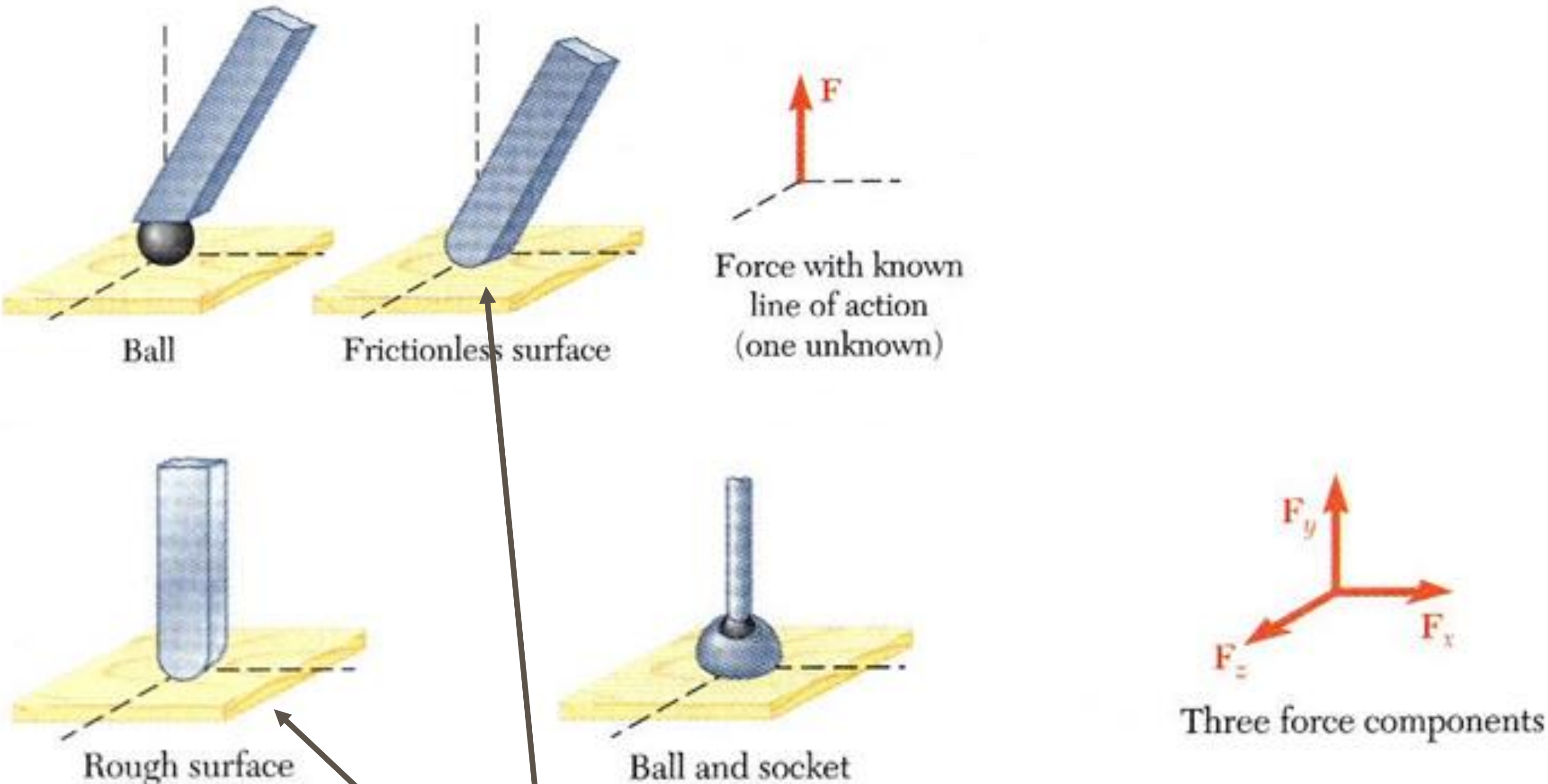
Lecture 8 Friction

Main Concepts: 1. The law of dry friction, 2. Coefficients of friction, and 3. Angles of friction

Reactions at Supports and Connections for a 3D Structure



What is the reaction from the rough surface

Introduction

- In preceding chapters, it was assumed that surfaces in contact were either *frictionless* (surfaces could move freely with respect to each other) or *rough* (tangential forces prevent relative motion between surfaces).

- Actually, no perfectly frictionless surface exists. For two surfaces in contact, tangential forces, called *friction forces*, will develop if one attempts to move one relative to the other.

- However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.

- The distinction between frictionless and rough is, therefore, a matter of degree.

