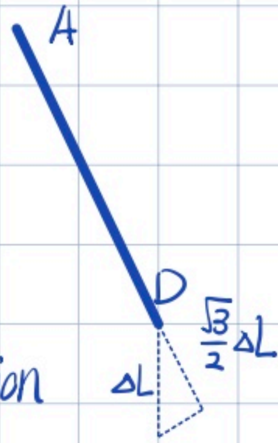
P1: (a) assume the displacement at point D is ΔL

then the elongation of truss element BD is ΔL



the elongation of truss element AD is $\frac{\sqrt{3}}{2}\Delta L$

(note: the displacement at transverse direction is rigid body motion)

Similarly, the elongation of truss element CD is $\frac{\sqrt{3}}{2}\Delta L$

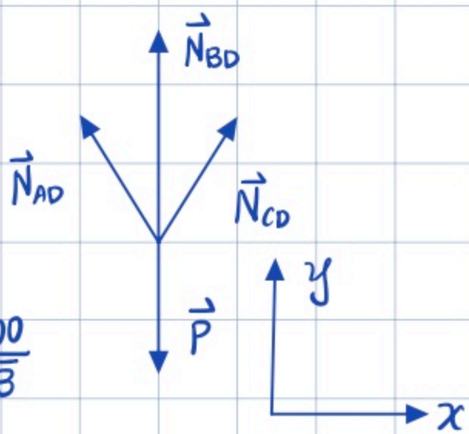
$$\Rightarrow \quad \epsilon_{BD} = \frac{\Delta L}{L}. \quad \epsilon_{AD} = \frac{\frac{\sqrt{3}}{2}\Delta L}{2/\sqrt{3}\,L} = \frac{3}{4}\cdot\frac{\Delta L}{L} \quad \epsilon_{CD} = \frac{\frac{\sqrt{3}}{2}\Delta L}{2/\sqrt{3}\,L} = \frac{3}{4}\frac{\Delta L}{L}$$

therefore, the internal axial forces are

$$|\vec{N}_{BD}| = \frac{EA\Delta L}{L}, \quad |\vec{N}_{AD}| = \frac{3}{4}\cdot\frac{EA\Delta L}{L}, \quad |\vec{N}_{CD}| = \frac{3}{4}\frac{EA\cdot\Delta L}{L}$$

i.e. $|\vec{N}_{AD}| = \frac{3}{4}|\vec{N}_{BD}|$, $|\vec{N}_{CD}| = \frac{3}{4}|\vec{N}_{BD}|$

$\Sigma F_y = 0 \Rightarrow$



$$-P + N_{BD} + \frac{3}{4} N_{BD} \cdot \frac{\sqrt{3}}{2} + \frac{3}{4} N_{BD} \frac{\sqrt{3}}{2} = 0$$

$$\left(1 + \frac{3\sqrt{3}}{4}\right) N_{BD} = P \qquad N_{BD} = \frac{4P}{4+3\sqrt{3}} = \frac{4 \times 2000}{4+3\sqrt{3}}$$

$$\Rightarrow \boxed{N_{BD} = 869.93\ N(T) \quad N_{AD} = 652.45 N(T) \quad N_{CD} = 652.45 N(T)}$$

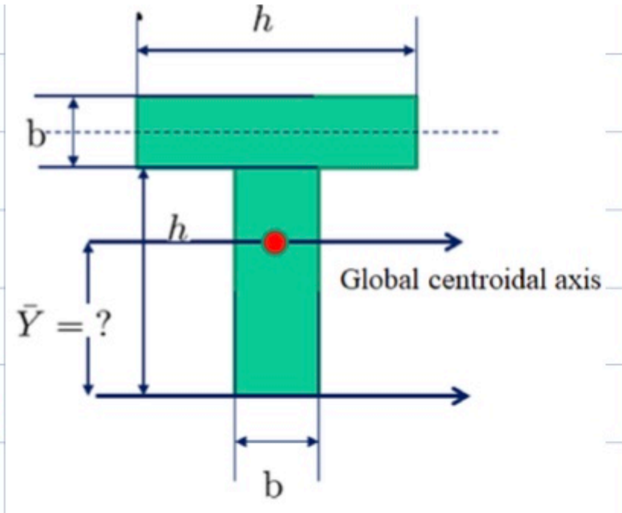
(b) for truss element BD, if $\sigma_{BD} = \sigma_{all} = 140\,MPa$

$$A_{BD} = \frac{N_{BD}}{\sigma_{BD}} = \frac{869.93}{140} = 6.214\ mm^2$$

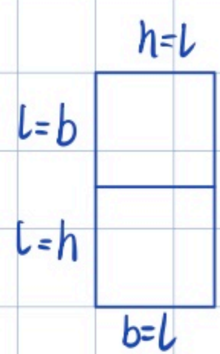
$$\boxed{d_{BD} = \sqrt{\frac{4A_{BD}}{\pi}} = \sqrt{\frac{4 \times 6.214}{\pi}} = 2.813\ mm}$$

