CE30 – Discussion 2

Equilibrium of Particles & Moments

Textbook: 2.4 – 3.2

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Homework 2

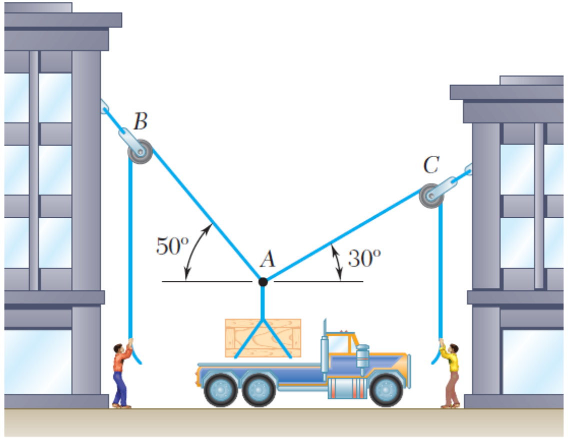
- Problems from the textbook:
  2.78, 2.83, 2.92, 2.107, 3.16, 3.45, and 3.47

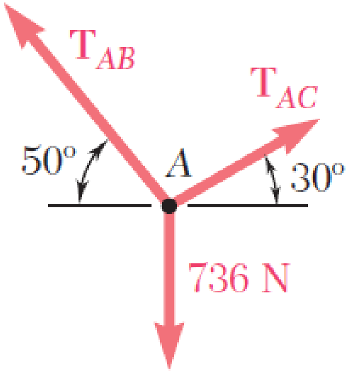
- Late Policy: **20% penalty** if submitted before Monday midnight
  - No credit after Monday!

- Submit regrade request only through Gradescope
  - Do not email Prof or GSIs

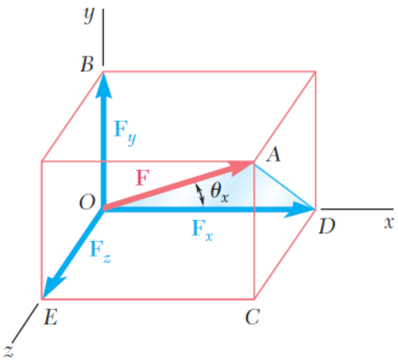
Free Body Diagrams

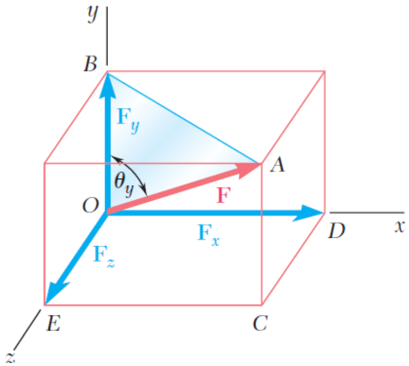
- A simplification of the engineering problem
  - Draw a simple sketch
  - Identify applied and unknown forces

