Lecture 5: Mesh Shape Representation

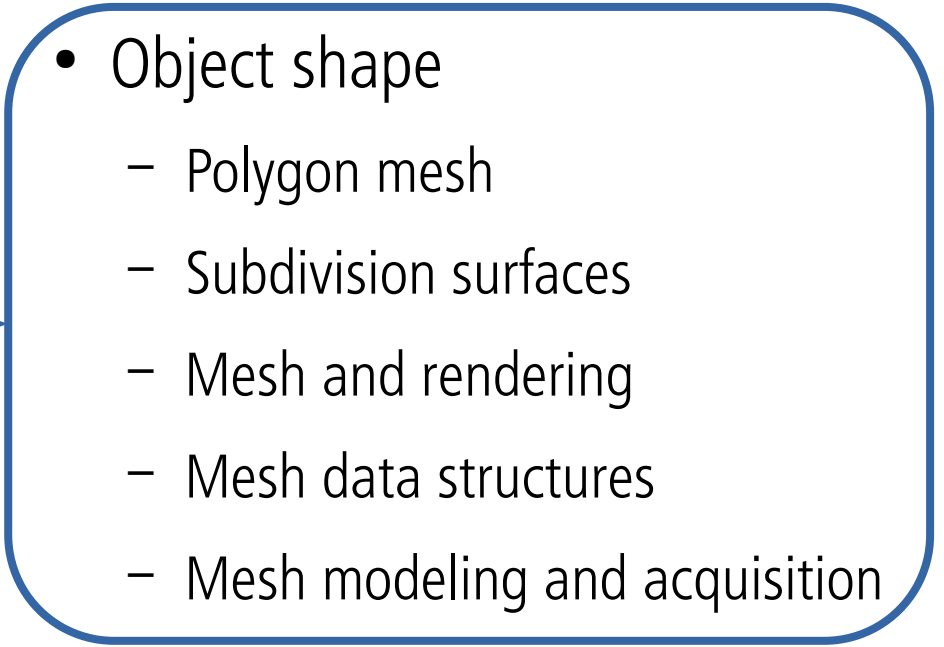
DHBW, Computer Graphics

Lovro Bosnar

1.2.2023.

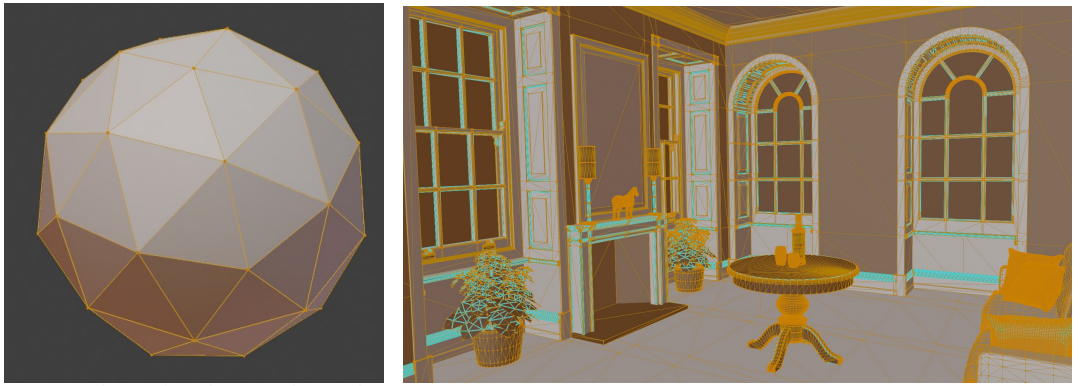
Syllabus

- 3D scene
  - Objects
    - Shape
    - Material
  - Lights
  - Cameras
- Rendering
- Image

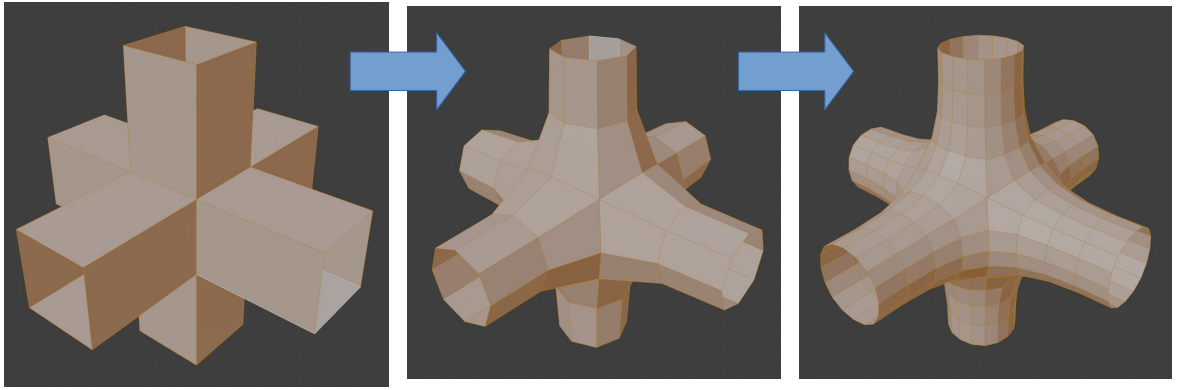


Recap: shape representations

- Points
  - Point clouds
  - Particle systems
- **Surfaces:**
  - **Polygonal mesh**
  - **Subdivision surfaces**
  - Parametric surfaces
  - Implicit surfaces
- Volumetric objects/solids
  - Voxels
  - Space partitioning data-structures



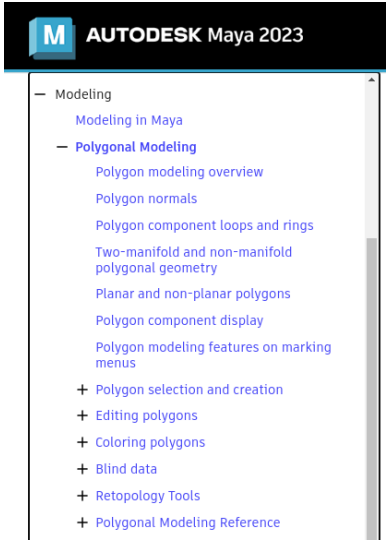
Examples of polygonal mesh

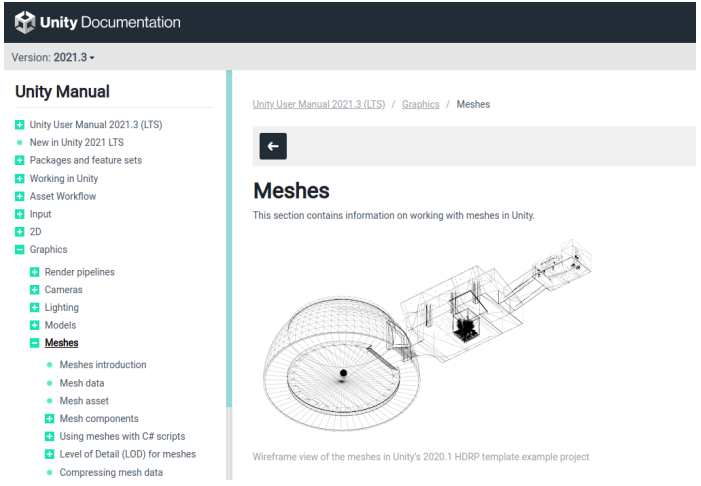


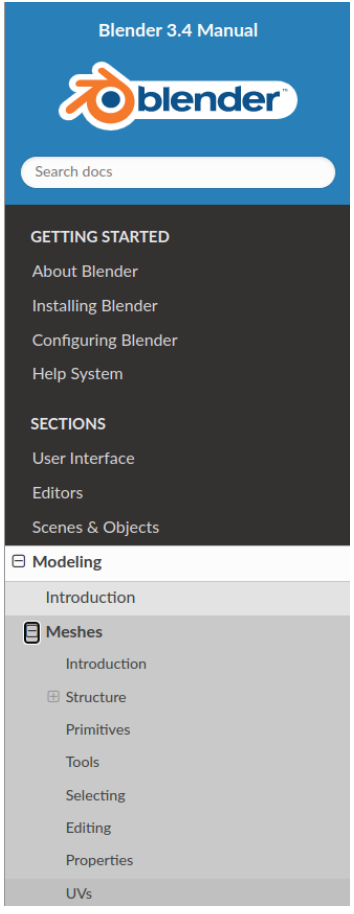
Subdivision surfaces

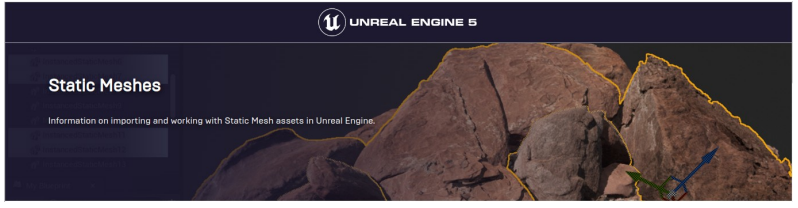
Polygon meshes

- **Polygon mesh** (shortly mesh) representation is one of the oldest, most popular and widespread geometry representation used in computer graphics
  - All DCC tools or game engines offer mesh representation that is used either for modeling or for rendering









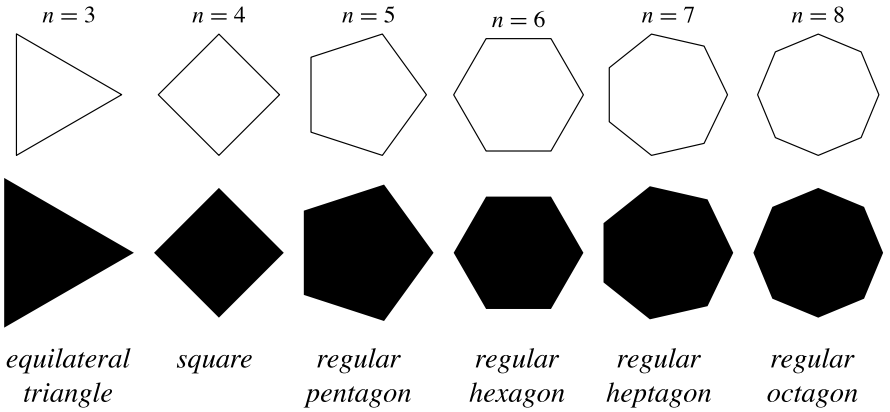
Blender: https://docs.blender.org/manual/en/latest/modeling/meshes/index.html
Maya: https://help.autodesk.com/view/MAYAUL/2023/ENU/
Houdini: https://www.sidefx.com/docs/houdini/nodes/lop/mesh.html
Unity: https://docs.unity3d.com/Manual/class-Mesh.html
Unreal: https://docs.unrealengine.com/4.26/en-US/WorkingWithContent/Types/StaticMeshes/

* Very often, mesh is commonly used for transporting models and scenes from DCC tools to game engines. DCC tools enable modeling using different shape representations, but in a lot of cases, all shapes are transformed to mesh representation and exported to other programs.

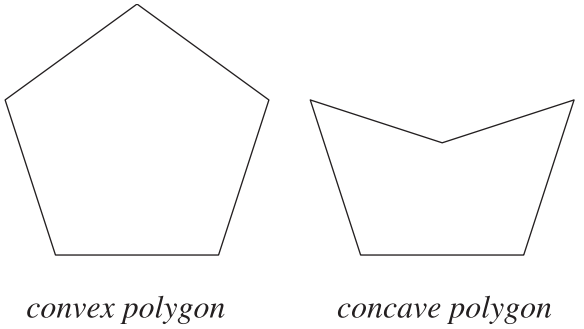
Polygon mesh

Polygon

- Polygon is planar shape defined by connecting array of points.
- **Vertices** (vertex, singular) - individual points
  - In 2D they are defined using two coordinates,e.g., (x,y)
  - In 3D they are defined using three coordinates, e.g., (x,y,z)
- **Edges** - lines connecting two vertices
- **Polygon (face) –** inner part of a connected and closed line of edges
  - Order of connecting vertices (**winding direction): clockwise** or **counterclockwise**
  - Face orientation, defined by **normal**, depends on winding direction



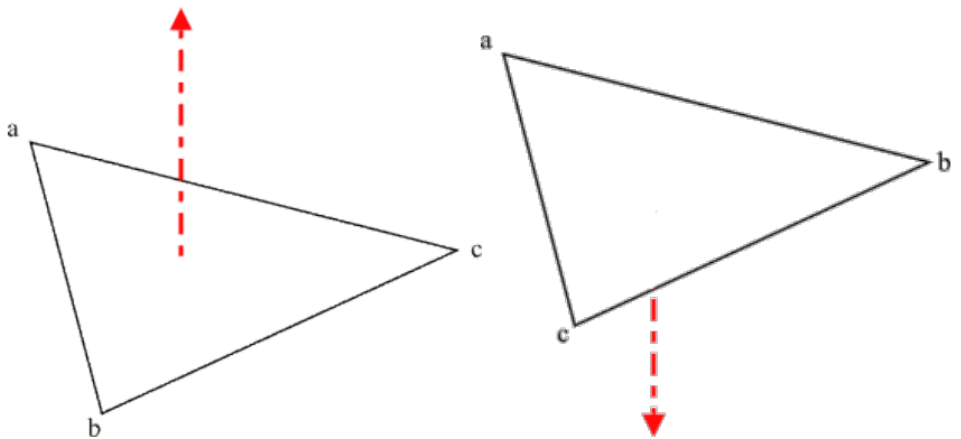
Regular polygons:
https://mathworld.wolfram.com/RegularPolygon.html



Convex polygon: connection of any two points completely lies inside polygon:
https://mathworld.wolfram.com/ConvexPolygon.html

Polygon

- Winding order: order of connecting vertices **clockwise** or **counterclockwise**
- Face orientation, defined by **normal**, depends on winding direction

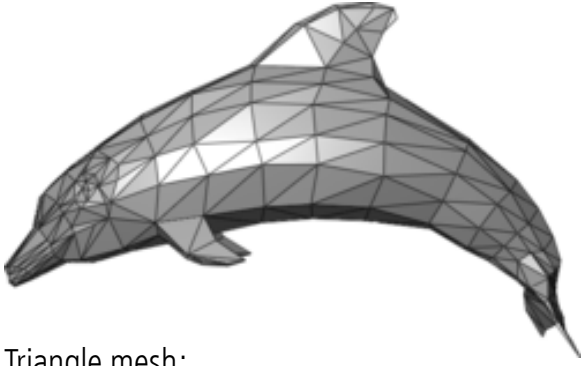


Left: anticlockwise. Right: clockwise

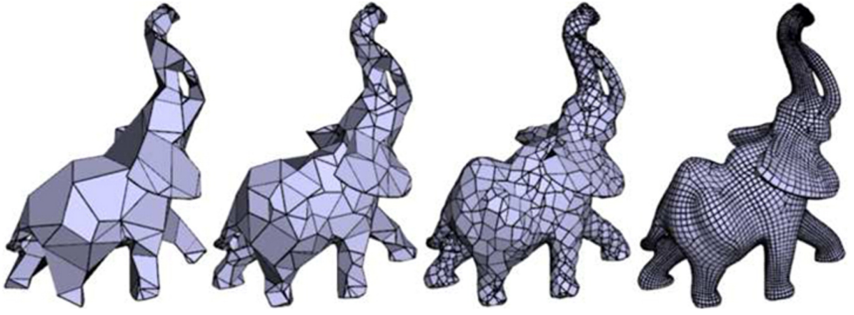
https://research.ncl.ac.uk/game/mastersdegree/graphicsforgames/indexbuffers/Tutorial%208%20-%20Index%20Buffers.pdf

Polygon mesh

- **Polygonal (surface) mesh: boundary representation of an object**
  - 2D surface embedded in 3D space
- Assumption: objects are hollow and can be represented by an **approximation of their surface using polygons**
  - Note that this representation is not good for example smoke and volumetric effects.
- Typical polygons: **triangles** and **quads**



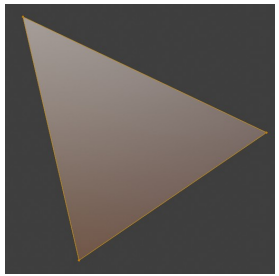
Triangle mesh:
https://en.wikipedia.org/wiki/Polygon_mesh

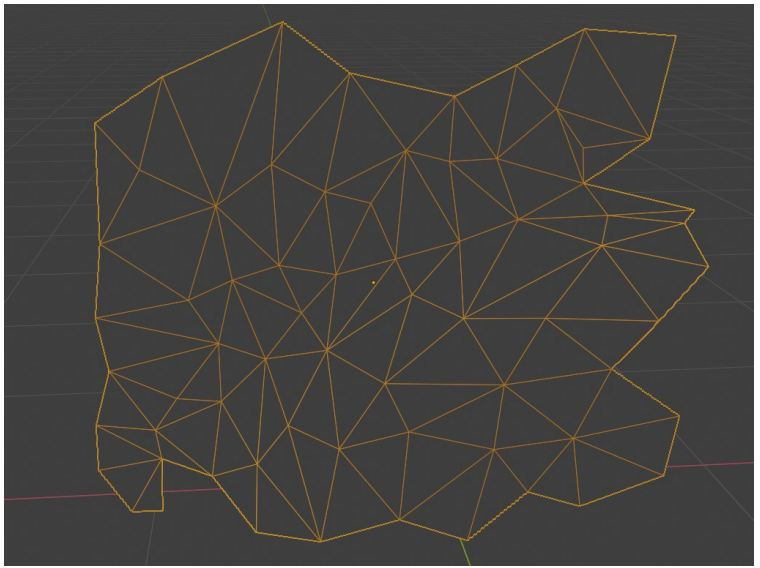


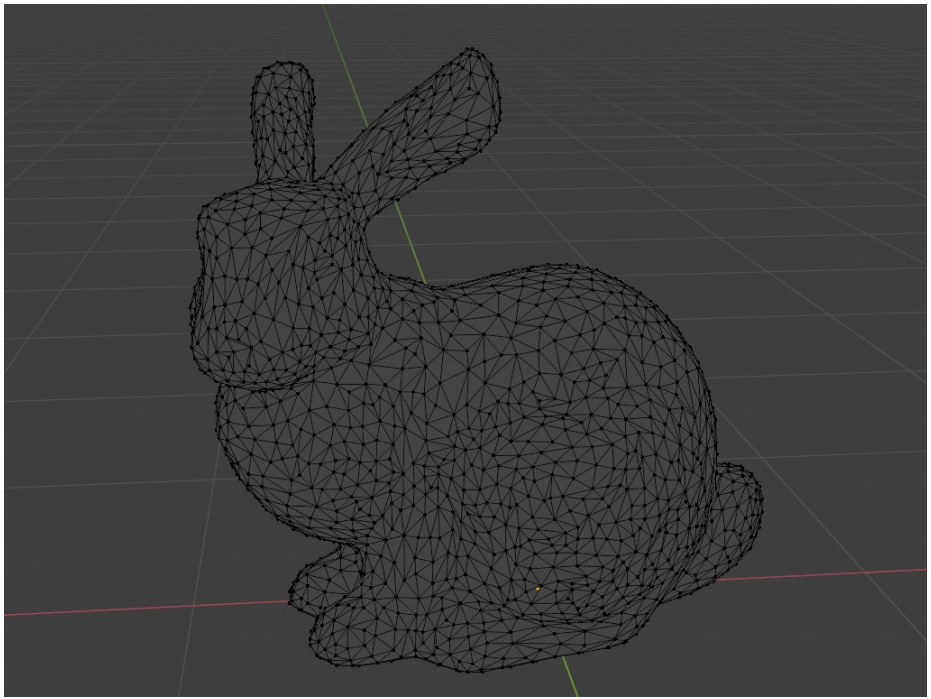
Quad mesh:
https://www.sciencedirect.com/science/article/abs/pii/S0097849312000623

Triangle mesh

- Polygon with three vertices → **triangle polygon**.
- **Triangle mesh** consists of many triangles joined along their edges to form a surface





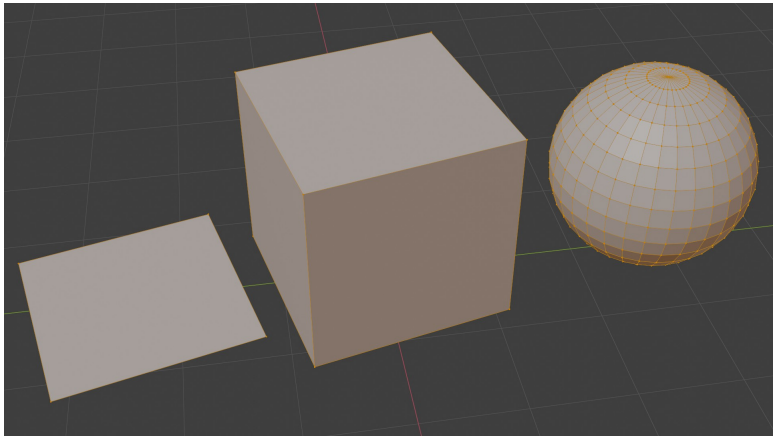


Triangle mesh

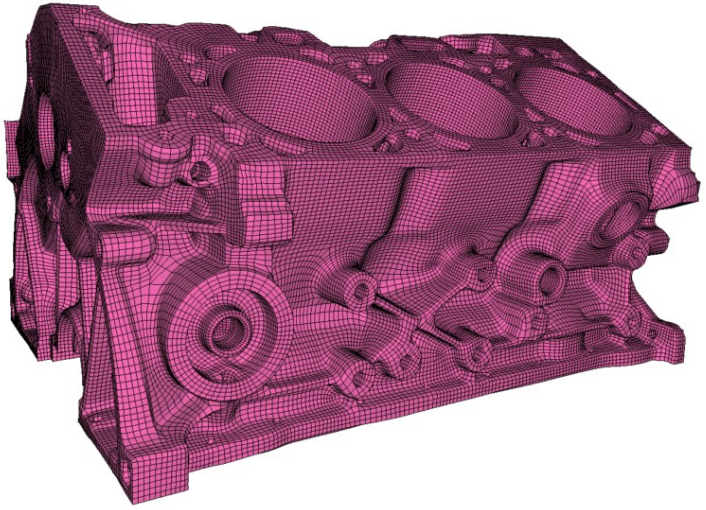
- Triangle mesh is foundational and most widely used data-structure for representation of a shape in graphics
  - All vertices lie in the same plane – **always coplanar**
  - GPU graphics rendering pipeline is optimized for working with triangles
  - Easy to define ray-triangle intersections needed for ray-tracing-based rendering
  - Easy to subdivide in smaller triangles – triangle is replace with several smaller triangles (e.g., smoothing)
  - Texture coordinates and other data are easily interpolated across triangle
- Different shape representations used in modeling and acquisition can be transformed to triangle mesh

