_z & 0 \\ N_x & N_y & N_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

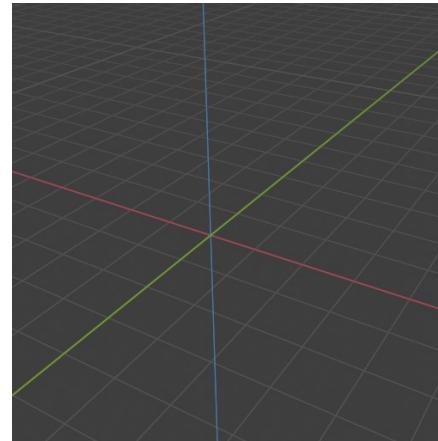
Row-major notation

# Matrix and coordinate system

- Matrix can represent basis of a coordinate system → **orientation matrix**
  - Each row of matrix represents an axis of coordinate system – orthogonal vectors → **orthogonal matrix**
  - Inverse of orthogonal matrix is equal to its transpose

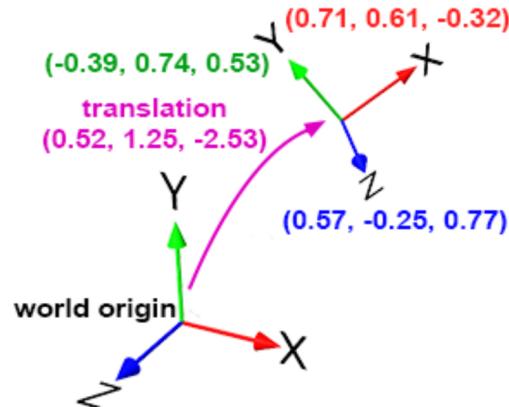
$$\begin{bmatrix} c_{00} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{bmatrix} \rightarrow \begin{array}{l} x - \text{axis} \\ y - \text{axis} \\ z - \text{axis} \end{array}$$

Row-major notation



# Transform, matrix and coordinate system

- Transformation: take points, vectors and normals defined with respect to one coordinate frame and map their coordinate values to another frame.
- Transformation of point from one coordinate system to another is done by **matching their origins and basis**.
- Transformation from one to other coordinate system is performed with transformation matrix [4x4]
  - By characterizing how the frame is transformed, we know how any point or vector specified in terms of that frame is transformed
  - Points and vectors are expressed in terms of current coordinate system frame. Applying transformation matrix to points and vectors is equivalent to applying transformation matrix to current coordinate system frame

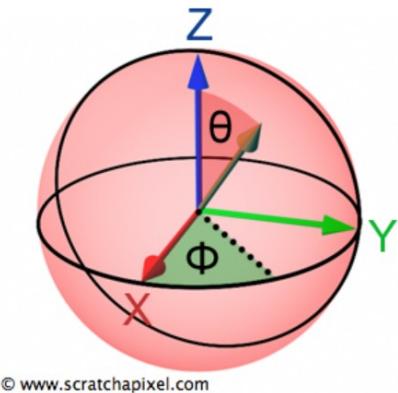


$$\begin{bmatrix} c_{00} & c_{01} & c_{02} & 0 \\ c_{10} & c_{11} & c_{12} & 0 \\ c_{20} & c_{21} & c_{22} & 0 \\ c_{30} & c_{31} & c_{32} & 1 \end{bmatrix} \rightarrow \begin{array}{l} x-axis \\ y-axis \\ z-axis \\ translation \end{array}$$

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# Spherical coordinate system

- Spherical coordinates simplify computation needed for rendering
- Representing vectors in spherical coordinate system
  - Two angles: altitude (polar) angle  $\Theta$  [ $0, \pi$ ] and azimuth angle  $\Phi$  in  $[0, 2\pi]$



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Vector representation in spherical coordinate system using polar and azimuth angle.

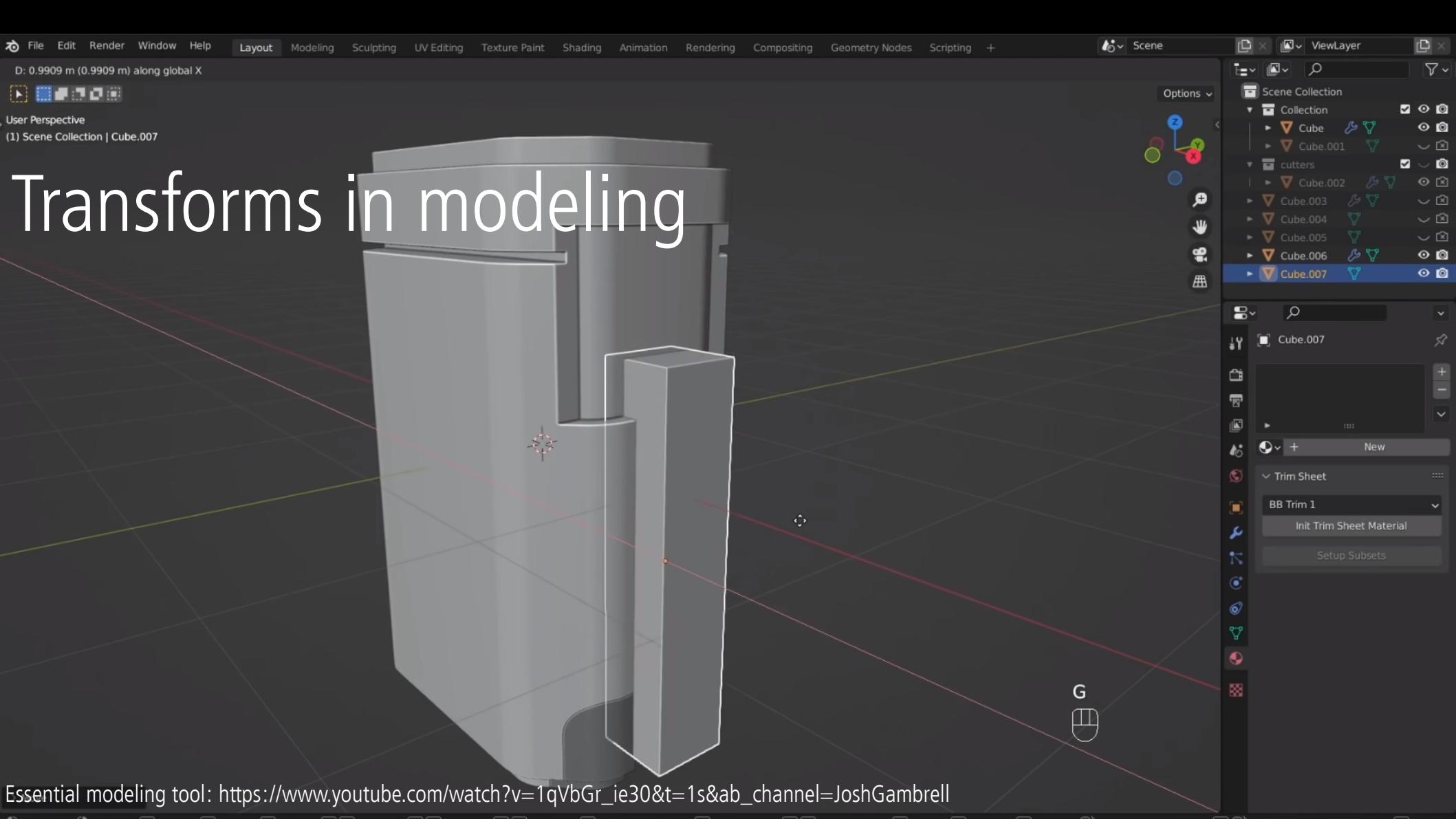
$$\begin{aligned}x &= \cos(\phi) \sin(\theta) \\y &= \sin(\phi) \sin(\theta) \\z &= \cos(\theta)\end{aligned}$$

Converting from spherical to Cartesian coordinate system

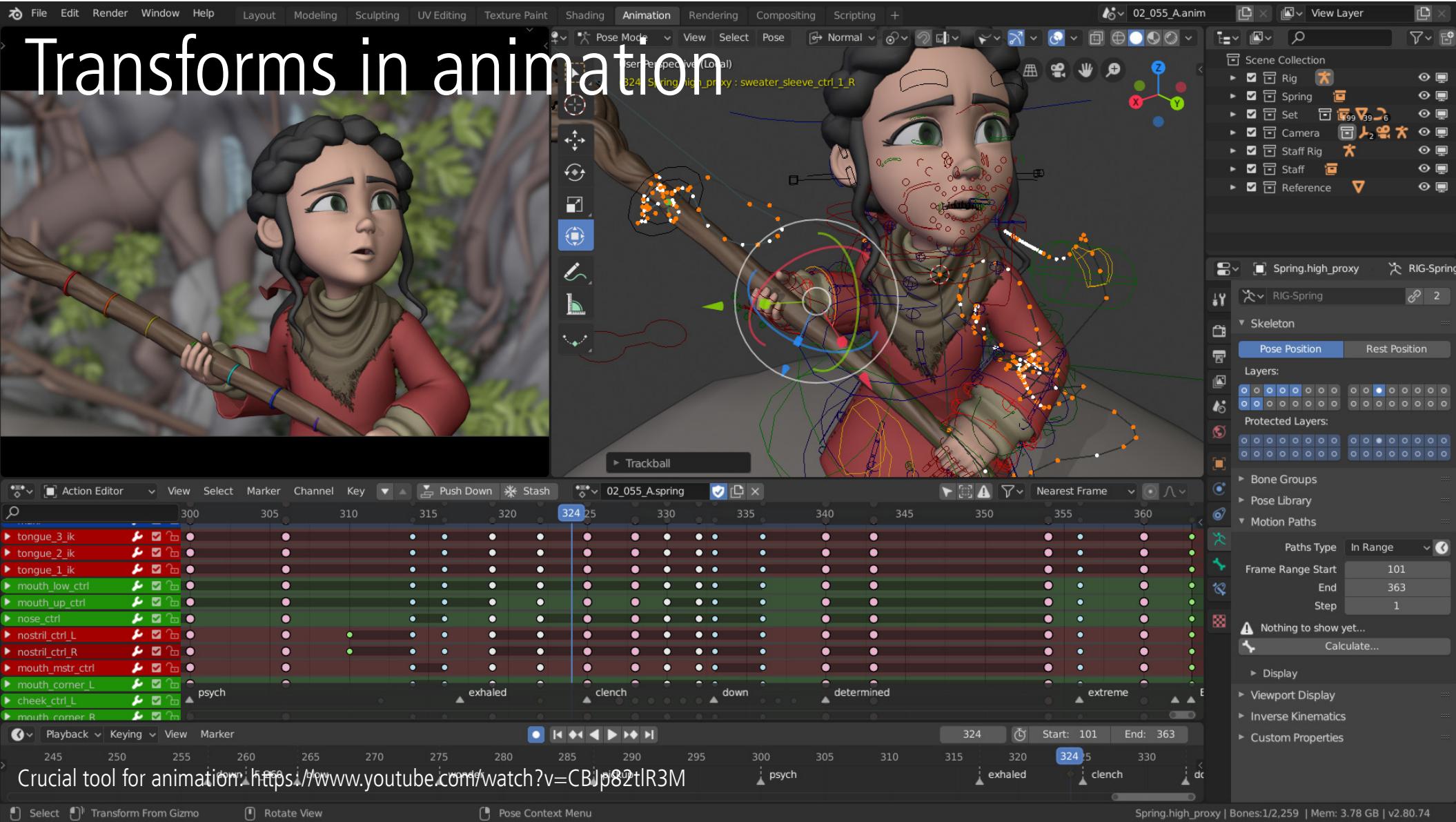
# Transforms

# Transforms

- Basic tool for manipulating 3D scene elements; points and vectors:
  - Position, orientate, reshape and animate objects, lights and cameras (3D scene modeling)
  - Essential for rendering computations
- **Linear transforms:** scaling, rotation
- **Affine transforms:** translation, rotation, scaling, reflection, shearing
  - Preserve parallelism of lines but not necessary lengths and angles
  - Require homogeneous point notation



