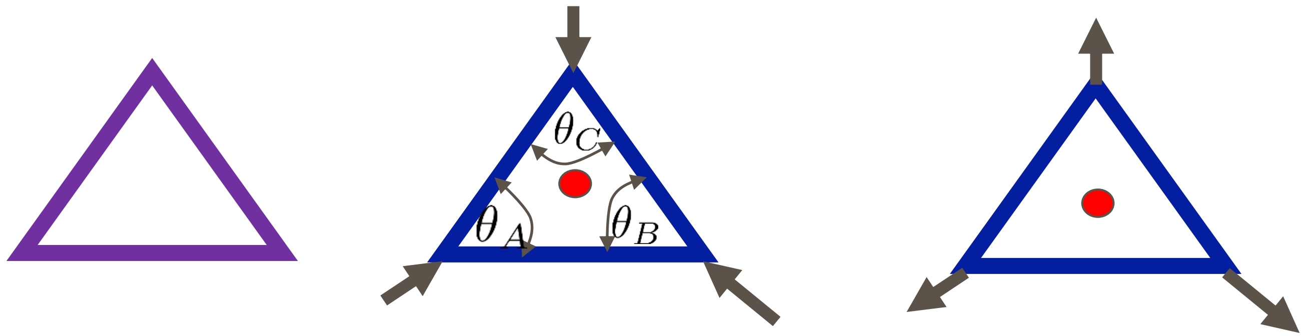
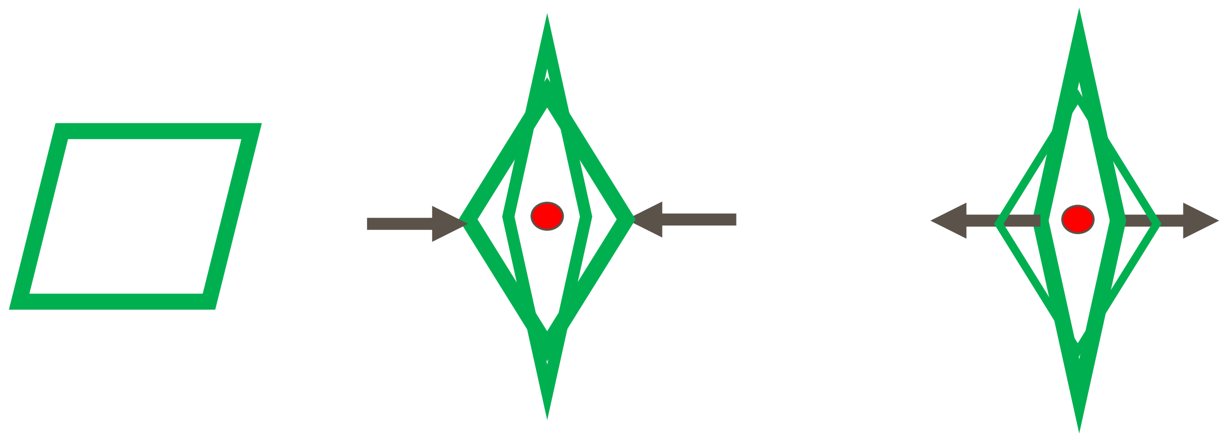
# Lecture 10 Truss Structure (II)

# What is rigid structure?

**A rigid structure is a structure that is intrinsically geometrical deformation-resistant under any set of equilibrium forces passing through its interior.**