Quad mesh

- Polygon with four vertices → **quadrilateral polygon**, shortly **quad**
- **Often used as a modeling primitive**
- Problem: not all quad vertices lie on a plane (not necessary coplanar)
  - More complex ray intersection testing
- For rasterization-based rendering converted to triangle mesh
- Ray-tracing rendering can define ray-plane intersection



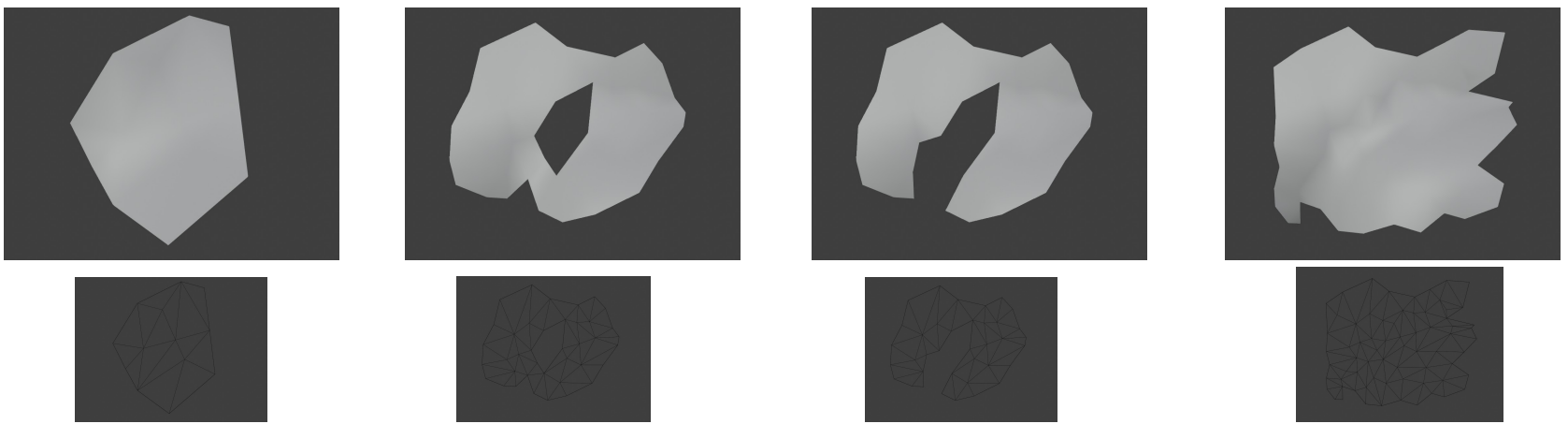
Blender mesh:
https://docs.blender.org/manual/en/latest/modeling/meshes/index.html



https://geometryfactory.com/products/igm-quad-meshing/

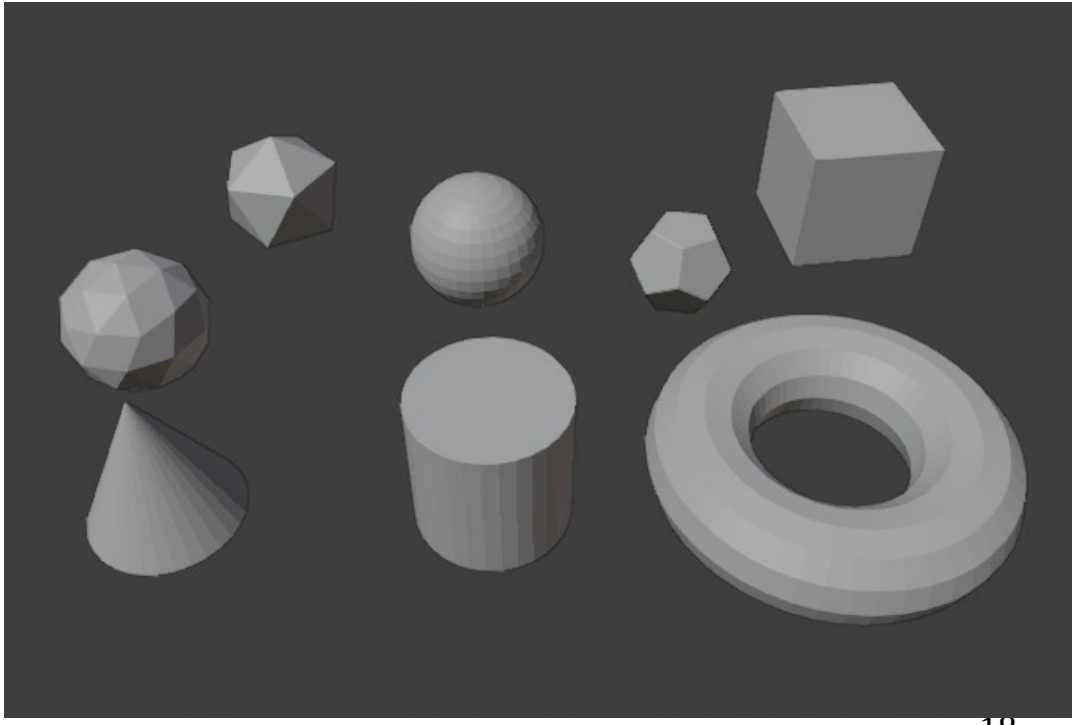
General polygon

- **General** polygon: polygon with more than four vertices
  - Can be quite complex, e.g., holes.
- **Face (atomic element)** of mesh can be any polygon.
  - Good practice: keep atomic elements as simple as possible (so that computation is easier) and combine those atomic elements into more complex shapes, e.g., convex or concave meshes or meshes with holes.



Polygon mesh representation

- Basic information needed for representing and storing any polygonal mesh:
  - **Vertex positions:** represented by coordinates **(x,y,z) → geometry**
  - **Vertex connectivity:** how are vertices connected to form polygon mesh **→ topology**

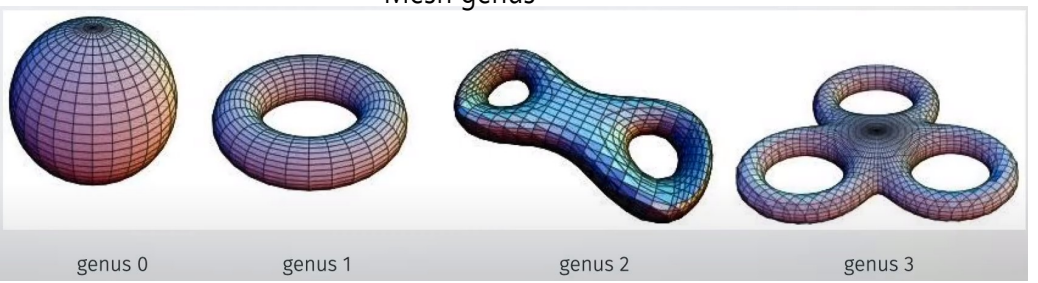


Properties of meshes

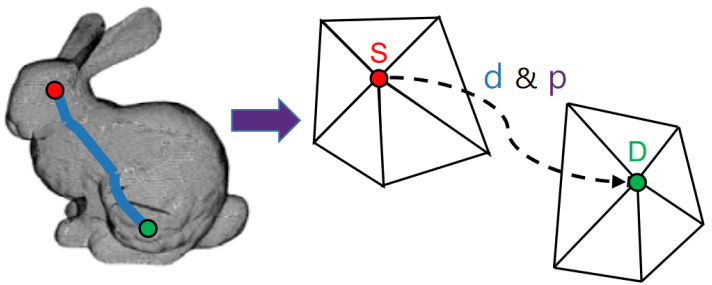
- **Manifold mesh**
  - **Mesh is manifold if the intersection of two polygons is empty, a common vertex or a common line**
  - Desired property of many algorithms and general in graphics
- **Mesh genus**
  - Intuition: "number of holes"
  - Important for computational topology
- **Meshes as graphs**
  - Vertices, edges and faces
  - Application of graph algorithms, e.g., shortes edge path between two vertices

Manifold properties

Mesh genus



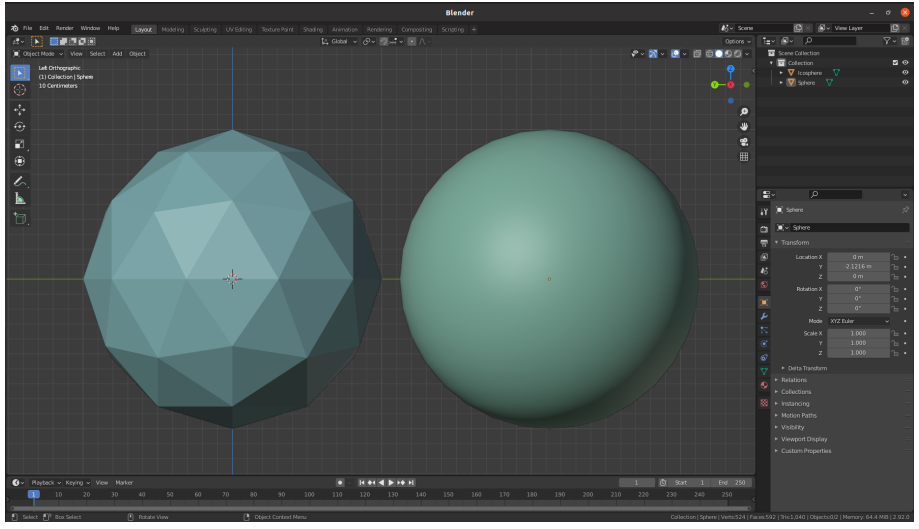
https://www.youtube.com/watch?v=V3Npa0uZYlE&list=PL4TptkuzgxxUVZ-_DiO33kp4_rkoAy1BC&index=6&ab_channel=ChristophGarth

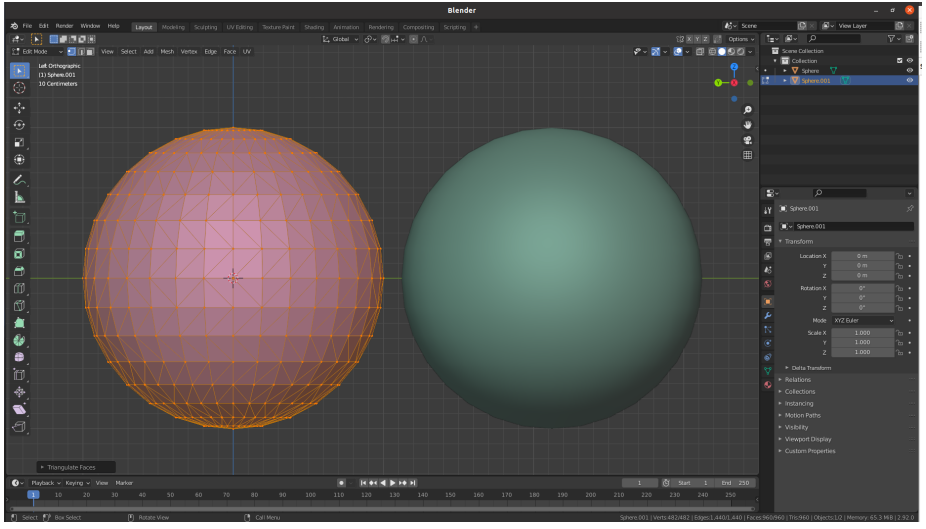


Mesh as graph: http://www.jatit.org/volumes/Vol95No18/12Vol95No18.pdf

Polygonal appoximation

- Example: ideal sphere vs polygon sphere
- **Polygonal approximation**: piece-wise linear (flat) approximation of continuous surface with polygonal mesh

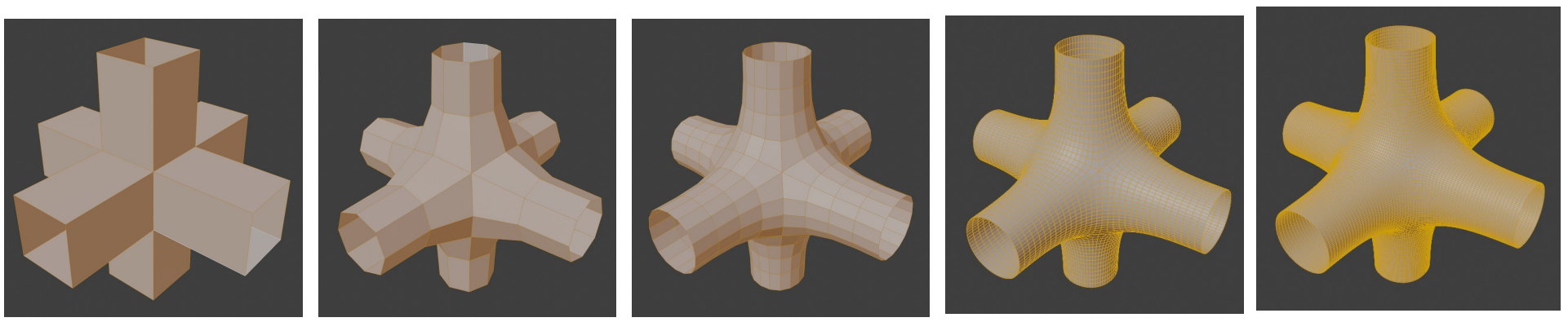




Subdivision surfaces

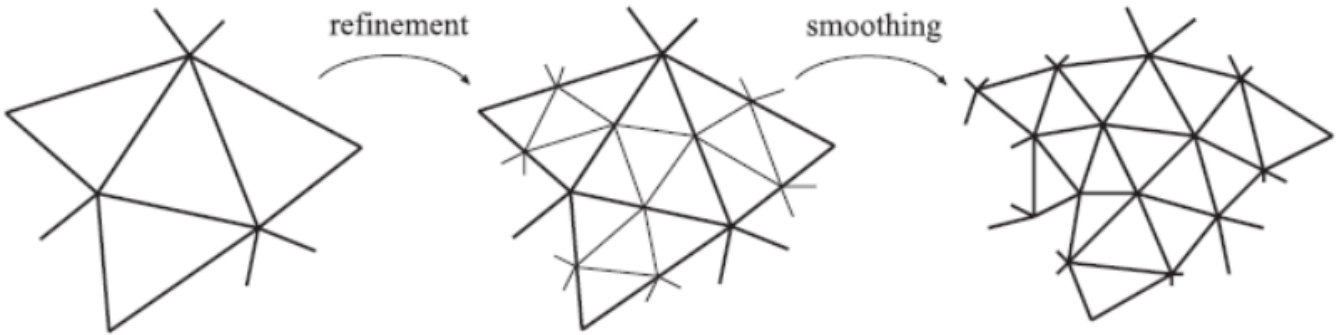
Representing smooth surfaces

- **Subdivision surfaces**: methods for defining smooth and continuous surfaces from meshes with arbitrary topology
  - Infinite level of detail: as many as needed triangles can be generated



Subidivsion surfaces

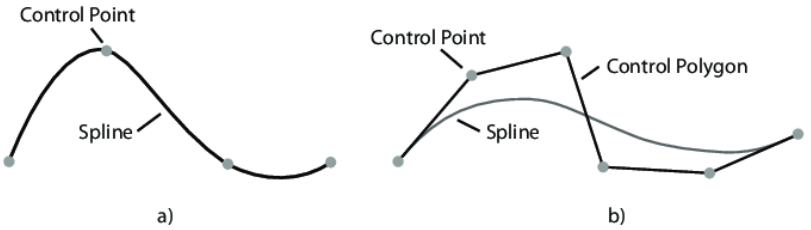
- Initial input is polygonal mesh: **control mesh (cage)**
- Two-phase process:
  - First, **refinement phase**, creates new vertices and reconnects them to create new smaller polygons
  - Second, **smoothing phase**, computes new positions for some or all vertices in the mesh



https://www.realtimerendering.com/

Subidivsion surfaces

- Different **subdivision schemes** are determined by **Refinement and Smoothing**:
  - **Loop subdivision**
  - **Catmull-Clark subdivision**
  - Doo-Sabin subdivision
- Choice of rules gives different surfaces:
  - Level of **continuity**
  - **Approximative or interpolating** surface

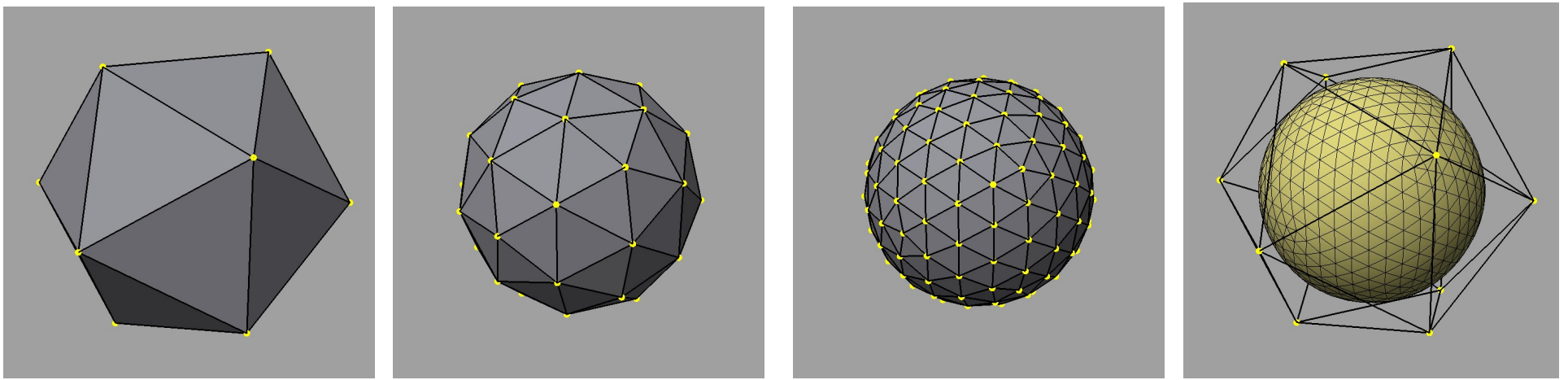


Interpolation – passing through control points (a)
Approximation – doesn't lie on control points (b)

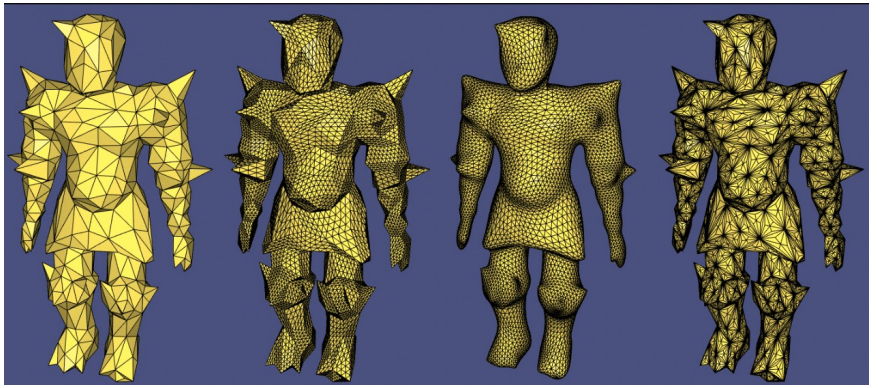
- Subdivision schemes characterization:
  - **Stationary vs non-stationary**: always use same rules at every subdivision step or change rules
  - **Uniform vs non-uniform**: use same rules for all vertices and edges or change rules
  - **Triangle vs polygon**: only triangles or arbitrary polygon

Loop subdivision

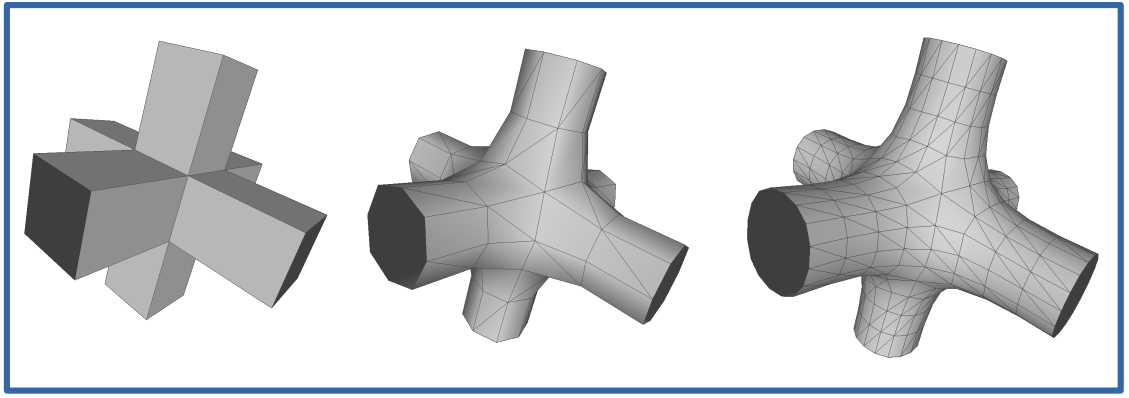
- Stationary, triangle based
- Starts with control mesh, updates each existing vertex and creates new vertex for each edge
  - At each step, triangle is subdivide into four new triangles. After n steps → 4^n triangles
- Resulting surface is approximative → smooth but shrinked
  - To reduce shrinking, more vertices must be defined in control mesh
  - Final surface is contained in convex hull of the original control points



Loop subdivision



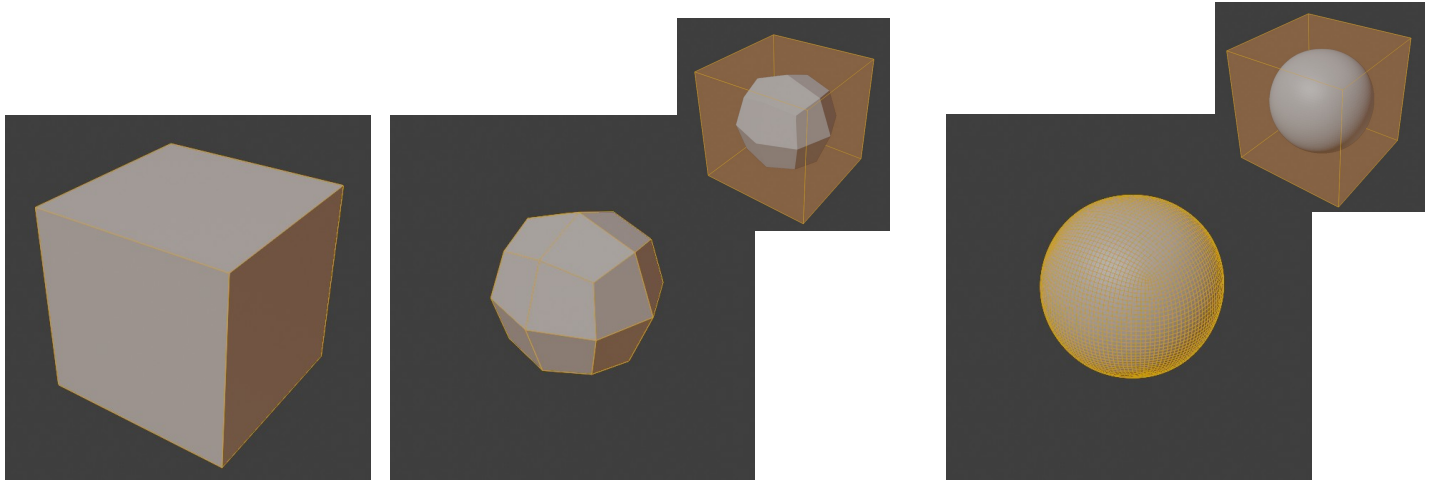
libigl subdivision: https://libigl.github.io/tutorial/ (original, upsample, **loop** and barycentric.

