

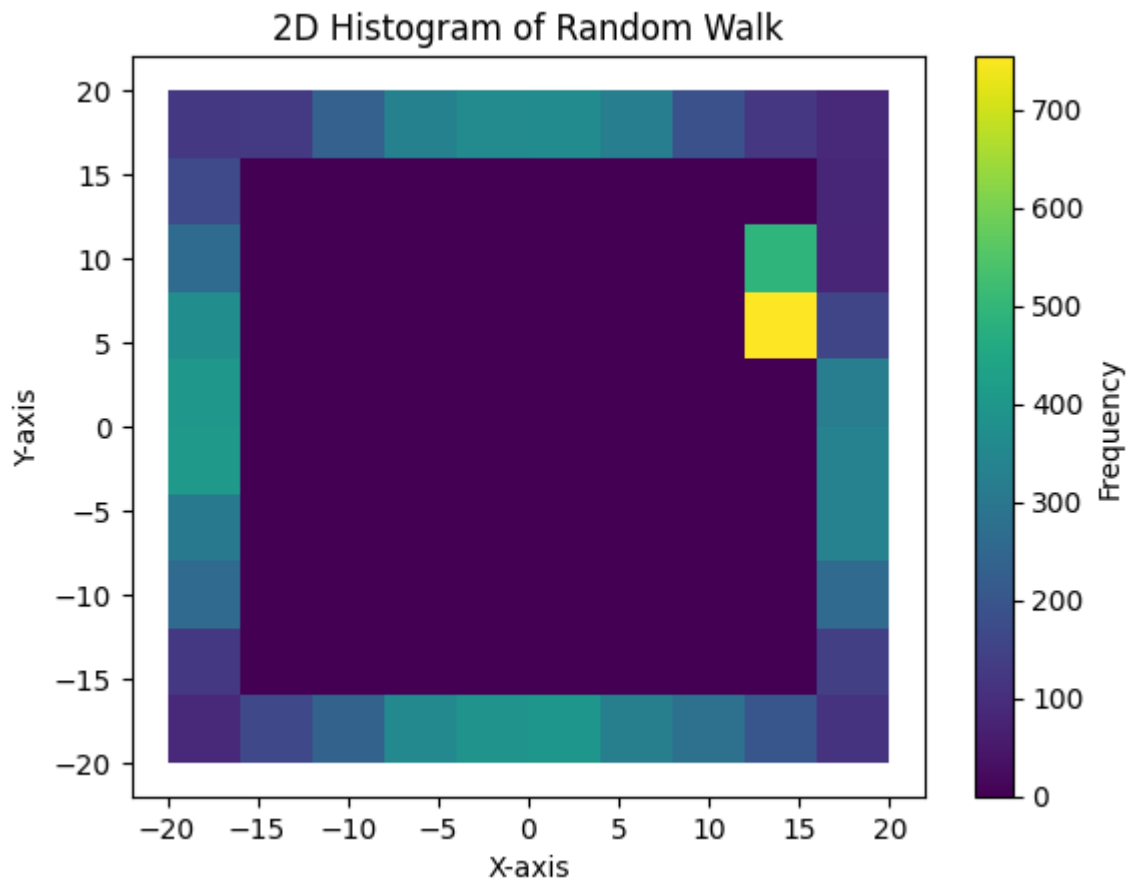
```
In [5]: import random
import numpy as np
import matplotlib.pyplot as plt
import math
```

