HW 11 Genevieve Bique

P11.11, P12.45, P12.49, P12.88; P13.3, P13.9

11.9 through *11.11* Two vertical forces are applied to the beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

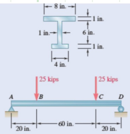
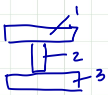


Fig. *P11.11*



	A	y_0	Ay_0
1	8	7.5	60
2	6	4	24
3	4	0.5	2
	Σ: 18		Σ: 86

$$y_0 = \frac{\Sigma A y_0}{\Sigma A} = \frac{86}{18} \quad y_0 = 4.778 \text{ in}$$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(8)(1)^3 + 8(2.722)^2$$

$$I_1 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2$$

$$I_2 = 21.63 \text{ in}^4$$

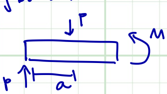
$$I_3 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(4)(1)^3 + 4(4.278)^2$$

$$I_3 = 73.54 \text{ in}^4$$

$$I = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4$$

$y_{top} = 3.22$ $\qquad \sigma = -\frac{MI}{y}$

$y_{bottom} = 4.77$



$M - P(a) = 0$

$M = Pa$

$= 25 \cdot 20 = 500$ kip·in

$$\sigma_{top} = -\frac{500 \times 3.22}{155.16} = -10.38 \text{ ksi} - \text{compression}$$

$$\sigma_{bottom} = -\frac{500 \times 4.77}{155.16} = 15.40 \text{ ksi} - \text{tension}$$

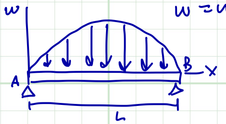
12.41 Using the method of Sec. 12.2, Solve Prob. 12.13.

12.42 Using the method of Sec. 12.2, Solve Prob. 12.14.

12.43 Using the method of Sec. 12.2, Solve Prob. 12.15.

12.44 Using the method of Sec. 12.2, Solve Prob. 12.17.

12.45 and *12.46* Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending Page 615



$$w = w_0 \sin \frac{\pi x}{L}$$

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$C_2 = 0$$

$$0 + 0 + C_1 L + 0 \quad C_1 = 0$$

a)

$M = 0$ at $x = 0$

$M = 0$ at $x = L$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

b) $\frac{dM}{dx} = V = 0$ at $x = L/2$

$$M_{max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2} = \frac{w_0 L^2}{\pi^2}$$

12.49 and 12.50 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

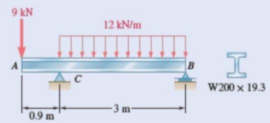


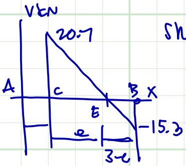
Fig. P12.49

$M_C = 0$ $(0.9)(9) - (1.5)(3)(12) + 3B = 0$

$B = 15.3$ kN

$MM_B = 0$ $(3.9)(9) - 3C + (1.5)(3)(12) = 0$

$C = 29.7$ kN



Shear A to C $V = -9$ kN

C^+ $V = -9 + 29.7 = 20.7$ kN $= V_{max}$

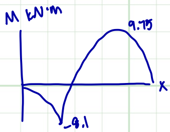
B $V = 20.7 - (3)(12) = -15.3$ kN

finding E where $V = 0$

$\frac{e}{20.7} = \frac{3-e}{15.3}$ $36e = (20.7)(3)$

$e = 1.725$ ft

$3 - e = 1.275$ ft



Areas of shear diagram

A→C $\int V dx = 0.9 \cdot 9 = 8.1$ kN·m

C→E $\int V dx = \frac{1}{2} \cdot 1.725 \cdot 20.7 = 17.853$ kN·m

E→B $\int V dx = \frac{1}{2} \cdot -1.275 \cdot 15.3 = -9.753$ kN·m

Bending moment

$M_A = 0$

$M_B = 0 - 8.1 = -8.1$ kN·m

$M_E = -8.1 + 17.8537 = 9.7537$ kN·m

$M_B = 9.753 - 9.753 = 0$

Maximum $|M| = 9.75 \times 10^3$ N·m $= 9.75$ kN·m @E

for W 200×19.3 → $S = 162 \times 10^3 \text{ mm}^3 = 162 \times 10^{-6} \text{ m}^3$

normal stress $\sigma = \frac{|M|}{S} = \frac{9.75 \text{ kN·m}}{162 \times 10^{-6}} = 60.2 \text{ MPa}$

