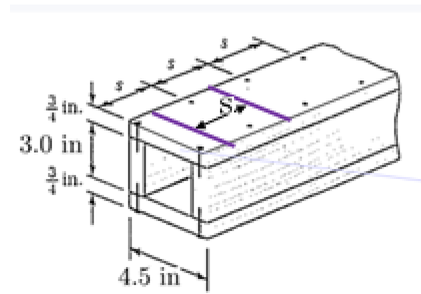
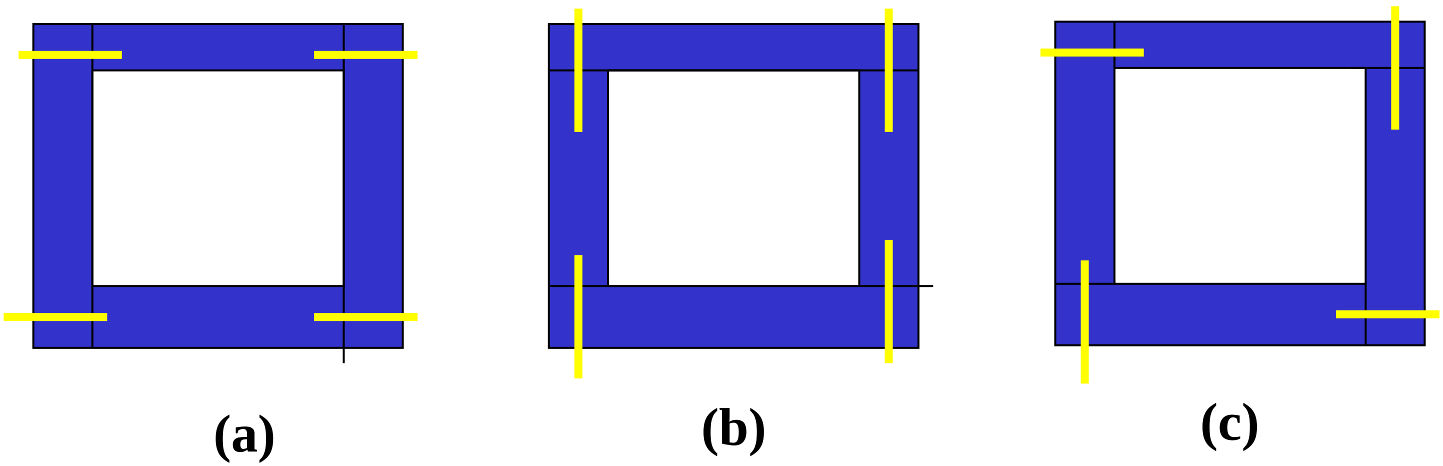
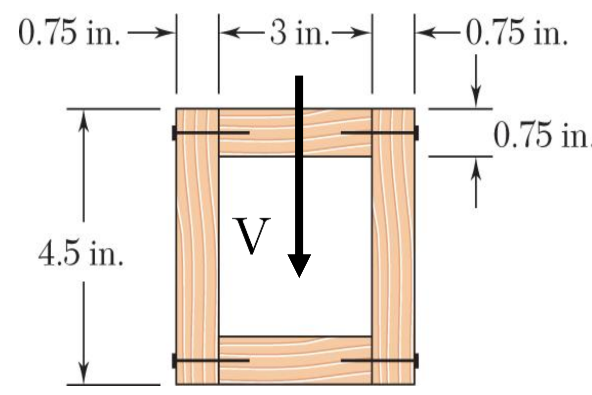
We have three designs:
Which one is the best design?





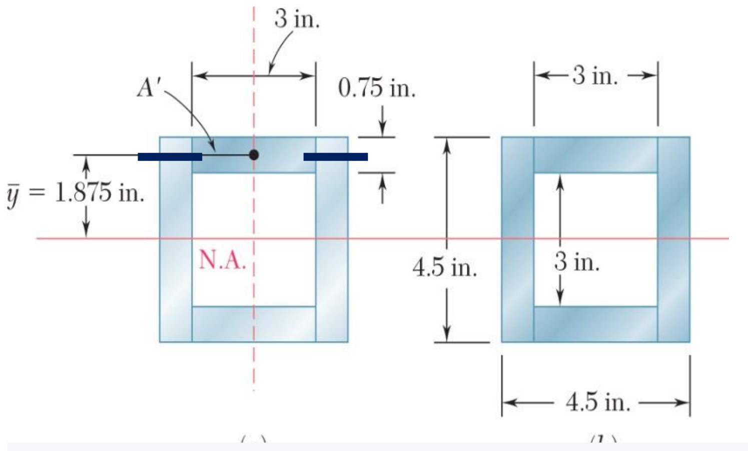
The square box beam is constructed from four planks as shown. Knowing that the spacing between nails is s=1.75 in. and the beam is subjected to a vertical shear of magnitude V = 600 lb, determine the shearing force in each nail for all three designs.



