Lecture 13: Rasterization-based rendering

DHBW, Computer Graphics

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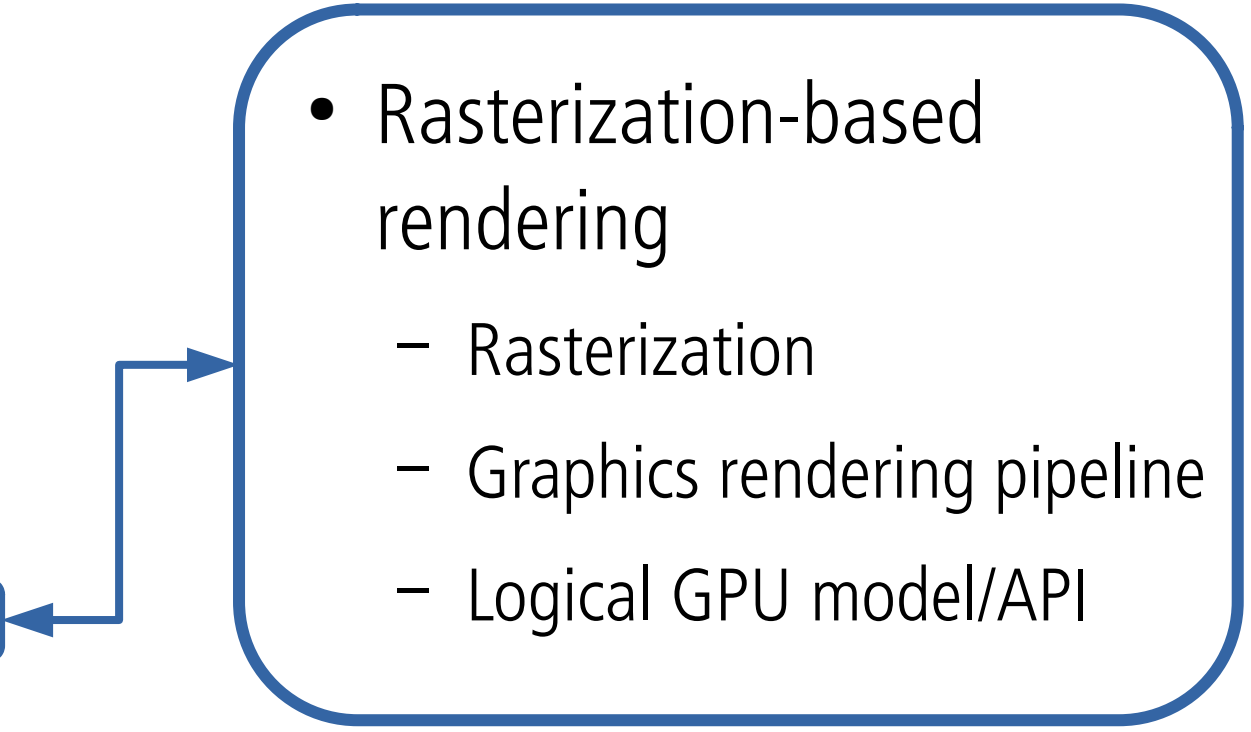
Syllabus

- 3D scene
  - Object
  - Light
  - Camera
- Rendering
  - Ray-tracing based rendering
  - Rasterization-based rendering
- Image and display



Rasterization

Rendering

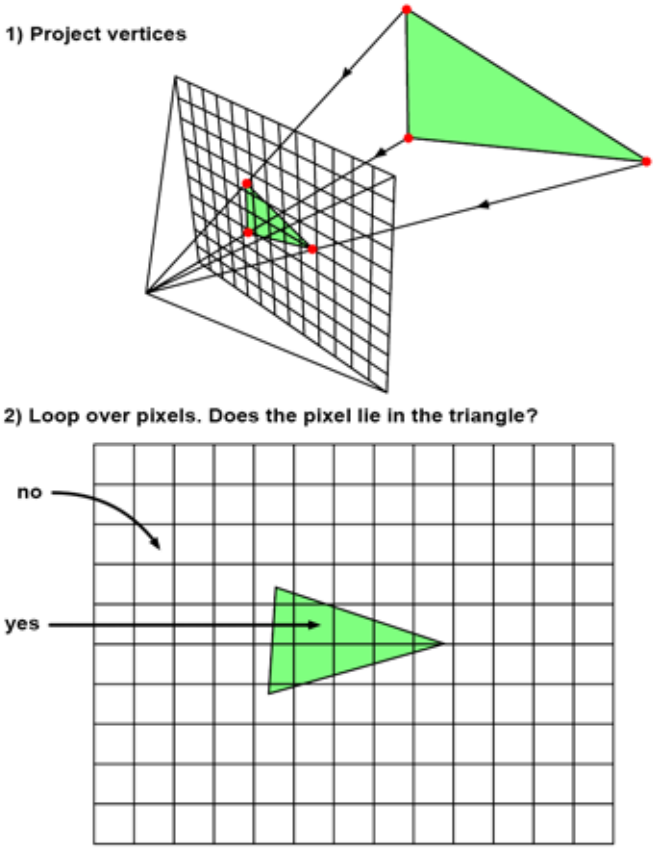
- **Rendering: process of creating 2D image from 3D scene**
- Two main **tasks of rendering**:
  - **Visibility solving**: compute objects visible from camera
  - **Shading**: compute colors of visible objects

Visibility problem

- Visibility is geometrical problem using object shape representation to determine which points in 3D scene are visible to each other
- We assume triangulated mesh as object shape representation for efficient visibility solving
  - Different shape representations can be converted to triangulated mesh using triangulation
- Main approaches for solving visibility:
  - Ray-tracing
  - Rasterization

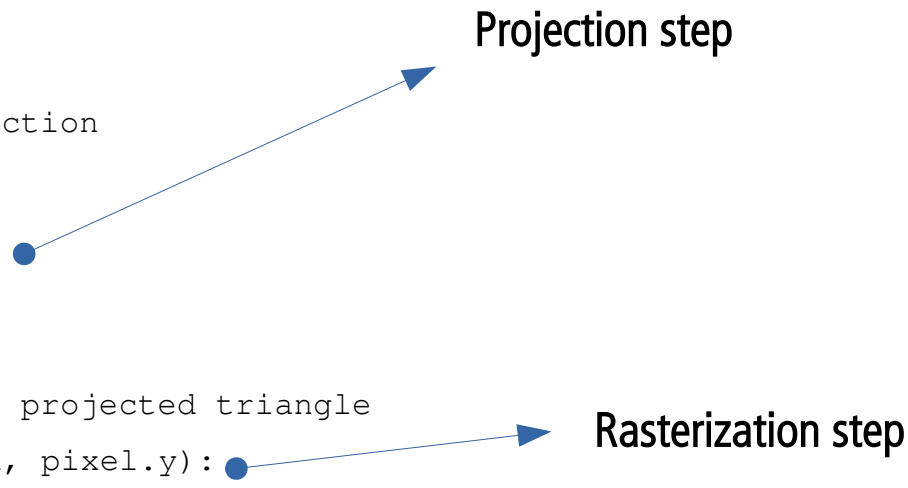
Rasterization: visibility solving

- To **find objects visible from camera, rasterization** can be decomposed in **two main stages**:
  - **Triangle projection**: project 3D vertices of mesh triangles onto image plane using perspective projection matrix
  - **Triangle rasterization**: for each pixel of image plane check if it is contained in projected triangle



Rasterization: algorithm

```
for each triangle in the scene:
    // STEP 1: project vertices using perspective projection
    vector v0 = perspectiveProjection(triangle.v0);
    vector v1 = perspectiveProjection(triangle.v1);
    vector v2 = perspectiveProjection(triangle.v2);
    for each pixel in the image:
        // STEP 2: compute if pixel is contained in the projected triangle
        if(pixelCointainedIn2DTriangle(v0,v1,v2,pixel.x, pixel.y):
            image(pixel.x, pixel.y) = triangle.color
```



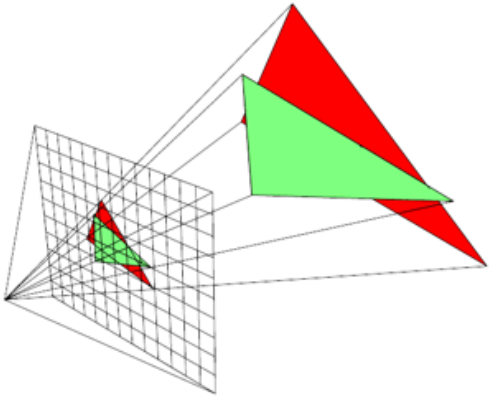
- Object centric approach: first loops over triangles, then loops over image pixels
- Note: looping over all triangles and all pixels is naive approach: further optimizations are introduced

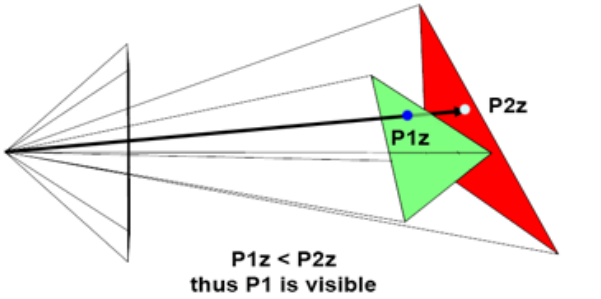
Rasterization: buffers

- **Rasterization**: decomposing triangles into pixels forming raster image
  - Formally: rasterization of triangles into an image or frame buffer
- **During rasterization rendering, pixel information is stored in buffers** → 2D array of pixels which has same size as final image
- **Frame buffer is 2D array, an image, containing the result of rendering ready for display**
  - Before rendering, frame-buffer is created with all pixels set to black color
  - During rendering, when triangles are rasterized, for each pixel that overlaps triangle store that triangle color in frame-buffer at that pixel location
  - After rendering, when all triangles are rasterized and pixels are processed, the frame-buffer will contain the final image of the scene

Rasterization: z-buffer

- **Multiple triangles may overlap the same pixel in image**
- To correctly solve visibility, rasterization must find the first visible triangle
- This is done using **z-buffer or depth buffer**
  - 2D array with same dimensions as image but instead of colors it contains array of floating point numbers
  - z-buffer stores the distance of each pixel to the nearest object in the scene
- **Visibility computation with z-buffer:**
  - Before rendering z-buffer is initialized to a very large number
  - When pixel overlaps triangle, compute distance to this triangle and compare it to the distance in z-buffer at that pixel position
  - If distance to triangle is smaller then value in z-buffer, then triangle is visible for that pixel: update z-buffer with this distance and frame-buffer with color of this triangle at this pixel position
- Alternative: Painter's algorithm





Rasterization: frame-buffer and z-buffer

```
color frame_buffer = new float[imageWidth, imageHeight];
float z_buffer = new float[imageWidth, imageHeight];
for (int i = 0; i < imageWidth * imageHeight; ++i)
    frame_buffer[i] = color(0,0,0);
    z_buffer[i] = INF;
for each triangle in scene:
    vector v0 = perspectiveProjection(triangle.v0);
    vector v1 = perspectiveProjection(triangle.v1);
    vector v2 = perspectiveProjection(triangle.v2);
    float z;
    if (pixelContainedIn2DTriangle(triangle.v0, triangle.v1, triangle.v2, pixel.x, pixel.y, z):
        if (z < z_buffer(pixel.x, pixel.y)):
            z_buffer(pixel.x, pixel.y) = z;
            frame_buffer(pixel.x, pixel.y) = triangle.color
```

Projection stage

Rasterization stage

Rasterization: projection stage

- Goal: transform triangle vertices (x,y,z) from camera space to raster (screen) space (i,j)
  - Objects are from world space transformed to camera space using world-to-camera matrix (e.g., inverse of transformation matrix constructed using camera look-at notation)
- World to camera:
- Camera space to screen space:
- Screen space to NDC space:
- NDC space to raster space:

TODO

- https://www.scratchapixel.com/lessons/3d-basic-rendering/rasterization-practical-implementation/projection-stage.html

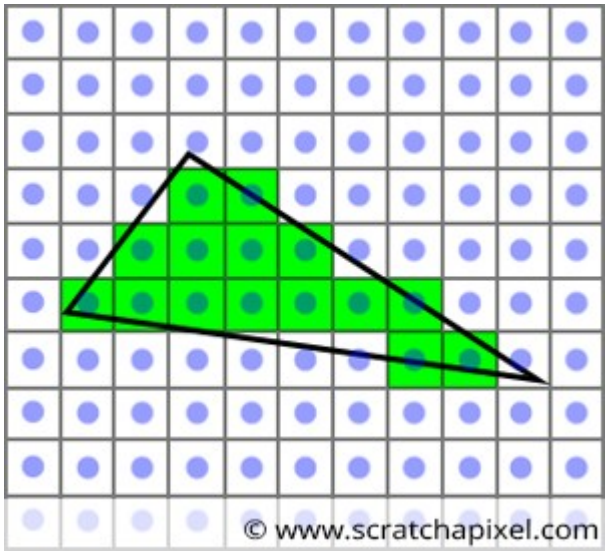
Rasterization: projection stage

- Show the whole way of transforming triangle vertex to pixel coordinate
  - https://www.scratchapixel.com/lessons/3d-basic-rendering/computing-pixel-coordinates-of-3d-point/mathematics-computing-2d-coordinates-of-3d-points.html
- Say that this is what gra vertex of triangulated mesh → ne is rendered

Merge with prev

Triangle rasterization

- Goal: convert projected triangle to a 2D image:
  - Loop over all pixels in 2D image to determine which pixels overlap the projected triangle
    - **Inside-outside (aka coverage) test**
    - Alternative: scanline rasterization → based on Bresenham algorithm for drawing lines
  - For each pixel that overlaps triangle, compute:
    - **Barycentric coordinates**
    - **Depth**
    - Etc.



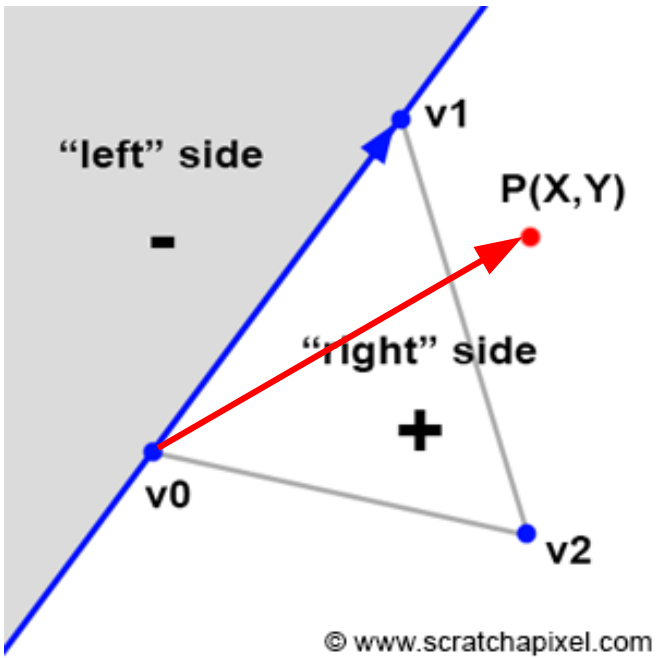
Inside-outside test: edge function

- Triangle edges can be seen as lines splitting planes
- Edge function returns (clockwise winding of triangle vertices):
  - positive number if point P is on left side of the line
  - Negative number if point is on the right side of the line
  - Zero if point is on the line

$$E_{01}(P) = (P.x - v0.x) * (V1.y - V0.y) - (P.y - V0.y) * (V1.x - V0.x).$$

Edge function interpretation:

- Magnitude of cross product between `(v1-v0)` and `(P-v0)`
- Determinant of 2x2 matrix defined with components of vectors `(v1-v0)` and `(P-v0)`

