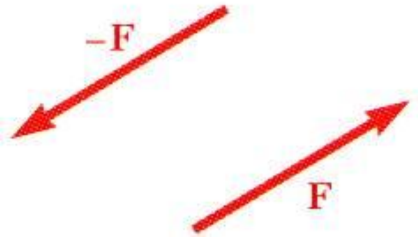
Moment of a Couple

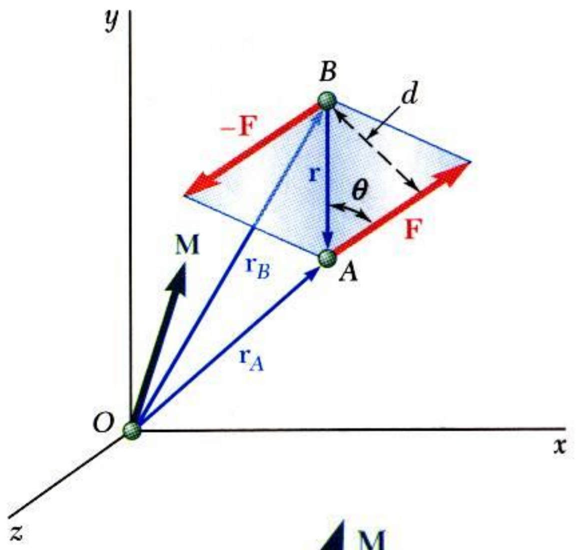
- Two forces **F** and **–F** having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.



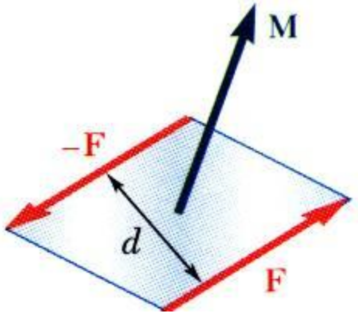
Select any point, say **O**.

- Moment of the couple,

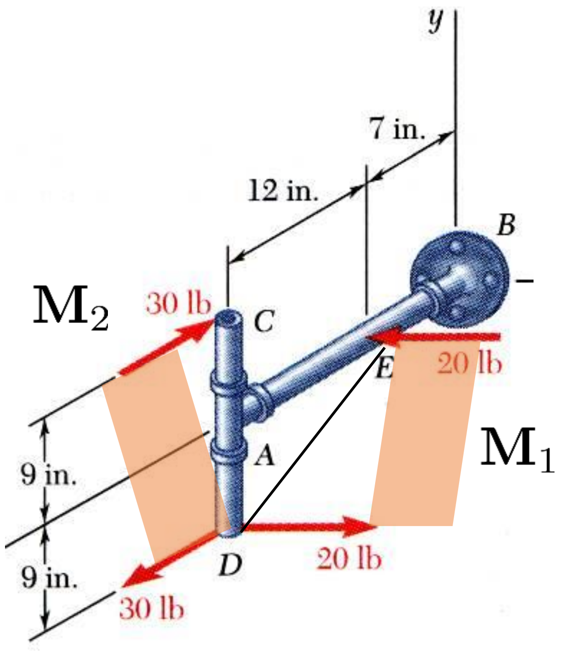
$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right)$$
$$= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$
$$= \mathbf{r}_{BA} \times \mathbf{F}$$
$$M = rF \sin\theta = Fd$$



- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.



Sample Problem 3.6



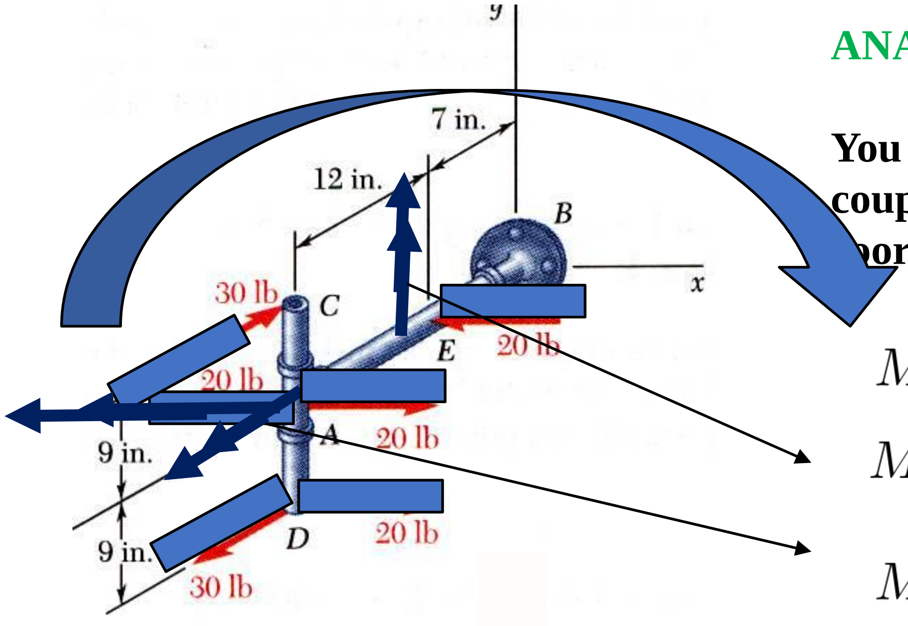
Determine the components of a single couple equivalent to the two couples shown.

STRATEGY:

Look for ways to add equal and opposite forces to the diagram that, along with already known perpendicular distances, will produce new couples with moments along the coordinate axes. These can be combined into a single equivalent couple.

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_R = M_{Rx}\mathbf{i} + M_{Ry}\mathbf{j} + M_{Rz}\mathbf{k}$$



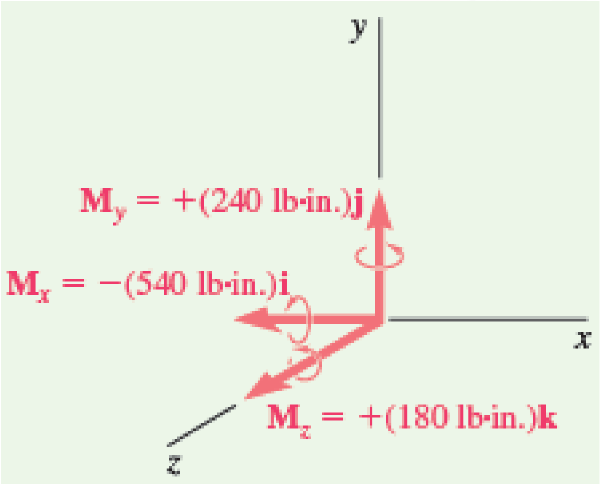
ANALYSIS:

You can represent these three couples by three couple vectors M_x, M_y, and M_z directed along the coordinate axes. The corresponding moments are

$$M_x = -(30lb)(18in.) = -540lb \cdot in$$

$$M_y = +(20lb)(12in) = +240lb \cdot in.$$

$$M_z = +(20lb)(9in) = +180lb \cdot in.$$



$$\boxed{\vec{M} = -\left(540 \text{ lb} \cdot \text{in.}\right)\vec{i} + \left(240 \text{lb} \cdot \text{in.}\right)\vec{j} + \left(180 \text{ lb} \cdot \text{in.}\right)\vec{k}}$$

Last Lecture

Equilibrium Equations for Rigid Bodies

- The necessary and sufficient condition for the static equilibrium of a body are that the resultant force and couple from all external forces form a system equivalent to zero,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

For any points

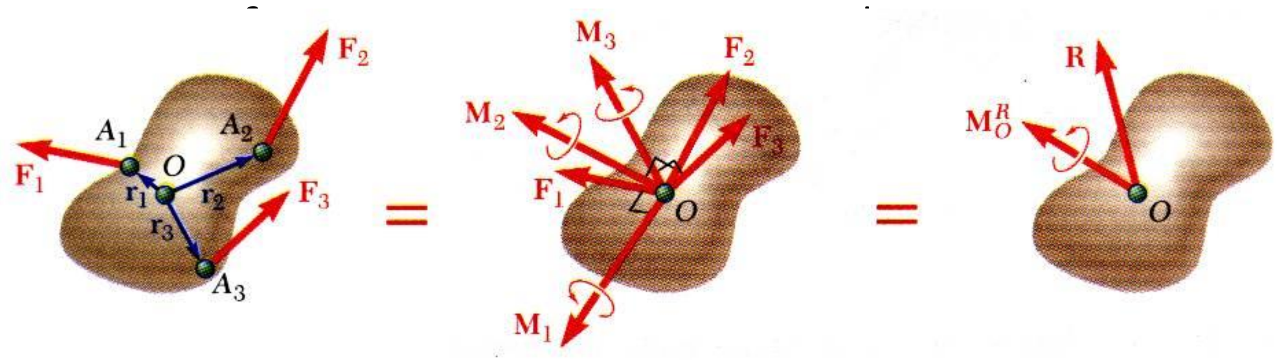
In two-dimensional space,

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

$$\sum_i F_{xi} = 0, \quad \sum_i F_{yi} = 0.$$

$$M_O = \sum_j \mathbf{r}_{Oj} \times \mathbf{F}_j = 0$$

Reducing



- A system of forces may be replaced by a collection of force-couple systems acting at a given point *O*

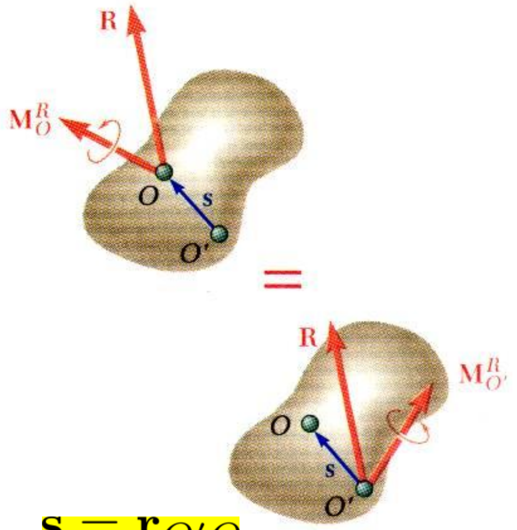
- The force and couple vectors may be combined into a resultant force vector and a resultant couple vector,

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum \left(\vec{r} \times \vec{F} \right)$$

- The force-couple system at *O* may be moved to *O′* with the addition of the moment of ***R*** about *O′* ,

$$\vec{M}_{O'}^R = \vec{M}_O^R + \vec{s} \times \vec{R}$$

- Two systems of forces are equivalent if they can be reduced to the same force-couple system.

