

# Lecture 7: Materials

DHBW, Computer Graphics

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# Syllabus

- 3D scene
    - Object
      - Shape
      - **Material**
    - Camera
    - Light
  - Rendering
  - Image and display
- Material
    - Material observation
    - Physics (optics) models for CG
    - Material models in CG
      - Scattering models

# Big picture



# Materials



<https://www.exp-points.com/asking-the-masters-material-art>



SUBSTANCE  
DESIGNER

Material is important for object appearance.  
Without material...



# Shape and material

- Rendering and creating objects in 3D scene to represent a real world objects requires:
  - Shape representation
- **Shape** is needed for:
  - **Modeling**:
    - Define object form, size, etc.
    - Place object correctly in the scene with respect to other objects
  - **Rendering**:
    - Determine which objects are occluded and areas into which shadow is cast by object → visibility solving



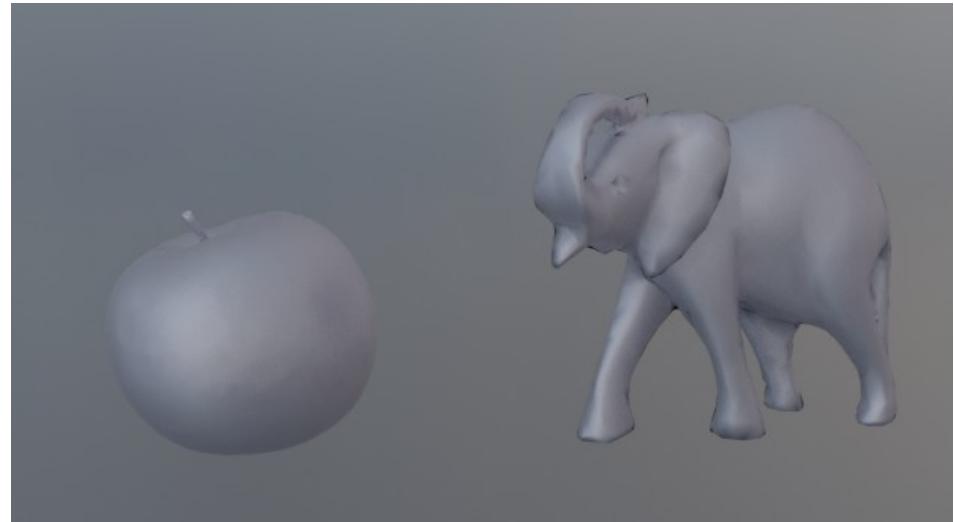
# Shape and material

- Rendering and creating objects in 3D scene to represent a real world objects requires:
  - Shape modeling
  - Visual appearance modeling → material
- **Material** is needed for:
  - Modeling: how object will look like
  - Rendering: light-object interaction (shading)



# Shape and material

- Material characteristics are independent of shape and position
  - Material is modeled separately
- Enough to model how specific material generally interacts with light
  - This model is then used for arbitrary shape



Aluminum apple and aluminum statue – in both cases aluminum properties are the same. Also, changing position of aluminum apple in space doesn't change aluminum properties.

"Let the form of an object be what it may, - light, shade and perspective will always make it beautiful" – John Constable

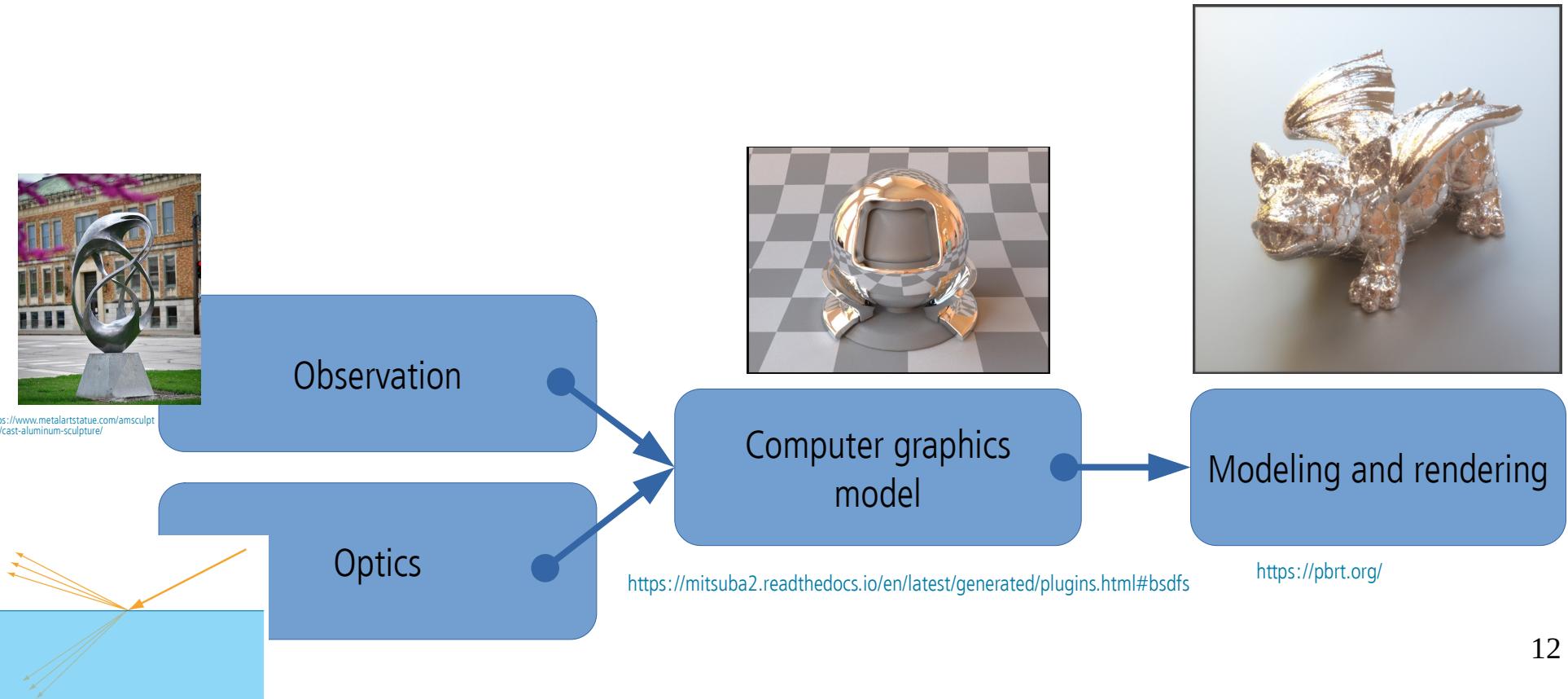
- Importance of material for appearance



- Metal
- Plastic
- Glass
- Wood
- Fabric
- Stone
- Clouds
- Water
- Tree bark
- Leaf
- Plaster
- Paper
- Leather
- Sky
- Etc.

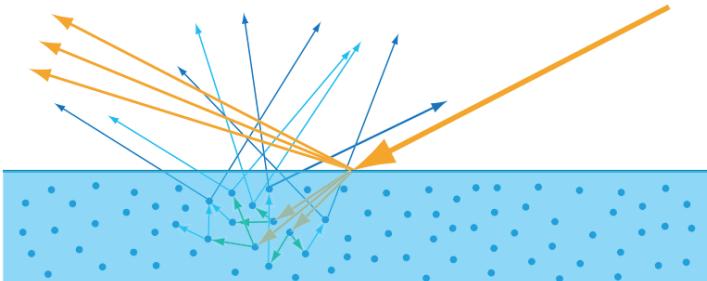


# Appearance modeling and rendering

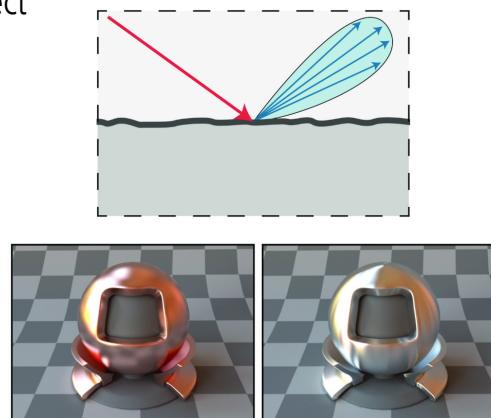


# Materials: real world vs models

- Real world materials are very complex are simplified using different **models**:
  - **Physical models**: best description of real world materials
    - Example: geometric optics
  - **Computer graphics models**: simplification for creation and computational purposes
    - Example: separating objects into shape and material
  - **Subjective, artist observation**: based on perception – subjective model of the world
    - Artist draw what they see, not what is physically-correct



Physical model of light scattering



Computer graphics material model



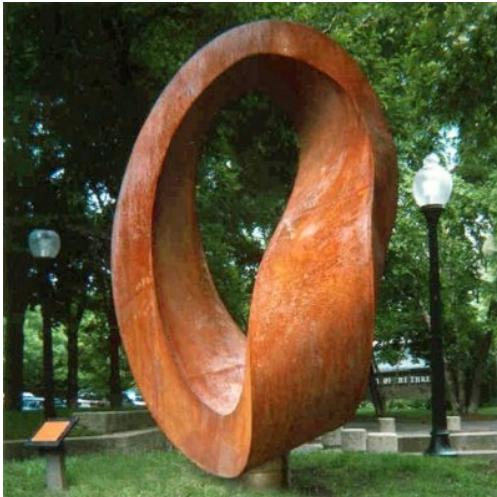
Artist model:  
<https://shelleyhannafineart.com/painting-silver-objects/>

# Material observation

Modeling begins with **observation**

# Observation

- Observation goals:
  - Understand **what makes each material look different** than other materials
  - Observe **characteristics which are responsible for object appearance**



# Observation

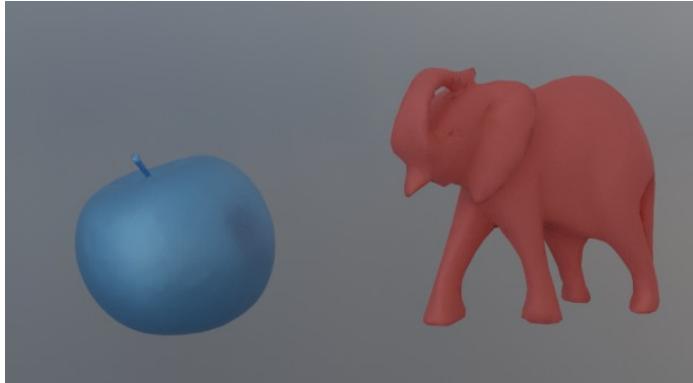
- Characteristics responsible for object appearance:
  - **Shape**: large scale form or geometry of object.
  - **Material**: for computer graphics modeling purposes: fine-scale geometrical variations and substance properties
  - **Illumination**: size, direction, color, etc.
  - **Sensor/Perception**: point of view, camera properties, etc.



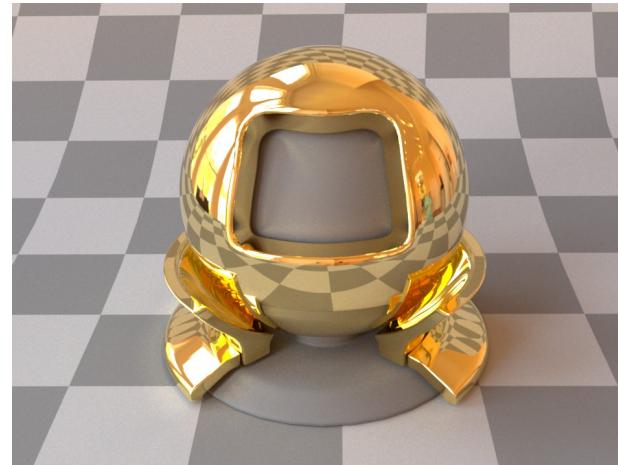
# Classifying materials

- Classifying materials enables us to **understand which characteristics are needed to be modeled** in order to obtain required appearance. We can classify any material by following variations:

Spectral: color



Directional

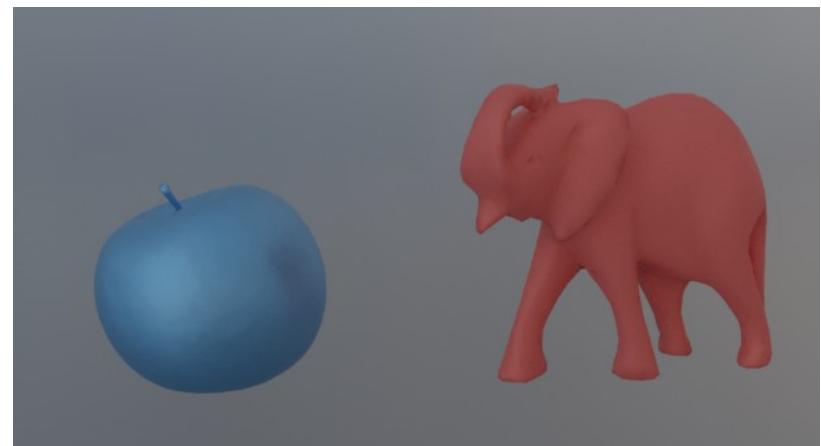
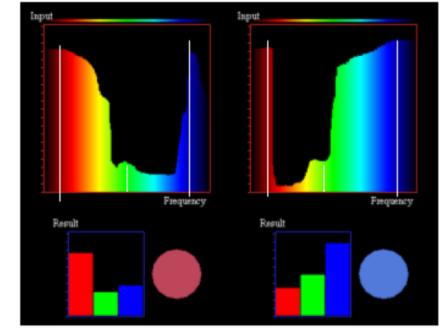
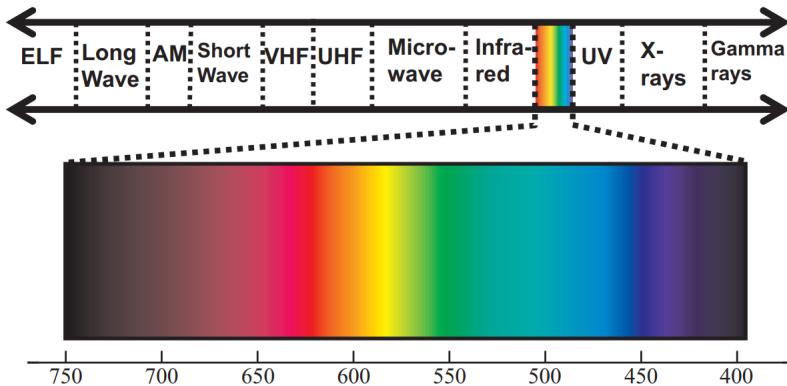


Texture



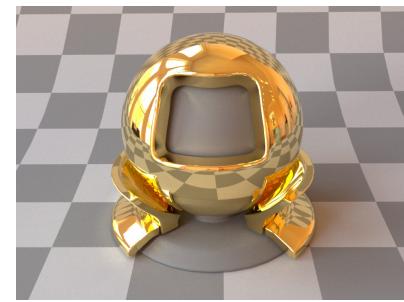
# Spectral/color material characteristic

- Light is a electromagnetic radiation in the spectrum of visible wavelength band 380-780nm
- Light scattered from object surface (its material) is described using:
  - Color is described with red, green and blue floating points (R, G, B) in [0,1]
  - Brightness is floating point value [0, inf]



# Directional material characteristic

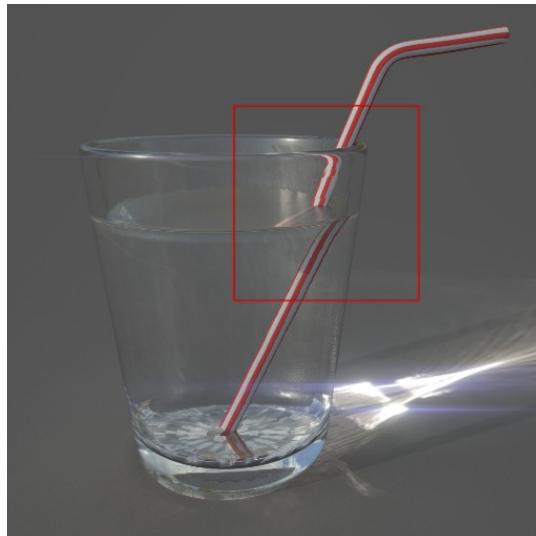
- Direction effects result from **directionality of light scattering** by the object
  - Changes in object appearance as we change view (or light or object position)
- Directional effects are “attached” to object (do not depend on environment)



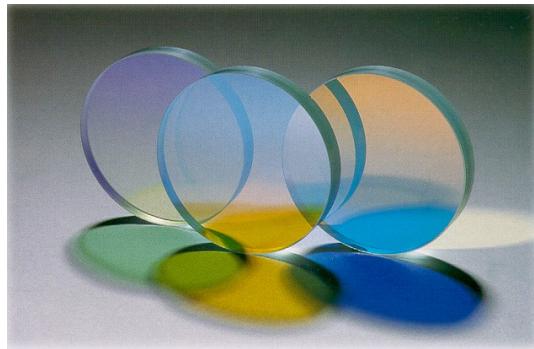
Shiny/glossy



Matte/diffuse



Transparent



Translucent

<https://substance3d.adobe.com/tutorials/courses/the-pbr-guide-part-1>

<https://polyhaven.com/>

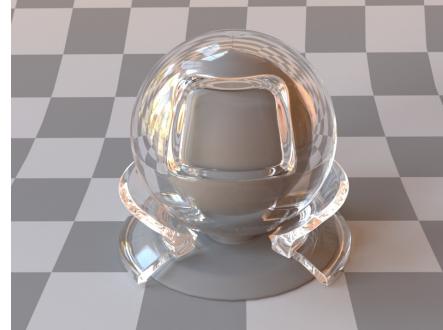
[https://en.wikipedia.org/wiki/Transparency\\_and\\_translucency](https://en.wikipedia.org/wiki/Transparency_and_translucency)

# Directional material characteristic

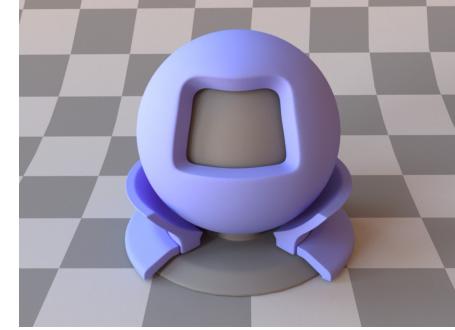
- In computer graphics, higher level description of scattering is used to describe directional – **surface scattering** - characteristics:
  - Reflective
  - Transmissive
  - Specular (mirror)
  - Glossy
  - Lambertian (diffuse)
  - Refractive
  - Retroreflective



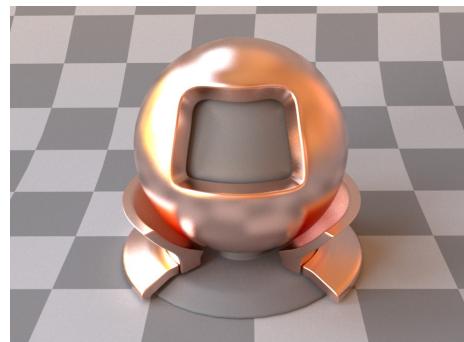
Reflective, specular (conductor, gold)



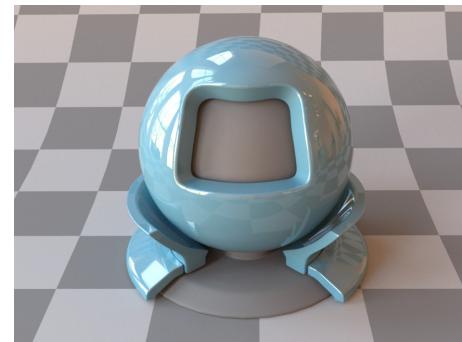
Transmissive, specular (dielectric, glass)



Reflective, Lambertian/diffuse  
(dielectric, plastic)



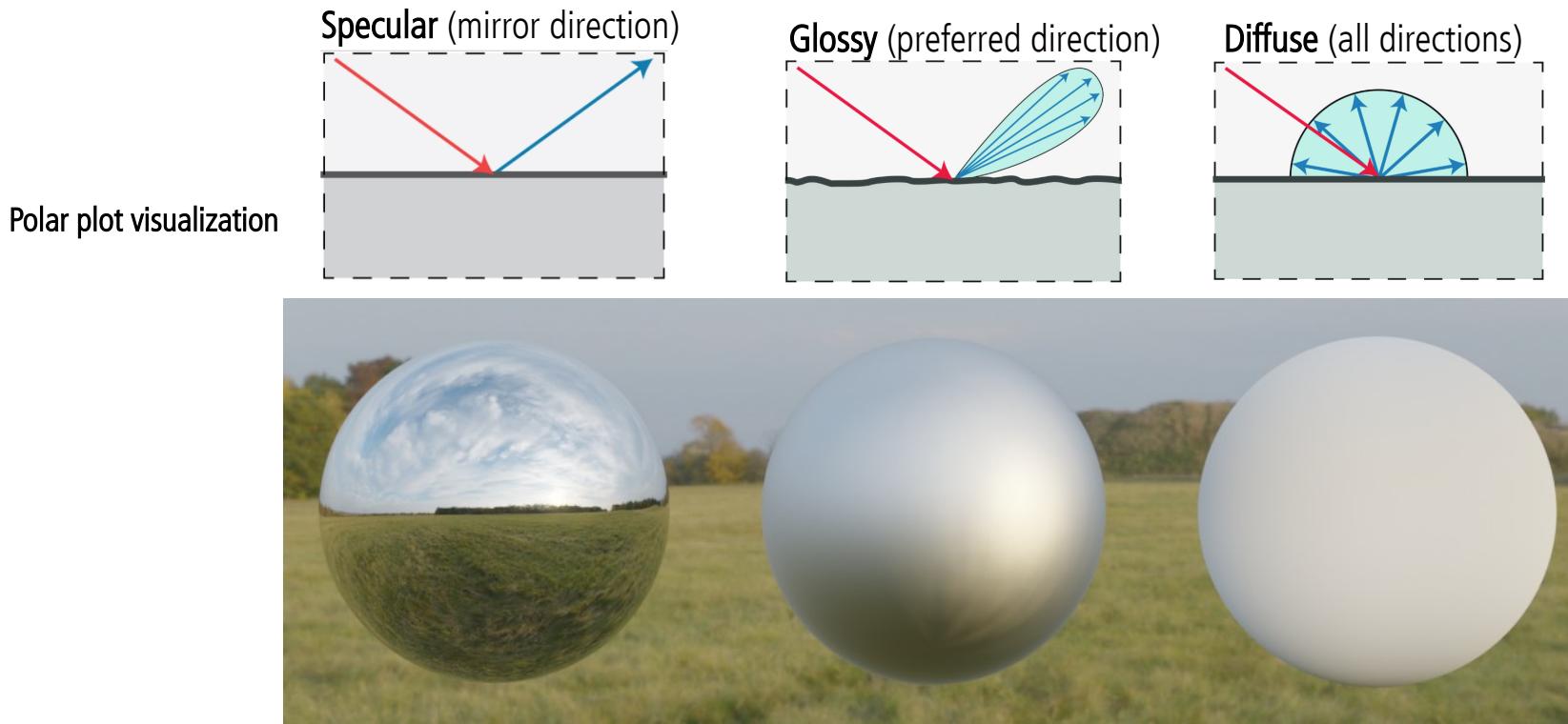
Reflective, glossy



Refractive, glossy (dielectric shiny plastic)

