Materials Torsion Testing

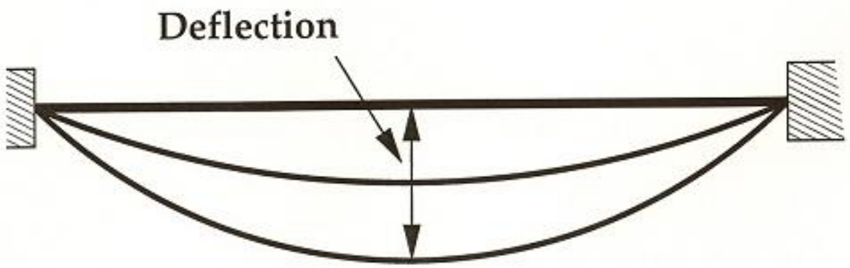
Lecture 24 Equilibrium of Beams

Definitions:

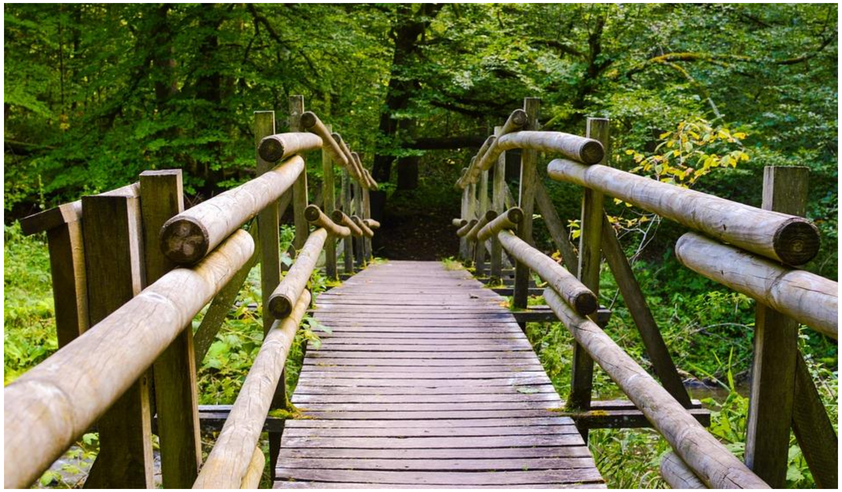
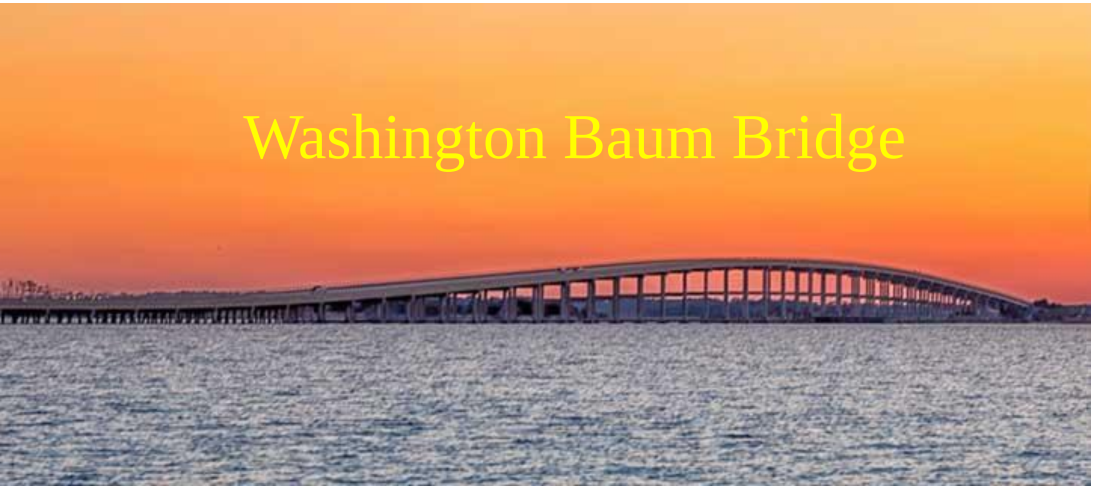
Beam is a **slender** structural member that can resists lateral or transverse load (force), or a member that is deformable laterally or has **transverse deformation**. The word "beam" comes from the word "baum" in German, which means a tree, or the trunk of a tree.

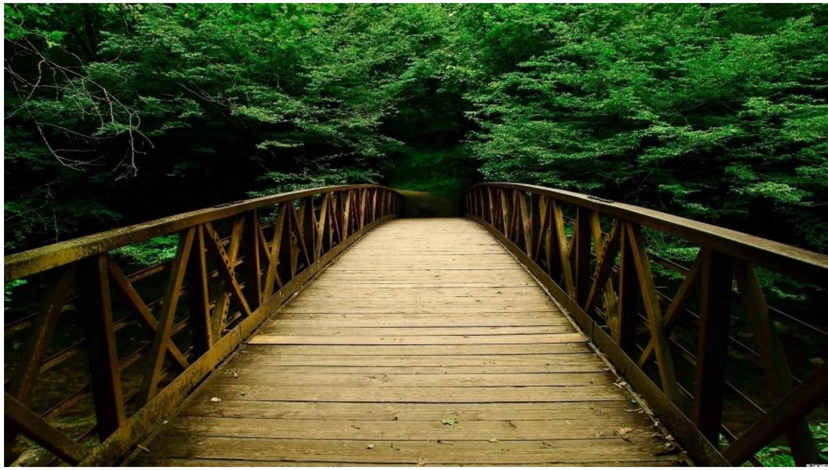
Slender: it means that the aspect ratio of the longitudinal dimension, L, and transverse dimension, b, is very large $\frac{L}{b} >> 10$.

Transverse deformation: is referred to the transverse displacement, which is usually called **deflection**.

