UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

Introduction to Solid Mechanics (ME85/C30)

Homework8 (Due on Friday midnight March 15th)

Problem 1. (30 points)

Consider an elastic bar with Young's modulus, $E = 10$, the cross section area $A = 1$, and the length of the bar $L = 1$. The bar has a built-in boundary condition at $x = 0$, i.e. $u(0) = 0$, and at $x = L$, the internal force $R(L) = 0$ as shown in Fig. 1.

The differential equation that governs the equilibrium of the bar has been derived as follows,

$$\boxed{\frac{d}{dx}\Big(EA\frac{du}{dx}\Big) + b(x) = 0, \quad 0 < x < L\ ,}$$

E = 10
A = 1
L = 1   x = L , R(L) = 0
x = 0 (aka u(0) = 0)

where $u(x)$ is the displacement field.

The bar is subjected a distributed load along its span, i.e.

$$\boxed{b(x) = p\sin\Big(\frac{2\pi x}{L}\Big)},$$

where $p = 1$ with a unit of force per unit length.

u(x) = displacement field
R(x) = internal force

The internal force is defined as $R(x) = \sigma A = EA\epsilon$, i.e.

$$\boxed{R(x) = EA\frac{du}{dx}}$$

$\frac{d}{dx}$ R(x) + b(x) = 0

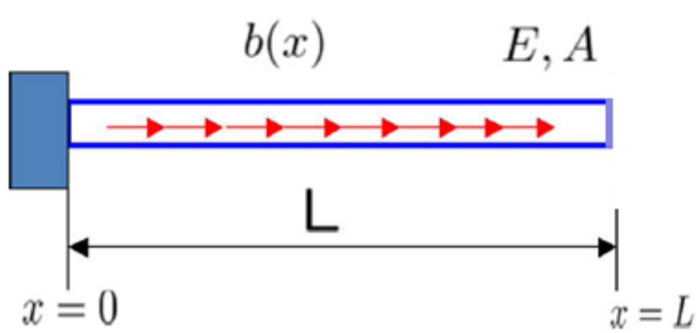


Figure 1: An elastic bar with the distributed load.

Use Matlab to find the displacement field and internal force/stress field, and compare them with the statically indeterminate system that has the same dimensions and the same material properties,

but with different boundary conditions: $u(0) = u(L) = 0$. Hint:
Go to class Bcourses website and go to the lecture folder, and then download a Matlab-P1 folder that contains the file: bar1d.m. You start your solution there. For all details, please refer to Lecture20F.pdf slide.

√ Problem 2. **P9.16** (10 points)

√ Problem 3. **P9.30** (10 points)

√ Problem 4. **P9.38** (10 points)

√ Problem 5. **P9.81** (10 points)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% C30/ME85 Introduction to Solid Mechanics (Fall 2023)
%
%
% -- Function to solve Tension-Compression Bar(1D)
%    using MATLAB ODE solver BVP4C
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function bar1d

% -- Define geometry
L = 1;                   % -- Length of bar

% -- Set solver parameters
nvar  =  2;              % -- Number of variables
np    = 10;              % -- Initial Number of points on [0,L]
xp    = linspace(0,L,np);% -- Initial Points at which to satisfy ODE

% -- Set initial solution for the solver
solinit = bvpinit(xp,zeros(1,nvar));

% -- Set options on the tolerance for the accuracy of the solution
%    Default: RelTol(1e-3), AbsTol(1e-6)
%
%    y'(x) = f(x,y(x)) + res(x)
%
%    norm( res(i)/f(i) ) <= RelTol
%
%    AND
%
%    norm( res(i)      ) <= AbsTol
%
options = bvpset('RelTol',1e-3,'AbsTol',1e-6);

% -- Invoke solver bvp4c is a built-in Matlab function
sol     = bvp4c(@bar1d_ode,@bar1d_bc,solinit,options);

% -- Plot solution
bar1d_plot(L,sol);

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% function [fxy] = bar1d_ode(x,y)
%
% -- Function defining ODE
%
% Inputs : x    -- position
%          y    -- vector of unknowns evaluated at x (disp,internal force)
%
```

