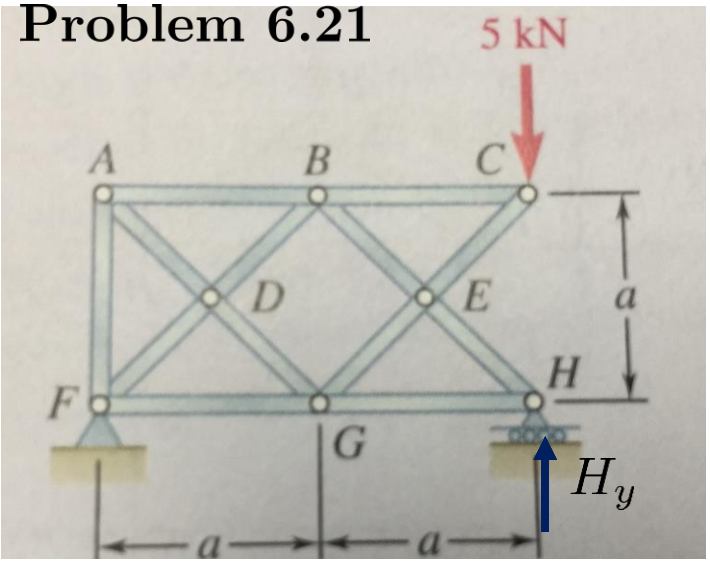
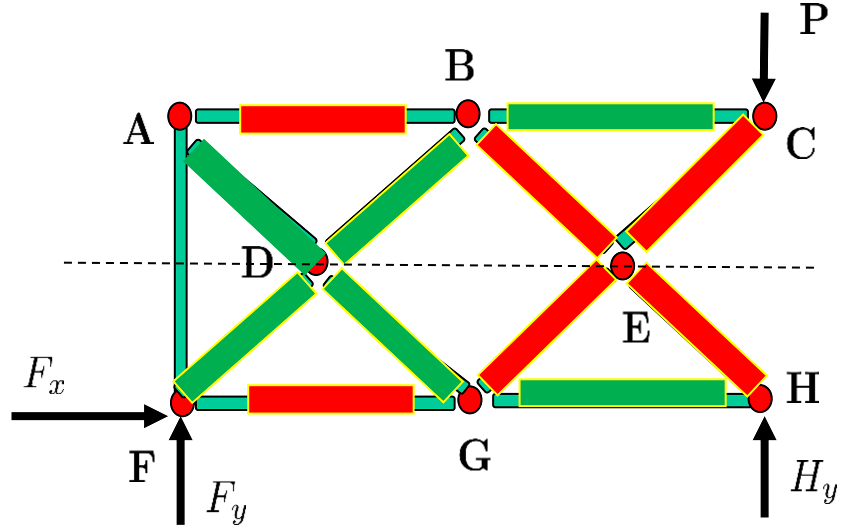
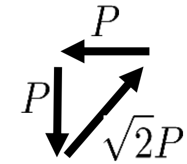






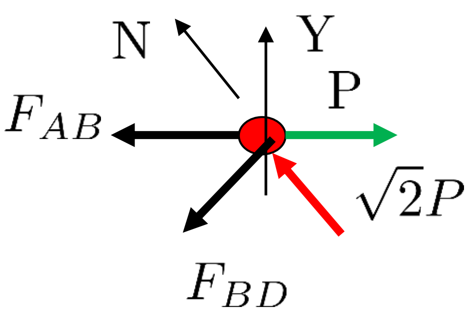
Joint C

$H_y = P$

$F_x = F_y = 0$

$$\sum M_F = 0$$

Joint B



$$-(2a) \times P + (2a) \times H_y = 0 \;\rightarrow\; H_y = P$$

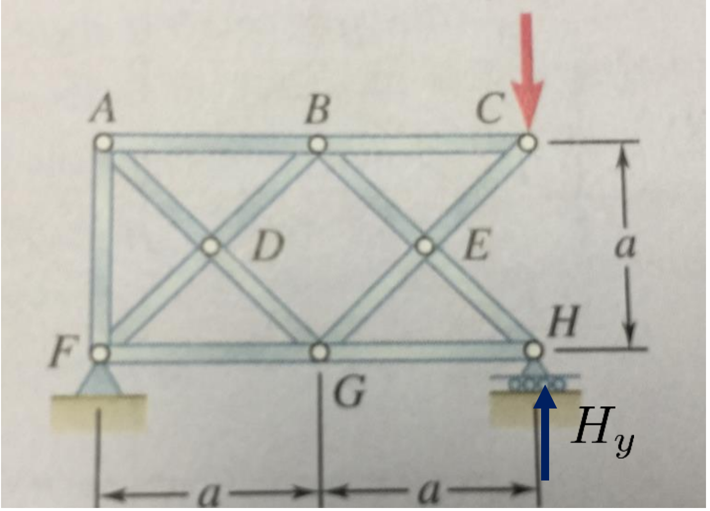
$$\sum F_Y = 0 \;\rightarrow\; \cos 45^o(\sqrt{2}P) - \cos 45^o F_{BD} = 0$$

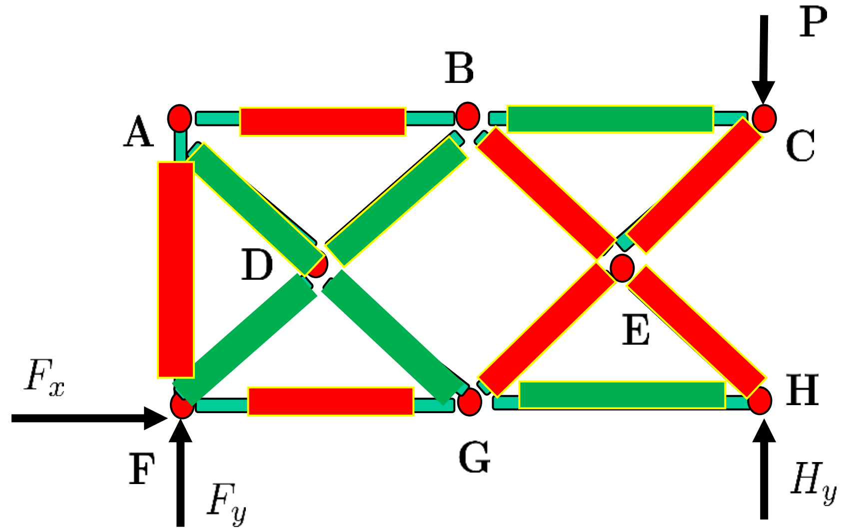
$$F_{BD} = \sqrt{2}P$$

$$\sum F_N = 0 \;\rightarrow\; \cos 45^o F_{AB} + \sqrt{2}P - \cos 45^o P = 0$$

$$F_{AB} = P - 2P = -P$$

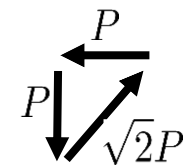
Problem 6.21





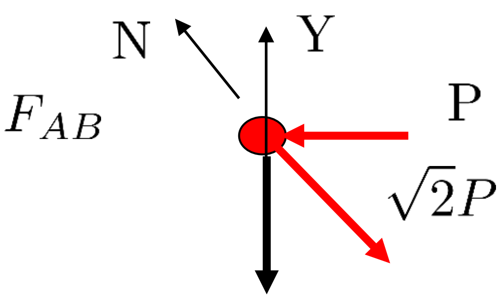
$F_x = F_y = 0$

$H_y = P$



Joint C

Joint A



$$\sum F_X = 0 \;\rightarrow\; F_{AD}\cos 45^o - P = 0$$

$$F_{AD} = \sqrt{2}P$$

$$\sum F_Y = 0 \;\rightarrow\; -F_{AF} - \sqrt{2}P\cos 40^o = 0$$

$$F_{AF} = -P$$

6.44 The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the force in member *DE* and in the counters that are acting under the given loading.

This is a tensegrity Structure.

