Correction

3.39 To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the moments about the y and z axes of the force exerted at B by portion AB of the rope are, respectively, 120 N·m and –460 N·m, determine the distance a.

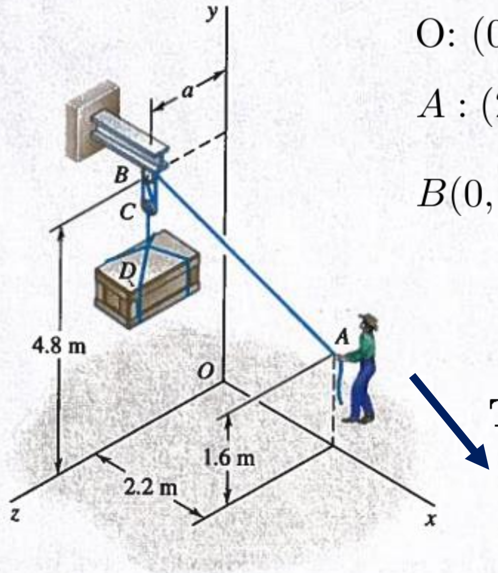


Fig. P3.39 and P3.40

O: (0,0,0)

$A:(2.2, 1.6, 0)$

$B(0, 4.8, a)$

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T}_{AB}$$

$$M_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$\mathbf{r}_{OA} = \mathbf{r}_A = 2.2\mathbf{i} + 1.6\mathbf{j}$$

$$\mathbf{r}_{BA} = \mathbf{r}_{A/B} = 2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k}$$

$$\begin{aligned} r_{BA} &= \sqrt{(2.2)^2 + (-3.2)^2 + (-a)^2} \\ &= \sqrt{15.08^2 + a^2} \end{aligned}$$

$$\mathbf{T}_{BA} = \frac{T_{BA}}{r_{BA}}\Big(2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k}\Big)$$

Q12. Considering the following three exact numerical numbers:

$$(1)100.913, \quad (2)73.152, \text{ and } (3)200.12$$

Which of the following satisfies the minimum accuracy requirements imposed by the textbook?

(**A**) $(a)100.9; \quad (b)73.1, \; (c)200$.

(**B**) $(a)1.0091 \times 10^2; \; (b)7.3 \times 10^1, \; (c)\ 2.0 \times 10^2$

(**C**) $(a)1.0091 \times 10^2; \; (b)7.31 \times 10^1, \; (c)\ 2.12 \times 10^2$

(**D**) $(a)100.91; \; (b)73.15, \; (c)\ 200.1$

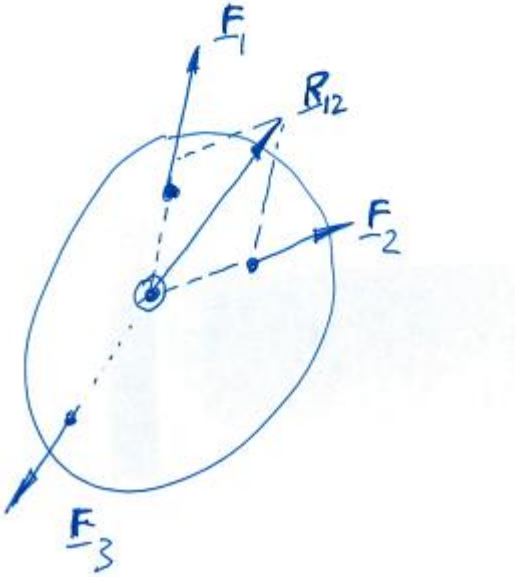
(Ans): (D)

Lecture 7
Equilibrium of Rigid Bodies (3D)

