

# Lecture 13: Rasterization-based rendering

DHBW, Computer Graphics

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# Syllabus

- 3D scene
    - Object
    - Light
    - Camera
  - Rendering
    - Ray-tracing based rendering
    - Rasterization-based rendering
  - Image and display
- A blue bracket-shaped callout box surrounds the 'Rasterization-based rendering' section of the syllabus. A blue arrow points from the word 'Rendering' in the main list to the start of the detailed rendering section. Another blue arrow points back from the end of the detailed rendering section to the 'Rasterization-based rendering' item in the main list.

  - Rasterization-based rendering
    - Rasterization: the core
    - Graphics rendering pipeline
    - Logical GPU model/API

# Rasterization

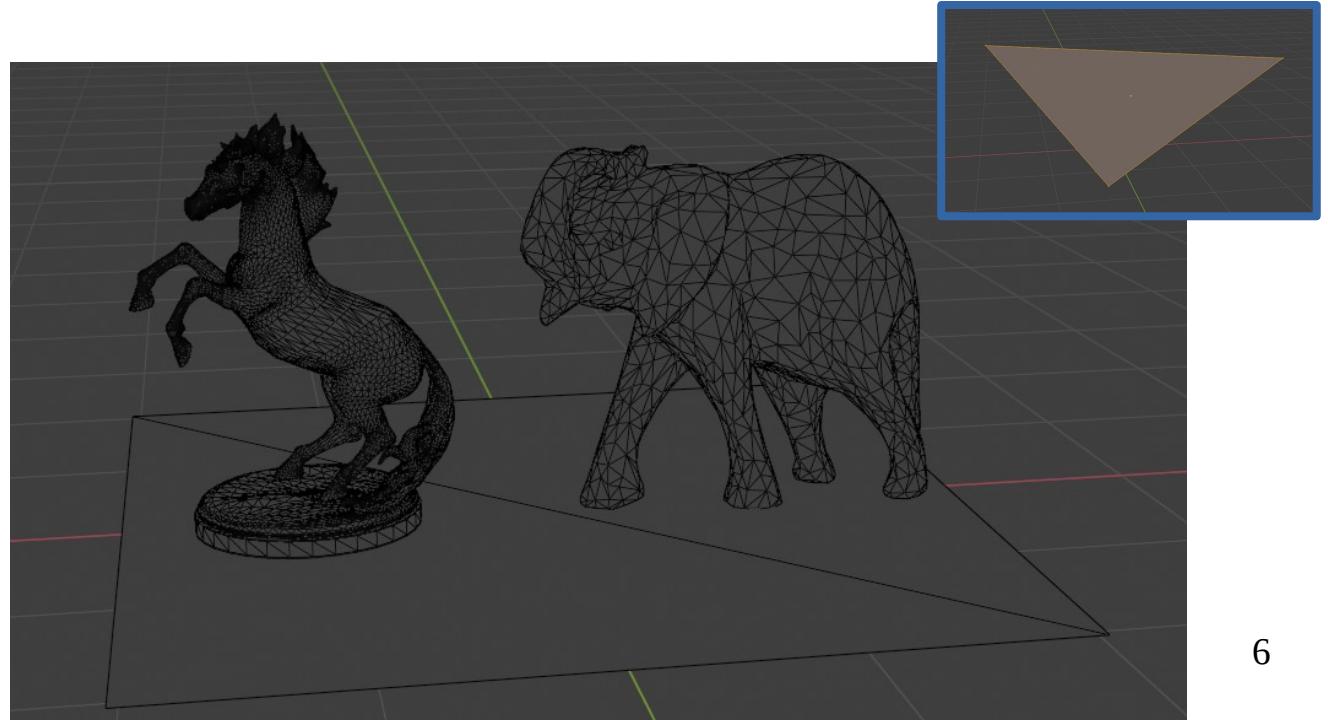
The core for visibility solving

# Rendering

- Rendering: process of creating 2D image from 3D scene
- Two main **tasks of rendering**:
  - **Visibility solving**: compute objects visible from camera
  - **Shading**: compute colors of visible objects

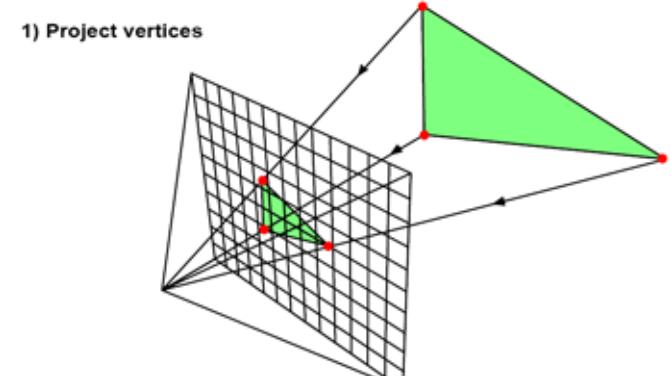
# Visibility problem

- Visibility is geometrical problem using **object shape representation** to determine which points in 3D scene are visible to each other
- **Triangulated mesh** is shape representation used for efficient visibility solving
  - Different shape representations can be converted to triangulated mesh using **triangulation**
- Main approaches for solving visibility:
  - Ray-tracing
  - Rasterization

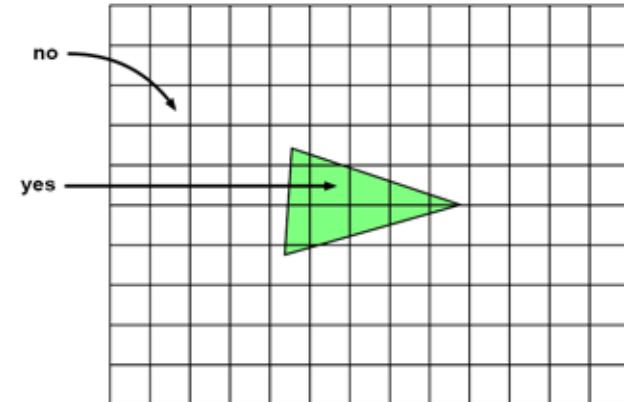


# Rasterization: visibility solving

- To find objects visible from camera, rasterization can be decomposed in **two main stages**:
  - Triangle projection:** project 3D vertices of mesh triangles onto image plane (aka screen) using perspective projection matrix
  - Triangle rasterization:** for each pixel of image plane check if it is contained in projected triangle



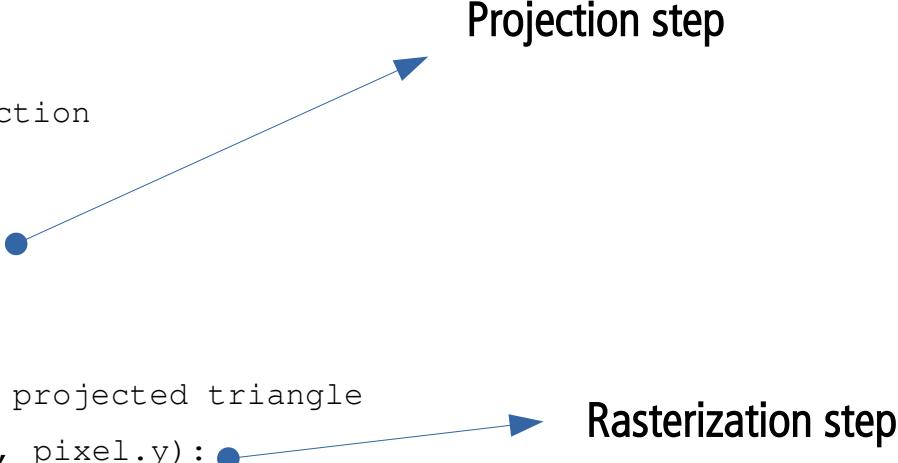
1) Project vertices



2) Loop over pixels. Does the pixel lie in the triangle?

# Rasterization: algorithm

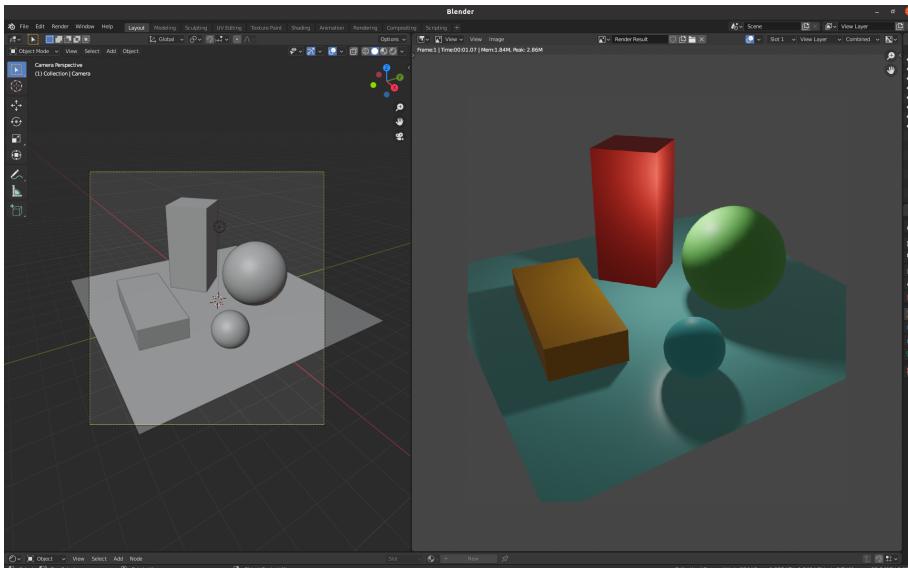
```
for each triangle in the scene:  
    // STEP 1: project vertices using perspective projection  
    vector v0 = perspectiveProjection(triangle.v0);  
    vector v1 = perspectiveProjection(triangle.v1);  
    vector v2 = perspectiveProjection(triangle.v2);  
  
    for each pixel in the image:  
        // STEP 2: compute if pixel is contained in the projected triangle  
        if(pixelContainedIn2DTriangle(v0,v1,v2,pixel.x, pixel.y):  
            image(pixel.x, pixel.y) = triangle.color
```



- Object centric approach: first loops over triangles, then loops over image pixels
- Note: looping over all triangles and all pixels is naive approach: further optimizations are introduced

# Rasterization: image and color buffer

- Rasterization: process of decomposing triangles into pixels → raster image
- During process of rasterization, pixel color is stored in **color buffer** – a 2D array of pixels
- After rasterization, **color buffer will contain image** ready for display



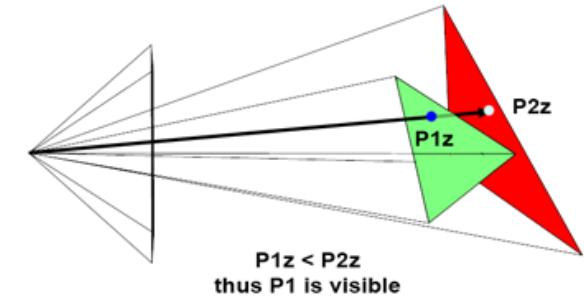
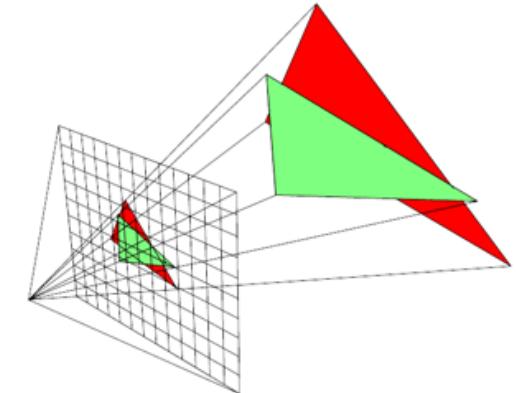
- Before rendering, color-buffer is created with all pixels set to black color
- During rendering, when triangles are rasterized, for each pixel that overlaps triangle store that triangle color in color-buffer at that pixel location
- After rendering, when all triangles are rasterized and pixels are processed, the color-buffer will contain the final image of the scene

# Rasterization: buffers

- During rasterization rendering, various pixel information is stored in buffers
  - **Buffer**: 2D array of pixels which has same size as final image
- **Framebuffer** generally consists of all buffers on a system
  - Multiple different buffers can be created during rasterization-based rendering which are used for achieving various effects
- **Common buffers:**
  - **Color buffer**: contains colors of rasterized triangles
  - **Depth buffer aka z-buffer**: contains depths of rasterized triangles

# Rasterization: z-buffer

- Multiple triangles may overlap the same pixel in image
- To correctly solve visibility, rasterization must find the first visible triangle
- This is done using **z-buffer or depth buffer**
  - 2D array with same dimensions as image but instead of colors it contains array of floating point numbers
  - z-buffer stores the distance of each pixel to the nearest object in the scene



$P1z < P2z$   
thus  $P1$  is visible

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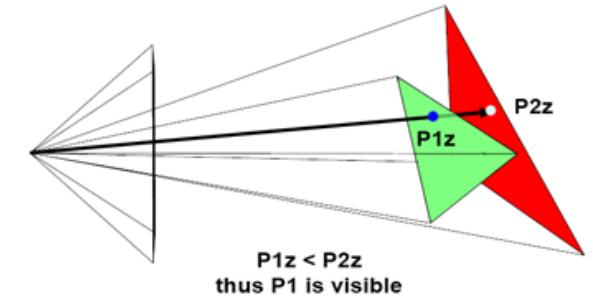
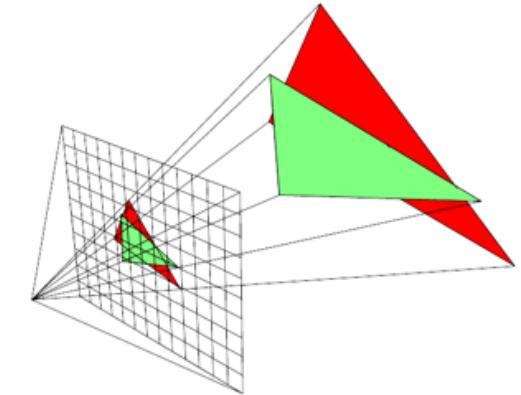
# Rasterization: z-buffer

- **Visibility computation with z-buffer:**

- Before rendering z-buffer is initialized to a very large number
- When pixel overlaps triangle, compute distance to this triangle and compare it to the distance in z-buffer at that pixel position
- If distance to triangle is smaller then value in z-buffer, then triangle is visible for that pixel:
  - Update z-buffer with this distance
  - Update frame-buffer with color of this triangle at this pixel position

- Alternative visibility computation: Painter's algorithm

- Sorting objects by distance



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# Rasterization: frame-buffer and z-buffer

```
color frame_buffer = new float[imageWidth, imageHeight];  
float z_buffer = new float[imageWidth, imageHeight];  
for (int i = 0; i < imageWidth * imageHeight; ++i)  
    frame_buffer[i] = color(0,0,0);  
    z_buffer[i] = INF;  
for each triangle in scene:  
    vector v0 = perspectiveProjection(triangle.v0);  
    vector v1 = perspectiveProjection(triangle.v1);  
    vector v2 = perspectiveProjection(triangle.v2);  
    float z;  
if (pixelContainedIn2DTriangle(triangle.v0, triangle.v1, triangle.v2, pixel.x, pixel.y, z):  
    if (z < z_buffer(pixel.x, pixel.y)):  
        z_buffer(pixel.x, pixel.y) = z;  
        frame_buffer(pixel.x, pixel.y) = triangle.color
```

The diagram illustrates the flow of data from the projection stage to the rasterization stage. It features two blue circular nodes connected by a blue arrow pointing from left to right. The top node is labeled "Projection stage" and the bottom node is labeled "Rasterization stage".

# Rasterization: projection stage

- Goal: Project triangles onto image plane (screen) and convert their coordinates from screen to raster space
  - When triangles are converted to same space as pixels it is possible to compare their coordinates which is needed to find which pixels are overlapping triangle
  - This way, whole triangle can be rendered knowing only vertex positions
- Projection stage is responsible for transforming object from world space to raster space:
  1. World space to camera space
  2. Camera space to screen (image plane) space
  3. Screen space to normalized device coordinate (NDC) space
  4. NDC space to raster space

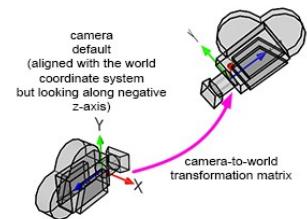
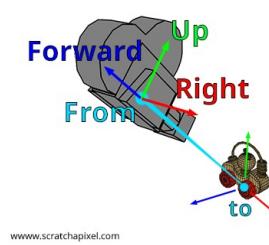
# Projection stage: (1) world to camera space

World space to camera space:

- Triangulated mesh vertex coordinates are defined in world space  $P_{\text{world}}(x, y, z)$
- Convert vertex coordinates  $P_{\text{world}}(x, y, z)$  from world to camera space  $P_{\text{camera}}(x, y, z)$  using **world-to-camera matrix**
- World-to-camera transformation matrix can be constructed using look-at notation

$$P_{\text{camera}} = P_{\text{world}} * M_{\text{world-to-camera}}$$

$$M_{\text{world-to-camera}} = \begin{matrix} Right_x & Right_y & Right_z & 0 \\ Up_x & Up_y & Up_z & 0 \\ Forward_x & Forward_y & Forward_z & 0 \\ T_x & T_y & T_z & 1 \end{matrix}$$



Forward = normalize(From - To)  
 Right = crossProduct(randomVec, Forward)  
 Up = crossProduct(forward, right)

Reading:

<https://www.scratchapixel.com/lessons/3d-basic-rendering/computing-pixel-coordinates-of-3d-point/mathematics-computing-2d-coordinates-of-3d-points.html>

# Projection stage: (2) camera to screen space

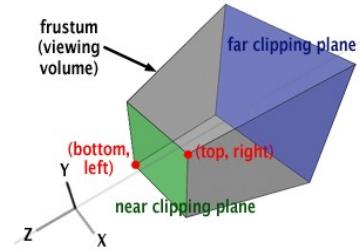
**Camera space to screen (image plane) space:** convert camera space vertex coordinates  $P_{\text{camera}}(x, y, z)$  to screen (image plane) coordinates  $P_{\text{screen}}(x, y, z)$

$$P_{\text{screen}}.x = \frac{\text{near} * P_{\text{camera}}.x}{-P_{\text{camera}}.z}$$

$$P_{\text{screen}}.y = \frac{\text{near} * P_{\text{camera}}.y}{-P_{\text{camera}}.z}$$

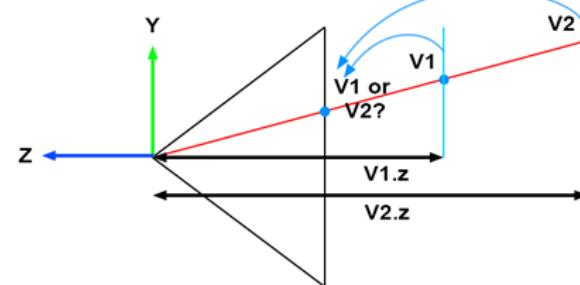
$$P_{\text{screen}}.z = -P_{\text{camera}}.z$$

- Perspective projection is used to transform points → **perform perspective divide**
- Image plane is positioned at near clipping plane at distance **near** → **multiply with near**



Points in screen space  $P_{\text{screen}}(x, y, z)$  are still 3D:

- (x,y) coordinates have undergone a **perspective distortion**
- z contains original distance from camera → this will be needed for resolving visibility and overlapping triangles



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If multiple points are projected on same point on screen, use z coordinate to solve visibility

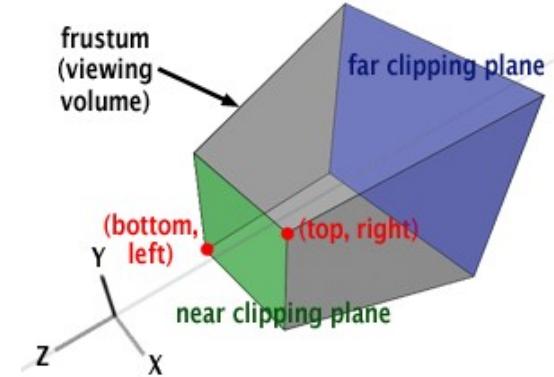
# Projection stage: (3) screen to NDC space

- NDC space coordinates are in range:
  - $[left, right] \rightarrow [l, r]$  for x coordinate
  - $[bottom, top] \rightarrow [b, t]$  for y coordinate
- GPU and real-time rendering standard (e.g., OpenGL, DirectX): NDC is  $[-1, 1]$  for both coordinates
- $(x, y)$  coordinates from screen space are converted NDC space using:

$$P_{NDC}.x = \frac{2P_{screen}.x}{r - l} - \frac{r + l}{r - l}$$

$$P_{NDC}.y = \frac{2P_{screen}.y}{t - b} - \frac{t + b}{t - b}$$

$$-1 < P_{NDC}.x, P_{NDC}.y < 1$$

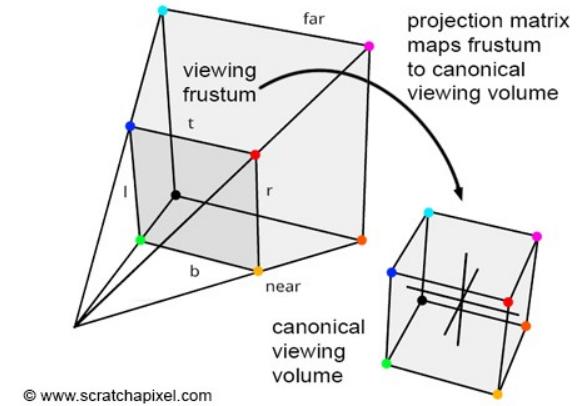


Left, right, bottom, top are coordinates of the image plane (screen)

- Z coordinate is remapped to  $[0,1]$  or  $[-1,1]$ 
  - If P lies on near clipping plane then remap to 0 (or -1)
  - If P lies on far clipping plane then remap to 1

# Note: Screen and NDC space

- Conversion of vertex coordinates from camera space to NDC space can also be done using **projection matrix**, it replaces:
  - Perspective divide operation
  - Remapping from screen to NDC space
- Projection matrix remaps viewing frustum into unit cube – canonical view volume**  
→ normalization of viewing frustum
  - Canonical view volume: cube with extreme points at  $(-1, -1, -1), (1, 1, 1)$
- GPU rendering uses projection matrix to convert vertex coordinates from screen space to clip space** and then to NDC and raster space
  - Clipping:** triangles overlapping viewing frustum planes are “chopped off” (Cohen-Sutherland algorithm)



$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective projection matrix is built using far (f), near (n), right (r), left (l), top (t), bottom (b) which can be calculated using angle of view and image aspect ratio.