# Surface scattering: scattering function

- Surface directional effects (and color) are modeled using **scattering function**. Most important light scattering models are:



# Material characteristics: texture

- Visual variations on the object surface, much smaller scale than size of the object but larger than the wavelength of light - **spatial variations**:
  - Directional or spectral (color)
  - Small scale geometric variations: bumps and pores
- Surfaces with spatial variation are observed as non-uniform: **texture or pattern** can be observed



Color variation



Directional variation

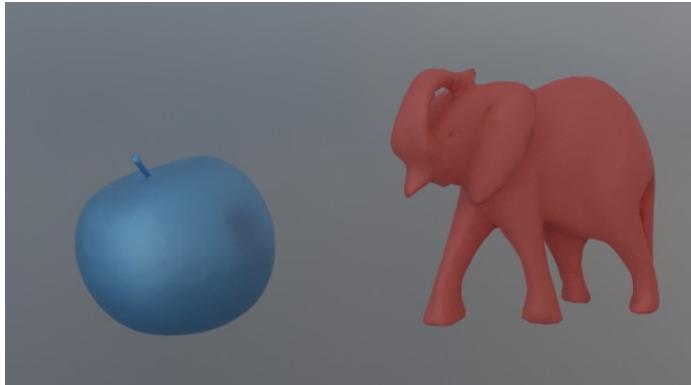


Small scale geometrical variation

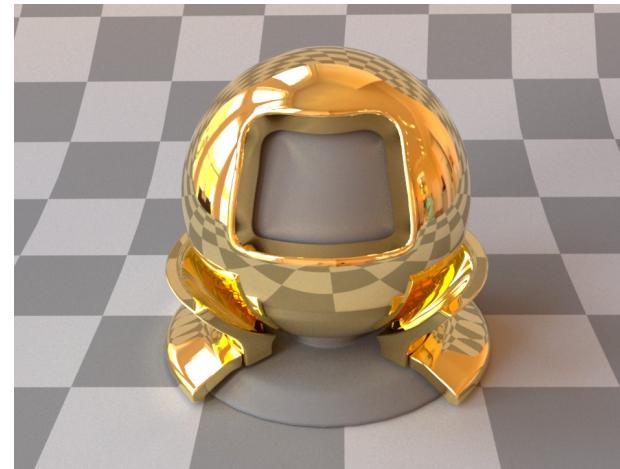
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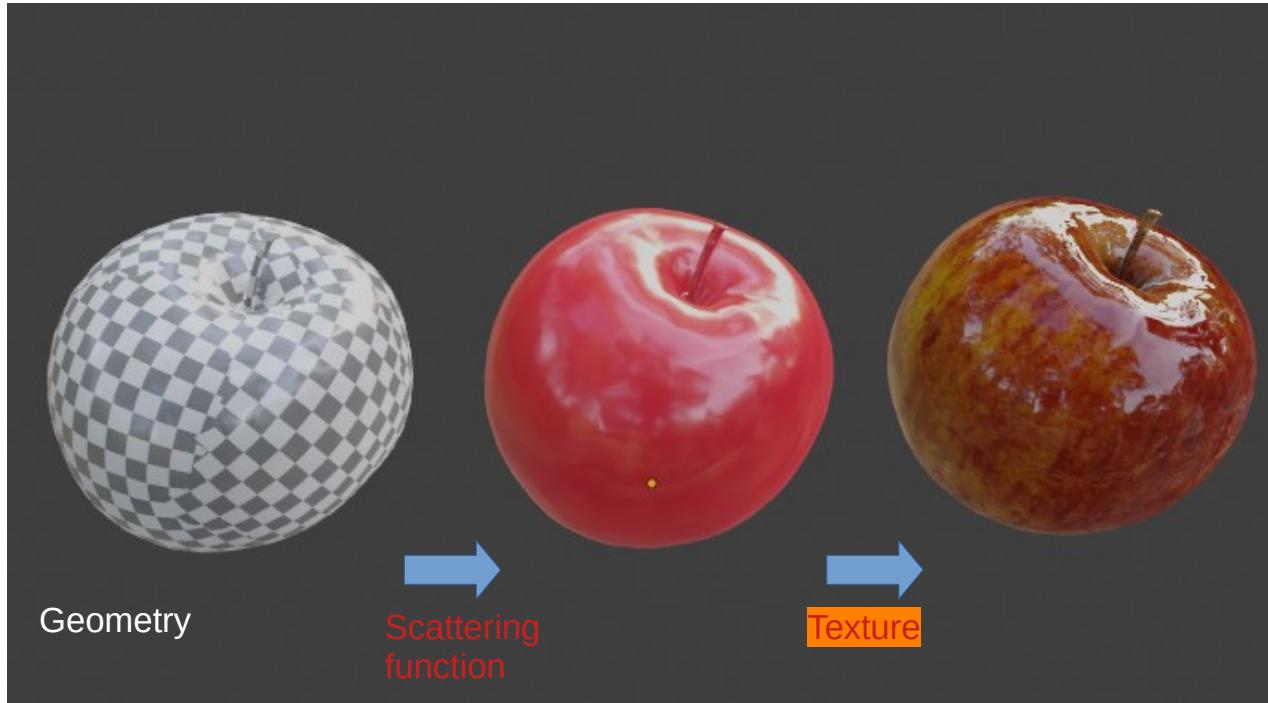
Directional



Texture



# Materials in computer graphics



**Scattering function**

- Directional effects
- Color

**Texture**

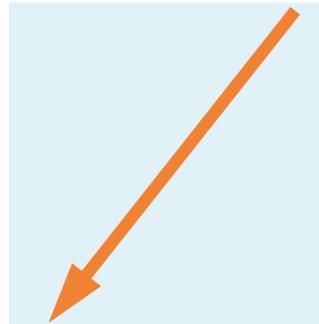
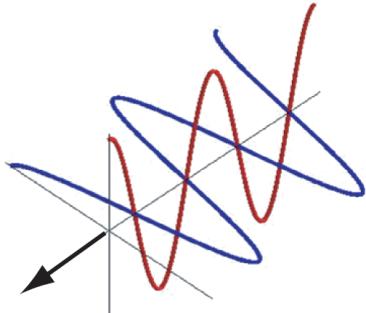
- Surface patterns

# Physical models for computer graphics

Modeling is founded on **optics**

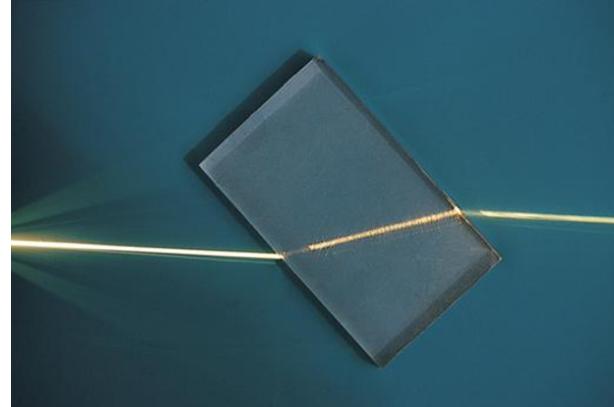
# Light-matter interaction

- Light is electromagnetic transverse wave → too complex for CG!
- Geometrical optics
  - Approximates light as rays
  - Describes light-matter interaction via **index of refraction** (IOR) – a complex number:
    - Real part determines speed of light → **scattering**
    - Imaginary part determines **absorption** of light



Light can be represented as ray.

[https://ssteinberg.xyz/2022/04/03/practical\\_plt/](https://ssteinberg.xyz/2022/04/03/practical_plt/)



Light scattering due to difference in IOR.

# Light-matter interaction

- Simplest light-matter interaction is light propagating through **homogeneous medium**.
  - **Uniform IOR**: light travels on a straight line.
  - It can only be **absorbed**: direction is same, intensity might be attenuated



Transparent media (water): straight line with same intensity (scattering only happens when light crosses air-glass-water homogeneous media since they have different (but constant) IORs.)



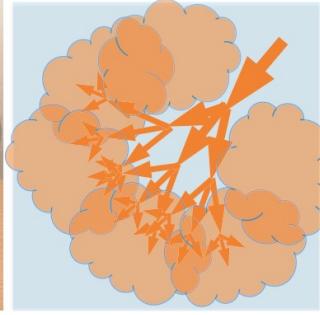
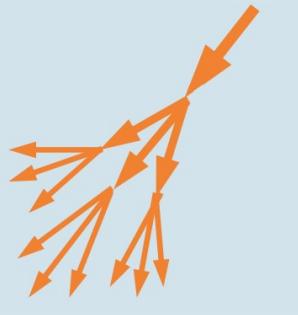
Clear absorbent media (tea): straight line with loss of intensity – selective absorption (color is changed)



Absorption becomes significant with distance.

# Light-matter interaction

- In **Heterogeneous medium** IOR varies which causes **light scattering**
  - Direction of light is changed but not the intensity
  - Light can scatter in all directions, mostly non-uniformly: forward or back scattering (in or reverse of incoming direction)



Microscopic particles cause varying of IOR and light to scatter continuously in all possible directions.

**Translucent or opaque materials** → light is scattered so much that we can not see (clearly) through the object



Longer distances cause more scattering (e.g., clean air)

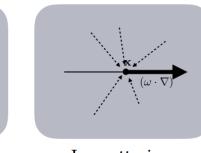
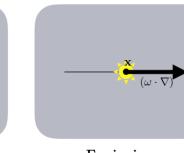
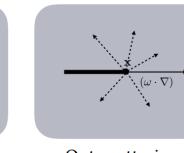
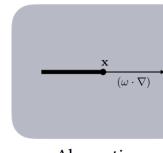
# Light-matter interaction: scattering and absorption

- Light traveling through medium, based on **index of refraction** will:
  - Absorb
  - Scatter
- Appearance depends on both scattering and absorption



# Participating media\*

- Participating media
  - Medium between objects
  - Simplest: vacuum
- Light traveling inside medium in CG is called **volumetric scattering**



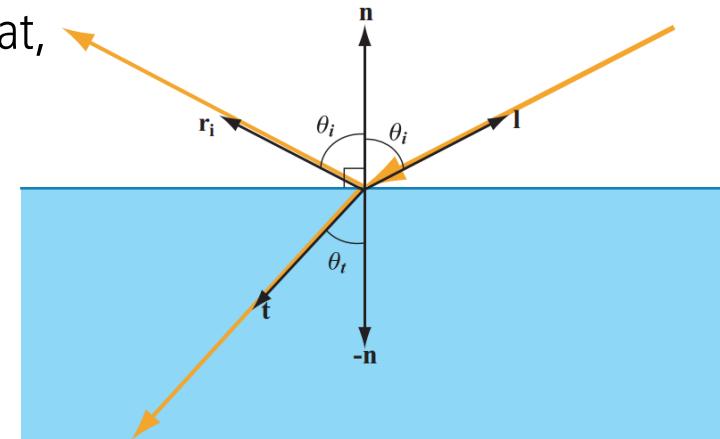
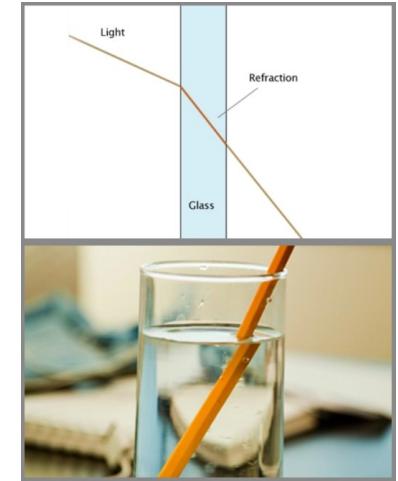
<https://cs.dartmouth.edu/wjarosz/publications/jarosz08radiance.html>

<https://studios.disneyresearch.com/2012/08/05/virtual-ray-lights-for-rendering-scenes-with-participating-media/>

<https://graphics.pixar.com/library/ProductionVolumeRendering/paper.pdf>

# Light-surface interaction

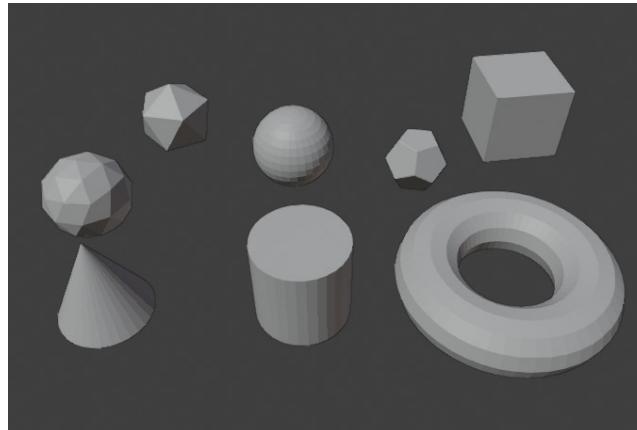
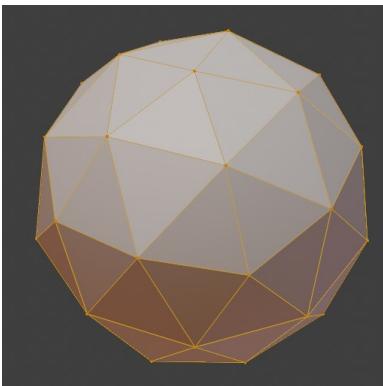
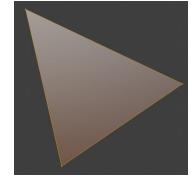
- Light **traveling between media of different index or refraction** - Maxwell's equations – too computationally heavy!
- Simplification and assumptions:
  - Interface between (volumes) media is perfectly (optically) flat, planar boundary\* → **object surface**
  - **Geometrical optics**: rays light representation, IOR
- Solution: **Fresnel equations and Snell's law**



\* Surface should be infinitely large, but in comparison with wavelength of light, surface real objects can be considered as such.

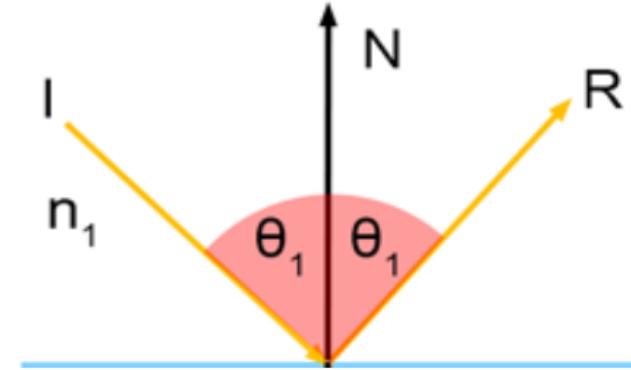
# Surface

- Surface: 2D interface separating volumes with different IOR (thin interface between media)
- In CG it is described with **shape representations**, e.g., polygonal mesh
- Behavior of light falling on surface depends on:
  - **Geometry** → surface orientation (normal)
  - **Substance (material)** → IOR



# Light-surface interaction: reflection

- Light falling on planar surface can **reflect**
  - **Direction:** law of reflection – angle of reflection is equal to angle of incidence.

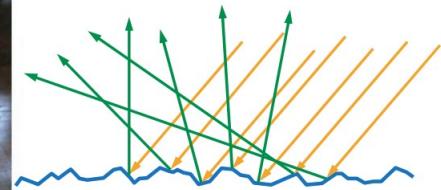
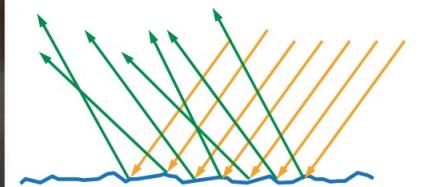


This is property of opaque objects:

- Metals
- Dielectrics (specular reflection)

# More physically correct surface reflection

- Real surfaces: small geometrical irregularities not visible by eye affecting reflection
- Surface model: large collection of tiny optically flat surfaces
  - **microfacets**. Final appearance is aggregate result of relevant facets.
    - Smaller deviation of those facets → **mirror-like surface reflection**
    - Larger deviation of those faces → **glossy surface reflection**



Similar shape, different reflection.

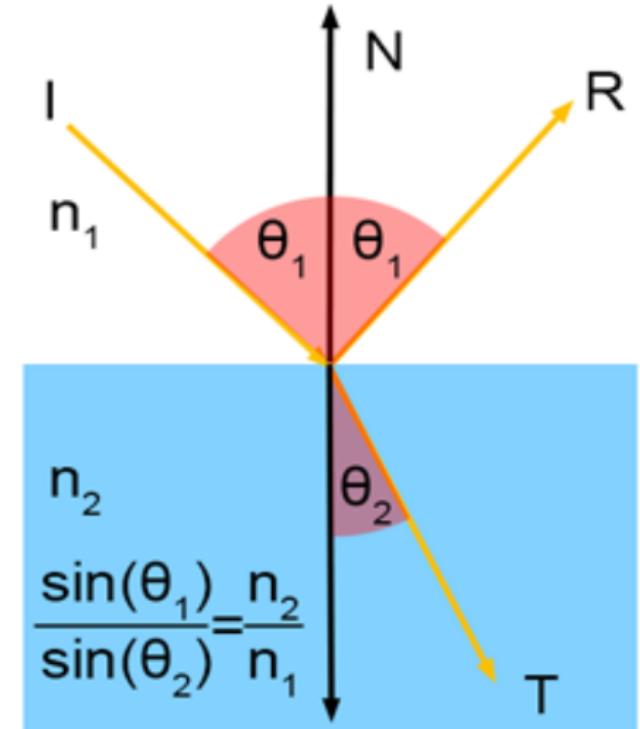
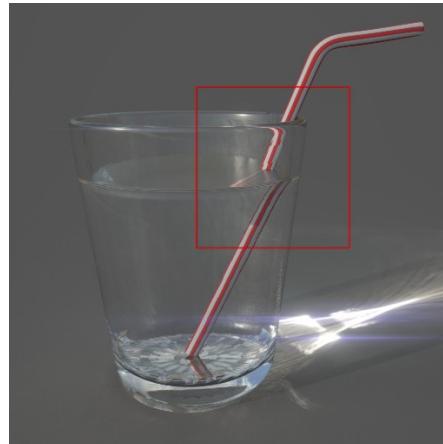
# Light-matter interaction: flat surface

- Light falling on planar surface can **refract**:

- Direction of refraction: Snell's law

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}.$$

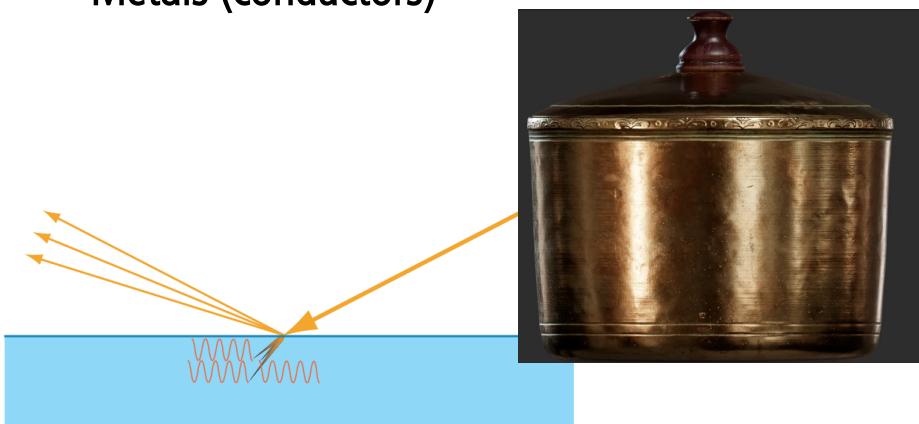
- Characteristic of dielectric objects:
  - Example: glass



# Surface refraction

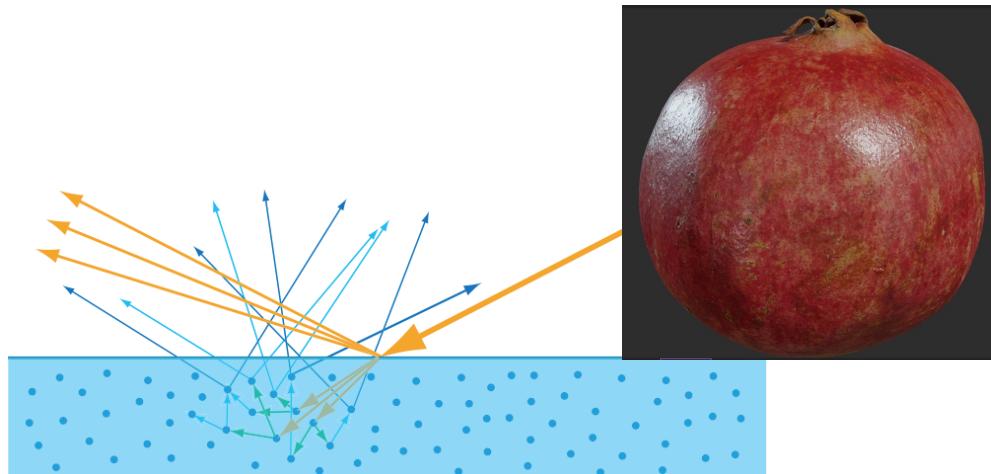
- Amount and direction of refracted light depends on material which we can separate in:

Metals (conductors)



In case of **metals**, most of the light is reflected and rest is immediately absorbed. That is why mirrors are made using metal foundation. Conductors are spectrally selective and thus reflection color may vary

Dielectrics (non-metals)



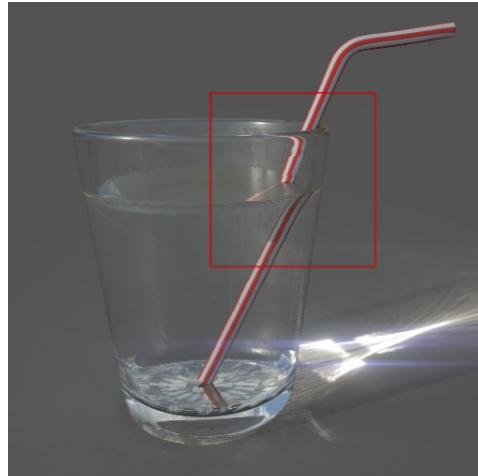
In case of **dielectrics**, light partially reflects and partially refracts. Refracted light is then absorbed and scattered inside surface (**sub-surface scattering (SSS)**) causing **diffuse** reflection.