This is a rated-R problem,

$$A_w = 4.5^2 - 3^2 = 11.25in^2$$

Design 1



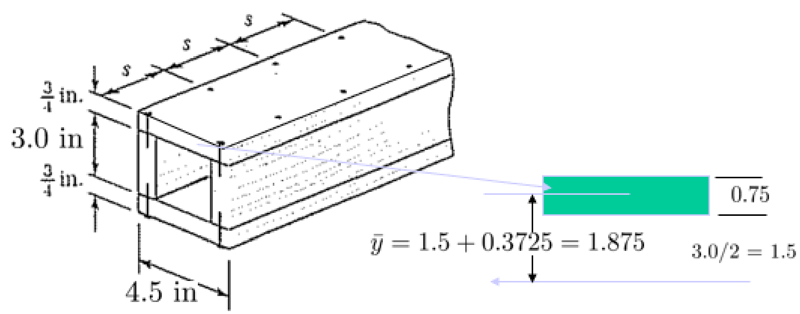
$$I = \frac{1}{12}(4.5\text{in})^4 - \frac{1}{12}(3\text{in})^4$$
$$= 27.42\text{in}^4$$

$$V = 600\ N$$

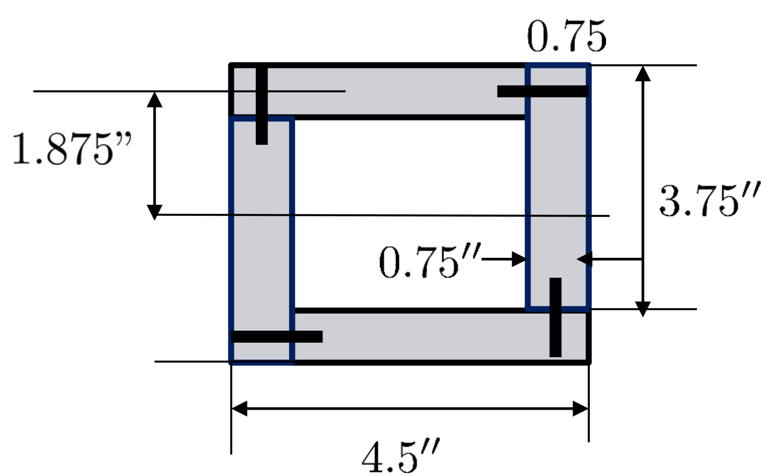
$$s = 1.75in$$

$$2F_{nail} = \Delta H = q\Delta x$$

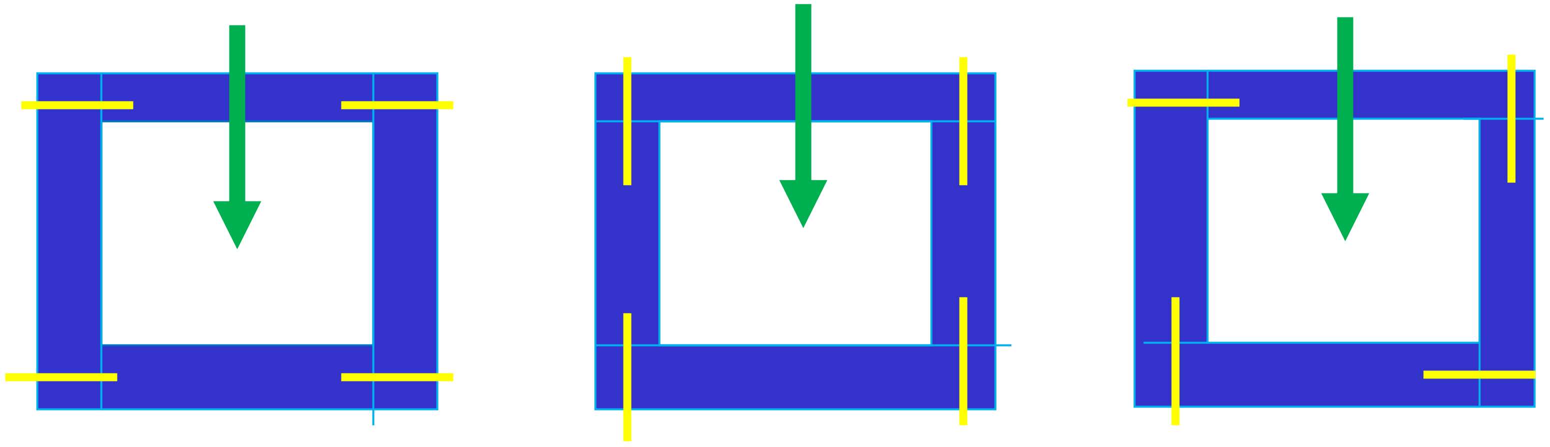
Design 2



Design 3



Summary



(a) (b) (c)