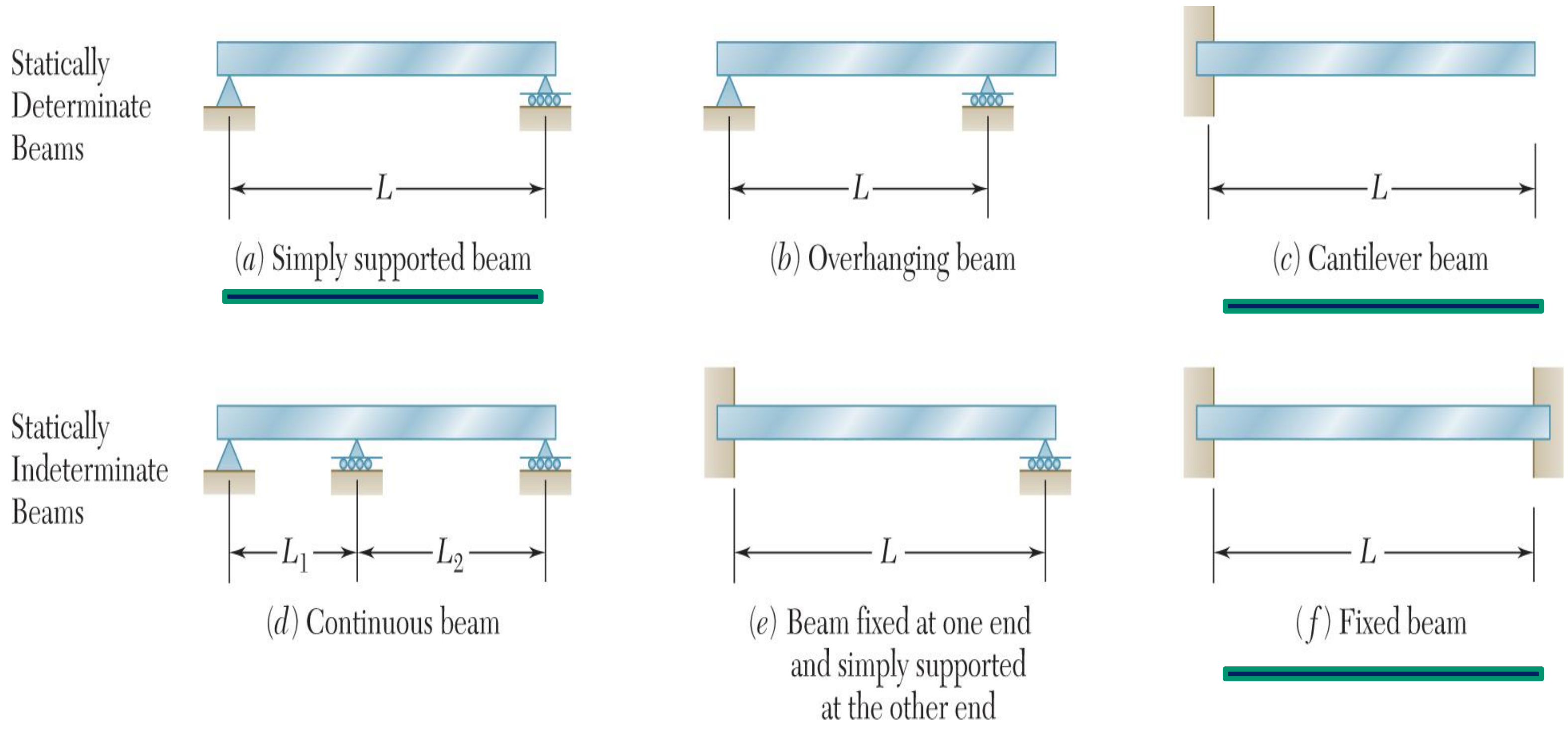


Examples of “Baum”