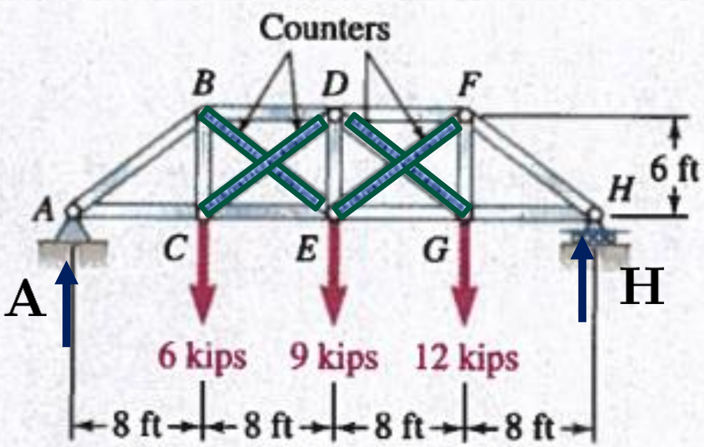


Fig. P6.44

$$\sum M_H = 0 \ \rightarrow$$

$$-32A + 24 \times 6(kips) + 16 \times (9kips) + 8 \times (12kips) = 0$$

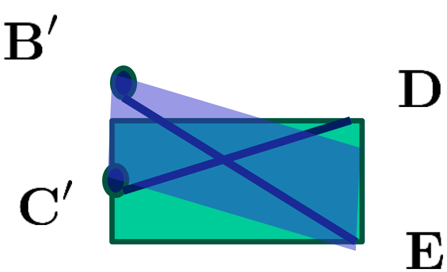
$$\sum M_A = 0 \ \rightarrow$$

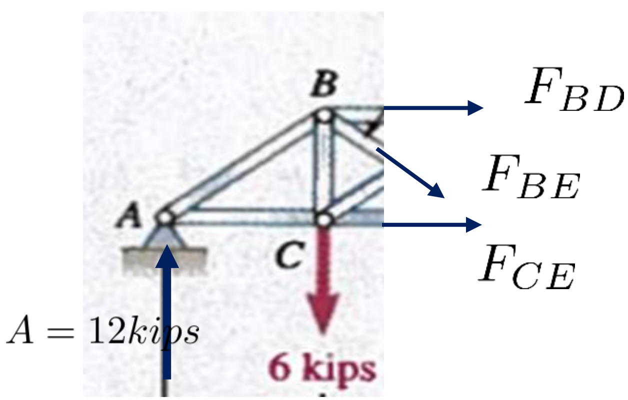
$$32H - 24 \times 12(kips) - 16 \times (9kips) - 8 \times (6kips) = 0$$

Reactions from free body of entire truss:

$+\circlearrowleft \Sigma M_H = 0$: $\mathbf{A} = 12$ kips $\uparrow$

$+\uparrow \Sigma F_y = 0$: $\mathbf{H} = 15$ kips $\uparrow$



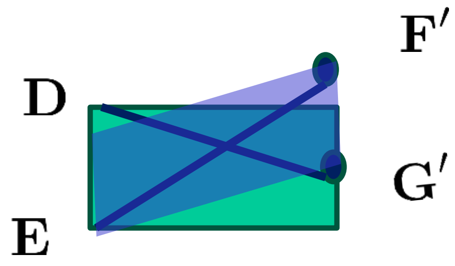


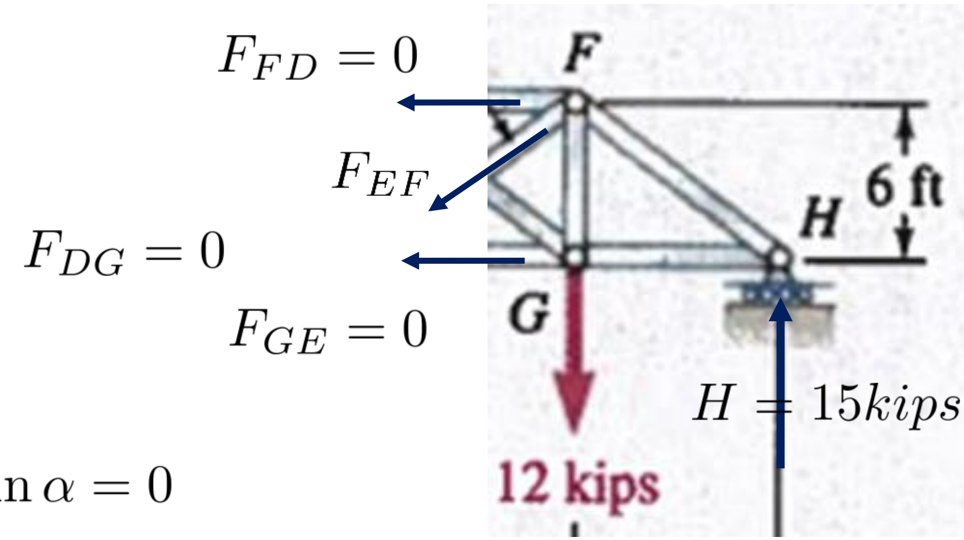
$$F_{CD} = 0$$

$$\sin\alpha = \frac{3}{5}$$

$$\sum F_y = 0 \ \ 12 - 6 - F_{BE}\sin\alpha = 0$$

$$F_{BE} = 10kips$$





$$\sum F_y = 0 \;\rightarrow\; 15 - 12 - F_{FE} \sin\alpha = 0$$

$$F_{FE} = 5kips$$

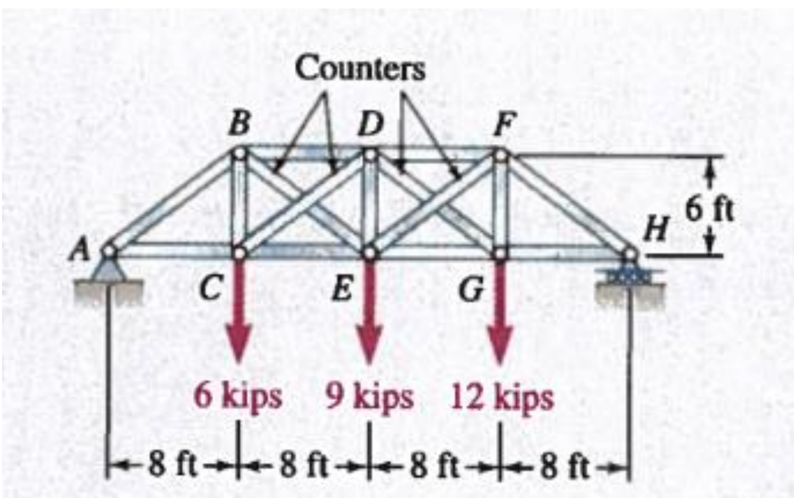
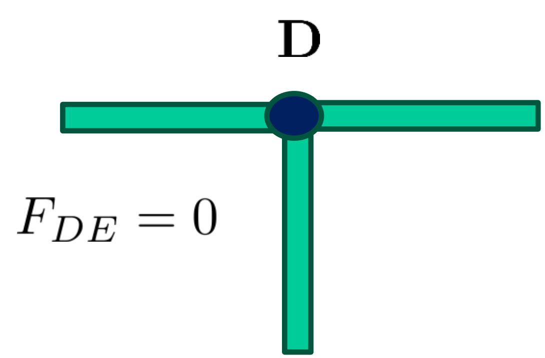


Fig. P6.44



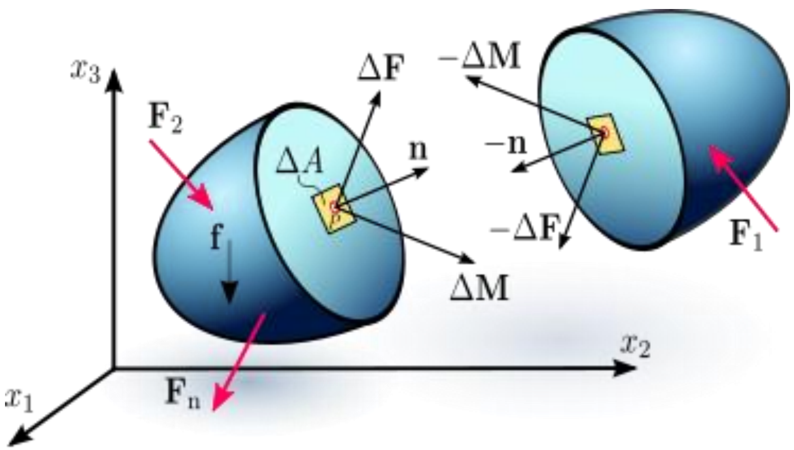
Lecture 16 Concept of Stress (I)

My Top Five Sources (List) of Stress

1.	Money	76 %
2.	Homework	70 %
3.	Midterm	66 %
4.	Roommates	59 %
5.	My Professor	55 %

What is (Mechanical) Stress ?

