CE30 – Discussion 9

Torsion

Textbook: 10.1, 10.2, 10.3

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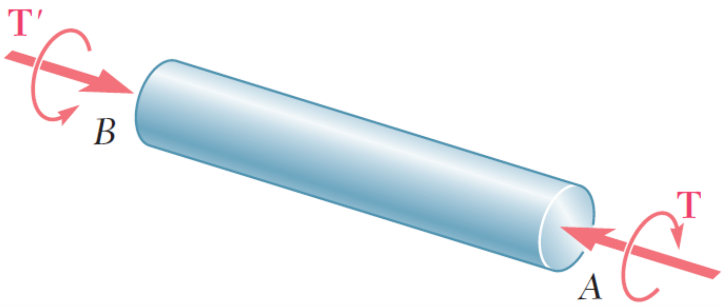
Announcements

- HW9 Problems from the textbook:

  10.7, 10.32, 10.41, 10.54

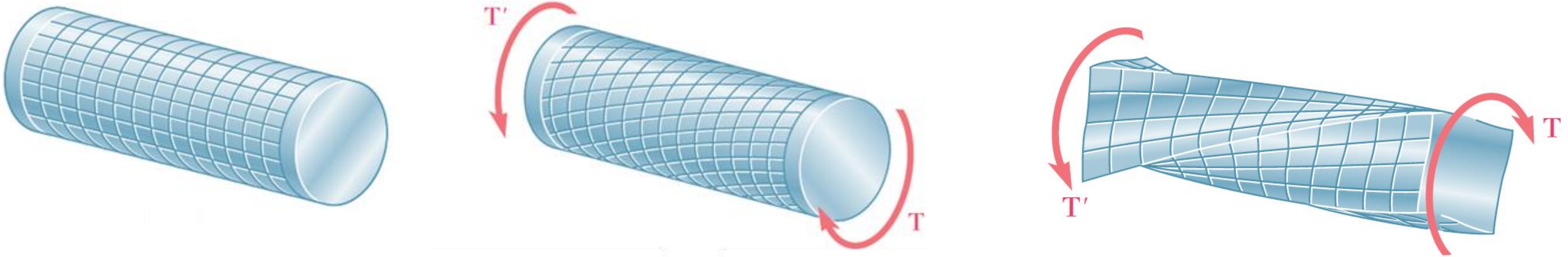
Members in Torsion

- Torsion: Twisting of an object, subjected to a moment
- A moment that causes twisting = "**Torque**"
- Torque is denoted by ***T*** and have units of (Force x distance)

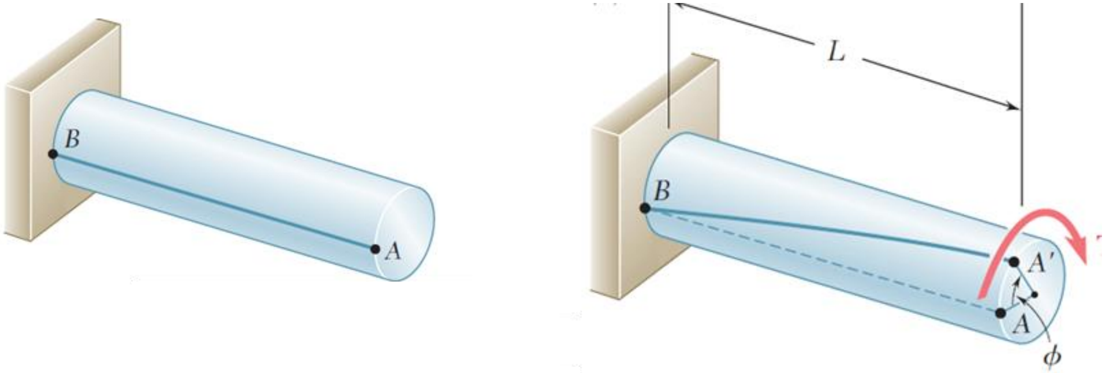


Torsion of Circular Bars

- Individual cross-sections do not get twisted (if axisymmetric)



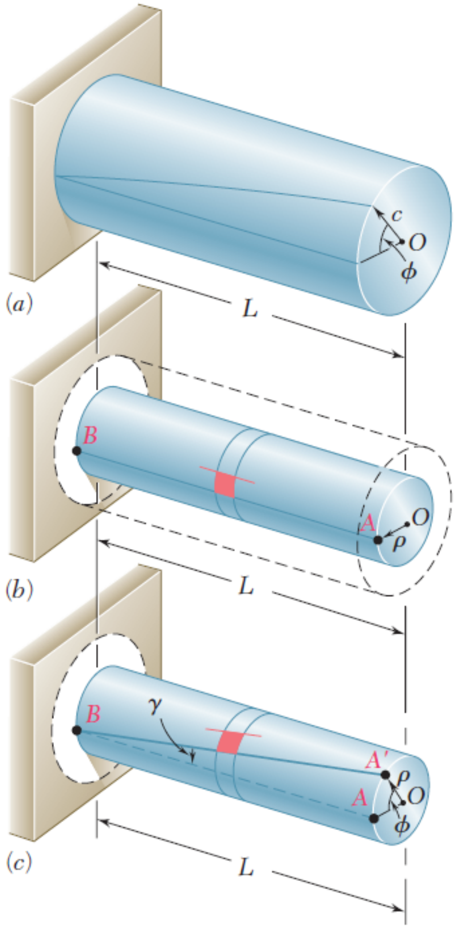
- Deformation:



Angle of Twist

ϕ

Torsion of Circular Bars



Shear Strain γ

- Measured by the change of angle
- Considering arc AA′, we get

$$\boldsymbol{\gamma = \frac{\rho\phi}{L}}$$

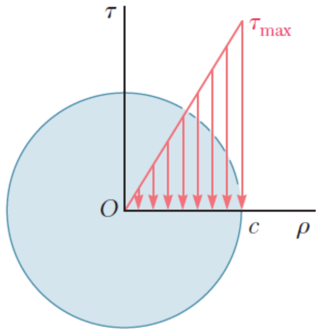
- Maximum would happen at the surface ($\rho = c$)

$$\gamma_{max} = \frac{c\phi}{L}$$

Shear Stress

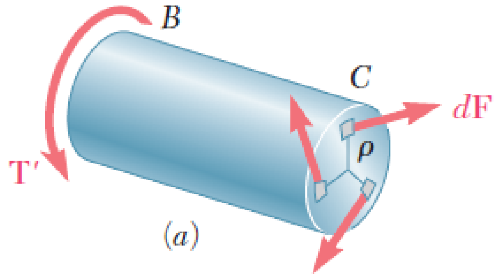
- Hooke's law for shear stress-strain:

$$\tau = G\,\gamma \qquad G\text{: Shear modulus}$$



Shear stress τ distribution

- Moment equilibrium at each cross section:



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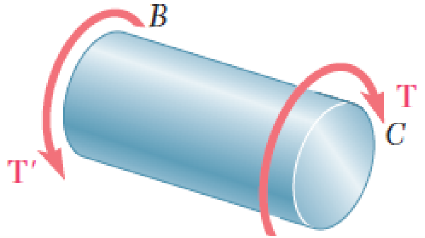


$$\int \rho dF = \int \rho(\tau dA) = T$$

Shear Stress

- Combining moment equilibrium with strain equation and stress distribution:

$$\boldsymbol{\tau} = \frac{\boldsymbol{T}}{\boldsymbol{J}}\boldsymbol{\rho}$$

$J =$ Polar moment of inertia

$$J = \int \rho^2 dA = I_x + I_y$$

- Polar moment of inertia (J) for circular shafts:

$$J = \frac{1}{2}\pi c^4$$

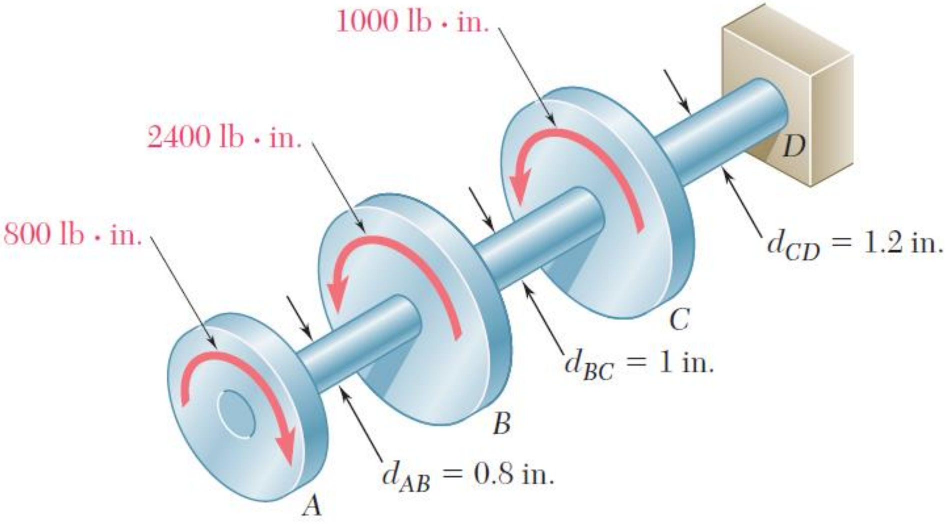
Solid shaft, radius c

$$J = \frac{1}{2}\pi(c_2^4 - c_1^4)$$

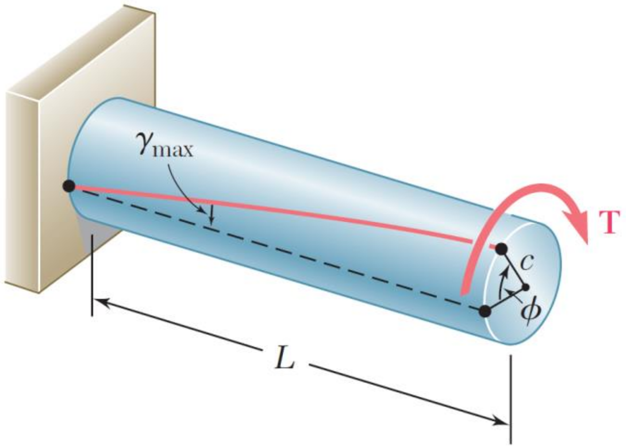
Hollow shaft with inner and outer radius c_1 and c_2

Practice – Similar to HW P10.7

Knowing that each of the shafts AB, BC, and CD consist of solid circular rods, determine (a) the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



Angle of Twist



Using the equations

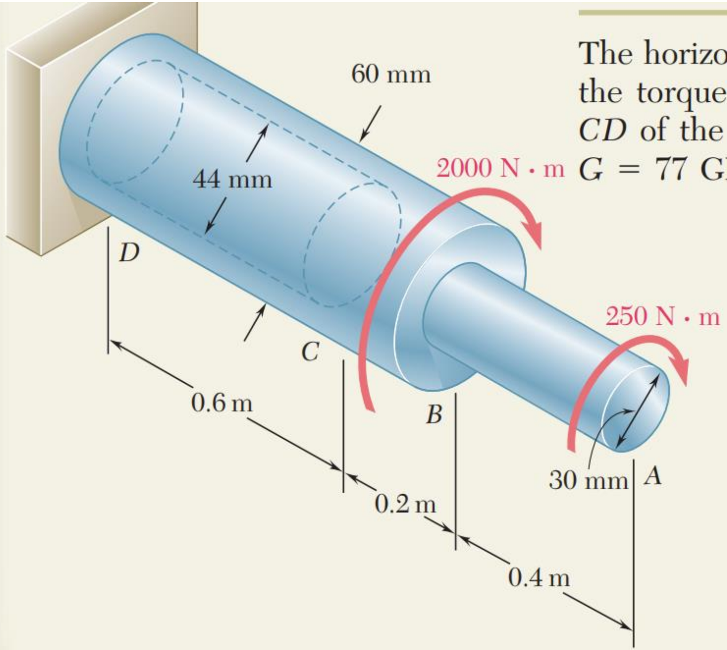
$$\tau_{max} = \frac{T}{J} c \qquad \gamma_{max} = \frac{c\phi}{L} \qquad \tau = G\,\gamma$$

We can solve for angle of twist

$$\boldsymbol{\phi = \frac{TL}{JG}}$$

- If the shaft has different sections/materials: $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

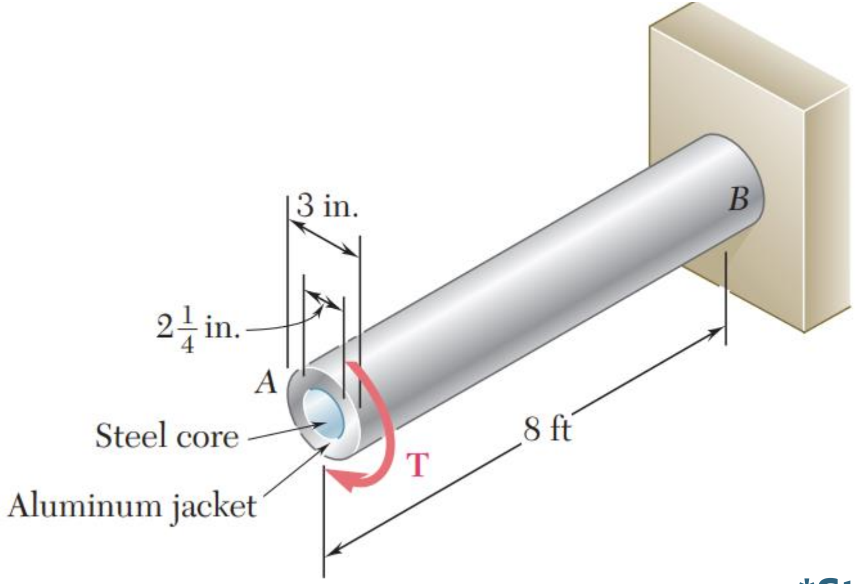
Practice – Similar to HW P10.32



The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77$ GPa, determine the angle of twist at end A.

Practice* – Similar to HW P10.41

A torque of magnitude $T = 35$ kip · in. is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 11.2×10^6 psi for the steel and 3.9×10^6 psi for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.



***Statically indeterminate problem!**