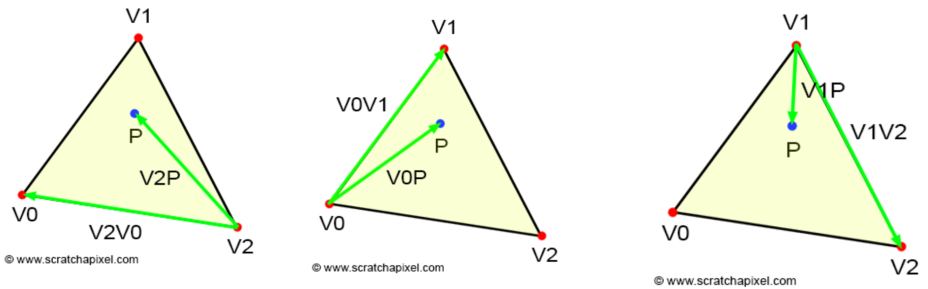


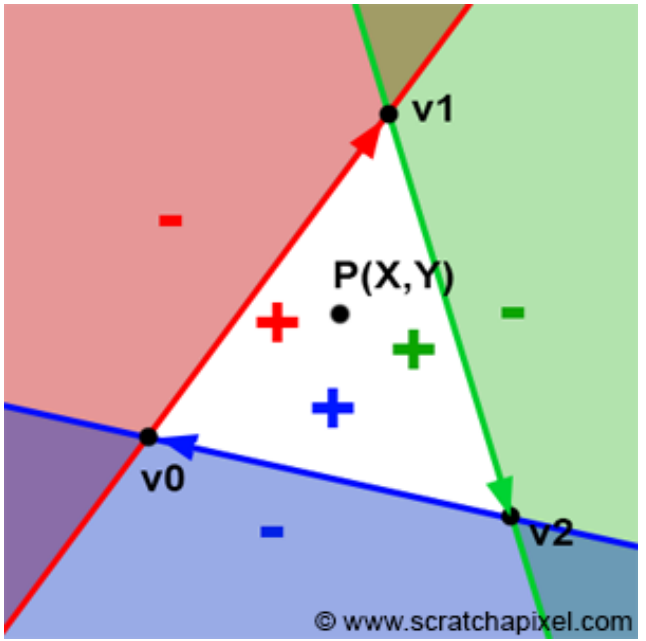
https://www.scratchapixel.com/lessons/3d-basic-rendering/rasterization-practical-implementation/rasterization-stage.html

Inside-outside test: edge function test

- Point must be tested for all edges
- If edge function returns positive value for all edges, then point is inside triangle
- Note: winding order of triangle matters
  - For counterclockwise winding, the sign of edge function is inverted

$$E_{01}(P) = (P.x - V0.x) * (V1.y - V0.y) - (P.y - V0.y) * (V1.x - V0.x),$$
$$E_{12}(P) = (P.x - V1.x) * (V2.y - V1.y) - (P.y - V1.y) * (V2.x - V1.x),$$
$$E_{20}(P) = (P.x - V2.x) * (V0.y - V2.y) - (P.y - V2.y) * (V0.x - V2.x).$$





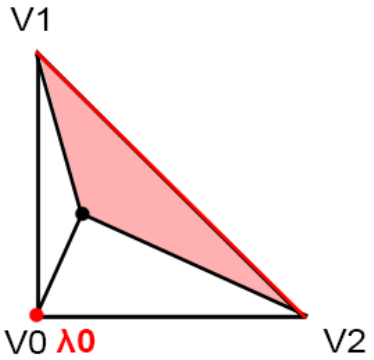
Barycentric interpolation and edge function

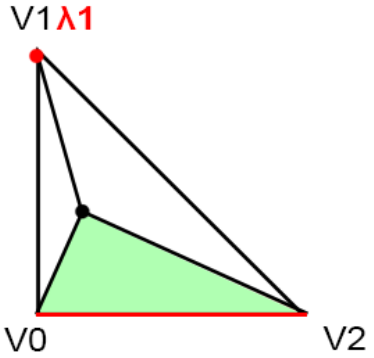
- Barycentric coordinates $(\lambda_1, \lambda_2, \lambda_3)$ can be computed as the ratio between area of sub-triangles and the whole triangle.
- Area of sub-triangle is calculated using edge function.

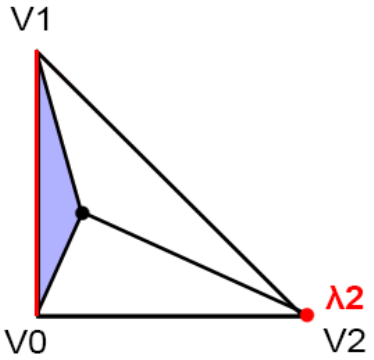
$$\lambda_0 = \frac{Area(V1, V2, P)}{Area(V0, V1, V2)},$$
$$\lambda_1 = \frac{Area(V2, V0, P)}{Area(V0, V1, V2)},$$
$$\lambda_2 = \frac{Area(V0, V1, P)}{Area(V0, V1, V2)}.$$

$$Area_{tri}(V1, V2, P) = \frac{1}{2} E_{12}(P),$$
$$Area_{tri}(V2, V0, P) = \frac{1}{2} E_{20}(P),$$
$$Area_{tri}(V0, V1, P) = \frac{1}{2} E_{01}(P).$$

$$E_{01}(P) = (P.x - V0.x) * (V1.y - V0.y) - (P.y - V0.y) * (V1.x - V0.x),$$
$$E_{12}(P) = (P.x - V1.x) * (V2.y - V1.y) - (P.y - V1.y) * (V2.x - V1.x),$$
$$E_{20}(P) = (P.x - V2.x) * (V0.y - V2.y) - (P.y - V2.y) * (V0.x - V2.x).$$







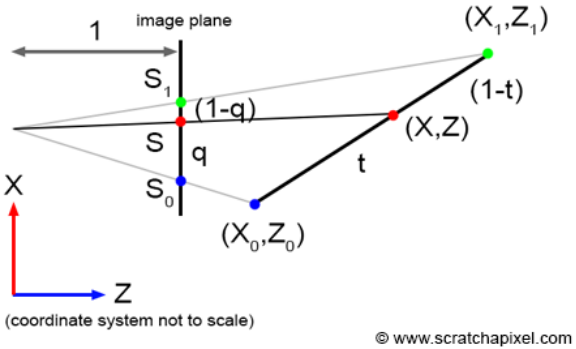
Depth and visibility

- **If pixel overlaps two or more triangles use z-buffer aka depth-buffer ➞ visibility solving**
  - Compute z-coordinate or depth of the point on the triangle that the pixel overlaps
  - Compare computed depth with depth value stored at z-buffer at that pixel location
  - If current depth value is smaller than value from z-buffer, update frame-buffer with color of current triangle and z-buffer with depth of current point on triangle
- **Once all triangles have been processed, depth buffer contains distance to visible object from camera**
  - Depth is useful for visibility solving, shading and post-processing (e.g., depth of field, fog, shadow mapping, etc.)

Depth and barycentric coordinates

- In projection stage, **original camera space z-coordinate is stored for each vertex of a triangle**
- **For current pixel overlapping triangle, z-value is found using barycentric interpolation**
- Since **perspective projection preserves lines but not distances**, the solution is to **compute the inverse of P z-coordinate**:

$$\frac{1}{P.z} = \frac{1}{V0.z} * \lambda_0 + \frac{1}{V1.z} * \lambda_1 + \frac{1}{V2.z} * \lambda_2.$$



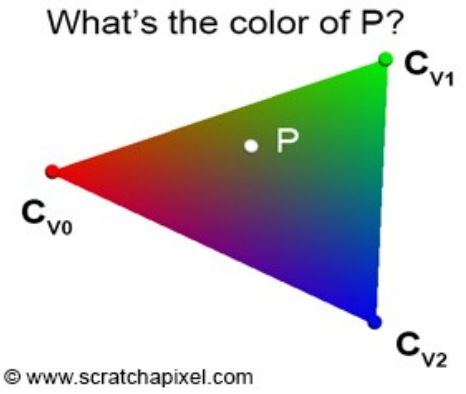
Perspective projection doesn't preserve distances.

Barycentric interpolation and vertex attributes

- **Rasterization stage ends once it is determined for all pixels if they overlap triangle** (inside-outside test)
- Triangle vertices can also have assigned **vertex attributes: colors, normals, texture coordinates, etc.**
- **Vertex attributes are used during shading** for computing color of pixels overlapping the triangle
  - Vertex attributes must be interpolated across surface of a triangle when it is rasterized
- **Barycentric coordinates** $(\lambda_1, \lambda_2, \lambda_3)$ **are used to interpolate vertex attributes** for any pixel overlapping the triangle
  - Example: interpolating color at point P where each vertex has color attribute: C_{v0}, C_{v1}, C_{v2}

$$C_P = \lambda_0 * C_{V0} + \lambda_1 * C_{V1} + \lambda_2 * C_{V2}.$$

$C_{v0} = (1,0,0)$
$C_{v1} = (0,1,0)$
$C_{v2} = (0,01,)$



Barycentric coordinates: perspective projection

- Due to perspective projection, it is required to preform **perspective correction** for vertex attribute interpolation
- Example: perspective correct color vertex attribute interpolation

$$C = Z(\frac{C_{v0}}{Z_{v0}}\lambda_0 + \frac{C_{v1}}{Z_{v2}}\lambda_1 \frac{C_{v2}}{Z_{v2}}\lambda_2)$$

  - Z – current pixel depth
  - Z_{v0}, Z_{v1}, Z_{v2} – triangle vertices depth
  - C_{v0}, C_{v1}, C_{v2} – triangle vertices colors

Rasterization-based rendering on GPU

- Rasterization is elegant algorithm for solving visibility:
  - Projecting vertices of triangulated mesh onto image plane
  - Looping over image pixels to find which pixels lie in projected triangle
- Both tasks are well suited for GPU and can be performed fast
  - GPU rendering uses rasterization approach for solving visibility
  - GPU rendering is conceptually described with graphics rendering pipeline
  - Graphics rendering pipeline is foundation for real-time rendering

Rasterization-based rendering on GPU

- Rasterization-based rendering technique is deeply integrated in GPU ha
rendering. Raytracing-based rendering is purely implemented on CPU.
- Rasterization-based rendering can also be completely implemented on
purposes this is useful to understand all the aspects of it. There are som
cases which also benefit from CPU implementation of rasterization based-rendering*.
- Almost all professional software which uses rasterization-based rendering is using GPU hardware implementation which can be further programmed using graphical APIs such as OpenGL, Vulkan, DirectX, Metal, etc.

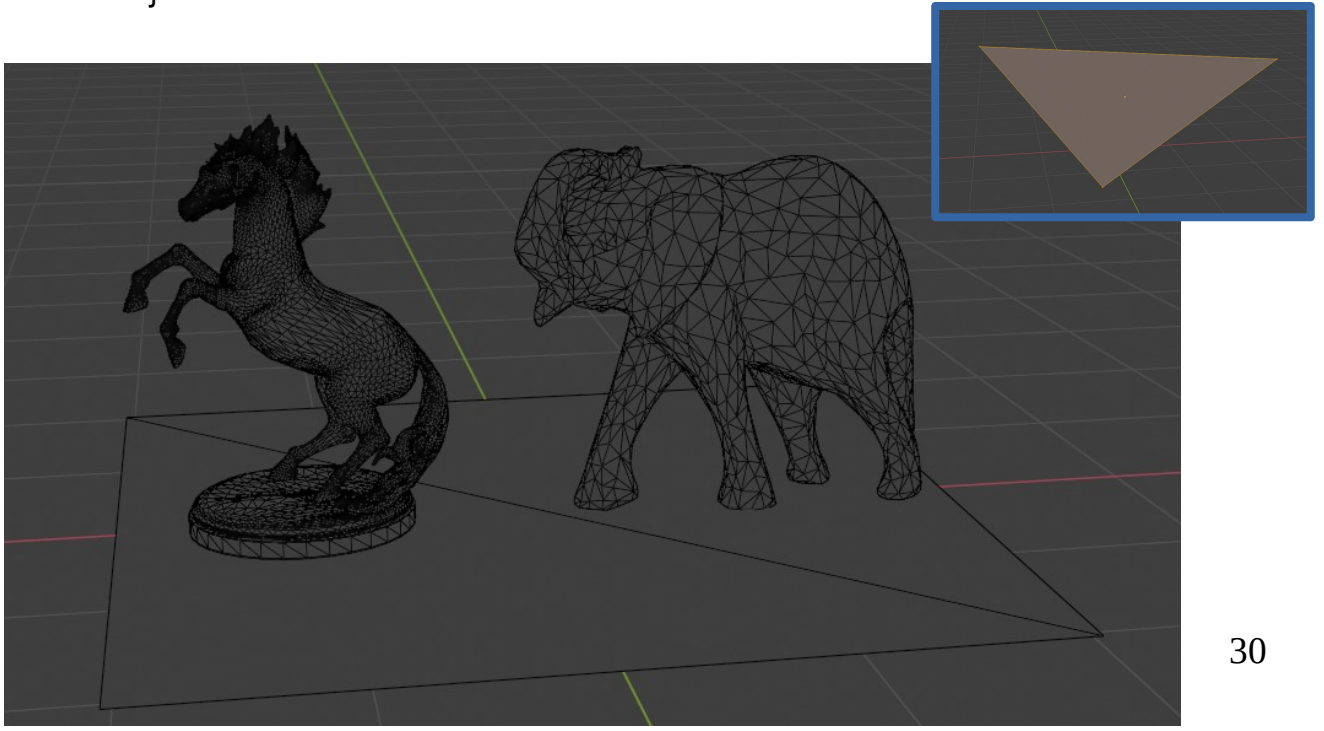
TODO

* Cases when objects that are rendered are smaller than pixel. Those are advanced topics and are related to point rendering (https://www.cg.tuwien.ac.at/research/publications/2022/SCHUETZ-2022-PCC/) or micropolygon rendering (https://docs.unrealengine.com/5.0/en-US/nanite-virtualized-geometry-in-unreal-engine/ or https://graphics.pixar.com/library/Reyes/)

Graphics rendering pipeline: conceptual overview

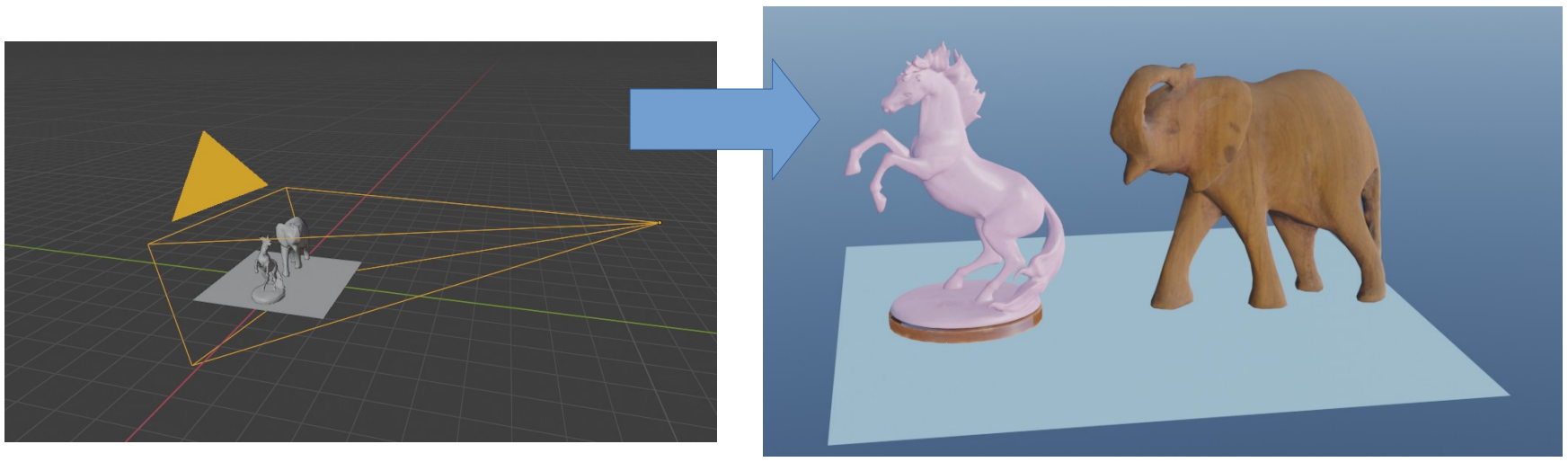
A intro note

- For graphics rendering pipeline, we assume that all objects in 3D scene, that is, their shape is triangulated mesh.
- When describing graphics rendering pipeline, we will then focus on one triangle or one vertex – but keep in mind that this is also done for all triangles for all objects/models in 3D scene



Graphics rendering pipeline

- Main function of graphics rendering pipeline (shortly pipeline): render a 2D image from 3D scene (objects, lights, cameras)
  - Underlying tool for real-time rendering
- Rendering:
  - Visibility calculation (which objects are visible from camera)
  - Shading calculation (light-matter calculation and light transport)



Virtual camera frustum and 3D objects.

Visibility: locations and shapes are determined by object geometry and camera placement.