- There are two types of friction: *dry* or *Coulomb friction* and *fluid friction*. Fluid friction applies to lubricated mechanisms. The present discussion is limited to dry friction between nonlubricated surfaces.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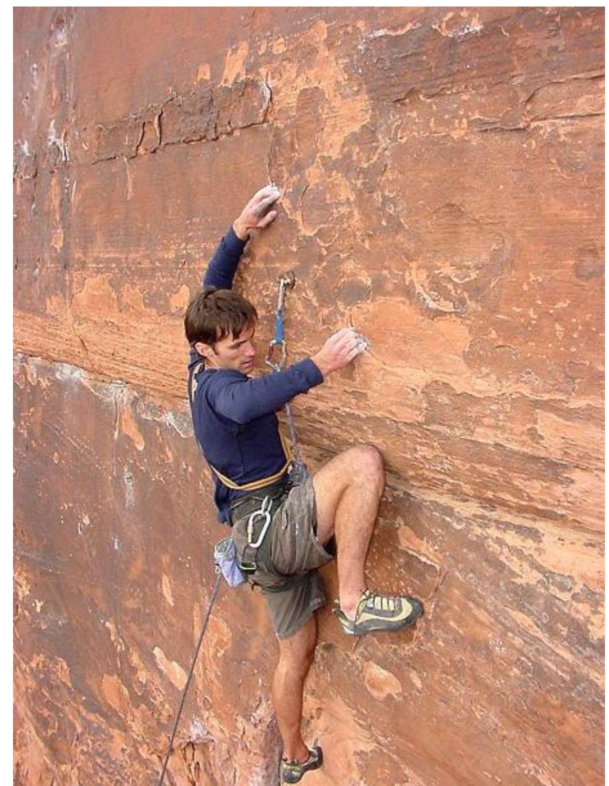
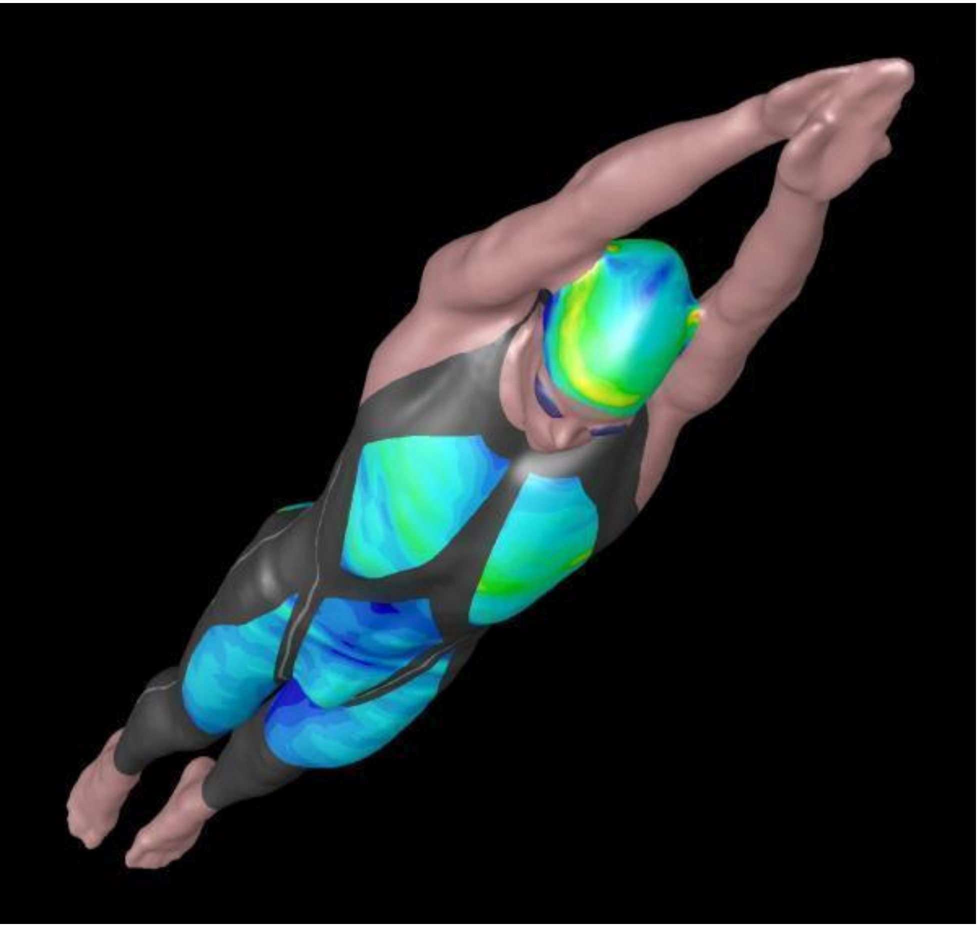
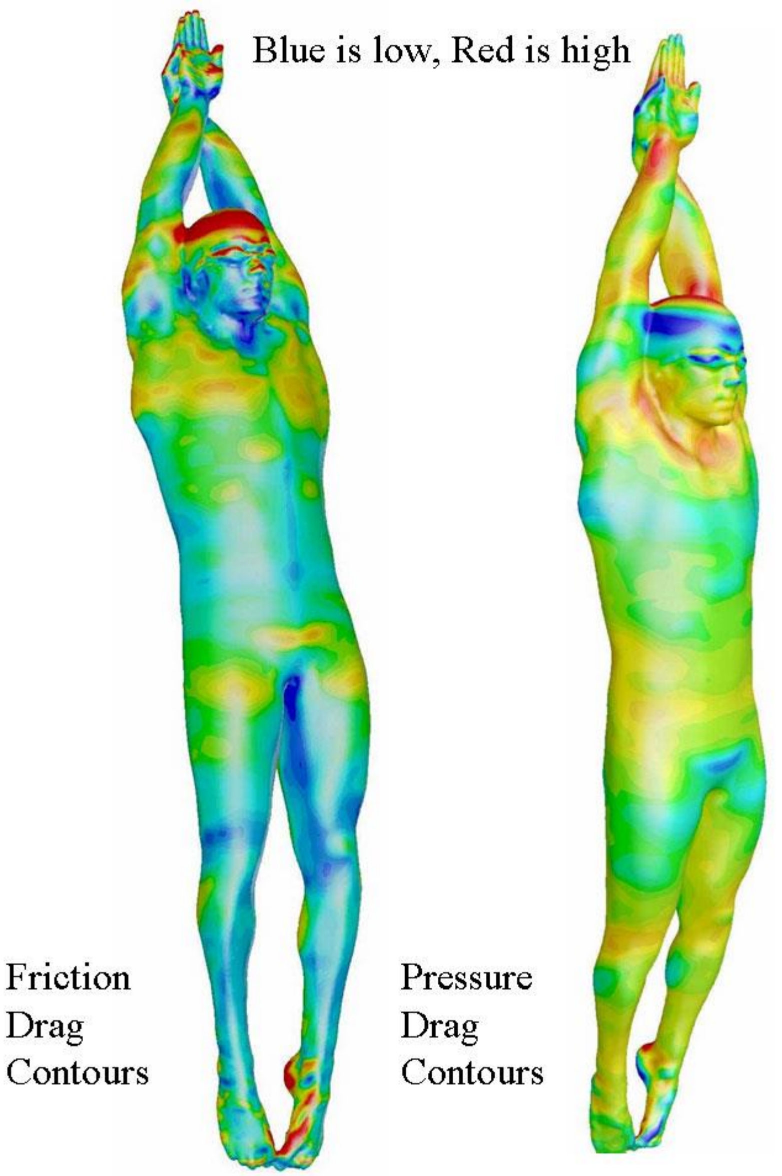


Speedy Speedo Swimsuit





Take Home Message:
Friction is a type of reaction force!

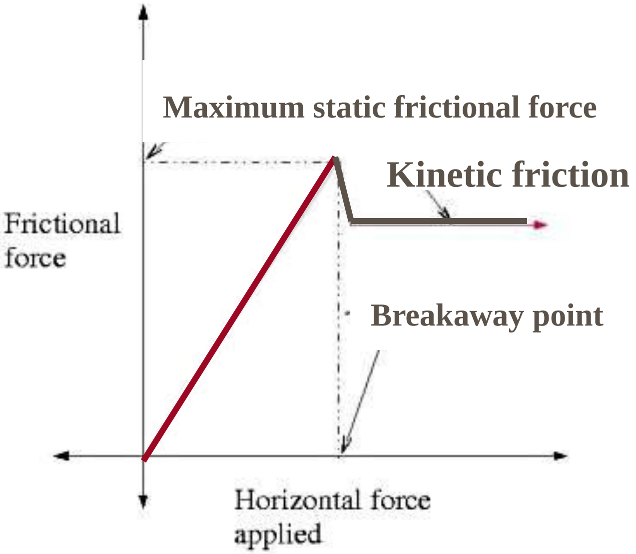
Coulomb's Law of Dry Friction

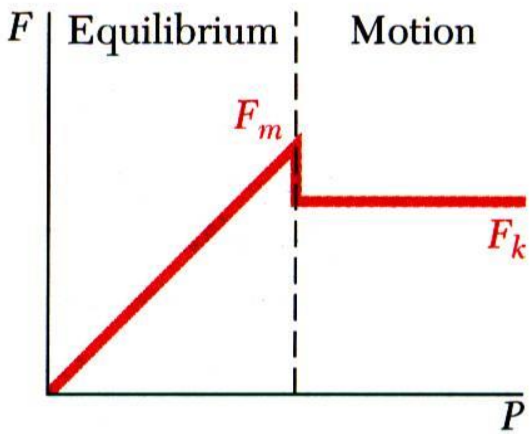
$|F| < \mu_s N$ before sliding and for impending motion