(a)

index	A_i	$\bar{y}_i$	$A_i \bar{y}_i$
1	hb	$\frac{h}{2}$	$\frac{1}{2}h^2 b$
2	bh	$h+\frac{b}{2}$	$bh(h+\frac{b}{2})$
sum	$2hb$		$\frac{1}{2}b^2h+\frac{3}{2}h^2b$ (5')



$$\boxed{\bar{Y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{\frac{1}{2}b^2h+\frac{3}{2}h^2b}{2hb} = \frac{1}{4}b+\frac{3}{4}h}$$ (5')



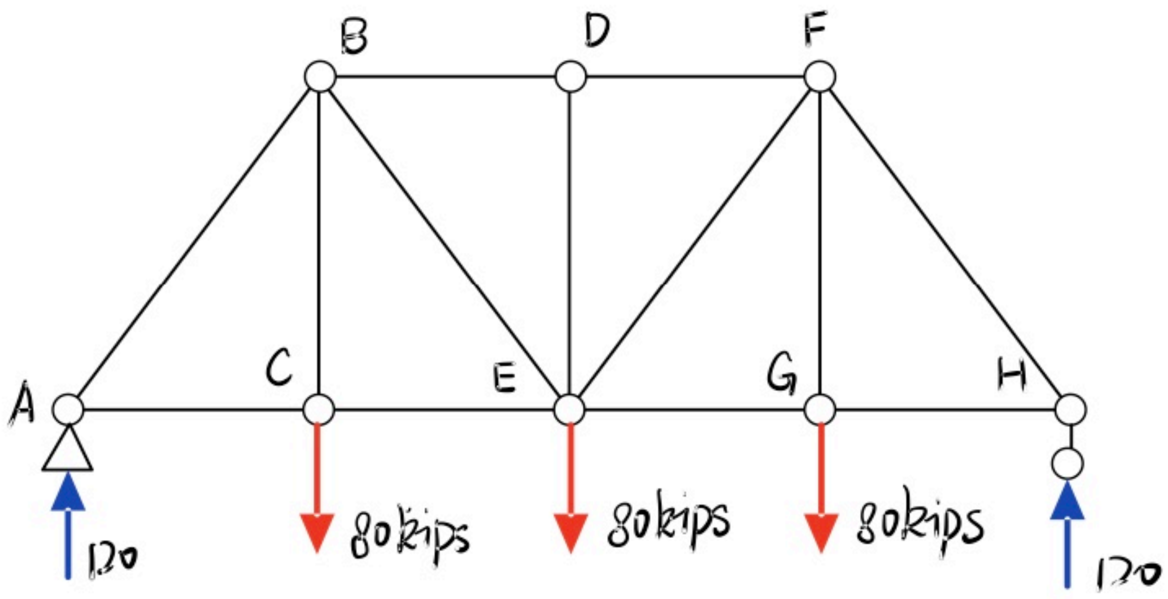
double check: if $b=h=l$ $\bar{Y}=\frac{1}{4}l+\frac{3}{4}l=l$ ✓

(b)

index	$I_{x,i}$	$\bar{y}_i$	$(\bar{Y}-y_i)^2 A_i$	$I_{x,i}+(\bar{Y}-y_i)^2 A_i$
1	$\frac{bh^3}{12}$	$\frac{h}{2}$	$\frac{1}{16}(b+h)^2 bh$	$\frac{1}{16}(b+h)^2 bh+\frac{bh^3}{12}$ (5')
2	$\frac{hb^3}{12}$	$h+\frac{b}{2}$	$\frac{1}{16}(b+h)^2 bh$	$\frac{1}{16}(b+h)^2 bh+\frac{hb^3}{12}$ (5')
sum				$\frac{1}{8}(b+h)^2 bh+\frac{(b^2+h^2)}{12}bh$

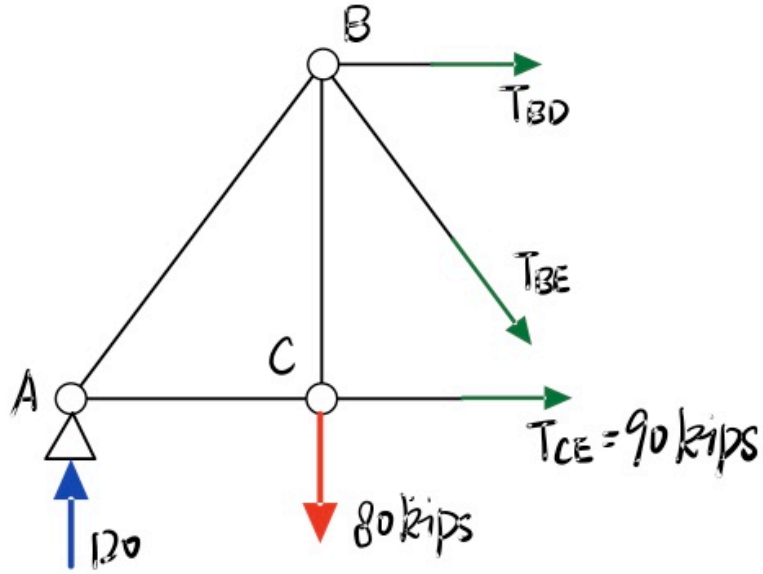
$$\boxed{I_x = \left[\frac{1}{8}(b+h)^2+\frac{1}{12}(b^2+h^2)\right]bh}$$ (5')

