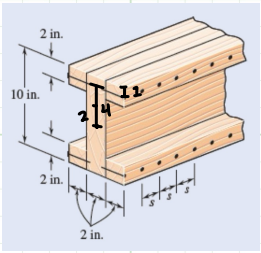
PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

MATLAB:

```
function [fxy] = beam1d_ode(x,y)

% % -- Define material property and geometry
% E = 30e6;     % Young's Modulus, in psi
% I = 255;       % Second moment of inertia, in in^4
% L = 36;       % Length of beam, in in
% w = 0;        % load, in lb/in

% -- Problem 3 point moment
E = 30e6;     % Young's Modulus, in psi
I = 256;       % Second moment of inertia, in in^4
L = 120;       % Length of beam, in in

% Define point moment
epL = 0.01; %how small can epL be? NOTE: THIS MIGHT BE 0.1 OR 0.01
if (x<=L/2+epL/2 && x>=L/2-epL/2)
    w = 1/epL^2;
%elseif (x<=L/2 && x>=L/2-epL)
 %   w = -1/epL^2;
else
    w = 0;
end

% -- Define differential function here
fxy = [ y(2)
        y(3)/(E*I)
        y(4)
        -w];
end
```

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

P13.26:



$V = 1200$ lb

$F_{nail} = 75$ lb

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(2)(2)^3 + (2(2))(4)^2$$

$$I_1 = 65.333 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3$$

$$= \frac{1}{12}(2)(10)^3$$

$$I_2 = 166.667 \text{ in}^4$$

$$I = 4I_1 + I_2$$

$$= 4(65.333) + 166.667$$

$$I = 428 \text{ in}^4$$

$$Q_1 = A_1 \bar{y}_1 = (2(2))(4) = 16 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(16)}{428} = 44.86 \text{ lb/in}$$

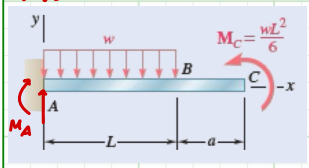
$$F_{nail} = qs$$

$$75 = 44.86(s)$$

$$\boxed{s = 1.672 \text{ in}}$$

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

P15.5:



$\Sigma F_y = 0$:

$R_A - WL = 0$

$R_A = WL$

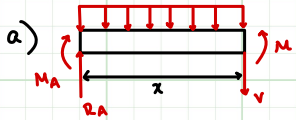
$\Sigma M_A = 0$

$-M_A - WL(\frac{L}{2}) + \frac{WL^2}{6} = 0$

$-M_A = \frac{WL^2}{2} - \frac{WL^2}{6} = \frac{2WL^2}{6}$

$M_A = -\frac{WL}{3}$

a)



$\Sigma M = 0$:

$M + (Wx)(\frac{x}{2}) - R_A(x) - M_A = 0$

$M = M_A + R_A(x) - \frac{Wx^2}{2}$

$M = -\frac{WL^2}{3} + WLx - \frac{Wx^2}{2}$

$EI\frac{d^2y}{dx^2} = -\frac{Wx^2}{2} + WLx - \frac{WL^2}{3}$

$EI\frac{dy}{dx} = -\frac{Wx^3}{6} + \frac{WLx^2}{2} - \frac{WL^2x}{3} + C_1$ ← $x=0$, $\frac{dy}{dx}=0$, $C_1=0$

$EIy = -\frac{Wx^4}{24} + \frac{WLx^3}{6} - \frac{WL^2x^2}{6} + C_2$ ← $x=0$, $y=0$, $C_2=0$

$EIy = -\frac{Wx^4}{24} + \frac{4WLx^3}{24} - \frac{4WL^2x^2}{24}$

$$y = \frac{W}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$$

b) y @ $x = L$:

$y(L) = \frac{W}{24EI}(-(L)^4 + 4L(L)^3 - 4L^2(L)^2)$

$$y(L) = \frac{-WL^4}{24EI}$$

c) $\frac{dy}{dx}$ @ $x = L$:

$\frac{dy}{dx} = \frac{W}{\cancel{24}EI}(\cancel{-4}x^3 - \cancel{12}Lx^2 - \cancel{8}L^2x)$ (6)