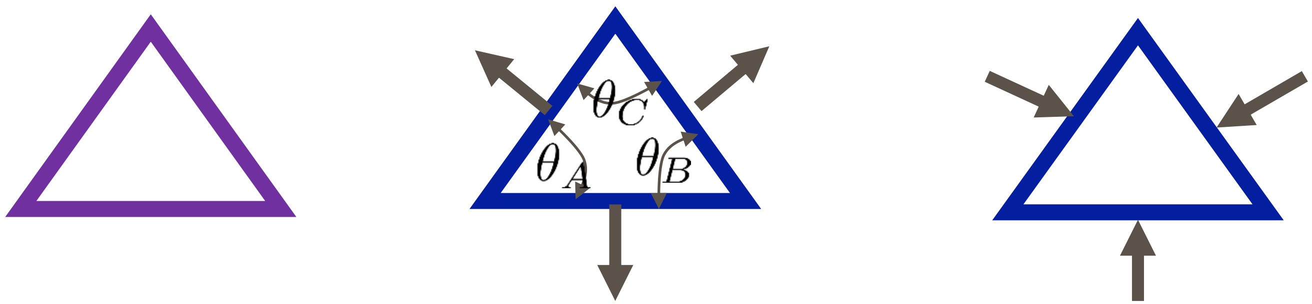

**Topological deformation-resistant**



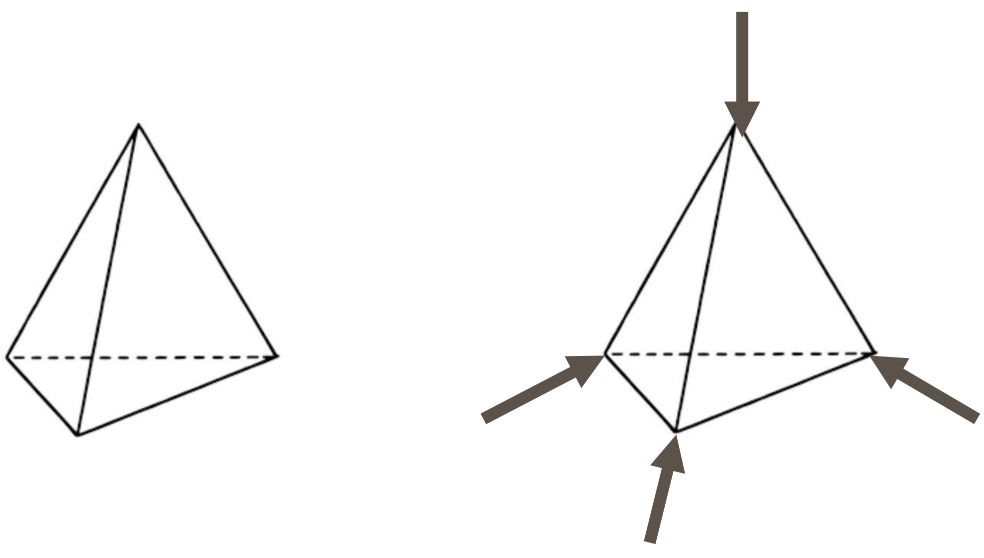
**Topologically deformation-compliant**

# What is rigid structure?

$$\delta\theta_A + \delta\theta_B + \delta\theta_C > 0$$



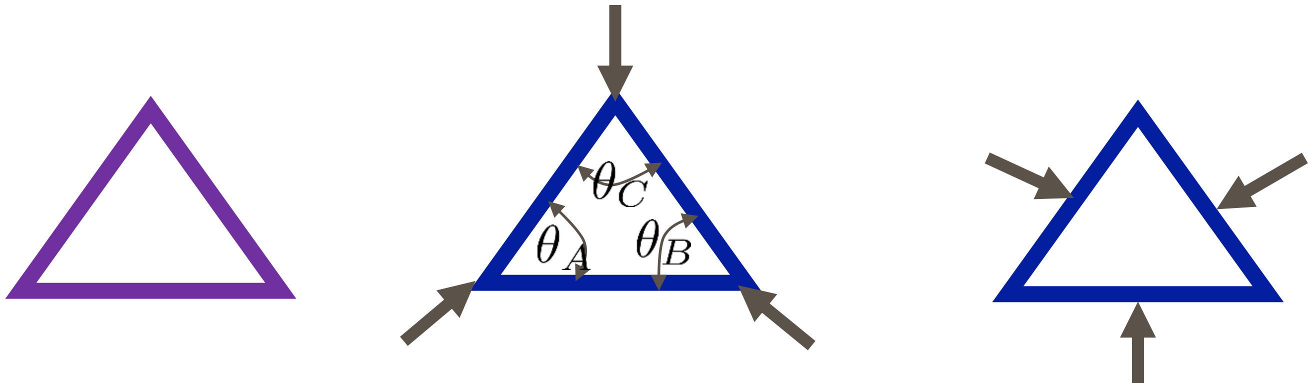

**The sum of all interior angles is equal to $\pi$.**



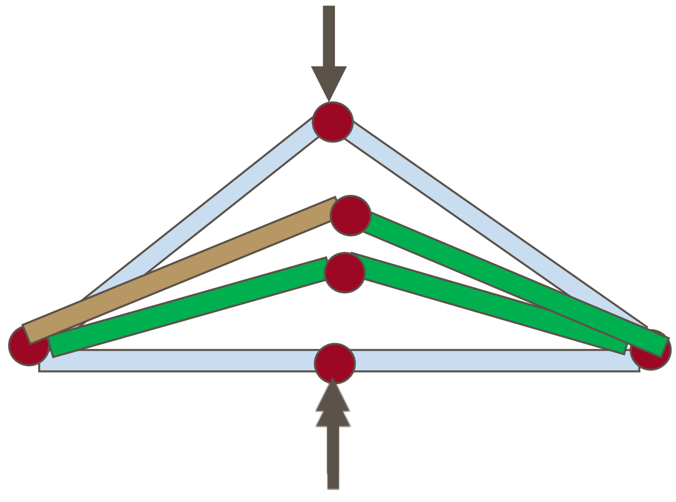
**The sum of all interior solid angles is equal to $4\pi$.**

