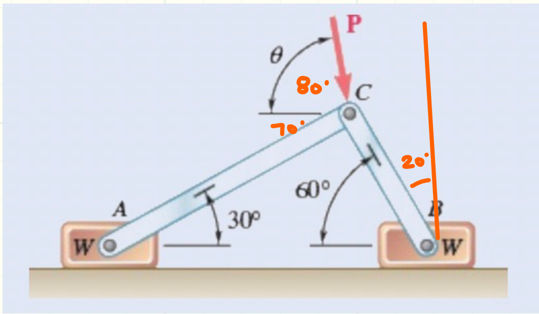
ANANYA VARMA	HOMEWORK 6	3/1/24

P4.95

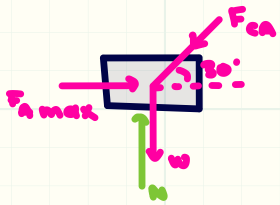


$$\frac{P}{\sin 90^\circ} = \frac{F_{CA}}{\sin 20^\circ} = \frac{F_{CB}}{\sin 70^\circ}$$

$F_{CA} = 0.342P$

$F_{CB} = 0.94P$

BLOCK A:



$\Sigma F_y = 0$

$N - F_{CA}\sin(30^\circ) - w = 0$

$N = 0.342P(\sin 30) + w$

$N = 0.171P + w$

$\Sigma F_x = 0$

$-F_{CA}\cos 30^\circ + F_f = 0$

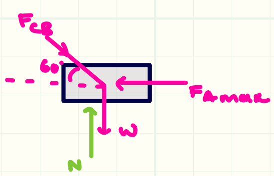
$-0.242P \cdot \cos 30^\circ + 0.3(0.171P + w) = 0$

$-0.2961P + 0.0513P + 0.3w = 0$

$-0.2448P = -0.3w$

$\boxed{P = 1.225w}$

BLOCK B



$\Sigma F_y = 0$

$N - F_{CB}\sin 60^\circ - w = 0$

$N = 0.814P + w$

$\Sigma F_x = 0$

$F_{CB}\cos 60^\circ - F_f = 0$

$0.94P \cdot \cos 60^\circ - 0.3(0.814P + w) = 0$

$0.47P - 0.2442P - 0.3w = 0$

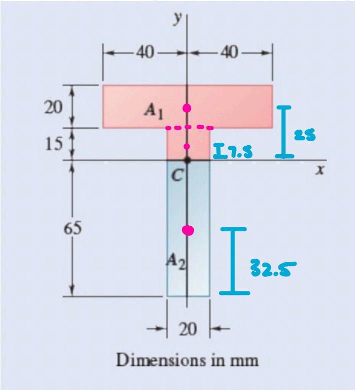
$0.2258P = 0.3w$

$\boxed{P = 1.329w}$

MIN OF TWO VALUES = LARGEST P VALUE.

$\boxed{P = 1.225w}$

P5.13



FIRST MOMENT A_1:

$Q_1 = \sum y \cdot A = 25(80 \cdot 20) + 7.5(15 \cdot 20)$

$= \boxed{42.25 \times 10^3 \text{ mm}^3}$

FIRST MOMENT A_2:

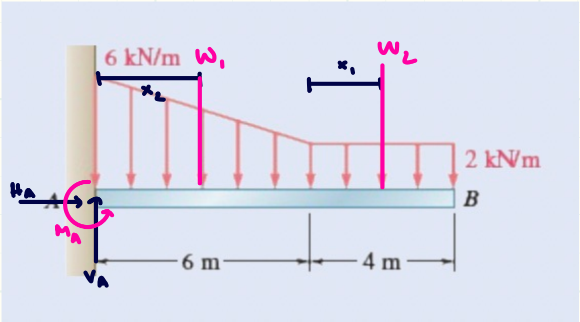
$Q_2 = \sum y \cdot A = 32.5(65 \times 20)$

$= \boxed{42.25 \times 10^3 \text{ mm}^3}$

$Q_x = 42.25 \times 10^3 \text{ mm}^3 - 42.25 \times 10^3 \text{ mm}^3 = 0$

↑ CENTROID ON X-AXIS.

