Macroeconomics - Growth

Unified Growth

Gerrit Meyerheim

Economics Department
LMU München

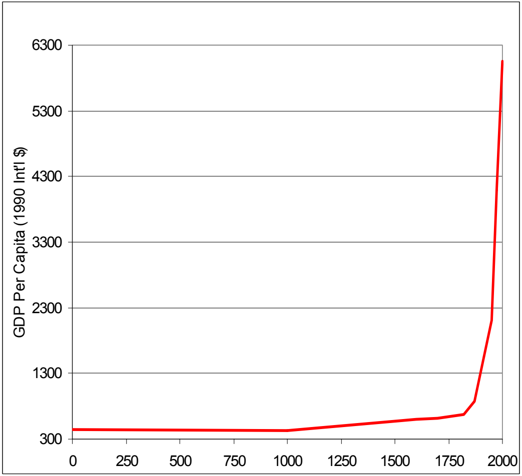
Winter Semester 20/21

Neo-Classical vs. Unified Growth

- Neo-classical growth theories characterize steady state growth (and convergence towards it).
- Endogenous growth models generate sustained positive growth rates.
- The main emphasis is on features that allow to overcome decreasing returns (expanding varieties, improved quality, increasing knowledge, productivity etc.).

Neo-Classical vs. Unified Growth

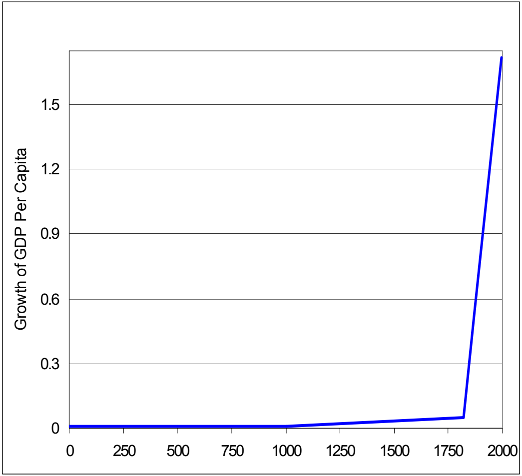
GDP Per Capita



(Source: Maddison, 2003, Galor, 2005)

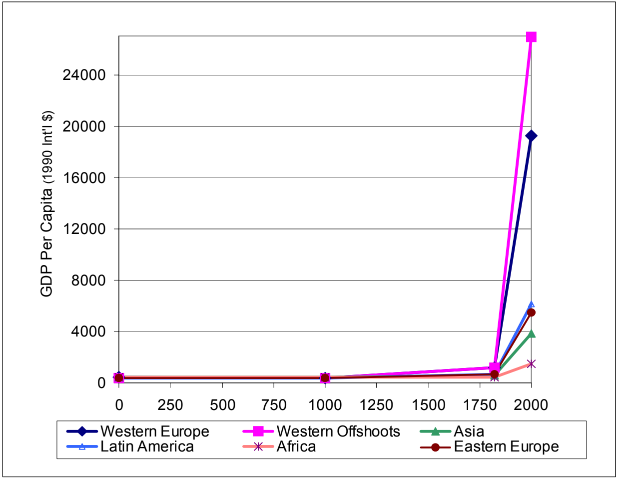
Neo-Classical vs. Unified Growth

Average Annual growth of GDP Per Capita



(Source: Maddison, 2001, 2003, Galor, 2005)

Neo-Classical vs. Unified Growth

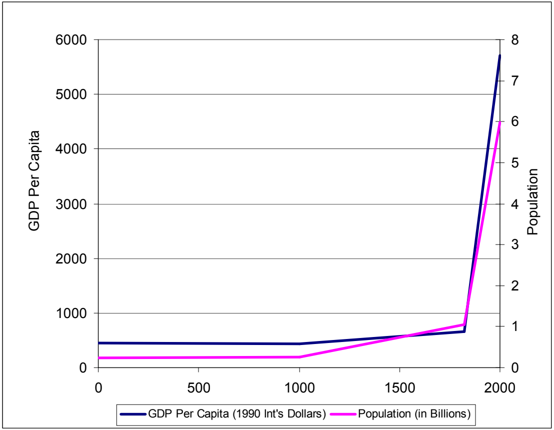


(Source: Maddison, 2001, 2003, Galor, 2005)

