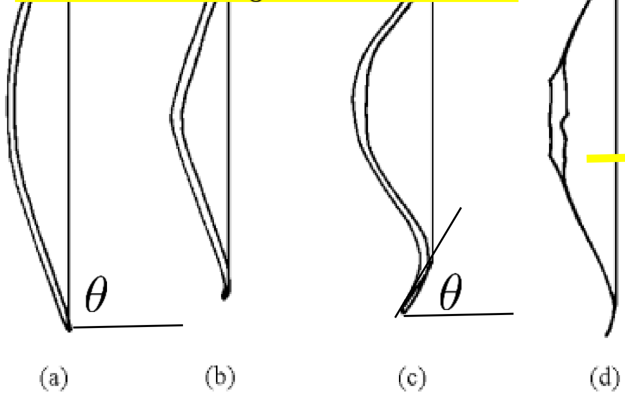
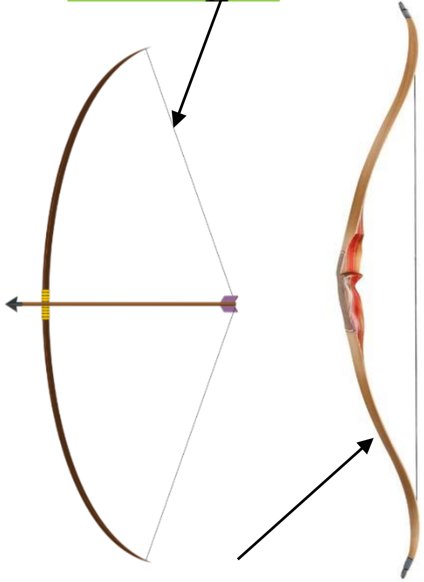
$$U = \frac{1}{EI}\int_0^L M^2(x)dx$$

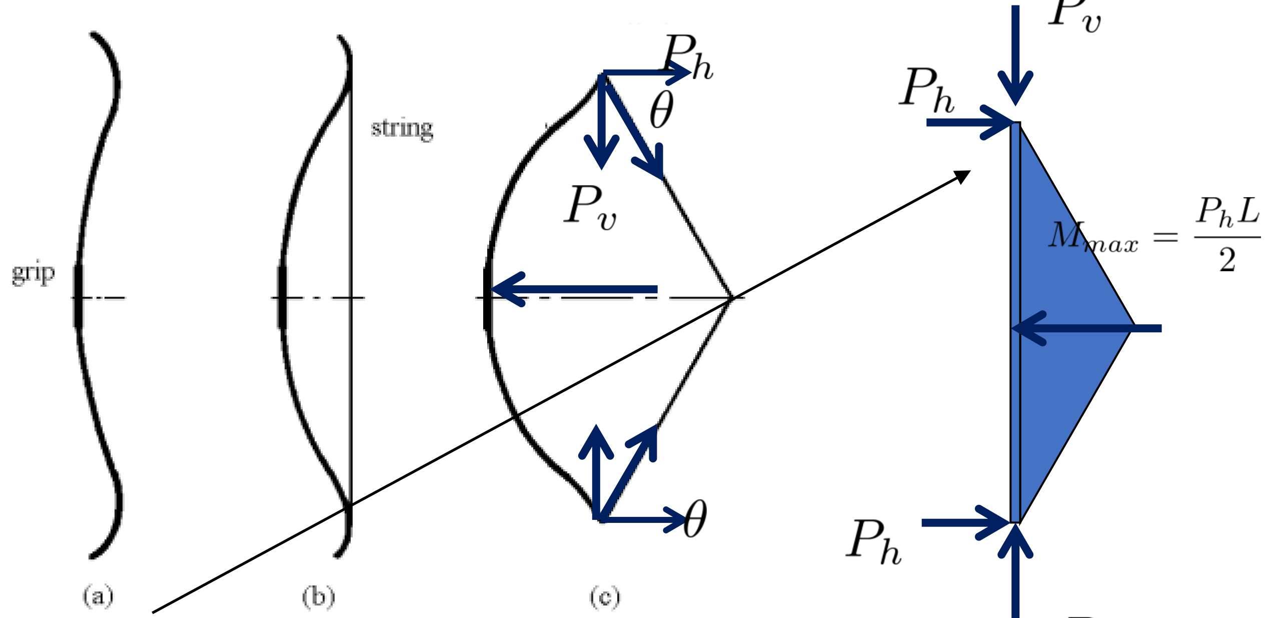


(a) and (b) non-recurve, (c) static-recurve, and (d) working recurve

$$P_v = \frac{4\pi^2 EI}{L^2}$$

$$P_v = \frac{\pi^2 EI}{L^2}$$



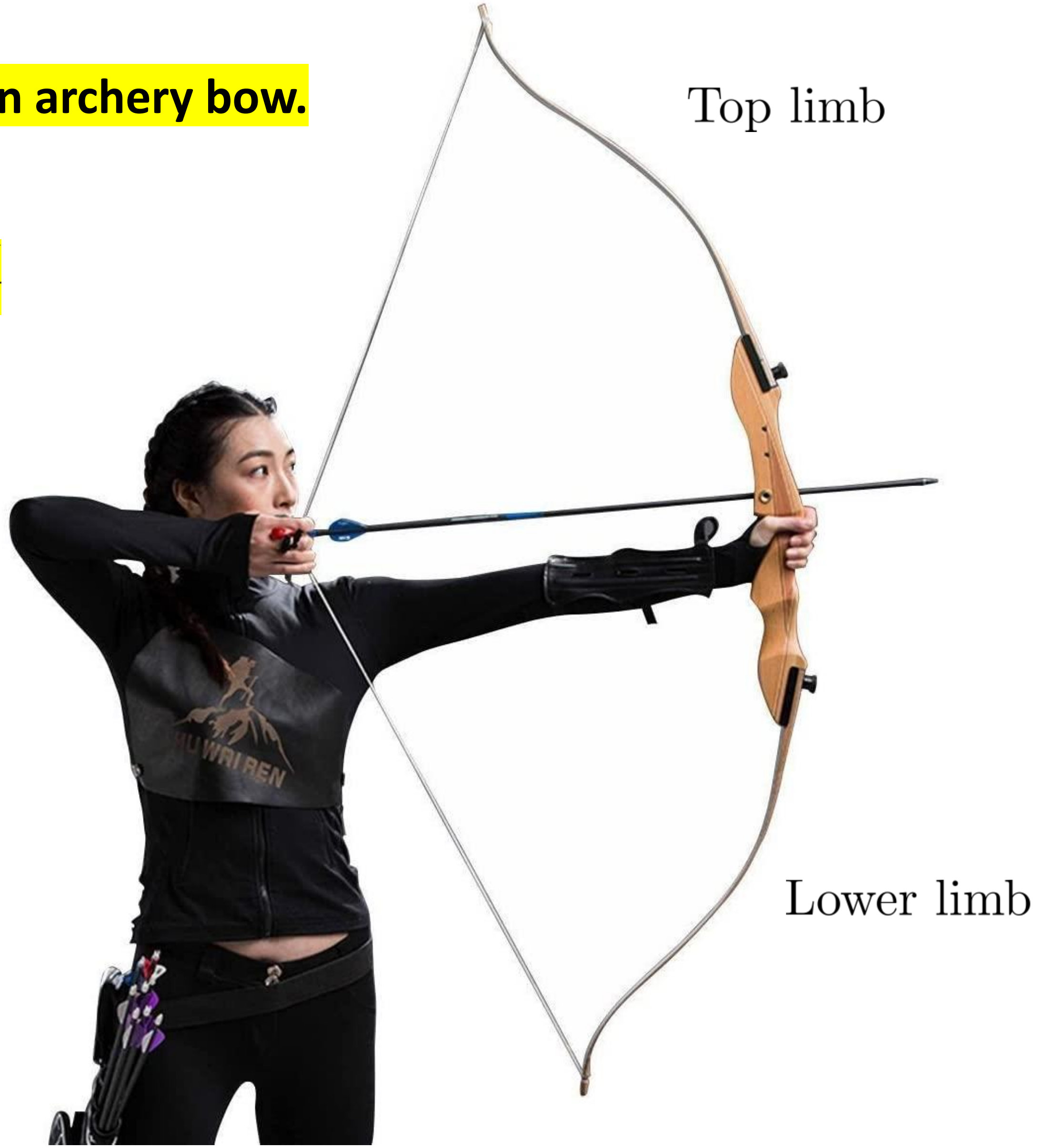


$$P_v = \frac{4\pi^2 EI}{L^2}$$

$P_h = P_v/\tan\theta$, where P_v is fixed by the value of the buckling load.

This is modern archery bow.

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$



Problem 1.

HW 14

As shown in Fig. 1, the Cauchy stress tensor of a plane stress state at a material point is given as

$$\begin{bmatrix} 4 & 4 \\ 4 & -2 \end{bmatrix} \quad MP_a$$

1. Draw its Mohr's circle;
2. Find all the principal stresses and maximum and minimum shear stresses;
3. The corresponding θ_p and θ_s;
4. Use the method of pole to draw the element in the physical space in the principal directions and in the maximum/minimum shear stress directions.