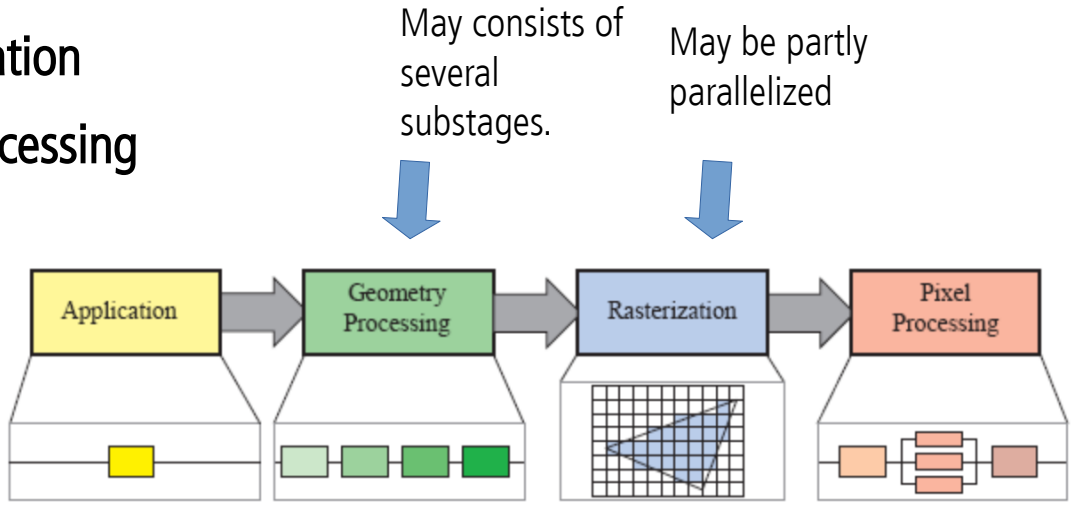
Shading: appearance of objects detpends on materials, lights, textures, etc.

Graphics rendering pipeline

- Graphics rendering pipeline is decomposed in smaller steps or stages:
  - Each stage performs part of larger task
  - Input of any given stage depends on the output of previous stage
  - Sequence of stages forms rendering pipeline
  - Pipeline stages, although working in parallel, are stalled until slowest stage is finished.
    - Slowest stage is said to be **bottleneck**. Stages which are waiting are called **starved**.

Graphics rendering pipeline: stages

- Graphics rendering pipeline can be coarsely divided in four stages:
  - **Application**
  - **Geometry processing**
  - **Rasterization**
  - **Pixel processing**



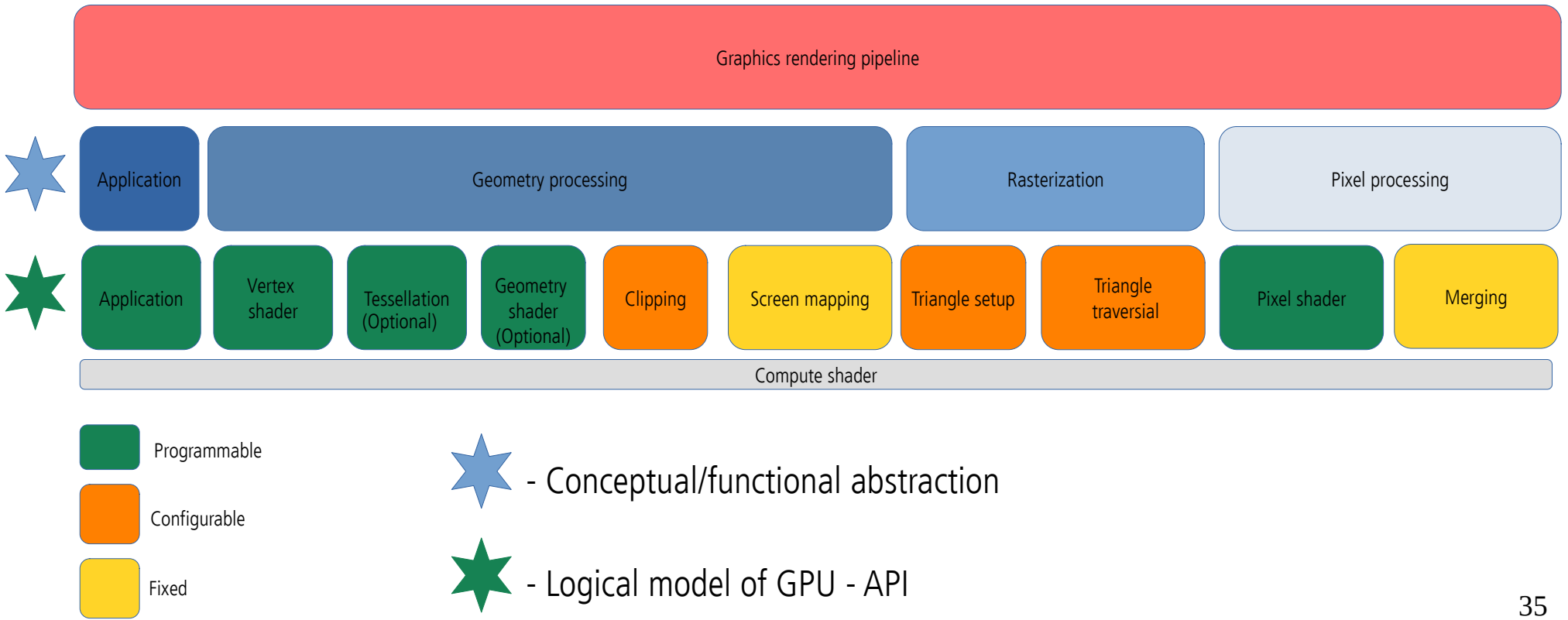
Certain stages are fixed, some are configurable to certain extent and some are fully programmable. Trend is towards programmability and flexibility.

https://www.realtimerendering.com/

Pipeline abstraction

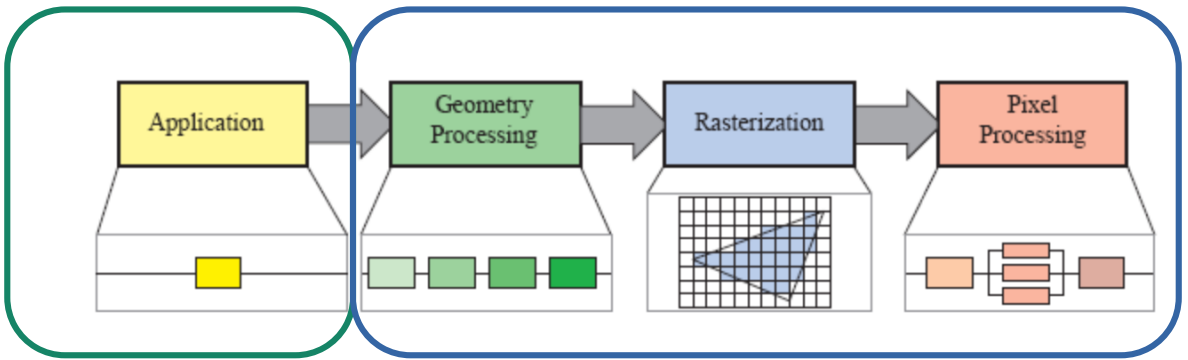
- **Functional/conceptual stages –** task to be performed but not how
  - **Physical/Implemented stages –** how are functional stages implemented in hardware and exposed to the user as API.
- **Logical model of GPU** – exposed to a programmer by API. Physical model is up to the hardware vendor.

Graphics rendering pipeline overview



Pipeline: CPU and GPU

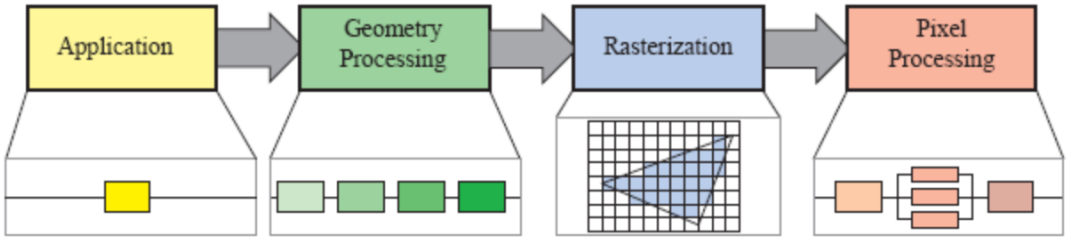
- CPU implements application stage.
  - CPUs are optimized for various data structures and large code bases, they can have multiple cores but in mostly serial fashion (SIMD processing is exception)
- GPU* implements conceptual geometry processing, rasterization and pixel processing stage.
  - GPUs are dedicated to large set of processors called **shader cores** – small processors that do independent and isolated task (no information sharing and shared writable memory) in a massively parallel fashion**.



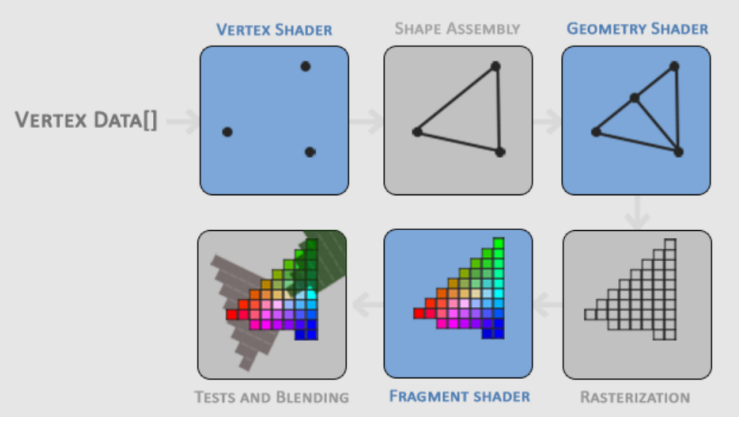
* GPU – graphics processing unit, term coined by NVIDIA to differentiate GeForce 256 from previous rasterization chips. From then on, this term is still used.

** Memory access and transfer is huge topic and efficient handling of data transfer from CPU memory to GPU memory is important for efficient rendering. The term latency describes how much processor must wait for data access.

Pipeline: speed run



- Application stage
  - Driven by application implemented in software running on CPU
  - e.g., for modeling tool application, user input is handled on application stage
- Geometry processing
  - Geometry handling (triangulated mesh): transformations, projections; what, how and where it is drawn
  - Implemented on GPU that contains many programmable shader cores
- Rasterization
  - Takes 3 vertices which form a triangle, finds all pixels inside triangle and forwards to next phase
  - Fixed implementation on GPU
- Pixel processing
  - Shading operation per pixel: calculating color and depth testing → programmable GPU shading cores
  - Per-pixel operations, e.g., blending new and old pixel color → implemented on GPU



https://learnopengl.com/Getting-started/Hello-Triangle

1. Application stage

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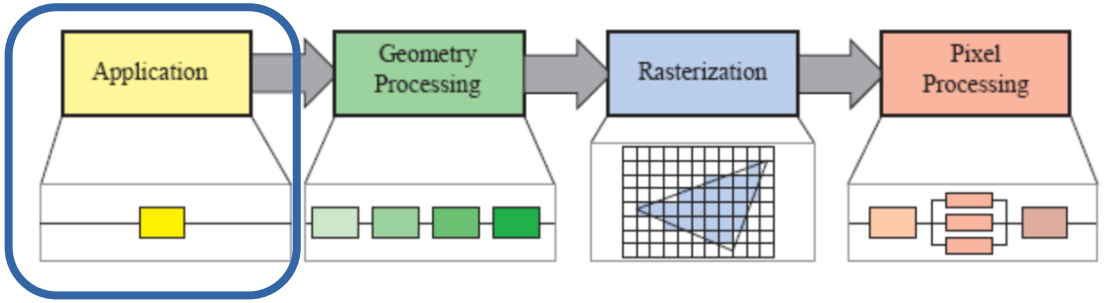
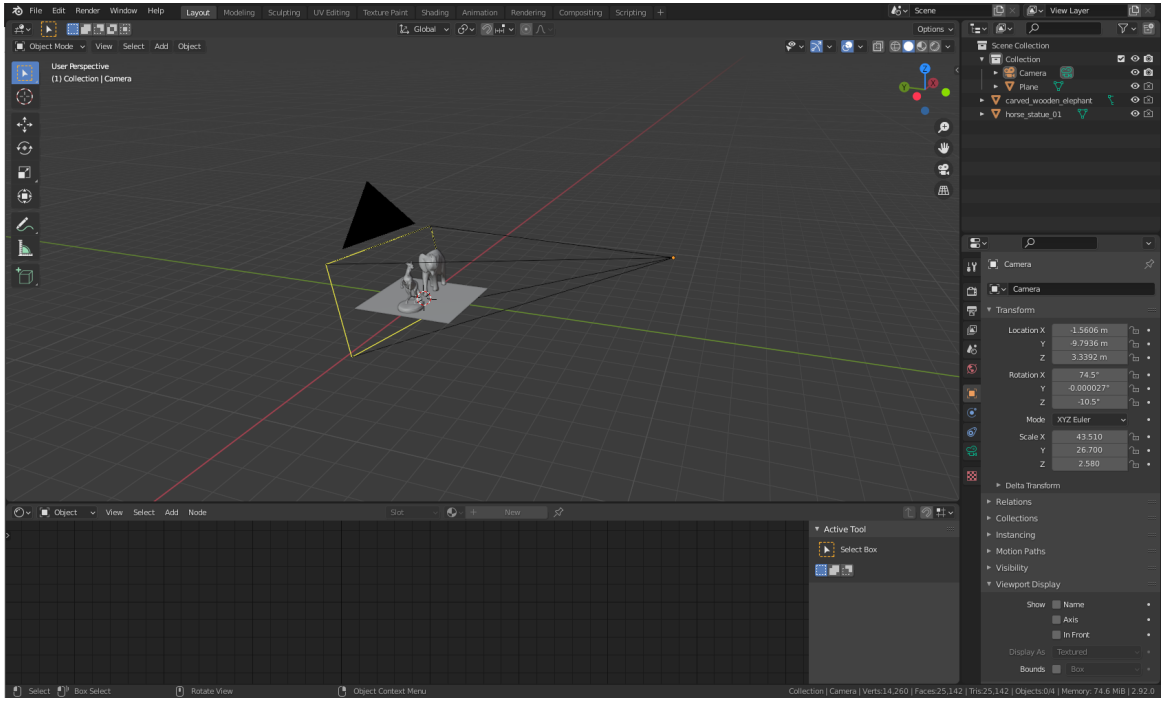
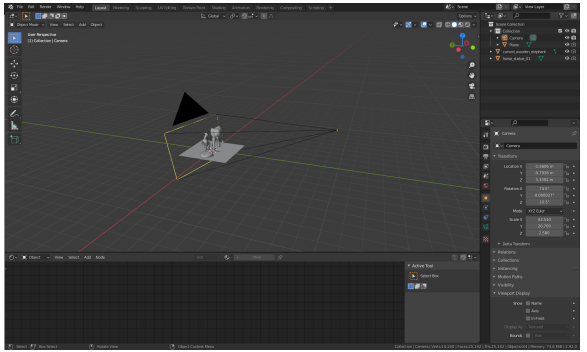


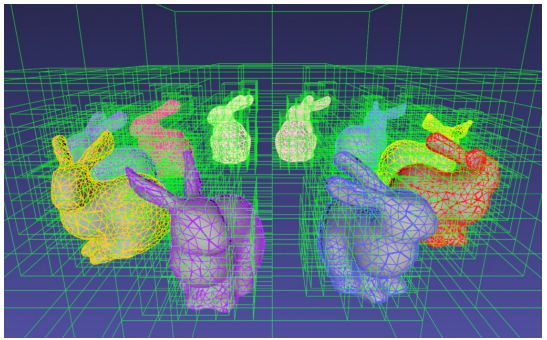
- Driven by application (e.g., modeling tool) and typically implemented on CPU (optionally on multiple threads).
  - Developer has full control over what happens in this stage and how it is implemented