Three-force member

For three non-parallel forces acting on a rigid body,

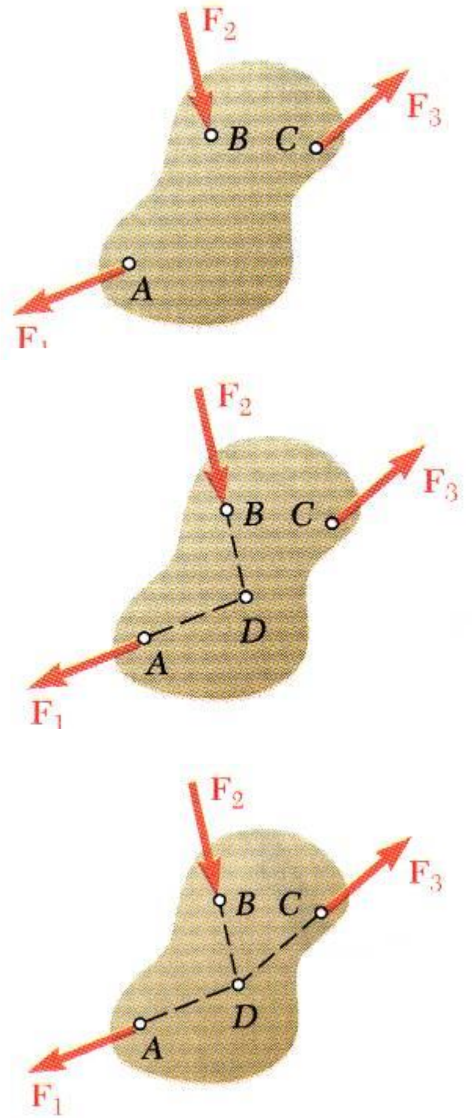
the line of action of the three forces must interect at on common point.



Why ?

because the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$, $\mathbf{R}_{12}$, will form a two-force system with the remaining force $\mathbf{F}_3$.

Equilibrium of a Three-Force Body



- Consider a rigid body subjected to forces acting at only 3 points.

- Assuming that their lines of action intersect, the moment of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ about the point of intersection represented by *D* is zero.

- Since the rigid body is in equilibrium, the sum of the moments of $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$ about any axis must be zero. It follows that the moment of $\boldsymbol{F_3}$ about *D* must be zero as well and that the line of action of $\boldsymbol{F_3}$ must pass through *D*.

- The lines of action of the three forces must be **concurrent** or parallel.

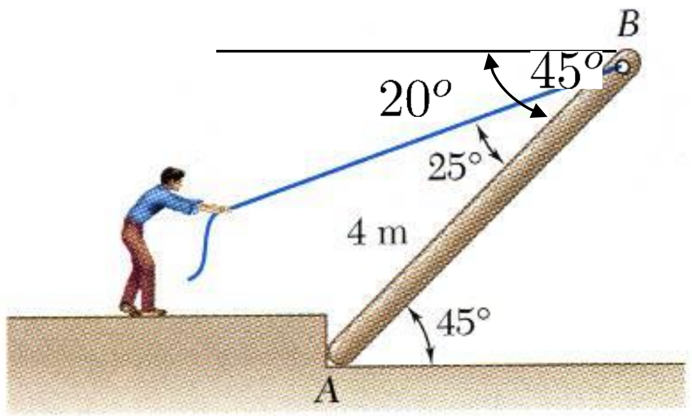
Definition:

If three **non-parallel** forces act on a rigid body in equilibrium, it is known as a three-force member.

Three-force member principle

If a three-force member is in equilibrium, the line of action of all three forces must intersect at a common point; and the total resultant is zero. In other words, any single force is the equilibrant of the two other forces.

Sample Problem 4.6



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

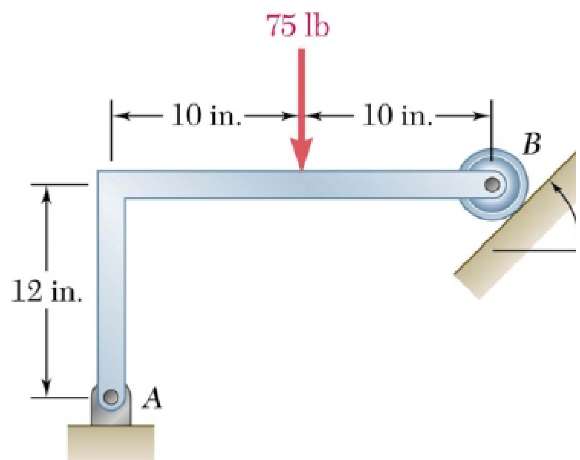
Find the tension in the rope and the reaction at *A*.

SOLUTION:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces.

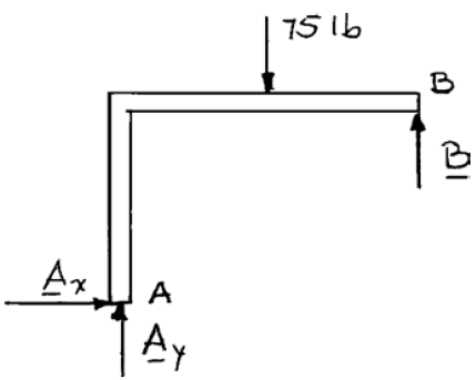
- Utilize a force triangle to determine the magnitude of the reaction force ***R***.