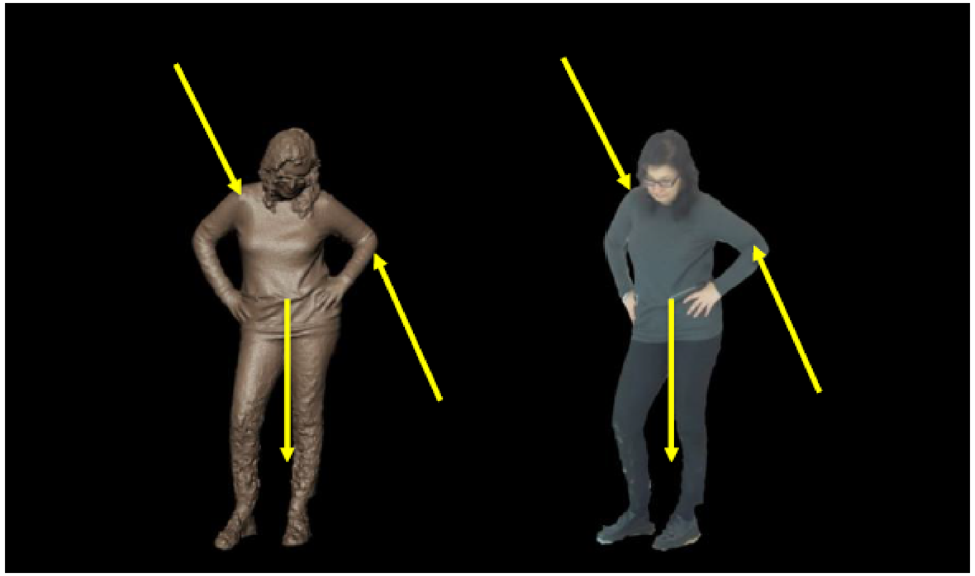


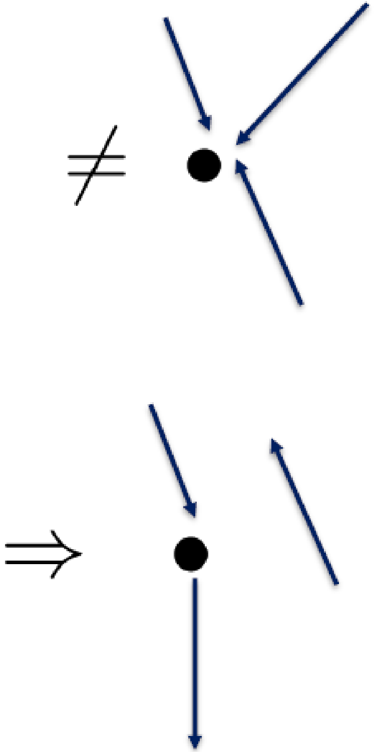
$\mathbf{s} = \mathbf{r}_{O'O}$

If you don't like point O.

Rigid Body Model

- Treatment of a body as a single particle is not accurate. In general, the size of the body and the specific points of application of the forces must be considered.

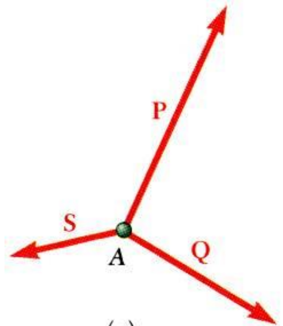




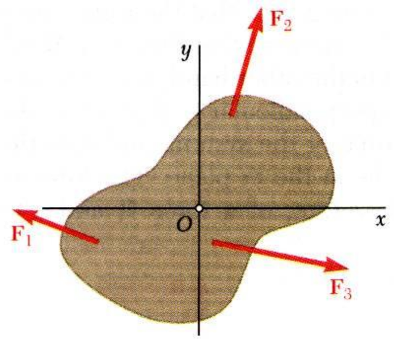
The forces may NOT be concurrent, but can be concurrent.

Special Case:
Reduction of a System of Forces to a single Force

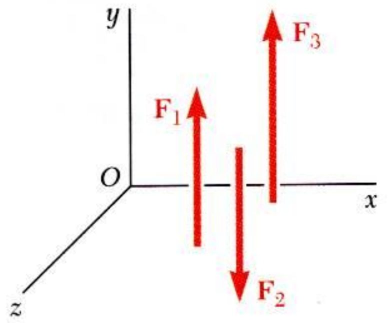
- If the resultant force and couple at *O* are mutually perpendicular, they can be replaced by a single force acting along a new line of action.



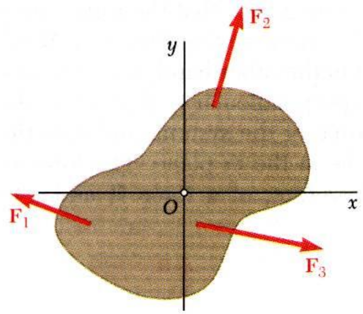
(a)



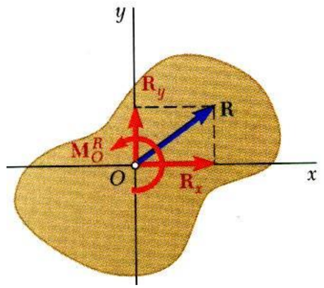
- The resultant force-couple system for a system of forces will be mutually perpendicular if:
  1) the forces are concurrent,
  2) the forces are coplanar, or
  3) the forces are parallel.

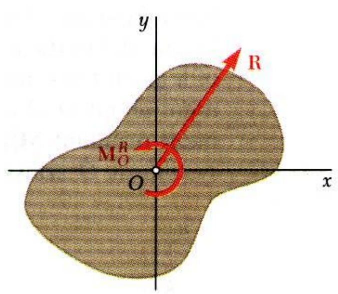


Special case II: All the forces are co-planar

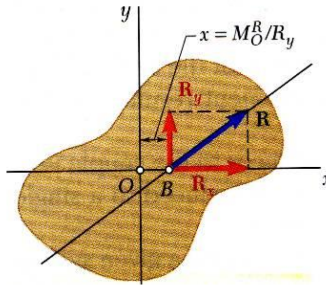


- System of coplanar forces is reduced to a force-couple system $\vec{R}$ and $\vec{M}_O^R$ that is mutually perpendicular.



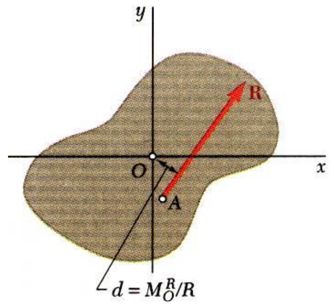


- System can be reduced to a single force by moving the line of action of $\vec{R}$ until its moment about O becomes $\vec{M}_O^R$



- In terms of rectangular coordinates,

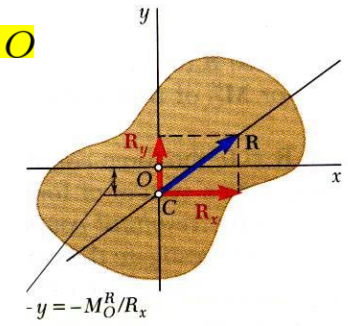
$$xR_y - yR_x = M_O^R$$



$\mathbf{s} = \mathbf{r}_{O'O}$

Recall $\mathbf{M}_{O'} = \mathbf{M}_O + \mathbf{s} \times \mathbf{R}$.
Let $\mathbf{r}_{OO'} = x\mathbf{i} + y\mathbf{j}$ such that $\mathbf{M}_{O'} = 0$, thus

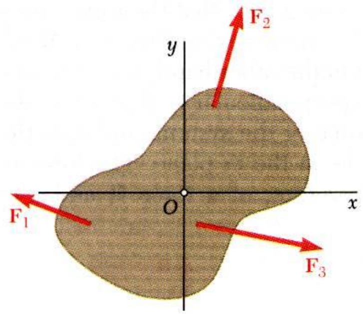
$$\mathbf{M}_{O'} = \mathbf{M}_O - \mathbf{r}_{O'O} \times \mathbf{R}\ .$$



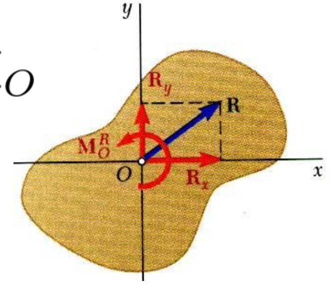
$\mathbf{s} = \mathbf{r}_{O'O}$

Recall $\mathbf{M}_{O'} = \mathbf{M}_O + \mathbf{s} \times \mathbf{R}$.
Let $\mathbf{r}_{OO'} = x\mathbf{i} + y\mathbf{j}$ and $\mathbf{M}_{O'} = 0$, thus