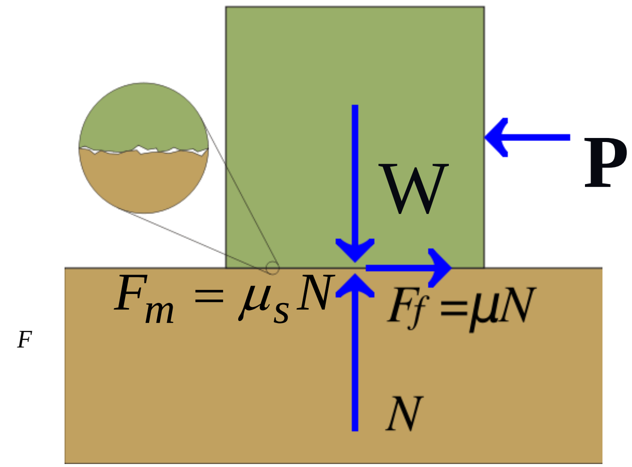
$|F| = \mu_k N$ after sliding begins

Friction is a passive external force (reaction force), which is equivalent to a support, and Charles Coulomb said that this "friction support" will fail at $F_m = \mu_s N$

Charles-Augustin de Coulomb (1736-1806)







- N is the normal force, W is the self weight, P is the pushing force, and Ff is the friction force. It has two different versions:

- As *P* increases, the static-friction force *F* increases as well until it reaches a maximum value F_m.

$$N = W$$

$$F_m = \mu_s N \qquad (1)$$

where μ_s is the coefficient of static friction

$$F_k = \mu_k N \qquad (2)$$

where μ_k is the coefficient of kinetic friction.

The Laws of Dry Friction and Coefficients of Friction

Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

- Maximum static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

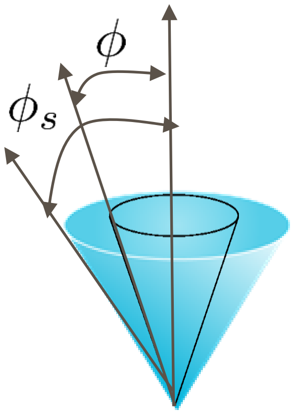
$$F_k = \mu_k N$$

$$\mu_k \cong 0.75\mu_s$$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

Angle of Friction

Equilibrium condition

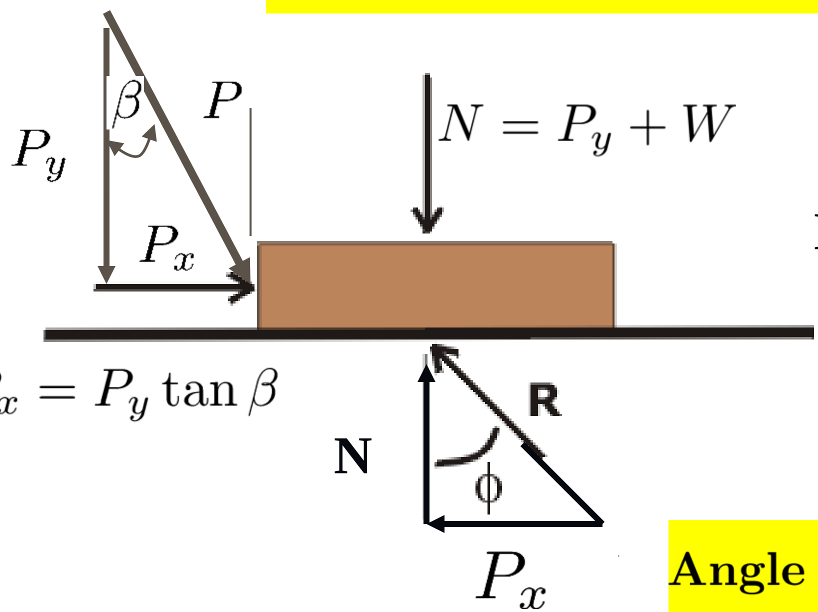


$\phi \le \phi_s$

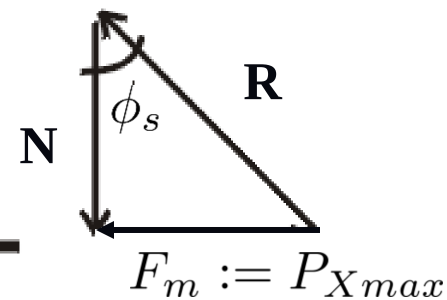
$$\textbf{Angle of Reaction}: \quad \tan\phi = \frac{P_x}{N} = \frac{P_x}{P_y + W}$$

$$\textbf{Angle of Friction}: \boxed{\tan\phi_s = \frac{\mu_s N}{N} = \mu_s}$$

$$\textbf{Kinetic Angle of Friction}: \tan\phi_k = \frac{\mu_k N}{N} = \mu_k$$



$P_x = P_y \tan\beta$



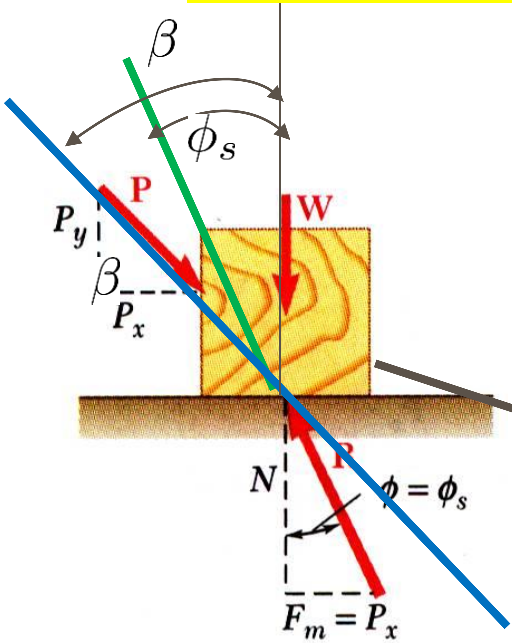
$$\textbf{Angle of Driven – force}: \tan\beta = \frac{P_x}{P_y}$$