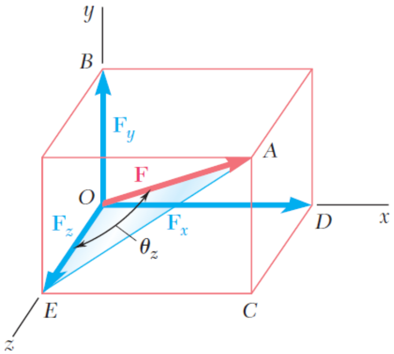




Forces in 3D







$$F_x = F\ cos\theta_x$$

$$F_y = F\ cos\theta_y$$

$$F_z = F\ cos\theta_z$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

Unit vector representation

$$F = \sqrt{F_x^2 + F_y + F_z^2}$$

Magnitude of **F**

Equilibrium of Particles

- Summations of forces in all directions should be zero

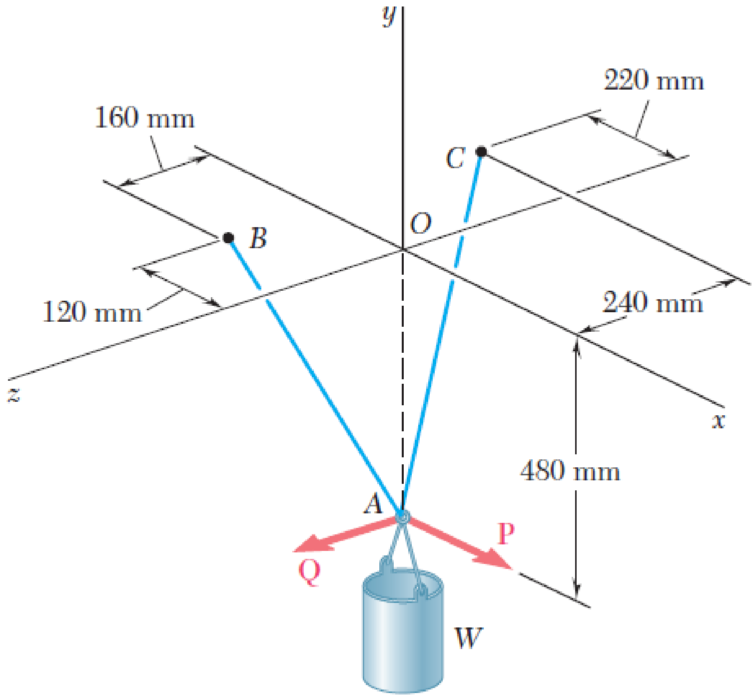
$$\sum \mathbf{F} = 0 \quad \text{or} \quad \sum F_x = 0 \;\;\&\;\; \sum F_y = 0 \;\;\&\;\; \sum F_z = 0$$

- For rigid bodies, we should also consider moment equilibrium (Chapter 4)

$$\sum \mathbf{F} = 0 \quad \& \quad \sum \mathbf{M}_O = 0$$

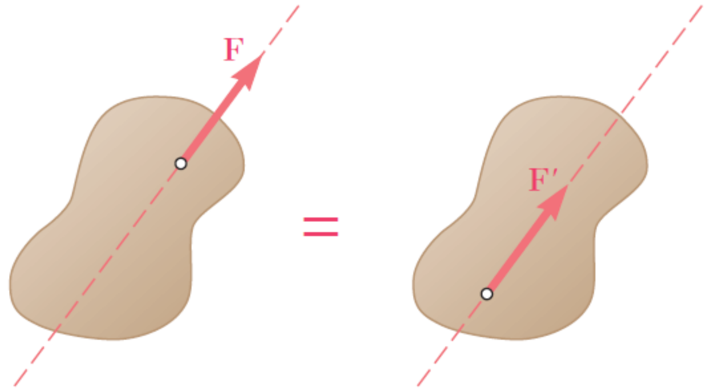
Practice

A container of weight $W = 360$ N is supported by cables AB and AC, which are tied to ring A. Knowing that $\mathbf{Q} = 0$, determine (*a*) the magnitude of the force $\mathbf{P}$ that must be applied to the ring to maintain the container in the position shown, (*b*) the corresponding values of the tension in cables AB and AC.



Equivalent Forces

- Principle of transmissibility
  - Equilibrium conditions do not change if we move the force in the line of action

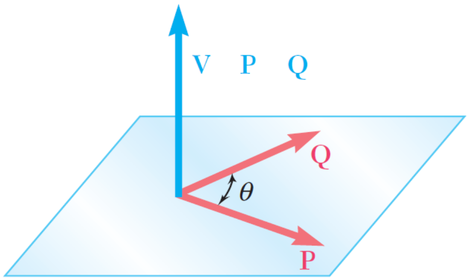


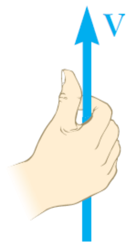
Vector Products

Cross product

$$\mathbf{V} = \mathbf{P} \times \mathbf{Q}$$

$$V = PQ \sin\theta$$



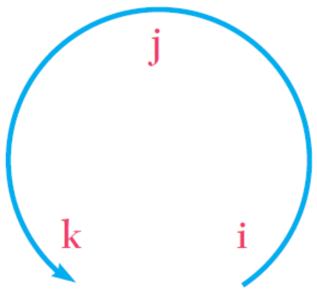


Right hand rule

1. Place your hand on the first vector s.t. your fingers points towards the vector's directions
2. Curl your fingers towards the second vector
3. Your thumb points to the resultant direction