Classification of Beam Supports

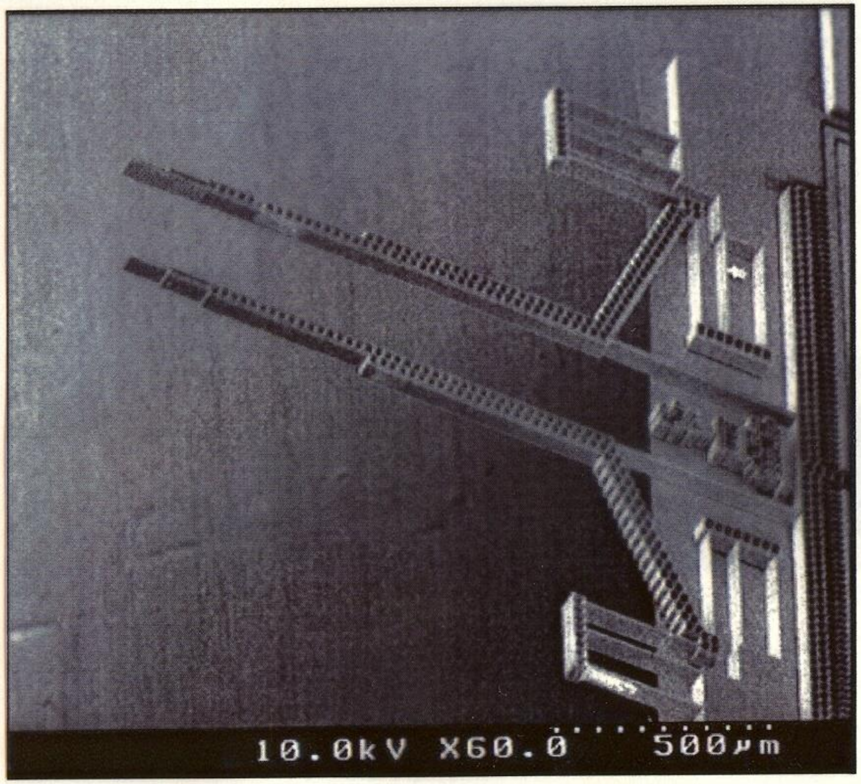


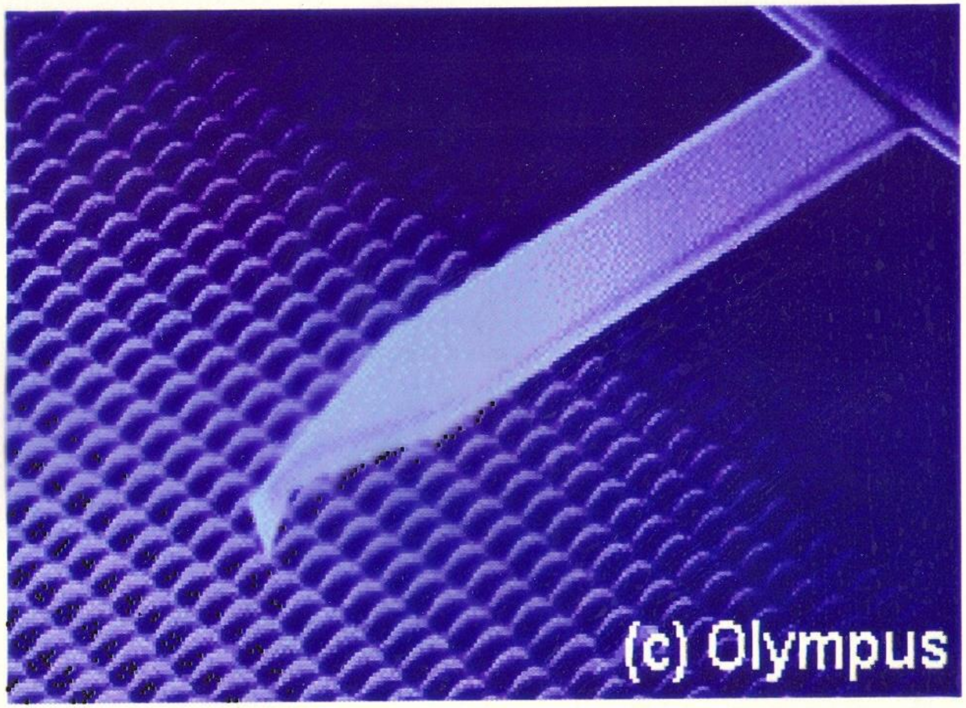
Examples of Beams (I) Boeing 777

Two cantilever beams

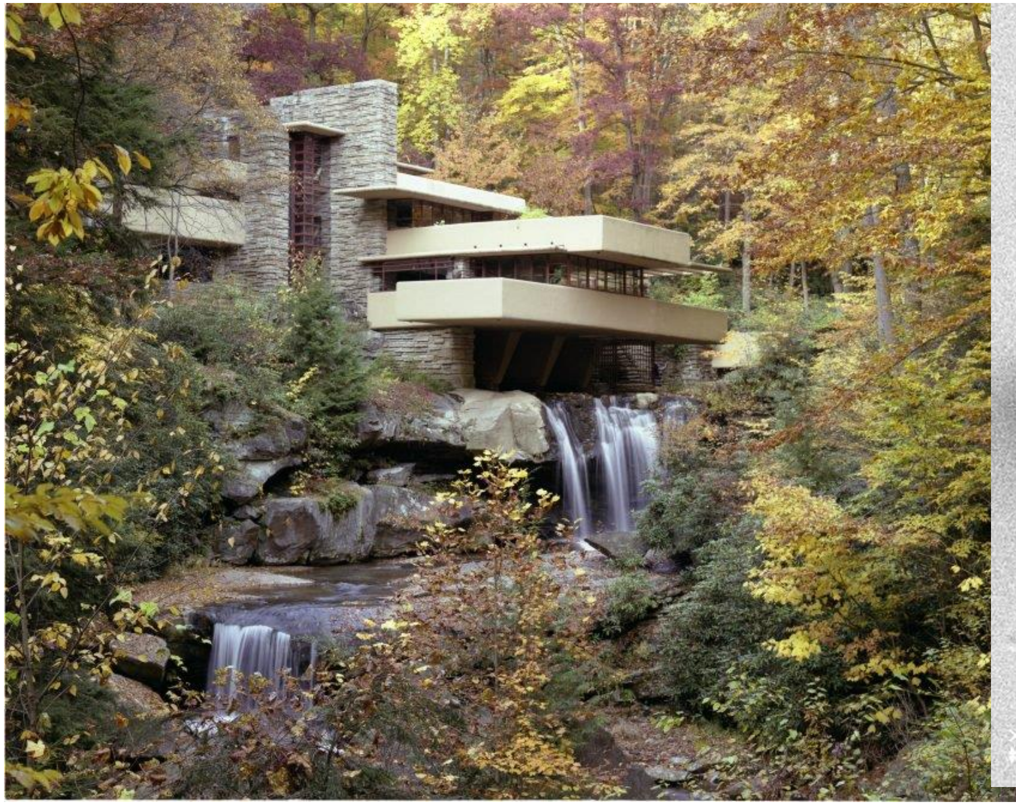
The No.1 landmark in bay area is a BEAM !

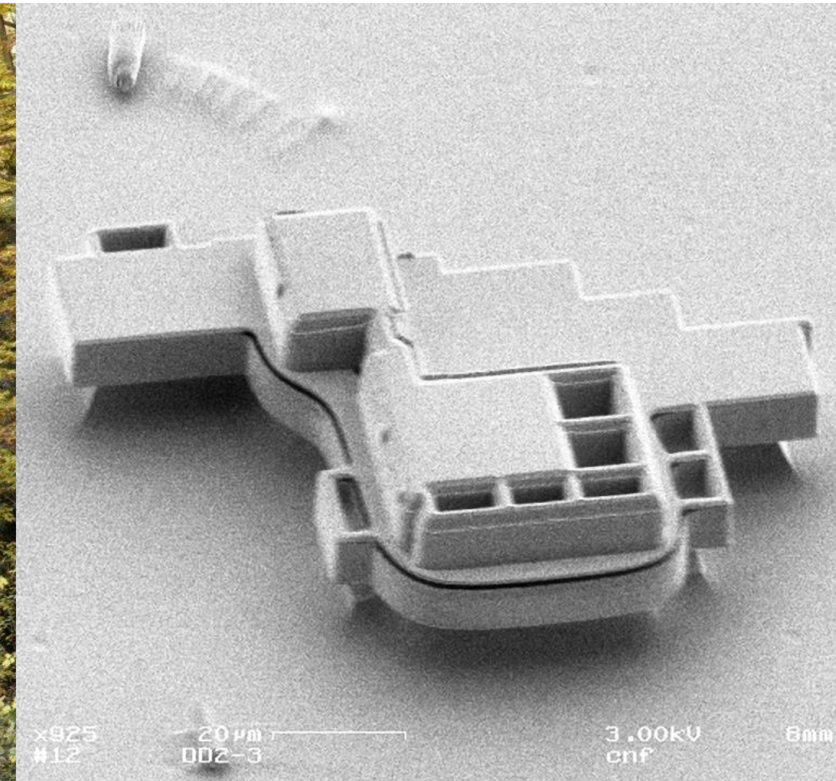
Golden Gate Bridge





Minute Cantilevers: (a) Nanomanipulators; (b) Atom force microscope (AFM).