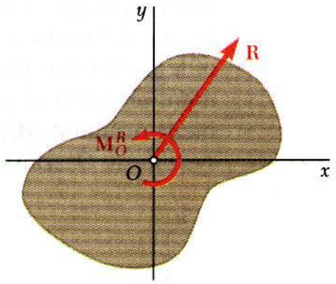
$\mathbf{r}_{O'O} = -\mathbf{r}_{OO'}$

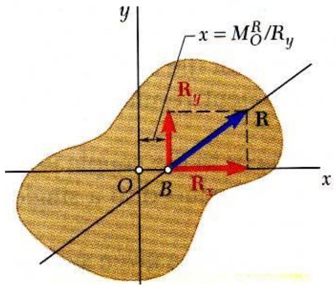


$$\mathbf{M}_{O'} = \mathbf{M}_O - \mathbf{r}_{O'O} \times \mathbf{R} = 0 \quad \rightarrow \quad \mathbf{r}_{OO'} \times \mathbf{R} = \mathbf{M}_O$$



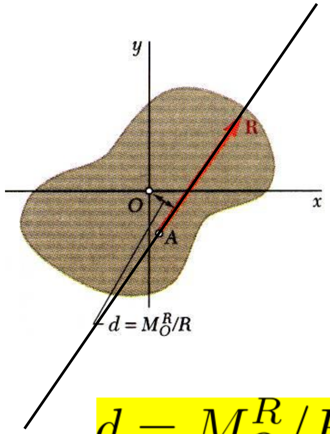
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ R_x & R_y & 0 \end{vmatrix} = M_{Oz}\mathbf{k}$$

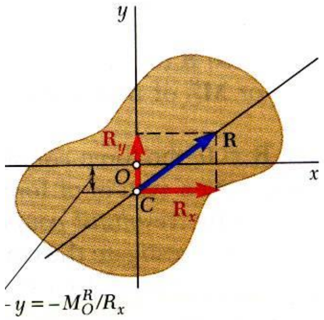




- In terms of rectangular coordinates,

$$xR_y - yR_x = M_O^R$$

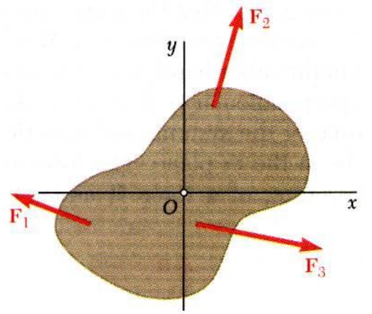




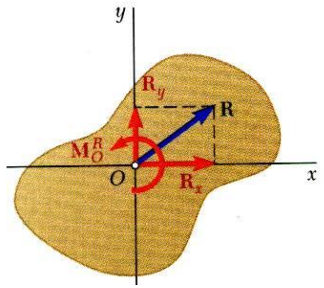
This is a linear equation.
If we choose any point on this line as **O**, we shall reduce the system into a resultant force.

$d = M_O^R / R$

$\mathbf{s} = \mathbf{r}_{O'O}$



$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ R_x & R_y & 0 \end{vmatrix} = M_{Oz}\mathbf{k}$$



- In terms of rectangular coordinates,

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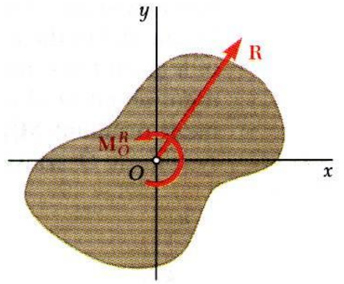
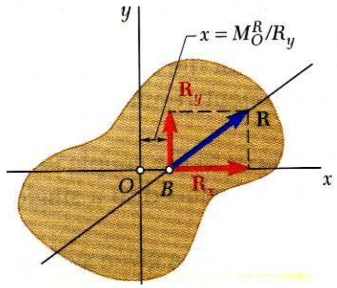
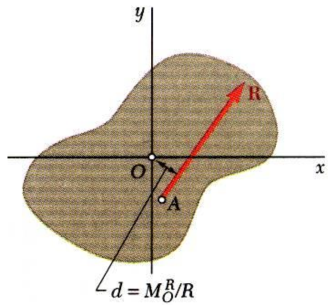
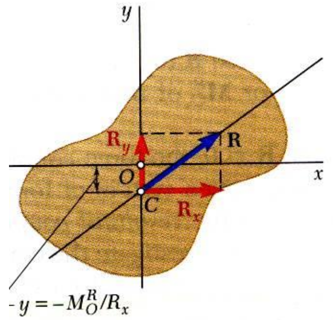


If $y = 0$, $x = M_O^R / R_y$.





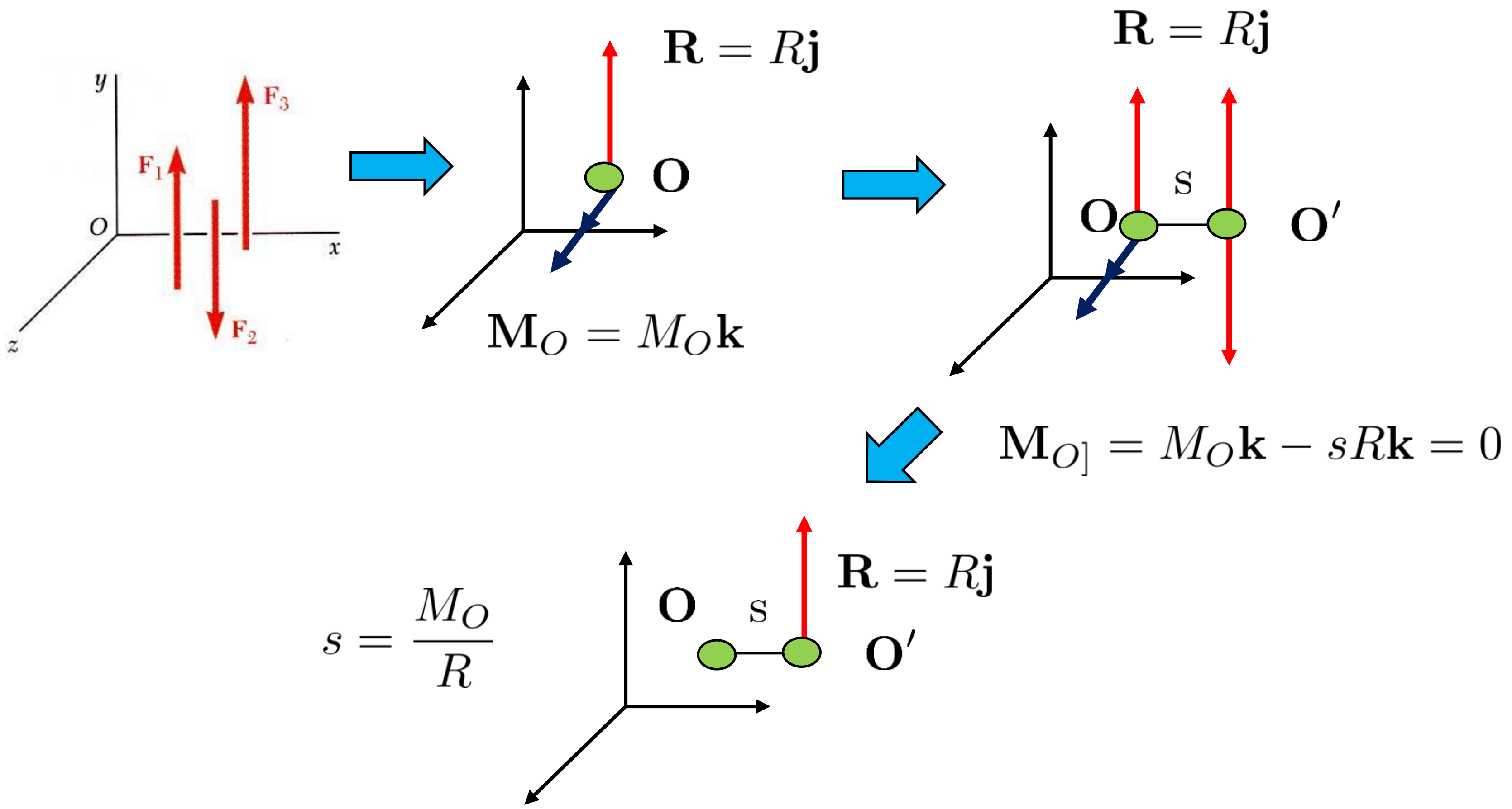
If $x = 0$, $y = -M_O^R / R_x$.



Special Case III: The forces are parallel.

- The resultant force-couple system for a system of forces will be mutually perpendicular if:

  3) the forces are parallel.



Lecture 5
Equilibrium of Rigid Bodies (2D)

Equilibrium Equations for Rigid Bodies

- The necessary and sufficient condition for the static equilibrium of a body are that the resultant force and couple from all external forces form a system equivalent to zero,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

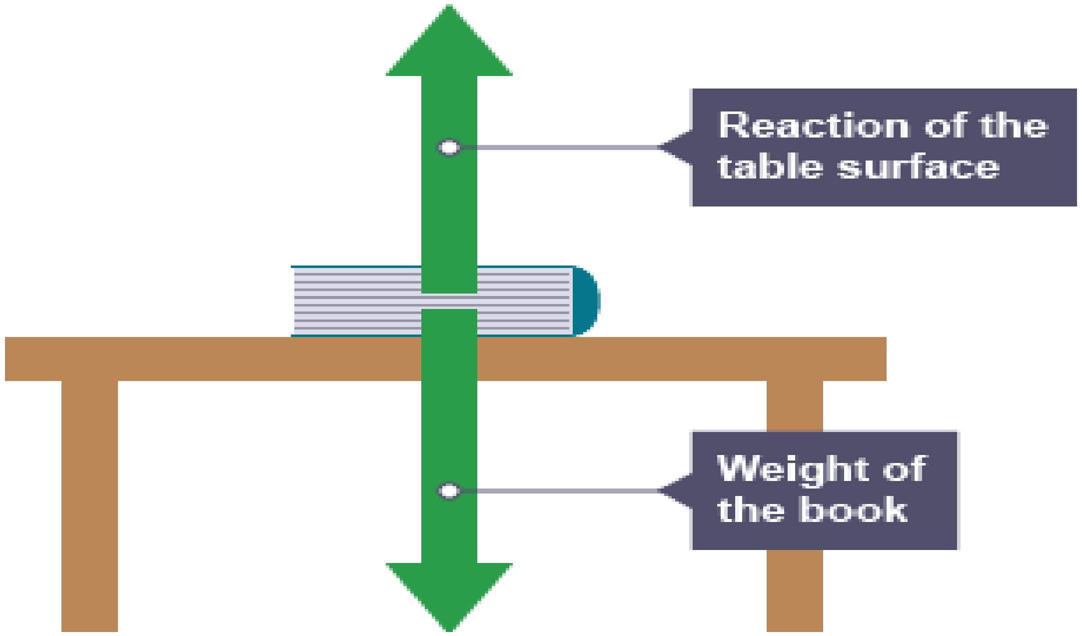
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

This is an engineering class !