```
% Outputs: fxy -- function f evaluated at x where f
%                 is defined through the relation,
%
%                       y' = f(y(x),x)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
function [fxy] = bar1d_ode(x,y)

% -- Define material property and geometry
E = 10;  % Young's Modulus
A = 1;  % Cross-sectional area
L = 1;  % Length of bar

% -- Define distributed load
b = sin(2*pi*x/(L));

% -- Define function
fxy = [y(2)/(E*A);
               -b];
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% function [res] = bar1d_bc(ya,yb)
%
% -- Function defining the BC
%
% Inputs : ya  -- y evaluated at end point x=a
%          yb  -- y evaluated at end point x=b
%
% Outputs: res -- vector of residuals defining how much
%                 the boundary conditions are not satisfied
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [res] = bar1d_bc(ya,yb)

% -- Boundary Conditions (BC)
%    u: displacement
%    f: force

ua = 0;     % -- Fixed      at x=a
fb = 0;     % -- Zero force at x=b
res= [ya(1)-ua;
      yb(2)-fb];
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% function bar1d_plot(L,sol)
%
```

```
% -- Function to plot solution
%
% Inputs : L   -- length of bar
%          sol -- solution structure
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function bar1d_plot(L,sol)

% -- Define discretization of plot
np   = 100;               % -- Number of points on [0,L]
xint = linspace(0,L,np);

% -- Evaluate solution at the points [xint]
Sxint = deval(sol,xint);

% -- Figure properties
FontSize = 16;
LineWidth=  2;

% -- Plot points
figure;

subplot(2,1,1);           % -- Plot displacement u(x)
plot(xint,Sxint(1,:),...
     'LineWidth',LineWidth);
grid on;
set(gca,'FontSize',FontSize);
xlabel('x',    'FontSize',FontSize);
ylabel('Displacement','FontSize',FontSize);

subplot(2,1,2);           % -- Plot force R(x)
plot(xint,Sxint(2,:),...
     'LineWidth',LineWidth);
grid on;
set(gca,'FontSize',FontSize);
xlabel('x',     'FontSize',FontSize);
ylabel('Internal force','FontSize',FontSize);