Method of Section

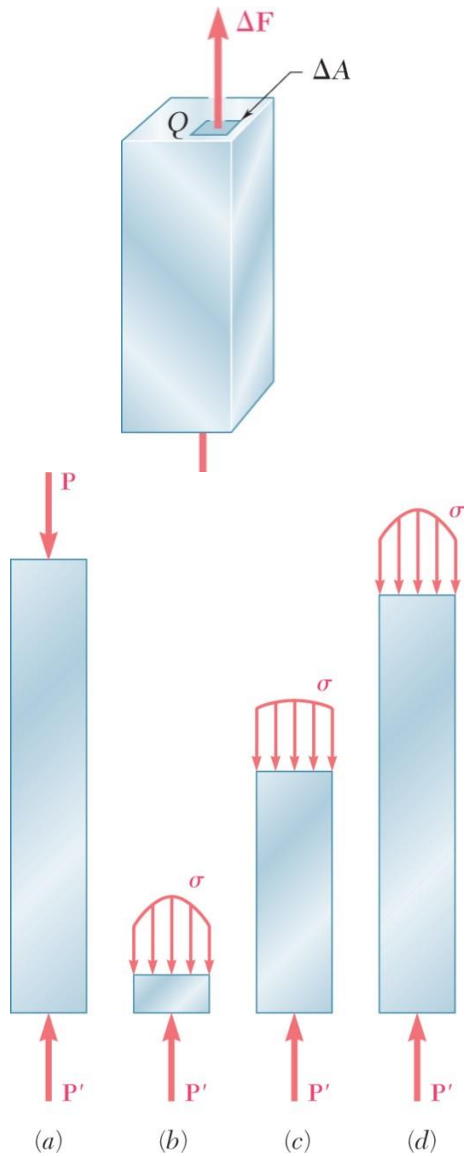


$$\mathbf{T}^{(n)} = \lim_{\Delta A \to 0} \frac{\Delta\mathbf{F}}{\Delta A} = \sigma\mathbf{n} + \tau\mathbf{t}$$

1. Stress is the <u>intensity</u> of <u>internal force;</u>

2. Stress is a distribution field (stress field).

1. Normal Stress



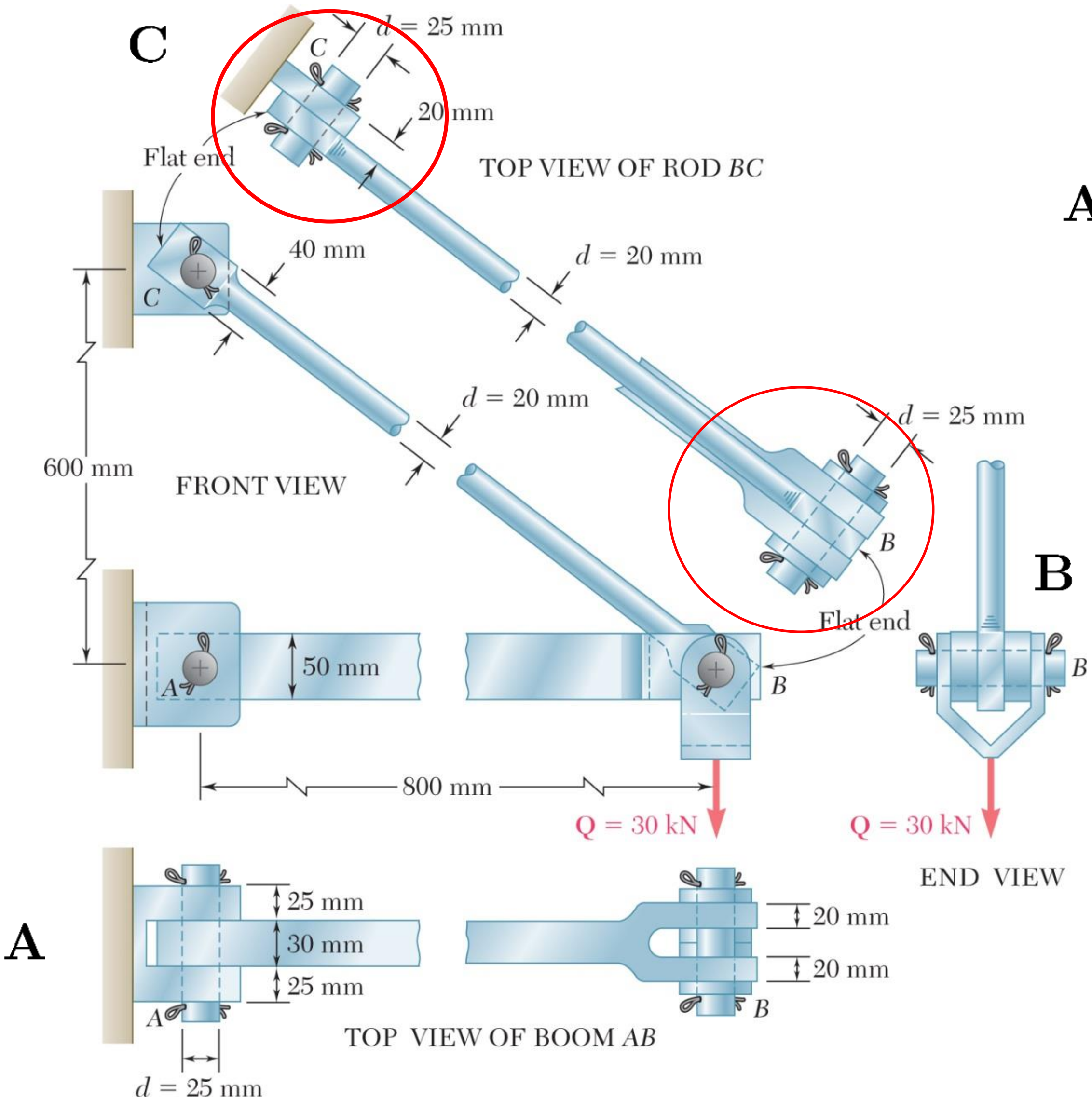
- The force intensity on that section is defined as the normal stress.