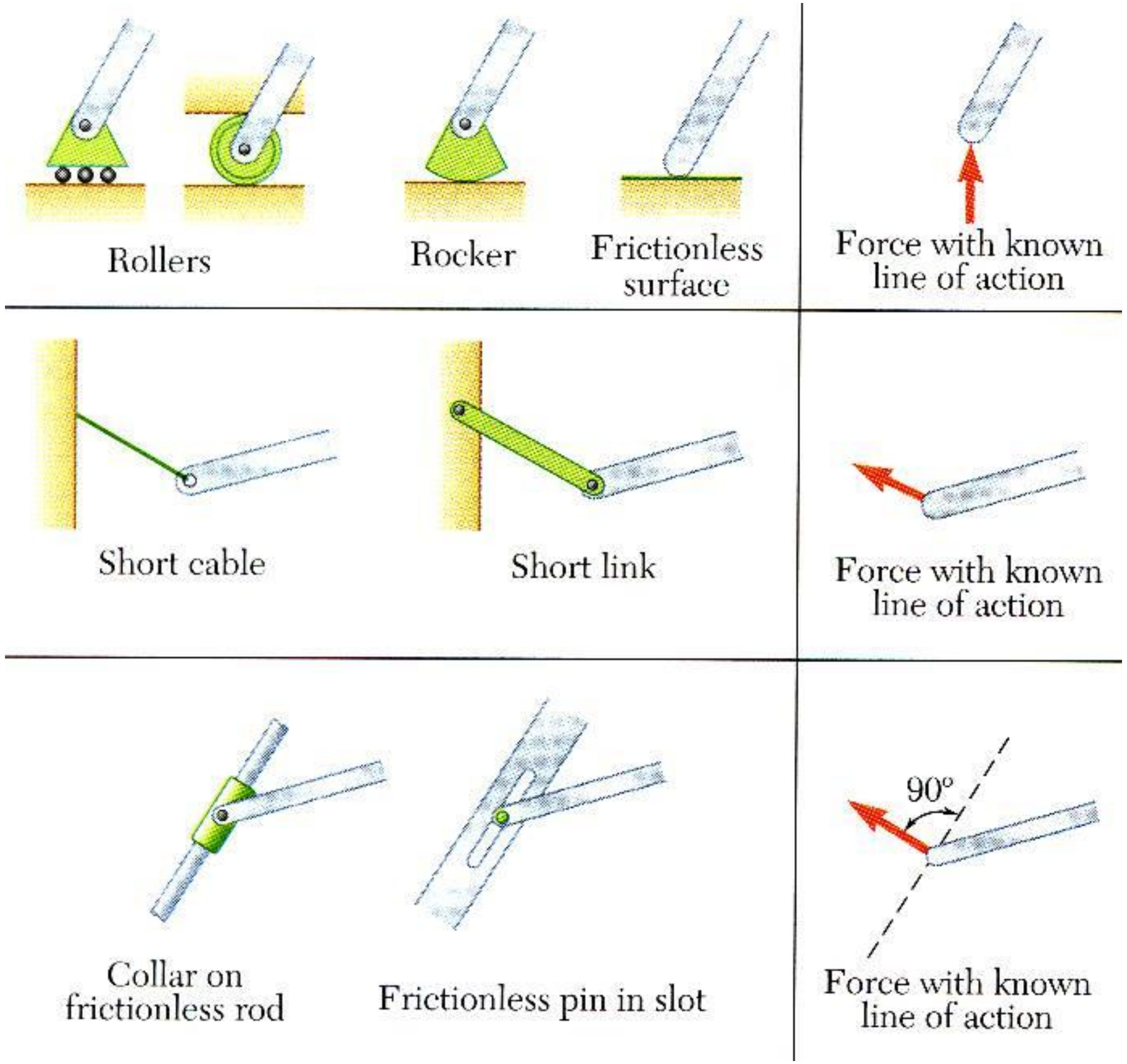
$$\sum_{i=1}^{n} F_{xi} = 0, \quad \sum_{i=1}^{n} F_{yi} = 0 \quad \text{and} \quad \sum_{j=1}^{m} M_{zj}^{O} = 0$$

What is REACTION (force) ?

Reaction is a type of **passive external forces**, and it usually arises due to the enforcement of a **displacement constraint.**



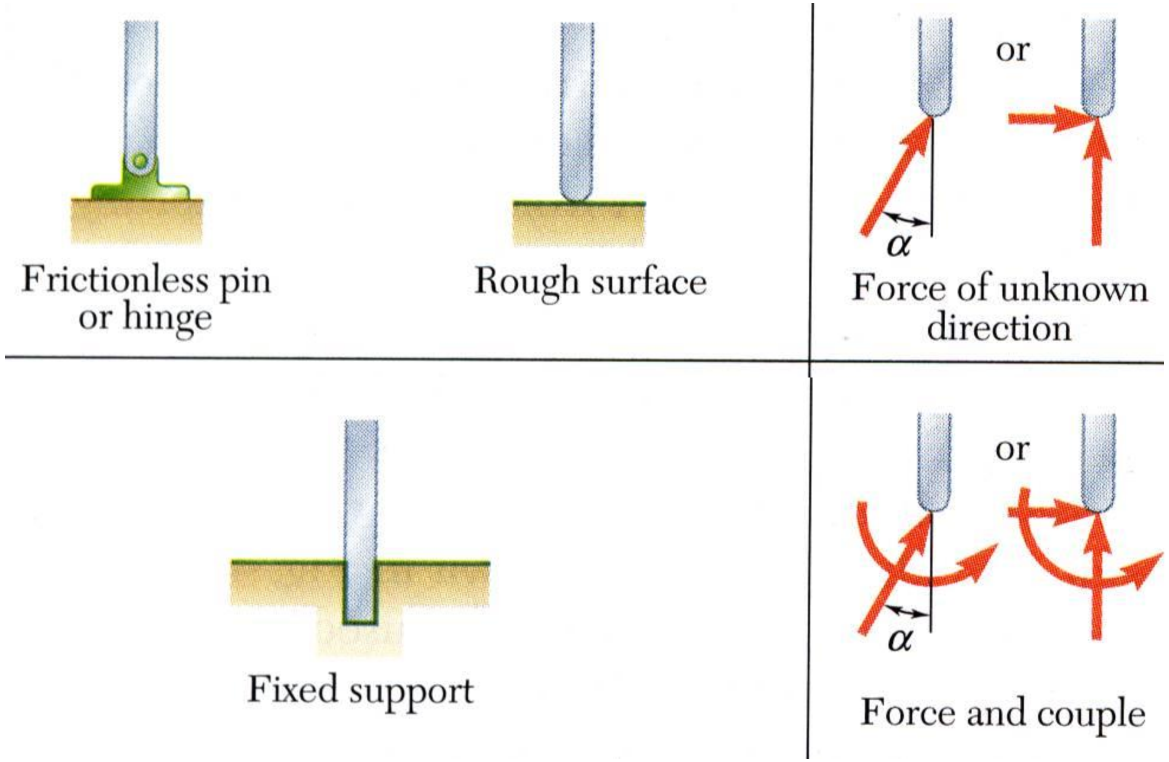
Reactions at Supports and Connections for a 2D Structure



- Reactions equivalent to a force with known line of action.

Technical sysmbols for displacement constraints. They specify directions of the reactions,

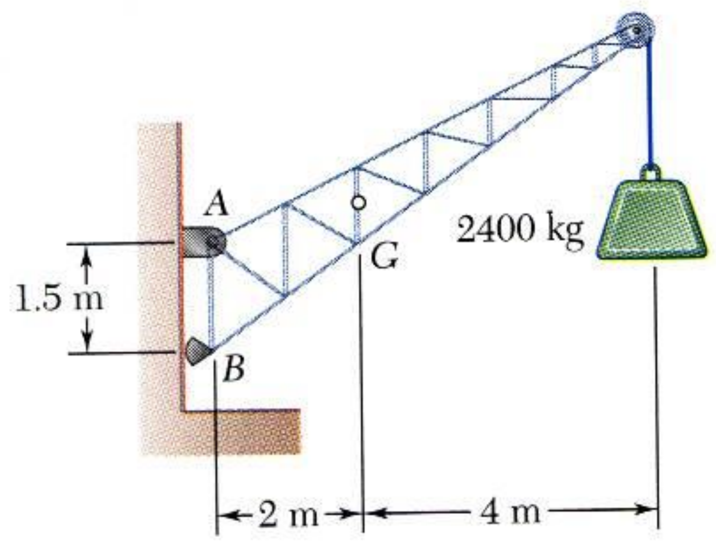
Reactions at Supports and Connections for a 2D Structure



- Reactions equivalent to a force of unknown direction and magnitude.

- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

Sample Problem 4.1



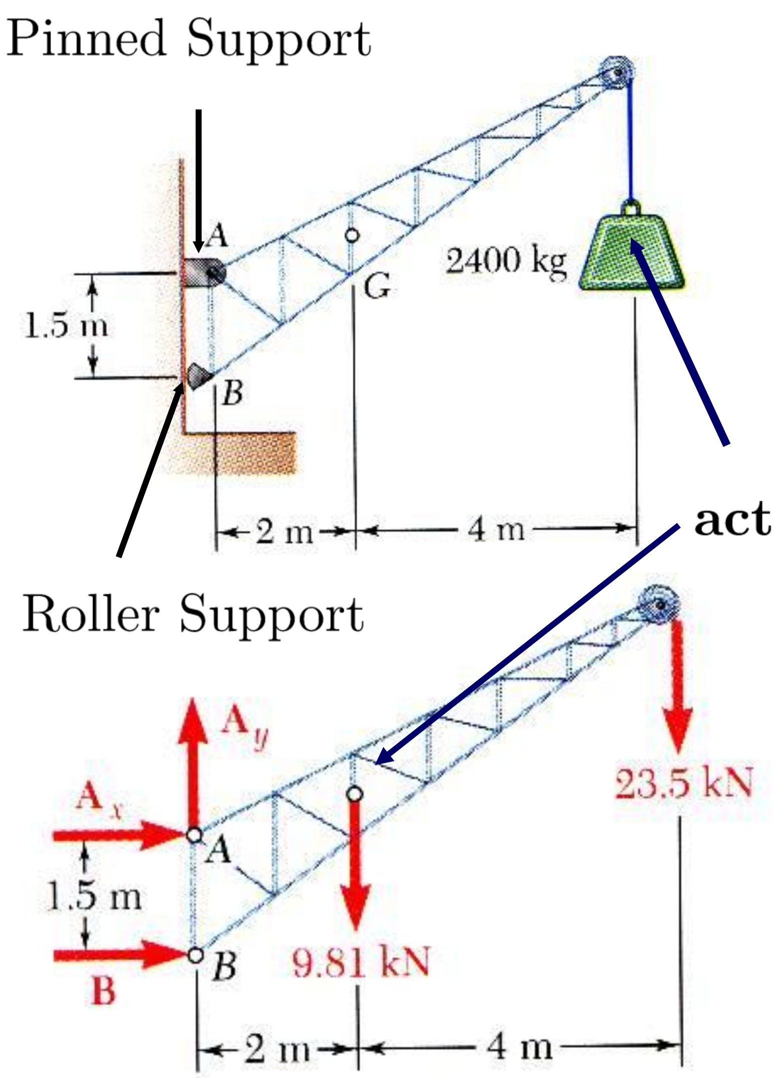
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the reactions at *A* and *B*.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Draw Free-Body Diagram

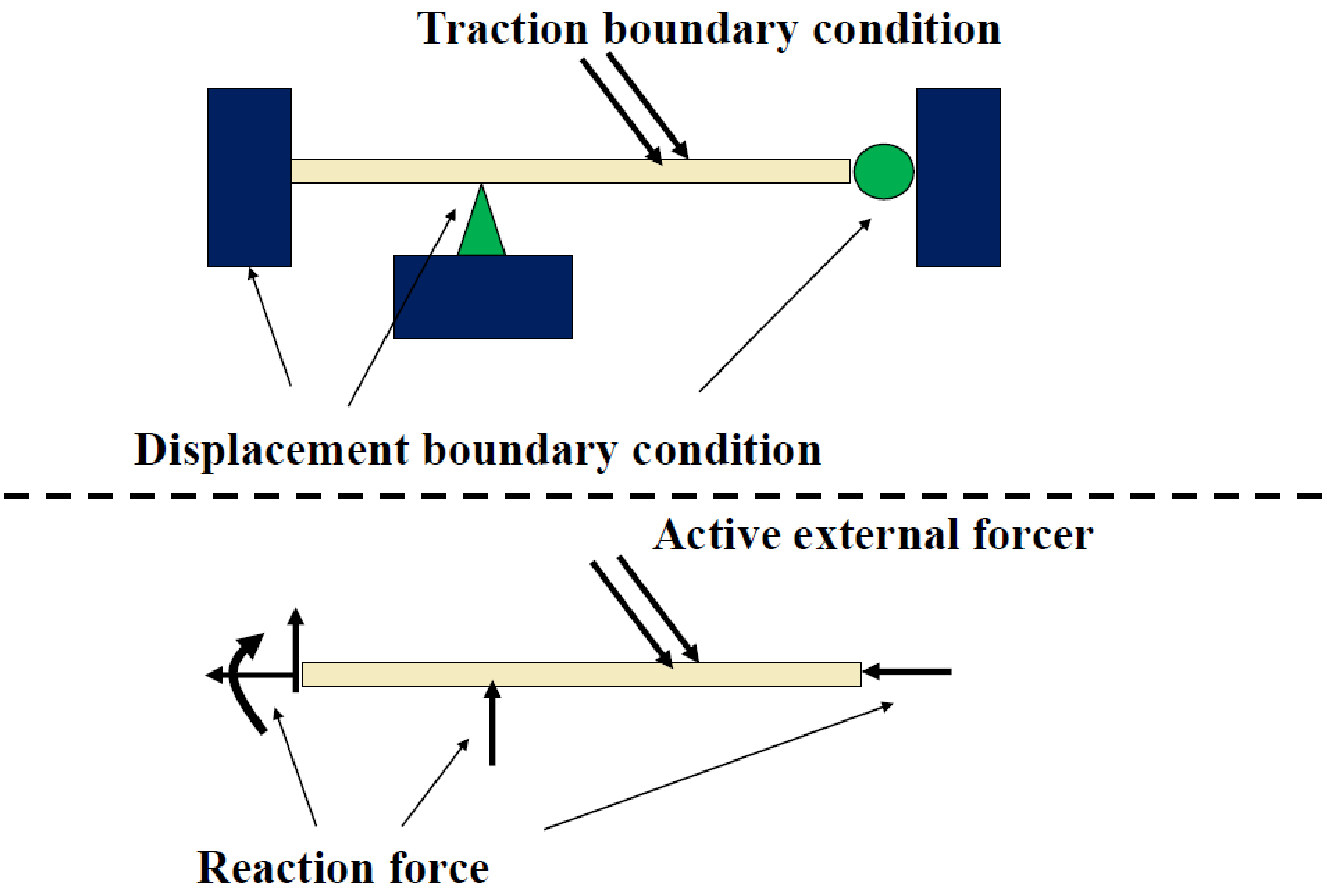


1. Select the extent of the free-body and detach it from the ground and all other bodies.

2. Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.

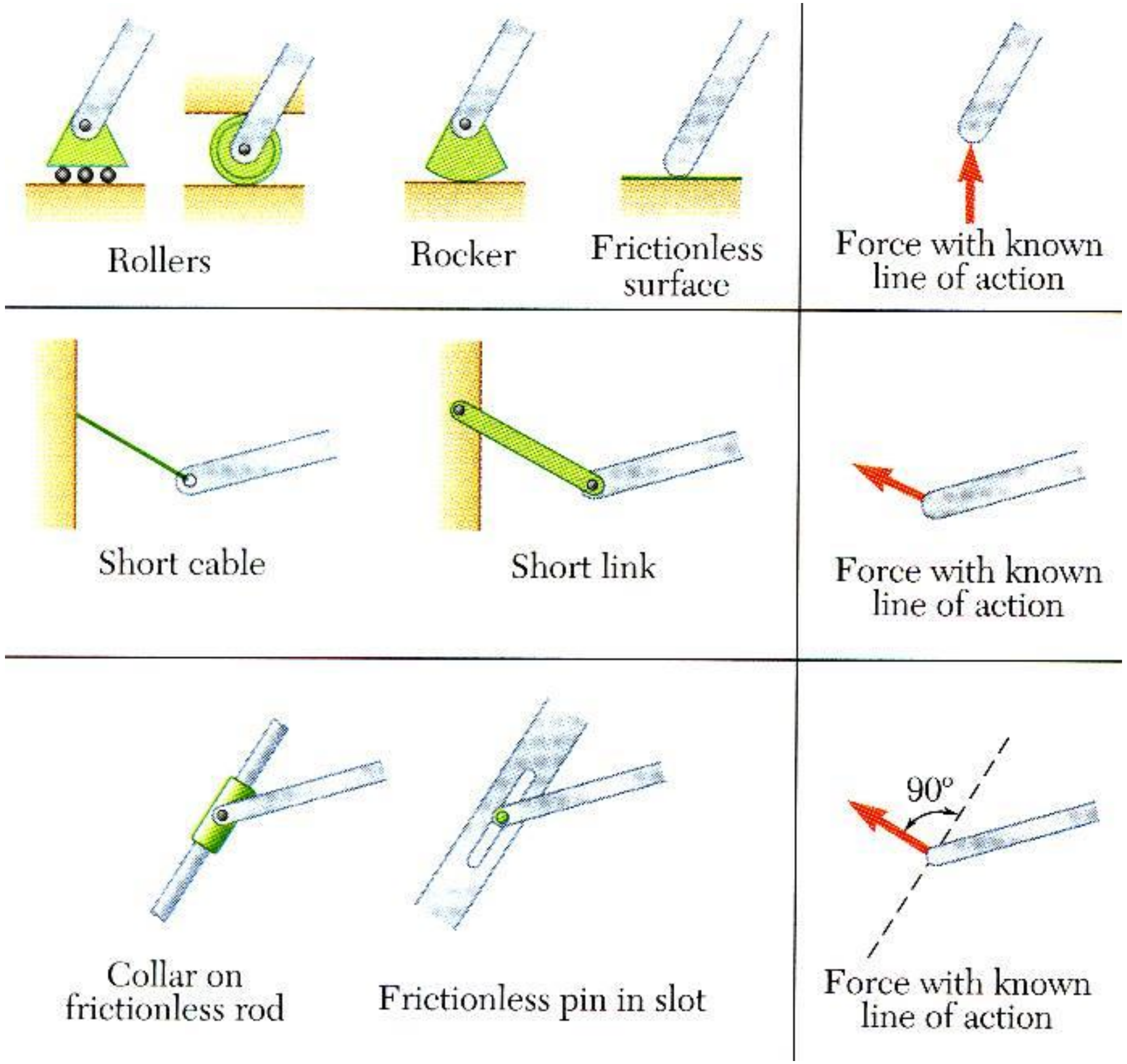
3. Indicate point of application and assumed direction of unknown applied forces. These usually consist of *reactions* through which the ground and other bodies oppose the possible motion of the rigid body.

