Star & Planet Formation Essay
March 28, 2023

Effect of planet migration on dust rings

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March 28, 2023

Key words. planetary systems – planet migration – planet-disk interactions – disks: dynamical evolution and stability

Contents

Nomenclature

α	Alpha Viscosity Parameter [$\alpha = cH/\nu$]
Δ	Width
ν	Viscosity
Ω	Angular Velocity
Σ	Surface Density
τ	Timescale
c	Speed of Sound
H	Scale Height
h	Aspect Ratio
M	Mass
P	Pressure
R	Radius
St	Stokes Number
t	Time
v	Velocity
Z	Metallicity

1. Introduction

Elaborate ring and gap structures are visible in exosolar disks in the (sub-) millimeter continuum. These structures can be explained by multiple or massive stationary planets as well as plant migration. As discussed by McClure (2021)[1] planet migration is assumed to be common and explains a diverse range of phenomena. Here it offers a compelling theory since the observed data fits well with the expected results from simulating the interactions of a a migrating planet and disk. In the following I will give an overview of the current research into these processes and detail some areas for further work.

[1] The lecture series.

2. Modeling Planet-Disk Interactions

Contemporary modeling of planet-disk interactions is done with FARGO-3D (Benítez-Llambay & Masset 2016), a coupled multifluid numerical simulation which has been expanded to include hydrodynamics (Masset 2000), multifluid (Benítez-Llambay et al. 2019), dust diffusion (Weber et al. 2019), the Hall effect and ambipolar diffusion (Krapp et al. 2018). For its application to disks the code solves the hydrodynamic equations[2] and magnetohydrodynamics, typically on a 2D cylindirical geometry. The position of planets are propagated via a Runge-Kutta N-body solver and dust is simulated as a pressureless fluid via Lagrangian particles.

Commonly, the gas surface density varies in the models varies as $\propto 1/R$ and the disk has a constant flaring index. Gas self-gravity is typically neglected. The dust is coupled to the gas via linear drag equations, however the effect of the dust on the gas is generally neglected (Garaud et al. 2004; Meru et al. 2019; Nazari et al. 2019). Additionally, the dust is not evolved in the simulations and the dust particles do not interact with each other. For a discussion of the effects of these assumptions see Wafflard-Fernandez & Baruteau (2020). While early papers such as Rosotti et al. (2016) had a fixed dust density at the inner boundary of the mesh, later papers such as Meru et al. (2019) and Nazari et al. (2019) let it vary by extrapolating it from a power-law (see Garaud (2007)). To avoid nonphysical results from producing dust at the inner boundary the maximum density is its initial value. This does not eliminate the production of dust entirely, but it constraints a worst case run-away model. The effect of potential dust production at this boundary is not further discussed in any of the above works, however since there is no back-reaction of the dust on the gas and its life-time is expected to be shorter than the planet migration time-scale (McClure 2021), the effects might be negligible. (Meru et al. 2019) The disc structure is commonly modeled in the 2D simulation as locally isotherm, however as pointed out by Miranda & Rafikov (2019) this assumption tends to result in an overestimation of contrast of ring and gap and incorrect planet positions. The difference is especially stark for structures consisting of large grains where $St \gtrsim 0.01$ (Kanagawa et al. 2020). Wafflard-Fernandez & Baruteau (2020) let the gas temperature vary by considering viscous heating, radioactive cooling and stellar irradiation for their disks.

