10.53 A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 45° with a plane parallel to the axis of the pipe. Knowing that the maximum allowable tensile stress in the weld is 12 ksi, determine the largest torque that can be applied to the pipe.

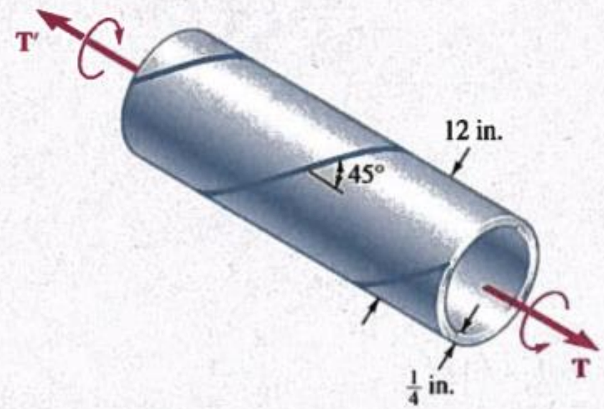
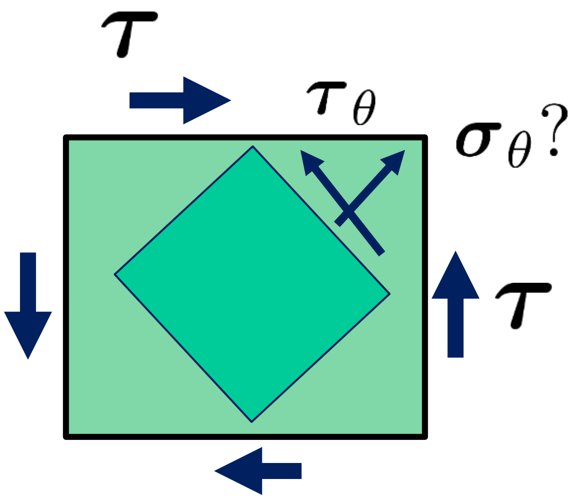
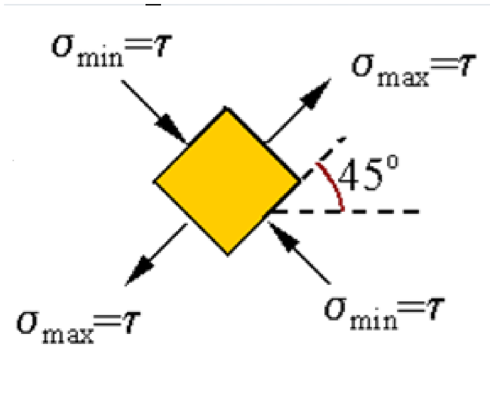


Fig. P10.53



$$\sigma_\theta = \tau \sin 2\theta$$

$$\tau_\theta = \tau \cos 2\theta;$$



$$\sigma_{45^o}^{allow} = 12ksi$$

$$\sigma_{45} = \tau_{\max}$$

hence, $$\tau_{\max} = 12 \text{ ksi} = 12 \times 10^3 \text{ psi}$$

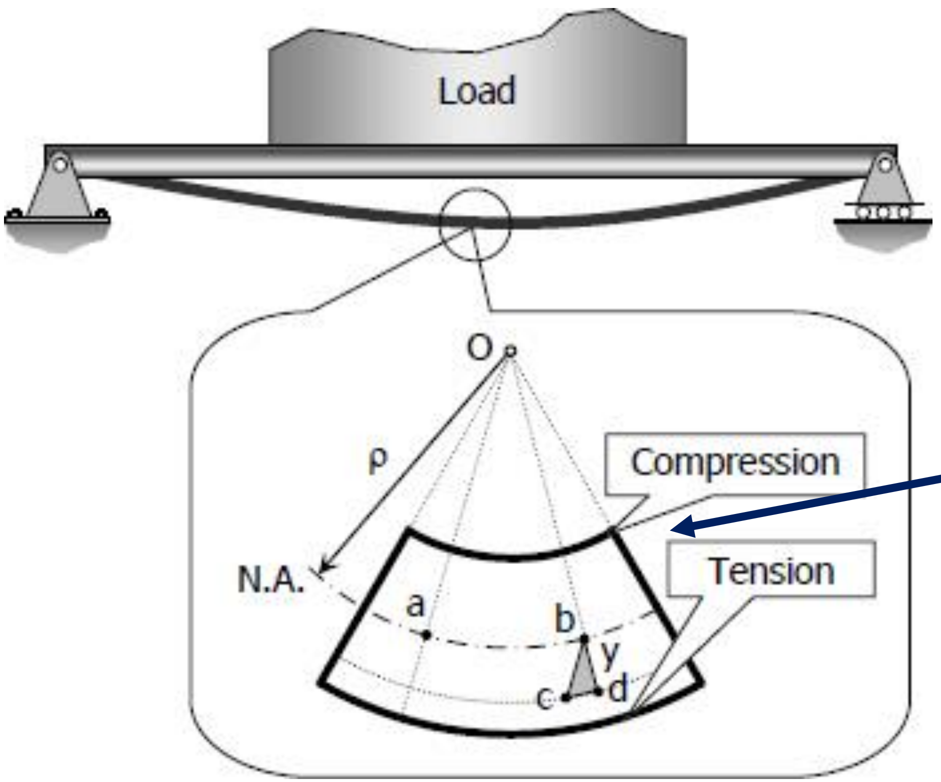
$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(12) = 6.00 \text{ in.}$$
$$c_1 = c_2 - t = 6.00 - 0.25 = 5.75 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(6.00)^4 - (5.75)^4] = 318.67 \text{ in.}$$

$$\tau_{\max} = \frac{Tc}{J} \qquad T = \frac{\tau_{\max} J}{c}$$

$$T = \frac{(12 \times 10^3)(318.67)}{6.00} = 637 \times 10^3 \text{ lb} \cdot \text{in.}$$