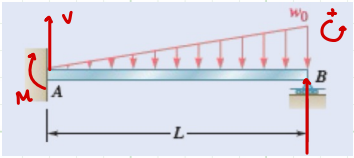
$\frac{dy}{dx} = \frac{W}{6EI}(-x^3 + 3Lx^2 - 2L^2x)$

$= \frac{W}{6EI}(-L^3 + 3L(L)^2 - 2L^2(L))$

$= \frac{W}{6EI}(-L^3 + 3L^3 - 2L^3) \longrightarrow$ $\frac{dy}{dx} = 0$

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

P15.19:



$\Sigma M_A = 0$:

$$-M + R_B(L-x) - \frac{1}{2}w_0(L-x)\cdot\frac{2}{3}(L-x) - \frac{1}{2}\cdot\frac{w_0 x}{L}(L-x)\cdot\frac{1}{3}(L-x) = 0$$

$$-M = -R_B(L-x) - \frac{1}{3}w_0(L-x)^2 - \frac{1}{6}\cdot\frac{w_0 x}{L}(L-x)^2 - \frac{2Lw_0(L-x)^2}{6L} - \frac{w_0 x(L-x)^3}{6L}$$

$$-M = -R_B(L-x) + \frac{w_0}{6L}\left(-2L(L-x)^2 - x(L-x)^2\right)$$

$$M = R_B(L-x) - \frac{w_0}{6L}\left(2L(L-x)^2 + x(L-x)^2\right)$$

$$= R_B(L-x) - \frac{w_0}{6L}\left(2L(L^2 - xL - xL + x^2) + x(L^2 - xL - xL + x^2)\right)$$

$$= R_B(L-x) - \frac{w_0}{6L}\left(2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2x^2L + x^3\right)$$

$$= R_B(L-x) - \frac{w_0}{6L}\left(x^3 - 3L^2x + 2L^3\right)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}\left(x^3 - 3L^2x + 2L^3\right)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) - \frac{w_0}{6L}\left(\frac{x^4}{4} - \frac{3L^2x^2}{2} + 2L^3x\right) + C_1$$

$$EI y = R_B\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) - \frac{w_0}{6L}\left(\frac{x^5}{20} - \frac{L^2x^3}{2} + L^3x^2\right) + C_1x + C_2$$

$\downarrow$ $y=0$, $x=L$

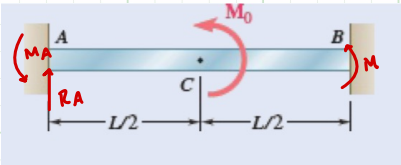
$$0 = R_B\left(\frac{L^3}{2} - \frac{L^3}{6}\right) - \frac{w_0}{6L}\left(\frac{L^5}{20} - \frac{L^5}{2} + L^5\right)$$

$$0 = R_B L^3\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{w_0 L^5}{6L}\left(\frac{1}{20} - \frac{1}{2} + 1\right)$$

$$\frac{11}{20}\cdot\frac{w_0 L^4}{6} = \frac{1}{3}R_B L^3 \longrightarrow \boxed{R_B = \frac{11 w_0 L}{40}}$$

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

P15.26:



$\sum M = 0$:

$$-R_A(x) - M_A + M_0\left(\frac{x}{2}\right)^0 = 0$$

$$EI\frac{d^2y}{dx} = R_A(x) - M_A - M_0\left(\frac{x}{2}\right)^0 = 0$$

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - M_A x - \frac{M_0 x}{2} + C_1 \longleftarrow @\ x=0,\ \frac{dy}{dx}=0,\ C_1 = 0$$

$\downarrow$ @ B, slope = 0, $\frac{dy}{dx}=0$ at $x=L$

$$0 = \frac{R_A L^2}{2} - M_A L - \frac{M_0 L}{2} \longrightarrow \frac{R_A L^2}{2} = M_A L + \frac{M_0 L}{2}$$

