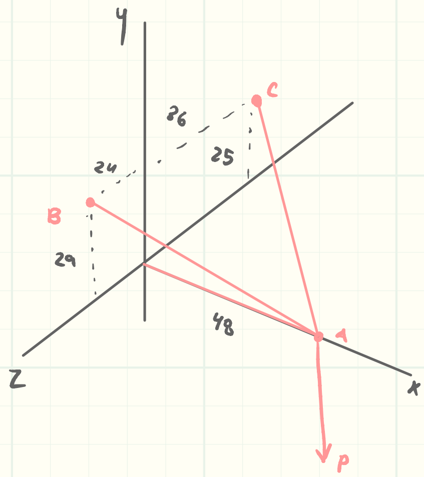
Katie Connors | HW #2 P2.78, P2.83, P2.92, P2.107, P3.16, P3.45, P3.47 | 2/2/24

2.78



$R_y = 0 \quad R_z = 0 \quad R_x = ?$

$T_{AB} = 183 \text{ lb}$

$T_{AC} = ?$

$A = (48, 0, 0)$

$B = (0, 29, 24)$

$C = (0, 25, -36)$

$$T_{AC} + T_{AB} + P = R$$

$$r_{AC} = -48i + 25j - 36k$$

$$r_{AB} = -48i + 29j + 24k$$

$$|r_{AC}| = \sqrt{(-48)^2 + 25^2 + (-36)^2} = 65$$

$$|r_{AB}| = \sqrt{(-48)^2 + 29^2 + 24^2} = 61$$

$$\vec{T}_{AB} = (183) \cdot \frac{-48i + 29j + 24k}{61} = -144i + 87j + 72k$$

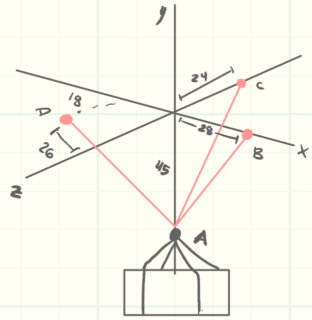
$$\vec{T}_{AC} = T_{AC} \cdot \frac{-48i + 25j - 36k}{65}$$

The z component of $\vec{T}_{AB}$ and $\vec{T}_{AC}$ should cancel out

$$T_{AC} \cdot \frac{36}{65} = 72$$

$$\boxed{T_{AC} = 130 \text{ lb}}$$

2.83



$W = ?$

$T_{AB} = 1378\text{ lb}$

$R_x = 0$

$R_z = 0$

$R_y = W$

$A = (0, -45, 0)$

$B = (28, 0, 0)$

$C = (0, 0, -24)$

$D = (-26, 0, 18)$

$r_{AB} = 28i + 45j + 0k \quad |r_{AB}| = \sqrt{28^2 + 45^2} = 53$

$r_{AC} = 0i + 45j - 24k \quad |r_{AC}| = \sqrt{45^2 + (-24)^2} = 51$

$r_{AD} = -26i + 45j + 18k \quad |r_{AD}| = \sqrt{(-26)^2 + 45^2 + 18^2} = 55$

$$\vec{T}_{AB} = (1378) \cdot \frac{28i + 45j + 0k}{53} = 728i + 1170j + 0k$$

$$\vec{T}_{AC} = T_{AC} \cdot \frac{0i + 45j - 24k}{51}$$

$$\vec{T}_{Ad} = T_{Ad} \cdot \frac{-26i + 45j + 18k}{55}$$

$$T_{AB}\,i + T_{AC}\,i + T_{Ad}\,i = R\,i = R_x = 0$$

$$728 + 0 - \frac{26}{55} T_{Ad} = 0$$

$$T_{AD} = 1540\text{ lb}$$

$$\vec{T}_{AB}\,k + \vec{T}_{AC}\,k + \vec{T}_{AD}\,k = R\,k = R_z = 0$$

$$0 \quad \frac{-24}{51} T_{AC} + \frac{18}{55} T_{AD} = 0$$

$$\frac{24}{51} T_{AC} = \frac{18}{55}(1540)$$

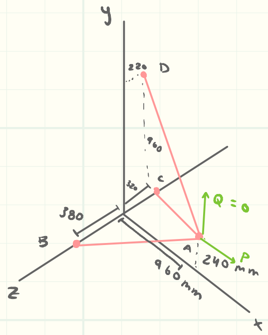
$$T_{AC} = 1071\text{ lb}$$

$$W = T_{AC}\,j + T_{AB}\,j + T_{AD}\,j$$

$$W = 1071\left(\frac{45}{51}\right) + 1378\left(\frac{45}{53}\right) + 1540\left(\frac{45}{55}\right)$$