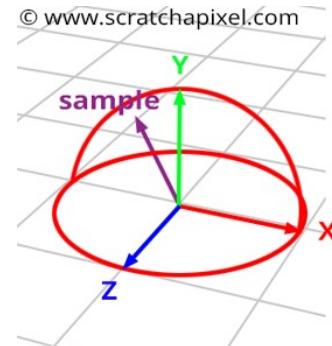
Essential modeling tool: [https://www.youtube.com/watch?v=1qVbGr\\_ie30&t=1s&ab\\_channel=JoshGambrell](https://www.youtube.com/watch?v=1qVbGr_ie30&t=1s&ab_channel=JoshGambrell)



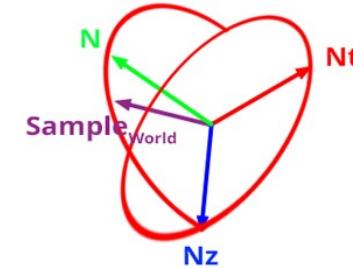
Crucial tool for animation: <https://www.youtube.com/watch?v=CBjIp82tlR3M>

# Transforms in rendering

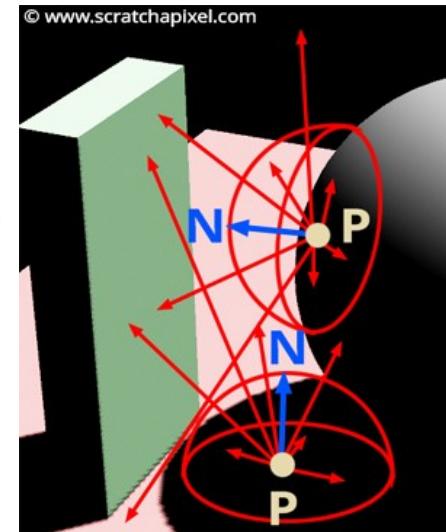
- All calculations must be preformed in the same coordinate system
  - Example: light-surface interaction calculation (shading)
  - Example: moving objects in camera space for easier calculations
- Another example is projecting objects onto plane which is used for rasterization-based rendering.



generate sample over hemisphere



convert sample to world space  
(N is up vector)



# Transforms in professional software

- Godot: [https://docs.godotengine.org/en/stable/classes/class\\_transform.html](https://docs.godotengine.org/en/stable/classes/class_transform.html)
- Unity: <https://docs.unity3d.com/ScriptReference/Transform.html>
- Unreal: <https://docs.unrealengine.com/4.27/en-US/BlueprintAPI/Math/Transform/>
- Blender: [https://docs.blender.org/manual/en/latest/scene\\_layout/object/properties/transforms.html](https://docs.blender.org/manual/en/latest/scene_layout/object/properties/transforms.html)
- GLM: <https://github.com/g-truc/glm>



A screenshot of the Godot Engine documentation page titled "Using 3D transforms". The page features a blue header with the Godot logo and navigation links. The main content area contains sections like "Introduction" and "Euler Angles", with explanatory text and code snippets.

A screenshot of the Blender 3.4 Manual page for "Transform". The page has a dark header with the Blender logo and navigation links. The main content area is titled "Transform" and includes sections for "Reference", "Mode: Object Mode", "Panel: Properties &gt; Object Properties &gt; Transform", and "Panel: 3D Viewport &gt; Sidebar &gt; Transform". Below this, there is a detailed description of the Transform panel's functionality.

The *Transform* panel in the Sidebar region allows you to view and manually/numerically control the position, rotation, and other properties of an object, in *Object Mode*. Each object stores its position, orientation, and scale values. These may need to be manipulated numerically, reset, or applied. In *Edit Mode*, it mainly allows you to enter precise coordinates for a vertex, or median position for a group of vertices (including an edge/face). As each type of object has a different set of options in its *Transform* panel in *Edit Mode*, see their respective descriptions in the *Modeling chapter*.

# Basic transformations

- Modeling elements, 3D scene and animations relies on transformations
  - Translation
  - Rotation
  - Scale
  - Shear

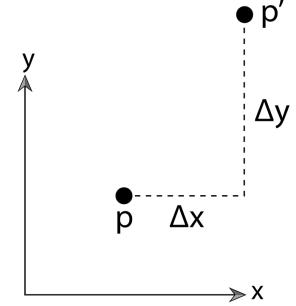
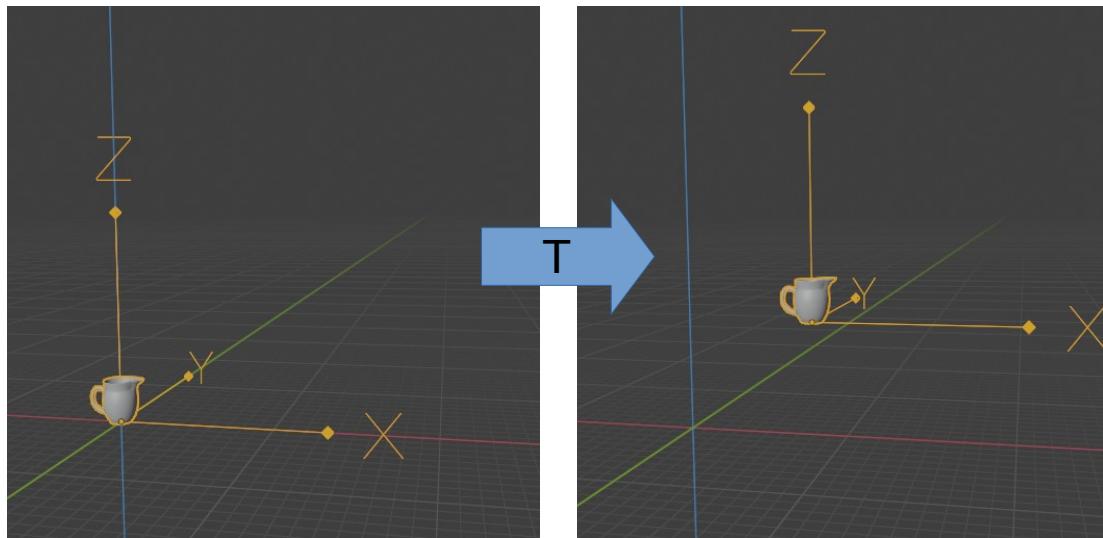
# Identity transform

- Identity transformation is default transformation
- Represented by **identity matrix (I)**
  - $A * I = A$ ,  $A$  – vector or point

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

# Translation transform

- Changes point from one location to another
  - Translation transform translates coordinates of point  $P(x, y, z)$  by  $(\Delta x, \Delta y, \Delta z)$
- Represented by **translation matrix T**



$$T(\Delta x, \Delta y, \Delta z) = \begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Translation matrix (T)

$$\begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{pmatrix}.$$

Translation matrix on point.

# Transforming points and vectors

- Transformation matrices are [ 4x4 ] matrices.
- Homogeneous notation is needed:
  - Points:  $P(x, y, z, 1)$
  - Vectors:  $V(x, y, z, 0)$
- Translation only affects points, leaving vectors unchanged!

$$\begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{pmatrix}.$$

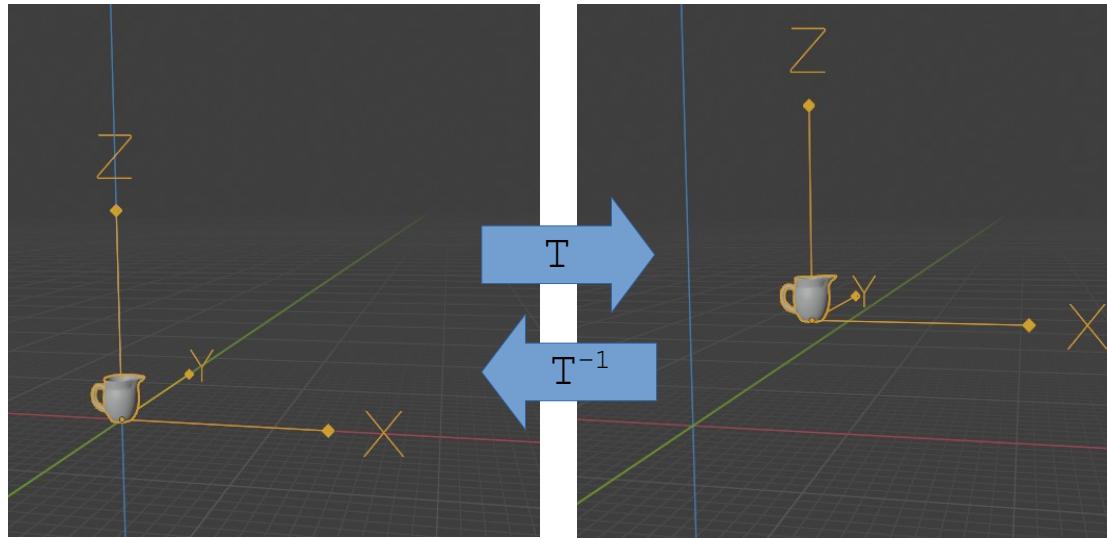
Translation matrix on point.

$$\begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}.$$

Translation matrix on vector – no change!

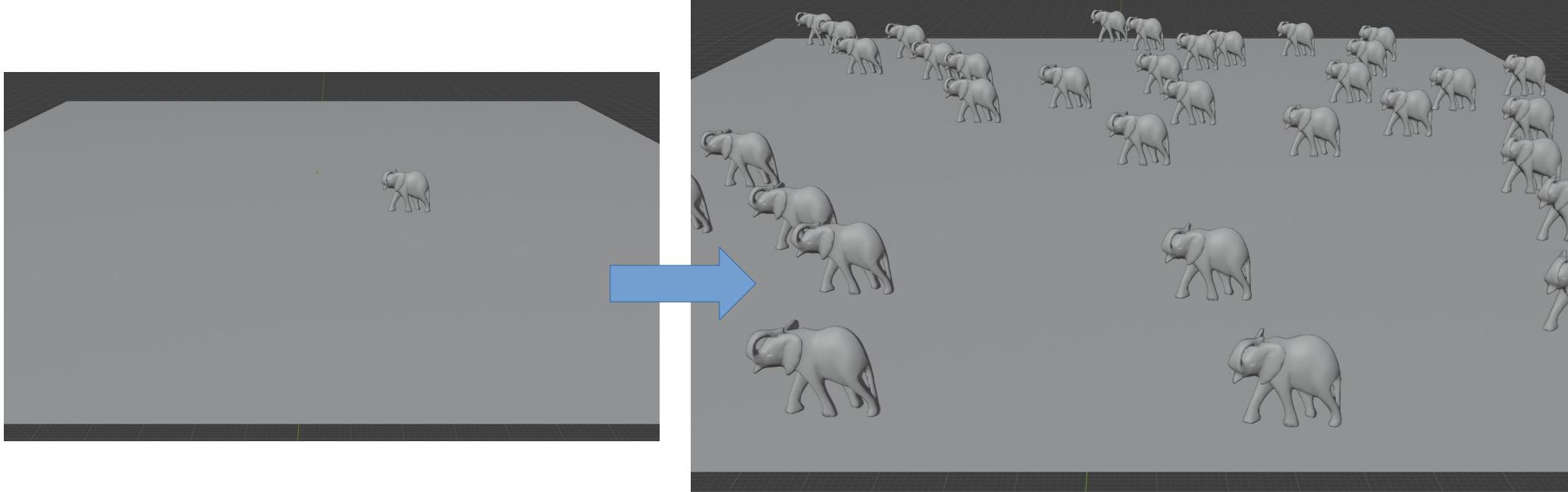
# Translation transform

- Inverse:  $T^{-1}(t) = T(-t)$
- **Rigid-body transform:** preserves distances between points and headedness