# Projection stage: (4) NDC to raster space

- Converting coordinates from NDC to raster space (integer pixel coordinates) requires remapping from  $[-1, 1]$  (float) to  $[imageWidth, imageHeight]$  (int)

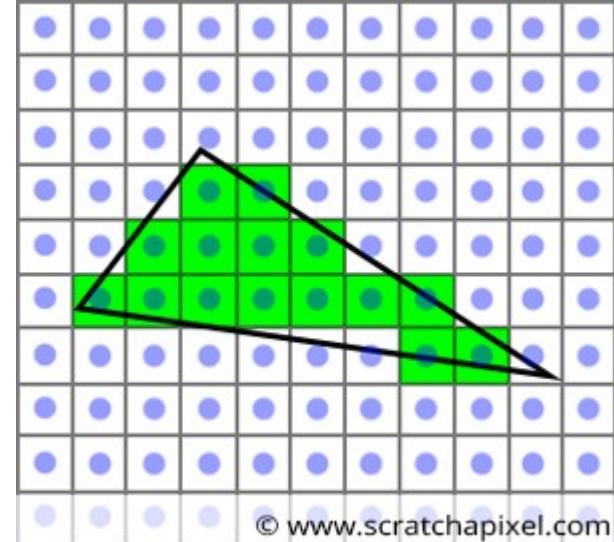
$$P_{raster}.x = \frac{P_{NDC}.x + 1}{2} imageWidth$$

$$P_{raster}.y = 1 - \frac{P_{NDC}.y}{2} imageHeight$$

In raster space the  $y$ -axis goes down while in NDC space it goes up. Therefore, change  $y$  direction during remapping process

# Triangle rasterization

- Triangles are now converted to same space as pixels it is possible to compare their coordinates which is needed to find which pixels are overlapping triangle
    - This way, whole triangle can be rendered knowing only vertex positions
  - Goal: convert projected triangle to a 2D image:
    - Loop over all pixels in 2D image to determine which pixels overlap the projected triangle
      - Inside-outside (aka coverage) test
      - Alternative: scanline rasterization → based on Bresenham algorithm for drawing lines
    - For each pixel that overlaps triangle, compute:
      - Barycentric coordinates
      - Depth
      - Etc.
- Needed for visibility and shading



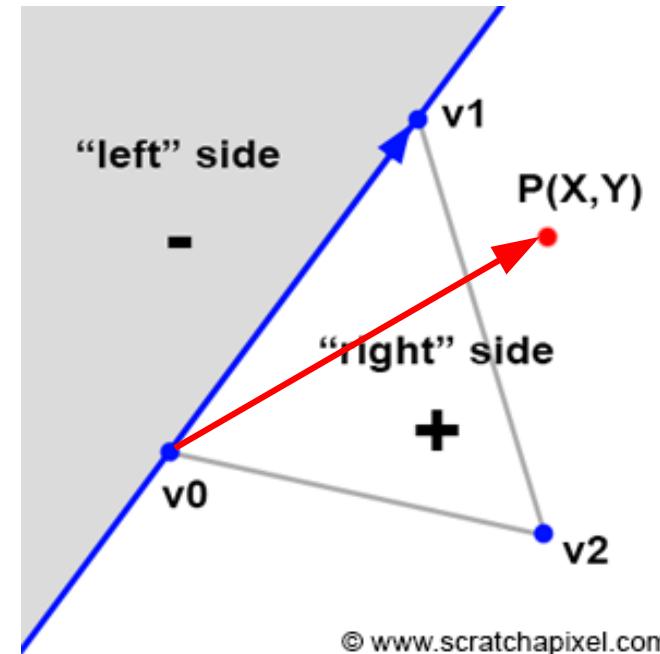
# Inside-outside test: edge function

- Triangle edges can be seen as lines splitting planes
- Edge function returns (clockwise winding of triangle vertices):
  - positive number if point P is on left side of the line
  - Negative number if point is on the right side of the line
  - Zero if point is on the line

$$E_{01}(P) = (P.x - v0.x) * (V1.y - V0.y) - (P.y - V0.y) * (V1.x - V0.x).$$

Edge function interpretation:

- Magnitude of cross product between  $(v1-v0)$  and  $(P-v0)$
- Determinant of 2x2 matrix defined with components of vectors  $(v1-v0)$  and  $(P-v0)$



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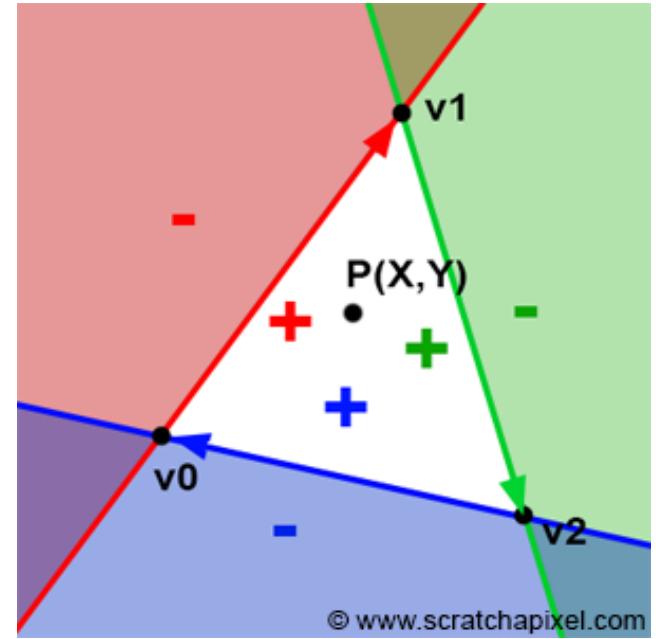
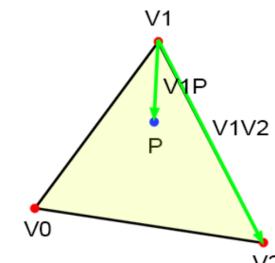
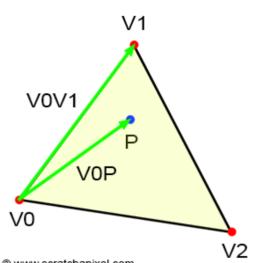
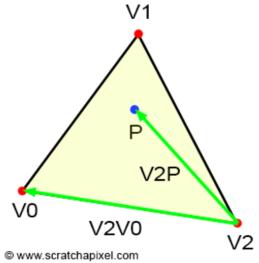
# Inside-outside test: edge function test

- Loop over all pixels in 2D image:
  - Pixel position  $P(x,y)$  must be tested for all 3 triangle edges
  - If edge function returns positive value for all edges, then pixel position is inside triangle
- Note: winding order of triangle matters
  - For counterclockwise winding, the sign of edge function is inverted

$$E_{01}(P) = (P.x - V0.x) * (V1.y - V0.y) - (P.y - V0.y) * (V1.x - V0.x),$$

$$E_{12}(P) = (P.x - V1.x) * (V2.y - V1.y) - (P.y - V1.y) * (V2.x - V1.x),$$

$$E_{20}(P) = (P.x - V2.x) * (V0.y - V2.y) - (P.y - V2.y) * (V0.x - V2.x).$$



# Barycentric interpolation and edge function

- Barycentric ( $\lambda_1, \lambda_2, \lambda_3$ ) coordinates can be used to define any point on the triangle.
  - Can be computed as the ratio between area of sub-triangles and the whole triangle.
  - Area of sub-triangle is calculated using edge function.

$$\lambda_0 = \frac{\text{Area}(V1, V2, P)}{\text{Area}(V0, V1, V2)},$$

$$\lambda_1 = \frac{\text{Area}(V2, V0, P)}{\text{Area}(V0, V1, V2)},$$

$$\lambda_2 = \frac{\text{Area}(V0, V1, P)}{\text{Area}(V0, V1, V2)}.$$

$$P = \lambda_0 * V0 + \lambda_1 * V1 + \lambda_2 * V2.$$

$$\text{Area}_{tri}(V1, V2, P) = \frac{1}{2} E_{12}(P),$$

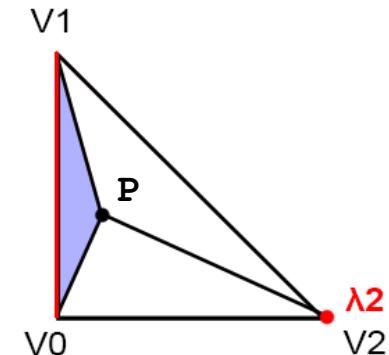
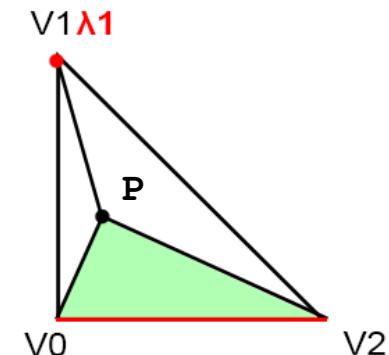
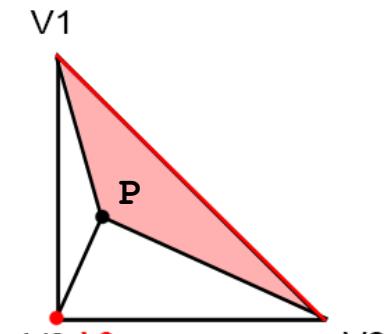
$$\text{Area}_{tri}(V2, V0, P) = \frac{1}{2} E_{20}(P),$$

$$\text{Area}_{tri}(V0, V1, P) = \frac{1}{2} E_{01}(P).$$

$$E_{01}(P) = (P.x - V0.x) * (V1.y - V0.y) - (P.y - V0.y) * (V1.x - V0.x),$$

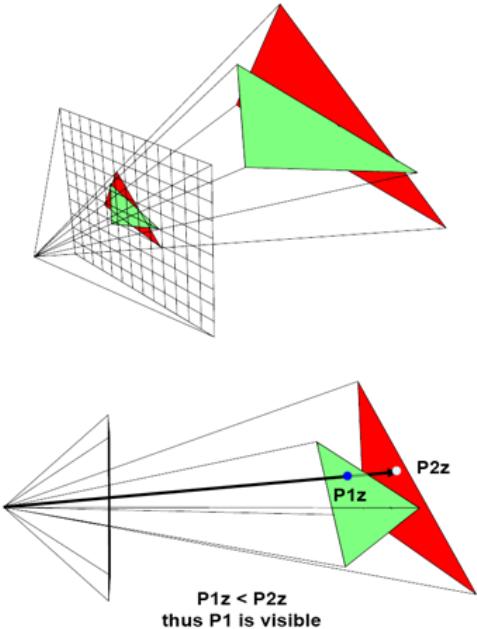
$$E_{12}(P) = (P.x - V1.x) * (V2.y - V1.y) - (P.y - V1.y) * (V2.x - V1.x),$$

$$E_{20}(P) = (P.x - V2.x) * (V0.y - V2.y) - (P.y - V2.y) * (V0.x - V2.x).$$



# Depth and visibility

- If pixel overlaps two or more triangles use z-buffer for visibility solving
  - Compute z-coordinate (depth) of the point on the triangle that the pixel overlaps
  - Compare computed depth with depth value stored at z-buffer at that pixel location
  - If current depth value is smaller than value from z-buffer, update frame-buffer with color of current triangle and z-buffer with depth of current point on triangle
- Once all triangles have been processed, depth buffer contains distance to visible object from camera
  - Depth is useful for visibility solving, shading and post-processing (e.g., depth of field, fog, shadow mapping, etc.)

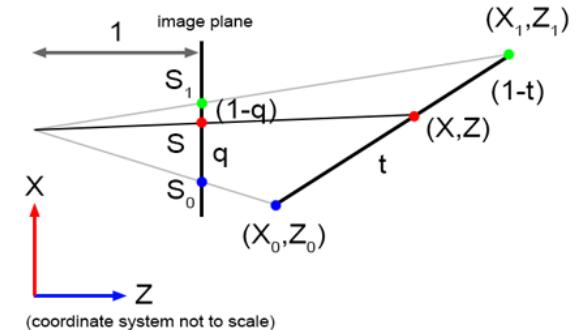


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# Depth and barycentric coordinates

- In projection stage, original camera space z-coordinate is stored for each vertex of a triangle
- For current pixel overlapping triangle, z-value is found using barycentric interpolation
- Since perspective projection preserves lines but not distances, the solution is to compute the inverse of P z-coordinate:

$$\frac{1}{P.z} = \frac{1}{V0.z} * \lambda_0 + \frac{1}{V1.z} * \lambda_1 + \frac{1}{V2.z} * \lambda_2.$$



Perspective projection doesn't preserve distances.

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# Barycentric interpolation and vertex attributes

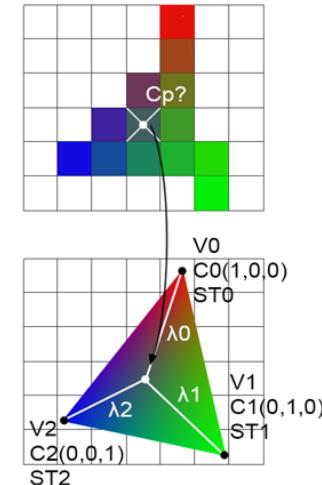
- Rasterization stage ends once it is determined for all pixels if they overlap triangle (inside-outside test)
- Triangle vertices can also have assigned **vertex attributes**: colors, normals, texture coordinates, etc.
- **Vertex attributes are used during shading** for computing color of pixels overlapping the triangle
  - Vertex attributes must be interpolated across surface of a triangle when it is rasterized
- **Barycentric coordinates** ( $\lambda_1, \lambda_2, \lambda_3$ ) are used to interpolate vertex attributes for any pixel overlapping the triangle
  - Example: interpolating color at point P where each vertex has color attribute:  $C_{v0}, C_{v1}, C_{v2}$

$$C_P = \lambda_0 * C_{V0} + \lambda_1 * C_{V1} + \lambda_2 * C_{V2}.$$

$$C_{v0} = (1, 0, 0)$$

$$C_{v1} = (0, 1, 0)$$

$$C_{v2} = (0, 0, 1)$$



# Barycentric coordinates: perspective projection

- Due to perspective projection, it is required to perform **perspective correction** for vertex attribute interpolation
- Example: perspective correct color vertex attribute interpolation

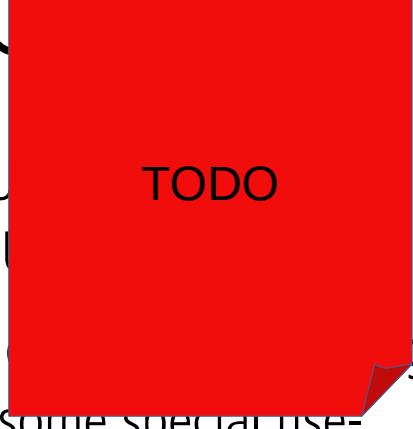
$$C = Z \left( \frac{C_{v0}}{Z_{v0}} \lambda_0 + \frac{C_{v1}}{Z_{v2}} \lambda_1 \frac{C_{v2}}{Z_{v2}} \lambda_2 \right)$$

- $Z$  – current pixel depth
- $Z_{v0}, Z_{v1}, Z_{v2}$  – triangle vertices depth
- $C_{v0}, C_{v1}, C_{v2}$  – triangle vertices colors

# Rasterization-based rendering on GPU

- Rasterization is elegant algorithm for solving visibility:
  - Projecting vertices of triangulated mesh onto image plane
  - Looping over image pixels to find which pixels lie in projected triangle
- Both tasks are well suited for GPU and can be performed fast
  - GPU rendering uses rasterization approach for solving visibility
  - GPU rendering is conceptually described with graphics rendering pipeline with rasterization as one module of the pipeline
  - Graphics rendering pipeline is foundation for real-time rendering

# Rasterization-based rendering on GPU

- Rasterization-based rendering technique is deeply integrated in GPU rendering. Raytracing-based rendering is purely implemented on CPU.  **TODO**
- Rasterization-based rendering can also be completely implemented on CPU. For learning purposes this is useful to understand all the aspects of it. There are some special use-cases which also benefit from CPU implementation of rasterization based-rendering\*.
- Almost all professional software which uses rasterization-based rendering is using GPU hardware implementation which can be further programmed using graphical APIs such as OpenGL, Vulkan, DirectX, Metal, etc.

\* Cases when objects that are rendered are smaller than pixel. Those are advanced topics and are related to point rendering (<https://www.cg.tuwien.ac.at/research/publications/2022/SCHUETZ-2022-PCC/>) or micropolygon rendering (<https://docs.unrealengine.com/5.0/en-US/nanite-virtualized-geometry-in-unreal-engine/> or <https://graphics.pixar.com/library/Reyes/>)

# Graphics rendering pipeline: conceptual overview

Using rasterization for rendering

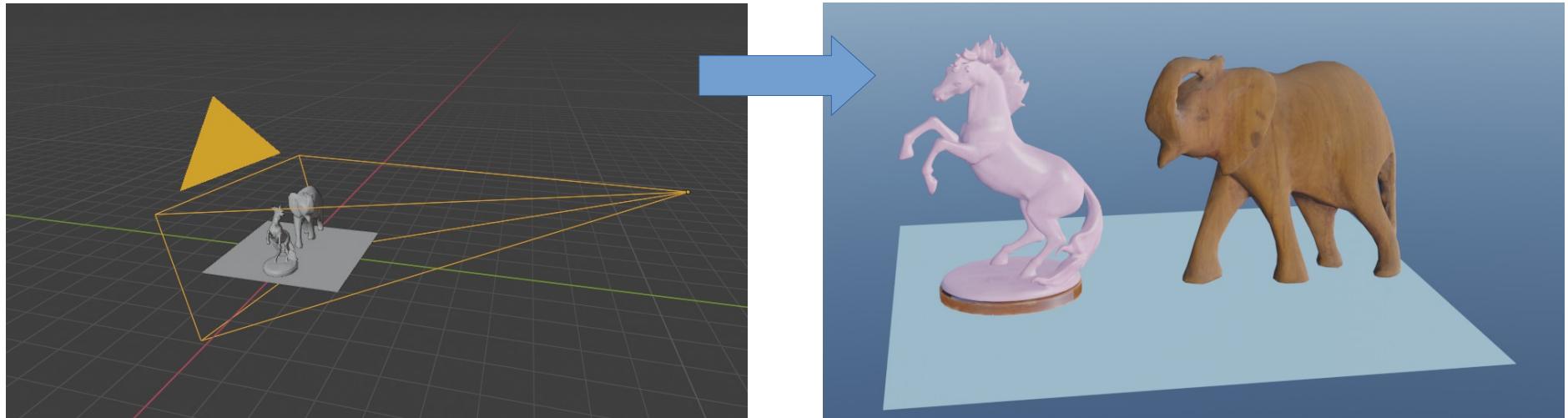
# Motivation

- Graphics rendering pipeline describes rasterization based rendering on graphics processing unit (GPU)
- It is used for:
  - Real-time rendering applications
  - Interactive rendering applications
  - Games, modeling, visualization
  - VR, XR, etc.

Supporting image

# Graphics rendering pipeline

- Main function of graphics rendering pipeline: generate a 2D image from 3D scene
- Rendering:
  - **Visibility calculation:** which objects are visible from camera
  - **Shading calculation:** computing color of visible objects that is light-matter calculation and light transport



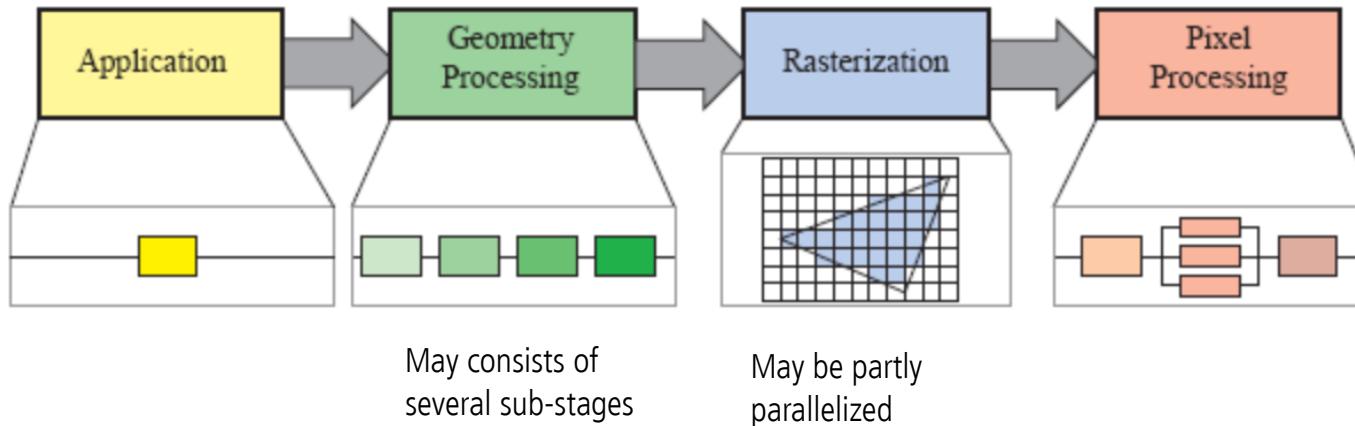
# Graphics rendering pipeline

- Graphics rendering pipeline is decomposed in smaller steps or stages
  - Each stage performs part of a larger task
  - Input of any given stage depends on the output of previous stage
- Sequence of stages forms rendering pipeline
  - Pipeline stages, although working in parallel, are stalled until slowest stage is finished
  - Slowest stage is said to be **bottleneck**
  - Stages which are waiting are called **starved**



# Graphics rendering pipeline: stages

- Graphics rendering pipeline can be **coarsely divided in four stages**.