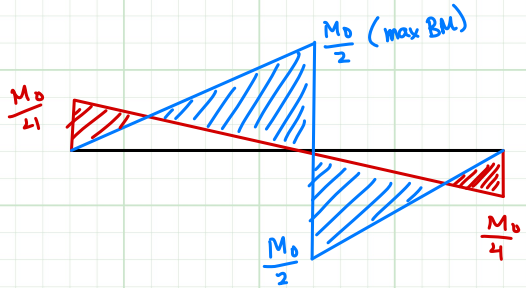
$$\textcircled{1}\quad R_A L = 2M_A + M_0$$

$$EIy = \frac{R_A x^3}{6} - \frac{M_A x^2}{2} - \frac{M_0 x^2}{8} + C_1 x + C_2 \longleftarrow @\ x=0,\ \frac{dy}{dx}=0,\ C_2 = 0$$

$\downarrow$ @ B, deflection = 0, $y=0$ at $x=L$

$$0 = \frac{R_A L^3}{6} - \frac{M_A L^2}{2} - \frac{M_0 L^2}{8} \longrightarrow \frac{R_A L^3}{6} = \frac{M_A L^2}{2} + \frac{M_0 L^2}{8}$$

$$\textcircled{2}\quad R_A L = 3M_A + \frac{3M_0}{4}$$



Substitute ① → ②:

$$2M_A + M_0 = 3M_A + \frac{3M_0}{4}$$

$$M_0 - \frac{3M_0}{4} = 3M_A - 2M_A$$

$$\boxed{\frac{M_0}{4} = M_A}$$

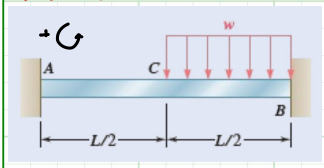
$$R_A L = 2M_A + M_0$$

$$R_A L = \frac{2M_0}{4} + M_0$$

$$R_A L = \frac{6M_0}{4L} \longrightarrow \boxed{R_A = \frac{3M_0}{2L}}$$

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43 P15.52

P15.43:



$Y = Y$

$$\frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI} = \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{wL^4}{128EI}$$

$$\frac{2V_c L^3}{24EI} = \frac{wL^4}{128EI}$$

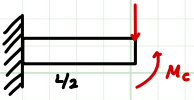
$$V_c = \frac{12wL}{128} \longrightarrow V_c = \frac{3wL}{32}$$

$$\frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} = -\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} + \frac{wL^3}{48EI}$$

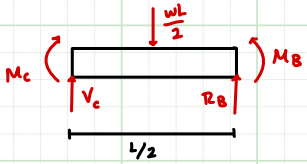
$$\frac{2M_c L}{2EI} = \frac{wL^3}{48EI} \longrightarrow M_c = \frac{wL^2}{48}$$

Section AC:



$$Y = \frac{M_c (\frac{L}{2})^2}{2EI} - \frac{V(\frac{L}{2})^3}{3EI} = \frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI}$$

$$\frac{dy}{dx} = \frac{M_c (\frac{L}{2})}{EI} - \frac{V_c (\frac{L}{2})^2}{2EI} = \frac{M_c L}{2EI} - \frac{V_c L^2}{8EI}$$



$$\sum F_y = 0:$$

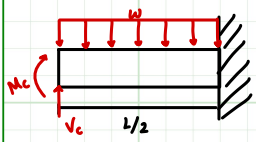
$$R_B + V_c - \frac{wL}{2} = 0$$

$$R_B + \frac{3wL}{32} - \frac{16wL}{32} = 0$$

$$\boxed{R_B = \frac{13wL}{32} \uparrow}$$

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

Section CB:



$$y = \frac{M_c\left(\frac{L}{2}\right)^2}{2EI} + \frac{V_c\left(\frac{L}{2}\right)^3}{3EI} - \frac{w\left(\frac{L}{2}\right)^4}{8EI}$$

$$= \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{wL^4}{128EI}$$

$$\frac{dy}{dx} = \frac{M_c\left(\frac{L}{2}\right)}{EI} + \frac{V_c\left(\frac{L}{2}\right)^2}{2EI} - \frac{w\left(\frac{L}{2}\right)^3}{6EI}$$