Lecture 27 Elastic Flexure Formula



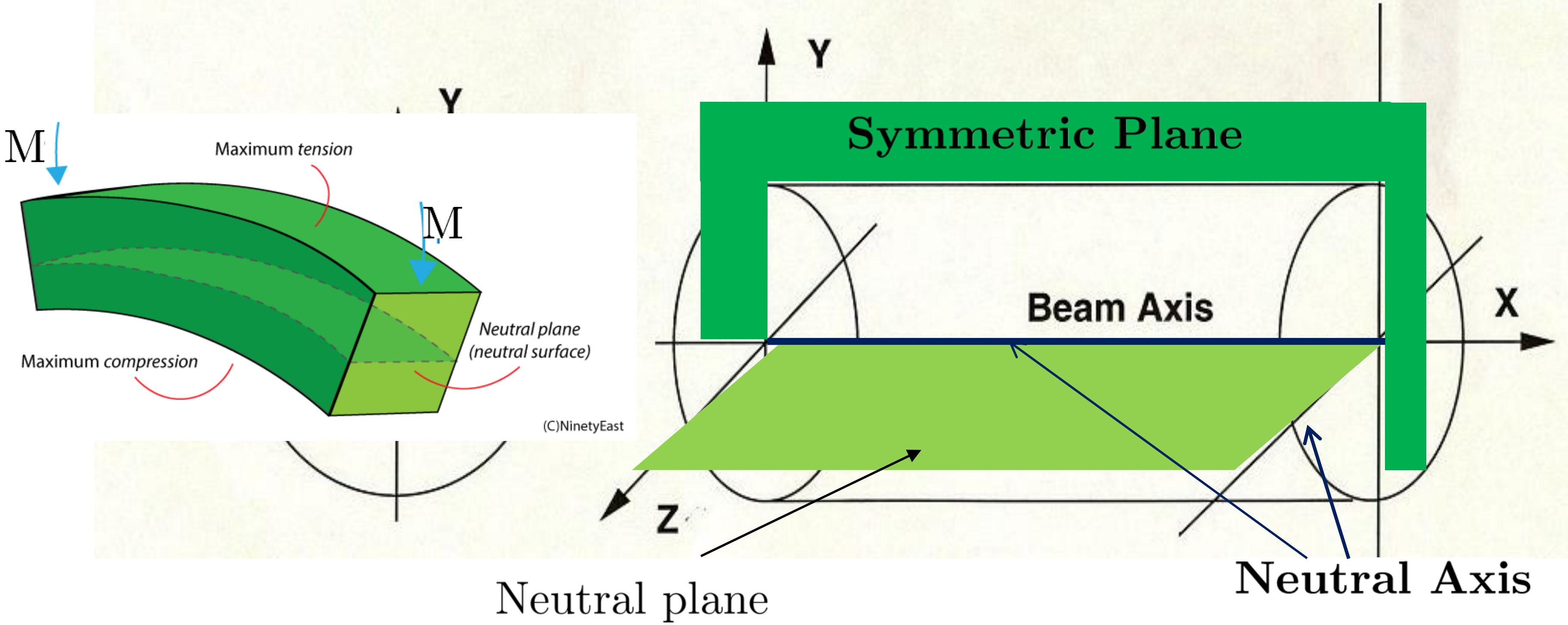
$$\sigma_x = -\frac{M_z(x)y}{I_z}$$

Assumptions of the Euler-Bernoulli Beam Theory

1. The cross section of a beam remain plane after the beam is subjected to bending (remain perpedicular to the beam axis);

2. There is no transverse interaction between horizontal fibers ($\sigma_{yy} = 0, \quad \sigma_{zz} = 0.$)

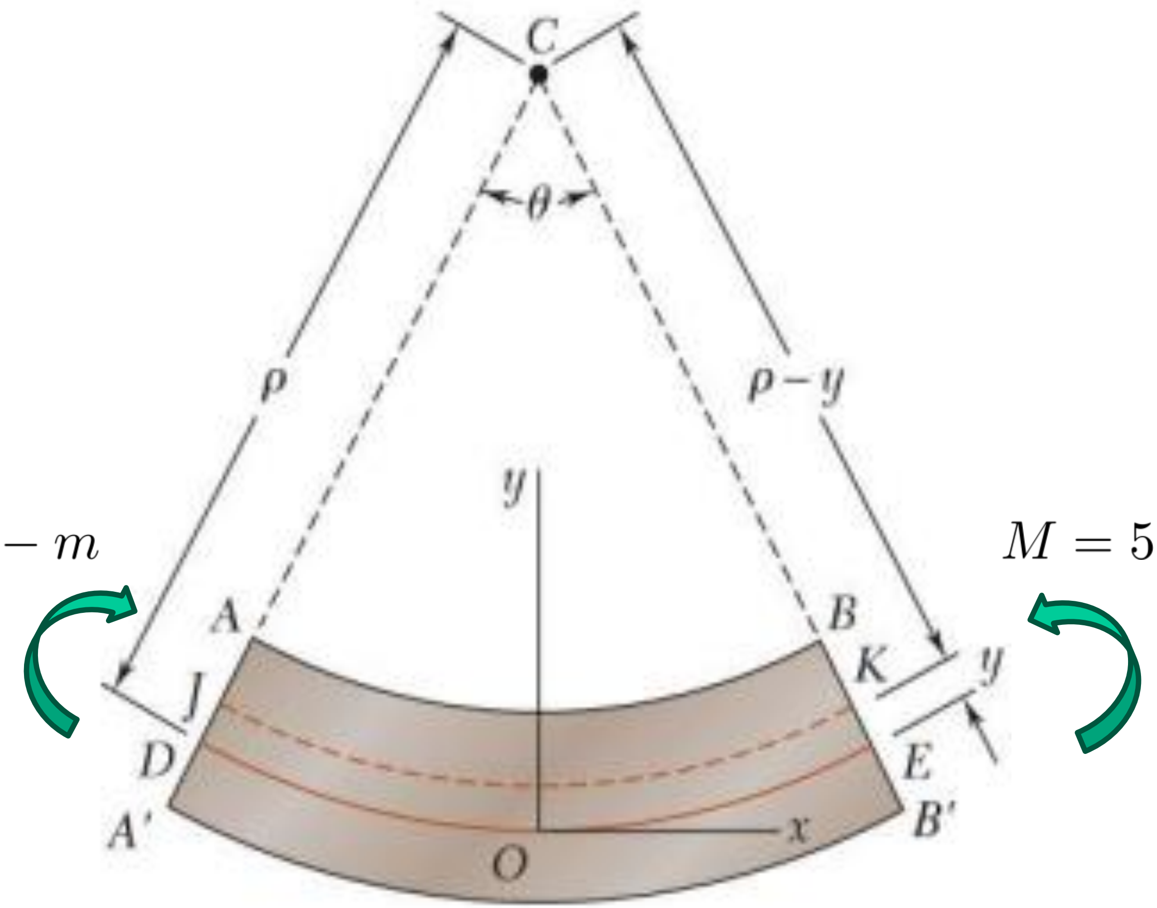
3. The beam axis is inextensible.



The purpose of the kinematic assumption is to find strain distribution, and then stress distribution, and the external loads are within the symmetric plane.

Today's lecture attendance passphrase: Pure Bending

$M = 5 \times 10^4 N - m$



$M = 5 \times 10^4 N - m$

Elastic Flexure Formula

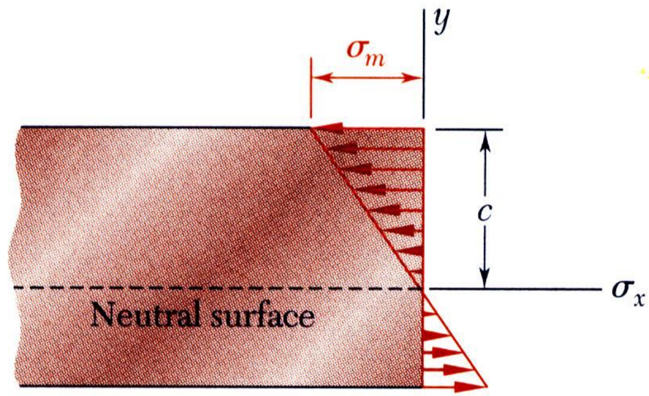
- For a linearly elastic material,

$$\sigma_x = -\sigma_m \left(\frac{y}{c}\right)$$

$$F_x = \int \sigma_x \, dA = 0$$

$$M_y = \int z\sigma_x \, dA = 0$$

$$M_z = \int -y\sigma_x \, dA = M$$



$$M_z = \int_A (-y\sigma_x)dA = \int_A (-y)\left(-\sigma_m \frac{y}{c}\right)dA$$

$$\begin{aligned} M_y &= \int_A z\sigma_x dA = -\frac{\sigma_m}{c}\int_A zy dA \\ &= 0, \end{aligned}$$

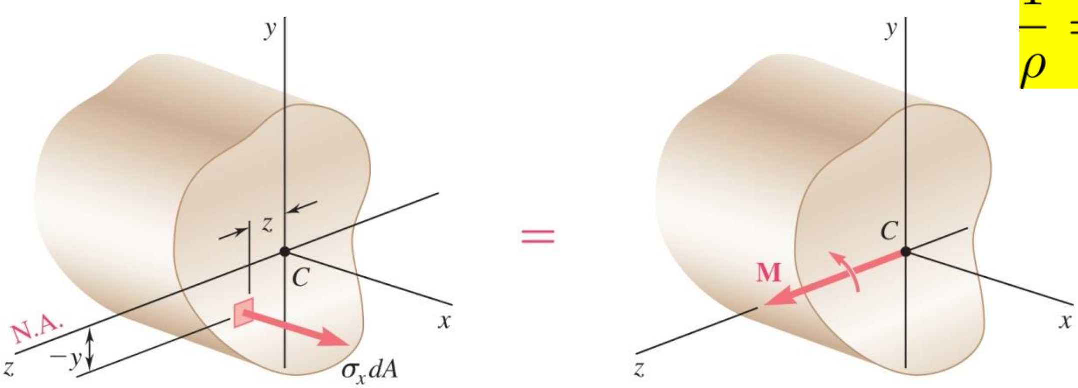
if y−z are centridoal axes.