end
```

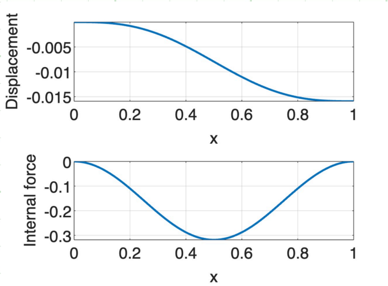
problem 1

$\boxed{\text{my graph}}$

At $L = 0$ and $L = 1$, $R(x) = 0$

$\because$ At $L = 0$, $u(0) = 0$; at $L = 1$, $u(1) = -0.015$

$\therefore$ bar is only fixed on left hand side

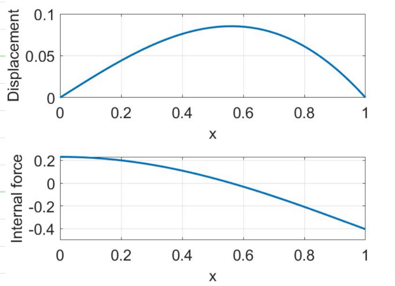


$\boxed{\text{graph given}}$

At $L = 0$, $R(x) \approx 0.2$; at $L = 1$, $R(x) = -0.4$

$\because$ At $L = 0$, $u(0) = 0$; at $L = 1$, $u(1) = 0$

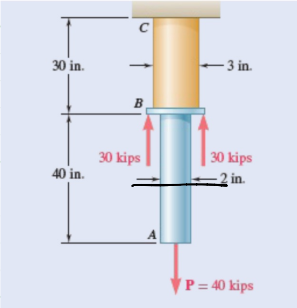
$\therefore$ bar is fixed on both ends



problem 2: 9.16

2 cylindrical rods joined at B
steel rod AB $E = 29 \times 10^6$ psi
brass rod BC $E = 15 \times 10^6$ psi

a) total deformation of composite rod ABC
b) deflectn of point B



$$\delta = \frac{PL}{EA}$$

$E_{AB} = 29 \times 10^6$ psi

$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2\text{in})^2 = 3.14\text{ in}^2$

$L_{AB} = 40$ in

P acting on mid section of rod

$P_{AB} = 40$ kips $= 40 \times 10^3$ lb

$$\delta_{AB} = \frac{(40 \times 10^3\text{lb})(40\text{in})}{(29 \times 10^6\text{psi})(3.14\text{ in}^2)} = 0.0176\text{in} = 1.76 \times 10^{-2}\text{ in}$$

$E_{BC} = 15 \times 10^6$ psi

$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(3\text{in})^2 = 7.07\text{ in}^2$

$L_{BC} = 30$ in

$P_{BC} = 40\text{kips} - 60\text{kips} = -20\text{kips} = -20 \times 10^3$ lb

$$\delta_{BC} = \frac{(-20 \times 10^3\text{lb})(30\text{in})}{(15 \times 10^6\text{psi})(7.07\text{ in}^2)} = -0.00568\text{ in}$$

$\delta_{BC} = -5.68 \times 10^{-3}$ in

$\delta_{AB} = 1.76 \times 10^{-2}$ in

$\delta = \delta_{AB} + \delta_{BC} = 1.76 \times 10^{-2}\text{ in} - 5.68 \times 10^{-3} = 0.01192$
$= 11.92 \times 10^{-3}$ in

$\delta_B = -\delta_{BC} = -(-5.68 \times 10^{-3}\text{in}) = 5.68 \times 10^{-3}$ in

a) $\delta = 11.92 \times 10^{-3}\text{ in} \downarrow$

b) $\delta_B = 5.68 \times 10^{-3}\text{ in} \uparrow$

#

problem 4: 9.30

9.30 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ Gpa and $E_b = 105$ Gpa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.

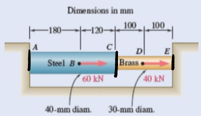


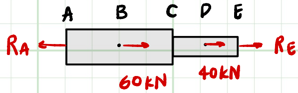
Fig. P9.30

$E_s = 200$ GPa

$E_b = 105$ GPa

a) A, E reactns

b) deflectn of pt. C



$\overline{AC}$

$E_s = E_{AC} = 200\,GPa = 200 \cdot 10^9$ Pa

$A_{AC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(40\,mm)^2 = \frac{\pi}{4}(0.04\,m)^2 = 4 \cdot 10^{-4} \pi\, m^2$

$EA = 200 \cdot 10^9\,Pa \cdot 4 \cdot 10^{-4} \pi m^2 = 251327412.3\,N = 251.327 \cdot 10^6\,N$

$\overline{CE}$

$E_B = E_{CE} = 105\,GPa = 105 \cdot 10^9$ Pa

$A_{CE} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(30\,mm)^2 = \frac{\pi}{4}(0.03\,m)^2 = 7.07 \cdot 10^{-4}\,m^2$

$EA = 105 \cdot 10^9\,Pa \cdot 7.07 \cdot 10^{-4}\,m^2 = 74235000 = 74.235 \cdot 10^6\,N$

a) $62.8\,kN \leftarrow$ at A