$$\boxed{W = 3375\text{ lb}}$$

2.92



$Q = 0 \quad P = ? \quad T_{AD} = 305\,N$

$A = (960, 240, 0)$

$B = (0, 0, 380)$

$C = (0, 0, -320)$

$D = (0, 960, -220)$

$r_{AD} = -960i + 720j - 220k \qquad |r_{AD}| = \sqrt{(-960)^2 + 720^2 + (-220)^2} = 1220$

$r_{AC} = -960i - 240j - 320k \qquad |r_{AC}| = \sqrt{(-960)^2 + (-240)^2 + (-320)^2} = 1040$

$r_{AB} = -960i - 240j + 380k \qquad |r_{AB}| = \sqrt{(-960)^2 + (-240)^2 + (380)^2} = 1060$

$$\vec{T}_{AD} = T_{AD}\frac{r_{AD}}{|r_{AD}|} = (305)\frac{-960i + 720j - 220k}{1220}$$

$$\vec{T}_{AC} = T_{AC} \cdot \frac{-960i - 240j - 320k}{1040}$$

$$\vec{T}_{AB} = T_{AB} \cdot \frac{-960i - 240j + 380k}{1060}$$

if $Q = 0$ then $\Sigma F_y = 0$

$$(305)\frac{720}{1220} + T_{AC}\left(-\frac{240}{1060}\right) + T_{AB}\left(-\frac{240}{1060}\right) = 0$$

$$180 = \frac{240}{1040}T_{AC} + \frac{240}{1060}T_{AB}$$

there is no force in the z direction so $\Sigma F_z = 0$

$$(305)\left(\frac{-220}{1220}\right) + T_{AC}\left(\frac{-320}{1040}\right) + T_{AB}\left(\frac{380}{1060}\right) = 0$$

$$55 + \frac{380}{1060}T_{AB} = \frac{320}{1040}T_{AC}$$

solve system of equations

$$T_{AC} = \frac{1040}{320}(55) + \left(\frac{1040}{320}\right)\left(\frac{380}{1060}\right)T_{AB}$$

$$180 = \frac{240}{1040}\left[\frac{1040}{320}(55) + \left(\frac{1040}{320}\right)\left(\frac{380}{1060}\right)T_{AB}\right] + \frac{240}{1060}T_{AB}$$

$$180 = 41.25 + \frac{57}{212}T_{AB} + \frac{12}{53}T_{AB}$$

$$138.75 = .49528\,T_{AB}$$

$$\rightarrow T_{AB} = 280.1429\ N$$

$$T_{AC} = \frac{1040}{320}\left[55 + \frac{380}{1060}(280.1429)\right]$$

$$\rightarrow T_{AC} = 505.1429$$

$$P = -[T_{AD}\,i + T_{AC}\,i + T_{AB}\,i]$$

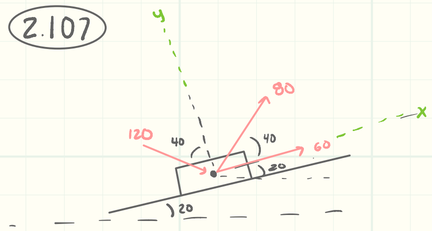
$$P = -\left[(305)\left(\frac{-960}{1220}\right) + 505.14\left(\frac{-960}{1040}\right) + 280.1429\left(\frac{-960}{1060}\right)\right]$$

$$P = 240 + 466.286 + 253.714$$

$$P = 959.999$$

$$\boxed{P = 960.0\,N}$$

(2.107)



using the xy coordinate from the slope

$R_x = 60 + 80\cos 40 + 120\sin 40$

$R_x = 198.418$

$R_y = 80\sin 40 - 120\cos 40$

$R_y = -40.5023$

$R = \sqrt{198.418^2 + (-40.5023)^2}$

$R = 202.5096744$

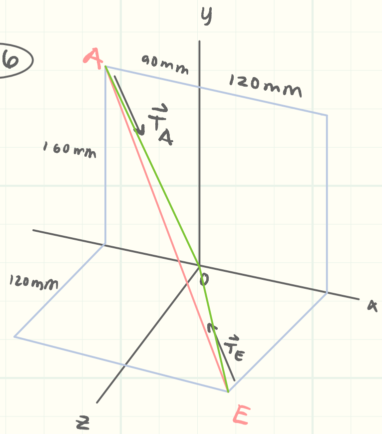
$\tan\theta = \dfrac{-40.5023}{198.418}$

