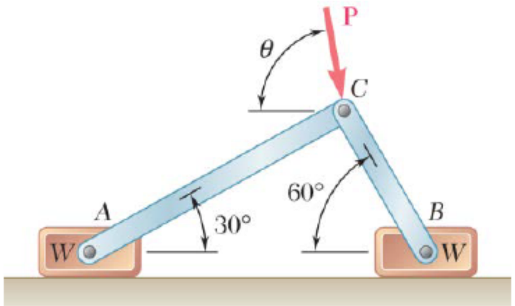
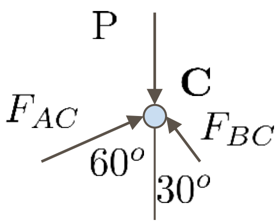


PROBLEM 4.95

Let $\theta = 90^o$.

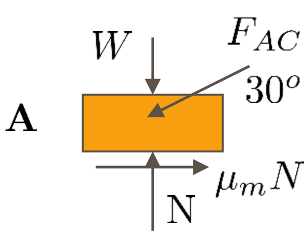
Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.



$$\sum F_x = 0; \quad F_{AC}\sin 60^o - F_{BC}\sin 30^o = 0 \ \rightarrow \ F_{BC} = \sqrt{3}F_{AC}$$

$$\sum F_y = 0; \quad F_{AC}\sin 30^o + F_{BC}\sin 60^o - P = 0 \ \rightarrow \ F_{AC} = \frac{P}{2}$$

$$F_{BC} = \frac{\sqrt{3}}{2}P$$



$$\sum F_x = 0 \quad \mu_m N - \cos 30^o F_{AC} = 0 \ \rightarrow \ N = \frac{\sqrt{3}P}{1.2}$$

$$\sum F_y = 0 \quad -W - \sin 30^o F_{AC} + N = 0 \ \rightarrow \ N = W + \frac{P}{4}$$

$$P = \frac{W}{1.443 - 0.25} = 0.8382W$$

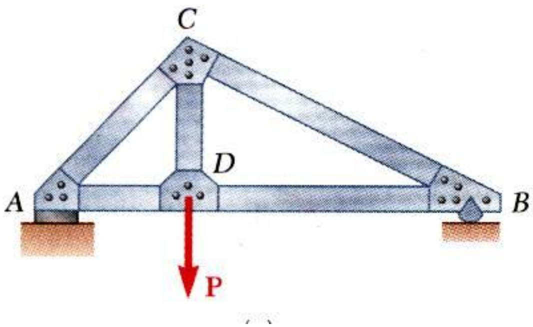
Lecture 9 Analysis of Structures (Truss)

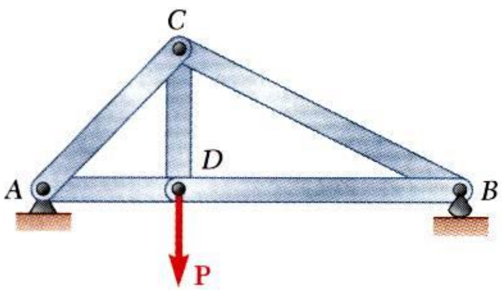
Objectives:

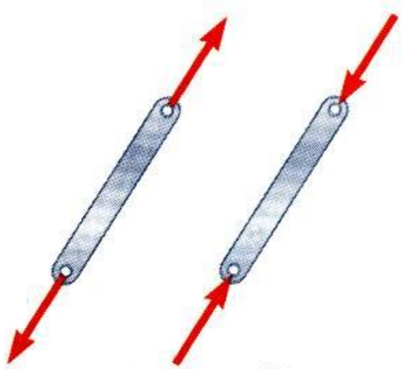
Method of Joints and

Method of Section

Definition of a Truss







- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only ***two-force members*** are considered.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

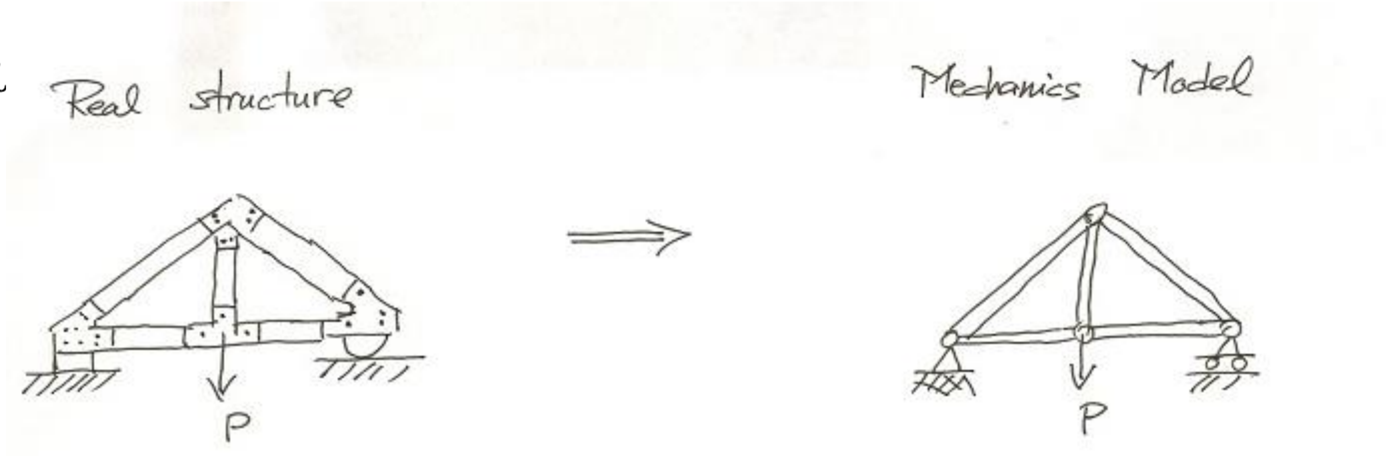
Static Analysis of Structures is basic part of Structural Analysis