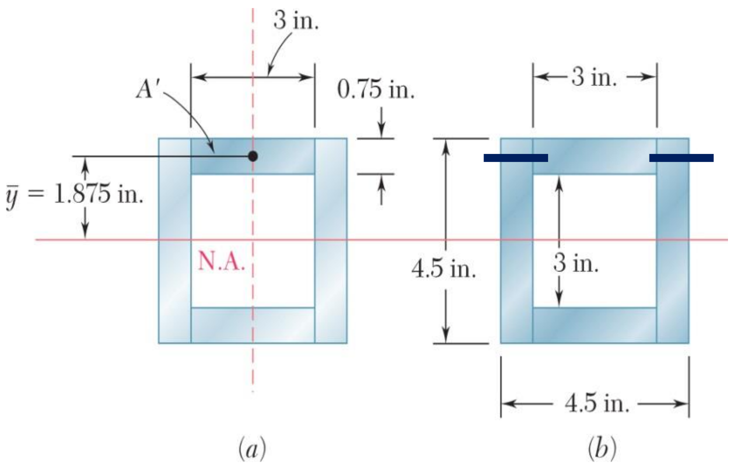
$F_{nail} = 80.8\ lb$ $\qquad$ $F_{nail} = 117.931\ lb$ $\qquad$ $F_{nail} = 98.275\ lb$

Which design is the optimal design ?

For the fixed F_{nail}, which design has the smallest $\Delta x = s$?

Design 1

$$\Delta H = q\Delta x$$



For the upper plank,

$$Q = A'\bar{y} = \boxed{(0.75\text{in.})(3\text{in.})}(1.875\text{in.})$$
$$= 4.22\,\text{in}^3$$

$$I = \tfrac{1}{12}(4.5\,\text{in})^4 - \tfrac{1}{12}(3\,\text{in})^4$$
$$= 27.42\,\text{in}^4$$

$$s = 1.75in$$

$$q = \frac{VQ}{I} = \frac{(600\,\text{lb})(4.22\,\text{in}^3)}{27.42\,\text{in}^4} = 92.3\frac{\text{lb}}{\text{in}}$$

$$f = \frac{q}{2} = 46.15\frac{\text{lb}}{\text{in}}$$
$$= \text{edge force per unit length}$$

Based on the spacing between nails, determine the shear force in each nail.

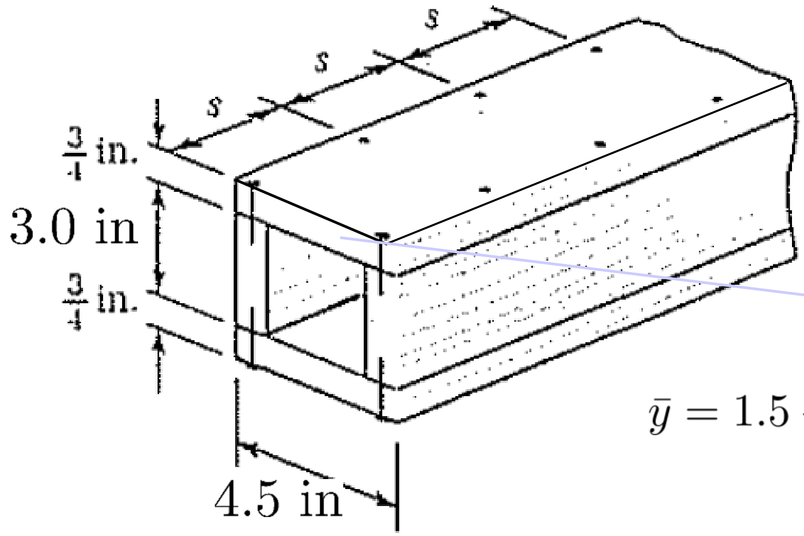
$$F = f\,s = \left(46.15\frac{\text{lb}}{\text{in}}\right)(1.75\,\text{in})$$

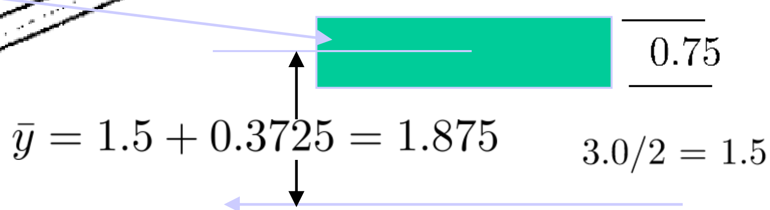
$$\boxed{F = 80.8\,\text{lb}}$$

Design 2:

$$V = 600lb \qquad I = 27.42in^4$$

$$A = 4.5 \times 0.75 = 3.375\ in^2 \qquad \Delta x = s = 1.75\ in$$





$$Q = \bar{y}A = (1.825)(3.375) = 6.1594in^3$$

$$2F_{nail} = qs$$

$$q = \frac{VQ}{I} = \frac{(600)(6.1594)}{27.42} = 134.778lb/in$$

$$F_{nail} = \frac{(134.778)(1.75)}{2} = 117.931\ lb$$

Design 3:

$V = 600lb$

$A = 3.75 \times 0.75 = 2.8125\ in^2$

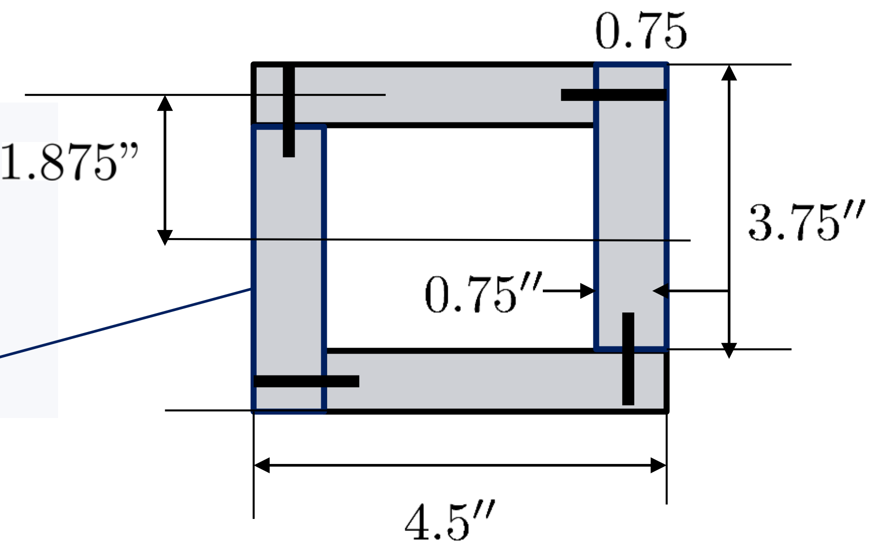
$s = 1.75\ in$

$I = 27.42in^4,\ \bar{y} = 1.875$

$Q = \bar{y}A = (1.875)(2.815) = 5.1328in^3$

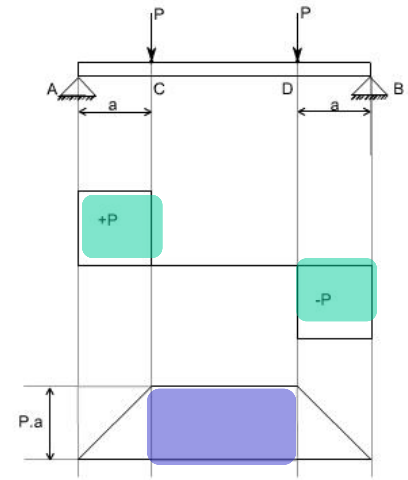
$$q = \frac{VQ}{I} = \frac{(600)(5.1328)}{27.42} = 112.315lb/in$$

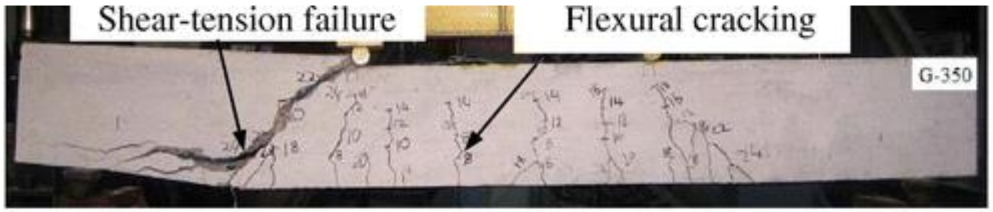
$$F_{nail} = \frac{(112.315)(1.75)}{2} = 98.275\ lb$$



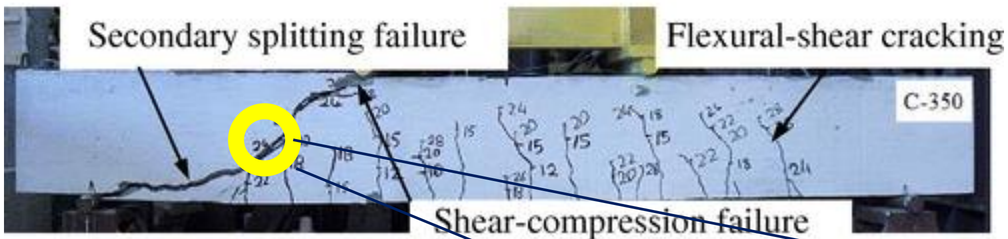
$\Delta H = 2F_{nail} = qs$

Steel Beam Shear Test

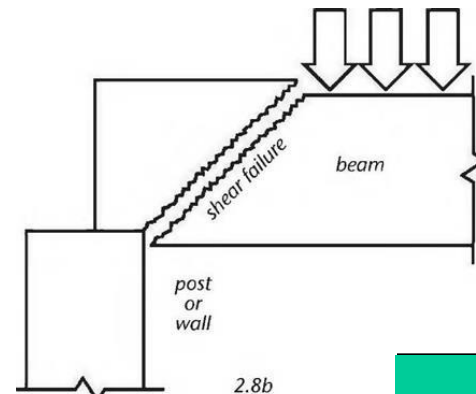


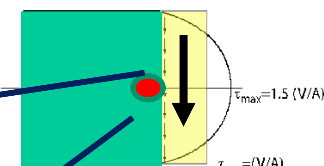


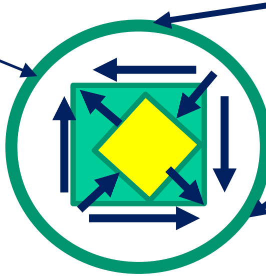
(a)



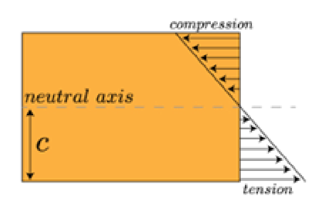
(b)







