Computer Vision Homework 4

Theory:
Q1.1
Answer:

Generalized homogenous coordinates can be expressed as:

$$0 = [x_1 \quad y_1 \quad 1][F]\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

As they all lie on the same horizontal plane in the diagram while x and y serves as cursor that slides over this plane. Here we have the x_1, y_1, x_2, y_2 all equal to 0 both cameras are pointing to the exact same point and this point happens to be the centre-point of the axis.

Therefore $0 = [x_1 \quad y_1 \quad 1][F]\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$ can be re-expressed as:

$$0 = [0 \quad 0 \quad 1][F]\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now F is a 3x3 matrix and so we can see that the last element (bottom right –> third row, third column AKA F_{33} will be left equal to 0)

Q1.2
Answer

From the previous question:

$$0 = [x_1 \quad y_1 \quad 1][F]\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

Now from the notes, F can be represented as:

$$F = K'^{-T} t_x R K^{-1}$$

Now epipolar line behaviour is governed by these variables: $t_x R K^{-1}$

Therefore representing alone as:

$$t_x R K^{-1} = l'$$

Now since camera 1 is where it is, it can be expressed as:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} K^{-1} = l$$

And for camera 2 we have:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} K^{-1} = l$$

Comparing the two cameras we realize that the epipolar line for camera 2 aligns with the X-direction as they both share equal first collumns, wheras camera 1 only experienced a translation in X and since it must match epipolar lines of camera 2, its lines are aligned with X-direction.

Q1.3
Answer

We begin by defining the two frames of reference. These two frames we call $essential_1$ and $essential_2$. Their essential matrix can be represented as:

$$essential_1 = [t_{x1}][R_1] \quad and \quad essential_2 = [t_{x2}][R_2]$$

That deals with the essential matrix side of things but now we have to deal with the fundamental matrix. The fundamental matrix can be expressed as:

$$fundamental_1 = K^T[essential_1]K^{-1}$$

$$fundamental_2 = K^T[essential_2]K^{-1}$$

Given this fundamental matrix, we estimate the relative pose between a frame and the inertial sensor and we do this for both frames. Here is how represent it:

$$Rel_{Pose_1} = [K][R_1]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}[-t_1] \quad and \quad Rel_{Pose_2} = [K][R_2]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}[-t_2]$$

And if we assume that they have a common 3D point we can express them as:

$$Rel_{Pose_1}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix} = \begin{bmatrix}x_1\\ y_1\\ z_1\\ 1\end{bmatrix} \quad and \quad Rel_{Pose_2}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix} = \begin{bmatrix}x_2\\ y_2\\ z_2\\ 1\end{bmatrix}$$

So to get from Rel_{Pos1} to Rel_{Pose_2} we do the following:

$$Rel_{Pose_2} * Rel_{Pose_1}^{-1}\begin{bmatrix}x_1\\ y_1\\ z_1\\ 1\end{bmatrix} = \begin{bmatrix}x_2\\ y_2\\ z_2\\ 1\end{bmatrix}$$

With that, we just plug in the equations for Rel_{Pose_1} and Rel_{Pose2}. We obtain:

