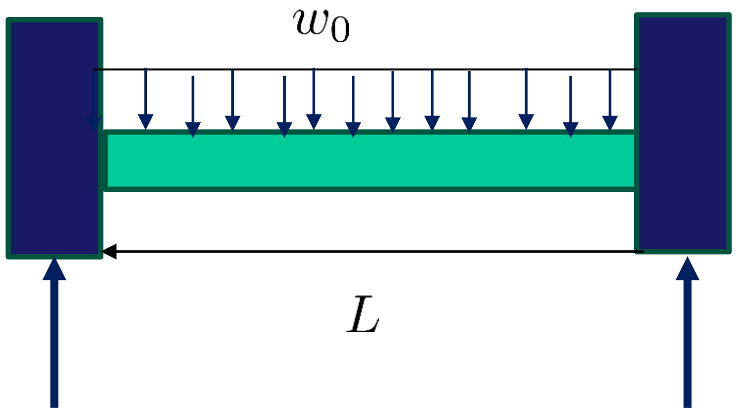
$y(0) = 0, y'(0) = 0$ $\qquad$ $y(L) = 0, y'(L) = 0$



$V(0) = \frac{w_0 L}{2}$

$$EIy^{(iv)} = -w_0$$

$$EIy''' = -w_0 x + C_1$$

$$C_1 = \frac{w_0 L}{2}$$

$$EIy'' = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2} + C_2,$$

$$EIy' = -\frac{w_0 x^3}{6} + \frac{w_0 L x^2}{4} + C_2 x + C_3$$

$$y'(0) = 0 \rightarrow C_3 = 0$$

$$-\frac{w_0 L^3}{6} + \frac{w_0 L^3}{4} + C_2 L = 0 \ \rightarrow \ C_3 = -\frac{w_0 L^2}{12}$$

$$EIy(x) = -\frac{w_0 x^4}{24} + \frac{w_0 L x^3}{12} - \frac{w_0 L^2 x}{12} + C_4$$

$$y(0) = 0 \rightarrow C_4 = 0$$

$$EIy^{(iv)}(x) = -w_0$$

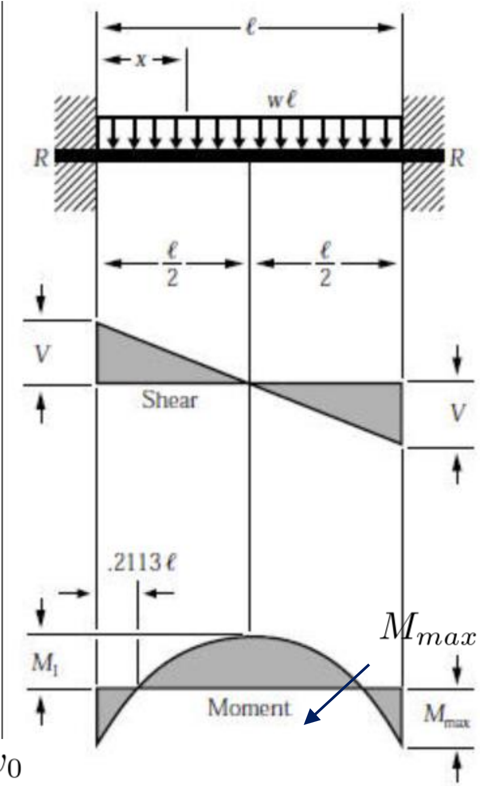
$$EIy'''(x) = V(x) = -w_0 x + \frac{w_0 L}{2}$$

$$EIy''(x) = M(x) = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2} - \frac{w_0 L^2}{12}$$