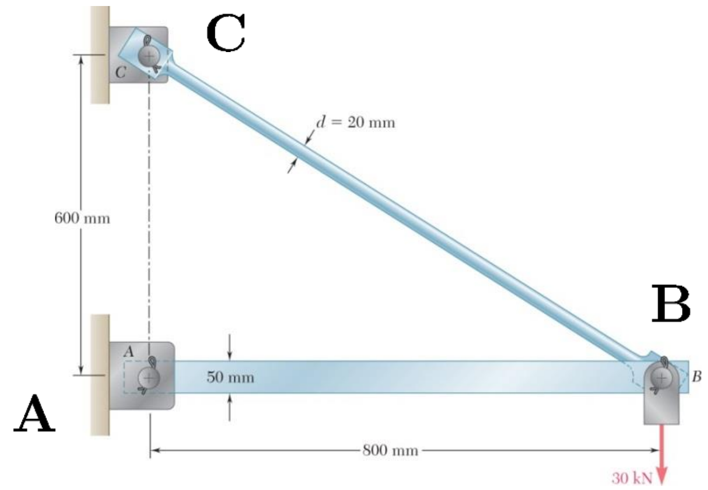
$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

- The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

Stress Analysis & Design Example

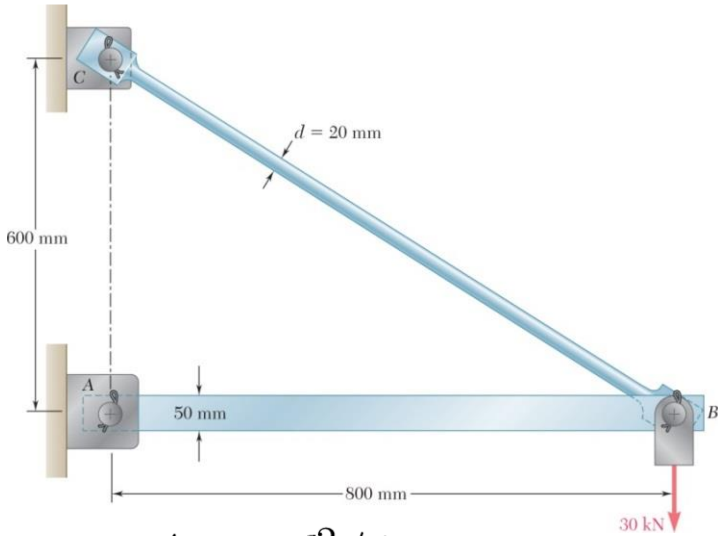




- Would like to determine the stresses in the members and connections of the structure shown.

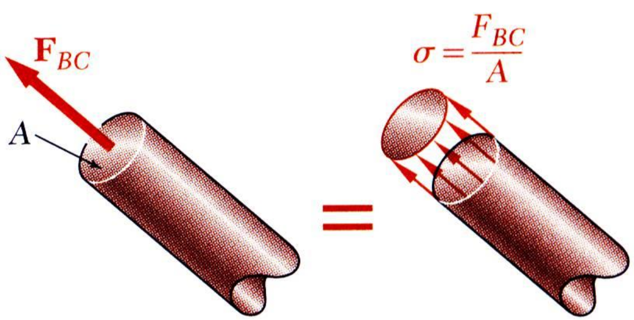
- Must consider maximum normal stresses in *AB* and *BC*, and the shearing stress and bearing stress at each pinned connection

Stress Analysis BC bar



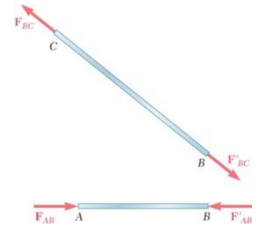
$$A = \pi d^2/4$$

$$d_{BC} = 20 \text{ mm}$$



- Based on the material properties of the steel, the allowable stress is

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

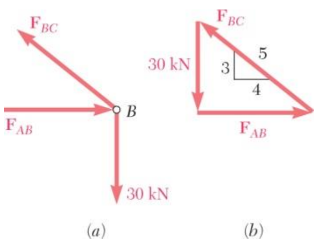


JointB

$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$



F_{AB} = 40 kN (compression)
F_{BC} = 50 kN (tension)

- At any section through member BC, the internal force is 50 kN

$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = 159 \text{ MPa}$$

- Conclusion: the strength of member *BC* is adequate

At the flat end of member BC

$$\sigma_{\text{all}} = 165\,\text{MPa}$$

- The rod is in tension with an axial force of 50 kN.

$$\sigma_{BC} = \frac{P}{A} = \frac{50\times10^{3}\,\text{N}}{314\times10^{-6}\,\text{m}^{2}} = 159\ \text{MPa}$$

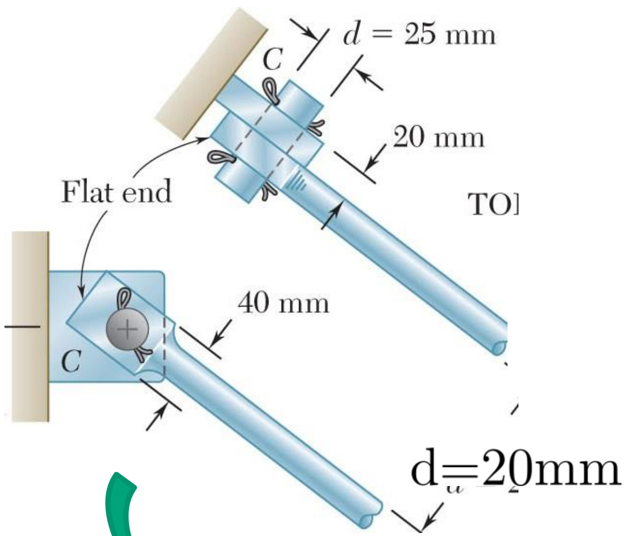


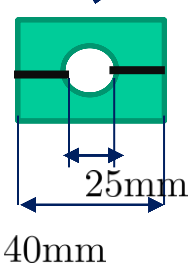
Fig. 8.21 (partial)

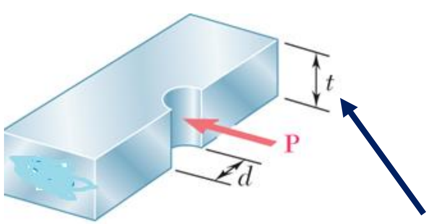
- At the flattened rod ends,

$$A = (20\text{mm})(40\text{mm} - 25\text{mm}) = 300\times10^{-6}\,\text{m}^{2}$$

$$\sigma_{BC,end} = \frac{P}{A} = \frac{50\times10^{3}\,\text{N}}{300\times10^{-6}\,\text{m}^{2}} = 167\,\text{MPa}$$

$$\sigma_{\text{all}} = 165\,\text{MPa}$$





$t = 20mm$

Now, we have a problem, Houston!

Design Issue: Factor of Safety (FS)

$$FS = \text{Factor of safety} > 1$$

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Given $\sigma_{ult} = 200MP_a$
and $\sigma_{all} = 165MP_a$

$$FS = \frac{200}{165} = 1.212 \ .$$

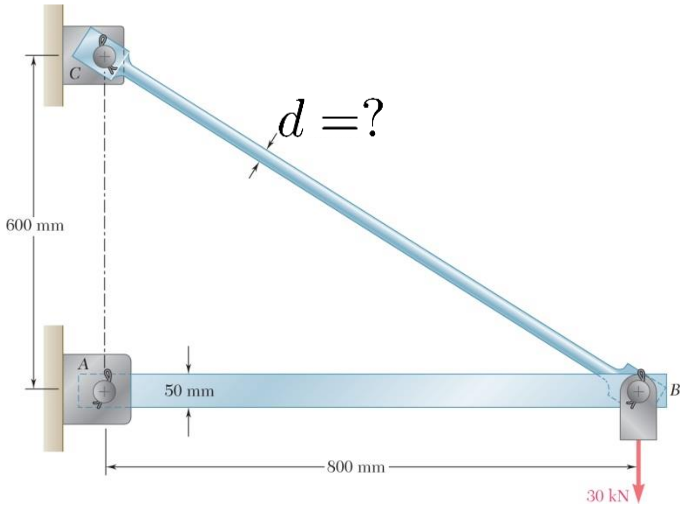
Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to integrity of whole structure
- risk to life and property
- influence on machine function

In practice, sometimes, allowable stress is not given, but FS is given.

Previously, d is given: $d = 20mm$.
Now we want to design its size.

Given $\sigma_{ult} = 200$ and FS $= 2$.



- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \sigma_{ult}/SF$$

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^3\,\mathrm{N}}{100\times10^6\,\mathrm{Pa}} = 500\times10^{-6}\,\mathrm{m}^2$$

$$A = \pi\frac{d^2}{4}$$

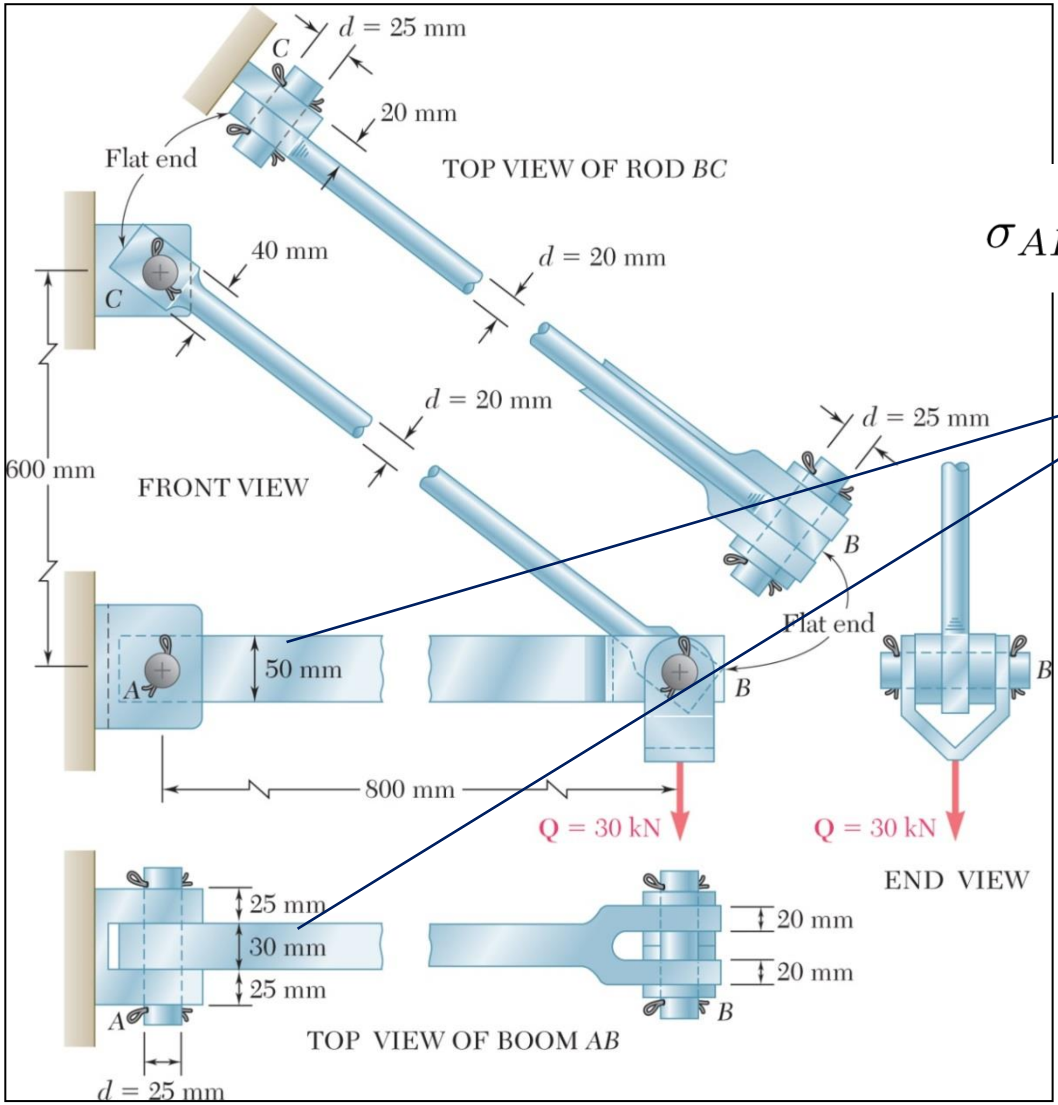
Find A ?

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\mathrm{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

- An aluminum rod 26 mm or more in diameter will be adequate

Member AB

- From a statics analysis:
  $F_{AB} = 40$ kN (compression)
  $F_{BC} = 50$ kN (tension)

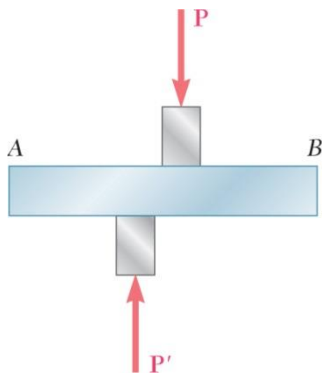


$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40 \times 10^3 N}{30 \times 50 \times 10^{-6} m^2}$$

$$= 26.7 MP_a$$

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

2. Shearing Stress

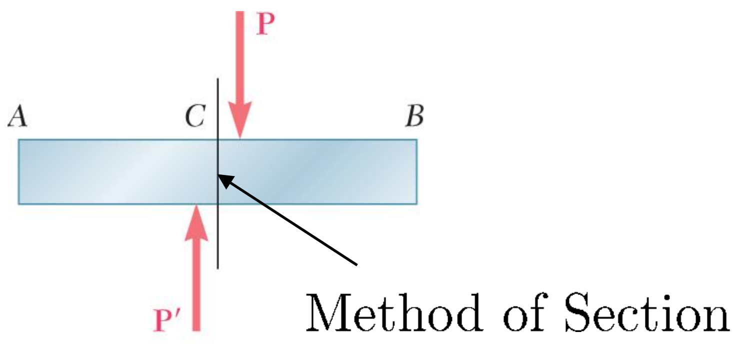


- Forces *P* and *P'* are applied transversely to the member *AB*.

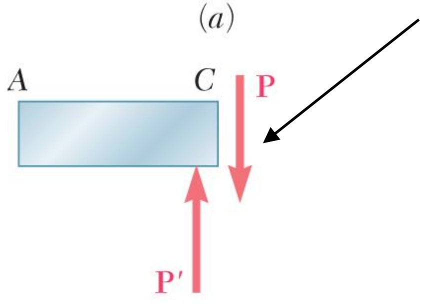
- Corresponding internal forces act in the plane of section *C* and are called *shearing* forces.

- The corresponding average shear stress is,

$$\tau_{\text{ave}} = \frac{P}{A}$$



(a)



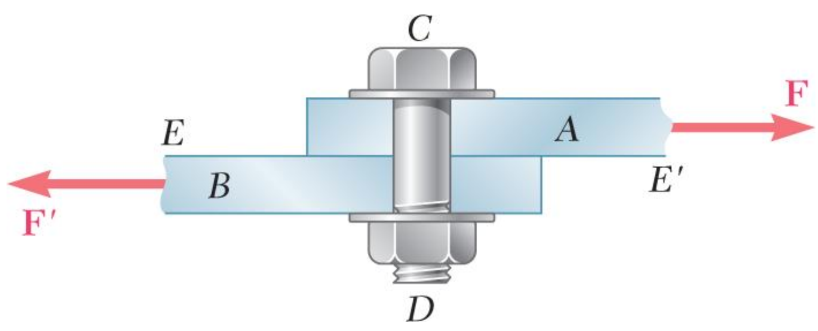
(b)

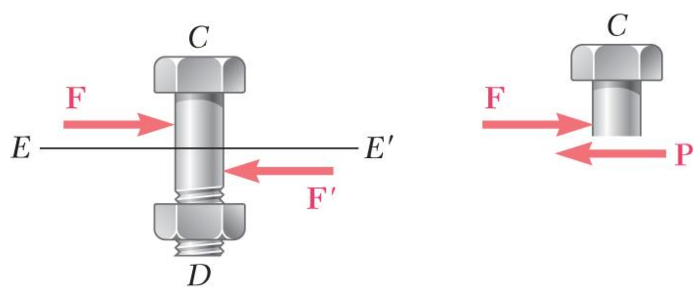
Caveat !!!

