Ch. 3 Neoclassical Growth Model

[3] ρ is again a measure of impatience → the higher ρ the more impatient

[4], [5] FOCs for c_t, c_{t+1} and k_{t+1}

[5] rule for the shadow price

[7] costate vars ≙ Lagrange multiplier ; Lagrange only for discrete time, not for continuous time

[8] $\dot{\lambda}(t)$ => compare with side 3

[9] conditions are derived later on

[11] 2nd line: time derivative on both sides

[14] shadow price → saving today also changes prices and implicit prices tomorrow

[15] in infinity, saving more has no value

[20] χ is a constant growth rate ; another interior continuous solution implies no growth

[21] first line equality holds only in the steady state

[24] again, these conditions are satisfied

no exogenous savings → isn't contained in law of motion

[27] $L(0)$ normalized to 1

↳ $\rho > n$: to account for time preferences → in order for discounting to be satisfied

[28] curvature does not influence the consumption decisions
↳ of the utility

[29] assets $A(t)$: what are the households choices to invest

[32] $f(k(t)) - c(t) \triangleq \dot{k} + \delta \cdot k(t)$

wage is in there via $f(k(t))$ and $f'(k(t))$ $w(t) = f - f'(k) \cdot k$

[33] incentive → borrow infinite assets to achieve infinite consumption

↳ feasibility constraint

[34] $\int_t^T r(t)$ → all the interest rates accruing over time

expenditure has to equal earnings

↳ you cannot hold debt as $T \to \infty$

↳ one has to hold some wealth or no wealth at all

[35] otherwise the feasibility of the system would be violated (since it is a representative household)

No-Ponzi game ≠ transversality

[36] assuming constant elasticity of substitution

[40] if benefits were linear, there does not have to be a steady state
↳ if $f'(k) > \delta + \rho$, then there is a constant growth rate

implications: we need ideas that step around diminishing returns to foster long-term growth

→ Inada: one point where benefits are equal to costs and not many

[38-40] very important for the exam

↳ what do assumptions imply for results and for implications?

[44] with any externalities, they would not coincide

corresponds to the 1st Fundamental Welfare Theorem

[46] Leibniz's rule → you can differentiate a function at the bounds of the integral

↳ not know by heart, just know what it's used for

[47] transversality implies No-Ponzi scheme to hold with equality

↳ not waste resources and not cheat are therefore equal

[49] expression that describes the entire consumption path over time

[51] wage plays a role for $c(t)$, as well as $a(0)$, the starting point

↳ not know by heart, but the intuition (importance of initial values for consumption path)

[54] because the production function is concave

[57] function originates at $(0,0)$

[58-59] this maximizes consumption → Golden Rule

↳ here: discounted utility of c is maximized, not simply c

[61] if no discount, graph is flat between k^* and k^{GR}

[62] income vs. expenditure $=0$ we the points on the line in [63]

[63] ↳ arrows imply the direction of the starting point

curve: all steady states but k^* is only value that also sets c to zero

↳ only down left or upper right are optimal

↳ there is just one combination of $c(0)$, $k(0)$ that is optimal

! certainly part of the exam! know the arrows (if you are given an initial condition)

[64] time consistent → does economy evolve to steady state for initial endowment? → saddle path

↳ consumption path on [51] is the saddle path

↳ stable and unique?

[65] two derivatives for k^* and c^* respectively; Taylor approximation to linearize the function

[68] eigenvalues can be used for diagonalizing a non-diagonal matrix

[70] transformed a very complicated system to a fairly solvable system

all eigenvalues negative → in order for a $z(t)$ to be ~~0~~ zero at a certain point

[72] only locally stable since we used a Taylor approximation

stable: it will always tend to a certain steady state

→ there is a unique stable saddle path that fulfills the optimality conditions

[74] neo-classical model → scope for policy analysts ↓ Solow: everything (esp. s) exogenous

$\frac{1}{\theta}$ → no consensus about the size

ρ → patience is hard to achieve

δ → data problem

n, g → somewhat of a choice

[77] no frictions on capital markets; no inequality

two assumptions that are important to keep in mind

↳ redistribution with unequal households may have positive effects

↳ only looks at capital taxation

[79] explicit functional forms are needed for a quantitative evaluation

[80] τ drives a wedge between investment and consumption → makes investment less attractive

↳ relative difference between countries is most important

[82] α and τ still have to be determined with actual data

↳ sevenfold increase of price of investment only leads to a 2/5 times lower steady state

↳ taxation is not enough to explain development differences

[84] it is not only physical capital → there has to be another source of accumulation that explains the differences

[85] idea of intertemporal savings and consumption decisions → tradeoff between utility and accumulation is at the heart of the model

↳ simple idea allows for lots of extensions

important: understanding the reasons for using technicalities (eigenvalues or Hamiltonians)