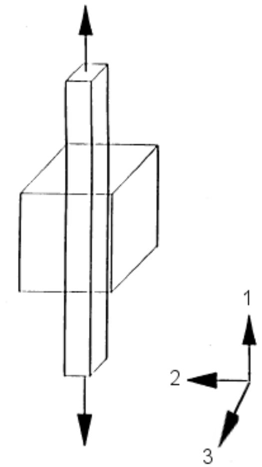
Lecture 20 Generalized Hooke's Law

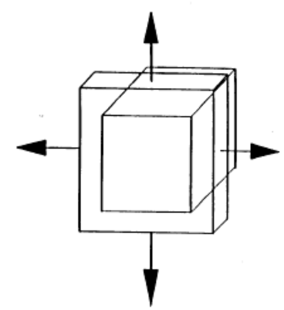
$$\sigma = E\epsilon \qquad \tau = G\gamma$$

$$\boldsymbol{\sigma} \ ? \ \boldsymbol{\epsilon}$$



UNIAXIAL

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} \qquad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{bmatrix}$$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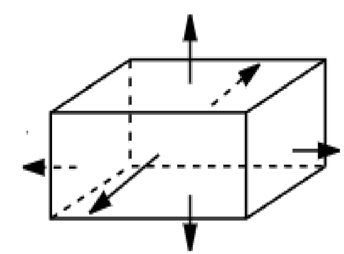
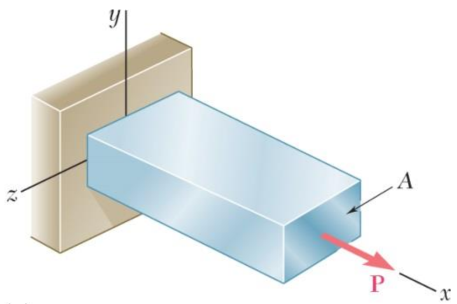
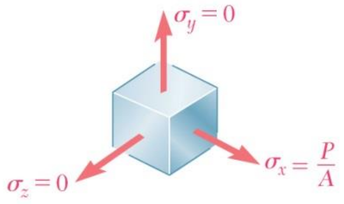

Hydrostatic Tension

$$\epsilon_{xy} = \frac{1}{2}\gamma_{xy}$$

Poisson's Ratio



(a)



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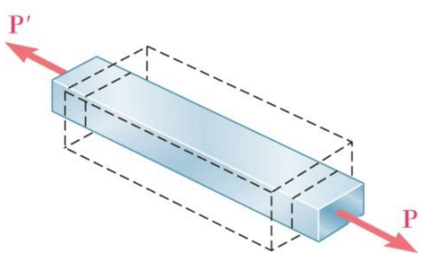


- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

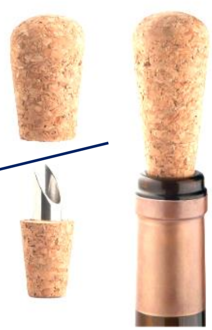
$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson's ratio is defined as

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Poisson's ratio:

material	poisson's ratio
rubber	~ 0.50
gold	0.42
saturated clay	0.40–0.50
magnesium	0.35
titanium	0.34
copper	0.33
aluminium-alloy	0.33
clay	0.30–0.45
stainless steel	0.30–0.31
steel	0.27–0.30
cast iron	0.21–0.26
sand	0.20–0.45
concrete	0.20
glass	0.18–0.3
foam	0.10–0.40
cork	~ 0.00
auxetics	negative



Siméon Denis Poisson
(21 June 1781 – 25 April 1840),

$$\nu = -\frac{lateral\ \ strain}{normal\ \ strain}$$

In general $0 < \nu < 1/2$,
but sometimes, $\nu < 0$.

Negative Poisson's ratio materials

Roderic Lakes

The original article.
" Foam structures with a negative Poisson's ratio", ***Science***, 235 1038-1040 (1987).

Normal Material

Today's Lecture Attendance Password is:
Auxetics

Question: How to find stress-strain relation in multiple dimensions ?

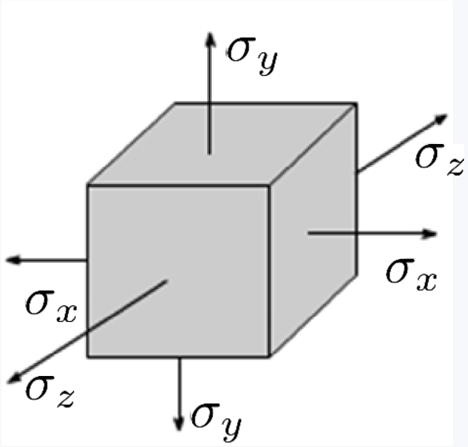
Can we do this ?

$$\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

No ! because this implies that $\sigma_y = 0 \ \rightarrow \ \epsilon_y = 0$.

This is NOT true,
because of Poisson's ratio effect,
even if $\sigma_y = 0, \ \epsilon_y \neq 0$!

Let's consider a unit cube under triaxial loading



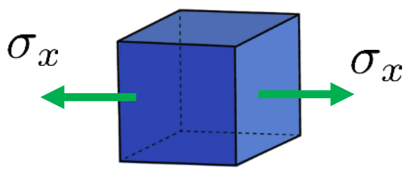
$$\begin{aligned} P_x &= \sigma_{xx}(1 \times 1) = \sigma_x \\ P_y &= \sigma_{yy}(1 \times 1) = \sigma_y \\ P_z &= \sigma_{zz}(1 \times 1) = \sigma_z \end{aligned}$$

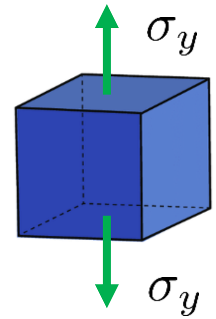
Use superposition method to decompose the tri-axial loading
into three uni-axial loadings

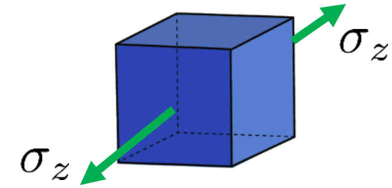
Step 1: $P_x = \sigma_{xx}, P_y = P_z = 0;$