- The shear stress distribution cannot be assumed to be uniform.
- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.

Shearing Stress Calculation Examples

Single Shear Connection

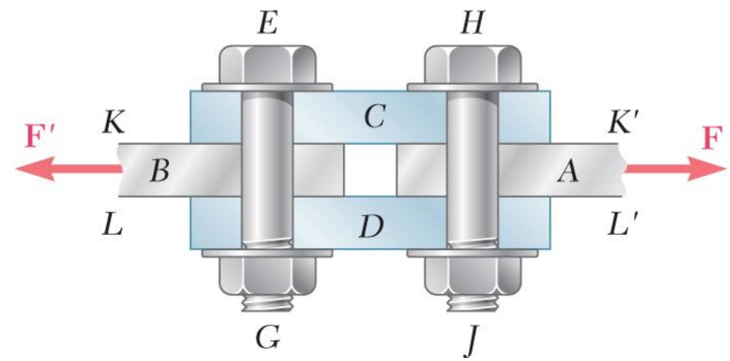


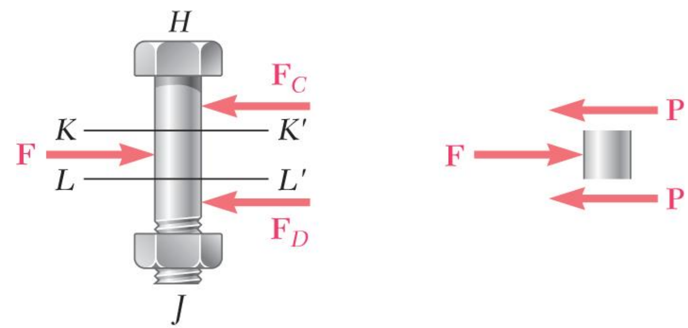


(a) (b)

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

Double Shear Connection





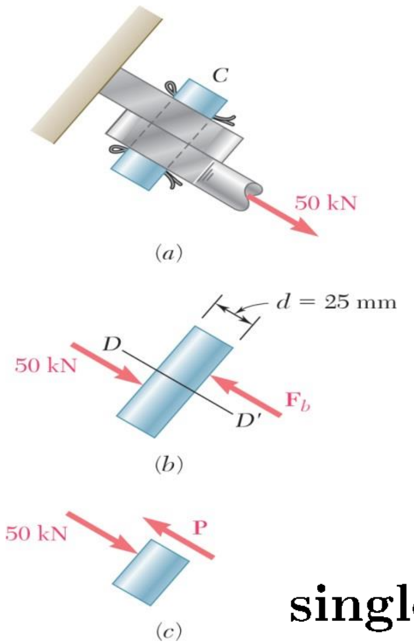
(a) (b)

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

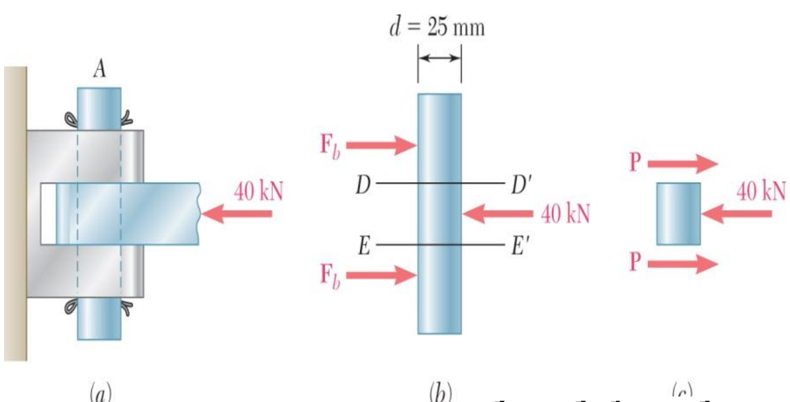
Today's Lecture Attendance Password is: Double Shear

Pin Shearing Stresses at C and A

Read by yourself



single shear



double shear

- The cross-sectional area for pins at *A*, *B*, and *C*,

$$A = \pi r^2 = \pi\left(\frac{25\,\text{mm}}{2}\right)^2 = 491\times10^{-6}\,\text{m}^2$$

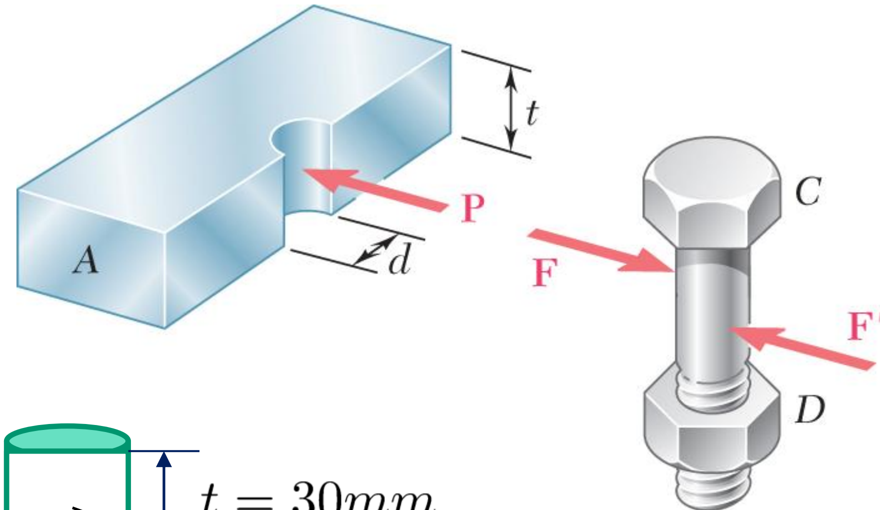
- The force on the pin at *C* is equal to the force exerted by the rod *BC*,

$$\tau_{C,ave} = \frac{P}{A} = \frac{50\times10^3\,\text{N}}{491\times10^{-6}\,\text{m}^2} = 102\,\text{MPa}$$

- The pin at *A* is in double shear with a total force equal to the force exerted by the boom *AB*,

$$\tau_{A,ave} = \frac{40kN}{2A} = \frac{40\text{kN}}{2\times491\times10^{-6}\text{m}^2} = 40.7\text{MPa}$$

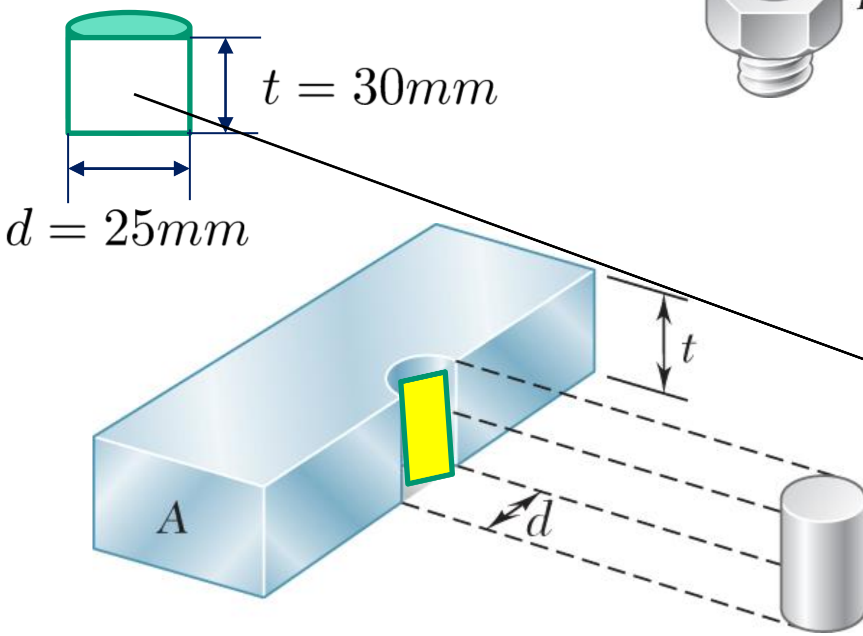
Bearing Stress in Connections



- Bearing stress is a contact stress.