Structural Members: Truss, Frame, and Machine

Definition of Truss

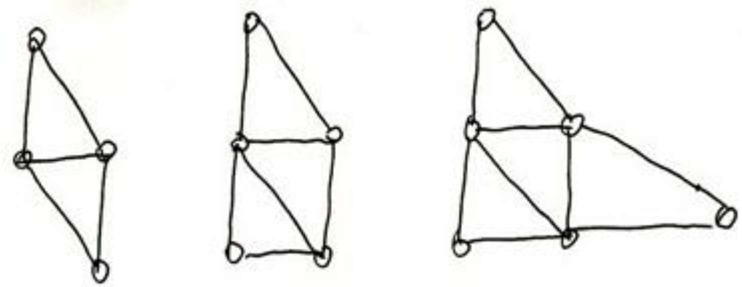
Truss is a structural member that only transmits axial loads.

R

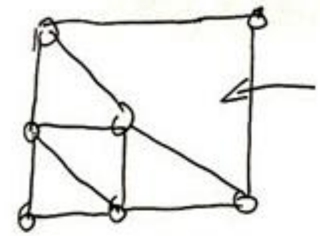


Truss structure only consists of two-force members.

2D simple trusses $m = 2n - 3$:

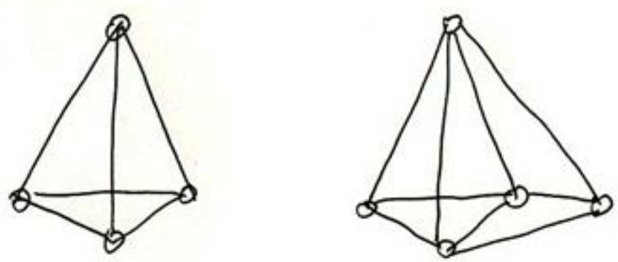


n=4 m=5; n=5 m=7; n=6 m=9;



A simple truss but not rigid

For space structures, a simple truss is made by tetradedrons,



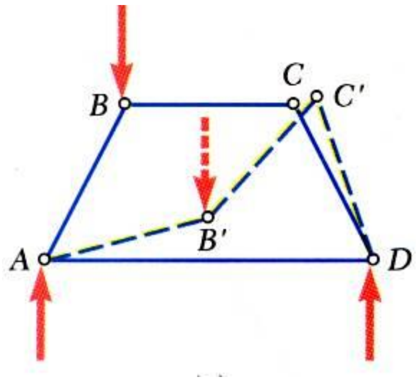
n=4 m=6

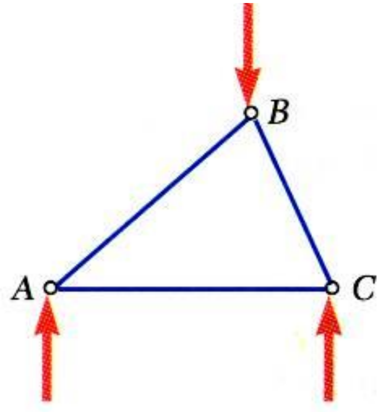
n=5 m=9

....

m=3n-6

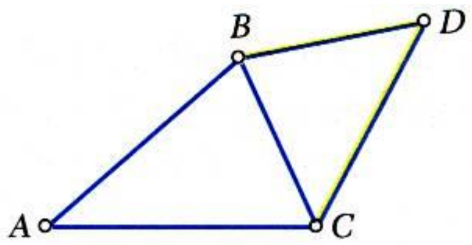
Rigid Truss and Simple Truss



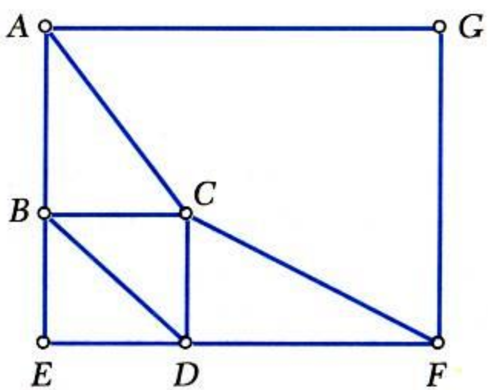


- A ***rigid truss*** will not collapse under the application of a load.