Step 2: $P_y = \sigma_{yy}, P_x = P_z = 0;$

Step 3: $P_z = \sigma_{zz}, P_x = P_y = 0;$







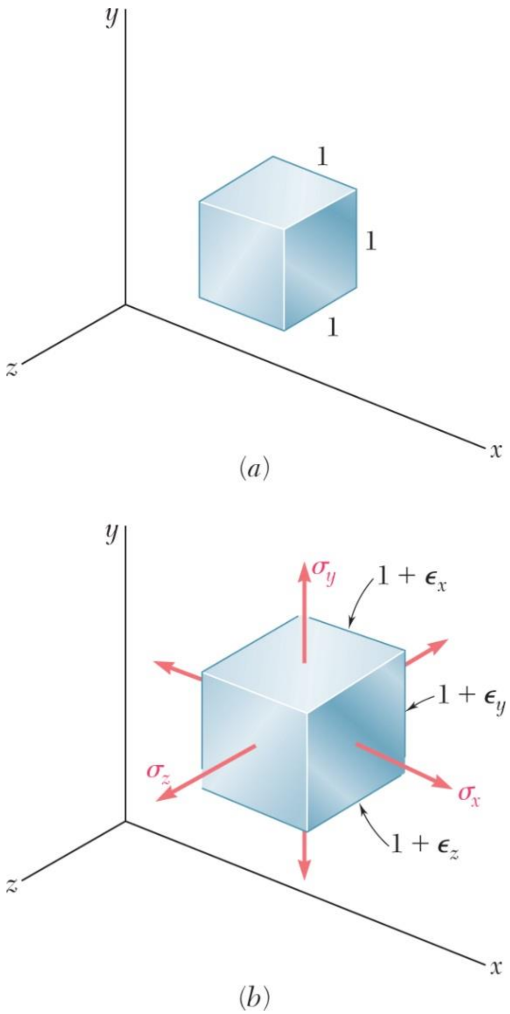
$$\begin{cases} \epsilon^1_{xx} &= \dfrac{\sigma_{xx}}{E} \\ \epsilon^1_{yy} &= -\nu\epsilon^1_{xx} = -\nu\dfrac{\sigma_{xx}}{E} \\ \epsilon^1_{zz} &= -\nu\epsilon^1_{xx} = -\nu\dfrac{\sigma_{xx}}{E} \end{cases} \qquad \begin{cases} \epsilon^2_{xx} &= -\nu\epsilon^2_{yy} = -\nu\dfrac{\sigma_{yy}}{E} \\ \epsilon^2_{yy} &= \dfrac{\sigma_{yy}}{E} \\ \epsilon^2_{zz} &= -\nu\epsilon^2_{yy} = -\nu\dfrac{\sigma_{yy}}{E} \end{cases} \qquad \begin{cases} \epsilon^3_{xx} &= -\nu\epsilon^3_{zz} = -\nu\dfrac{\sigma_{zz}}{E} \\ \epsilon^3_{yy} &= -\nu\epsilon^3_{zz} = -\nu\dfrac{\sigma_{zz}}{E} \\ \epsilon^3_{zz} &= \dfrac{\sigma_{zz}}{E} \end{cases}$$

$$\epsilon_{xx} = \epsilon^1_{xx} + \epsilon^2_{xx} + \epsilon^3_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \tag{1}$$

$$\epsilon_{yy} = \epsilon^1_{yy} + \epsilon^2_{yy} + \epsilon^3_{yy} = -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \tag{2}$$

$$\epsilon_{zz} = \epsilon^1_{zz} + \epsilon^2_{zz} + \epsilon^3_{zz} = -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E} \tag{3}$$

Generalized Hooke's Law



- For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the ***principle of superposition***. This requires:

  1) strain is linearly related to stress
  2) deformations are small

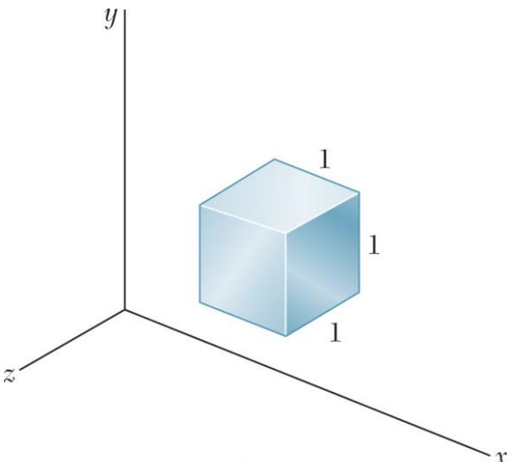
- With these restrictions:

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

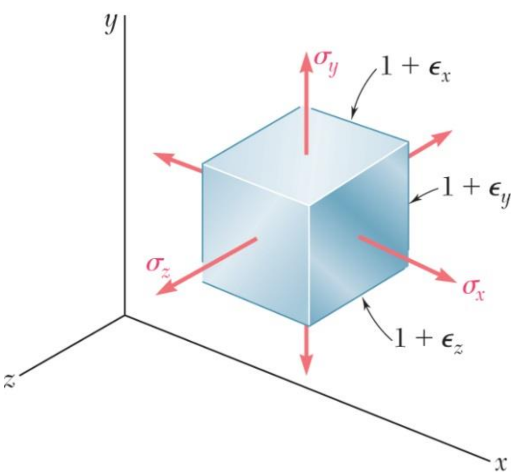
$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

Dilatation: Bulk Modulus



(a)



(b)

$$e = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z) - 1 = (1+\epsilon_x+\epsilon_y+\epsilon_z) - 1$$

$$= \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\begin{aligned}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{yy} &= -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{zz} &= -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}
\end{aligned}$$

$$e = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \frac{1}{E}(1-2\nu)(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Let $\sigma_x = \sigma_y = \sigma_z = -p$