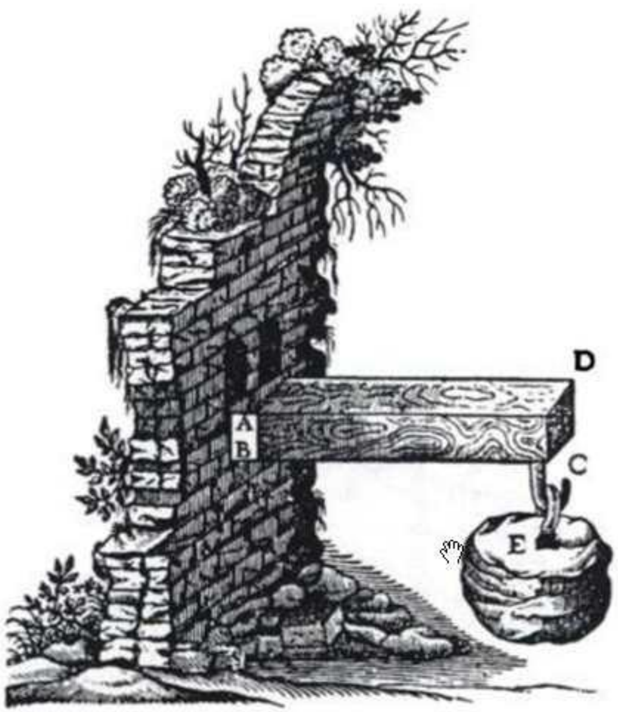




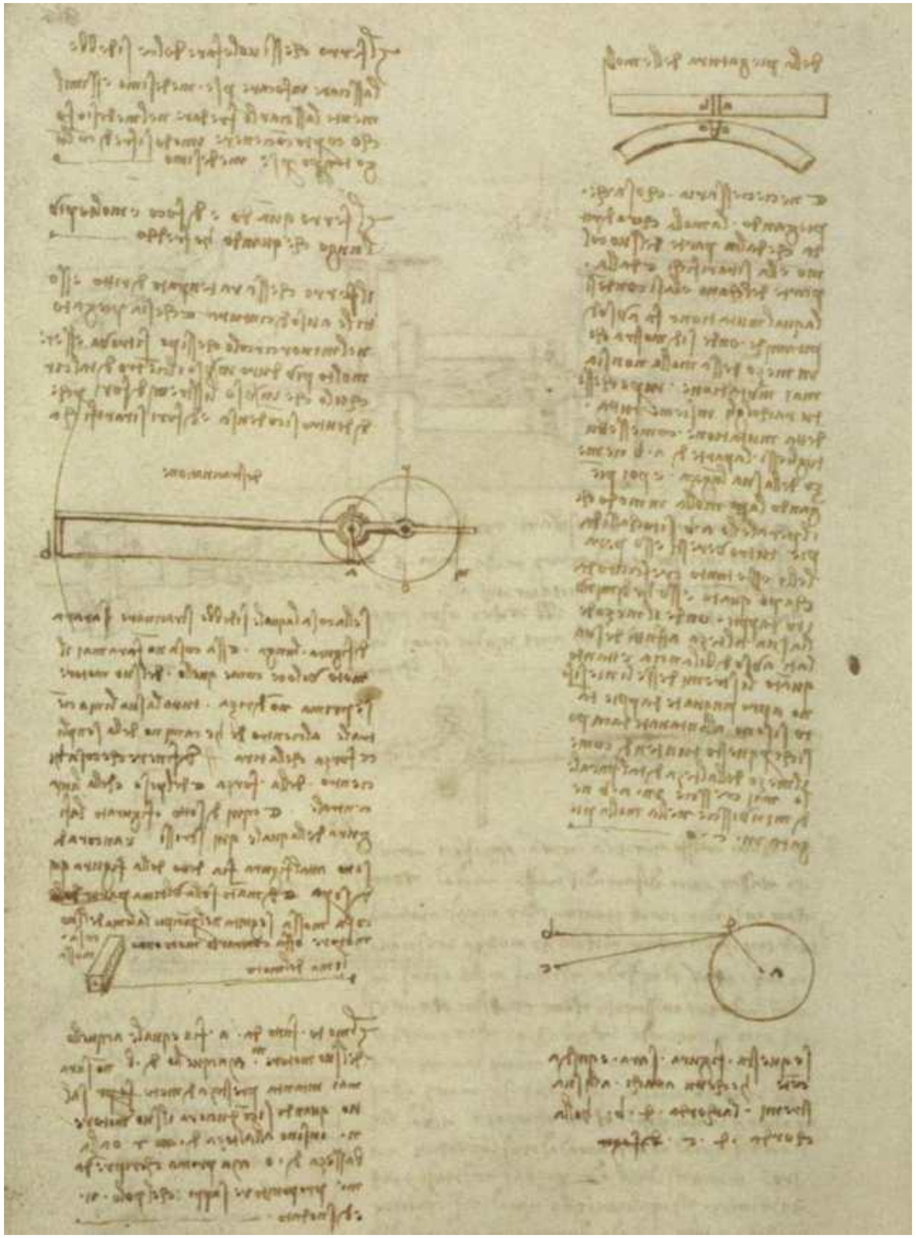
(a) Frank Lloyd Wright's Master piece: Fallingwater

"Why Fallingwater? Wright employed the cantilever: a horizontal structure for distributing force, "the true earth-line of human life" (Wright). (b) Minature (1 millionth scale) cantilevers are used to measure forces in devices etched from silicon using ultra high precision lithography .

Galileo Galilei is often credited with the first published theory of the strength of beams in bending, but with the discovery of The Codex Madrid in the National Library of Spain in 1967 it was found that Leonardo da Vincis work (published in 1493) had not only preceded Galileos work by over 100 years, but had also, unlike Galileo, correctly identified the stress and strain distribution across a section in bending.



Galileo Galilei
(1564 – 1642)



Leonardo da Vinci
(1452-1519)

The Codex Madrid in the National Library of Spain

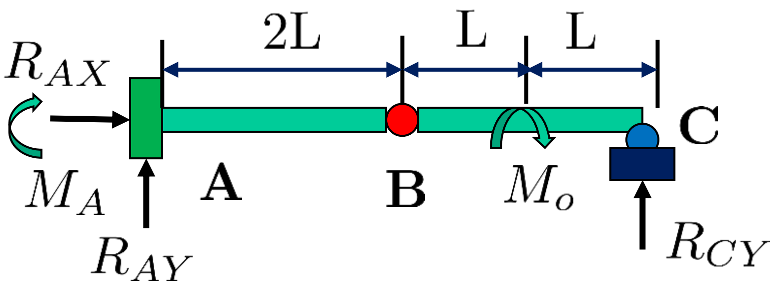
People who invented beam theory

Jacob Bernoulli (1623-1708)

Leonhard Euler (1707-1783)

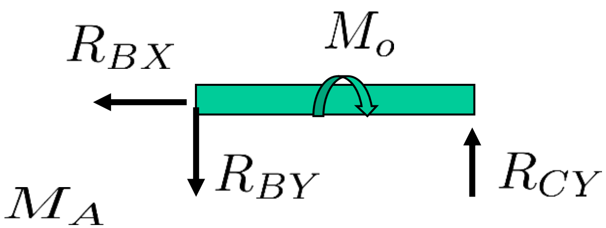
The Euler-Bernolli beam theory

Example I: How to find reaction forces ?



The point B is a hinge, which means that it can rotates, i.e. it cannot transmit moment.

Free-body diagram of BC



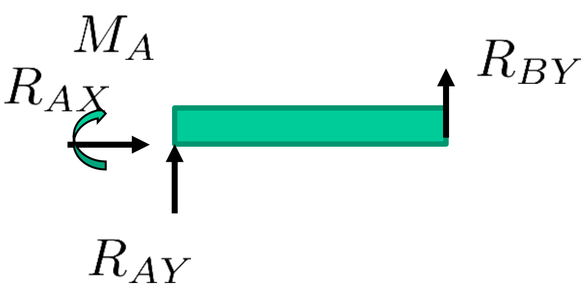
$$\sum F_x = 0, \ \rightarrow \ R_{BX} = 0;$$

$$\sum F_y = 0, \ \rightarrow \ -R_{BY} + R_{CY} = 0 \ \rightarrow \ R_{BY} = R_{CY}$$

$$\sum M_B = 0, \ \rightarrow \ 2LR_{CY} - M_o = 0;$$

$$\rightarrow \ \boxed{R_{CY} = M_o/(2L)}$$

Free-body diagram of AB



$$\boxed{M_A = M_0}$$

$$\sum F_x = 0 \ \rightarrow \ R_{AX} = 0;$$

$$\sum F_y = 0 \ \rightarrow \ R_{AY} + R_{BY} = 0 \ \rightarrow \ R_{AY} = -R_{BY}$$

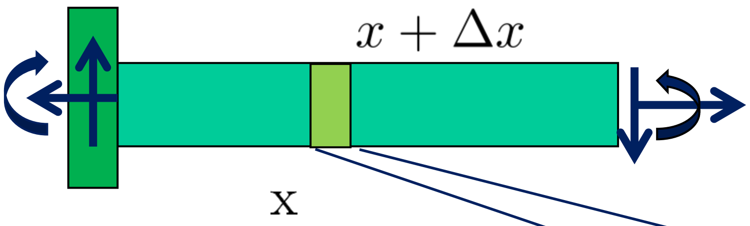
$$\sum M_A = 0 \ \rightarrow \ 2LR_{BY} - M_A = 0$$

$$M_A = 2LR_{BY} = 2LR_{CY} = 2L\frac{M_0}{2L} = M_0.$$

$$\boxed{R_{AY} = -R_{BY} = -M_0/(2L).}$$

Internal forces (stress resultants) in beams:

How external forces being transmitted from one end to another end ?