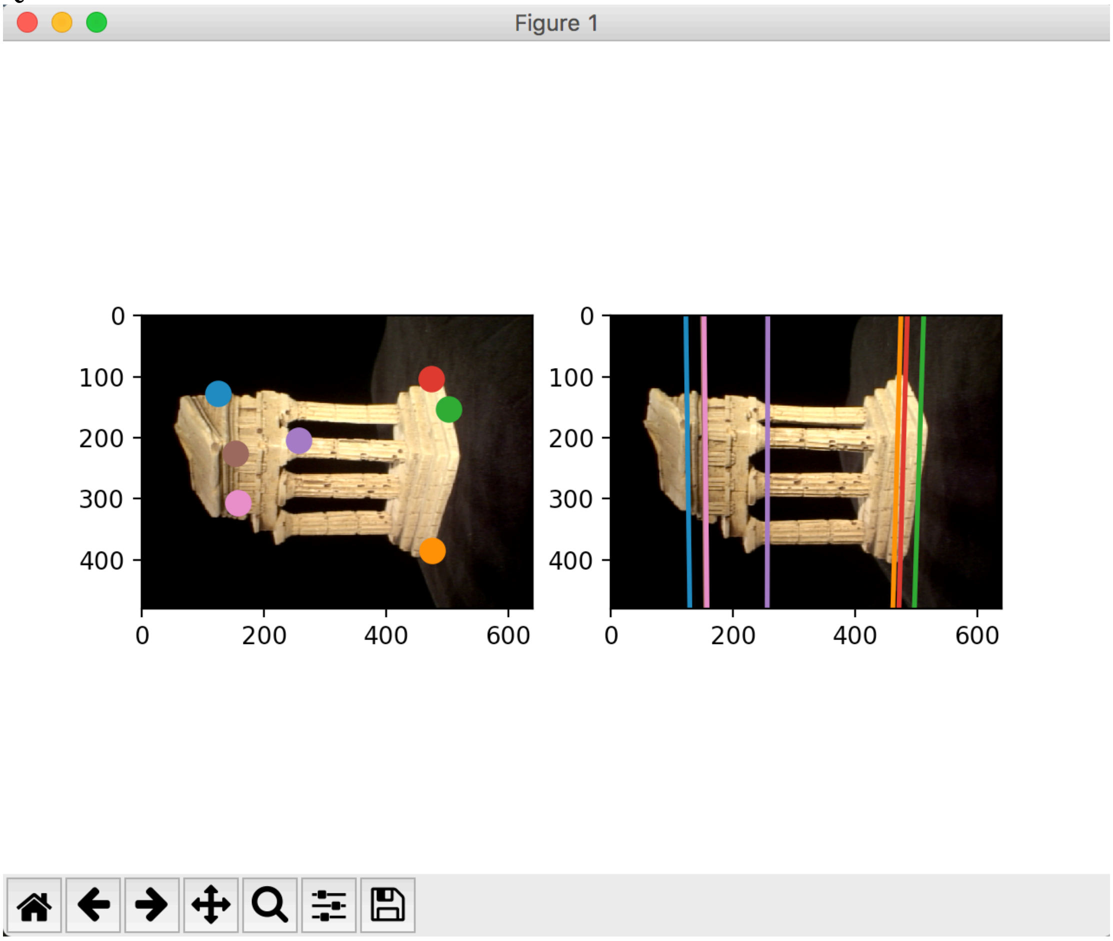
$$[K][R_2]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}[-t_2] \quad * \quad \left[[K][R_1]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}[-t_1]\right]^{-1}$$

Simplifying we get $Rel_{Pose_2} * Rel_{Pose_1}^{-1}$:

$$= \begin{bmatrix} & K & & 0\\ & & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}Rel_{Pose_2} * Rel_{Pose_1}^{-1} & R_2(t_2 - t_1)\\ 0 \quad 0 \quad 0 & 1\end{bmatrix}\begin{bmatrix} & K & & 0\\ & & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{bmatrix}^{-1}$$