$$M_z = \frac{\sigma_m}{c}\int_A y^2 dA = \frac{\sigma_m I_z}{c}.$$

Substituting $\dfrac{\sigma_m}{c} = \dfrac{M_z(x)}{I_z} \rightarrow \sigma_x = -\sigma_m \dfrac{y}{c}$

$$\frac{1}{\rho} = \frac{\epsilon_m}{c} = \frac{M_z}{EI_z}$$

$$\boxed{\sigma_x = -\frac{M_z(x)y}{I_z}}$$



Summary

1. Kinematic Assumption : $\epsilon_x = -\epsilon_m \left(\frac{y}{c} \right)$ $\qquad \epsilon_m = \frac{c}{\rho}$

2 From equilibrium $\rightarrow$ $\frac{\sigma_m}{c} = \frac{M_z}{I_z}$

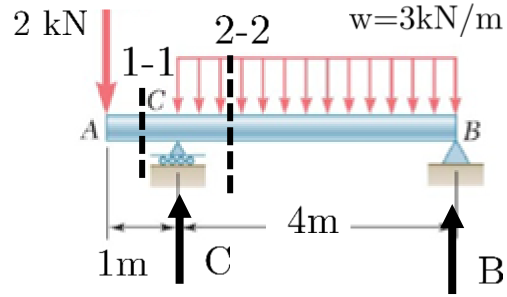
3. Elastic Flexure Formula : $\boldsymbol{\sigma}_x = -\frac{M_z y}{I_z}$

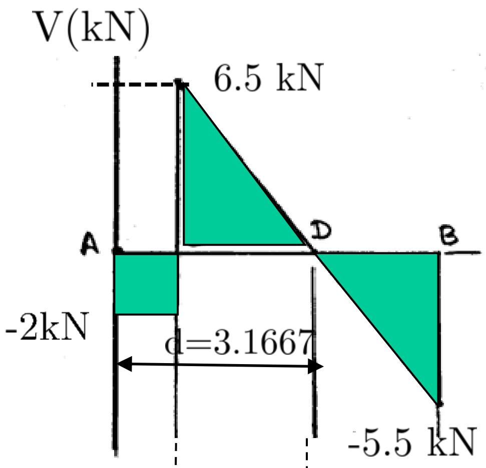
4. We define $S := I_z/c$ as the section modulus.

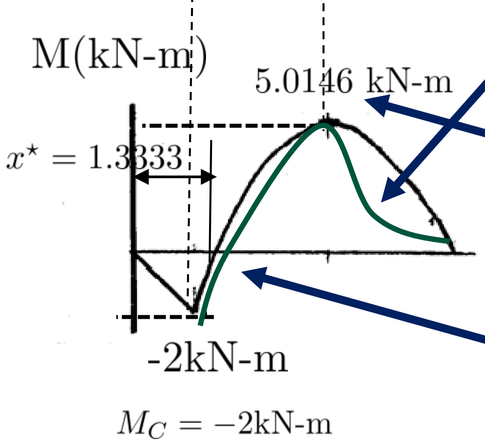
5. Curvature : $\kappa = \frac{1}{\rho} = \frac{\epsilon_m}{c}.$ $\qquad \frac{1}{\rho} = \frac{M}{EI}$

	Pure Bending	Torsion
Member	Bar (rod)	Shaft
Internal force	Bending Moment	Torque T
Constitutive law	$\sigma = E\epsilon$	$\tau = G\gamma$
Kinematic Assumption	Only allow axial deformation	The cross section of the shaft remains a plane after the twist.
Relation between internal force and stress	$\sigma = -\frac{M(x)y}{I_z}$	$\tau = \frac{T\rho}{J}$
Deformation		$\Delta\phi = \frac{TL}{GJ}$
Flexibility & Stiffness		$f = \frac{L}{GJ}$ & $k = \frac{JG}{L}$

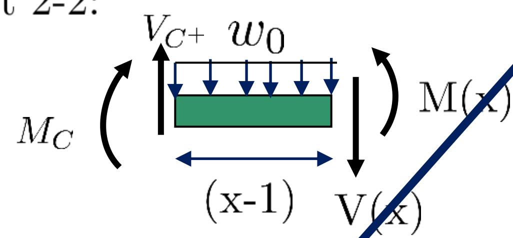
Why can we draw moment diagram like this?







Cut 2-2:



$C^+ \le x \le 5$

$$\sum F_y = 0 \;\to\; -V(x) + V_{C+} - w_0(x-1) = 0 \;\to$$

$$V(x) = 9.5 - 3x kN, \; 1^+ < x < 5$$

Let $V(d) = 9.5 - w_0 d = 0 \;\to$ $d = 3.1667m$

$d = 9.5/3 = 3.16667m$

$V_B = V(5) = 9.5 - 3 \times 5 = -5.5kN \quad V_B \neq R_B$

$$\sum M_x = 0 \;\to\; -M_c - (x-1)V_{C+} + w_0\frac{(x-1)^2}{2} + M(x) = 0 \;\to$$

$$\mathbf{M(x) = -\frac{3}{2}(x-1)^2 + 6.5(x-1) - 2}$$ kN-m

At which point $M(x) = M_{max}$?

$M(d) = -\frac{3}{2}(3.1667-1)^2 + 6.5(3.1667-1) - 2 = 5.04167$kN-m

Let $M(x^*) = -\frac{3}{2}(x^*-1)^2 + 6.5(x^*-1) - 2 = 0 \;\to\; x^* = 1.333m$

11.3 Using an allowable stress of 155 MPa, determine the largest bending moment M_x that can be applied to the wide-flange beam shown. Neglect the effect of the fillets.