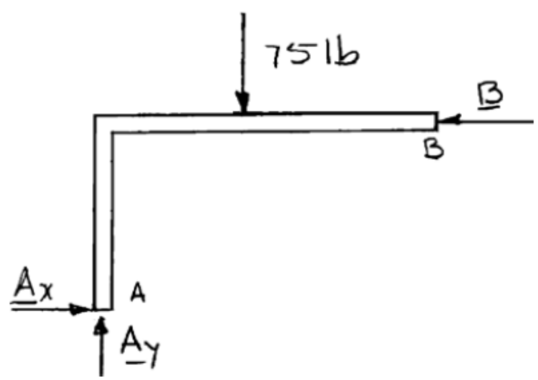
PROBLEM 4.13

Determine the reactions at A and B when (*a*) $\alpha = 0$, (*b*) $\alpha = 90°$, (*c*) $\alpha = 30°$.

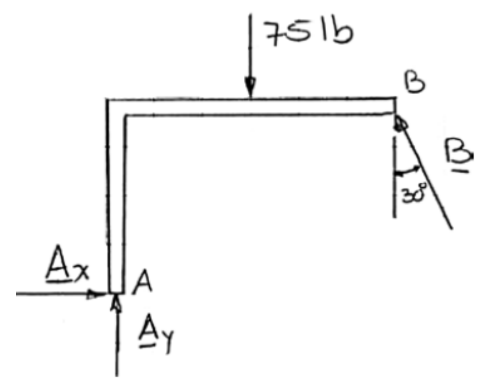
$\alpha = 0$



$\alpha = 90^o$



$\alpha = 30^o$



Main Takeaways

1. **Start solving statics problems by drawing the freebody diagram;**

2. **Identify boundary support conditions;**

3. **Inspect for two-force body or three-force body, and**

4. **Judiciously select the point for moment equations.**

Equilibrium of a Rigid Body in Three Dimensions

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum\left(\vec{r} \times \vec{F}\right) = 0$$

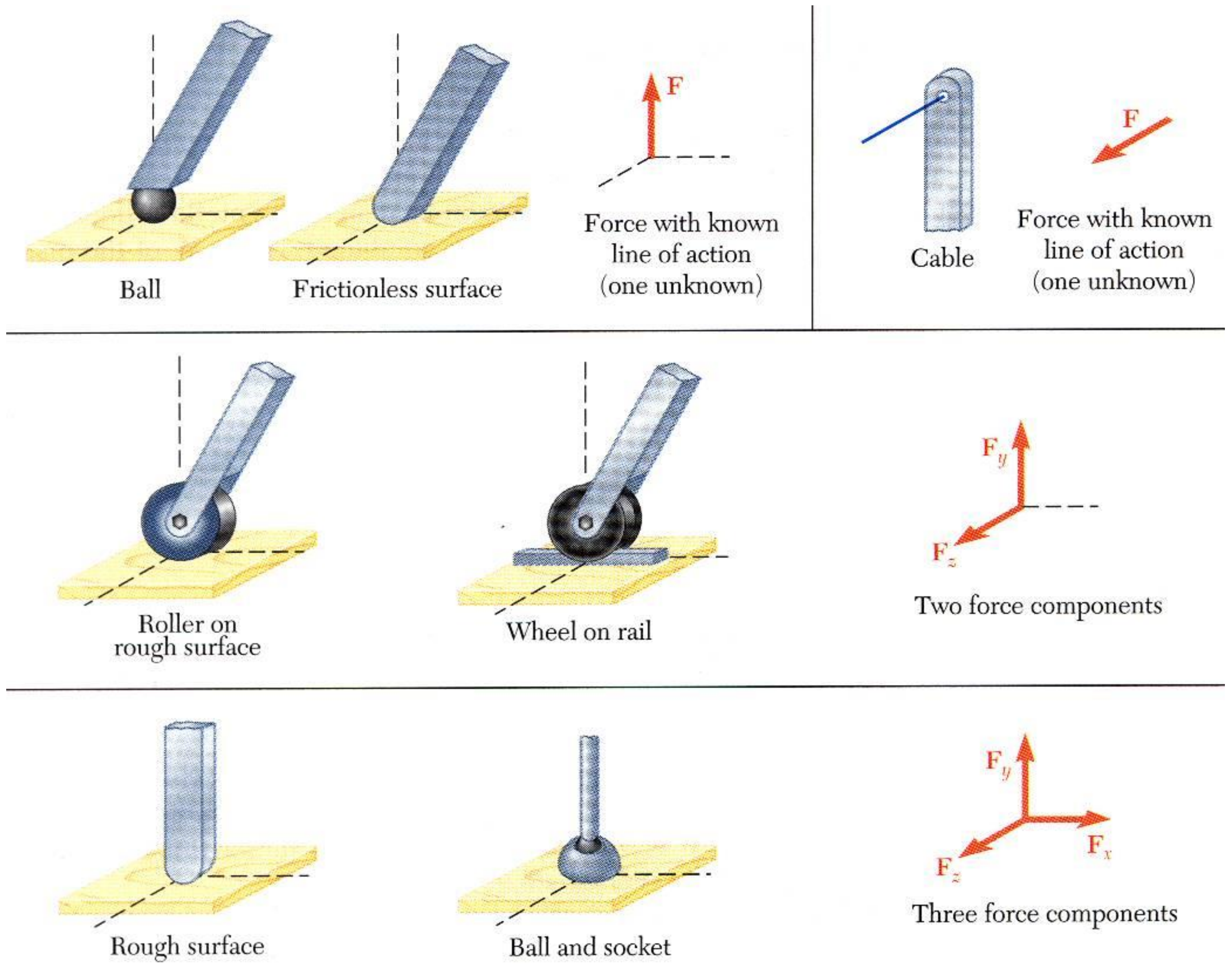
- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