$-3p$

$$e = -\frac{3(1-2\nu)}{E}p \quad \Rightarrow \quad p = -Ke, \quad \text{where} \quad K = \frac{E}{3(1-2\nu)}$$

- For element subjected to uniform hydrostatic pressure,

$$e = -p\frac{3(1-2\nu)}{E} = -\frac{p}{K}$$

$$K = \frac{E}{3(1-2\nu)} = \text{bulk modulus}$$

- Subjected to uniform pressure, dilatation must be negative, therefore

$$0 < \nu < \frac{1}{2}$$

How about $\nu = 0.5$ **?**

Remark : Why ν cannot be greater than 1/2 ?

$$e = -\frac{3(1-2\nu)}{E}p \quad \Rightarrow \quad K = \frac{E}{3(1-2\nu)}$$

$-1 < \nu \le 0.5$, Why ? if $\nu > 0.5$
$e = 3(2\nu - 1)p/E > 0;$

That is: compression load will cause material expansion !

Remark 1: Why ν cannot be greater than $1/2$?

$$e = \frac{3(1-2\nu)}{E}(-p) \quad \Rightarrow \quad K = \frac{E}{3(1-2\nu)}$$

1. How about $\nu = 0.5$?

The material is incompressible!

2. What types of materials $\nu = 0.5$?

Which material is most compressible?

$$\nu = 0!$$

Remark 2: Inverse relation

$$
\begin{aligned}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{yy} &= -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{zz} &= -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}
\end{aligned}
$$

The inverse relationship,

$$
\begin{aligned}
\sigma_{xx} &= \lambda(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) + 2G\epsilon_{xx}, \quad \tau_{xy} = 2G\epsilon_{xy} \\
\sigma_{yy} &= \lambda(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) + 2G\epsilon_{yy}, \quad \tau_{yz} = 2G\epsilon_{yz} \\
\sigma_{zz} &= \lambda(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) + 2G\epsilon_{zz}, \quad \tau_{xz} + 2G\epsilon_{xz}
\end{aligned}
$$

where λ and G are Lamé constants,

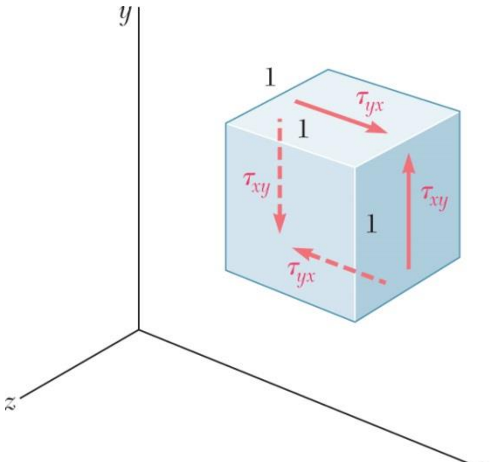
$$
\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)}.
$$

Remark 3: If we consider thermal strain,

$$\begin{aligned}
\epsilon^e_{xx} &= \epsilon_{xx} - \epsilon^T = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon^e_{yy} &= \epsilon_{yy} - \epsilon^T = -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon^e_{zz} &= \epsilon_{zz} - \epsilon^T = -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}
\end{aligned}$$

$$\begin{aligned}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} + \alpha\Delta T \\
\epsilon_{yy} &= -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} + \alpha\Delta T \\
\epsilon_{zz} &= -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E} + \alpha\Delta T
\end{aligned}$$

Hooke's Law in Shear $\tau_{xy} = f(\gamma_{xy})$

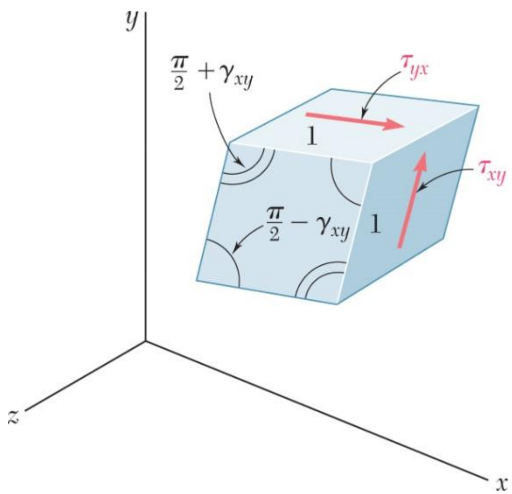


- Hooke's law in shear

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where G is the shear modulus.

γ_{xy} means the change of angle between x-axis and y-axis.



Obviously, $\gamma_{xy} = \gamma_{yx}$.

Define Mathematical strain,

$$\begin{bmatrix} \epsilon_{xy} & \epsilon_{yz} & \epsilon_{zx} \end{bmatrix} := \frac{1}{2}\begin{bmatrix} \gamma_{xy} & \gamma_{yz} & \gamma_{zx} \end{bmatrix}.$$

Summary: Generalized Hooke's Law

$$\begin{bmatrix} \epsilon_{xx} - \epsilon_T \\ \epsilon_{yy} - \epsilon_T \\ \epsilon_{zz} - \epsilon_T \\ \epsilon_{yz} \\ \epsilon_{zx} \\ \epsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

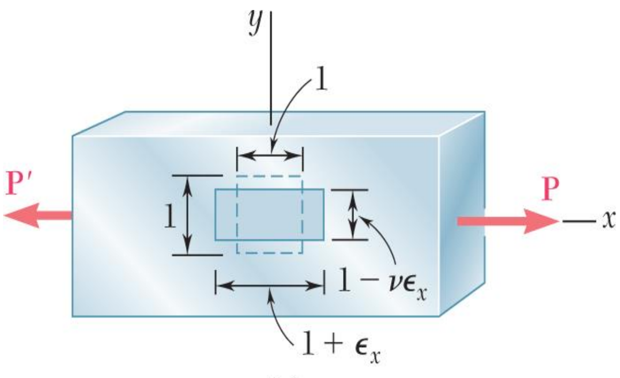
Mathematical Shear Strains

$$\tau_{xy} = 2G\epsilon_{xy}, \quad \tau_{yz} = 2G\epsilon_{yz}, \quad \tau_{zx} = 2G\epsilon_{zx} \ .$$

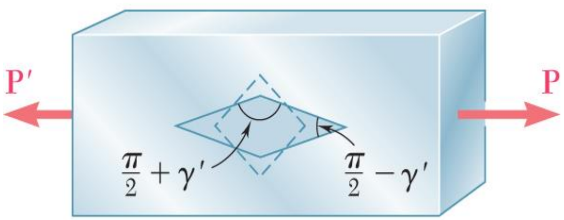
Physical shear strain: $\gamma_{xy} = 2\epsilon_{xy}$

There is no thermal shear strain in uniform temperature change!