4. Include the dimensions necessary to compute the moments of the forces.



Free-body Diagram

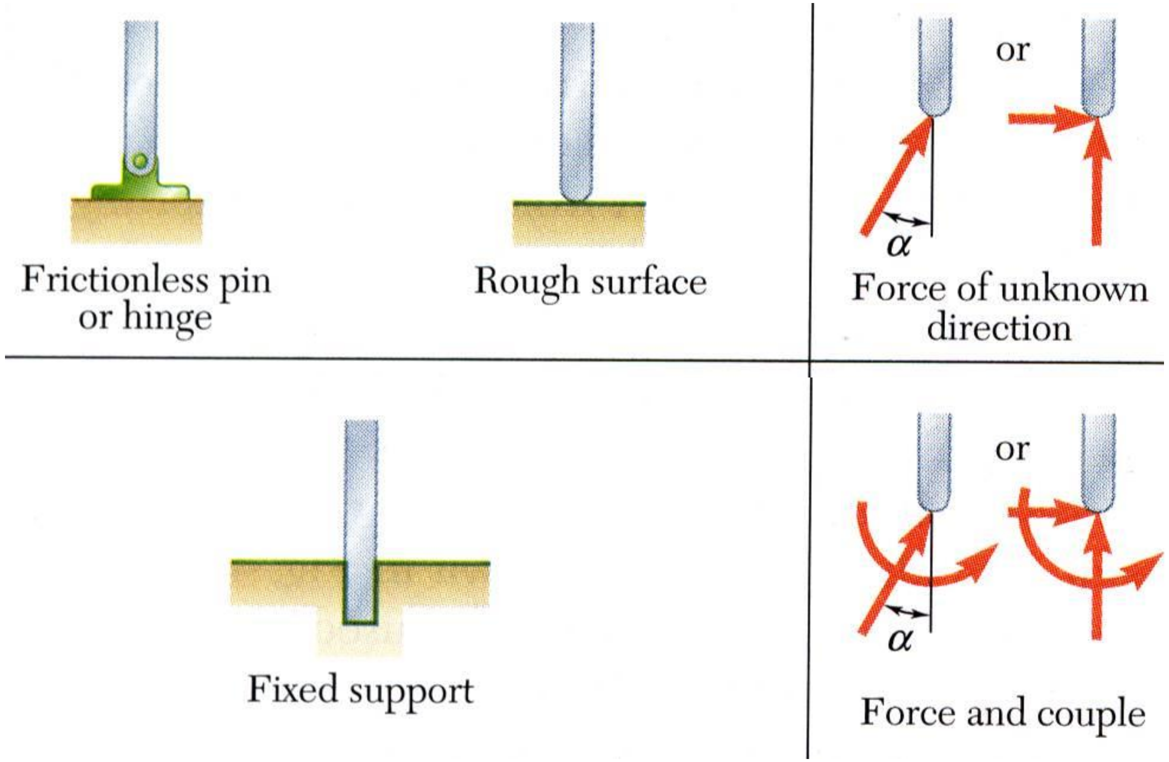
Reactions at Supports and Connections for a 2D Structure



- Reactions equivalent to a force with known line of action.

Technical sysmbols for displacement constraints. They specify directions of the reactions,

Reactions at Supports and Connections for a 2D Structure



- Reactions equivalent to a force of unknown direction and magnitude.

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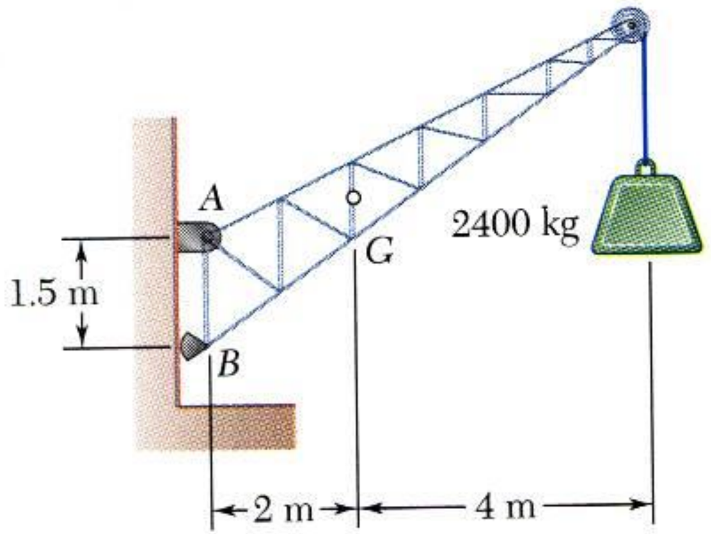
Sample Problem 4.1



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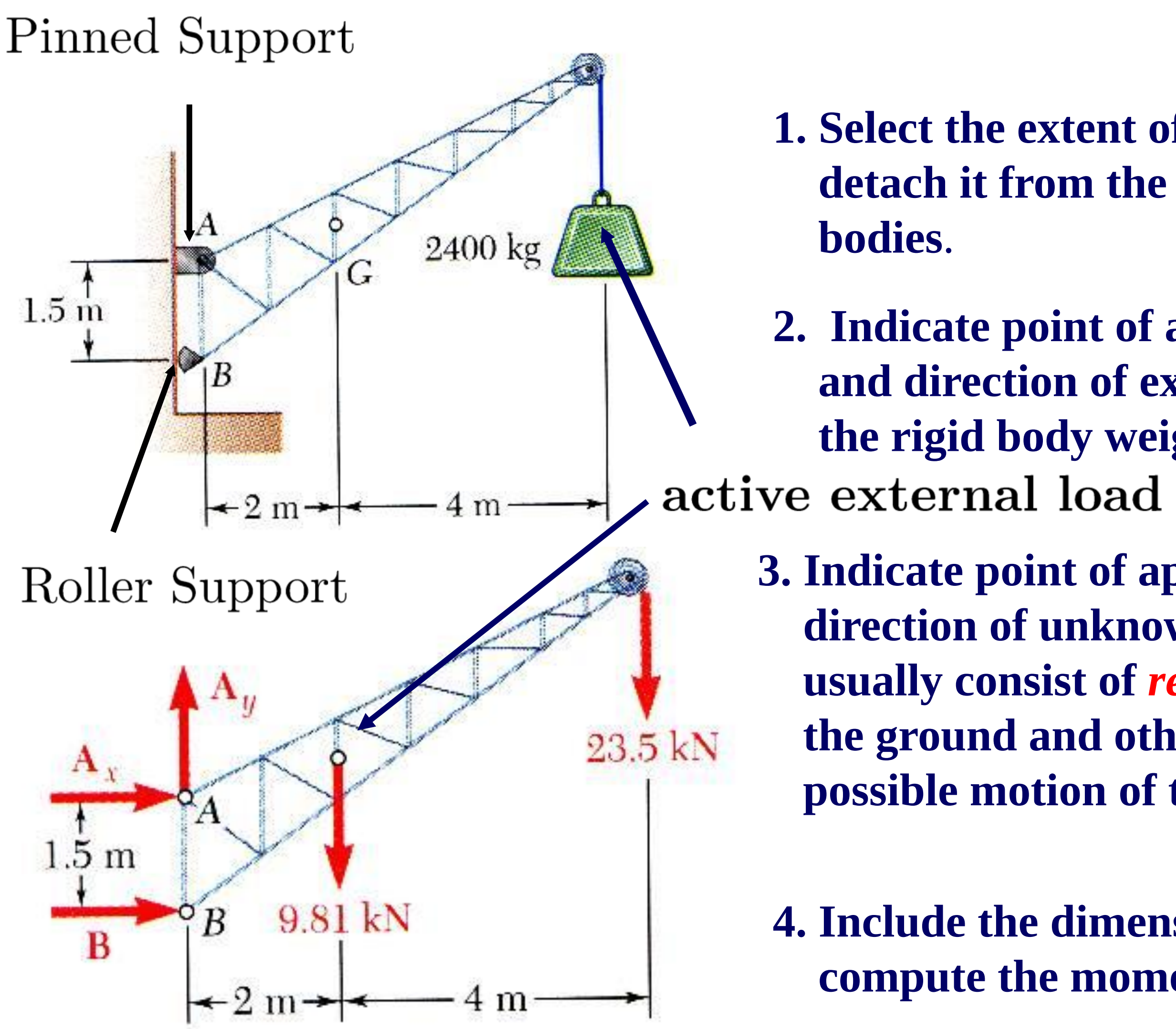
Determine the reactions at *A* and *B*.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Draw Free-Body Diagram



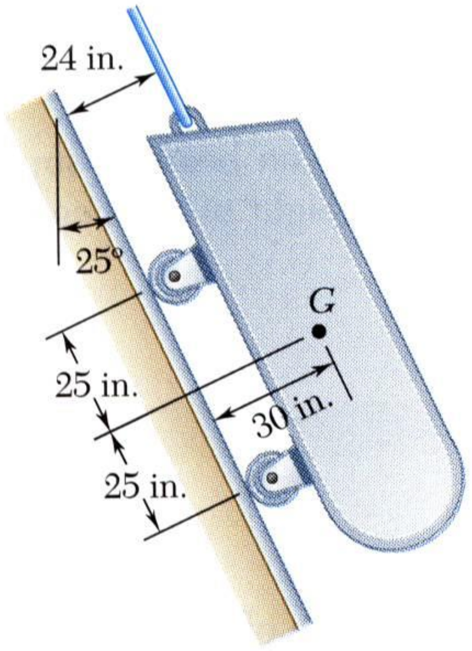
1. Select the extent of the free-body and detach it from the ground and all other bodies.

2. Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.

3. Indicate point of application and assumed direction of unknown applied forces. These usually consist of *reactions* through which the ground and other bodies oppose the possible motion of the rigid body.

4. Include the dimensions necessary to compute the moments of the forces.

Sample Problem 4.3



A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at at *G*. The cart is held in position by the cable.

Determine the tension in the cable and the reaction at each pair of wheels.

SOLUTION:

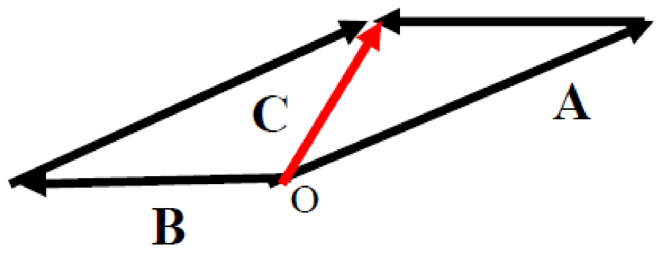
- Create a free-body diagram for the car with the coordinate system aligned with the track.
- Determine the reactions at the wheels by solving equations for the sum of moments about points above each axle.
- Determine the cable tension by solving the equation for the sum of force components parallel to the track.
- Check the values obtained by verifying that the sum of force components perpendicular to the track are zero.

C30/ME85 Million Dollar Questions

Rules: How to play?

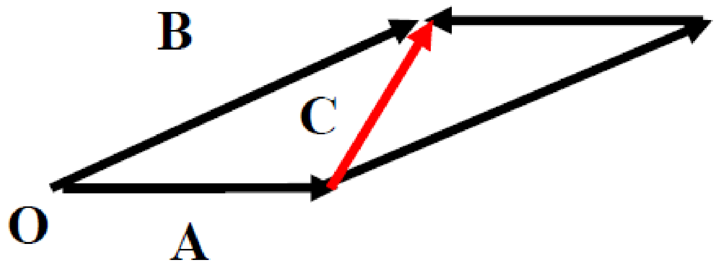
1. **Anyone can volunteer to participate;**
2. **Answering correctly can get 0.5 overall points;**
3. **Answer wrong without penalty;**

(**Q1**)In the following parallelogram, the vector $\mathbf{C}$ is:



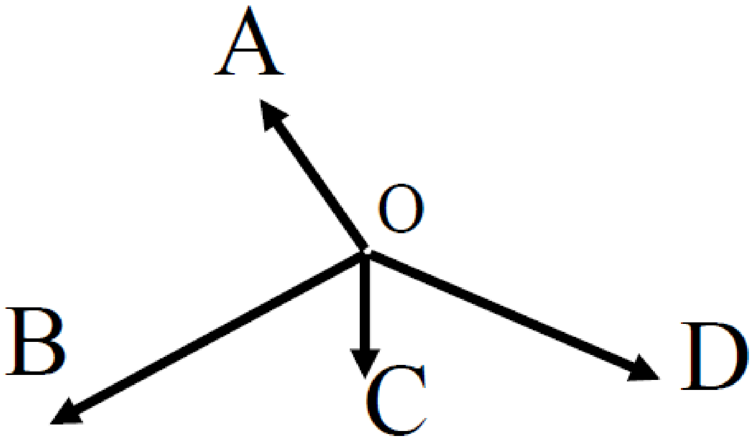
(**a**): $\mathbf{C} = \mathbf{A} + \mathbf{B}$;
(**b**): $\mathbf{C} = \mathbf{A} - \mathbf{B}$;
(**c**): $\mathbf{C} = \mathbf{B} - \mathbf{A}$;
(**d**): $\mathbf{C} = \mathbf{B} + \mathbf{B}$.

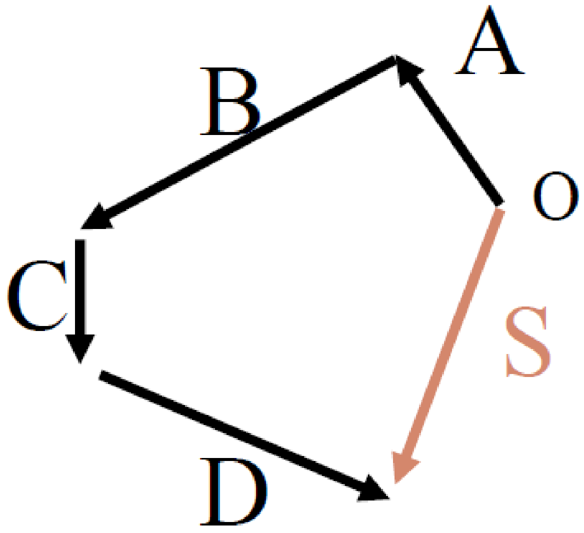
($\mathbf{Q2}$) In the following parallelogram, the vector $\mathbf{C}$ is:



$(\mathbf{a})$: $\mathbf{C} = \mathbf{A} + \mathbf{B}$;
$(\mathbf{b})$: $\mathbf{C} = \mathbf{A} - \mathbf{B}$;
$(\mathbf{c})$: $\mathbf{C} = \mathbf{B} - \mathbf{A}$;
$(\mathbf{d})$: $\mathbf{C} = \mathbf{B} + \mathbf{B}$.

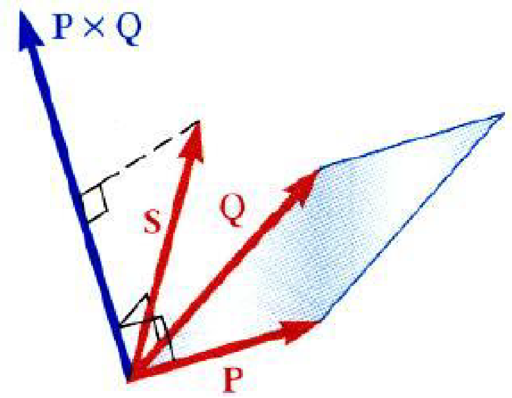
(Q3) Assume that the current force vectors, A,B,C, and D are in equilibrium. Which of the following equations is NOT correct?





(a) S = A + B + C + D;
(b) S= -(A + B + C+ D);
(c) S= 0;
(d) S = A – B + C –D.

(Q4) For the mixed triple product, which of the following is NOT correct:

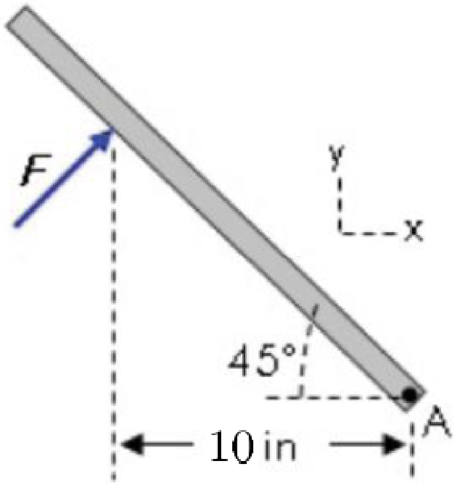


(a) $\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = \mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S})$;

(b) $\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = \mathbf{Q} \cdot (\mathbf{S} \times \mathbf{P})$;

(c) $\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = \mathbf{P} \cdot (\mathbf{S} \times \mathbf{Q})$;

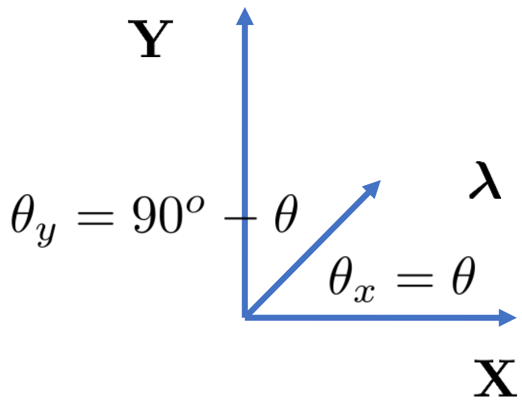
(d) $\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = -\mathbf{P} \cdot (\mathbf{S} \times \mathbf{Q})$;



(Q5) The moment about point A due to the force **F** is $\mathbf{M}_A = 43\mathbf{k}$ lb-in. Determine the magnitude of the force **F**. (**F** is perpendicular to the bar.)

(a) 4.00 lb;
(b) 3.04 lb;
(c) 5.04 lb
(d) 10.0 lb

Q6. Which of the following expressions is the correct expression of the unit vector $\boldsymbol{\lambda}$?