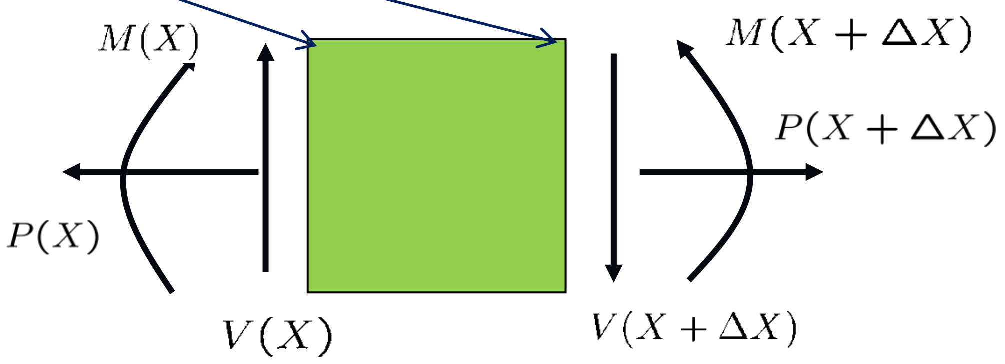


We call internal force as Stress resultant.
There are three stress resultants:

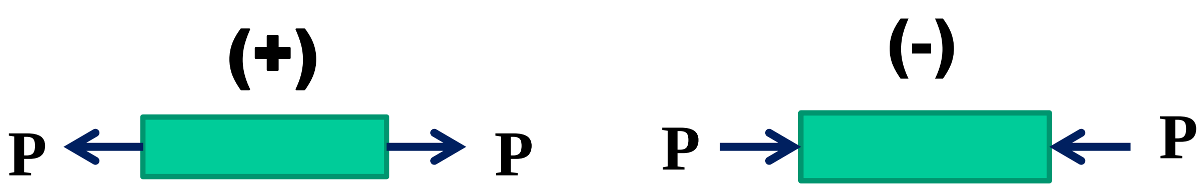
$$P = \int_A \sigma dA$$

$$V = -\int_A \tau dA$$

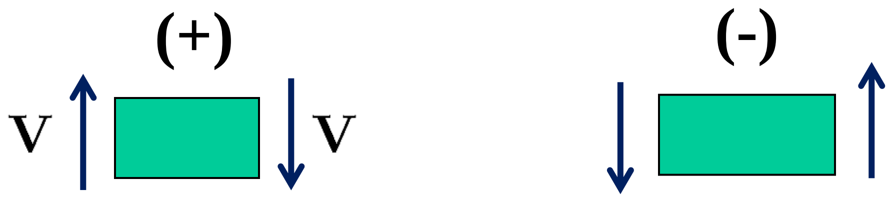
$$M_z = -\int_A y\sigma dA$$



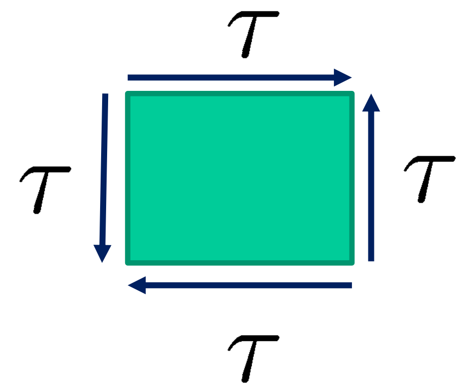
(1) Axial force sign convention



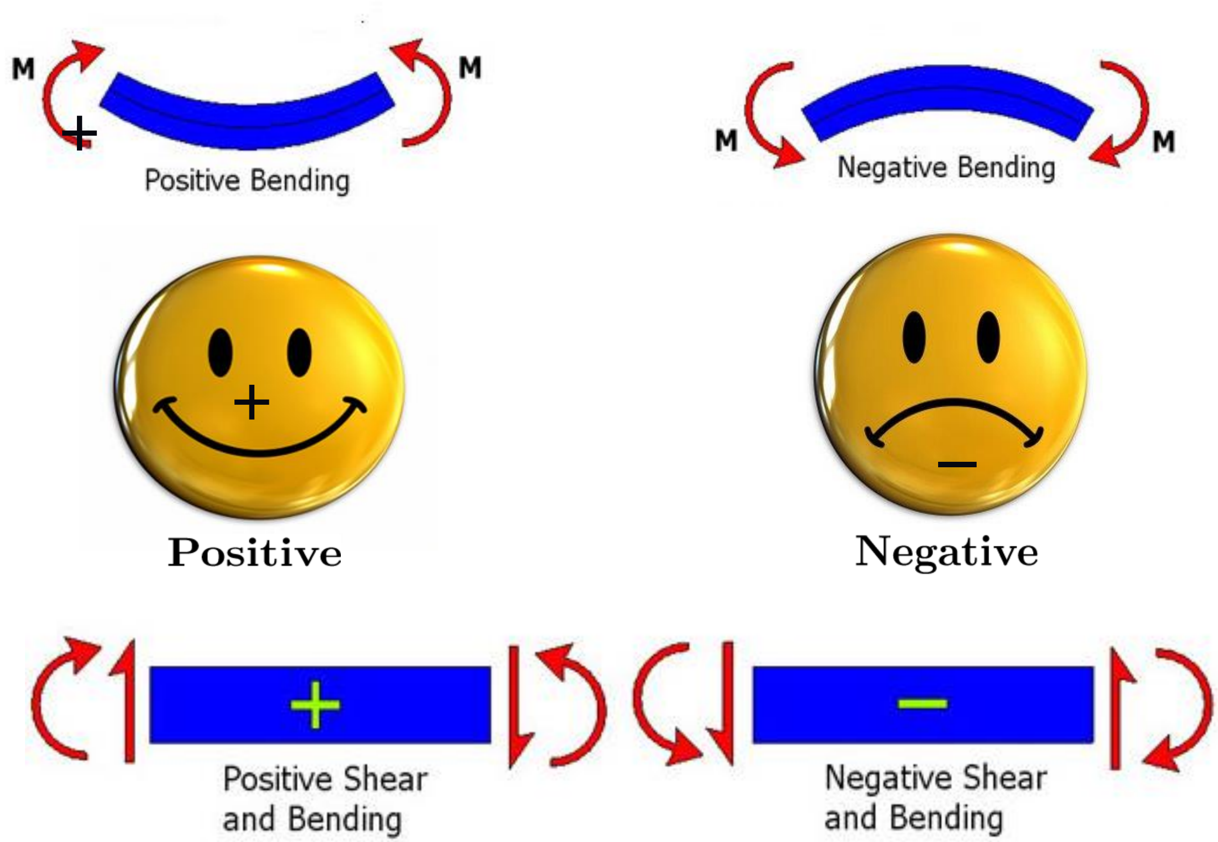
(2) Shear force convention



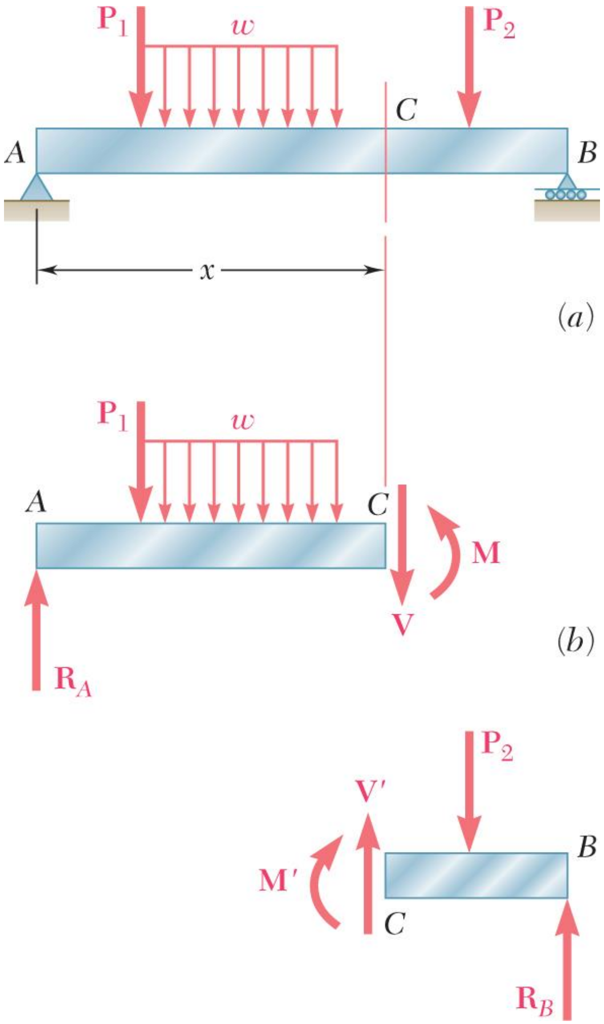
Compare with



(3) Moment convention



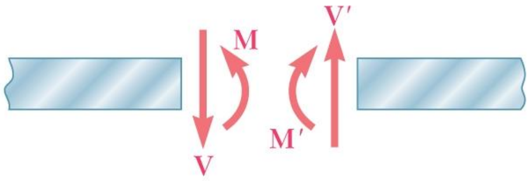
How to draw Shear and Bending Moment Diagrams ?



- Determination of maximum normal and shearing stresses requires identification of maximum internal shear force and bending couple.

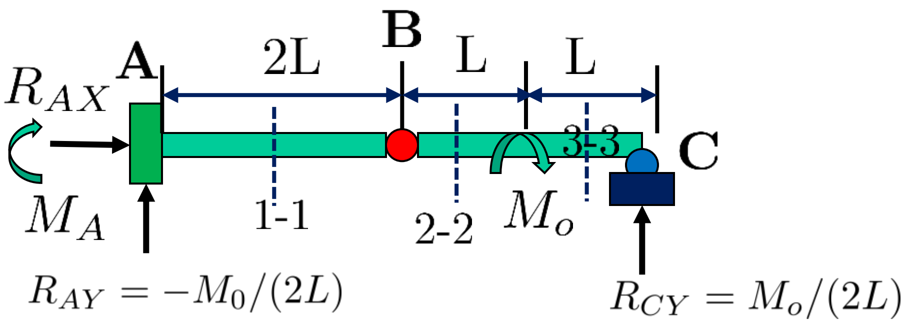
- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.