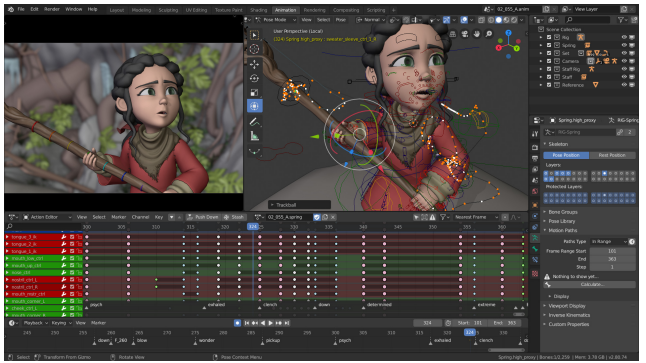
1. Application stage

- Application stage includes tasks such as:
  - taking care of user input from keyboard, mouse, etc. for interaction
  - animation
  - physics simulation
  - collision detection – detection of collision between two objects and generating response
  - 3D scene acceleration structures (e.g., culling algorithms)
  - Composition
  - Other tasks depending on application which subsequent stages of pipeline can not handle





https://www.kitware.com/octree-collision-imstk/



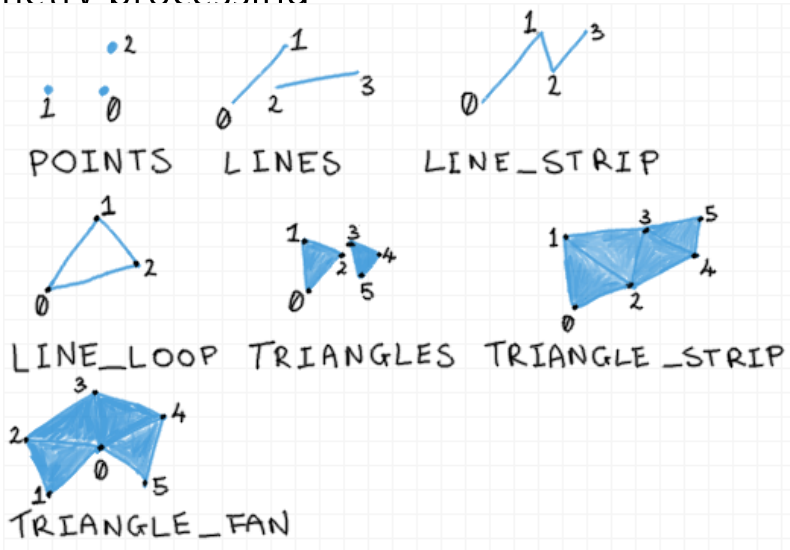
https://www.blender.org/features/

1. Application stage

- On application stage the following is defined:
  - Objects information
    - Geometry: vertices, normals, texture coordinates
      - Transformation matrix
    - Material parameters (e.g., color, roughness, normal map, etc.)
  - Camera information
    - Transformation matrix (e.g., from look at notation)
    - Camera parameters (e.g., focal length)
  - Light information
    - Position and/or direction
    - Intensity, color

1. Application stage

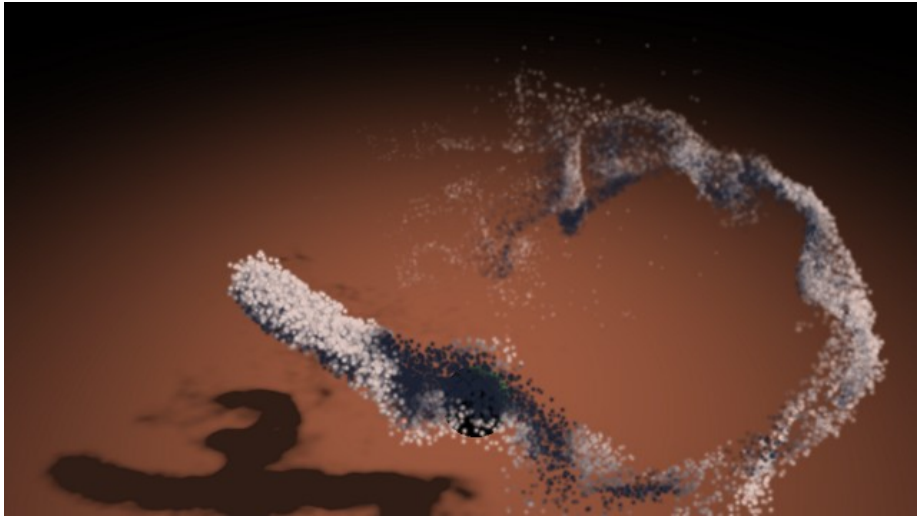
- Application phase outputs:
  - geometry to be rendered – **rendering primitives:** points, lines and triangles
  - Vectors, matrices, textures describing object, light and camera parameters
- Efficiency of this stage is propagated to further stage: geometry processing
  - e.g., amount of geometry (triangles) sent to GPU



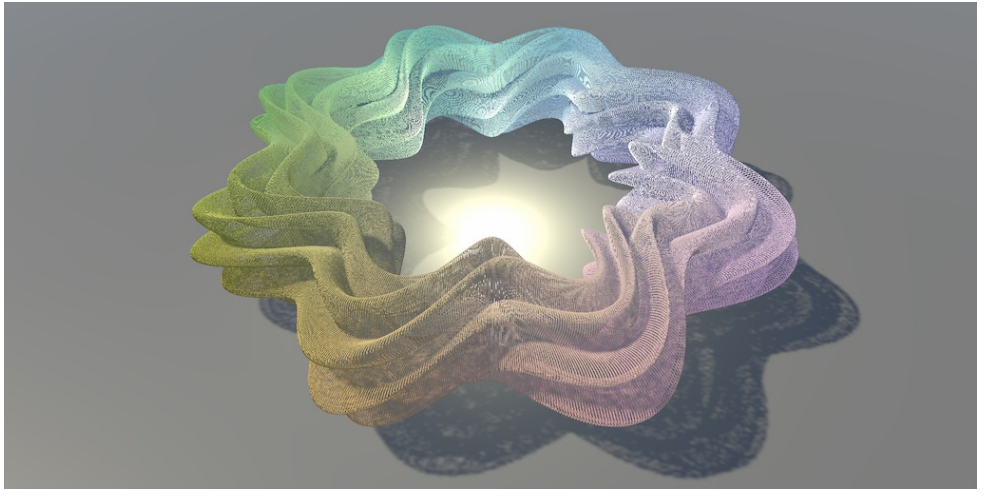
https://antongerdelan.net/opengl/vertexbuffers.html

1. Application stage

- Some work on application stage can be sent to GPU for processing – **compute shader**.
  - Treat GPU as highly parallel general processor



https://arm-software.github.io/opengl-es-sdk-for-android/compute_particles.html



https://catlikecoding.com/unity/tutorials/basics/compute-shaders/

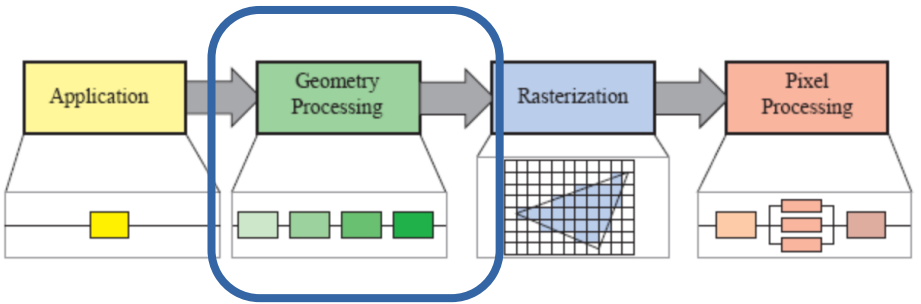
Compute shader

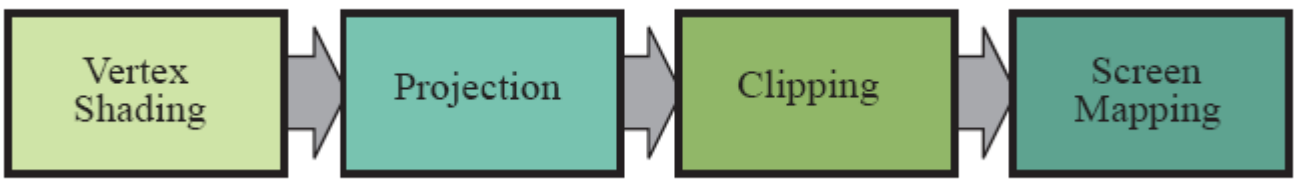
- Compute shader is form of GPU computing, not locked into a location in the graphics pipeline.
  - It is like shader we discussed: it has some input data, can access buffers (e.g., texture) for input and output.
  - Using hardware this way is called GPU computing – CUDA and OpenCL are used to control the GPU as massive parallel processor.
- It is closely tied to the process of rendering graphics API: it is used alongside vertex, pixel and other shaders.
- It is used in:
  - Post-processing: modifying rendered image with certain operations
  - Particle systems: computing behavior of particles
  - Mesh processing: facial animation
  - Culling
  - Image filtering
  - Improving depth precision
  - Shadows calculation
  - Depth of field
  - Replacing tessellation hull shaders.
- With compute shaders, GPU can be used more than implementing traditional graphics pipelines:
  - Training neural networks

2. Geometry processing stage

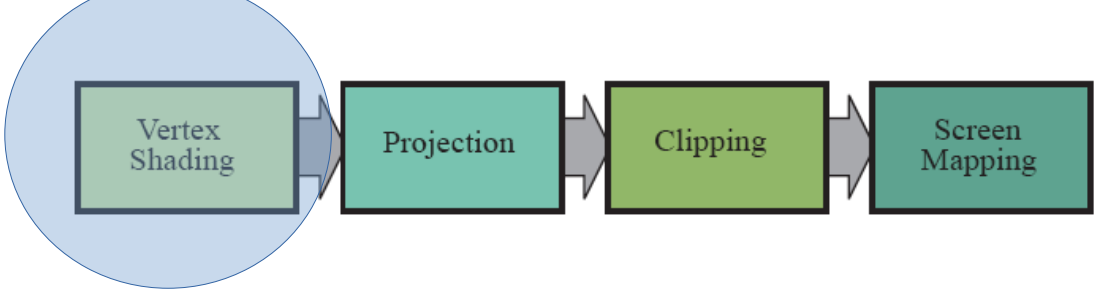
2. Geometry processing stage

- Responsible for most of the **per-triangle** and **per-vertex** geometry operations
  - Deals with transformations, projections and all other geometry handling
- Computes what is drawn, how and where
- Divided into following functional stages:
  - 2.1. **Vertex shading (vertex shader)**
  - 2.2. **Projection**
  - 2.3. **Clipping**
  - 2.4. **Screen mapping**





2.1. Vertex shading



- First stage on GPU, completely under programmer's control.
- Before this stage, data manipulation from application stage is done by **input assembler**
  - For example, an object can be represented by array of positions and array of colors. Input assembler would create object's primitives (e.g., triangle) by creating vertices with positions and colors.
- First stage to process a triangle mesh. It provides a way to modify, create or ignore values associated with each triangle's vertex (e.g., color, normal, texture coordinates and position).
  - Note that normals do not have to be triangle normals rather normals of smooth shape which triangle mesh represents.
  - Note that vertex shader can not create new vertices or destroy existing ones

2.1. Vertex shading

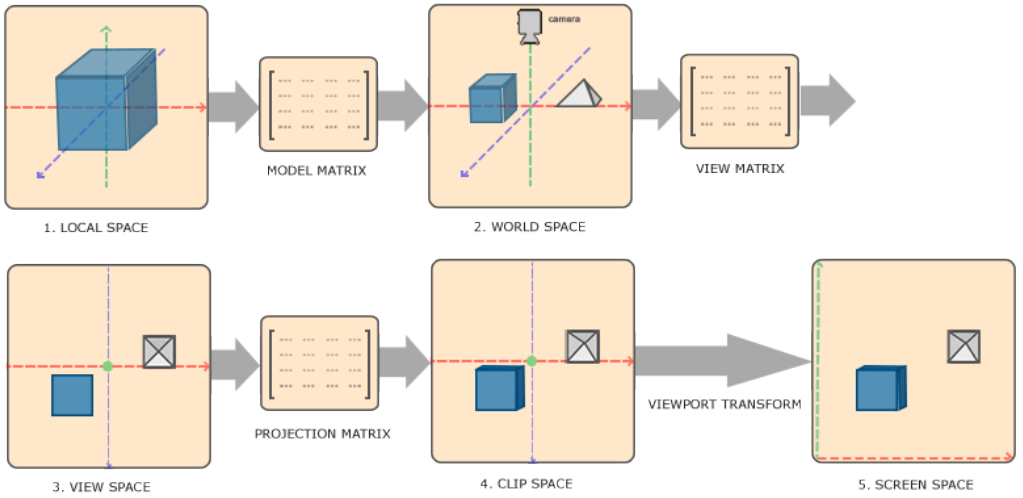
- Two main tasks:
  - Compute **position of vertex** given input from application stage
  - Evaluate/set-up any additional **vertex data output** desired by programmer such as normal and texture coordinates

Vertex shading applications

- An example of vertex shader task is animating objects using transformations on vertices.
  - Vertex blending – RTR 4.4
  - Morphing – RTR 4.5.
- Object generation: creating mesh once and then deforming it by vertex shader
- Animating characters using skinning and morphing
- Procedural deformations such as cloth or water
- Instancing objects: copying same object in different positions in the scene.
- Particle creation: using degenerate (no area) mesh down the pipeline and using those as positions where instancing takes place
- VFX: lens distort, heat haze, water ripples, page curls
- Terrain modeling by applying height fields given by texture

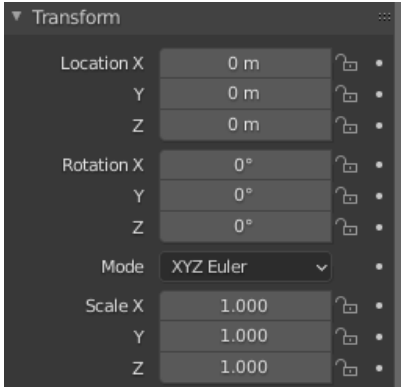
2.1.1 Vertex shading: computing vertex position

- **Vertex positions** of a model is minimal information that has to be passed from application stage to vertex shading stage.
- On the way to rasterization stage, vertex shading performs transformation of a model in several different spaces or coordinate systems:
  - **Model/local space**
  - **World space**
  - **View/camera space**

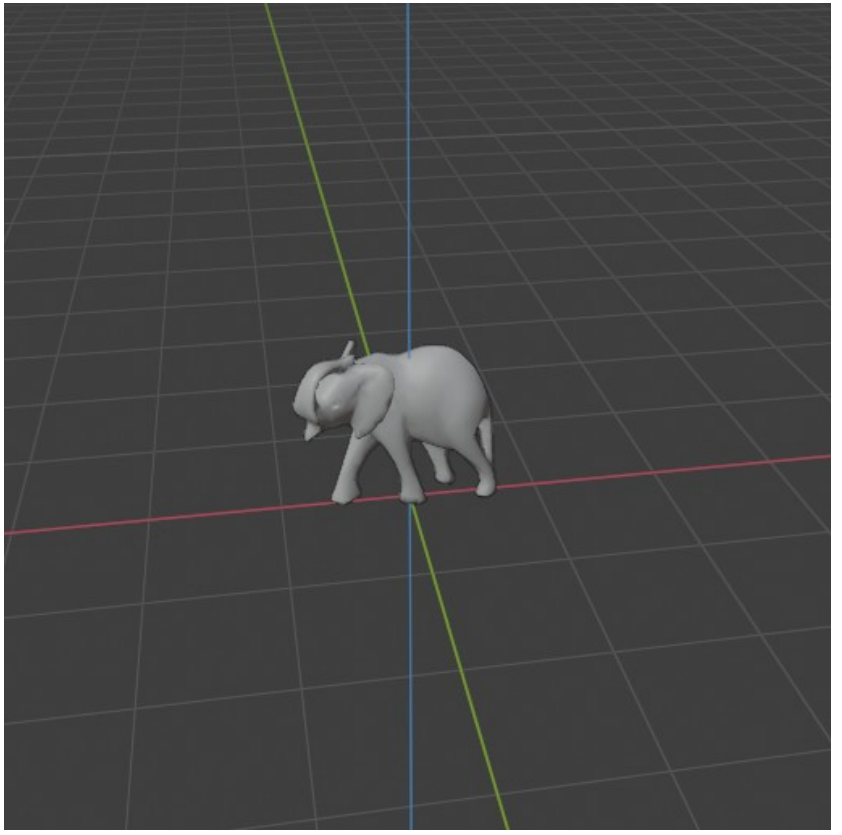


Model space

- Originally, model (vertex positions) given to vertex shading stage is in its own space – **model space**.
  - Model space → model has not been transformed at all.
- Each model can be associated with **model transform –** for positioning and orienting
- Model transform is applied on model's vertices and normals.
  - Coordinates of a object are also called model **coordinates**.