The migrating planets in the reference works have a wide range of masses from 12 to 60 and $\backsim$ 95 $M_\oplus$ with accretion onto the planet generally being neglected. Meru et al. (2019)

[2] Continuity equations, Navier-Stokes equations and conservation of energy

and Nazari et al. (2019) decouple the planet migration from the simulation by determining the Type I migration timescale[3] τ_I by fitting the simulated orbital decay to:

$$R_\mathrm{p} = R_\mathrm{p,0}\mathrm{e}^{-t/\tau_I} \quad (1)$$

In Meru et al. (2019) the Type I migration timescale was validated for planet masses $M_P < 60M\oplus$ and tested against the expected timescale from linear theory $\tau_{I,\mathrm{lin}}$, see Equation 2 (Kanagawa et al. 2020), which differed by no more than a factor of 2. In subsequent simulations the planet migration was then independent from the simulation, evolving with Equation 1 immediately from the beginning of the simulation. Wafflard-Fernandez & Baruteau (2020) do not constrain the migration of the planet in this way and focus on Type III migration of a ∽ 95 $M_\oplus$ planet in a massive disk.

$$\tau_{I,\mathrm{lin}} = 1.68\left(\frac{c}{3}\right)\left(\frac{M_\mathrm{p}/M_*}{5\times10^{-5}}\right)^{-1}\left(\frac{\Sigma_\mathrm{un,p}}{1\ \mathrm{g/cm^2}}\right)^{-1} \left(\frac{h_\mathrm{p}/R_\mathrm{p}}{0.05}\right)^{2}\left(\frac{M_*}{1M_\odot}\right)^{1/2}\left(\frac{R_\mathrm{p}}{50\mathrm{AU}}\right)^{1/2}\mathrm{Myr} \quad (2)$$

3. Results

In the following I will be referring mainly to the results from Meru et al. (2019); Nazari et al. (2019) for Type I migration and Wafflard-Fernandez & Baruteau (2020) for Type III migration. See the relevant papers for an in-depth description of the results.

4. Discussion

4.1. Disk Structure

Gas and dust are cleared from the disk by falling towards the star at the radial dust velocity $v_\mathrm{d} = \frac{St}{\Sigma_g\Omega}\frac{\partial P}{\partial R}$ whereas a migrating planet in the simulations of Meru et al. (2019) and Nazari et al. (2019) is moving at the radial planet velocity $v_\mathrm{p} = \frac{R_p}{\tau_I}$. Furthermore, planets affect the disk material around them. Most important is their effect on the pressure profile in the disk. Rosotti et al. (2016) identifies two mechanisms for ring and gap formation in the disk: A strong enough pressure perturbation can create a pressure maximum which leads to a change in the direction of drift since the radial drift velocity v_{drift} relates to the pressure field as:

$$v_{drift} \propto \frac{\partial \log P}{\partial \log R} \quad (3)$$

In case the pressure perturbation is too small to create a local pressure maximum it can still lead to a local minimum in the pressure gradient. Then, again with Equation 3, the radial drift velocity is at a minimum, leading to a build up of gas and dust due to the gas-dust coupling if the flow is steady-state which can be seen in Figure 2. (Meru et al. 2019) In the case of a stationary planet a pressure maximum is produced outside of the planets orbit. Inside the orbit no pressure maximum is present, however a inflection point might exist.

The resulting gas surface density profiles are shown in Figure 1 where a planet is migrating inwards from $R = 1$ at Type I migration rate. Inside the planet there is a density maximum with a less pronounced maximum outside the planet which develops into an inflection point as the planet migrates.

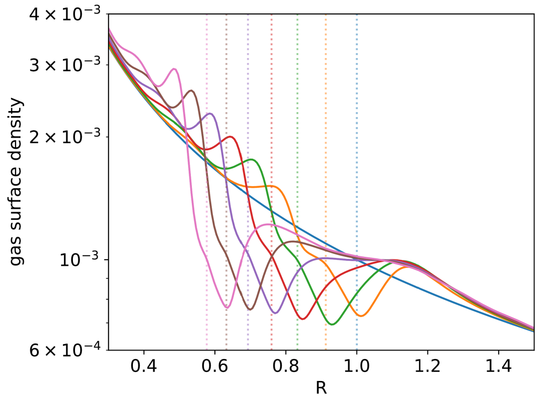

Fig. 1. Gas Surface density evolution for a 60 $M_\oplus$ planet, initially at $R = 1$, migrating at Type I rate. The dotted lines are the planet locations corresponding to the density profile of the same color. (Meru et al. 2019)

Nazari et al. (2019) confirm this result and identify three cases for inward migration: a slow ($v_p << v_d$), intermediate ($v_p \approx v_d$) and fast ($v_p >> v_d$) migrating planet. The regime into which a specific planet falls is dependent not only on its migration rate but also on the grain size. For the critical case $v_p = v_d$ there is a critical grain size and Stokes number (see Nazari et al. (2019)) which will be discussed further in subsection 4.4.