This is a rated-R problem,

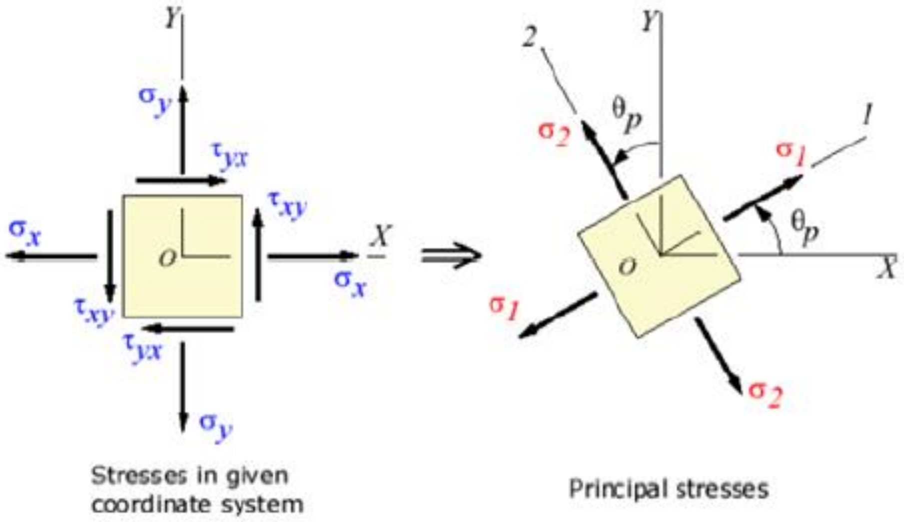
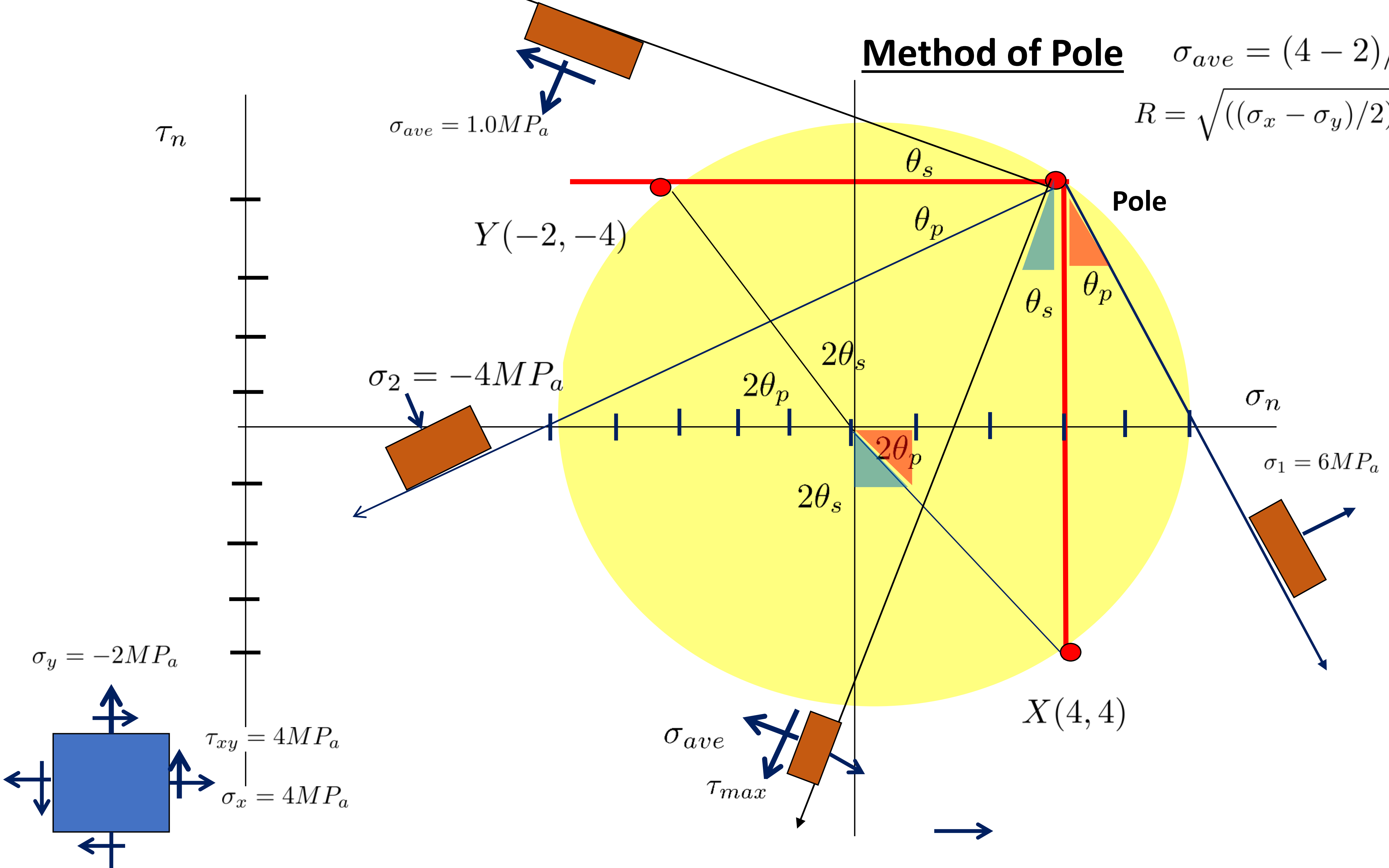


Figure 1: Initial orientation of the infinitesimal element and the element in the principal direction.

Method of Pole

$$\sigma_{ave} = (4-2)/2 = 1MP_a$$

$$R = \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2} = 5MP_a$$



An angle at the circumference of a circle is half the angle at the center **standing on the same arc.**

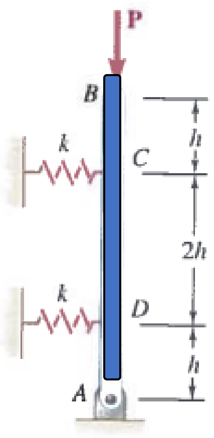
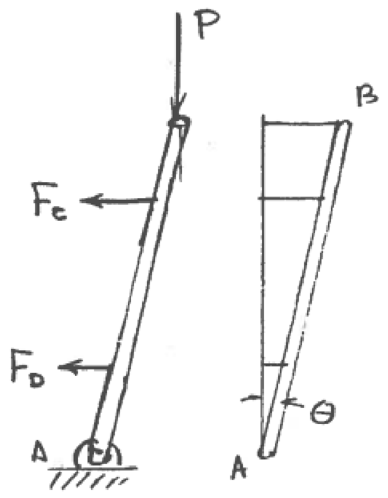


Fig. P16.5

16.5 The rigid rod AB is attached to a hinge at A and to two springs, each of constant $k = 2$ kips/in., that can act in either tension or compression. Knowing that $h = 2$ ft, determine the critical load.

SOLUTION

Let θ be the small rotation angle.



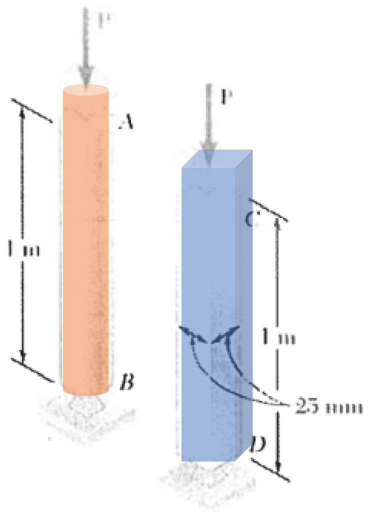
$$x_D \approx h\theta$$
$$x_C \approx 3h\theta$$
$$x_B \approx 4h\theta$$
$$F_C = kx_C \approx 3kh\theta$$
$$F_D = kx_D \approx kh\theta$$
$$+\circlearrowleft \Sigma M_A = 0: \quad hF_D + 3hF_C - Px_B = 0$$
$$kh^2\theta + 9kh^2\theta - 4hP\theta = 0, \quad P = \frac{5}{2}kh$$

Data: $\quad k = 2.0$ kip/in. $\quad h = 2$ ft $= 24$ in.

$$P = \frac{5}{2}(2.0)(24)$$

$P = 120.0$ kips ◀

Square strut has a higher critical load.



PROBLEM 16.11

Determine the radius of the round strut so that the round and square struts have the same cross-sectional area and compute the critical load for each. Use $E = 200$ GPa.

For square strut, $A = 25^2 = 625 \text{ mm}^2$

$$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$$

For round strut, $\frac{1}{4}\pi d^2 = A$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(625)}{\pi}} = 28.2 \text{ mm}$$

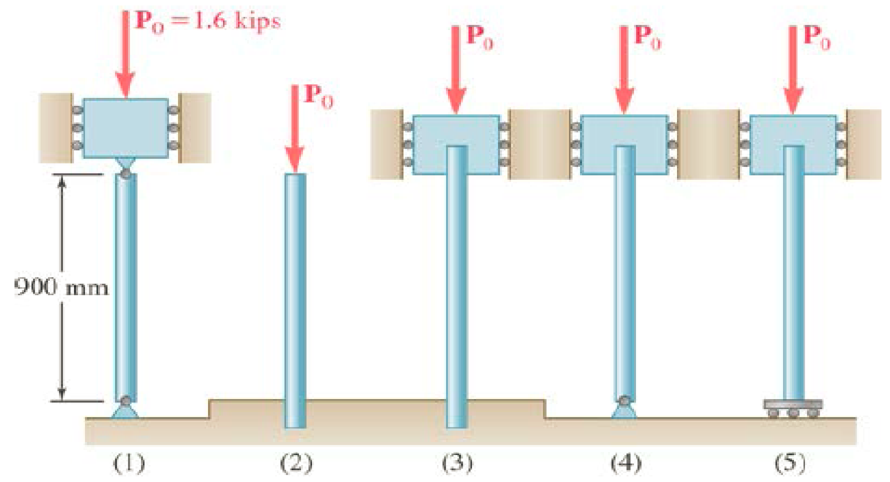
$$c = \frac{1}{2}d = 14.10 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = 31.085 \times 10^3 \text{ mm}^3 = 31.085 \times 10^{-9} \text{ m}^4$$

Critical loads: $P_{cr} = \frac{\pi^2 EI}{L^2}$

Round strut. $P_{cr} = \frac{\pi^2(200 \times 10^9)(31.085 \times 10^{-9})}{(1)^2} = 61.4 \times 10^3 \text{ N}$

Square strut. $P_{cr} = \frac{\pi^2(200 \times 10^9)(32.552 \times 10^{-9})}{(1)^2} = 64.3 \times 10^3 \text{ N}$



PROBLEM 16.24

Each of the five struts shown consists of a solid steel rod. (*a*) Knowing that the strut of Fig. (1) is of a 0.8-in. diameter, determine the factor of safety with respect to buckling for the loading shown. (*b*) Determine the diameter of each of the other struts for which the factor of safety is the same as the factor of safety obtained in part Use $E = 29 \times 10^6$ psi.