Q1.4
Answer

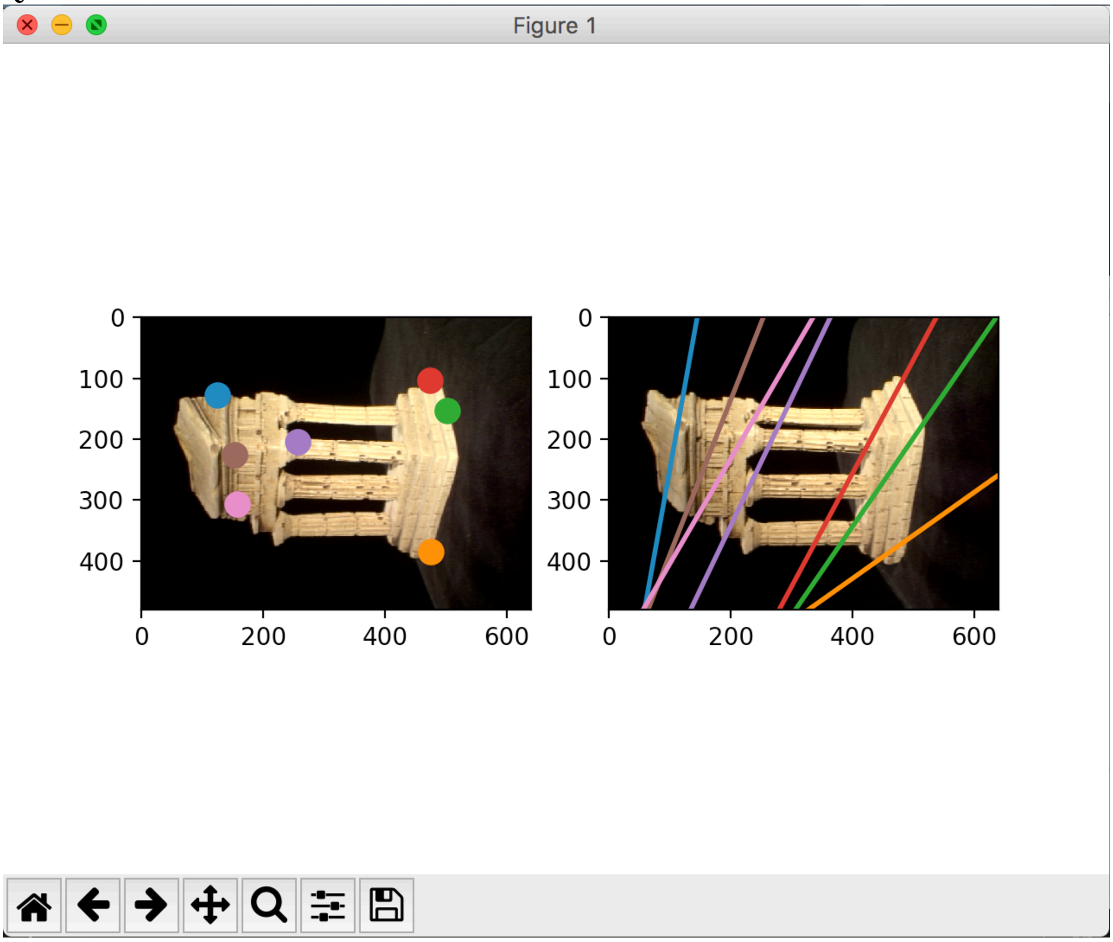
Q2.1



The Matrix Itself is:

```
[[ 0.00000031  0.00001421  0.25928411]
 [ 0.00003141 -0.00000086 -0.00731607]
 [-0.26998576  0.00396151  1.        ]]
```

Q2.2



The matrix itself is:

```
[[-0.00000582 -0.00002039  0.02350023]
 [ 0.00002014  0.00002909 -0.00254987]
 [-0.02093698 -0.01042157  1.        ]]
```

Q3.1

Answer

K1

```
[[1520.4     0.    302.32]
 [   0.   1525.9   246.87]
 [   0.      0.      1.  ]]
```

K2

```
[[1520.4     0.    302.32]
 [   0.   1525.9   246.87]
 [   0.      0.      1.  ]]
```

F

```
[[    2.28833486   103.73365453  1257.32657554]
 [  229.19518074    -6.2669569      9.44236901]
 [-1253.73634946    38.62817185     1.        ]]
```

Q3.2
Answer:

Matrix A

```
[[-1520.4      0.      -65.33      0.]
 [    0.  -1525.9    -86.87      0.]
 [-1519.5    39.01   -78.31    -41.21]
 [-1519.5 -1448.74   -48.34  -1518.5]]
```

Q3.3
Answer:

Sum of Square Errors:

```
1932.9671050418237
```

Selected C2 Matrix:

```
[[ 0.99927017 -0.03817801  0.0012546  -0.00765526]
 [ 0.0371901   0.96486513 -0.26010031  1.        ]
 [ 0.00871959  0.25995714  0.96558079  0.18279574]]
```

Q4.1

Answer:

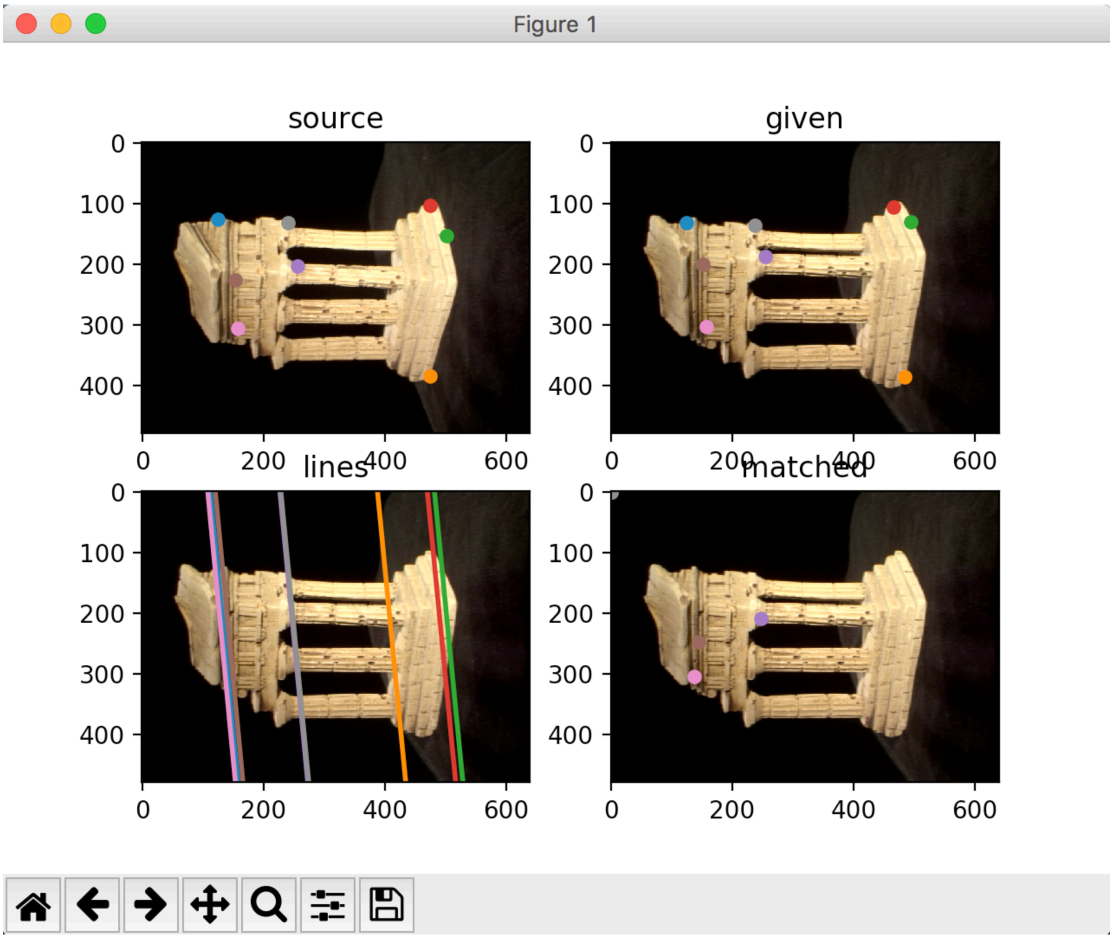


Figure 1
source
given
lines
matched
0
100
200
300
400
0
200
400
600

Q4.2

Answer

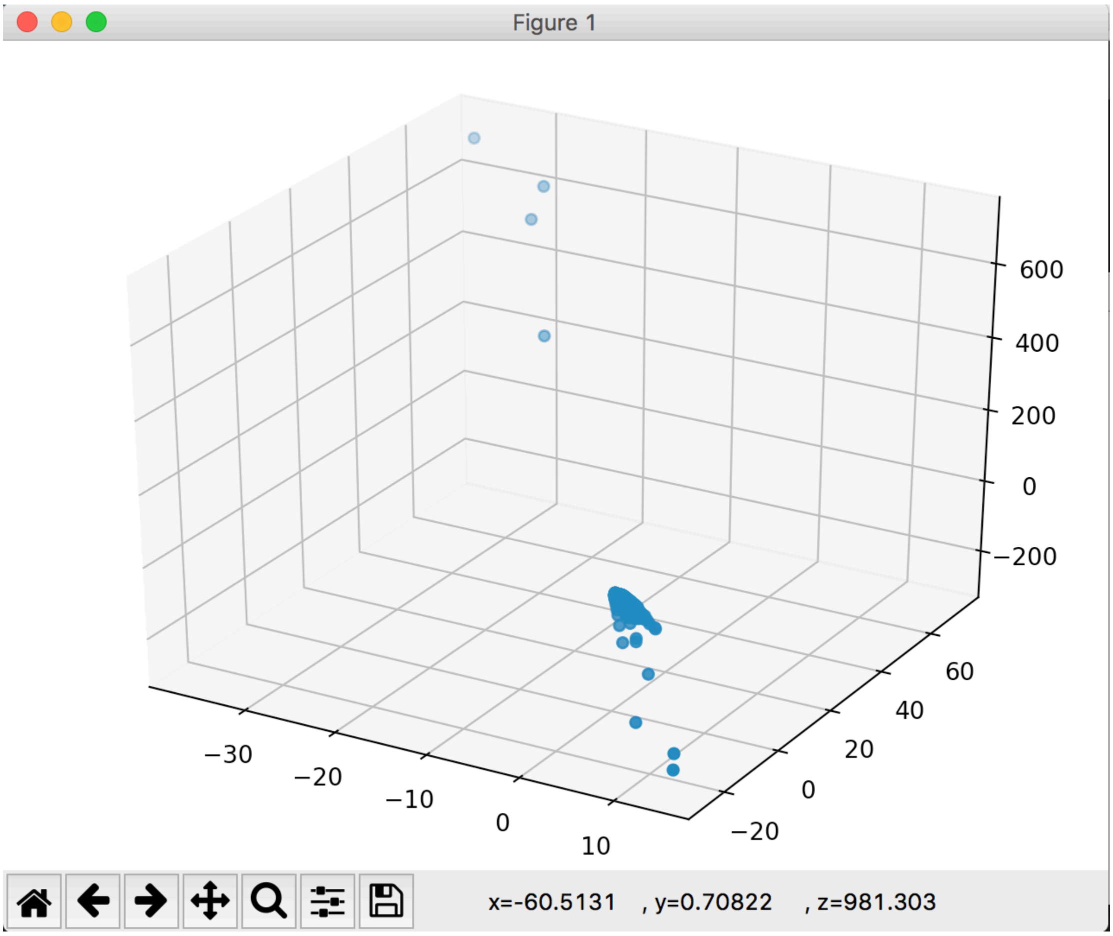


Figure 1
600
400
200
0
−200
60
40
20
0
−20
−30
−20
−10
0
10
x=-60.5131 , y=0.70822 , z=981.303