- A ***simple truss*** is constructed by successively adding two members and one connection to the basic triangular truss.



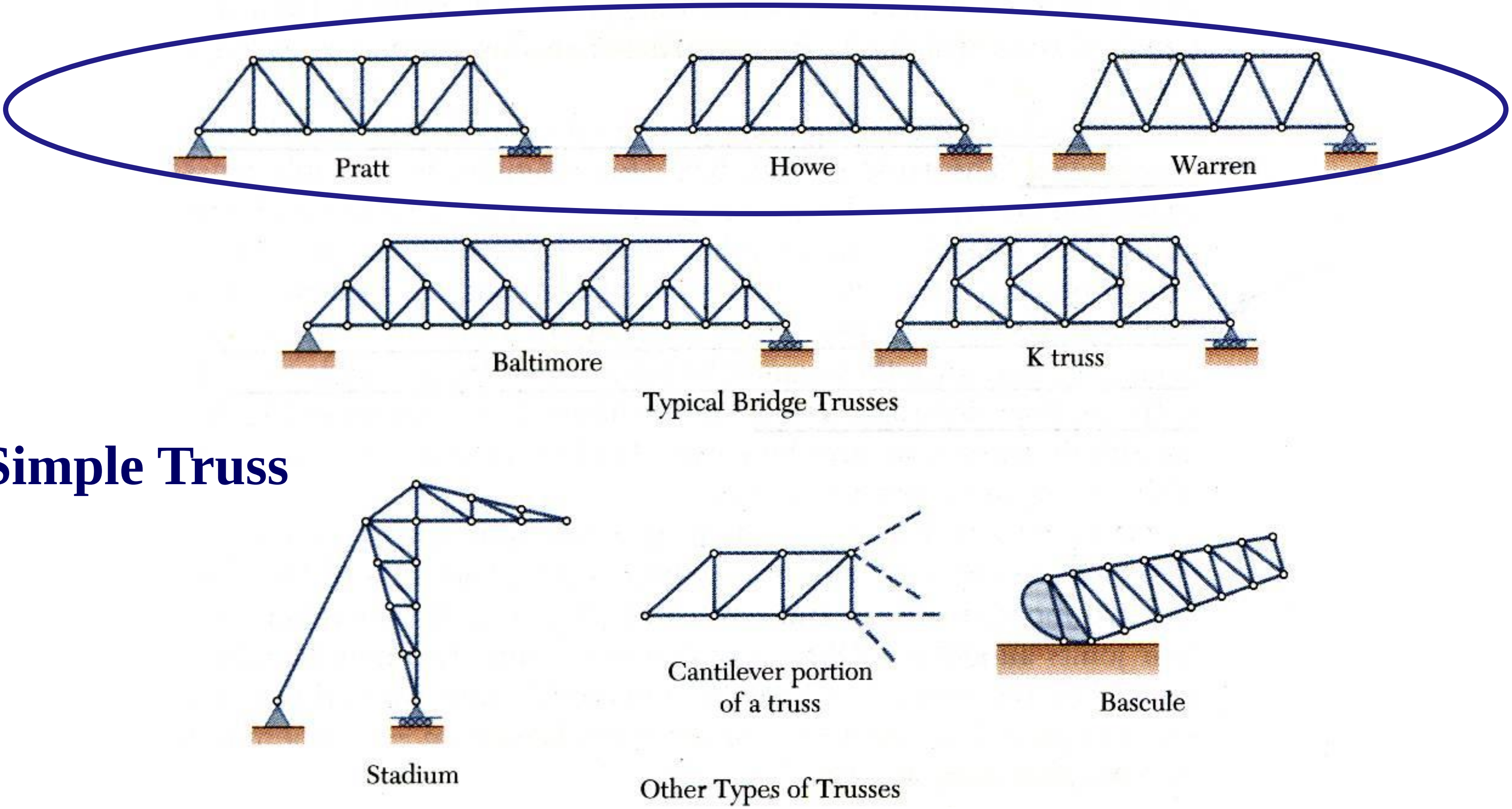
- In a 2D simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.



A simple truss
that is not rigid.