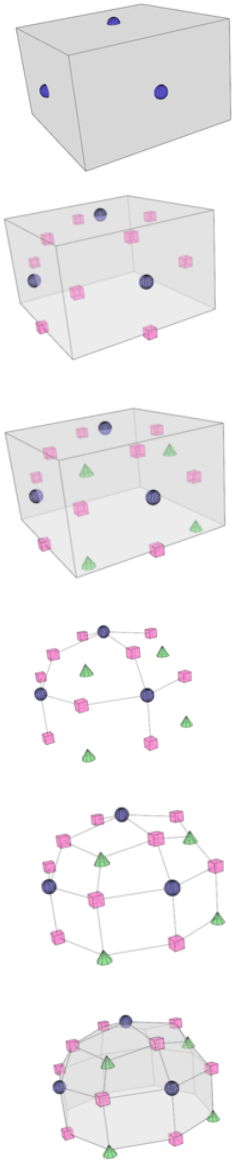


Meshlab loop subdivision:
https://pymeshlab.readthedocs.io/en/latest/filter_list.html#meshing_surface_subdivision_loop
https://github.com/cnr-isti-vclab/meshlab

Catmull-Clark

- Method of arbitrary polygons
- Starts with control mesh, recursive process defining rules for*:
  - Computing new face point
  - Computing new edge point
  - Updating vertices, creating new edges and faces





*https://graphics.pixar.com/library/Geri/paper.pdf

https://en.wikipedia.org/wiki/Catmull%E2%80%93Clark_subdivision_surface

Catmull-Clark subdivision

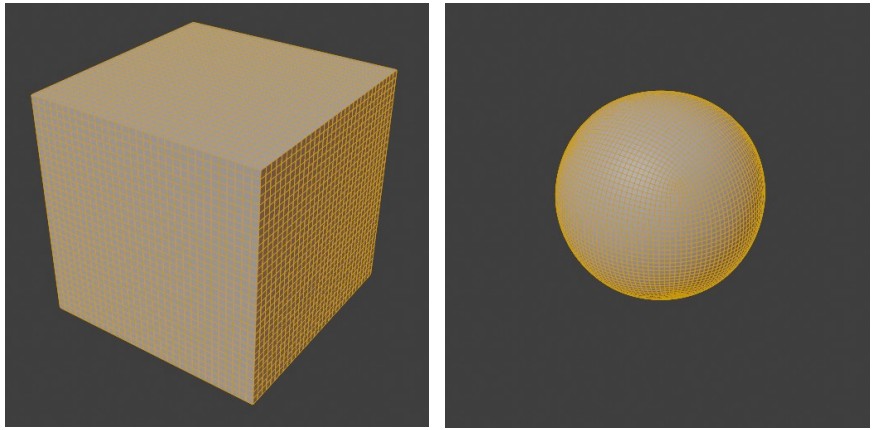
- Most commonly used subdivision surface method
  - Used in all Pixar feature films from Toy Story 2 onward
  - Used from making models for games
- Tends to generate more symmetrical surfaces

Catmull-Clark Subdivision in Blender (OpenSubdiv backend):
https://docs.blender.org/manual/en/latest/modeling/modifiers/generate/subdivision_surface.html

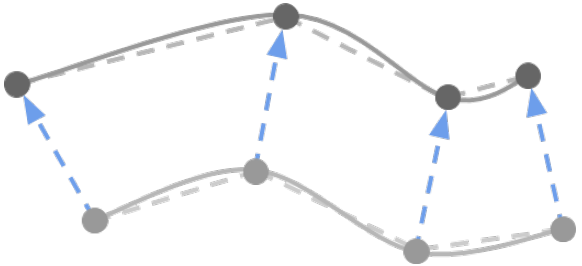
OpenSubdiv: State of the art subdivision library for production:
https://graphics.pixar.com/opensubdiv/docs/intro.html

Surface details

- **Flat and curved surfaces are too smooth** and thus not realistic while lacking detail
- Detail on surface can be added with:
  - **Geometrical enhancements**: displacement
  - **Material modeling**:
    - Scattering function parameters variation: e.g., color patterns
    - small scale surface variations: surface normal perturbation

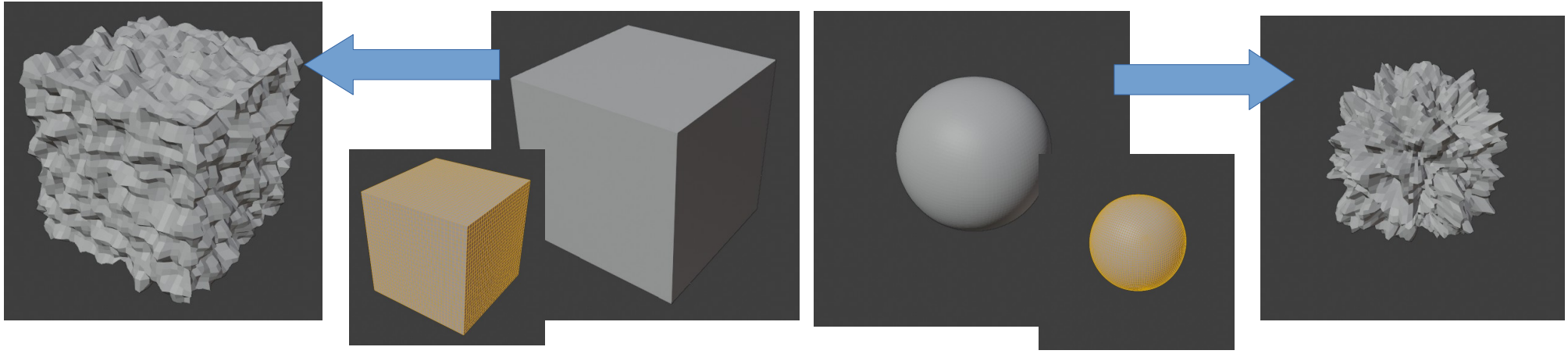


Displaced subdivision



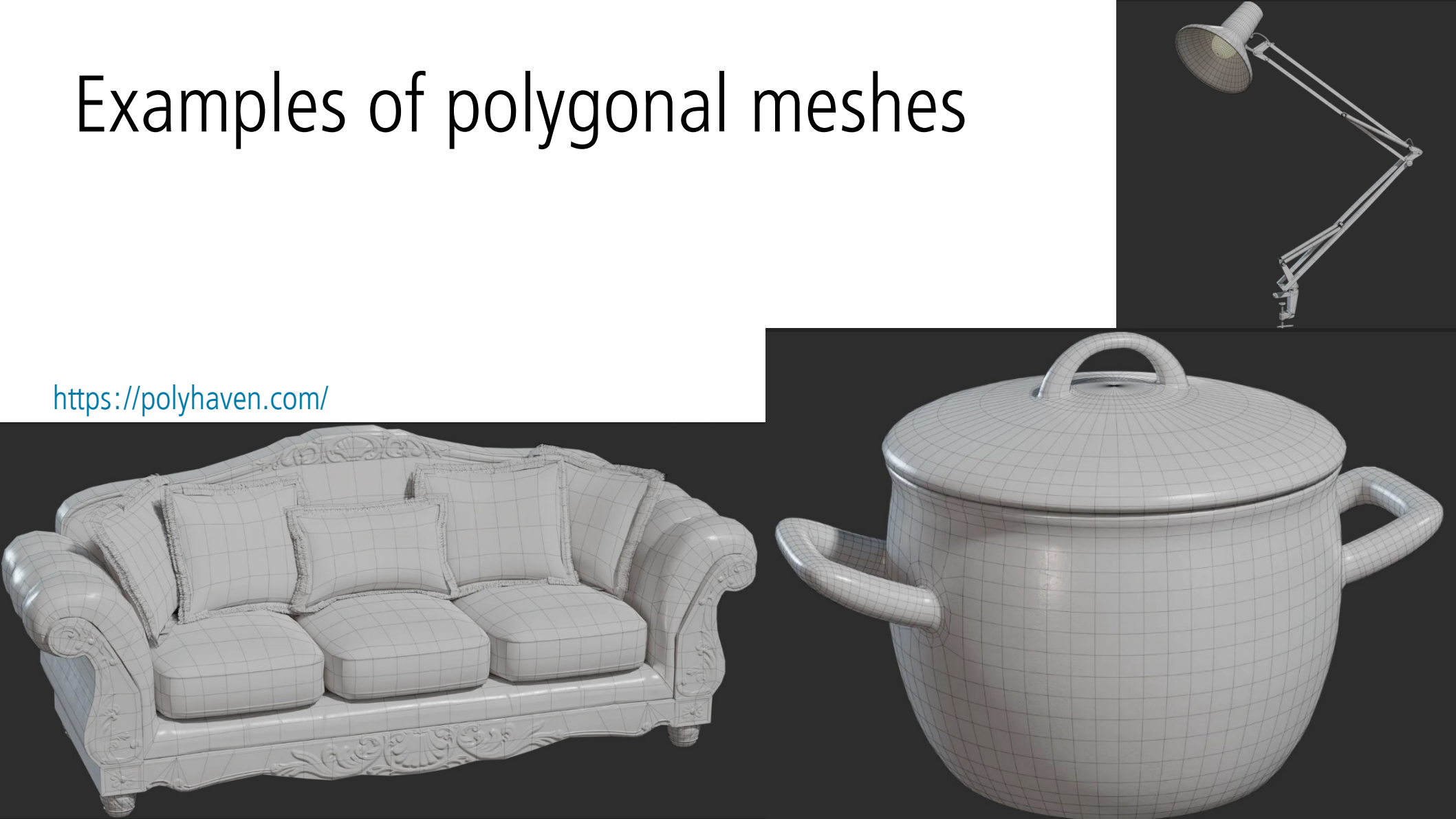
https://www.alanzucconi.com/2019/07/03/interactive-map-01/

- Detail can be added on subdivided surface using **displacement mapping**
- Often displacement mapping moves vertices (v) in direction of surface normal vector (N) for distance (d):
  - `v' = v + d * N`
- Generally, displacement can be done using arbitrary vector (M)
  - `v' = v + M`
- Scalar distance values (d) or vector values (M)used for displacement are described with image or procedural texture



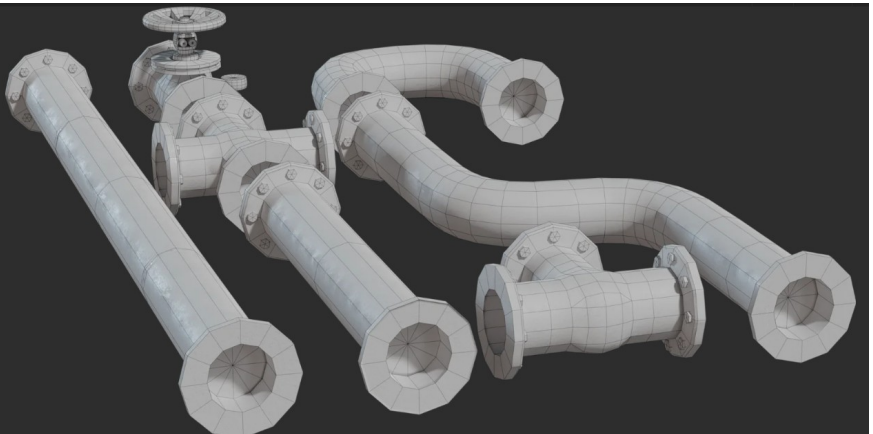
Examples of polygonal meshes

https://polyhaven.com/



Examples of polygonal meshes

https://polyhaven.com/



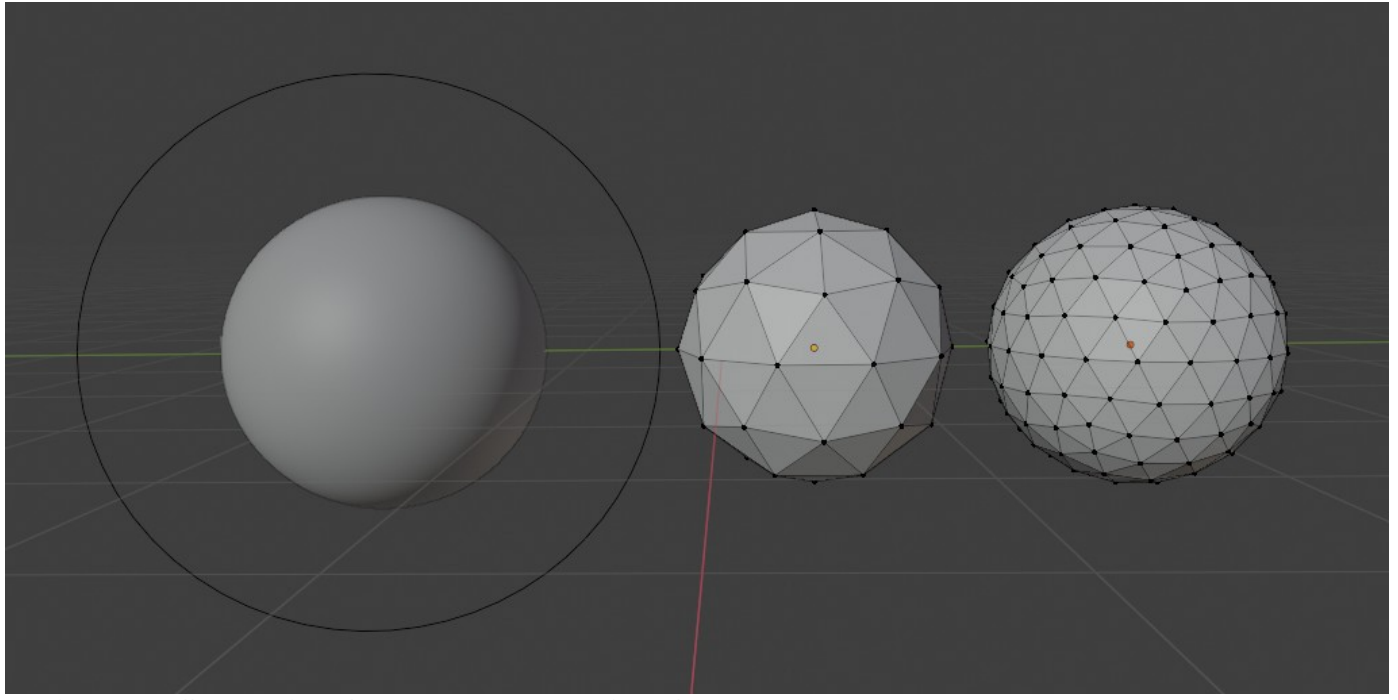
Mesh and rendering

Mesh and rendering

- Rendering uses object shape to solve **visibility problem**
  - What is visible from camera
  - Which surfaces are visible to each other → which might reflect light onto each other (light transport, shading)
- **Rasterization and ray-tracing based rendering are optimized for working with triangulated mesh**
- Any shape representation can be transformed into triangulated mesh using **tessellation**, specifically, **triangulation**:
  - Uniform
  - Adaptive

Uniform tessellation

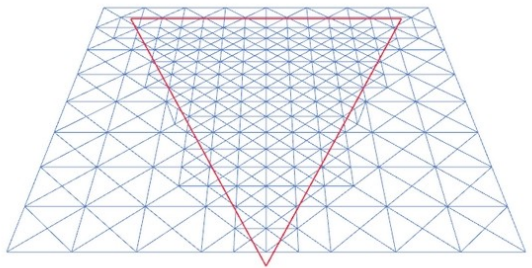
- uniformly sized triangles are created for whole surface.

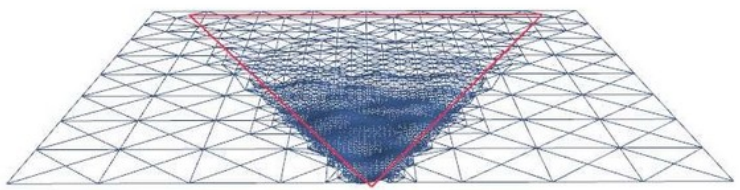


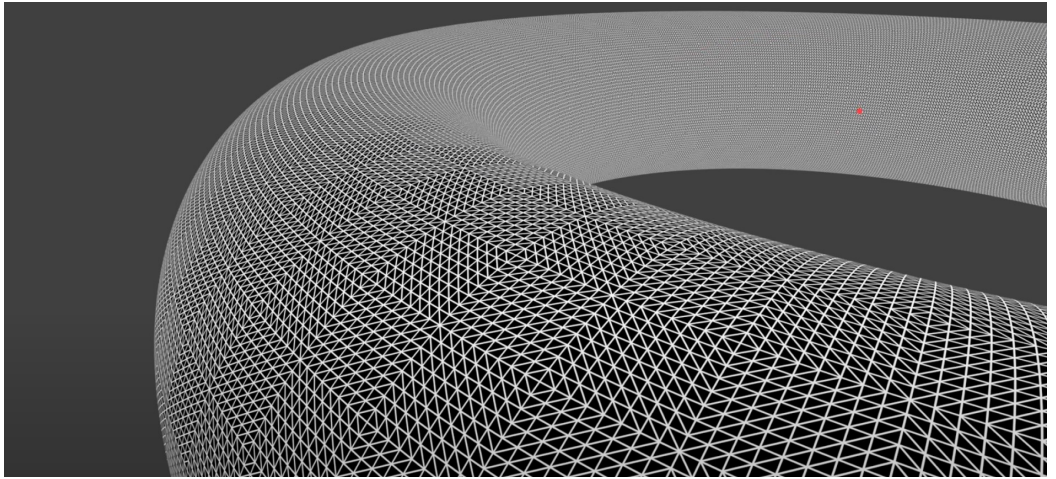
Uniform tessellation of implicit representation of a sphere.

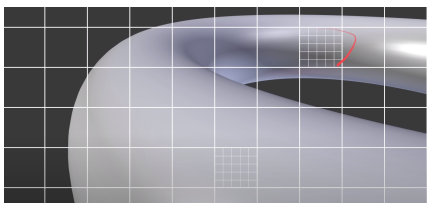
Non-uniform (adaptive) tessellation

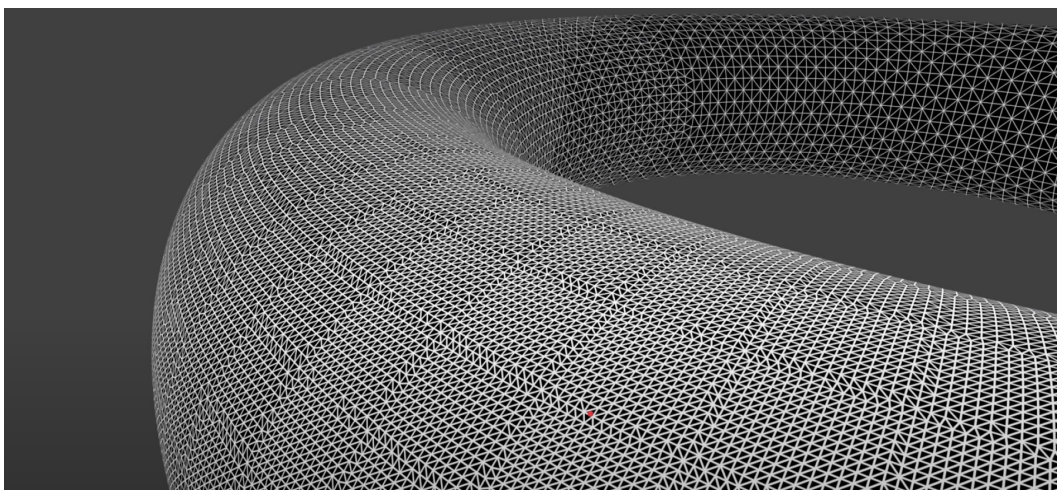
- Triangles of varying sizes are created for surface, based on:
  - Distance to surface







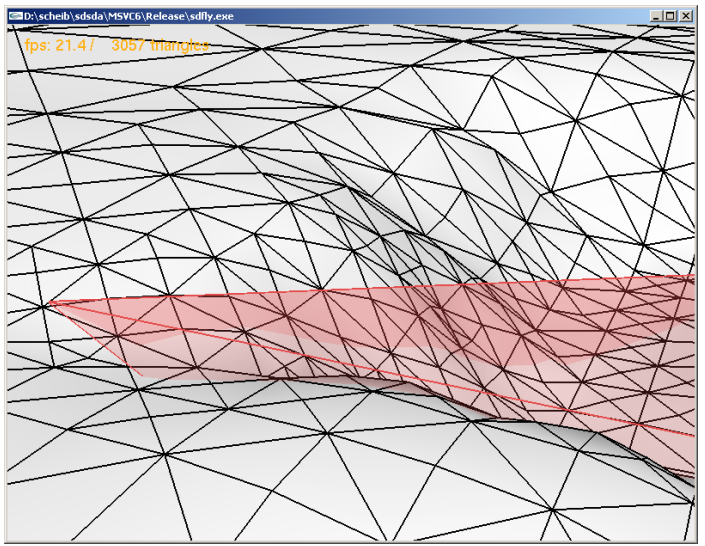


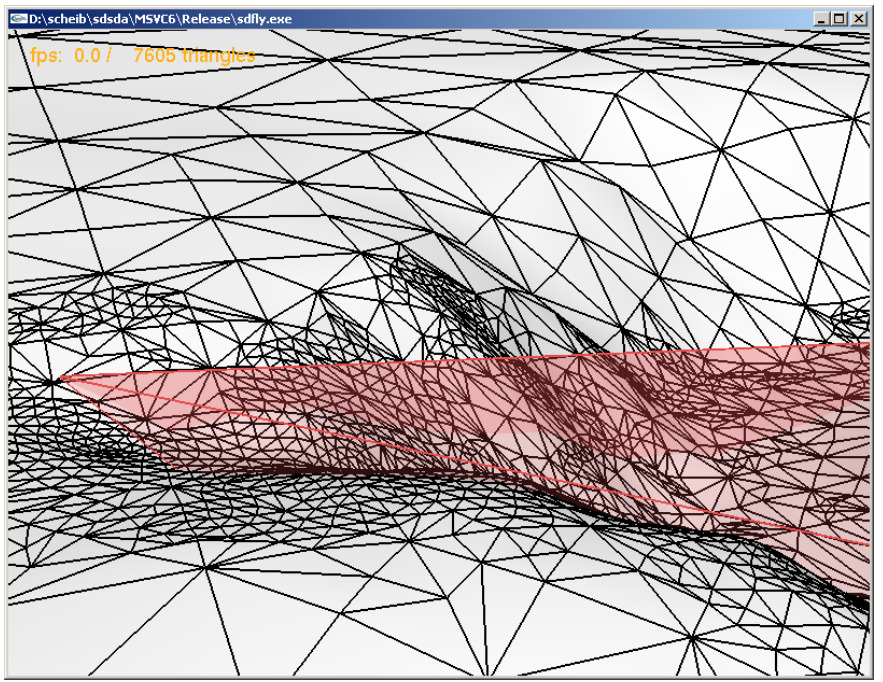


https://www.hindawi.com/journals/tswj/2014/979418/
https://www.blenderguru.com/tutorials/introduction-microdisplacements