Relation Among E, ν, and G



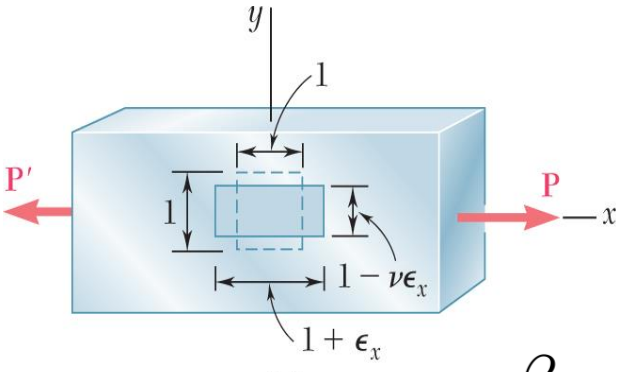
(a)



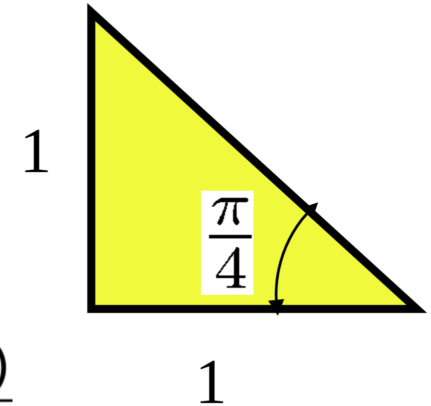
(b)

- An axially loaded slender bar will elongate in the axial direction and contract in the transverse directions.
- An initially cubic element oriented as in top figure will deform into a rectangular parallelepiped. The axial load produces a normal strain.
- If the cubic element is oriented as in the bottom figure, it will deform into a rhombus. Axial load also results in a shear strain.
- Components of normal and shear strain are related,

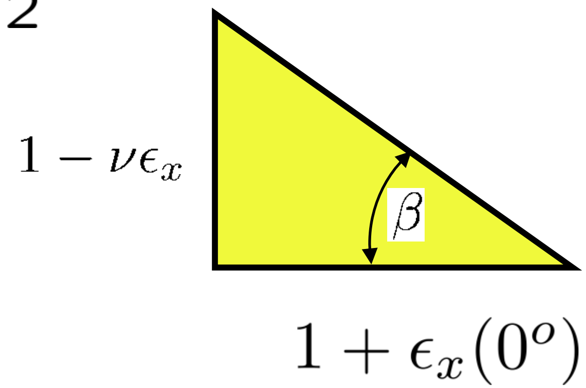
$$\frac{E}{2G} = (1 + \nu)$$

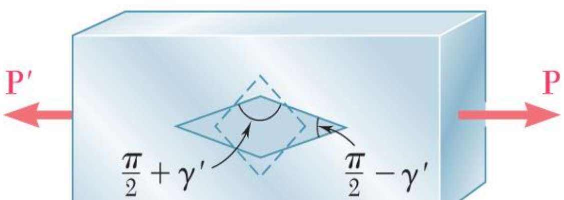


(a)



$$\beta = \frac{\pi}{4} - \frac{\gamma(45^o)}{2}$$





(b)

$$\tan\beta = \tan(\pi/4 - \gamma/2) = \frac{\tan(\pi/4) - \tan(\gamma/2)}{1 + \tan(\pi/4)\tan(\gamma/2)} = \frac{1 - \tan\frac{\gamma}{2}}{1 + \tan\frac{\gamma}{2}}$$

$$\tan\beta = \frac{1 - \frac{\gamma(45^o)}{2}}{1 + \frac{\gamma(45^o)}{2}} = \frac{1 - \nu\epsilon_x(0)}{1 + \epsilon_x(0)}$$

$$\tan\beta = \frac{1-\frac{\gamma}{2}}{1+\frac{\gamma}{2}} = \frac{1-\nu\epsilon_x}{1+\epsilon_x}$$

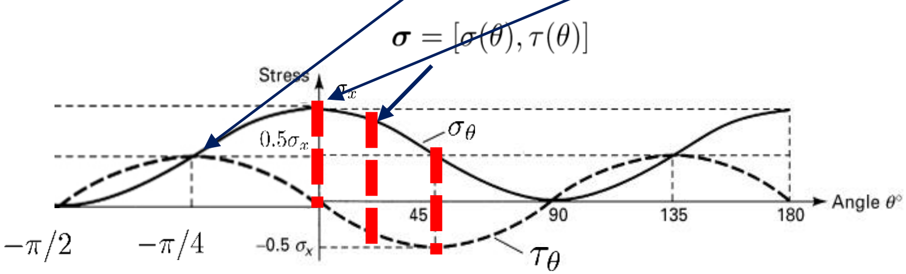
$$\gamma(45^o) = \frac{(1+\nu)\epsilon_x}{1+\epsilon_x(1+\nu)/2} \rightarrow \gamma = (1+\nu)\epsilon_x(0^o),$$

when $\epsilon_x << 1$.

$$\gamma(45^o) = \frac{\tau(45^o)}{G} = (1+\nu)\epsilon_x(0^o) = (1+\nu)\frac{\sigma_x}{E} = (1+\nu)\frac{2\tau(45^o)}{E}$$

Since $\tau(45^o) = \dfrac{P}{2A}$ and $\sigma_x(0) = P/A$,

we have $\sigma_x(0) = 2\tau(45^o)$, and hence



$$\frac{1}{G} = \frac{2(1+\nu)}{E}, \quad \rightarrow \quad G = \frac{E}{2(1+\nu)}\ .$$

Midterm Logistics

The Midterm Exam will be held on next Monday (March 11th)
From 1:00 pm to 2:00 pm at 50 Birge Hall.