When $W \neq 0$, $\phi < \beta$

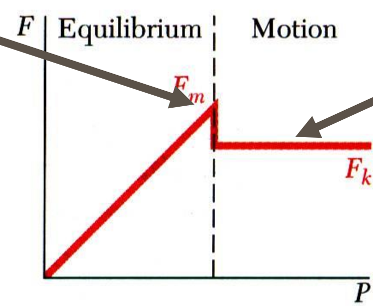
$$\tan\phi = \frac{P_x}{W + P_y} = \frac{1}{W/P_y + 1}\tan\beta$$

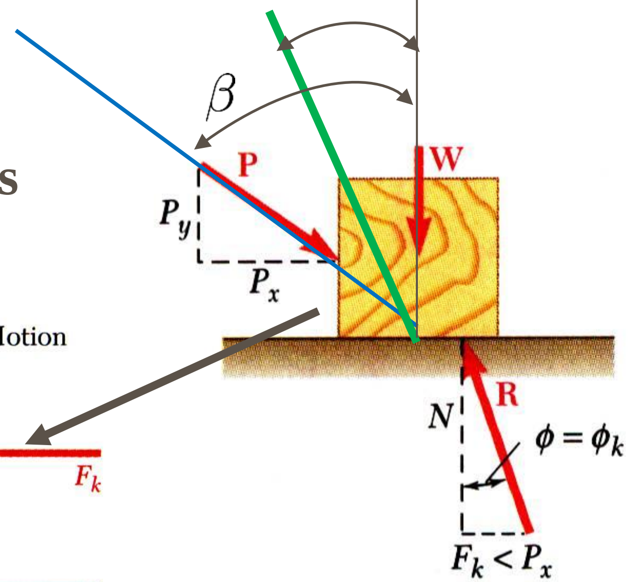
$$\textbf{Static Angle of Friction}: \tan\phi_s = \frac{\mu_s N}{N} = \mu_s$$

$$\textbf{Kinetic Angle of Friction}: \tan\phi_k = \frac{\mu_k N}{N} = \mu_k$$



Critical Cases





- Motion impending

$$\phi = \phi_s$$

- Motion

$$\phi = \phi_k$$

$$\textbf{Angle of Driven - force}: \tan\beta = \frac{P_x}{P_y}$$

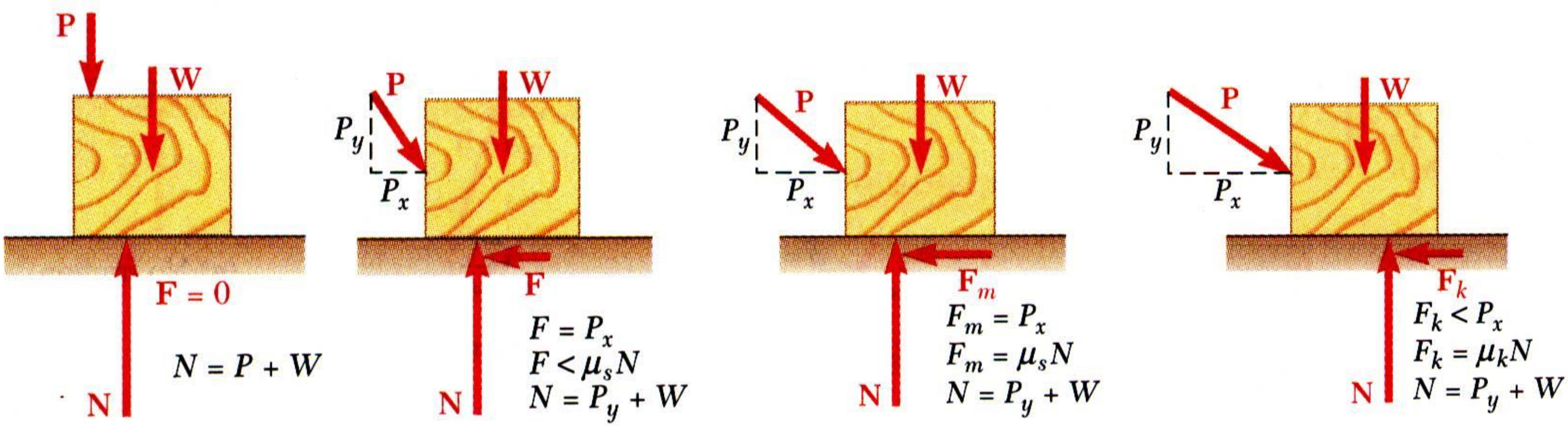
When $W \neq 0$, $\phi < \beta$

$$\tan\phi = \frac{P_x}{W + P_y} = \frac{1}{W/P_y + 1}\tan\beta$$

The Laws of Dry Friction. Coefficients of Friction

- Four situations can occur when a rigid body is in contact with a horizontal surface:

$$\tan \phi_s = \frac{\mu_s N}{N} = \mu_s$$



- No friction, ($P_x = 0$)
- No motion, ($P_x < F_m$)
- Motion impending, ($P_x = F_m$)
- Motion, ($P_x > F_m$)