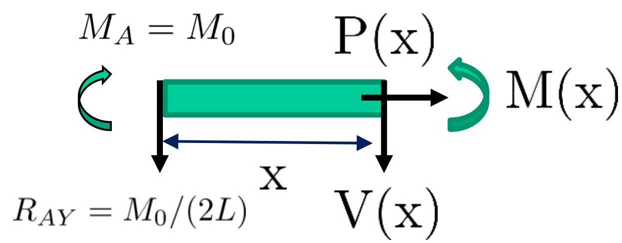
- Sign conventions for shear forces *V* and *V'* and bending couples *M* and *M'*



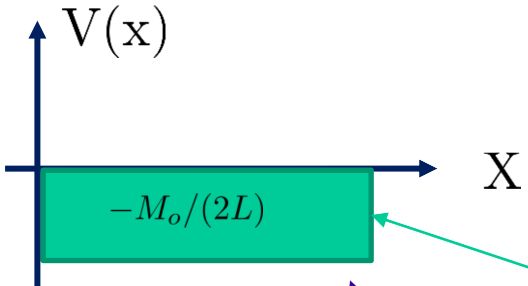
(a) Internal forces
(positive shear and positive bending moment)

Example Ib: How to find internal forces ?





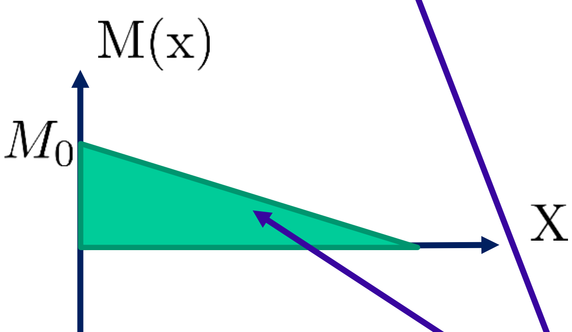
1. (Cut 1-1): For $0 < x < 2L$



$$\sum F_x = 0 \;\rightarrow\; P(X) = 0;$$

$$\sum F_y = 0 \;\rightarrow\; -R_{AY} - V(x) = 0 \;\rightarrow\; V(x) = -R_{AY} = -M_0/(2L)$$

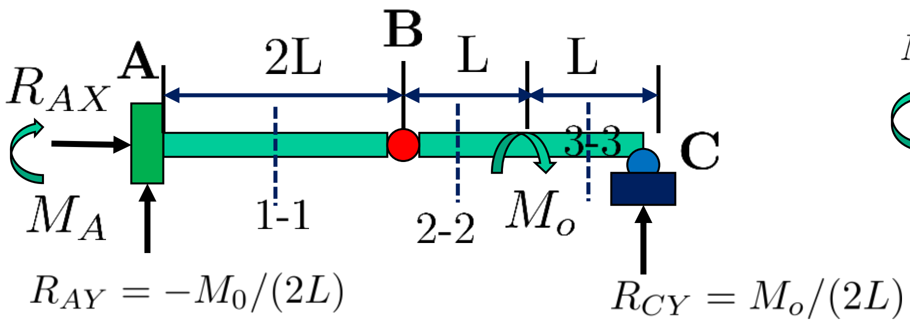
$$\sum M_x = 0 \;\rightarrow\; xR_{AY} - M_A + M(x) = 0$$

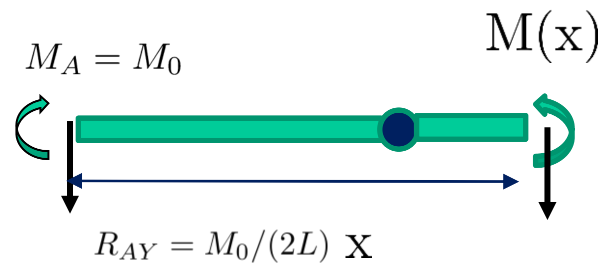


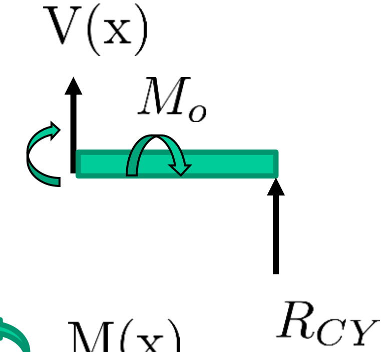
$$M(x) = M_0 - \frac{M_0}{2L}x = \left(1 - \frac{x}{2L}\right)M_0.$$

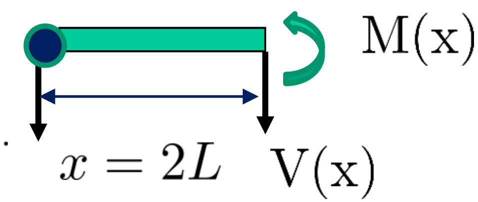
Any observation ?