# Translation transform and instancing

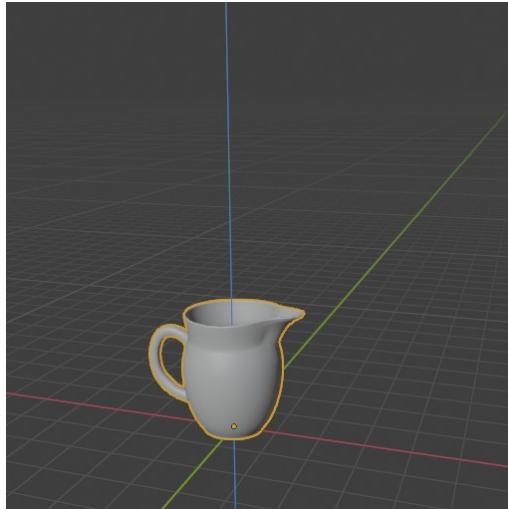
- Different transformations can be assigned to same object in order to place copies of object in the scene - **instancing**



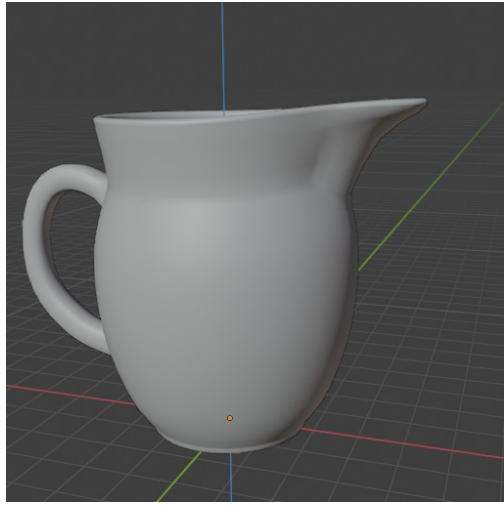
# Scale transform

- Enlarging or diminishing objects
  - Multiply points  $P$  or vectors  $v$  components  $(x, y, z)$  by scale  $(s_x, s_y, s_z)$
  - Represented by **scaling matrix  $S$**
- If all scaling factors are same: **uniform** (isotropic) scaling, else **non-uniform** (anisotropic).

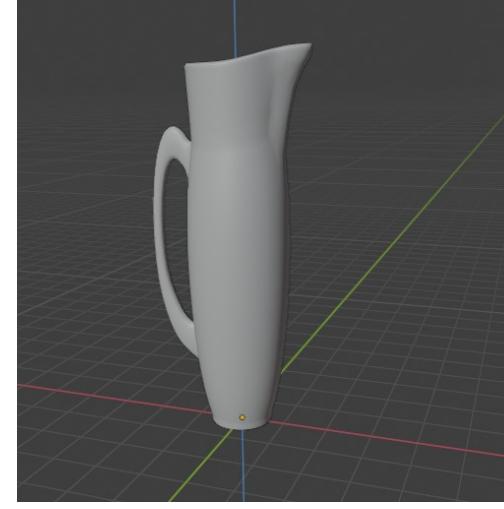
$$S(x, y, z) = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$



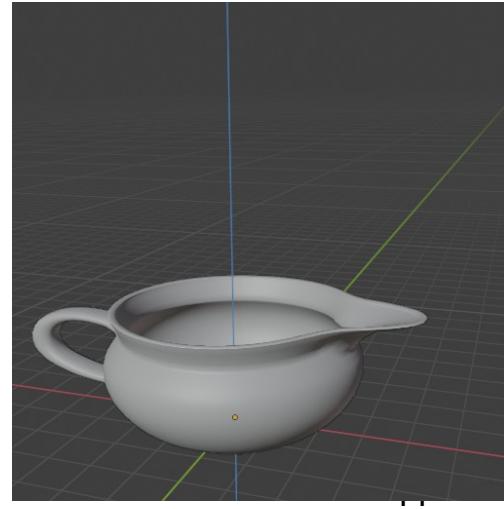
Original



Uniform



Anisotropic (z)

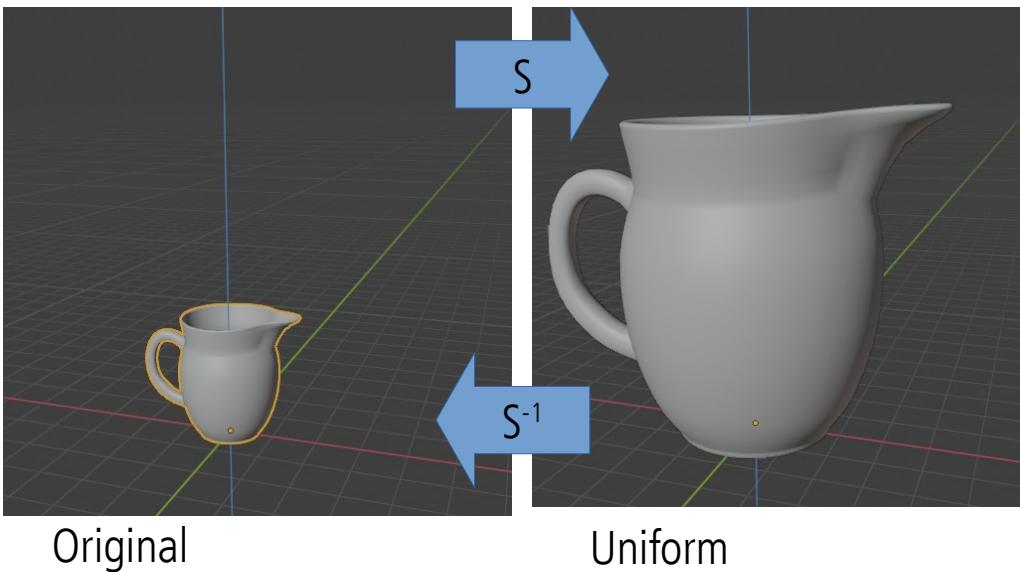


Anisotropic (x,y)

# Scale transform

- Inverse

$$\mathbf{S}^{-1}(x, y, z) = \mathbf{S} \left( \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \right).$$

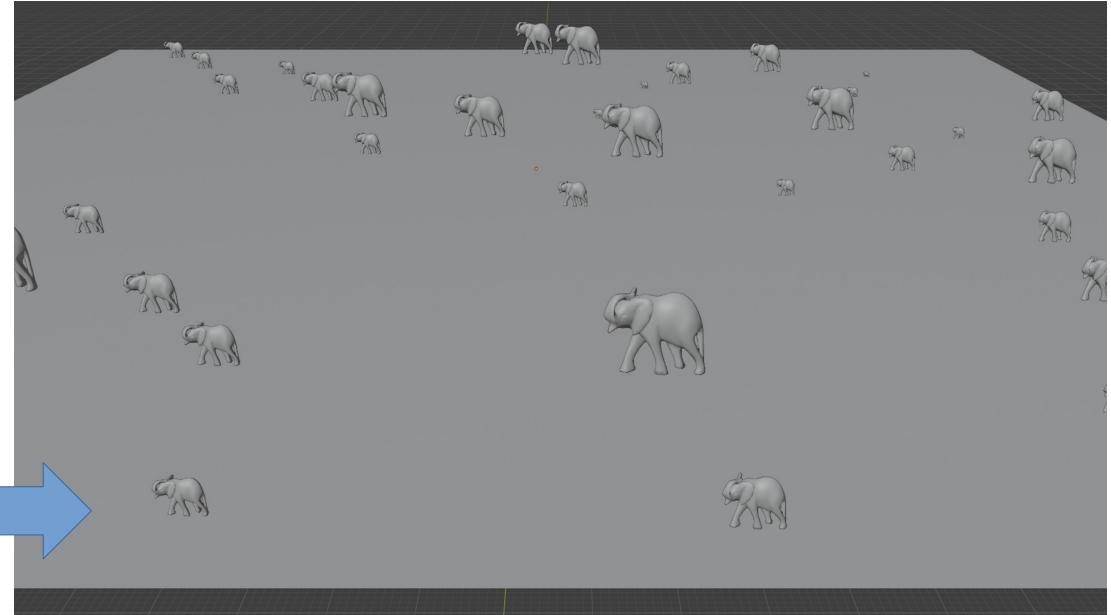
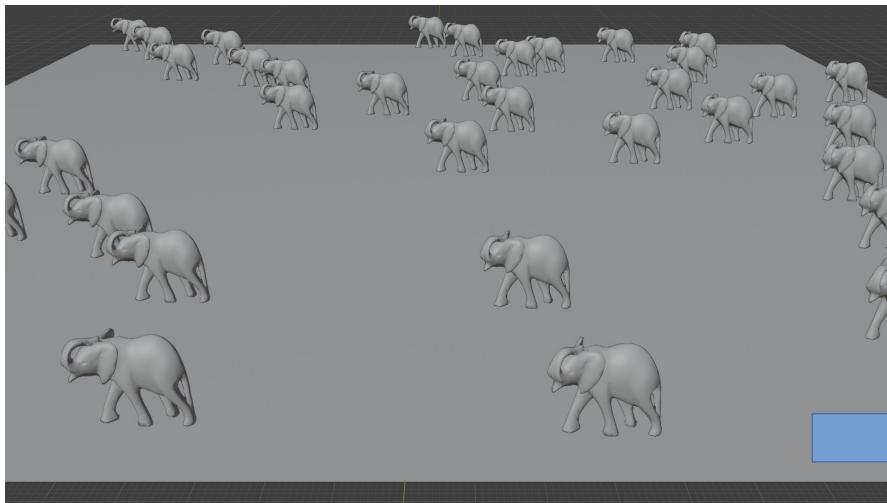


Negative value of scaling factor gives **reflection (mirror)** matrix

- Triangle with clockwise orientation will have counter-clockwise orientation after reflection