## 平面扩散

```
In [26]: def randomwalk():
P=[0,0]
while ((P[0]<20 and P[0]>-20 and P[1]<20 and P[1]>-20 ) and not ((P[0]<=14 and
u=random.uniform(0,1)
if u<0.25:
P[0]+=1
elif u<0.5:
P[0]-=1
elif u<0.75:
P[1]+=1
else:
P[1]-=1
return P[0],P[1]
```

```
In [27]: def Simulate(times):
DataX=[]
DataY=[]
for t in range(times):
x,y=randomwalk()

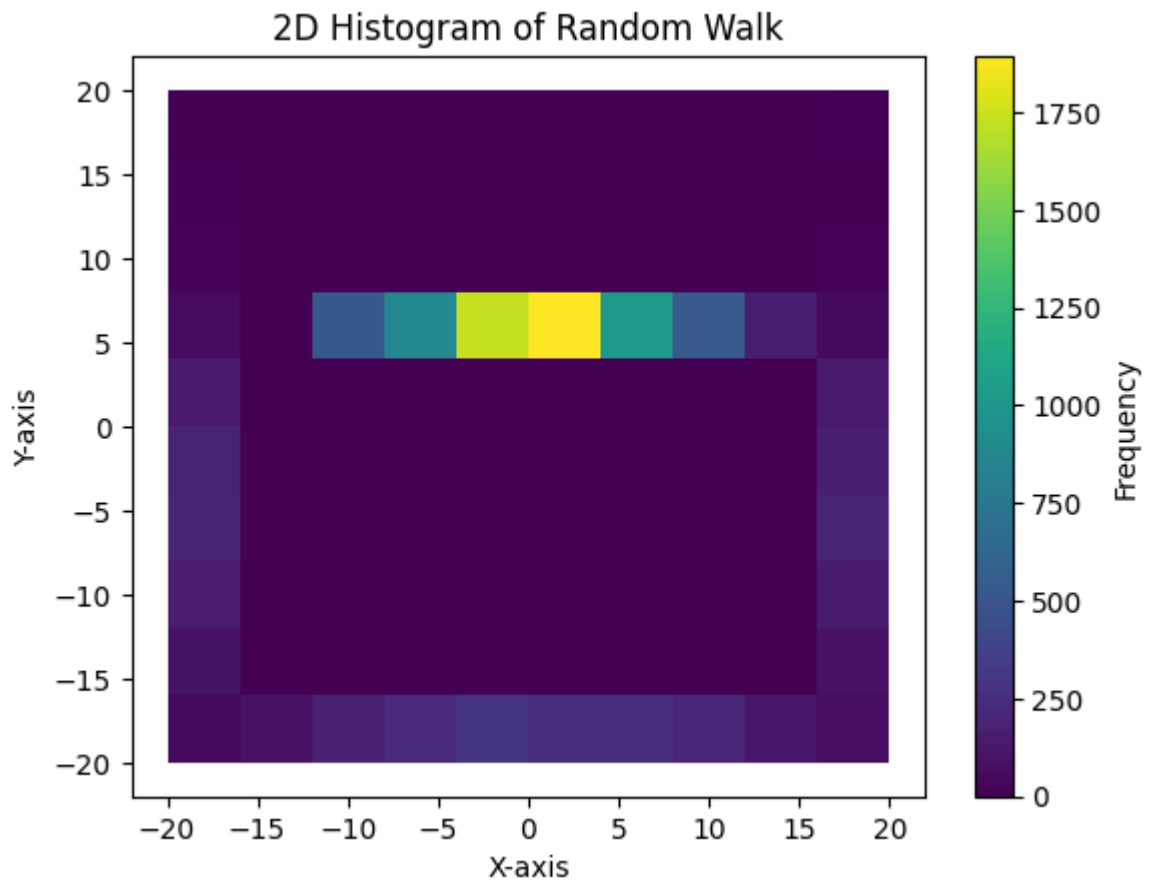
DataX.append(x)
DataY.append(y)
plt.hist2d(DataX,DataY)
plt.colorbar(label='Frequency')
plt.title('2D Histogram of Random Walk')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.xlim(-22, 22)
plt.ylim(-22, 22)
plt.show()
Simulate(10000)
```



```
In [28]: def randomwalk2():
P=[0,0]
while(((P[0]<20 and P[0]>-20 and P[1]<20 and P[1]>-20 ) and not((P[0]<=12 and
u=random.uniform(0,1)
if u<0.25:
P[0]+=1
elif u<0.5:
P[0]-=1
elif u<0.75:
P[1]+=1
else:
P[1]-=1
return P[0],P[1]
```

```
In [29]: def Simulate(times):
DataX=[]
DataY=[]
for t in range(times):
x,y=randomwalk2()

DataX.append(x)
DataY.append(y)
plt.hist2d(DataX,DataY)
plt.colorbar(label='Frequency')
plt.title('2D Histogram of Random Walk')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.xlim(-22, 22)
plt.ylim(-22, 22)
plt.show()
Simulate(10000)
```