Example Ib: How to find internal forces ?



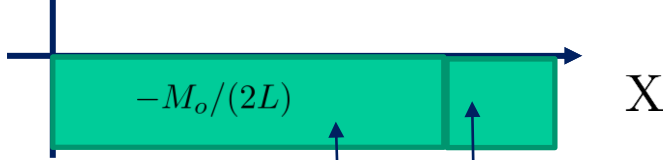






2. (Cut 2-2): For $2L < x < 3L$ $\quad R_{BY} = M_0/(2L)$.

V(x)

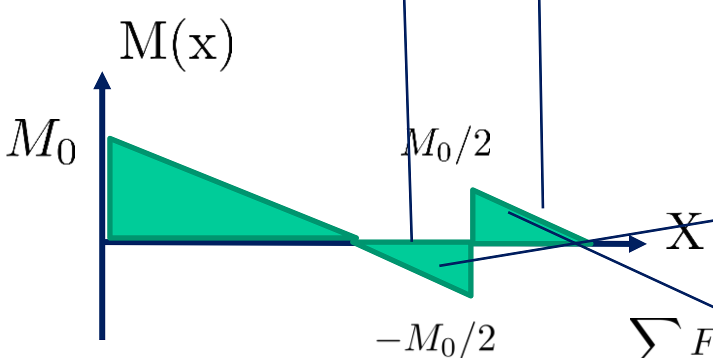


$$\sum F_y = 0 \;\rightarrow\; -R_{BY} - V(x) = 0 \;\rightarrow$$

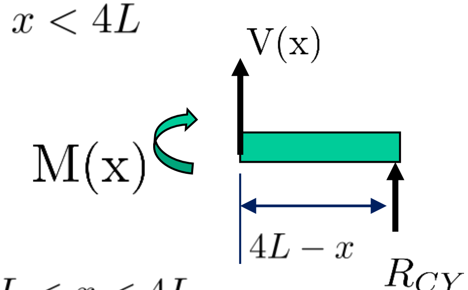
$$V(x) = -R_{BY} = -M_0/(2L)$$

$$\sum M_x = 0 \;\rightarrow\; (x - 2L)R_{BY} + M(x) = 0$$

$$M(x) = \left(2L - x\right)\frac{M_0}{2L}, \quad 2L < x < 3L.$$



3. (Cut 3-3): For $3L < x < 4L$

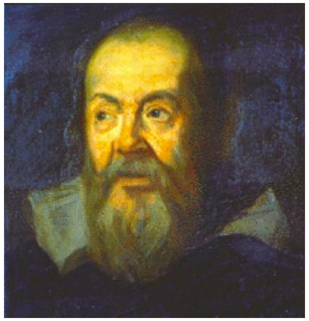


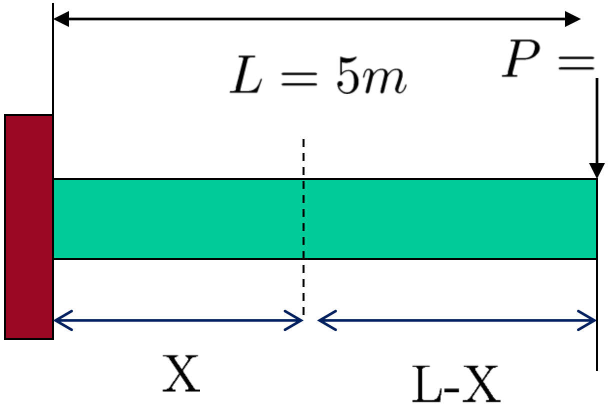
$$\sum F_y = 0 \;\rightarrow\; R_{CY} + V(x) = 0 \;\rightarrow$$

$$V(x) = -R_{CY} = -M_0/(2L)$$

$$\sum M_x = 0 \;\rightarrow\; (4L - x)R_{CY} - M(x) = 0 \;\rightarrow\; M(x) = \left(4L - x\right)\frac{M_0}{2L}, \quad 3L < x < 4L.$$

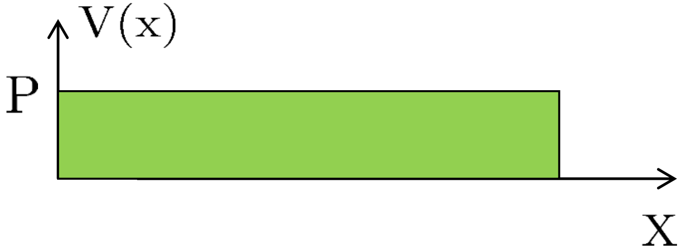
Example II. Galileo's Problem



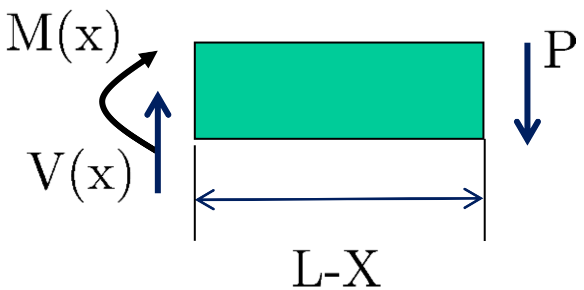


$$\sum F_y = 0 \;\rightarrow$$

$$V(x) - P = 0 \;\rightarrow\; V(x) = P;$$

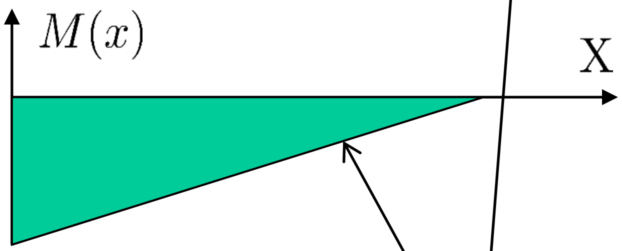


Free-body diagram



$$\sum M_x = 0 \;\rightarrow$$

$$-M(x) - P(L-x) = 0 \;\rightarrow\; M(x) = P(x-L);$$



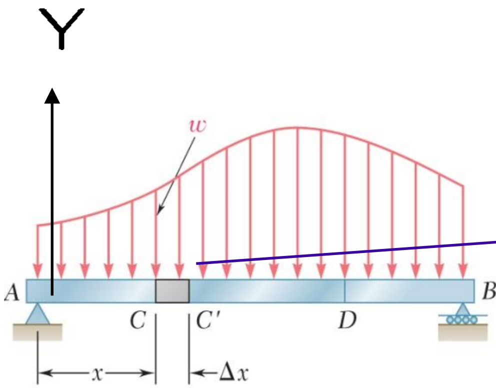
$$M_{max} = -PL$$

$$M_{\max} = M(0) = -PL = -5\times10^{4}\, N\cdot m$$

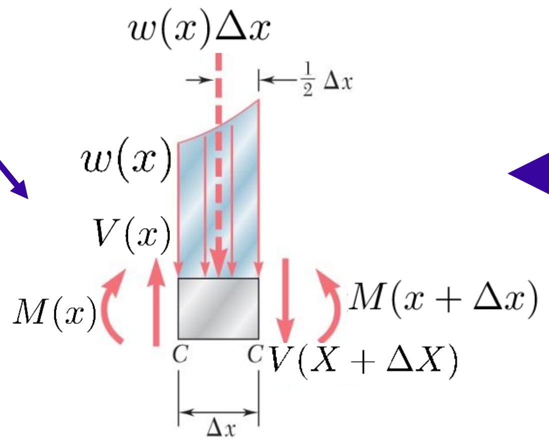
What is the slop of this line ?