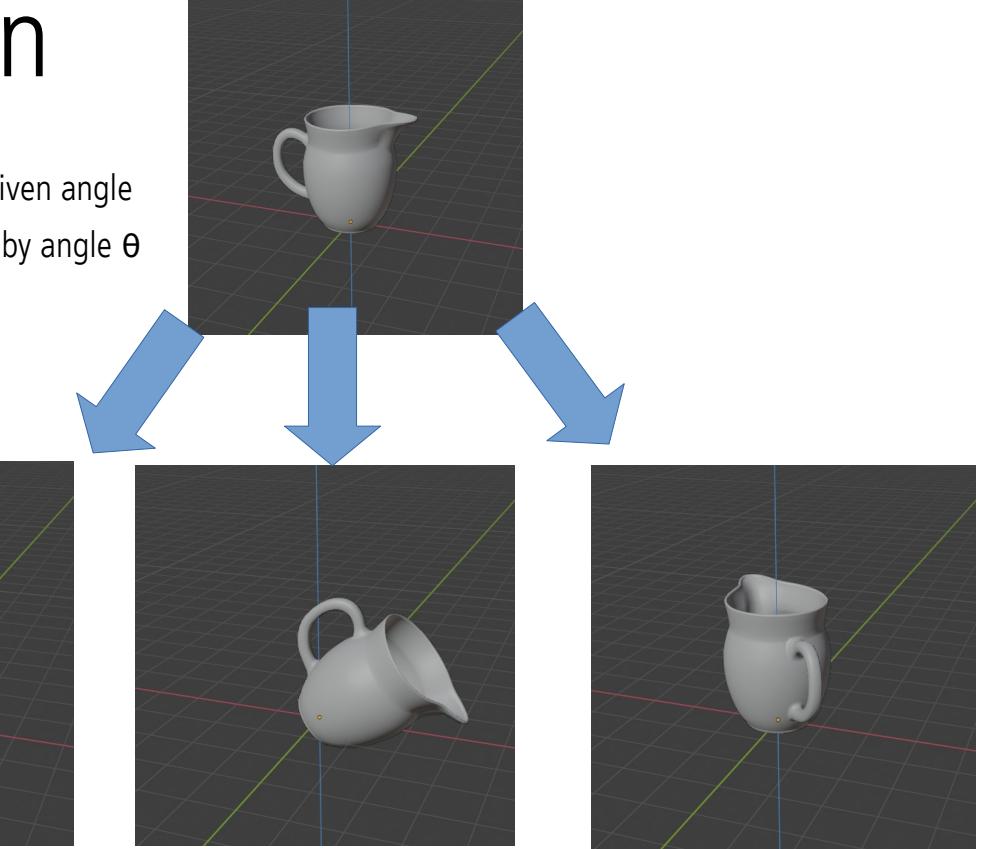
# Scale transform and instancing

- Application of different scale and translation matrices for **instancing**



# Rotation transformation

- Rotates objects around arbitrary axis from origin to any direction for a given angle
  - Most common rotations: rotation around X, Y and Z coordinate axes by angle  $\theta$
  - Represented by **rotation matrices**  $R_x(\theta)$ ,  $R_y(\theta)$ ,  $R_z(\theta)$
  - Clockwise vs counter-clockwise rotation by angle  $\theta$



left-handed coordinate system, the matrix for clockwise rotation

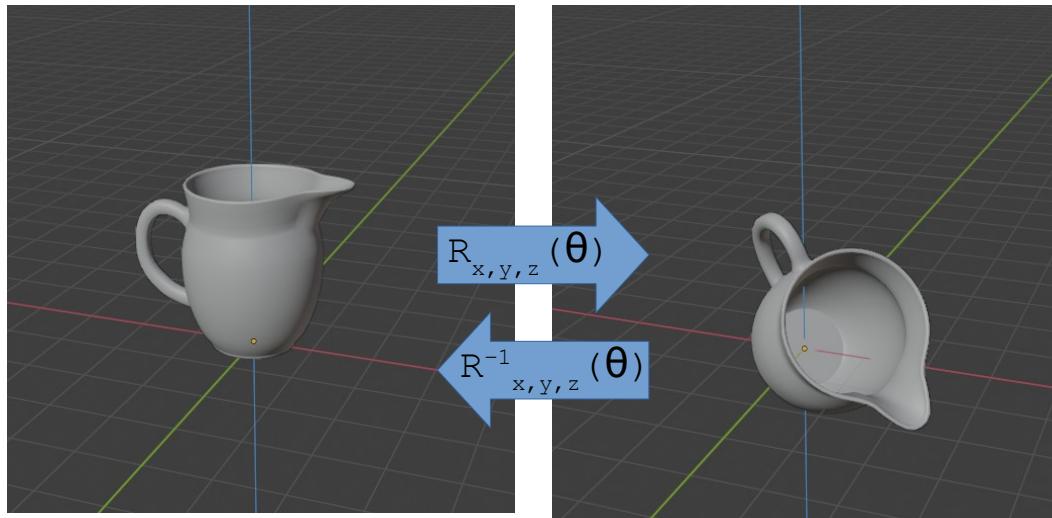
$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

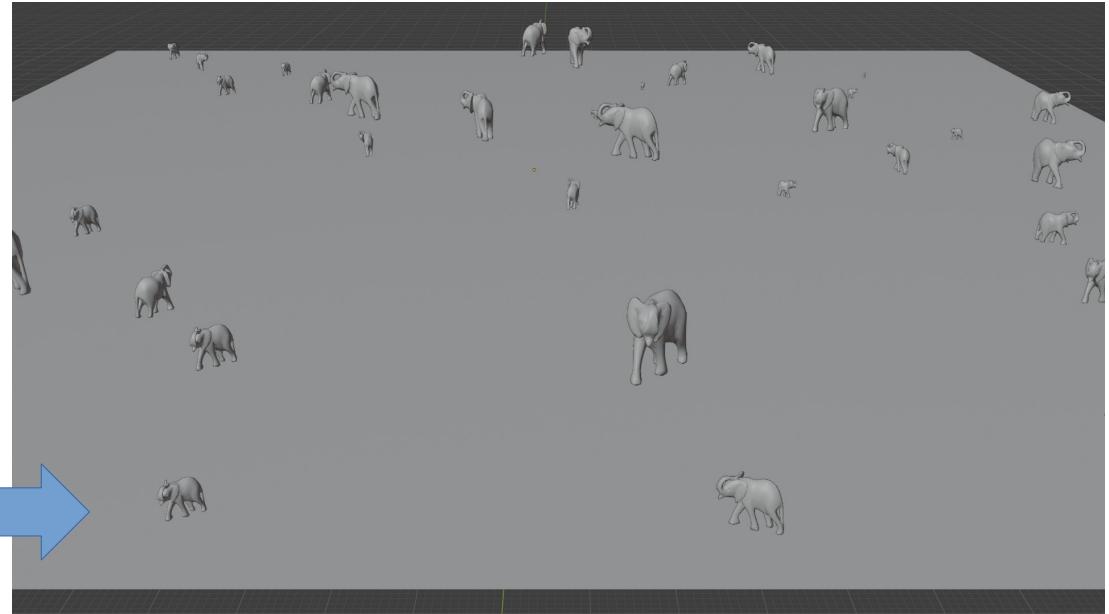
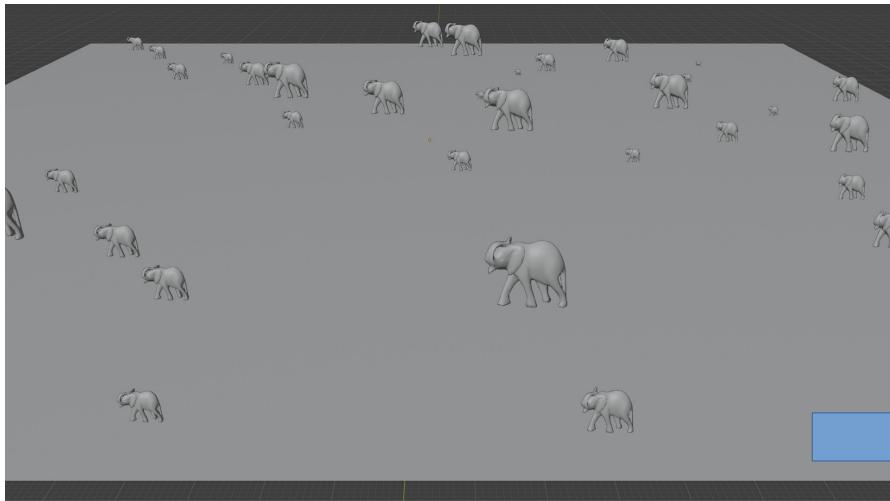
# Rotation transformation

- **Rigid-body transform:** preserve distance between points and headedness
- **Inverse**  $\mathbf{R}_a^{-1}(\theta) = \mathbf{R}_a(-\theta) = \mathbf{R}_a^T(\theta)$ ,
- **Rotation around arbitrary axis**
  - Rotation around arbitrary axis  $(x, y, z)$  is represented by rotation matrix  $\mathbf{R}_{x, y, z}(\theta)$  – a combination of  $\mathbf{R}_x(\theta)$ ,  $\mathbf{R}_y(\theta)$ ,  $\mathbf{R}_z(\theta)$
  - Matrix consist of unit vectors → **orthogonal matrix**

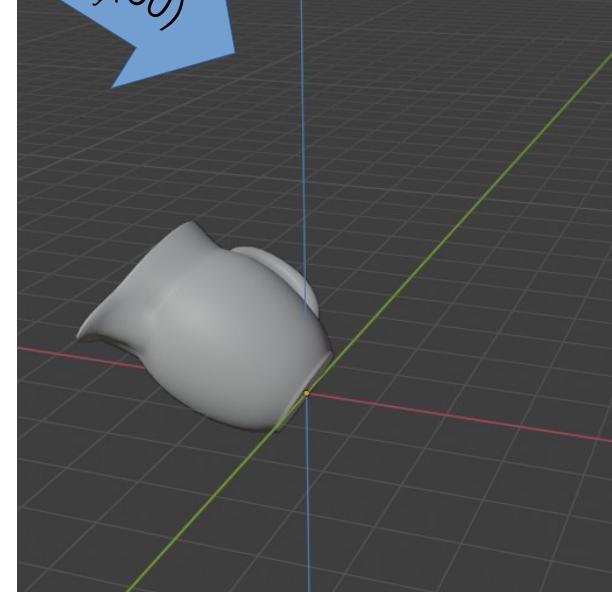
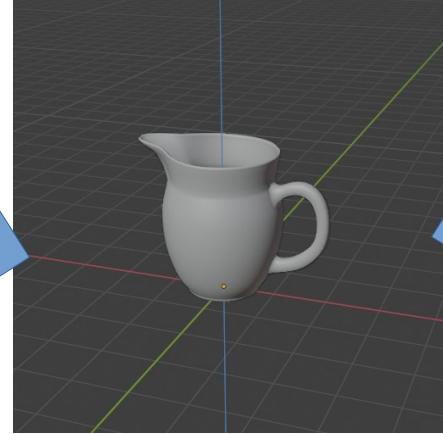
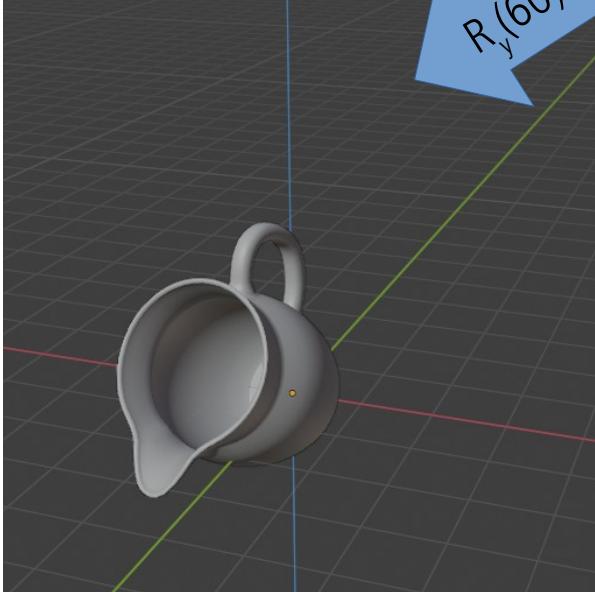


# Rotate transform and instancing

- Application of different scale, translation and rotation matrices for **instancing**