列维飞行 积分可以得到:

$$\int_0^t \frac{2}{\pi^2 + x^2} = \frac{2}{\pi} \arctan(x) \Big|_0^t = \frac{2}{\pi} \arctan(t)$$

```
In [45]: def randomwalk3():
P=[0,0]
theta=random.uniform(0,360.001)
while(theta>360):
    theta=random.uniform(0,360.001)
x=random.uniform(0,1)
y=math.tan(math.pi/2*x)
x0=y*math.cos(theta/180*math.pi)
y0=y*math.sin(theta/180*math.pi)
P[0]+=x0
P[1]+=y0
return P[0],P[1]
def random_walk_2D(num_steps):
x = np.zeros(num_steps + 1)
y = np.zeros(num_steps + 1)

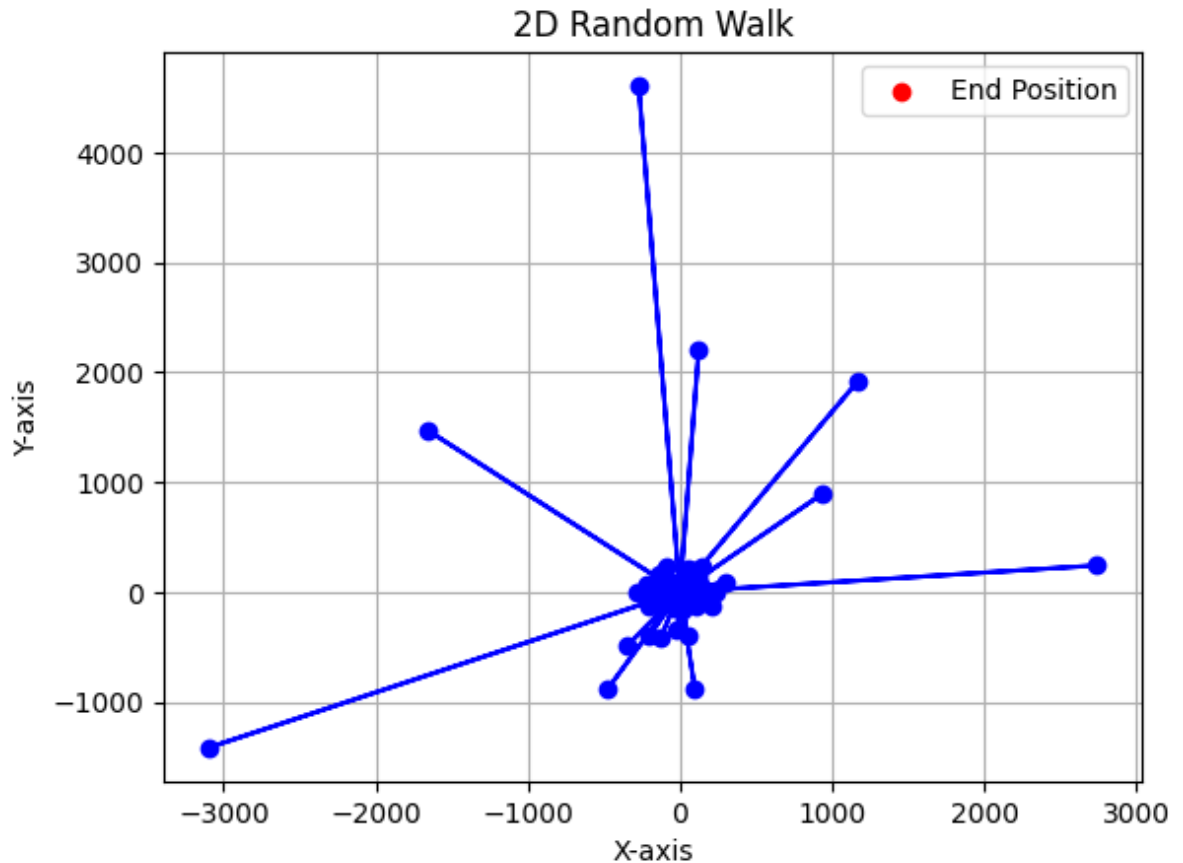
for i in range(1, num_steps + 1):
    # 随机选择方向
    angle = np.random.uniform(0, 2 * np.pi)
    x[i],y[i]=randomwalk3()

return x, y
def plot_random_walk(x, y):
plt.plot(x, y, marker='o', linestyle='-', color='b')
plt.scatter(x[-1], y[-1], color='r', label='End Position')
plt.title('2D Random Walk')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
```

```
plt.legend()
plt.grid(True)
plt.show()

# 模拟随机游走
num_steps = 10000
x, y = random_walk_2D(num_steps)

# 绘制最终位置分布图
plot_random_walk(x, y)
```