Non-uniform (adaptive) tessellation

- triangles of varying sizes are created for surface, based on:
  - Distance to surface
  - Amount of surface details (surface curvature)

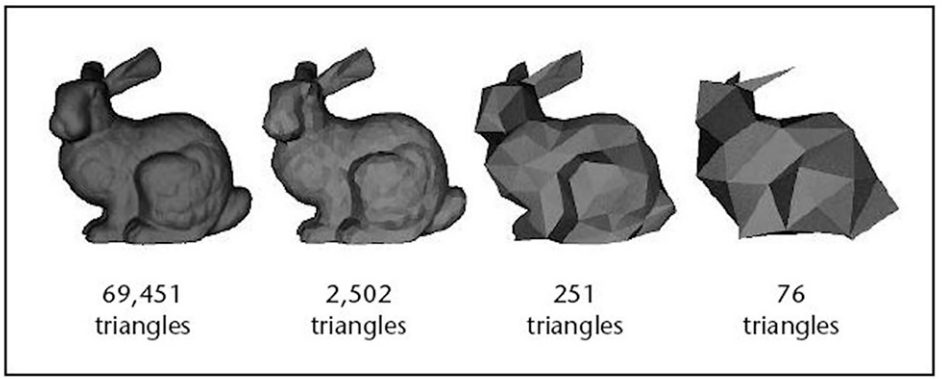




Curved parts of surface are represented with more triangles while flat parts can be represented with less triangles:
https://gamma.cs.unc.edu/SDSDA/

Level of detal

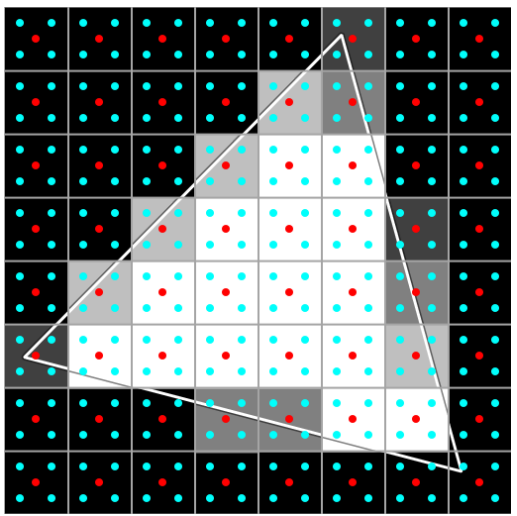
- Depending on camera distance, mesh can contain more or less details
- **Static**: multiple object shapes with different amount of polygons (polygonal resolutions) are stored and used depending on camera distance.
- **Dynamic**: amount of polygons in object shape is computed and used on the fly
  - https://www.gamedeveloper.com/programming/real-time-dynamic-level-of-detail-terrain-rendering-with-roam



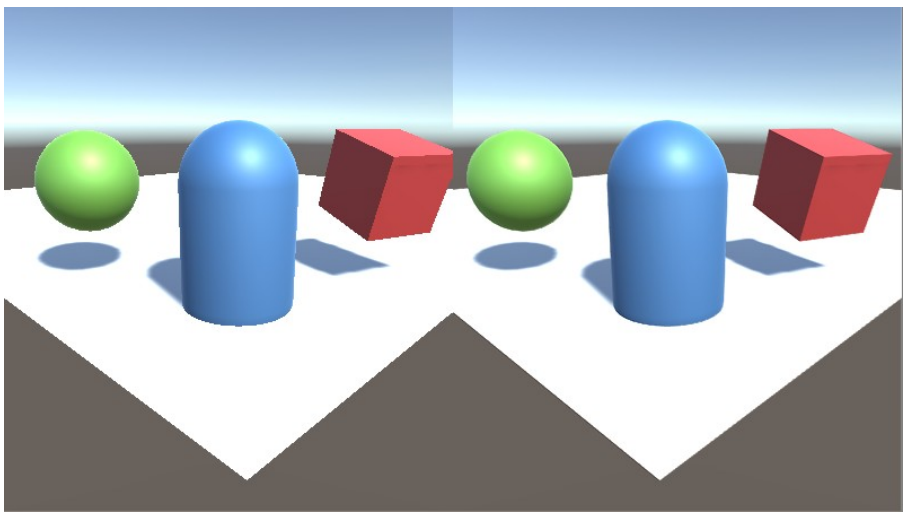
Static level of detail: https://3dstudio.co/3d-lod-level-of-detail/

Mesh and aliasing

- Triangle surface is continuous, to be displayed on raster image discretization is performed during rendering
  - Due to discretization, aliasing problems might appear
  - Solution: multiple rays/samples per pixel



https://www.beyond3d.com/content/articles/122/4



https://docs.unity3d.com/560/Documentation/Manual/PostProcessing-Antialiasing.html

Vertex attributes

- Besides vertex positions (geometry) and connectivity information (topology), additional information is used for rendering.
- This information is stored per vertex - **vertex attributes*** :
  - Normal
  - Color
  - Texture coordinate
  - Any kind of information that can be encoded and used for rendering: weights, temperature, etc.

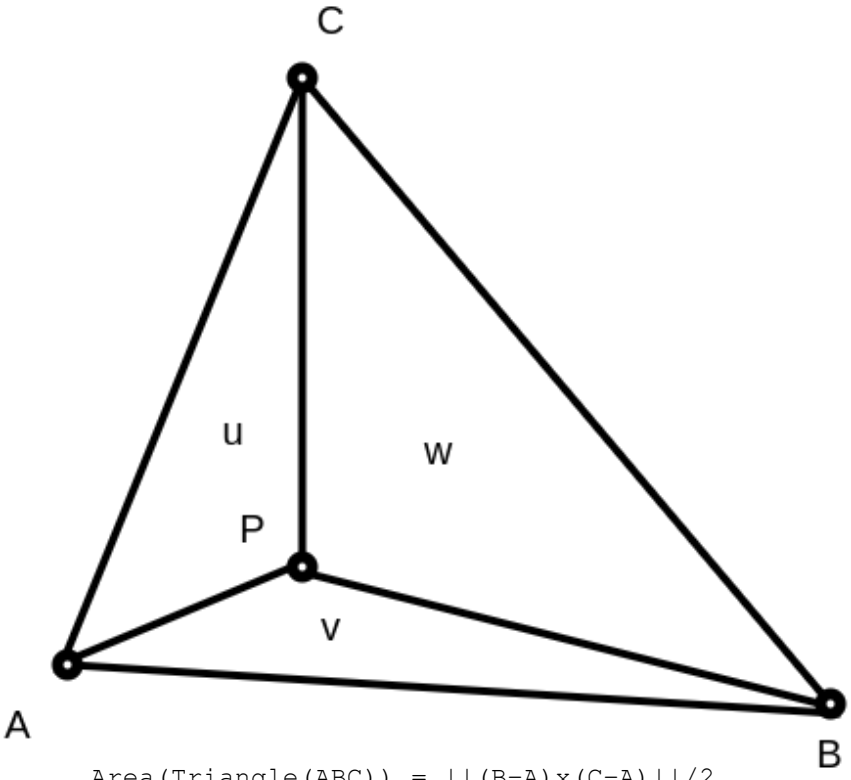
* Those are in general called primitive variables. Primitive is generic term in computer graphics which describes object which is understandable by program.

Vertex attributes

- Vertex attributes are, during rendering (shading), interpolated across triangle or quad
  - Example ray-tracing: ray hitting inside of a polygon
  - Example rasterization: color of pixel inside polygon
- Interpolation depends on polygon types:
  - **Triangles: barycentric interpolation**
  - **Quads: bilinear interpolation**

Barycentric interpolation

- Any point `P` on triangle is described as:
  - `P = uA + vB + wC`, where `A,B,C` are triangle vertices
  - `u,v,w` are **barycentric (areal) coordinates**
    - `u + v + w = 1`
    - When `0 <= u,v,w <=1` `P` inside or on triangle edge. Otherwise P is outside of triangle.
- Barycentric coordinates: `u,v,w` are proportional to area of sub-triangles defined by P
- Interpolating vertex data
  - We know `P` and `A,B,C`. Calculate, u,v and w and use this factors for interpolating



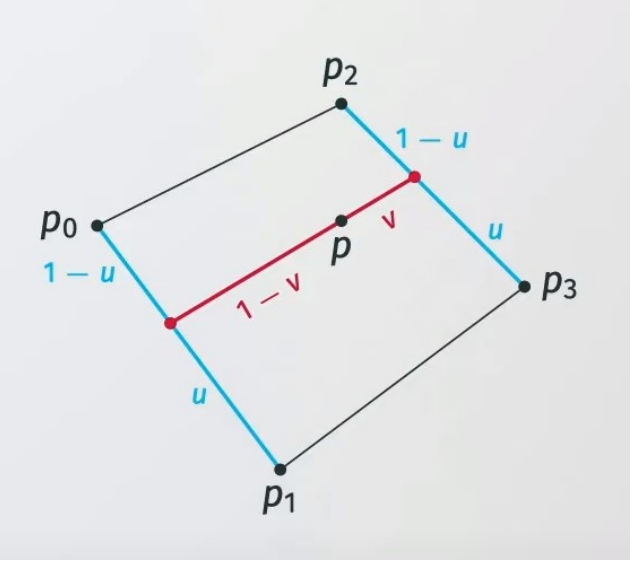

```
Area(Triangle(ABC)) = ||(B-A)x(C-A)||/2

u = Area(Triangle(CAP)) / Area(Triangle(ABC))
v = Area(Triangle(ABP)) / Area(Triangle(ABC))
w = Area(Triangle(BCP)) / Area(Triangle(ABC))
```

Bilinear interpolation

- Any point on quad can be found with bilinear interpolation
  - Thus, and value stored per vertex of quad can be interpolated in the same way

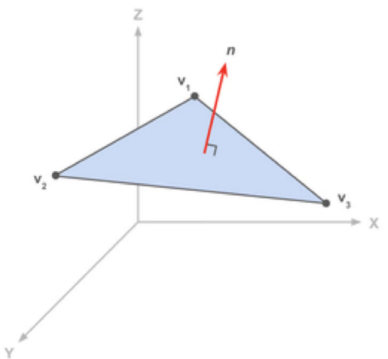
$$f(p) = (1-v)\left((1-u)f(p_0) + uf(p_1)\right) + v\left((1-u)f(p_2) + uf(p_3)\right)$$

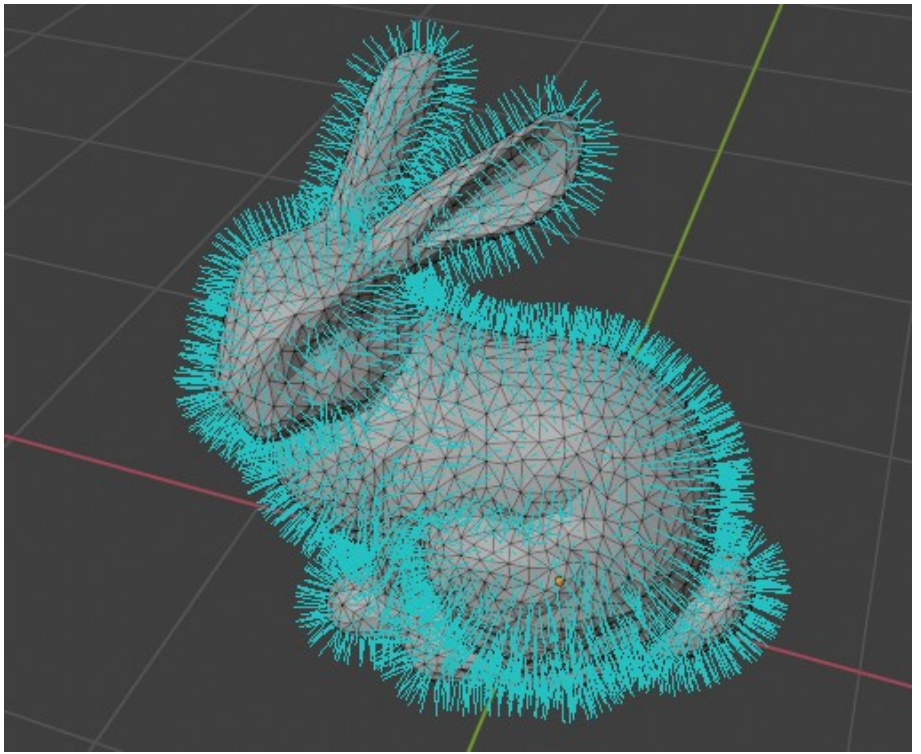
$$u, v \in [0, 1]$$



Vertex attributes: normals

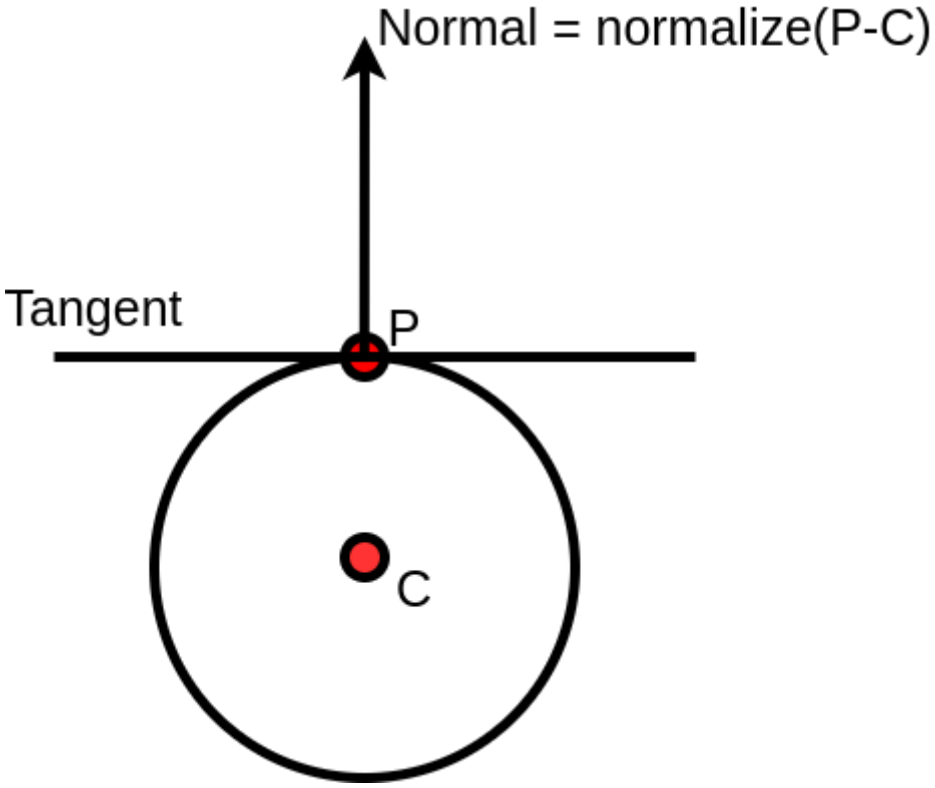
- Orientation of surface in each point is determined by normal vector.
- Normal vector is **core information for rendering (shading) and modeling**.
- Mesh normal can be defined per:
  - face
  - per vertex





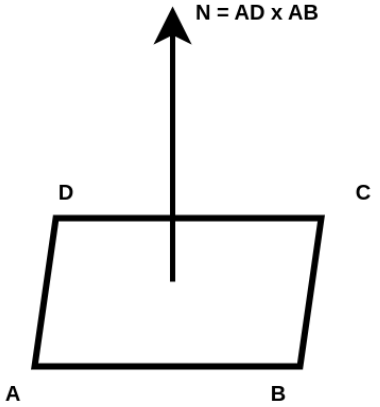
Computing normals

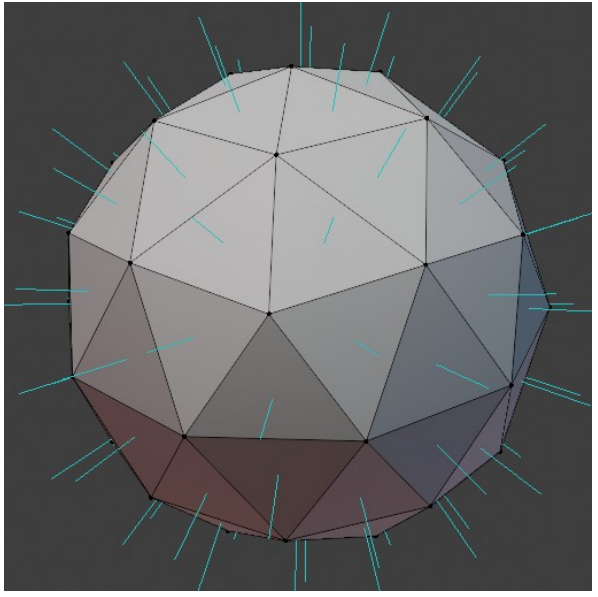
- Normal in surface point is **vector perpendicular to tangent** in that surface point
- Computation of normals depends on shape representation
  - Sphere normal
  - Mesh face normals can be calculated using triangle or quad polygon edges.

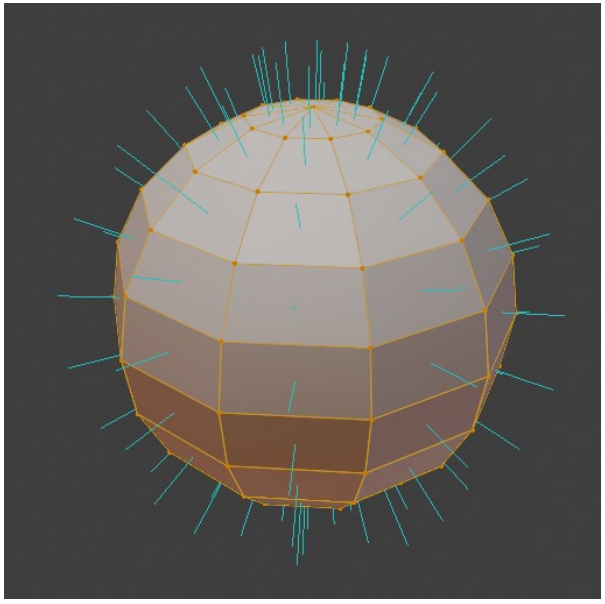


Normals: per face

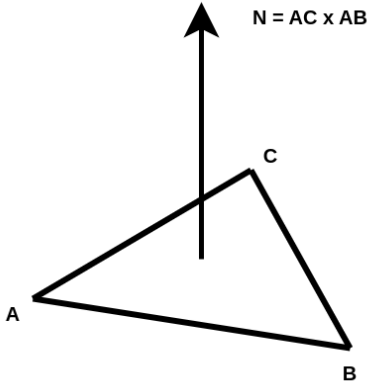
- Cross product between triangle or quad edges
- Winding order of vertices defines orientation of normal
- For right-handed coordinate system, polygons with counter clockwise winding will be front facing





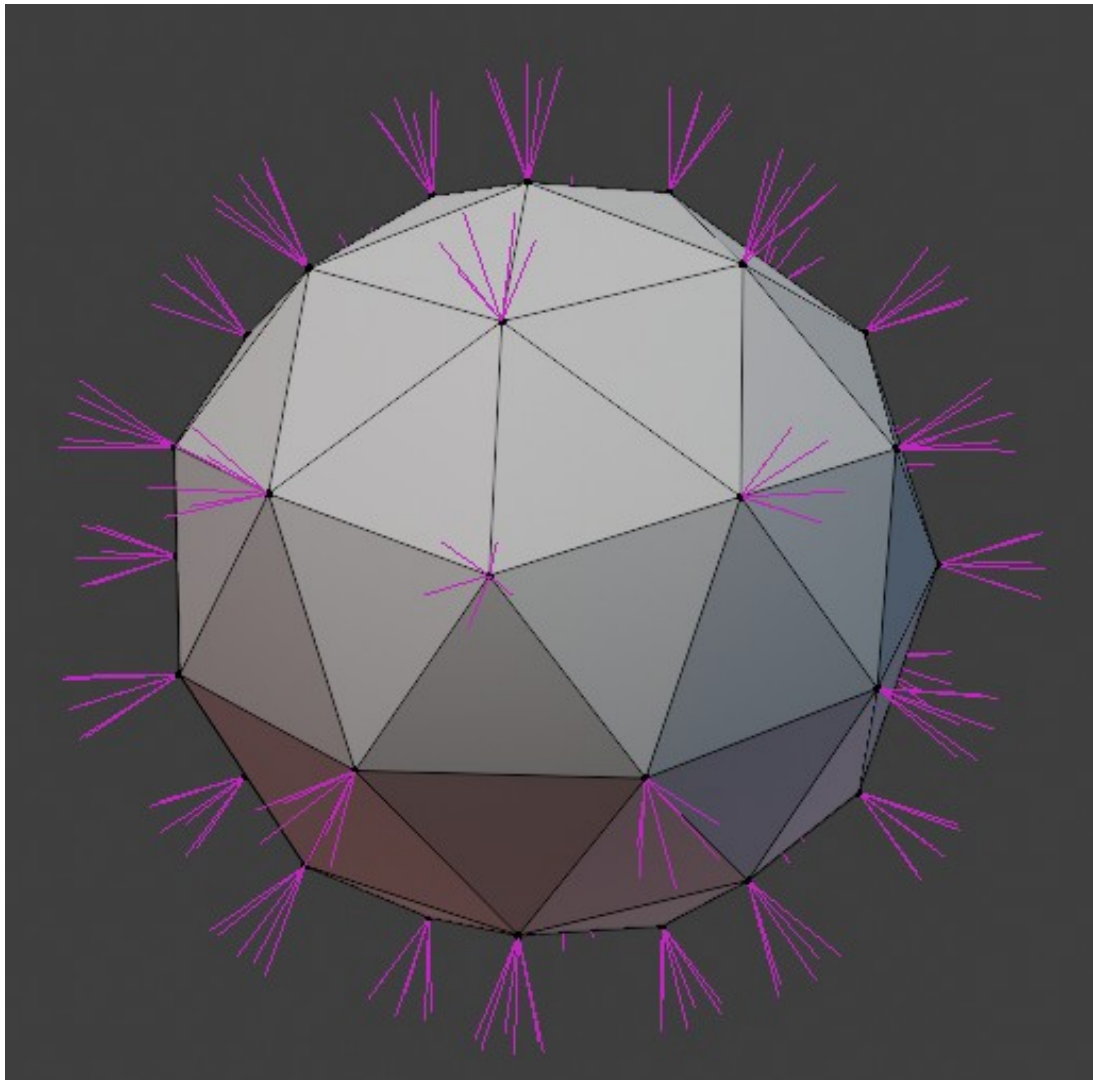


```
N = (B-A).crossProduct(C-A);
```



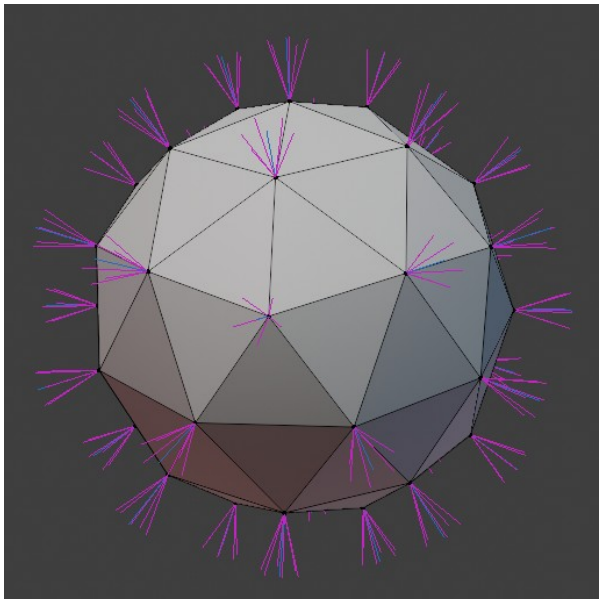
Normals: per vertex

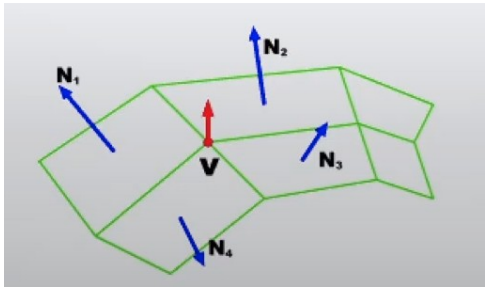
- Multiple normals, one for each face, can be stored per vertex.



