Hw 14 problem 1, # 16.5, 16.11, 16.17, 16.24, 16.55

1

**Problem 1.**

As shown in Fig. 1, the Cauchy stress tensor of a plane stress state at a material point is given as

$$\begin{bmatrix} 4 & 4 \\ 4 & -2 \end{bmatrix} \; MP_a$$

**1.** Draw its Mohr's circle;
**2.** Find all the principal stresses and maximum and minimum shear stresses;
**3.** The corresponding $\theta_p$ and $\theta_s$;
**4.** Use the method of pole to draw the element in the physical space in the principal directions and in the maximum/minimum shear stress directions.

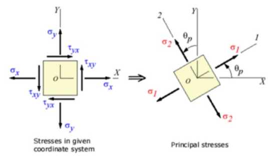


Figure 1: Initial orientation of the infinitesimal element and the element in the principal direction.

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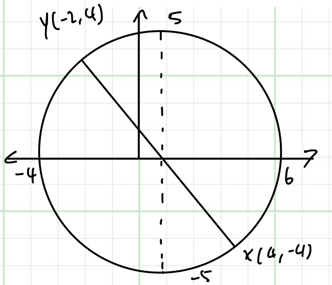



3 $\tan(2\theta_p) = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{2(4)}{4-(-2)} = \dfrac{4}{3}$

$\tan^{-1}\left(\frac{4}{3}\right) \rightarrow 2\theta_{P_{max}} = 53.13$

$2\theta_{P_{min}} = 53.13 + 180 = 233.13$

$\theta_{\sigma_{max}} = \theta_{P_{max}} = \dfrac{53.13}{2} = 26.565°$

$\theta_{min} = \theta_{P_{min}} = \dfrac{233.13}{2} = 116.565°$

$\tan(2\theta_s) = \dfrac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} = -\dfrac{4-(-2)}{2(4)} = -0.75$

$\tan^{-1}(-0.75) = 2\theta_s = 143.06°$

2 $\sigma_{ave} = \dfrac{4-2}{2} = 1\,MPa = C$

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\dfrac{4-(-2)}{2}\right)^2 + 4^2} = 5$

$\sigma_{min} = 1 - 5 = -4$

$\sigma_{max} = 1 + 5 = 6$

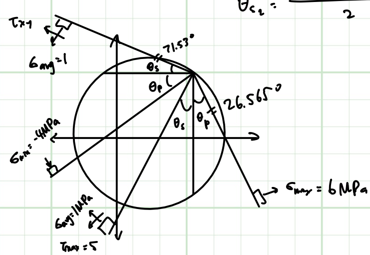
$\tau_{max} = R = 5$

$\tau_{min} = -R = -5$

$\theta_s = \dfrac{143.06}{2} = 71.53°$

$\theta_{s_2} = \dfrac{143.06 + 180}{2} = 161.53$

Method of Pole



**16.5** The rigid rod *AB* is attached to a hinge at *A* and to two springs, each of constant $k = 2$ kips/in., that can act in either tension or compression. Knowing that $h = 2$ ft, determine the critical load.

16.5

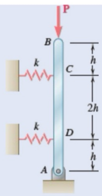



**Fig. P16.5**

$x_D = h\theta$

$x_C = 3h\theta$

$x_B = 4h\theta$

$F_C = kx_C = 3kh\theta$

$F_D = kx_D = kh\theta$

$\Sigma M_A = 0$

$hF_D + 3hF_C - Px_B = 0$

$kh^2\theta + 9kh^2\theta - 4hP = 0$

$P = \frac{5}{2}kh = \frac{5}{2}(2)(24) =$ 120 kips

16.11

**16.11** Determine the radius of the round strut so that the round and square struts have the same cross-sectional area, and compute the critical load for each. Use $E = 200$ GPa.

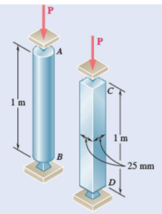



**Fig.** *P16.11*

$$A_{CD} = 25^2 = 625 \text{ mm}^2$$

$$A_{AB} = \pi r^2$$

$$r = \sqrt{\frac{625}{\pi}} = 14.104 \text{ mm}$$

CD:

$$I = \frac{25^4}{12} = 32.55 \times 10^{-9} \, m^4$$

$$P_{cr} = \frac{\pi^2 EI}{(L)^2} = \frac{\pi^2 (200 \times 10^6)(32.55 \times 10^{-9})}{1^2}$$

$$P_{cr} = 64.3 \text{ kN}$$

AB:

$$I = \frac{\pi}{4}(14.104)^4 = 31.02 \times 10^{-9} \, m^4$$

$$P_{cr} = \frac{\pi^2 (200 \times 10^6)(31.02 \times 10^{-9})}{1} = 61.24 \text{ kN}$$

16.17

**16.17** Knowing that a factor of safety of 2.6 is required, determine the largest load **P** that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