Vector Products

Product of unit vectors



$$\mathbf{i} \times \mathbf{i} = 0$$

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$

$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

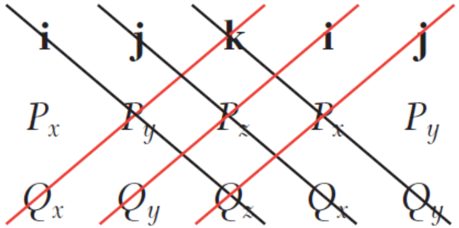
Vector Products

- How to compute a cross product $\mathbf{V} = \mathbf{P} \times \mathbf{Q} = (V_x, V_y, V_z)$

$$
\begin{aligned}
V_x &= P_y Q_z - P_z Q_y \\
V_y &= P_z Q_x - P_x Q_z \\
V_z &= P_x Q_y - P_y Q_x
\end{aligned}
$$

- Do not memorize this! Instead use a determinant:

$$
\mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}
$$



Practice

$$\boldsymbol{x} = (1, 1, 0)$$

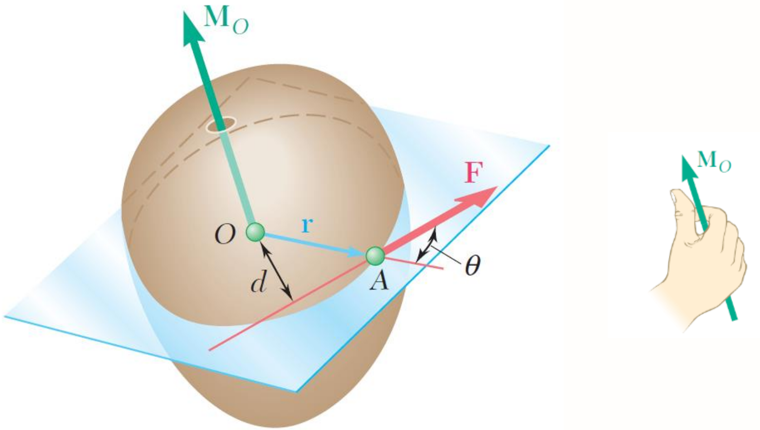
$$\boldsymbol{y} = (-1, 1, 0)$$

$$\boldsymbol{x} \times \boldsymbol{y} = ?$$

Moment

- Moment of a force **F** about a point *O*

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$



- Magnitude

$$M_O = rF\sin\theta = Fd$$

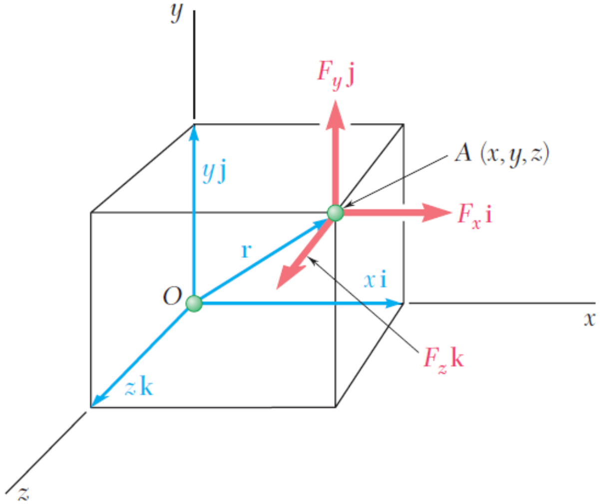
Moment

- Moment is a vector quantity

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