# Surface refraction

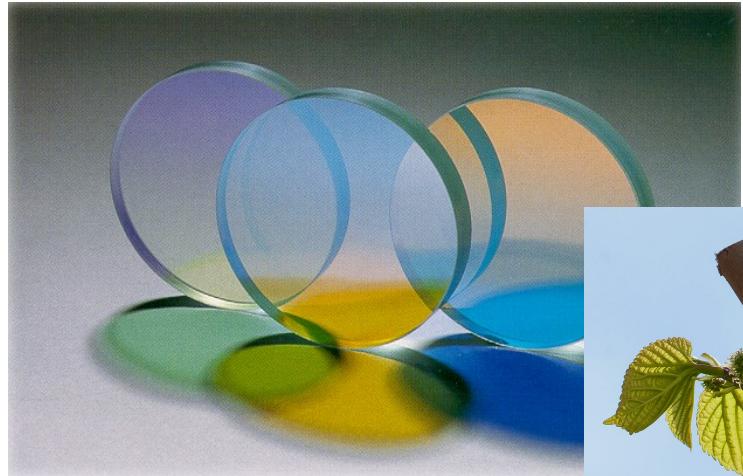
- Refracted light on **dielectric surface** can:
  - Sub-surface scatter, absorb and re-emit: **opaque** objects
  - Transmit; pass through object: **transparent** objects
  - Sub-surface scatter, absorb, re-emit and transmit: **translucent** objects



Opaque



Transparent



Translucent

<https://www.hippopx.com/en/query?q=translucent>



# Light-matter interaction: flat surface

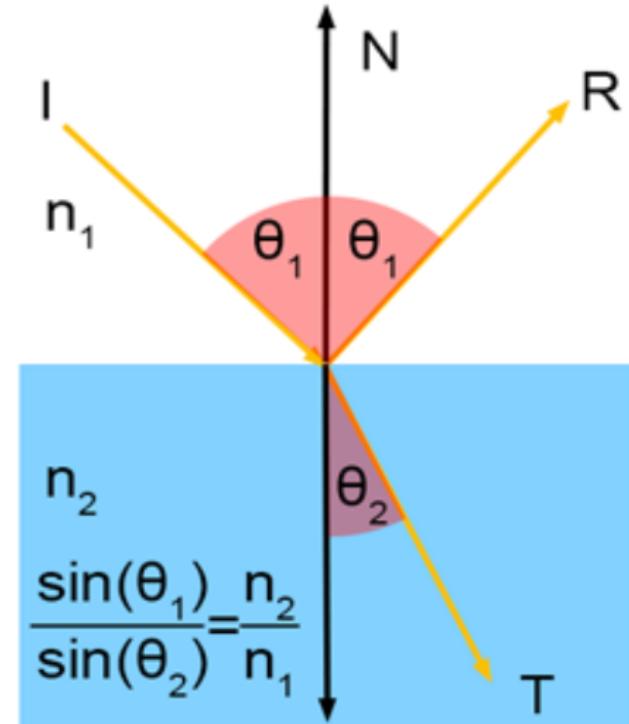
- Amount of light reflected/refracted (transmitted) is described with **Fresnel's equations**

$$F_{R\parallel} = \left( \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} \right)^2,$$

$$F_{R\perp} = \left( \frac{\eta_1 \cos \theta_2 - \eta_2 \cos \theta_1}{\eta_1 \cos \theta_2 + \eta_2 \cos \theta_1} \right)^2.$$

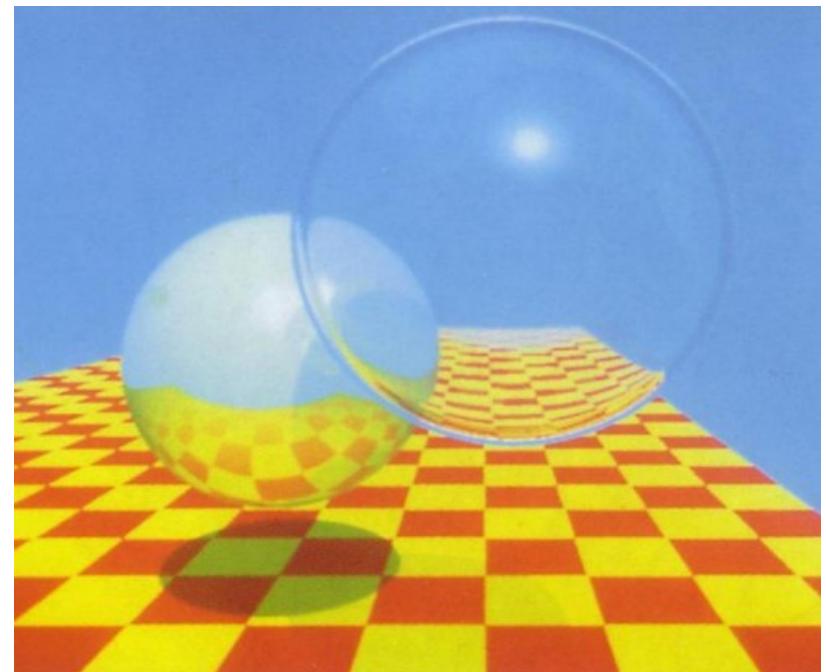
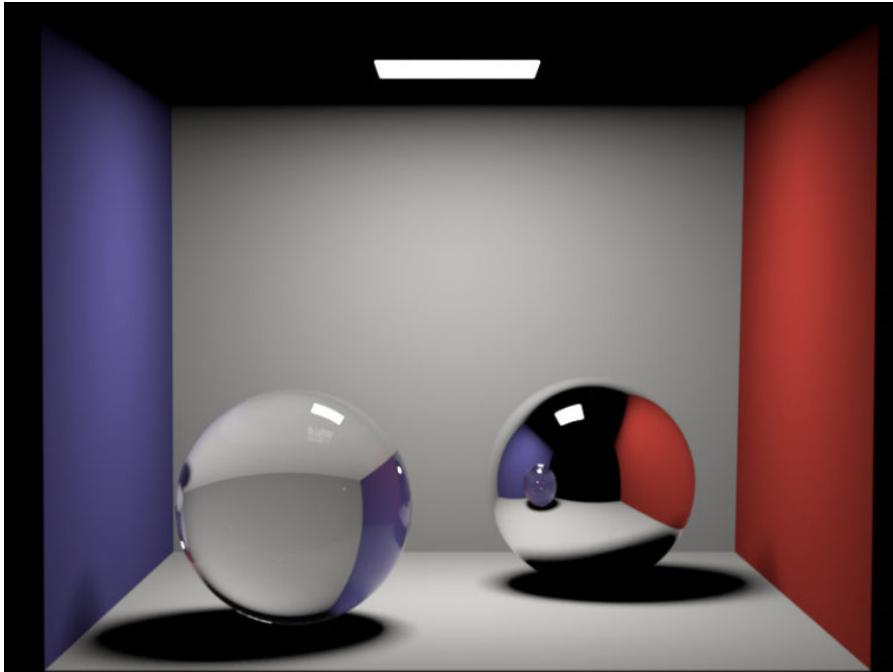
Amount of reflected light  $\rightarrow F_R = \frac{1}{2}(F_{R\parallel} + F_{R\perp}).$

Amount of refracted light  $\rightarrow F_T = 1 - F_R.$



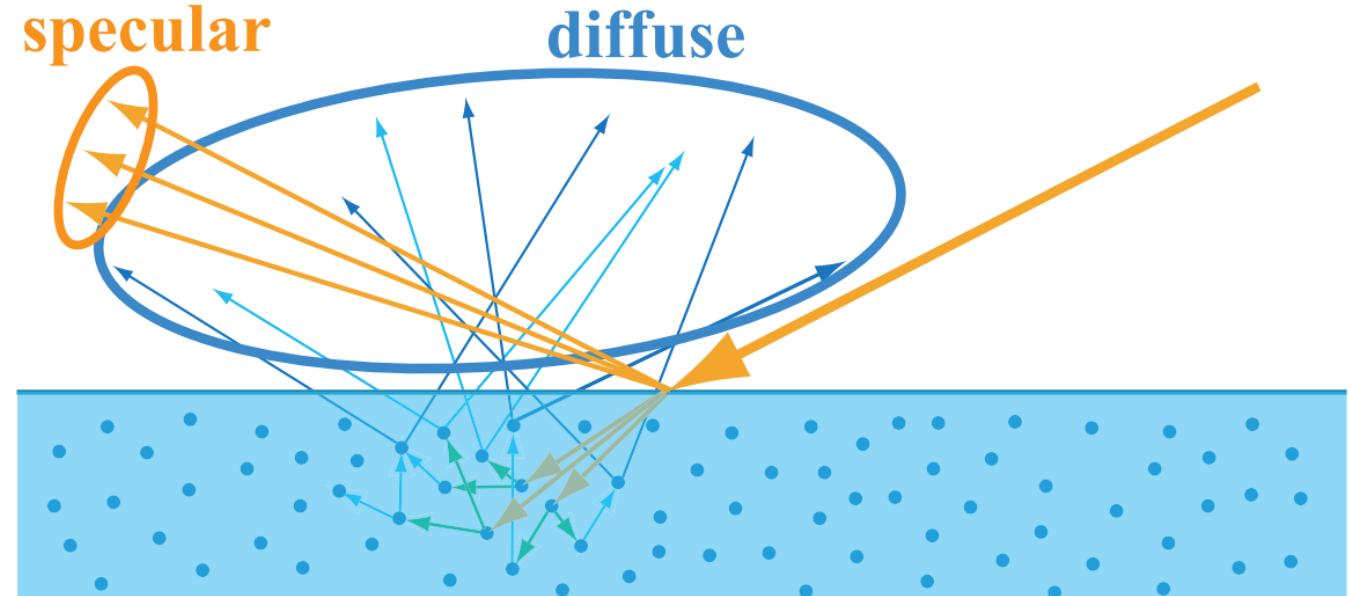
# Light-matter interaction: flat surface

- Surface reflection and refraction is crucial for realistic synthesis
  - Whitted ray-tracing for realistic image synthesis



# Diffuse and specular reflection

- Two fundamental **light-surface** scattering processes:
  - **Diffuse**: incoming light is distributed in all directions
  - **Specular/glossy**: incoming light is reflected into single or preferred direction

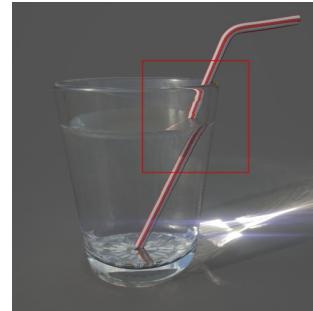


# Light-matter interaction: summary

- Light traveling between objects can be scattered/absorbed
  - **Volumetric scattering** in participating media
- Light falling on object surface – **surface scattering**:
  - **Reflect** (metals and dielectrics)
  - **Refract** (only dielectrics)
- Depending on dielectric material, refracted light can:
  - **Transmit** (transparent, translucent surfaces)
  - **Sub-surface scatter and absorb** (opaque and translucent object volume) → **volumetric scattering**



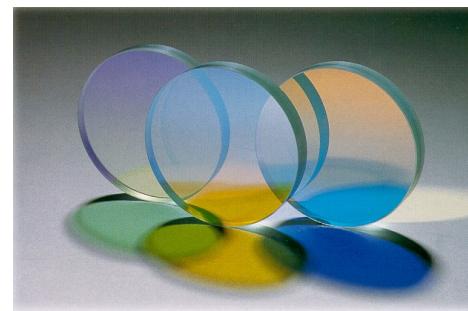
Opaque dielectric: reflect + refract → SSS/diffuse



Transparent dielectric: reflect + refract → transmit



Participating media



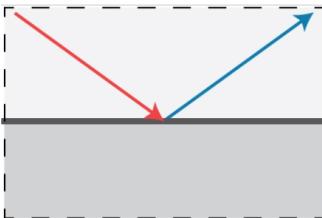
Translucent dielectric: reflect + refract → transmit



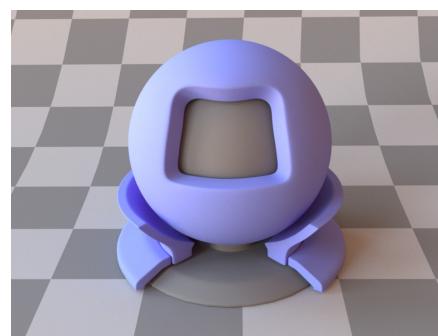
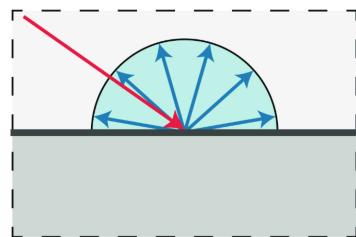
Metal: reflect

# Material modeling

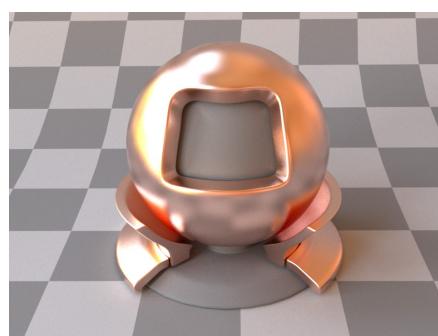
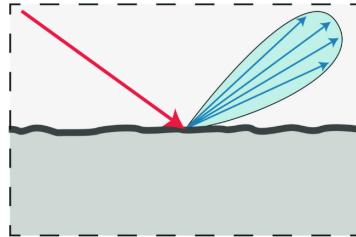
- Four scattering models for this lecture: local surface models



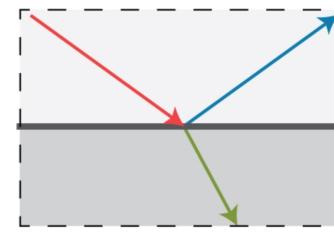
Perfect specular reflection.  
Light is reflected into only one direction.



Diffuse reflection.  
Light is reflected into all directions equally.

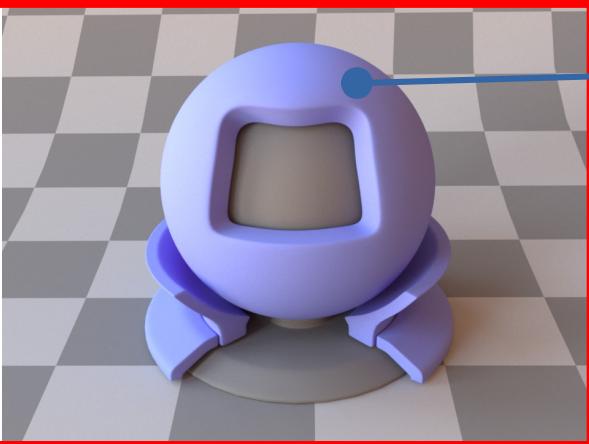


Glossy reflection.  
Light is reflected into preferred direction.



Perfect specular reflection and transmission.

# Modeling material characteristics



**Scattering function** describes small scale light-object interactions:

- Color: light absorbtion
- Directional effects: how light scatters

Uniform parameters of scattering function → smooth surfaces.

**Texture** describes surface variations visible by eye:

- Varying scattering function parameters (e.g., color)
- Varying surface details (e.g., normals)

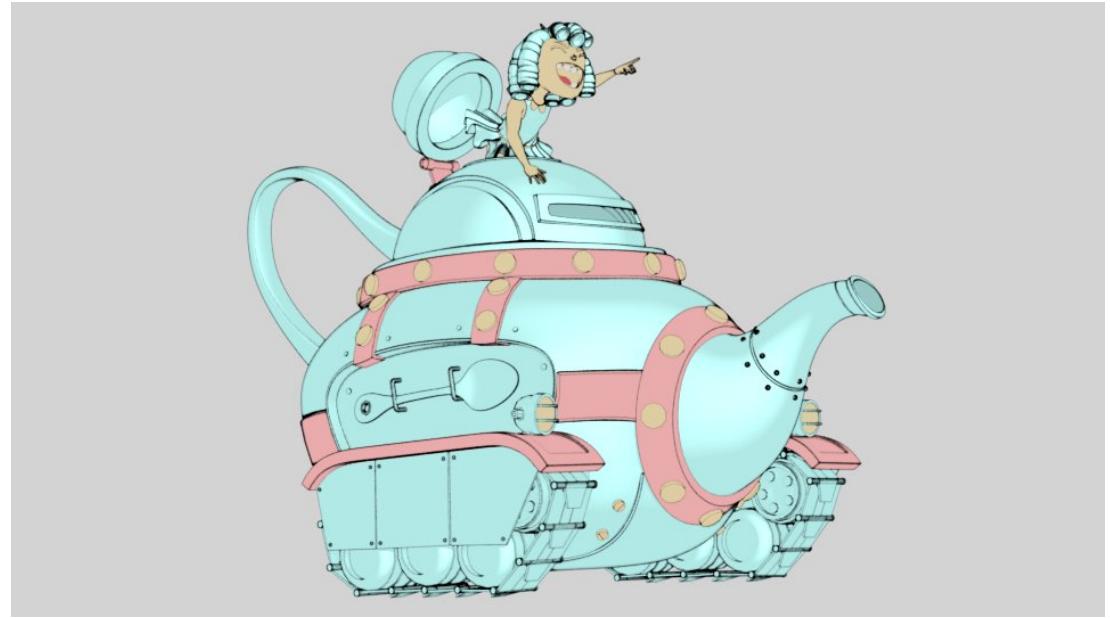
# Modeling appearance

# Diversity of appearance models

- Appearance models, depending on application, can range from **photo-real** to **stylized**.



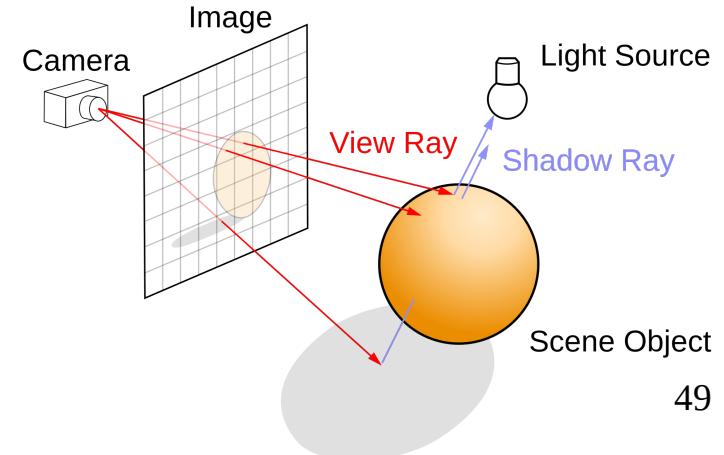
<https://www.artstation.com/artwork/rANRe5>



"Rolling Teapot" - Model by Brice Laville, concept by Tom Robinson, render by Esteban Tovagliari - RenderMan 'Rolling Teapot' Art Challenge:  
<https://appleseedhq.net/gallery.html#https%3A%2F%2Fappleseedhq.net%2Fimg%2Frenders%2Frolling-teapot.jpg>

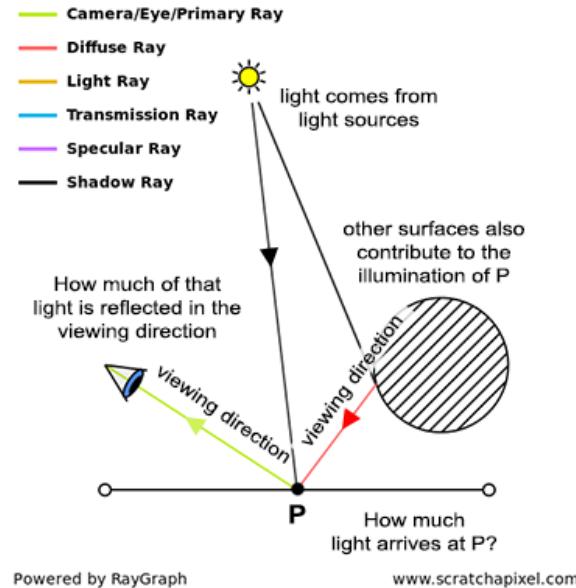
# Material and rendering

- Rendering: computing **intensity** and **color** of pixels of virtual image plane in virtual camera
- For each pixel of virtual image plane, **camera (viewing) ray** is generated and traced into scene
  - This simulates light entering the camera along set rays
- **View rays are tested for intersection** with objects in the scene
  - Closest intersected object surface is found → **shading point**
  - Ignore participating media for now
- Compute intensity and color of the surface intersected by viewing ray → **shading**



# Materials and shading

- Shading uses:
  - Material model
  - Viewing position
  - Object shape
  - Light information
- Final color of surface is calculated by:
  - Summing all **incoming light** that falls onto surface
    - Light transport – also uses material information
  - Calculating color and intensity of light reflected into camera using **material description** - shading

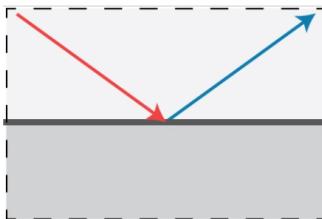


Powered by RayGraph

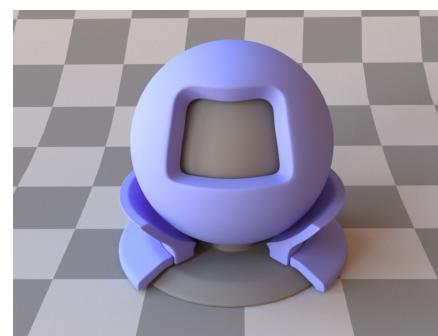
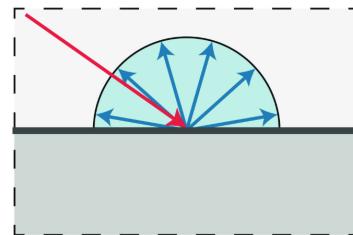
[www.scratchapixel.com](http://www.scratchapixel.com)

# Material modeling

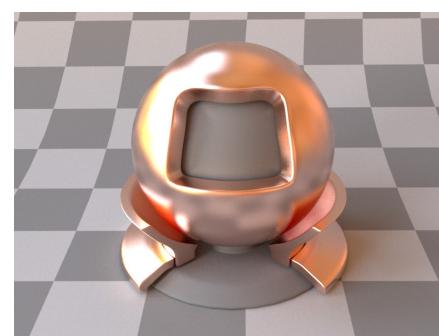
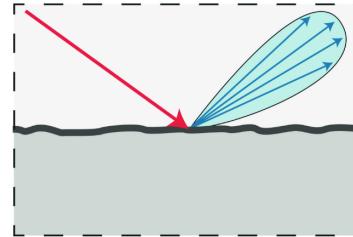
- Model interaction of light in shading point → **scattering model**.
  - Four scattering models for this lecture: local and surface-related



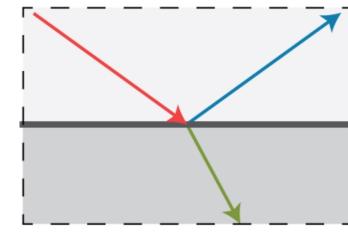
Perfect specular reflection.  
Light is reflected into only one direction.



Diffuse reflection.  
Light is reflected into all directions equally.



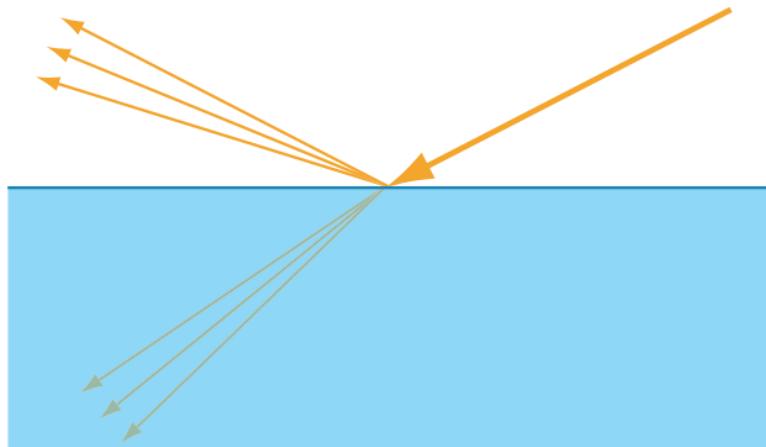
Glossy reflection.  
Light is reflected into preferred direction.