For the slow case the dust interior to the planet drains onto the star, however the dust exterior to it is trapped in a pressure maximum, resulting in a ring. This is the same result that is expected for a stationary planet Rosotti et al. (2016) for which the exterior ring develops on the same timescale as the interior is cleared of dust due to its radial drift. In case of a fast migrating planet the process is the reverse. The exterior dust does not keep up with the pressure maximum or inflection point whereas the dust interior to the planet is swept up in the resulting pressure maximum. As the radial dust velocity depends on the dust grain size larger dust particles are better able to keep up with the pressure maximum or inflection point exterior to the planet. This leads to a differentiation of the different dust grains in radial direct exterior to the planet (somewhat visible in Figure 2). This could have some implications for the formation and growth of particles in these zones which is noteworthy since this growth would occur in a particle trap inhibiting further inward migration, thus counteracting the radial drift problem. In the intermediate case two rings are produced on either side of the planet due to the same mechanisms as above. (Nazari et al. 2019)

Wafflard-Fernandez & Baruteau (2020) analyse a single planet undergoing periods of runaway inward migration. They confirm the theory of ring formation due to migrating planets and expand on the transition process at the end of each phase of runaway migration. When runaway migration halts gas is able to enter the horseshoe region of the planets co-orbital ring. The gas then exerts a positive co-rotation torque on the planet through secondary U-turns. This further decelerates the planet which leads to its wakes shocking the gas exterior to its orbit. These shocks rebuild the pressure maximum or inflection point which traps dust in it, building a new ring outside the planets orbit. During the following phase of (non-) migration the aforementioned processes enhance the rings. Since the runaway Type III migration timescale is very short compared to the infall timescale of the dust the rings can persist even after the planet has migrated away during another phase of runaway migration as can be seen in Figure 2. This process could explain massive disks with multiple gaps and rings, which were previously only ex-

[3] Both papers consider only Type I migration

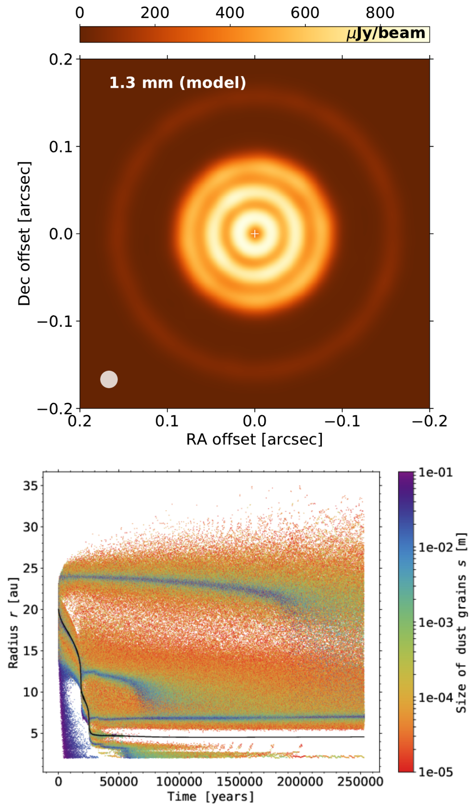

Fig. 2. Top: Simulated 1.3 *mm* continuum emission for an intermittently migrating $M_\oplus$ planet after 59.9 kyr (669 planet orbits) after convolution with a beam (0".02) for a system with a planet-star mass ratio of $q_p = 4.3\times10^{-4}$, $\alpha = 10^{-4}$ and a dust grain size of $0.01 - 10$ *cm* distributed as $n(s) \propto s^{-3.5}$. The planet is outside the innermost ring. Bottom: Space-time diagram for the Lagrange particles (dust) in the same simulation. The planet position is shown as a black line. (Wafflard-Fernandez & Baruteau 2020)