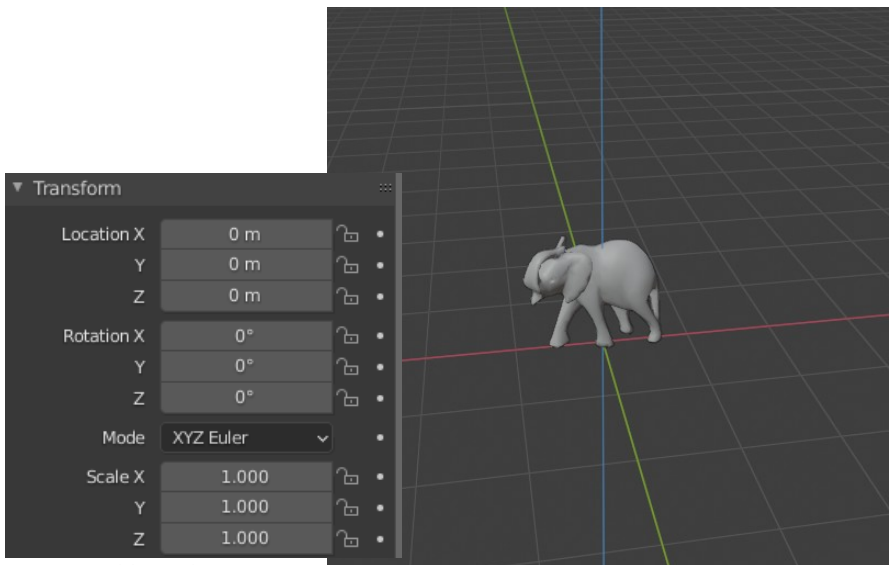


World coordinates



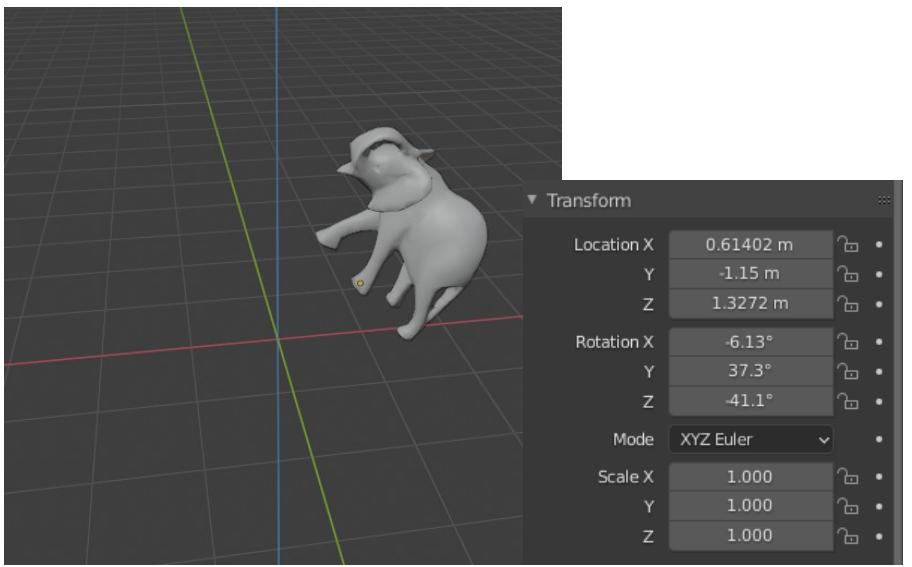
World space

- After model transform has been applied on model coordiantes, the model is said to be in **world coordinates** or **world space**.
  - World space = scene
- World space is unique and after all models have been transformed with their respective model transforms, they all exist in the same space – world space.
- Model transform is defined on application stage and performed by **vertex shader**.



World coordinates

Model transform



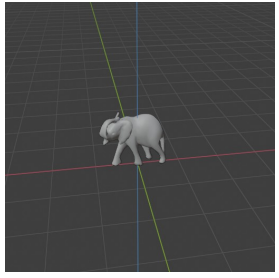
World coordinates

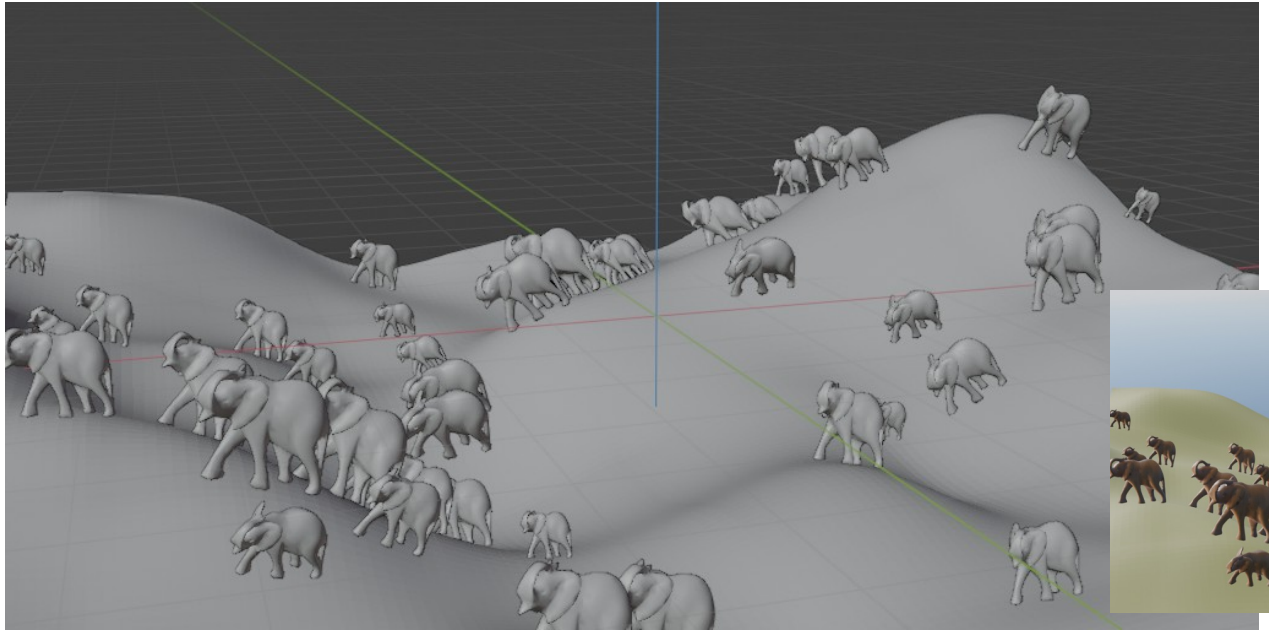
Model transform

- Model transform is 4x4 matrix containing:
  - Translation
  - Rotation
  - Scaling
  - Other transforms
- Various libraries provide functions which build transformation matrix given translation, rotation and scaling vectors.
  - Example: https://github.com/g-truc/glm
- Model transform matrix applied on **model coordinates** gives **world space coordinates**
- Inverse of transform matrix applied on **world coordinates** gives **model coordinates**

Model transform

- Each model can have multiple transforms
  - This allows copying the same model across the 3D scene without specifying additional geometry – **instancing**
  - Same model can have different locations, orientations and sizes in the same scene.





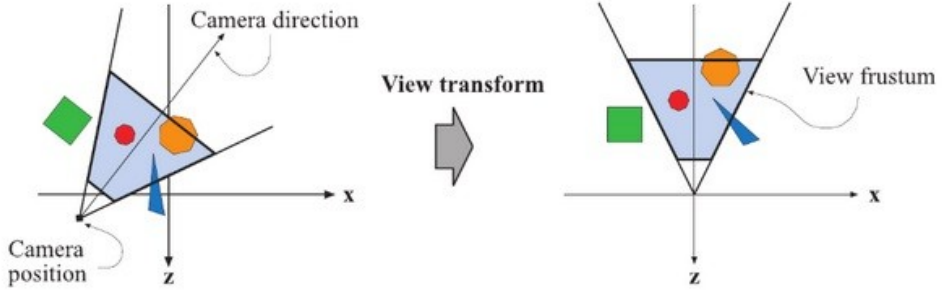
Instancing

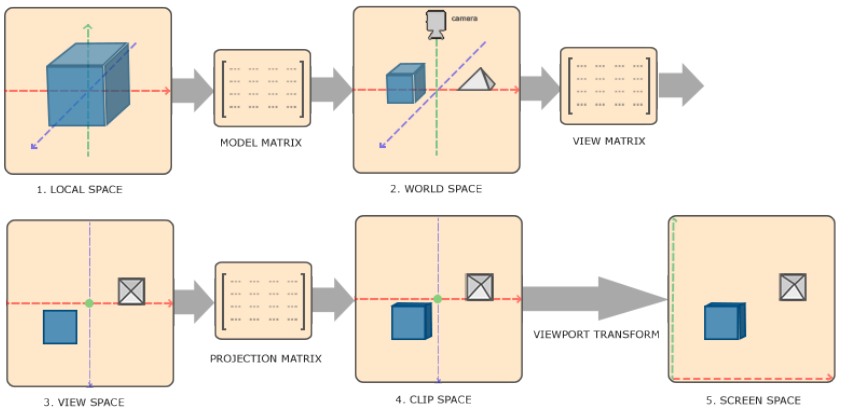
- Very much used for environment art and object placing repeating objects in 3D scene

https://www.exp-points.com/junliang-zhang-procedural-environment-houdini-ue4

Camera (view, eye) space

- Only objects visible from camera are rendered.
- Camera has a **location and orientation in world space.**
- Further pipeline stages (projection and clipping) require camera and all objects to be in **view (camera) space**.
- **View transform** is applied to all objects and camera
  - Result: camera is placed in world origin and aimed in negative z axis with y pointing up and x pointing right*.
- View transform is defined on application stage and performed by vertex shader.





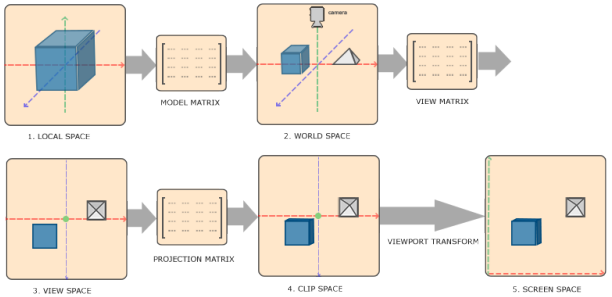
https://www.realtimerendering.com/

* This convention is called negative z axis convention. Another convention is positive z axis convention. All are fine but must be consistently used when decided. Actual position and direction after view transform are dependent on underlying API.

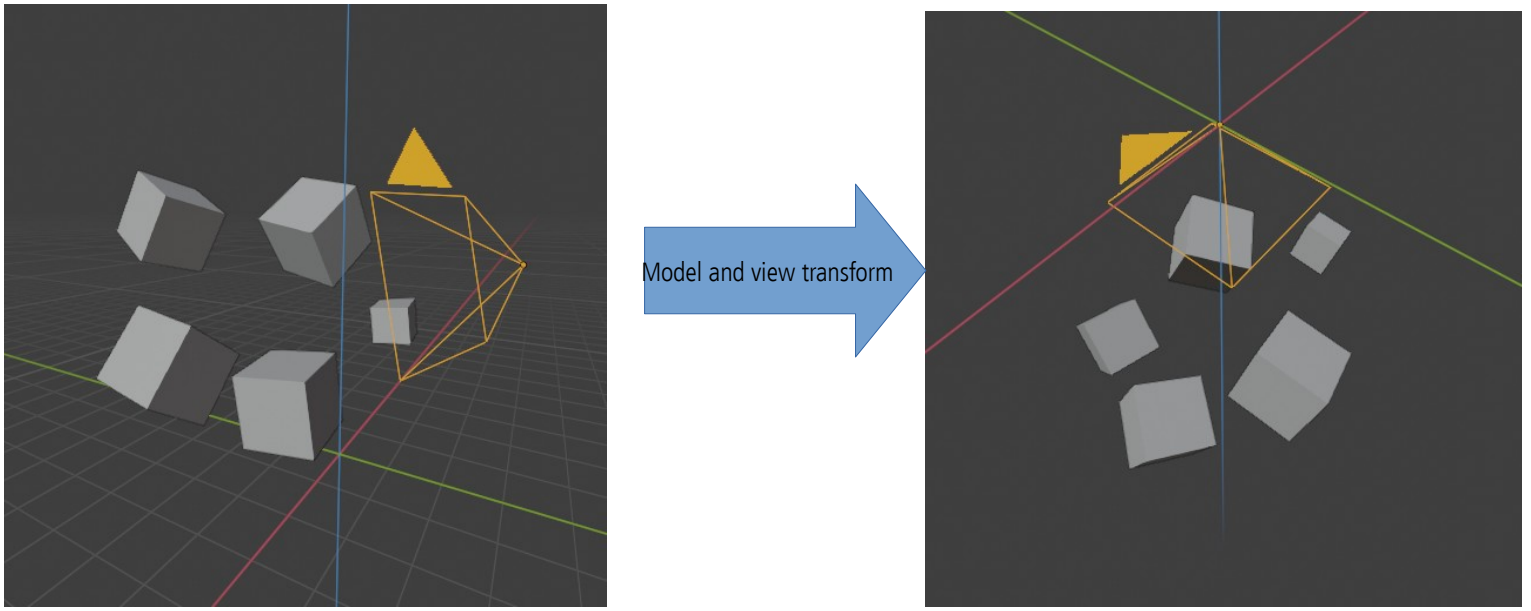
Digression: camera

- Graphics rendering pipeline doesn't have notion of camera as ray-tracing based rendering did
- Instead of using camera, we define matrices which transform all objects in 3D scene as they would be seen from camera
  - For this **view transform** is used
- Example:
  - Moving camera to another position → transforming all objects in 3D scene

Camera (view, eye) space



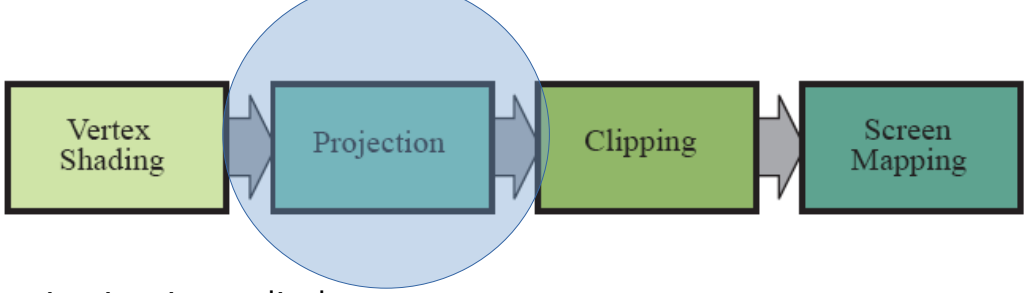
- After **view** transform, the model is said to be in **camera (view or eye) space.**
- Model and view transform can be implemented as one 4x4 matrix for efficient multiplication with vertex and normal vectors.
  - Note: programmer has full control over how the position and normals are computed.



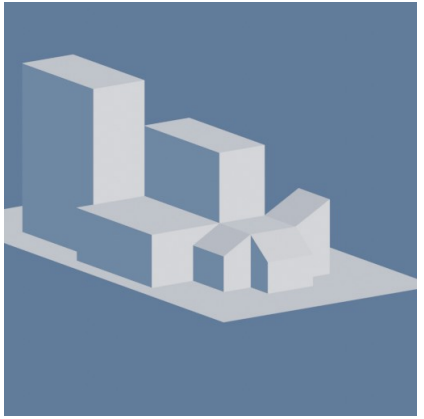
Camera

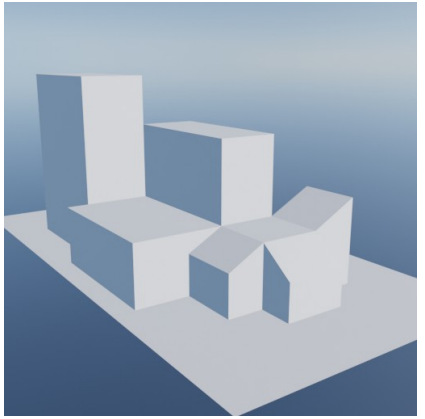
- Various libraries exist for computing view matrix
  - https://github.com/g-truc/glm/blob/master/manual.md#-52-glm-replacements-for-glu-functions
- Often, view matrix is constructed using look at notation

Projection



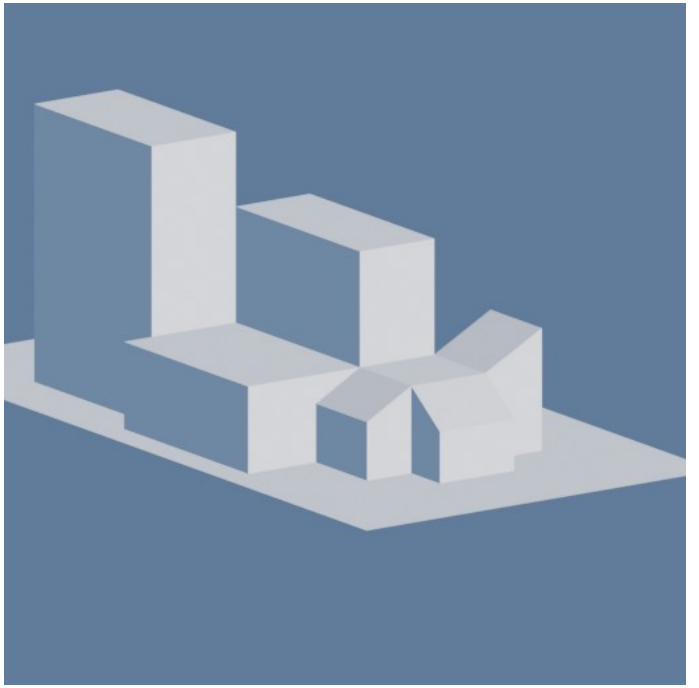
- After models are transformed to camera space, a projection is applied.
- Two commonly used projection methods:
  - **Orthographic**
  - **Perspective**
- Projection is represented as matrix and can be combined with other geometry transforms: model and camera.
  - Projection matrix is defined in application stage and applied in vertex shader on object vertices.