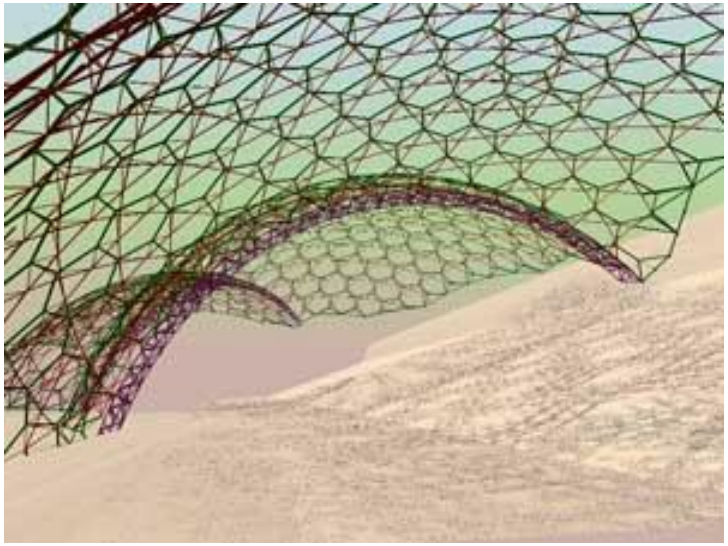
Examples of Simple Truss Structures

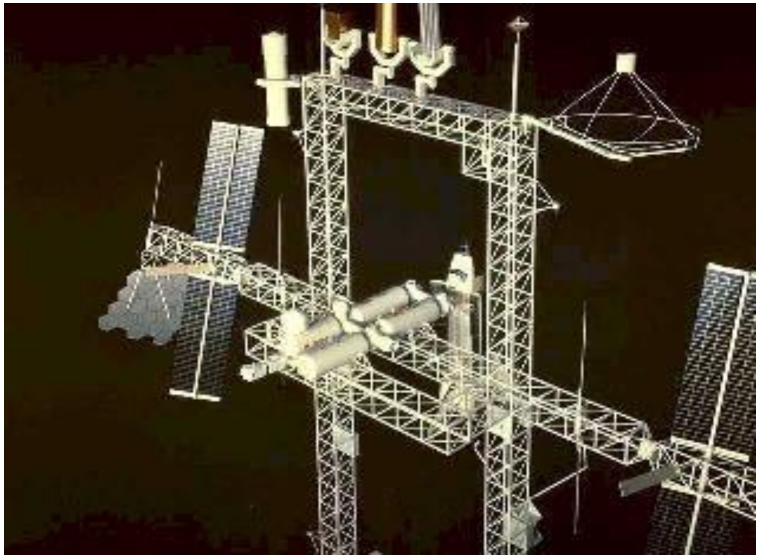
http://en.wikipedia.org/wiki/Truss_bridge