$$\sigma_x = -\frac{M_z(x)y}{I_z}$$

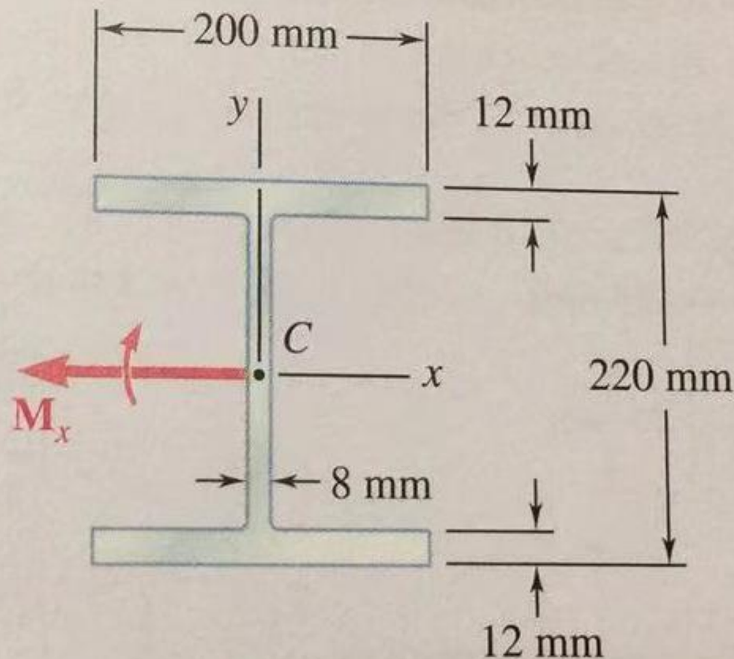
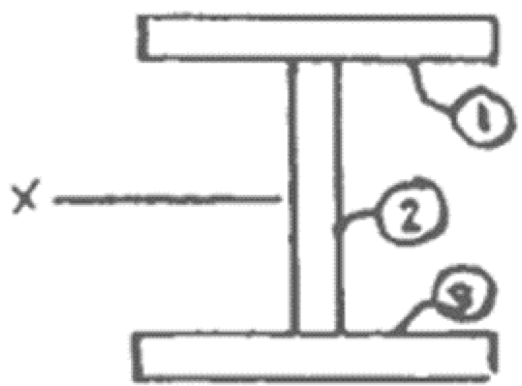
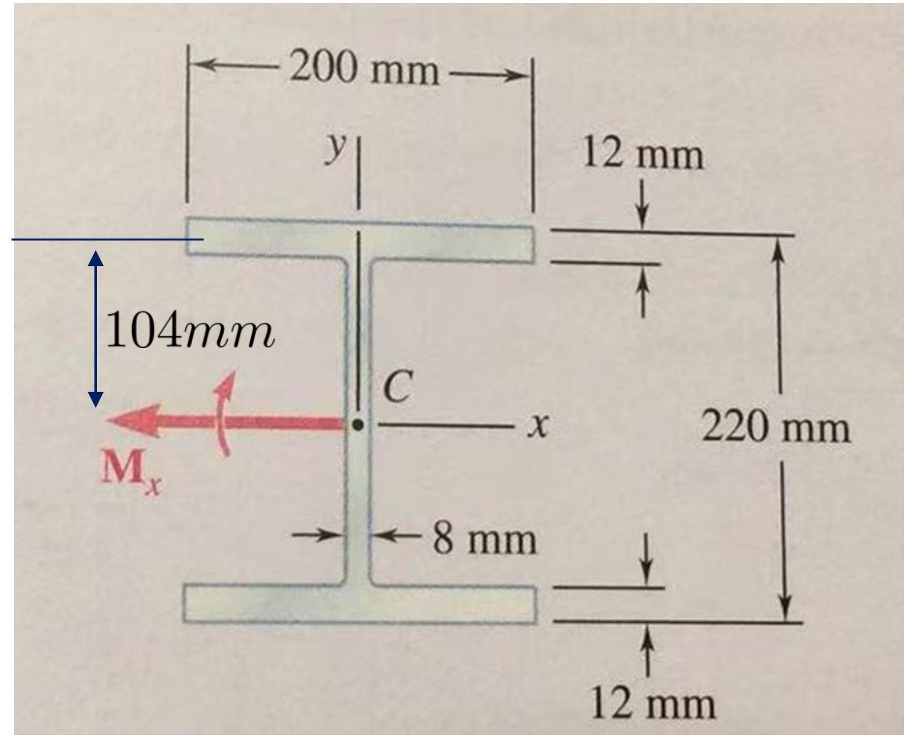


Fig. P11.3

SOLUTION

Moment of inertia about x-axis:





$$I_1 = \frac{1}{12}(200)(12)^3 + (200)(12)(104)^2$$
$$= 25.9872 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(8)(196)^3 = 5.0197 \times 10^6 \text{ mm}^4$$

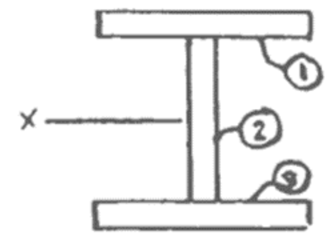
$$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4 \quad I = I_1 + I_2 + I_3 = 56.944 \times 10^6 \text{ mm}^4 = 56.944 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(220) = 110 \text{ mm} = 0.110 \text{ m}$$

$$M = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

SOLUTION

Moment of inertia about x-axis:



$$I_1 = \frac{1}{12}(200)(12)^3 + (200)(12)(104)^2$$

$$= 25.9872 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(8)(196)^3 = 5.0197 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 56.944 \times 10^6 \text{ mm}^4 = 56.944 \times 10^{-6} \text{ m}^4$$

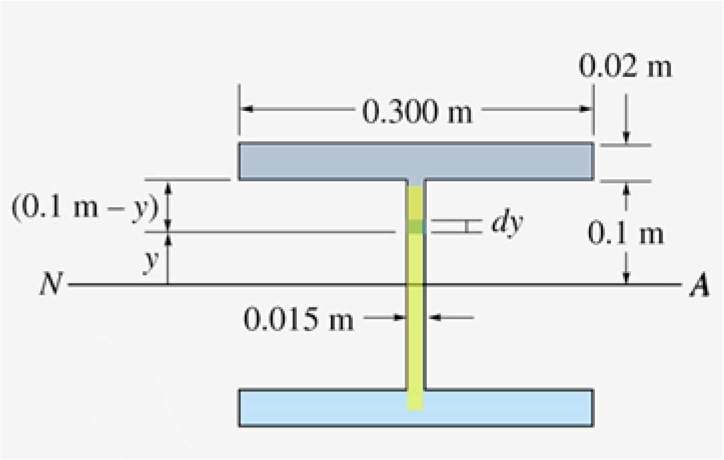
$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(220) = 110 \text{ mm} = 0.110 \text{ m}$$

$$M = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

$$M_x = \frac{(56.944 \times 10^{-6})(155 \times 10^6)}{0.110} = 80.2 \times 10^3 \text{ N} \cdot \text{m}$$

How much bending moment does the web sustain?

This Problem is Rated-R



$$\sigma_x = -\frac{My}{I_z}$$

$$\begin{aligned} M_w &= -\int_{A_w} \sigma_x y dA = \frac{M}{I_z}\int_{A_w} y^2 dA \\ &= M\frac{I_{A_w}}{I_z} \end{aligned}$$

$$\begin{aligned} I &= \frac{1}{12}(0.30)(0.24)^3 - \frac{1}{12}(0.285)(0.20)^3 \\ &= 155.6 \times 10^{-6} m^4 \end{aligned}$$

$$I_{A_w} = \tfrac{1}{12}(0.015)(0.2)^3$$
$$= 4.444 \times 10^{-6} m^4$$

$$\frac{M_w}{M} = \frac{I_{A_w}}{I_z} = \frac{4.4444}{155.6} = 2.856\%$$

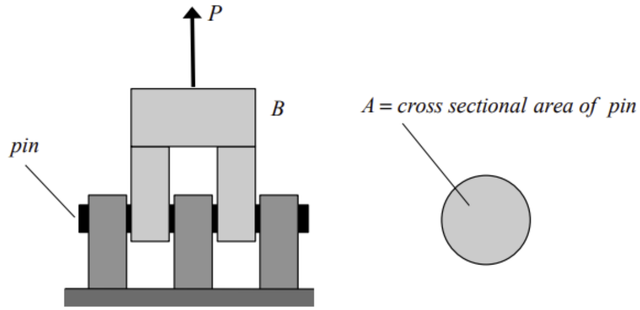
What are the takeaways?

1. **Web sustains main part of shear force, while**
2. **Flange resists main part of bending moment.**


WHO WANTS TO BE A
MILLIONAIRE

Q1. Conceptual question 3.1 Consider the hinge shown below that is supported by a single pin whose cross-sectional area is A. A load P is applied to end B of the hinge.

What is the **maximum s**hear stress in the pin?



