① The Solow Model

- $\dot{K}(t)$ = investment - depreciation [slide 38]
- National accounting identity: $I(t) = S(t)$

[slide 40] 3rd line + 4th line: lacks "times $k(t)$"
↳ fundamental law of motion in per capita units

[slide 46] → ε_k: elasticity of production w.r.t. capital

[slide 48] efficiency units are always denoted as small letters with tilde

[slide 53] always a convergence to k^* (either from the left or the right)

② OLG model

- utility maximization ≠ Solow
- New: population with heterogeneity

[slide 5] $L_1, \dots, L_4$ are population masses ≠ Solow: households not stacked over each other
↳ generation is born at time t and is alive in periods t and $t+1$
↳ heterogeneity across cohorts, but not within cohorts

[slide 7] exogenous pop. growth rate n ; individuals only work in the first period of life
savings not exogenous but endogenous

[slide 8] superscripts 1 and 2 for young and old ind., respectively
$U(c_t^1, c_{t+1}^2) = u(c_t^1) + \beta u(c_{t+1}^2)$ β → discount factor for time preference (measure of impatience)

[slide 10] consumption today vs. consumption tomorrow ⇒ Euler equation
↳ return on capital has to be sufficiently high to compensate for impatience $(1+r_{t+1})$ vs. $\beta \in [0;1]$

[slide 12] incentive to save more with higher r vs. less savings now will generate the same future savings as with a lower r (substitution and income effect)

[slide 13] CIES utility in more detail in Ch. 3

[slide 14] same intuition as before

[slide 15] $\frac{\partial s_t}{\partial w_t} > 0$ again , $\frac{\partial s_t}{\partial(1+r_{t+1})} \gtrless 0$ ⇒ depends on θ (which can also be >1)
↳ setting $\theta = 1$ makes the equations much easier
↳ θ → coefficient of relative risk aversion → willingness to trade off consumption today and tomorrow

[18] R_t = return to capital = $f'(k_t)$

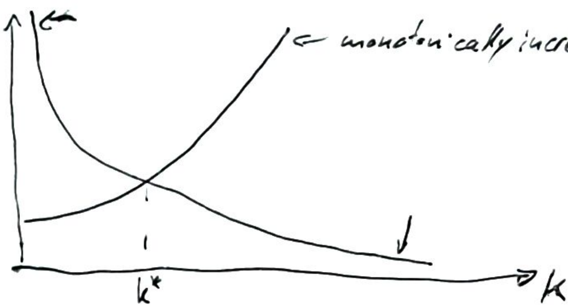
[19] $k_{t+1} = \dots$ ⇒ law of motion

[22] $\delta = 1$ makes the problem much simpler even though it is very unrealistic

↳ only a good assumption if it allows for enormous simplification

[23] denominator multiplied and k_t^- divided on both sides

[24] not exam-material, not be able to explain but Intermediate Value Theorem



log utility → just a horizontal line → also just one steady state

[28] multiple equilibria if function is not monotonic

→ exam: explaining those graphs (and possible adjustments and changes that could occur)

↳ poverty trap: several steady states → which one is reached depends on initial capital accumulation

→ only exogenous push could help if initial k is too low for higher steady state

[29] externality arises since generations cannot contract with each other → steady state qu.

[30] assuming no information frictions, competitive equilibrium no production externalities

↳ but still: social planner does better

↳ pecuniary externality: young would possibly like to interact with former generation to alter their savings decision

↳ planner has a total resource constraint but no budget constraint [31]

[32] social planner can choose consumption and capital allocation

↳ with full depreciation, same result as before (path is the same)

↳ problem is that steady state is different

[34] c^{ss} is consumption p.c. in the steady state [correction]

[35] if k is too large that decreasing it would incur a higher return than keeping it that high

↳ would increase the return more than the decrease

[36] because of the complementarity in the production function

[38] only difference: BCs have changed

r_{t+1} the same for s_t and d_t

↳ problem did not fundamentally change

↳ higher accumulation if d_t was higher than the personal s_t choice

[39] d_{t+1} ~~[illegible]~~ → depends on n this time; with negative n one would lose out

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