plained by massive or multiple planets such as by Dong et al. (2017); Zhang et al. (2018). For an in-depth analysis of the transient stopping process due to the gas see Kanagawa (2019) and Wafflard-Fernandez & Baruteau (2020) and for an exploration of these process for low mass planet migration see McNally et al. (2017, 2018).

In the case of outward migration (Bitsch & Kley 2010; Ayliffe & Bate 2010) it is expected that a single ring forms in the pressure maximum exterior to the planets orbit. (Meru et al. 2019) However, as Weber et al. (2019) finds, in the case of fast outward migration the planet destroys the pressure trap and allows the exterior dust to migrate inwards past the planet, destroying the ring.

4.2. Planet Mass

The main effect of planet mass on the planet-disk interaction is via the migration rate as more massive planets migrate faster. This leads to stronger pressure maxima for more massive planets which in turn lead to more dust being trapped. Conversely, there is a minimum planet mass below which planets do not open detectable gaps. For stationary planets this critical mass was identified by Crida et al. (2006); Rosotti et al. (2016) to be $10 - 20M_\oplus$ depending on the wavelength used for observation. This enables observations to limit the mass of a planet with relations derived in Rosotti et al. (2016); $M_\mathrm{p} \propto (\Delta_\mathrm{gap}/R_\mathrm{p})^{3/1.143}$ or $M_\mathrm{p} \propto (\Delta_\mathrm{ring}/R_\mathrm{p})^{1/0.32}$ which depend only on the relative sizes of the visible structures c.f. Dong & Fung (2017). Determining the critical mass for a migrating planet will require additional simulations, however Meru et al. (2019) provides some speculations: They show that the location of the pressure maximum is not significantly affected by the migration rate, however they also find that the gap becomes asymmetric due to migration as is visible in Figure 3. The asymmetry would lead to an overestimation of R_P whose mitigation will require further simulations. To resolve this problem Kanagawa et al. (2020) determine empirical formula for the position of the planet in the gap for a rapidly inwards migrating planet. By comparing the time scale of gap formation t_{gap} and planet migration t_{mig} from previous work (Kanagawa & Szuszkiewicz 2020; Kanagawa et al. 2018);

$$t_\mathrm{gap} = \left(\frac{\Delta_\mathrm{gap}}{2R_\mathrm{p}}\right)^2 \left(\frac{h_\mathrm{p}}{R_\mathrm{p}}\right)^{-2} \alpha^{-1}\Omega_{\mathrm{K,p}}^{-1}\,, \tag{4}$$

$$t_\mathrm{mig} = \left[1 + 0.04\left(\frac{M_\mathrm{p}}{M_*}\right)^2 \left(\frac{h_\mathrm{p}}{R_\mathrm{p}}\right)^{-5} \alpha^{-1}\left(1 - e^{-t/t_\mathrm{gap}}\right)\right]\tau_I\,, \tag{5}$$

an empirical formula can be fitted to the simulation results of Kanagawa et al. (2020) which gives an expression for the position of the planet relative to the gap:

$$\frac{R_\mathrm{gap} - R_\mathrm{p}}{h_\mathrm{p}} = 6.05\exp\left[-\left(\frac{t_\mathrm{mig}}{t_\mathrm{gap}}\right)^{0.25}\right] \tag{6}$$

As these equations are functions of the planet mass as well as disk properties, they can be used to constrain these parameters based on observations.