$$y'(x) = \theta(x) = -\frac{w_0}{24EI}\left(4x^3 - 6Lx^2 + 2L^2 x\right)$$

$$y(x) = -\frac{w_0}{24EI}\left(x^4 - 2Lx^3 + L^2 x^2\right)$$

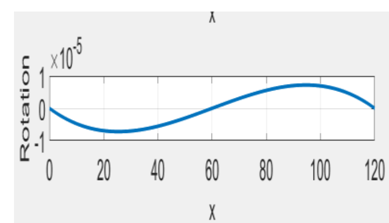
$$V(0) = \frac{w_0 l}{2}$$



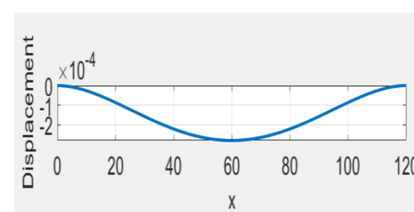
$$-\frac{w_0 l}{2}$$

$$M_{max} = \frac{w_0 L^4}{24EI}$$

$$M(0) = -\frac{L^3 w_0}{12EI}$$

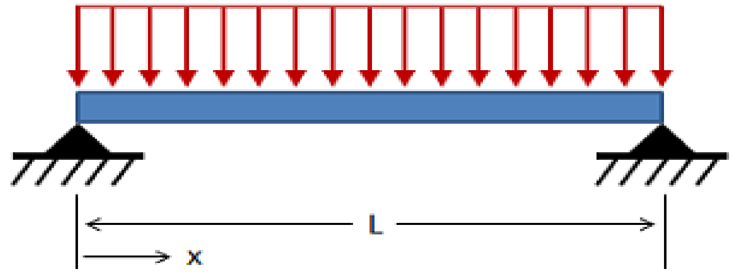


$$y_{max} = -\frac{w_0 l^4}{384EI}$$



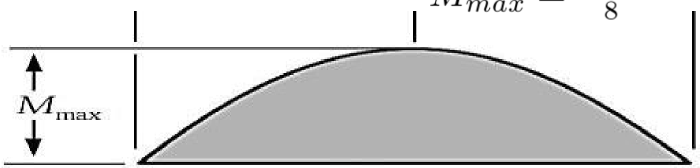
Compare

$$EIy^{(iv)}(x) = -w_0$$

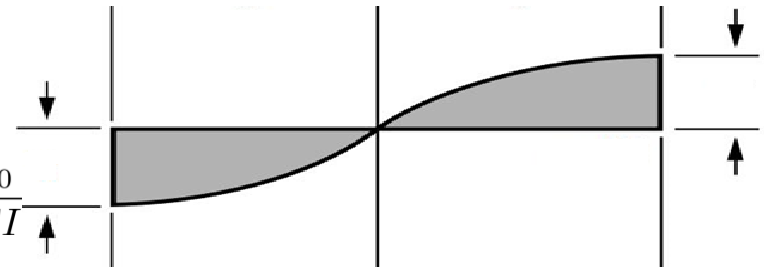


$$EIy'''(x) = V(x) = -w_0 x + \frac{w_0 L}{2}$$

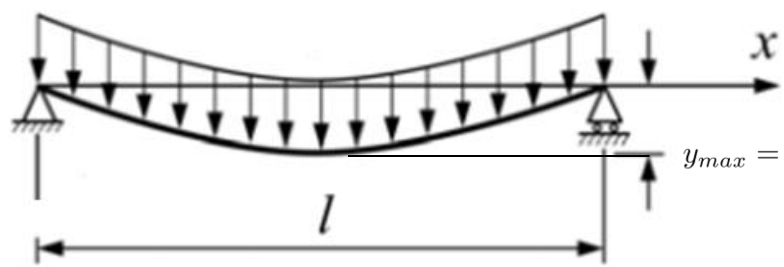
$$EIy''(x) = M(x) = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2}$$

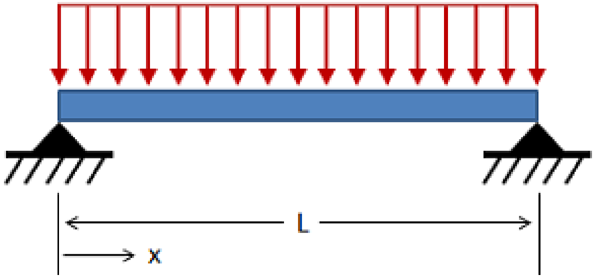


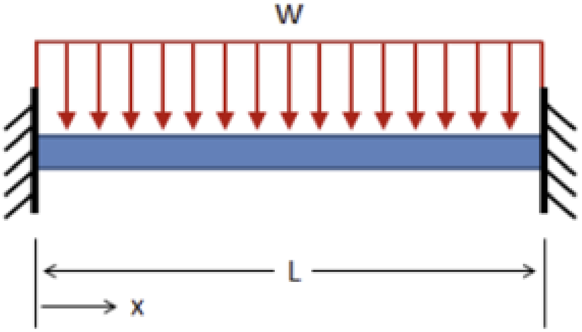
$$y'(x) = \theta(x) = -\frac{w_0}{24EI}\left(4x^3 - 6Lx^2 + L^3\right)$$