# Combining rotation matrices

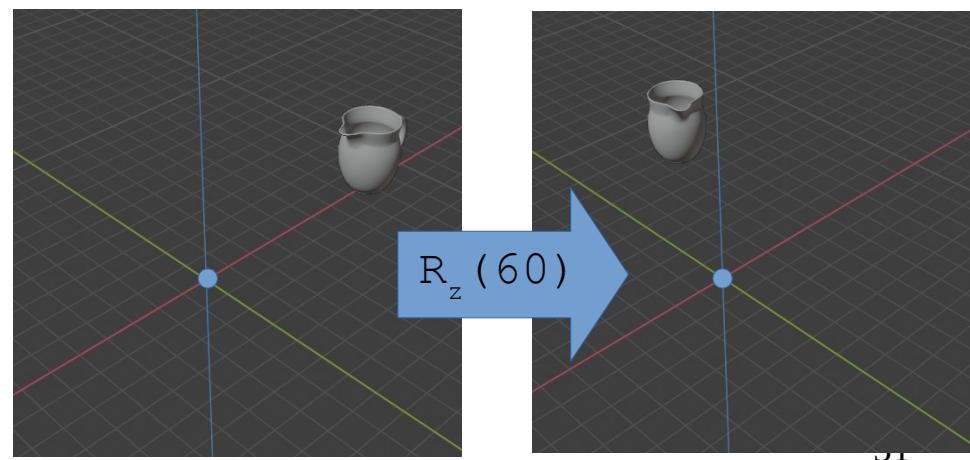
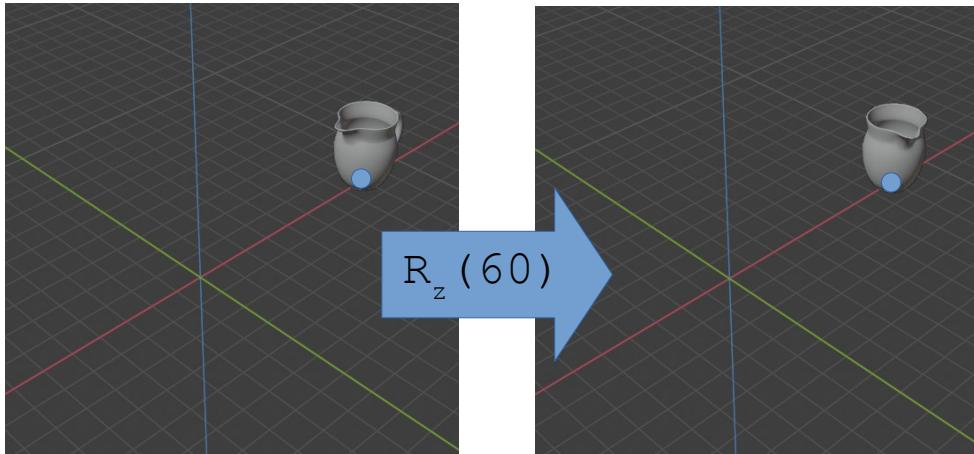


$R_y(60) * R_z(80)$

$R_z(80) * R_y(60)$

# Rotation and translation

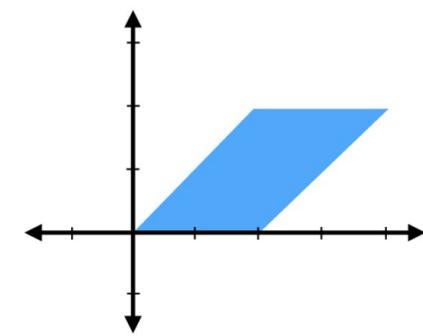
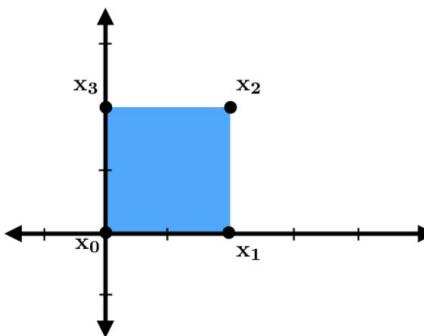
- If object is not in world origin, rotation origin can be local or world coordinate system
  - Pivot point
- For orienting object which is not in world origin
  - object is first translated so that pivot point in the center and then rotation is performed after which object is again transformed so that pivot point is in the same position as before.



# Shear transform

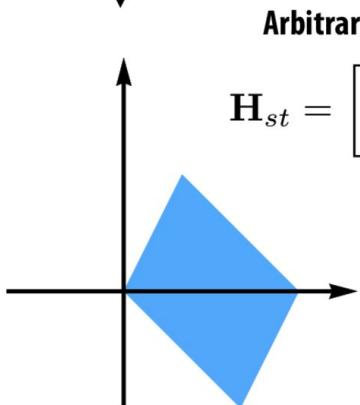
- 6 basic shearing matrices for 3D  
case: xy, xz, yx, yz,  
zx, zy

## Shear



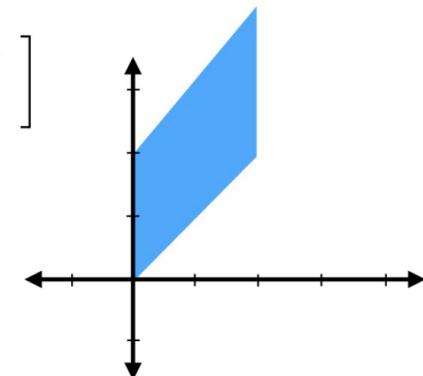
Shear in x:

$$\mathbf{H}_{xs} = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$



Arbitrary shear:

$$\mathbf{H}_{st} = \begin{bmatrix} 1 & s \\ t & 1 \end{bmatrix}$$

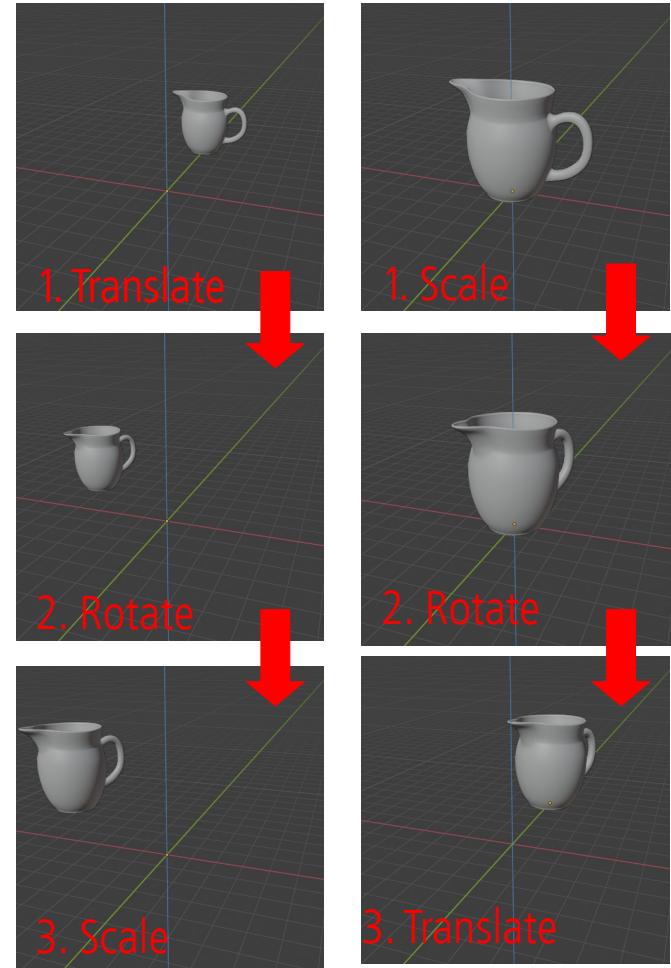
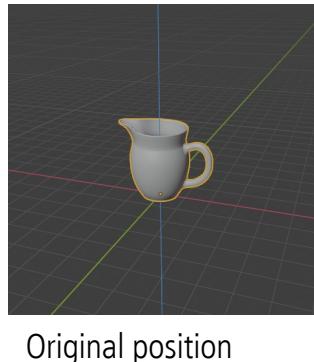


Shear in y:

$$\mathbf{H}_{ys} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

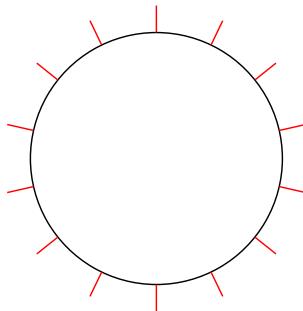
# Concatenation of transforms

- Matrix multiplication is non-commutative:  
**order of multiplication matters**
- Example: objects in 3D scene must be scaled, rotated and translated
  - Matrix is concatenated into one matrix which is used for multiplication of points P
  - Such matrix must be composed as :
    - $C = \text{TRS}$ ,  $C * P = \text{TRS} * P$
    - Scaling is applied first, then rotation and finally translation
- Concatenation of only translation and rotation matrices results in **rigid-body transform**.

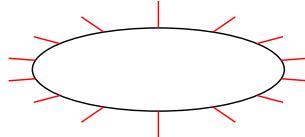


# Special care: transformations of normals

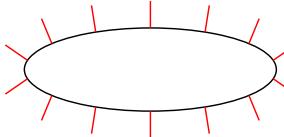
- Normal can not be transformed with the same matrix used for transforming points and vectors.
  - It has to be multiplied with transpose of the inverse of that matrix:  $P' = P * (M^{-1})^T$



Original



Wrong



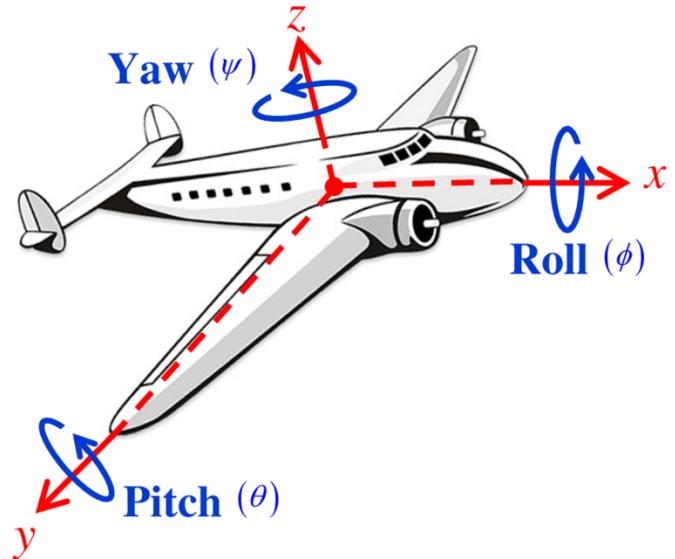
Correct

# Special transforms

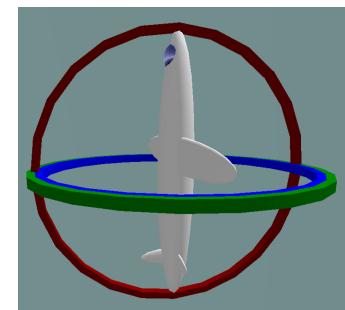
- Euler transform
- Rotation about an arbitrary axis
- Quaternions
- Orthographic projection
- Perspective projection
- Look-at matrix transform

# Euler transform

- Represented by Euler matrix, a concatenation of  $R_x(\theta)$ ,  $R_y(\theta)$ ,  $R_z(\theta)$  – rotation matrices around X, Y and Z axes
- Establish default view, e.g., facing positive X with Z up.
- Angles of rotation: **yaw**, **pitch**, **roll**
- Problems:
  - Gimbal lock: e.g., when pitch and roll become aligned, changes to roll and yaw result in same rotation
  - Two different sets of Euler angles can give same orientation
  - Interpolation between two Euler angle sets is not as simple as interpolating each angle
- Good: angles can be easily extracted from Euler matrix → matrix decomposition



[https://www.researchgate.net/publication/335854843\\_Special\\_Othogonal\\_Group\\_SO3\\_Euler\\_Angles\\_Angle-axis\\_Rodriguez\\_Vector\\_and\\_Unit-Quaternion\\_Overview\\_Mapping\\_and\\_Challenges](https://www.researchgate.net/publication/335854843_Special_Othogonal_Group_SO3_Euler_Angles_Angle-axis_Rodriguez_Vector_and_Unit-Quaternion_Overview_Mapping_and_Challenges)