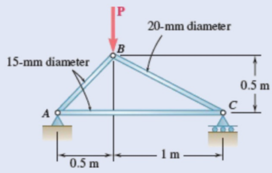



**Fig. P16.17**

$L_{BC} : \sqrt{1^2 + 0.5^2} = 1.118$ m

$L_{AB} : \sqrt{0.5^2 + 0.5^2} = 0.71$ m

$I_{BC} : \frac{\pi}{64}(20)^4 = 7.854 \times 10^{-9}$ m$^4$

$I_{AB} : \frac{\pi}{64}(15)^4 = 2.485 \times 10^{-9}$ m$^4$

$P_{cr} : \frac{\pi^2 EI}{L^2}$

$P_{cr,AB} : \frac{\pi^2 (200 \times 10^9)(2.485 \times 10^{-9})}{0.71^2} = 9.812$ kN

$P_{cr,BC} : \frac{\pi^2 (200 \times 10^9)(7.854 \times 10^{-9})}{1.118^2} = 12.40$ kN

$F_{GH} : \frac{P_{cr}}{F_s}$

$F_{GH,AB} : \frac{9.812}{2.6} = 3.773$ kN

$F_{GH,BC} : \frac{12.40}{2.6} = 4.770$ kN

$\Sigma F_x = 0 \quad \frac{0.5}{0.71} F_{AB} - \frac{1}{1.118} F_{BC} = 0$

$F_{BC} : 0.7905\, F_{AB}$

$\Sigma F_y = 0$

$\frac{0.5}{0.71} F_{AB} + \frac{0.5}{1.118} F_{BC} - P = 0$

$P = 1.061\, F_{AB}$

$P : \frac{1.061\, F_{BC}}{0.7906} = 1.342\, F_{BC}$

$P = 1.061\,(3.773) = 4.000$ kN

16.24

**16.24** Each of the five struts shown consists of a solid steel rod. (*a*) Knowing that strut (1) is of a 0.8-in. diameter, determine the factor of safety with respect to buckling for the loading shown. (*b*) Determine the diameter of each of the other struts for which the factor of safety is the same as the factor of safety obtained in part *a*. Use

$E = 29 \times 10^6$ psi.

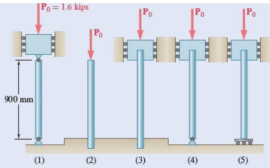



**Fig. P16.24**

a)

$P_0 : 1.6 \times 10^3 \text{ lb}$

$E : 29 \times 10^6 \ \frac{\text{lb}}{\text{in}^2}$

$L : 900 \text{ mm}$

$P_{cr} : \frac{\pi^2 EI}{L^2}$

$I : \frac{\pi}{64}(d_1{}^4) = \frac{\pi}{64}(0.8)^4 = 0.020 \text{ in}$

$P_{cr} : \frac{\pi^2 (29 \times 10^6)(0.020)}{36^2} = 4439.04 \text{ lb}$

$FOS = \frac{P_{cr}}{P_0} = \frac{4439.04}{1600} = 2.78$

$d_1 = 0.800 \text{ in}$

b) $P_{cr} : \frac{\pi^2 E (0.049) d_2{}^4}{4L^2}$

$d_2 : \sqrt[4]{\frac{4439.04 (4)(36)^2}{\pi^2 (29 \times 10^6)(0.049)}} = 1.131 \text{ in}$

$d_3 : \sqrt[4]{\frac{4439.04 (36^2)}{\pi^2 (29 \times 10^6)(0.049)(4)}} = 0.566 \text{ in}$

$d_4 : \sqrt[4]{\frac{4439.04 (36)^2}{\pi^2 (29 \times 10^6)(0.049)(2)}}$

$d_4 : 0.669 \text{ in}$

$d_5 : \sqrt[4]{\frac{4439.04 (36)^2}{\pi^2 (29 \times 10^6)(0.049)}}$

$d_5 = 0.800 \text{ in}$

16.55

**16.55** (*a*) Considering only buckling in the plane of the structure shown and using Euler's formula, determine the value of $\theta$ between 0 and $90^\circ$ for which the allowable magnitude of the load **P** is maximum. (*b*) Determine the corresponding maximum value of $P$ knowing that a factor of safety of 3.2 is required. Use $E = 29 \times 10^6$ psi.

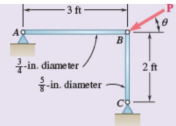



Fig. P16.55

a) $P_{cr,AB} = \dfrac{\pi E I}{L_e^2}$

$I = \dfrac{\pi (\frac{3}{4})^4}{64} = 15.53 \times 10^{-3}\ in^4$

$L_e = 3(12) = 36\ in$

$P_{cr,AB} = \dfrac{\pi^2 E I}{L^2} = \dfrac{\pi^2 (29\times10^6)(15)(53)}{36^2} = 3430.11\ lb$

$P_{cr,BC} = \dfrac{\pi^2 (\frac{5}{8})^4 (\frac{\pi}{64})(29\times10^6)}{24^2} = 3721.90\ lb$

$\theta = \tan^{-1}\left(\dfrac{3721.90}{3430.11}\right) = 47.33^\circ$

b) $P_{cr} = \sqrt{3430.11^2 + 3721.90^2} = 5061.44\ lbs$

$P = \dfrac{P_{cr}}{FS} = \dfrac{5061.44}{3(2)} = 1581.7\ lb$