$$y(x) = -\frac{w_0}{24EI}\left[x^4 - 2Lx^3 + L^3 x\right]$$



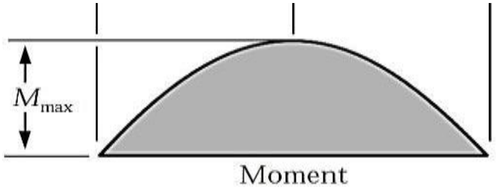




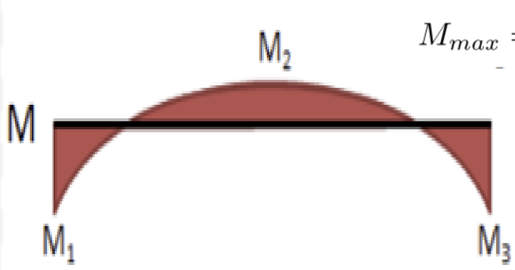
x=0, y(0) =0;
x=0, M(0) =0

x=0, y(0) =0;
x=0, $\theta(0)$ =0

$$M_{max} = \frac{w_0 l^2}{8}$$



$$M_{max} = \frac{w_0 L^2}{24}$$



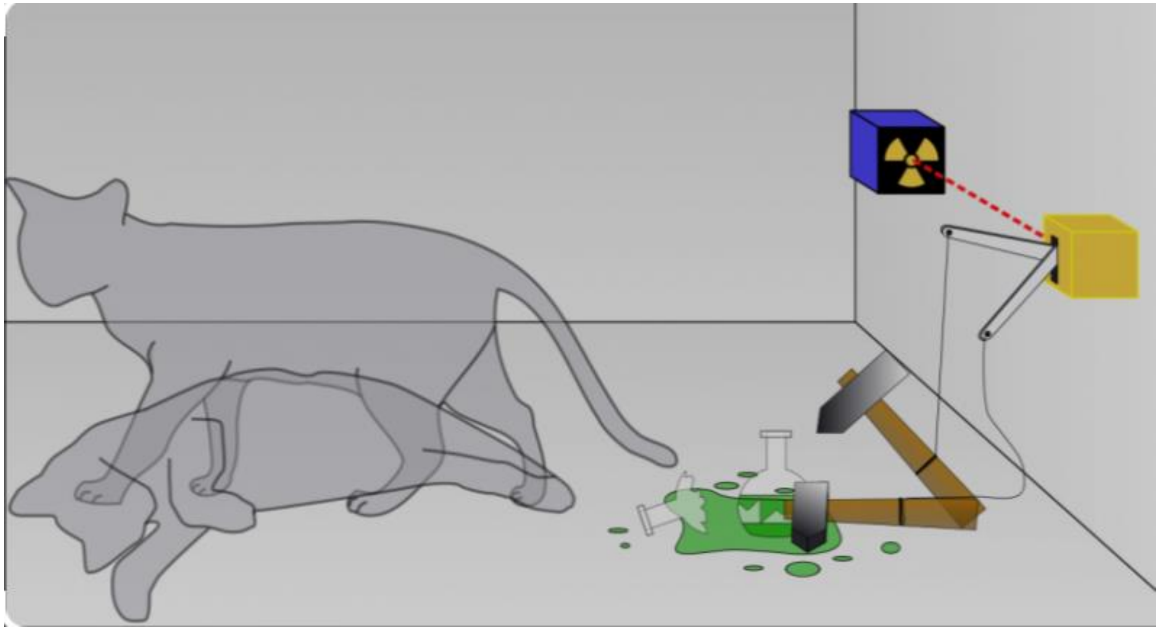
$$M(x) = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2}$$

$$M(x) = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2} - \frac{w_0 L^2}{12}$$