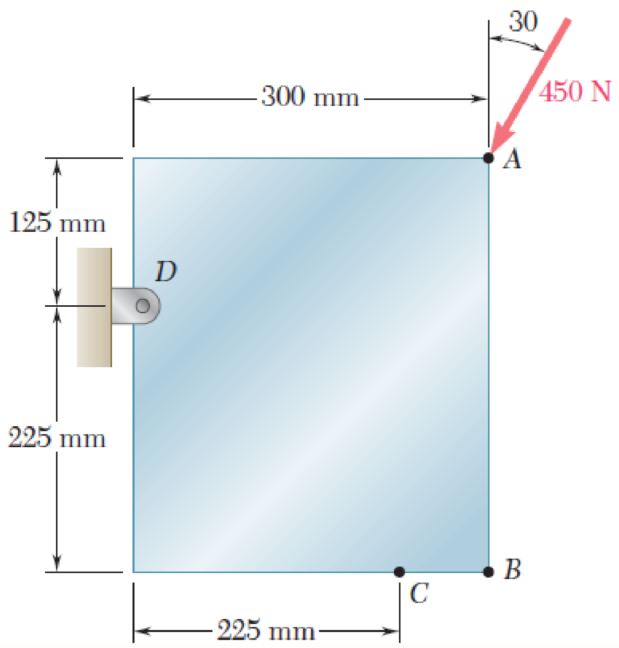
- Easier to compute by looking at components

$$\begin{aligned} M_x &= yF_z - zF_y \\ M_y &= zF_x - xF_z \\ M_z &= xF_y - yF_x \end{aligned}$$



Practice

A 450-N force is applied at A as shown. Determine (a) the moment of the 450-N force about D, (b) the smallest force applied at B that creates the same moment about D.



Vector Products

- Scalar (dot) product

$$\mathbf{P} \cdot \mathbf{Q} = P\,Q\ \cos\theta$$

- Scalar triple product

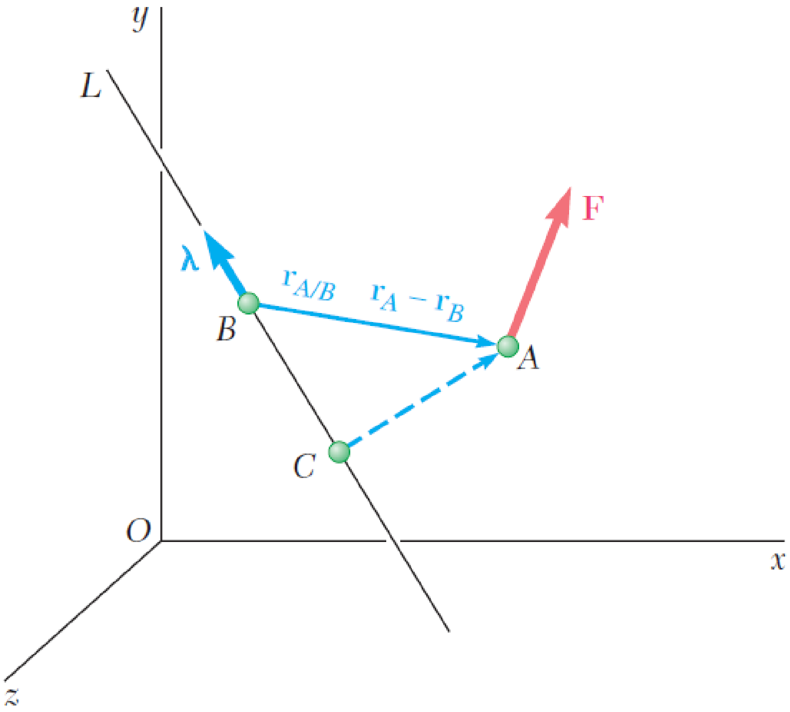
$$\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Moment

- Moment of a force **F** about an axis *L*

$$M_{BL} = \boldsymbol{\lambda} \cdot \mathbf{M}_B = \boldsymbol{\lambda} \cdot (\mathbf{r}_{A/B} \times \mathbf{F})$$

$$M_{BL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$



Practice

The jib crane is oriented so that the boom DA is parallel to the x axis. At the instant shown, the tension in cable AB is 13 kN. Determine the moment about each of the coordinate axes of the force exerted on A by the cable AB.