(**A**) $\boldsymbol{\lambda} = \cos\theta\mathbf{i} + \cos\theta\mathbf{j};$

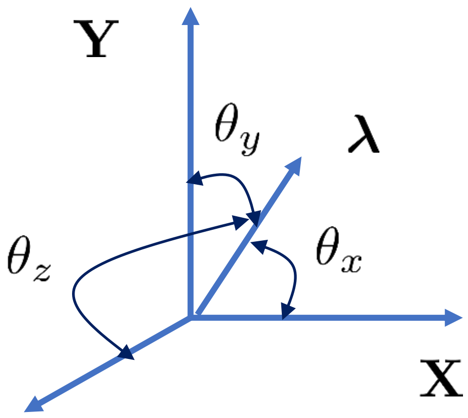
(**B**) $\boldsymbol{\lambda} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j};$

(**C**) $\boldsymbol{\lambda} = \sin\theta\mathbf{i} + \cos\theta\mathbf{j};$

(**D**) $\boldsymbol{\lambda} = \sin\theta\mathbf{i} + \sin\theta\mathbf{j};$

(Ans): (B)

Q7. Which of the following expressions is the correct expression of the unit vector $\boldsymbol{\lambda}$?



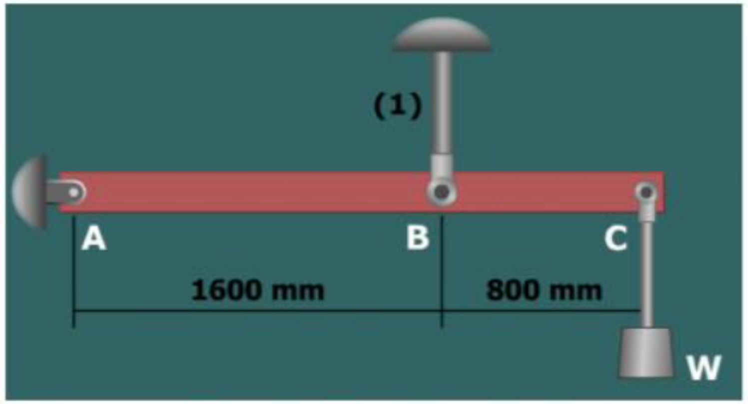
(**A**) $\boldsymbol{\lambda} = \cos\theta_x \mathbf{i} + \cos\theta_y \mathbf{j} + \cos\theta_z \mathbf{k}$

(**B**) $\boldsymbol{\lambda} = \cos\theta_x \mathbf{i} + \sin\theta_y \mathbf{j} + \sin\theta_z \mathbf{k}$;

(**C**) $\boldsymbol{\lambda} = \sin\theta_x \mathbf{i} + \sin\theta_y \mathbf{j} + \cos\theta_z \mathbf{k}$;

(**D**) $\boldsymbol{\lambda} = \sin\theta_x \mathbf{i} + \sin\theta_y \mathbf{j} + \sin\theta_z \mathbf{k}$.

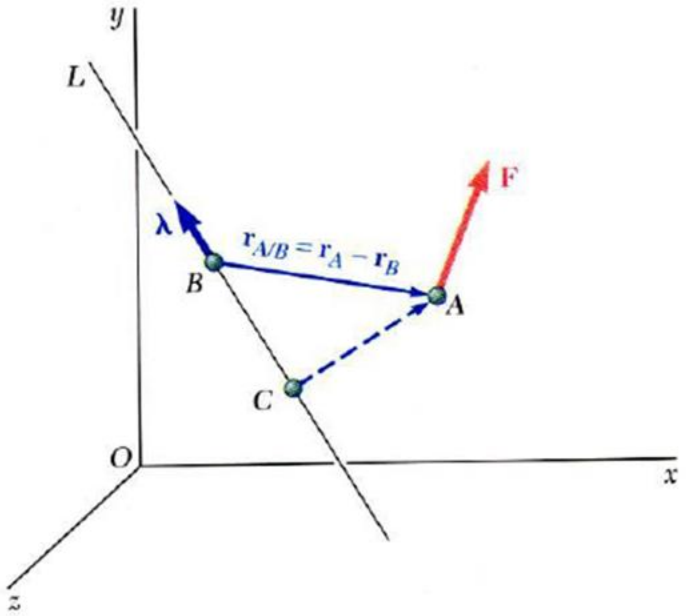
(Ans): (A)



(Q8) Rigid bar ABC supports a weight of W = 50 kN. Bar ABC is pinned at A and supported at B by rod (1). What is the axial force in rod (1) ?

(A) 1000 kN;
(B) 75 kN;
(C) 50 kN;
(D) 200 kN

Ans:(B)



- Moment of a force about an arbitrary axis

is shown in the Figure.,

$$\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F}$$

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}$$

$$M_{BL} = \boldsymbol{\lambda} \cdot (\mathbf{r}_{BA} \times \mathbf{F})$$

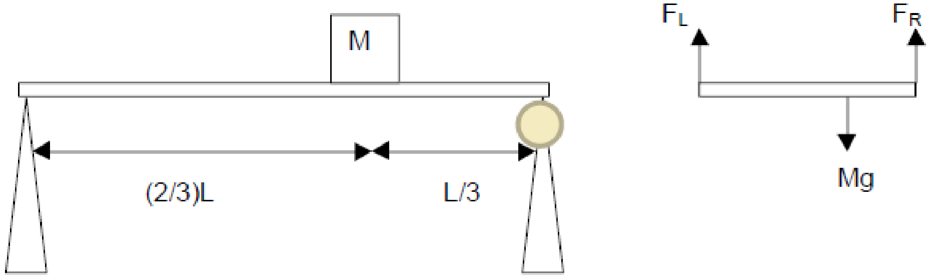
$$M_{CL} = \boldsymbol{\lambda} \cdot (\mathbf{r}_{CA} \times \mathbf{F})$$

(Q9) Which of the following statements is correct:

(a) $\mathbf{M}_B = \mathbf{M}_C$;
(b) $M_{BL} = M_{CL}$;
(c) $\mathbf{r}_{BA} \cdot \mathbf{F} = \mathbf{r}_{CA} \cdot \mathbf{F}$;
(d) $\mathbf{r}_{BA} \cdot \boldsymbol{\lambda} = \mathbf{r}_{CA} \cdot \boldsymbol{\lambda}$;

Ans (b)

Q (10). A color TV of mass M is placed on a very light board supported at the ends, as shown. The free-body diagram shows directions of the forces, but not their correct relative sizes.



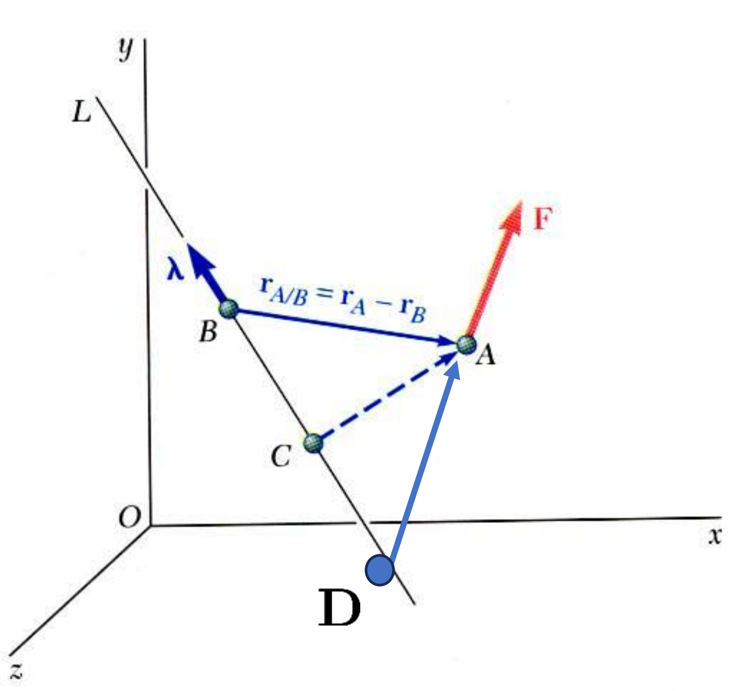
What is the ratio $\frac{F_R}{F_L}$?

A) 2/3
B) 1/3
C) 1/2
D) 2
E) 1

Ans: (D)

$(\mathbf{Q11})$ Assume that : $\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} \neq \mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F} \neq \mathbf{M}_D = \mathbf{r}_{DA} \times \mathbf{F}$

Which of the following is correrct:



(A) $\boldsymbol{\lambda} \cdot \mathbf{M}_B = \boldsymbol{\lambda} \cdot \mathbf{M}_C \neq \boldsymbol{\lambda} \cdot \mathbf{M}_D$

(B) $\boldsymbol{\lambda} \cdot \mathbf{M}_B \neq \boldsymbol{\lambda} \cdot \mathbf{M}_C = \boldsymbol{\lambda} \cdot \mathbf{M}_D$

(C) $\boldsymbol{\lambda} \cdot \mathbf{M}_B = \boldsymbol{\lambda} \cdot \mathbf{M}_C = \boldsymbol{\lambda} \cdot \mathbf{M}_D$

(D) $\boldsymbol{\lambda} \cdot \mathbf{M}_B \neq \boldsymbol{\lambda} \cdot \mathbf{M}_C \neq \boldsymbol{\lambda} \cdot \mathbf{M}_D$

(Ans): (C)

- Moment of a force about an arbitrary axis,

Q12. Considering the following three exact numerical numbers:

$$(1)100.913, \quad (2)73.152, \text{ and } (3)200.12$$

Which of the following satisfies the minimum accuracy requirements imposed by the textbook?

(**A**) $(a)100.9; \quad (b)73.1, \ (c)200$.

(**B**) $(a)1.0091 \times 10^2; \ (b)7.3 \times 10^1, \ (c) \ 2.0 \times 10^2$

(**C**) $(a)1.0091 \times 10^2; \ (b)7.31 \times 10^1, \ (c) \ 2.12 \times 10^2$

(Ans): (D)

(**D**) $(a)100.91; \ (b)73.15, \ (c) \ 200.1$