P5.53



$\Sigma F_y = 0$

$V_A - \frac{1}{2}(6)(6+2) - (4 \times 2) = 0$

$V_A = 32\ kN$

$\boxed{\text{VERTICAL REACTION AT A } V_A = 32KN}$

$\Sigma F_x = 0$

$H_A = 0$

$\boxed{\text{HORIZONTAL REACTION AT A } H_A = 0}$

$\Sigma M_A = 0$

$(2 \times 4) \times (\frac{4}{2} + 6) + \frac{1}{2}(6)(6+2) \times \bar{x}_2 - M_B$

$\bar{x}_2 = \left(\frac{2 \times 2 + 6}{6 + 2}\right) \times \frac{6}{3} = \frac{60}{24} m$

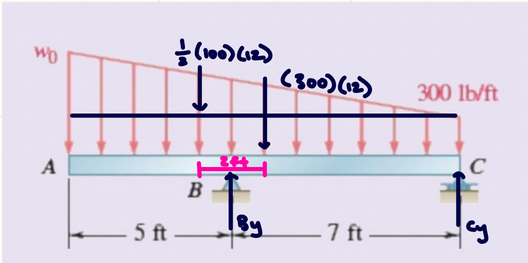
$124 - M_B = 0$

$M_A = 124\ KN \cdot m$

$\boxed{\text{MOMENT REACTION AT A } M_A = 124\ KN \cdot m}$

$\boxed{\begin{array}{l} M_A = 124.0\ KN \cdot m \circlearrowleft + \\ A = 32.0 KN \uparrow \end{array}}$

P5.81



$\sum F_y = 0$

$B_y + C_y = \frac{1}{2}(100)(12) + (300)(12) = 0$

$= 4200 \text{ lb}$

$+\circlearrowleft M_C = 0$

$(300 \times 2)8 - 7B_y + (3600 \times 6) = 0$

$7B_y = 26400$

$\boxed{B_y = 3771.43 \text{ lb}}$

$+\circlearrowleft M_B = 0$

$7C_y - (3600 \times 1) + (600 \times 1) = 0$

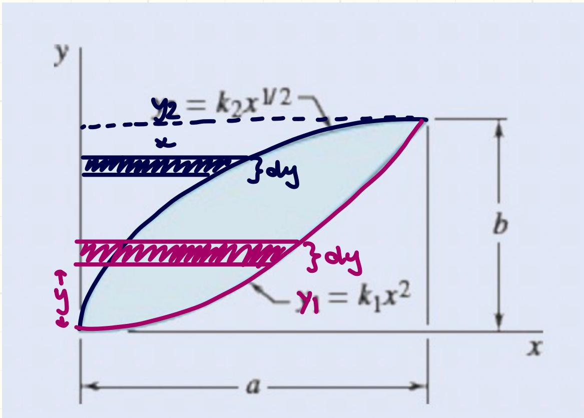
$7C_y = 3000$

$\boxed{C_y = 428.57 \text{ lb}}$

$B_y + C_y = 4200$

$3771.43 + 428.57 = 4200$ ✓

P7.12



$y_2 = k_2 x^{1/2}$ $\qquad x = y_2^2 / k_2$

$b = k_2 a^{1/2}$ $\qquad x = a y_2^2 / b^2$

$k_2 = b / a^{1/2}$ $\qquad x = a y_2^2 / b^2$

$dA = x\,dy$

$dI_x = y^2\,dA \longrightarrow y^2\,(x\,dy)$

$\hookrightarrow dI_x = y^2 \left(\frac{a y^2}{b^2} \right) dy$

$dI_x = \frac{a}{b^2} y^4\,dy \longrightarrow I_x = \int dI_x$

$I_x = \frac{a}{b^2} \int_0^b y^4\,dy = \frac{a}{b^2} \Big|_0^b \frac{y^5}{5} \longrightarrow I_x = \frac{ab^3}{5}$

$I_{x_{RECTANGLE}} = \frac{ab^3}{3} \longrightarrow (I_x)_y = (I_x)_{RECT} - (I_x)$