Differential Equation Approach to Moment and Shear

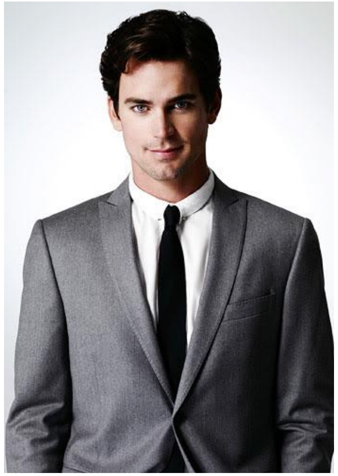
Representative element approach



(a)



(b)



White-collar approach

We study the equilibrium condition:

$$(1) \quad \sum F_y = 0 \qquad\qquad V(x) - V(x+\Delta x) - w(x)\Delta x = 0$$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = -w(x) \quad \longrightarrow \quad \boxed{\frac{dV}{dx} = -w(x)}$$

$$\boxed{V(x) - V(0) = -\int_0^x w(x)dx}$$

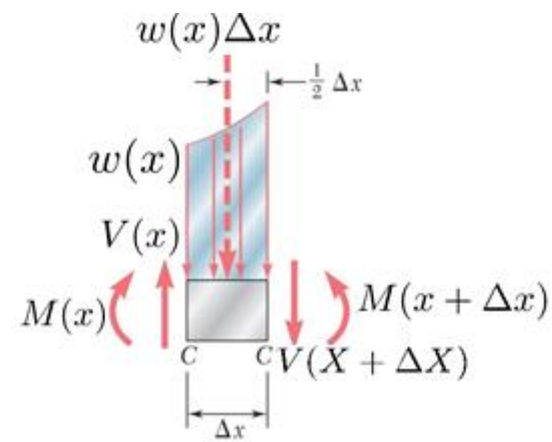
w(x) is downward.

(2) $\sum M_{x+\Delta x} = 0$ $\quad (M(x+\Delta x) - M(x) - V(x)\Delta x + (w\Delta x)\frac{\Delta x}{2} = 0;$

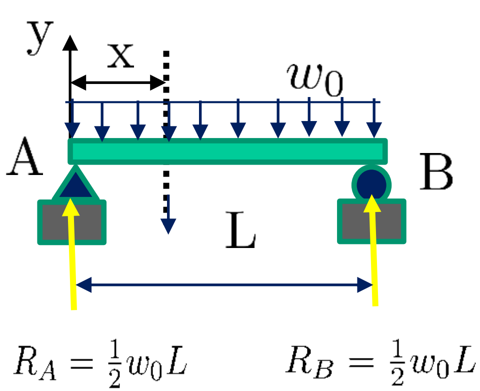
$$\frac{M(x+\Delta x) - M(x)}{\Delta x} = V(x) - \frac{1}{2}w(x)\Delta x \quad \rightarrow \quad \boxed{\frac{dM}{dx} = V(x)}$$

Therefore,

$$\boxed{M(x) - M(0) = \int_0^x V(x')dx'}$$



Example III



[Solution]

By symmetry $R_A = R_B$.

$$R_A = R_B = \frac{1}{2}w_0L$$

$V(0) = R_A$ and $M(0) = 0$.

What is $V(L)$?

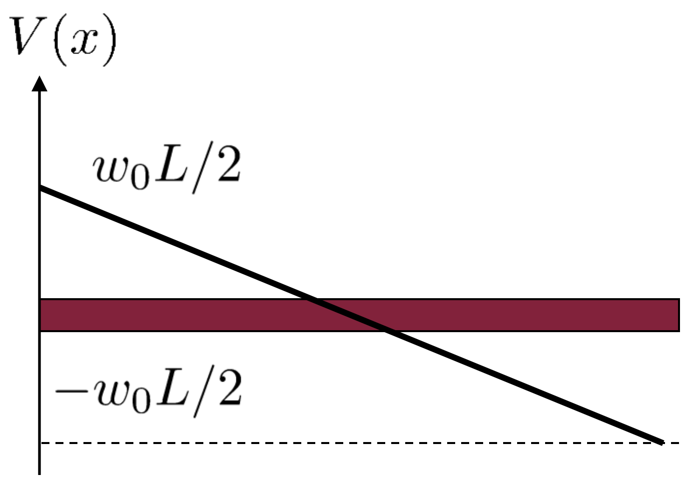
$$V(L) = -R_B = -\tfrac{1}{2}w_0L$$

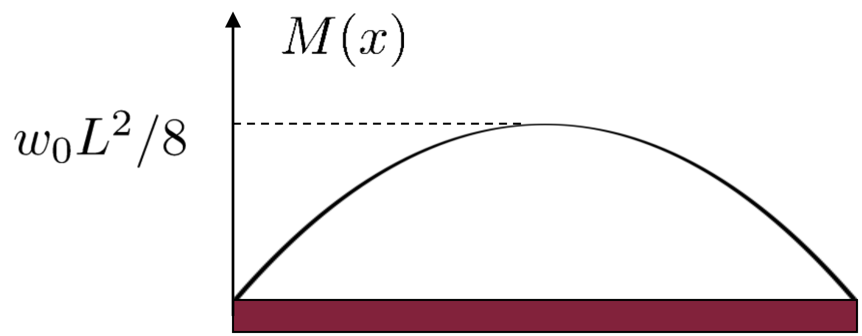
$$R_A = R_B = \frac{1}{2} w_0 L$$

$V(0) = R_A$ and $M(0) = 0.$

$$V(x) - V(0) = -\int_0^x w_0 dx = -w_0 x \quad \rightarrow \quad V(x) = -w_0 x + \frac{w_0 L}{2}$$

$$M(x) - M(0) = \int_0^x V(x) dx = -\frac{w_0 x^2}{2} + \frac{w_0 L}{2} x$$





Summary:

$$\boxed{\frac{dV}{dx} = -w(x)} \longrightarrow \boxed{V(x) - V(0) = -\int_0^x w(x)dx}$$

$$\boxed{\frac{dM}{dx} = V(x)} \longrightarrow \boxed{M(x) - M(0) = \int_0^x V(x')dx'}$$

Main Takeaways:

For bending $\dfrac{dV}{dx} + w(x) = 0;$

For uniaxial tension : $\dfrac{dR}{dx} + f_x = 0\ ;$

For torsion : $\dfrac{dT}{dx} + m_x = 0;$