$$\tan \beta = \frac{P_x}{P_y}$$

$$\text{Define :} \quad \tan \phi = \frac{P_x}{N} = \frac{P_x}{P_y + W}$$

$$\beta \geq \phi$$

Angles of Friction

$$\text{Define :} \quad \tan\phi = \frac{P_x}{N}$$

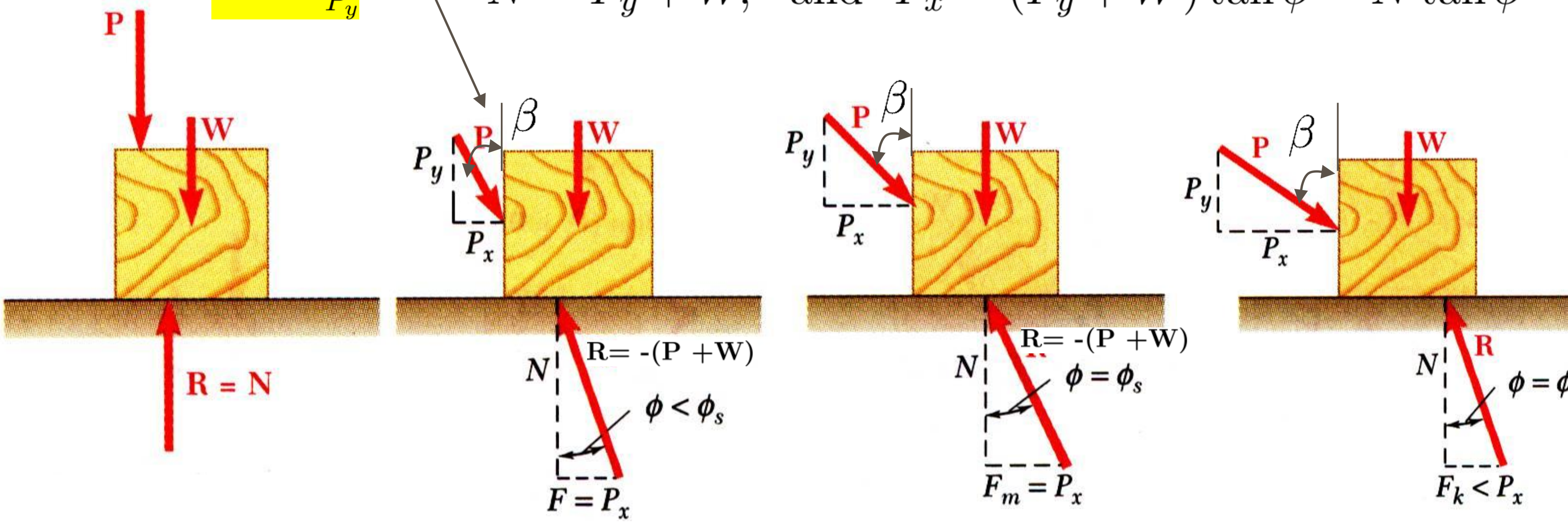
$$P_{max} = F_m = \mu_s N$$

$$\tan\phi_s = \frac{\mu_s N}{N} = \mu_s$$

Caveat: β may be greater than ϕ_s
Even if $\phi < \phi_s$.

$$\tan\beta = \frac{P_x}{P_y}$$

$$N = P_y + W, \quad \text{and} \quad P_x = (P_y + W)\tan\phi = N\tan\phi$$



- No friction
- No motion
- Motion impending
- Motion

$$\mathbf{R} = -(\mathbf{P} + W)$$

$$\tan\phi = \frac{P_x}{W + P_y} = \frac{\tan\beta}{W/P_y + 1}$$

$$\tan\phi = \frac{P_x}{N}$$

$$\phi < \phi_s$$

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\phi = \phi_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

$$\phi_s > \phi = \phi_k$$

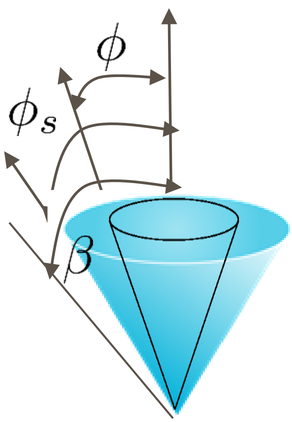
Angles of Friction

When $\phi < \phi_s$, the body is in equilibrium.

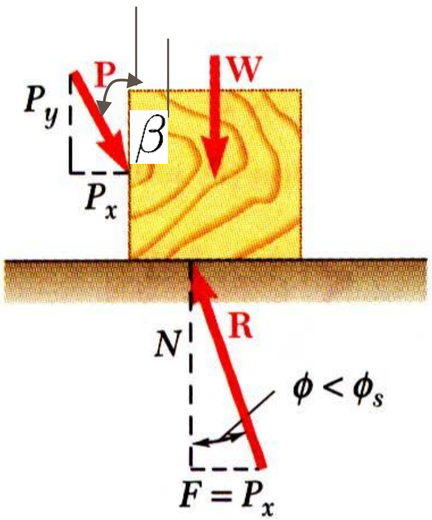
- It is sometimes convenient to replace normal force *N* and friction force *F* by their resultant ***R***:

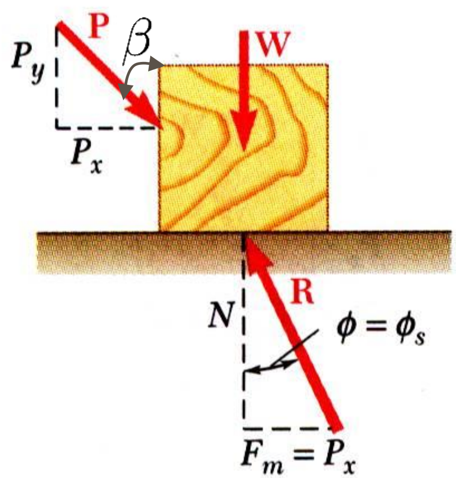
$$N = P_y + W, \quad \text{and} \quad P_x = P_y \tan\beta$$

$$P_x = N \tan\phi$$



$\phi < \phi_s$





- No motion

$$\tan\phi = \frac{P_x}{N}$$

$$\phi < \phi_s$$

- Motion impending

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

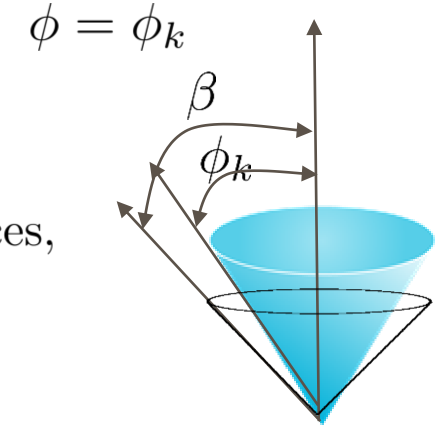
$$\phi = \phi_s$$

β may be larger than ϕ_s,
it depends on the value of W

When $W \neq 0$, $\phi < \beta$

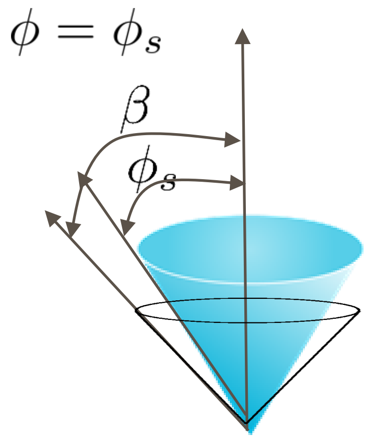
The angle of friction is defined as

$$tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N} = \mu_s \ ,$$



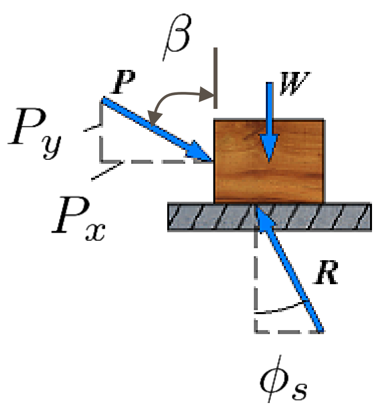
for static friction. If motion actually takes places,