# What is rigid structure?

$$\delta\theta_A + \delta\theta_B + \delta\theta_C > 0$$

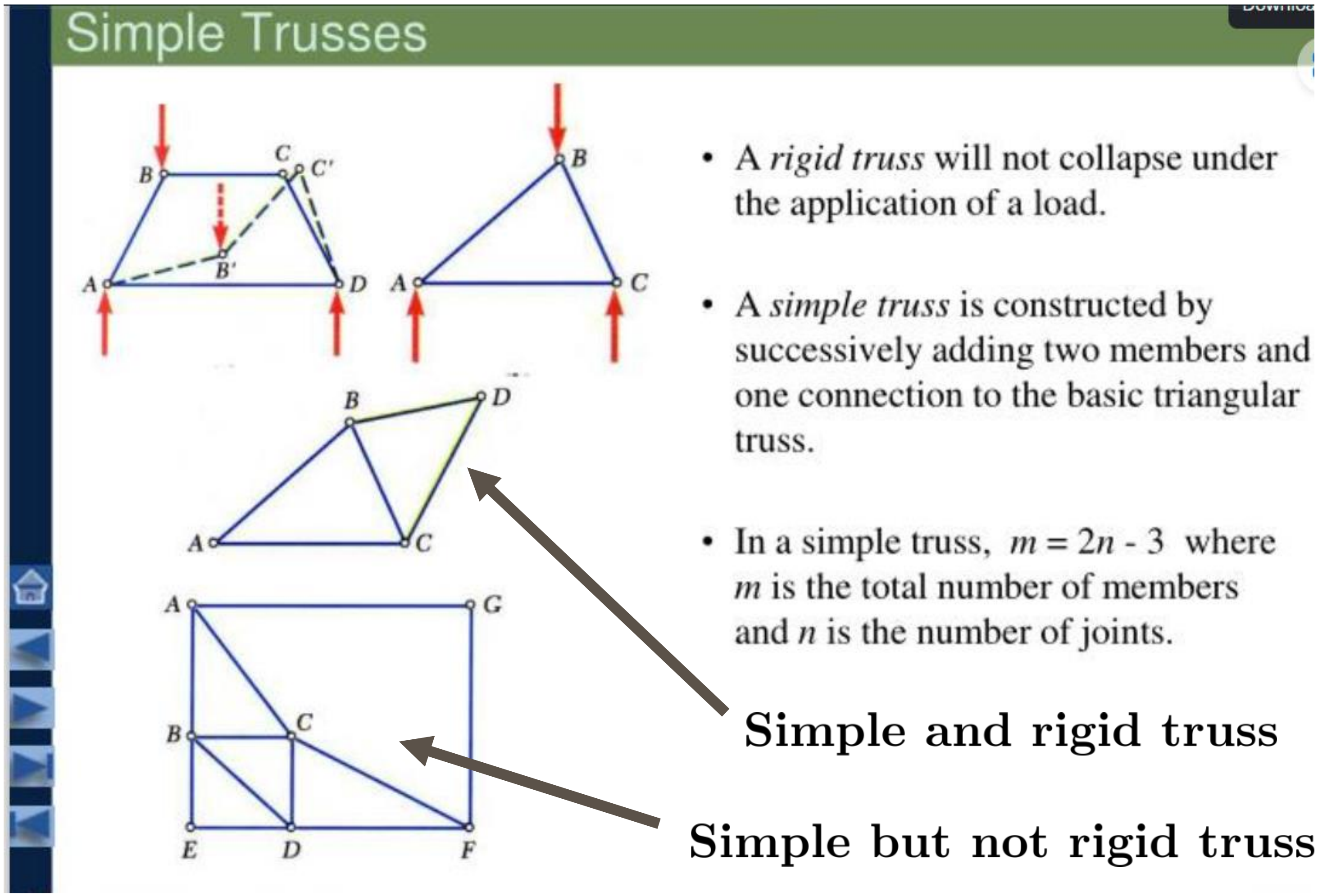


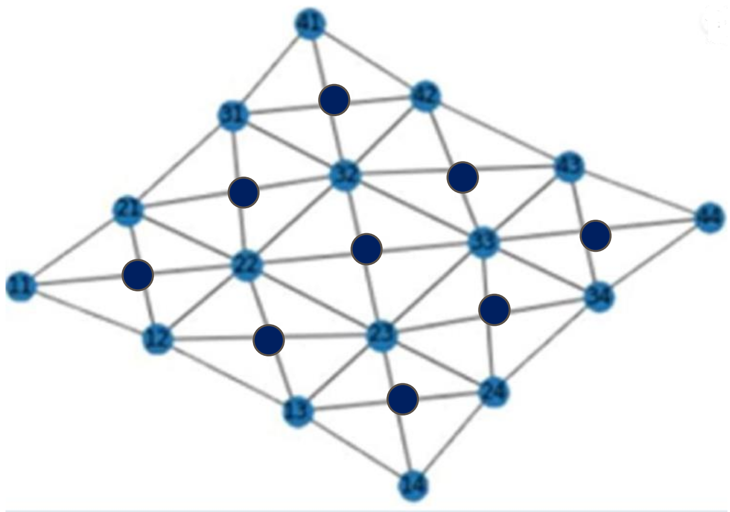

**The sum of all interior angles is equal to $\pi$.**



**This is NOT a rigid structure.**

# A rigid simple truss requires all its sub-units are rigid triangles or rigid tetrahedrons.







**Planar truss vs. Space truss**

**Geodesic Dome**

**(A curved rigid Structure)**

## *Tensegrity Structure*

**The word 'tensegrity' is a contraction of 'tensional integrity'.**
**Tensegrity describes the integrity of structures as being based on a synergy between balanced tension and compression components.**

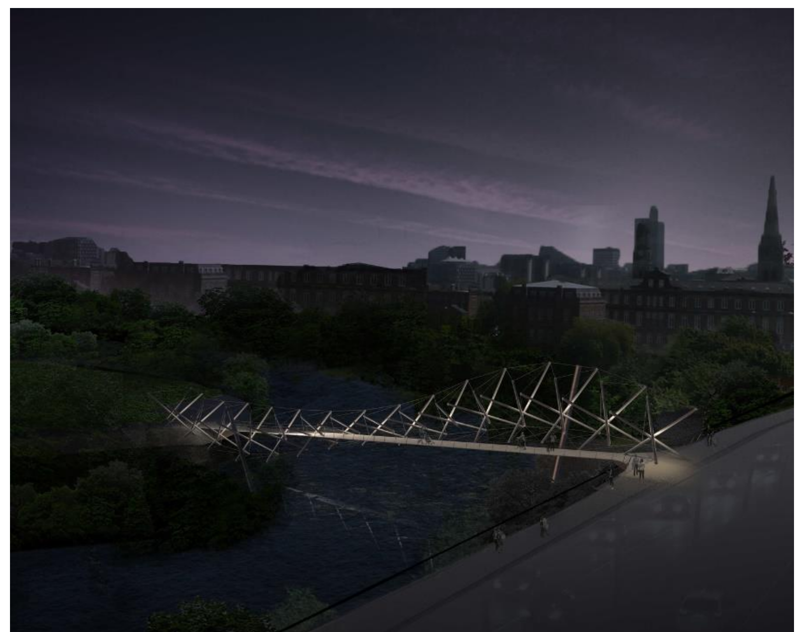
- ***Richard Buckminster Fuller***
- ***(exerpt from Synergetics, p. 372.)***

