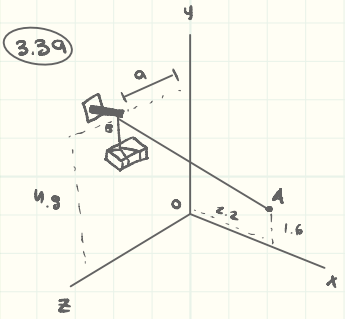
Katie Connors | HW 3 P3.39, P3.40, P3.49, P3.60 P3.71, P3.75, P4.1 | Due 2/9/24

3.39



A = (2.2, 1.6, 0)
B = (0, 4.8, a)
O = (0, 0, 0)

$M_{oy} = 120\ N\cdot m$ $\quad M_{oy} = \lambda_{oy} \cdot M_o$ $\quad \lambda_{oy} = (0,1,0)$

$M_{oz} = -460\ N\cdot m$ $\quad M_{oz} = \lambda_{oz} \cdot M_o$ $\quad \lambda_{oz} = (0,0,1)$

$r_{BA} = (2.2, -3.2, -a)$

$r_{OA} = (2.2, 1.6, 0)$

$$|r_{BA}| = \sqrt{2.2^2 + (-3.2)^2 + (-a)^2} = \sqrt{15.08 + a^2}$$

$$\vec{T}_{BA} = \frac{T_{BA}}{|r_{BA}|}(2.2i, -3.2j, -ak)$$

$$M_o = \vec{r}_{OA} \times \vec{T}_{BA}$$

$$= (2.2i, 1.6j, 0k) \times \frac{T_{BA}}{|r_{BA}|}(2.2i, -3.2j, -ak)$$

$$M_o = \left(-1.6a\frac{T_{BA}}{|r_{BA}|} - 0\right)i + \left(0 + 2.2a\frac{T_{BA}}{|r_{BA}|}\right)j + \left(-7.04\frac{T_{BA}}{|r_{BA}|} - 3.52\frac{T_{BA}}{|r_{BA}|}\right)$$

$$M_o = \frac{T_{BA}}{|r_{BA}|}(-16a\,i, 2.2a\,j, -10.56k)$$

$$M_{oy} = (0,1,0) \cdot \frac{T_{BA}}{|r_{BA}|}(-16a, 2.2a, -10.56)$$

$$M_{oy} = \frac{T_{BA}}{|r_{BA}|}(2.2a)$$

$$M_{oz} = (0,0,1) \cdot \frac{T_{BA}}{|r_{BA}|}(-16a, 2.2a, -10.56)$$

$$M_{oz} = -10.56\frac{T_{BA}}{|r_{BA}|}$$

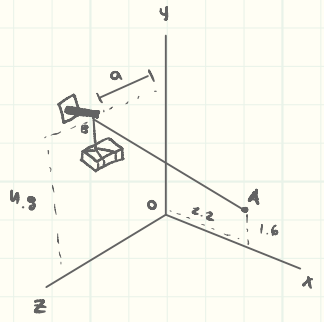
$$\frac{120\ N\cdot m}{-460\ N\cdot m} = \frac{M_{oy}}{M_{oz}} = \frac{2.2a\left(\frac{T_{BA}}{|r_{BA}|}\right)}{-10.56\left(\frac{T_{BA}}{|r_{BA}|}\right)} = \frac{2.2a}{-10.56}$$

$$\left(\frac{120}{-460}\right)\left(\frac{-10.56}{2.2}\right) = a$$

$$a = 1.25217\ m$$

$$\boxed{a = 1.252\ m}$$

(3.40)