Stages can be:

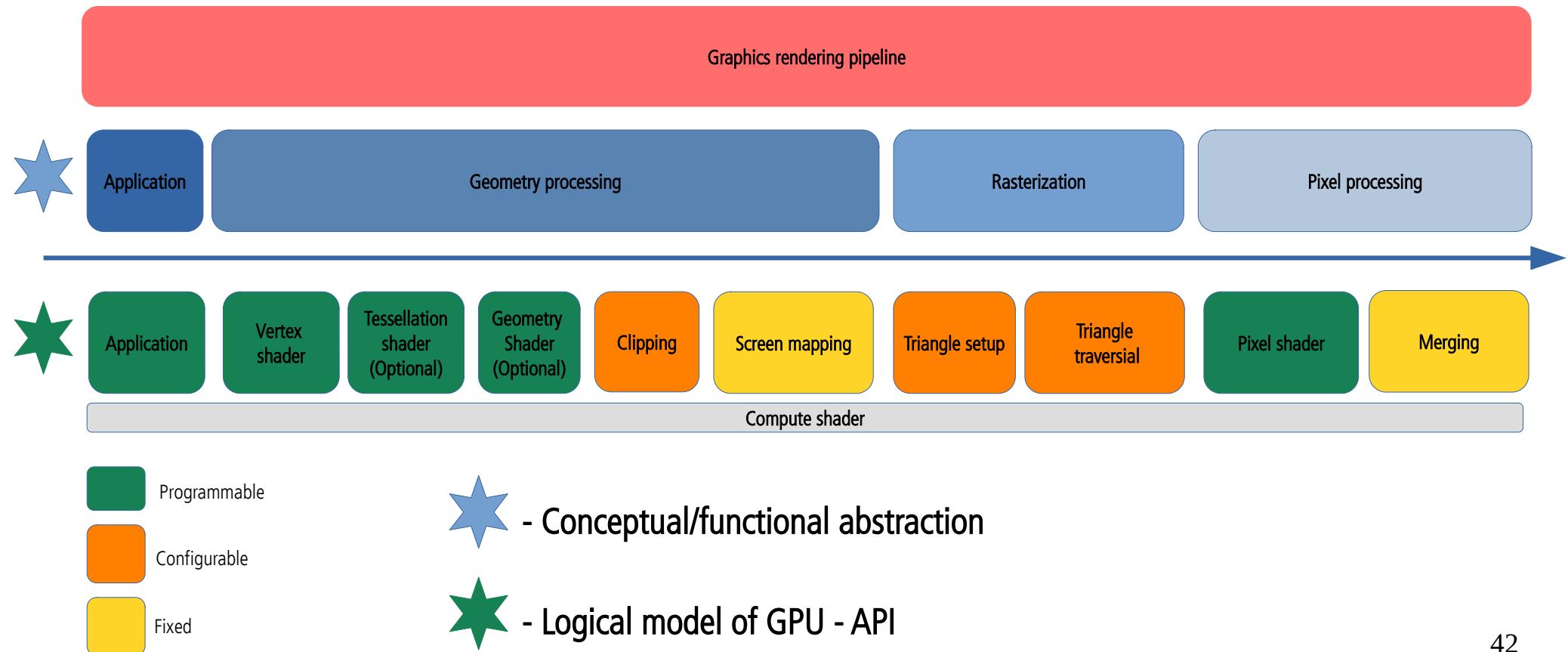
- **Fixed** (no programmer control)
- **Partially configurable** (control over parameters)
- **Fully programmable (shaders)**

Trend is towards programmability and flexibility.

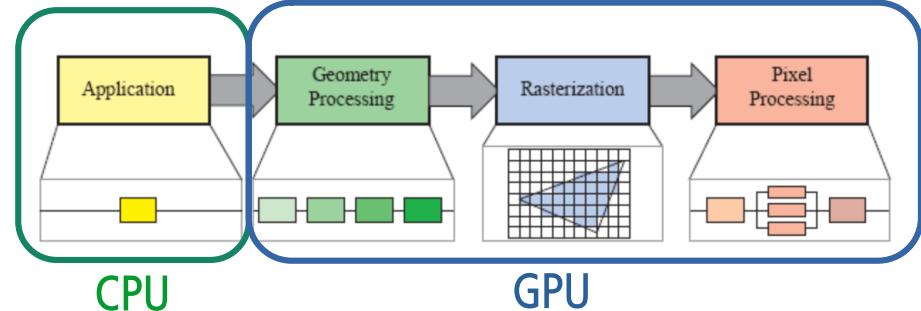
# Pipeline abstraction

- Pipeline can be described using different abstractions levels:
  - **Functional/conceptual stages** – describes tasks that rendering pipeline must perform but not how
  - **Physical/Implemented stages** – describes how are functional stages implemented in hardware and exposed to the user as API. Physical model is up to the hardware vendor (e.g., nvidia, AMD, Intel, etc.)
  - **Logical model of GPU** –describes rendering pipeline interface exposed to a programmer by API

# Graphics rendering pipeline: conceptual vs logical (API) model



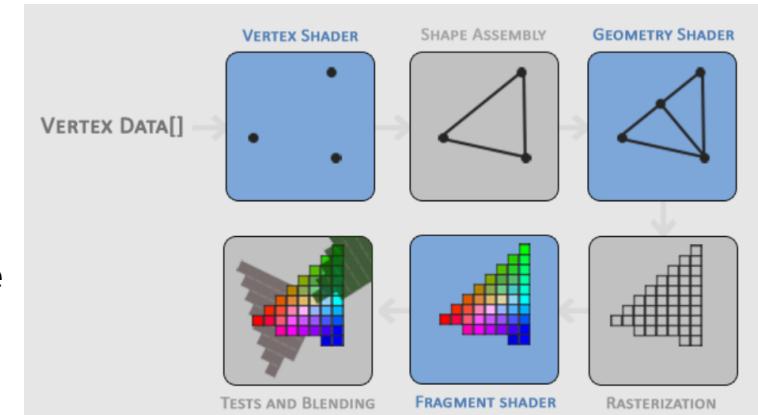
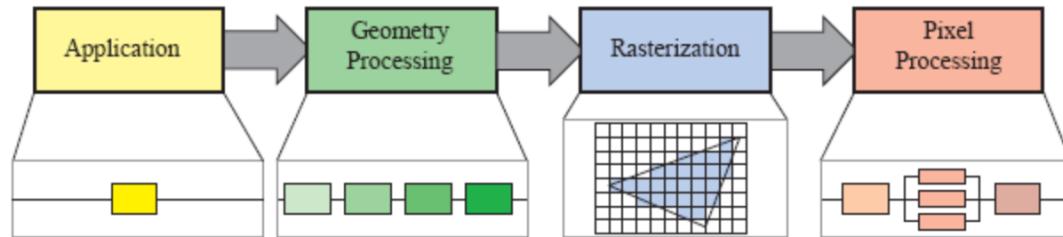
# Pipeline: CPU and GPU



- **CPU implements application stage.**
  - CPUs are optimized for various data structures and large code bases, they can have multiple cores but in mostly serial fashion (SIMD processing is exception)
- **GPU implements** what is conceptually described as **geometry processing, rasterization and pixel processing stage.**
  - GPUs chips contain large set of processors called **shader cores** – small processors that do independent and isolated task (no information sharing and shared writable memory) in a massively parallel fashion.
  - Different types of programmable shaders enable controlling GPU rendering.

# Pipeline: speed run

- Application stage (CPU)
  - Driven by application implemented in software running on CPU
  - 3D scene elements are defined here: objects (geometry, material), cameras, lights
  - Examples: 3D modeling tool application. User input is handled on application stage
- Geometry processing (GPU)
  - Per-triangle or per-vertex operations: transformations, projections; what, how and where it is drawn
  - Implemented on GPU that contains many programmable shader cores
- Rasterization (GPU)
  - Takes 3 vertices which form a triangle, finds all pixels inside triangle and forwards them to next phase
  - Fixed implementation on GPU
- Pixel processing (GPU)
  - Shading operation per pixel: calculating color and depth testing → programmable GPU shading cores
  - Per-pixel operations, e.g., blending new and old pixel color → partially configurable



<https://learnopengl.com/Getting-started>Hello-Triangle>

# Example of pipeline in application

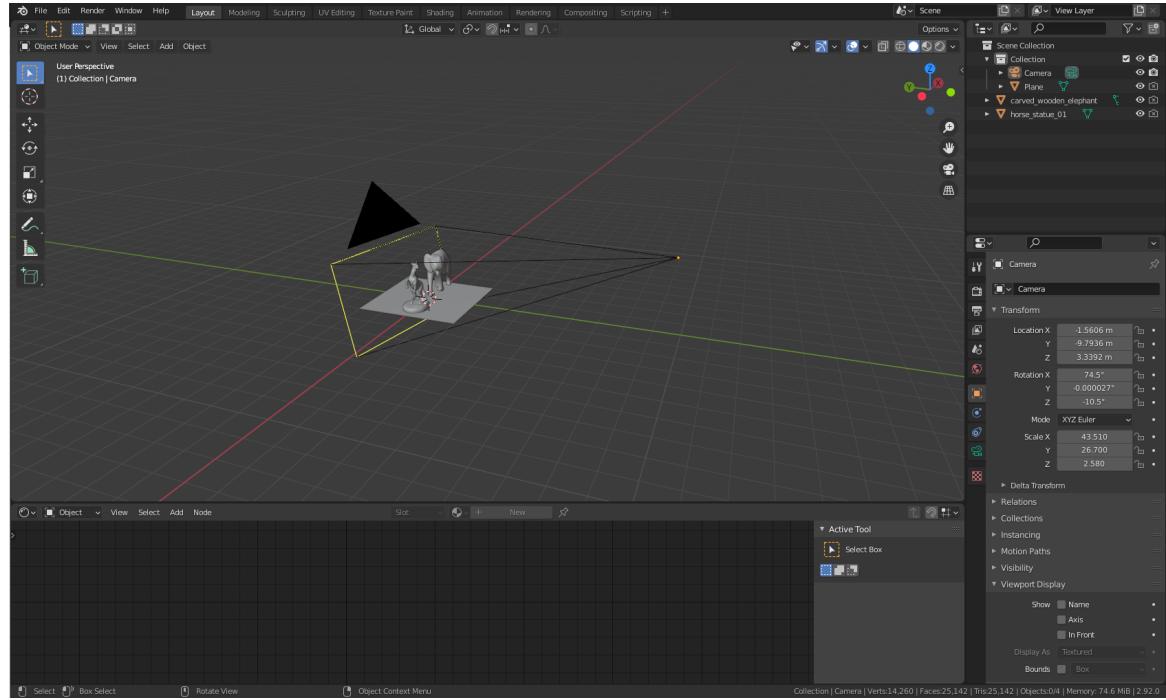
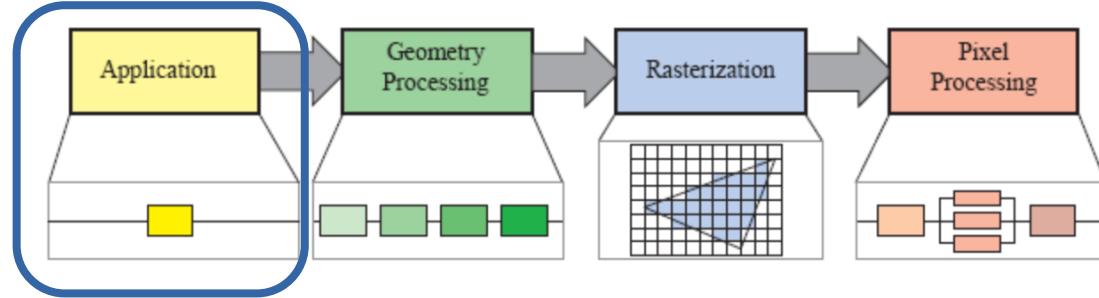
- Example: CAD program
- Application stage:
  - Enables user to select and move parts of the model. Selection and movement is done using mouse pointer. Thus, application stage must translate mouse movement into transformation matrix (e.g., translation or rotation) which is used in subsequent stage
  - Enables user to move camera in 3D scene to view the model from different angles. Thus, camera parameters are provided by application.
  - For each frame (CAD programs are real-time when in modeling mode) application must provide information about primitives of the model.
- Geometry processing
  - Application provided parameters of the camera which includes projection matrix. Next, application, for each primitive, finds vertices in world space.
  - Object is put in view space and optionally shading per vertices is preformed using provided light and material properties.
  - Projection is applied on the object transforming it in unit cube space that represents what the eye sees.
  - Finally, vertices are mapped into the window on the screen
- Rasterization
  - All primitives coming from geometry stage are rasterized: all pixels (fragments) inside a primitive are found and sent for pixel processing.
- Pixel processing
  - Color of each pixel obtained from rasterization is computed using material information (colors, textures, shading equations) and visibility is resolved using z-buffer algorithm with optional discard and stencil testing.
  - Each object is processed and final image is displayed.

re-read

# 1. Application stage

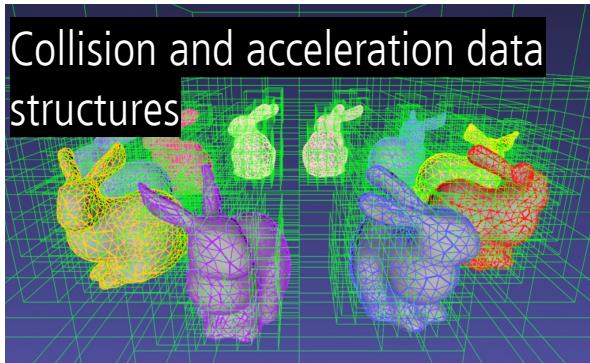
# 1. Application stage

- Driven by application (e.g., 3D modeling application)
- Typically implemented on CPU (optionally on multiple threads)
- Developer has full control over what happens in this stage and how it is implemented
- Software-based implementation: not subdivided into stages



# Application stage tasks

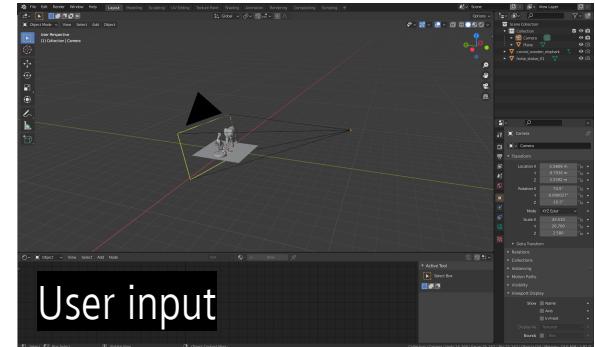
- Application stage includes tasks such as:
  - Taking care of user input from keyboard, mouse, etc. for interaction
  - Animation
  - Physics simulation
  - Collision detection – detection of collision between two objects and generating response
  - 3D scene acceleration structures (e.g., culling algorithms)
  - Compositing
  - Other tasks depending on application which subsequent stages of pipeline can not handle



<https://www.kitware.com/octree-collision-imstk/>

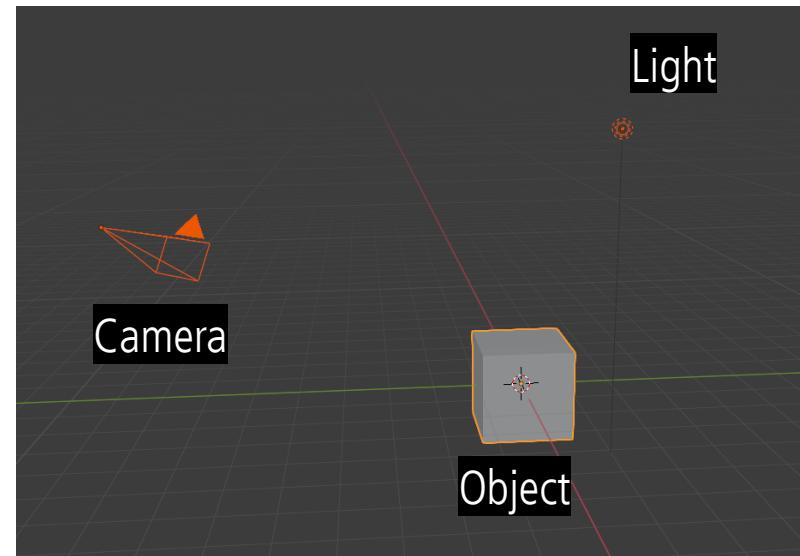


<https://www.blender.org/features/>



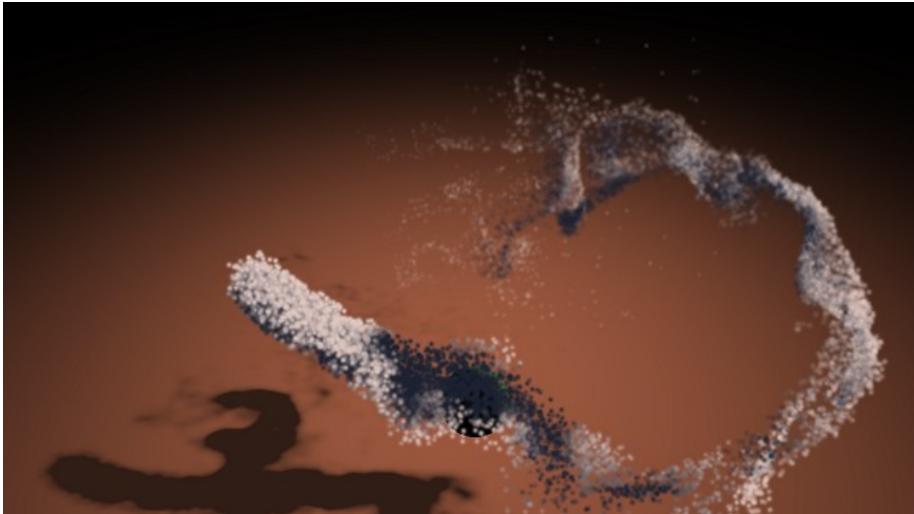
# Application stage: scene definition

- On application stage elements of 3D scene to be rendered are defined:
  - **Objects**:
    - **Geometry** (shape representation): triangulated mesh, vertices, normals, texture coordinates per vertex
    - **Transformation matrices**
    - **Material** parameters (e.g., color, roughness, normal map, etc.) → per vertex
  - **Cameras**
    - Transformation matrix (e.g., transformation build using look at notation)
    - Camera parameters (e.g., focal length)
  - **Lights**
    - Position and/or direction (transformation matrices or vectors)
    - Intensity, color
    - Shape, size (geometry)



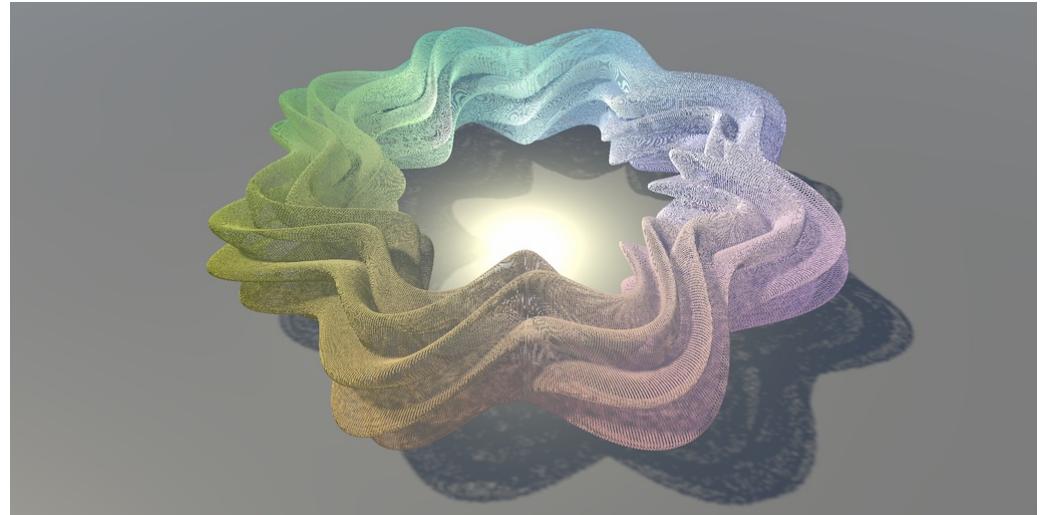
# Application stage: compute shader

- Some work on application stage can be sent to GPU for processing – **compute shader**.
  - Idea treat **GPU as highly parallel general processor** rather than rendering pipeline processor



Particle simulation using compute shader:

[https://arm-software.github.io/opengl-es-sdk-for-android/compute\\_particles.html](https://arm-software.github.io/opengl-es-sdk-for-android/compute_particles.html)



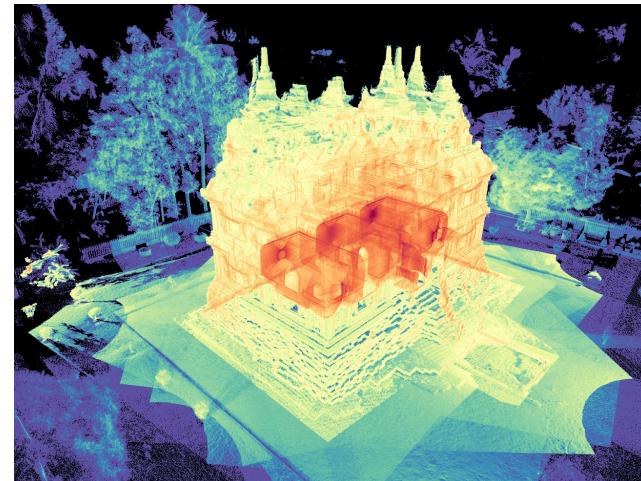
Compute shaders for point clouds animation:

<https://catlikecoding.com/unity/tutorials/basics/compute-shaders/>

# Compute shader

[https://vkguide.dev/docs/gpudriven/compute\\_shaders/](https://vkguide.dev/docs/gpudriven/compute_shaders/)  
[https://vulkan-tutorial.com/Compute\\_Shader](https://vulkan-tutorial.com/Compute_Shader)  
<https://learnopengl.com/Guest-Articles/2022/Compute-Shaders/Introduction>

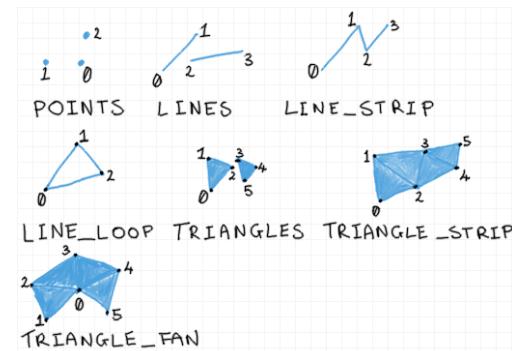
- Compute shader is form of GPU computing which is not necessary rendering
  - CUDA and OpenCL are used to control the GPU as massive parallel processor
- It is closely tied to the graphics rendering but not locked into a specific location in the graphics pipeline
  - It is used alongside vertex, pixel and other shaders
- Can be used for:
  - Post-processing: modifying rendered image with certain operations
  - Particle systems: computing behavior of particles
  - Mesh processing: facial animation
  - Culling
  - Image filtering
  - Improving depth precision
  - Shadows calculation
  - Depth of field computation
  - Computing tessellation



Rendering point clouds with compute shaders:  
[https://www.cg.tuwien.ac.at/research/publications/2021/SC\\_HUETZ-2021-PCC/](https://www.cg.tuwien.ac.at/research/publications/2021/SC_HUETZ-2021-PCC/)