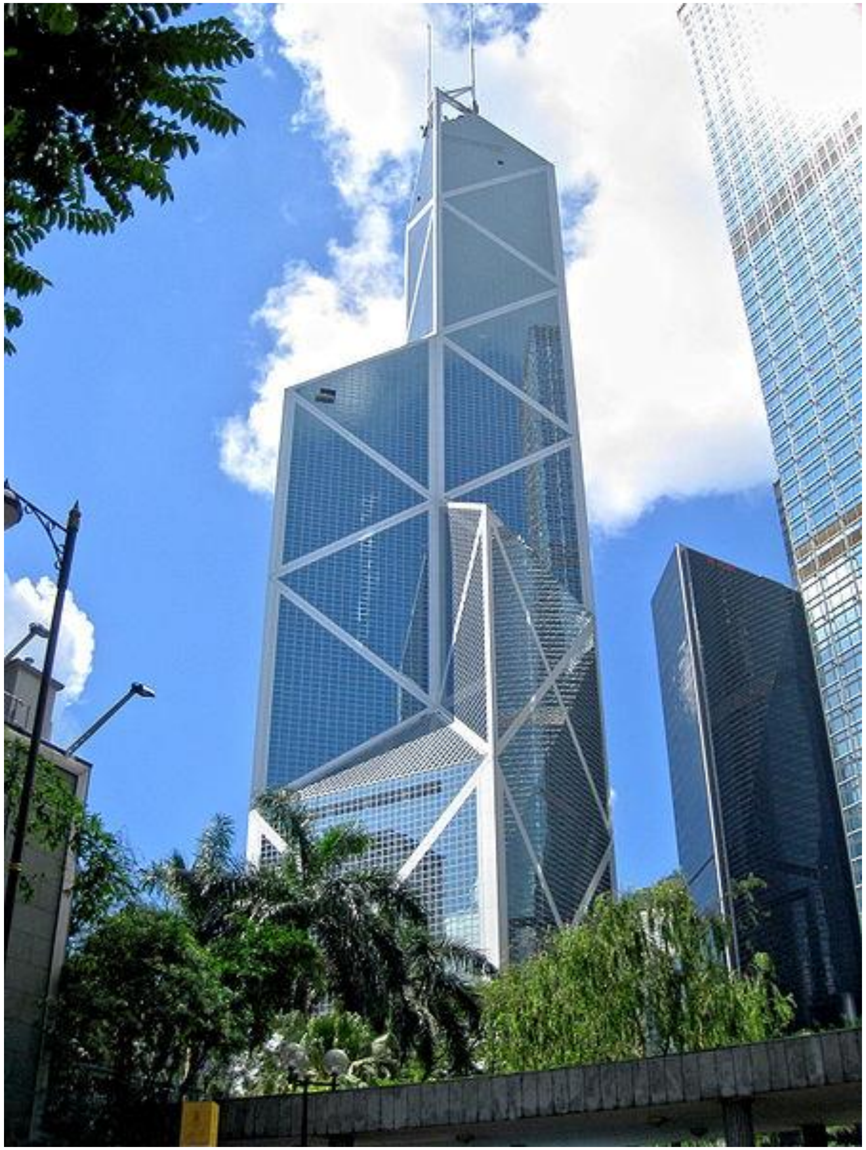
Simple Truss

Gallery of Truss Structures





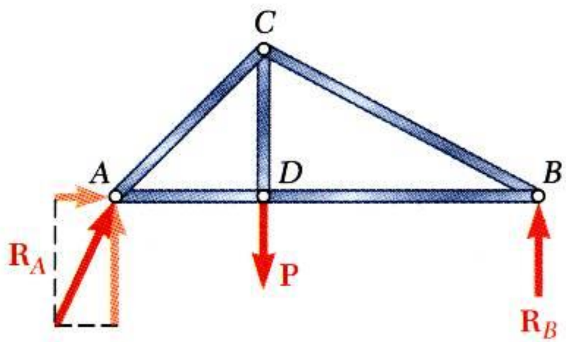
The Old Oakland Bay Bridge

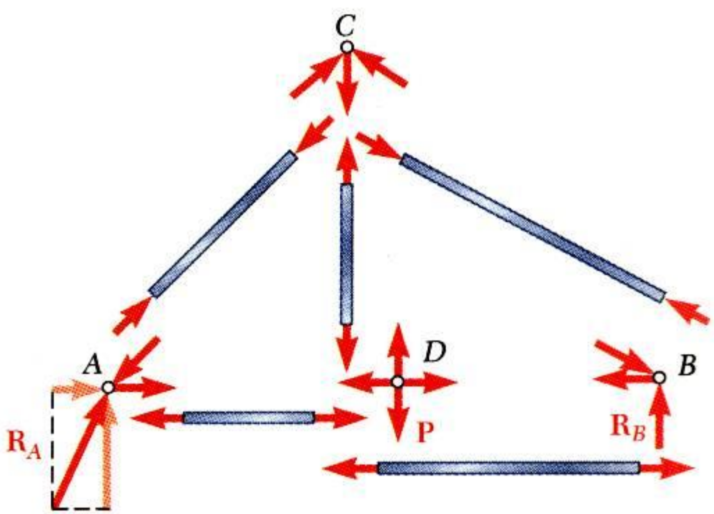


Hong Kong China Bank

Houston Reliant Astrodome

Method of Joints

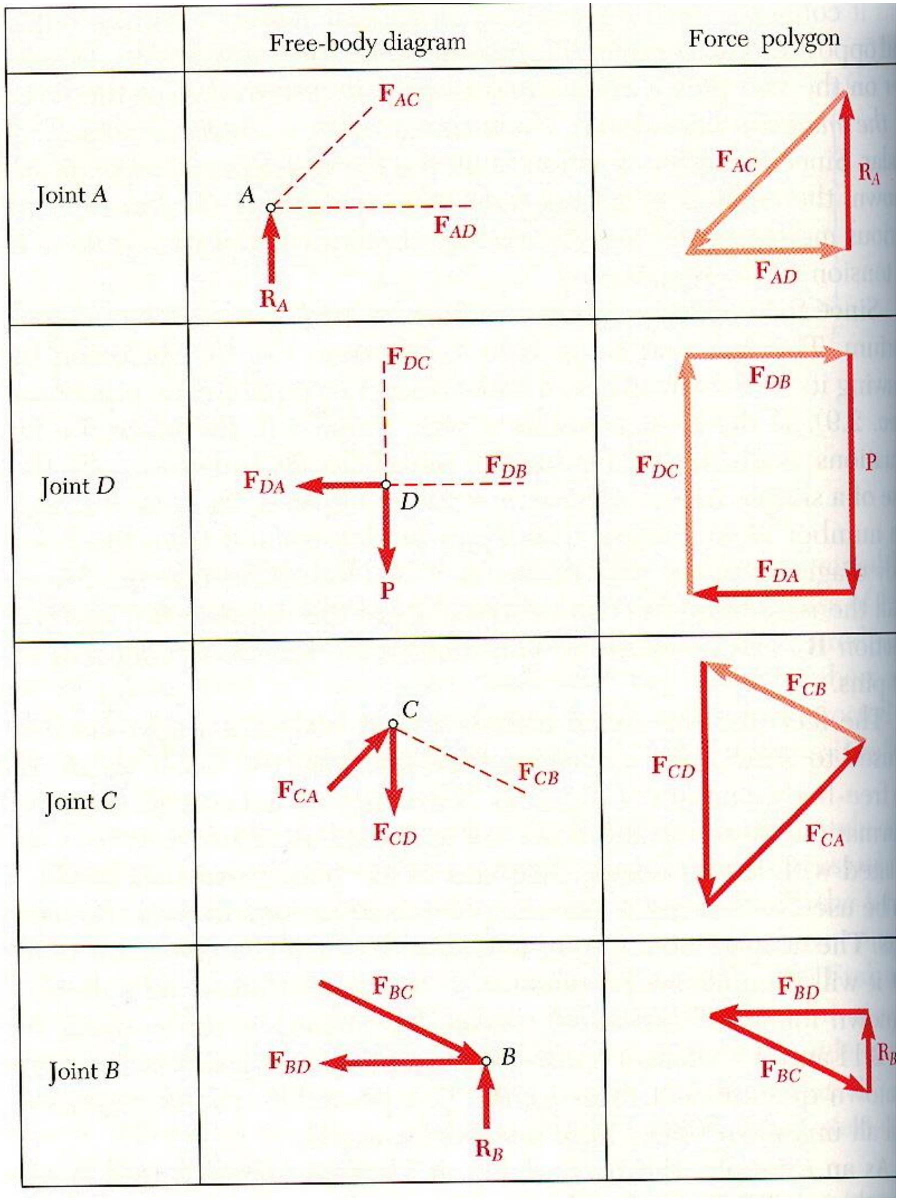




- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

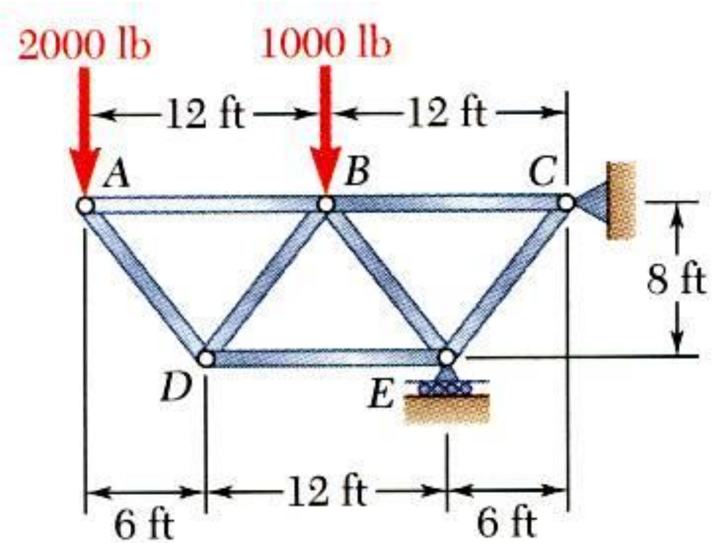
Summary

1. Draw the free-body diagram of the entire structure and determine the reaction forces;
2. Start from an joint with only or less than two unknowns;
3. Draw free-body diagram of the joint and solve two equilibrium equations;
4. Repeat this procedure until forces in all members have been solved.
5. When the reaction of an internal force is (assumed) towards an joint, the internal force is (assumed) in compression; whereas the reaction of an internal force is (assumed) away from the joint, it is (assumed) in tension.