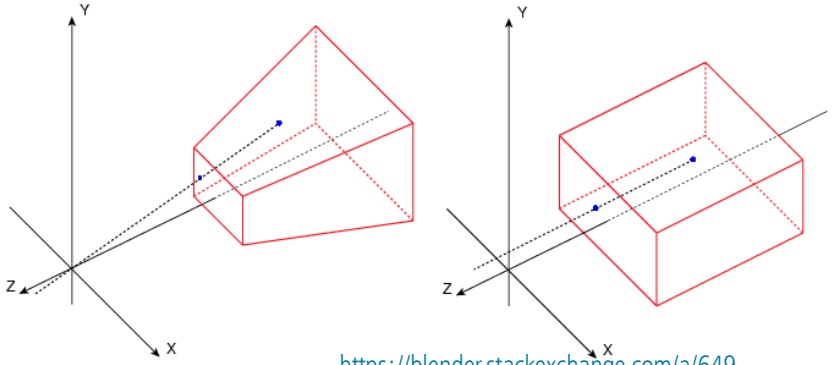




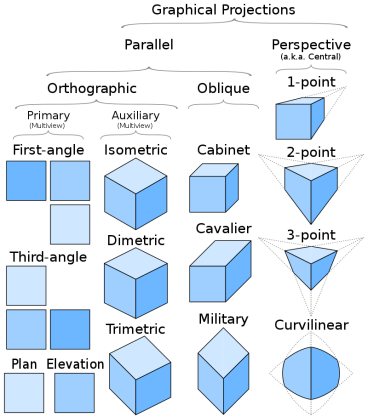
Orthographic projection

- Orthographic projection is just one type of parallel projection.
- View volume of orthographic projection is rectangular box
- Characteristics:
  - parallel lines remain parallel
  - Objects maintain the same size regardless of distance from camera





https://blender.stackexchange.com/a/649



Orthographic projection

- **Simple orthographic projection** projects onto plane z = 0
  - View volume of orthographic projection is rectangular box
  - z coordinate is simply set to 0 → information of depth is lost!

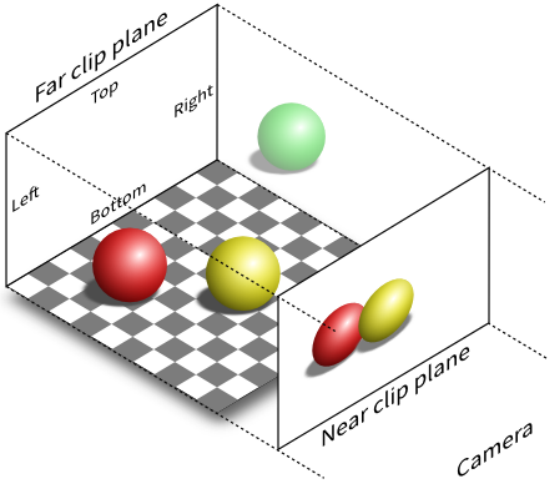
$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Pv = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ 0 \\ 1 \end{bmatrix}$$

Orthographic projection

- **General orthographic projection** is expressed using orthographic frustum
  - Extremes are viewing frustum **near** (n), **far** (f), **left** (l), **right** (r), **top** (t) and **bottom** (b)
  - This matrix scales and translates axis aligned bounding box formed by these planes into axis-aligned cube centered around origin
  - In OpenGL*, minimum corner is (-1,-1,-1) and maximum (1,1,1) → **canonical view volume**

$$P = \begin{bmatrix} \frac{2}{\text{right}-\text{left}} & 0 & 0 & -\frac{\text{right}+\text{left}}{\text{right}-\text{left}} \\ 0 & \frac{2}{\text{top}-\text{bottom}} & 0 & -\frac{\text{top}+\text{bottom}}{\text{top}-\text{bottom}} \\ 0 & 0 & \frac{-2}{\text{far}-\text{near}} & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

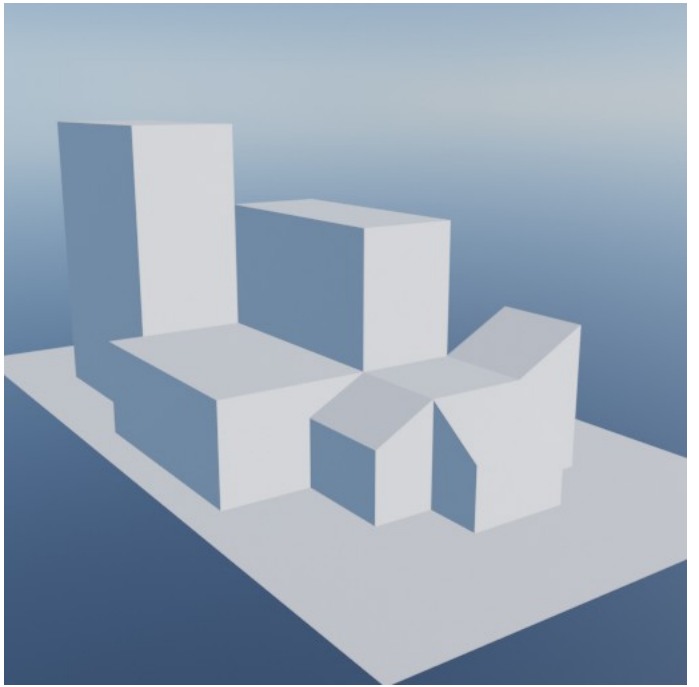


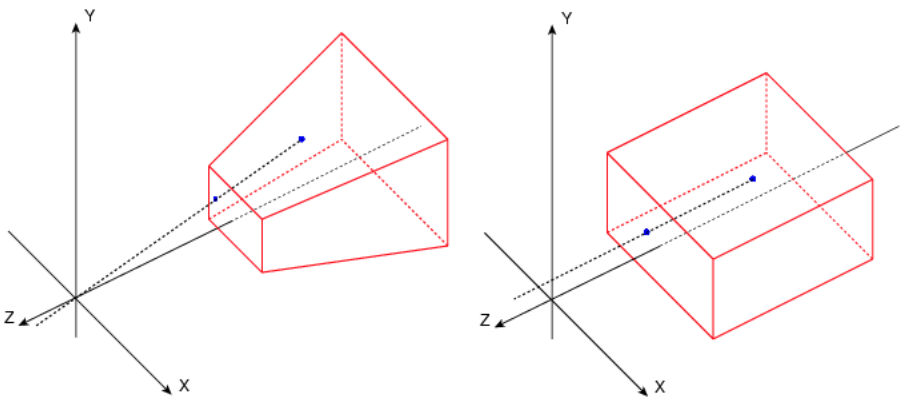
https://stackoverflow.co

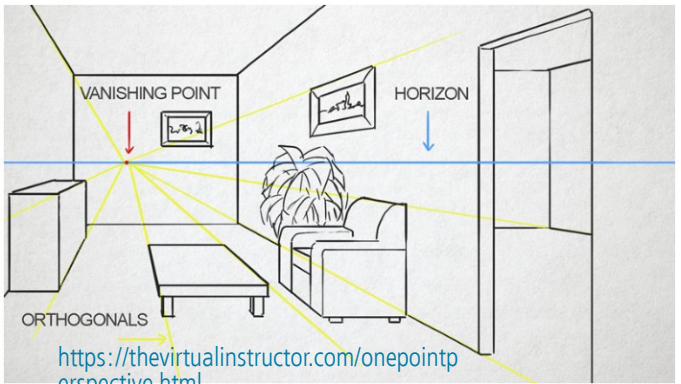
* DirectX has bounds (-1,-1,0) and (1,1,1)

Perspective projection

- The view volume is called **fustum**
  - Truncated pyramid with rectangular base
- Characteristics:
  - The further the object, the smaller appears
  - Parallel lines converge at single point → mimics how humans perceive objects







https://thevirtualinstructor.com/onepointperspective.html

Perspective projection

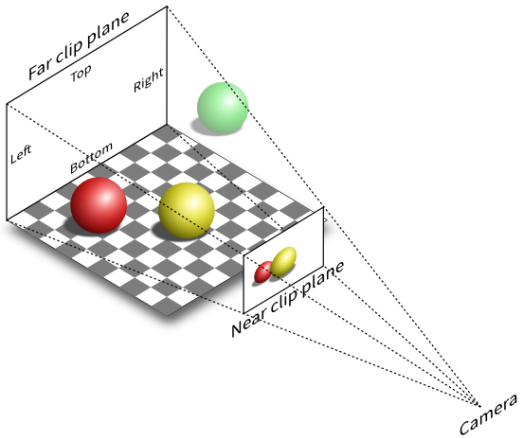
- **Simple perspective projection** is projecting points on a plane `z = 1` → information on depth is lost!
- After multiplication with perspective matrix, homogeneous coordinate w_c will be equal to vertex coordinate z. Other stays the same
- Perspective divide gives vertex on a plane

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \frac{1}{w_c} \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix}$$

Perspective projection

- General perspective projection transforms viewing frustum to cube for which minimum corner is (-1,-1,-1) and maximum (1,1,1) → **canonical view volume**
  - General perspective projection matrix is defined with viewing frustum **near** (n), **far** (f), **left** (l), **right** (r), **top** (t) and **bottom** (b)
- Construction of this matrix is supported by various libraries:
  - OpenGL Utility Library (GLU) as `gluPerspective()` https://registry.khronos.org/OpenGL-Refpages/gl2.1/xhtml/gluPerspective.xml
  - GLM: https://github.com/g-truc/glm/blob/master/manual.md#-52-glm-replacements-for-glu-functions



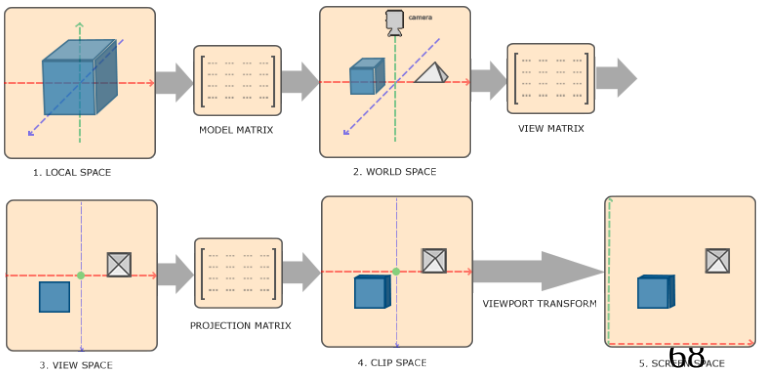
$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Digression: camera

- Graphics rendering pipeline doesn't have notion of camera as ray-tracing based rendering did
- To simulate camera projection (i.e., perspective projection) all objects in 3D scene are transformed with **projection matrix**

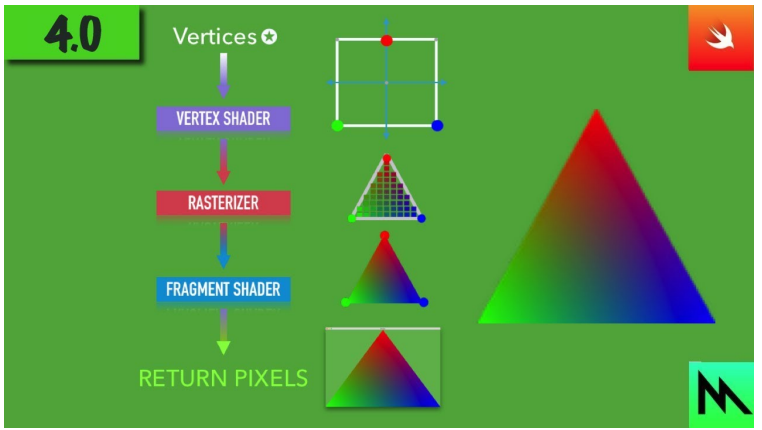
Projection

- After projection coordinates are in **clip space - clip coordinates**
  - These are homogeneous coordinates (x,y,z,w)
    - Perspective division with w to obtain (x′, y′, z′) occurs later
  - Vertex shader must output this type for next, clipping stage
- z-coordinate is not stored in generated image but as a **depth-buffer/z-buffer**
  - Model is projected from three to two dimensions.



2.1.2 Vertex shading: computing additional vertex output data

- **Shading** can also be performed on this stage*
  - Computing color using on vertex data (position, normal, material) and light
  - Result is stored as color per vertex which is later interpolated and used cross triangle and pixels
  - Vertex shading is performed using vertex shader using vertex data defined on application stage.



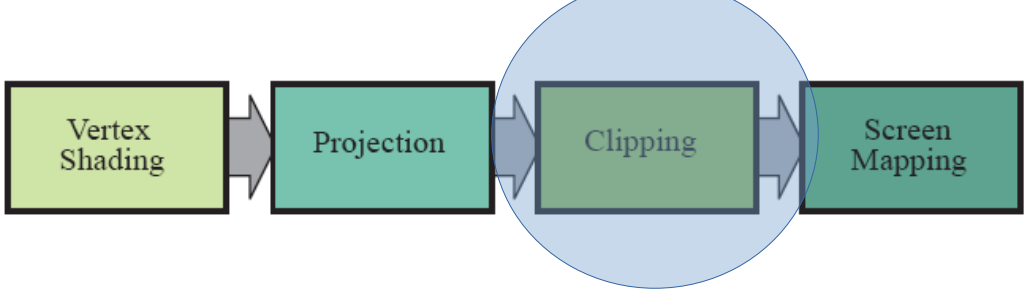
https://www.youtube.com/watch?v=F7bpcyPhiH8&ab_channel=2etime

* Used when simpler and more faster shading is needed. This is why programmable vertex processing unit was named "vertex shader" also the name: "vertex shading" performed by this stage which is kept even today. In modern GPU-s and API-s, some or all shading takes place in pixel processing stage. Vertex shading stage is more general and doesn't have to perform equation at all – depending on programmer. Vertex shader is more ge neral unit dedicated to setting up data associated with each vertex.

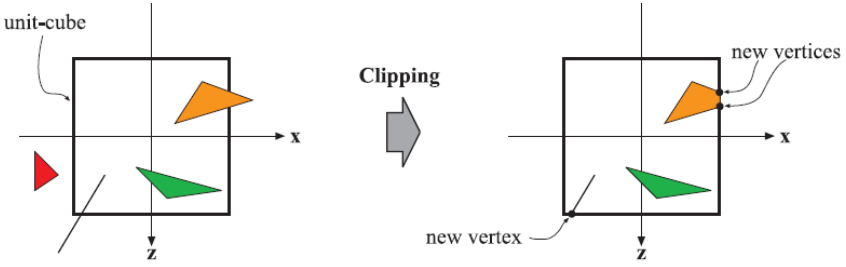
Optional vertex processing

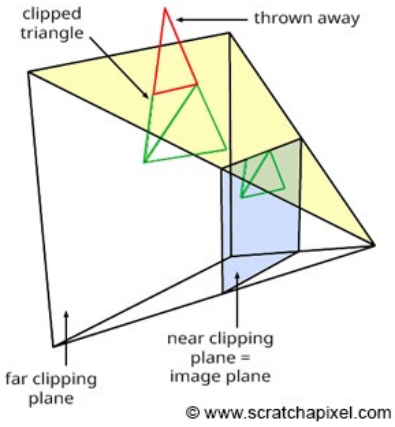
- Standard use of GPU's pipeline is to send data through vertex shader, then rasterize the resulting triangles and process those in the pixel shader.
- Optional processing are:
  - **Tessellation** – curved surface can be generated with appropriate number of triangles. Sub-stages: hull shader, tessellator and domain shader
  - **Geometry shader** – takes in various primitives (e.g., triangles) and creates new vertices. Often used for particle generation – e.g., a set of vertices are given and square can be created for more detailed shading.
  - **Stream output** – instead of sending vertices down the pipeline, these can be outputted for further processing to CPU or GPU.
- Usage of those depends on GPU hardware (not all GPU supports those stages) and application.