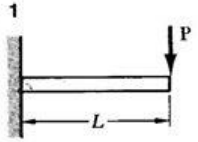
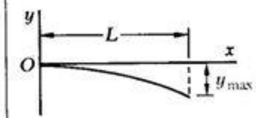
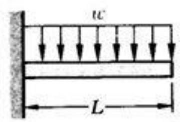
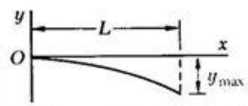
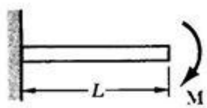
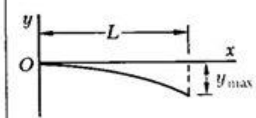
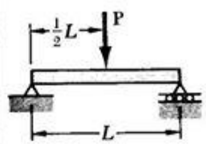
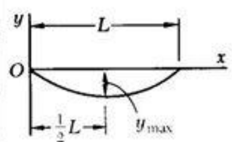
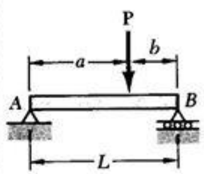
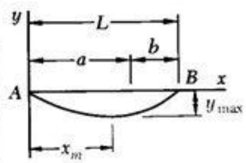
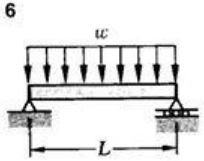
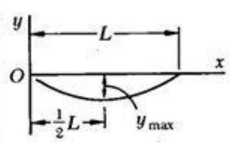
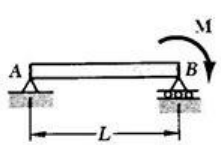
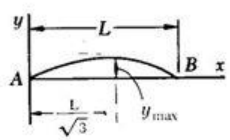
For the same external load, the statistically indeterminant system has lower value of the maximum internal force. However, …..

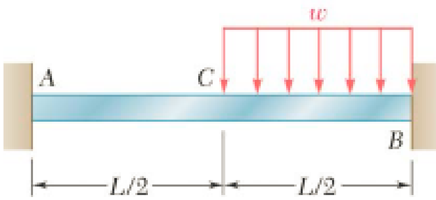
Lecture 32 Beam Deflection (III)

1. Superposition method;
2. Statically indeterminant problem;



Superposition of living and dead cats

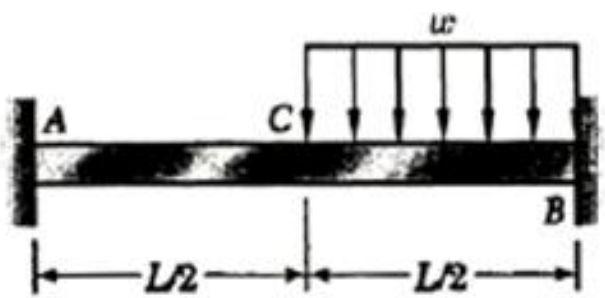
Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$	$y = \dfrac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$	$y = -\dfrac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$	$y = -\dfrac{M}{2EI}x^2$
4 		$-\dfrac{PL^3}{48EI}$	$\pm\dfrac{PL^2}{16EI}$	For $x \leq \frac{1}{2}L$: $y = \dfrac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\dfrac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\dfrac{L^2 - b^2}{3}}$	$\theta_A = -\dfrac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\dfrac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \dfrac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\dfrac{Pa^2b^2}{3EIL}$
6 		$-\dfrac{5wL^4}{384EI}$	$\pm\dfrac{wL^3}{24EI}$	$y = -\dfrac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\dfrac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\dfrac{ML}{6EI}$ $\theta_B = -\dfrac{ML}{3EI}$	$y = -\dfrac{M}{6EIL}(x^3 - L^2x)$