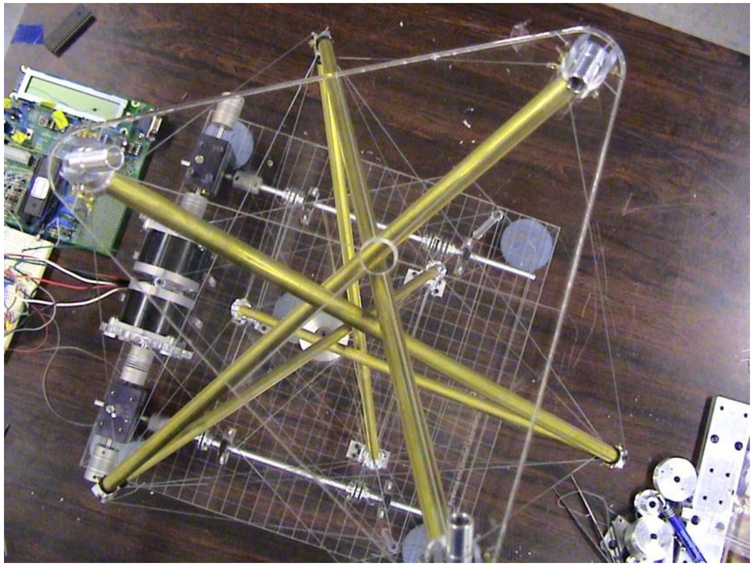
This is a story of statics, and
a story of American ingenuity.

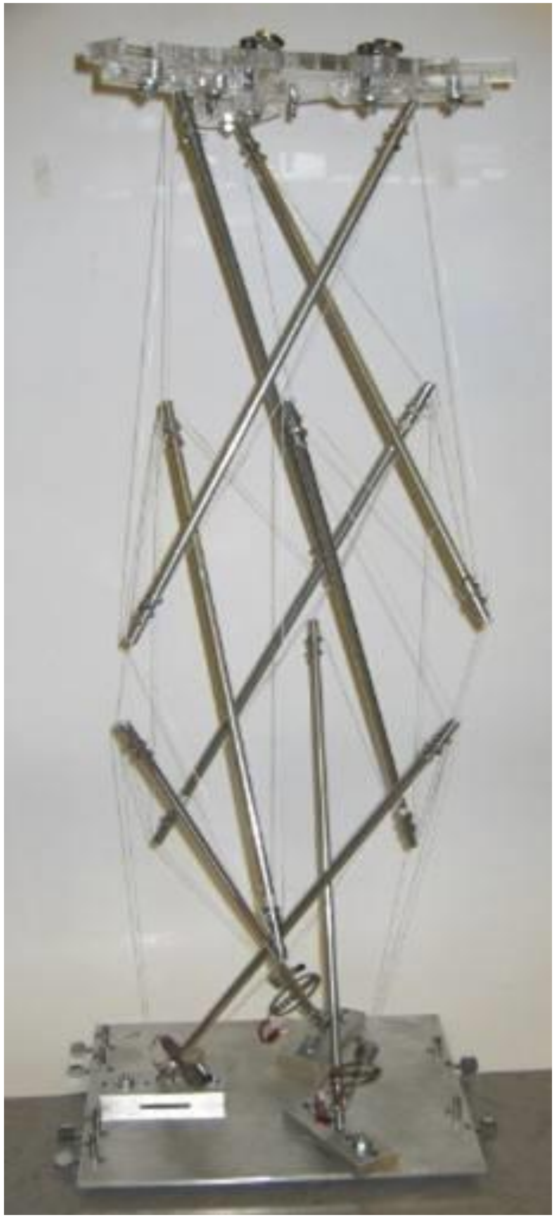
**Needle Tower**

**At Washington DC**
**Smithsonian Museum**


Ali Brown







Pre-stressed tensegrity structure
as novel meta-materials

**Today's Lecture Attendance Password: Tensegrity**

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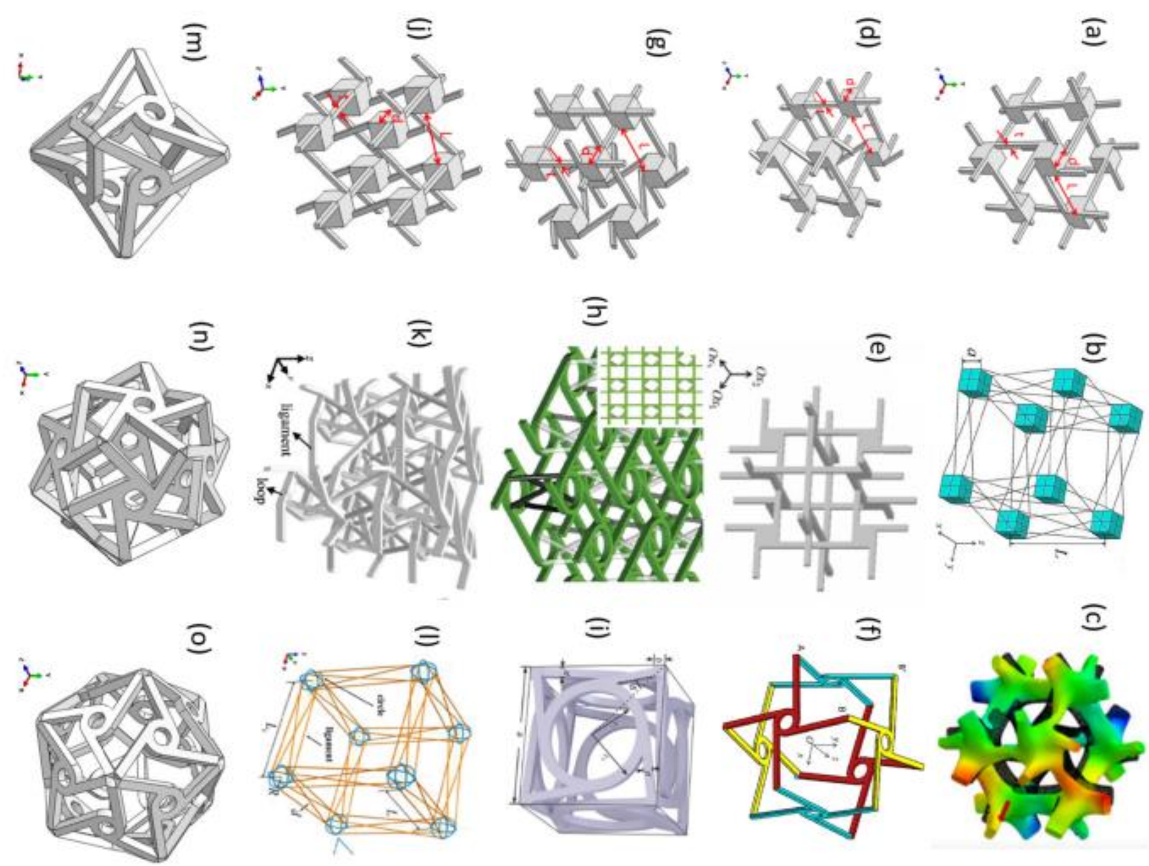
# Tensegrity Metamaterials: Toward Failure-Resistant Engineering Systems through Delocalized Deformation