double check: if $b=h=l$ $I_x=[\frac{1}{8}\times 4l^2+\frac{1}{12}\times 2l^2]l^2=\frac{2}{3}l^4$ ✓



$\Sigma M_H = 0, \quad 80\times 9 + 80\times 18 + 80\times 27 = R_A \times 36, \quad R_A = 120$ kips

$\Sigma F_y = 0 \quad 120 - 80 - 80 - 80 + R_H = 0, \quad R_H = 120$ kips



$\Sigma M_B = -120\times 9 + T_{CE}\times 12 = 0$

$T_{CE} = 90$ kips

$\sigma_{CE} = \dfrac{T_{CE}}{A_{CE}} \leq 21$ ksi

$A_{CE} \geqslant \dfrac{T_{CE}}{21}$ in^2

$= 4.2857$ in^2

A: because stress matrix must be symmetric

correct answers are $\boxed{\text{(d)}}$

B: (a) ✗ elongation is in the unit of [L], while strains are in the unit of [1]

$\boxed{\text{(b) ✓}}$ By definition

(c) ✗ shear strain can only characterize the change of shape

(d) ✗ the unit of relative displacement is [L] while the unit of strain is [1]

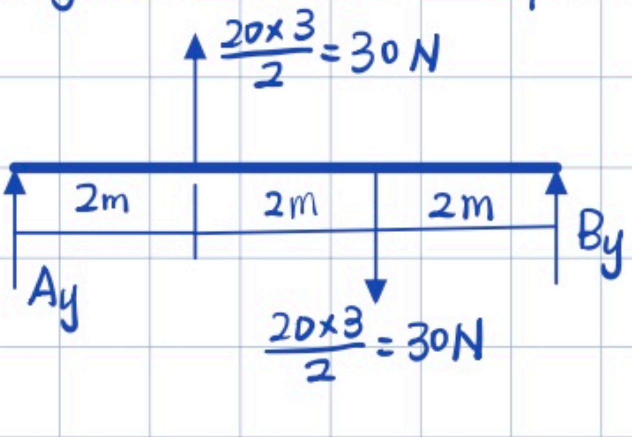
C.

$$I_{AA'} = I_{x'} + A d^2 = I_{x'} + \frac{\pi r^2}{2} \cdot a^2 = \frac{\pi r^4}{8}$$

$$I_{x'} = \frac{\pi r^4}{8} - \frac{\pi r^2}{2} a^2, \quad I_y = \frac{\pi r^4}{8}$$

$$J_c = I_{x'} + I_y = \frac{\pi r^4}{4} - \frac{\pi r^2}{2} a^2 \qquad \boxed{\text{(b)}}$$

D. The system can be equated as

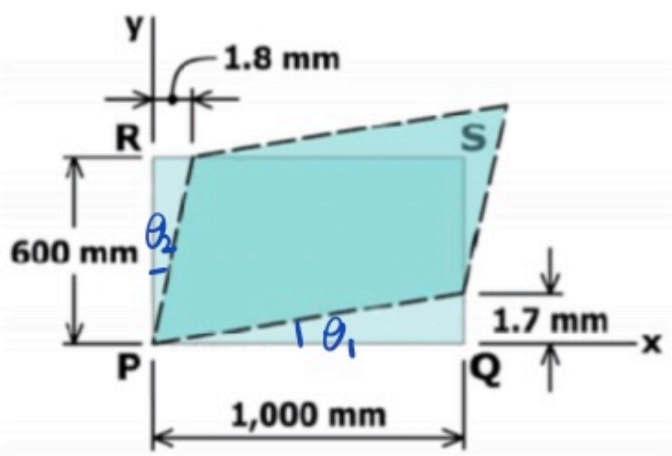


take moment at point B,

$$-A_y \times 6 + 30\times 2 - 30\times 4 = 0$$

$$6A_y = -60, \quad \boxed{A_y = -10\ N\downarrow \ (a)}$$

E



because

$\theta_1 \approx \tan\theta_1 = 1.7 \times 10^{-3}$

$\theta_2 \approx \tan\theta_2 = 3 \times 10^{-3}$

$\Delta\theta = \theta_1 + \theta_2 = 4.7 \times 10^{-3}$

$\boxed{\gamma_{xy} = 4.7 \times 10^{-3}, \text{ choose (C)}}$