Students with disabilities should take the exam should go
University Hall No. 1 Proctoring Service.
Please contact them immediately.

It is a close-book & close-notes exam, but a one-page
8x11 size cheat sheet (both sides) is allowed.

The grade of midterm consists of 30% of the overall grade.

Midterm Review (Spring 2024)

(1) Equilibrium of Rigid Bodies (2D) (Chapter 4)

$$\sum F_x = 0, \quad \sum F_y = 0, \text{ and } \sum M_0 = 0;$$

Key Points

1. Boundary conditions for supports (reactions);
2. Catergory and identify structure types (statically indeterminant, partially constrianed, etc) ;
3. Two-force member;
4. Three-force member;
5. How to calculate friction force.

(2) Equilibrium of a rigid body in 3D (Chapter 4)

$\sum \mathbf{F}_i = 0$, and
$\sum \mathbf{M}_O = \sum \mathbf{r}_{Oi} \times \mathbf{F}_i = 0$;

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum F_x = 0;$$

$$\sum M_x = 0, \sum M_y = 0, \ \sum M_z = 0;$$

Key Points

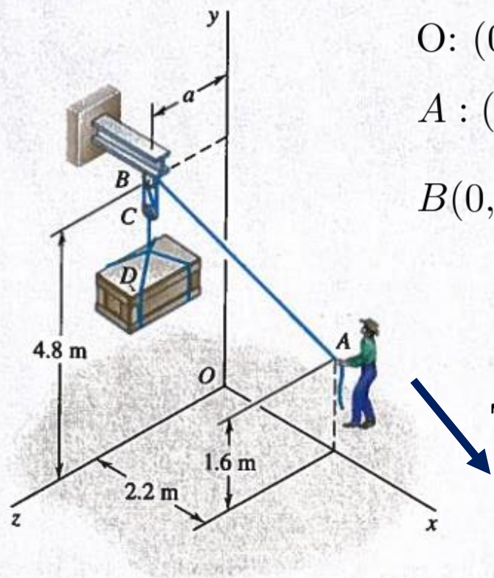
1. Reactions corresponding to different supports;
2. Representation of force vector in 3D

Let $i = A$ and $\mathbf{F}_A$ pointing from A to B: $\quad \mathbf{F}_A = |\mathbf{F}| \boldsymbol{\lambda}_{AB}$,

3. Position vector: $\mathbf{r}_{OA} = \mathbf{r}_A - \mathbf{r}_O$; and $\boldsymbol{\lambda}_{AB} = \mathbf{r}_{AB}/|\mathbf{r}_{AB}|$;
4. Calculate the moment about one point:

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_A = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_{Ax} & F_{Ay} & F_{Az} \end{bmatrix}$$

3.39 To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook *B*. Knowing that the moments about the *y* and *z* axes of the force exerted at *B* by portion *AB* of the rope are, respectively, 120 N·m and –460 N·m, determine the distance *a*.



O: $(0,0,0)$

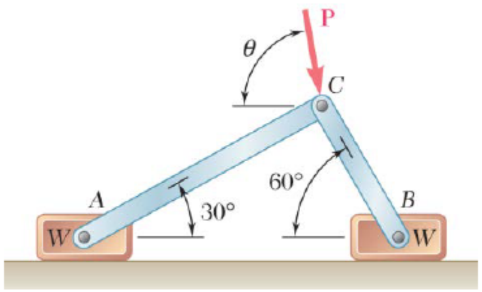
$A:(2.2, 1.6, 0)$

$B(0, 4.8, a)$

$\mathbf{T}_{AB}$

Fig. P3.39 and P3.40

These problems are rated R



PROBLEM 4.95

Two slender rods of negligible weight are pin-connected at *C* and attached to blocks *A* and *B*, each of weight *W*. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of *P* for which equilibrium is maintained.

(3) Structures (Chapter 6)

Key Points

1. Truss structure → Method of joints, and Method of Section;
2. Frame and Machine; → multiple force members; free-body diagram;
3. Two-force member and three-force member.

4.53 A 4 × 8-ft sheet of plywood weighing 40 lb has been temporarily propped against column *CD*. It rests at *A* and *B* on small wooden blocks and against protruding nails. Neglecting friction at all surfaces of contact, determine the reactions at *A*, *B*, and *C*.

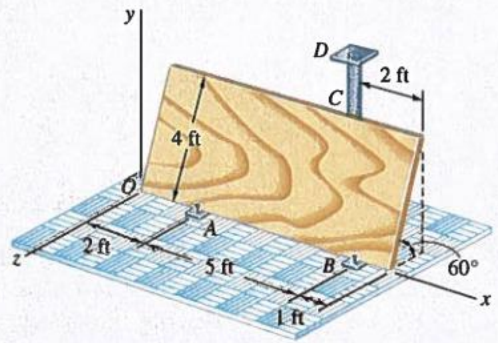
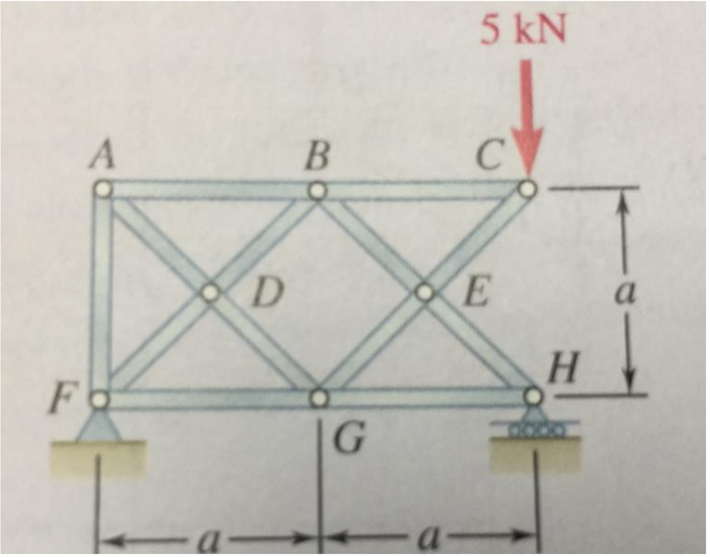