$37.2\,kN \leftarrow$ at E

b) $46.3\,\mu m \rightarrow$

$\overline{AB}$

$P = R_A$

$L = 180\,mm = 0.18\,m$

$PL = 0.18\,R_A\,m$

$\delta = \frac{PL}{EA}$

$\delta = \frac{0.18\,R_A\,m}{251.327 \cdot 10^6\,N} = 7.1620 \cdot 10^{-10} R_A$

$\overline{BC}$

$P = R_A - 60 \cdot 10^3 N$

$L = 120\,mm = 0.12\,m$

$PL = 0.12m\,(R_A - 60 \cdot 10^3 N)$

$\delta = \frac{0.12m\,(R_A - 60 \cdot 10^3 N)}{251.327 \cdot 10^6\,N} = \frac{0.12m\,R_A - 7200\,N \cdot m}{251.327 \cdot 10^6\,N} = 4.7747 \cdot 10^{-10} R_A - 2.8648 \cdot 10^{-5}$

$\overline{CD}$

$P = R_A - 60\cdot 10^3\ N$

$L = 100\ mm = 0.1\ m$

$PL = 0.1\ m(R_A - 60\cdot 10^3\ N)$

$$\delta = \frac{0.1\ m(R_A - 60\cdot 10^3\ \cancel{N})}{74.235\cdot 10^6\ \cancel{N}} = 1.347\cdot 10^{-9} R_A - 8.082\cdot 10^{-5}$$

$\overline{DE}$

$P = R_A - 100\cdot 10^3\ N$

$L = 100\ mm = 0.1\ m$

$PL = 0.1\ m(R_A - 100\cdot 10^3\ N)$

$$\delta = \frac{0.1\ m(R_A - 100\cdot 10^3\ N)}{74.235\cdot 10^6\ N} = 1.347\cdot 10^{-9} R_A - 1.347\cdot 10^{-4}$$

$$\delta = \delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0$$

$$= 7.1620\cdot 10^{-10} R_A + 4.7797\cdot 10^{-10} R_A - 2.8648\cdot 10^{-5} + 1.347\cdot 10^{-9} R_A - 8.082\cdot 10^{-5} + 1.347\cdot 10^{-9} R_A - 1.347\cdot 10^{-4}$$

$$= 3.89\cdot 10^{-9} R_A - 2.44\cdot 10^{-4} = 0$$

$R_A = 62724.94 = 62.8\cdot 10^3\ N = 62.8\ kN$ — a) $R_A = 62.8\ kN \leftarrow$

$R_E = R_A - 100\cdot 10^3 = 62.8\cdot 10^3\ N - 100\cdot 10^3\ N$

$= -37200\ N = -37.2\ kN$ — $R_E = 37.2\ kN \leftarrow$

$\delta_C = \delta_{AB} + \delta_{BC} = 1.164\cdot 10^{-9} R_A - 26.848\cdot 10^{-6}$

$= 1.164\cdot 10^{-9}(62.8\cdot 10^3) - 26.848\cdot 10^{-6}$

$= 4.63\cdot 10^{-5}\ m = 46.3\cdot 10^{-6}\ m = 46.3\ \mu m$ — b) $\delta_C = 46.3\ \mu m \rightarrow$

problem 4: 9.38

9.38 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C. Page 450

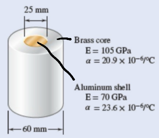


Fig. P9.38

unstressed at 15°C
axial deformations
stress of aluminum when 195°C

$\delta_T = \alpha(\Delta T)L$

$\delta_P = \dfrac{PL}{AE}$

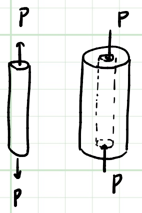
$\delta = \delta_T + \delta_P = 0$

$\alpha(\Delta T)L + \dfrac{PL}{AE} = 0$

$P = -AE\alpha(\Delta T)$

$\sigma = \dfrac{P}{A} = -E\alpha(\Delta T)$

tensile compressive



$\Delta T = 195° - 15° = 180°$

$\delta_T = \alpha(\Delta T)L$

brass: $(\delta_T)_b = \alpha_b L(\Delta T)$

aluminum: $(\delta_T)_a = \alpha_a L(\Delta T)$

$\delta = (\alpha_a - \alpha_b)L(\Delta T)$

brass: $E_b = 105\text{ GPa} = 105 \cdot 10^9\text{ Pa}$

$A_b = \frac{\pi}{4}(25\text{ mm}^2) = 490.87\text{ mm}^2$

$= 490.87 \cdot 10^{-6}\text{ m}^2$

$(\delta_P)_b = \alpha_b L(\Delta T)$

$(\delta_P)_b = \dfrac{PL}{E_b A_b}$

aluminum: $E_a = 70\text{ GPa} = 70 \cdot 10^9\text{ Pa}$

$A_a = \frac{\pi}{4}(60\text{mm}^2 - 25\text{mm}^2)$