$(I_x)_{y_2} = \frac{ab^3}{3} - \frac{ab^3}{5} = \frac{2ab^3}{15}$

$y_1 = k_1 x^2 \longrightarrow x = (y_1 / k_1)^{1/2}$

$\frac{b}{a^2} = k_x \longrightarrow x = \frac{a}{\sqrt{b}} y_1^{1/2}$

$dI_x = y^2\,dA = y^2\,(x\,dy)$

$dI_x = y^2 \left(\frac{a}{b^{1/2}} y^{1/2} \right) dy = y^{5/2} \cdot \frac{a}{b^{1/2}}\,dy$

$I_x = \int dI_x = \frac{a}{b^{1/2}} \int_0^b y^{5/2}\,dy \longrightarrow \frac{a}{b^{1/2}} \Big|_0^b \frac{y^{7/2}}{(7/2)} \longrightarrow \frac{2a}{7\sqrt{b}} y^{7/2} \Big|_0^b$

$I_x = \frac{2}{7} ab^3$

$(I_x)_{y_1} = (I_x)_{RECT} - I_x$

$= \frac{ab^3}{3} - \frac{2}{7} ab^3$

$(I_x)_{y_1} = \frac{ab^3}{21}$

$(I_x)_{SHADED} = (I_x)_{y_2} - (I_x)_{y_1}$

$= \frac{2ab^3}{15} - \frac{ab^3}{21}$

$\boxed{(I_x)_{SHADED} = \frac{3}{35} ab^3}$ ← MOMENT OF INERTIA OF SHADED REGION

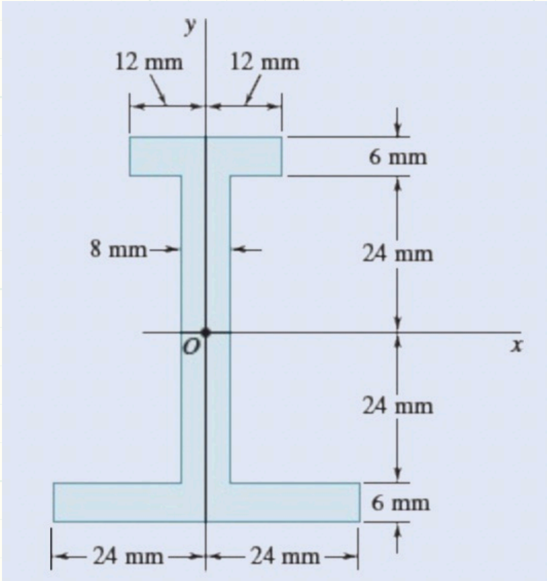
$A_{y_1} = A_{RECT} - A \longrightarrow ab - \frac{a}{\sqrt{b}} \int_0^b y^{1/2}\,dy \longrightarrow ab - \frac{a}{\sqrt{b}} \Big|_0^b \frac{y^{3/2}}{(3/2)} \longrightarrow ab - \frac{2}{3} ab \longrightarrow A_{y_1} = \frac{ab}{3}$

$A_{y_2} = A_{RECT} - A \longrightarrow ab - \frac{a}{b^2} \int_0^b y^2\,dy \longrightarrow ab - \frac{a}{b^2} \Big|_0^b \frac{y^3}{3} \longrightarrow ab - \frac{ab}{3} \longrightarrow A_{y_2} = \frac{2ab}{3}$

$A_{SHADED} = (A_{y_2}) - (A_{y_1}) = \frac{2ab}{3} - \frac{ab}{3} = \frac{ab}{3}$

RADIUS OF GYRATION $= \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{3}{35} ab^3}{\frac{ab}{3}}} \Rightarrow \boxed{k_x = \frac{3b}{\sqrt{35}}}$ ← RADIUS OF GYRATION OF SHADED REGION

P7.25



$$\text{AREA} = 6 \cdot 24 + 48 \cdot 8 + 6 \cdot 48$$
$$= 816 \text{ mm}^2$$

$$I_y = \frac{6 \times 24^3}{12} + \frac{48 \times 8^3}{12} + \frac{6 \times 48^3}{12} = 64256 \text{ mm}^4 \longrightarrow 64.56 \times 10^3 \text{ mm}^4$$