The economic transition:

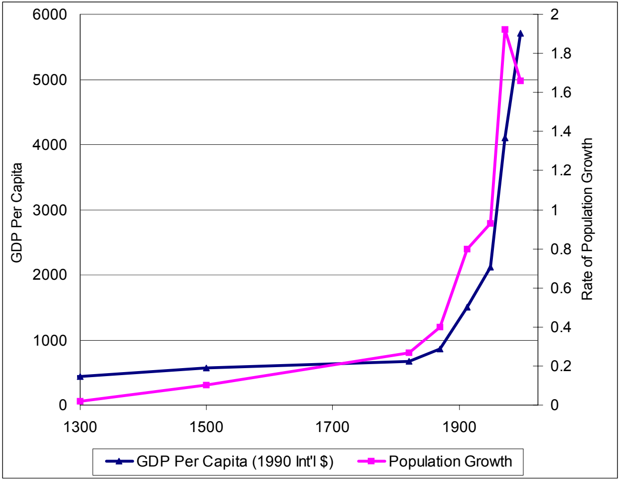
- Economic development was "glacial" for most of human history.
- "Structural break" only in the late middle ages, onset of a phase of sustained, ongoing economic growth.

Evolution of World Population and Income per Capita:



(Source: Maddison, 2001, Galor, 2005)

Neo-Classical vs. Unified Growth



(Source: Maddison, 2001, Galor, 2005)

Neo-Classical vs. Unified Growth

Taxonomy of regimes:

- Malthusian regime:
  - Stationary income per capita.
  - Balanced births and deaths (self-equilibrating, stationary population: positive check, preventive check).
  - Positive relationship between per capita income and fertility.
- Post-Malthusian regime:
  - Modestly growing income per capita.
  - Positive relationship between per capita income and fertility.
- Modern Growth regime
  - Permanently growing income per capita.
  - Negative relationship between per capita income and fertility.

Neo-Classical vs. Unified Growth

Unified Growth Theory:

Models of the underlying behavioral and technological structures that can simultaneously account for the distinct phases of development and that deliver implications for the contemporary growth process of developed and underdeveloped countries.

(More technical):

Models that allow for multiple steady states and the endogenous transition between them.

Malthusian Models of Development

- Ashraf, Q. and O. Galor (2011): "Dynamics and Stagnation in the Malthusian Epoch", *American Economic Review* 101(5), 2003–2041.
- Central hypothesis of Malthusian models: improvements in the technological environment during the pre-industrial era generated only temporary gains in income per capita, eventually leading to a larger, but not significantly richer, population.
- Here: overlapping generations model with decreasing returns to scale production and preference for children.

Production:

The production function is given by

$$Y_t = (AX)^{\alpha} L_t^{1-\alpha} \quad \alpha \in (0,1) \ ,$$

where X is the fixed amount of land, L_t population and A technology.
There are no returns to land, income is given by income per capita

$$y_t = \left(\frac{AX}{L_t}\right)^{\alpha}$$

Individuals:

The utility function contains a preference for children n_t

$$u_t = (c_t)^{1-\gamma} n_t^{\gamma} \quad \gamma \in (0,1)$$

Note: these are the same preferences as log-utility. (why?)

The budget constraint is given by

$$\rho n_t + c_t \leq y_t \quad 0 < \rho < \gamma$$

Optimization:

Maximizing utility with respect to the budget constraint gives

$$c_t = (1-\gamma) y_t$$

$$n_t = \frac{\gamma}{\rho} \cdot y_t$$

Population dynamics:

Population evolves according to

$$L_{t+1} = n_t L_t = \frac{\gamma}{\rho} y_t L_t = \frac{\gamma}{\rho} Y_t$$

$$L_{t+1} = \frac{\gamma}{\rho} \cdot (AX)^{\alpha} L_t^{1-\alpha} \equiv \phi(L_t; A)$$

A steady state for a given technology A and $L_0 > 0$ is given by

$$\bar{L} = \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha}} AX$$

$$\bar{P}_d = \frac{\bar{L}}{X} = \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha}} A$$