V



Lecture 30 Differential Equation for Beam Deflection

Objectives: Study the deformation (deflection) of beams

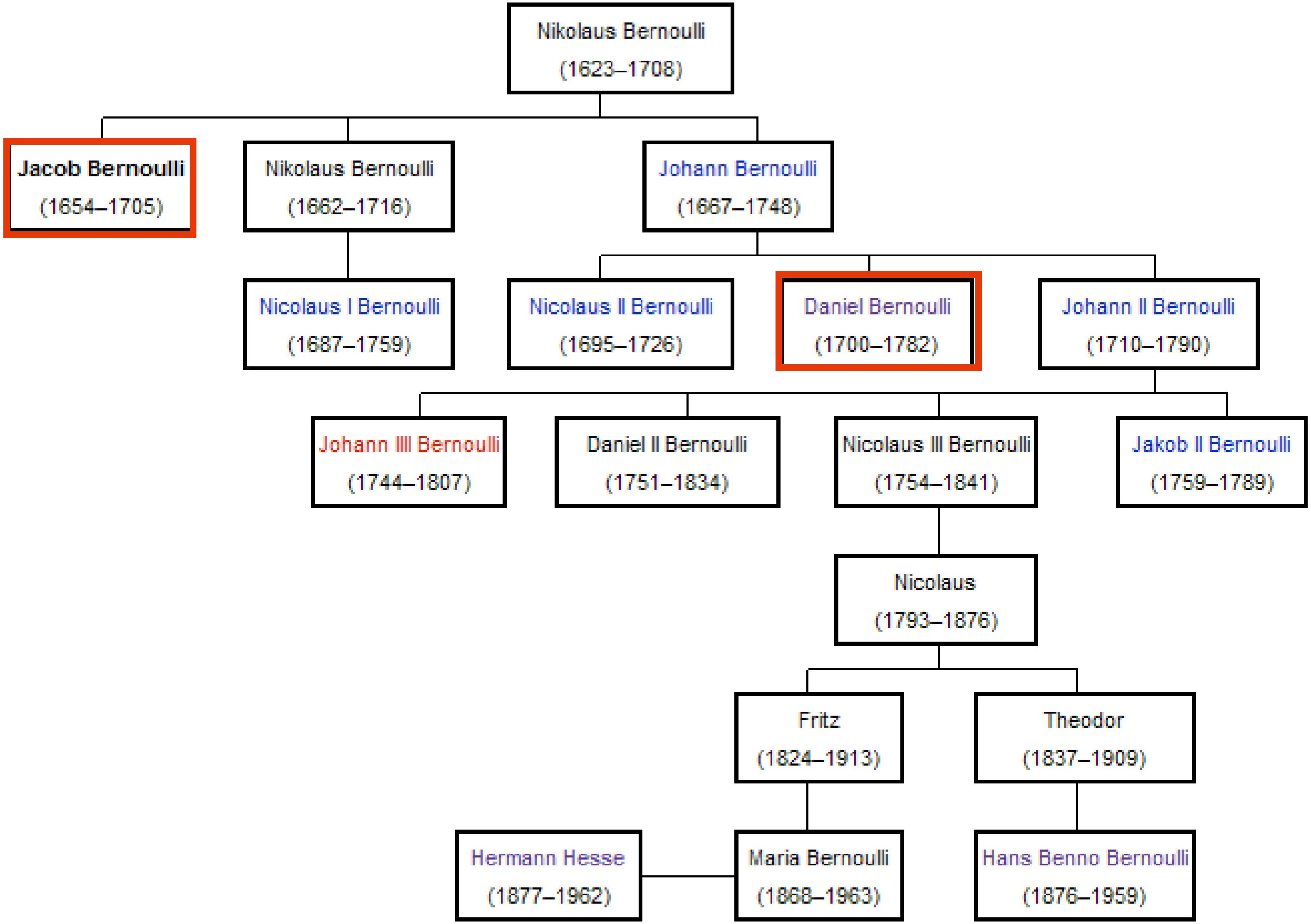
Bar:	Elongation	$\Delta = \frac{PL}{EA}$;
Shaft	Angle of Twist	$\Phi = \frac{TL}{GJ}$
Beam	Deflection	$y = ?$