$A = (2.2, 1.6, 0)$
$B = (0, 4.8, a)$
$O = (0, 0, 0)$

$F = 195\ N$
$M_{OY} = 132\ N \cdot m$

$$r_{BA} = (2.2, -3.2, -a)$$
$$|r_{BA}| = \sqrt{2.2^2 + (-3.2)^2 + (-a)^2}$$
$$|r_{BA}| = \sqrt{15.08 + a^2}$$

$$r_{OA} = (2.2, 1.6, 0)$$

$$\lambda_{OY} = (0, 1, 0)$$

$$\vec{T}_{BA} = (195)\frac{r_{BA}}{|r_{BA}|} = (195)\frac{(2.2i, -3.2j, -aK)}{|r_{BA}|}$$

$$M_O = \vec{r}_{OA} \times \vec{T}_{BA}$$

$$M_O = (2.2, 1.6, 0) \times \frac{1}{|r_{BA}|}(429i, -624j, -195aK)$$

$$M_O = \left(\frac{-312a}{|r_{BA}|} - 0\right)i + \left(\frac{429a}{|r_{BA}|}\right)j + (-13728 - 686.4)K$$

$$M_{OY} = \lambda_{OY} \cdot M_O$$
$$M_{OY} = (0,1,0) \cdot \left[\left(\frac{-312a}{|r_{BA}|} - 0\right)i + \left(\frac{429a}{|r_{BA}|}\right)j + (-13728 - 686.4)K\right]$$

$$M_{OY} = \frac{429a}{|r_{BA}|}$$

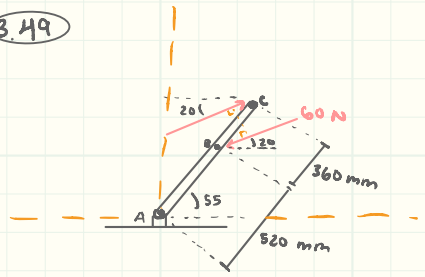
$$132 = \frac{429a}{\sqrt{15.08 + a^2}}$$

$$132^2(15.08 + a^2) = 429^2 a^2$$
$$262753.92 - 132^2 a^2 = 429^2 a^2$$
$$262753.92 = 166617 a^2$$

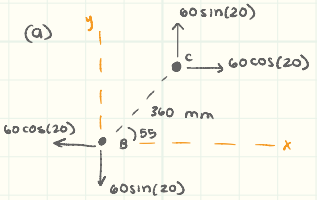
$$a = 1.255578$$

$$\boxed{a = 1.2558\ m}$$

3.49



(a)



$M_y = r_{BC} \times F_y = (360\cos(55)\,i + 360\sin(55)\,j + 0k) \times (0i + 60\sin(20)\,j + 0k)$

$M_y = 0i + 0j + [(360\cos(55))(60\sin(20)) - 0]\,k$

$M_y = 4237.37\ N\cdot mm\ \hat{k} = 4.23737\ N\cdot m\ \hat{k}$

$M_x = r_{BC} \times F_x = (360\cos(55)\,i + 360\sin(55)\,j + 0k) \times (60\cos(20)\,i + 0j + 0k)$

$M_x = 0i + 0j + [-(360\sin(55))(60\cos(20))]\,k$

$M_x = -16626.62\ N\cdot mm\ \hat{k} = -16.6266\ N\cdot m\ \hat{k}$

$\Sigma M = 4.23737 - 16.6266 = -12.38925\ N\cdot m\ \hat{k}$

$M = -12.389\ N\cdot m\ \hat{k}$

(b) using ⊥ distance between the forces

$\sin(55-20) = \frac{d}{360}$

$d = 360\sin(55-20) = 206.488\ mm$

$.206488\ m$

$M = Fd = (60N)(.206488m)$

$M = 12.38925\ N\cdot m$

but the moment is going into the page so

$M = -12.389\ N\cdot m\ \hat{k}$

(c) summing Moments about point A

$r_{AC} = 880\cos(55)\,i + 880\sin(55)\,j + 0k$

$M = r_{AC} \times F = (880\cos(55)\,i, 880\sin(55)\,j, 0k) \times (60\cos(20)\,i, 60\sin(20)\,j, 0k)$