Population dynamics in a Malthusian economy:

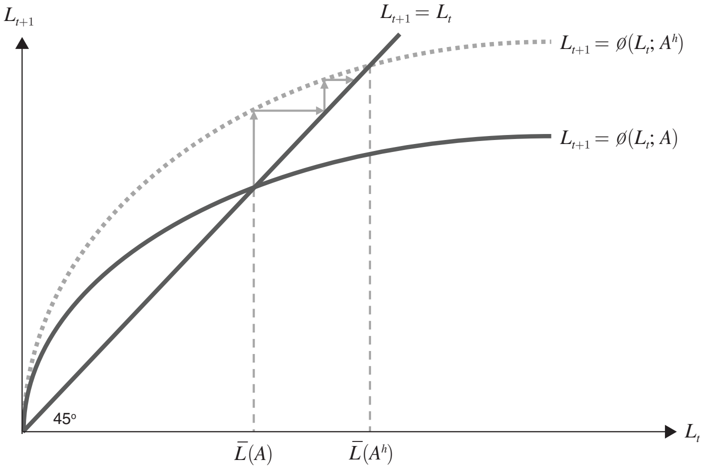


Figure 1. The Evolution of Population Size

Malthusian Models of Development

Income dynamics:

The evolution of income is given by

$$y_{t+1} = \left(\frac{AX}{L_{t+1}}\right)^{\alpha} = \left(\frac{AX}{n_t L_t}\right)^{\alpha} = \frac{y_t}{n_t^{\alpha}} = \left(\frac{\rho}{\gamma}\right)^{\alpha} y_t^{1-\alpha} \equiv \psi(y_t)$$

A steady state for a given $y_0 > 0$ is given by

$$\bar{y} = \frac{\rho}{\gamma}$$

Implications:

1. Within country: an increase in productivity increases population but not income per capita in the long run.
2. Across countries: higher technology implies higher population density, but not income per capita.

Income dynamics in a Malthusian economy:

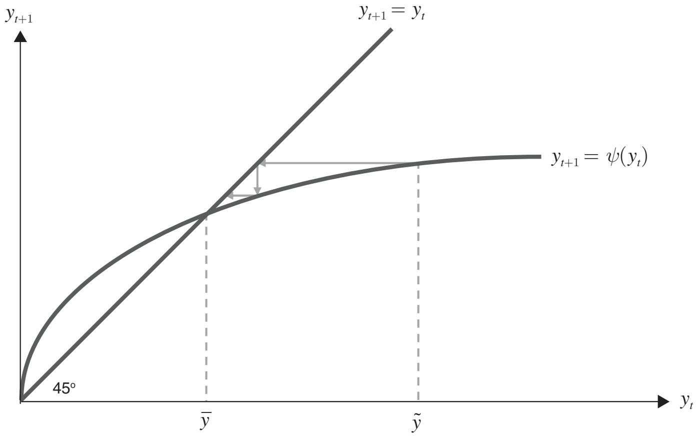
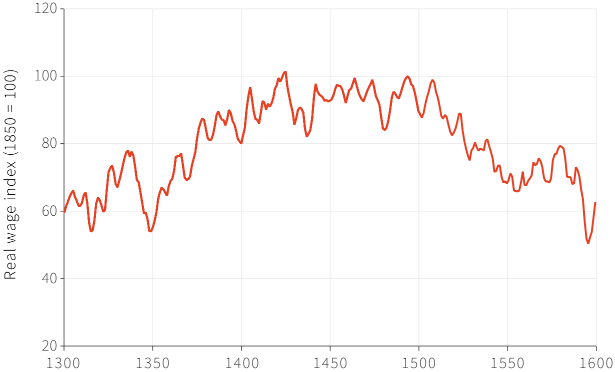