4.3. Viscosity

As is visible from Figure 2 rings and gaps can persist for some time even without the planet present. Their total lifetime is constrained by the turbulent, viscous interactions with the disk in the absence of other bodies. As Wafflard-Fernandez & Baruteau (2020) show, the lifetimes of structures in low viscosity disks ($\alpha \lessapprox 10^{-5}$) are significantly longer than those for more viscous disks ($\alpha \gtrapprox 10^{-3}$) which is expected. (Masset & Casoli 2010; Weber et al. 2019) However, even in the presence of additional, small migrating planets, which would be otherwise below the critical mass discussed in subsection 4.2, these structures can be prematurely disrupted.

4.4. Dust

Larger dust grains migrate faster, thus "changing" the regime into which the migrating planet falls. Nazari et al. (2019) identify a strong dependence of the dust trapped in the pressure maxima or inflection points on the maximum dust size. As mentioned above, large dust migrates inwards faster so that in disks with large grains planet migration might seem slower than it actually is.

In disks with high metallicity and large dust grains ($Z \gtrsim 0.3$, $St \gtrsim 0.03$) the dust might also affect the migration of planets by inducing small scale dust vortices in the co-orbital region of the migrating planet. These vortices then consequently also form at the edges of the dust gap carved out by the planet, widening it. The interaction of the vortices and the planet are chaotic and can halt or reverse the migration of the planet. Since the process is stochastic the exact position of the planet is case dependent, but due to vortices developing on either side of the gap the co-rotation torques on the planet scatter it back into the gap whenever it gets close to either edge of the gap. (Hsieh & Lin 2020) However, the results of this analysis are not yet confirmed and rely on the assumption of efficient vortex formation. In Hsieh & Lin (2020) dust-gas interactions are simulated while dust-dust interactions are neglected. Yang & Zhu (2019) consider the dust-dust as well as dust-gas interaction and find more efficient vortex formation, increasing the likelihood and decreasing the threshold (Z, St) for stochastic planet scattering. It is worth noting that the critical resolution that Hsieh & Lin (2020) identify as a requirement to observe this dynamic behaviour is $\backsim 4^2$ that of other simulations[4] such as Nazari et al. (2019); Wafflard-Fernandez & Baruteau (2020) and would have not been caught by the resolution test of Meru et al. (2019).

4.5. Accretion

Accretion into the planet affects both its migration rate as well as the surrounding pressure field by removing matter from the planets surroundings. Dürmann & Kley (2015) find however, that the assumption to neglect these effects, mentioned in section 2, is generally valid. The migration rate is generally slightly reduced due to weaker co-rotation torques induced by the reduced density and the increased inertia of the growing planet, and the now larger gap resulting from the larger Hill sphere of the planet. (Dürmann & Kley 2015, 2017)

4.6. Gas Self-Gravity

Wafflard-Fernandez & Baruteau (2020) find that their simulations are significantly different when the gas' self gravity is considered. Including self-gravity mainly affects the initial migration rate, some transient behaviour and the final state of the system. Furthermore, only considering the axisymmetric part of the gas self-gravity is, while initially a good approximation, similar to simulations completely neglecting it. The missing back-reaction of the gas on itself when the gas acts on the planet leads to a missing forced torque when self-gravity is neglected which explains to this discrepancy. (Pierens & Huré 2005; Baruteau & Masset 2008; Zhu & Baruteau 2016)

5. Observation

No unambiguous observations of a migrating planet producing rings exist at this point. Hence, the strategy in past works relied on simulating observations, based on the modeled disks. As explained above the signatures of the disks are best visible in the (sub-) millimeter continuum due to the grain sizes and temperatures. Of specific interest are observations from the Atacama Large Millimeter/submillimeter Array (ALMA), either from the recent Disk Substructures at High Angular Resolution Project (DSHARP) or future observations focused on disks. To this end all Meru et al. (2019); Nazari et al. (2019) and Wafflard-Fernandez & Baruteau (2020) simulate observations at different wavelengths based on their simulations. The results of one of these simulated observations by Nazari et al. (2019) are shown in Figure 3.