$$tan\phi_k = \frac{F_k}{N} = \frac{\mu_s N}{N} = \mu_k \ .$$

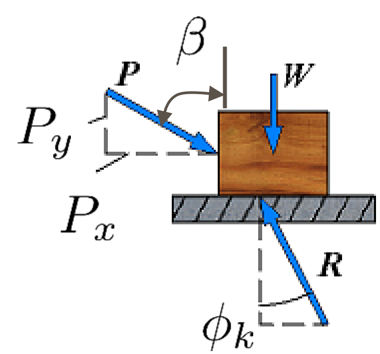


It is true that $F_k < F_m$

$$N = P_y + W, \quad \text{and} \quad P_x = P_y \tan\beta = N \tan\phi$$



Motion pending



Motion
$P_x > F_k$

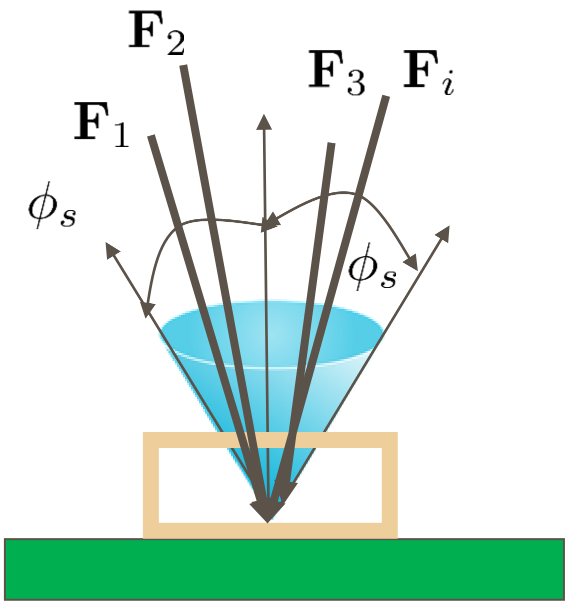
$$F_m = N\tan\phi_s = P_x = N\tan\phi$$

$$\phi_s = \phi \leq \beta$$

$$F_k = N\tan\phi_k < (W + P_y)\tan\beta$$

$$\phi_k = \phi \leq \beta$$

For multiple forces, it is possible that
for a fixed i, $\beta_i > \phi_s$,
and rest of j, $\beta_j < \phi_s$ $i \neq j$,
the block still stays in equilibrium.



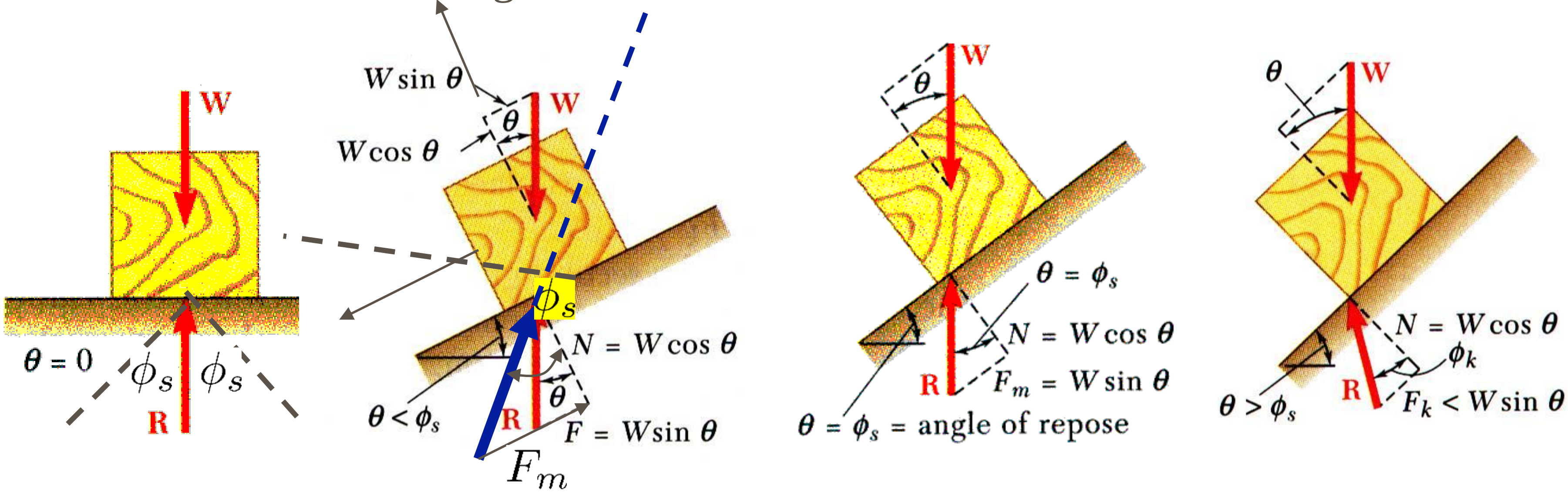
Today's Lecture Attendance Password: **Angle of Friction**

Angles of friction in a slope

$F = W \sin\theta$ and $N = W\cos\theta$

$W = N/\cos\theta \;\rightarrow\; F = N\tan\theta$

- Consider block of weight *W* resting on board with variable inclination angle θ.