$\theta = -11.537°$

$20 - 11.537 = 8.463°$

$\boxed{R = 202.5 \text{ lb} \quad \text{and} \quad \theta = 8.463° \text{ from horizontal}}$

3.16



$T_{AE} = 435$ N

A = (-90, 160, 0)

E = (120, 0, 120)

(a) M_o ON CORNER A

$r_{AE} = 210\,\hat{x} - 160\,\hat{y} + 120\,\hat{z}$

$|r_{AE}| = \sqrt{210^2 + (-160)^2 + 120^2} = 290$

$\vec{T}_A = (435) \cdot \dfrac{210\hat{x} - 160\hat{y} + 120\hat{z}}{290}$

$\vec{T}_A = 315\hat{x} - 240\hat{y} + 180\hat{z}$

→ find moment

$M_o = \vec{r}_{OA} \times \vec{T}_A$

$\vec{r}_{OA} = -90\hat{x} + 160\hat{y} + 0\hat{z}$

$M_o = (-90\hat{x} + 160\hat{y} + 0\hat{z}) \times (315\hat{x} - 240\hat{y} + 180\hat{z})$

$M_o = (160 \cdot 180 - (-240)\cdot(0))\hat{x} + (0 \cdot 315 - (-90)\cdot 180)\hat{y}$
$+ ((-90)\cdot(-240) - 315 \cdot 160)\hat{z}$

mm → $M_o = 28800\,\hat{x} + 16200\,\hat{y} - 28800\,\hat{z}$

m → $\boxed{M_o = (28.8\,N\cdot m)i + (16.2\,N\cdot m)j - (28.8\,N\cdot m)k}$

(b) M_o ON CORNER E

$r_{EA} = -210\,i + 160\,j - 120\,k$

$|r_{EA}| = \sqrt{(-210)^2 + 160^2 + (-120)^2} = 290$

$\vec{T}_E = (435) \cdot \dfrac{-210i + 160j - 120k}{290}$

$\vec{T}_E = -315i + 240j - 180k$

→ find moment

$M_o = \vec{r}_{OE} \times \vec{T}_E$

$\vec{r}_{OE} = 120\,i + 0\,j + 120\,k$

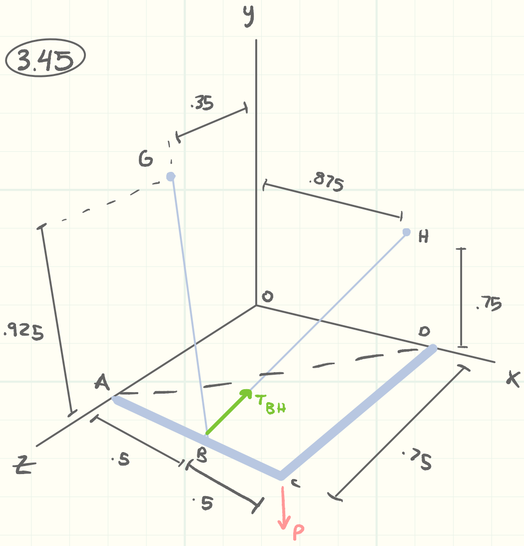
$M_o = (120i + 0j + 120k) \times (-315i + 240j - 180k)$

$M_o = (0(-180) - 120\cdot 240)i + (120\cdot(-315) - 120(-180))j +$
$(120\cdot 240 - 0\cdot(-315))k$

mm → $M_o = -28800\,i - 16200\,j + 28800\,k$

m → $\boxed{M_o = -(28.8\,N\cdot m)i - (16.2\,N\cdot m)j + (28.8\,N\cdot m)k}$

3.45



$T_{GB}, T_{BH} = 450N$

M_{AD} by the portion BH of the cable

$A = (0, 0, .75)$
$B = (.5, 0, .75)$
$C = (1, 0, .75)$
$D = (1, 0, 0)$
$G = (0, .925, .35)$
$H = (.875, .75, 0)$

$$M_{AD} = \lambda_{AD} \cdot (\vec{r}_{AB} \times \vec{T}_{BH})$$

$\rightarrow r_{AB} = .5i + 0j + 0k$

$r_{BH} = (.875 - .5)i + .75j - .75k$

$r_{BH} = .375i + .75j - .75k$

$|r_{BH}| = \sqrt{.375^2 + .75^2 + (-.75)^2} = 1.125$

$$\vec{T}_{BH} = T_{BH} \cdot \frac{r_{BH}}{|r_{BH}|}$$

$$\vec{T}_{BH} = (450) \cdot \frac{.375}{1.125} i + \frac{.75}{1.125} j - \frac{.75}{1.125} k$$