The morphologies discussed in subsection 4.1 are clearly visible for the different migration rates and might also be visible for ALMA observations such as shown in Figure 2 for a complex

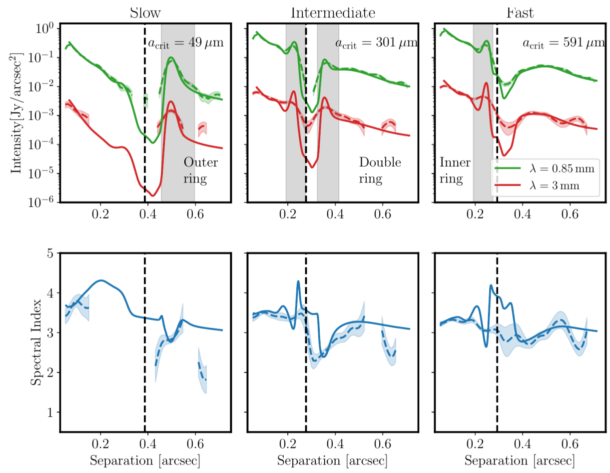

Fig. 3. Simulated intensity (top) and spectral index (bottom) profile by Nazari et al. (2019) with a maximum grain size of 1mm for a migrating 30 $M_\oplus$ planet. The dashed line indicates the planet position in the gap.

ring system. Depending on the visible ring structure a migrating planet can be detected, see Figure 4. As discussed before a ring interior to a gap would immediately confirm a migrating planet as $v_p >> v_d$ in this case. An exterior ring might be either slowly migrating or stationary and ALMA could not differentiate. In the case of two rings around a single gap the decision requires further data to differentiate it from multiple stationary planets (Dipierro et al. 2015; Long et al. 2018) or multiple gaps being opened by a single planet due to a low viscosity disk (Dong et al. 2017; Bae et al. 2017; Fedele et al. 2018). Nazari et al. (2019) identify the spectral index of the two rings as a possible criterion. Due to the radial velocity of the dust and the radial velocity of the planet the grain sizes that are in the inner ring will be smaller that the ones outside the ring. Hence rings due to intermediate rate migration can be distinguished by their spectral indices from other processes resulting in two rings around a gap. Figure 3 shows how the spectral index could be used in this way, c.f. Figure 12 of Pinilla et al. (2021a) for a stationary planet. This result is confirmed by Weber et al. (2019); Wafflard-Fernandez & Baruteau (2020). A false negative could occur if the inner ring is optically thick. (Nazari et al. 2019) Furthermore, false positives and false negatives could occur due to shadowing such as in the disks explored by Wolff et al. (2020); Stolker et al. (2016); Benisty et al. (2018); Muro-Arena et al. (2020), however for most expected shadowing cases the asymmetries would be obvious indicators of other processes as long as the different disks are of similar age and composition.

Occlusion by the disk itself becomes important for large disk masses in which moderately fast migration is expected but the disk is optically thick. In these cases the spectral index can not be used to identify a migrating planet. For even more massive disks however, most planets migrate very fast, thus producing only an interior ring which does no longer require a spectral index measurement to confirm migration. (Nazari et al. 2019)

Some of the recent progress has already been applied to explain observations, however comprehensive studies the morphology of the imaged disks with rings and gaps is still outstanding. Weber et al. (2019) uses the developed theory to constrain the migration rate of a possible planet around HL Tau as predicted by Dong et al. (2018). The results of Nazari et al. (2019) lead them to belief that the structures in Elias 24 (see Cieza et al. (2017)) are not, like previously thought by e.g. Zhang et al. (2018), due to a stationary planet but rather due to a migrating