Leonhard Euler

Jacob Bernoulli

Daniel Bernoulli

Bernoulli family tree

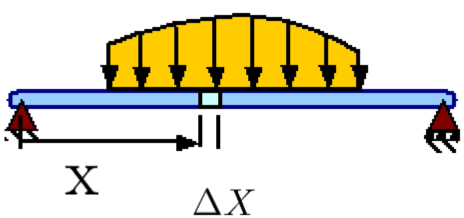


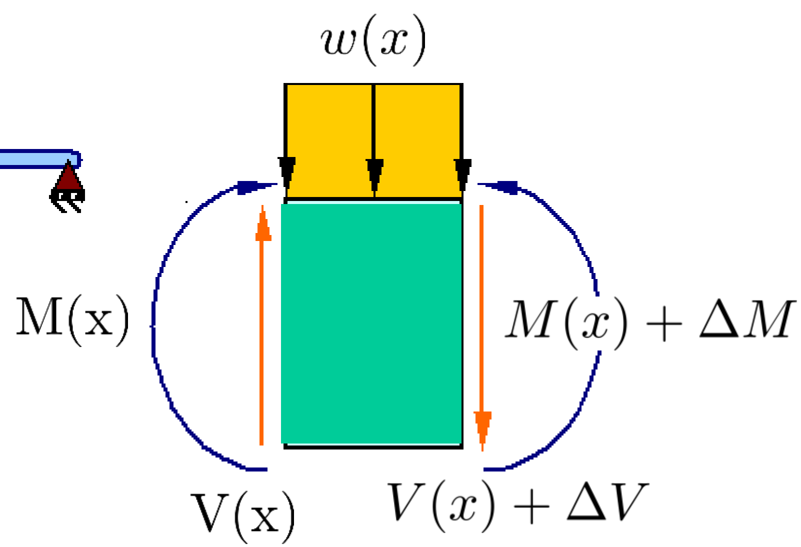
Family tree of the Bernoulli family

Recall: From equilibrium

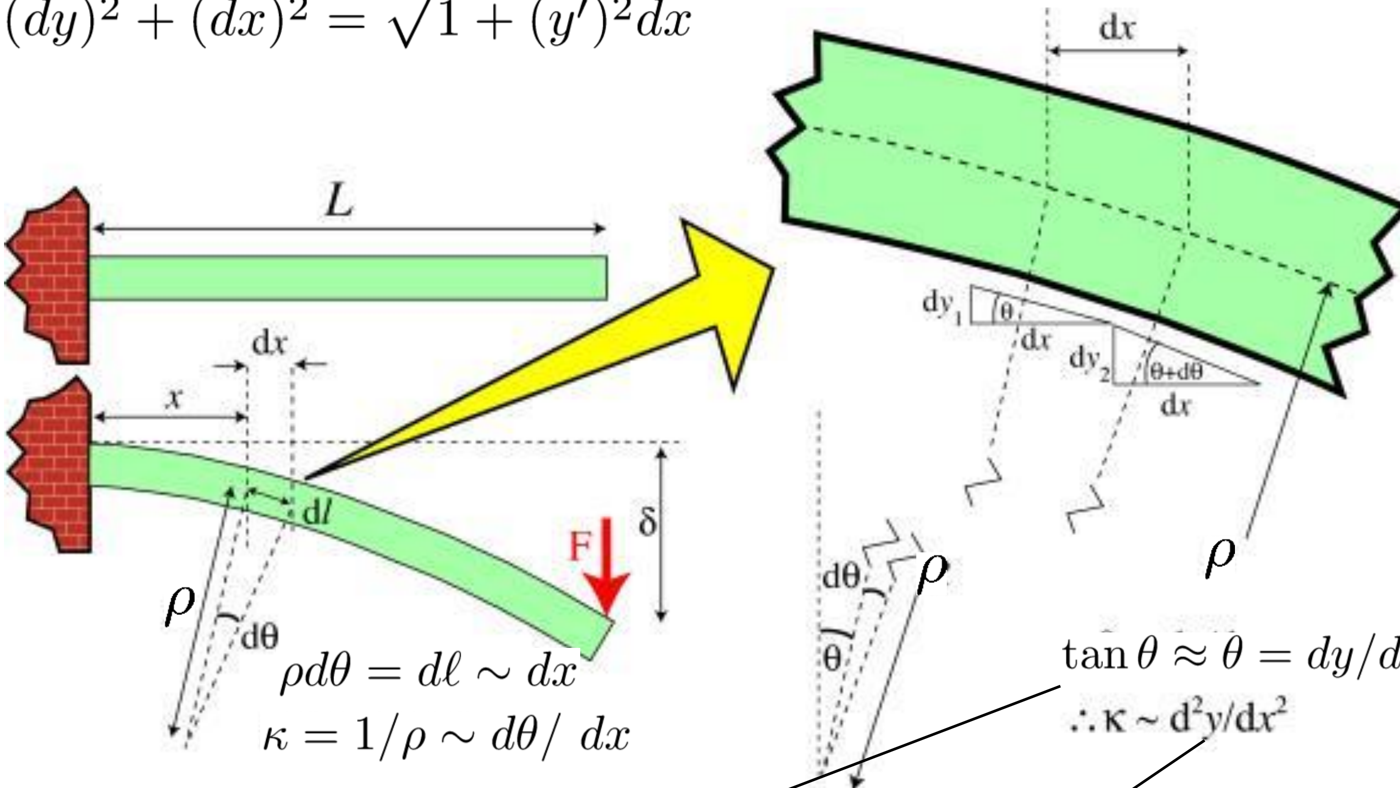
$$\frac{dV}{dx} = -w(x); \qquad \frac{dM}{dx} = V(x);$$