At each joint, one has a closed force polygon: $\mathbf{F}_{CB} + \mathbf{B}_Y + \mathbf{F}_{DB} = 0$

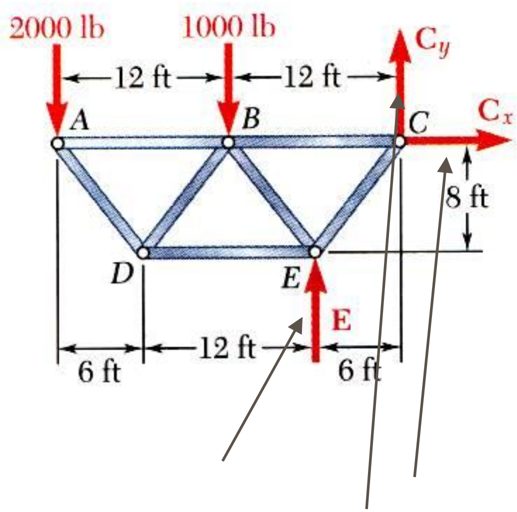
Sample Problem 6.1



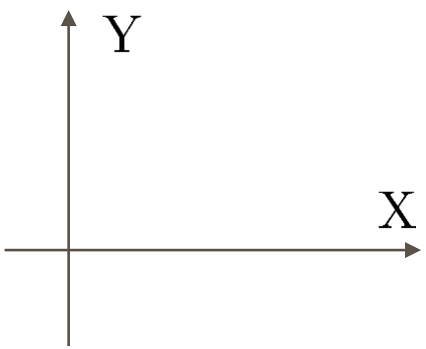
Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.



External forces



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.

$$\sum M_C = 0$$
$$= (2000\,\text{lb})(24\,\text{ft}) + (1000\,\text{lb})(12\,\text{ft}) - E(6\,\text{ft})$$
$$\boxed{E = 10{,}000\,\text{lb}\uparrow}$$

$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -2000\,\text{lb} - 1000\,\text{lb} + 10{,}000\,\text{lb} + C_y$$

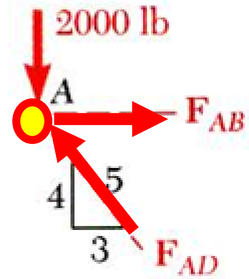
$$C_y = -7000, \quad \rightarrow \quad \boxed{C_y = 7000\,\text{lb}\downarrow}$$

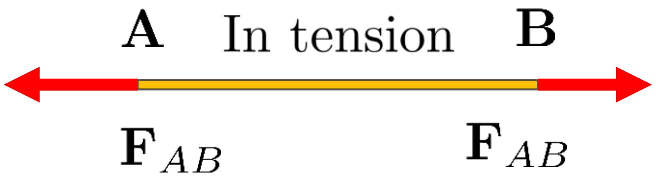
All the external reactions are assumed along the positive direction of coordinate axes.