Perfect specular reflection and transmission.

# Local scattering models

- Local (direct) models: describe what happens when light falls on one point and scatters from that one point.
- Surface scattering can be well described (approximated) with local models.

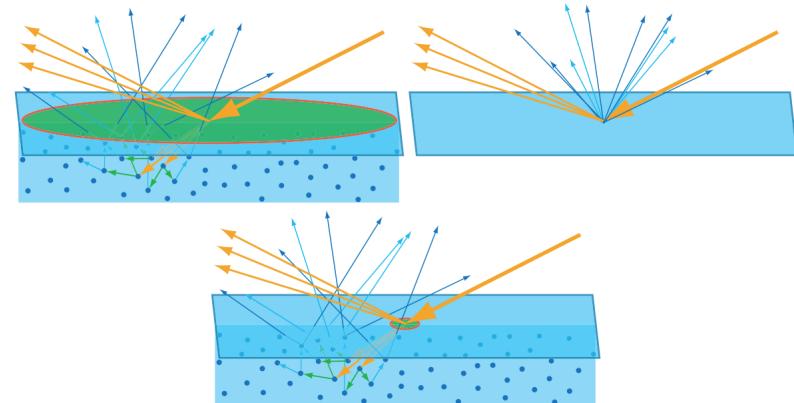


**Local models** describe what happens at one point of intersection with object surface

Macroscopically, non-optically flat surface can be treated as flat surfaces reflecting with multiple directions

# Global scattering models

- Volumetric scattering phenomena requires **global scattering model**
- Can be approximated with local model (e.g., SSS with diffuse model)



**Volumetric scattering** inside surface (SSS) or in participating media requires global models: light path is not exiting the same point where it entered.

# Material modeling: texture

- Same parameters of scattering model in each point of 3D surface results in **homogeneous surface material**: smooth surface
- Scattering model parameters are varied using **texture**.



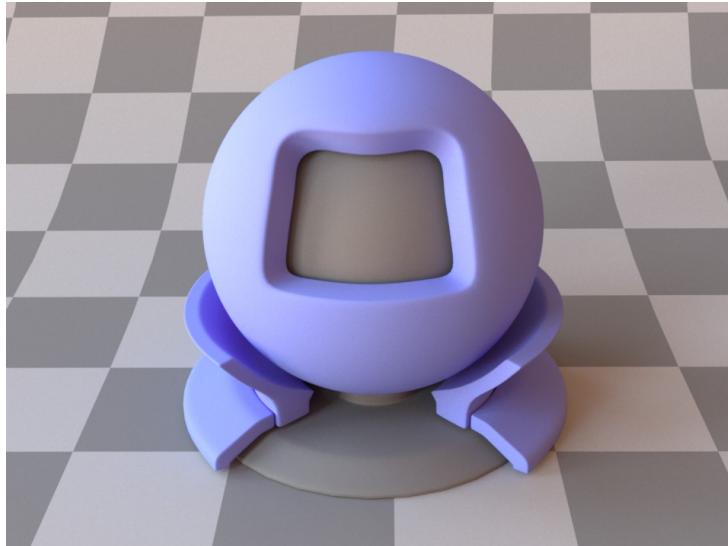
Homogeneous material: scattering model has the same parameters in all points of the surface.



In CG texture is used to vary scattering parameters over surface

# Elements of material model

- Material modeling in computer graphics is separated into:
  - **Scattering** → description of light-matter interaction at a point (directional effects and color)
  - **Texture** → variation of small-scale geometry and scattering properties across 3D object surface



Diffuse scattering model only.



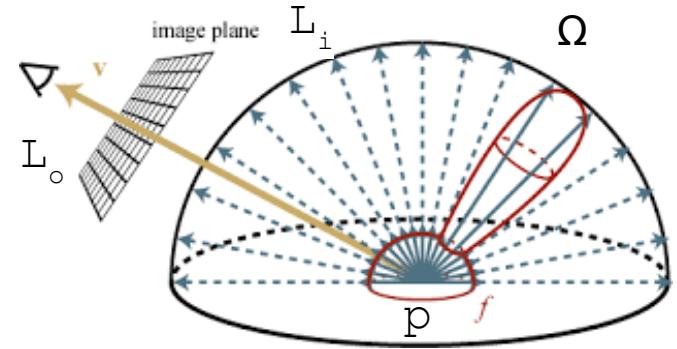
Diffuse scattering model with texture.

# Shading and light

- Rendering equation – basis for physically based rendering
  - Describes global illumination

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega} f(p, \omega_o, \omega_i) L_i(p, \omega_i) (\omega_i \cdot n) d\omega_i$$

Outgoing light in point $p$ in direction $\omega_o$ .	Emitted light in point $p$ (only if light source)	Scattering model	Incoming light in point $p$ from light direction $\omega_i$ . Calculated using light transport	Attenuation (cosine) factor: surface orientation towards light
--	--	------------------	--	---



For case when camera ray intersects the surface then:

- $L_o$  is shading results, a color
- $\omega_o$  is equal to view direction
- $\omega_i$  is equal to light direction

Note that this is recursive equation and for each light direction direction this equation must be evaluated again.

# Shading and light

- Reflectance equation. Special case of rendering equation.
  - Describes global illumination but only for reflective surfaces.
- Shading “collects” all incoming light and shading point and multiplies it with scattering model.

$$L_o(p, v) = \int_{l \in \Omega} f(l, n, v) L(p, l) (n \cdot l)^+ dl$$

Collect light over whole hemisphere above shading point, placed on normal in shading point

Incoming light at shading point

Color in shading point.

Scattering model

Light attenuation due to surface orientation (cosine factor)

# Shading and light

- Simplification: **direct illumination** – take in account only light coming from light sources (e.g., point lights).

$$c_{shaded(p,v)} = \sum^n f(l_i, n, v) c_{light_i}(n \cdot l_i)^+$$

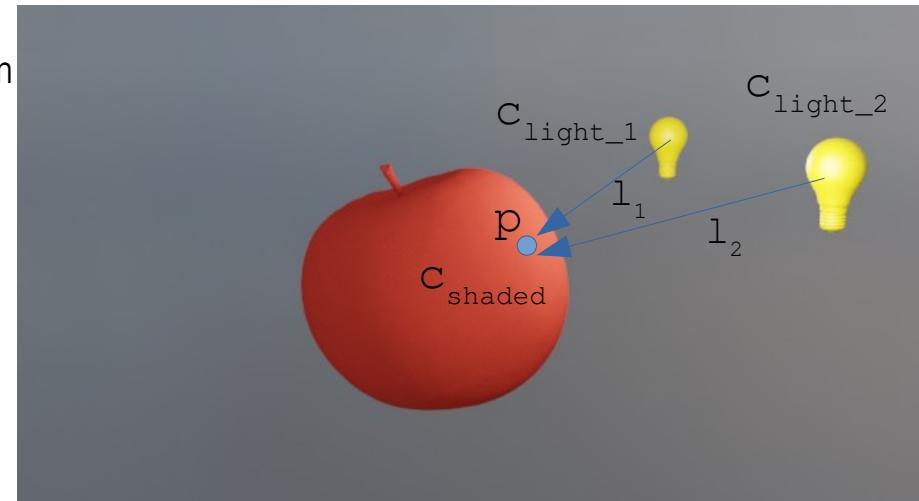
Color in  
shading  
point

Sum over all  
lights in 3D  
scene

# Scattering model

# Current light color

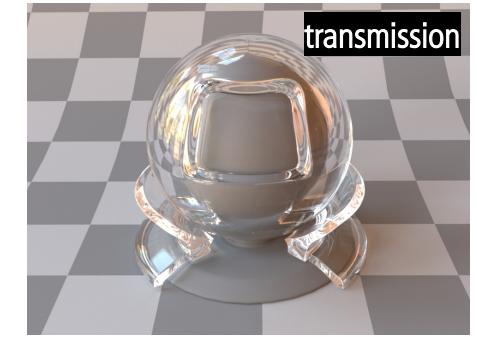
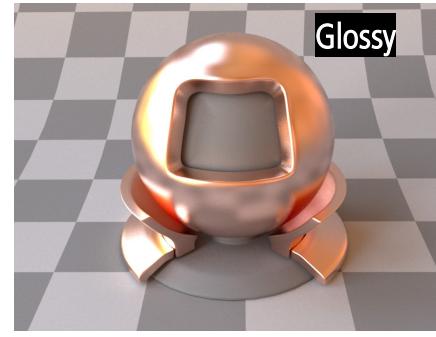
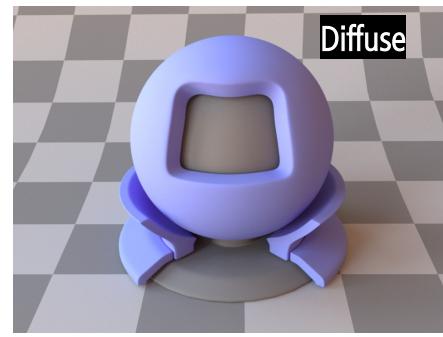
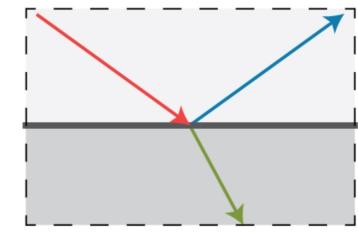
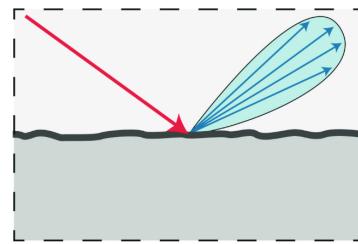
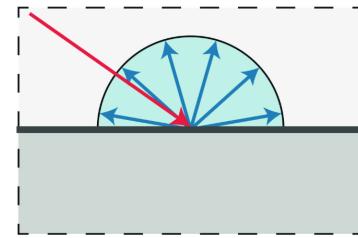
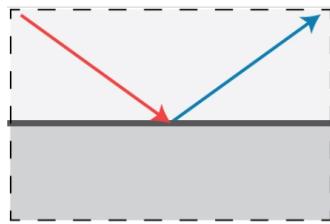
## Attenuation



# Scattering models

# Surface scattering:

## Reflection and transmission



# Shape and brightness

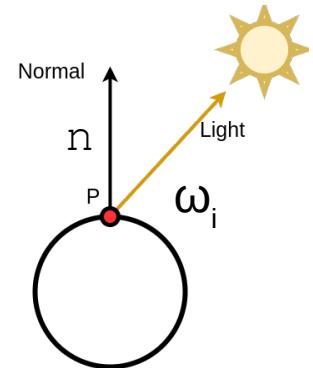
Amount of light falling on surface depends on:

- Shape
- Light position/direction:

Alignment of surface normal towards light determines amount of surface brightness

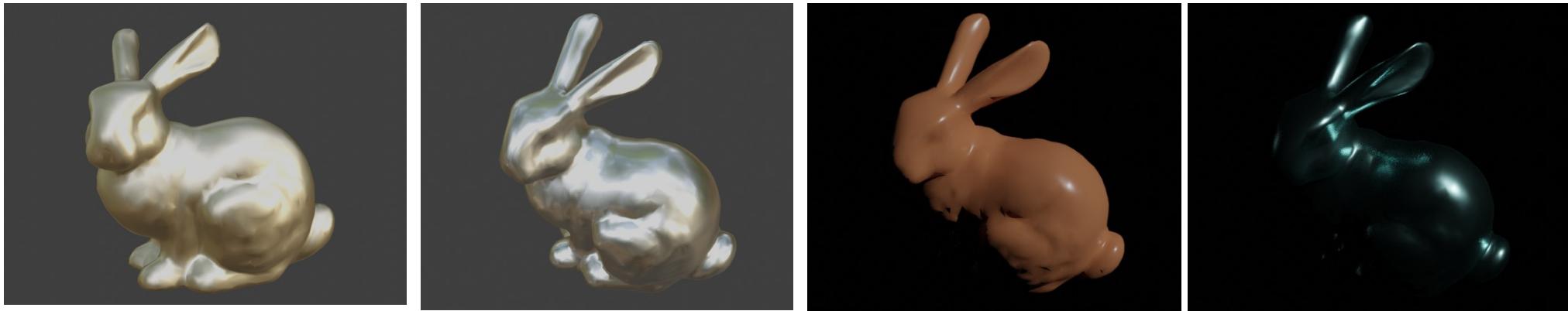
- Light attenuation (cosine factor)

$$(\omega_i \cdot n)$$



# Scattering model

- Observing objects we can see that they have different appearance, although similar in shape
  - This particular look of objects is defined by how light scatters when it falls on surface point.
  - This behavior is defined by **surface scattering function** - surface response to light



# Surface scattering function

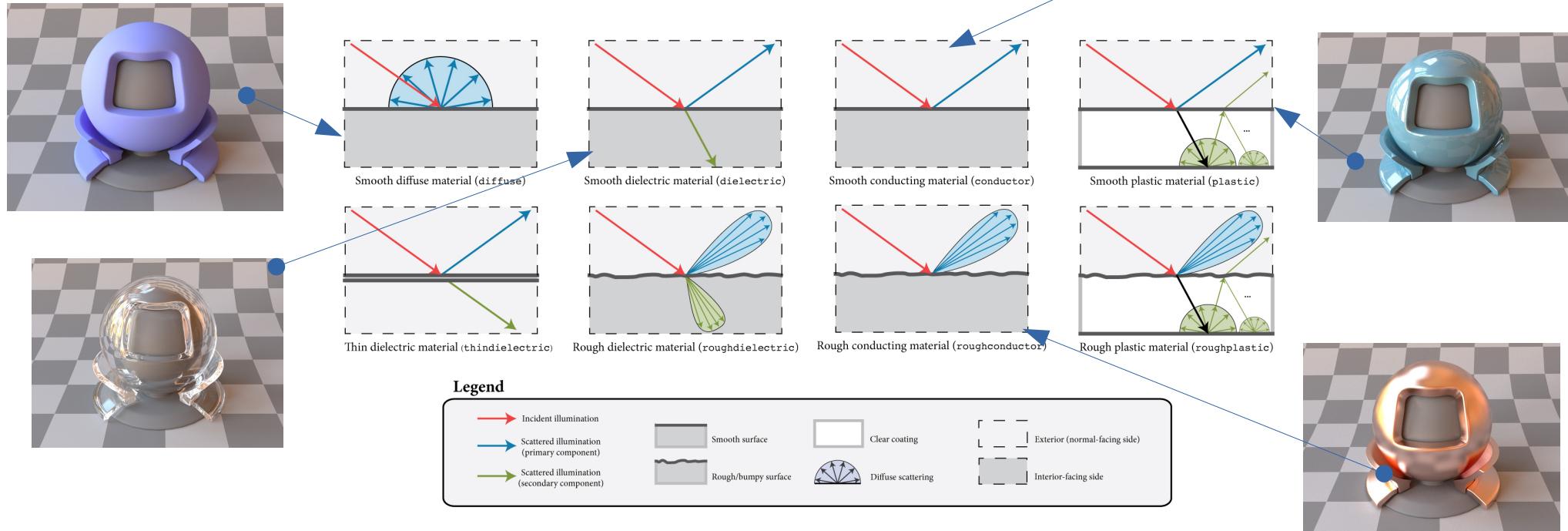
- Scattering function can be separated in reflection and transmission
  - Model describing reflection is called “bidirectional **reflectance** distribution function” – **BRDF**.
  - Model describing transmission is called “bidirectional **transmission** distribution function” – **BTDF**.
  - Model describing both reflectance and transmission is called “bidirectional **scattering** distribution function” – **BSDF**.



- **Reflective** – all light is scattered above surface
- **Transmissive** – all light is scattered below surface
  - Refractive – special case of transmissive

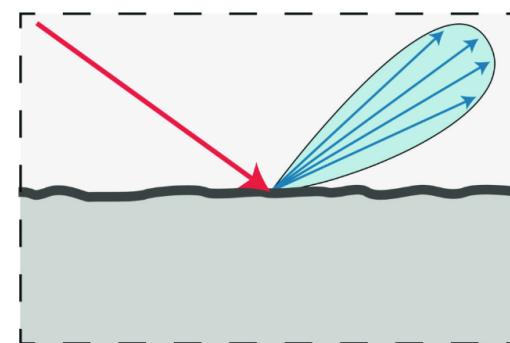
# Visualizing BRDF/BTDF

- The way surface reflects light can be visualized using **lobes – polar plot**
  - Given incident light direction, describes surface light reflection



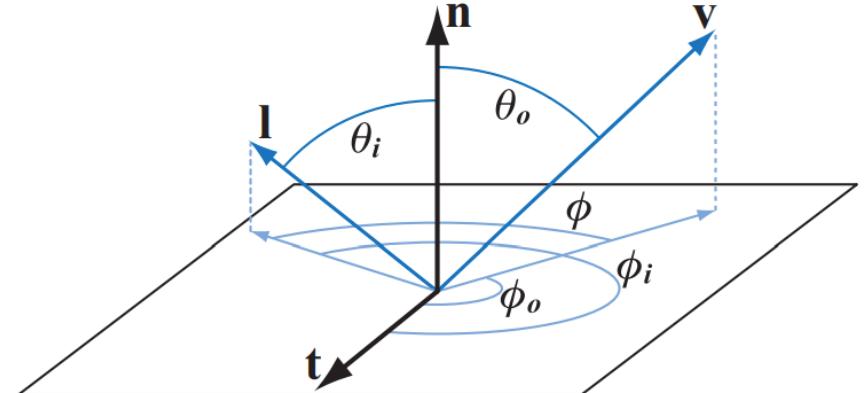
# BRDF

- Describes **reflectance phenomena**: redirection of light hitting surface (shading point) back outward
- **Mathematical approximation of light interaction and microscopic structure of material:**
  - Surface reflection
  - Re-emitted light from sub-surface scattering approximated with local model (e.g., diffuse model)
- BRDF is evaluated at shading point



# BRDF notation

- BRDF  $f(v, n, l)$  describes light reflection on surface which depends on:
  - Incoming – unit length vector - **light direction** ( $l$ )
  - Outgoing – unit length vector - **view direction** ( $v$ )
  - **Shading point normal** ( $n$ )
- Incoming ( $l$ ) and outgoing ( $v$ ) directions have 2 degrees of freedom, two angles relative to surface normal:
  - Elevation ( $\theta$ )
  - Azimuth ( $\phi$ )
- Dimensionality of BRDF: 4



# BRDF in shading

$$c_{shaded}(p, v) = \sum_{i=1}^n f(l_i, n, v) c_{light_i} (n \cdot l_i)^+ \quad \text{Direct (local) illumination.}$$

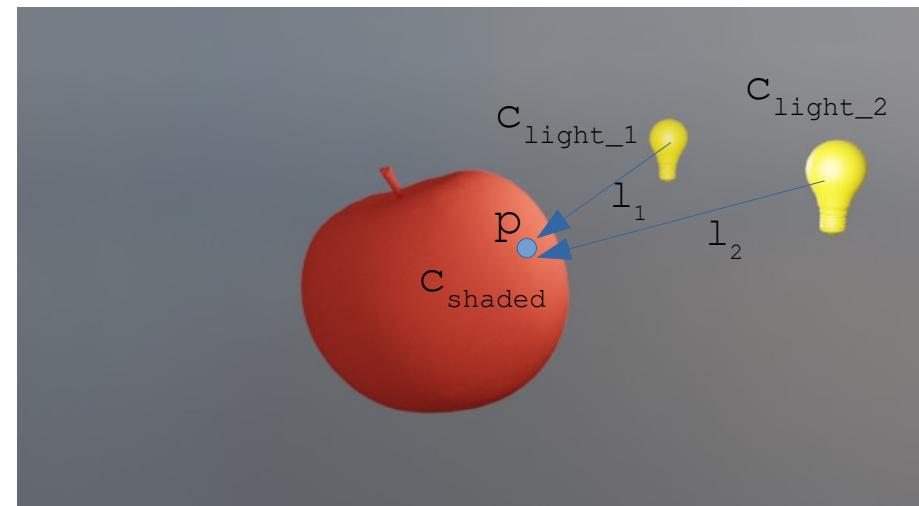
Color in shading point

Scattering model → BRDF

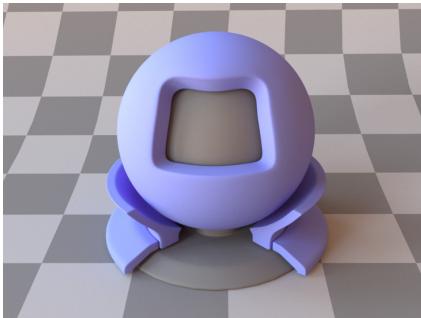
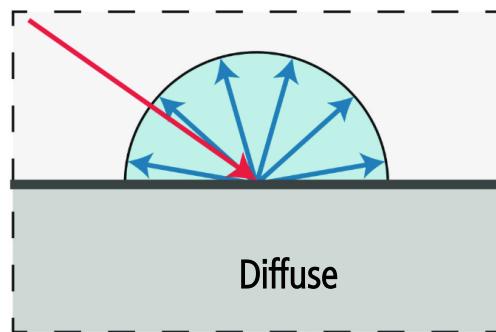
Sum over all lights in 3D scene

Current light color

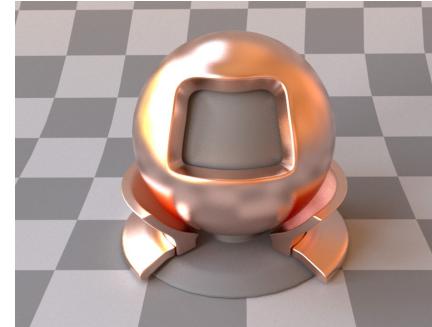
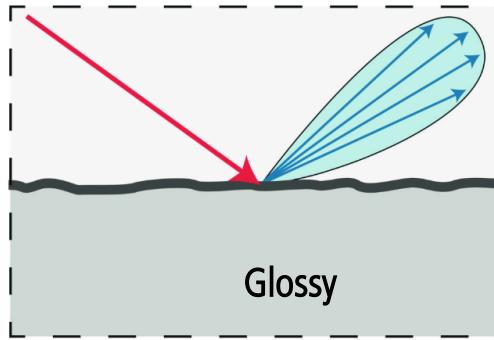
Attenuation



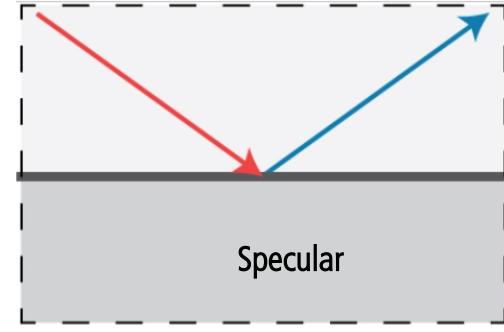
# Base BRDFs



- Light is scattered in all possible directions.
- Independent of viewing direction.
- Equally bright from all directions.
- Example: rough wood



