CE30 – Discussion 8

Statically Indeterminate Problems

Textbook: 9.2, 9.3, 9.4

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Spring 2024
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Announcements

- Computer project with MATLAB

- HW8 Problems from the textbook:
  9.16, 9.30, 9.38, 9.81

Stress-Strain

- Stress-Strain related through elastic modulus:

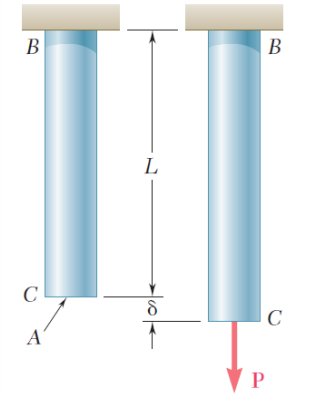
$$\sigma = E\,\epsilon$$

- Axial deformation of a bar

$$\sigma = \frac{P}{A}$$

$$\epsilon = \frac{\delta}{L}$$

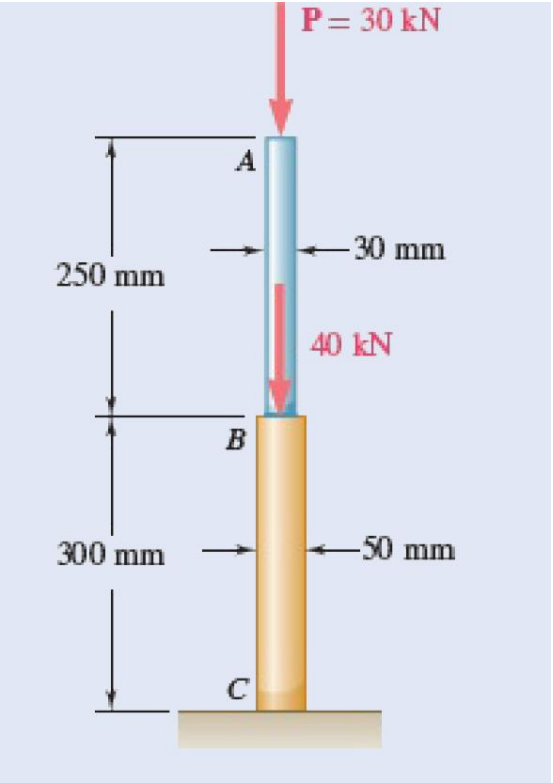
$$\delta = \frac{PL}{AE}$$



Practice – Similar to HW P9.16

Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel ($E = 200\ \text{GPa}$) and rod *BC* of brass ($E = 150\ \text{GPa}$). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.

Page 431



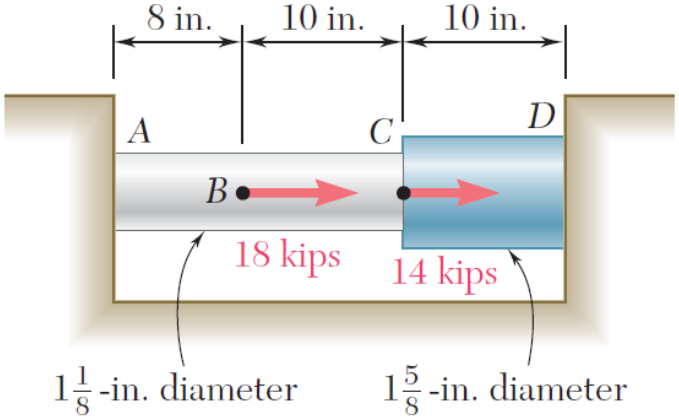
Statically Indeterminate Problems

- Reactions and/or internal forces can't be determined by statics only
  - **(# Unknowns) > (# Equations)**

- Introduce additional equations: **Compatibility**
  - Use the geometry of the problem
  - Find the relation between deformations (compatibility equations)