Jens Bauer, Julie A. Kraus, Cameron Crook, Julian J. Rimoli,* and Lorenzo Valdevit*



## (1) System Constraints (2D Truss)

Simple Truss : $\mathbf{m+3=2n} \quad \rightarrow \quad m+r=2n$ *General Trusses*

Case (a): $\quad m(members)+r(reactions)<2n(nodes)$

— Partially constrained;

Case (b): $\quad m+r>2n$

— Statically indeterminate (over constrained);

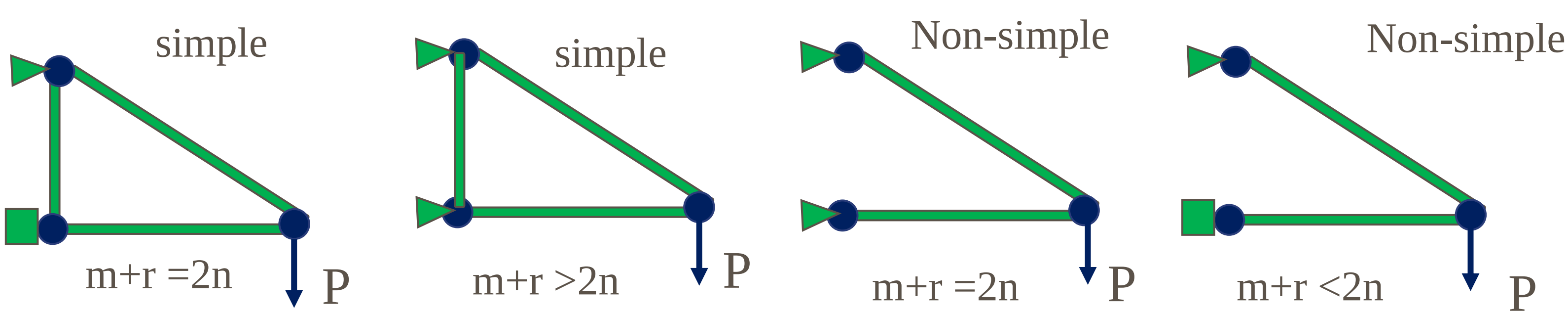
Case (c) $m+r=2n$ **is a necessary condition** for statically
is a necessary condition for statically determinate system,
but the condition is not sufficient.

For some structures that satisfy $m+r=2n$, the structures can be improperly constrainted.
To find out whether or not a structure is completely constrainted and statically determined, one has to be able to determine all the reactions as well as internal forces.