Solid circular cross section: $c = \frac{1}{2}d = 0.40$ in.

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.40)^4 = 20.106 \times 10^{-3}\ \text{in}^4$$

$L_e = L$

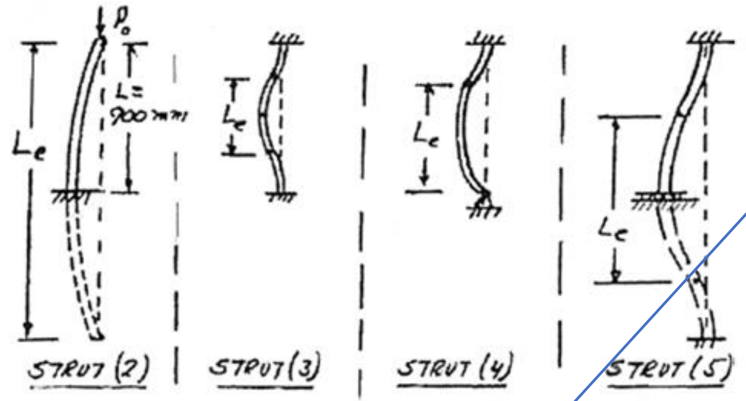
For strut (1), $L_e = L = 36$ in. $\quad E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$$EI = (29 \times 10^3)(20.106 \times 10^{-3}) = 583.07\ \text{kip} \cdot \text{in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(583.07)}{(36)^2} = 4.4403\ \text{kips}$$

(*a*) $$F.S. = \frac{P_{cr}}{P_0} = \frac{4.4403\ \text{kips}}{1.6\ \text{kips}}$$

F.S. $= 2.78$



$L_e = 2L \quad L_e = L/2 \quad L_e = 0.7L \quad L_e = ?$

(*b*) For the same factor of safety, the struts must have the same critical load.

$$P_{cr} = \frac{\pi^2 EI_i}{L_i^2} \quad \text{where } i = 1, 2, 3, 4, \text{ and } 5$$

For $i = 2, 3, 4$, and 5, $\quad \frac{I_i}{L_i^2} = \frac{I_1}{L_1^2} \quad$ or $\quad \frac{I_i}{I_1} = \frac{L_i^2}{L_1^2}$

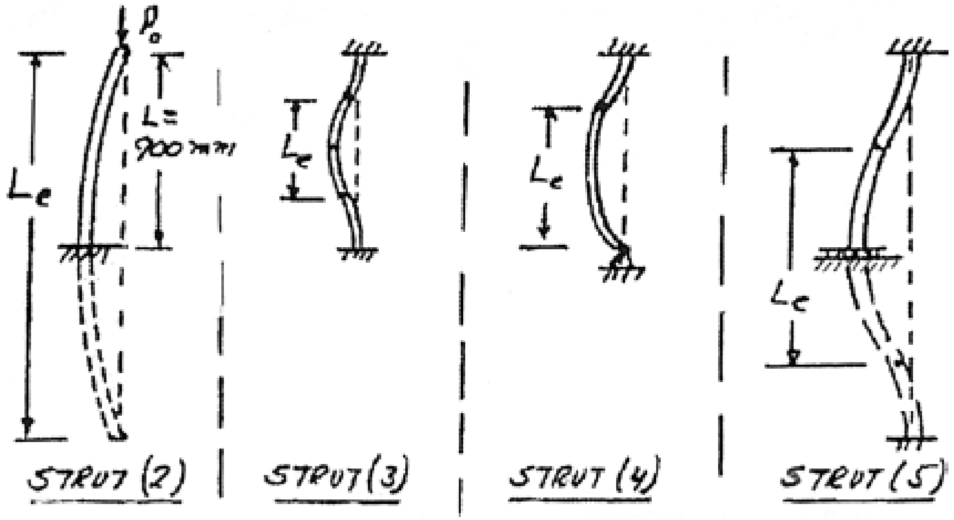
$$\frac{I_i}{I_1} = \frac{L_i^2}{L_1^2}$$

Since I is proportional to d^4, $\quad \frac{d_i^4}{d_1^4} = \frac{L_i^2}{L_1^2}$

$$P_{cr}^i = P_{cr}^1 = 2.78P_0, \quad i = 2, 3, 4, 5$$

or $\quad \frac{d_i}{d_1} = \sqrt{\frac{L_i}{L_1}}$, where L_i is the effective length.

$L_e = L$ $\quad L_e = 2L$ $\quad L_e = L/2$ $\quad L_e = 0.7L$ $\quad L_e = L$



<u>Strut (1):</u> $L_1 = 36$ in. $\quad d_1 = 0.800$ in. $\quad 900\ mm \rightarrow 35.4in$

<u>Strut (2):</u> $L_2 = 2L_1 = 72$ in. $\quad (L_e = 2L)$

$$\frac{d_2}{0.80} = \sqrt{\frac{72}{36}} \qquad \frac{d_i}{d_1} = \sqrt{\frac{L_i}{L_1}}. \qquad d_2 = 1.131 \text{ in.} \blacktriangleleft$$

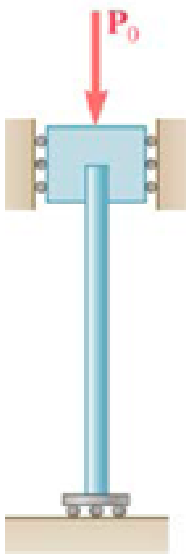
<u>Strut (3):</u> $L_3 = \frac{1}{2}L_1 = 18$ in. $\quad (L_e = L/2)$

$$\frac{d_3}{0.80} = \sqrt{\frac{18}{36}} \qquad d_3 = 0.566 \text{ in.} \blacktriangleleft$$

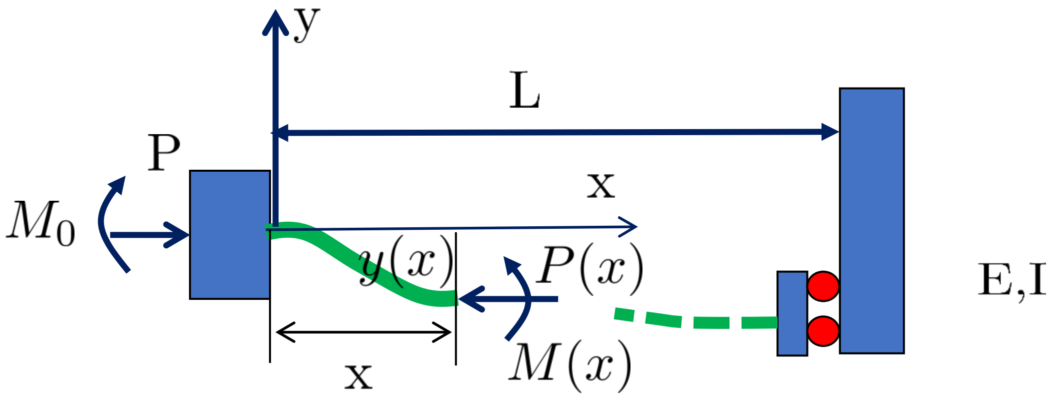
<u>Strut (4):</u> $L_4 = 0.669 \quad L_1 = 629.1$ mm $\quad (L_e = 0.7L)$

$$\frac{d_4}{0.80} = \sqrt{\frac{25.2}{36}} \qquad d_4 = 0.669 \text{ in.} \blacktriangleleft$$

<u>Strut (5):</u> $L_5 = L_1 = 36$ in. $\quad (L_e = L)$ $\qquad d_5 = 0.800$ in. $\blacktriangleleft$



(5)



This problem is rated R

$$y(0) = 0;$$
$$y'(0) = 0;$$
$$y'(L) = 0;$$

$$\sum M_z \Big|_{@x} = 0 \quad \rightarrow \quad M(x) - M_0 - P|y(x)| = 0$$

$$EIy''(x) + Py(x) = M_0 \quad \rightarrow \quad y''(x) + \lambda^2 y(x) = \frac{M_0}{EI}, \quad \lambda^2 = \frac{P}{EI}$$

$$y(x) = y_h(x) + y_p(x), \quad y_h(x) = A\sin\lambda x + B\cos\lambda x, \quad y_p(x) = \frac{M_0}{P};$$

$$y(x) = A\sin\lambda x + B\cos\lambda x + \frac{M_0}{P};$$

$$y'(x) = A\lambda\cos\lambda x - B\lambda\sin\lambda x;$$

$$y'(0) = 0 \rightarrow A = 0$$

$$y'(L) = 0 \rightarrow \sin\lambda L = 0$$

$$\rightarrow \lambda L = \pi$$

$$\rightarrow P_{cr} = \frac{\pi^2 EI}{L}$$

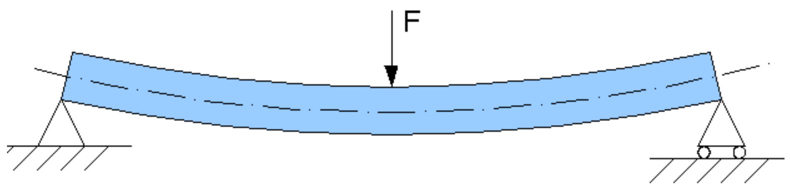
$$\rightarrow L_e = L$$

Today's Lecture Passcode is: Million Dollar Questions

Question 1

For a simply supported beam whose middle section is subjected to a vertical load F.
Which section of the beam has the maximum bending moment?