$M = 0i + 0k + [(880)\cos(55)(60)\sin(20) - (880)\sin(55)(60)\cos(20)]\,k$

$M = -30284.836\ N\cdot mm\ \hat{k} = -30.2848\ N\cdot m\ \hat{k}$

$r_{AB} = 520\cos(55)\,i + 520\sin(55)\,j + 0k$

$M = r_{AB} \times F_z = (520\cos(55)\,i, 520\sin(55)\,j, 0k) \times (-60\cos(20)\,i, -60\sin(20)\,j, 0k)$

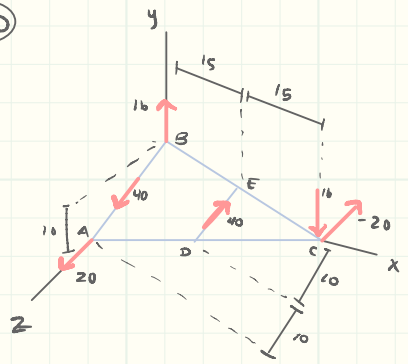
$M = 0i + 0j + [(520)\cos(55)(-60)\sin(20) - (520)\sin(55)(-60)\cos(20)]\,k$

$M = 17895.5848\ N\cdot mm\ \hat{k} = 17.8956\ N\cdot m\ \hat{k}$

$\Sigma M = -30.2848 + 17.8956 = -12.389\ N\cdot m\ \hat{k}$

$M = -12.389\ N\cdot m\ \hat{k}$

3.60



$E = (15, 5, 0)$
$C = (30, 0, 0)$
$B = (0, 10, 0)$

$M_{16} = (30i, 0j, 0k) \times (0i, -16j, 0k) = 0i + 0j - 480k = -480\,k$

$M_{20} = (30i, 0j, -20k) \times (0i, 0j, -20k) = 0i - (30)(-20)j + 0k = 600j$

$M_{40} = r_{BE} \times \vec{F}$

$\vec{F} = \frac{F}{|r_{DE}|}(0i + 5j - 10k)$

$|r_{DE}| = \sqrt{5^2 + (-10)^2} = \sqrt{125} = 5\sqrt{5}$

$\vec{F} = \frac{40}{5\sqrt{5}}(0i + 5j - 10k) = \frac{8}{\sqrt{5}}(0i + 5j - 10k)$

$M_{40} = (15i, -5j, 0k) \times \frac{8}{\sqrt{5}}(0i + 5j - 10k) = \frac{8}{\sqrt{5}}[50i + 150j + 75k]$

$M_{40} = \frac{400}{\sqrt{5}}i + \frac{1200}{\sqrt{5}}j + \frac{600}{\sqrt{5}}k$

$M_T = M_{16} + M_{20} + M_{40} = \frac{400}{\sqrt{5}}i + \left(\frac{1200}{\sqrt{5}} + 600\right)j + \left(\frac{600}{\sqrt{5}} - 480\right)k$

$M_T = 178.88i + 1136.656j - 211.67\,k$

$|M_T| = 1169.9541$ lb·in

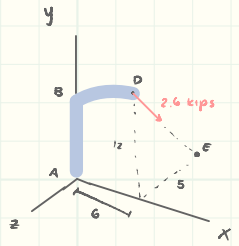
$\lambda = \frac{178.88}{1169.95}i + \frac{1136.656}{1169.9541}j - \frac{211.67}{1169.95}k = .1529i + .9715j - .1809k$

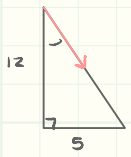
$\cos\theta_x = .1529$ $\quad \cos\theta_y = .9715$ $\quad \cos\theta_z = -.1809$