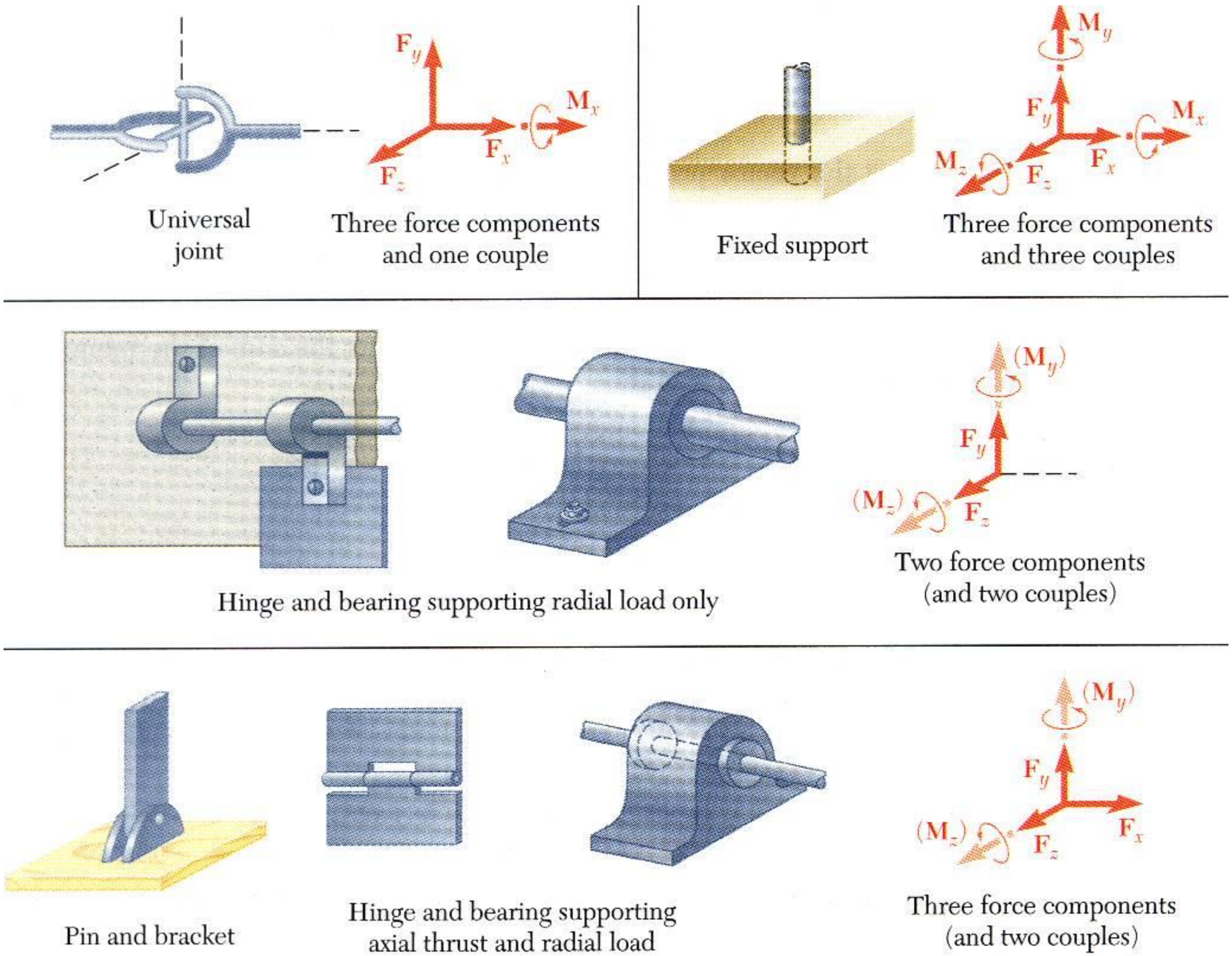
- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