# Application stage output

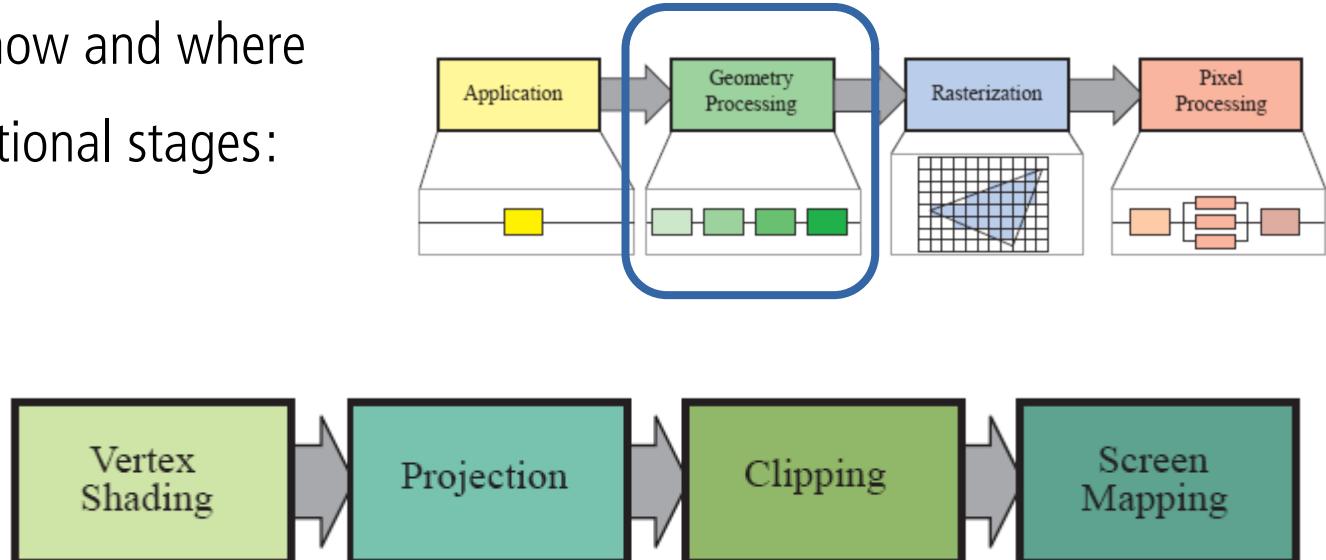
- Application stage outputs data to geometry processing stage:
  - Object geometry to be rendered → rendering primitives: points, lines and triangles
  - Vectors, matrices, textures describing object materials, light and camera parameters
- Efficiency of this stage is propagated to further into pipeline
  - Example programmer defines amount of geometry (triangles) sent to GPU and how optimized description is



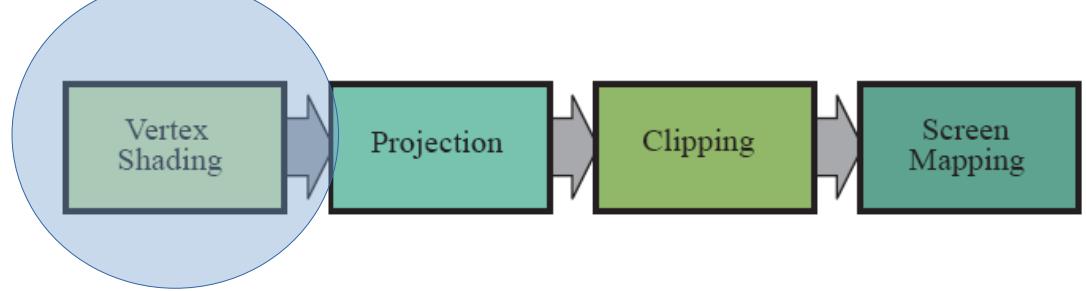
## 2. Geometry processing stage

# 2. Geometry processing stage

- Responsible for most of the **per-triangle** and **per-vertex** geometry operations
  - Deals with **transformations**, **projections** and all other geometry handling
- Computes what is drawn, how and where
- Divided into following functional stages:
  - 2.1. **Vertex shading**
  - 2.2. **Projection**
  - 2.3. **Clipping**
  - 2.4. **Screen mapping**



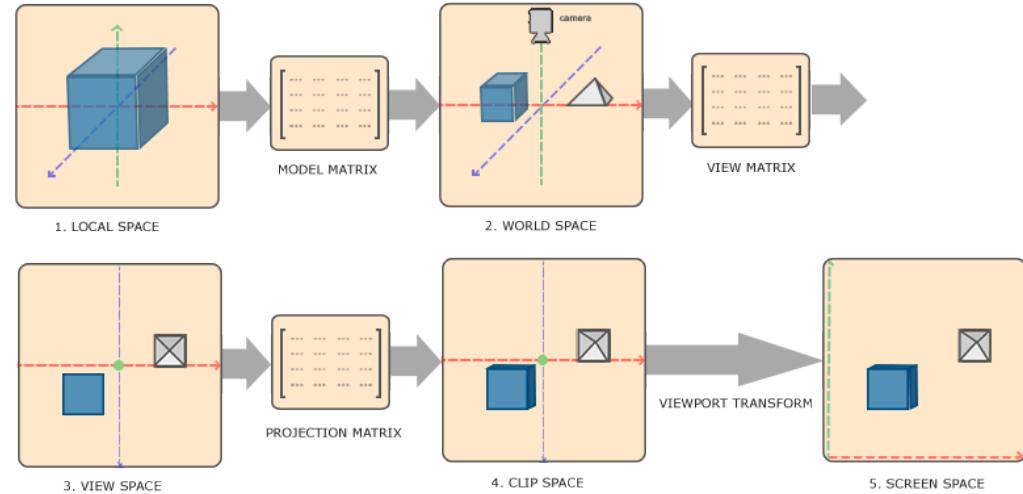
# 2.1 Vertex shading



- Two main tasks:
  - 2.1.1 **Compute position of vertex** inputted from application stage to space needed for projection
  - 2.1.2 **Evaluate/set-up vertex attributes**: any additional vertex data output desired by programmer such as normal, texture coordinates, colors, vertex shading, etc.

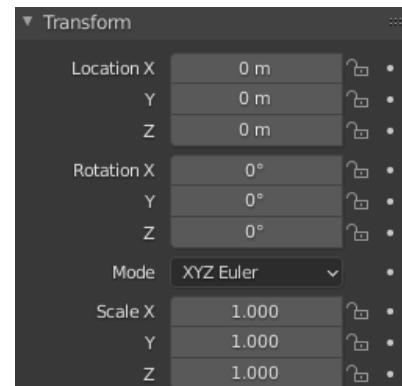
## 2.1.1 Vertex shading: compute vertex position

- **Vertex positions** of a model are minimal information that has to be passed from application stage to vertex shading stage.
- On the way to rasterization stage, **vertex shading** performs transformation of a model in **several different spaces** or coordinate systems:
  - Model/local space
  - World space
  - View/camera space

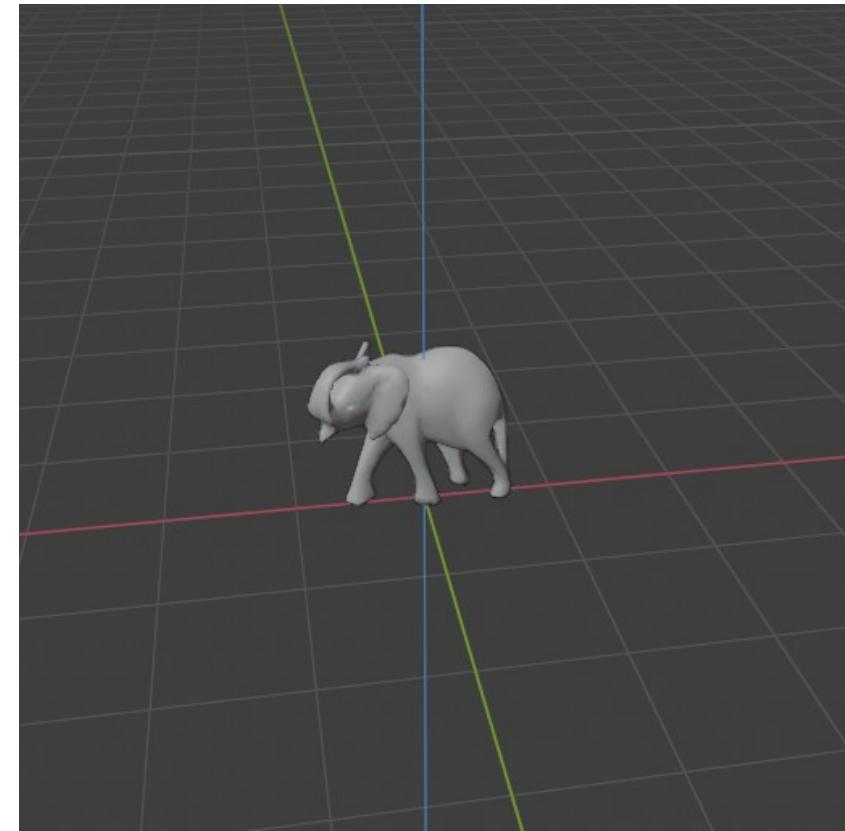


# Model space

- Originally, model (vertex positions) resides in its own **model space**.
  - Model has not been transformed at all
- Each model can be associated with **model transform**
  - Transformation matrix used for positioning and orienting
  - Used for model-to-world transformation
- **Model transform is applied on model's vertices and normals**
  - Coordinates of a object are also called **model coordinates**

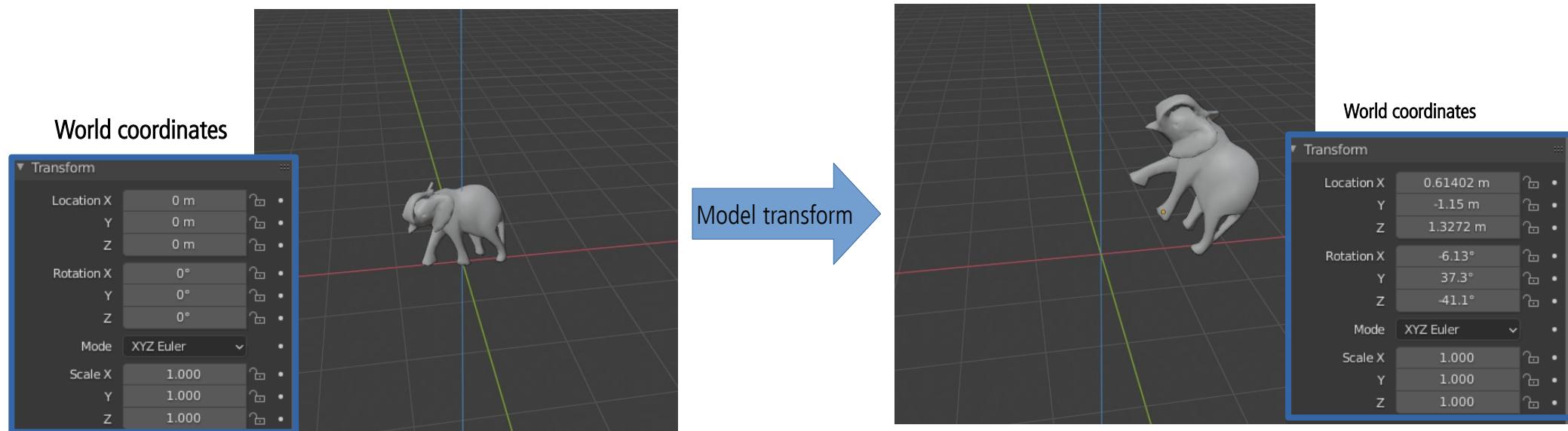


World coordinates



# World space

- After model transform has been applied on model coordinates, the model is said to be in **world coordinates** or **world space**
- **World space is unique** and after all models have been transformed with their respective model transforms, they all exist in this same space
- Model transform is defined on application stage, sent to GPU and performed by vertex shader.



# Model transformation matrix

- Defines **model-to-world** or **world-to-model** transformation of mesh vertices
  - Model transform matrix applied on **model coordinates** gives **world space coordinates**
  - Inverse of transform matrix applied on **world coordinates** gives **model coordinates**
- Model transform is **4x4 matrix** which can describe:
  - Basic transformations: Translation (T), Rotation (R), Scaling (S)
  - Complex transformation: Euler, Quaternions, look-at transformation matrix, etc.
- Build transformation transformation matrices is supported by many math libraries
  - Example: <https://github.com/g-truc/glm>

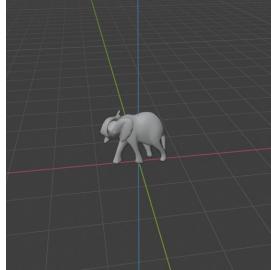
$$\mathbf{T}(\Delta x, \Delta y, \Delta z) = \begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\mathbf{S}(x, y, z) = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

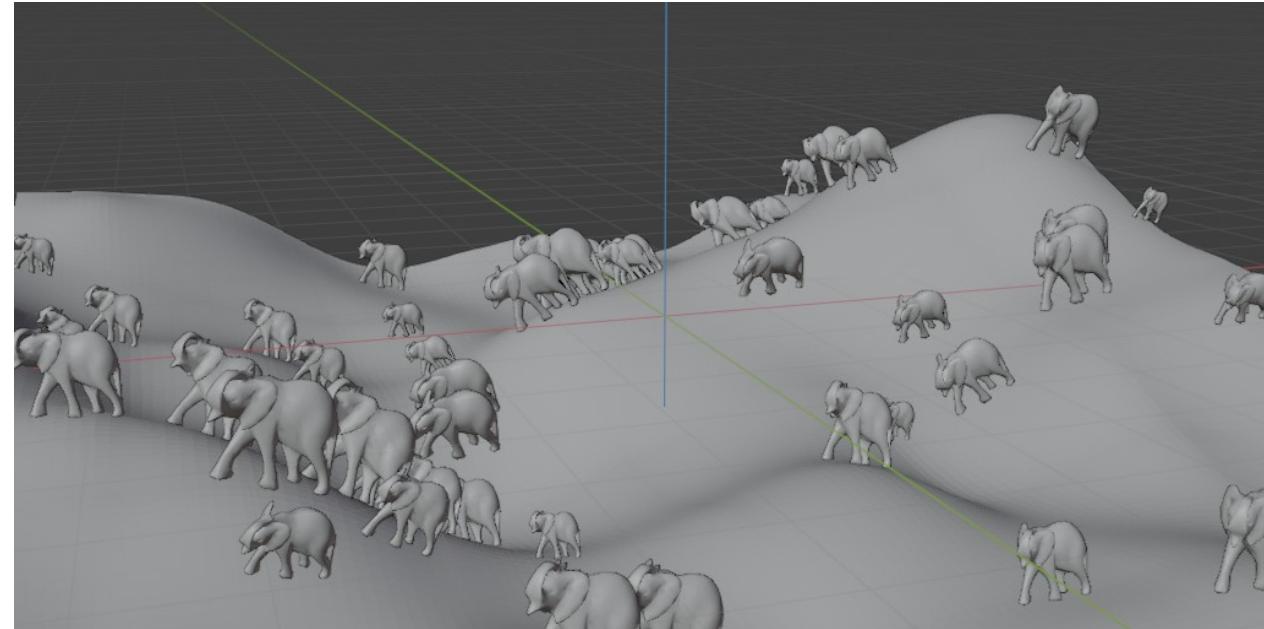
$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

# Model transform: instancing

- Each model can have multiple transforms
  - This allows **copying the same model across the 3D scene without specifying additional geometry** – instancing
  - Same model can have different locations, orientations and sizes in the same scene.



Same geometry is instanced using translation, rotation and scaling.



# Instancing

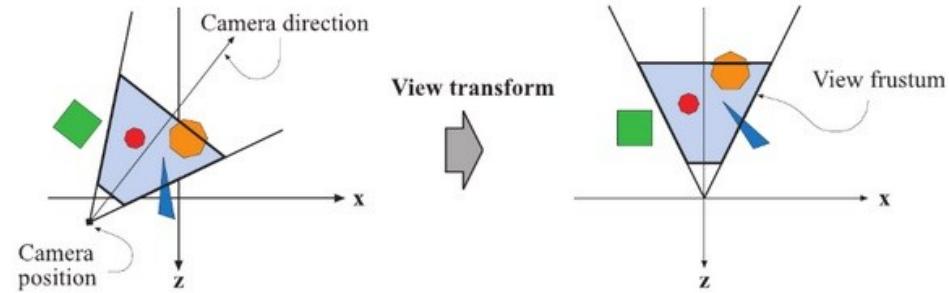
- Often used for repeated placement of same geometry with variations

Example: environment art



# Camera (view, eye) space

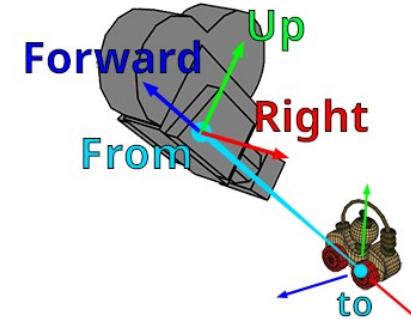
- Camera has a location and orientation in world space
- Only objects visible from camera are rendered
  - Further pipeline stages (projection and clipping) require camera and all objects to be in view (camera) space
- View transform is applied to all objects and camera:
  - After transformation, camera is placed in world origin and aimed in negative z axis with y pointing up and x pointing right.
- View transform is defined on application stage and performed by vertex shader.



**Negative z axis convention.** Another convention is positive z axis convention. Actual position and direction after view transform are dependent on underlying API.

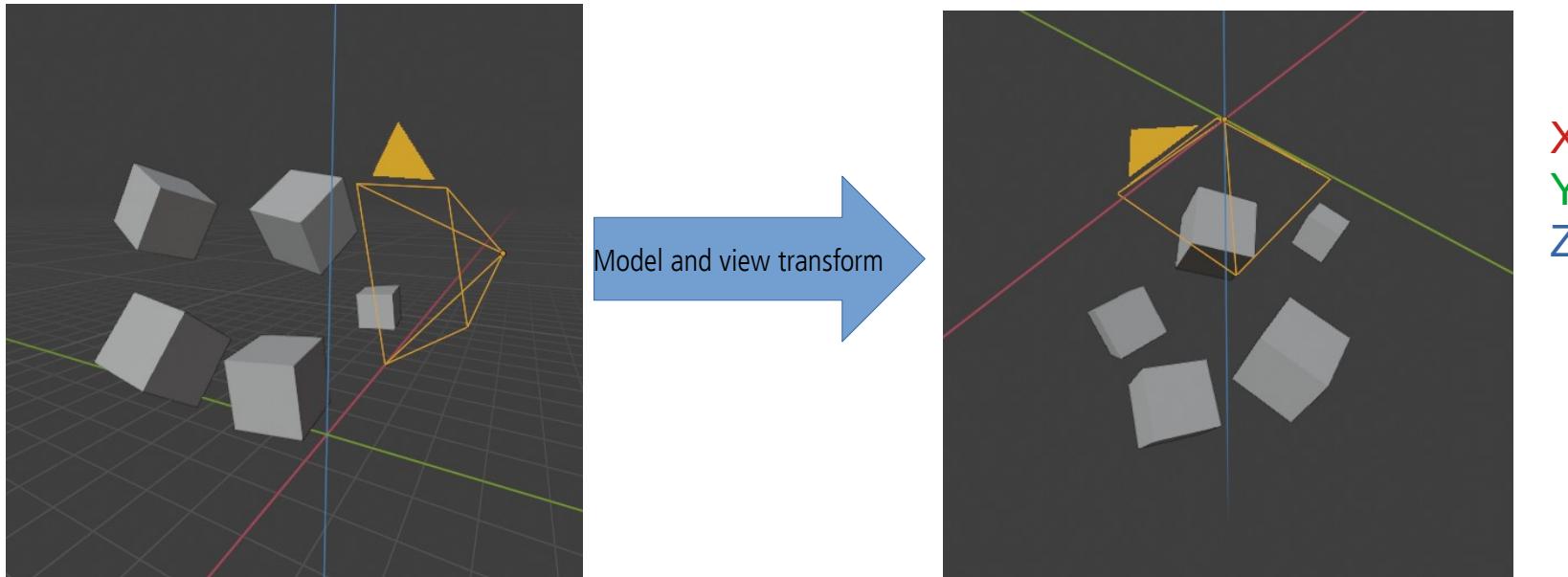
# Camera in graphics pipeline

- Graphics rendering pipeline doesn't use camera as described in ray-tracing based rendering
- Camera position and orientation is used to create **view transform matrix** which is used **transform all objects in 3D scene from world into camera space**
  - Camera space is used later in sub-subsequent rendering stages: projection
- Various libraries exist for computing view matrix
  - <https://github.com/g-truc/glm/blob/master/manual.md#-52-glm-replacements-for-glu-functions>
- Often, **view matrix** is constructed using **look-at notation**



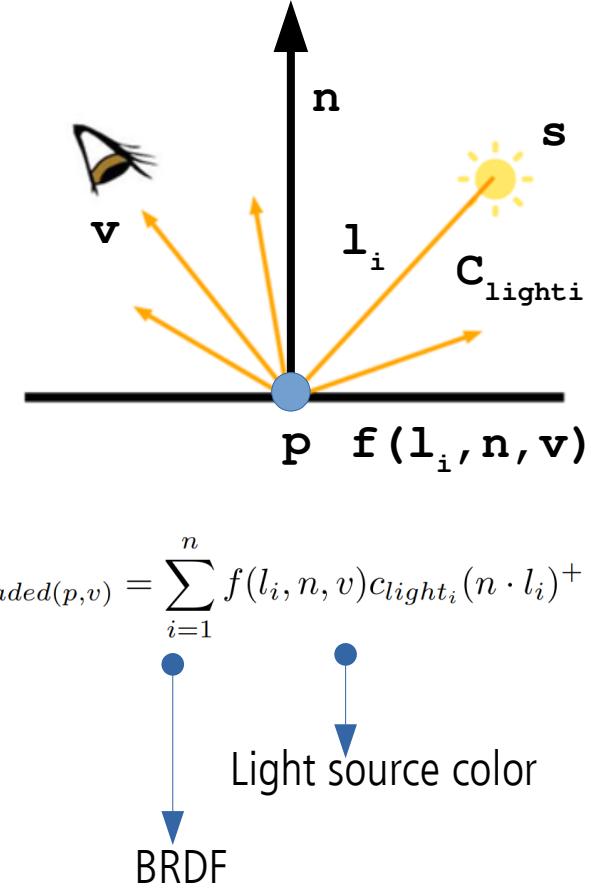
# Camera (view, eye) space

- After **view** transform, the model is said to be in **camera (view or eye) space**.
- **Model and view transform can be implemented as one  $4 \times 4$  matrix** for efficient multiplication with vertex and normal vectors.
  - Note: programmer has full control over how the position and normals are computed.