Fig. P4.53

Problem 6.21



These problems are rated-R

(4) Distributed Forces: (Chapter 5 and 7)

Key Points

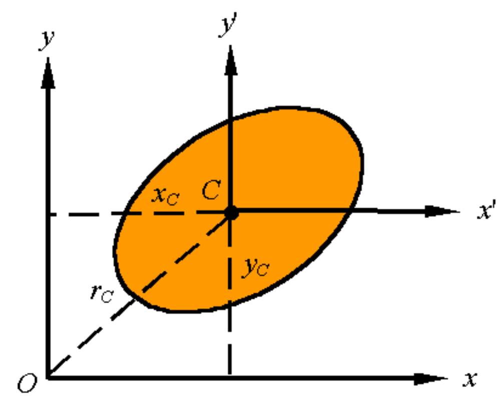
1. **First moments,**

$$Q_x = \int_A y dA, \quad Q_y = \int_A x dA$$

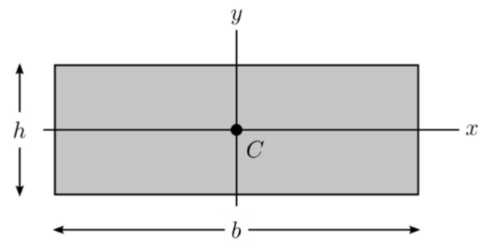
2. **Centroid and Center of Gravity,**

$$\bar{x} = \frac{Q_y}{A} = \frac{\int_A x dA}{A}; \quad \bar{y} = \frac{Q_x}{A} = \frac{\int_A y dA}{A};$$

3. **Centroidal Axes:**



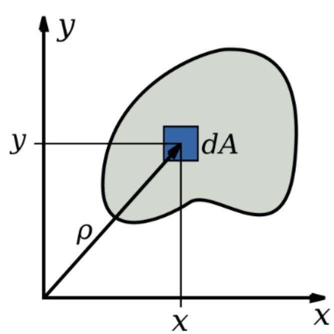
4. Moment of Inertia and Polar Moment of Inertia of an Area



$$I_{xx} = \int_A y^2 dA$$

$$I_{yy} = \int_A x^2 dA$$

$$\boxed{I_\rho = J = \int_A \rho^2 dA = I_{xx} + I_{yy}}$$



For rectangular region : $I_{xx} = \dfrac{bh^3}{12}, \quad I_{yy} = \dfrac{hb^3}{12}$

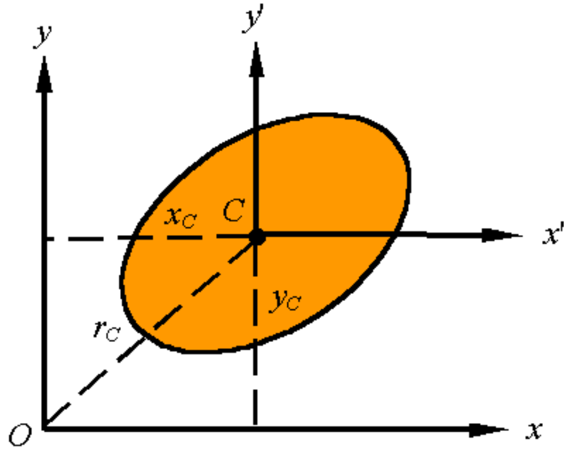
For circular region : $J = \dfrac{\pi c^4}{2} = \dfrac{\pi d^4}{32}$

5. Parallel Axis Theorem

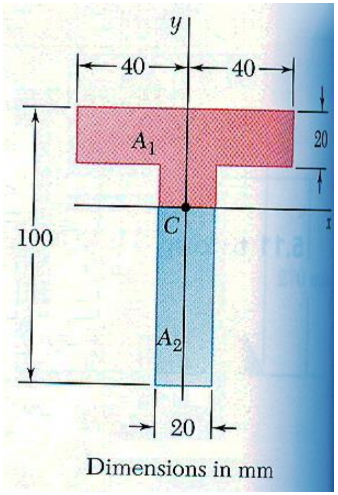
$$I_x = I_{Cx} + y_c^2 A$$

$$I_y = I_{Cy} + x_c^2 A$$

$$J_O = J_C + r_c^2 A$$

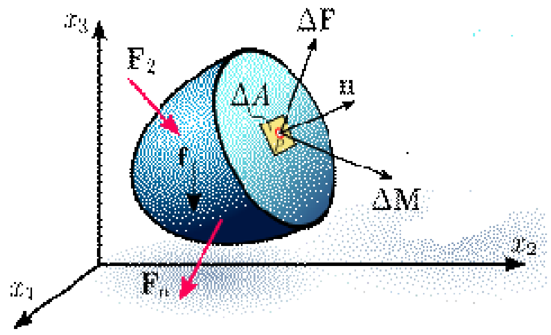


This problem is rated-R



Find the moment of inertia about the global centroid axis?

