LMU München, Winter Term 2020/2021
Macroeconomics - Growth
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Macroeconomics - Growth, Problem Set 2: Overlapping-Generations Models (OLG Models)

Problem 1: The Canonical Overlapping-Generations Model

Consider an overlapping-generations model. Time is discrete and runs to infinity. Each individual lives for two periods. All individuals born at time t live for dates t and $t+1$. Preferences over consumption at times t and $t+1$ are given by the utility function

$$\ln(c_t^1) + \beta \ln(c_{t+1}^2) \quad \beta \in (0,1) \; .$$

Markets are perfectly competitive and factors earn their marginal product. Individuals can only work in the first period of their lives. They supply one unit of labor inelastically, receiving the equilibrium wage w_t, which they allocate between consumption and saving. In the second period of life they solely consume their savings.

Output is produced according to a Cobb-Douglas production function using inputs of capital K_t and labor L_t (with $K_0, L_0 > 0$):

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \quad \alpha \in (0,1) \; .$$

Capital depreciates fully after one period. Population grows at rate $n > 0$.

1.1 Write down the two period budget constraints and derive the Euler equation in consumption per capita.

1.2 Show that optimal savings per capita are given by

$$s_t = \frac{\beta}{1+\beta} \cdot w_t \; .$$

1.3 Derive the accumulation of capital per capita in this economy and show that the steady state level of capital per capita is given by

$$k^* = \left[\frac{\beta}{1+\beta} \cdot \frac{1-\alpha}{1+n} \right]^{\frac{1}{1-\alpha}} \; .$$

1.4 Draw a diagram of the evolution of the capital stock per capita (be careful to label all curves, axes and values correctly!).

Show graphically, supported by theoretical arguments:

a) How an increase in the rate of population growth n would affect the steady state level of capital per capita. Give a verbal explanation for the result.

b) How a drop in the discount factor β would affect the steady state level of capital per capita. Give a verbal explanation for the result.

Part 2: Overlapping-Generations Models with Endogenous Labor Supply

Consider an overlapping generations model. Time is discrete and runs to infinity. Each individual lives for two periods. All individuals born at time t live for dates t and $t+1$. Markets are perfectly competitive and factors earn their marginal product. Individuals can only work in the first period of their lives. They are endowed with one unit of labor, which they elastically supply to the market and receive the equilibrium wage w_t. The resulting income is allocated between consumption and saving. In the second period of life they solely consume their savings. Preferences over consumption and free time are given by

$$\ln\left(c_t^1\right) + \beta \ln\left(c_{t+1}^2\right) + \phi \ln\left(1 - l_t\right) \quad \{\phi, \beta\} \in (0,1) \ ,$$

where c_t^1 and c_{t+1}^2 are consumption in period t and $t+1$ while l_t is the fraction of the unit of (potential) labor supplied to the market, i.e. $(1 - l_t)$ is time spent not working. Output is produced according to a Cobb-Douglas production function using inputs of capital K_t and total labor supply L_t (with $K_0, L_0 > 0$)

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \quad \alpha \in (0,1) \ .$$

Capital depreciates fully after one period; population N_t grows at constant, exogenous rate $n > 0$.

2.1 Explain (analytically or verbally) why working full-time is never optimal.

2.2 Write down the two period budget constraints, derive the Euler equation in consumption per capita and show that optimal labor supply and savings per capita are given by

$$l_t = \frac{1+\beta}{1+\beta+\phi} \quad \text{and} \quad s_t = \frac{\beta}{1+\beta+\phi} \cdot w_t \ .$$

2.3 Show that the steady state of the capital stock per capita in this economy is given by

$$k^* = \frac{1}{1+\beta+\phi} \left[\frac{\beta}{(1+\beta)^{\alpha}} \cdot \frac{1-\alpha}{1+n} \right]^{\frac{1}{1-\alpha}} \ .$$

2.4 Draw a diagram of the evolution of the capital stock per capita (be careful to label all curves, axes and values correctly!).

Show analytically and graphically how an increase in the preference for leisure "ϕ" would affect the steady state level of capital per capita. Give a (short) verbal explanation for the result.

Problem 3: A Numerical Implementation of the OLG-Model

Consider a standard overlapping-generations growth model. Each individual lives for two periods (childhood and adulthood) and has exactly one child (i.e. there is no population growth). Consumption takes place at the end of adulthood. Preferences are given by

$$(1-\delta)\ln c_t^i + \delta \ln e_t^i \ ,$$

where c_t^i is consumption of individual i and e_t^i is educational spending on on its child. The budget constraint is given by

$$c_t^i + e_t^i \leq w_t^i \ ,$$

where w_t^i is wage income obtained in competitive labor markets as a linear function of human capital h_t^i

$$w_t^i = A h_t^i \ ,$$

with $\delta A > 1$. The human capital of the child is given by

$$h_{t+1}^i = \begin{cases} \left(e_t^i\right)^\gamma & \text{if } e_t^i \geq 1 \\ \bar{h} & \text{if } e_t^i < 1 \ , \end{cases}$$

with $1 > \delta A \bar{h}$.

3.1 What are the two possible steady states of h^i in the model? Why are there two steady states?

3.2 Assume parameter values of $\delta = 0.3$, $A = 4.5$, $\gamma = 0.9$ and $\bar{h} = 0.6$. Simulate the model over 50 periods of twenty years length for two individuals with initial human capital stock

$$\begin{aligned} h_0^1 &= 0.25 \\ h_0^2 &= 0.75 \ . \end{aligned}$$

3.3 Plot the paths of human capital over time (make sure you have labels on your graph).

3.4 What is the crucial factor that determines which of the two steady states is achieved?

3.5 What could be added to the model in order to enable breakout from the lower steady state?