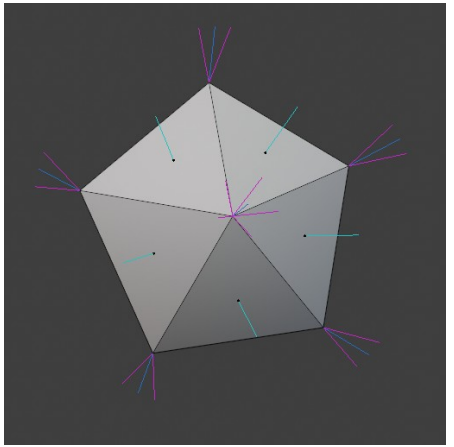
Normals: per vertex

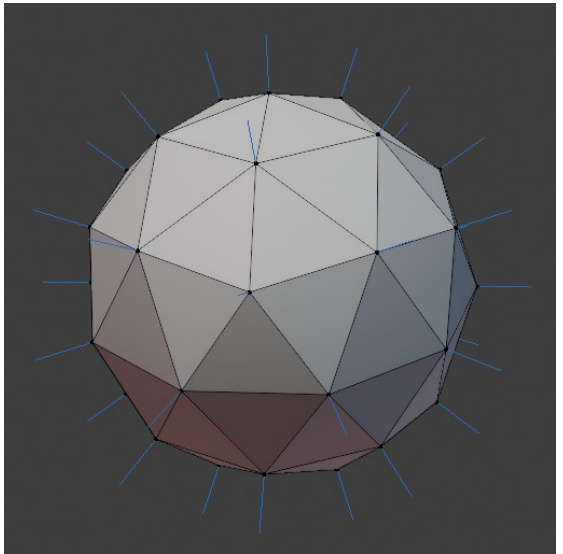
- One normal per vertex is calculated as:
  - Calculate normal for each face connected to vertex
  - Average calculated face normals





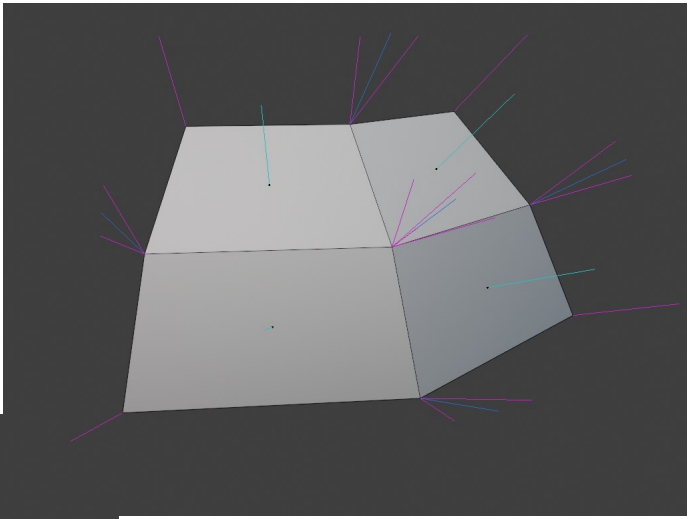
$$\vec{N}_v = \frac{\sum_{i=1}^{k} \vec{N}_i}{\left\| \sum_{i=1}^{k} \vec{N}_v \right\|}$$

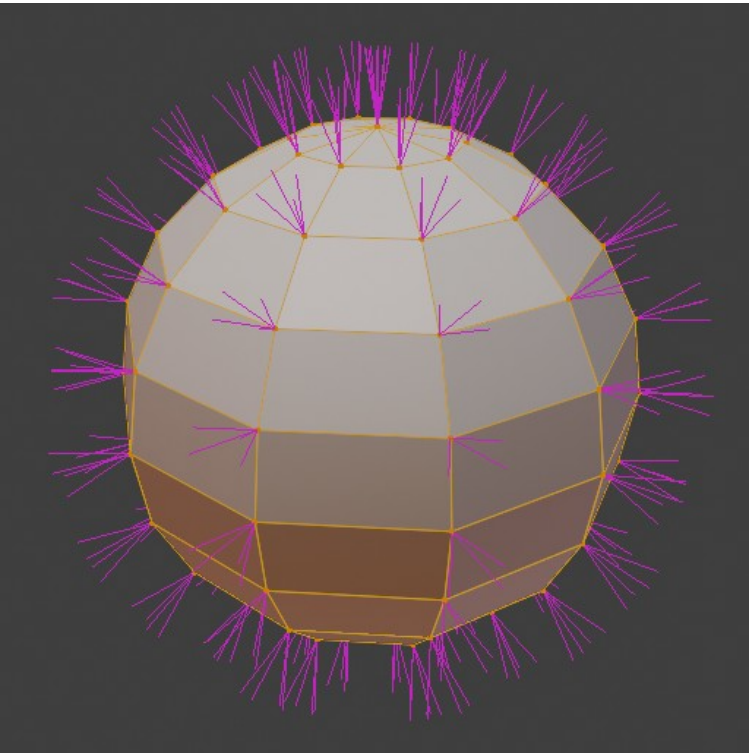


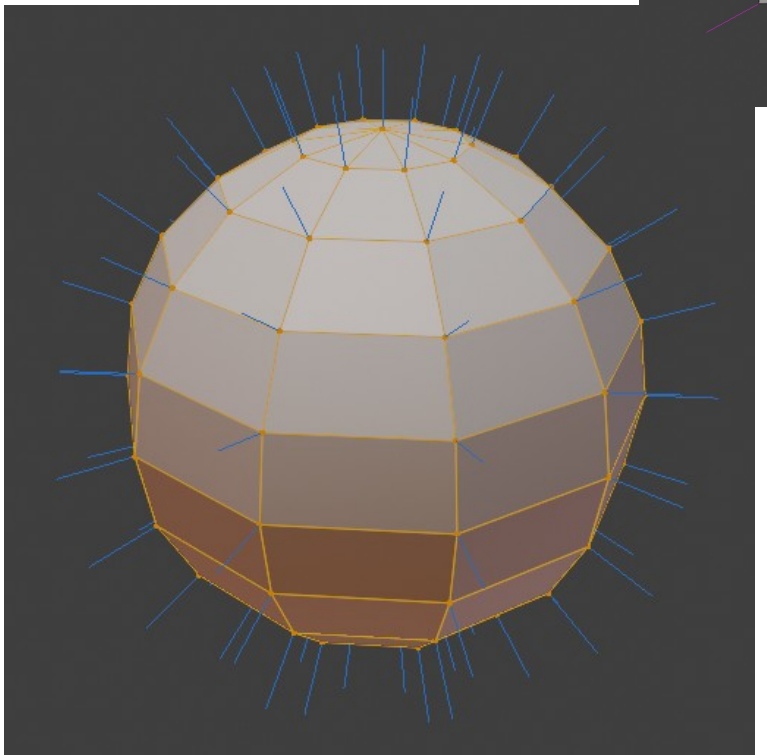


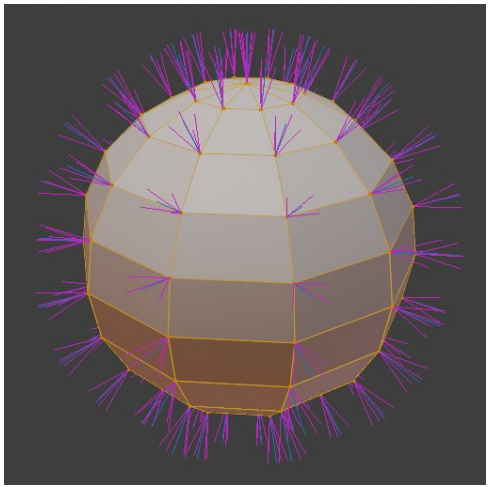
Normals: per vertex

- One normal for each face per vertex vs one normal per vertex



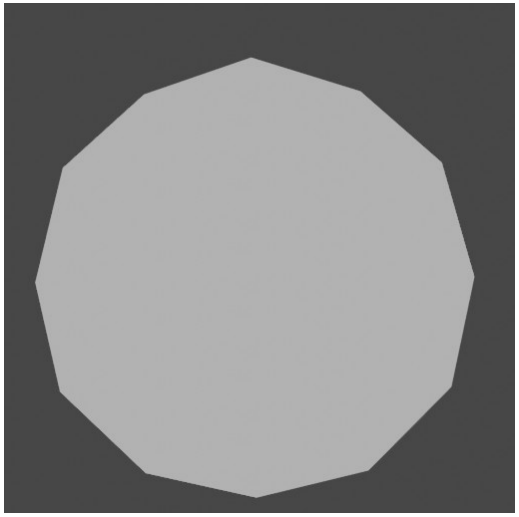


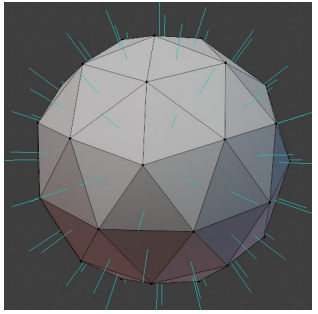


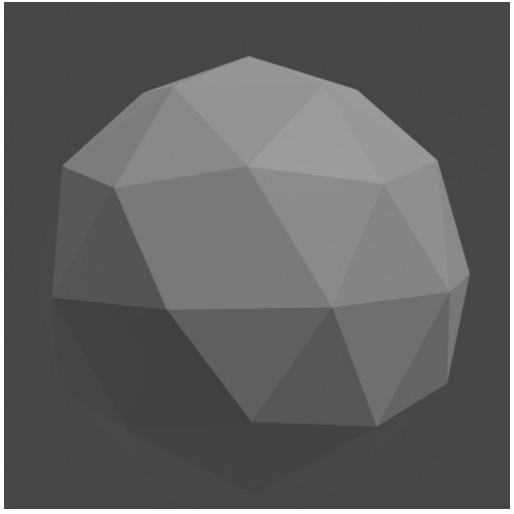


Using normals: shading

- Appearance of object depends both on shape and material.
  - **Normal is main shape information used in shading**: evaluation of scattering function depends on normal
- **Constant shading:** normal information is not used for shading
- **Flat shading:** evaluation of scattering model using per-face normals (no interpolation)

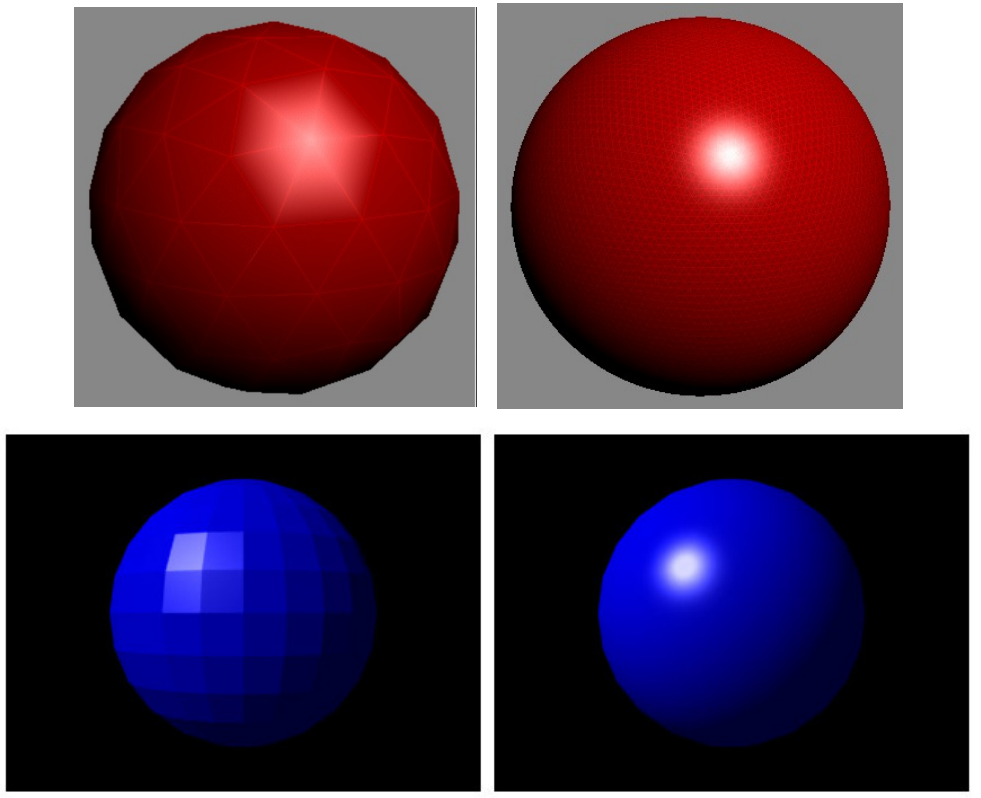






Using normals: shading

- **Gouraud (smooth) shading**: shading is calculated at vertices and resulting color is interpolated across triangle
  - Problem: unnatural highlights for larger polygons. Largest highlights appear at vertices of mesh and less on polygon during interpolation
  - "**Per-vertex**" light calculation
- **Phong (smooth) shading**
  - Normals per vertex are linearly interpolated over triangles and used for shading.
  - "**Per-pixel**" light calculation
  - Note: phong shading vs phong lighting model



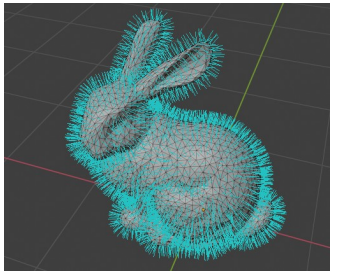
Smooth shading:

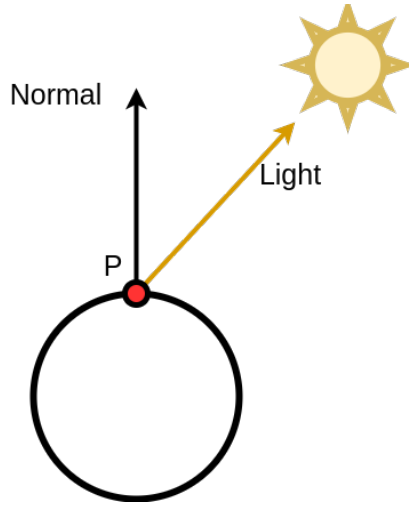
- Geometry is not changed: note faceted silhouette
- Illusion of smooth surface is created during shading

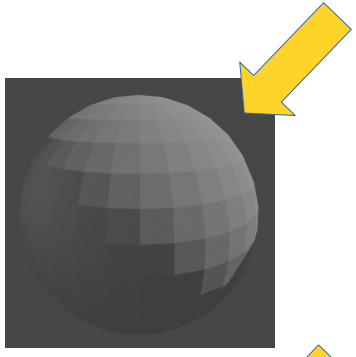
Flat vs smooth shading

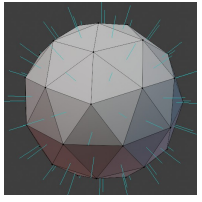
Using normals: shading

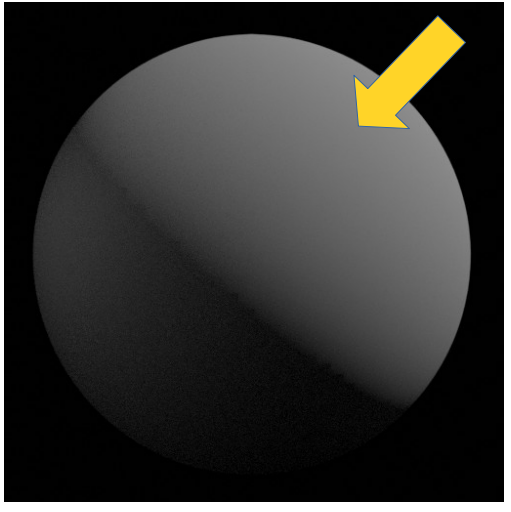
- Amount of light incident on surface depends on cosine of the angle between light direction and surface normal
  - **Lambert's Cosine law:** `cos(θ) = dot(N, L)`

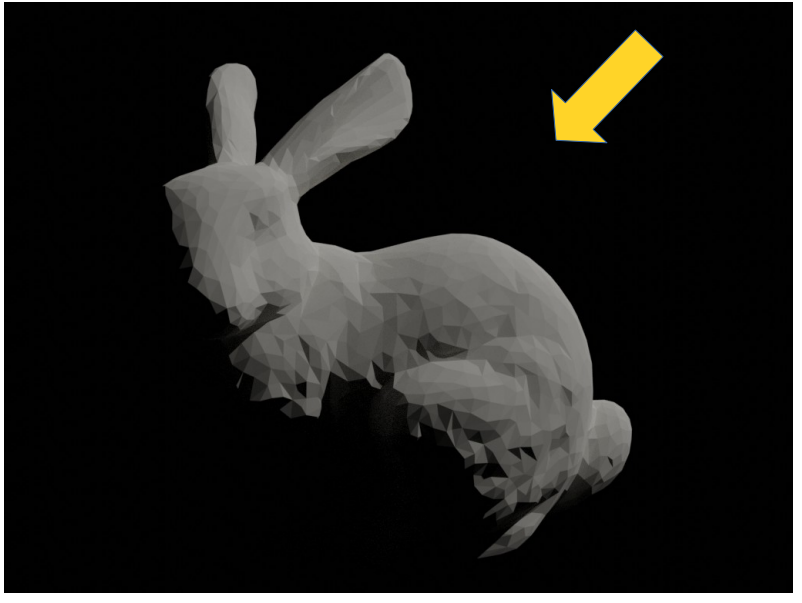






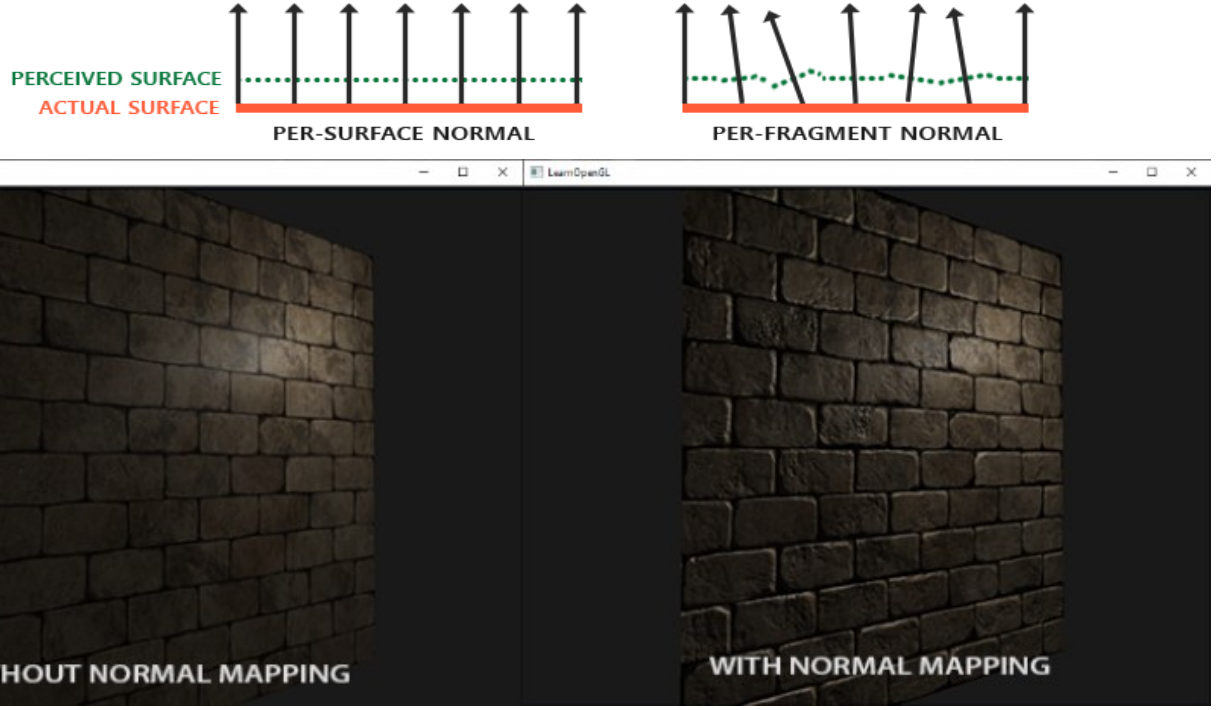






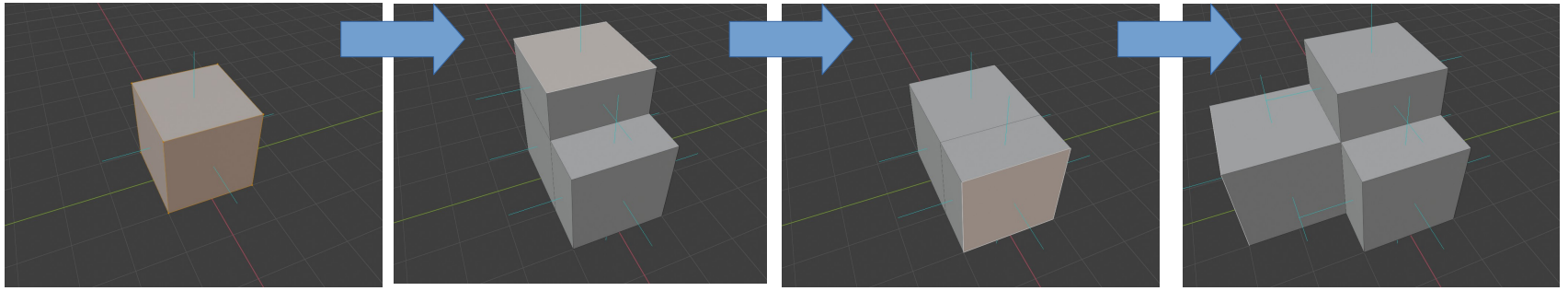
Using normals: surface details

- Face or vertex normals result in smooth and too perfect surface
- These normals, during rendering, can be perturbed for each point on surface which give appearance of surface details
  - Bump/normal mapping