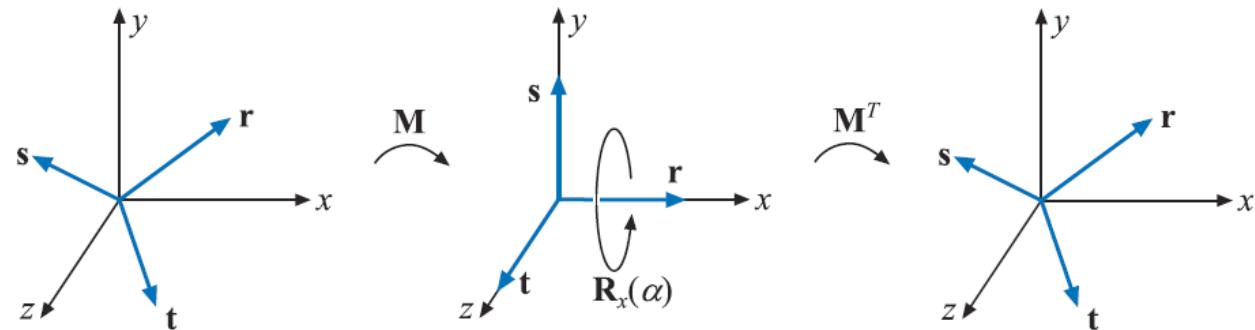


Gimbal lock: one degree of freedom (rotation angle) is lost.  
57  
[https://en.wikipedia.org/wiki/Gimbal\\_lock](https://en.wikipedia.org/wiki/Gimbal_lock)

# Rotation about arbitrary axis

Rotate point by some angle  $\theta$  around arbitrary rotation axis  $r$ :

- 1) Translate space so that rotation axis  $r$  passes through the origin
- 2) Rotate space about X axis so that rotation axis  $r$  lies in the XZ plane
- 3) Rotate space about Y axis so that rotation axis  $r$  lies along Z axis
- 4) Perform rotation around Z axis for angle  $\theta$
- 5) Return to original space using inverse of steps (3), (2), and (1)



# Quaternions

- Practical use: **describing rotation and orientation**
- Orientation is represented by a **rotation around particular axis**

– Quaternion:  $q = s + xi + yj + zk$

- $s$  – scalar

- $i, j, z$  – three spatial axes

- $(x, y, z)$  represent axis

- **Unit quaternion can represents any 3D rotation**

– Rotating point  $p$  using quaternion:  $q * p * q^{-1}$

– Inverse of unit quaternion:  $q^{-1} = s - xi - yj - zk$

– Often, quaternion is transformed into matrix and then multiplied with point

- **Interpolation between two quaternions is stable and constant**

– GLM: <https://glm.g-truc.net/0.9.0/api/a00135.html>

– Blender: <https://docs.blender.org/api/current/mathutils.html#mathutils.Quaternion>

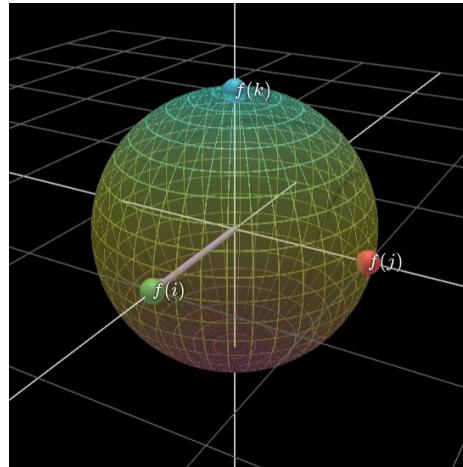
– Unity: <https://docs.unity3d.com/ScriptReference/Quaternion.html>

$$\begin{bmatrix} 1 - 2q_yq_y - 2q_zq_z & 2q_xq_y - 2q_wq_z & 2q_xq_z + 2q_wq_y & 0 \\ 2q_xq_y + 2q_wq_z & 1 - 2q_xq_x - 2q_zq_z & 2q_yq_z - 2q_wq_x & 0 \\ 2q_xq_z - 2q_wq_y & 2q_yq_z + 2q_wq_x & 1 - 2q_xq_x - 2q_yq_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Quaternions

- Intuitive representation:
  - $q = \cos(\theta/2) + \sin(\theta/2) * (xi + yj + zk)$
  - Rotation around axis defined with  $(x, y, z)$  for angle  $\theta$

```
q = cos(-30°) + sin(-30°)(1.00i + 0.00j + 0.00k)  
p = [i × j × k × ijk sphere ×]  
q⁻¹ = cos(30°) + sin(30°)(1.00i + 0.00j + 0.00k)  
f(p) = q · p · q⁻¹
```



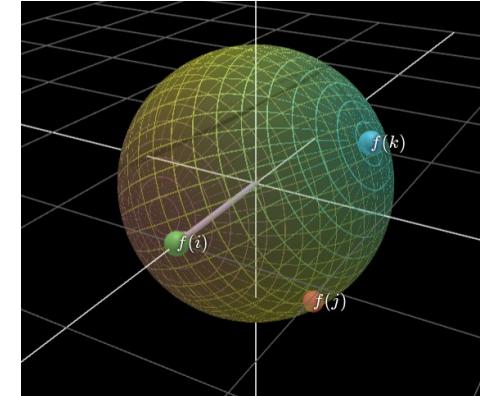
<https://eater.net/quaternions/video/doublecover>

<https://danceswithcode.net/engineeringnotes/quaternions/quaternions.html>

<https://personal.utdallas.edu/~sxb027100/dock/quaternion.html>

<https://danceswithcode.net/engineeringnotes/quaternions/quaternions.html>

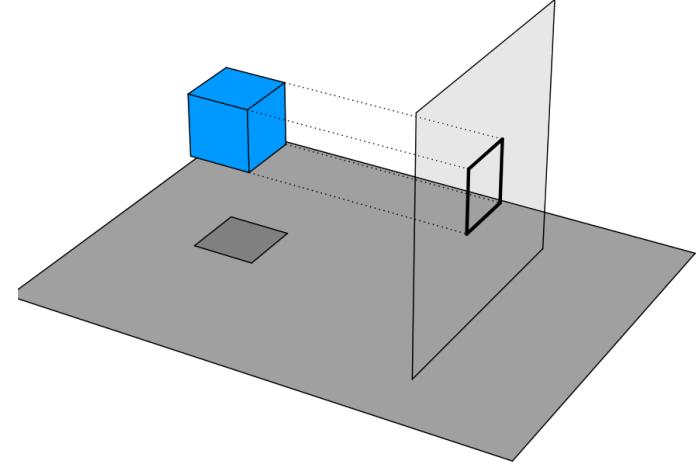
<https://paroj.github.io/gltut/Positioning/Tut08%20Quaternions.html>



```
q = cos(0°) + sin(0°)(1.00i + 0.00j + 0.00k)  
p = [i × j × k × ijk sphere ×]  
q⁻¹ = cos(0°) + sin(0°)(1.00i + 0.00j + 0.00k)  
f(p) = q · p · q⁻¹
```

# Orthographic projection

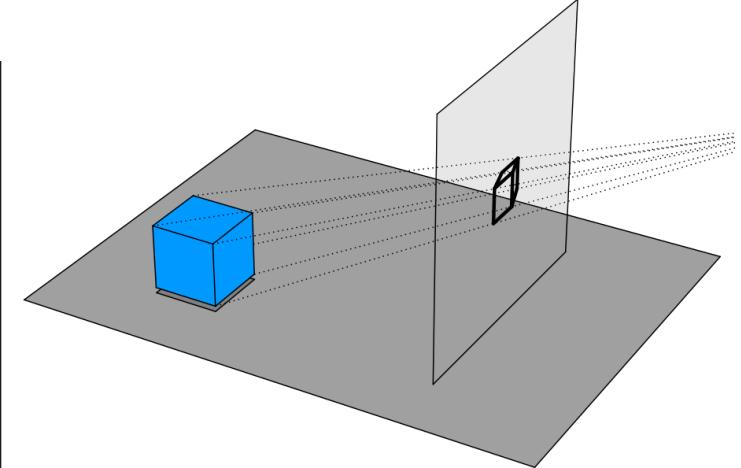
- Projects points  $P(x,y,z)$  on plane  $z = 0$ 
  - Used for projecting one object onto another
  - Used for simulating orthographic camera for rasterization-based rendering.
    - Additional parameters will be introduced: viewing frustum, focal length



$$Pv = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ 0 \\ 1 \end{bmatrix}$$

# Perspective projection

- Projects points  $P(x,y,z)$  on plane  $z = 1$ 
  - Used for simulating perspective camera for rasterization-based rendering.
    - Additional parameters will be introduced: viewing frustum, focal length
  - Perspective division: foreshortening effect

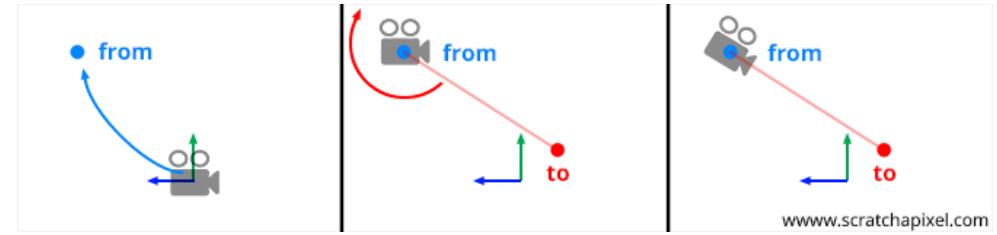


$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \frac{1}{w_c} \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix}$$

Perspective division:  
division:  $x/z$  and  $y/z$

# Look-at transform



- Often used for transforming camera in 3D scene
- Look-at methods gives look-at transform which defines transformation matrix

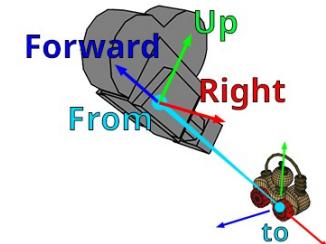
$Right_x$	$Right_y$	$Right_z$	0
$Up_x$	$Up_y$	$Up_z$	0
$Forward_x$	$Forward_y$	$Forward_z$	0
$T_x$	$T_y$	$T_z$	1

Row-major, right-handed coordinate system.

Forward = normalize(from - to)

Right = crossProduct(tmp, forward), tmp = (0,1,0)

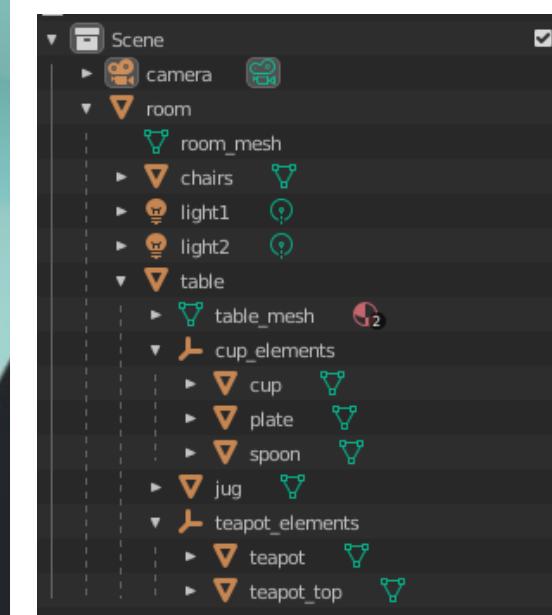
Up = crossProduct(forward, right)



# Scene organization

# Introduction

- Elements of a 3D scene: lights, cameras and objects coupled with transformations
- Production scenes are often complex, containing large amount of scene elements
- To simplify scene organization, editing, rendering, storage and transfer **scene graphs** are used