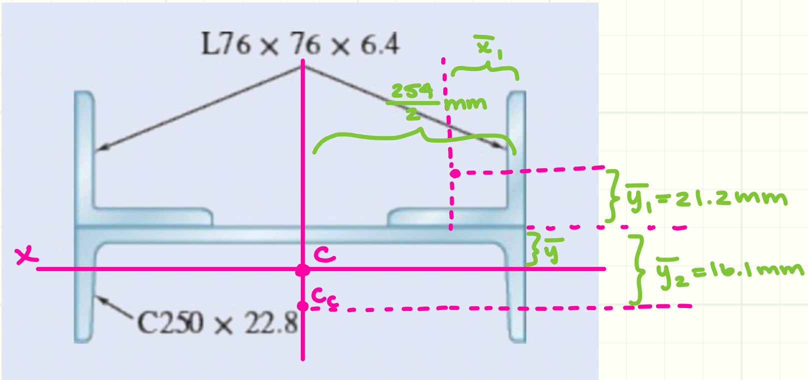
$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{64256}{816}} = 8.874 \text{ mm}$$

$$I_x = \left(\frac{24 \times 6^3}{12} + 24 \times 6 \times (24+3)^2\right) + \left(\frac{8 \times 48^3}{12}\right) + \left(\frac{48 \times 6^3}{12} + 48 \times 6 \times (24+3)^2\right)$$

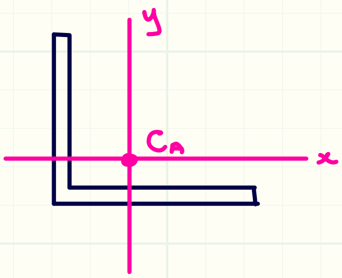
$$\boxed{I_x} = 389952 = \boxed{389.952 \times 10^3 \text{ mm}^4}$$

$$\boxed{k_x} = \sqrt{\frac{389.952 \times 10^3}{816}} = \boxed{21.86 \text{ mm}}$$

P7.47



L76 × 76 × 6.4



$A = 929\ mm^2$

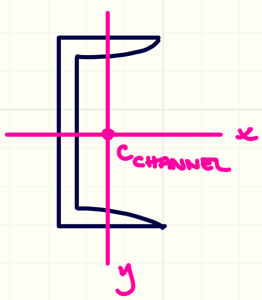
$I_{x_1} = 0.512 \times 10^6\ mm^4$

$I_{y_1} = 0.512 \times 10^6\ mm^4$

$\bar{x} = 21.2\ mm$

$\bar{y} = 21.2\ mm$

C250 × 22.8



$A = 2890\ mm^2$

$I_{x_2} = 28 \times 10^6\ mm^4$

$I_{y_2} = 0.945 \times 10^6\ mm^4$

$\bar{x} = \bar{y} = 16.1\ mm$

DEPTH = 254mm

CENTROID C:

$$\bar{y} = \frac{1858(21.2) + 2890(-16.1)}{1858 + 2890} = \frac{-7139.4}{4748} = -1.50366$$

NEGATIVE y DIRECTION

MOMENT OF INERTIA ABOUT C:

$$(I_x)_L = \bar{I}_{x_1} + A \cdot dx^2$$
$$= (0.512 \times 10^6) + (929)((21.2 + 1.50366))^2$$
$$= 0.990859 \times 10^6\ mm^4$$

$$(I_y)_L = \bar{I}_{y_1} + A \cdot dy_1^2$$
$$= (0.512 \times 10^6) + (929)((127 - 21.2))^2$$
$$= 10.9109 \times 10^6\ mm^4$$

MOMENT OF INERTIA FOR CHANNEL C:

$$(I_x)_c = \bar{I}_{x_2} + A_c\, dx_2^2$$
$$= (0.945 \times 10^6) + (2890)((16.1 - 1.50366))^2$$
$$= 1.56072 \times 10^6\ mm^4$$

$$(I_y)_c = I_{y_2} = 28 \times 10^6\ mm^4$$

MOMENT OF INERTIA FOR TOTAL SHAPE:

$$I_x = 2 \times (I_x)_L + (I_x)_c = \boxed{3.54 \times 10^6\ mm^4}$$

$$I_y = 2 \times (I_y)_L + (I_x)_c = \boxed{49.8 \times 10^6\ mm^4}$$