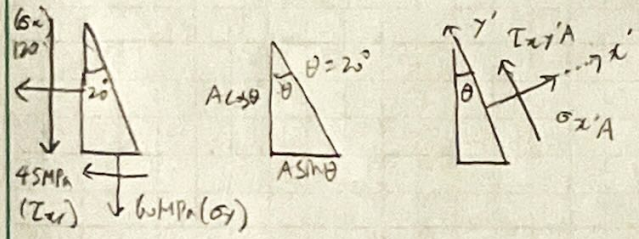
HW 13

P14.2



$\Sigma F_{x'} = 0 = \sigma_{x'}A - (\sigma_x A\cos\theta)\cos\theta - (\tau_{xy}A\cos\theta)\sin\theta - (\tau_{xy}A\sin\theta)\cos\theta - (\sigma_y A\sin\theta)\sin\theta)$

$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$\sigma_{x'} = \frac{(120+60)}{2} + \frac{(120-60)}{2}\cos 40° + 45\sin 40° = 14.19\ MPa$

$\Sigma F_{y'}: \tau_{x'y'}A + (\sigma_x A\cos\theta)\sin\theta - (\tau_{xy}A\cos\theta)\cos\theta + (\tau_{xy}A\sin\theta)\sin\theta - (\sigma_y A\sin\theta)\cos\theta = 0$

$\tau_{x'y'} = \tau_{xy}(\cos^2\theta - \sin^2\theta) + \sigma_y\sin\theta\cos\theta - \sigma_x\cos\theta\sin\theta$

$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -\frac{(120-60)}{2}\sin 40° + 45\cos 40° = 15.19\ MPa$

$\sigma = 14.19\ MPa$

$\tau = 15.19\ MPa$

P14.7

$\sigma_x = 18\ ksi \quad \sigma_y = -12\ ksi \quad \tau_{xy} = 8\ ksi$

a) $\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x-\sigma_y)} = \frac{2(8)}{18-(-12)} = \frac{8}{15}$

$2\theta_p = 28.07° \Rightarrow \theta_p = 14.04°$

Principle Planes: 14.04°, 104.0°

b) $\sigma_{max/min} = \frac{\sigma_x+\sigma_y}{2} \pm R = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_{max} =$ 20.0 ksi $\quad \sigma_{min} =$ -14.00 ksi

P14.23

$r_o = 51mm$, $t = 6mm$

$J = \frac{\pi}{2}(r_o^4 - (r_o - t)^4) = 4.19 \times 10^{-6} m^4$

$I = \frac{1}{2}J = 2.09 \times 10^{-6} m^4$

$F_x = 10kN$

$M_y = (10 \times 10^3 N)(0.200m) = 2000 N \cdot m$

$M_z = -(10 \times 10^3 N)(0.150m) = -1500 N \cdot m$

$T = M_y = 2000 N \cdot m$

$c = r_o = 51mm$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000 N \cdot m)(51 \times 10^{-3} m)}{4.19 \times 10^{-6} m^4} = 24.4 MPa$

$\sigma_y = \frac{|M_z|c}{I} = \frac{(1500 N \cdot m)(51 \times 10^{-3} m)}{2.09 \times 10^{-6} m^4} = 36.6 MPa$

$\sigma_x = 0$

$\sigma_y = 36.6 MPa$

$\tau_{xy} = 24.4 MPa$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 MPa$

$R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}} = 30.5 MPa$

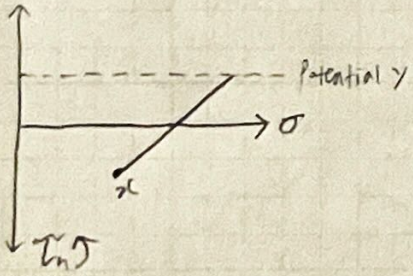
$\sigma_{max} = 12.18 MPa$

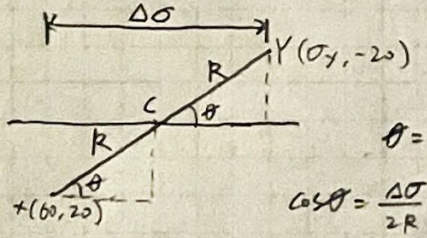
$\sigma_{min} = -48.7 MPa$

$\tau_{max} = 30.5 MPa$

P14.40

$\tau_{max} = R = 75 MPa$ $\sigma_x = 60 MPa$ $\tau_{xy} = 20 MPa$ $X(60, 20)$ $Y(\sigma_y, -20)$