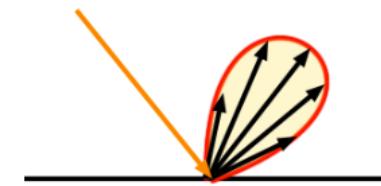
- Scattered light is concentrated around particular direction (lobe).
- Dependent of viewing direction.
- Appears blurred.
- Examples: brushed copper



- Light is scattered in single direction (mirror-reflection direction).
- Dependent of viewing direction.
- Perfectly sharp reflection.
- Example: smooth gold

# Base BRDFs\*

- Real materials have complex lobes (reflections)
- While modeling, several lobes can be combined with different weights and parameters
- Wide range of materials can be described with these three basic reflection types:
  - Specular
  - Diffuse
  - Glossy (rough specular)



**specular reflection**



**diffuse reflection**



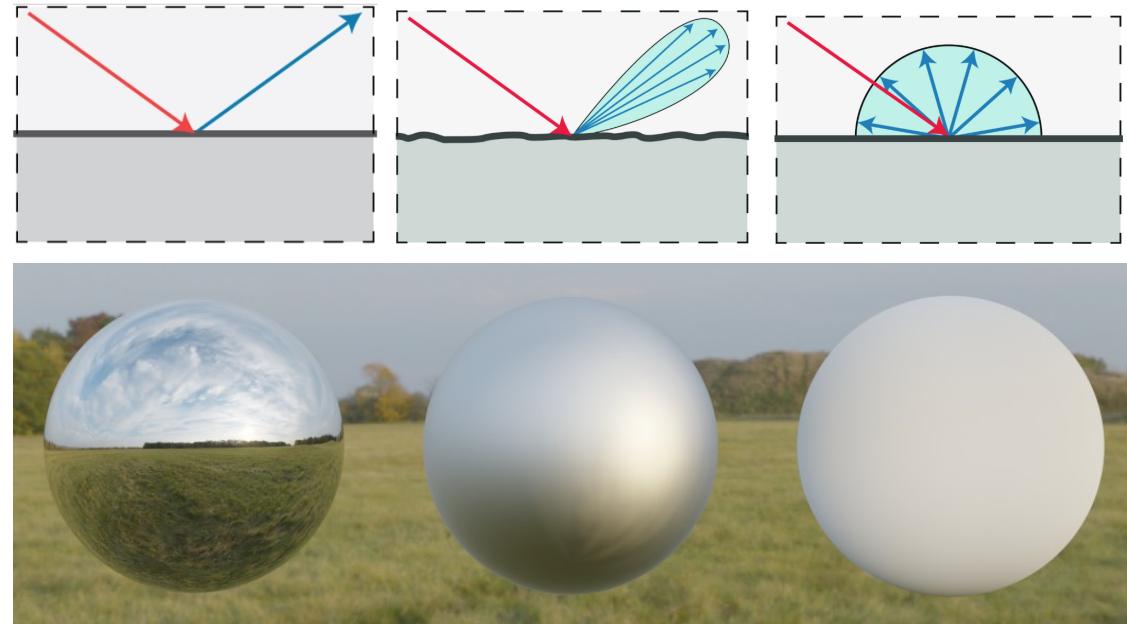
**diffuse + specular**

# Scattering models

- Modeling approaches:
  - Empirical
    - Simulate observed scattering phenomena
  - Data-based
    - Scattering is measured from real world and stored in tables
  - Physically-based
    - Based on physical interaction of light with matter

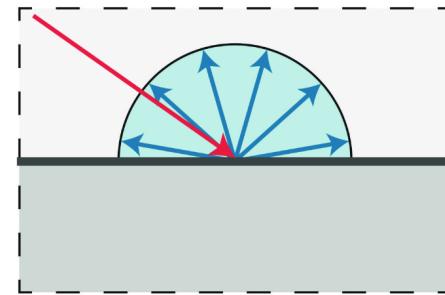
# Empirical models

- Models based on **observation** of scattering phenomena rather than physical correctness
- **Phenomenological models**: describe the quantitative properties of real-world surfaces by mimicking them
- Easy to implement and use
- Types:
  - **Lambertian** → diffuse
  - **Specular** → mirror
  - **Phong and Blinn-Phong** → glossy



# Lambertian (diffuse) model

- Reflected light is equal in all directions and linearly depends on incoming light
- Simplest BRDF model:
  - Doesn't depend view direction
  - Constant reflectance value: diffuse color (albedo): (R, G, B)
- Local model can be used to approximate sub-surface scattering
- The basis for more complex models



$$f(v, l) = \frac{\text{albedo}}{\pi}$$



# Lamberitan in shading

$$c_{shaded}(p, v) = \sum_{i=1}^n \frac{albedo}{\pi} c_{light_i} (n \cdot l_i)^+$$

Color in shading point

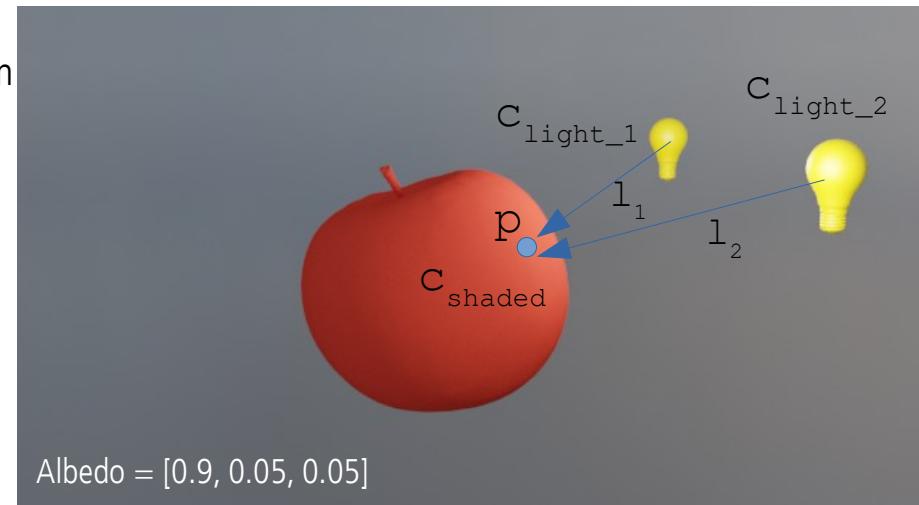
Lambertian scattering model

Sum over all lights in 3D scene

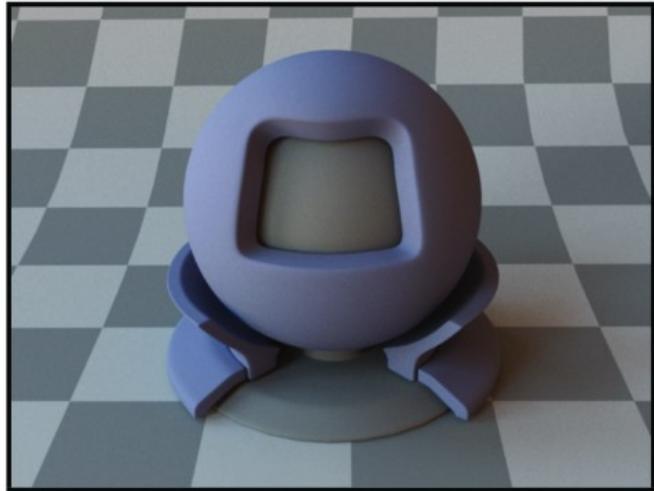
Current light color

Attenuation

Note that surface orientation will attenuate or increase incoming light making surface darker or brighter (attenuation factor)

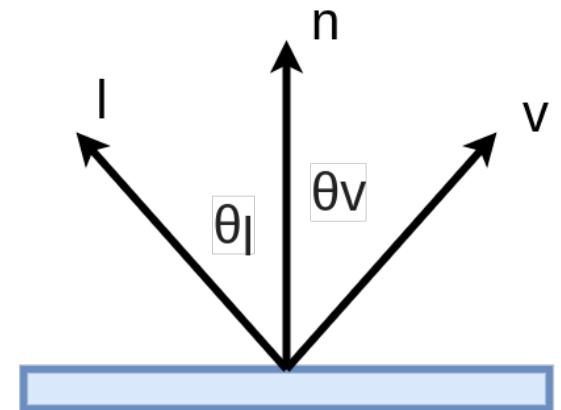
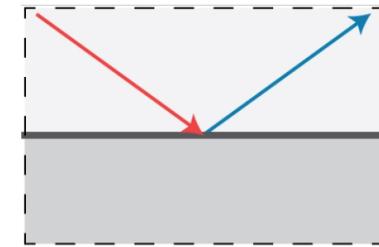


# Lambertian (diffuse) model examples



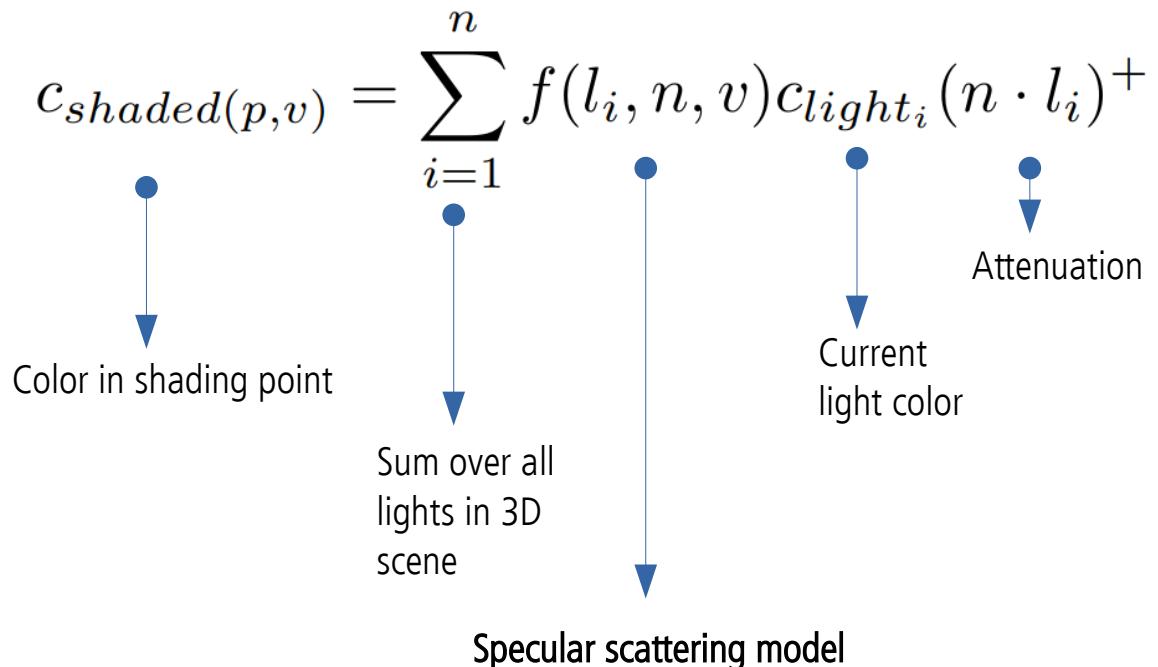
# Specular (mirror) model

- Ideal reflection where incoming light is reflected completely in a single outgoing direction
  - Mirror reflectance direction  $\rightarrow \Theta_l = \Theta_v$
- Depends on view and light directions
- Single parameter: **reflectivity** – a constant (R,G,B)

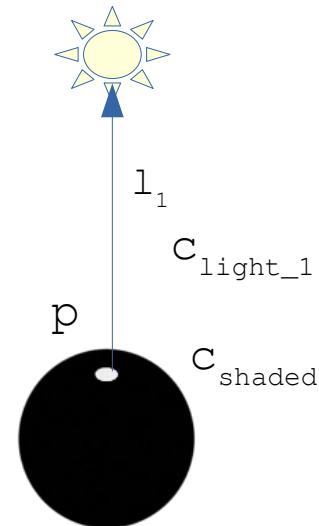
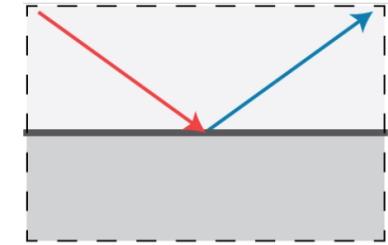


$$f(v, l) = \begin{cases} \text{reflectivity} & \text{if } v = l - 2(l \cdot n)n \\ 0 & \text{otherwise} \end{cases}$$

# Specular model in shading



$$f(v, l) = \begin{cases} reflectivity & \text{if } v = l - 2(l \cdot n)n \\ 0 & \text{otherwise} \end{cases}$$



Reflects only light source!

# Specular model in shading

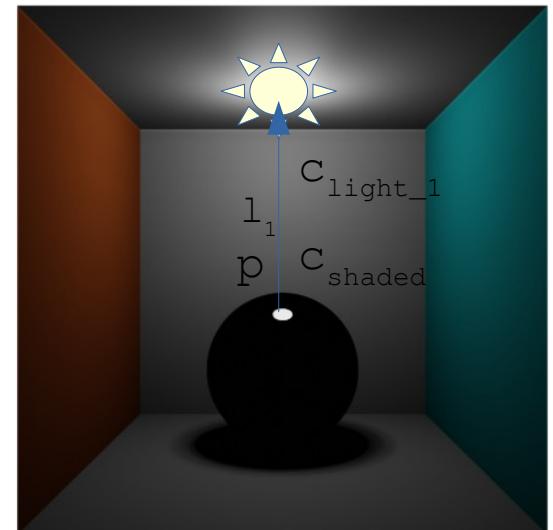
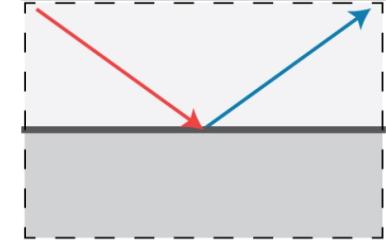
$$c_{shaded}(p, v) = \sum_{i=1}^n f(l_i, n, v) c_{light_i} (n \cdot l_i)^+$$

↓      ↓      ↓      ↓      ↓

Color in shading point      Sum over all lights in 3D scene      Current light color      Attenuation

Specular scattering model

$$f(v, l) = \begin{cases} reflectivity & \text{if } v = l - 2(l \cdot n)n \\ 0 & \text{otherwise} \end{cases}$$



Drawback of direct illumination

- Only light sources are considered for incoming light

# Specular model in shading

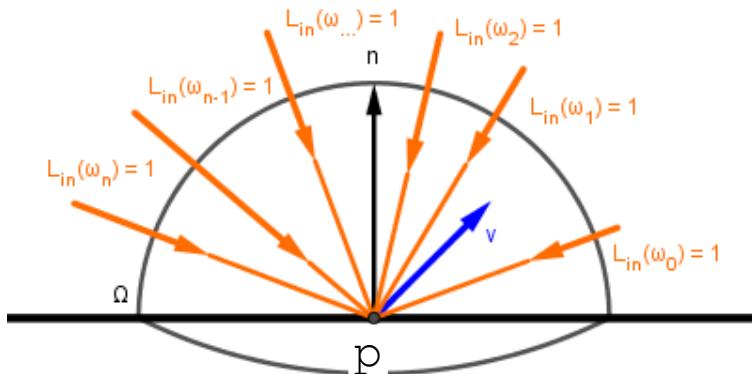
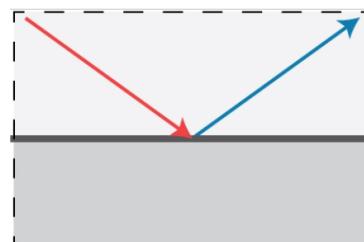
- Ideal reflection where incoming light is reflected completely in a single outgoing direction – **mirror reflectance direction**
  - Light has to be “gathered” from all directions above shading point:
    - Light sources → direct illumination
    - Surfaces in 3D scene → indirect illumination

$$L_o(p, v) = \int_{l \in \Omega} f(l, n, v) L(p, l) (n \cdot l)^+ dl$$

Reflectance equation

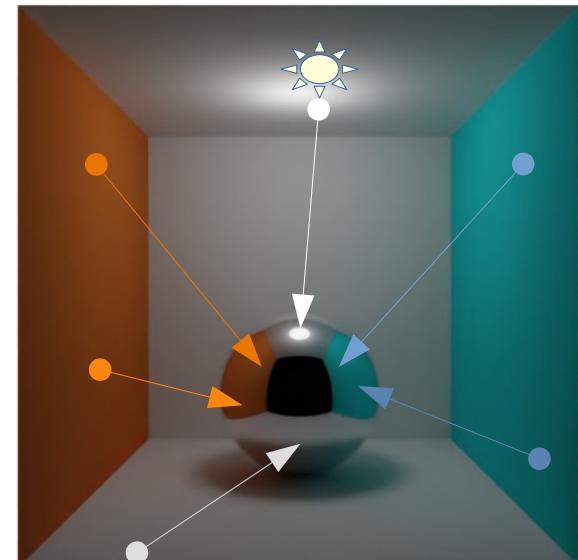


$$f(v, l) = \begin{cases} \text{reflectivity if } & v = l - 2(l \cdot n)n \\ 0 & \text{otherwise} \end{cases}$$



**Global illumination:**

- Direct illumination
- Indirect illumination



# Specular and diffuse reflection

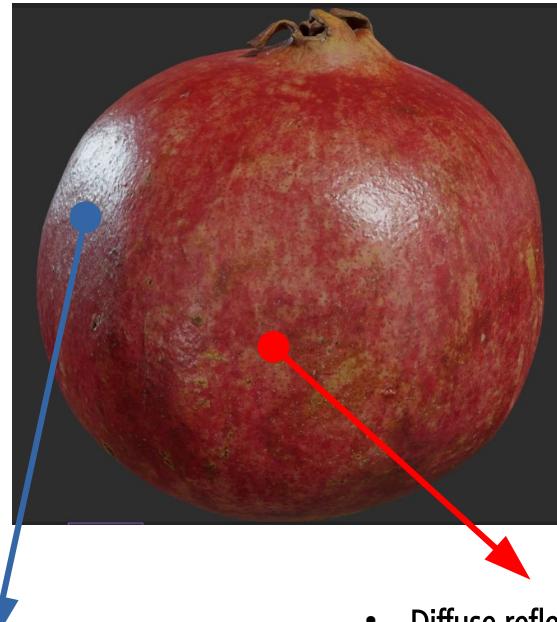
Metals



- Reflects almost all specularly

- Colored specular reflection (highlights)
- Specular reflectivity parameter determines color for metals (conductors) – approximation of spectral selectivity

Dielectrics



- Specular highlight is the same color as light source color.

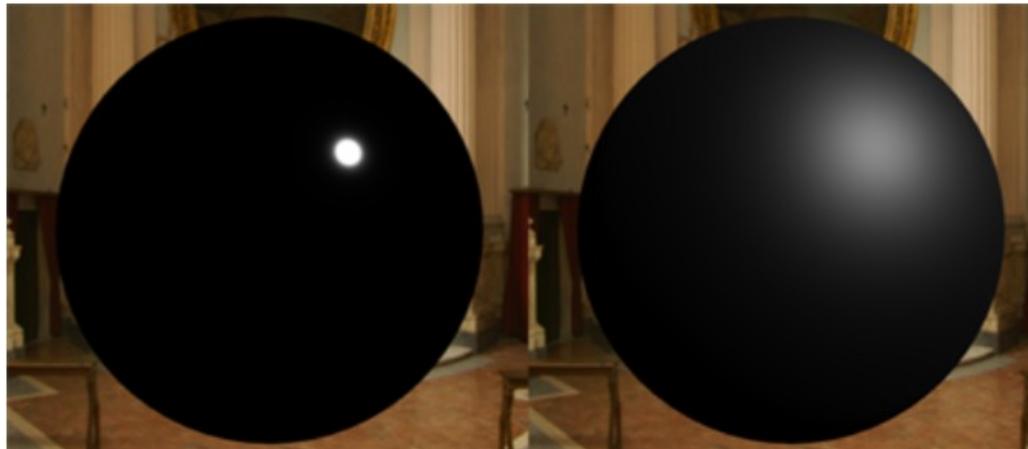
- Diffuse reflection gives color to the dielectric materials.
- It is a result of re-emitted light which entered surface and partially absorbed.

