LMU München, Winter Term 2020/2021
Macroeconomics - Growth
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Macroeconomics - Growth, Problem Set 1: The Solow Model

Problem 1: The Canonical Solow Model

Consider a standard version of the Solow Model in continuous time. Output is produced according to the aggregate production function

$$Y(t) = K(t)^{\alpha}\left[A(t)L(t)\right]^{1-\alpha} \quad \alpha \in (0,1)\ .$$

The total capital stock at time t is denoted by $K(t)$, the number of workers by $L(t)$ and the level of labor-augmenting technology by $A(t)$, with $\{K(0), L(0), A(0)\} > 0$. Markets are perfectly competitive and factors are paid their marginal products. Capital depreciates at rate $\delta \in (0,1)$ and the exogenous savings rate is given by $s \in (0,1)$. Population and technology grow at rates n and g, respectively. The law of motion of capital is given by

$$\dot{K}(t) = sY(t) - \delta K(t)\ .$$

1.1 Show that the steady state of capital per effective unit of labor is given by

$$k^* = \left[\frac{s}{n+g+\delta}\right]^{\frac{1}{1-\alpha}}\ .$$

1.2 Show that the golden-rule level of capital per effective unit of labor is given by

$$k^{GR} = \left[\frac{\alpha}{n+g+\delta}\right]^{\frac{1}{1-\alpha}}\ .$$

Show that this implies a savings rate of

$$s^{GR} = \alpha$$

and give a verbal intuition for the result.

1.3 Assume that $s > n + g + \delta$. Draw a diagram of the evolution of the capital stock per effective unit of labour (be careful to label all curves, axes and values correctly!).

Show analytically and graphically how a rise in the capital share "α" would affect the steady state level of capital per effective unit of labour. Give a (short) verbal explanation for the result.

Problem 2: The Solow Model with a Fixed Factor

Consider a modified version of the Solow Model in continuous time. Output is produced according to the aggregate production function

$$Y(t) = K(t)^{\alpha} L(t)^{\beta} Z^{1-\alpha-\beta} \quad \alpha + \beta < 1 \ .$$

The total capital stock at time t is denoted by $K(t)$, the number of workers by $L(t)$ and the exogenous **fixed** amount of land by Z, with $\{K(0), L(0), Z\} > 0$. Markets are perfectly competitive and factors are paid their marginal products. Capital depreciates at rate $\delta \in (0,1)$ and the exogenous savings rate is given by $s \in (0,1)$. The law of motion of capital is given by

$$\dot{K}(t) = sY(t) - \delta K(t) \ .$$

2.1 Is the production function constant returns to scale and does it satisfy the Inada conditions?

2.2 Suppose there is no population growth. Derive that the steady-state capital-labour ratio k^* is given by

$$k^* = \left[\frac{s}{\delta} \cdot z^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}} \ ,$$

where z is the land-labour ratio.

2.3 Now suppose that population grows at rate

$$\frac{\dot{L}(t)}{L(t)} = n \ .$$

Derive the growth rate of land per worker. What is the return to land and labour in this economy as as $t \to \infty$? Give a verbal intuition for your results.

2.4 Draw a diagram of the evolution of the capital stock per capita (be careful to label all curves, axes and values correctly!). Show graphically how an increase in the size of land Z would affect the steady state capital-labour ratio. Give a verbal explanation for the result.

Problem 3: A Numerical Implementation of the Solow Model

Consider the Solow Model without technological progress in discrete time. Assume that production is given by a Cobb-Douglas production function such that the evolution of the entire economy is given by

$$
\begin{aligned}
Y_t &= K_t^{\alpha} L_t^{1-\alpha} \\
K_{t+1} &= s \cdot Y_t + (1-\delta) K_t \\
L_{t+1} &= (1+n) L_t
\end{aligned}
$$

3.1 What is the steady-state capital-labor ratio of the economy?

3.2 Assume that $\alpha = 0.4$, $s = 0.33$, $\delta = 0.1$ and $n = 0.001$. Simulate the economy for 50 years, starting in the year 1950. What additional values do you need? Does the numerical result match the analytical result derived beforehand?

3.3 Now simulate the economy for countries differing in their initial capital stock. What differences do you observe with respect to timing and steady state?

3.4 Repeat the exercise for countries differing in their savings rate. Again, what differences do you observe with respect to timing and steady state?