- Corresponding average force intensity is called the bearing stress,

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

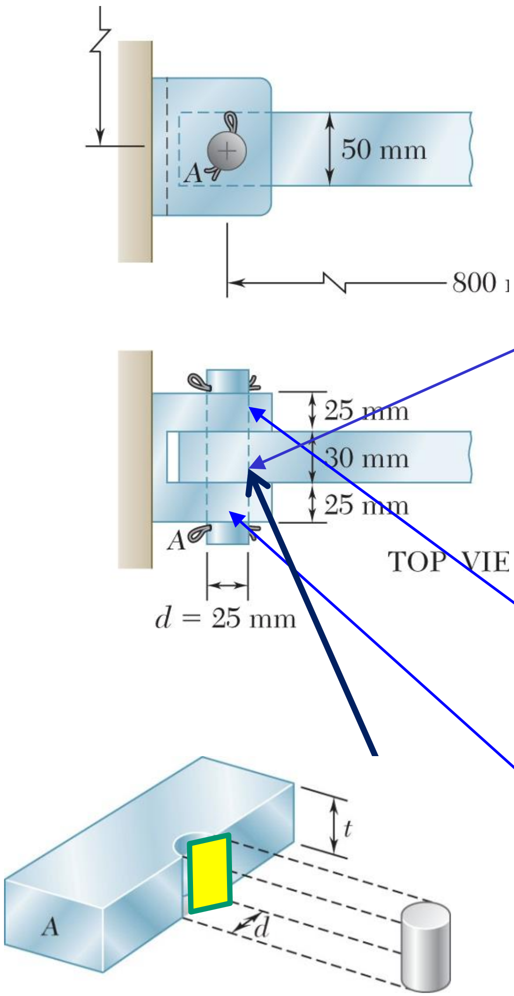


- The bearing stress in the pin,

we have t = 30 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\text{kN}}{(30\,\text{mm})(25\,\text{mm})} = 53.3\,\text{MPa}$$

Bearing Stresses in Boom and Bracket
(Bearing stress at the point A)

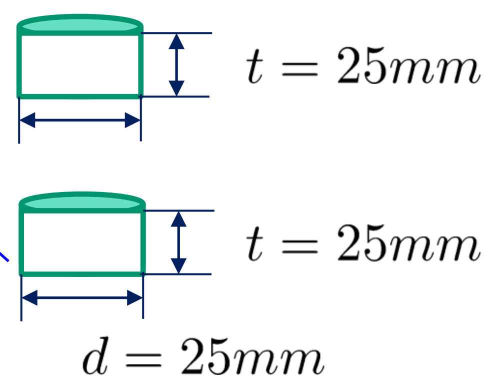


- To determine the bearing stress at *A* in the boom *AB*, we have t = 30 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\mathrm{kN}}{(30\,\mathrm{mm})(25\,\mathrm{mm})} = 53.3\,\mathrm{MPa}$$

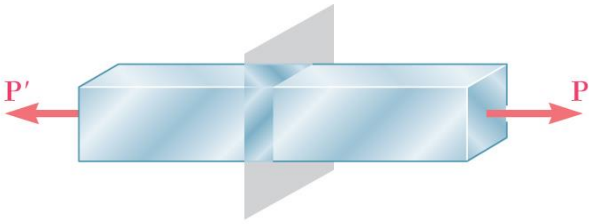
- To determine the bearing stress at *A* in the bracket, we have $t = 2(25\ \mathrm{mm})$ = 50 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\mathrm{kN}}{(50\,\mathrm{mm})(25\,\mathrm{mm})} = 32.0\,\mathrm{MPa}$$

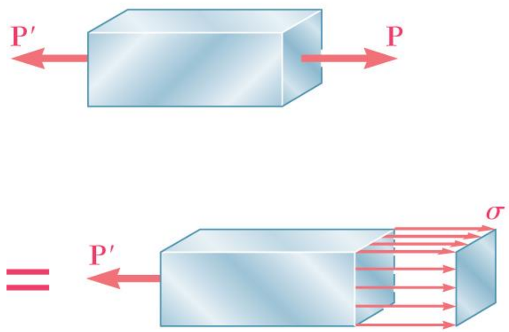


double shear

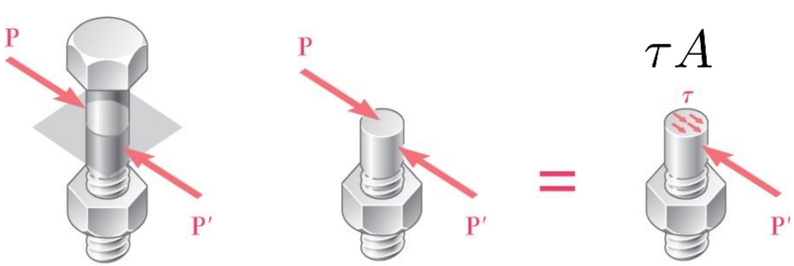
Summary:



(a)



(b)



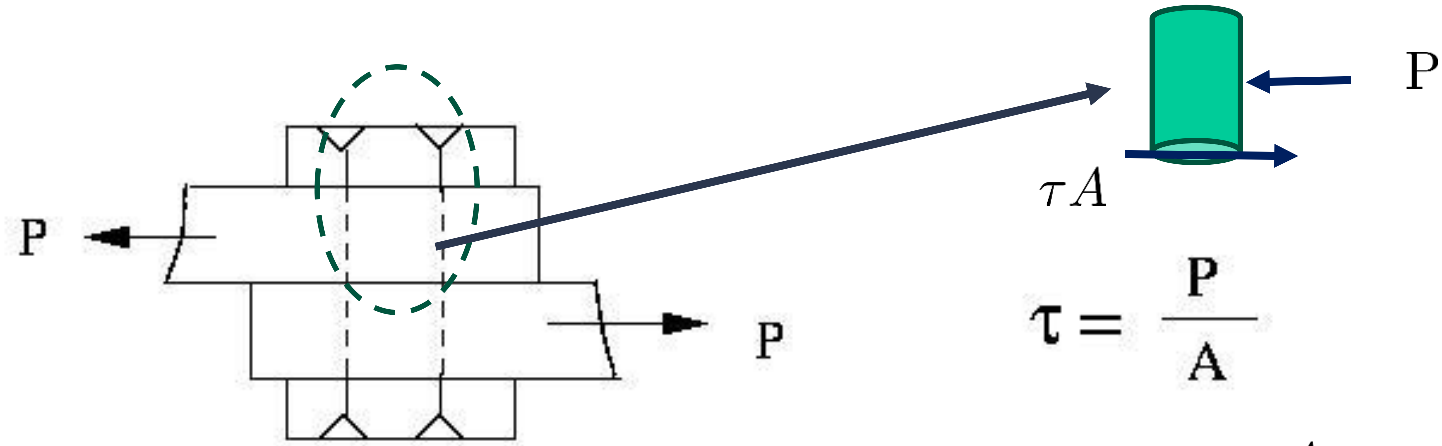
(a) (b)

- Axial forces on a two force member result in only normal stresses on a plane cut perpendicular to the member axis.