# Phong model

- Idea: wide range of materials can be described with combination of **diffuse** and **specular** components:

$$\text{surface\_color} = \text{diffuse}() * K_d + \text{specular}() * K_s$$

- Problem: some surfaces are not perfectly reflecting light in mirror specular reflection: sharp reflection
- **Glossy highlights**: reflection of light source (or another object) on object surface is “blurred”

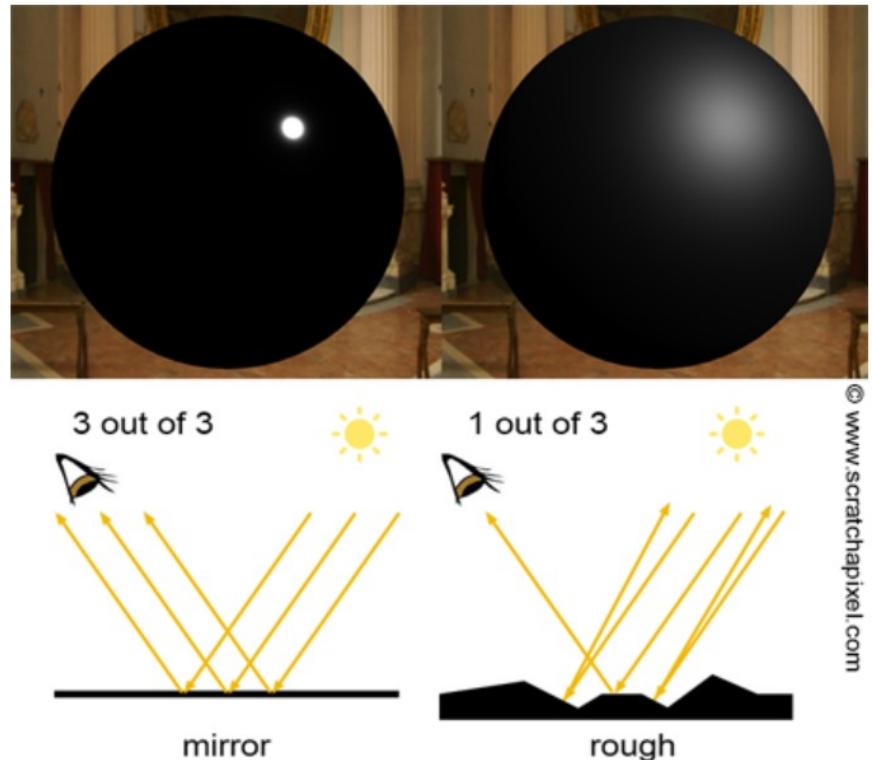


Perfect specular reflection

Blurred specular reflection: glossy

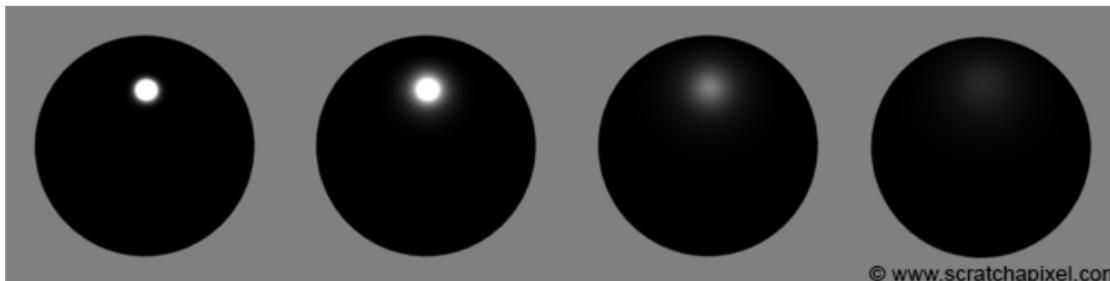
# Phong model: glossy observations

- **Glossy reflection** is a result of rough surface which acts as "broken mirror"
- Such surface can be represented as collection of small mirrors → **micro-facets**
- Only fraction of light rays are reflected in eye direction by rough surface causing dimmer/blurred light reflection
- Brightness of glossy reflection decreases as angle between view and ideal reflection increases
  - This simulates decreasing number of microfacets reflecting in eye direction

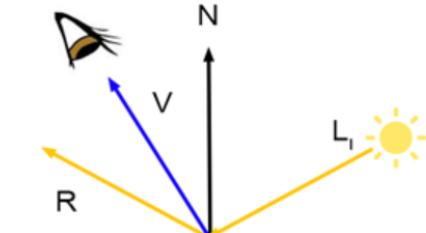


# Phong model: glossy observations

- Maximum reflection is when viewer ( $V$ ) aligns with reflection ( $R$ )
- Reflection decreases as angle between viewer ( $V$ ) and reflection ( $R$ ) distance increases
  - Reflection spreads across larger area
  - Highlight brightness decreases as distance of object points from original reflected light position increases



$$R = \text{reflect}(L_i, N)$$
$$\text{specular} = (V \cdot R)^n$$



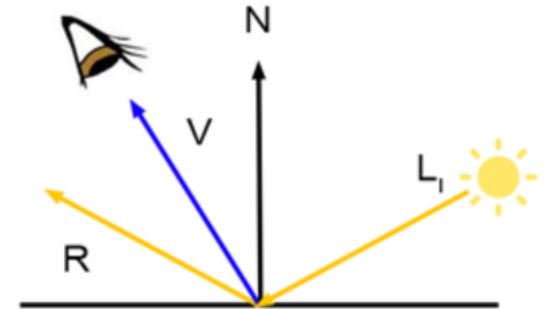
© www.scratchapixel.com

# Phong model: specular component

- Glossy surface can be simulated by:
  - Computing ideal specular reflection ( $R$ ) of incident light ray ( $L$ )
  - Computing dot product between reflected ray and view direction
  - Raising dot product to power of  $n$

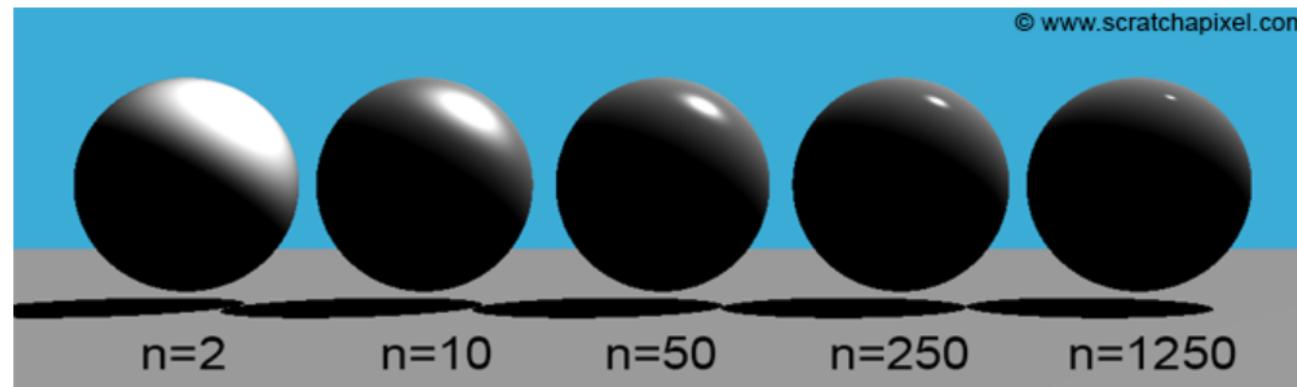
$$\text{Specular} \approx (V \cdot R)^n.$$

$$R = \text{reflect}(L, N)$$
$$\text{specular} = (V \cdot R)^n$$



© www.scratchapixel.com

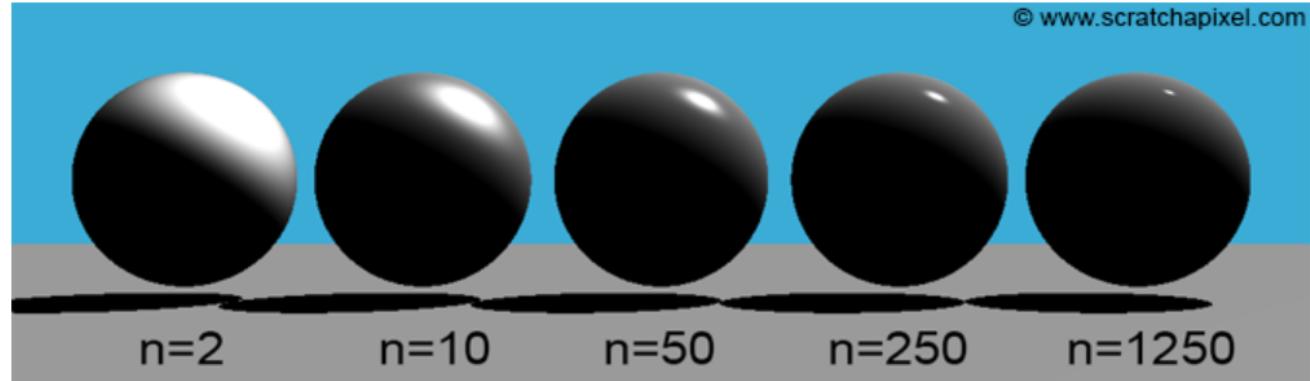
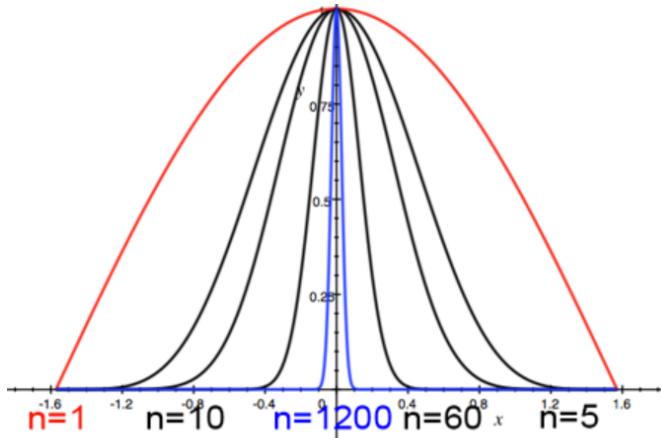
$$R = 2(N \cdot L)N - L.$$



# Phong model: specular component

- **Empirical model:** parameters doesn't have physical meaning
  - Parameters are tweaked by artist/user until desired appearance is achieved
  - Specular highlights become "sharper" as  $n$  grows

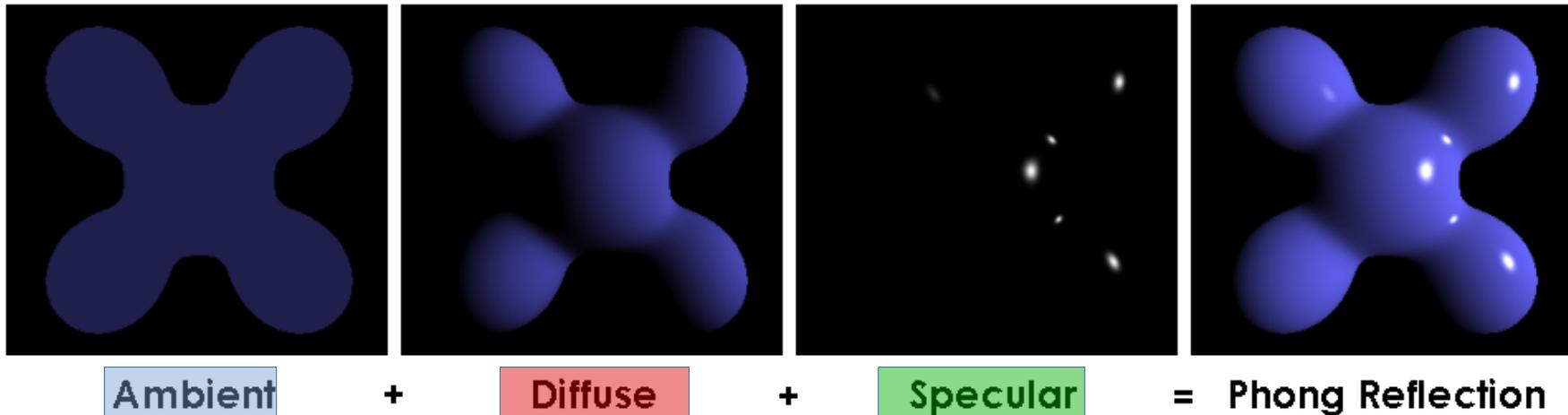
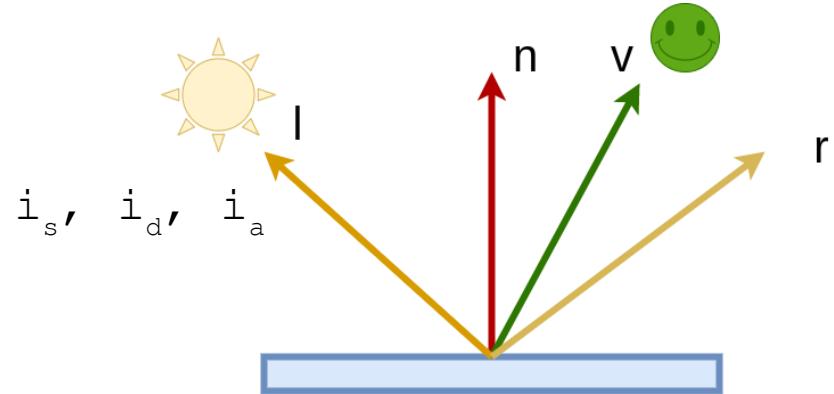
$$\text{Specular} \approx (V \cdot R)^n.$$



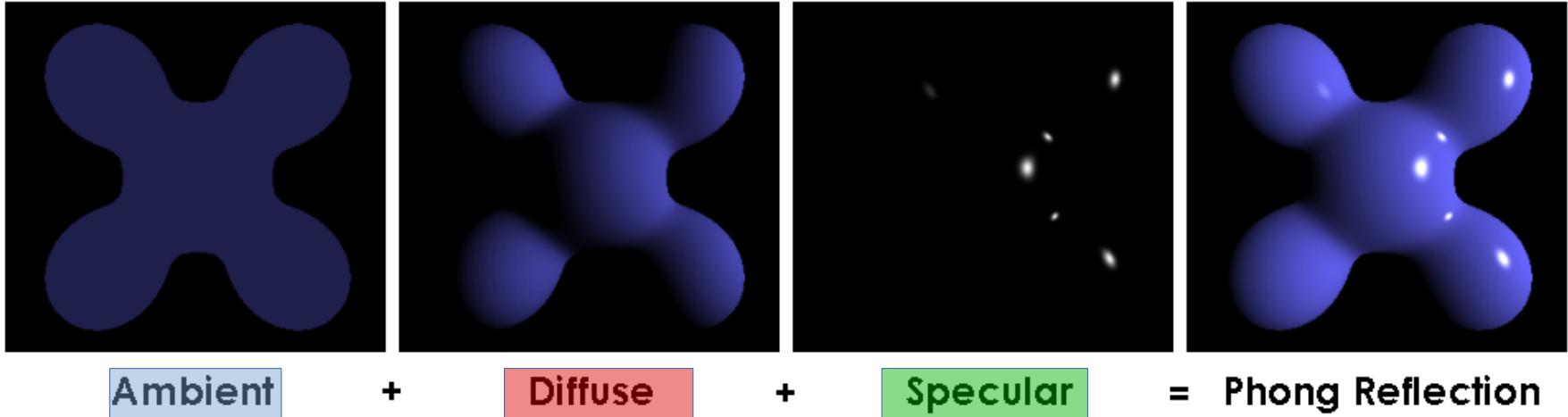
# Phong direct illumination model

- Describes local shading (direct illumination)

$$L_o = k_a i_a + \sum_j^n (k_d(l_j \cdot n)i_{j,d} + k_s(r_j \cdot v)^\alpha i_{j,s})$$



$$L_o = k_a i_a + \sum_j^n (k_d(l_j \cdot n)i_{j,d} + k_s(r_j \cdot v)^\alpha i_{j,s})$$



**Ambient:** small amount of light that comes from around the scene

- Parameter:  $k_a$  (R,G,B) – ratio of ambient reflection over incoming light  $i_a$  (R,G,B)

**Diffuse** reflection from rough surfaces; large highlights

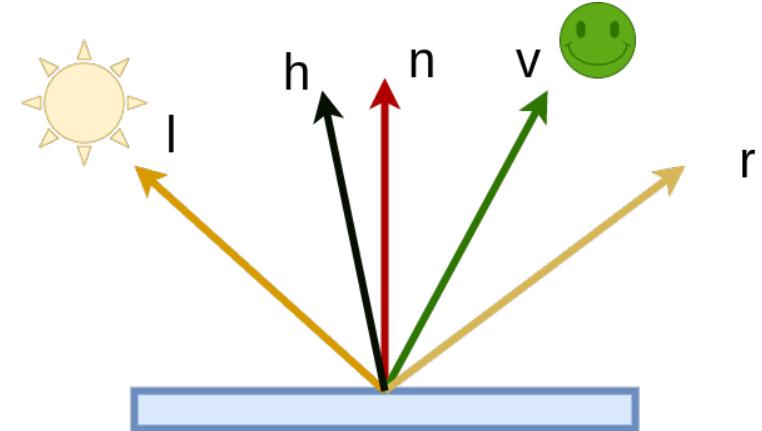
- Parameter:  $k_d$  (R,G,B) – ratio of diffuse reflection over incoming light  $i_{j,d}$  (R,G,B)

**Specular** reflection from shiny surfaces; small highlights

- Parameter:  $k_s$  (R,G,B) – ratio of specular reflection over incoming light  $i_{j,s}$  (R,G,B)
- Parameter:  $\alpha$  – shininess constant: larger value causes smooth and mirror-like surfaces with small specular highlights

# Blinn-Phong reflection model

- Introducing half vector ( $h$ ) between light ( $l$ ) and view ( $v$ ) vectors
  - Maximum reflection:  $v = r$ , that is:  $n = h$
  - Reflection decreases when angle between  $N$  and  $H$  increases
- Parameters:
  - Lambertian ( $k_L$ ), range:  $[0, 1]$
  - Glossy ( $k_G$ ), range:  $[0, 1]$
- Energy conserving if  $k_L + k_G \leq 1$

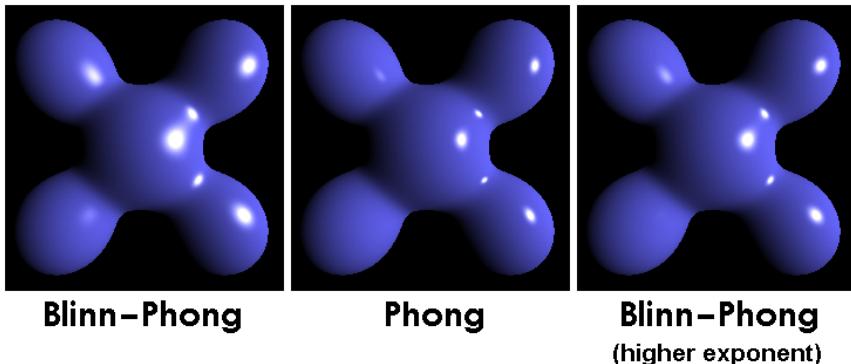


$$f(v, l) = \frac{k_L}{\pi} + k_G \frac{8+s}{8\pi} z^2$$

$$z = \max(0, h \cdot n)$$

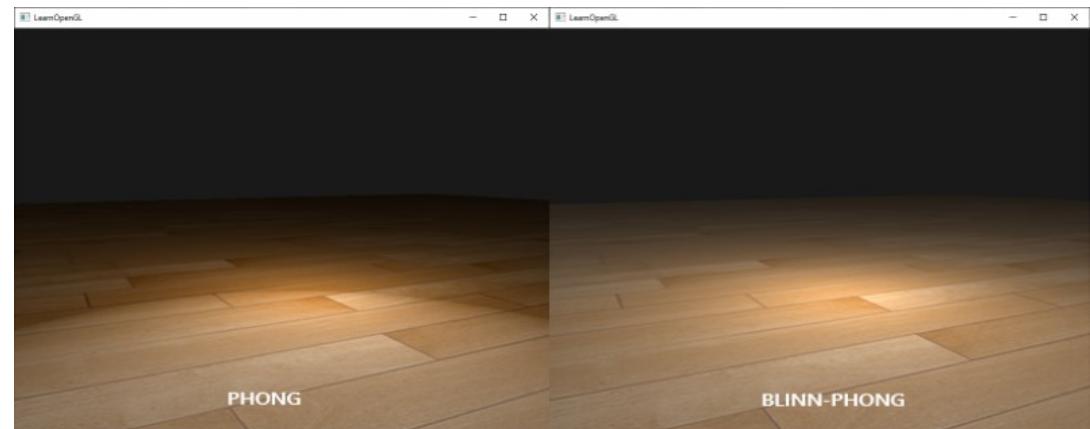
$$h = \frac{v + l}{\|v + l\|}$$

# Blinn-Phong reflection model



[https://en.wikipedia.org/wiki/Blinn%20%26%20Phong\\_reflection\\_model](https://en.wikipedia.org/wiki/Blinn%20%26%20Phong_reflection_model)

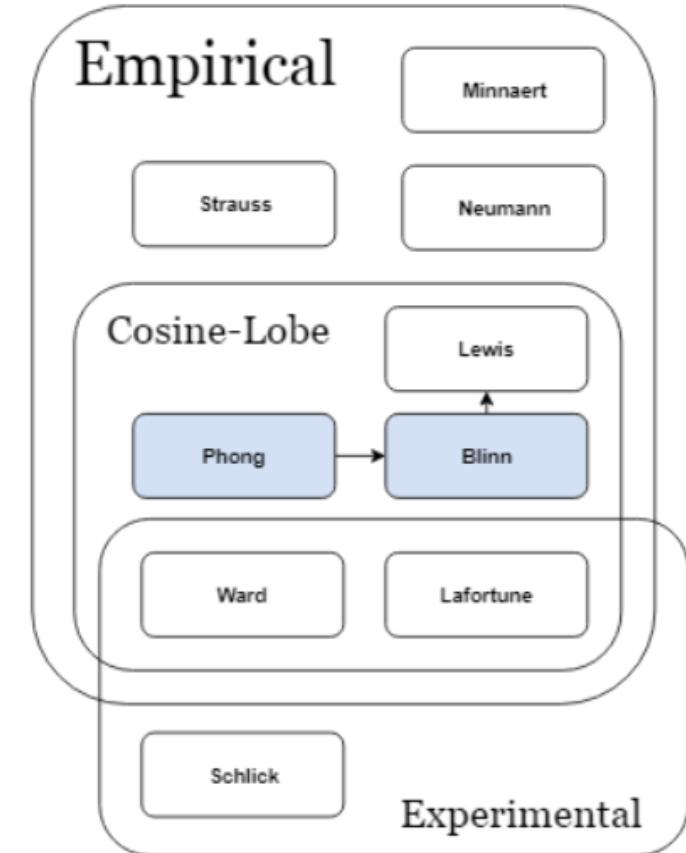
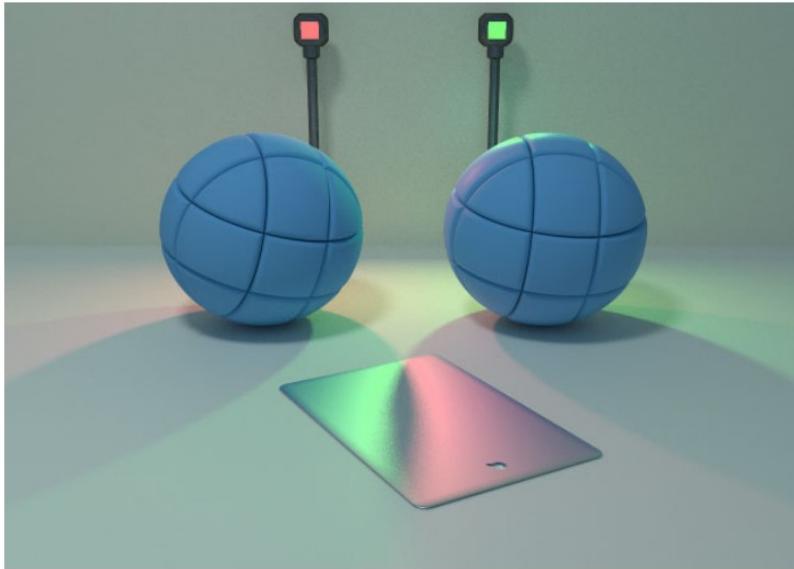
- Phong and Blinn-Phong models are often perceived as “plastic”.
- Physically-based microfacet models solve this problem



Example in OpenGL:  
<https://learnopengl.com/Advanced-Lighting/Advanced-Lighting>

# Other empirical models

- **Lafortune model**
  - Generalization of Phong's model
  - Richer appearance with multiple lobes



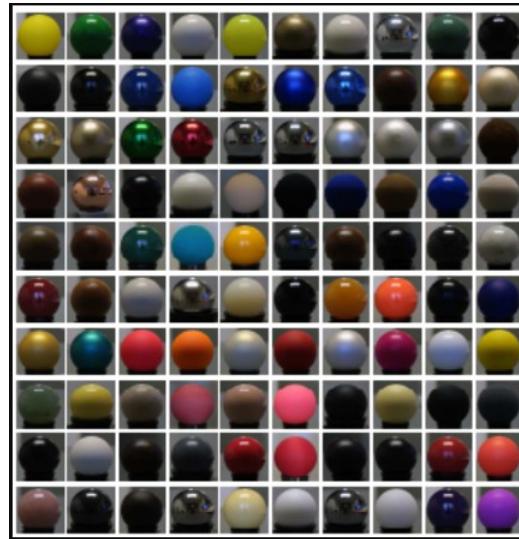
<https://cglearn.eu/pub/advanced-computer-graphics/physically-based-shading>

# Scattering models

- Modeling approaches:
  - Empirical
    - Simulate observed scattering phenomena
  - Data-based
    - Scattering is measured from real world and stored in tables
  - Physically-based
    - Based on physical interaction of light with matter