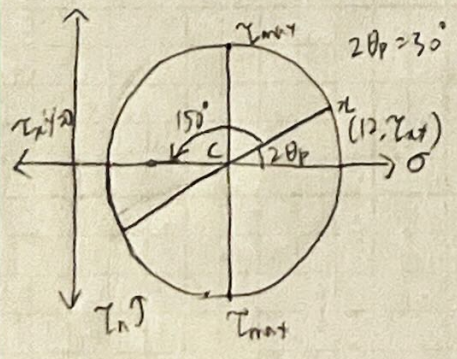
$\theta = \tan^{-1}\left(\frac{\tau_{xy}}{R}\right) = 14.93°$

$\cos\theta = \frac{\Delta\sigma}{2R} \Rightarrow \Delta\sigma = 2R\cos\theta = 144.9 MPa$

$\sigma_y = \sigma_x + \Delta\sigma = 205 MPa$

P14.43

$X: (12, \tau_{xy})$ $Y: (2, -\tau_{xy})$ $\theta = 75°$



a) $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$\tau_{xy} = \frac{(\sigma_x - \sigma_y)}{2}\tan 2\theta_p = 2.89 MPa$

$\tau_{xy} = -2.89 MPa$

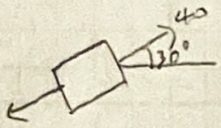
b) $\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 7 MPa$

$R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} = 5.77 MPa$

$\sigma_{max} = 12.77 MPa$

$\sigma_{min} = 1.226 MPa$

P14.46



$\sigma_{x'} = 40\,MPa$

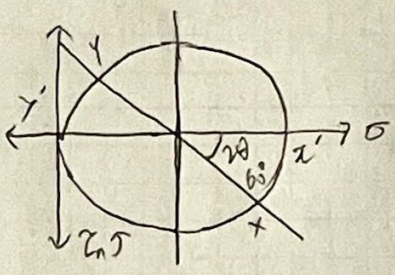
$\sigma_{y'} = 0$

$\tau_{x'y'} = 0$

$\sigma_{ave} = 20\,MPa$

$$R = \sqrt{\frac{(\sigma_{x'} - \sigma_{y'})^2}{4} + \tau_{x'y'}^2}$$

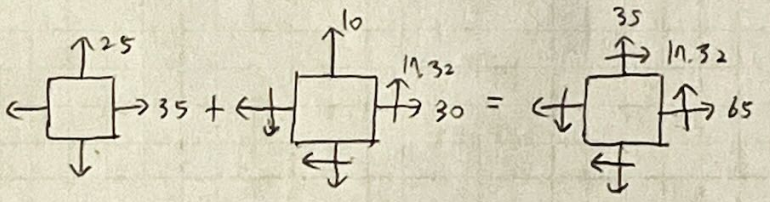
$= 20\,MPa$



$\sigma_x = R(1+\cos 2\theta) = 30\,MPa$

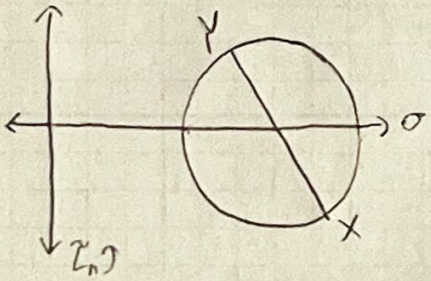
$\sigma_y = R(1-\cos 2\theta) = 10\,MPa$

$\tau_{xy} = R\sin 2\theta = 17.32\,MPa$



$X_1: (65, 17.32\,MPa)$

$Y_1: (35, -17.32\,MPa)$



$$2\theta_p = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$2\theta_p = 49.1° \Rightarrow \theta_p = 24.6°$

Principle Planes: 24.6°, 114.6°

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 50\,MPa$$

$$R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} = 22.9\,MPa$$

$\sigma_{max} = 72.9\,MPa$

$\sigma_{min} = 27.1\,MPa$