A. $\tau = P/A$;

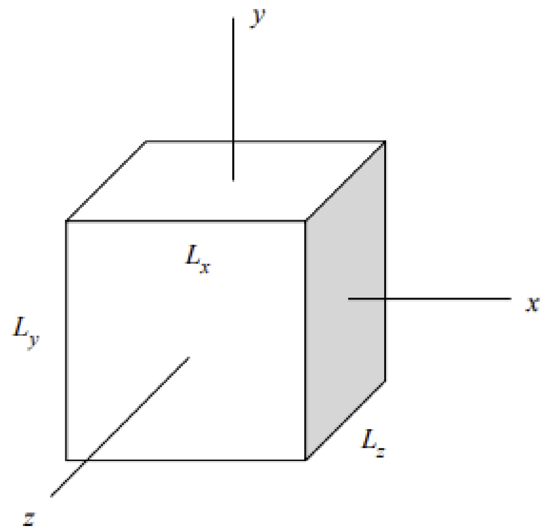
B. $\tau = P/2A$;

C. $\tau = P/6A$;

D. $\tau = P/3A$.

Ans. (D)

Q2.



Ans. (A)

A cube of dimensions $\left(L_x, L_y, L_z\right)$ experiences a state of stress with uniform components of stress though out the cube. The material of the cube has a Young's modulus of E and a Poisson's ratio of $\nu = 0.4$. As a result of the loading on the cube, it is known that $\sigma_y = \sigma_z = \sigma_x / 2 > 0$. As a result of this loading (circle the correct answer):

a) The dimension L_z is *increased*.

b) The dimension L_z remains the *same*.

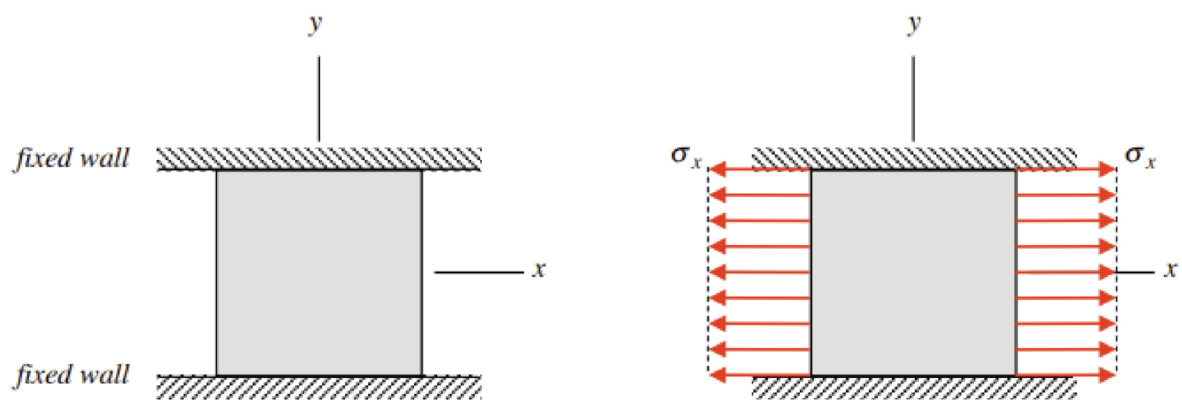
c) The dimension L_z is *decreased*.

d) More information is needed to answer this question.

$$
\begin{aligned}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{yy} &= -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{zz} &= -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}
\end{aligned}
$$

Q3.

Ans. (C)



(a)

(b)

A square homogeneous block made up of a material with a Poisson's ratio of $\nu = 0.3$ is placed between two smooth, rigid walls. Initially, the temperature of the block in Figure (a) above is increased by an amount that produces a compressive normal stress of $\sigma_y = -20\ ksi$. After that, the block is given an additional tensile stress component σ_x, as shown in Figure (b) above, with this stress, in turn, reducing the y-component of stress to $\sigma_y = -5\ ksi$. Determine the value of σ_x.

$$\epsilon_{yy} - \alpha\Delta T = -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E}$$

$$\epsilon_y = 0$$

(**A**) $\sigma_x = 40ksi$

(**B**)$\sigma_x = 20ksi$

(**C**)$\sigma_x = 50ksi$

(**D**) $\sigma_x = 30ksi$

$$(Step\ 1) \quad -\alpha\Delta T = \frac{\sigma_{yy}^1}{E}$$

$$(Step\ 2) \quad -\alpha\Delta T = -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}^2}{E}$$

$$\sigma_x?$$

Q4. When the Poisson's ratio vanishes i.e. $\nu = 0$, what is the relation between the Young's modulus E and the bulk modulus K?

(A): $E = 2K$;

(B): $E = K/2$;

(**C**) :E=3K;

(D): $E = K/3$.

Ans: (C)

$$K = \frac{E}{3(1-2\nu)}$$

Q5. When the Poisson's ratio vanishes i.e. $\nu = 0$, what is the relation between ϵ_x and stress components?

(A): $\epsilon_x = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)$;

(B): $\epsilon_x = \frac{1}{E}(\sigma_x + \sigma_z)$;

(C): $\epsilon_x = \frac{1}{E}(\sigma_x + \sigma_y)$;

(D): $\epsilon_x = \frac{\sigma_x}{E}$.

Ans: (D)

Q6. Under the plane strain condition, if $\sigma_x = -\sigma_y$, what can be said of σ_z?

(A): $\sigma_z = \sigma_x + \sigma_y$;

(B): $\sigma_z = 0$;

(C): $\sigma_z = 2\sigma_x$;

(D): $\sigma_z = -\sigma_x$.

Ans: (B)

Q7. Under the plane strain condition, for the incompressible materials i.e. $\nu = 0.5$, what can be said of σ_z?

(A) $\sigma_z = \sigma_x + \sigma_y$;

(B) $\sigma_z = (\sigma_x + \sigma_y)/2$;

(C) $\sigma_z = 0$;

(D) $\sigma_z = \sigma_x$.

Ans: (B)

Q8. When the Poisson's ratio vanishes i.e. $\nu = 0$, what is the relation between the Young's modulus E and the shear modulus G?

(**A**): $E = 2G$;

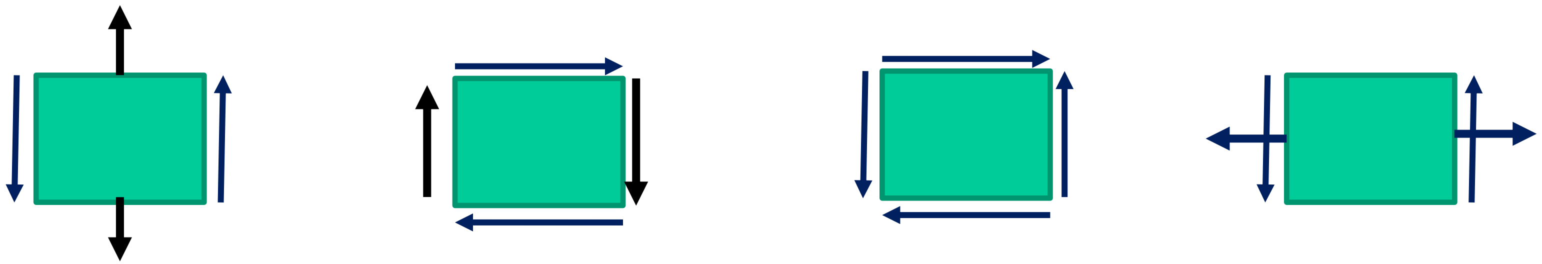
(**B**): $E = G/2$;

(**C**) :E=3G;

(**D**): $E = G/4$.

Ans: (A)

(Q9) Which of the following stress states is possible?