# Data-based models

- BRDF measurement → **gonioreflectometer**
  - Measurements are stored in look-up table
  - View and light direction is used for indexing the table
- BRDF measurement of real-world can be used for:
  - Evaluation of phenomenological and physically based models
  - Modeling of material
- Different real-world materials have been measured:
  - Isotropic BRDFs
  - Anisotropic BRDFs
  - Texture characteristics
  - Sub-surface scattering
- Problems:
  - Costly for rendering
  - Memory



Isotropic materials: <https://www.merl.com/brdf/>



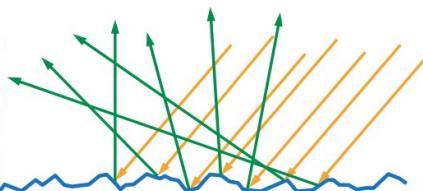
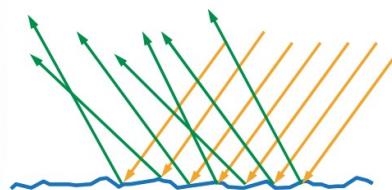
Anisotropic materials: Brushed aluminium, Yellow satin,  
Purple satin, Red velvet  
<http://people.csail.mit.edu/addy/research/brdf/>

# Scattering models

- Modeling approaches:
  - Empirical
    - Simulate observed scattering phenomena
  - Data-based
    - Scattering is measured from real world and stored in tables
  - **Physically-based**
    - Based on physical interaction of light with matter

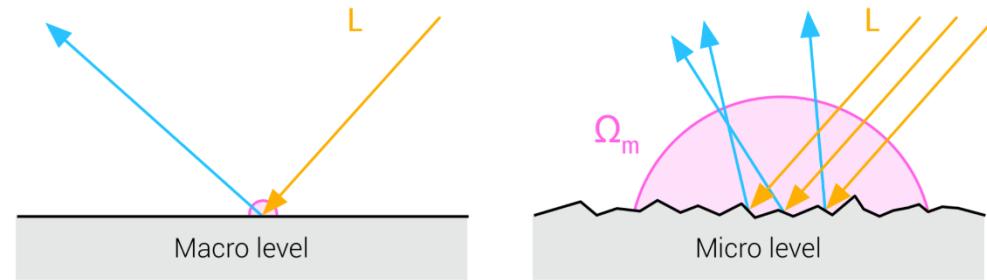
# Physically-based models

- Real-world surfaces have geometrical detail on multiple and thus **very small scales**
- Small scale geometry is invisible to the eye directly, but its cumulative response is visible in light reflection
  - Micro-facet theory – an approach of modeling small-scale surface geometry



# Microfacet-based models

- Small-scale surface irregularities (under one pixel) are expensive to model explicitly → model with BRDF!
- Small scale irregularities are represented as optically-flat facets with **microsurface normals** → **microfacet-based models**.
  - **Geometric optics assumption**: microfacets are much larger than light wavelength → they only cause **light redirection**
  - **Wave optics** describes phenomena of light scattering where surface irregularities are comparable to light wavelength\*



<https://google.github.io/filament/Filament.html>

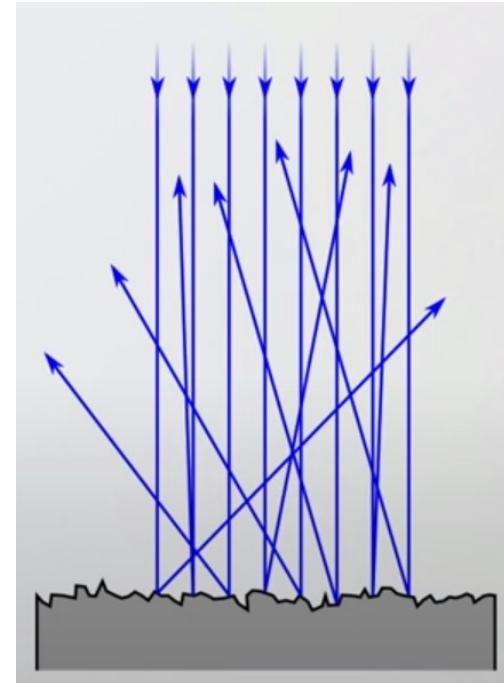
# Microfacet-based models

- Cook-Torrance reflection model: model surface statistically as large number of microfacets

$$R_{Cook-Torrance} = R_s + R_d$$

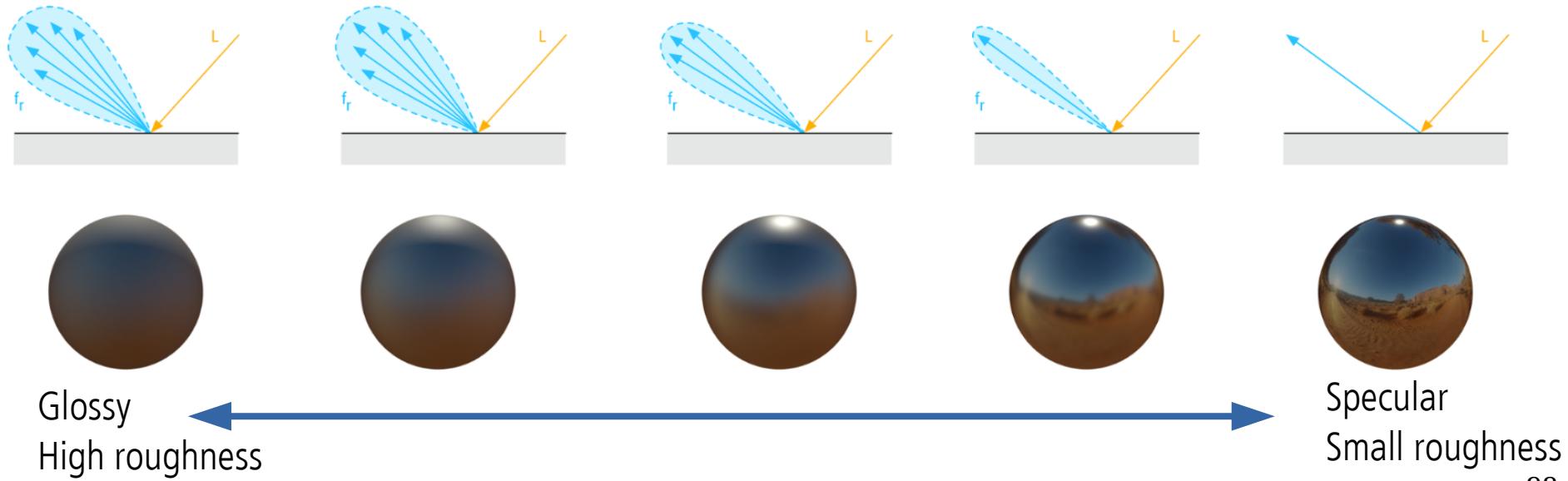
$$R_s = \frac{D \cdot G \cdot F}{dot(N, V) \cdot dot(N, L)}$$

- $R_s$  – Specular component
  - D – distribution of microfacet normals
  - F – Fresnel term
  - G – geometry term
- $R_d$  – Diffuse (Lambertian) reflection model



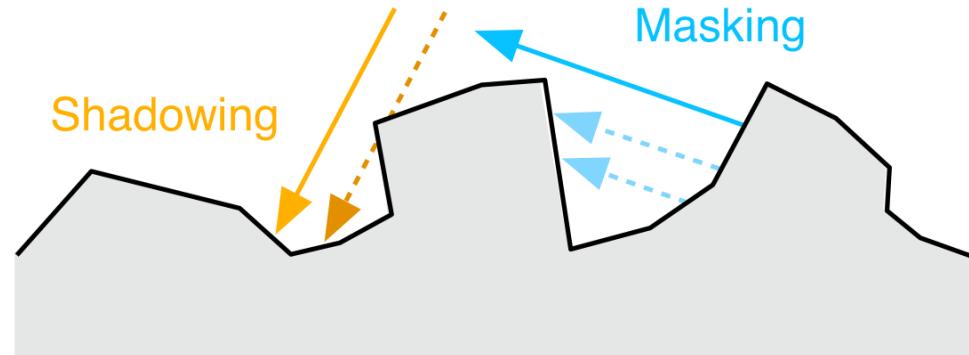
# D – distribution of microfacet normals

- Statistical distribution of microfacet normal orientation. Distributions: Gaussian, Beckmann, GGX
- Spread of this distribution is called **roughness** and it determines the regularity of microfacets
  - Higher roughness → more blurred surface since micronormals will be more irregular (**glossy-diffuse**).
  - Small roughness → mirror-like surface (**specular**).



# G – geometry term

- Statistically describes attenuation at microfacet level
  - Probability of microfacets shadowing or mask one another
- Choice of geometry term model (G) depends on distribution of microfacet normals (D)
  - Often Smith's model is used for geometry term model.



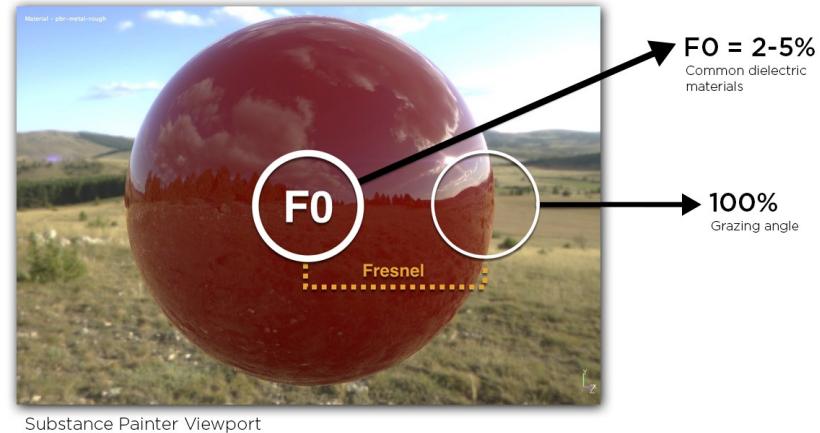
# F – Fresnel term

- Determines amount of reflected vs refracted light
- Depends on:
  - Index of refraction
  - Angle between surface normal and view direction
- Specular reflection is high at grazing angles
  - Large angle between normal and view direction → larger amount of reflection
- Fresnel equations are well approximated with Schlick's approximation

$$F = F_0 + (1 - F_0) * (1 - (\vec{n} \cdot \vec{v}))^5$$

$$F_0 = \frac{(n - 1)^2}{(n + 1)^2}$$

Reflection at normal incidence



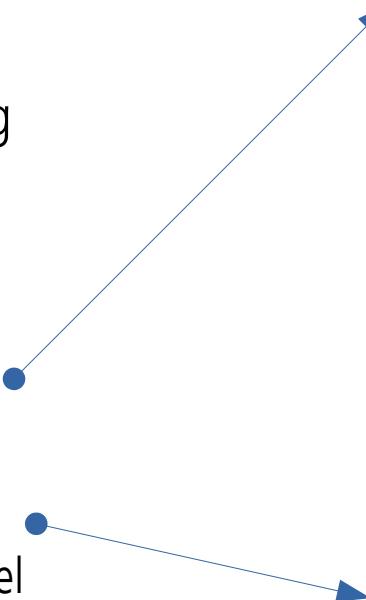
<https://substance3d.adobe.com/tutorials/courses/the-pbr-guide-part-1>

<https://blog.selfshadow.com/publications/s2020-shading-course/>

<https://graphicscompendium.com/raytracing/11-fresnel-beer>

# Microfacet-based models

- Microfacet-based BRDFs are state of the art physically-based reflection models used in professional modeling and rendering software.
- Fundamental models are:
  - **Torrance–Sparrow, Cook-Torrance** for specular/glossy surfaces
  - **Oren-Nayar** better approximation of diffuse reflection than Lambertian model



# Physically-based models

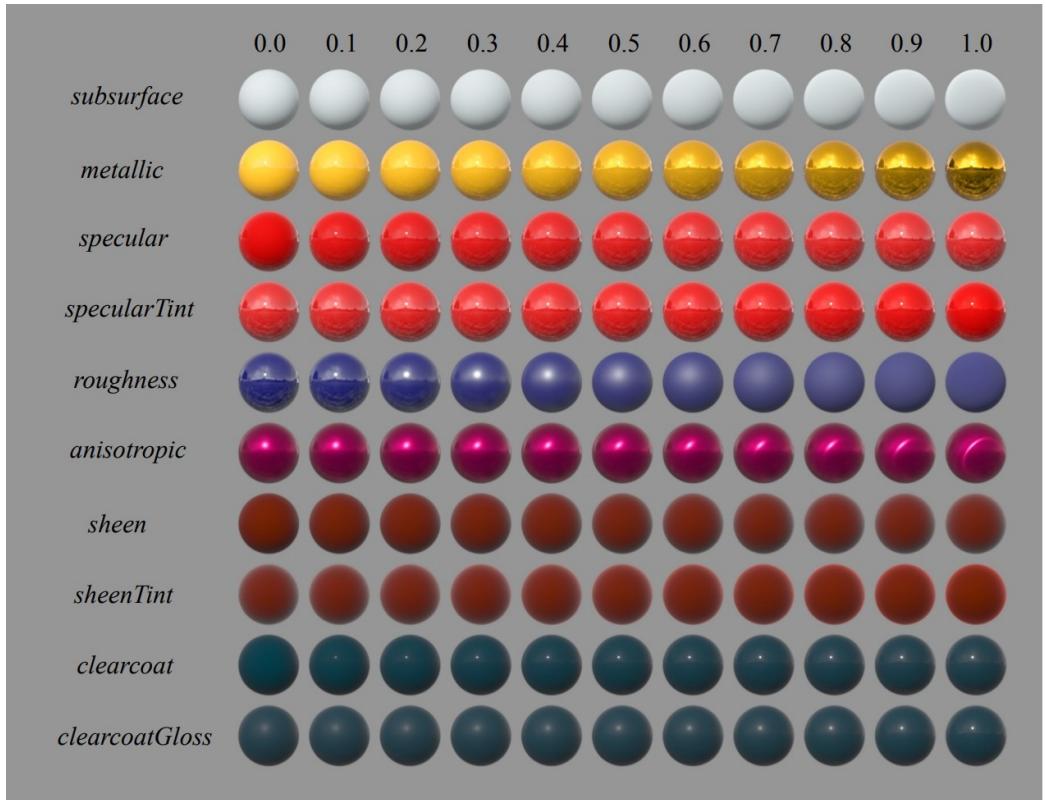
- Physically based models require:
  - **Helmholz reciprocity:**  $f(l, v) = f(v, l)$  – input and output can be switched and the function value will stay the same
  - **Conservation of energy\***: outgoing energy can not be greater than incoming energy.
    - BRDF which significantly violates this property leads to too bright and thus not realistic surfaces.
    - Note that BRDF can have arbitrary large values in certain directions if the distribution it describes is highly non-uniform (e.g., specular highlights)



**White furnace test:** white sphere is illuminated with white light from all directions. If energy conserving is satisfied, the sphere will disappear in white background.  
<https://boksajak.github.io/files/CrashCourseBRDF.pdf>

# Physically-based models

- Disney BRDF
  - Universal BRDF with few parameters capable of producing wide range of surfaces
- Principled BRDF → Blender:  
[https://docs.blender.org/manual/en/latest/render/shader\\_nodes/shader/principled.html](https://docs.blender.org/manual/en/latest/render/shader_nodes/shader/principled.html)

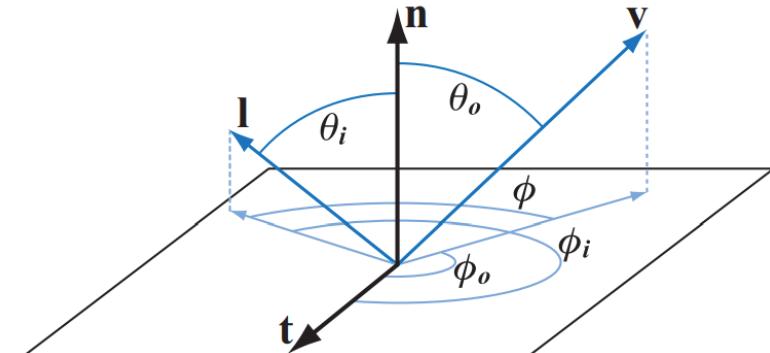
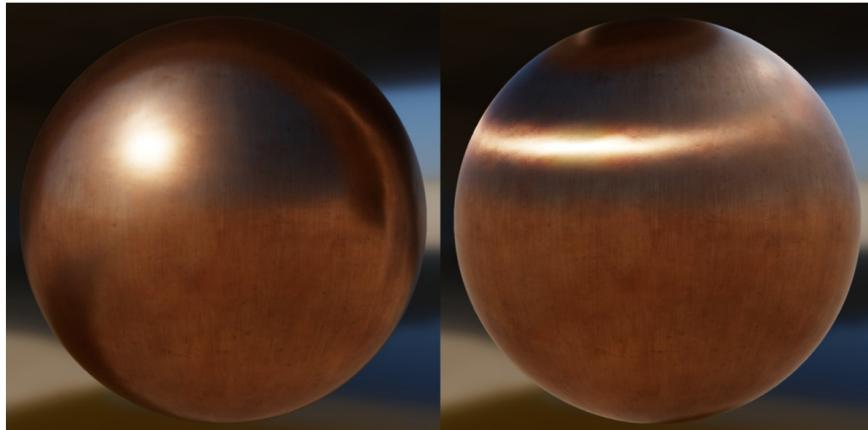


Disney BRDF:

[https://media.disneyanimation.com/uploads/production/publication\\_asset/48/asset/s2012\\_pbs\\_disney\\_brdf\\_notes\\_v3.pdf](https://media.disneyanimation.com/uploads/production/publication_asset/48/asset/s2012_pbs_disney_brdf_notes_v3.pdf)

# Isotropic and anisotropic BRDF

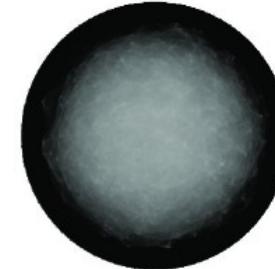
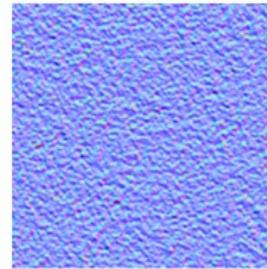
- **Isotropic BRDF:** rotating light and view directions around the surface normal does not affect the BRDF.
  - Incoming and outgoing directions have the same relative angles between them: such BRDF can be parameterized with three angles.
- **Anisotropic BRDF:** reflection behavior changes when light and view vectors are rotated around normal
  - Ward anisotropic BRDF: good for modeling surfaces with anisotropic structure, e.g., hair, grooves in metals



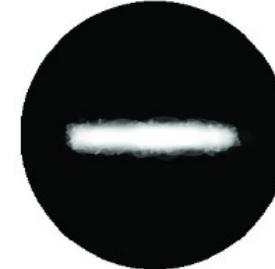
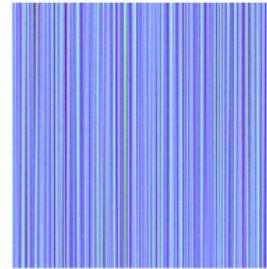
# Isotropic and anisotropic BRDF

- Anisotropic light reflection is present due to the directional underlying surface structure

Isotropic



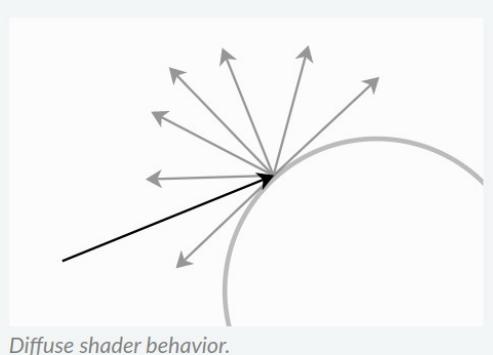
Anisotropic



# Blender is our friend



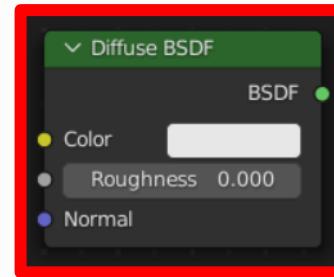
Lambertian reflection.



Diffuse shader behavior.



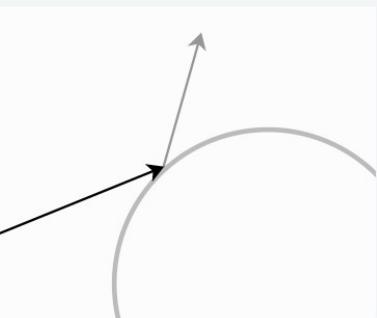
Oren-Nayar reflection.



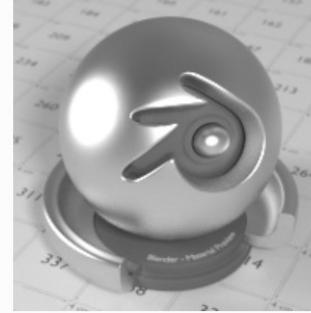
- [https://docs.blender.org/manual/en/latest/render/shader\\_nodes/shader/diffuse.html](https://docs.blender.org/manual/en/latest/render/shader_nodes/shader/diffuse.html)
- [https://docs.blender.org/manual/en/latest/render/shader\\_nodes/shader/glossy.html](https://docs.blender.org/manual/en/latest/render/shader_nodes/shader/glossy.html)
- [https://docs.blender.org/manual/fr/2.79/render/blender\\_render/materials/properties/diffuse\\_shaders.html?highlight=diffuse%20shaders](https://docs.blender.org/manual/fr/2.79/render/blender_render/materials/properties/diffuse_shaders.html?highlight=diffuse%20shaders)



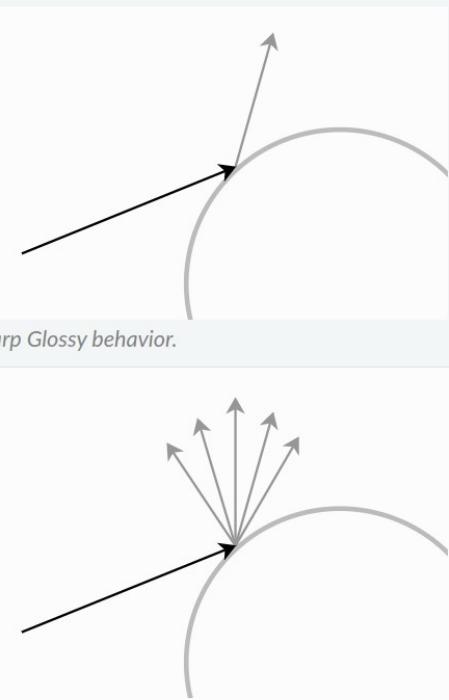
Sharp Glossy example.



Sharp Glossy behavior.