[4] at roughly the same cell aspect ratio

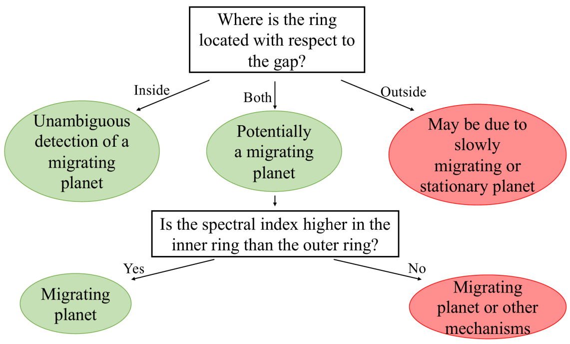

Fig. 4. Description of the summary figure. (Nazari et al. 2019)

one which is supported by the brightness of the rings. Nevertheless, Jiang & Ormel (2021) explore a process that does not rely on pressure maxima to explain the structure of the system. In both cases however, an in-depth and comparative analysis is still outstanding. Recently the developed models have been used to constrain the companion planet masses of disks which are part of DSHARP by Jorquera et al. (2021).

6. Future Work & Recommendations

In the following I will discuss possible developments and further work in this field.

6.1. Observation

As pointed out by Nazari et al. (2019) the capabilities of ALMA have made high spatial resolution probing of disks possible, however its maximum wavelength is limited by both technology and the weather. Long wavelength observation with Next Generation Very Large Array (ngVLA) would allow for probing of the grain distributions in the disks, the spectral index and currently occluded regions. ngVLA can confirm the validity of the developed models (threshold for gap development in the case of stationary planets) for small planet masses ($5-10\ M_{\oplus}$) whose gaps and rings will be resolvable. (van der Marel et al. 2018; Ricci et al. 2018) Reaching out into different fields might also yield additional constraints on the processes by considering the end results of planet migration such as explored in Unterborn et al. (2018). Generally, expanding observational capabilities would not only benefit research in this field but also other research at (sub-) millimeter wavelengths.

6.2. Models

Currently no models account for dust evolution, which might especially influential in the pressure traps which are the main target for detecting a migrating planet. If observation time is allotted to disk surveys probing for migrating plates using the methods described by Nazari et al. (2019) the probed bands have to be determined carefully and evolution of the dust in the traps might affect their visibility. This might be especially severe since the processes in the dust traps are very dynamic and not fully understood as current models have poorly validated assumptions due to: potential small scale vortices discussed in subsection 4.4, non-isotherm disks as discussed in section 2, and accreting planets discussed in subsection 4.5. All of these phenomena would be especially severe in the dust traps. Further simulations focusing on the dust traps might verify these assumptions. This could either be an extension of the existing FARGO-3D code with new physics and (potentially) requiring a complex mesh and/or a new multiphysics model focused on the dust traps. However, even if above processes can be decoupled from the evolving disk, a new multiphysics model would be complex and involve evolving boundary conditions. Concerning the mesh, there is a good case to be made to move to a time-variant, multi-resolution mesh due to the the different scales involved. A strong candidate for evolving mesh geometries is localized wavelet compression which is ideally suited for turbulent, multi-species and compressible flows (see e.g.: Farge & Schneider (2015) as well as Schneider & Vasilyev (2010); Bergdorf & Koumoutsakos (2006); De Stefano et al. (2008))[5]. Its disadvantage, loss of resolution at the highest frequencies, might not be very significant as those are not resolvable by neither ALMA nor ngVLA. Naturally, fully 3D simulations would be beneficial but until detectors can map the disks at much higher spatial resolutions or there is a need to determine the behavior in dust traps with high accuracy (e.g. to probe for particle growth) it is probably too expensive.

6.3. Multi-Planet Systems

Disks can form multiple planets that might migrate simultaneously. These migrations might be coupled or involve chaotic interactions. Currently no work explores these processes since the base process is still poorly understood. However, if large scale disk surveys with the aim to discover migrating planets are conducted some systems will probably include multiple planets (Rowe et al. 2014), hence the need to at least qualitatively explore the expected morphologies of these systems like was done for stationary planets by Zhang et al. (2018).