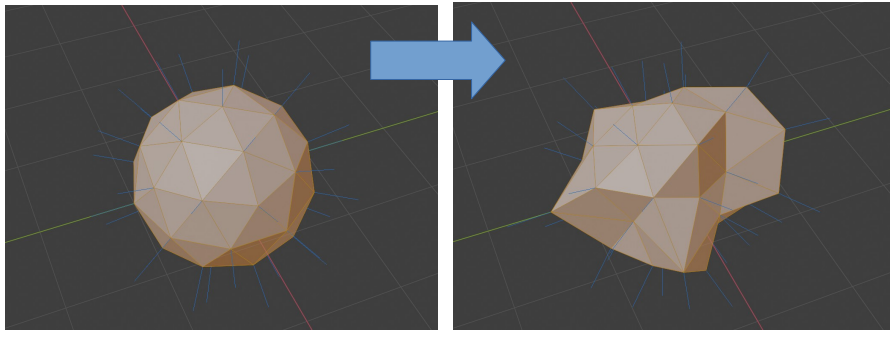


https://learnopengl.com/Advanced-Lighting/Normal-Mapping

Using normals: geometric manipulation



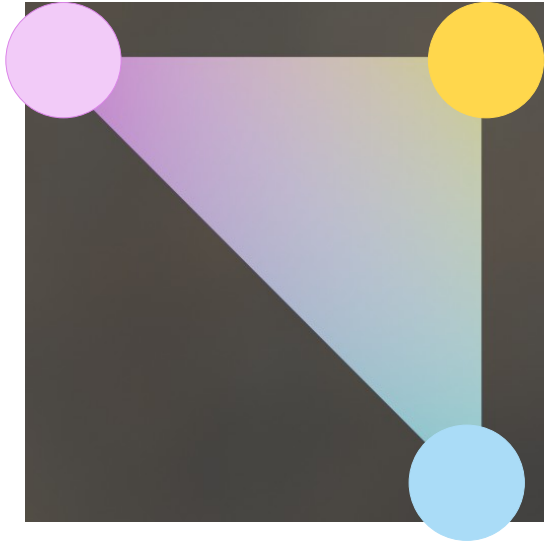
Example: **extrusion**. Fundamental modeling operation moving face in direction of normal.

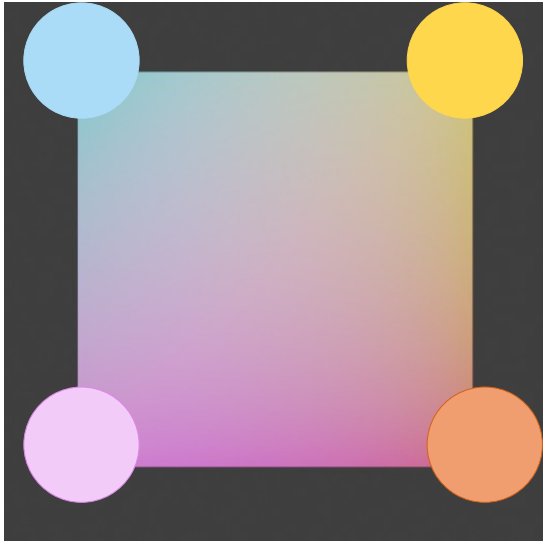


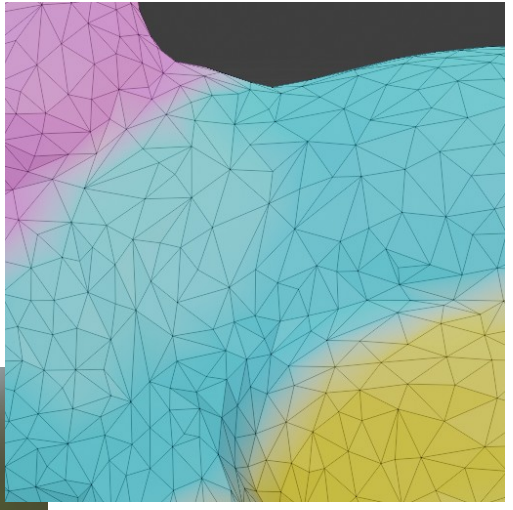
Example: **vertex displacement**. Vertices are moved in normal direction.

Vertex attributes: vertex colors

- Similarly as normals are defined per vertex and interpolated over triangle, the same can be done with color
- **The simplest way to introduce variation over object surface** – a form of texture
- This method works fine for meshes with finer structure → more colors can be assigned to more vertices.
  - When it is not possible to have fine mesh, then texture is used.



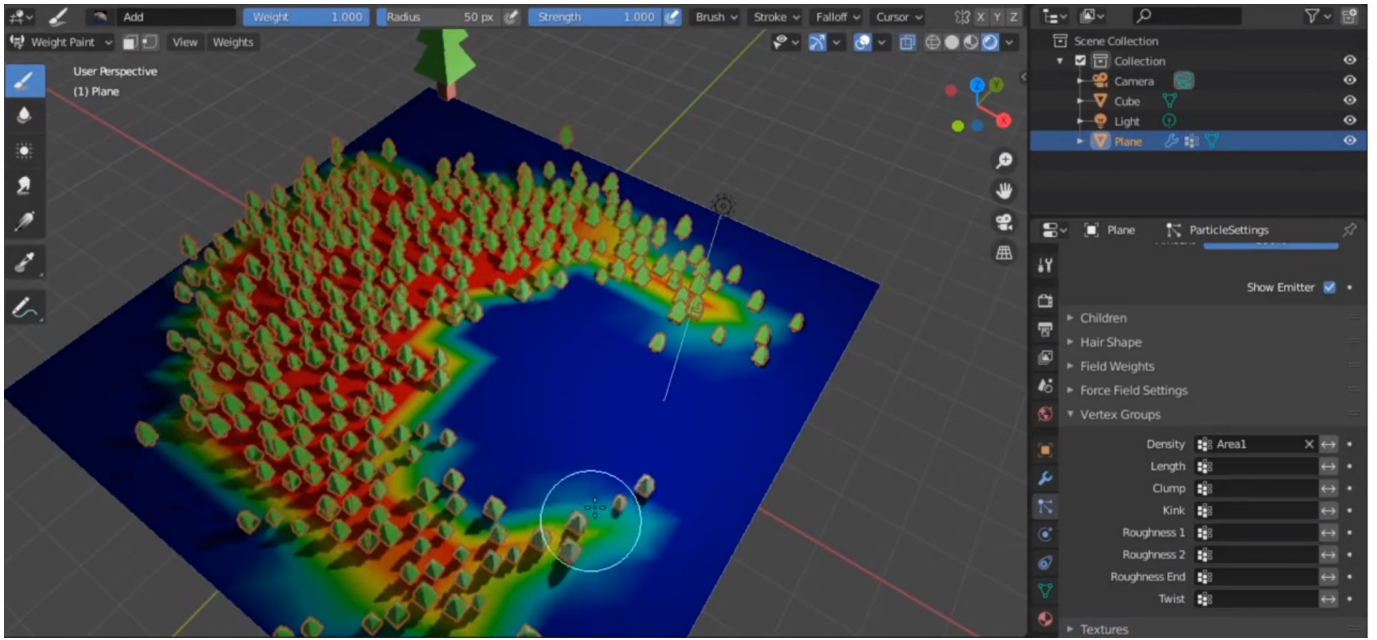




https://docs.blender.org/manual/en/latest/sculpt_paint/vertex_paint/index.html

Vertex attributes: vertex weights

- Vertex attributes can be any information that can be used for rendering
- Example: vertex weight designating density for instancing objects (e.g., particles)



https://www.youtube.com/watch?v=oPenYcM6Usw&ab_channel=HelperGround

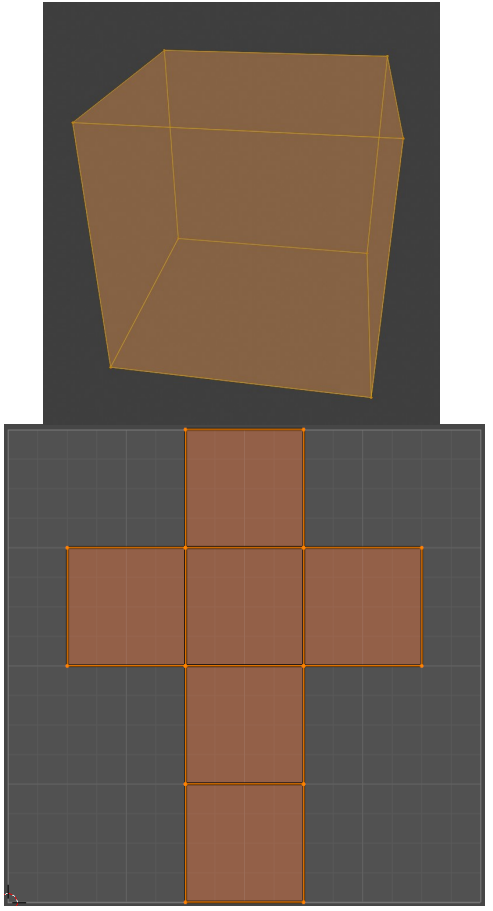
Vertex attributes: texture coordinates

- To add more details on object, often images and procedural patterns are used → **textures**.
- The problem of applying texture image on 3D object is often quite complex than on flat plane.
  - A a way of mapping a 2D image/pattern to 3D shape can be done by "unwrapping" mesh onto 2D plane.

https://polyhaven.com/a/wine_bottles_01

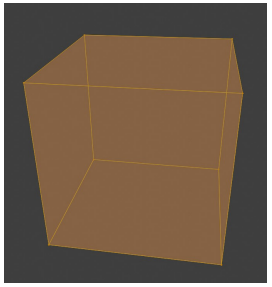
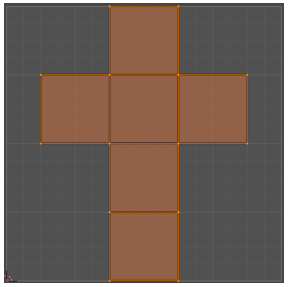
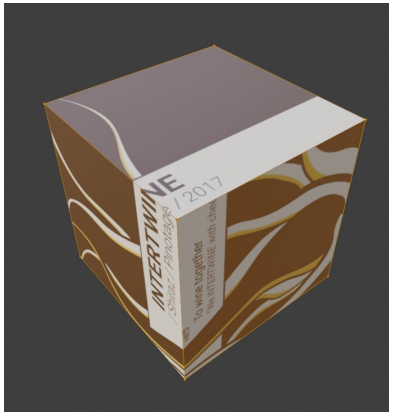
Texture coordinates

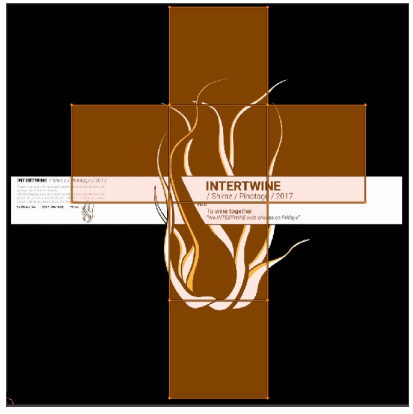
- Representing texture coordinates:
  - Texture coordinate per vertex: list of (u,v) coordinates equal to the number of vertices. Example: **texture_coordinates** = {{0,0.5}, {1,0.5}, {0.5,1}, {0,0.5}, {0.5,0}, {1,0.5}}
  - In UV space it is possible that multiple vertices have same texture coordinates thus connectivity information can be used to reduce number of texture coordinates to be written down

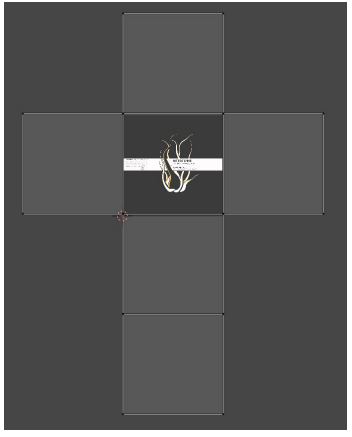


Mesh unwrapping: intuition

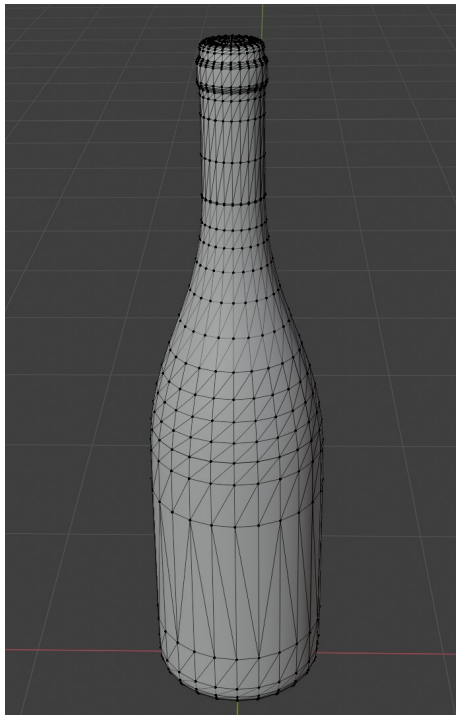
- Mesh is unwrapped into 2D plane we can imagine it lies in 2D coordinate system.
- Vertices of unwrapped mesh now have coordinates in this 2D coordinate system (u,v) → **texture coordinates***
  - (u,v) are in [0,1] range
  - Unwrapped faces can be manipulated, but then texture might get deformed
  - Whole unwrapped mesh can be translated or rotated or scaled to achieve different texture positioning
  - Several faces can overlap

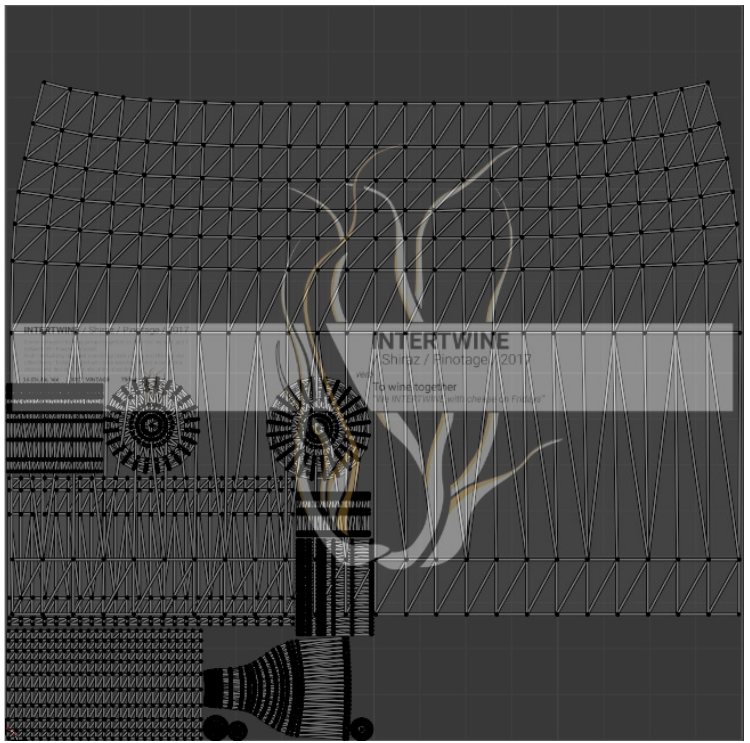








Texture coordinates: mesh unwrap





* Note: this is one way of calculating texture coordinates. Texture coordinates can also be calculated on the fly during rendering or other methods that we will discuss later.

Polygon Mesh: datastructures, storing and transfer

Datastructures and representation

- There are multiple data structures for polygonal meshes that offer trade-off between run-time complexity and memory requirements
  - Rendering
  - Modeling
  - Algorithms on meshes
  - Efficient topology traversal
  - Efficient mesh updates
  - Storage and compression
  - Etc.

Mesh data structure

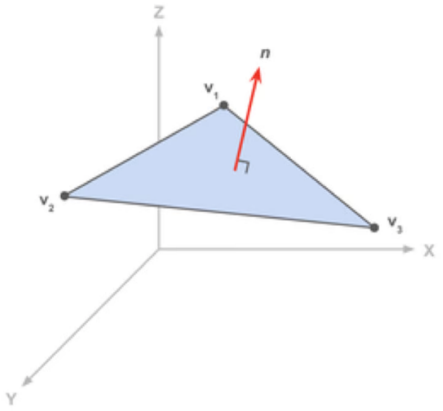
- Mesh data structure must at least describe:
  - **Geometry**: position of vertices specified using coordinates
  - **Connectivity/topology** information for vertices → how are edges and faces formed from the vertices. Options:
    - For each polygon, list all vertices that define it
    - Polygon topology: specify edges of mesh in terms of pairs of vertices (connection between vertices and edges)
    - Mesh topology: for each edge, state which polygons are incident connection between edges and polygons (connection between edges and polygons)
  - Other information not explicitly given can be calculated from given information
- Optional: Vertex attributes
  - Normals, colors, texture coordinates, etc.

Explicit face representation

- **Triangle soup**: each triangle is explicitly represented using vertex coordinates → topology is not represented explicitly
  - Triangle: 3 vertices each having 3 coordinates `(x,y,z)`.
  - n triangles: `n * 3 * 3 = 9 * n` floats → high redundancy!
- Data format used in production:
  - **STL** (**ST**ereo**L**itography)

t	v_i	v_j	v_k
0	(1.0, 1.0, 1.0)	(-1.0, 1.0,-1.0)	(1.0, 1.0, 1.0)
1	(1.0, 1.0, 1.0)	(-1.0,-1.0, 1.0)	(1.0,-1.0,-1.0)
2	(1.0, 1.0, 1.0)	(1.0,-1.0, 1.0)	(-1.0, 1.0,-1.0)
3	(1.0,-1.0,-1.0)	(-1.0,-1.0, 1.0)	(-1.0, 1.0,-1.0)

Example: STL file format




```
facet normal ni nj nk
    outer loop
        vertex v1x v1y v1z
        vertex v2x v2y v2z
        vertex v3x v3y v3z
    endloop
endfacet
```

```
foreach triangle
    REAL32[3] – Normal vector
    REAL32[3] – Vertex 1
    REAL32[3] – Vertex 2
    REAL32[3] – Vertex 3
    UINT16    – Attribute byte count
end
```

STL file format (ASCII and binary type) stores triangles by explicitly listing vertices and normals: https://all3dp.com/1/stl-file-format-3d-printing/

Shared vertex representation

- AKA: indexed face set
  - Reduce redundancy
- Vertex coordinates are stored in **vertex array** (`3 * n` floats)
- Triangles are stored in **triangle/face array containing vertex indices** (`3 * n` integers)
- Data formats used in production:
  - OBJ
  - OFF

v	*x*	*y*	*z*
0	1.0	1.0	1.0
1	-1.0	1.0	-1.0
2	-1.0	-1.0	1.0
3	1.0	-1.0	1.0

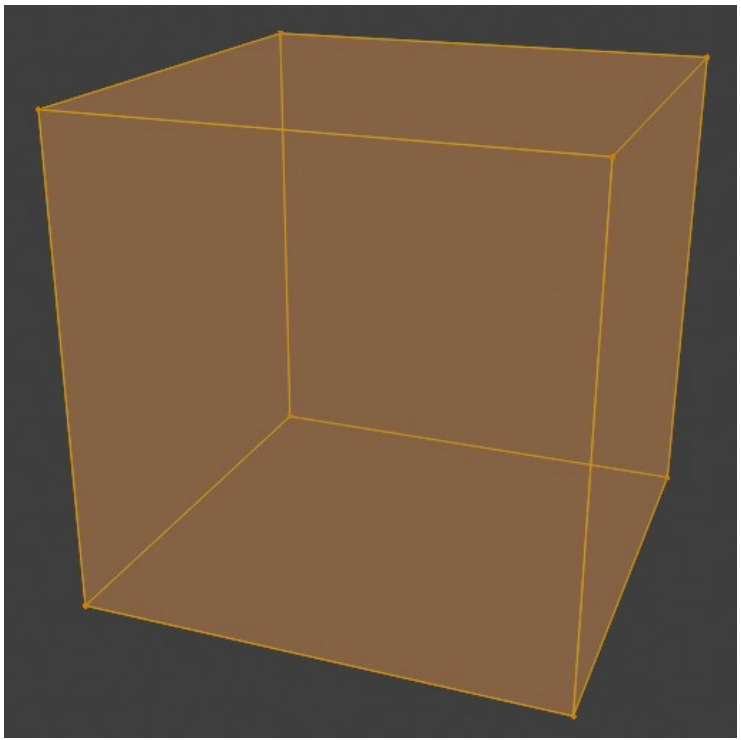
vertex array

t	*i*	*j*	*k*
0	0	1	2
1	0	2	3
2	0	3	1
3	3	2	1

triangle array

Example: shared vertex representation

- **Renderman Interface Bytestream (RIB) standard***
- 8 vertices: **vertex array – geometry information**
  - `vert_array = {{-1,1,1},{1,1,1},{1,1,-1},{-1,1,-1},{-1,-1,1},{1,-1,1},{1,-1,-1},{-1,-1,-1}}`
- 6 quads for representing faces. Each quad requires 4 vertices: **face index array – topology information**.
  - `face_index_array = {4, 4, 4, 4, 4, 4}`
  - `size(face_index_array)` = number of polygons used for representing a shape
- For each face, which indices of vertex array are used: **vertex index array – topology information**.
  - `vertex_index_array = {0, 1, 2, 3, 0, 4, 5, 1, 1, 5, 6, 2, 0, 3, 7, 4, 5, 4, 7, 6, 2, 6, 7, 3}`
  - `size(vertex_index_array)` = sum of all values in face index array
  - Note: each face of the cube shares some vertices with other faces