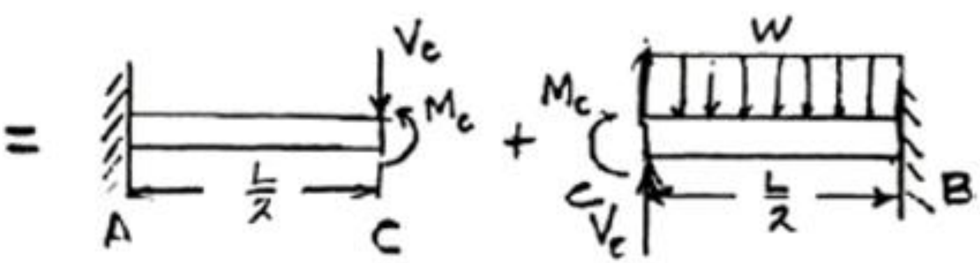


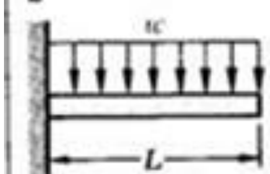
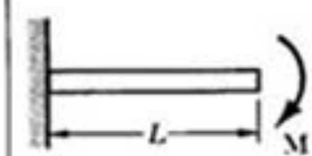
PROBLEM 15.43

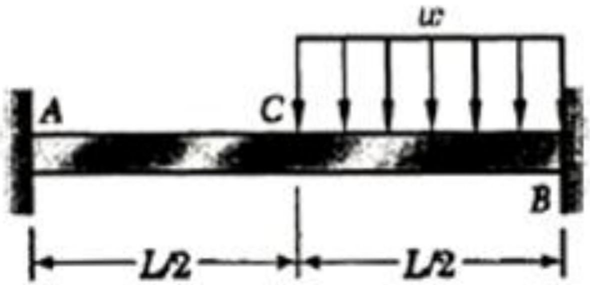
For the beam shown, determine the reaction at B.

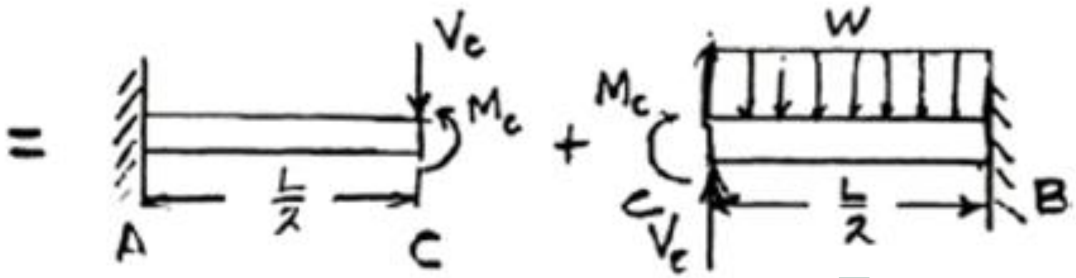
This problem is rated R!

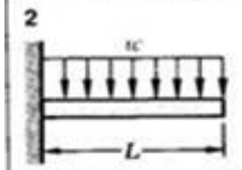
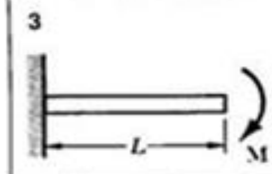




	y_B	θ_B
1 	$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$
2 	$-\frac{wL^4}{8EI}$	$-\frac{wL^3}{6EI}$
3 	$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$





	y_B	θ_B
1  B	$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$
2 	$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$
3 	$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$

$$y_c = -\frac{V_c(L/2)^3}{3EI} + \frac{M_C(L/2)^2}{2EI} \qquad y_c = +\frac{V_c(L/2)^3}{3EI} + \frac{M_C(L/2)^2}{2EI} - \frac{w(L/2)^4}{8EI}$$

$$V_c = \frac{3wL}{32}$$

$$\theta_c = -\frac{V_c(L/2)^2}{2EI} + \frac{M_C(L/2)}{EI} \qquad \theta_c = -\frac{V_c(L/2)^2}{2EI} - \frac{M_C(L/2)}{EI} + \frac{w(L/2)^3}{6EI}$$

$$M_c = \frac{wL}{48}$$

Today's Lecture Attendance Password is: Superposition

Problem 1.

This is a MATLAB homework problem. Consider an elastic beam with Young's modulus, $E = 30 \times 10^6 Psi$, the cross section moment inertia $I_z = 256 in^4$, and the length of the beam $L = 120 in$. The beam has a built-in boundary conditions at $x = 0$, i.e. $y0) = 0$ and $\theta(0) = 0$; and at $x = L$, $y(L) = 0$ and $\theta(L) = 0$ as shown in Fig. 1.