(5) Concept of Stress (Chapter 8)



Key Points

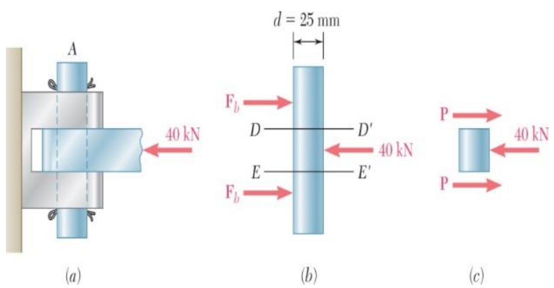
1. Traction vector:

$$\mathbf{T}^{(n)} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A} = \underbrace{\sigma \mathbf{n}}_{\text{normal stress}} \quad + \quad \underbrace{\tau \mathbf{t}}_{\text{shear stress;}}$$

2. Stress Tensor

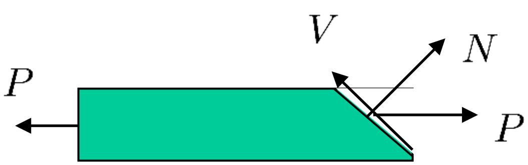
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}; \quad \sigma_{xy} = \sigma_{yx}$$

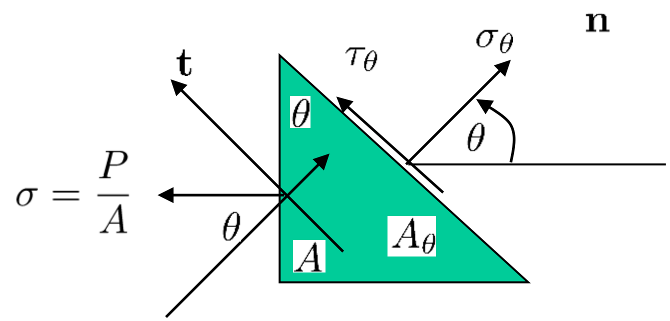
3. Double shear: $\tau = P/2A$;



$$A_\theta \cos\theta = A, \quad \to \quad A_\theta = A/\cos\theta;$$

Wedge Method



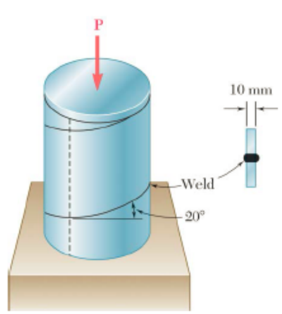


$$\sum F_n = 0, \quad \to \quad -P\cos\theta + \sigma_\theta A_\theta = 0;$$

$$\sigma_\theta = \frac{P\cos\theta}{A_\theta} = \frac{P}{A}\cos^2\theta$$

$$\sum F_t = 0, \quad \to \quad P\sin\theta + \tau_\theta A_\theta = 0;$$

$$\tau_\theta = -\frac{P\sin\theta}{A_\theta} = -\frac{P}{A}\sin\theta\cos\theta$$

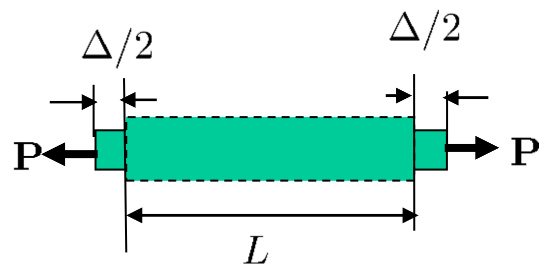


PROBLEM 8.31

A steel pipe of 400-mm outer diameter is fabricated from 10-mm thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

This problem is rated-R

(6) Strains (Chapter 9)



Key Points

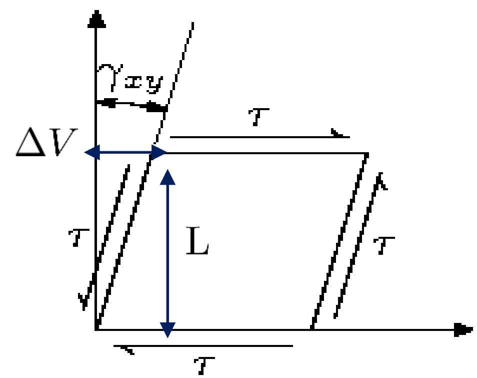
1. Normal Strain:

$$\text{Average normal strain } \epsilon = \frac{\Delta}{L}$$

Mathematical definition (pointwise):

$$\epsilon = \frac{du}{dx}$$

2. Shear strain is the change of angle γ:



3. Thermal strain

$$\epsilon_T = \alpha \Delta T$$

Is thermal strain a normal strain or shear strain ?

$$\gamma \approx \tan\gamma = \frac{\Delta V}{L}$$

(7) Stress-strain relation (Chapter 9)

Key Points:

Hooke's Law in Tension

$$\sigma = E\epsilon,$$

Hooke's Law in shear

$$\tau = G\gamma \ .$$

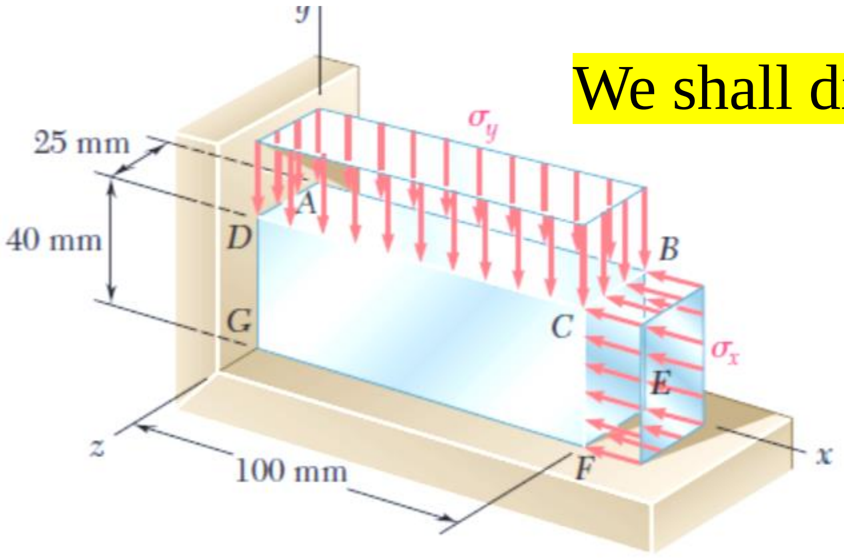
Generalized Hooke's Law in 3D,

$$\begin{aligned}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{yy} &= -\nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \\
\epsilon_{zz} &= -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}
\end{aligned}$$

PROBLEM 9.81

This Problem is rated-R

The block shown is made of a magnesium alloy, for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.