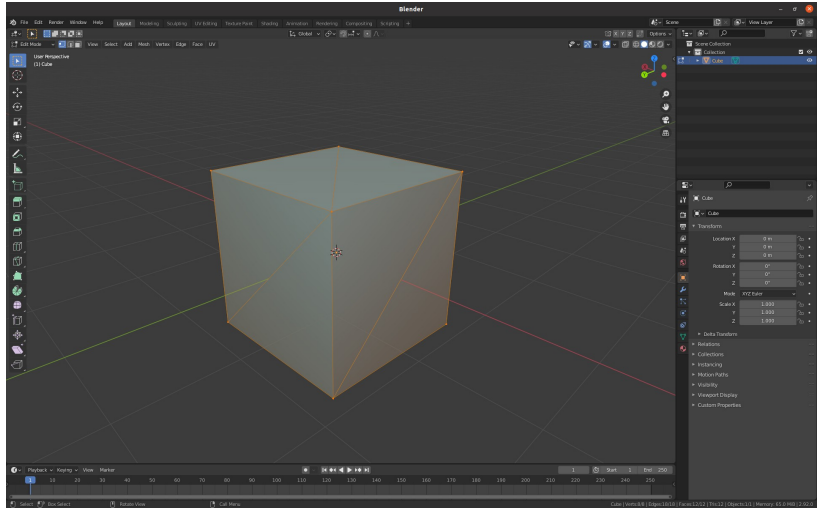


* https://renderman.pixar.com/resources/RenderMan_20/ribBinding.html
https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-polygon-mesh/polygon-mesh-file-formats

Example: OBJ file format

- Less redundancy than explicit representation (e.g., STL)
- Separates geometry (vertices) and topology (connectivity)
- This representation is costly for computation
- Each line starts with letter representing type of data:
  - `v` – vertices
  - `vt` – texture coordinates
  - `vn` – normals
  - `f` – faces (index of vertex, vertex texture coordinate, vertex normal

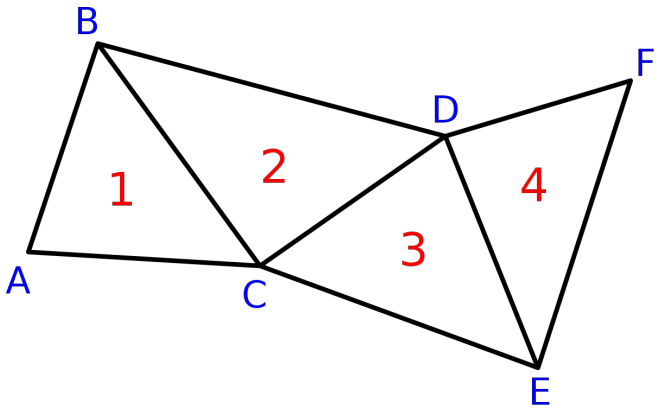
OBJ file format is further extensible:
https://en.wikipedia.org/wiki/Wavefront_.obj_file



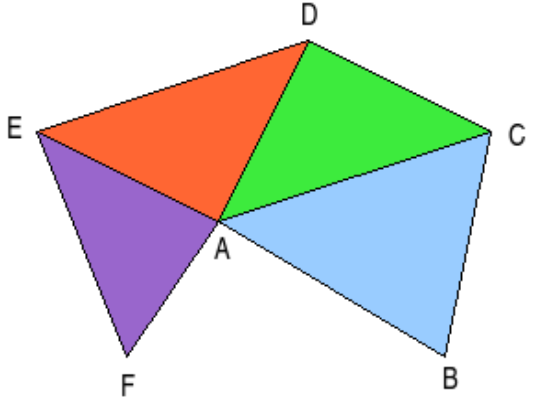
```
# Blender v2.92.0 OBJ File: ''
# www.blender.org
o Cube_Cube.002
v -1.000000 -1.000000 1.000000
v -1.000000 1.000000 1.000000
v -1.000000 -1.000000 -1.000000
v -1.000000 1.000000 -1.000000
v 1.000000 -1.000000 1.000000
v 1.000000 1.000000 1.000000
v 1.000000 -1.000000 -1.000000
v 1.000000 1.000000 -1.000000
vt 0.625000 0.000000
vt 0.375000 0.250000
vt 0.375000 0.000000
vt 0.625000 0.250000
vt 0.375000 0.500000
vt 0.625000 0.500000
vt 0.375000 0.750000
vt 0.625000 0.750000
vt 0.375000 1.000000
vt 0.125000 0.750000
vt 0.125000 0.500000
vt 0.875000 0.500000
vt 0.625000 1.000000
vt 0.875000 0.750000
vn -1.0000 0.0000 0.0000
vn 0.0000 0.0000 -1.0000
vn 1.0000 0.0000 0.0000
vn 0.0000 0.0000 1.0000
vn 0.0000 -1.0000 0.0000
vn 0.0000 1.0000 0.0000
s off
f 2/1/1 3/2/1 1/3/1
f 4/4/2 7/5/2 3/2/2
f 8/6/3 5/7/3 7/5/3
f 6/8/4 1/9/4 5/7/4
f 7/5/5 1/10/5 3/11/5
f 4/12/6 6/8/6 8/6/6
f 2/1/1 4/4/1 3/2/1
f 4/4/2 8/6/2 7/5/2
f 8/6/3 6/8/3 5/7/3
f 6/8/4 2/13/4 1/9/4
f 7/5/5 5/7/5 1/10/5
f 4/12/6 2/14/6 6/8/6
```

Example: specific data structures

- Data structures for minimal redundancy and memory efficient representation by implicit topology:
  - Triangle/quad strip
  - Triangle fan
- Complex meshes are represented as set of strips and fans
  - Used for hardware rendering (OpenGL - https://www.khronos.org/opengl/wiki/Primitive)



Triangle strip: implicit representation of strip-like sequence of triangles. https://en.wikipedia.org/wiki/Triangle_strip



Triangle fan: set of triangles sharing one vertex
https://www.khronos.org/opengl/wiki/Primitive

Storing and transferring mesh

- Single polygon mesh in a 3D scene can can be quite large (10^5-10^6 vertices is not unusual)
- Storing vertices and connectivity information must be performed efficiently
  - All vertices must be stored
  - Different techniques exist which try to minimize the amount of data needed for representing connectivity
- Different standards, formats and API specifications for storing and transferring mesh data
  - Compression level
  - Human readability
  - Additional object data (textures, materials, etc.)
  - Can contain various metadata (e.g., physical behavior of object)
  - Storing whole scenes (multiple objects)
  - Etc.

Storing and transferring mesh

- Mesh data is often transferred between different modeling, rendering and interactive tools and environments.
  - Example: Blender to Unity
- Interface between different tools and specification of how mesh should be stored is defined by standards and formats
  - Renderman Interface Bytestream: https://renderman.pixar.com/resources/RenderMan_20/ribBinding.html
  - OBJ: http://paulbourke.net/dataformats/obj/
  - OFF: https://shape.cs.princeton.edu/benchmark/documentation/off_format.html
  - FBX: https://code.blender.org/2013/08/fbx-binary-file-format-specification/
  - glTF: https://www.khronos.org/gltf/
  - USD: https://graphics.pixar.com/usd/release/index.html
- Other 3D file formats:
  - https://all3dp.com/2/most-common-3d-file-formats-model/
  - https://www.sidefx.com/docs/houdini/io/formats/geometry_formats.html
  - https://docs.fileformat.com/3d/fbx/

Tendency is towards standardization of whole scene description. Which, besides mesh polygons, include materials, lights, cameras, different shape representations, etc. See: USD

Importing/exporting mesh

- DCC and games engines provide user with "importer/exporter" – feature which enables importing and exporting different file formats
  - Blender: https://docs.blender.org/manual/en/3.4/files/import_export.html
  - Houdini: https://www.sidefx.com/docs/houdini/io/formats/index.html
  - Godot: https://docs.godotengine.org/en/stable/tutorials/assets_pipeline/import_process.html
  - Unity: https://docs.unity3d.com/Manual/ImportingAssets.html
- If one writes its own rendering program, parsing file formats for import can be not that easy. Luckily, different libraries can be used for this purposes:
  - Assimp: https://github.com/assimp/assimp

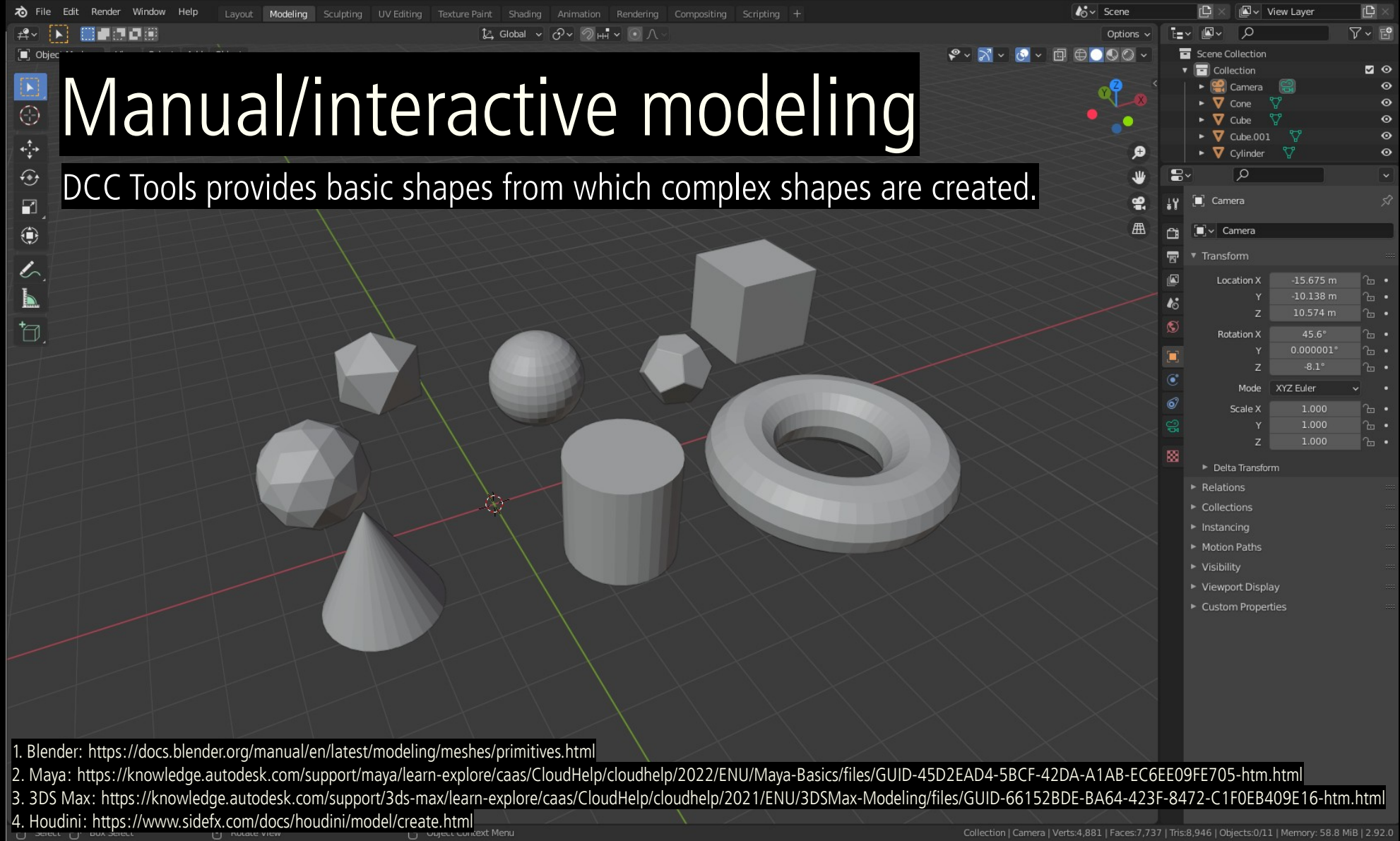
Practical note: units

- Units are important for preserving consistent scale among different modeling/rendering tools in which object might be transferred.
- Example: Blender. Mesh, that is, vertices are defined in a coordinate system. In this coordinate system we can have two same objects where one is larger and another is smaller. By relation/proportion we can distinguish the scale and we might not care about units.
  - Since positions of vertices are relative to coordinate system, the axis of coordinate system must specify units.
- But what happens if one of this objects in exported into another tool, for example Unity? Unity might use different coordinate system unit scale and object might be too big or too small in this coordinate system!
- Unit is defined by user who models the object and it must be taken care of during transfer.

Mesh polygons: modeling and acquisition

Manual/interactive modeling

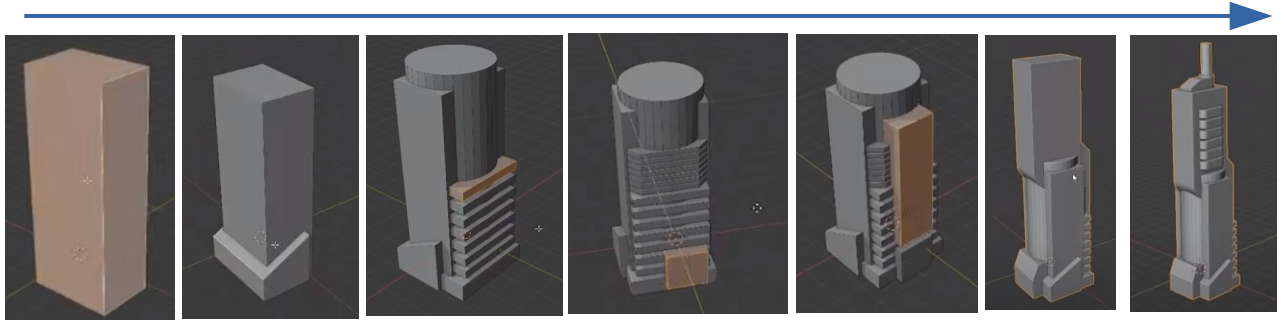
DCC Tools provides basic shapes from which complex shapes are created.



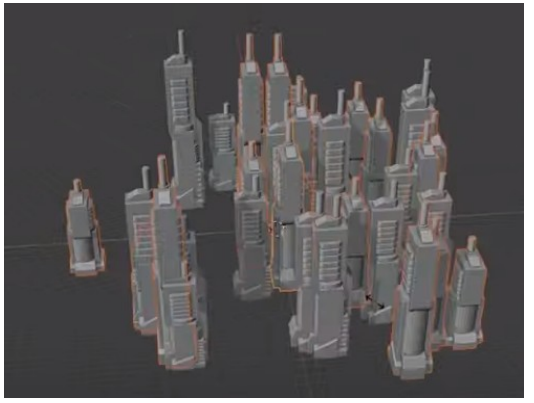
1. Blender: https://docs.blender.org/manual/en/latest/modeling/meshes/primitives.html
2. Maya: https://knowledge.autodesk.com/support/maya/learn-explore/caas/CloudHelp/cloudhelp/2022/ENU/Maya-Basics/files/GUID-45D2EAD4-5BCF-42DA-A1AB-EC6EE09FE705-htm.html
3. 3DS Max: https://knowledge.autodesk.com/support/3ds-max/learn-explore/caas/CloudHelp/cloudhelp/2021/ENU/3DSMax-Modeling/files/GUID-66152BDE-BA64-423F-8472-C1F0EB409E16-htm.html
4. Houdini: https://www.sidefx.com/docs/houdini/model/create.html

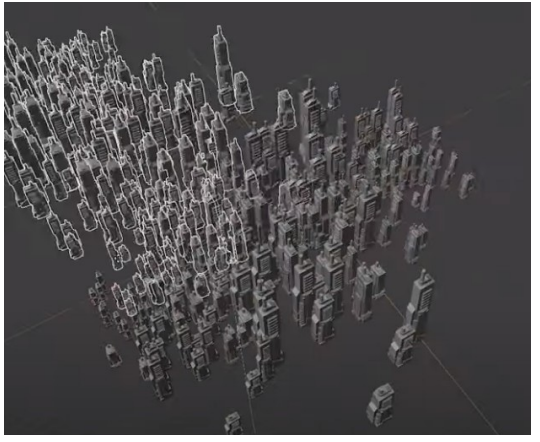
Practical tip: complex shapes from base shapes

- Anything can be decomposed in simple forms[1,2]: box, sphere, cylinder, torus, cones, etc.



https://www.youtube.com/watch?v=Q0qKO2JYR3Y&ab_channel=BlenderSecrets

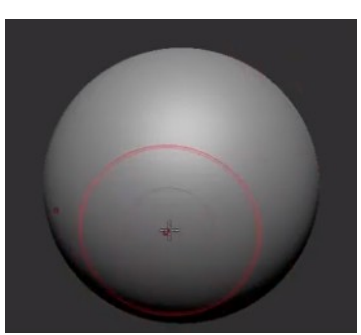
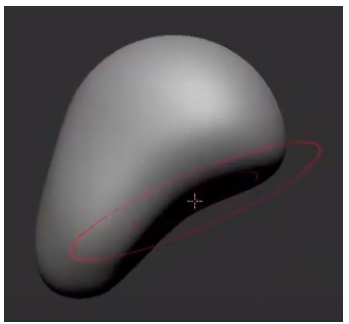
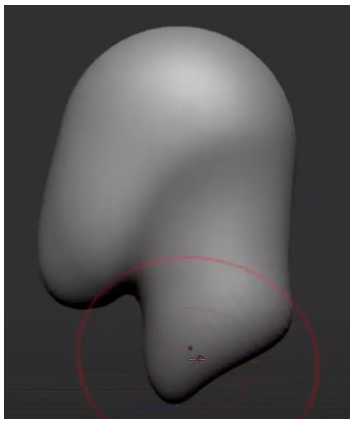


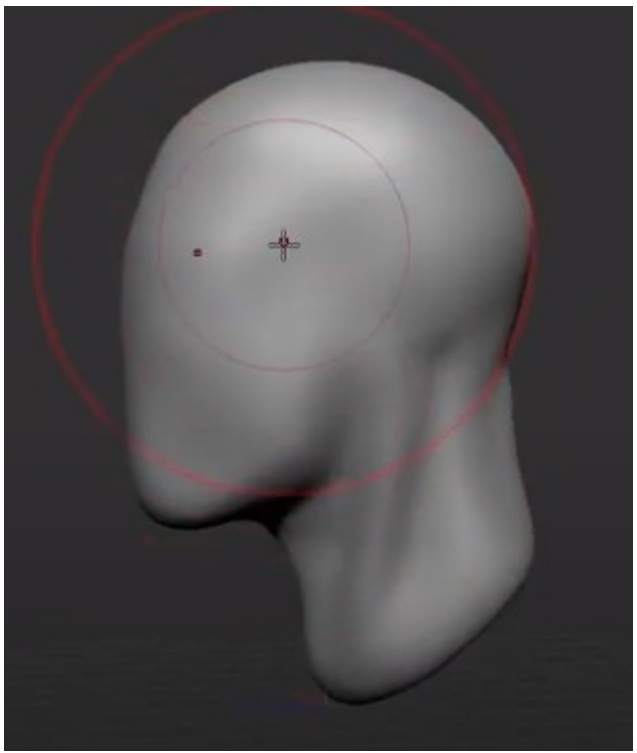


1. http://www.thedrawingwebsite.com/2015/02/18/practicing-your-draw-fu-forms-forms-are-like-sentences/
2. https://www.youtube.com/watch?v=6T_-DiAzYBc&list=RDCMUClM2LuQ1q5WEc23462tQzBg&start_radio=1&rv=6T_-DiAzYBc&t=1343&ab_channel=Proko

Practical tip: complex shapes from base shapes

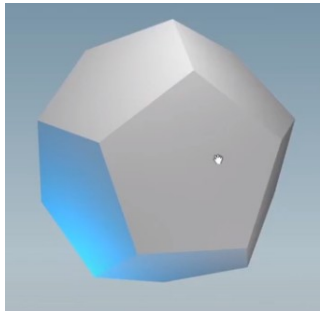
- Sculpting base shapes.