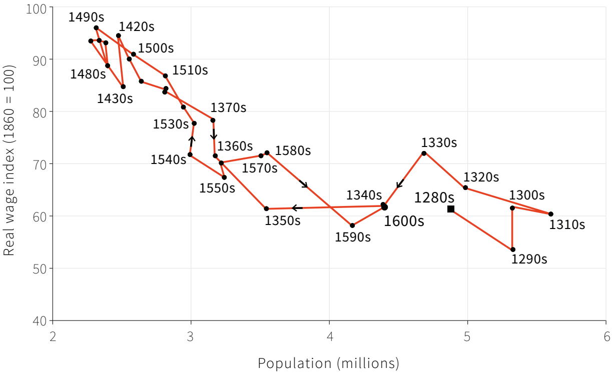
Figure 2. The Evolution of Income per Worker

Example: wages in England 1280-1600



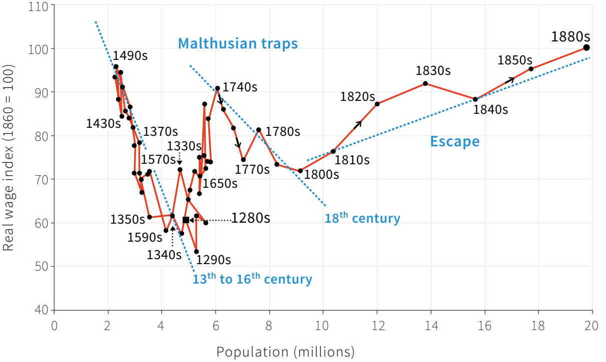
(Source: Allen, 2001, The Great Divergence in European Wages and Prices from the Middle Ages to the First World War)

Example: wages and population in England 1280-1600



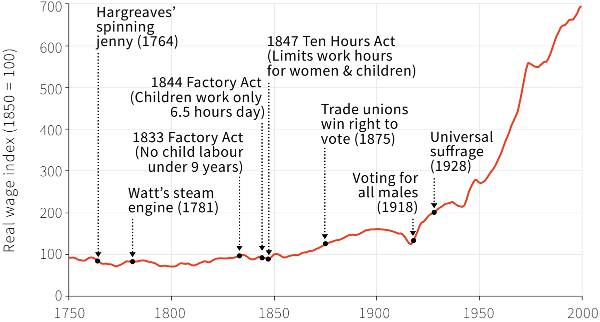
(Source: Allen, 2001, The Great Divergence in European Wages and Prices from the Middle Ages to the First World War)

The end of the Malthusian era:



(Source: Allen, 2001, The Great Divergence in European Wages and Prices from the Middle Ages to the First World War)

Exit from the Malthusian trap:



(Source: Allen, 2001, The Great Divergence in European Wages and Prices from the Middle Ages to the First World War)

The "Textbook" Unified Growth Model

- O. Galor, and D. Weil (2000): "Population, Technology and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond", *American Economic Review* 90(4), 806–828.
- Technological progress (income growth) as the driving force behind the transition.
- A higher demand for human capital changes relative returns.
- Population size as main determinant of innovation.
- Non-convexities: fixed factor of production (land), subsistence consumption.

The "Textbook" Unified Growth Model

- Two-period OLG in discrete time (childhood $t-1$, adulthood t).
- Production:

$$\begin{aligned} Y_t =& H_t^{\alpha}(A_t X)^{1-\alpha} \\ y_t =& h_t^{\alpha} x_t^{1-\alpha} = y(h_t, x_t) \end{aligned}$$

- Income generation: no returns on land, wages are given by the average product.
- Income growth through technological change:

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t}$$

$$\frac{\partial y(h_t, x_t)}{\partial g_t} > 0$$

The "Textbook" Unified Growth Model

- Preferences

$$u_t = (1-\gamma)\ln(c_t) + \gamma \ln(n_t h_{t+1})$$

- Budget constraints

$$\begin{aligned} c_t + w_t h_t n_t(\tau^q + \tau^e e_{t+1}) &\leq w_t h_t \\ c_t &\geq \tilde{c} \end{aligned}$$

- Education

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

with

$$h(\cdot) > 0, h_e(\cdot) > 0, h_{ee}(\cdot) < 0, h_g(\cdot) < 0, h_{gg}(\cdot) > 0$$

$h_{eg}(\cdot) > 0 \forall (e_{t+1}, g_{t+1}) > 0$ (erosion effect declines with education)

Optimization:

Individuals optimally choose the number and quality of children (and residually, through balanced budget, their consumption):

$$\{n_t, e_{t+1}\} = \arg\max\,(1-\gamma)\ln[w_t h_t\,[1 - n_t(\tau^q + \tau^e e_{t+1})]] + \gamma \ln[n_t h(e_{t+1}, g_{t+1})]$$

subject to

$$\begin{aligned} w_t h_t\,[1 - n_t(\tau^q + \tau^e e_{t+1})] &\geq \tilde{c} \\ \{n_t, e_{t+1}\} &\geq 0 \end{aligned}$$

Define a level of potential income $\tilde{z} = \frac{\tilde{c}}{1-\gamma}$ such that the subsistence level just binds.

$$n_t\,(\tau^q + \tau^e e_{t+1}) = \begin{cases} \gamma & \text{if } z_t \geq \tilde{z} \\ 1 - \frac{\tilde{c}}{w_t h_t} & \text{if } z_t \leq \tilde{z} \end{cases}$$

The "Textbook" Unified Growth Model

Taking the FOC with respect to e_{t+1}, substitung in the optimal result for n_t, and collecting terms gives

$$G(e_{t+1}, g_{t+1}) = (\tau^q + \tau^e e_{t+1}) h_e(e_{t+1}, g_{t+1}) - \tau^e h(e_{t+1}, g_{t+1}) \begin{cases} = 0 & \text{if } e_{t+1} > 0 \\ \leq 0 & \text{if } e_{t+1} = 0 \end{cases}$$

<u>Assume:</u> $G(0,0) = \tau^q h_e(0,0) - \tau^e h(0,0) < 0$.

Lemma

If (A1) is satisfied, then the level of education chosen by members of generation t for their children $t+1$ is a nondecreasing function of g_{t+1},

$$e_{t+1} = e(g_{t+1}) \begin{cases} = 0 & \textit{if } g_{t+1} \leq \bar{g} \\ > 0 & \textit{if } g_{t+1} > \bar{g} \end{cases}$$

where $\bar{g} : G(0, \bar{g}) = 0$, $\bar{g} > 0$ and $e'(g_{t+1}) > 0 \ \forall g_{t+1} > \bar{g}$.

The "Textbook" Unified Growth Model

Assume: $e''(g_{t+1}) < 0 \quad \forall g_{t+1} > \bar{g}$.

Substituting back into optimal fertility gives

$$n_t = \begin{cases} n^b(g_{t+1}) = \frac{\gamma}{\tau^q + \tau^e e(g_{t+1})} & \text{if } z_t \geq \tilde{z} \\ n^a(g_{t+1}, z_t) = \frac{1 - \frac{\tilde{c}}{z_t}}{\tau^q + \tau^e e(g_{t+1})} & \text{if } z_t \leq \tilde{z} \end{cases}$$

Note that income is given by

$$z_t = w_t h_t = h_t^{\alpha} x_t^{1-\alpha} := z(e_t, g_t, x_t)$$

Characterization of the different regimes:

- $\frac{\partial n_t}{\partial g_{t+1}} \leq 0$ and $\frac{\partial e_{t+1}}{\partial g_{t+1}} \geq 0$
- if $z_t < \tilde{z}$ (i.e. subsistence binds), $\frac{\partial n_t}{\partial z_t} > 0$ and $\frac{\partial e_{t+1}}{\partial z_t} = 0$;
- if $z_t \geq \tilde{z}$ (i.e. subsistence does not bind), $\frac{\partial n_t}{\partial z_t} = \frac{\partial e_{t+1}}{\partial z_t} = 0$;

Intuition: as long as subsistence consumption binds, income gains feed into higher fertility, but not higher education. Once subsistence is overcome, income gains only feed into higher consumption.

Time path of macroeconomic variables:

Technological progress is given by

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = g(e_t, L_t)$$

with $g(0, L_t) > 0$, $g_i(\cdot, \cdot) > 0$, and $g_{ii}(\cdot, \cdot) < 0$.