(A) At the right boundary.
(B) At the left boundary.
(C) In the middle section of the beam.
(D) Every section has the same bending moment

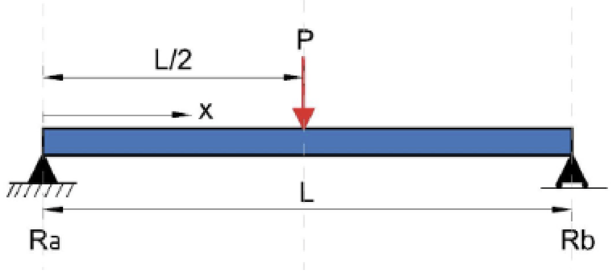


Ans. (C)

Question 2

For a simply supported beam whose middle section is subjected to a vertical load,
Which section of the beam has the maximum shear force?

(A) At the left end of the beam.
(B) At the right end of the beam.
(C) Every section has the same amount of shear force.
(D) In the middle section of the beam.

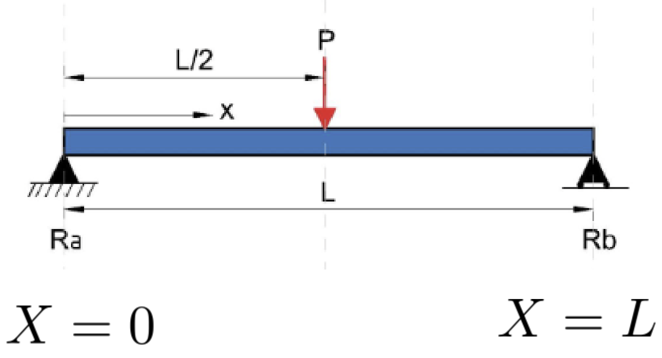


Ans. (C)

Question 3

For a simply supported beam whose middle section is subjected to a vertical load P,
What is the value of the bending moment at x=0, i.e. M(0)?

(A) PL.
(B) 0.
(C) 0.5PL.
(D) 0.25PL.

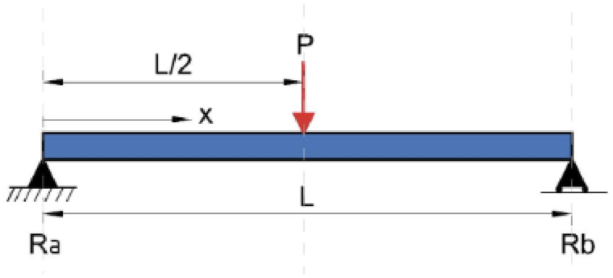


Ans. (B)

Question 4

For a simply supported beam whose middle section is subjected to a vertical load P,
What is the value of the bending moment at x=L, i.e. M(L)?

(A) PL.
(B) 0.5 PL.
(C) 0.
(D) 0.25PL.

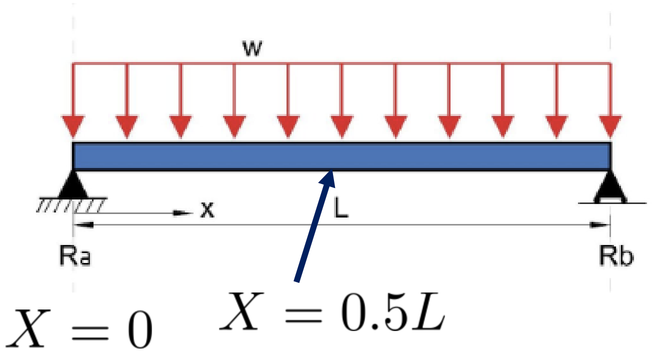


Ans. (C)

Question 5

For a simply supported beam that is subjected uniform load distribution w,
What is the value of the shear force at X=0.5L?

(A) wl
(B) 0.
(C) 0.5 wl.
(D) 0.25 wl.

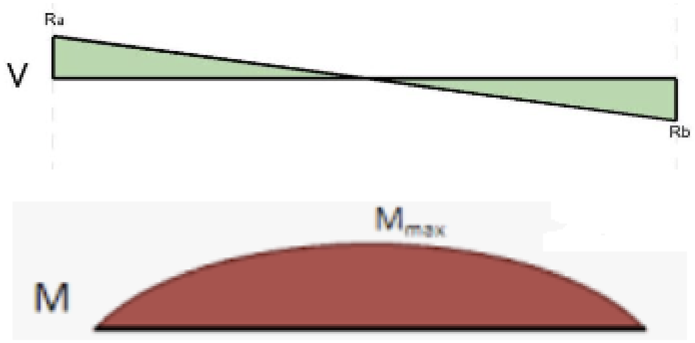


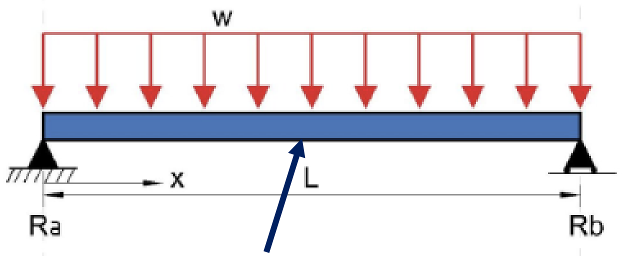
Ans. (B)

Question 6

For a simply supported beam that is subjected uniform load distribution w,
Which section has the maximum bending moment?

(A) X=0.
(B) X=0.25L.
(C) X=0.5L.
(D) X=L.

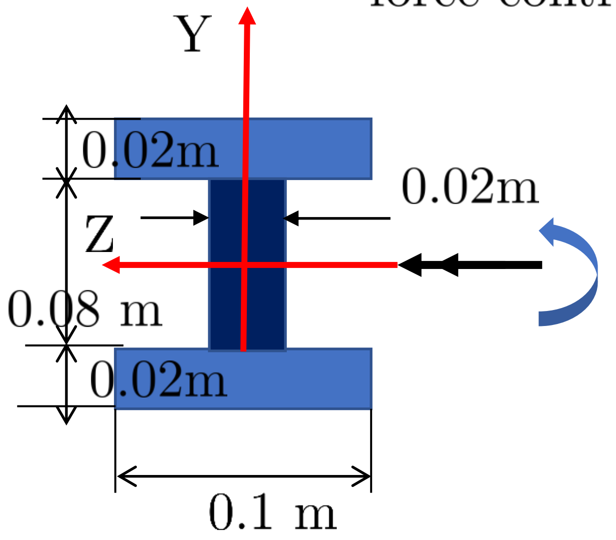




$X = 0$ $X = 0.5L$

Ans. (C)

Q7. An I-beam under pure bending. Assume that the internal moment at the cross section is 10 N-m, what is the net axial force contribution from the web section ?



a. $F_x = 10N$;

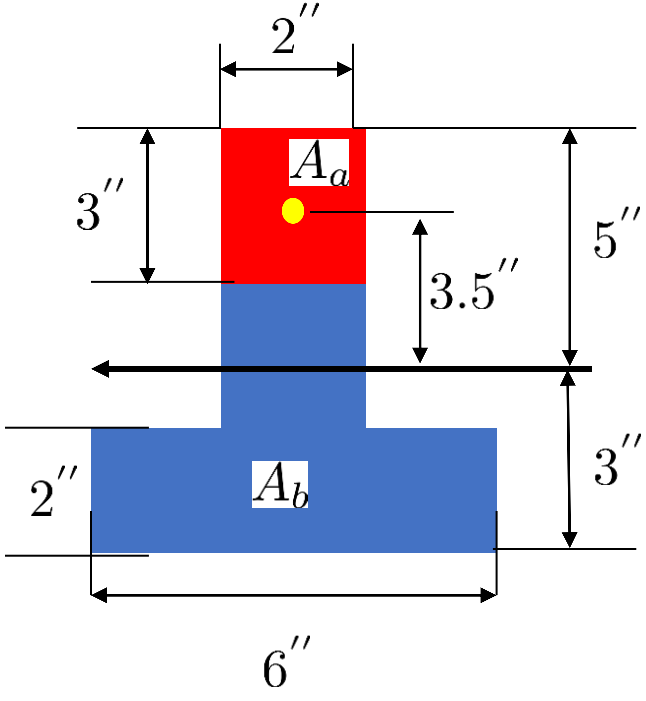
b. $F_x = -10N$;

c. $F_x = 0$;

d. $F_x = 20N$, and

e. None of above.

Ans: (c)



Q8. Which of the following is the correct first moment Q_b for the shaped area A_b ?