Today's Lecture Attendance Password is: 3D Problem

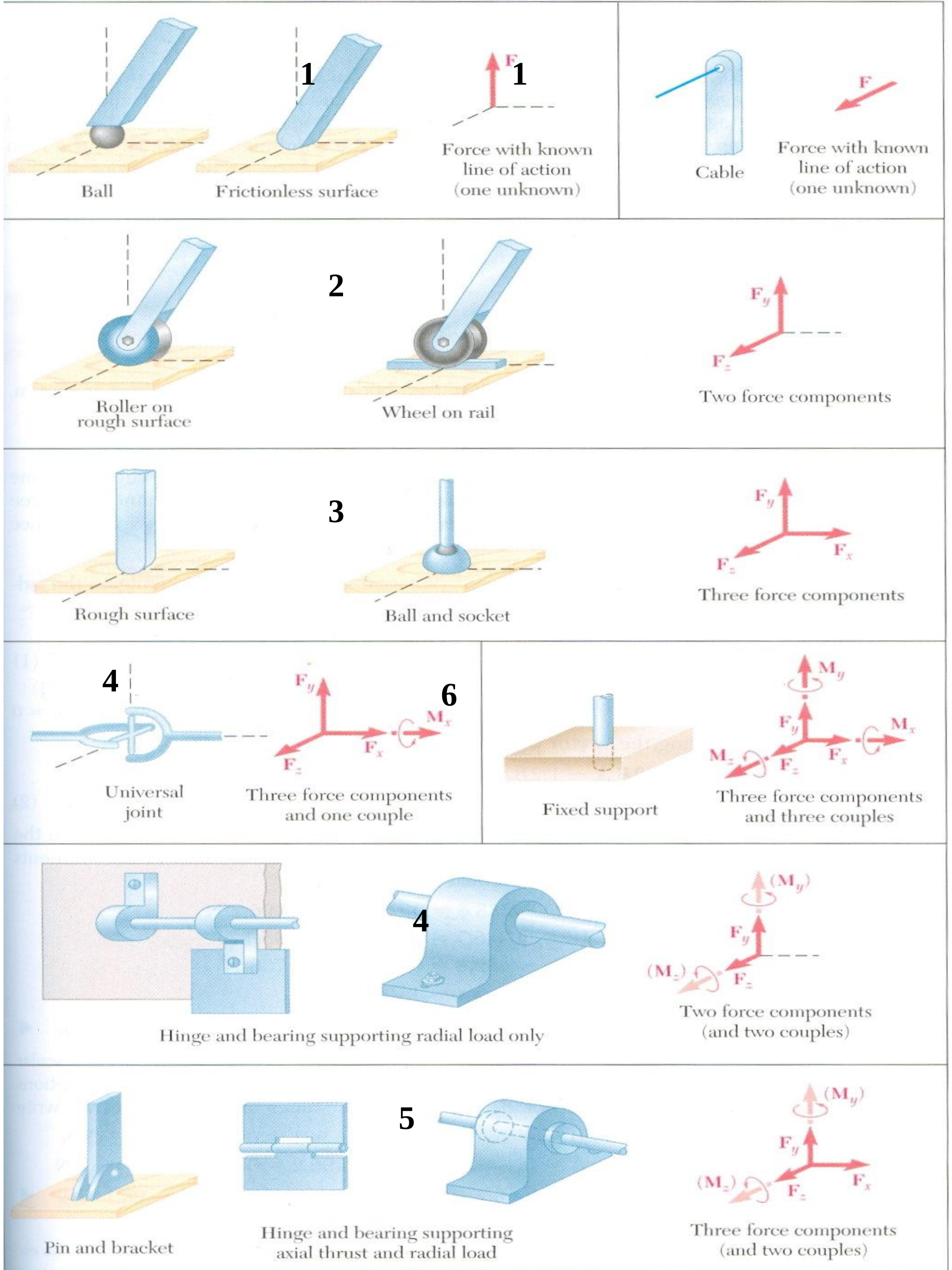
Reactions at Supports and Connections for a 3D Structure