If an internal force is (assumed) towards an joint, the internal force is (assumed) in compression; and if an internal force is (assumed) away from the joint, the internal force is (assumed) in tension.

Joint A

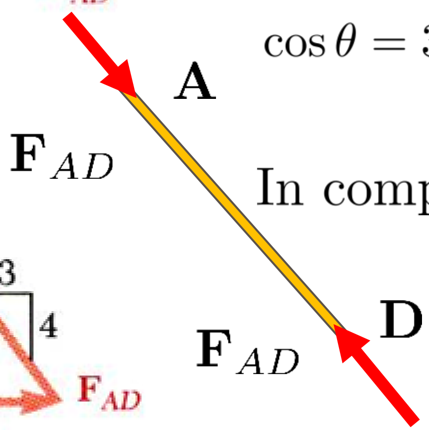
Intuitive approach



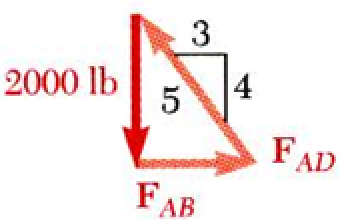


$\sin\theta = 4/5$

$\cos\theta = 3/5$



In compression



$$\sum F_y = 0 \;\rightarrow\; F_{AD}\sin\theta - 2000 = 0$$

$$F_{AD} = 2000/\sin\theta = 2500$$

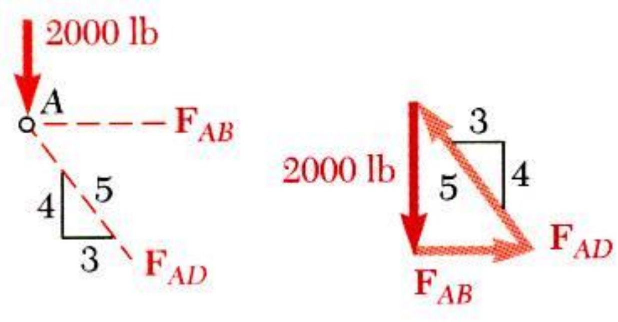
$$\sum F_x = 0 \;\rightarrow\; F_{AB} - F_{AD}\cos\theta = 0$$

$$F_{AB} = F_{AD}\cos\theta = 1500$$

$$F_{AB} = 1500lb(T) \qquad F_{AD} = 2500lb(C)$$

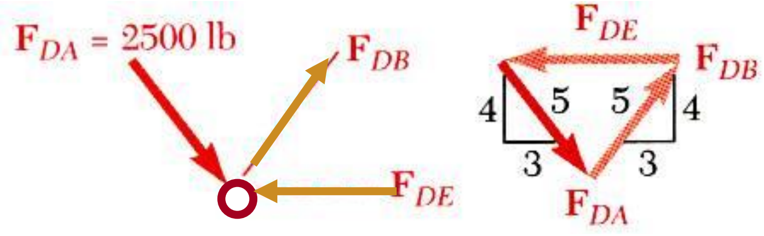
All these are assumed directions, if the actual results are negative number, then the assumed tension is compression, or the assumed compression is tension.

Internal forces going towards the joint are in compression, and internal forces going out the joint are in tension.



Joint D

- There are now only two unknown member forces at joint D.

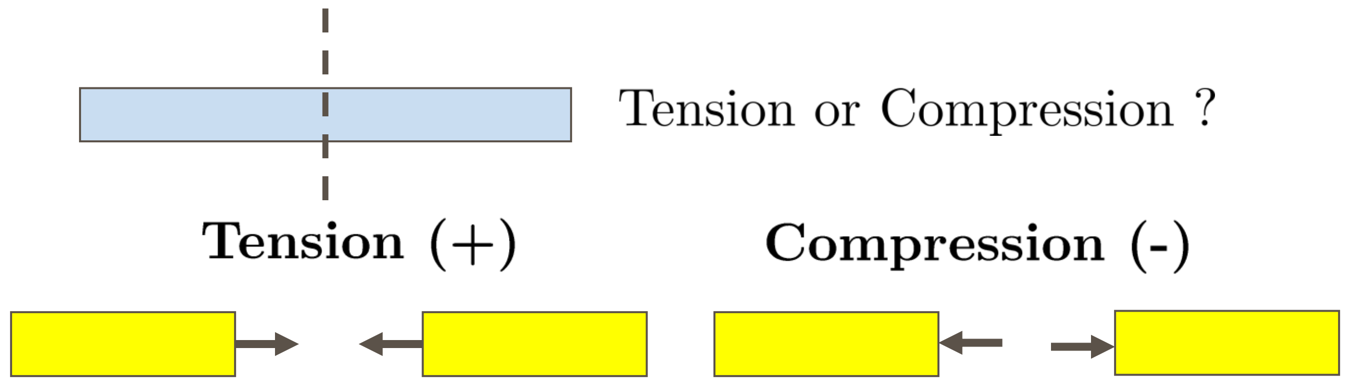


$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$\boxed{\begin{aligned} F_{DB} &= 2500\,\text{lb}\ \ T \\ F_{DE} &= 3000\,\text{lb}\ \ C \end{aligned}}$$