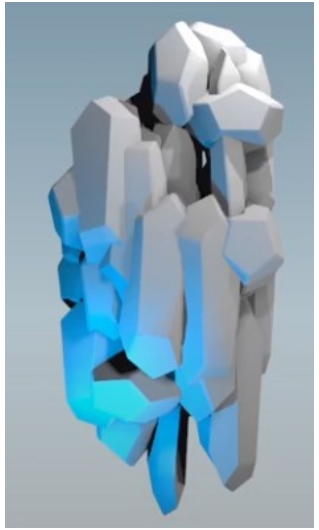




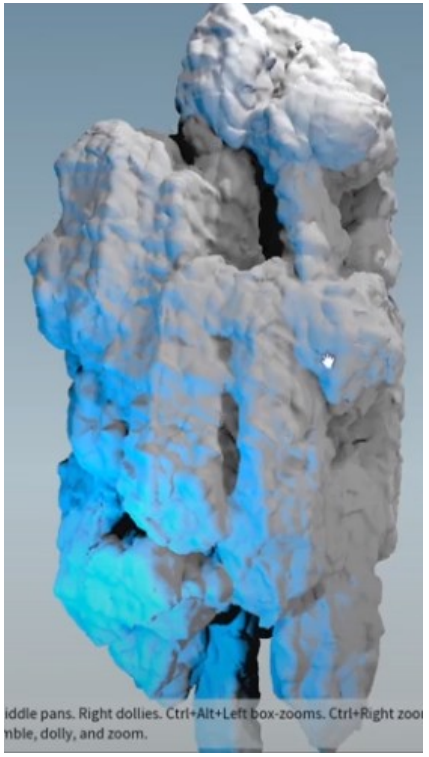


https://www.youtube.com/watch?v=p4Y1hVCMMXw&ab_channel=RealEnay

Practical tip: complex shapes from base shapes





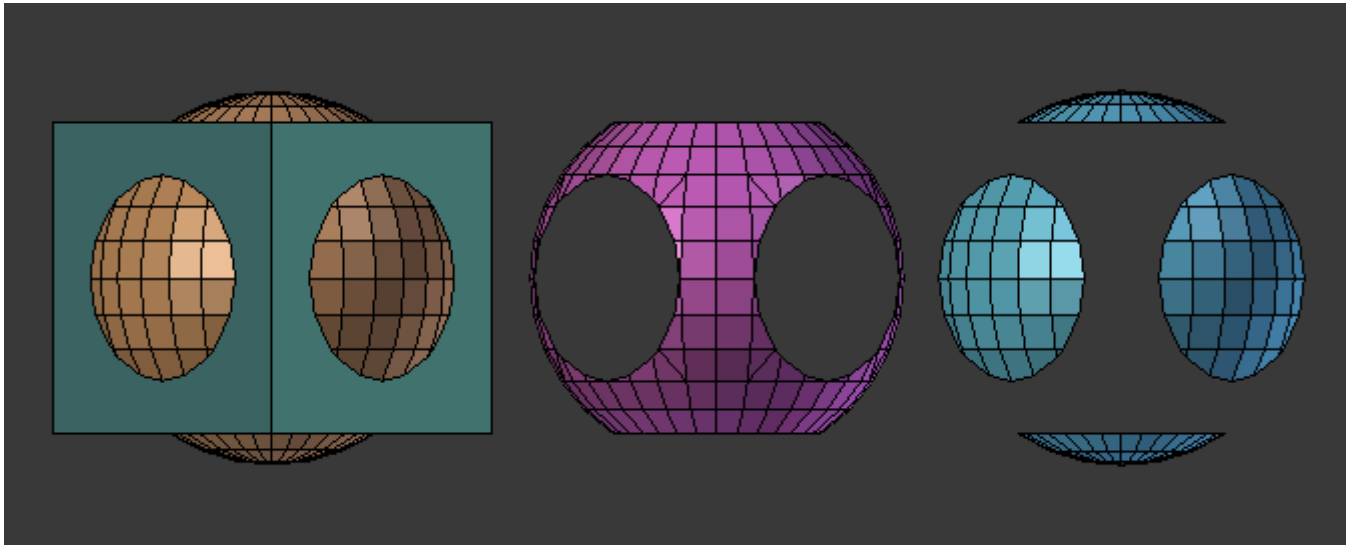


- Procedural modeling

https://www.youtube.com/watch?v=fxOxygaEOFk&ab_channel=SimonHoudini

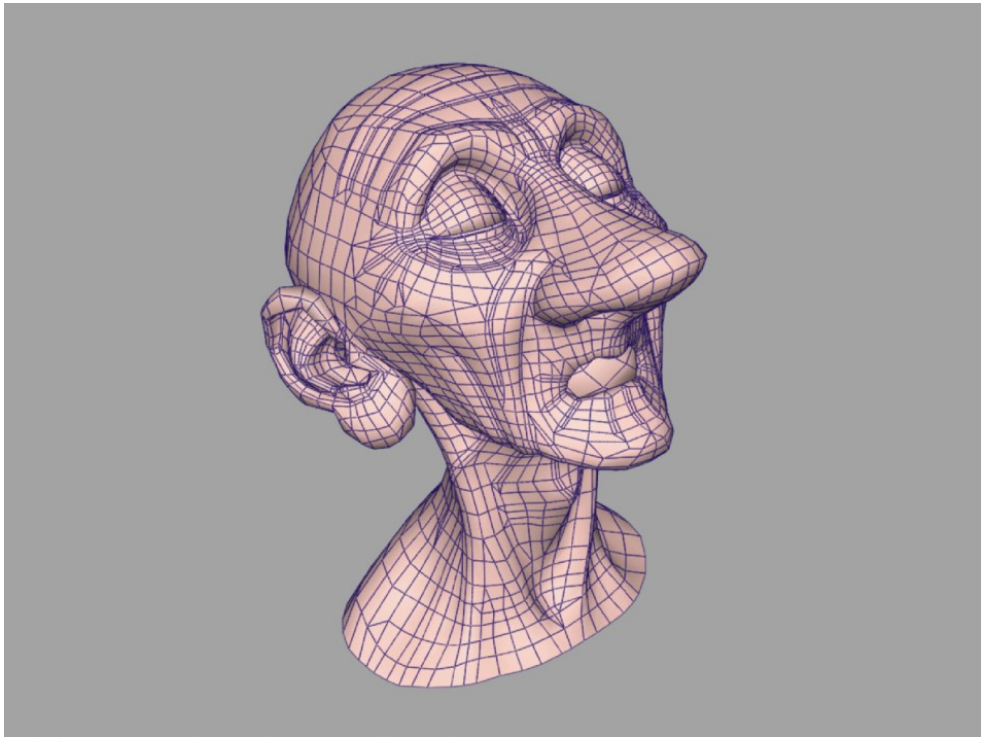
Constructive solid geometry

- Boolean operations applied on meshes:
  - Union
  - Intersection
  - Difference



Union, intersection and difference Boolean operators applied on meshes:
https://docs.blender.org/manual/en/latest/modeling/modifiers/generate/booleans.html

Subidvision surfaces and displacement

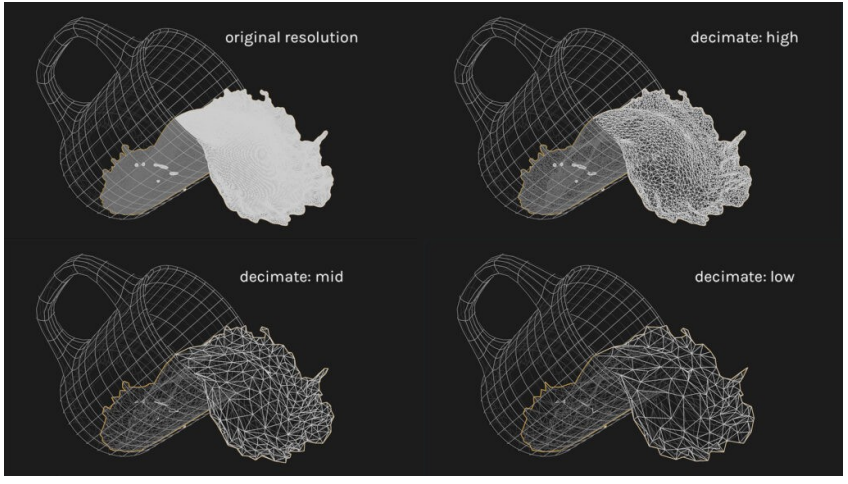


Subdivision surfaces for smooth surfaces in Pixar:
https://graphics.pixar.com/library/Geri/paper.pdf
https://www.fxguide.com/fxfeatured/pixars-opensubdiv-v2-a-detailed-look/

Blender displacement of subdivided surface:
https://docs.blender.org/manual/en/latest/render/materials/components/displacement.html

Simulations

- Polygonal model can be generated using simulations (e.g., fluid simulation using voxel representation) which is then tessellated into polygons for rendering.



Fluid simulation in Blender:
https://andi-siess.de/designing-a-book-cover-as-3d-vector/

Houdini fluid simulation: https://www.sidefx.com/tutorials/crashing-wave/

Procedural modeling

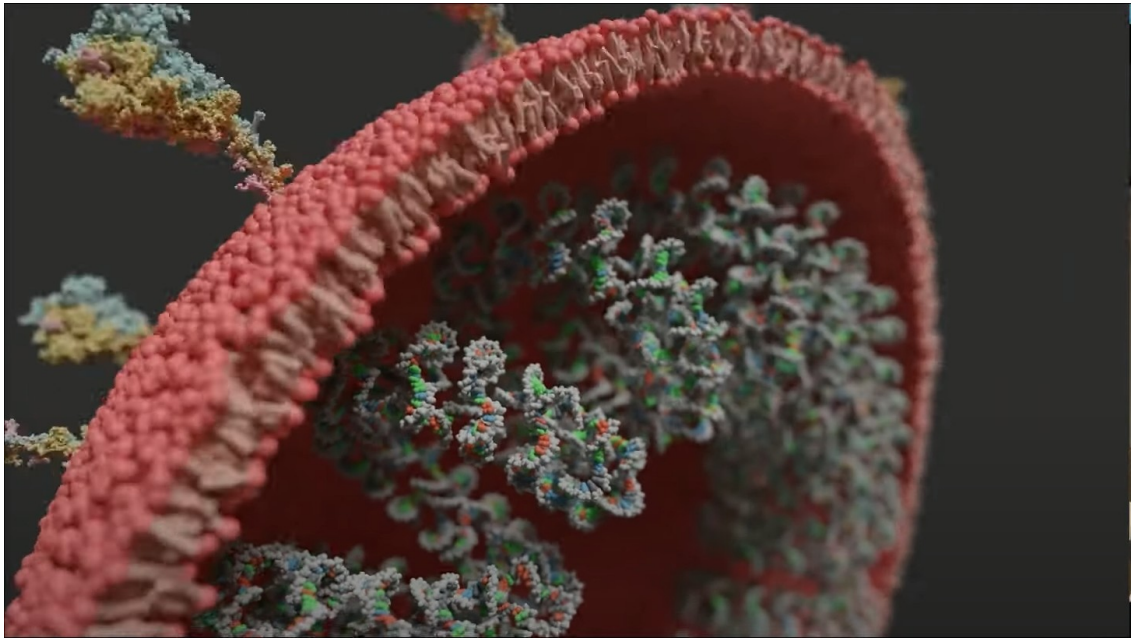
- Similarly to simulations, polygonal mesh can be created by writing programs that create such data – procedural modeling

Procedural modeling in Houdini:
https://entagma.com/category/tutorials/

Blender procedural modeling using geometry nodes:
https://blenderartists.org/t/procedural-abandoned-house-with-geometry-nodes/1363024

Transforming data into visualizations

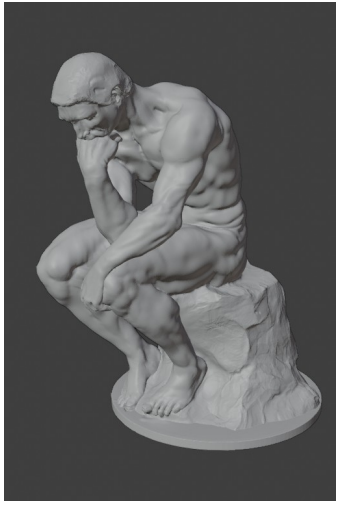
- Data found in different forms can be converted to polygon data. Example: visualizing protein data using spheres.

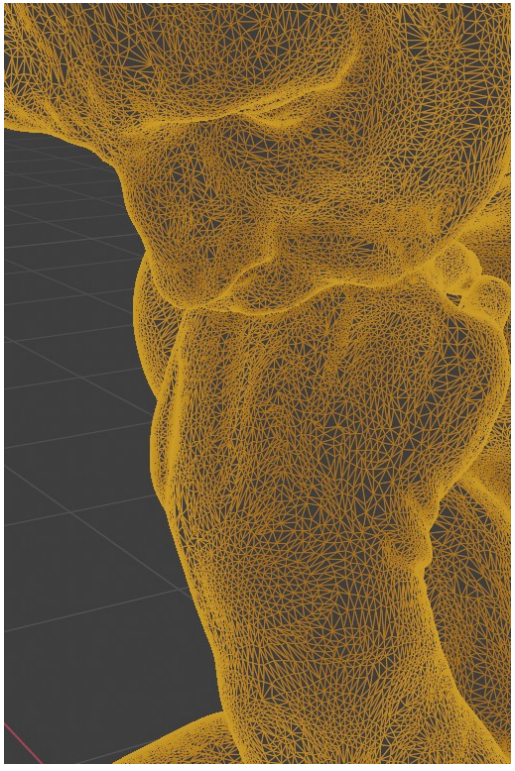


Visualizing viruses in Blender:
https://www.youtube.com/watch?v=adhTmwYwOiA&ab_channel=Blender

Scanning and photogrammetry

- Polygonal model can be generated using scanning and photogrammetry which results in point clouds and then is transformed into polygons for rendering.

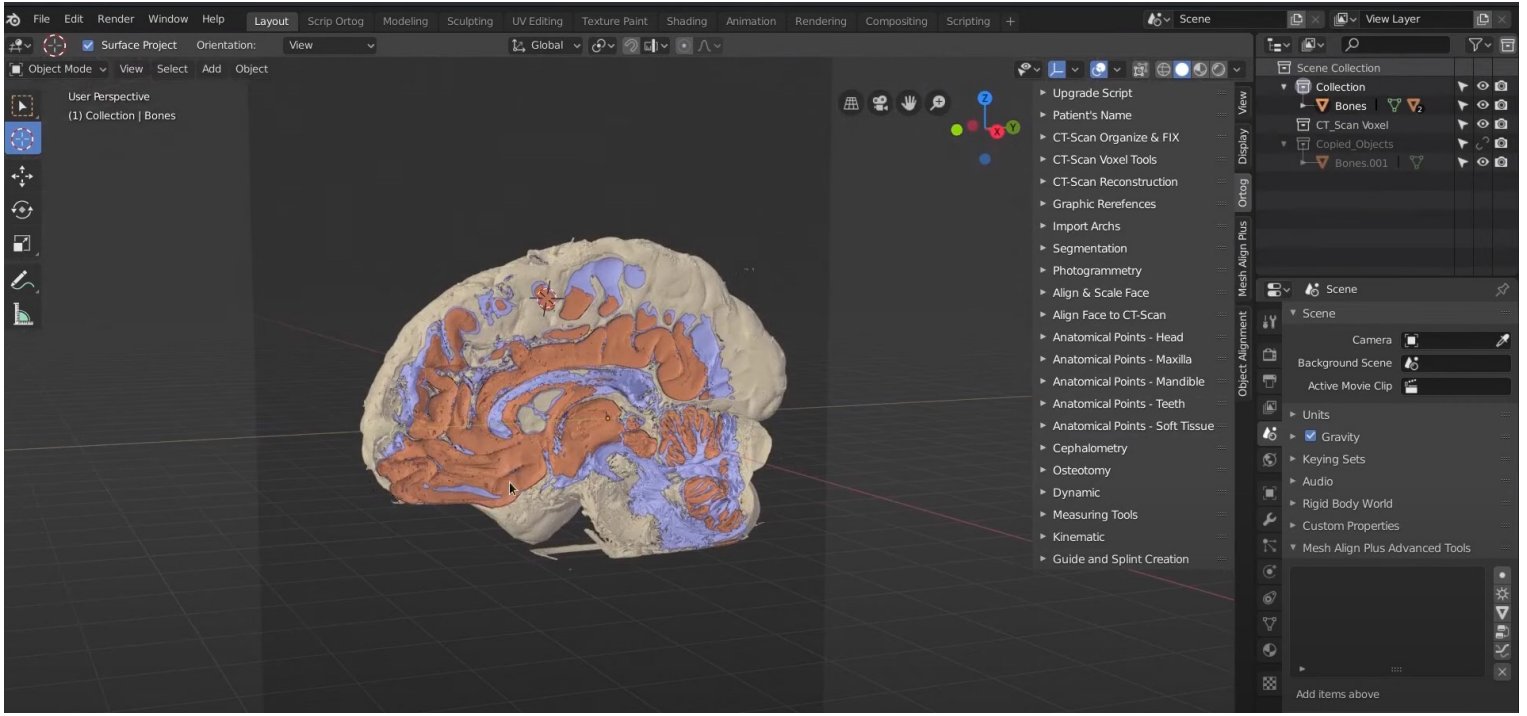




Scan the world: https://www.myminifactory.com/scantheworld/
Sans Factory: https://www.scansfactory.com/
Mega Scans: https://quixel.com/megascans/home/
Art Station 3D scanning and photogrammetry: https://www.artstation.com/channels/photogrammetry_3d_scanning?sort_by=popular

Data from volumetric scans

- Generating surface from volume data scanned with CT or MRI.

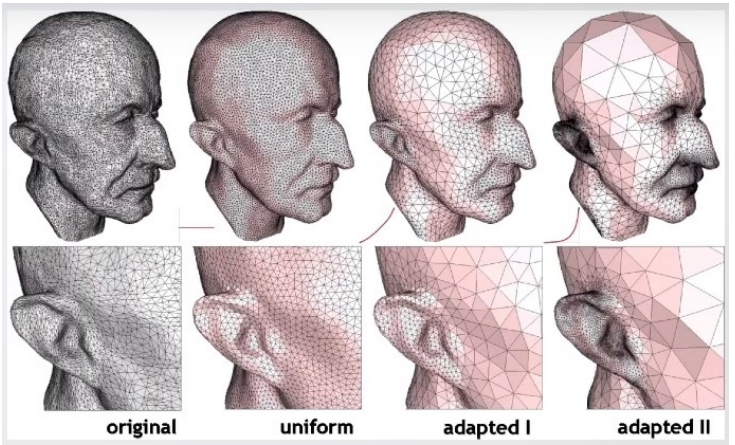


MRI to mesh in Blender: https://www.blendernation.com/2019/07/15/converting-a-video-into-a-dicom-and-a-3d-mesh/

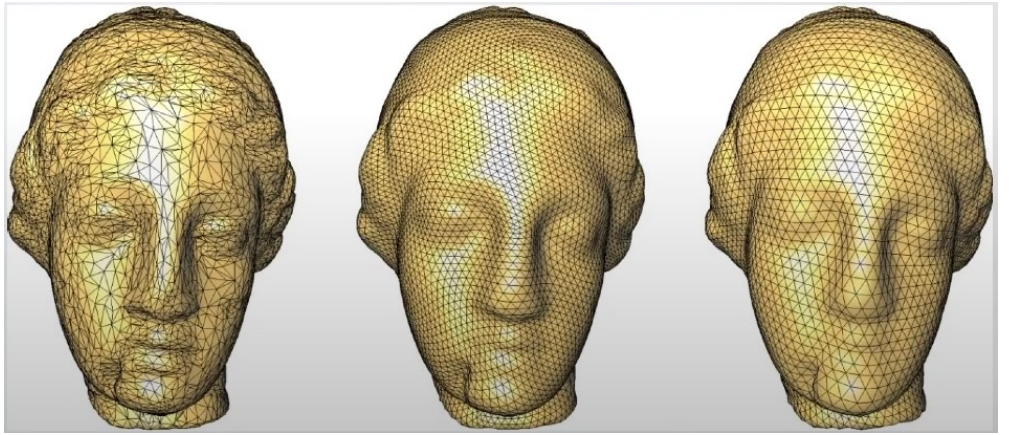
Mesh polygons: outlook

Deeper into topic

- Polygonal mesh is highly used and researched in computer graphics. We have covered foundations and other topics are out of scope:
  - Remeshing: https://docs.blender.org/manual/en/latest/modeling/modifiers/generate/remesh.html
  - Simplification, level of detail: https://docs.blender.org/manual/en/latest/modeling/modifiers/generate/decimate.html
    - replacing the mesh with the simpler one which has the similar shape (topological or geometrical)
  - Consolidation, mesh reparation: https://www.whiteclouds.com/blog/how-to-repair-mesh/

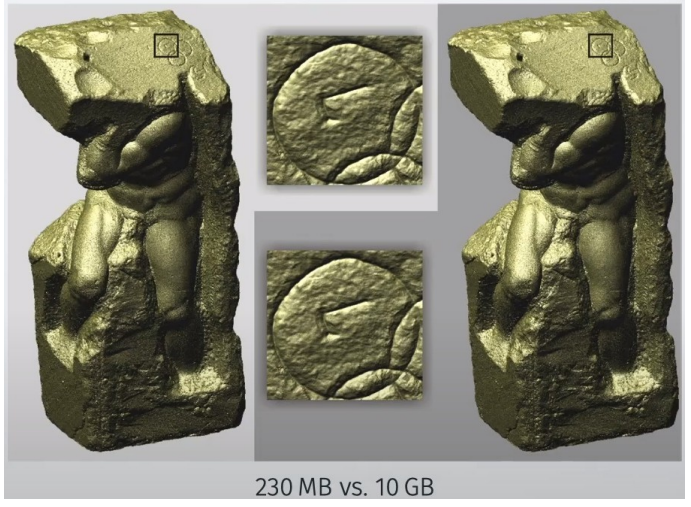


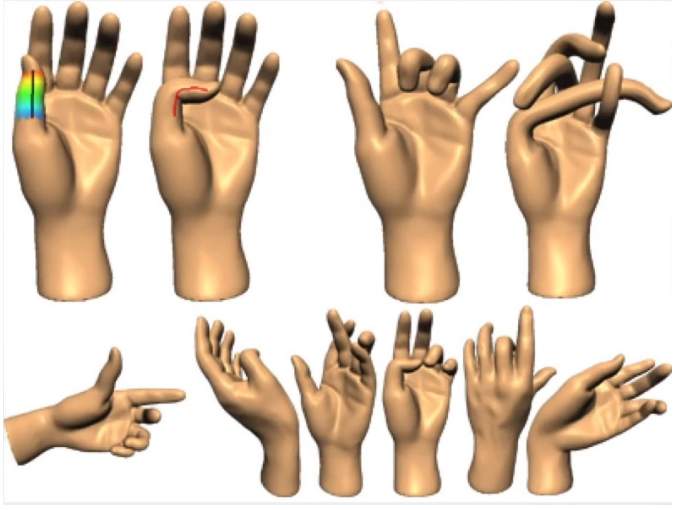
Remeshing


Simplification

Deeper into topic

- Topics to explore (out of scope)
  - Mesh compression (http://meshcompression.org/)
  - Animation, skinning and deformation
    - https://catlikecoding.com/unity/tutorials/mesh-deformation/





Exploring meshes

- **Blender**
  - Modeling with meshes
  - Procedural meshes with Python and geometry nodes
- **Meshlab:** open-source tool for mesh manipulation and processing
  - Mesh smoothing and sharpening
  - Re-meshing: subdivide, re-sample, simplify,
  - Topological operations: fill holes, fix self-intersections
  - Boolean operations
- **libigl and CGAL** libraries for mesh processing
- **Sources of mesh data**:
  - https://casual-effects.com/data/index.html
  - https://polyhaven.com/models
  - https://sketchfab.com/
  - http://graphics.stanford.edu/data/3Dscanrep/
- **More information:**
  - https://www.realtimerendering.com/#polytech

Mesh shape representation: verdict

- Pros:
  - Most common surface representation
  - Triangle meshes are great for rendering
  - A lot of effort has been made to represent various shapes with meshes
  - Lot of research has been done to convert other shape representations to mesh representation
  - Graphics hardware is adapted and optimized to work with (triangle) meshes → fast rendering
- Cons:
  - Not Guaranteeing smoothness
  - Triangle meshes are not efficient or intuitive for manual/interactive modeling
  - Not every object is well suited to mesh representation:
    - Shapes that have geometrical detail at every level (e.g., fractured marble)
    - Some objects have structure which is unsuitable for mesh representation, e.g., hair which has more compact representations

Reading material

- https://www.youtube.com/watch?v=V3Npa0uZYlE&list=PL4TptkuzgxxUVZ-_DiO33kp4_rkoAy1BC&index=6&ab_channel=ChristophGarth
- https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-polygon-mesh/introduction.html
- https://www.pbr-book.org/3ed-2018/Shapes
- Real-time rendering book
  - Chapters: 16 and 17
- Computer graphics practices and principles book
  - Chapters 8, 9, 23 and 25