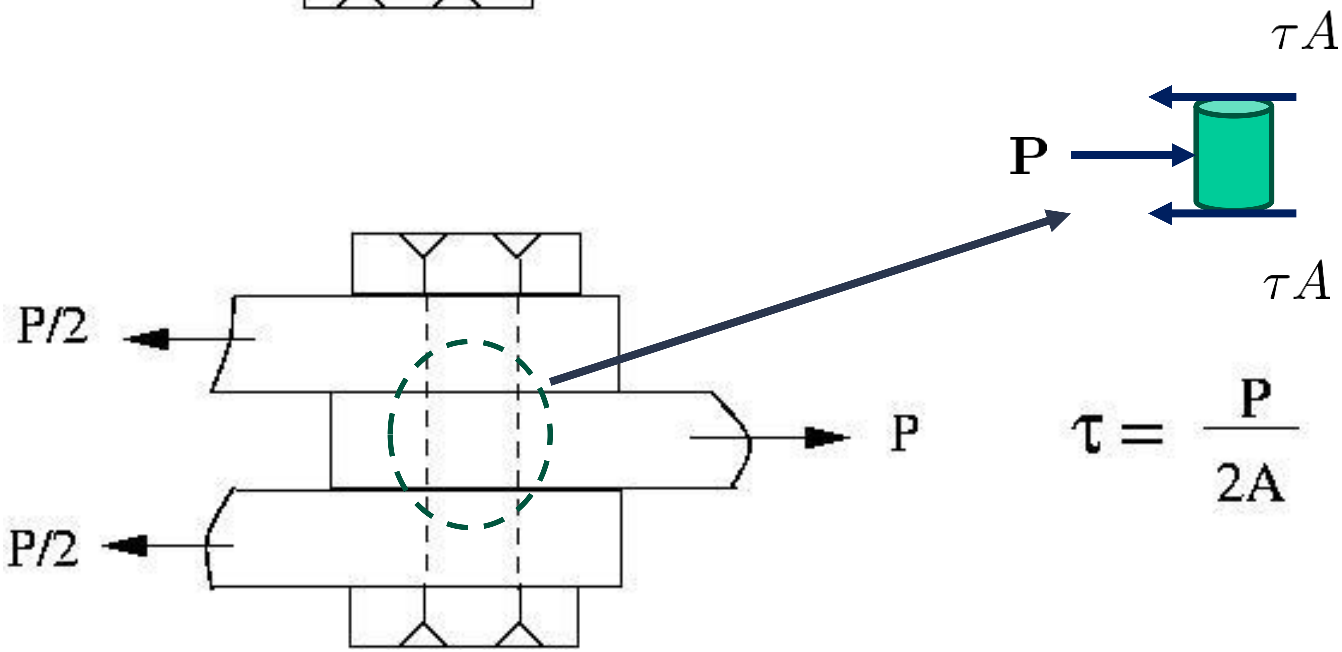
- Transverse forces on bolts and pins result in *only* shear stresses on the plane perpendicular to bolt or pin axis.

In the sense of average

$$\tau = \frac{P}{A}$$

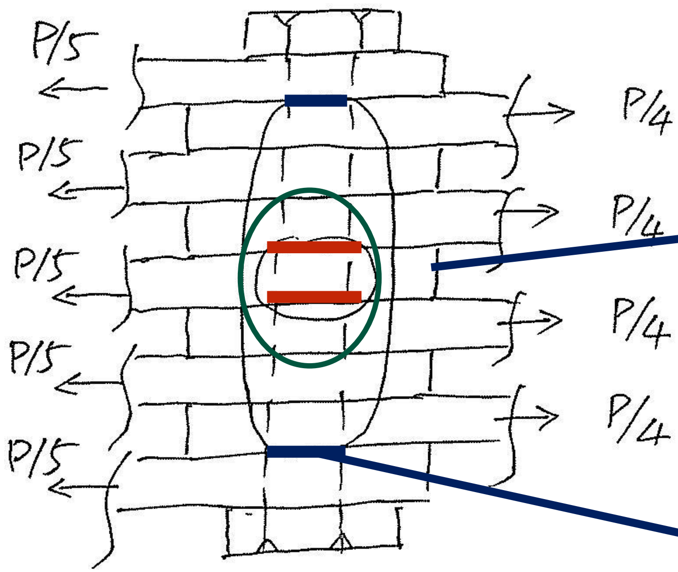


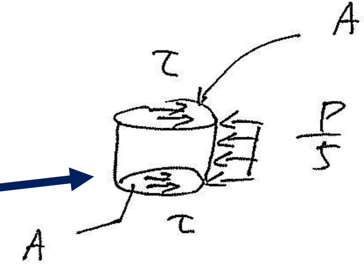
$$\tau = \frac{P}{A}$$



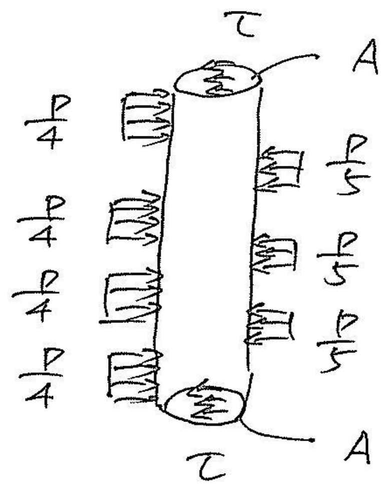
$$\tau = \frac{P}{2A}$$

Another Example : Five-fold shear





$$\tau_{av} = \frac{P}{10A}$$



$\longrightarrow \; x$

$$\Sigma F_x = 0$$

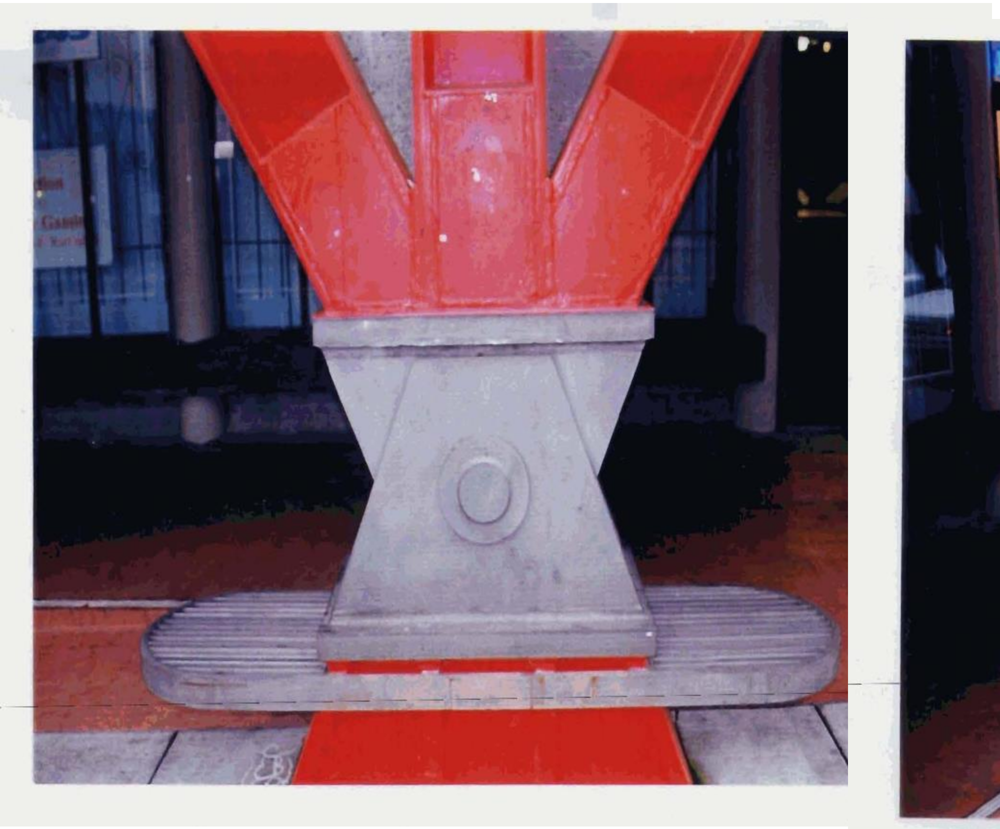
$$4 \times \frac{P}{4} - 3\frac{P}{5} - 2\tau \cdot A = 0$$

$$2\tau A = \frac{2P}{5}$$

$$\boxed{\tau_{ave} = \frac{P}{5A}} \longleftarrow \text{five-fold shear}$$

Maximum shear stress in this pin

Five-fold Shear Connection



A Stress-free Weekend in Berkeley