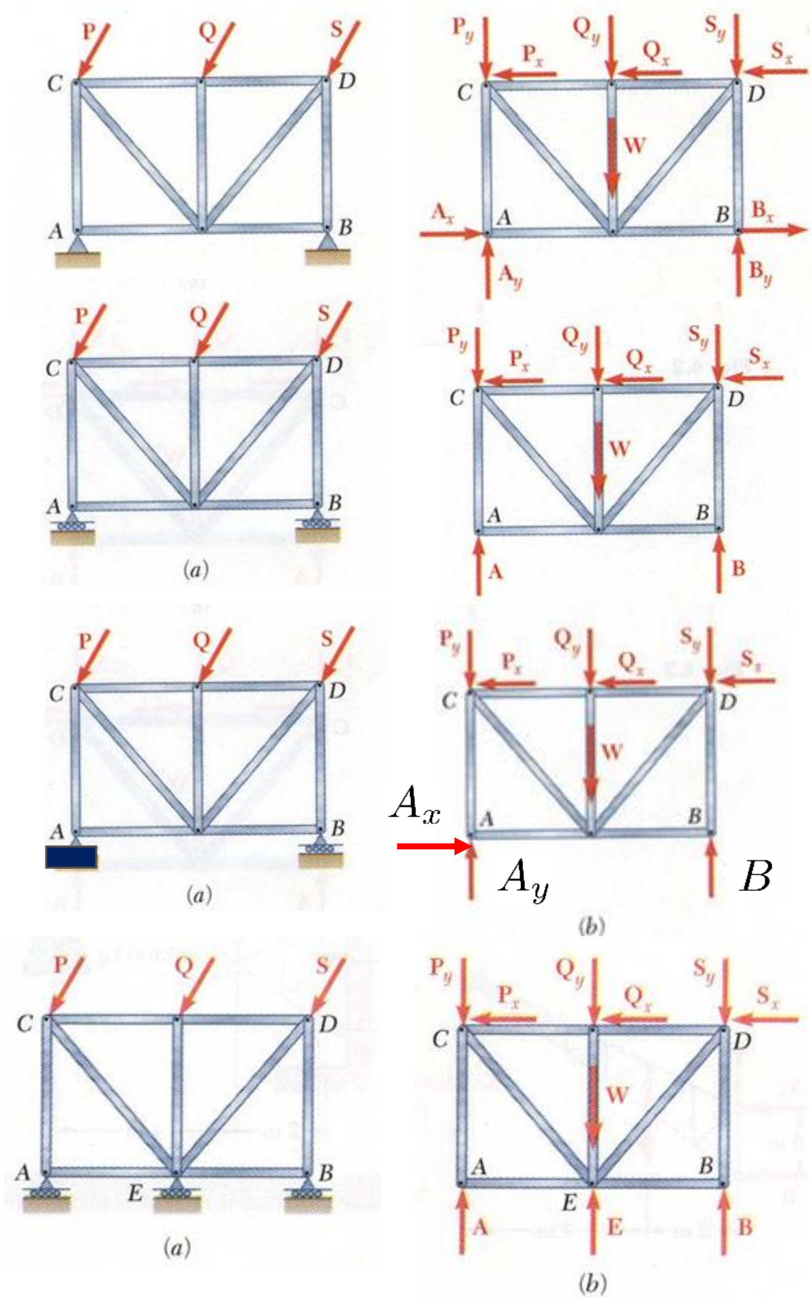
# Category of Truss Structures

1. **Statically indeterminant** (overly constrained) : $m+r > 2n$;
2. **Statically determinant**: $m+r = 2n$;
   (necessary condition but not sufficient)
3. **Partially constrained**: $m+r < 2n$;
   (you do not have enough constraints to satisfy all the equations, which means some nodes are free to move)
4. ***Improperly constrained*** *(*such that the equilibrium state may not be maintained, even though you have mathematically enough constraints, but physically not)

**Simple truss: $2n=m+3$,**
This tells you that you CAN solve simple truss problem only when $r=3$. Otherwise, you cannot solve it.



# System Constraints




$$m = 9, n = 6 \rightarrow \boxed{2n - m >? <? =? r}$$

$$2n - m = 3$$

- More unknowns than equations (2n < m + r;  over-constrained) $(r = 4)$

- Fewer unknowns than equations, partially constrained: 2n > m+r $(r = 2)$

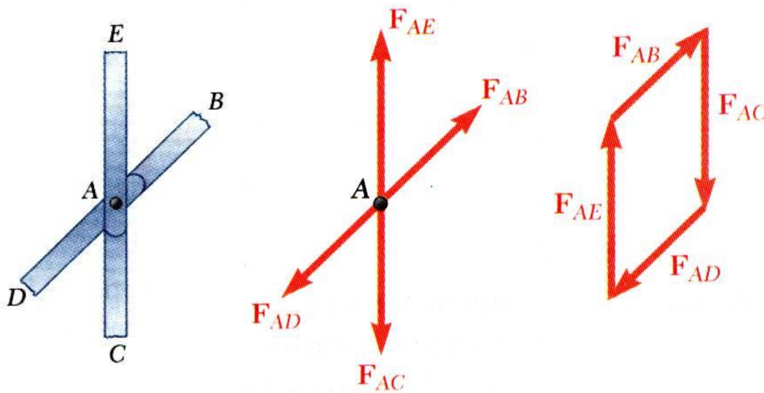
- Equal number in unknowns and equations 2n = m+r $(r = 3)$

- Equal number unknowns and equations but it may be improperly constrained

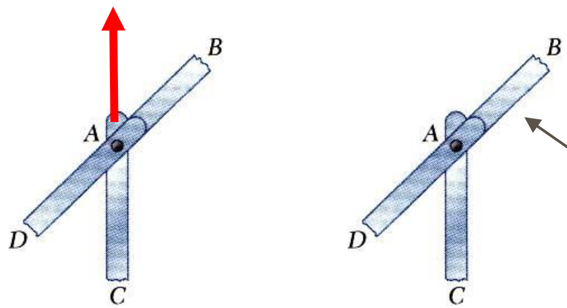
$$2n - m = r, \quad r = 3$$

# (2) Zero-force Member

General Case

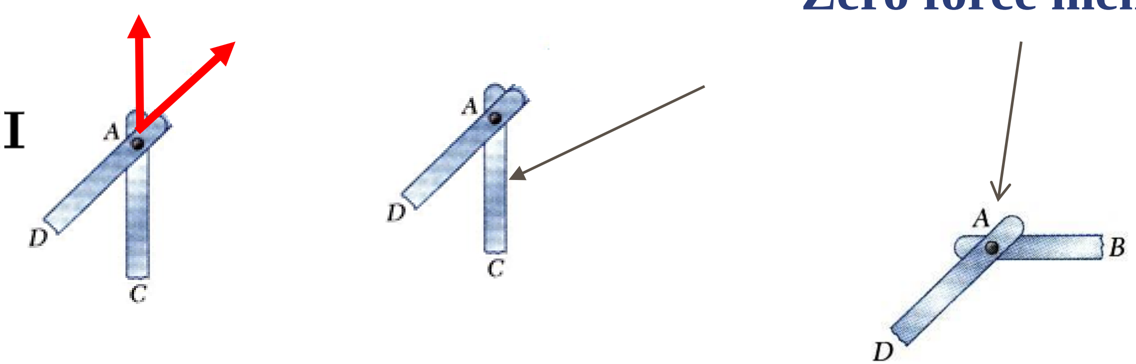



Special Case I

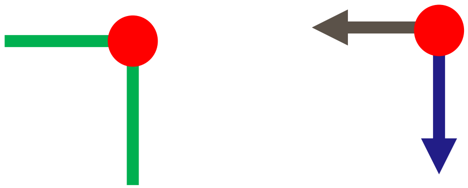



Zero force member

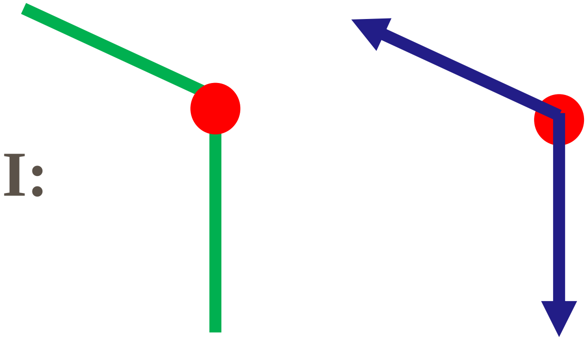
Special Case II



I:

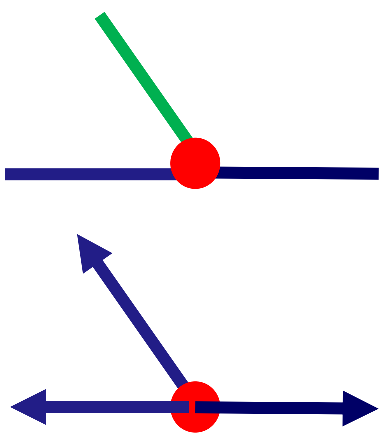


III:

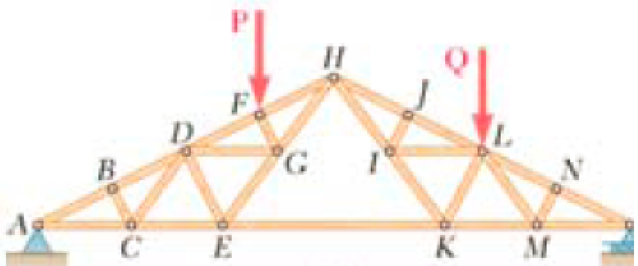


**Zero force member**

II:



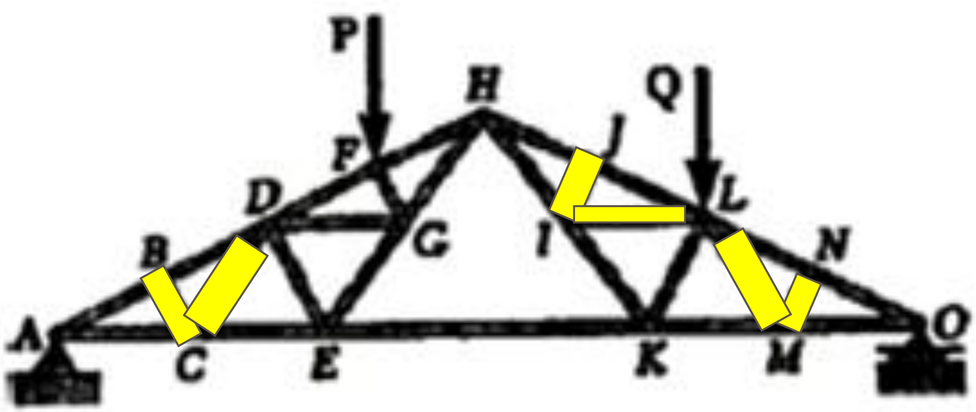
II



(a)

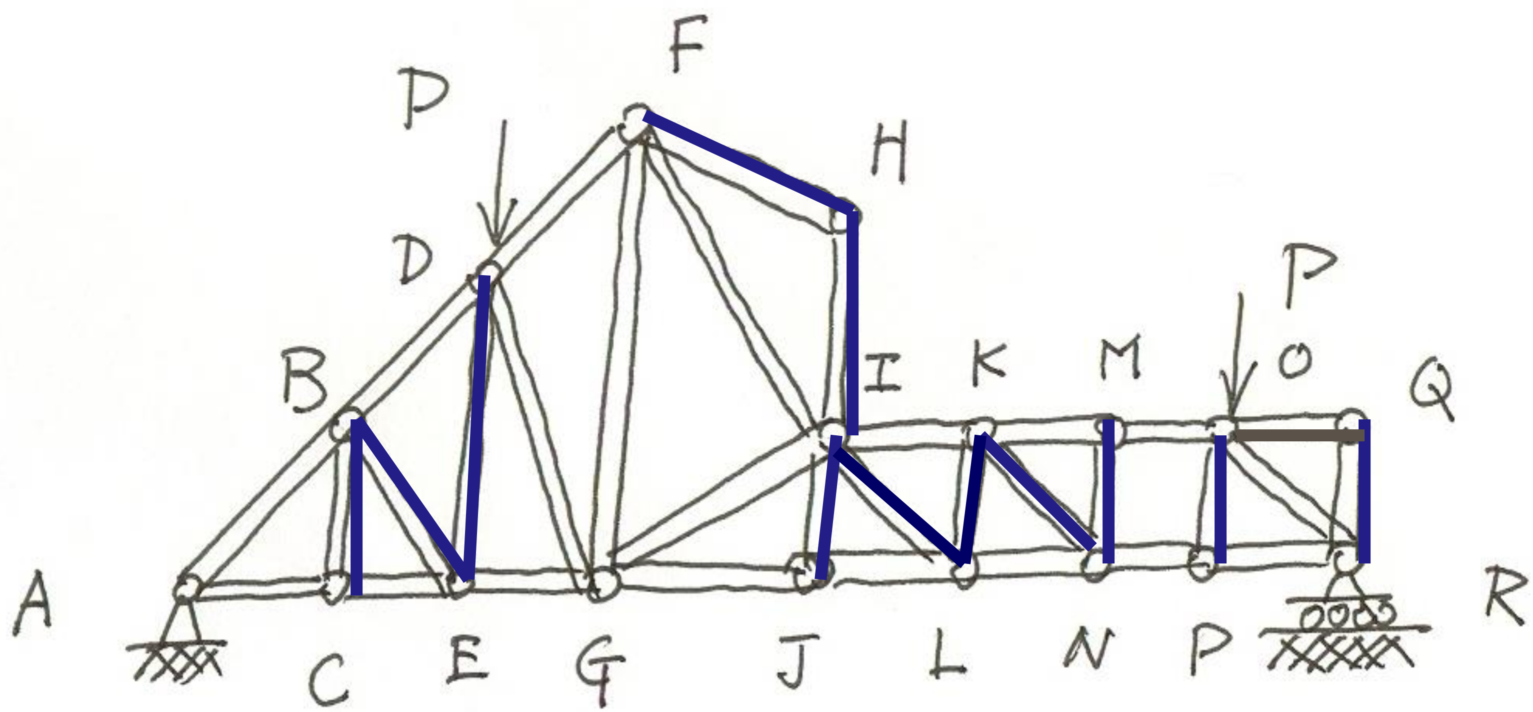
## PROBLEM 6.21

For the given loading, determine the zero-force members in the truss shown.

