Serie de Taylor

$$f(x_{i+1}) = f(x_i)+hf'(x_i)+\frac{h^2}{2!}f''(x_i)+\ldots+\frac{h^n}{n!}f^{(n)}(x_i)$$

Euler

$$\begin{aligned} y_{i+1} &= y_i + \phi h \\ \phi &= f(x_i, y_i) \\ h &= x_{i+1} - x_i \end{aligned}$$

Euler Modificado

Ecuación predictiva

$$y^0_{i+1} = y_i + f(x_i, y_i)h$$

Ecuación correctiva

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y^0_{i+1})}{2}h$$

Runge-Kutta 2do Orden

$$y_{i+1} = y_i + \frac{h}{2}(k_0 + k_1)$$

$$\begin{aligned} k_0 &= f(x_i, y_i) \\ k_1 &= f(x_i + h, y_i + hk_0) \end{aligned}$$

Runge-Kutta 3er Orden

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 4k_2 + k_3)$$

$$\begin{aligned} k_1 &= f(x_i, y_i) \\ k_2 &= f(x_i + \frac{h}{2}, y_i + \frac{h}{2}k_1) \\ k_3 &= f(x_i + h, y_i - hk_1 + 2hk_2) \end{aligned}$$

Runge-Kutta 2do Orden (Sistema de ecuaciones)

$$\begin{aligned} y_{j,i+1} &= y_{j,i} + (\frac{1}{3}k_{j,1} + \frac{2}{3}k_{j,2})h \\ k_{1,j} &= f_j(x_i, y_{1,i}, y_{2,i}, \ldots, y_{n,i}) \\ k_{2,j} &= f_j(x_i + \frac{3}{4}h, y_{1,i} + \frac{3}{4}hk_{1,1}, y_{2,i} + \\ & + \frac{3}{4}hk_{1,2}, \ldots, y_{n,i} + \frac{3}{4}hk_{1,j}) \end{aligned}$$

Diferencias finitas

$$\begin{aligned} \frac{d^2y}{dx} &= \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \\ \frac{dy}{dx} &= \frac{y_{i+1} - y_i}{2h} \end{aligned}$$

Sustitución (Ecuaciones orden superior)

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$$

$$y(0) = 0,\ y'(0) = 1,\ y''(0) = -1.$$

$$z_1 = y;\ f_1 = \frac{dz_1}{dx} = z_2$$

$$\frac{d}{dx}\left[\frac{d}{dx}\left(\frac{dz_1}{dx}\right)\right] - 3\frac{d}{dx}\left(\frac{dz_1}{dx}\right) - z_1\frac{d_z1}{dx} = 0$$

$$\frac{d}{dx}\left(\frac{dz_2}{dx}\right) - 3\frac{dz_2}{dx} - z_1z_2 = 0;\ f_2 = \frac{dz_2}{dx} = z_3$$

$$\frac{dz_3}{dx} - 3z_3 - z_1z_2 = 0;\ f_3 = \frac{dz_3}{dx} = 3z_3 + z_1z_2$$

Condiciones iniciales

$$z_1(0) = 0,\ \ z_2(0) = 1\ \ z_3(0) = -1$$