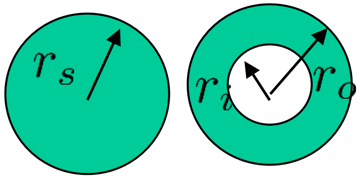
(a) $\boldsymbol{\sigma} = \begin{bmatrix} 0 & \tau \\ 0 & \sigma \end{bmatrix}$ (b) $\boldsymbol{\sigma} = \begin{bmatrix} 0 & -\tau \\ \tau & 0 \end{bmatrix}$ (c) $\boldsymbol{\sigma} = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}$ (d) $\boldsymbol{\sigma} = \begin{bmatrix} \sigma & \tau \\ 0 & 0 \end{bmatrix}$

Ans (C)

Q(10) Consider a solid cylinder and hollow cylinder that have the same cross-section area, i.e.

$$\pi r_s^2 = \pi(r_o^2 - r_i^2) \approx \pi(2r_{ave})t, \quad r_{ave} = \frac{1}{2}(r_o + r_i), t = r_o - r_i$$

Which section has the larger polar moment of inertia?



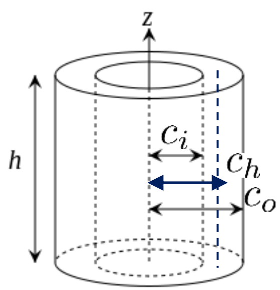
(A) The solid cylinder has larger polar moment of inertia.

(B) The hollow cylinder has larger polar moment of inertia.

(C) Both cylinders have the same polar moment of inertia.

Ans (B)

Using a hollow cylinder !



$$J_h = \frac{\pi}{2}(c_o^4 - c_i^4), \quad A_h = \pi(c_o^2 - c_i^2), \quad 2c_h = (c_0 + c_i)$$

$$\text{and} \;\; 2c_h^2 \approx c_i^2 + c_0^2$$

$$\text{If} \;\; c_o = c_i + t, \;\; \rightarrow \;\; J_h = \frac{\pi}{2}(c_o^2 + c_i^2)(c_o + c_i)t \approx 2\pi c_h^3 t; \;\; A_h \approx 2\pi c_h t$$

If we have a solid cylinder with radius c_s, we let $A_s = A_h$, i.e.

$$\pi c_s^2 = 2\pi c_h t \;\; \rightarrow \;\; c_s^2 = 2c_h t, \;\; \rightarrow \;\; c_h = \frac{2c_s^2}{t}$$

$$t << 1 \;\; \rightarrow \;\; c_h >> c_s \quad \Rightarrow \quad \frac{J_h}{J_s} = \frac{2\pi c_h^3 t}{\pi c_s^4/2} = \frac{32 c_s^2}{t^2} >> 1$$

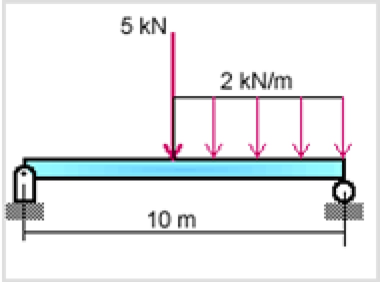
$$\frac{dV}{dx} = -w(x) \longrightarrow V(x) - V(0) = -\int_0^x w(x)dx$$

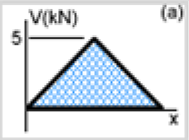
$$\frac{dM}{dx} = V(x) \longrightarrow M(x) - M(0) = \int_0^x V(x')dx'$$

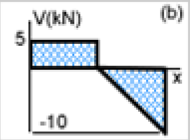
Question 11

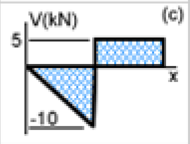
1

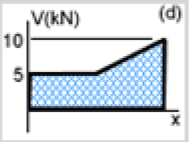
A 1 m-long beam has a load of 5 kN applied at its center and a 2 kN/m distributed load applied to the right half as shown here. The correct shear diagram for this beam is:



A) 

B) 

C) 

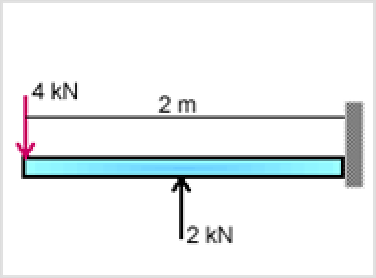
D) 

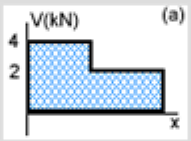
Ans (B)

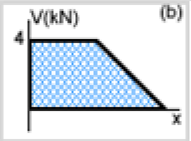
Question 12

3

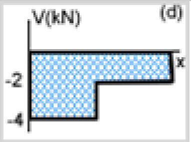
A 2-m-long, cantilevered beam has a 4 kN load applied at its free end and a 2 kN load applied at the beam midpoint as illustrated here. The correct shear diagram for this beam is:



A) 

B) 

C) 

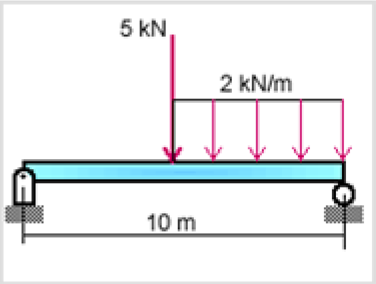
D) 

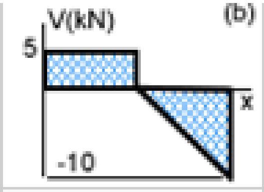
Ans: (D)

Question 13

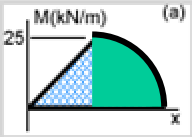
2

A 1 m-long beam has a load of 5 kN applied at its center and a 2 kN/m distributed load applied to the right half as shown here. The correct moment diagram for this beam is:

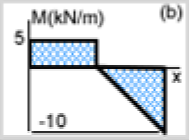




A)

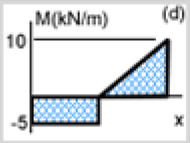


B)



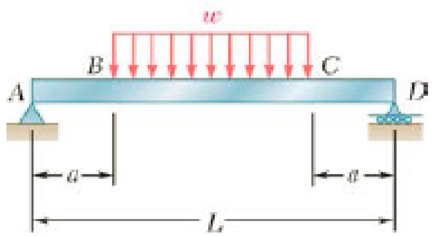
C)

D)



Ans: (E)

(E) None of the above.

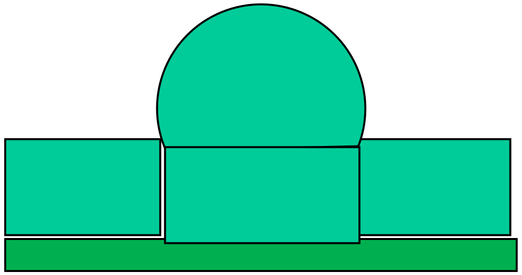


PROBLEM 12.4

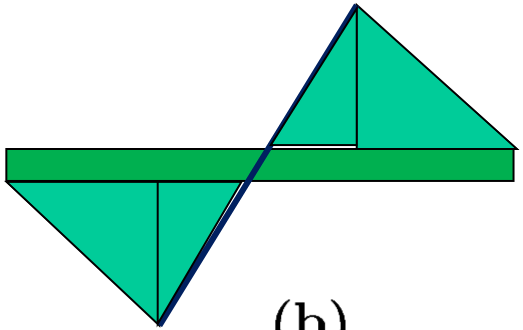
Question 14

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

Which is the correct shear diagram ?