The differential equation that governs the equilibrium of the bar has been derived as follows,

$$\frac{d^2}{dx^2}\Big(EI\frac{d^2y}{dx^2}\Big) + w(x) = 0, \quad 0 < x < L \ ,$$

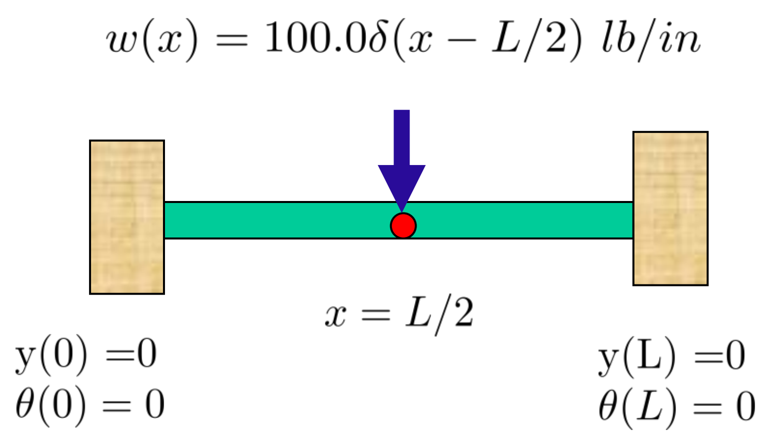
where $y(x)$ is the deflection field.

The beam is subjected a concentrated load at $x = L/2$, i.e.

$$w(x) = P\delta(x - L/2)$$

where $P = 100 lb$.

Modify the template MATLAB code, *beam_model.m*, to find shear diagram, moment diagram, rotation diagram, and the deflection profile.



Problem 1.

Matlab-code-HW12.zip	Monday	Monday	SHAO...	4 KB	✓
Matlab-P1.zip	Mar 5, 2024	Mar 5, 2024	SHAO...	75 KB	✓

$E =$ ... $20in.$ ... $= L,$ ... ws,

where $y(x)$ is the deflection field.

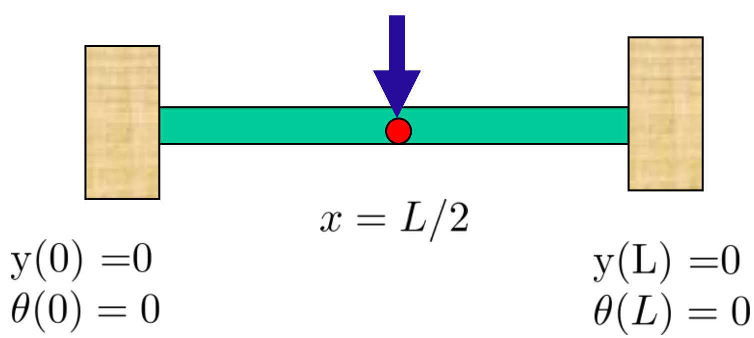
The beam is subjected a concentrated load at $x = L/2$, i.e.

$$w(x) = P\delta(x - L/2)$$

where $P = 100lb$.

Modify the template MATLAB code, *beam_model.m*, to find shear diagram, moment diagram, rotation diagram, and the deflection profile.

$$w(x) = 100.0\delta(x - L/2)\ lb/in$$



$$\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) = -w(x) \text{ can be decomposited into four first order ODE :}$$

$$\frac{d\mathbf{y}}{dx} = \mathbf{f}(\mathbf{y}, x) \quad \Longrightarrow$$

$$\theta = \frac{dy}{dx} \qquad (1)$$

$$\kappa = \frac{d\theta}{dx} = y'' = \frac{M(x)}{EI} \qquad (2)$$

$$\frac{dM}{dx} = V(x) \qquad (3)$$

$$\frac{dV}{dx} = -w(x) \qquad (4)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \Longrightarrow \begin{array}{l} y(x) \\ \theta = \dfrac{dy}{dx} \\ M(x) = EI_z\dfrac{d^2y}{dx^2} \\ V(x) = EI_z\dfrac{d^3y}{dx^3} \end{array} \Longrightarrow \frac{d}{dx}\begin{bmatrix} y \\ \theta \\ M \\ V \end{bmatrix} = \begin{bmatrix} \theta \\ \frac{M}{E(x)I_z(x)} \\ V \\ -w(x) \end{bmatrix}$$