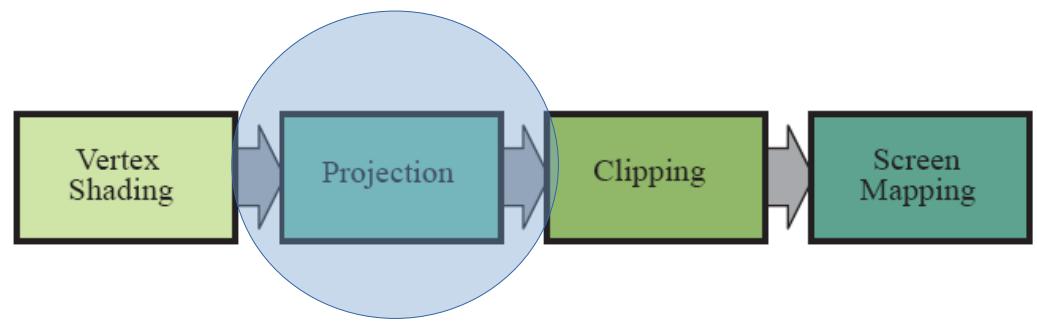


## 2.1.2 Computing/setting-up vertex attributes

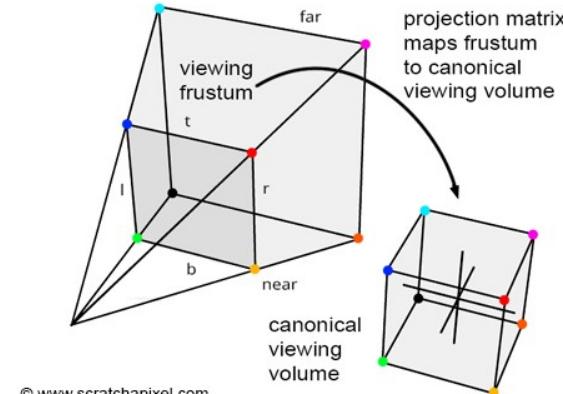
- Vertex attributes: colors, texture coordinates, normals, etc.
- Shading can also be performed on this stage → **vertex shading**
  - Computing color is done by **evaluating shading equation** using **vertex data** (position, normal, material) and **light information** defined on application stage
  - Shading equation example: **direct illumination equation**
  - Resulting color is stored as **color per vertex** which is later **interpolated** and used across triangle and pixels
- Vertex shading was traditionally used thus the name “vertex shader”
  - Modern GPUs perform shading during pixel processing stage and vertex shader is more general unit dedicated to setting up data associated with each vertex



# 2.2 Projection



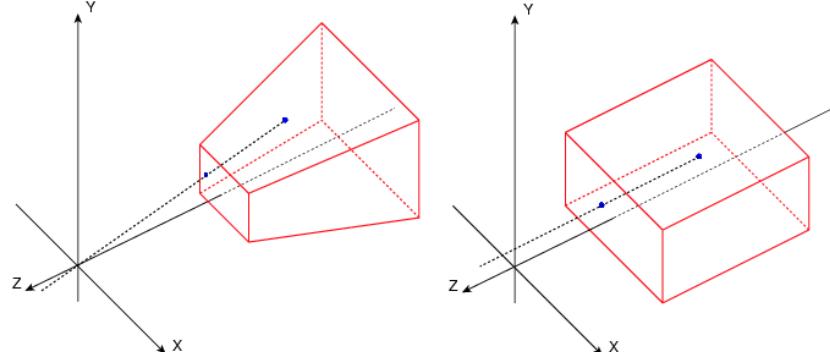
- After models are transformed to camera space, a **projection** is applied transforming vertices to clip space
- **Projection is represented as 4x4 matrix** and can be combined with other geometry transforms: model and view
  - Projection matrix is defined in application stage and applied during geometry processing stage (on GPU) on object vertices
- **Projection transforms the view volume into canonical view volume**
  - A unit cube with extreme points at  $(-1, -1, -1), (1, 1, 1)$
  - Doing so makes clipping stage easier



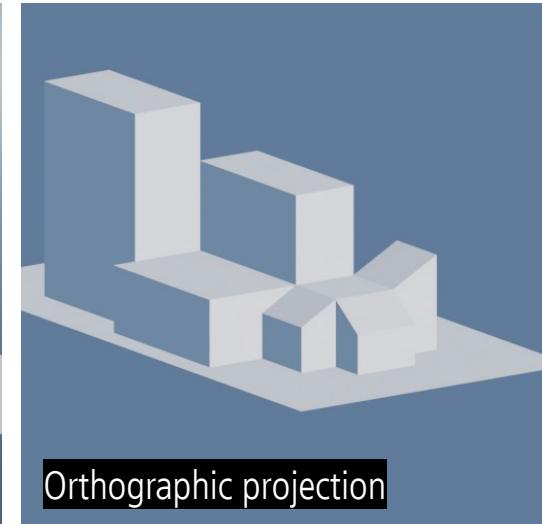
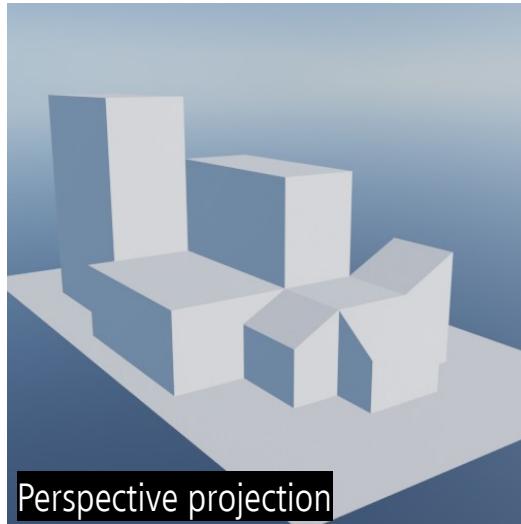
© www.scratchapixel.com

# 2.2 Projection

- Two commonly used projections are:
  - Orthographic projection
  - Perspective projection

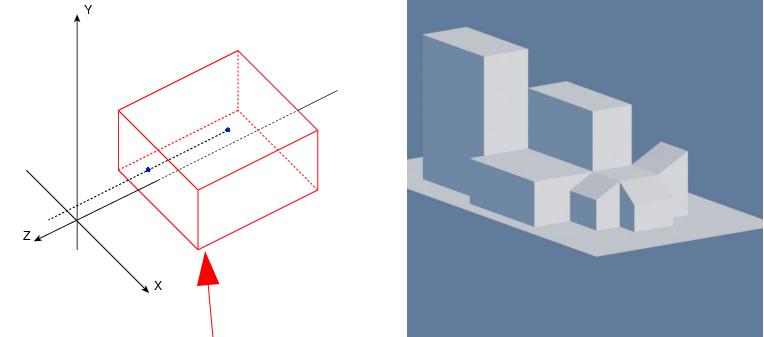


<https://blender.stackexchange.com/a/649>



# Orthographic projection

- One type of parallel projection
- View volume is rectangular box
- Characteristics:
  - parallel lines remain parallel
  - Objects maintain the same size regardless of distance from camera
- Expressed as matrix:
  - Simple orthographic projection: projects onto plane  $z = 0$
  - $z$  coordinate is simply set to 0 → store original value in depth buffer

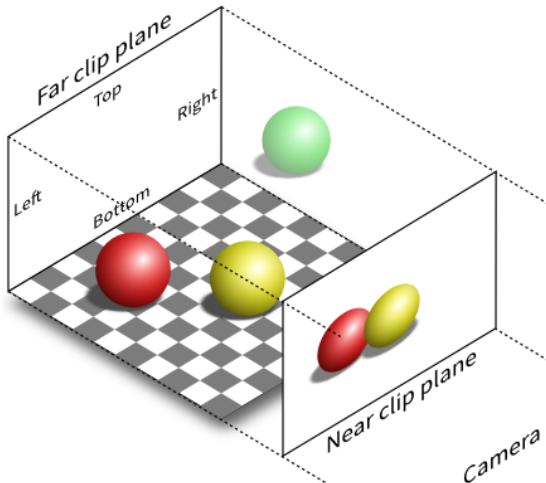


Orthographic projection

$$Pv = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ 0 \\ 1 \end{bmatrix}$$

# Orthographic projection

- General orthographic projection is expressed using **orthographic frustum**
  - Extremes are viewing frustum **near** (n), **far** (f), **left** (l), **right** (r), **top** (t) and **bottom** (b)
  - This projection **transforms rectangular box view volume** into unit cube → **canonical view volume**
    - OpenGL: (-1,-1,-1), (1,1,1),
    - DirectX: (-1,-1,0) and (1,1,1)

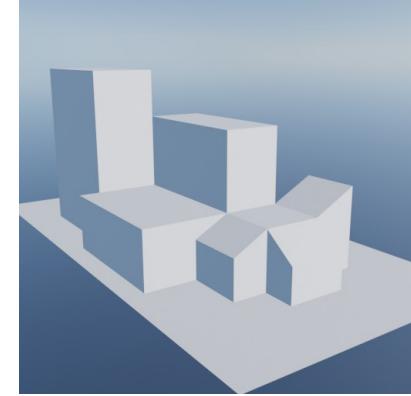
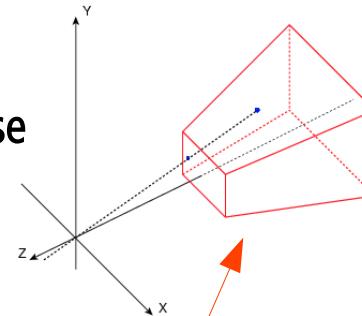


$$P = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix **scales** and **translates** axis aligned bounding box formed by these planes into axis-aligned cube centered around origin.

# Perspective projection

- View volume (frustum) is truncated pyramid with rectangular base
- Mimics how human visual system forms image
  - The further the object, the smaller appears → foreshortening effect
  - Parallel lines converge at single point
- Expressed as matrix



Orthographic projection

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Perspective projection

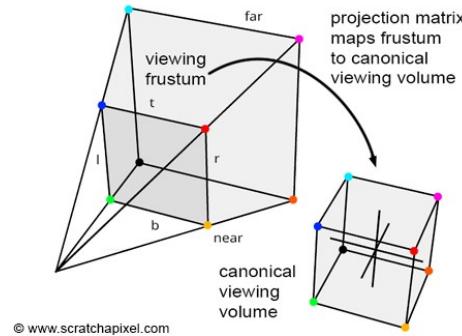
- Simple perspective projection is a matrix projecting points on a plane  $z = 1$
- After multiplication with perspective matrix:
  - Homogeneous coordinate  $w_c$  will be equal to z vertex coordinate
  - Other vertex coordinates stay the same
- Converting to Euclidean space: dividing by homogeneous coordinate results in Perspective divide
  - Vertex coordinates  $(x,y)$  have undergone perspective distortion
  - Vertex coordinate  $z$  is equal to 1. Its original value is stored in depth buffer

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \frac{1}{w_c} \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix}$$

# Perspective projection

- General perspective projection is **matrix** transforming viewing frustum to **canonical view volume**
  - A unit cube with extremes at  $(-1, -1, -1)$  and  $(1, 1, 1)$
- Construction of this matrix depends on **viewing furstum** and is supported by various libraries:
  - Opengl: `glFrustum(float left, float right, float bottom, float top, float near, float far);`
  - OpenGL Utility Library (GLU): `void gluPerspective(float fovy, float aspect, float zNear, float zFar);`
  - GLM: <https://github.com/g-truc/glm/blob/master/manual.md#-52-glm-replacements-for-glu-functions>

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



# Perspective projection

- General perspective projection matrix is defined with viewing frustum **near** (n), **far** (f), **left** (l), **right** (r), **top** (t) and **bottom** (b)
- Camera's **field of view** (FOV) and **image aspect ratio** are used to compute left, right, bottom and top

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

OpenGL perspective projection matrix: column-major order

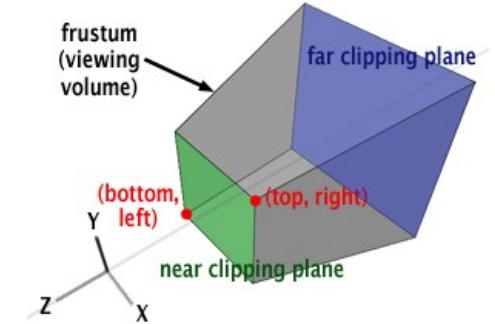
$$\text{aspect ratio} = \frac{\text{width}}{\text{height}}$$

$$\text{top} = \tan\left(\frac{\text{FOV}}{2}\right) * \text{near}$$

$$\text{bottom} = -\text{top}$$

$$\text{right} = \text{top} * \text{aspect ratio}$$

$$\text{left} = \text{bottom} = -\text{top} * \text{aspect ratio}$$

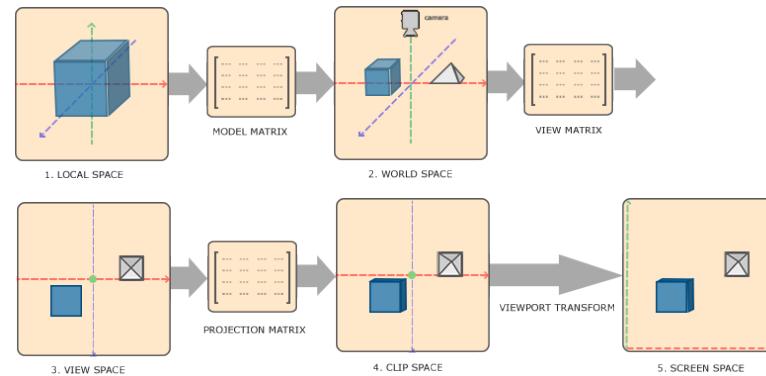


# Camera in graphics pipeline

- In contrast to ray-tracing based rendering, graphics rendering pipeline uses camera information (field of view and image aspect ratio) to construct **projection matrix**
- **Projection matrix is applied on all objects in 3D scene to project them on image plane**
  - This process simulates camera projection, e.g. perspective projection
  - Remember that prior to projection step, objects have been transformed to camera space
- Graphics pipeline then further does computation on projected vertex coordinates

# Projection and clip space

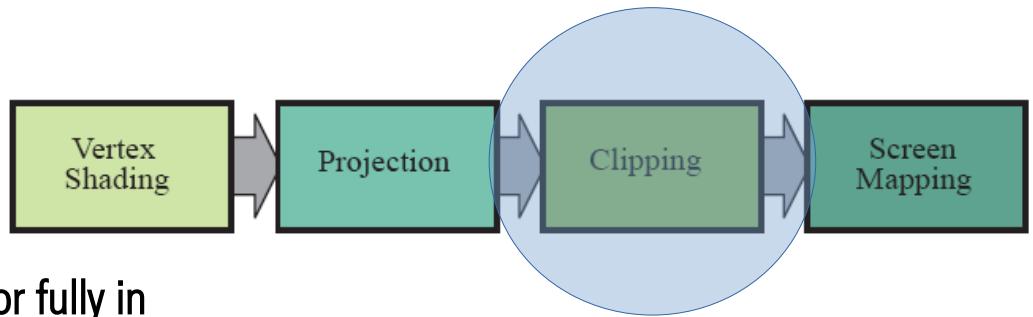
- At this point, objects have been transformed from local to world to camera space and with projection to **clip space**
- After projection step vertex coordinates are in **clip space - clip coordinates**
  - These are still homogeneous coordinates ( $x, y, z, w$ )
  - Division with  $w$  - **Perspective division** - to obtain Euclidean coordinates ( $x', y', z'$ ) occurs later
  - This coordinates are inputted to next stage: **clipping**



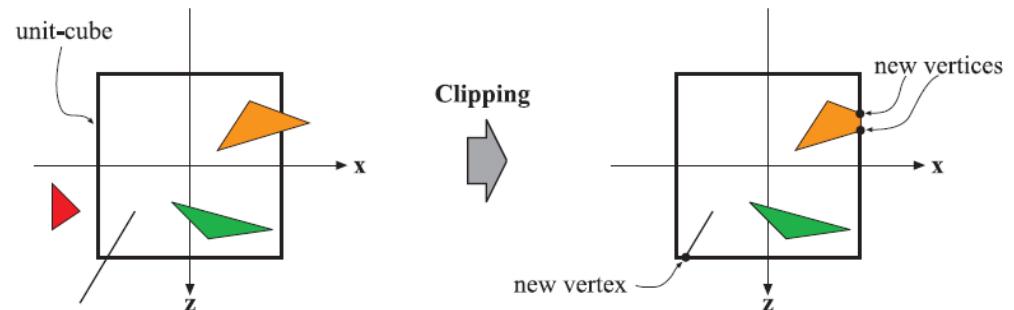
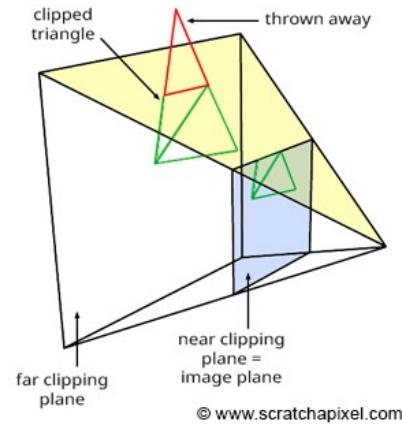
# Optional vertex processing

- Every GPU pipeline has the described vertex processing
- However, there are **few optional stages that can take place on GPU**:
  - **Tessellation** – generating additional triangles to better represent curved surfaces
    - Sub-stages: hull shader, tessellator and domain shader
  - **Geometry shader** – takes in various primitives (e.g., points, triangles) and creates new vertices
    - Often used for particle generation – e.g., for each input vertex representing particle position generate additional vertices to create square on which texture can be mapped
  - **Stream output** – instead of sending vertices down the pipeline, use them for further processing on CPU/GPU
    - Often used for particle simulation
- Note: not all GPU hardware support those stages

# 2.3 Clipping

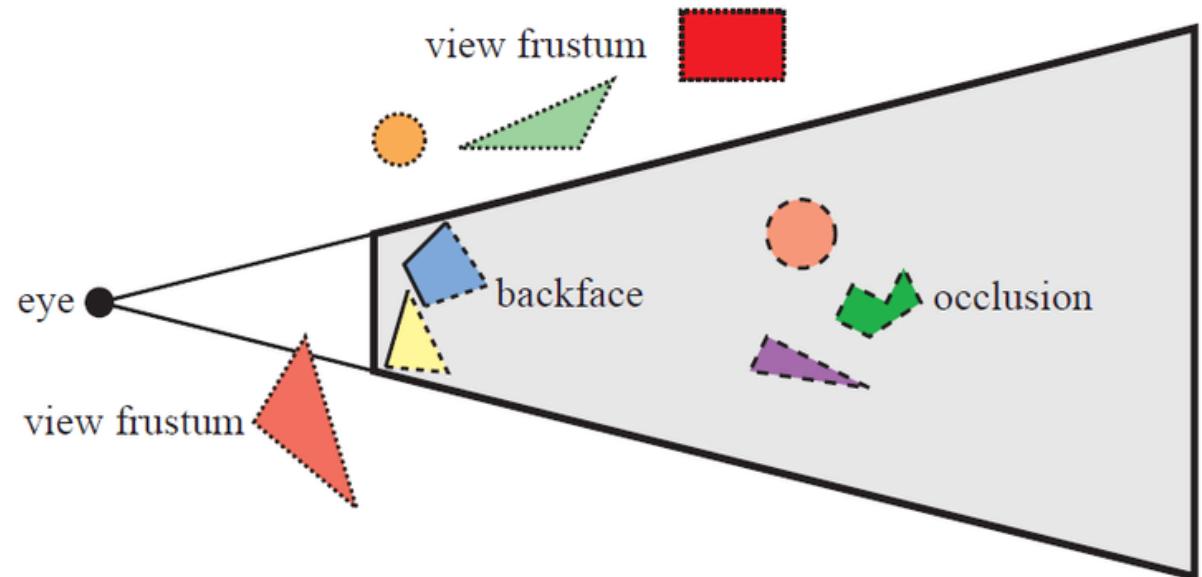


- After projection, only primitives that are **partially or fully in view volume** need to be passed to the rasterization stage and pixel processing for drawing on screen.
- Primitive that is **fully in view volume** will be passed further without clipping.
- **Primitives that are partially in view volume** require clipping.
  - After projection, **clipping of primitive is done against unit cube**.
  - Vertices that are outside of view volume are removed. New vertices are created on clipping position.