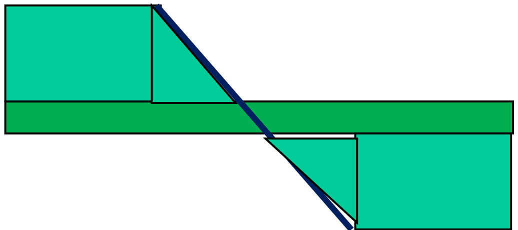


(a)



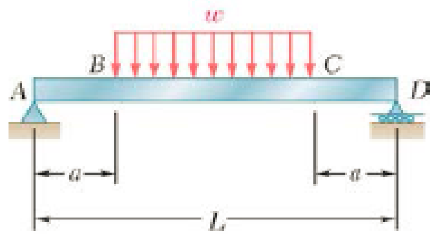
(b)

Ans: (c)



(c)

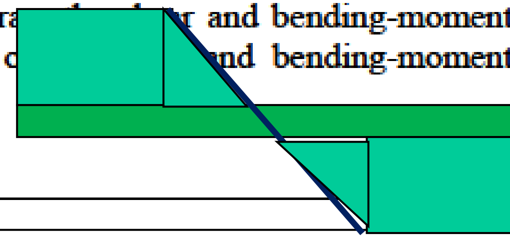
(d)



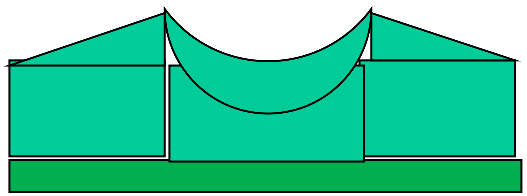
PROBLEM 12.4

Question 15

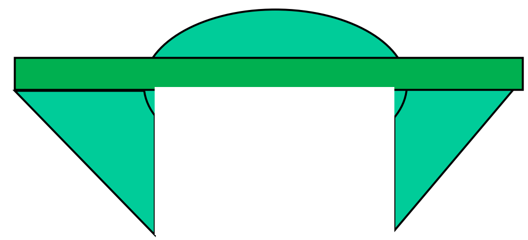
For the beam and loading shown, (*a*) dra... r and bending-moment diagrams, (*b*) determine the equations o... nd bending-moment curves.



Which is the correct moment diagram ?

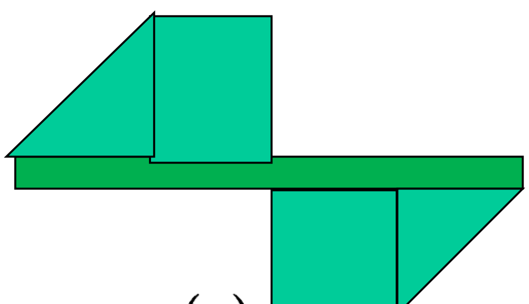


(a)

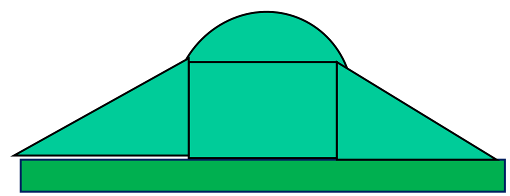


(b)

Ans: (d)



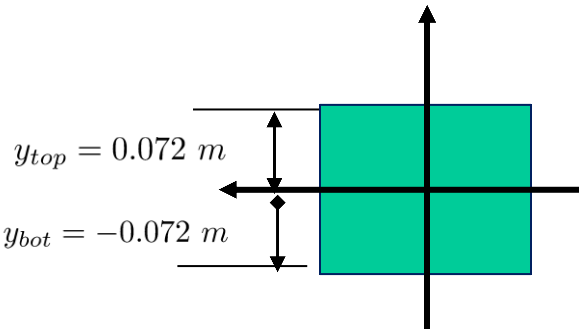
(c)



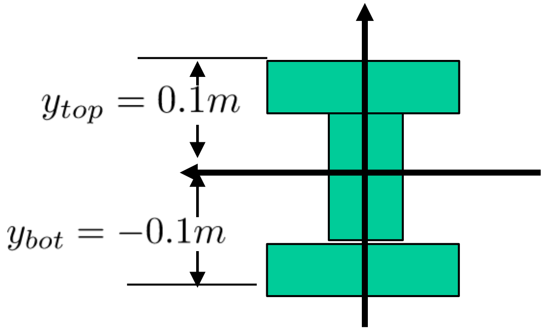
(d)

Question 16

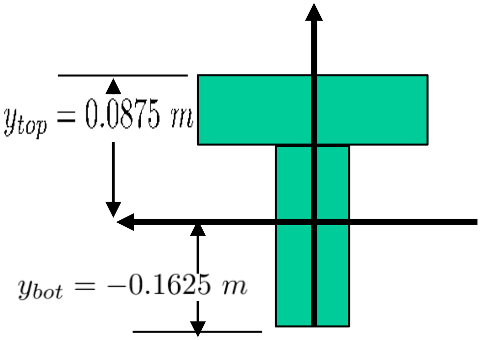
Three beams has the same length, same area of section, and the same maximum bending moment (assumed to be negative)



$I_z = 3.33 \times 10^{-5} m^4$



$I_z = 11.03 \times 10^{-5}\ m^4$



$I_z = 11.3512 \times 10^{-5} m^4$

Which beam has the smallest tensile stress ?

(a) Sqaure beam;

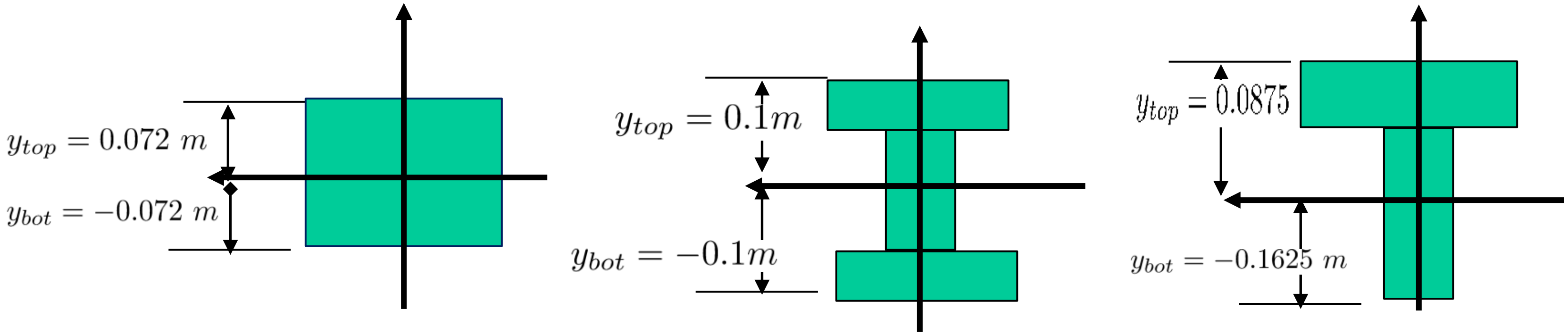
(b) I-beam;

(c) T-beam

Ans: (c)

Question 17

Three beams has the same length, same area of section, and the same maximum bending moment (assumed to be negative)



$I_z = 3.33 \times 10^{-5} m^4$

$I_z = 11.03 \times 10^{-5}\ m^4$

$I_z = 11.3512 \times 10^{-5} m^4$

Which beam has the smallest compressive stress ?