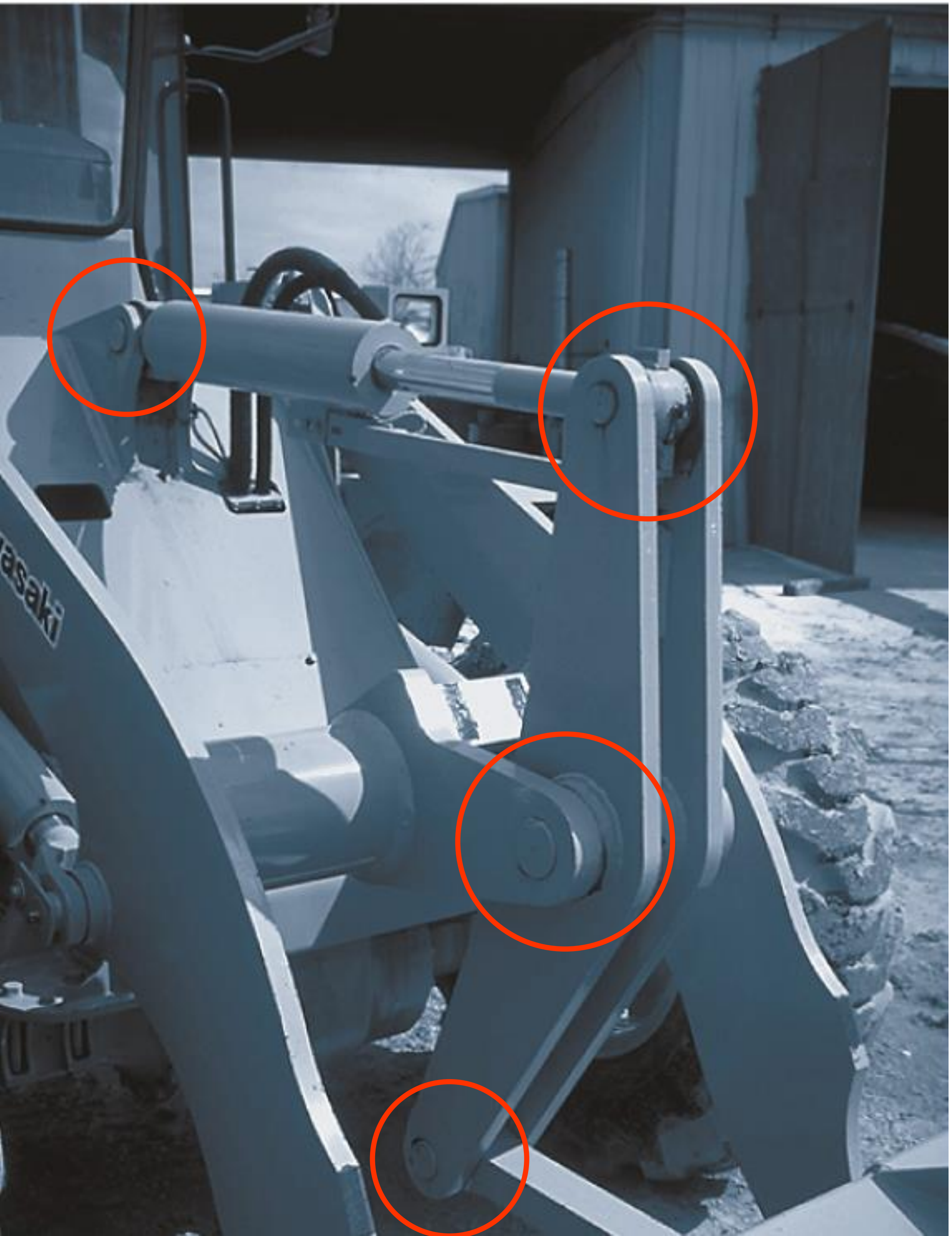
Reactions at Supports and Connections for a 3D Structure



Boundary Support Summary



Rocker Bearing used to Support the Roadway of a Bridge



Pin connections allow rotation. Reactions at pins are forces and NOT MOMENTS.