# Culling

- Whole scene processing is avoided using:
  - Frustum culling
  - Occlusion culling
  - Back face culling

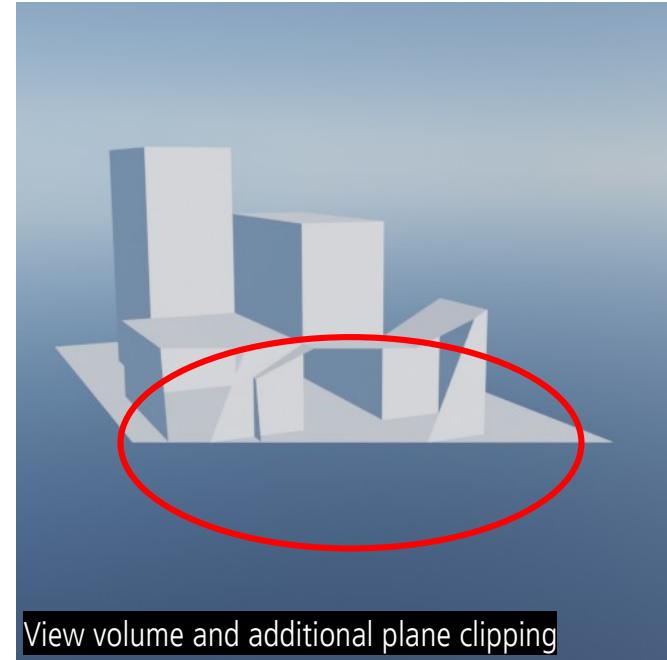
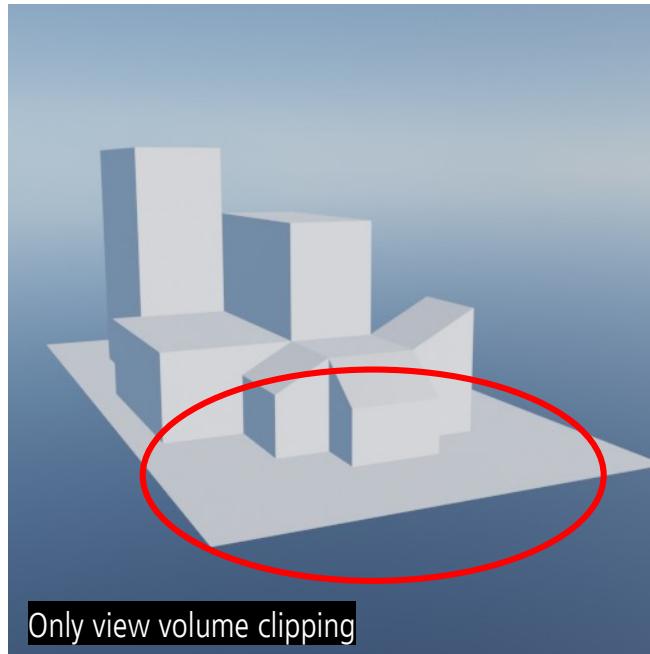


# 2.3 Clipping

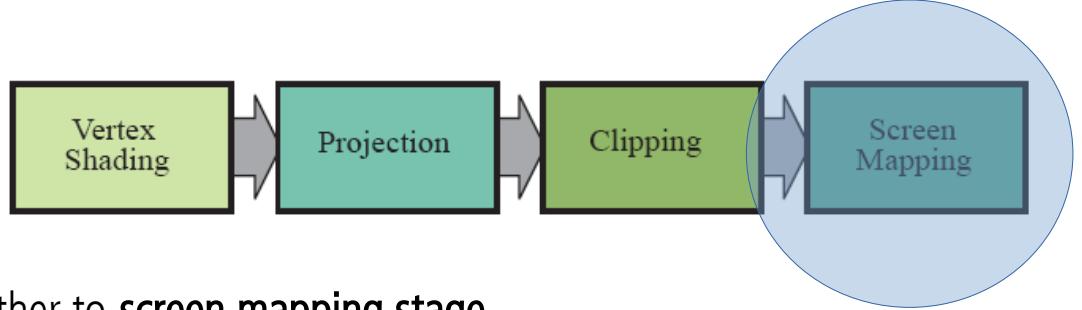
- Clipping uses homogeneous coordinates - clip coordinates  $(x,y,z,w)$  resulted from projection step.
  - Note that forth coordinate ( $w$ ) contains depth value after multiplication with projection matrix
  - Depth value is needed for correct interpolation of vertex attributes over triangle and clipping
- After clipping, perspective division is performed:
  - Division of  $(x, y, z)$  with  $w$  which contains depth
  - Resulting triangle positions are in **canonical view volume** a 3D normalized device coordinates (NDC)
    - This view volume ranges from  $(-1, -1, -1)$  to  $(1, 1, 1)$ .
- Clipping is fixed stage in rendering pipeline → user doesn't have influence, it is done automatically

# 2.3 Clipping

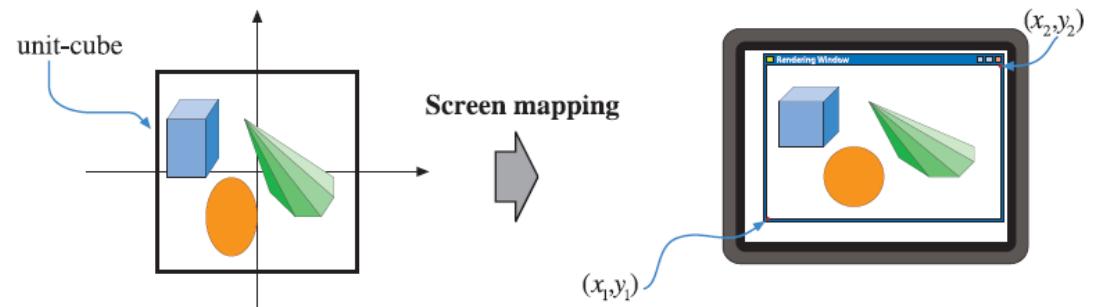
- Clipping is normally done with respect to six clipping planes of viewing frustum (view volume)
- Additional clipping planes can be introduced by programmer to chop the visibility of objects: **sectioning**



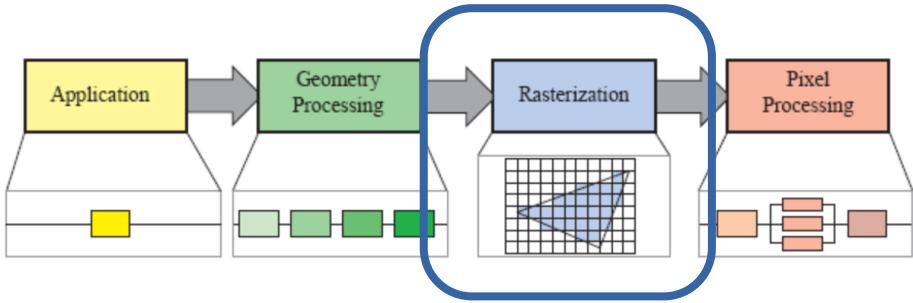
## 2.4 Screen mapping



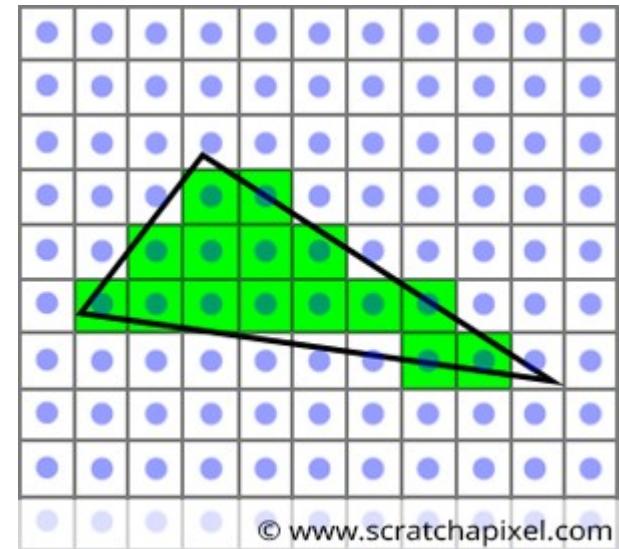
- Only primitives inside view volume are passed further to **screen mapping stage**
- $(x, y, z)$  coordinates entering this stage are still 3D in NDC space  $(-1, -1, -1)$  to  $(1, 1, 1)$
- $(x, y)$  coordinates are transformed to **screen coordinates**.
  - Screen mapping transforming  $(x, y)$  to screen with dimensions  $(x_1, y_1)$  and  $(x_2, y_2)$ .
- $(x, y)$  screen coordinates with z coordinates are called **window coordinates**.
  - z coordinates are mapped to  $[-1, 1]$  in OpenGL or  $[0, 1]$  in DX.
- Window coordinates are passed to the rasterizer stage.



# 3. Rasterization stage



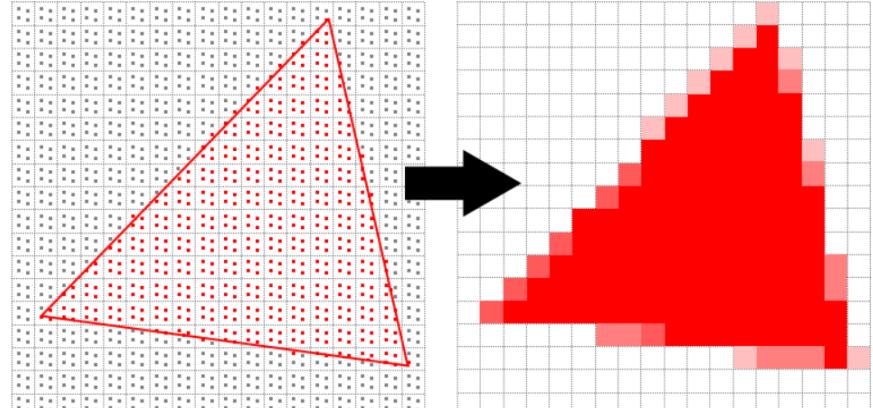
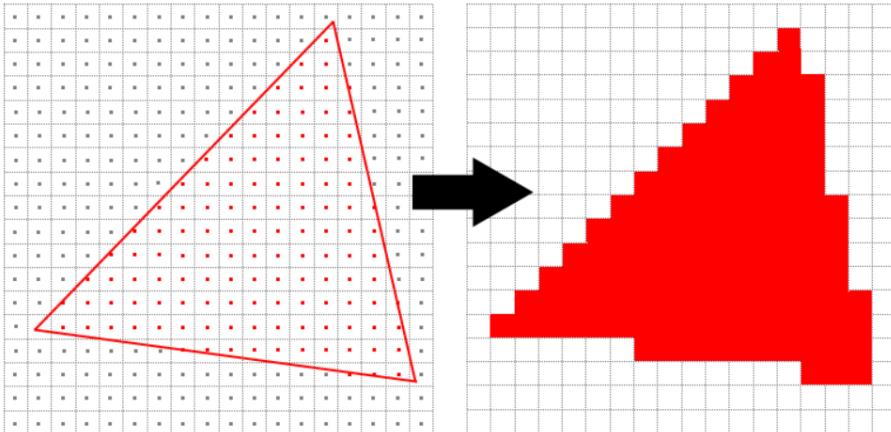
- Given transformed and projected vertices (with associated vertex attributes data for shading) the goal is to **find all pixels which are inside the primitive** (e.g., triangle) to be used in pixel processing stage.
  - Typically takes input of 3 vertices forming a triangle, finds all pixels that are considered inside the triangle and forwards those further for pixel processing
- Rasterization (aka screen conversion) is **conversion from 2D vertices in screen space into pixels on the screen**.
  - These 2D vertices have associated z value (depth) and vertex attributes for shading (e.g., normals)



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# 3. Rasterization stage

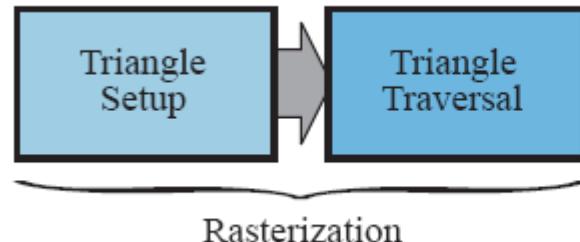
- To test if triangle is overlapping a pixel then a **pixel point sampling** is needed:
  - Only center of pixel: if center of pixel is inside triangle → pixel is considered inside triangle
  - Multiple samples per pixel are desired to evade **aliasing** problems → **super-sampling** or **multi-sampling anti-aliasing**.
    - Each tested pixel sample inside triangle is called fragment and it contributes to color of the pixel
- Another approach is to define pixel overlaps triangle if at least part of pixel overlaps triangle.



MSAAx4

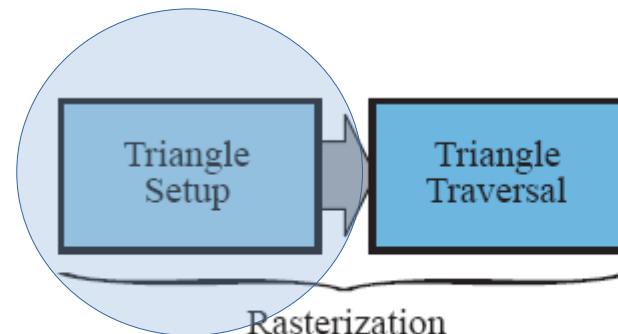
# 3. Rasterization stage

- Two functional sub-stages:
  - 3.1. Triangle setup
  - 3.2. Triangle traversal
- Process of “synchronization” between geometry processing and pixel processing
- This stage can also process lines and points but triangles are most often used



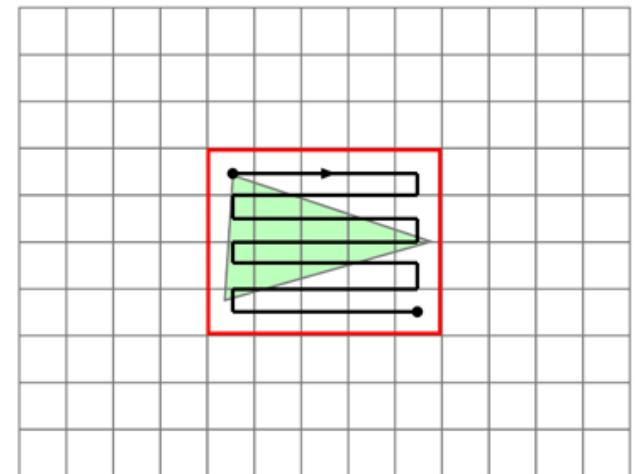
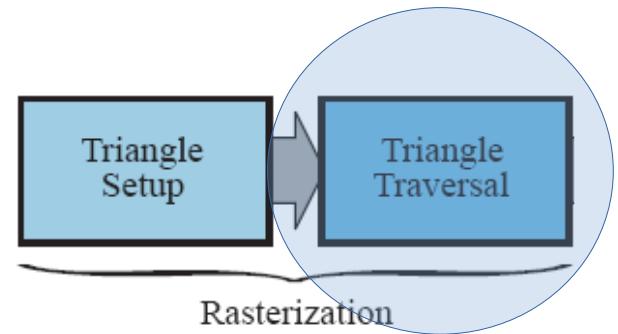
# 3.1 Rasterization: triangle setup

- In this step all data for the triangle needed for subsequent stages are computed using triangle information
  - Edge function needed for inside outside test
  - Barycentric coordinates and data needed for interpolating depth values and vertex attributes
- Computed data is needed for:
  - Triangle traversal and inside outside test
  - Interpolation of shading data produced by geometry stage (e.g., normals)
- Fixed-function hardware is used for this stage → programmer has no control over it.



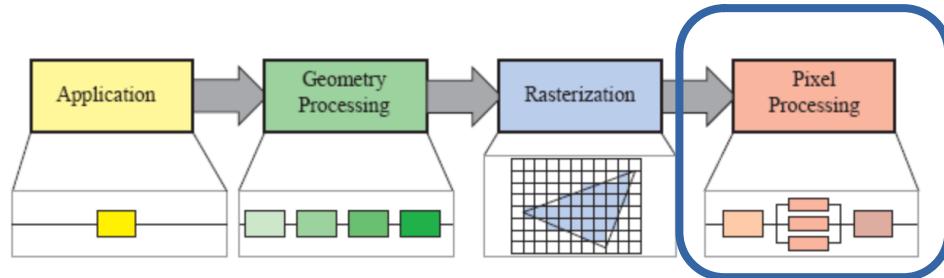
## 3.2 Rasterization: triangle traversal

- **Triangle traversal:** each image pixel center or sample is checked if covered by a triangle: **inside-outside test**
- **Fragment** is generated for part of the pixel that overlaps the triangle
- **Fragment properties** are generated by **interpolating vertex attributes** data among three triangle vertices
  - Note: due to perspective distortion, **perspective-correct interpolation** is needed
  - **Properties:** fragment depth and any shading data from geometry processing stage stored per vertex
- All pixels, that is **fragments**, inside primitive are sent to **pixel processing stage**.

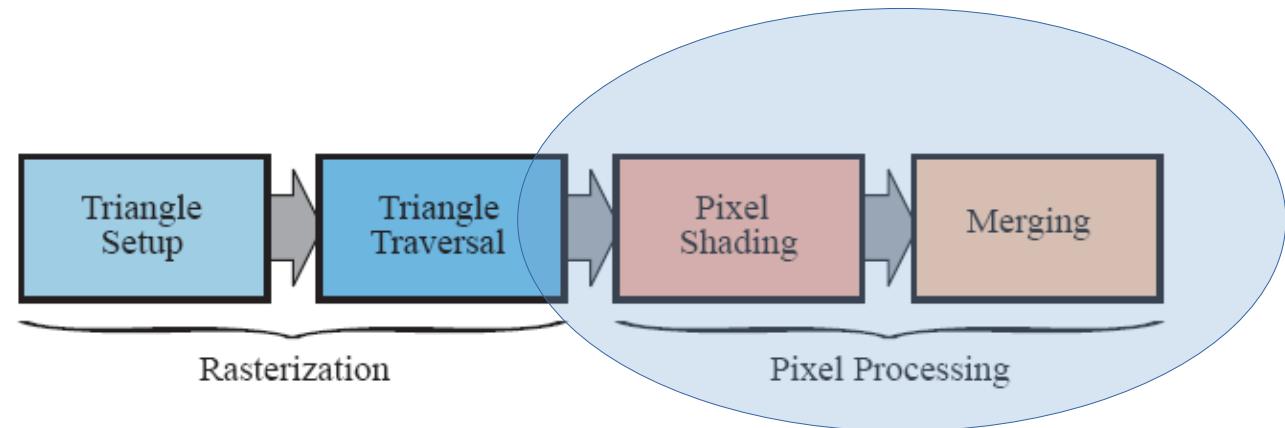


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# 4. Pixel processing stage

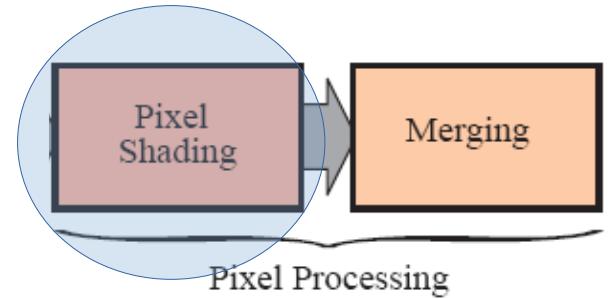


- Per-pixel or per-fragment computations are done on pixels or samples (fragments) inside a primitive
- Depth is taken in account for visibility (depth testing) as well as blending of new and old pixel color is computed
- Pixel processing stage can be divided into:
  - 4.1 Pixel shading
  - 4.2 Pixel merging



# 4.1 Pixel shading

- Per-pixel/fragment shading is performed here
  - Interpolated vertex attributes and shading data from previous stage are used to compute color
- Pixel shading stage is performed by **programmable GPU cores**
  - Programmer supplies a **program for pixel (fragment) shader** that contains any desired computations
  - A **shader program** is executed per-pixel/per-fragment to determine its color
  - To compute object color, **shader program** defines **shading equations**: e.g., direct illumination equation using scattering function and **texture** (image or procedural)
- Result of this stage is one or more colors for each pixel/fragment that are passed further the pipeline

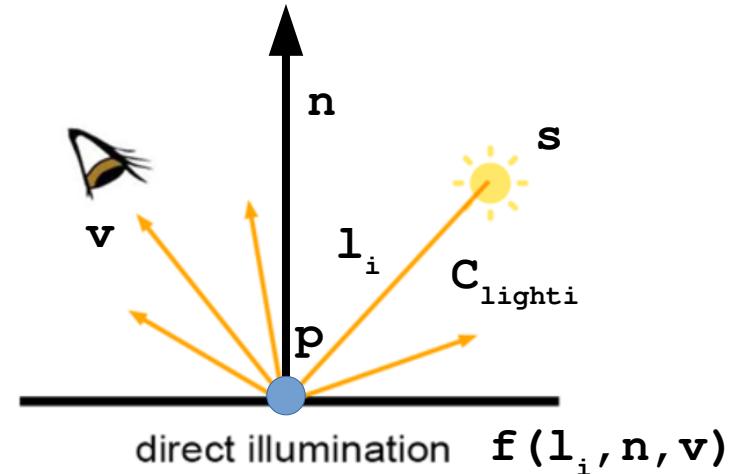


# Pixel shading: computing object color

- In pixel/fragment shader **shading equation** is employed to compute the color
- Shading equation often describes **direct illumination**. To compute color it uses:
  - Light position ( $s$ ) and color ( $c_{\text{light}}$ )
  - Camera view direction ( $v$ )
  - Object material (BRDF, texture):  $f(l_i, n, v)$
  - Object shape: normal ( $n$ )

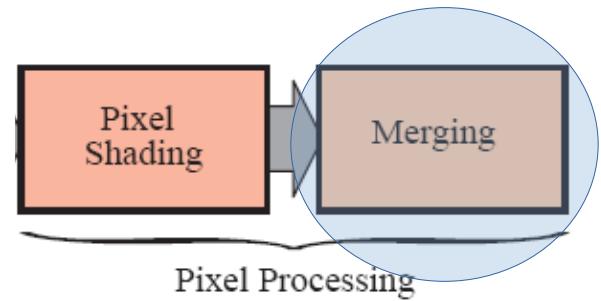
$$c_{\text{shaded}}(p, v) = \sum_{i=1}^n f(l_i, n, v) c_{\text{light}_i} (n \cdot l_i)^+$$

↓  
Light source color  
↓  
BRDF



# 4.2 Pixel merging

- Information for each pixel generated in pixel/fragment shader is stored in **color buffer**
  - Color buffer: **Rectangular array of colors (r, g, b)** same size as image
- Task of merging stage is to **combine new fragment color produced by pixel shading stage with the color currently stored in color buffer.**
- This stage is also called **raster operations pipeline (ROP)** or **render output unit**.
  - It performs **raster operations** or **blend operations**
- This stage is not fully programmable, but highly configurable for achieving various effects (e.g., transparency).
  - Tendency is towards more programmability



# Pixel merging: color blending and visibility

- Color blending: blending of new and current fragment color in color buffer
  - For opaque surfaces blending is not needed: fragment color simply replace the previously stored color
  - For transparent surfaces blending is needed: new and old fragment color in color buffer must be combined
  - Color blending is useful for compositing
  - Blending can perform a large number of operations involving color and alpha values
    - Combinations of multiplication, addition, subtractions, min, max, bit-wise logic, etc.
- Resolving visibility: depth testing
  - Color buffer should contain only color of triangles visible from camera point of view
  - Done using z-buffer/depth-buffer

# Pixel merging: frame buffers

- Pixel merging stage can use following buffers for computations:
  - z-buffer
  - Alpha channel (part of color buffer)
  - Stencil buffer
- **Frame buffer** generally consists of all buffers on a system.
- Different buffers that are generated can be used for **compositing** purposes.

# z-buffer

- When the whole scene has been rendered, the **color buffer** should contain **colors of the primitives** (e.g., triangles) in the scene **visible from camera point of view**.
- For most graphics hardware this is achieved using, **z-buffer (depth buffer)**.
  - Z-buffer has same **size and shape as color buffer**, but for each pixel it stores **z-value to the closest triangle**.
- Depending on **depth value of current triangle for current pixel and depth value for current pixel in z-buffer**, color-buffer and z-buffer are updated:
  1. When a triangle is being rendered at a certain pixel, z-value at that pixel is being computed and compared to the z-buffer
  2. If new z-value is smaller then value in z-buffer then that pixel is rendered closer to camera than previous pixel: color buffer and z-buffer must be updated with color and z-value of that pixel.
  3. Otherwise, color and z-buffer are not changed.

# z-buffer example

- Z-buffer has the same size and shape as color buffer, but for each pixel it stores **z-value to the closest primitive**.
- After rendering is done, **z-buffer contains depth values for all triangles visible to the camera**
  - Grayscale image where each pixel represents depth from camera to triangle



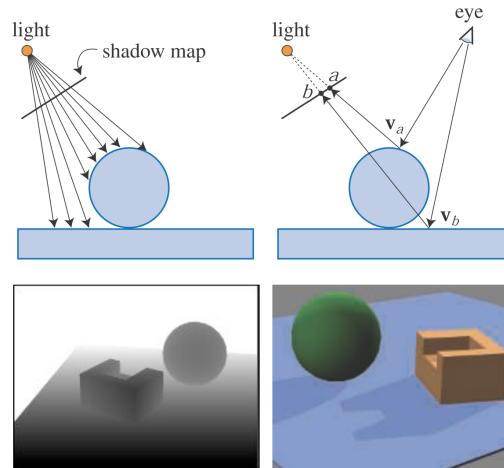
Color buffer.