A $Q_b = 21\ in^3$

B $Q_b = 24\ in^3$

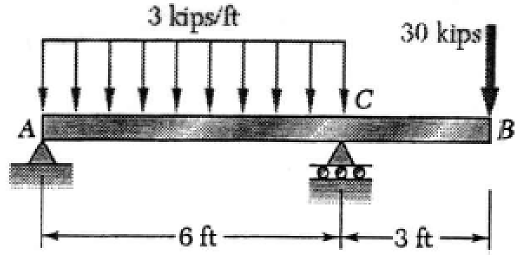
C $Q_b = -24\ in^3$

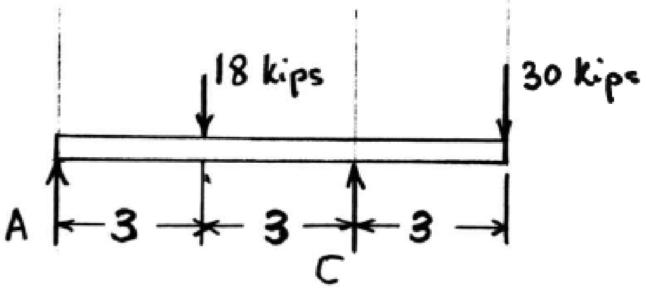
D $Q_b = -21 in^3$

Ans: (D)

Question 9.

12.7 and *12.8* Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.





Reactions

$$\frac{dV}{dx} = -w(x)$$

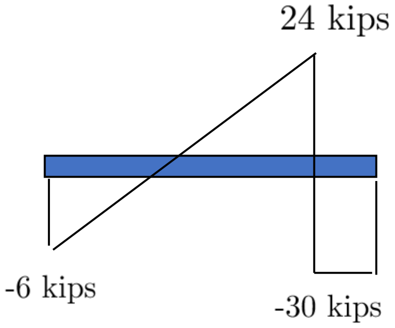
$\circlearrowleft \Sigma M_c = 0: \quad -6A + (3)(18) - (3)(30) = 0$

$A = -6$ kips ie. 6 kips $\downarrow$

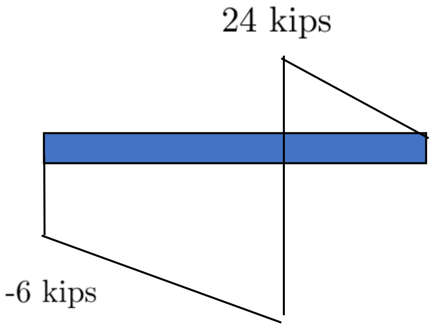
$\circlearrowleft \Sigma M_A = 0: \quad 6C - (3)(18) - (9)(30) = 0$

$C = 54$ kips $\uparrow$

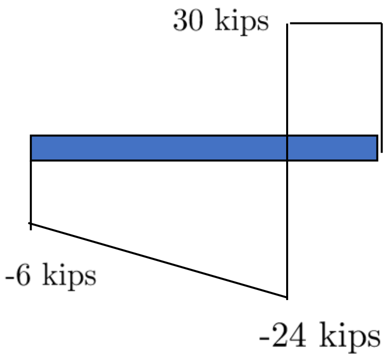
Which is the correct shear diagram ?



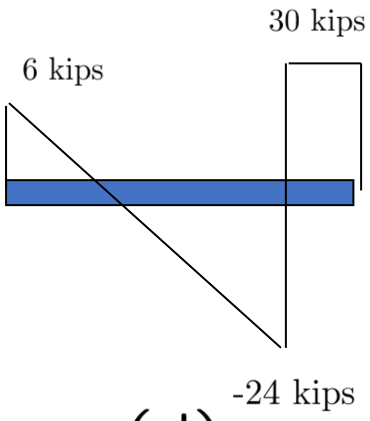
(a)



(b)



(c)

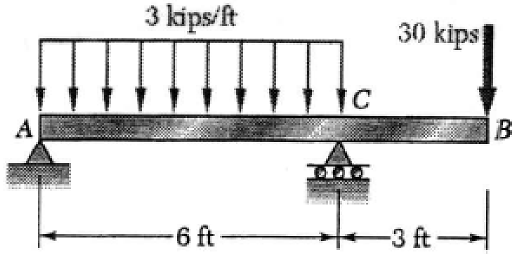


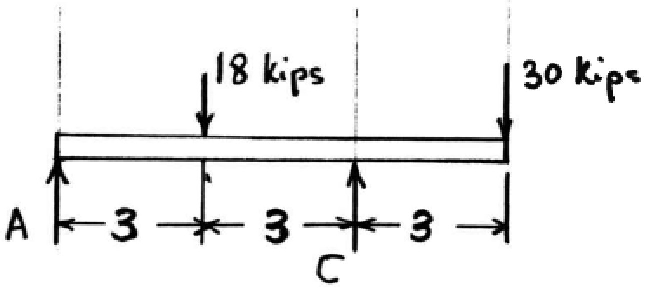
(d)

Ans (c)

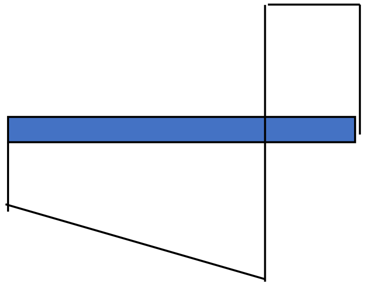
Question 10.

12.7 and *12.8* Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.





$$\frac{dM}{dx} = V(x), \quad \frac{d^2M}{dx^2} = -w(x)$$



Reactions

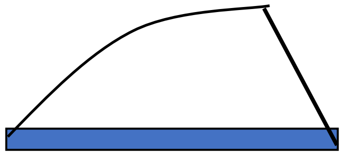
$\circlearrowleft \Sigma M_C = 0: \quad -6A + (3)(18) - (3)(30) = 0$

$A = -6$ kips ie. 6 kips ↓

$\circlearrowleft \Sigma M_A = 0: \quad 6C - (3)(18) - (9)(30) = 0$

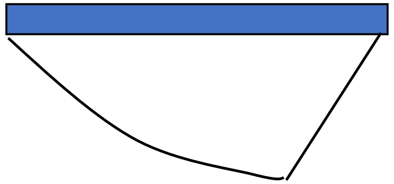
$C = 54$ kips ↑

Which is the correct moment diagram ?



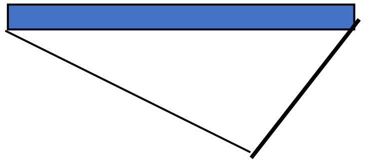
-90 kips-in

(a)



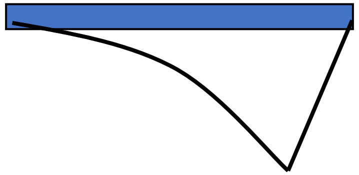
-94 kips-in

(b)



-54 kips-in

(c)



-90 kips-in

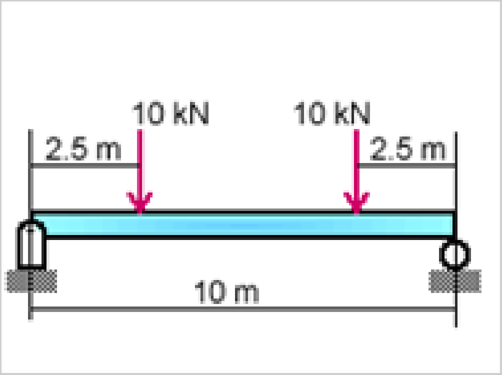
(d)

Ans: (d)

Question 11

6

Consider the simply loaded and supported beam shown here. The shear at the center of this beam is:



A) 0 kN

B) 10 kN

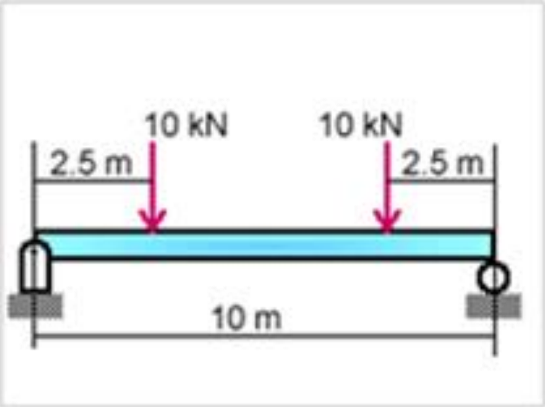
C) 20 kN

D) 50 kN

Ans: (A)

Question 12

Consider the beam loaded and supported as shown here. The maximum bending moment is:



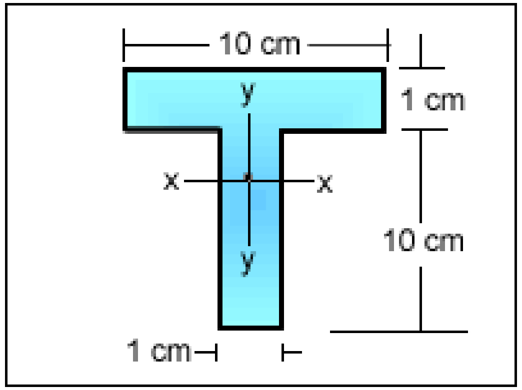
- A) 0 kN-m
- B) 10 kN-m
- C) 25 kN-m
- D) 50 kN-m

Ans: (c)

Question 13

1

A column has the cross-section shown here. If buckling occurs, it will occur about which axis?