$= \frac{\pi}{4}(2975\text{ mm}^2) = 2336.56\text{ mm}^2 = 2.336 \cdot 10^{-3}\text{ m}^2$

$(\delta_P)_a = \alpha_a L(\Delta T)$

$(\delta_P)_a = \dfrac{PL}{E_a A_a}$

$\delta = (\delta_P)_b + (\delta_P)_a$

$\delta = (\alpha_a - \alpha_b)L(\Delta T) = \dfrac{PL}{E_b A_b} + \dfrac{PL}{E_a A_a}$

$(\alpha_a - \alpha_b)\cancel{L}(\Delta T) = KP\cancel{L}$

$(\alpha_a - \alpha_b)(\Delta T) = KP$

$$P = \frac{(\alpha_a - \alpha_b)(\Delta T)}{\frac{1}{E_b A_b} + \frac{1}{E_a A_a}} = \frac{(23.6 \cdot 10^{-6}/\cancel{°C} - 20.9 \cdot 10^{-6}/\cancel{°C})(180\cancel{°C})}{\frac{1}{105 \cdot 10^9\text{ Pa} \cdot 490.87 \cdot 10^{-6}\text{ m}^2} + \frac{1}{70 \cdot 10^9\text{ Pa} \cdot 2.336 \cdot 10^{-3}\text{ m}^2}}$$

$$= \frac{4.86 \cdot 10^{-4}}{2.5517 \cdot 10^{-8}} = 19046\text{ N} = 19.05 \cdot 10^3\text{ N}$$

$$\sigma_a = \frac{-P}{A_a} = \frac{-19.05 \cdot 10^3\text{ N}}{2.336 \cdot 10^{-3}\text{ m}^2} = -8154965.753\text{ Pa} = -8.15 \cdot 10^6\text{ Pa} = -8.15\text{ MPa}$$

$\sigma_a = -8.15\text{ MPa}$ ✱

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problem 5 : 9.81

9.81 The block shown is made of a magnesium alloy for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

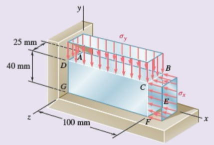

Fig. P9.81

$E = 45$ GPa
$\nu = 0.35$
$\sigma_x = -180$ MPa

a) σ_y mag : $\Delta h = 0$
b) ΔA of face ABCD
c) ΔV of block

a) $\delta_y = 0, \ \varepsilon_y = 0, \ \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\sigma_y = \nu\sigma_x = (0.35)(-180 \cdot 10^6) = -63 \cdot 10^6 \text{ Pa}$$

$$\sigma_y = -63.0 \text{ MPa}$$

$$\varepsilon_x = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = 0 \rightarrow \sigma_y = \nu\sigma_x$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x - \nu\sigma_y}{E} = \frac{157.95 \times 10^6}{45 \times 10^9} = -3.510 \cdot 10^{-3} \ *$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = \frac{-\nu}{E}(\sigma_x + \sigma_y) = \frac{(0.35)(-243 \times 10^6)}{45 \cdot 10^9} = +1.890 \times 10^{-3}$$

b) $A_0 = L_x L_z$

$$A = L_x(1 + \varepsilon_x) L_z(1 + \varepsilon_z) = L_x L_z(1 + \varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$$

$$\Delta A = A - A_0 = L_x L_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x + \varepsilon_z)$$

$$\Delta A = (100\text{ mm})(25\text{ mm})(-3.510 \cdot 10^{-3} + 1.890 \cdot 10^{-3}) \quad \Delta A = -4.05 \text{ mm}^2 \ *$$

c) $V_0 = L_x L_y L_z$

$$V = L_x(1 + \varepsilon_x) L_y(1 + \varepsilon_y) L_z(1 + \varepsilon_z)$$

$$= L_x L_y L_z(1 + \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x + \varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\Delta V = V - V_0 = L_x L_y L_z(\varepsilon_x + \varepsilon_y + \varepsilon_z) = (100\text{ mm})(40\text{ mm})(25\text{ mm})(-3.5 \cdot 10^{-3} + 1.8 \cdot 10^{-3}) = -162 \text{ mm}^3 \ *$$