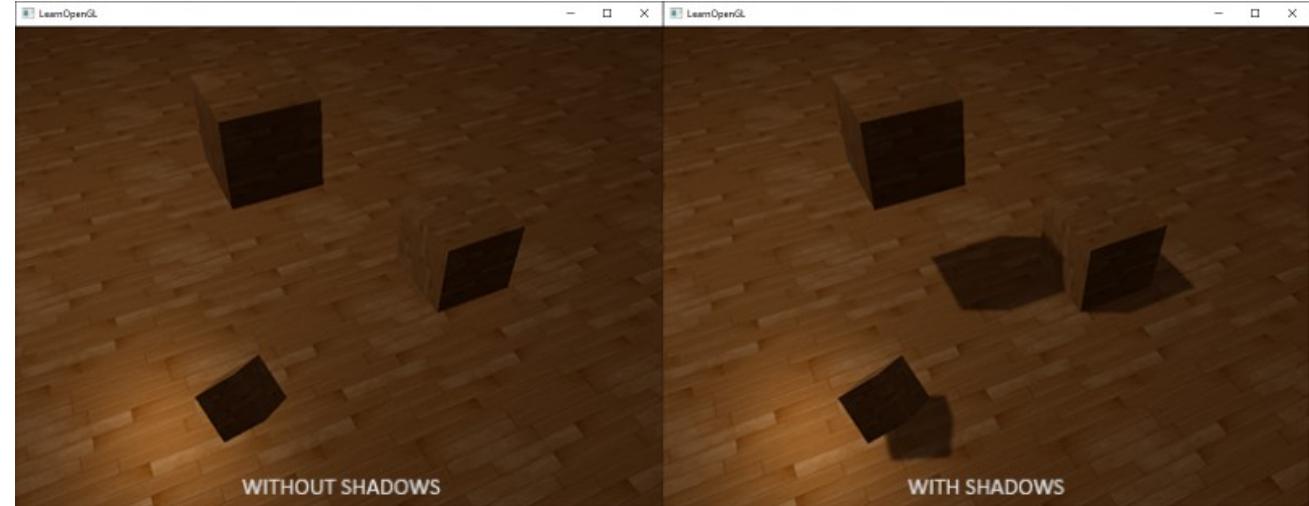
z-buffer.

# z-buffer example

- Z-buffer can be rendered not only from camera's point of view but also from light's point of view enabling shadow mapping technique



Rendering from light's point of view  
fills z-buffer with depth to closest  
visible objects to the light.

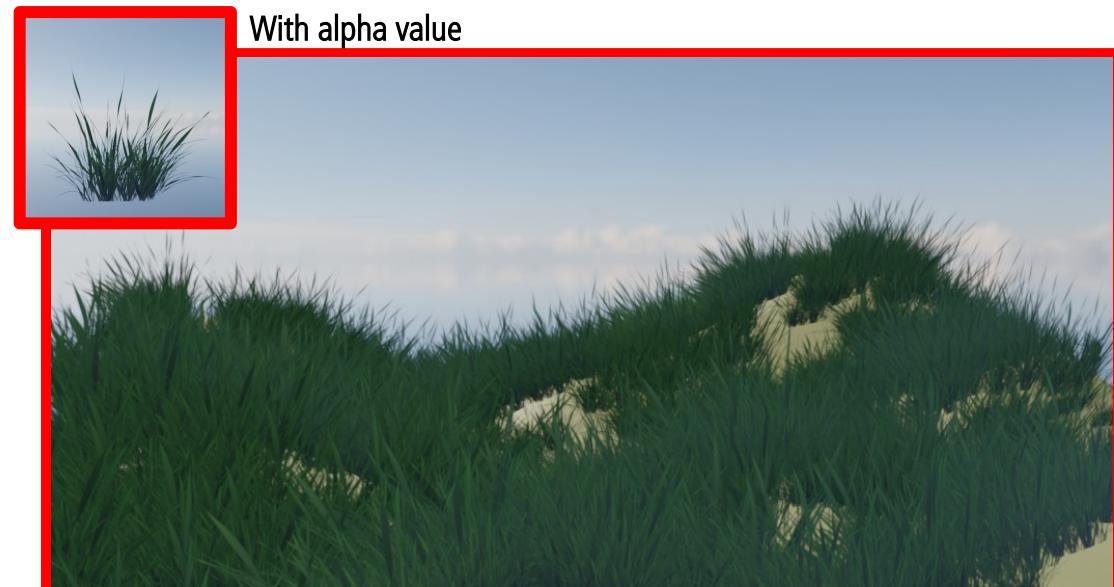


# z-buffer properties

- Z-buffer algorithm is simple and of  **$O(n)$**  complexity, where n is number of triangles
- **Order independent:** allows triangles to be rendered in any order.
- z-Buffer **stores only single depth value for each pixel**
  - Can not be used for partially transparent objects
- Partially transparent objects must be rendered after all opaque triangles using end to front order
- **early-z:** many GPUs perform some merge testing before pixel shader is executed
  - z value of fragment is available after rasterization and it can be used for testing visibility and culling before pixel shader
  - Pixel shader can change z-depth of the fragment or discard whole fragment. In this case, early-z is not possible.

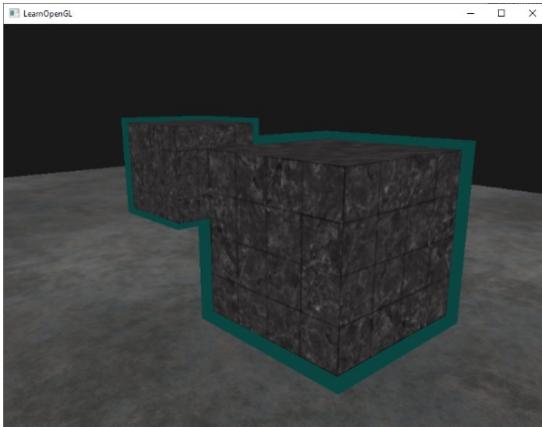
# Alpha channel

- Alpha channel is associated with color buffer and stores opacity, **alpha**, value for each pixel.
- Alpha value can be used for selective discarding of pixels using **alpha test feature** or pixel (fragment) shader.
  - This ensures that **fully transparent pixels/fragments do not affect the z-buffer**.



# Stencil buffer

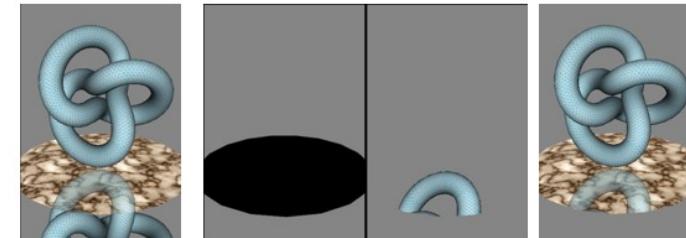
- Stencil buffer is **offscreen buffer** that **records locations of rendered primitives**
  - Buffer contents are used to control rendering into color and z-buffer
  - Example: filled rectangle is rendered into stencil buffer. This buffer is then used to render scene primitives into the color buffer only where the rectangle is present
- Stencil buffer is used in combination with different operators for **generating special effects**



Stencil buffer can be used to generate object outlines:  
<https://learnopengl.com/Advanced-OpenGL/Stencil-testing>



Stencil buffer can be used to render in color buffer only on specific locations.  
<https://www.ronja-tutorials.com/post/022-stencil-buffers/>



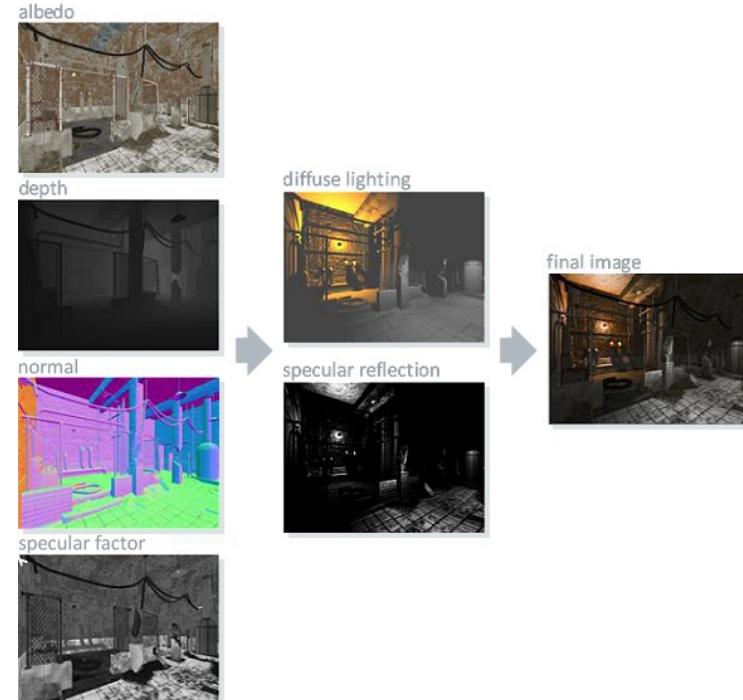
Problem: want to limit reflection to marbled floor

Stencil and stenciled reflection

result

# Multiple render targets

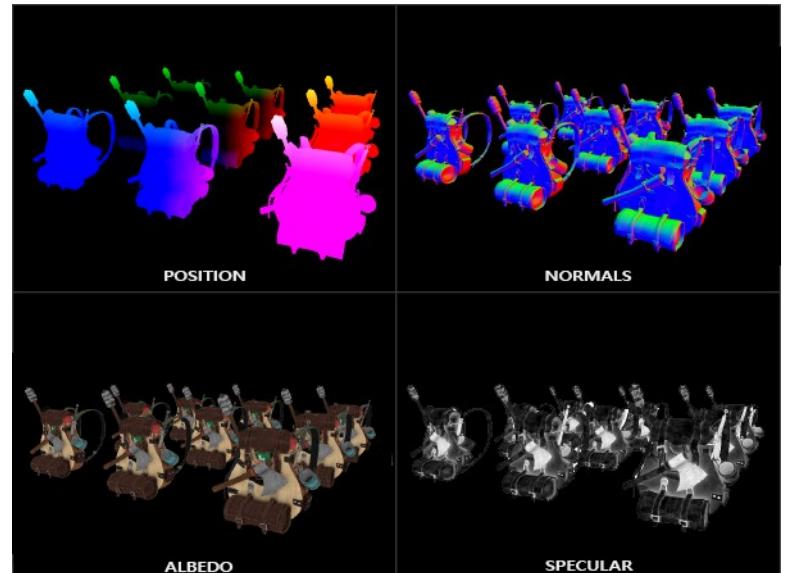
- Instead of storing results of pixel shader program directly to color buffer and z-buffer, multiple sets of values can be generated for each fragment and saved to different buffers each called **render targets**.
- Single rendering pass can generate multiple rendering targets:
  - Color image
  - Object identifies
  - Depth values
  - Surface normals
  - Etc.



# Forward vs deferred rendering

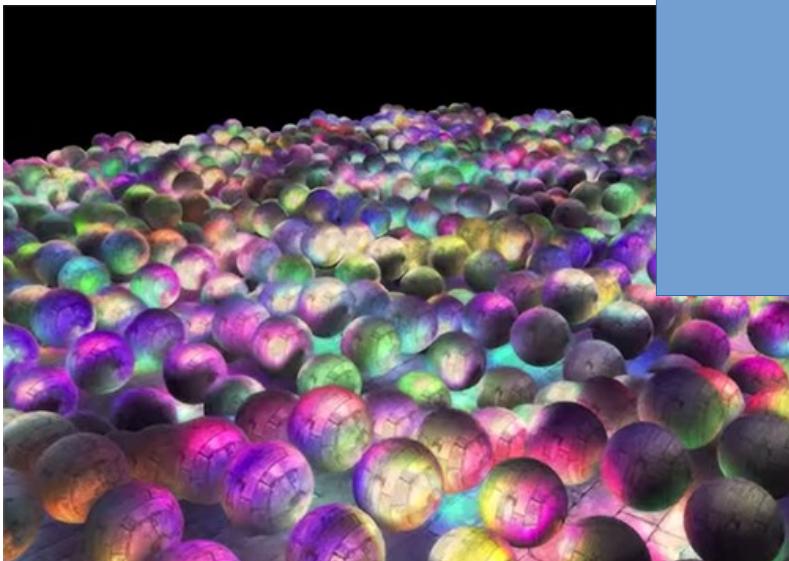
- Rendering pipeline discussed for now is called forward rendering
- Multiple rendering targets inspired different type of rendering pipeline called **deferred rendering (shading)**, a multi-pass pipeline:
  - First pass resolves **visibility** and stores data relevant for shading in buffers, e.g., object's location buffer, normals buffer, albedo buffer, material properties, z-buffer, etc. → **G (geometric) buffer**
  - Successive passes can efficiently use G buffer information for shading

**G-buffer** is full-screen image in which relevant data for shading is stored.

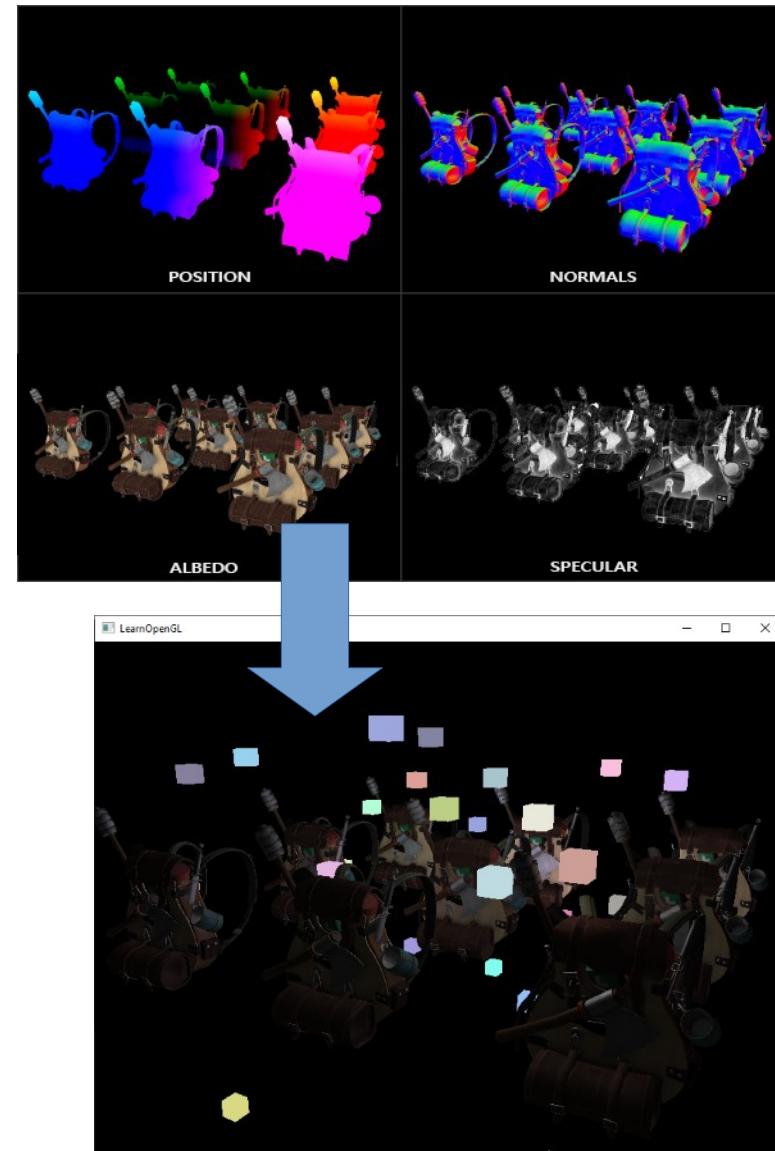


# Deferred shading

- Often, computation done in fragment shader is expensive.
- With deferred shading, shading is applied only actually visible fragments.
- This way, large number of lights with complex shading can be computed more efficiently.
- Problems:
  - Since only one surface is stored per pixel, basic deferred shading



RTR BOOK



# Displaying pipeline output

- Triangles that have reached and passed rasterizer stage are visible from camera point of view and their colors will be stored in color buffer.
- Display device (screen) will show color buffer after all operations are done.
- **Double buffering method:**
  - Rendering is performed in a **back buffer**.
  - Contents of back buffer are swapped with the contents of **front buffer** – buffer which is shown on display device.
  - Swapping occurs during **vertical trace** – a time when it is safe to do so.

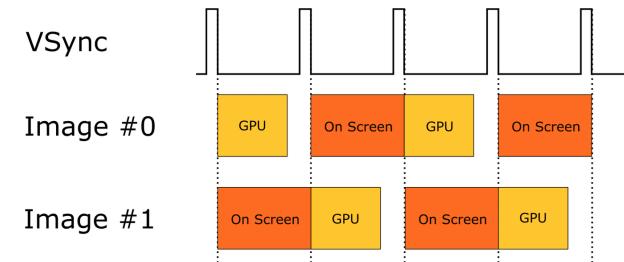
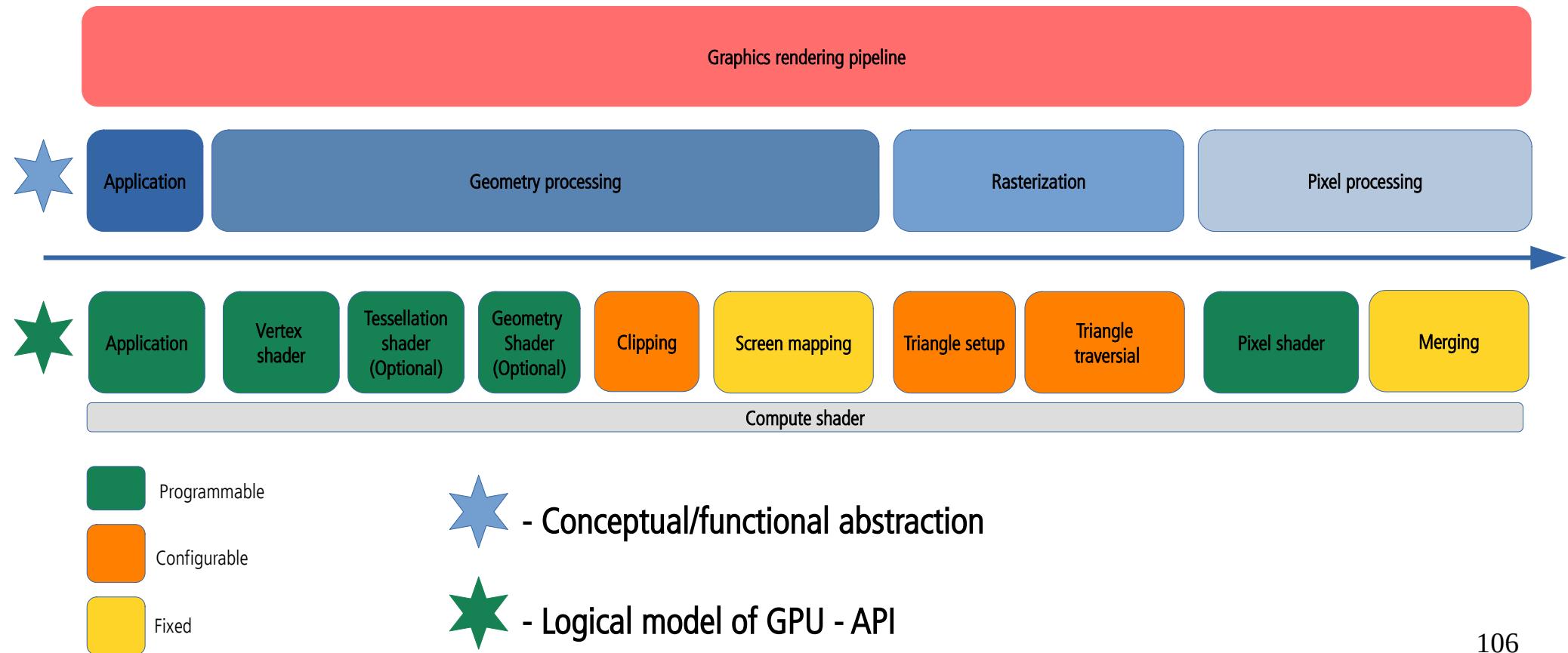


Figure 4: Correct double buffering behavior

# Logical GPU model

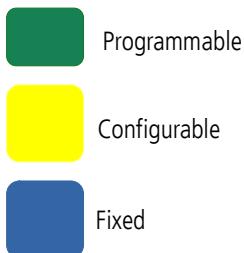
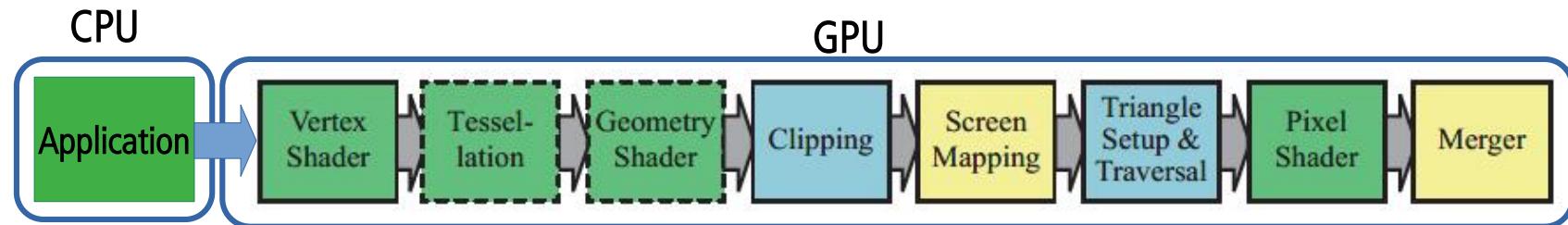
Programming API

# Graphics rendering pipeline: conceptual vs logical (API) model



# Logical GPU model: API

- Logical GPU rendering model is exposed to programmer as API
- Underlying hardware implementation of conceptual graphics pipeline varies by hardware vendor (e.g., Nvidia, AMD, etc.)