# Skeleton - Hierarchical Representation

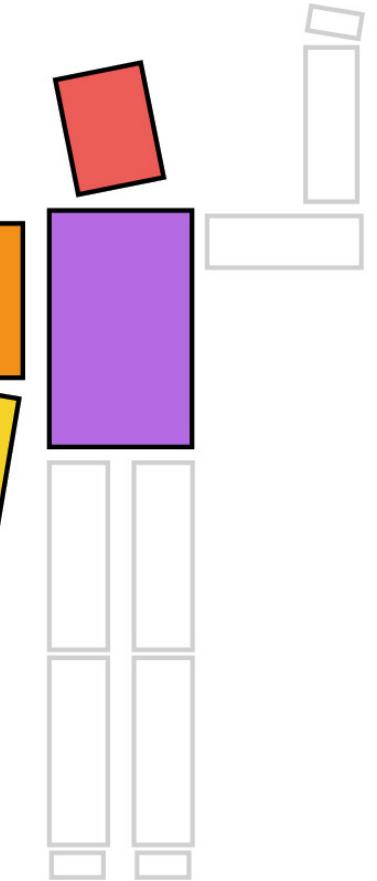
Complex objects are represented in hierarchical fashion.

```
translate(0, 10);
drawTorso();
pushmatrix(); // push a copy of transform onto stack
    translate(0, 5); // right-multiply onto current transform
    rotate(headRotation); // right-multiply onto current transform
    drawHead();
popmatrix(); // pop current transform off stack
pushmatrix();
    translate(-2, 3);
    rotate(rightShoulderRotation);
    drawUpperArm();
    pushmatrix();
        translate(0, -3);
        rotate(elbowRotation);
        drawLowerArm();
        pushmatrix();
            translate(0, -3);
            rotate(wristRotation);
            drawHand();
            popmatrix();
        popmatrix();
    popmatrix();
....
```

right hand

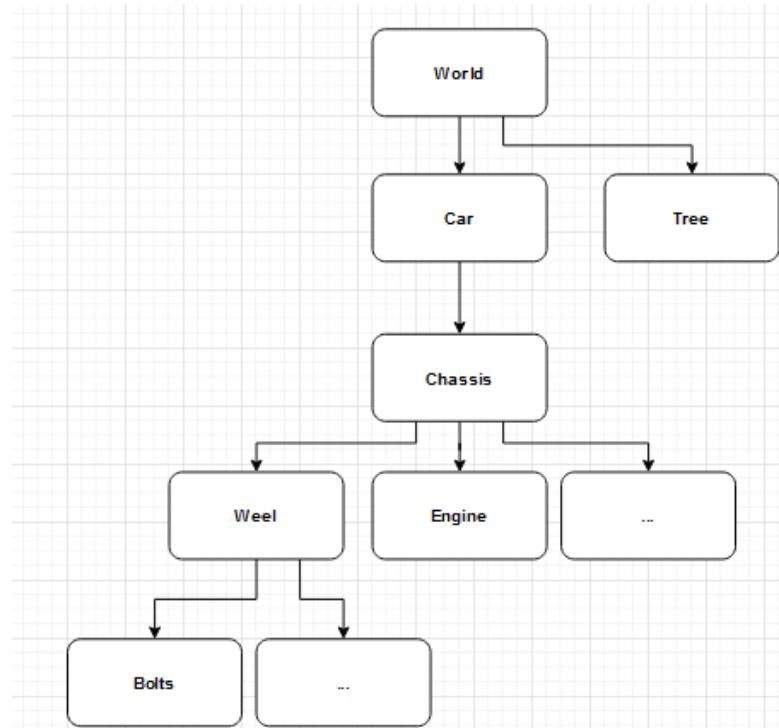
right lower arm group

right arm group



# 3D scene as scene-graph

- 3D scene representation has inherent tree-like structure
- 3D scene can be represented with **scene-graph**
  - Defines structure and hierarchy of 3D scene
  - Hierarchical data structure for organizing and structuring storing whole 3D scene, with all its elements



<https://learnopengl.com/Guest-Articles/2021/Scene/Scene-Graph>

Scene Import

+ Filter nodes

- World
- GridMap
- GIProbe
- DirectionalLight
- Cublo
- Elevator1
  - Mesh
  - CollisionShape
  - AnimationPlayer
- Elevator2
  - Mesh
  - CollisionShape
  - AnimationPlayer
- Princess
  - Mushroom

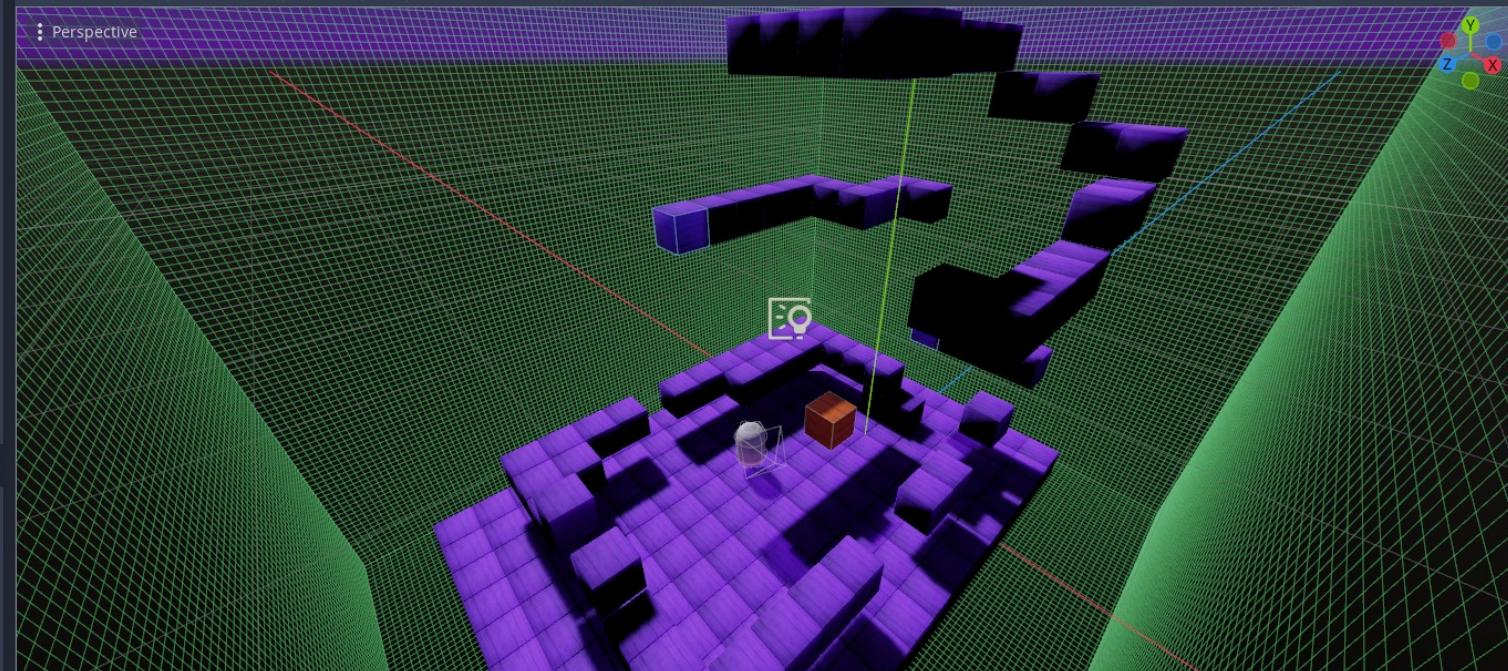
FileSystem

res://

Search files

Favorites:

- res://
  - models
  - player
  - cubelib.tres
  - cubeRB.tscn
  - default\_env.tres
  - gl\_probe\_data.res
  - Icon.png
  - level.gd
  - level.tscn



## Output:

```
--- Debugging process started ---
Godot Engine v3.4.2.stable.official.45eaa2daf - https://godotengine.org
OpenGL ES 3.0 Renderer: GeForce GTX 1660 Ti/PCIe/SSE2
OpenGL ES Batching: ON

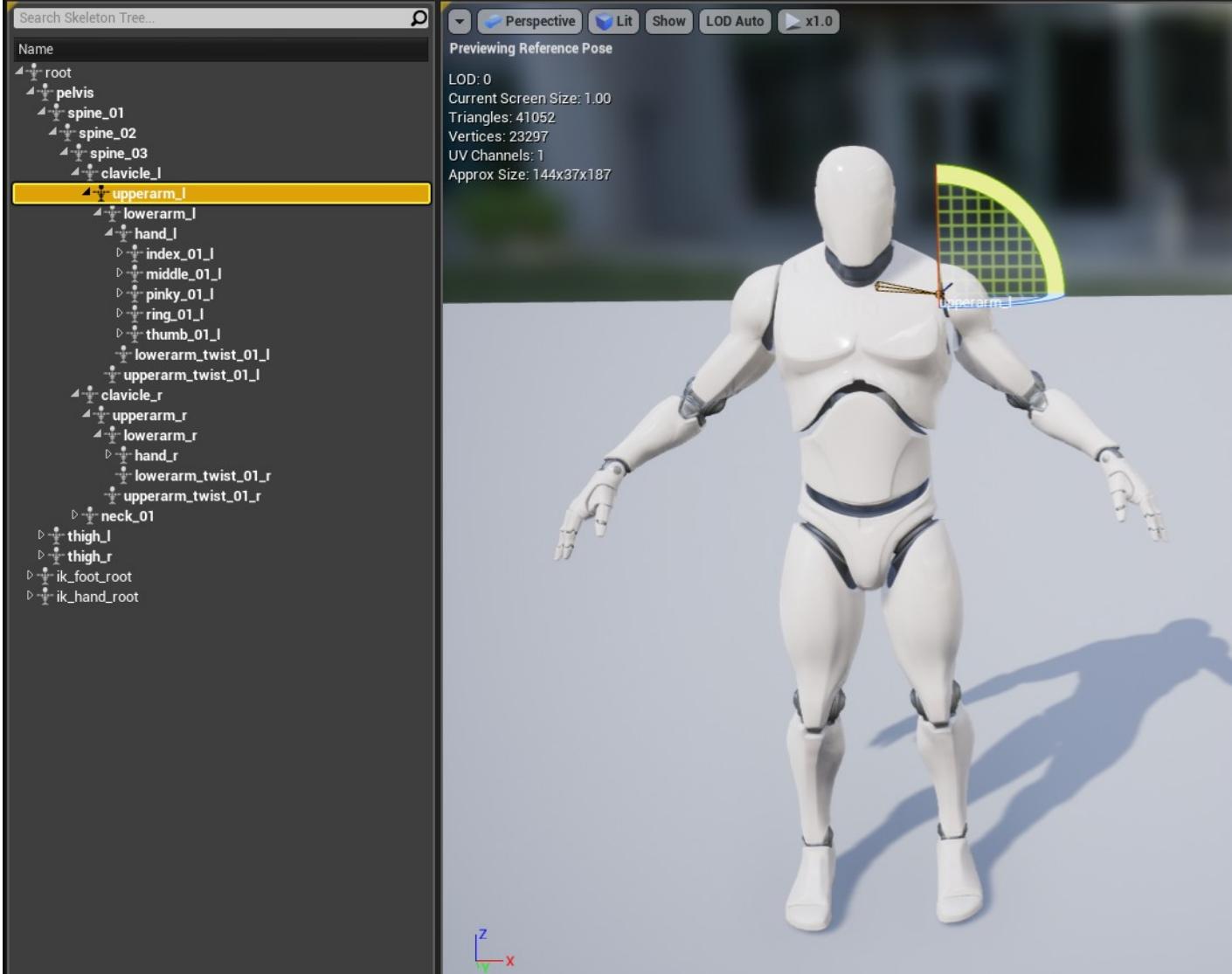
--- Debugging process stopped ---
```

Copy Clear

Output Debugger Audio Animation

3.4.2.stable

Scene graph in Godot: [https://github.com/godotengine/godot-demo-projects/tree/master/3d/rigidbody\\_character](https://github.com/godotengine/godot-demo-projects/tree/master/3d/rigidbody_character)