Population growth is given by

$$L_{t+1} = n_t L_t$$

Analysis of the dynamical system:

Note: if $g_L(e_t, L_t) = 0 \quad \forall L_t > 0$, the relationship between e and g is independent of x, that is

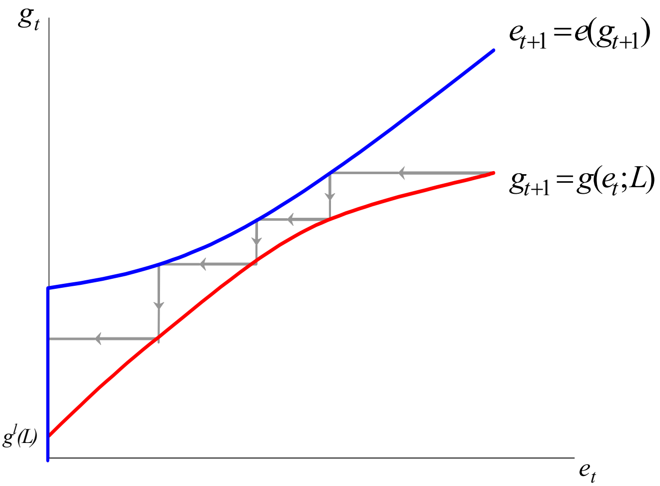
$$\{g_t, e_t\}_{t=0}^{\infty} : \; g_{t+1} = g(e_t, L) \quad \text{and} \quad e_{t+1} = e(g_{t+1})$$

As a result the dynamic system is characterized by three regimes.

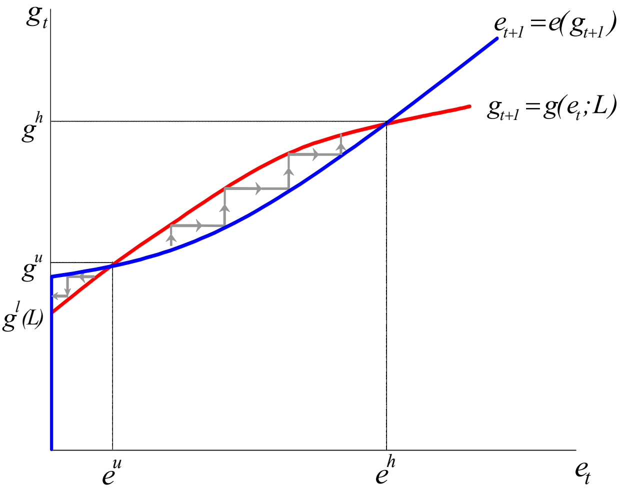
If population is allowed to play a role, there is an endogenous transition across three regimes.

The "Textbook" Unified Growth Model

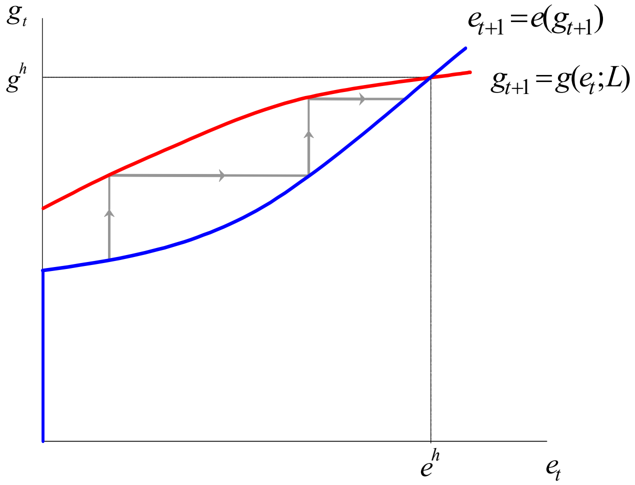
Small population size:



Intermediate population size:



Large population size:



Summary

- Unified endogenous growth model in which the evolution of population, technology, and output growth is largely consistent with the observed process of development in the long-run.
- The model generates an endogenous takeoff from a Malthusian regime, through a Post-Malthusian regime, to a demographic transition and a Modern Growth regime.
- Key elements: effect of technological change on the return to education, scale effects (population size).
- Shortcomings:
    1. The results are driven by strong assumptions
        - subsistence consumption
        - scale effects
        - functional form of human capital
    2. The model cannot account for the observed drop in fertility.
    3. No uncertainty concerning child survival, no role of mortality.