$$\frac{d^2 M}{dx^2} = -w(x);$$





$$d\ell = \sqrt{(dy)^2 + (dx)^2} = \sqrt{1 + (y')^2}dx$$



$$\rho d\theta = d\ell \sim dx$$

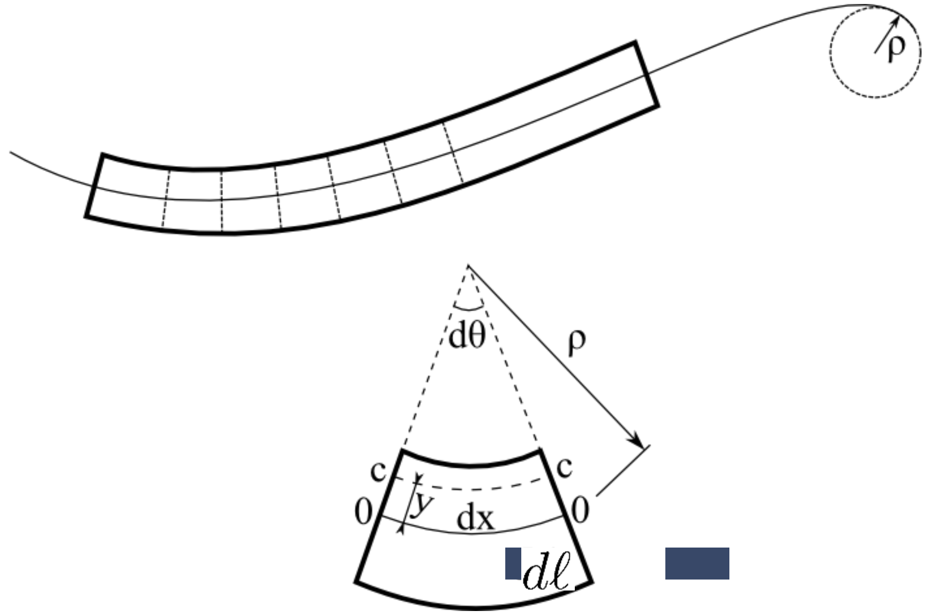
$$\kappa = 1/\rho \sim d\theta/\ dx$$

$$\tan\theta \approx \theta = dy/dx$$

$$\therefore \kappa \sim d^2y/dx^2$$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{d\ell} \approx \frac{d\theta}{dx} \quad \Rightarrow \quad \kappa \approx \frac{d^2y}{dx^2} = \frac{M(x)}{EI}, \qquad (I)$$

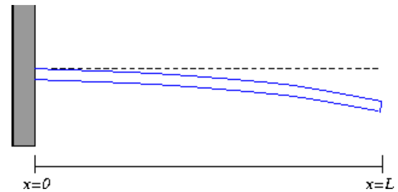
$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = -w(x) \quad \Rightarrow \quad \frac{d^2}{dx^2}(EI\frac{d^2y}{dx^2}) = -w(x), \quad (II)$$

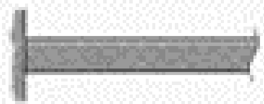
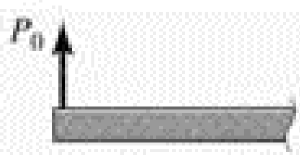
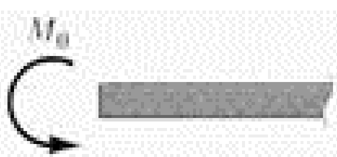


$$\frac{d^2}{dX^2}\left(EI\frac{d^2y}{dX^2}\right) = -w(x), \quad 0 < X < L$$

How to integrate beam deflection ?

Boundary Conditions



Type	Symbol*
Fixed End	 X=0
Simple Support	
Free End	
Concentrated Force	
Concentrated Couple	

$$\mathbf{1}:\ y(0)=0,\ \theta(0)=y'(0)=0;$$

$$\mathbf{2}:\ y(0)=0,\ \kappa(0)=M(0)=EIy''(0)=0;$$

$$\mathbf{3}:\ M(0)=EIy''(0)=0,\ V(0)=EIy'''(0)=0;$$

$$\mathbf{4}:\ M(0)=EIy''(0)=0,\ V(0)=EIy'''(0)=P_0;$$

$$\mathbf{5}:\ M(0)=EIy''(0)=-M_0,\ V(0)=EIy'''(0)=0;$$

Equation of the Elastic Curve (II) $EIy^{(iv)}(x) = -w(x)$:

$$(1) V(x) = EIy'''(x) = \int_0^x -w(t_1)dt_1 + C_1, \text{ so } C_1 = V(0) = EIy'''(0);$$