### 雅可比迭代

```
In [42]: import numpy as np

def jacobi_method(A, b, max_iterations=100000, tol=1e-3):
    n = len(b)
    x = np.zeros(n)
    for k in range(max_iterations):
        x_new = np.zeros(n)
        for i in range(n):
            x_new[i] = (b[i] - np.dot(A[i, :i], x[:i]) - np.dot(A[i, i+1:], x[i+1:])) / A[i, i]
        if np.linalg.norm(x_new - x, ord=np.inf) < tol:
            return x_new
        x = x_new
    raise ValueError("Jacobi method did not converge within the specified tolerance.")

A3 = np.array([[4, -1, 0],
               [-1, 4, -1],
               [0, -1, 4]])
b3 = np.array([15, 10, 10])

solution_3 = jacobi_method(A3, b3)
print("Jacobi Method - 3阶线性方程组的解:", solution_3)
```

Jacobi Method - 3阶线性方程组的解: [4.91043091 4.64233398 3.66043091]

## Monte Carlo方法

```
In [44]: import numpy as np

def monte_carlo_method(A, b, num_samples=100000):
    n = len(b)
    x_samples = np.random.rand(num_samples, n)
    b_samples = np.dot(x_samples, A.T)
    x_estimate = np.linalg.lstsq(A, b_samples.T, rcond=None)[0].mean(axis=1)
    return x_estimate

A3 = np.array([[4, -1, 0],
               [-1, 4, -1],
               [0, -1, 4]])
b3 = np.array([15, 10, 10])
# 3阶线性方程组的例子
A10 = np.random.rand(10, 10)
b10 = np.random.rand(10)

solution_3 = jacobi_method(A3, b3)
print("Jacobi Method - 3阶线性方程组的解:", solution_3)

solution_3 = monte_carlo_method(A3, b3)
print("Monte Carlo Method - 3阶线性方程组的解:", solution_3)
# 10阶线性方程组的例子
A10 = np.random.rand(10, 10)
b10 = np.random.rand(10)

solution_10 = monte_carlo_method(A10, b10)
print("Monte Carlo Method - 10阶线性方程组的解:", solution_10)

Jacobi Method - 3阶线性方程组的解: [4.91043091 4.64233398 3.66043091]
Monte Carlo Method - 3阶线性方程组的解: [0.50001453 0.50013696 0.49994444]
Monte Carlo Method - 10阶线性方程组的解: [0.5003056 0.50101161 0.49920796 0.5006717
1.0 0.49919134 0.50175129
0.49985776 0.49884955 0.50002169 0.49938059]
```

Jacobi Method 迭代次数有限，面对10阶迭代已经力不从心