$$= \frac{-M_c L}{2EI} - \frac{V_c L^2}{8EI} + \frac{wL^3}{48EI}$$

$\sum M_B = 0$:

$$M_B - M_c - V_c\left(\frac{L}{2}\right) + \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$M_B - \frac{wL^2}{48} - \frac{3wL}{32}\left(\frac{L}{L}\right) + \frac{wL^2}{8} = 0$$

$$M_B - \frac{wL^2}{48} - \frac{3wL^2}{64} + \frac{wL^2}{8} = 0$$

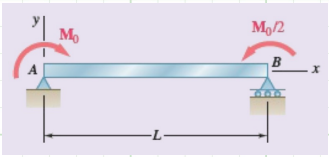
$$M_B - \frac{4wL^2}{192} - \frac{9wL^2}{192} + \frac{24wL^2}{192} = 0$$

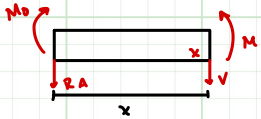
$$M_B = \frac{-11wL^2}{192}$$

$$\boxed{M_B = \frac{11wL^2}{192}\ )}$$

PROBLEMS: MATLAB, P13.26, P15.5, P15.19, P15.26, P15.43
P15.52

P15.52 :





$\Sigma M_x = 0$

$M + R_A L - M_0 x = 0$

$M = M_0 - \frac{M_0}{2L} x$

$= \frac{M_0 L}{L} - \frac{M_0}{2L} x$

$M = \frac{M_0}{L}(L - \frac{1}{2}x)$

$\Sigma M_B = 0$

$-M_0 + \frac{M_0}{2} + R_A L = 0$

$R_A L = \frac{2M_0}{2} - \frac{M_0}{2}$

$R_A = \frac{M_0}{2L} \downarrow$

a) $EI \frac{d^2y}{dx^2} = \frac{M_0}{L}(L - \frac{1}{2}x)$

$EI \frac{dy}{dx} = \frac{M_0}{L}(Lx - \frac{1}{4}x^2) + C_1$

$EIy = \frac{M_0}{L}(\frac{1}{2}Lx^2 - \frac{1}{12}x^3) + C_1 x + C_2$

- $x = 0$ and $y = 0 \longrightarrow C_2 = 0$
- $x = L$ and $y = 0 \longrightarrow C_1 = \frac{-5}{12} M_0 L$

$0 = \frac{M_0}{L}(\frac{1}{2}L(L)^2 - \frac{1}{12}L^3) + C_1 L$

$= \frac{M_0 L^3}{2L} - \frac{M_0 L^3}{12L} + C_1 L$

$\frac{M_0 L}{12} - \frac{6 M_0 L}{12} = C_1$

$y = \frac{M_0}{EIL}(\frac{1}{2}Lx^2 - \frac{1}{12}x^3 - \frac{5}{12}L^2 x)$

$\frac{dy}{dx} = \frac{M_0}{EIL}(Lx - \frac{1}{4}x^2 - \frac{5}{12}L^2) = 0$

$Lx - \frac{1}{4}x^2 - \frac{5}{12}L^2 = 0$

$\frac{1}{4}x^2 - Lx + \frac{5}{12}L^2 = 0$

$x = 2L - 2\sqrt{L^2 - 4(\frac{1}{4})(\frac{5}{12}L^2)}$

$= 2(1 - \sqrt{\frac{3}{12}})L$

$x = 0.472\,L$

b) $L = \sqrt{\frac{EI|y|}{0.0940\,M_0}}$

$= \sqrt{\frac{(200 \cdot 10^9)(347 \cdot 10^{-6})(1.8 \cdot 10^{-3})}{0.0940\,(80 \cdot 10^3)}}$

$L = 4.07$ m

$y(x) = \frac{M_0 L^2}{EI}\left[(\frac{1}{2})(0.472)^2 - (\frac{1}{12})(0.472) - (\frac{5}{12})(0.471)\right]$

$y = 0.0940 \frac{M_0 L^2}{EI} \downarrow$