Degrees of Freedom

4.53 A 4 × 8-ft sheet of plywood weighing 40 lb has been temporarily propped against column *CD*. It rests at *A* and *B* on small wooden blocks and against protruding nails. Neglecting friction at all surfaces of contact, determine the reactions at *A*, *B*, and *C*.

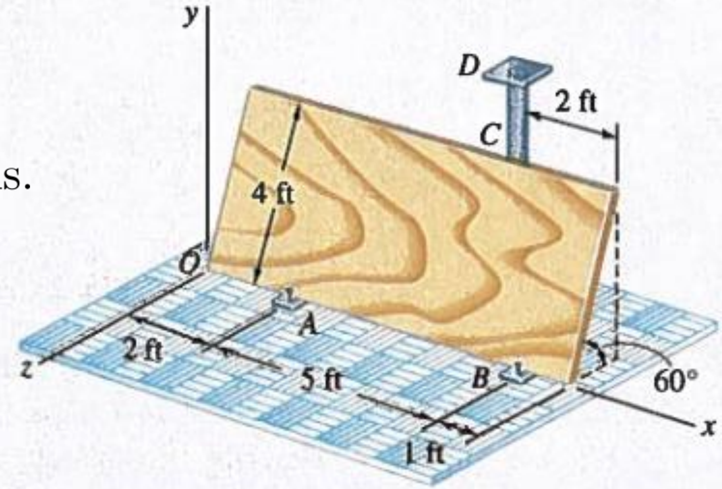
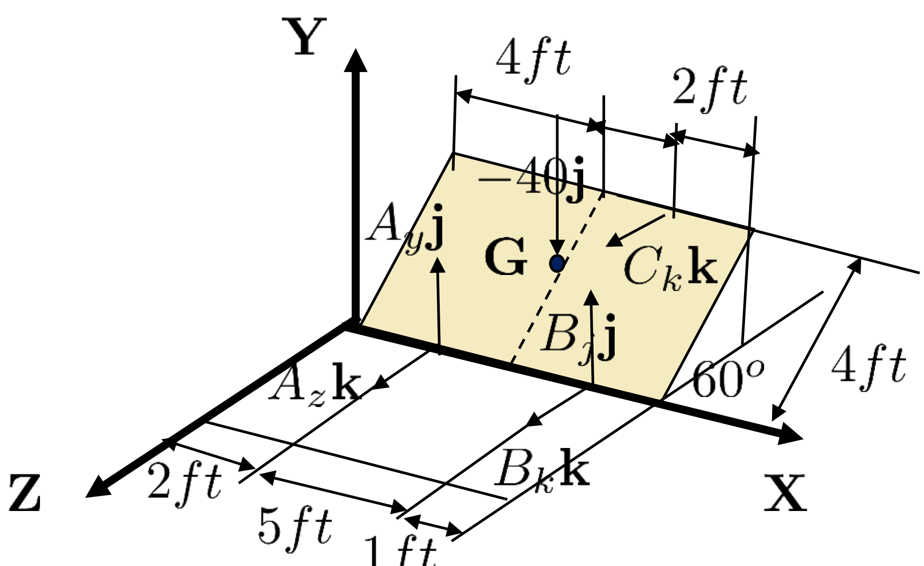


Fig. P4.53

Freebody-Diagram

We have five unknows and six equations.

The plywood sheet is free to move in x-direction, but ($\sum F_x = 0$).