2.2 Clipping



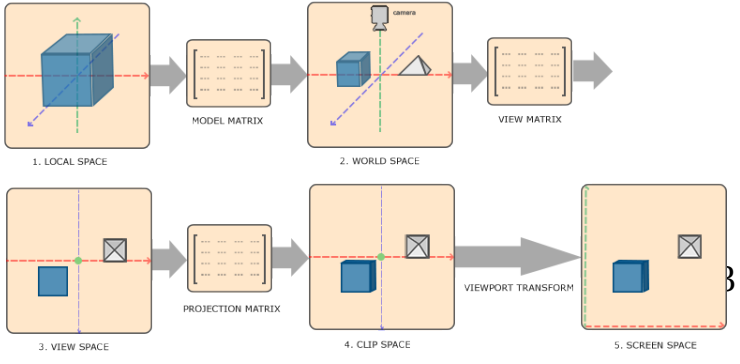
- Only primitives that are partially of fully in view volume need to be passed to the rasterization stage and pixel processing for drawing on screen.
- Primitive that is fully in view volume will be passed further without clipping
- Primitives that are partially in view volume require clipping.
  - After projection, clipping of primitive is done against unit cube.
  - Vertex that are outside of view volume is removed. New vertices are created on clipping position.





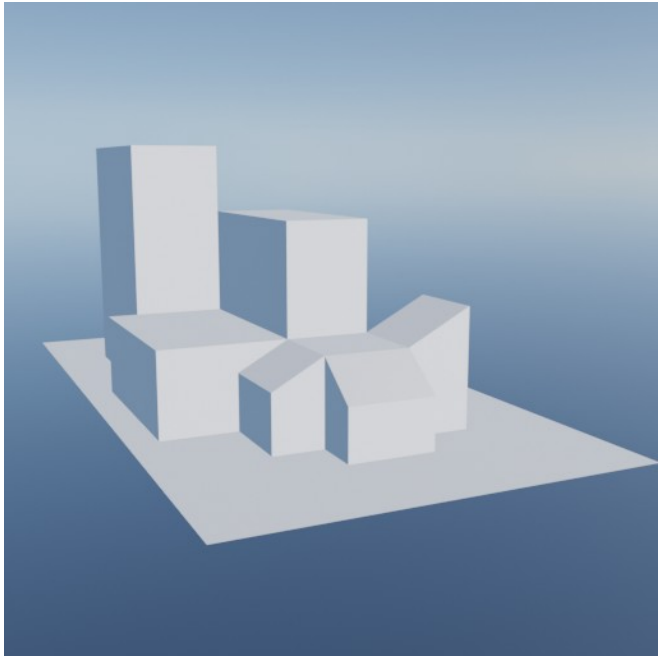
2.3 Clipping

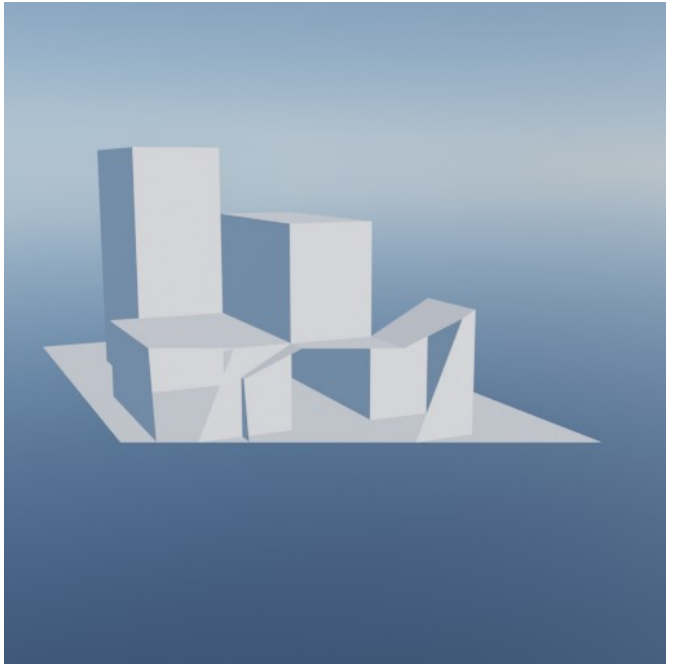
- Clipping uses clip coordinates (x,y,z,w) resulted from projection step.
  - Fourth coordinate (w) is used for correct interpolation over triangle and clipping if perspective projection is used.
- Finally perspective division is performed
  - Resulting triangle positions are in **normalized device coordinates (NDC)** – a **canonical view volume** that ranges from (-1, -1, -1) to (1,1,1).
- Clipping is fixed stage in rendering pipeline → user doesn't have influence, it is done automatically



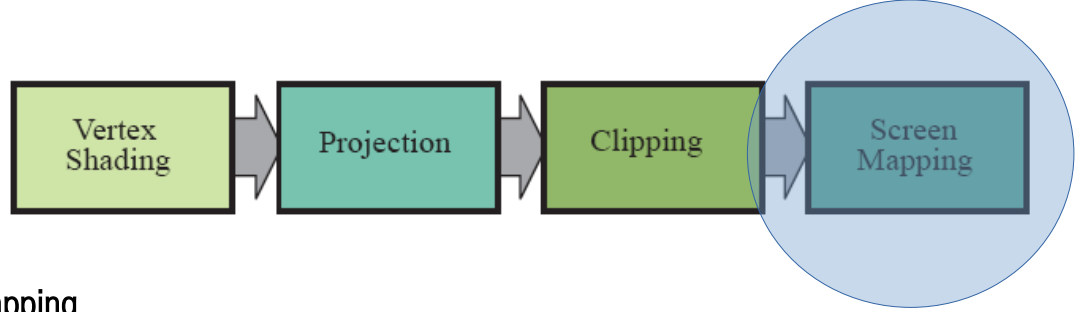
2.3 Clipping

- Additional clipping planes can be introduced by programmer to chop the visibility of objects: **sectioning**

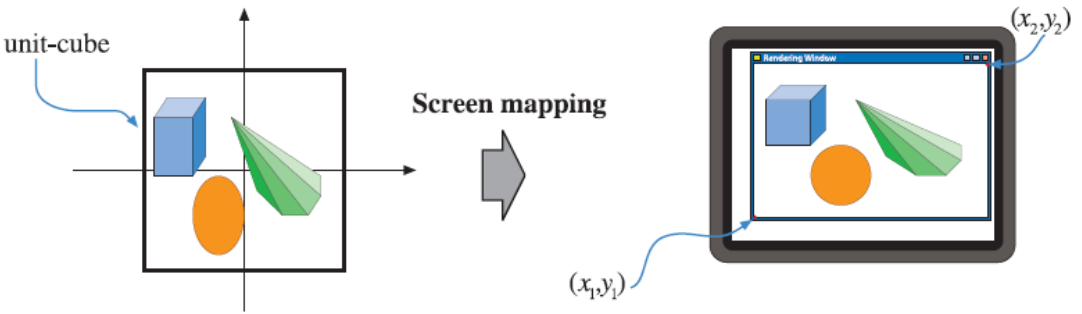




2.4 Screen mapping

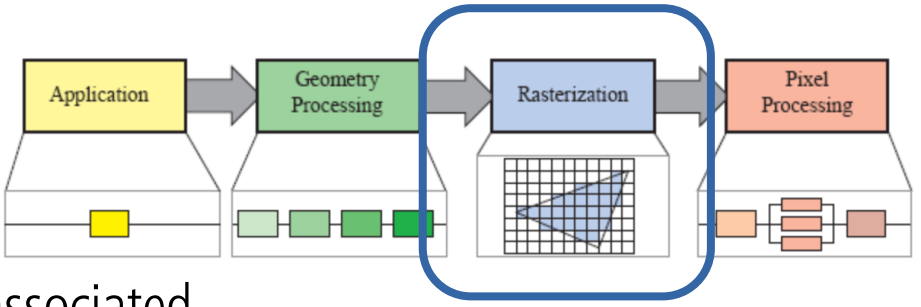


- Clipped primitives inside view volume are passed on **screen mapping**.
  - Coordinates entering this stage are still 3D → NDC.
- x and y coordinates are transformed to **screen coordinates**.
  - Screen mapping is translation followed by scaling to map (x,y) to screen with dimensions (x1,y1) and (x2, y2).
  - Pixel position: `d = floor(c), c = d + 0.5`
- Screen coordinates with z coordinates are called **window coordinates**.
  - Z coordinates are mapped to [-1,1] in OpenGL or [0,1] in DX.
- Window coordinates are passed to the rasterizer stage.

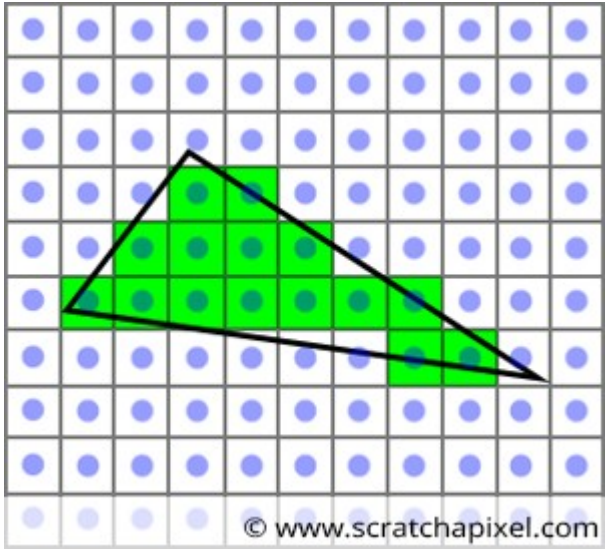


https://www.realtimerendering.com/

3. Rasterization stage



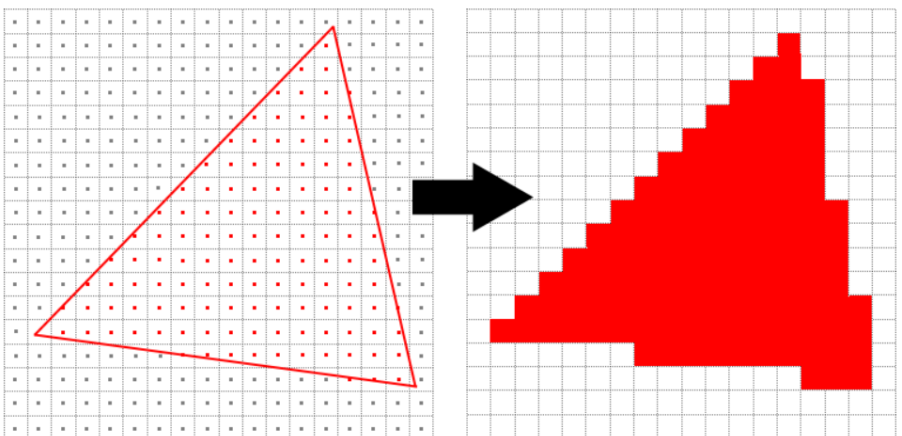
- Given transformed and projected vertices (with associated shading data) the goal is to find all pixels which are inside the primitive (e.g., triangle*) to be used in pixel processing stage → **rasterization**.
  - Typically takes input of 3 vertices forming a triangle, finds all pixels that are considered inside the triangle and forwards those further
  - Rasterization (screen conversion) is **conversion from 2D vertices in screen space into pixels on the screen**.
    - These 2D vertices have associated z value (depth) and shading information (e.g., normals)



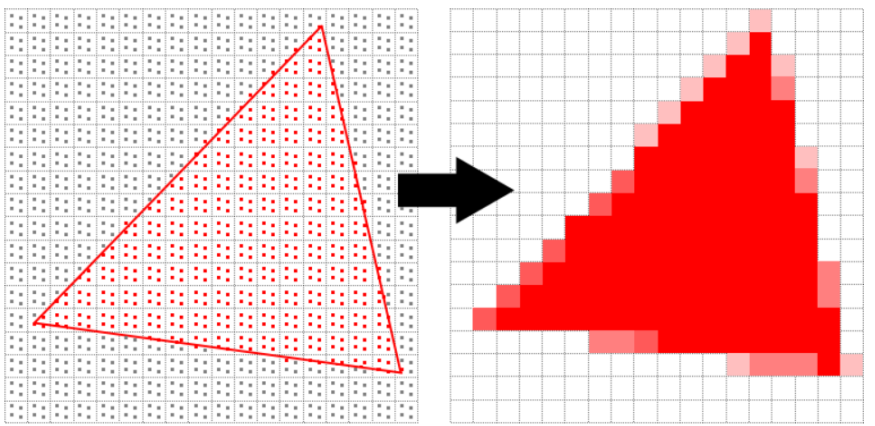
* Point and line primitives sent down the pipeline also create fragments for covered pixels.

3. Rasterization stage

- Test if triangle is overlapping a pixel, depends on GPU pipeline:
  - **Point sampling** - only center of pixel is used for testing.
    - If center of pixel is inside triangle → pixel is considered inside triangle
    - **Multiple samples per pixel** are desired to evade **aliasing** problems – **multi-sampling or anti-aliasing.**
  - Another approach is to define pixel overlaps triangle if at least part of pixel overlaps triangle.



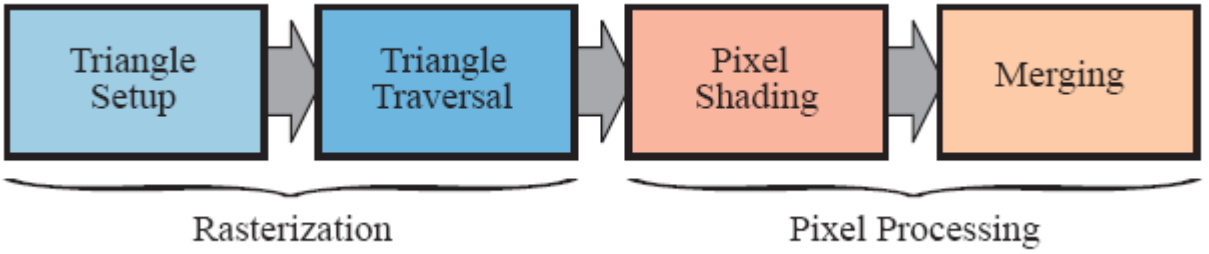
https://vulkan-tutorial.com/Multisampling



MSAAx4

3. Rasterization stage

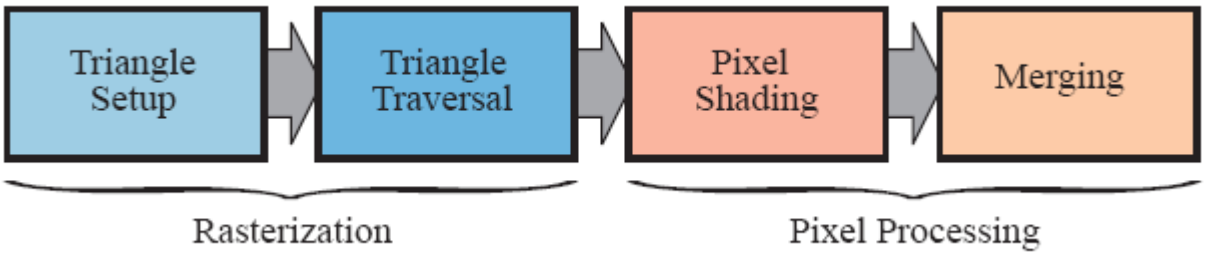
- Two functional sub-stages*:
  - 3.1.Triangle setup
  - 3.2 Triangle traversal
- Synchronization point between geometry processing and pixel processing



* Those can also process points and lines, but triangle is most often used primitive and thus the name.