Scene graph in Unreal:

<https://docs.unrealengine.com/4.27/en-US/AnimatingObjects/SkeletalMeshAnimation/Persona/VirtualBones/>

# Scene graph

- Arrangement between user who builds 3D scene and renderer
- **User oriented data structure** for modeling and organizing scene elements and their relationships in hierarchy
  - Edited and created by user: artists and designers
- **Support for rendering**
  - Scene graph is traversed to render the scene

# Scene graph

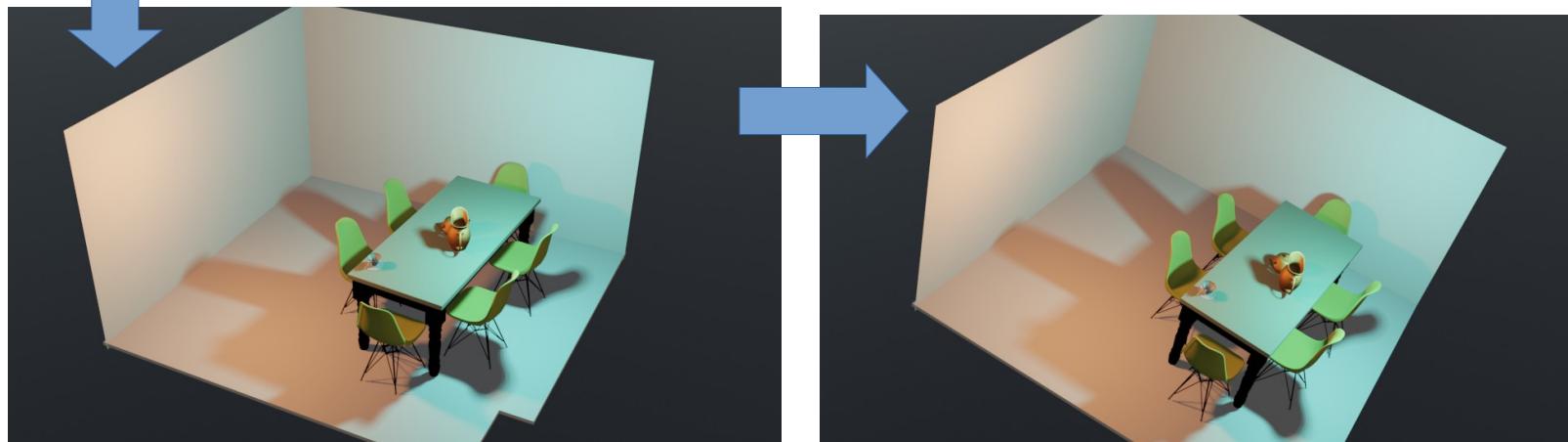
- Scene graph:
  - **Root node** – starting point for whole scene
    - Data: type of coordinate system, scene units, etc.
  - **Internal nodes** – organize scene into hierarchy.
    - Often those are transformation information (where and how are objects positioned)
  - **Leaf nodes** – contain elements of the scene: objects, camera, lights.
    - As objects in the scene can repeat, these nodes can be duplicated



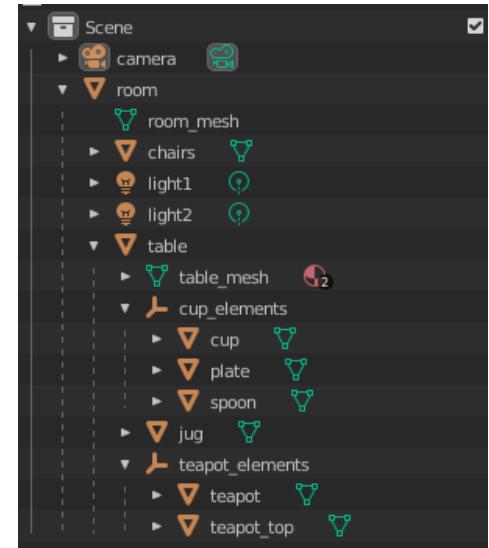
# Internal nodes

- Often: transformation nodes defining positions of sub-trees: translations, rotations, scaling, etc.
- Other types:
  - Grouping – contain nodes without any other function
  - Conditional – type of grouping node which enables activation only the particular children node
  - Level of detail – contains children nodes where each has a copy of objects with varying level of detail and only one is active depending on camera distance
  - Billboard – grouping node which orientates all children node towards camera
- As materials and textures are often shared between different objects, they can be stored as internal node which is references by children nodes

# Internal nodes: transformations



- Internal nodes with transformation affects whole sub-tree



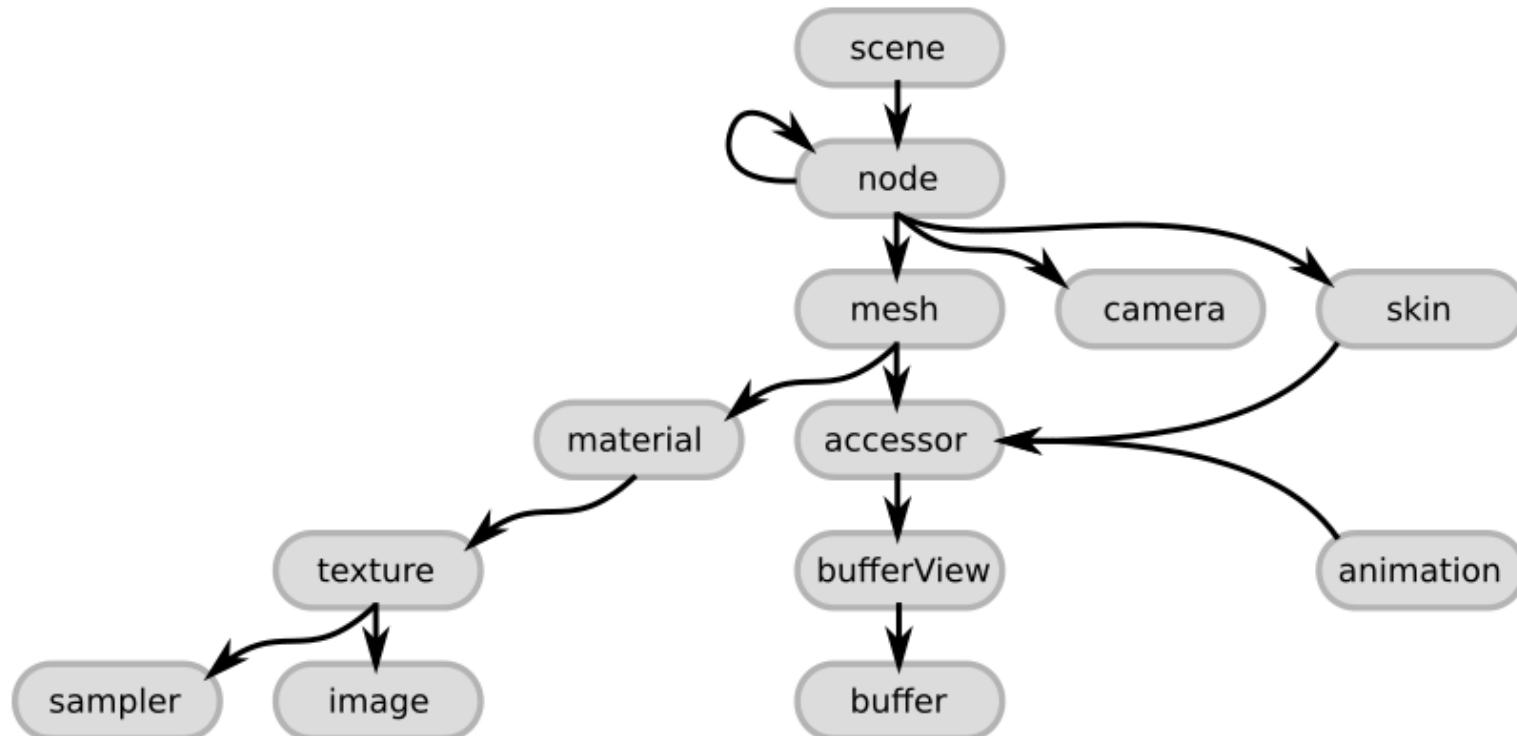
# Scene graph and scene transfer

- Different modeling, rendering and interaction software have different scene graph formats for description and storage
  - Transfer requires **standardized formats** and **importer/exporter** functions
  - Different formats exists which differ by **scene elements support**
- **Standardized scene description** that can be shared between different rendering, modeling and interaction tools

# Scene graph and scene transfer

- Tendency towards standardized formats is required
- Popular scene description formats:
  - glTF: <https://github.com/KhronosGroup/glTF>
  - USD: <https://graphics.pixar.com/usd/release/index.html>

# Scene graph example: glTF



# Scene graph example: glTF

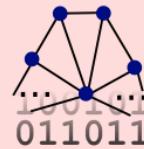
## .gltf (JSON) file

```
"scenes": [ ... ],  
"nodes": [ ... ],  
"cameras": [ ... ],  
"animations": [ ... ],  
...  
  
"buffers": [  
  {  
    "uri": "buffer01.bin",  
    "byteLength": 102040  
  },  
  ...  
  
  "images": [  
    {  
      "uri": "image01.png"  
    },  
    ...  
  ]
```

The JSON part describes the general scene structure, and elements like cameras and animations.

Additionally, it contains links to files with binary data and images:

.bin files



Raw data for geometry, animations and skins

.jpg or .png files

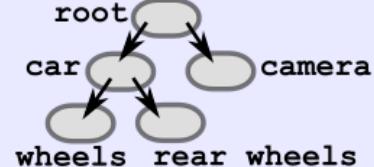


Images for the textures of the models

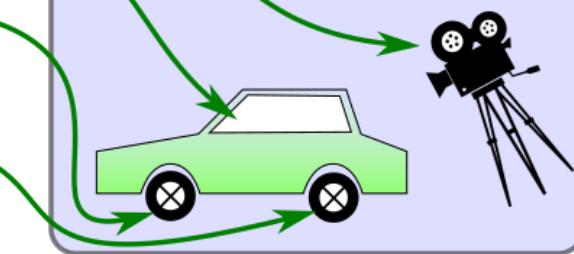
## gltf nodes

```
"nodes": [  
  {  
    "children": [ 1, 2 ]  
  },  
  {  
    "camera": 0,  
    "matrix" : [ ... ]  
  },  
  {  
    "mesh": 0,  
    "children": [ 3, 4 ]  
  },  
  {  
    "mesh": 2,  
    "rotation" : [...],  
    "translation" : [...]  
  },  
  {  
    "mesh": 2,  
    "rotation" : [...]  
    "translation" : [...]  
  }]
```

## Scene structure



## Scene



# Example: scene graph

- Scene graph can be stored in various formats:
  - Example: appleseed XML <https://github.com/appleseedhq/appleseed/wiki/Project-File-Format>

```
• <project> !
  o <scene> !
    - <assembly> *
      - <assembly> *
      - <assembly_instance> *
        - <transform> *
      - <bsdf> *
      - <color> *
      - <edf> *
      - <light> *
        - <transform> *
      - <material> *
      - <object> *
      - <object_instance> *
        - <assign_material> *
        - <transform> *
      - <surface_shader> *
      - <texture> *
      - <texture_instance> *
    o <assembly_instance> *
      - <transform> *
    o <camera> *
      - <color> *
    o <environment> ?
      - <environment_edf> ?
      - <environment_shader> ?
      - <texture> *
      - <texture_instance> *
  o <rules> +
    - <render_layer_assignment> ?
  o <output> !
    - <frame> ?
  o <configurations> !
    - <configuration> *
```

# Summary

- Questions: [https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/4\\_transforms](https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/4_transforms)

# Literature

- <https://github.com/lorentzo/IntroductionToComputerGraphics>