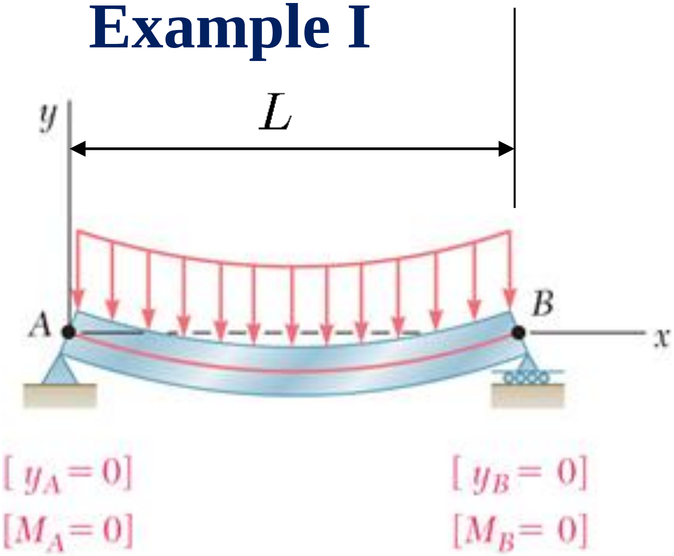
$$(2) \quad M(x) = EIy''(x) = \int_0^x \int_0^{t_2} -w(t_1)dt_1 dt_2 + C_1 x + C_2,$$
$$\text{so} \quad C_2 = M(0) = EIy''(0);$$

$$(3) \quad EI\theta = EIy'(x) = \int_0^x \int_0^{t_3} \int_0^{t_2} -w(t_1)dt_1 dt_2 dt_3 + C_1 \frac{x^2}{2} + C_2 x + C_3,$$
$$\text{so} \quad C_3 = EI\theta(0);$$

$$(4) \quad EIy(x) = \int_0^x \int_0^{t_4} \int_0^{t_3} \int_0^{t_2} -w(t_1)dt_1 dt_2 dt_3 dt_4 + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2}$$
$$+ C_3 x + C_4, \qquad \text{so} \quad C_4 = EIy(0);$$

Remark: In a given problem, one cannot know all four boundary conditions at one end. One can only find two boundary conditions at a given end.

Example I



$$w(x) = w_0 \qquad EIy^{(iv)} = -w_0$$

[Solution]

$$EIy''' = -w_0 x + C_1$$

$$EIy'' = -\frac{w_0 x^2}{2} + C_1 x + C_2, \quad (C_2 = M(0) = 0)$$

Based on $M(L) = 0$,

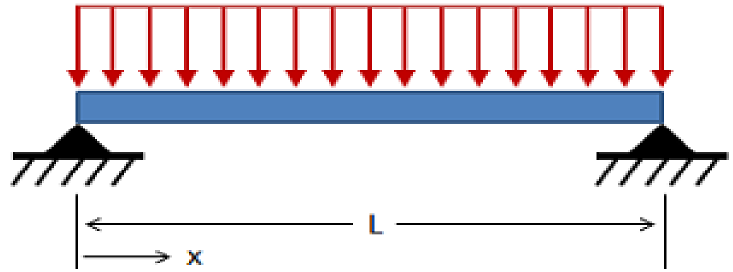
$$\rightarrow \quad -w_0 L^2/2 + C_1 L = 0, \quad \rightarrow \quad C_1 = \frac{1}{2} w_0 L = V(0)$$

$$EIy' = -\frac{w_0 x^3}{6} + \frac{w_0 L x^2}{4} + C_3$$

$$EIy(x) = -\frac{w_0 x^4}{24} + \frac{w_0 L x^3}{12} + C_3 x + C_4 \qquad y(0) = 0, \quad \rightarrow \quad C_4 = 0$$

$$EIy(L) = -\frac{w_0 L^4}{24} + \frac{w_0 L^4}{12} + C_3 L = 0 \quad \rightarrow \quad C_3 = -\frac{w_0 L^3}{24} = \theta(0)$$

$$EIy^{(iv)}(x) = -w_0$$

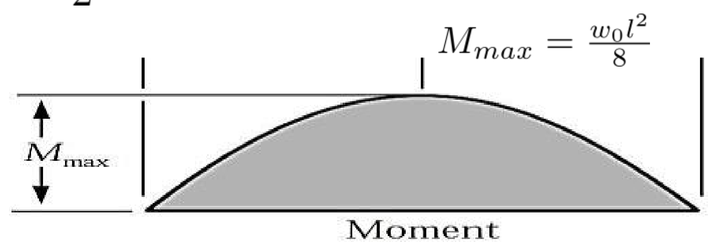


$$EIy'''(x) = V(x) = -w_0 x + \frac{w_0 L}{2}$$

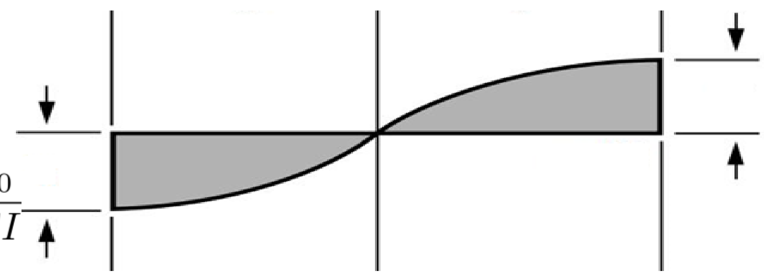
$$-\frac{w_0 l}{2}$$

$$V(0) = \frac{w_0 l}{2}$$

$$EIy''(x) = M(x) = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2}$$



$$y'(x) = \theta(x) = -\frac{w_0}{24EI}\left(4x^3 - 6Lx^2 + L^3\right)$$



$$y(x) = -\frac{w_0}{24EI}\left[x^4 - 2Lx^3 + L^3 x\right]$$