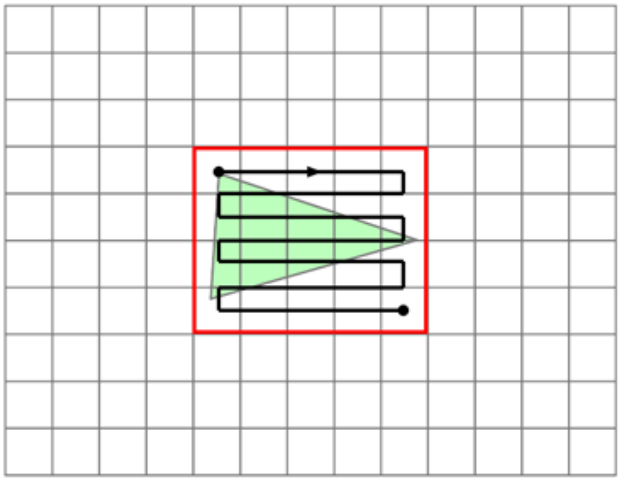
3.1 Rasterization: triangle setup

- Data for the triangle are are computed here (e.g., differentials, edge equations)
- This data is needed for:
  - Triangle traversal
  - Interpolation of shading data produced by geometry stage (e.g., normals)
- Fixed-function hardware is used for this stage → programmer has no control over it.



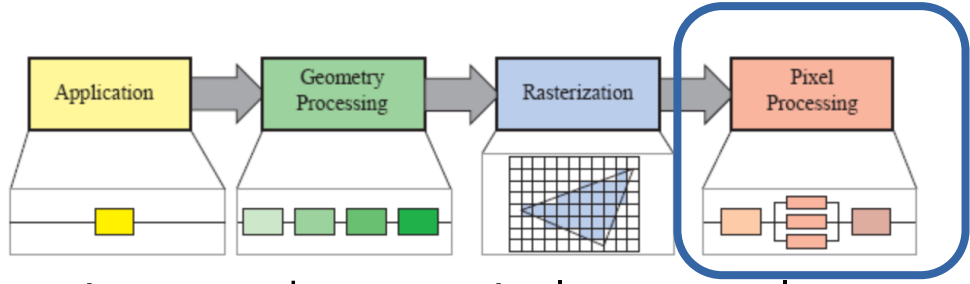
3.2 Rasterization: triangle traversal

- Each pixel sample is checked if covered by a triangle.
  - Finding which samples/centers for which pixels are inside a triangle is called **triangle traversal**
- **Fragment** is generated for part of the pixel that overlaps the triangle
- Fragment properties are generated by interpolating data among three triangle vertices, taking in account perspective – perspective-correct interpolation*.
  - Properties: fragment depth and any shading data from geometry processing stage.
- All pixels, that is fragments, inside primitive are sent to pixel processing stage.

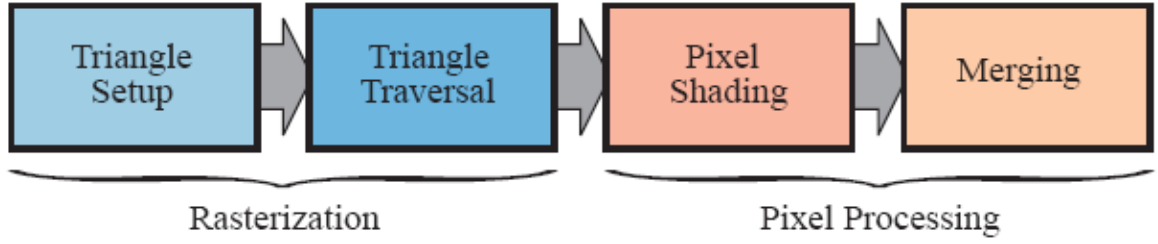


* Another type of interpolations are available; such as screen-space interpolation where perspective is not taken in account.

4. Pixel processing stage

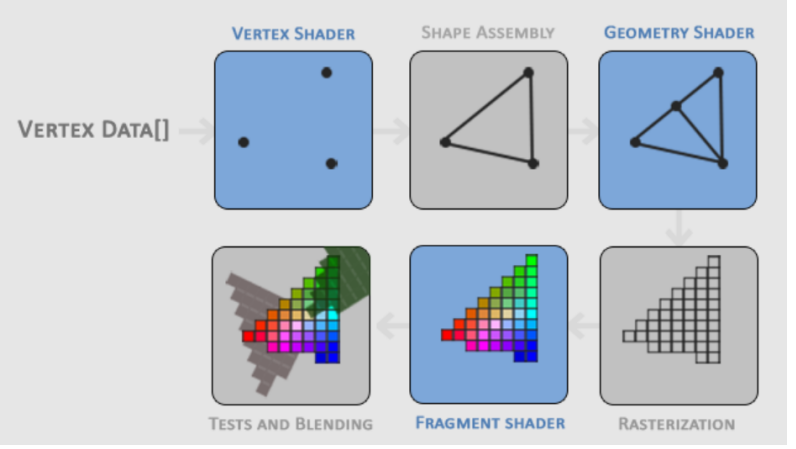


- Stage in which per-pixel or per-sample computations are done on pixels or samples inside a primitive.
  - A **shader program** is executed per-pixel/per-sample to determine its color and if color is visible (depth testing) as well as blending of pixel colors with newly computed with old color.
- Pixel processing stage can be divided into:
  - 4.1 Pixel shading
  - 4.2 Pixel merging



4.1 Pixel processing: pixel shading

- Any **per-pixel (fragment) shading** performs here
  - For computation, **interpolated shading data** from previous stage is used
- Pixel shading stage is performed by **programmable GPU cores**
  - Programmer supplies a **program for pixel (fragment) shader** that contain any desired computations.
  - Most commonly, **shading equations (scattering function)** and **texturing** (image or procedural) are defined here
- **Result of this stage is one or more colors for each pixel/fragment** that are passed further the pipeline



4.2 Pixel processing: pixel merging

- Information for each pixel (generated in pixel shader) is stored in **color buffer –** rectangular array of of colors `(r,g,b)`.
- This stage is also called **raster operations pipeline (ROP)** or **render output unit***.
  - It performs **raster operations** or **blend operations**
- This stage is not fully programmable, but highly configurable* for achieving various effects (e.g., transparency).

* DirectX calls this stage output merger. OpenGL calls this stage per-sample operations.

*Some APIs introduce more programmability, e.g. have support for raster order views – pixel shader ordering – which enables programmable blending capabilities (Real-Time Rendering Book).

Pixel merging: blending

- This stage is responsible to combine - **blend** - fragment color produced by pixel shading stage with the color currently stored in color buffer
  - Opaque surfaces blending is not needed: fragment color simply replace the previously stored color
  - Blending is important for transparent objects and compositing operations
- Color blending in particular can be set up to perform a large number of operations: combinations of multiplication, addition, subtractions, min, max, bitwise logic involving color and alpha values

Pixel merging: visibility

- This stage is responsible for resolving visibility
  - After rendering 3D scene, color buffer contains only color of primitives visible from camera point of view
  - Done using **z-buffer/depth-buffer**

Pixel merging: frame buffers

- This stage uses following buffers for computations:
  - **Z-buffer**
  - **Alpha channel** (part of color buffer)
  - **Stencil buffer**
- **Frame buffer** generally consists of all buffers on a system.

z-buffer

- When the whole scene has been rendered, the **color buffer** should contain **colors of the primitives** (e.g., triangles) in the scene **visible from camera point of view**.
- For most graphics hardware this is achieved using, **z-buffer (depth buffer).**
  - Z-buffer is same size and shape as color buffer, but for each pixel it stores **z-value to the closest primitive**.
- Z-value and color of pixel are updated with z-value and color of pixel being rendered
  - When a primitive is being rendered at a certain pixel, z-value at that pixel is being computed and compared to the z-buffer: if new z-value is smaller then one in z-buffer then that pixel is rendered closer to camera than previous pixel - which means updating color and z-buffer. Otherwise, color and z-buffer are not changed.

https://docs.chaos.com/display/VFBlender/Z-Depth

* Many algorithms require a specific order of execution. Often example is drawing transparent objects. In the standard pipeline, the fragment results are sorted in merger stage before being processed. DirectX API introduced rasterizer order views (ROV) to enforce order of exectution.

z-buffer

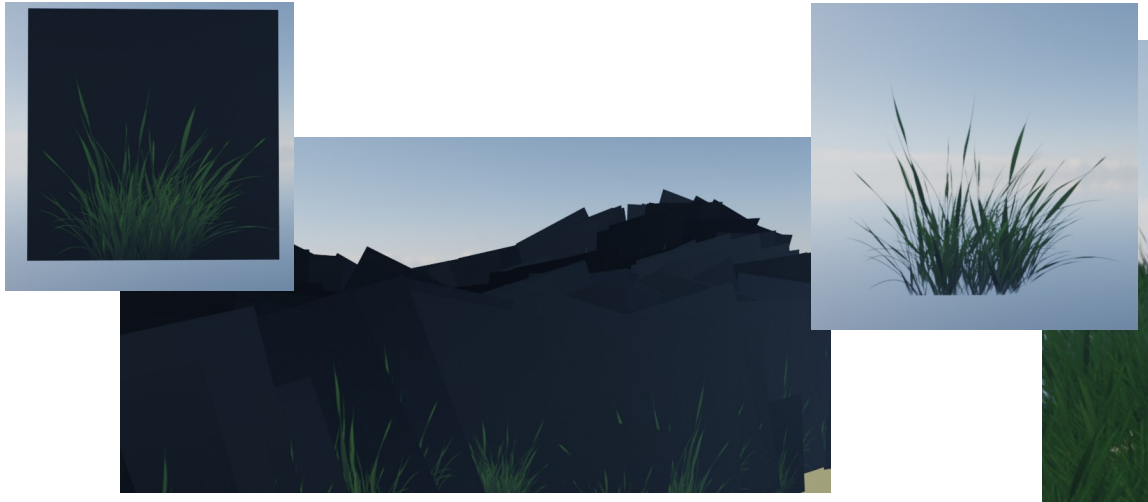
- Z-buffer algorithm is simple and of **O(n)** complexity, where n is number of primitives
- Z-buffer algorithm works for any primitive for which z-value can be computed, allows primitives to be rendered in any order and thus very much used – **order independent**.
- Z-Buffer stores only single depth value for each pixel – in the case of partially **transparent** objects; those must be rendered after all opaque primitives with end to front order* (or using order-independent algorithm), thus transparency is major weakness of z-buffer.

z-buffer

- Let's recap the pipeline for now:
  - Fragments are generated in rasterization stage from 3 vertices forming triangle
  - Fragment is then run through pixel shader which computes color and blending information
  - Finally, in merging stage, this fragment is tested for visibility using z-buffer
- In third step, fragment can be discarded and all processing that was done was unnecessary.
- Therefore, many GPUs perform some merge testing before pixel shader is exectued.
  - Z value of fragment is available after rasterization and it can be used for testing visibility and culling if hidden before pixel shader → **early-z** technique
  - Note that pixel shader can change z-depth of the fragment or discard whole fragment. In this case, early-z is not possible.

Alpha channel

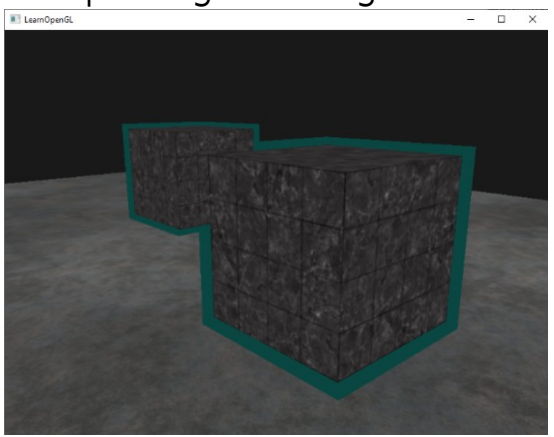
- Frame buffer also contains information about **alpha value – alpha channel.**
- Alpha channel is associated with color buffer and stores opacity value for each pixel.
- Alpha value can be used for selective discarding of pixels using **alpha test** feature of pixel (fragment) shader.
  - Using this test, we can ensure that fully transparent pixels/fragments do not affect the z-buffer
- In modern APIs, alpha test can be done already in pixel shader



https://learnopengl.com/Advanced-OpenGL/Blending

Stencil buffer

- Stencil buffer is the type of so called **offscreen buffer*** – buffer that records locations of rendered primitives but not for directly showing on a screen rather used for pixel merging stage.
  - For example, filled rectangle is rendered into stencil buffer. This buffer is then used to render scene primitives into the color buffer only where the rectangle is present
- Stencil buffer is used in combination with different operators which are offers powerful tool for generating special effects by compositing rendering results.



https://learnopengl.com/Advanced-OpenGL/Stencil-testing

https://www.ronja-tutorials.com/post/022-stencil-buffers/

* This type of buffer is often used for advanced rendering (shading) techniques not only for stencil buffer.

Multiple render targets

- Instead of sending results of pixel shader's program to just color and z-buffer, multiple sets of values can be generated for each fragment and saved to different buffers – **render targets**.
- Multiple rendering targets functionality is powerful aid in performing rendering algorithms more efficiently.
  - Single rendering pass can generate a color image in one target, object identifiers in second and world space distances in third.
  - This inspired different type of rendering pipeline called **deferred shading –** visibility pass and shading are done in separate passes. First pass stores data about object's location and material at each pixel. Successive passes can efficiently apply illumination and other effects.
- Different buffers that are generated can be used for **compositing** purposes.

Digression: Note on pixel shader

- Pixel shader can write to a render target at only the fragment location handed to it – reading of current results from neighboring pixels is not possible – computations are performed only at given pixel.
- To solve this problem, output image can be created with all required data and accessed by a pixel shader in a later pass.
- Also, pixel shader is provided with the amounts by which any interpolated values change per pixel along x and y direction – gradient (derivative) information.
  - This is, for example, useful for texture filtering – where it is important to know how much of texture image covers a pixel.
- Graphics APIs (e.g., OpenGL, Vulkan and DirectX) are constantly evolving enabling more flexibility*.

* DX11 introduced a buffer type that allows write access to any location – unordered access view (UAV). OpenGL calls this buffer shader storage buffer object (SSBO).

Displaying pipeline output

- Primitives that have reached and passed rasterizer stage (remember that a lot of those are discarded) are visible from camera point of view
- Display device will show color buffer after all operations are done.
- As this takes some time (visible to human eye), a technique called **double buffering** is used.
  - Rendering of screen is performed in a **back buffer –** a off screen buffer.
  - Contents of back buffer are swapped with the contents of **front buffer –** buffer which is shown on display device.
  - Swapping process occurs* during **vertical trace** – a time when it is safe to do so.

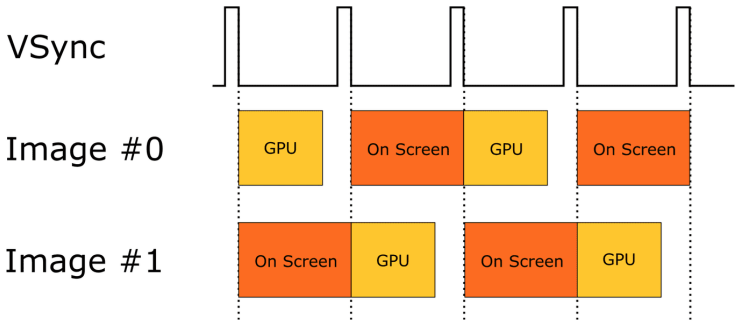


Figure 4: Correct double buffering behavior

https://developer.samsung.com/sdp/blog/en-us/2019/07/26/vulkan-mobile-best-practice-how-to-configure-your-vulkan-swapchain

Graphics rendering pipeline in practice

Production

Example of pipeline in application

- Example: CAD program
- Application stage:
  - Enables user to select and move parts of the model. Selection and movement is done using mouse pointer. Thus, application stage must translate mouse movement into transformation matrix (e.g., translation or rotation) which is used in subsequent stage
  - Enables user to move camera in 3D scene to view the model from different angles. Thus, camera parameters such as position and view direction must be updated by application.
  - For each frame (CAD programs are real-time when in modeling mode) application must provide information for next – geometry stage: camera position, lighting and primitives of the model.
- Geometry processing
  - Application provided parameters of the camera which includes projection matrix. Next, application, for each object, calculates a matrix which describes location and orientation of the object.
  - Object is put in view space and optionally shading per vertices is preformed using provided light and material information.
  - Projection is applied on the object tranforming it in unit cube space that represents what the eye sees. Primitives outside of that cube are discarded. Primitives intersecting this cube are clipped.
  - Finally, vertices are mapped into the window on the screen
- Rasterization
  - All primitives coming from geometry stage are rasterized: all pixels (fragments) inside a primitive are found and sent for pixel processing.
- Pixel processing
  - Color of each pixel obtained from rasterization is computed using material information (colors, textures, shading equations) and visibility is resolved using z-buffer algorithm with optional discard and stencil testing.
  - Each object is processed and final image is displayed.

- Show graphics pipeline rendering example in Blender EEVEE