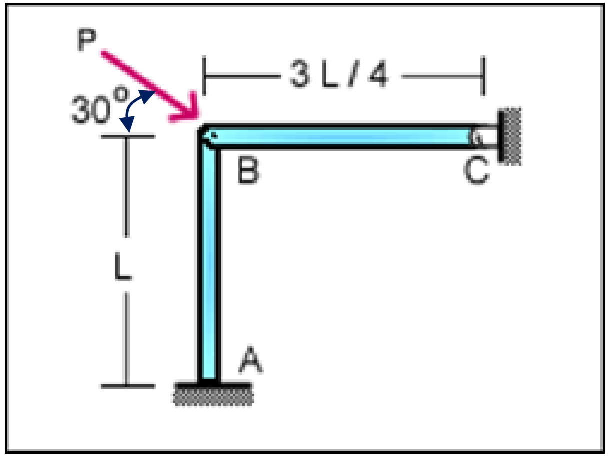
- A) x-x
- B) y-y
- C) not applicable
- D) not applicable

Ans: (B)

Question 14

5

The members of the structure shown here identical cross-sections. As P increases, which member, AB or BC, will fail first by buckling? Under the same force $\mathbf{P}$



Ans: (B)

- **A)** AB
- **B)** BC
- **C)** not applicable
- **D)** not applicable

$$L^e_{BC} = 3L/4, \quad L^e_{AB} = 0.7L$$

$$P_{BC} = \sqrt{3}P/2, P_{AB} = P/2$$

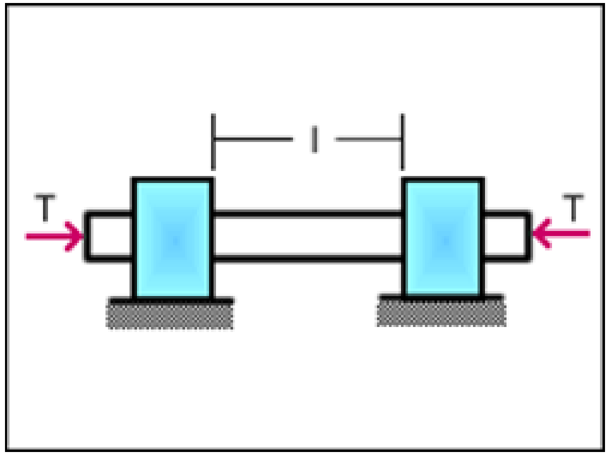
$$P = 2P^{AB}_{cr} = 2\frac{\pi^2 EI}{(0.7L)^2} = 4.08P_{cr}$$

$$P = \frac{2}{\sqrt{3}}P^{BC}_{cr} = \frac{2}{\sqrt{3}}\frac{\pi^2 EI}{(0.75L)^2} = 2.05P_{cr}$$

Question 15

10

A 4-cm-diameter, steel (E = 200 GPa) shaft in a steam turbine mounted between two bearings separated by $1.0m$ is subjected to a 10 kN thurst force. What is the critical Euler's stress in the shaft?



Ans: (B)

$(\mathbf{A})\ 3.16GP_a;$
$(\mathbf{B})\ 0.19GP_a;$
$(\mathbf{C})\ 0.578GP_a;$
$(\mathbf{D})\ 0.437GP_a$

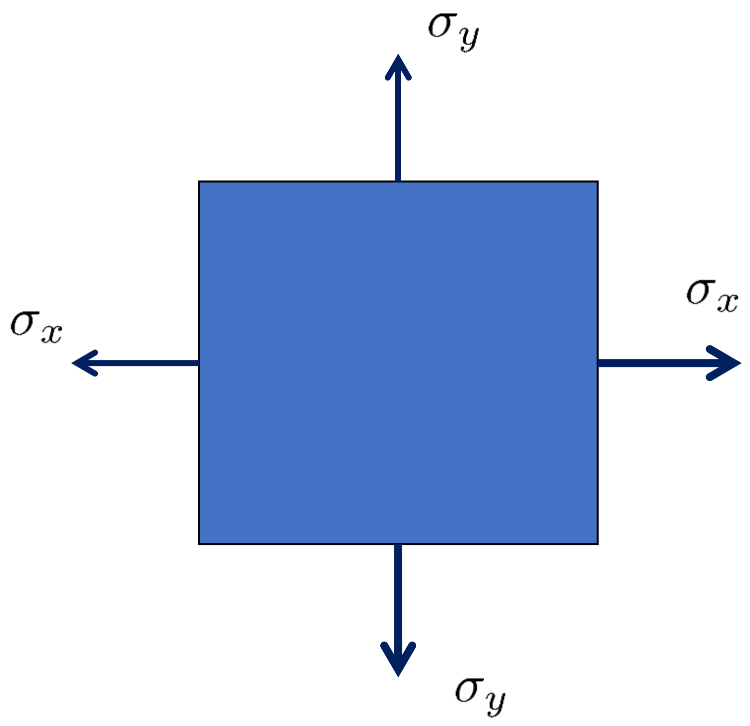
Hints:

$$r = \frac{R}{2}, \quad P_{cr} = \sigma_{cr} A, \text{ and } \sigma_{cr} = \frac{\pi^2 E}{(\ell/r)^2};$$

Question 16

The figure shows the state of plane stress at certain point in a stressed body. The magnitudes of normal stresses in x and y direction are $\sigma_x = 100MP_a$, $\sigma_y = 20MP_a$ and $\tau_{xy} = 0$. What is the maximum shear stress ?

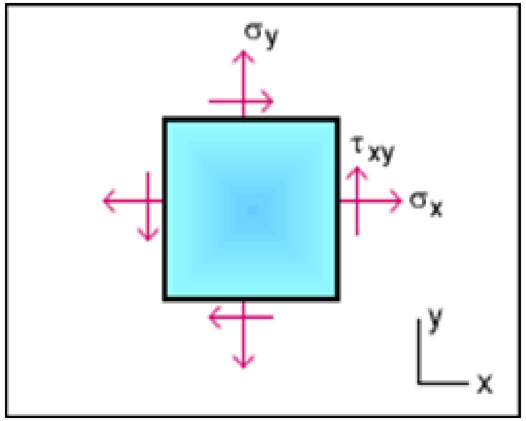
- (A) $120MP_a$
- (B) $80MP_a$;
- (C) $60MP_a$;
- (D) $40MP_a$.



Ans: (D)

Question 17

The general stress element shown is loaded such that σ_x = 20 MPa, σ_y = -20 MPa, τ_{xy} = 10 MPa. What is the orientation of the principal stress plane with respect to the x-axis?



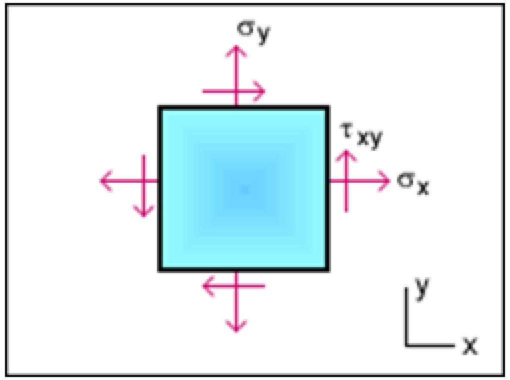
$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

- **A)** 26.6 º CW
- **B)** 13.3 º CW
- **C)** 13.3 º CCW
- **D)** 26.6 º CCW

Ans: (c)

Question 18

The general stress element shown is loaded such that σ_x = 20 MPa, σ_y = -20 MPa, τ_{xy} = 10 MPa. What is the maximum shear stress acting on this element?



$$R = \sqrt{(\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$$

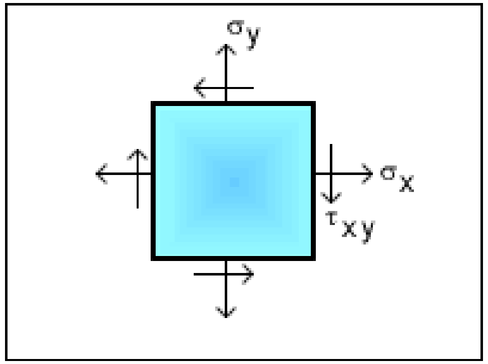
- **A)** 22.4 MPa
- **B)** 63.2 MPa
- **C)** 80.7 MPa
- **D)** 108 MPa

Ans: (A)

Question 19

9

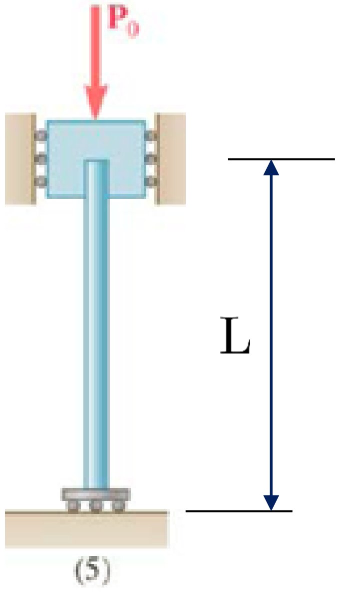
What is the maximum shear stress acting on the stress element shown here when σ_x = 10 MPa, σ_y = 5 MPa, and τ_{xy} = 5 MPa?



$$R = \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$$

- A) 3.74 MPa
- B) 5.61 MPa
- C) 7.71 MPa
- D) 9.32 MPa

Ans: (B)



Q20. Consider the column in the figure.
What is the effective length of the column?

(A) $L_e = 0.7L$;

(B) $L_e = 2L$;

(C) $L_e = L$;

(D) $L_e = 0.5L$;

Ans: (C)

Q21. Two beams, one of circular cross-sections and the other of square cross-sections, have equal areas of cross-sections and the same length. If both beams are used as the simply supported column, which beam or column has the higher critical load?

A. Both beams have the same critical load;

B. Square beam has the higher critical load;

C. Circular beam has the higher critical load;

D. None of above.

Ans: (B)

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad I = \begin{cases} \frac{a^4}{12} & \rightarrow \quad a = \sqrt{A} \rightarrow I = \frac{A^2}{12} \\ \frac{\pi R^4}{4} & \rightarrow \quad R = \sqrt{A/\pi} \rightarrow I = \frac{A^2}{4\pi} \end{cases}$$

Q22. Two beams, one of circular cross-sections and the other of square cross-sections, have equal areas of cross-sections. When both beams are subjected to pure bending, i.e. having the same bending moment, which of the following statements is correct?