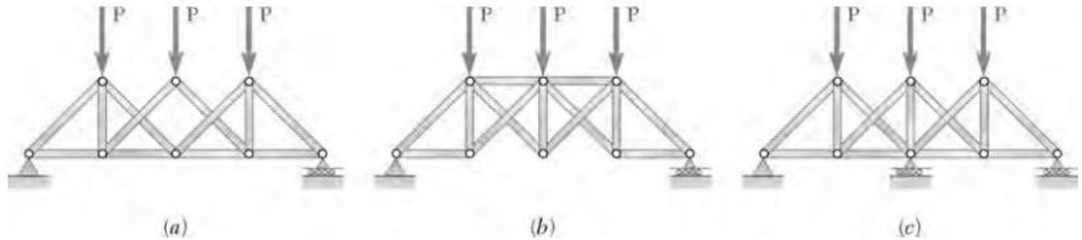


(a)

P6.31 Find the zero-force member



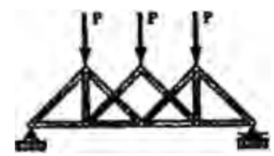
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)




**Structure (a)**

Number of members:

$$m = 12$$



Number of joints:

$$n = 8$$

Reaction components:

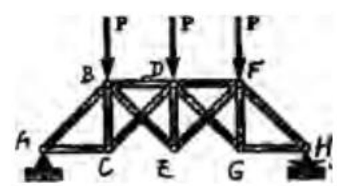
$$r = 3$$

$$m + r = 15, \quad 2n = 16$$

Thus: $\quad m + r < 2n$ ◁

Structure is partially constrained ◀

**Structure (b)**




$$m = 13, \; n = 8$$

$$r = 3$$

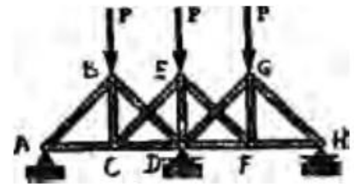
$$m + r = 16, \quad 2n = 16$$

Thus: $\quad m + r = 2n$ ◁

To verify that the structure is actually completely constrained and determinate, we observe that it is a simple truss (follow lettering to check this) and that it is simply supported by a pin-and-bracket and a roller. Thus:

Structure is completely constrained and determinate ◀

**Structure (c)**




$$m = 13, \quad n = 8$$
$$r = 4$$
$$m + r = 17, \;\; 2n = 16$$

Thus: $$m + r > 2n \qquad \triangleleft$$

Structure is completely constrained and indeterminate ◀

This result can be verified by observing that the structure is a simple truss (follow lettering to check thus), therefore rigid, and that its supports involve 4 unknowns.

**6.47 and 6.48** Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

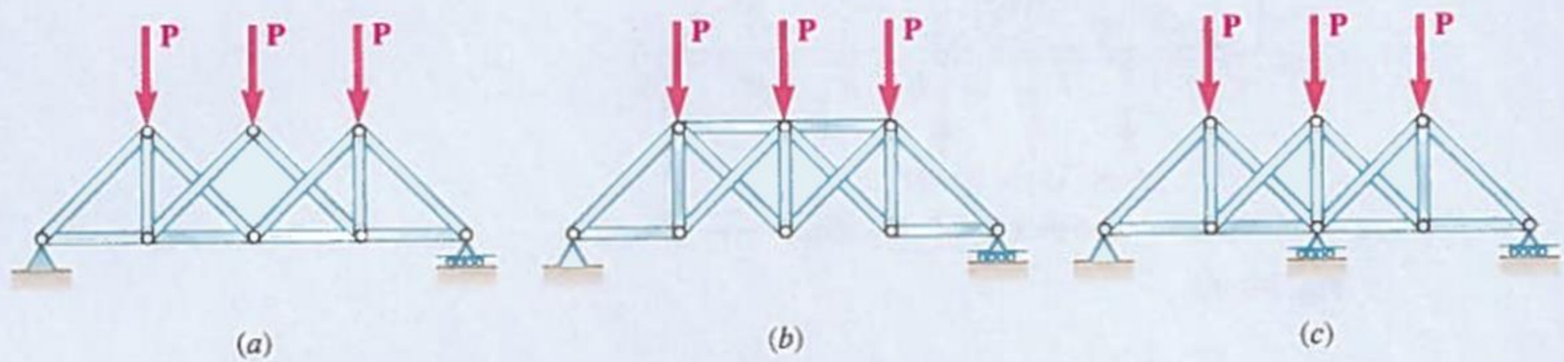



**Fig. P6.47**

$m = 12, r = 3, n = 8$
$15 < 16 \rightarrow m + r < 2n$

$m = 13, r = 3, n = 8$
$16 = 16 \rightarrow m + r = 2n$

$m = 13, r = 4, n = 8$
$17 > 16 \rightarrow m + r > 2n$

# Summary

**1. Tensegrity**, or tensile integrity, describes a system of isolated, compressed components within a network of chords that are under continuous tension.

**2. Statically indeterminant system** (overly constrained) : $m + r > 2n$;
**3. Statically determinant system**: $m + r = 2n$; (necessary condition but not sufficient)
**4. Partially constrained system**: $m + r < 2n$; (you do not have enough constraints to satisfy all the equations, which means some nodes are free to move)
**5. Improperly constrained system** (such that the equilibrium state may not be maintained, even though you have enough constraints mathematically, but not physically,)
**6. Zero force member:** Some members of a truss have zero force under a given external load.