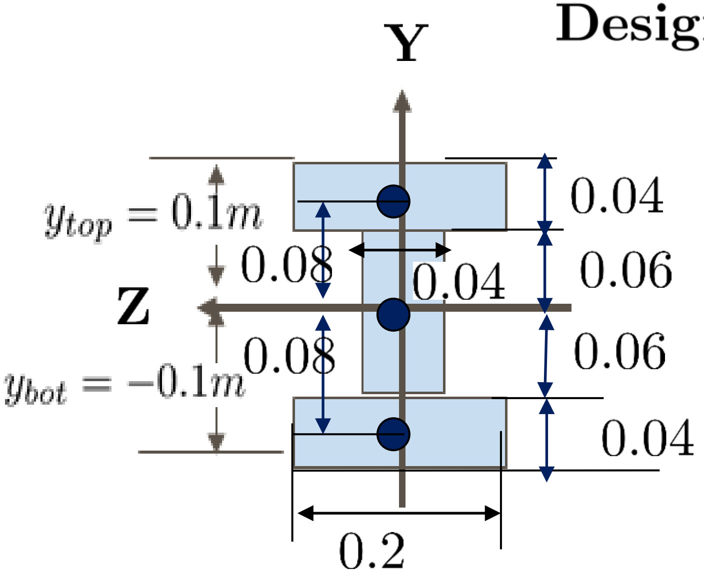
(a) Sqaure beam;

(b) I-beam

(c) T-beam

Ans: (b)

Design II: I-beam (two flange and a web)



$$\begin{aligned} A &= 2 \times (0.2 \times 0.04) + 0.04 \times 0.12 \\ &= 0.0208 \approx 0.02 m^2 \end{aligned}$$

$$I_z = 2I_f + I_w$$

$$I_w = \frac{1}{12}(0.04) \times (0.120)^3 = 0.576 \times 10^{-5} m^4$$

$$\begin{aligned} I_f &= I_{fc} + d^2 A = \frac{0.2 \times (0.04)^3}{12} + (0.08)^2 \times (0.04) \cdot (0.2) \\ &= 5.227 \times 10^{-5} m^4 \end{aligned}$$

$$I_z = 2I_f + I_w = 2 \times 5.227 \times 10^{-5} + 0.576 \times 10^{-5} = 11.03 \times 10^{-5} m^4$$

At the top surface

y_{top}

$$\sigma_{max} = -\frac{(-5 \times 10^4) \times (0.1)}{11.03 \times 10^{-5}} = 45.33 MP_a, \quad \text{in tension}$$

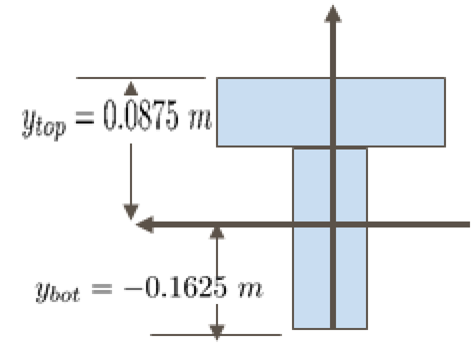
At the bottom surface

y_{bot}

54.94 % reduction

$$\sigma_{max} = -\frac{(-5 \times 10^4) \times (-0.1)}{11.03 \times 10^{-5}} = -45.33 MP_a, \quad \text{in compression}$$

T-beam design (Cont'd)



$$I_z = 11.35 \times 10^{-5} m^4$$

At the top surface

$$\sigma_{max} = -\frac{(-5 \times 10^4) \times (0.0875)}{11.35 \times 10^{-5}} = 38.54 MP_a, \quad \text{in tension}$$

61.19 % reduction

At the lower surface

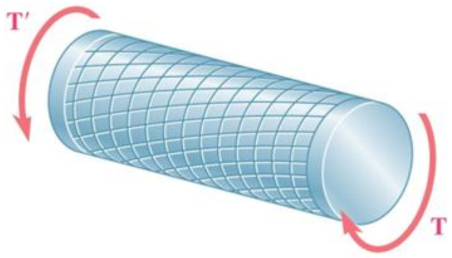
$$\sigma_{max} = -\frac{(-5 \times 10^4) \times (0.1625)}{11.35 \times 10^{-5}} = -71.58 MP_a, \quad \text{in tension}$$

28.85 % reduction

$$S = \frac{I}{c} = \frac{11.35 \times 10^{-5}}{0.0875} = 1.297 \times 10^{-3} m^3$$

Question 18

A circular shaft is subjected to a constant torque. Where is the location in a circular section where the maximum shear stress occurs?



(A) At the center of the circular section;
(B) At the periphery of the circular section.
(C) At the location that the radial distance of ¼ of the radius c.
(D) Along the middle horizontal line of the circular section.

Ans. (B)

Question 19

A rectangular beam is subjected to pure bending, with a positive bending moment. Where is the location in a cross-section where the maximum extension stress occurs?

(A) At the neutral axis.
(B) At the upper surface.
(C) At the lower surface.
(D) At the centroid of the section.

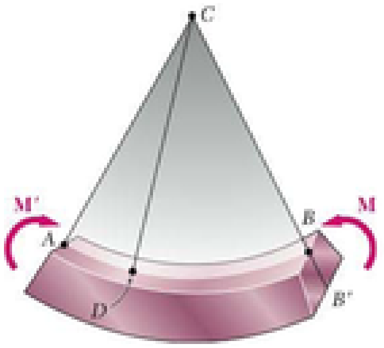


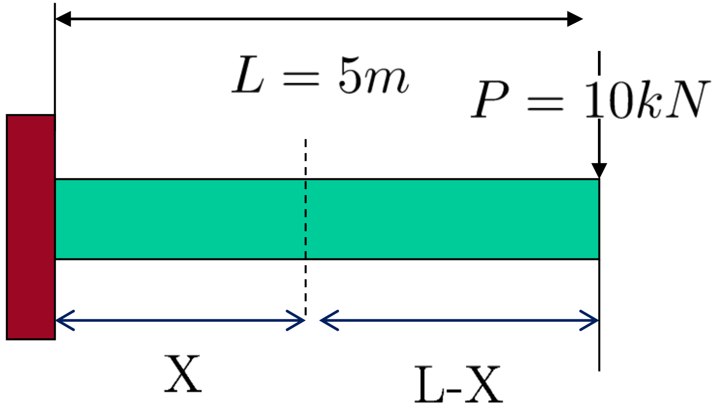
Fig.9

Ans: (C)

Question 20

For a cantilever beam whose tip is subjected to a vertical load, Which section of the beam has the maximum bending moment?

(A) At the tip of the beam.
(B) At the fixed end of the beam.
(C) In the middle section of the beam.
(D) Every section has the same amount of bending moment.

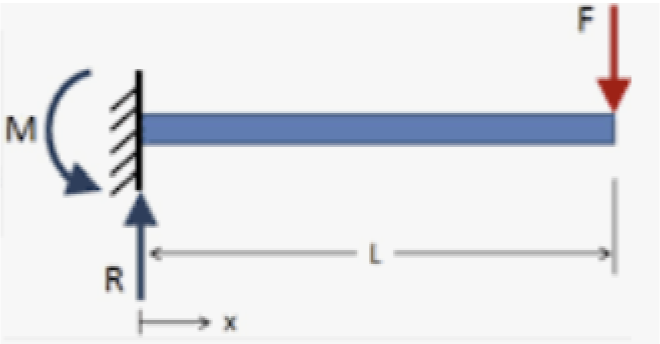


Ans. (B)

Question 21

For a cantilever beam whose free end is subjected to a vertical load, which section of the beam has the maximum shear force?

(A) At the tip of the beam.
(B) At the fixed end of the beam.
(C) In the middle section of the beam.
(D) Every section has the same amount of shear force

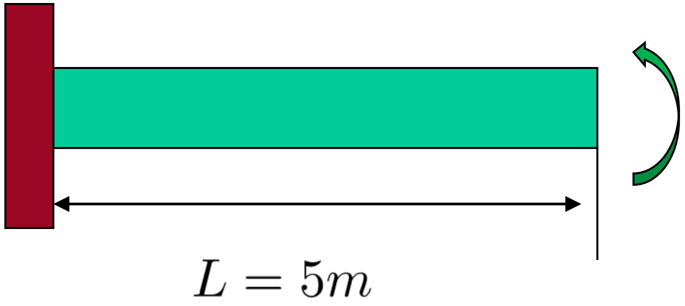


Ans. (D)

Question212

For a cantilever beam whose free end is subjected to an external bend moment, which section of the beam has the maximum shear force?

(A) At the tip of the beam.
(B) At the fixed end of the beam.
(C) Every section has the same amount of shear force.
(D) In the middle section of the beam.



Ans. (C)


Happy Spring Break !