$\rightarrow \vec{T}_{BH} = 150i + 300j - 300k$

$r_{AD} = 1i + 0j - .75k$

$|r_{AD}| = \sqrt{1^2 + 0^2 + (-.75)^2} = 1.25$

$$\lambda_{AD} = \frac{r_{AD}}{|r_{AD}|} = \frac{1}{1.25} i + \frac{0}{1.25} j - \frac{.75}{1.25} k$$

$\rightarrow \lambda_{AD} = .8i + 0j - .6k$

$$r_{AB} \times \vec{T}_{BH} = \begin{vmatrix} i & j & k \\ .5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix} = 0i + 150j + 150k$$

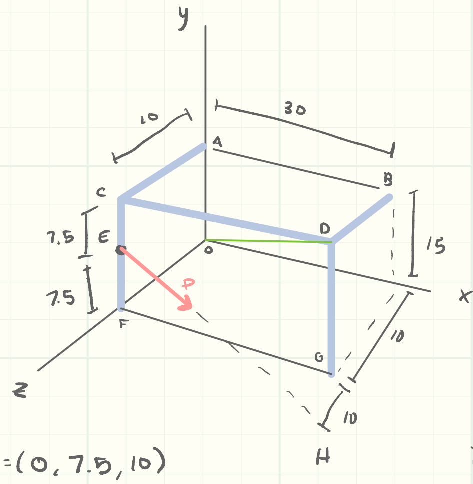
$M_{AD} = \lambda_{AD} \cdot (r_{AB} \times \vec{T}_{BH})$

$M_{AD} = (.8i + 0j - .6k) \cdot (0i + 150j + 150k)$

$M_{AD} = (.8)(0) + 0(150) + (-.6)(150) = -90$

$$\boxed{M_{AD} = -90\ N \cdot m}$$

3.47



$E = (0, 7.5, 10)$
$O = (0, 0, 0)$
$D = (30, 15, 10)$
$H = (30, 0, 20)$

$$r_{EH} = 30i - 7.5j + 10k$$

$$|r_{EH}| = \sqrt{30^2 + (-7.5)^2 + 10^2} = 32.5$$

$$\vec{T}_{PH} = (520)\left(\frac{30}{32.5}i - \frac{7.5}{32.5}j + \frac{10}{32.5}k\right)$$

$$\vec{T}_{PH} = 480i - 120j + 160k$$

$$\vec{r}_{OE} = 0i + 7.5j + 10k$$

$$\vec{r}_{OE} \times \vec{T}_{PH} = \begin{vmatrix} i & j & k \\ 0 & 7.5 & 10 \\ 480 & -120 & 160 \end{vmatrix}$$

$$\vec{r}_{OE} \times \vec{T}_{PH} = [(7.5)(160) - (10)(-120)]i + [(10)(480) - (0)(160)]j + [(0)(-120) - (7.5)(480)]k$$

$$\vec{r}_{OE} \times \vec{T}_{PH} = 2400i + 4800j - 3600k$$

$$M_{OD} = \lambda_{OD} \cdot (\vec{r}_{OE} \times \vec{T}_{PH})$$

$$r_{OD} = 30i + 15j + 10k$$

$$|r_{OD}| = \sqrt{30^2 + 15^2 + 10^2} = 35$$

$$\lambda_{OD} = \frac{r_{OD}}{|r_{OD}|} = \frac{30}{35}i + \frac{15}{35}j + \frac{10}{35}k$$

$$M_{OD} = \left(\frac{30}{35}i + \frac{15}{35}j + \frac{10}{35}k\right) \cdot (2400i + 4800j - 3600k)$$

$$M_{OD} = 2057.14 + 2057.14 - 1028.57$$

$$M_{OD} = 3085.714285$$

$$\boxed{M_{OD} = 3085.71 \text{ in} \cdot \text{lbs}}$$