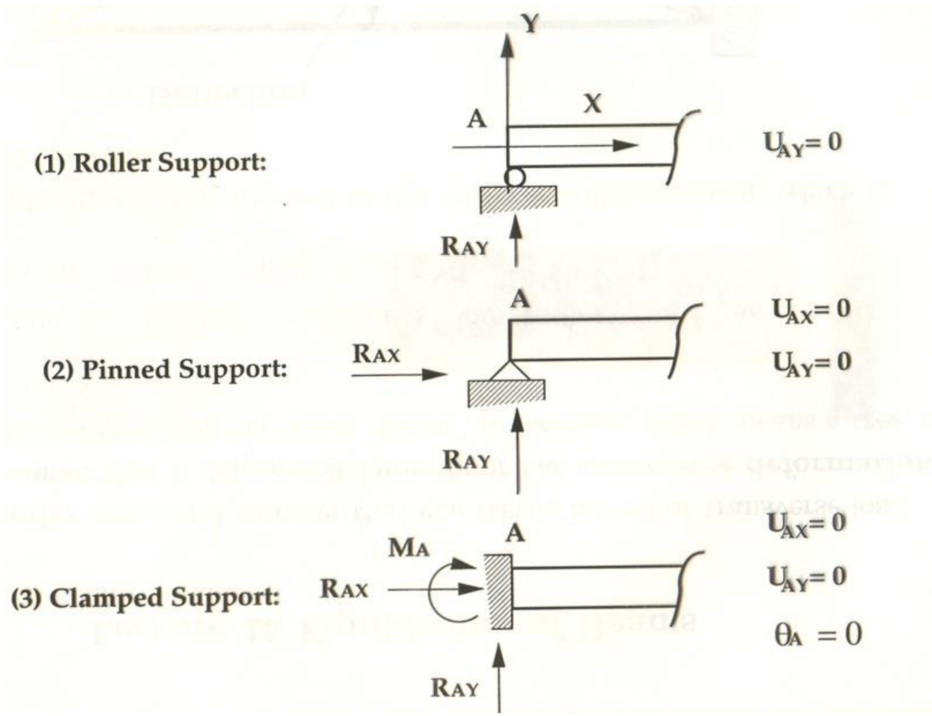
one obtains the canonical form of the first-order vector ODE,

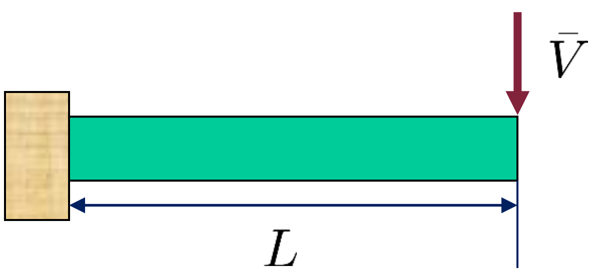
$$\frac{d}{dx}\mathbf{y} = \frac{d}{dx}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} y_2 \\ \frac{y_3}{E(x)I_z(x)} \\ y_4 \\ -w(x) \end{bmatrix} \quad \rightarrow \quad \mathbf{f} = \begin{bmatrix} y_2 \\ \frac{y_3}{E(x)I_z(x)} \\ y_4 \\ -w(x) \end{bmatrix}$$