12.88 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

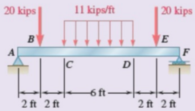
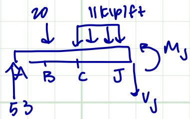


Fig. *P12.88*

By symmetry $R_A = R_F$

$\Sigma F_y = 0 \quad R_A - 20 - 6 \cdot 11 - 20 + R_F = 0$

$R_A = R_F = 50$ kips



max bending moment @ center of beam

$\Sigma M_J = 0 \quad -7 \cdot 53 + 5 \cdot 20 + 1.5 \cdot 3 \cdot 11 + M_J = 0$

$M_J = 221.5$ kip·ft $= 2658$ kip·in

$S_{min} = \frac{M}{\sigma} = \frac{2658}{24} = 110.75 \text{ in}^3$

shape	S in3
W 24×68	154
W 21×62	127 ⇒ use W 21×62
W 18×76	146
W 16×77	134
W 14×82	123
W 12×96	131

13.3 Three boards are nailed together to form a beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 175$ mm and that the allowable shearing force in each nail is 400 N, determine the allowable shear when $w = 120$ mm.

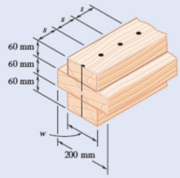


Fig. P13.3

$$\text{axis=neutral from top} = \frac{60\times120\times30 + 60\times200\times90 + 60\times120\times150}{60\times120 + 60\times200 + 60\times120}$$

$$I = \frac{bh^3}{12} + A(y-\bar{y})^2 \qquad = 90\text{mm}$$

$$I = \frac{60^3\times200}{12} + 2\left(\frac{60^3\times120}{12} + 60\times120\times60^2\right)$$

$$= 59760000\text{ mm}^4$$

$$q = \frac{VQ}{I} \quad F = 400\text{N} \quad S = 75\text{mm}$$

$$Q = A\bar{y} = 60\times120\times60 = 432000\text{ mm}^3$$

$$q = \frac{V\times432000\text{ mm}^3}{59.76\times10^6} = 7.2289\times10^{-3}V$$

$$q = \frac{F_{nail}}{s} = \frac{400}{75} = 7.2289\times10^{-3}V$$

$V = 738$N

13.9 through 13.12 For the beam and loading shown, consider section *n–n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

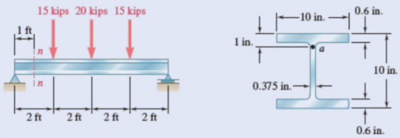


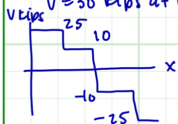
Fig. P13.9

By symmetry, $R_A = R_B$

$\Sigma F_y = 0 \quad R_A + R_B - 15 - 20 - 15 = 0$

$R_A = R_B = 25$ kips

$V = 30$ kips at n-n

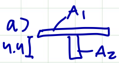


moment of inertias:

	A in²	d in	Ad in⁴	Ī in⁴
top flng	6	4.7	132.54	0.18
web	3.30	0	0	21.30
bottom flng	6	4.7	132.54	0.18
			Σ= 256.08	21.66

$I = \Sigma A d^2 + \Sigma \bar{I} = 286.74\ in^4$

a)



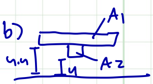
	A	$\bar{y}$	$A\bar{y}$ in²
1	6	4.7	28.2
2	1.65	2.2	3.63
			Σ = 31.83

$Q = \Sigma A y = 31.83\ in^3$

$t = 0.375$ in

$\tau_{max} = \frac{V Q_{max}}{I t} = \frac{25 \times 31.83}{286.74 \times 0.375} = 7.40$ ksi

b)



	A	$\bar{y}$ in	$A\bar{y}$ in²
1	6	4.7	28.2
2	0.15	4.2	0.63
			Σ = 28.83

$Q = \Sigma A y = 28.83\ in^3$

$t = 0.375$ in

$\tau = \frac{VQ}{It} = \frac{25 \times 28.83}{286.74 \times 0.375} = 6.70$ ksi