- No friction
- No motion
- Motion impending
- Motion

$$N\tan\theta = F < F_m = N\tan\phi_s$$

Equilibrium condition: $\theta < \phi_s$

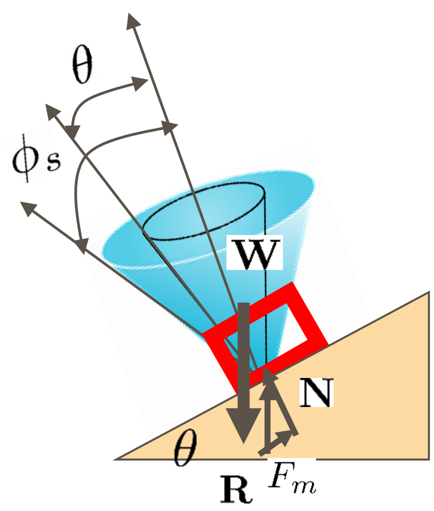
In this case: $\phi = \beta = \theta$

$$\theta = \phi_s$$

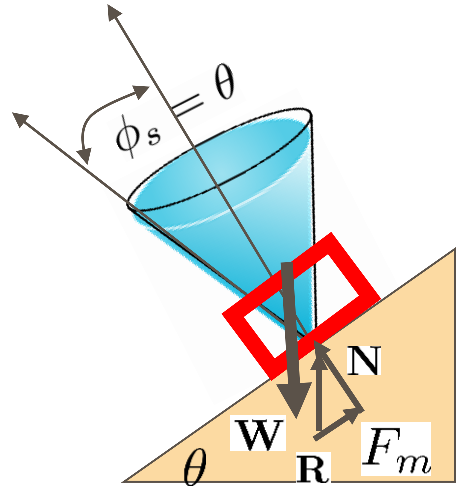
$$\phi_k \le \theta \le \phi_s$$

$$F_m^k < F_m^s$$

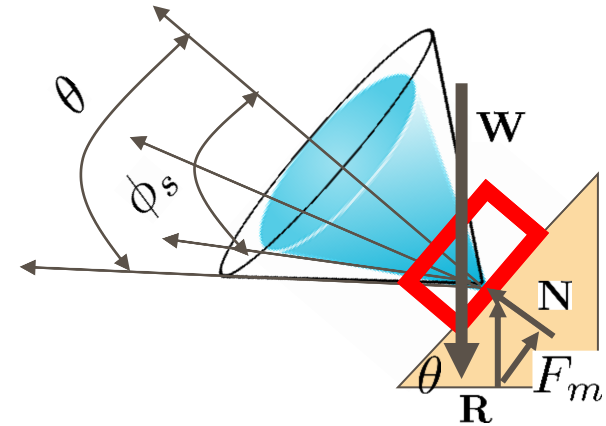
$$\mu_k < \mu_s \;\rightarrow\; \phi_k < \phi_s$$



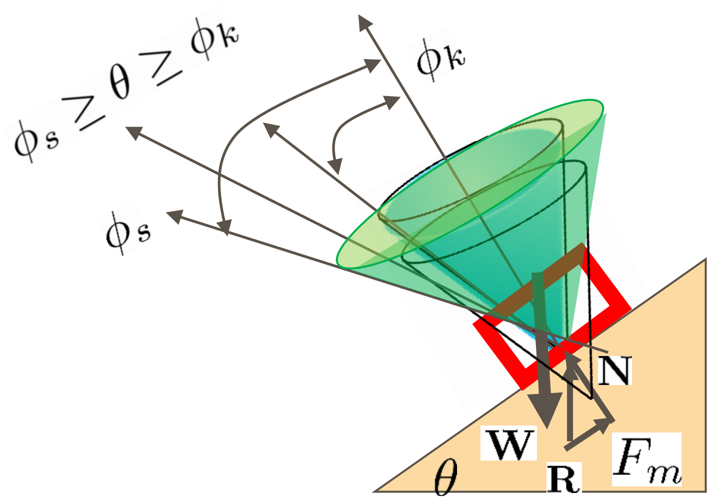
$\theta \leq \phi_s$,
the block stays still.



$\theta = \phi_s$,
Motion pending.



$\theta > \phi_s$,
the block falls down.



$\phi_s \geq \theta \geq \phi_k$
The block is unstable.

Measurement [edit]

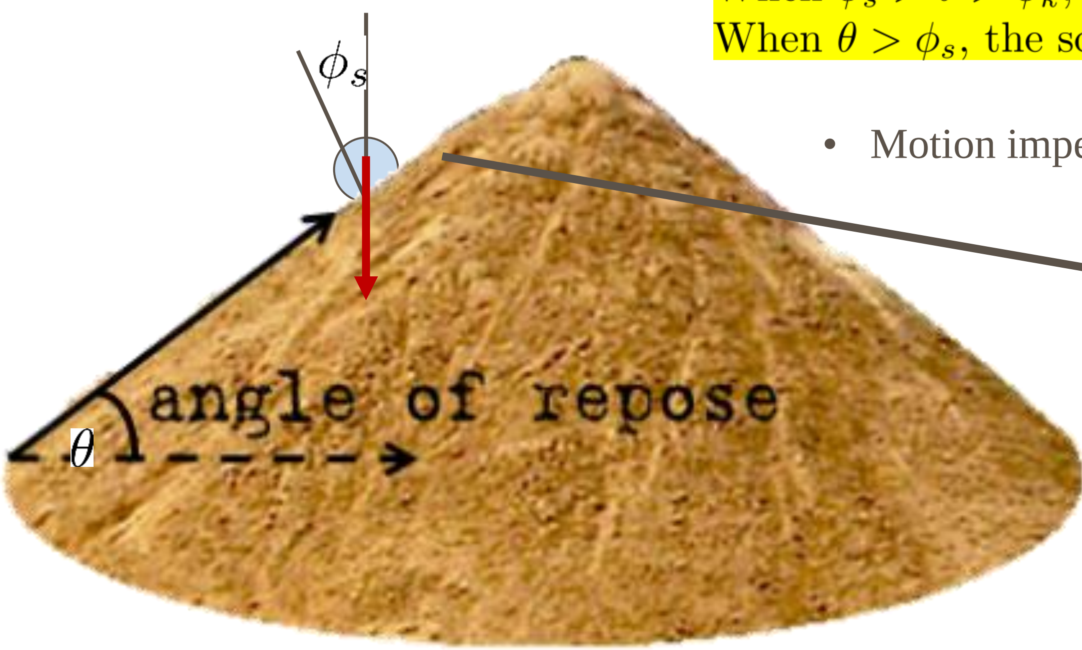
There are numerous methods for measuring angle of repose and each produces slightly different results. Results are also sensitive to the exact methodology of the experimenter. As a result, data from different labs are not always comparable. One method is the triaxial shear test, another is the direct shear test.

If the coefficient of static friction is known of a material, then a good approximation of the angle of repose can be made with the following function. This function is somewhat accurate for piles where individual objects in the pile are minuscule and piled in random order.[4]

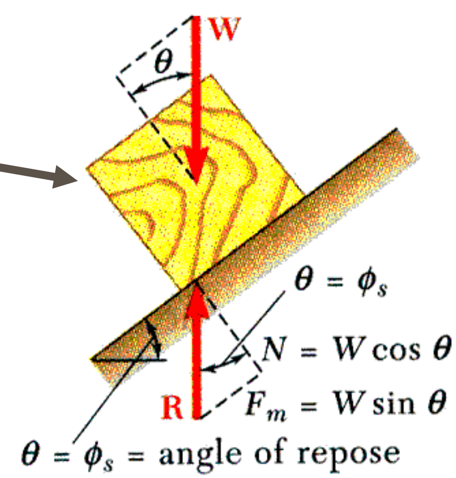
$$\tan(\theta) \approx \mu_s$$

where, μ_s is the coefficient of static friction, and θ is the angle of repose.