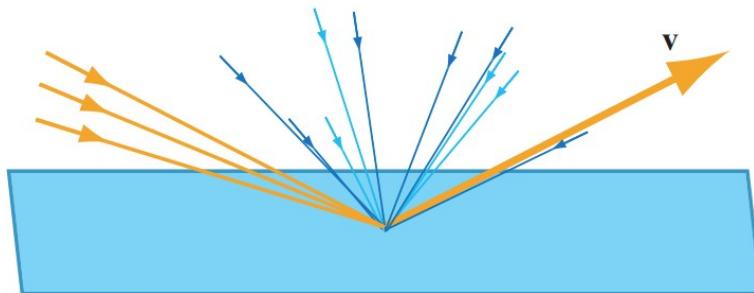
Rough Glossy example.



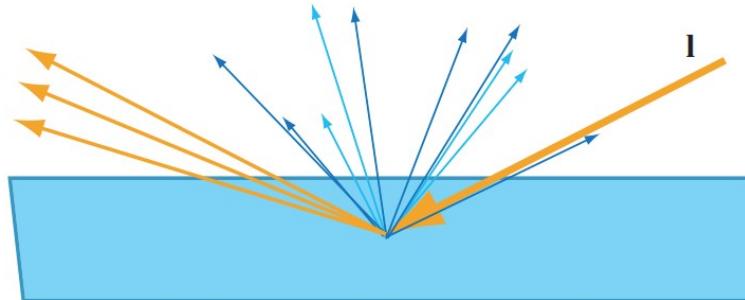
Rough Glossy behavior.

# Note: bidirectionality of BRDF

- “Bidirectional” in BRDF means that given incoming and outgoing direction, we can compute **amount of reflected light** in outgoing direction.
- Further, **bidirectionality** can be used for:
  - Given outgoing (view) direction, it specifies the relative contributions of incoming light
  - Given incoming light direction, it specifies distribution of outgoing light



Contributions of incoming light given view direction



Distribution of outgoing light directions given incoming light direction

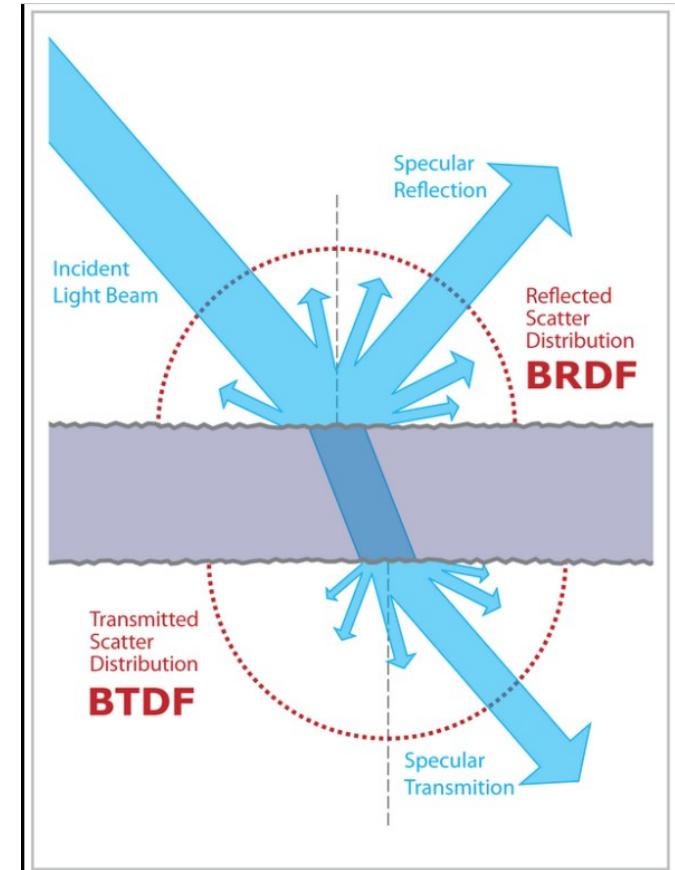
# BTDF



Reflection



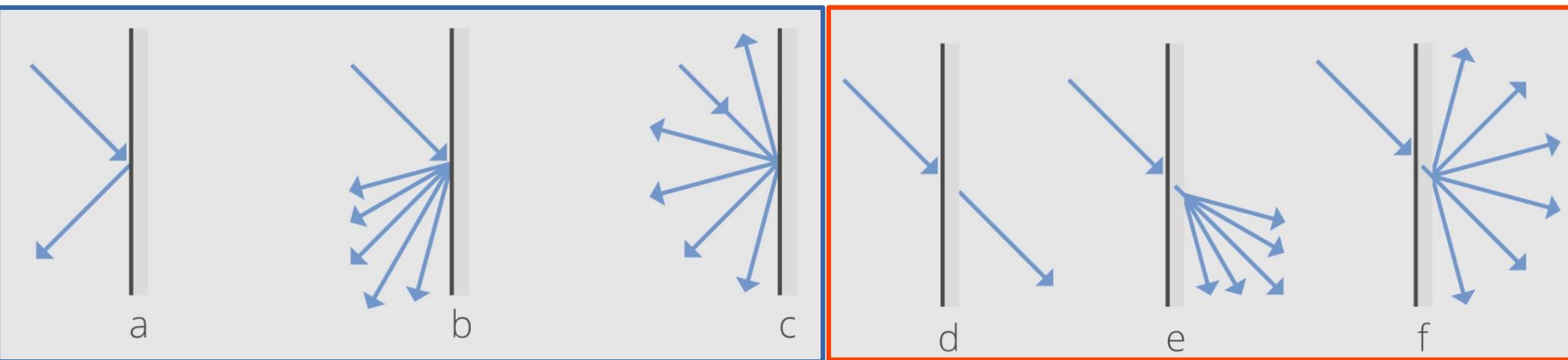
Transmission



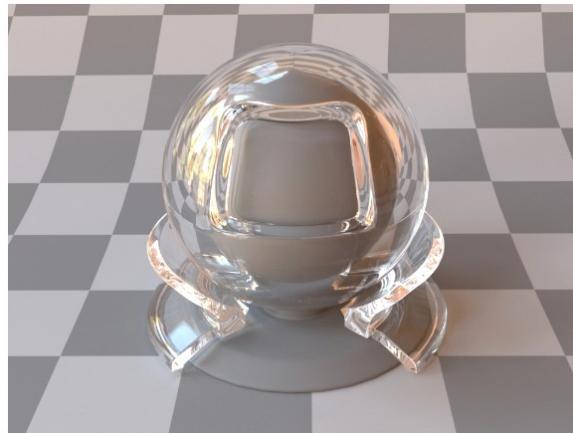
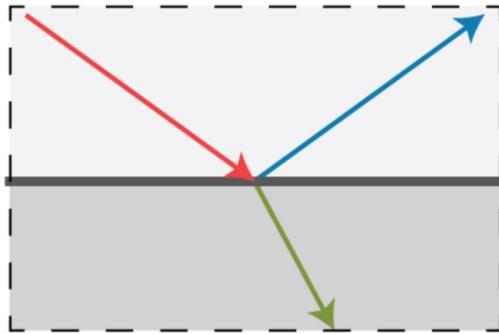
- Scattering function can be separated in reflection and transmission
  - Model describing reflection is called “bidirectional **reflectance** distribution function” – **BRDF**.
  - Model describing transmission is called “bidirectional **transmission** distribution function” – **BTDF**.

# Surface reflection and refraction/transmission

- Specular, glossy and diffuse surface reflection models are similar for refraction/transmission.

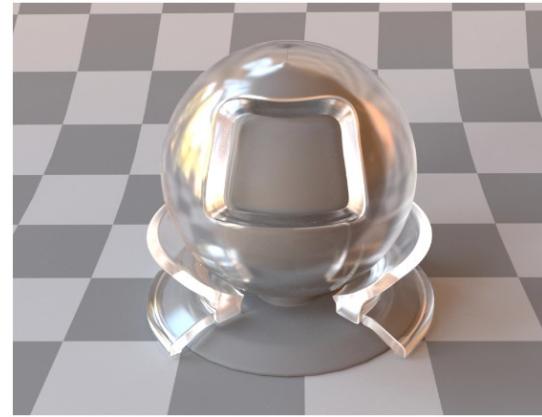
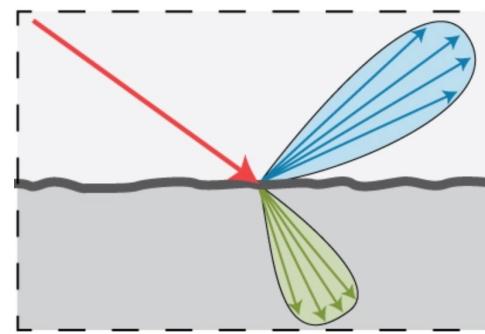


# Common BTDF types



**Perfect specular transmission.**

- Smooth dielectric, e.g., glass



**Glossy transmission.**

- Rough dielectric, e.g., frosted glass
- <https://www.cs.cornell.edu/~srm/publications/EGSR07-btdf.html>

# BTDF: specular transmission

- Light passing from one transparent to another transparent medium with different IOR
  - Refraction – light changes direction – illusion of disproportional/broken object
- Transmission direction depends on:
  - IOR of both media:  $n_1$  and  $n_2$
  - Incident direction:  $\theta_1$

Snell's law: refracted angle

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}.$$

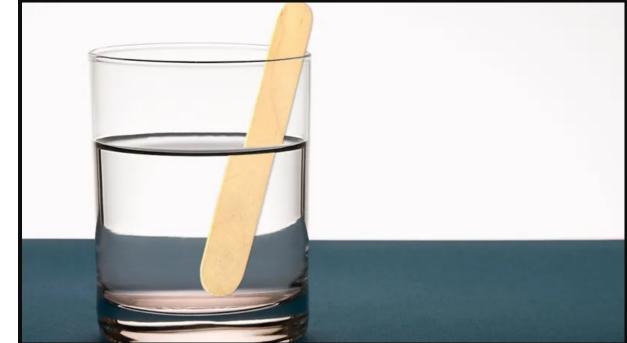
Refraction/transmission direction:

$$T = \eta I + (\eta c_1 - c_2)N.$$

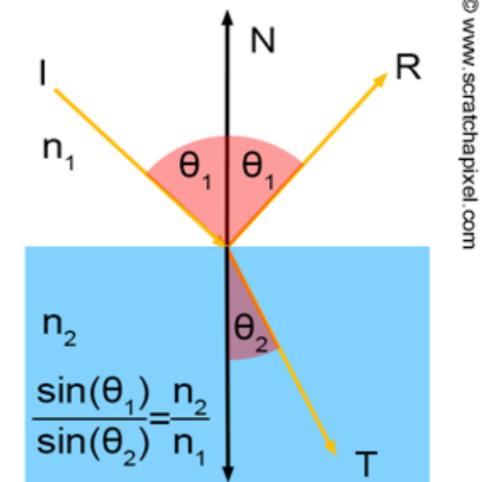
$$\eta = \frac{n_1}{n_2},$$

$$c_1 = \cos(\theta_1) = N \cdot I,$$

$$c_2 = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2(\theta_1)} \rightarrow \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (1 - \cos^2(\theta_1))}$$



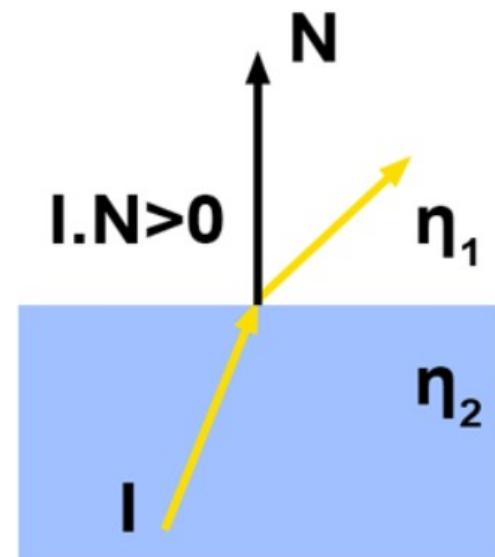
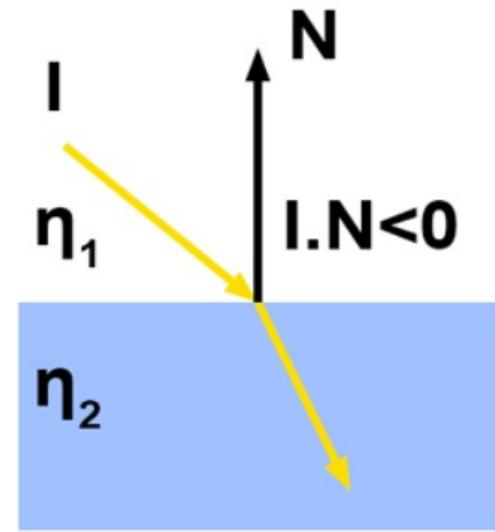
<https://www.britannica.com/science/refraction>



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# BTDF: specular transmission\*

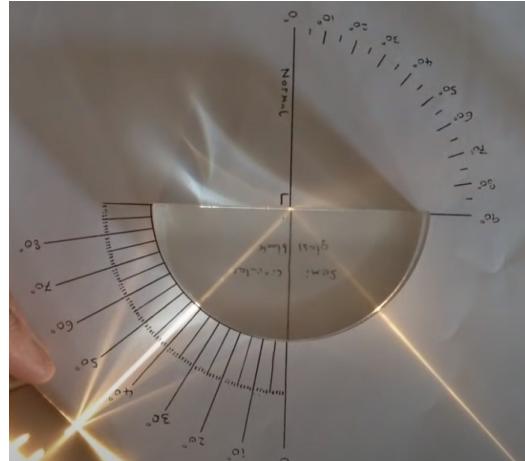
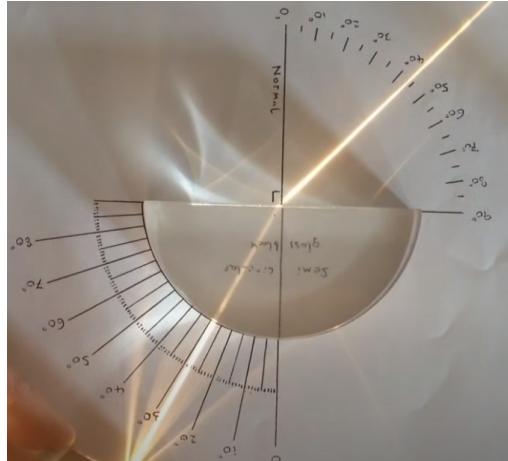
- 3D objects represent surfaces encapsulating media with different IOR
- Space around 3D objects is often considered vacuum → lowest IOR
- Light can travel:
  - From lower to higher IOR medium, e.g., light hitting surface from outside
  - From higher to lower IOR medium, e.g., light leaves volume of water
- If light travels from inside object (from higher to lower IOR) then normal direction must be flipped and then used for computing normal direction
  - Check the sign of dot product between incident ray direction and normal



# BTDF: specular transmission

- When angle of incidence is larger than **critical angle**, then 100% of the light is reflected
  - Happens when light ray passes from higher to lower IOR (e.g., glass-water)
  - Total internal reflection** test: term under square root is negative

$$\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (1 - \cos^2(\theta_1))}$$



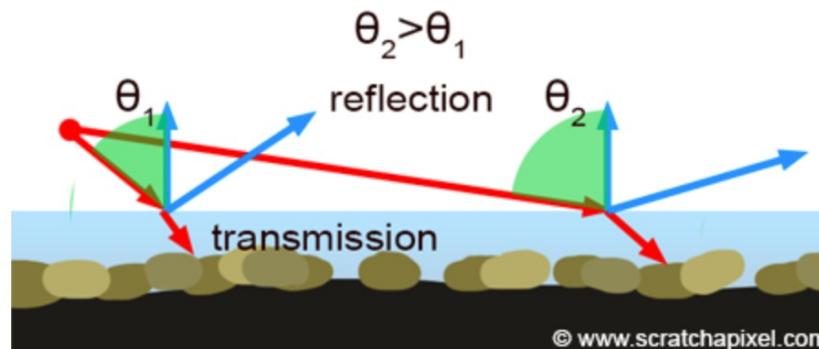
$$\theta_{critical} = \arcsin\left(\frac{n_1}{n_2}\right)$$

# BRDF and BTDF

- How much light is reflected and refracted?
  - Fresnel equations
  - Approximation: Schlick's equation
- Amount of reflected/refracted light depends on:
  - Media IOR:  $\eta_1$  and  $\eta_2$
  - Angle of incidence → amount of transmitted light increases when angle of incidence decreases
- Amount of reflected/transmitted light (in case when total internal reflection doesn't occurs)

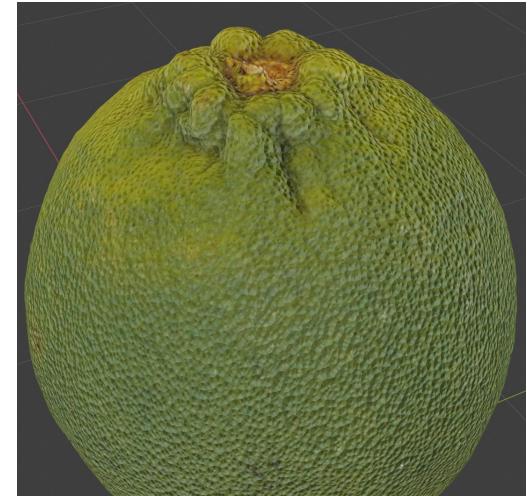
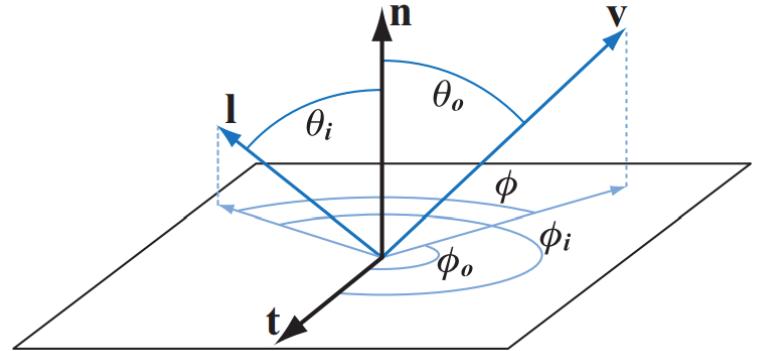


<https://shanesimmsart.wordpress.com/2018/03/29/fresnel-reflection/>



# Normal and BRDF (BTDF)

- BRDF evaluation depends on surface normal
  - Normal defines **basis** – local coordinate system of BRDF
- Note that **perturbing the normal**, would tilt the basis and reflected light would be different!
- This enables modeling small **scale geometrical details**
- Variation of surface normal is task for **texturing**



# Scattering models parameters

- Scattering models have various parameters: color, roughness, etc.
- **Uniform parameters** result in overly smooth and perfect surface – not realistic.
- If BRDF depends on position on which is evaluated then it is called **spatially varying BRDF**
  - Variation of parameters over surface is done using **texturing**



# Scattering models: practical tip

- In graphics, **various scattering models have been developed** (and still are!) to represent surface even more correctly or efficiently.
  - [https://docs.blender.org/manual/en/latest/render/shader\\_nodes/shader/index.html](https://docs.blender.org/manual/en/latest/render/shader_nodes/shader/index.html)
- Choice of the model depends on application and desired appearance.
- Practical tips:
  - **When modeling a material in DCC Tool**, you will be often offered with multiple implementations of basic (or more advanced) models that you further combine to achieve desired material description. In this case, it is good that you are familiar with how they work and their parameters because a lot of time is actually spent on “tweaking” parameters to achieve desired appearance. Understanding parameters of scattering models help very much with upcoming topic: texturing. This is huge and important topic.
  - If you are more interested in **developing your own scattering models** to achieve different appearance (not necessary photo-realistic, rather non-photorealistic which will be discussed later) then understanding of existing scattering models is great foundation to build on: you will see that advancements of scattering models just added more complexity to basic ones.

# More into topic

- Various BSDFs are available in modeling tools for material creation. Core scattering models are deeply integrated into renderer source code and user is provided with an interface to those for combining and creation of complex material.
  - Cycles/EEVEE (Blender): [https://docs.blender.org/manual/en/latest/render/shader\\_nodes/shader/index.html](https://docs.blender.org/manual/en/latest/render/shader_nodes/shader/index.html)
  - Appleseed: <https://appleseed.readthedocs.io/projects/appleseed-maya/en/master/shaders/shaders.html#materials>
- Similarly as for object shape (e.g., mesh), materials are meant to be transferable between applications. Note that it is up to applications renderer which scattering models are supported. Therefore, it is often a case that material defined in one modeling tool can not be easily fully and exactly transferred to another application.
  - This requires matching supported scattering functions between applications. Example is Blender to Unity
- In order enable easier communication and transfer, standardized BSDF is created and supported by different applications.
  - Principled BSDF: Blender to Godot: [https://docs.godotengine.org/en/3.0/tutorials/3d/spatial\\_material.html](https://docs.godotengine.org/en/3.0/tutorials/3d/spatial_material.html)
  - MaterialX: <https://materialx.org/>
- Tendency is towards integrations of material modeling tool standards into game engines for easier transfer
  - Example: <https://substance3d.adobe.com/plugins/substance-in-unreal-engine/>

# More into topic

- Note that BSDFs describe interaction of light with surface (opaque or transparent)
- Other objects have specific appearance due to sub-surface scattering. Example for such material is wax. Such material is not transparent, but it is important what happens under surface since some light scatters outside and influences appearance – such material is called **translucent** and requires **BSSRDF\***.
  - For simpler applications these materials can be approximated with diffuse BRDF (note that diffuse scattering is actually a result of sub-surface scattering and re-emitting as well as surface roughness)
  - Different approaches take phenomenological approach where they model what we observe in reality. They result in realistic appearance but are not physically correct:  
<https://www.ea.com/frostbite/news/approximating-translucency-for-a-fast-cheap-and-convincing-subsurface-scattering-look>
  - Finally, physically based approaches model actual scattering of light inside of surface and its absorption, reflection and re-emission.
- Next to translucent surfaces, there are many phenomena for which it is important to model light scattering inside a volume. These are called **volumetric rendering** approaches and they rely both on scattering function and light transport.
  - Volumetric rendering is highly researched and developed field:  
<http://advances.realtimerendering.com/s2015/The%20Real-time%20Volumetric%20Cloudscapes%20of%20Horizon%20-%20Zero%20Dawn%20-%20ARTR.pdf>
- Often cloth is important material to render. **BRDF for cloth** (e.g., sheen BRDF) are developed and investigated.
- <https://www.realtimerendering.com/#visapp>

\* [https://www.pbr-book.org/3ed-2018/Volume\\_Scattering/The\\_BSSRDF](https://www.pbr-book.org/3ed-2018/Volume_Scattering/The_BSSRDF)

# More into topic

- We discussed light scattering on geometric optics. **Wave optics** is active research area.
  - Required if physical simulation of wave-effects such as diffraction is needed.
  - Color is then not described with RGB triplets rather using spectral representation and wavelength dependence
  - More information: [https://ssteinberg.xyz/2022/04/03/practical\\_plt/](https://ssteinberg.xyz/2022/04/03/practical_plt/)

# Summary questions

- [https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/7\\_material](https://github.com/lorentzo/IntroductionToComputerGraphics/tree/main/lectures/7_material)

# Literature

- <https://github.com/lorentzo/IntroductionToComputerGraphics/wiki>
- <https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-to-shading/what-is-shading-light-matter-interaction.html>