We shall discuss it on Wednesday

(8) Axially Deformed Bars (Chapter 9)

Key Points

1. Basic formulas:

$$\sigma = \frac{P}{A};\ \epsilon = \frac{du}{dx};\ du = \epsilon dx\ \rightarrow\ u(x) - u(0) = \int_0^x \epsilon dx;$$

which leads to

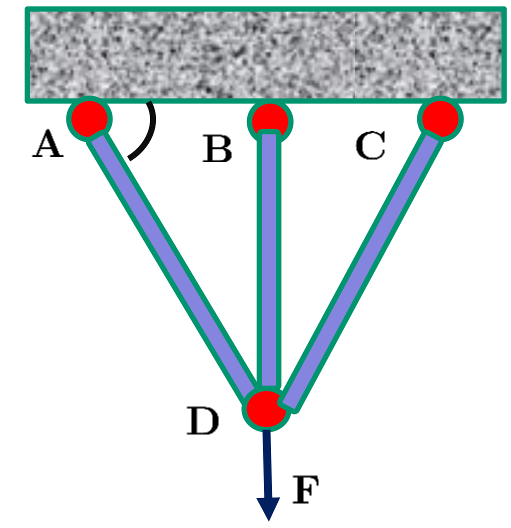
$$u(x) - u(0) = \int_0^x \frac{P(x)}{EA} dx \quad \rightarrow$$

$$\Delta = \frac{L}{EA} P = fP \quad \text{(two force members)}$$

2. Statically indeterminant system:

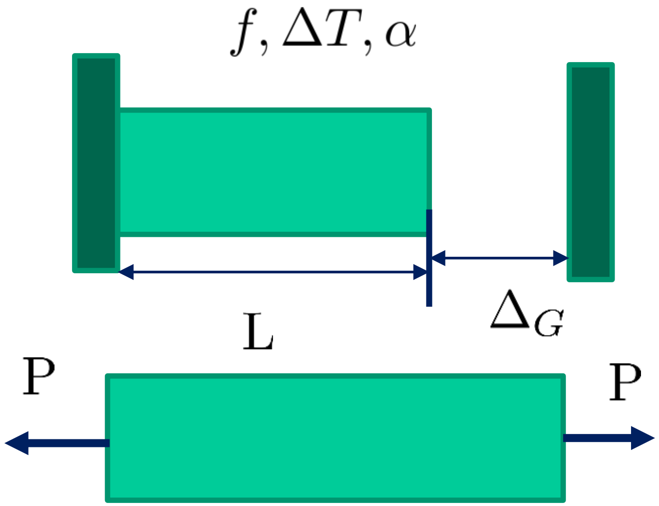
Force method, Superposition method

3. Thermal strain related statically inde-termined problems

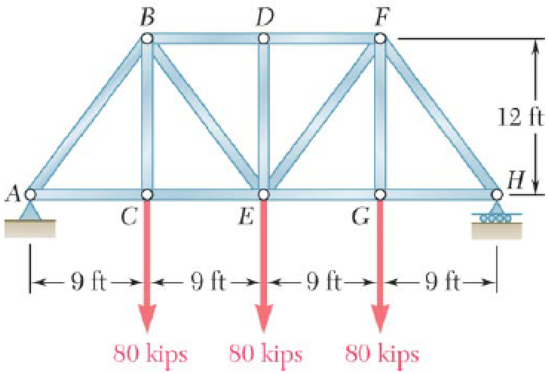


These problems are rated R

Find the force inside the bar.



This problem is rated R.



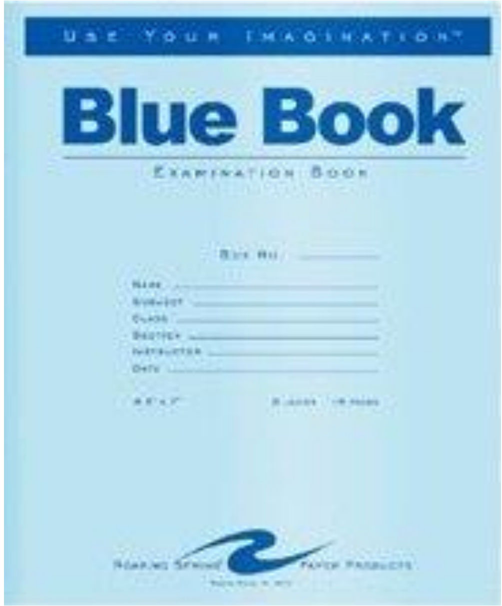
PROBLEM 8.10

Knowing that the average normal stress in member CE of the Pratt bridge truss shown must not exceed 21 ksi for the given loading, determine the cross-sectional area of that member that will yield the most economical and safe design. Assume that both ends of the member will be adequately reinforced.

We shall discuss this problem on Wednesday

Things to do and Exam-taking Tips

1. Review all your graded HWs and Lecture Notes;
2. Do all the rated-R problems by hand;
3. Bring your calculator, pen and pencil, and scratch papers;
4. Arrive on time: (1:00 pm-2:00 pm);
5. No books/Electronic devices and one-page cheat sheet on both sides;
6. Buy and bring your blue book;
7. Get a haircut and take a hot bath;
8. Have a good dinner and good sleep;
9. Be cool, relax, and take it easy;
10. All the students with disabilities go to University Hall No. 1.
11. We shall have another Million-Dollar Question Session on this Friday.



Any Questions?

1. Prepare an exam is all about discipline

Do two problems per day for one hour in five days.

2. How to calm exam anxiety?

Taking an exam is all about mindset.

Imagine that you have already been successful in life and you are writing your personal memoir. What would you like to put down in the section on how you were preparing for exams in college as a role model?

Exam-taking Motto

1. I come today to show off my knowledge, to have fun, and to get an A!
2. Veni, vidi, vici (I came, I saw, and I conquered). came

You were a genius before, and you can be a genius again!

<u>We need a midterm exam slogan</u>