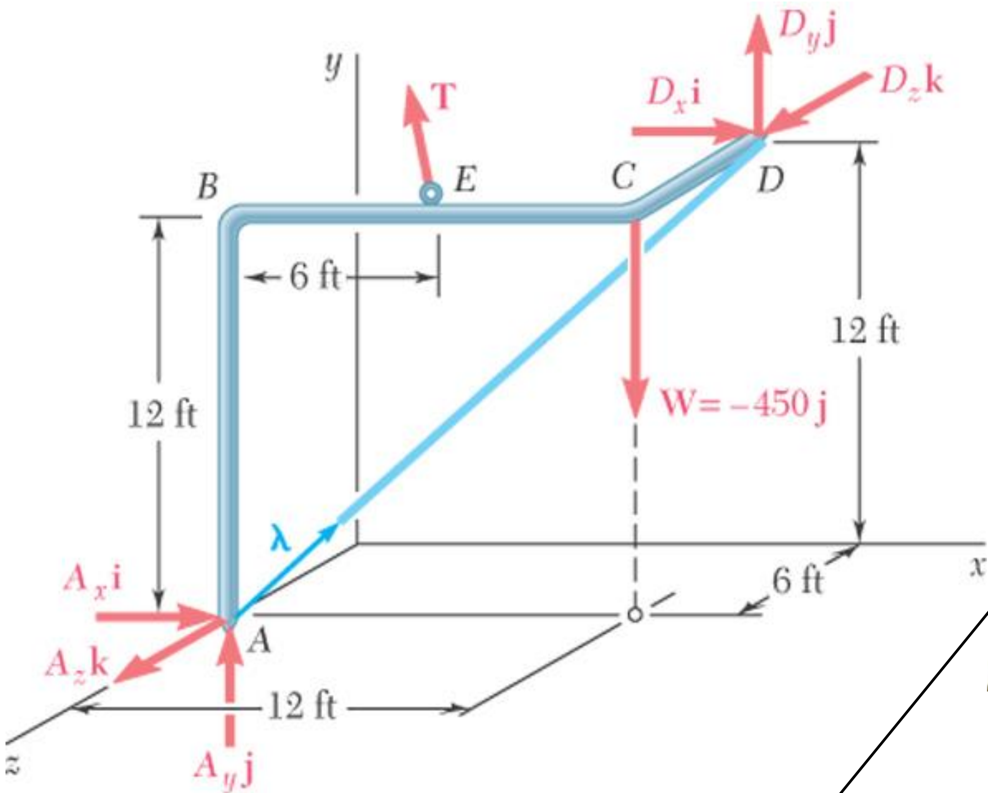
A:(2,0,0)

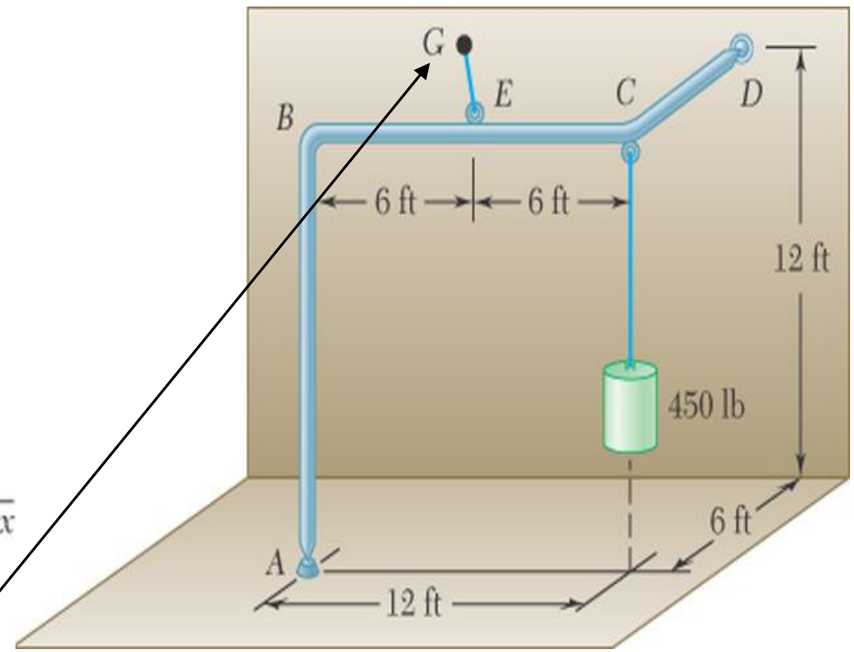
B:(7,0,0)

C : $(6, 2\sqrt{3}, -2)$

G : $(4, \sqrt{3}, -1)$

Sample Problem 4.10





Determine (a) where G should be located if the tension in the cable is to be minimum,

(b) the corresponding minimum value of the tension.