In low viscosity systems such as Fedele et al. (2018) with multiple planets, disrupted structures can be used to identify (migrating) planets whose impact on the disk would otherwise not be observable, see subsection 4.3.

6.4. Planet Detection, Mass & Orbit

A fully developed theory of the effect of a migrating planet on disks allows the indirect detection of planets which would otherwise not be visible. This will develop our understanding not only of planet formation and evolution but also of the wider statistics behind these processes, and with that further constrain the probability for habitable planets.

Several aspects of planet migration are not sufficiently well understood such as the final orbit and mass of a migrating planet. Debras et al. (2020) investigates planet migration to explain the highly eccentric orbits of some hot Jupiters, Ndugu et al. (2020); Piaulet et al. (2021) find implications for the masses and composition of planets, and further research may also help explain smaller phenomena closer to home such as discussed by Madeira et al. (2021).

6.5. Dust Traps

A more advanced theory of particle evolution, composition and particle-particle interactions in dust traps would not only be beneficial for this research but also support developments in planet formation, disk evolution and solar system research.

5 Or see this video for a less dense explanation.

7. Conclusion

In this work an overview of the current state of research into the interactions of migrating planets and disks was developed. The interactions have implications on planet formation as well as disk evolution and structure. Disks perturbed by planets migrating at a fast or intermediate rate have unambiguous features visible to ALMA and ngVLA. Furthermore, a single intermittently migrating planet can cause multiple rings and gaps. There is general agreement about the structure and dynamics of disks and migrating planets, however some smaller scale processes are not fully explored and might have implications for the larger structure. Further disk surveys by ALMA and ngVLA, expanding observational capabilities as well as validating model assumptions and expanding them are promising areas for future work.

Acknowledgements. I would like to thank Dr. M.K. McClure for the inspiring lecture series as well as Pooneh Nazari and Ardjan Sturm for their explanations during the lecture reviews and example sessions.

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Further Reading

I have again included a small section for further reading. It is not part of the essay but rather was a bit of a repository of some explanations that did not make the cut, and tangents I went on while researching for this essay. I have left them here in the appendix in case I ever need to reference them in the future. Please do not include these comments in the grading as that would be unfair to the other students.

- Modeling
  - Dust - gas interactions & modeling: Garaud et al. (2004), Garaud (2007)
  - Radiative gas: Pierens & Raymond (2016). In Wafflard-Fernandez & Baruteau (2020) radiative diffusion is neglected due to the added complexity and computational effort of solving the associated Monte Carlo simulations or the generally less expensive but also less accurate radiative transfer equations. For a discussion see Ayliffe & Bate (2010); Samadi et al. (2020) and Krumholz et al. (2020).
  - Wavelet compression: Lord et al. (2012); Fröhlich & Schneider (1999); Schneider et al. (1997) I am naturally aware that this is very unlikely to happen since it would be expensive to develop and a usable solution already exists. However, it is *super* cool.
- Planet Migration
  - Outwards: Regály (2020); Paardekooper (2014); Lyra et al. (2010), strong temperature gradients might be responsible: Lega et al. (2015); Bitsch et al. (2018); Masset (2017); Guilera et al. (2019); Chametla & Masset (2020); Chrenko & Nesvorný (2020), or dynamic scattering/ interactions: Veras & Armitage (2004); Chrenko et al. (2017); Pierens & Raymond (2016) (and Dempsey et al. (2021))
  - Type III: Lin & Papaloizou (2010); Lin (2012); Baruteau & Masset (2013)
- Dust:
  - General: Hughes & Armitage (2012) and one of the earliest mentions: Lecavelier Des Etangs (1998)
  - Dust Traps: probably most interesting in relation to planet migrations: Fu et al. (2014); Montesinos et al. (2020), otherwise: van der Marel et al. (2013); Baruteau & Masset (2013); Zhu et al. (2014); Pinilla et al. (2021b); Benac et al. (2020); Longarini et al. (2021); Jiang & Ormel (2021); Norfolk et al. (2021)