Ans: (C)

A. Both beams have the same maximum tensile stress;

B. Square beam has the higher maximum tensile stress;

$$\sigma_{max} = \frac{M}{S}$$

C. Circular beam has the higher maximum tensile stress;

D. None of above.

$$S = \frac{I}{c} = \begin{cases} \frac{a^4}{12}/(a/2) & \rightarrow \quad S \rightarrow \frac{A^{3/2}}{6} \\ \frac{\pi R^4}{4}/R & \rightarrow \quad S \rightarrow \frac{A^{3/2}}{4\sqrt{\pi}} \end{cases}$$

Q23. Two sections, one is a circular section and another is a square section, have equal areas of cross-sections. When both are subjected to the same shear force, which of the following statements is correct?

A. Both beams have the same maximum shear stress;

B. Square beam has the higher maximum shear stress;

C. Circular beam has the higher maximum shear stress;

D. None of above.

Ans: (B)

$$\tau = \frac{VQ}{It}, \; I = \begin{cases} \frac{A^2}{12} \\ \\ \frac{A^2}{4\pi} \end{cases} \; Q = \begin{cases} \frac{a^3}{8} \to \frac{A^{3/2}}{8} \\ \\ \frac{2R^3}{3} \to 2\left(A/\pi\right)^{3/2}/3 \end{cases} \; t = \begin{cases} a = \sqrt{A} \\ 2R = 2\sqrt{A/\pi} \end{cases}$$

$$\tau = \frac{VQ}{It} = \begin{cases} \frac{3}{2}\frac{V}{A} \\ \\ \frac{4}{3}\frac{V}{A} \end{cases}$$

Q24. A body is subjected to a tensile stress $\sigma_x = 1200MPa$ on the vertical plane and the tensile stress $\sigma_y = 600MPa$ on the horizontal plane at right angles to the former. It is also subjected to the shear stress $\tau_{xy} = 400MPa$ on the same planes. The maximum normal stress will be:

$$\sigma_1 = \sigma_{ave} + R$$

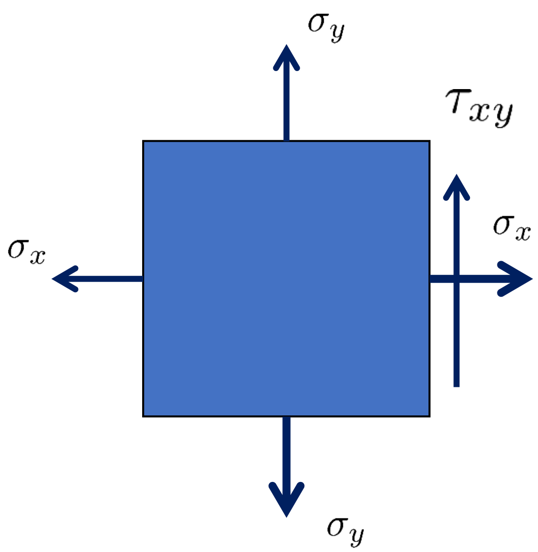
A. 400 MPa

B. 800 MPa

C. 900 MPa

D. 1400 MPa

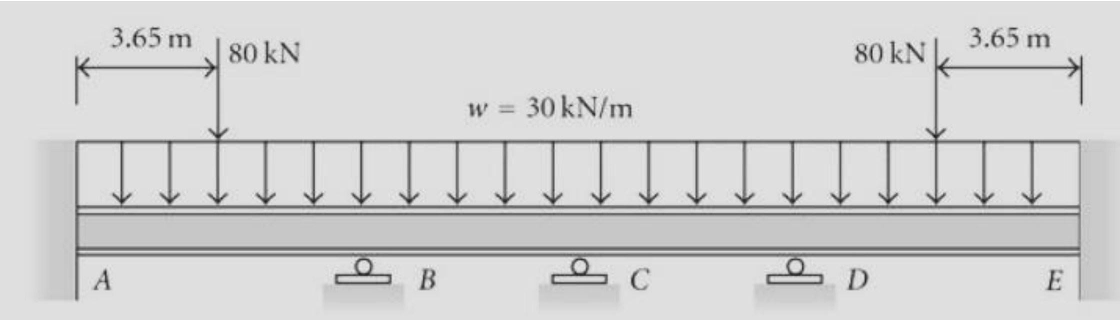
Ans: (D)



Q25.

A beam supported on more than two supports is called

A) Simply supported beam
B) Fixed-beam
C) Overhanging beam
D) Continuous beam



Ans. (D)

Q26. A body is subjected to a tensile stress σ_x on the vertical plane and it is also subjected to the shear stress $\tau_{xy} = 400 MPa$ on the same planes. The maximum normal stress will be:

$\sigma_y = 0$

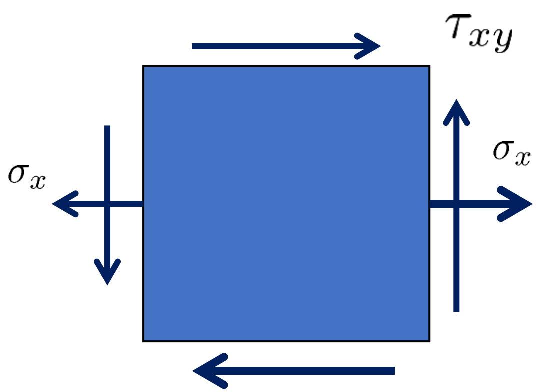
A. $\sigma_x/2 + \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2}$

B. $\sigma_x/2 - \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2}$

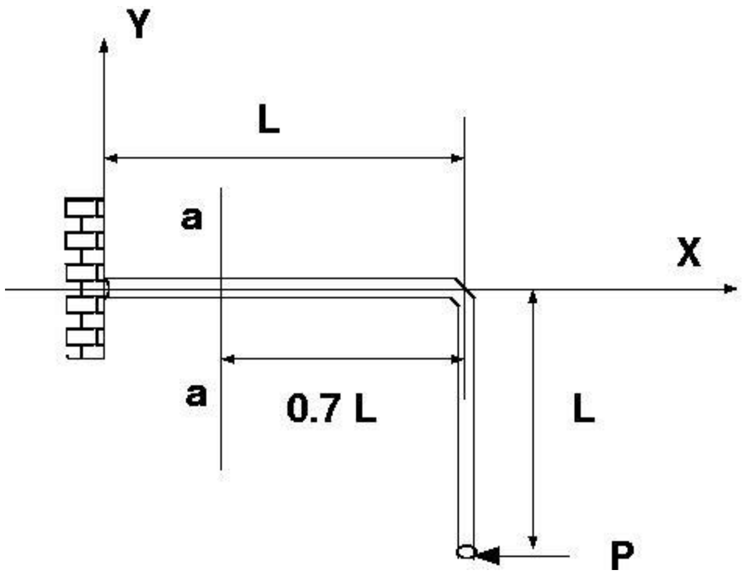
C. $\sqrt{(\sigma_x/2)^2 + \tau_{xy}^2}$

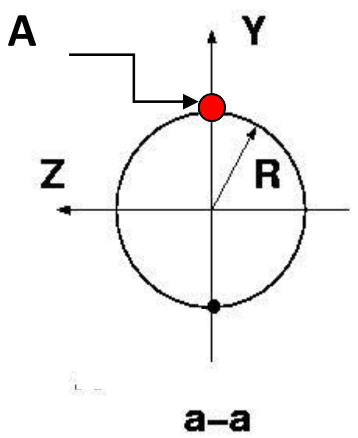
D. σ_x.

Ans: (A)

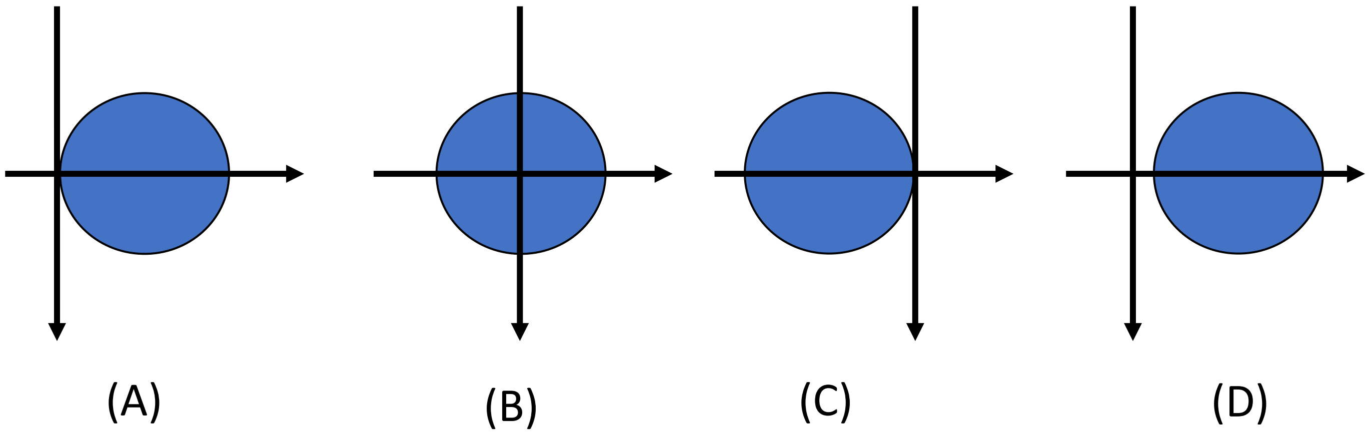


Q27. What is the correct Mohr's circle at the point A at section a-a?