Matlab-code-HW12.zip Yesterday Yesterday SHAOF... 4 KB

beam_T1

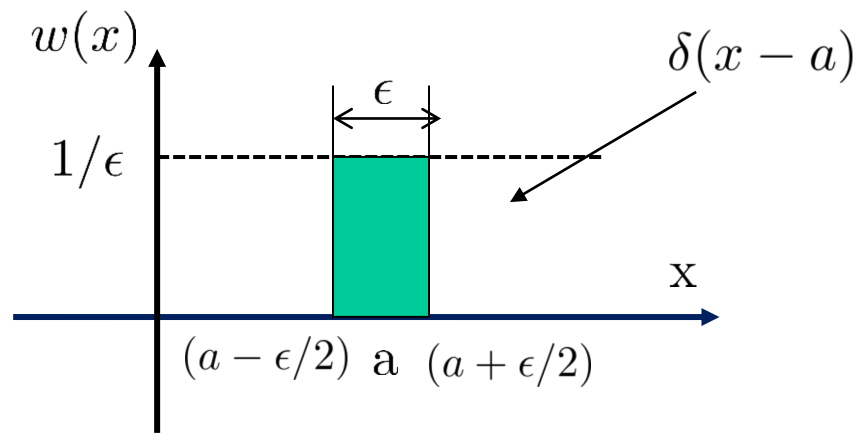
beam_T2





$$\begin{bmatrix} v(0)-0 \\ \theta(0)-0 \\ M(L)-0 \\ V(L)-\bar{V}_L \end{bmatrix} \Rightarrow \quad \boldsymbol{g}(\boldsymbol{y}(0),\boldsymbol{y}(L)) \;:=\; \begin{bmatrix} g_1(\boldsymbol{y}(0),\boldsymbol{y}(L)) \\ g_2(\boldsymbol{y}(0),\boldsymbol{y}(L)) \\ g_3(\boldsymbol{y}(0),\boldsymbol{y}(L)) \\ g_4(\boldsymbol{y}(0),\boldsymbol{y}(L)) \end{bmatrix} = \begin{bmatrix} y_1(0)-0 \\ y_2(0)-0 \\ y_3(L)-0 \\ y_4(L)-\bar{V}_L \end{bmatrix}$$

How to represent the Dirac delta function ?



The Dirac delta function can be defined as,

$$\delta(x-a) := \lim_{\epsilon\to 0}\begin{cases} \dfrac{1}{\epsilon}, & a-\epsilon/2 \le x \le a+\epsilon/2 \\ 0, & \text{otherwise} \end{cases}$$

```
function [fxy] = beam1d_ode(x,y)

% % -- Define material property and geometry
% w = 0;       % load, in lb/in
%
E = 30e6;    % Young's Modulus, in psi
I = 256;      % Second moment of inertia, in in^4
a = 120/2;      % Length of beam, in in

% Define point moment
epL = .1; %how small can epL be?
if (x<= a+epL/2 && x>=a-epL/2)
    w = 1/epL;
else
    w = 0;
end

% -- Define differential function here
fxy = [ y(2)
        y(3)/(E*I)
        y(4)
        -w];
end
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% C30/ME85 Matlab Solver for Beam
%
%  HW12 Problem Template
%
% -- Function to solve Beam problem
%    using MATLAB ODE solver BVP4C
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [sol]=beam_JP

% -- Define geometry
% L = 1000;                   % Question 1 -- Length of the beam
% L = 36;                       % Question 2 -- length of the beam, in in
L = 120; % Question 3 -- length in in.

% -- Set solver parameters
nvar  =  4;                % -- Number of variables
np    = 1001;              % -- Initial Number of points on [0,L]
xp    = linspace(0,L,np);% -- Initial Points at which to satisfy ODE

% -- Set initial solution for the solver
solinit = bvpinit(xp,zeros(1,nvar));
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [res] = beam1d_bc(ya,yb)

% -- Boundary Conditions (BC)
%    u: displacement
%    f: force

% ua = 0;      % -- Fixed       at x=a
% ma = 0;      % -- Zero moment at x=a
% ub = 0;      % -- Fixed       at x=b
% mb = 0;      % -- Zero moment at x=b
% res= [ya(1)-ua; ya(3)-ma;
%       yb(1)-ub; yb(3)-mb];

% % -- Problem 2 cantilever beam
% ua = 0;      % -- Fixed at x=a
% ta = 0;      % -- No rotation at x=a
% mb = 0;      % -- Zero moment at x=b
% Vb = -10e3; % -- -10 kip shear load at x=b
% res= [ya(1)-ua; ya(2)-ta;
%       yb(3)-mb; yb(4)-Vb];

% -- Simply supported beam
ua = 0;      % -- Fixed at x=a
ma = 0;      % -- No rotation at x=a
ub = 0;      % -- Fixed at x=b
mb = 0;      % -- No rotation at x=b
res= [ya(1)-ua; ya(3)-ma;
      yb(1)-ub; yb(3)-mb];

end
```