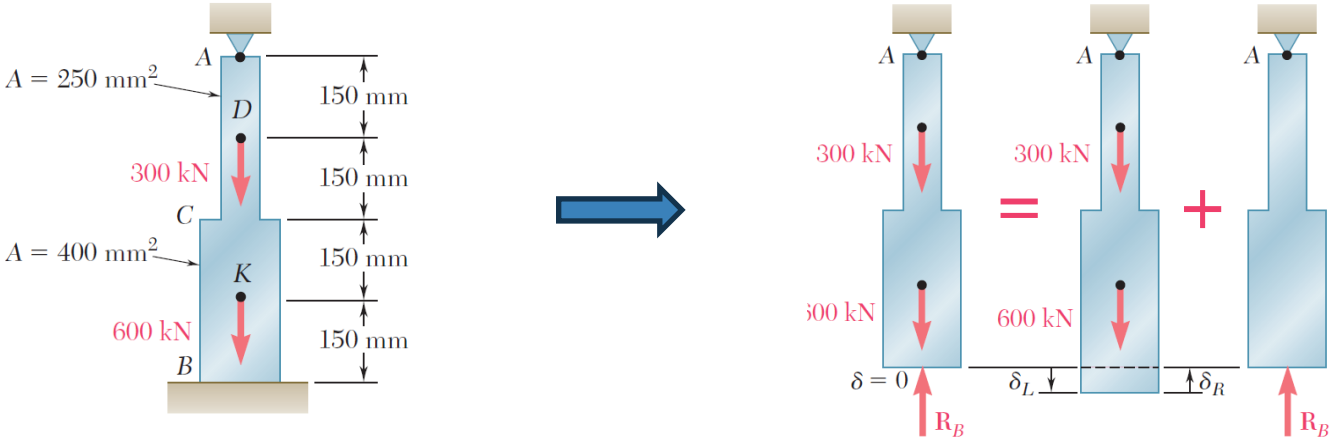
Practice – Similar to HW P9.30

Two cylindrical rods, CD made of steel ($E = 29 \times 10^6$ psi) and AC made of aluminum ($E = 10.4 \times 10^6$ psi), are joined at C and restrained by rigid supports at A and D. Determine (a) the reactions at A and D, (b) the deflection of point C.



Statically Indeterminate Problems

- **Method of Superposition**:
  - Decompose the problem by choosing a redundant reaction
  - Apply compatibility



Thermal Expansion

- Temperature change induces *thermal strain*

$$\epsilon_T = \alpha \, \Delta T$$

α = Coefficient of thermal expansion

ΔT = Temperature change

- If the structure is fixed, thermal expansion might cause internal stress

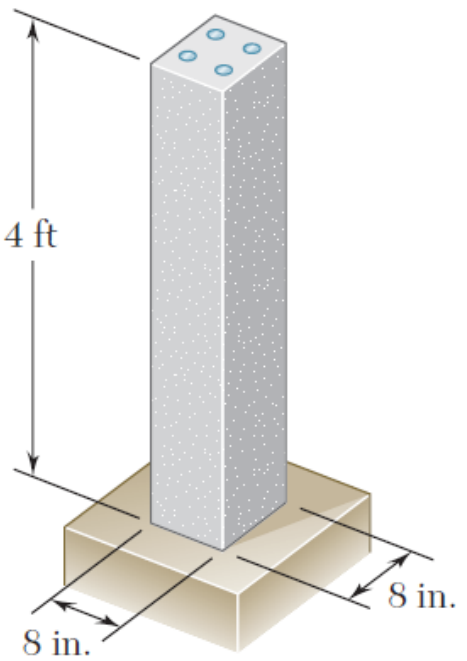
Temperature change
ΔT

Resultant thermal stress

$$\sigma_T = \frac{P}{A} = E\alpha(\Delta T)$$

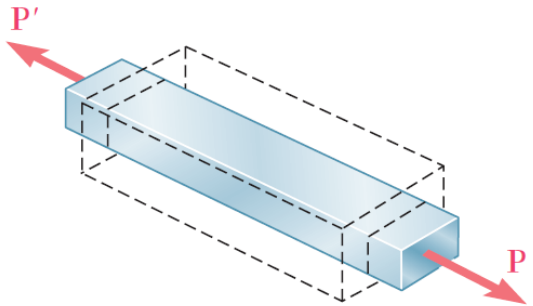
Practice – Similar to HW P9.38

A 4-ft concrete post is reinforced by four steel bars, each of $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°F$ and $E_c = 3.6 \times 10^6$ psi and $\alpha_c = 5.5 \times 10^{-6}/°F$, determine the normal stresses induced in the steel and in the concrete by a temperature rise of 80°F.



Poisson's Effect

- Materials expand or contract perpendicular to the direction of loading



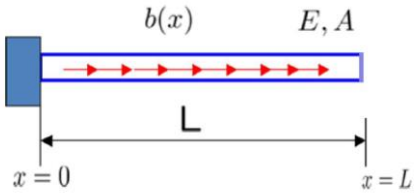
- Quantified by **Poisson's ratio** (a material property)

$$\nu = -\frac{\epsilon_y}{\epsilon_x}$$ (when loaded in x direction)

ϵ_x: Axial Strain

ϵ_y: Lateral Strain

Computer Project: Axial Deformation of 1D Bar



u = Displacement

Given ODE + BC, solve for u

Governing ODE

$$EA\frac{d^2u}{dx^2} + b(x) = 0 \qquad 0 < x < L$$

Boundary Conditions

$$u(0) = 0 \qquad R(L) = 0$$

Loading is given as:

$$b(x) = p\sin(\frac{2\pi x}{L})$$

Internal force in the bar

$$R(x) = EA\,\frac{du}{dx}$$

Your task:

1. Implement boundary conditions, material and geometric properties to the code
2. Plot displacements and internal force, compare with exact solution