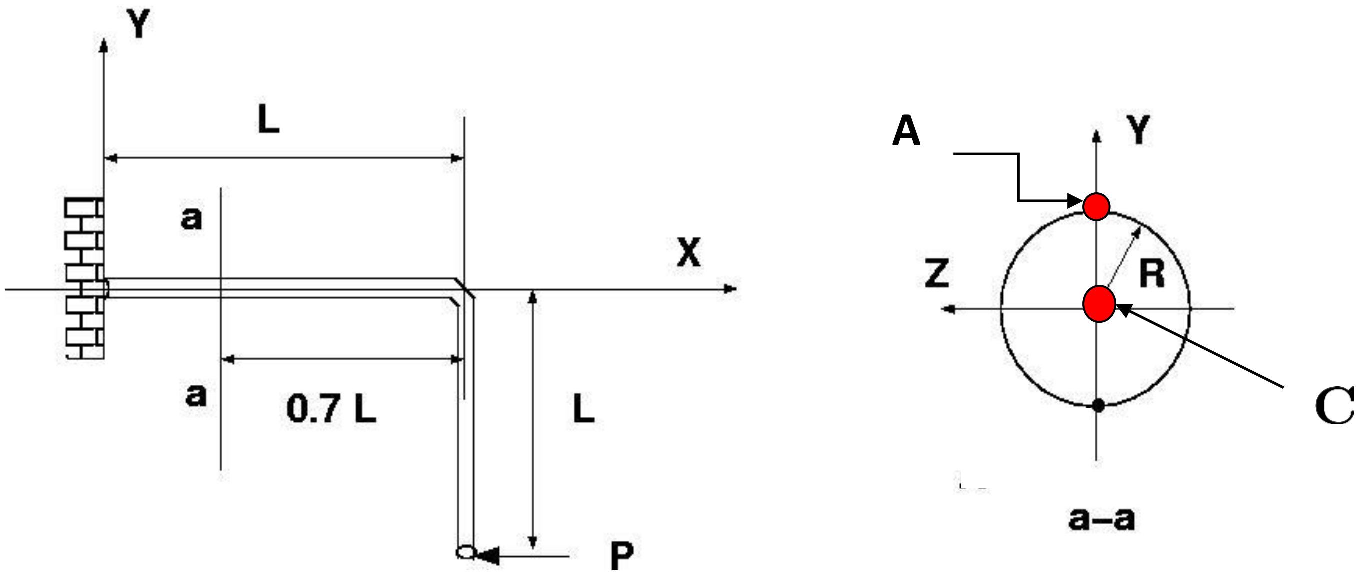




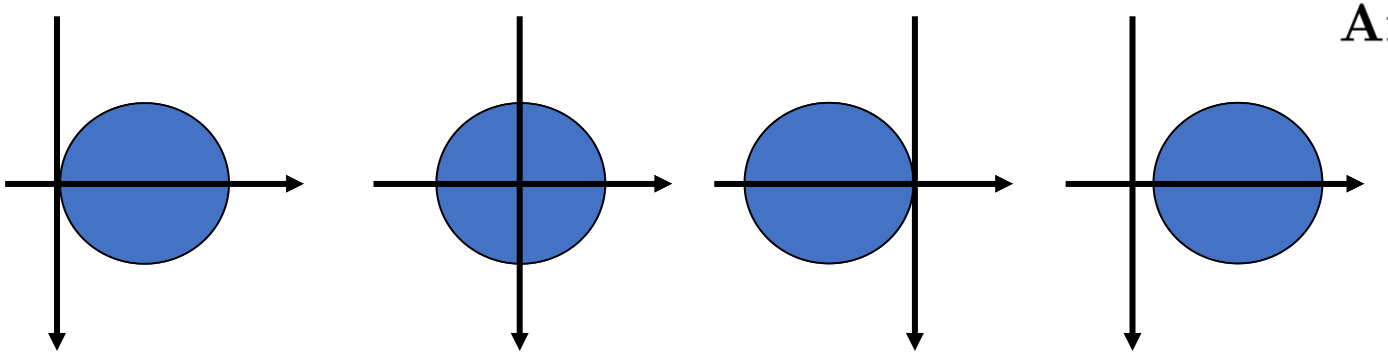
Ans. (A)



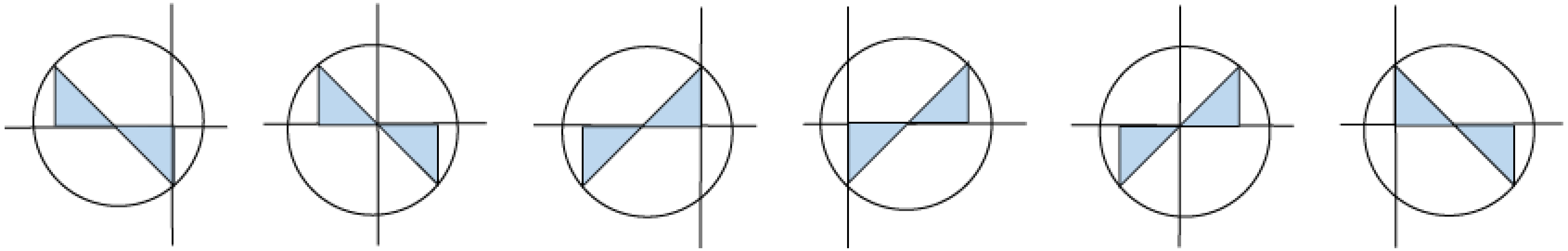
Q27. What is the correct Mohr's circle at the point B at section a-a?



Ans. (C)



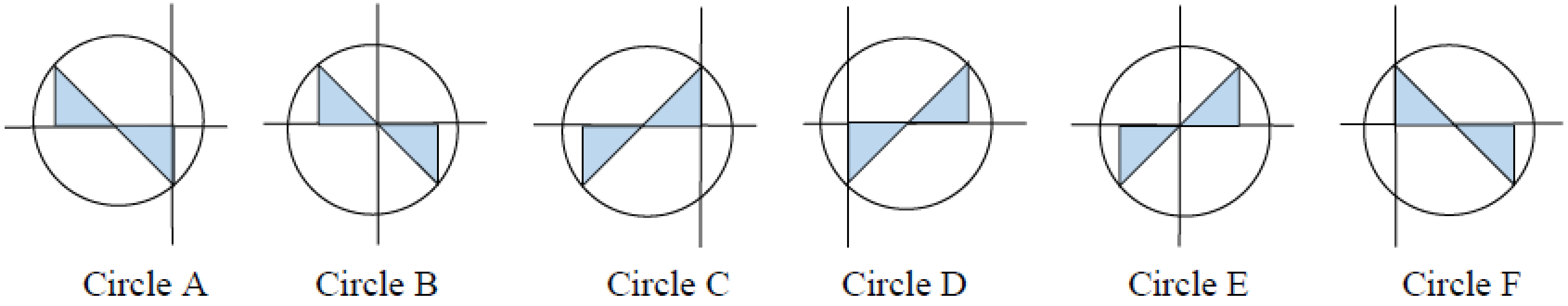
(A) (B) (C) (D)



Q28: The stress state: $\sigma_x = 0,\ \sigma_y = 80MP_a$, and $\tau_{xy} = 60MP_a$ is associated with which Mohr's circle above:

(A) Circle A;
(B) Circle B;
(C) Circle C;
(D) Circle D;
(E) Circle E;
(F) Circle F

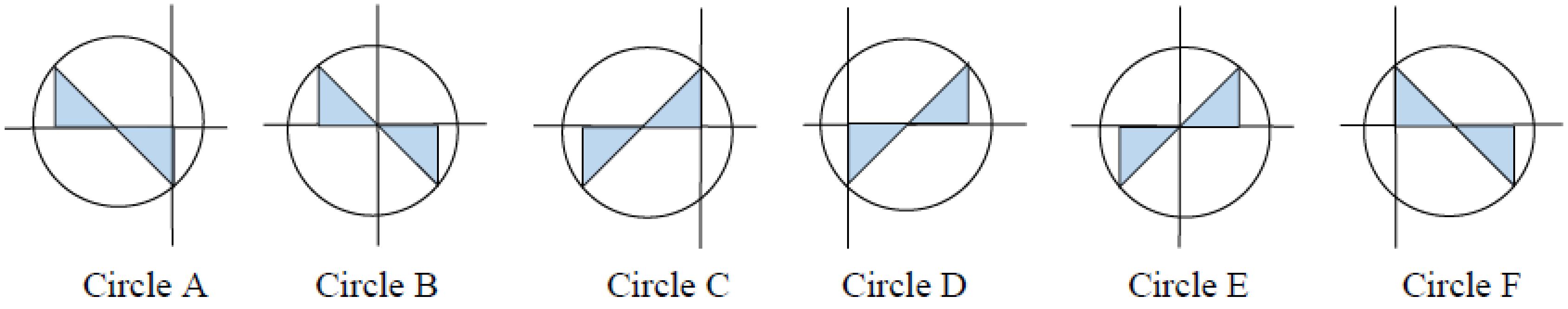
Ans (D)



29. The stress state: $\sigma_x = -80,\ \sigma_y = 0$, and $\tau_{xy} = 60MP_a$ is associated with which Mohr's circle above:

(A) Circle A;
(B) Circle B;
(C) Circle C;
(D) Circle D;
(E) Circle E;
(F) Circle F

Ans (C)



30. The stress state: $\sigma_x = 80MP_a$, $\sigma_y = -80MP_a$, and $\tau_{xy} = 60MP_a$ is associated with which Mohr's circle above:

(A) Circle A;
(B) Circle B;
(C) Circle C;
(D) Circle D;
(E) Circle E;
(F) Circle F

Ans (B)

This Friday's Lecture is the Last Lecture

Please join us