$\theta_x = 81.205°$ $\quad \theta_y = 13.702$ $\quad \theta_z = 100.42$

$M = 1169.951$ lb·in
$\theta_x = 81.2°$
$\theta_y = 13.70°$
$\theta_z = 100.4°$

3.71





$\tan\theta = \frac{5}{12}$

$\theta = 22.6198°$

$M_A = \vec{r}_{AD} \times \vec{F}$

$r_{AD} = 6i + 12j + 0k$

$\vec{F} = 0i - 2.6\cos(22.6198)j - 2.6\sin(22.6198)k$

$\vec{F} = 0.00\hat{i} - 2.40\text{ kips}\hat{j} - 1.000\text{ kips}\hat{k}$

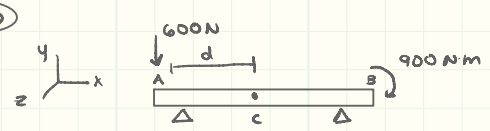
$M_A = (6i, 12j, 0k) \times (0i, -2.6\cos(22.6198)j, -2.6\sin(22.6198)k)$

$M_A = \left((12)(-2.6\sin(22.6198))i, (0-(6)(-2.6\sin(22.6198)))j, (6)(-2.6\cos(22.6198))k\right)$

$M_A = -12.00\text{ kips}\cdot\text{in}\hat{i} + 6.00\text{ kips}\cdot\text{in}\hat{j} - 14.40\text{ kips}\cdot\text{in}\hat{k}$

3.75

(a)



$M_B = 900\ N\cdot m$ in equilibrium so $\Sigma M = 0$

$F_A = 600\ N$

$R = \Sigma F = -600N\ \hat{y}$ $\boxed{R = -600N\ \hat{y}}$

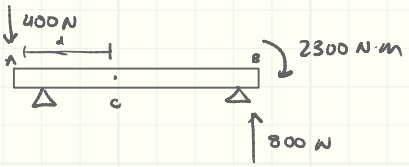
$M_C^R = -(600N)d = -600d$

$\Sigma M_C = 0 = -600d + 900$

$600d = 900$

$\boxed{d = 1.500m}$

(b)



$R = \Sigma F = 800 - 400 = 400N\ \hat{y}$

$\boxed{R = 400N\ \hat{y}}$

$\Sigma M_C = 0 = -400d - 800(4-d) + 2300$

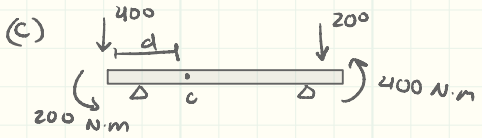
$0 = -400d - 3200 + 800d + 2300$

$3200 - 2300 = 400d$

$2.25 = d$

$\boxed{d = 2.250m}$

(c)



$R = \Sigma F = -400 - 200 = -600N\ \hat{y}$ $\boxed{R = -600N\ \hat{y}}$

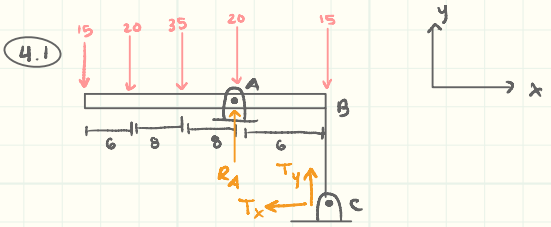
$\Sigma M_C = 0 = -400d - 200 + 200(4-d) - 400$

$600 = -400d + 800 - 200d$

$600d = 200$

$\boxed{d = .333\ m}$

4.1



(a) $\Sigma M_B = 0$

$\Sigma M_B = 0 = -15(28) - 20(22) - 35(14) - 20(6) + R_A(6)$

$0 = -1476 + R_A(6)$

$R_A = 246 \text{ lb } \hat{y}$

(b) $\Sigma M_A = 0$

$\Sigma F_x = 0 \rightarrow T_x = 0$

$\Sigma M_A = 0 = -15(22) - 20(16) - 35(8) + 15(6) - T_y(6)$

$0 = -930 + 90 - 6(T_y)$

$840 = -6(T_y)$

$T_y = -140 \text{ lb}$

The Tension in BC is 140.0 lbs