# Graphics stages

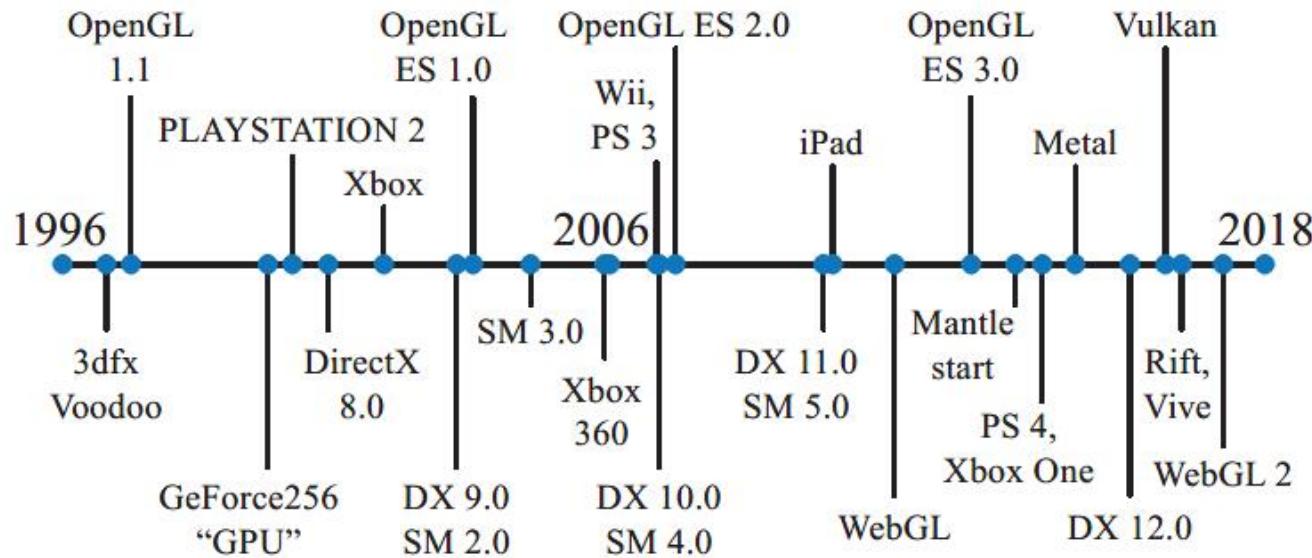
- Application stage
  - Specification of 3D scene: objects (geometry, material, transformations ,etc.), lights (color, transformations, etc.), cameras (transformations, focal length, field of view, etc.)
  - Specification of shader program (e.g., vertex and fragment shaders)
  - Configuration of graphics pipeline state (e.g., depth testing)
  - Draw (rendering) commands (i.e., sending information to GPU for rendering)
- Driver:
  - Translation of API instructions to hardware operations
  - Compilation of shader programs
- Graphics hardware (GPU)
  - Geometry processing, rasterization, pixel processing
  - Visibility resolving, clipping, shading, etc.



# Programming API

- Application stage (**CPU**) can be written in various languages: C, C++, Python, Rust, etc.
- **Graphics API for accessing GPU hardware:**
  - OpenGL (cross-platform), <https://www.khronos.org/>
    - WebGL (browser-based graphics)
    - OpenGL ES (embedded systems)
  - Vulkan (cross-platform), <https://www.khronos.org/>
  - DirectX (Windows), <https://devblogs.microsoft.com/directx/>
  - Metal (iOS), <https://developer.apple.com/metal/>
- **Focus: OpenGL**

# Evolution of graphics API and hardware



# GPU stages

## Application

- Definition of a 3D scene
- Triangle vertices and vertex attributes

## Geometry processing

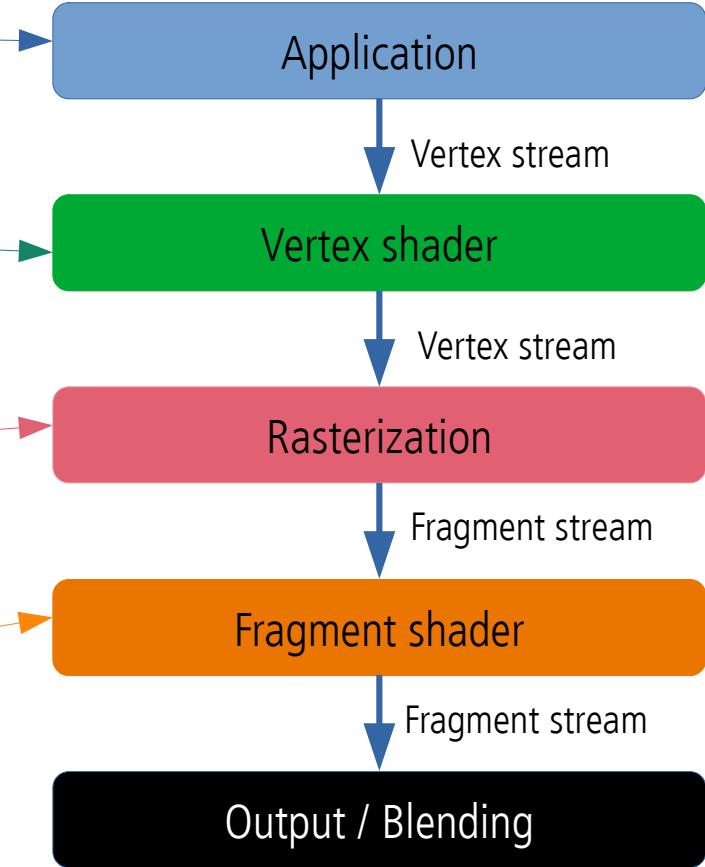
- Transformation and projection of triangulated mesh vertices
- Object → world → camera → screen → clip coordinates

## Rasterization

- Resolving visibility
- Generating fragment for each pixel covering triangle

## Pixel/Fragment processing

- Shading
- Color computation

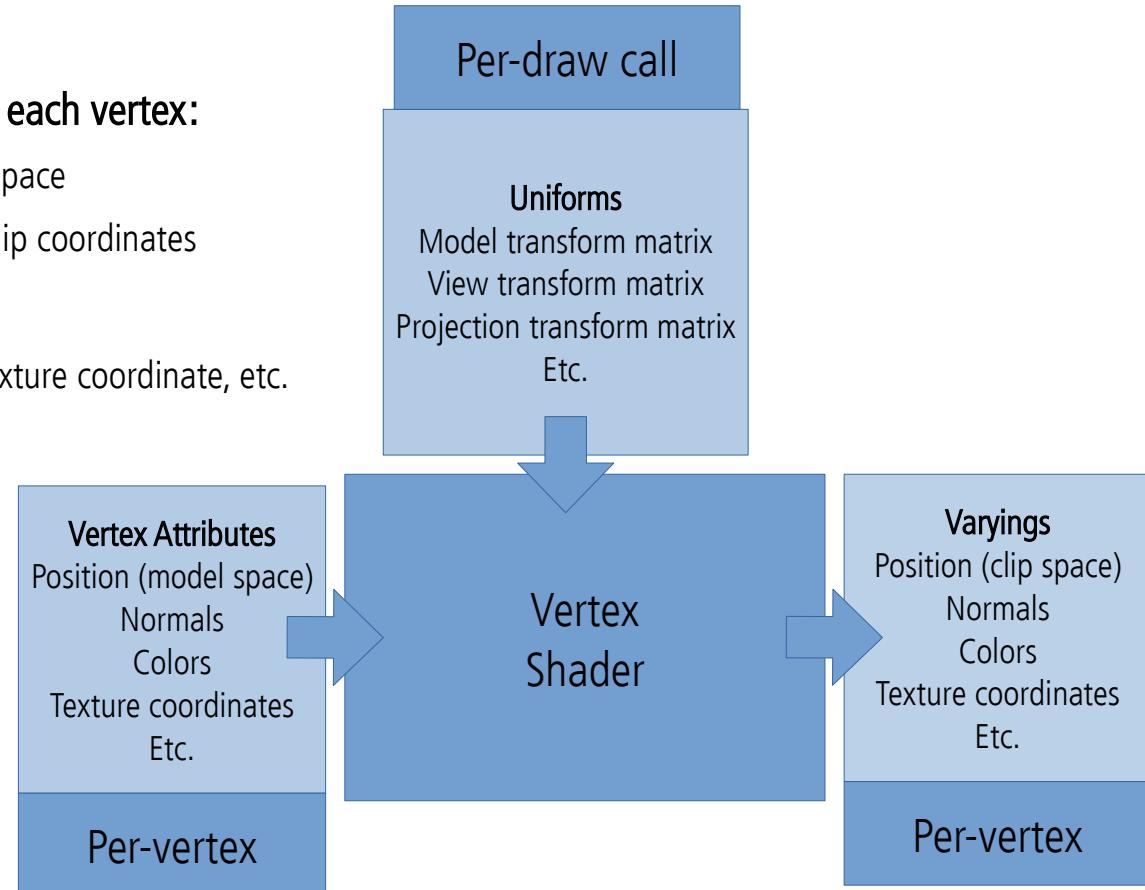


# Shader programming

- **Modern graphics** programming relies on programmable vertex and fragment shaders for rendering
  - **Unified shader design**: vertex, pixel, geometry, tessellation shaders share common programming model – same instruction set architecture (ISA)
  - GPU with cores which supports unified shader design: **unified shader architecture**
- **Shading languages:**
  - GLSL (OpenGL, Vulkan)
  - HLSL (DirectX)
- **Life cycle:**
  - Written and compiled on application stage
  - Linked at shader stages
  - Run when rendering objects

# Vertex shader

- Responsible for geometry processing, executes per each vertex:
  - Transformation of mesh vertices from model to world space
  - Transformation of mesh vertices from world space to clip coordinates
- Inputs are specified by application per vertex:
  - Vertex attributes: vertex coordinates, normals, color, texture coordinate, etc.
  - Constants – uniforms: e.g., transformation matrices
- Outputs: vertex variables – **varyings**:
  - Position in clip space (`gl_Position`)
  - Vertex variables (e.g., colors, normals, etc.)



# Vertex shader program

```
1 #version 330 core
2
3 // Input (vertex attributes).
4 layout (location = 0) in vec3 vertexPosition;
5 layout (location = 1) in vec3 vertexNormal;
6
7 // Output (varyings).
8 out vec3 normal;
9
10 // Uniforms (constant parameters).
11 uniform mat4 model;
12 uniform mat4 view;
13 uniform mat4 projection;
14
15 void main()
16 {
17     // Compute transformation of vertex coordinates
18     // from world to clip space.
19     gl_Position = projection * view * vec4(FragPos, 1.0);
20
21     normal = mat3(transpose(inverse(model))) * vertexNormal;
22 }
23 }
```

# Application stage: preparing uniforms for vertex shader program

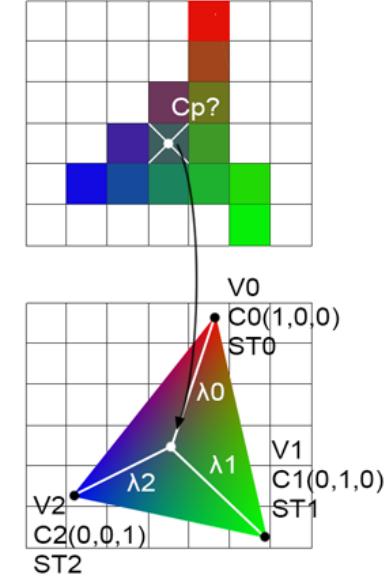
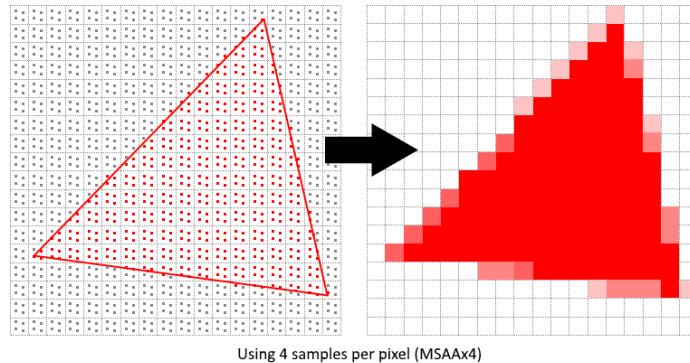
```
1 // Compile vertex shader program, link program.  
2 GLint vertexShaderProgram = buildShaderProgram( ... );  
3  
4 // Create transformation matrix.  
5 GLfloat* modelMatrix = createModelMatrix( ... );  
6 GLfloat* viewMatrix =createViewMatrix( ... );  
7 GLfloat* projectionMatrix = createProjectionMatrix( ... );  
8  
9 // Use shader program for draw calls.  
10 glUseProgram( vertexShaderProgram );  
11  
12 // Obtain index of uniform in shader program.  
13 GLint modelMatrixLocation = glGetUniformLocation( vertexShaderProgram, "model" );  
14 GLint viewMatrixLocation = glGetUniformLocation( vertexShaderProgram, "view" );  
15 GLint projectionMatrixLocation = glGetUniformLocation( vertexShaderProgram, "projection" );  
16  
17 // Set model matrix uniform.  
18 glUniformMatrix4fv( modelMatrixLocation, GL_FALSE, modelMatrix );  
19 glUniformMatrix4fv( viewMatrixLocation, GL_FALSE, viewMatrix );  
20 glUniformMatrix4fv( projectionMatrixLocation, GL_FALSE, projectionMatrix );  
21  
22
```

# Application stage: preparing geometry for vertex shader program

```
1 // 3 vertices of a triangle.  
2 float vertices[] =  
3 {  
4     0.0, 0.0, 0.0,  
5     1.0, 0.0, 0.0,  
6     0.0, 1.0, 0.0  
7 };  
8 // Create buffer in GPU memory.  
9 GLint vertexBuffer;  
10 glGenBuffers( 1, &vertexBuffer );  
11 // Bind vertexBuffer.  
12 glBindBuffer( GL_ARRAY_BUFFER, vertexBuffer );  
13 // Upload vertices to buffer.  
14 glBufferData (GL_ARRAY_BUFFER, vertices, GL_STATIC_DRAW );  
15 // Get vertex position in vertex shader.  
16 GLint vertexIndex = glGetAttribLocation ( vertexShaderProgram, "vertexPosition" );  
17 // Associate buffer with shader attribute and set interpretation.  
18 glVertexAttribPointer( vertexIndex, GL_FLOAT, 3, false, 0, 0 );  
19 // Enable vertex attribute.  
20 glEnableVertexAttribArray( vertexIndex );  
21  
22
```

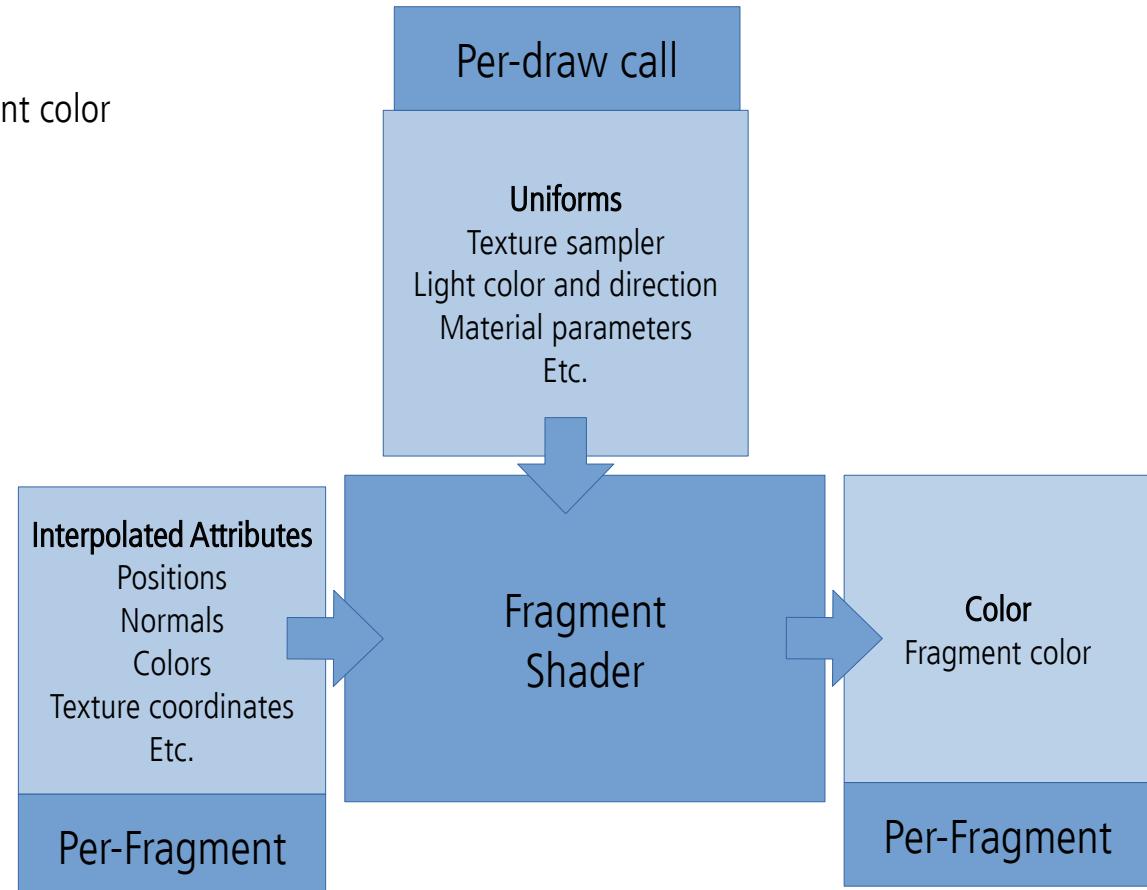
# Rasterization

- Given transformed and projected vertices (with associated vertex attributes data for shading) the goal is to **find all pixels which are inside the primitive** (e.g., triangle) to be used in pixel processing stage.
- For each pixel, multiple samples (points) are created and tested if are inside triangle → **fragments generation**
- Rasterization outputs fragments with:
  - Pixel coordinates
  - Interpolated varyings (vertex attributes) using barycentric interpolation
  - Interpolated depth value



# Fragment shader

- Responsible for shading, that is, computation of fragment color which is then combined into pixel color of color buffer
- Fragment shader input:
  - Interpolated vertex attributes from vertex shader
  - Texture samplers
  - Uniforms (constant parameters)
- Fragment shader output:
  - Fragment color



# Fragment shader program

```
1  #version 330 core
2
3
4  // Fragment shader output.
5  out vec4 fragColor;
6
7  // Input (interpolated from vertex shader output).
8  in vec3 normal;
9
10 // Uniforms, constant input.
11 uniform vec3 lightDirection;
12 uniform vec3 lightColor;
13 uniform vec3 objectColor;
14
15 void main()
16 {
17     vec3 nnormal = normalize( normal );
18     float diffuse = max( dot(nnormal, lightDirection), 0.0 );
19     FragColor = vec4( diffuse * objectColor * lightColor , 1.0 );
20 }
```

# Output and blending

- Resolving visibility
  - Color buffer should contain only color of triangles visible from camera point of view
  - Done using **z-buffer/depth-buffer**
- Blending new fragment color and existing pixel color in color buffer
- Alpha value testing
- Stencil testing

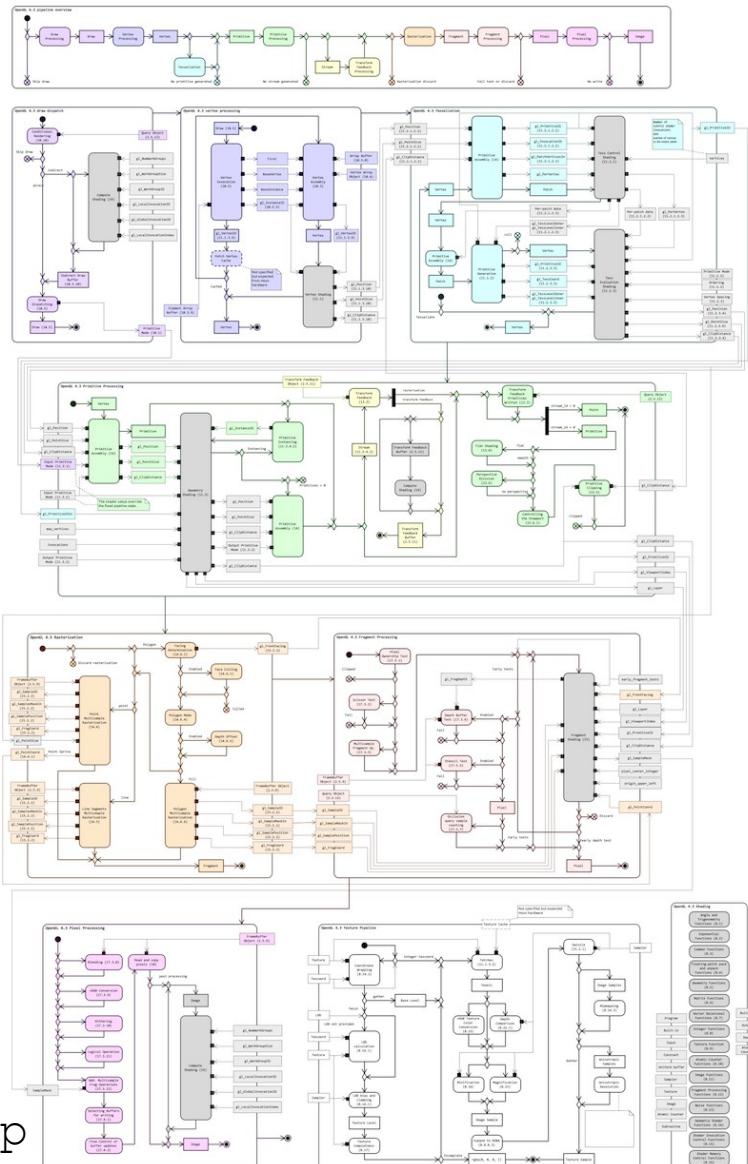
# Rendering effects

- Rasterization-based rendering requires additional work for achieving light phenomena effects: shadows, reflections, refractions, etc.
  - Achieved using combination of vertex and fragment shaders
- AO
- Shadow mapping
- Environment mapping
- Etc.

Supporting images

# More into topic

- Scratchapixel:
  - <https://www.scratchapixel.com/lessons/3d-basic-rendering/computing-pixel-coordinates-of-3d-point/perspective-projection.html>
  - <https://www.scratchapixel.com/lessons/3d-basic-rendering/rasterization-practical-implementation/overview-rasterization-algorithm.html>
  - <https://www.scratchapixel.com/lessons/3d-basic-rendering/perspective-and-orthographic-projection-matrix/projection-matrix-introduction.html>
- OpenGL:
  - <https://graphicscompendium.com/opengl/>
  - [https://www.khronos.org/opengl/wiki/Rendering\\_Pipeline\\_Overview](https://www.khronos.org/opengl/wiki/Rendering_Pipeline_Overview)
  - <https://learnopengl.com/Lighting/Basic-Lighting>
- Graphics pipeline in general:
  - <https://alaingalvan.gitbook.io/a-trip-through-the-graphics-pipeline/>
  - <https://www.g-truc.net/post-0580.html>
  - <https://fgiesen.wordpress.com/2011/07/09/a-trip-through-the-graphics-pipeline-2011-index/>
- Vulkan: <https://vulkan-tutorial.com/Introduction>
- More links: <https://www.realtimerendering.com/>



OpenGL 4.3. Pipeline map

# Summary questions

- [https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/13\\_rendering\\_rasterization](https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/13_rendering_rasterization)

# Repository

- <https://github.com/lorentzo/IntroductionToComputerGraphics>