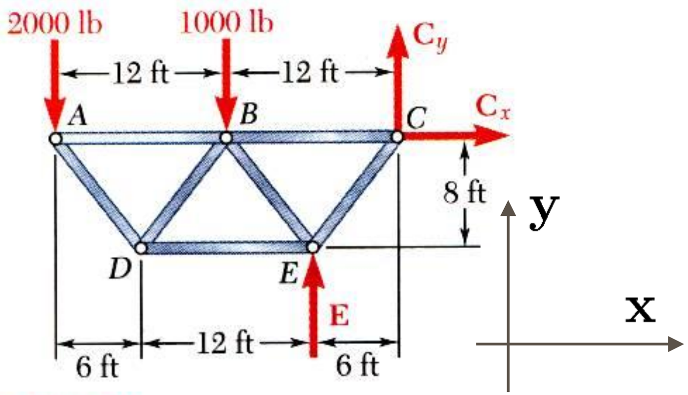
Internal Force Sign Convention

External force is a vector, and its direction is well defined; but Internal force is a **different animal**, and its direction depends on whether it is in tension or in compression.



In standard engineering computations, we always assume that the internal force is in tension (away from the joint). If the final result is positive, it is indeed in tension; if the final result is negative it is then in compression.

Internal force is a different animal than the external force !

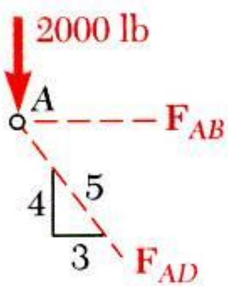


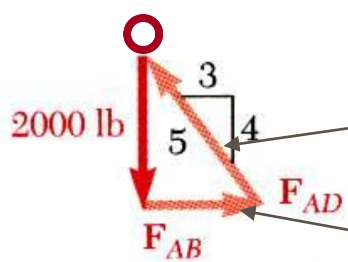
Internal force convention:

Internal forces towards the joint are in compression (-), and internal forces away from the joint are in tension (+).

$$\frac{2000\,\text{lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$\boxed{\begin{aligned} F_{AB} &= 1500\,\text{lb}\ \ T \\ F_{AD} &= 2500\,\text{lb}\ \ C \end{aligned}}$$





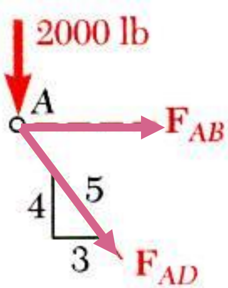
Assume in compression

Assume in tension

These two negative signs mean different things !

?

Do This $\rightarrow$



$$\sum F_y = 0; \rightarrow -2000 - F_{AD}\frac{4}{5} = 0;$$

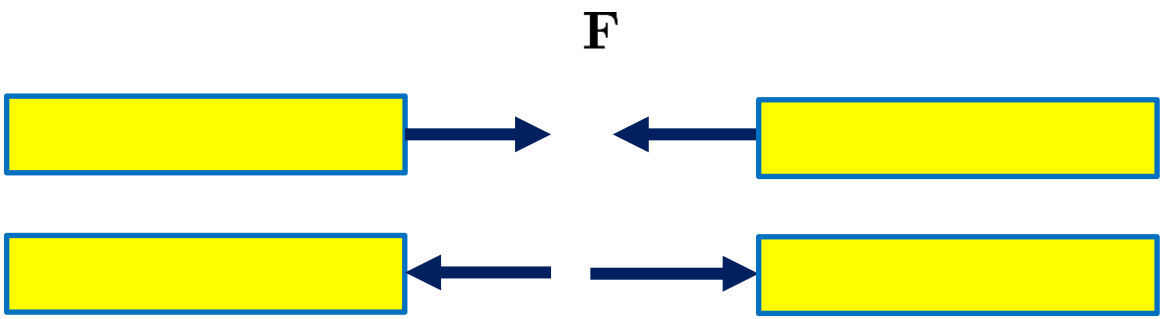
$$\rightarrow F_{AD} = -\frac{5}{4} \times 2000 = -2500\ (C)$$

$$\sum F_x = 0; \rightarrow F_{AB} + F_{AD}\frac{3}{5} = 0 \rightarrow F_{AB} = -\frac{3}{5}(-2500)$$

$$F_{AB} = 1500\ (T)$$

There are two different states that the internal force acts.

Example: Find internal force inside the bar



Internal force is an evil twin!