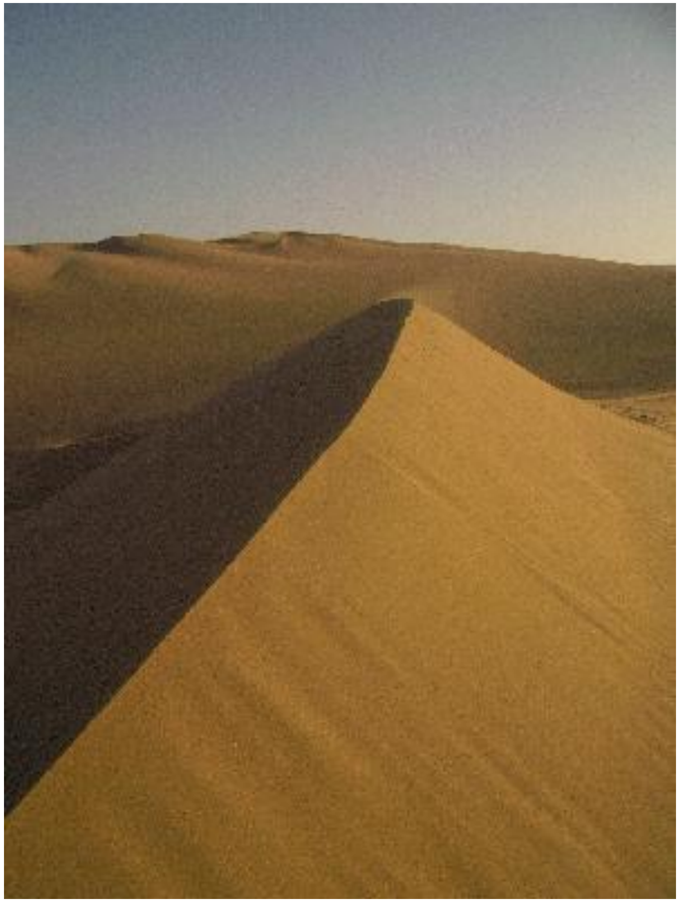
When $\theta < \phi_k$, the soil pile will be stable;
When $\theta = \phi_s$, the soil pile is in critical situation, and
When $\phi_s > \theta > \phi_k$, the soil pile will be unstable.
When $\theta > \phi_s$, the soil pile cannot form.



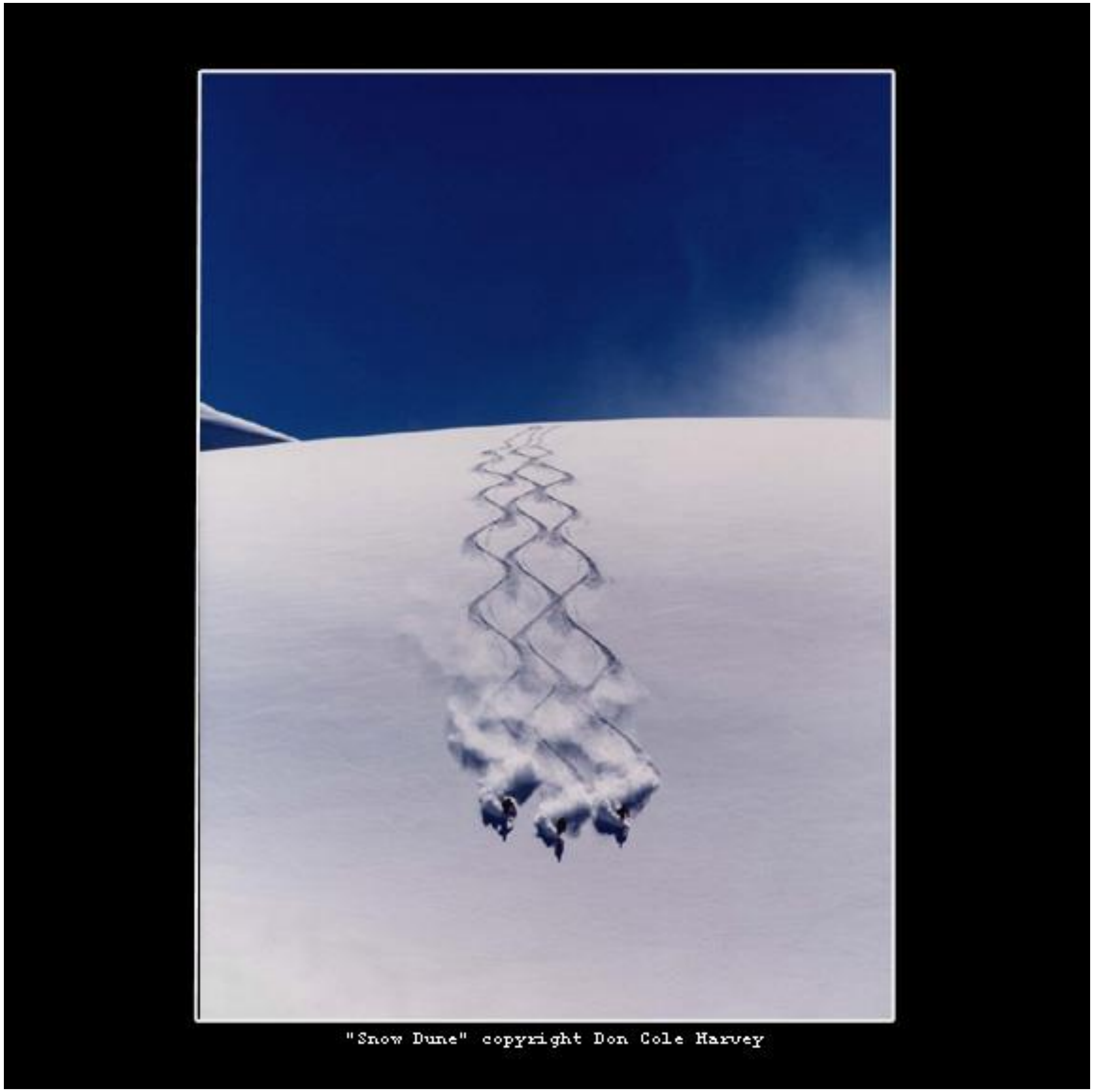
- Motion impending



Angles of friction and angle of sanddune



Snow Dune



Unstable Snow Cover (Avalanche): $\phi_k \leq \theta \leq \phi_s$

Snow Dune

Unstable Snow Cover (Avalanche): $\phi_k \le \theta \le \phi_s$

Take Home Message:
Friction is a type of reaction force !

Perspectives

The fundamental understanding of friction and adhesion forces between dry and lubricated surfaces is crucial in fields as widespread as earthquake dynamics and human joints. From a technological point of view, with the miniaturization of moving components in many technological devices, such as micro and nano-electromechanical systems (MEMS) and magnetic hard disk drives, it has become of primary importance to study surface forces like friction, viscous drag and adhesion at micro and nanoscale. Despite to its scientific and technological importance, there are still no generally accepted explanations for the basic laws of friction and for the origin of diverse adhesion forces.