Lecture 5
Equilibrium of Rigid Bodies (2D)

Equilibrium Equations for Rigid Bodies

The necessary and sufficient condition for the static equilibrium of a body is that the resultant force and couple from all external forces form a system equivalent to zero,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

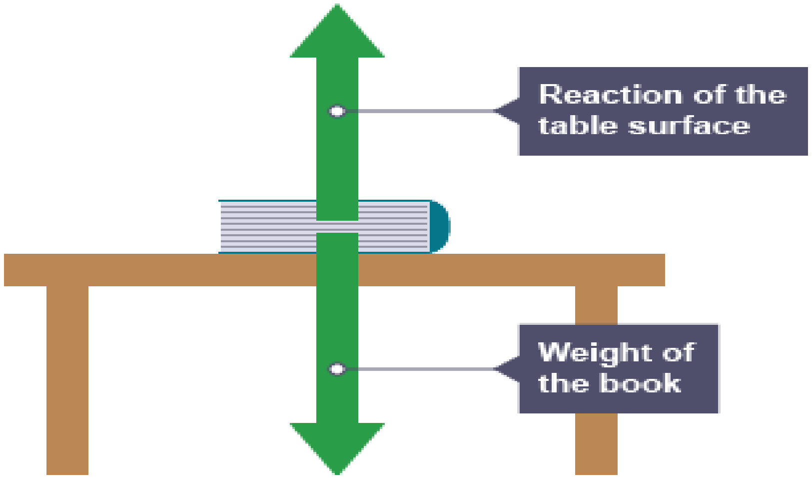
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

This is an engineering class!

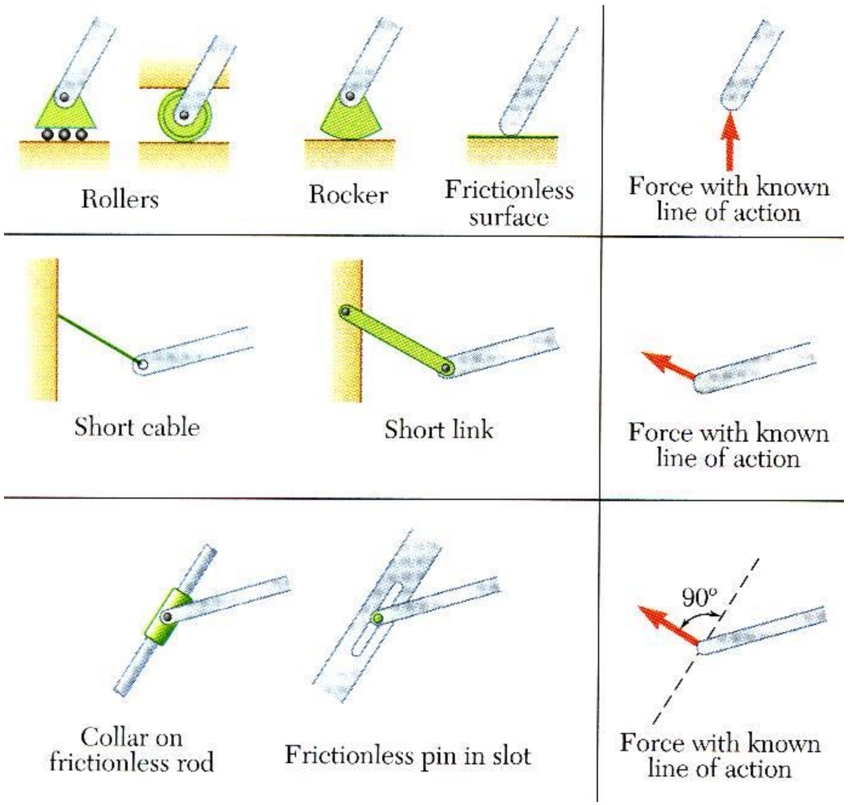
$$\sum_{i=1}^{n} F_{xi} = 0, \quad \sum_{i=1}^{n} F_{yi} = 0 \quad \text{and} \quad \sum_{j=1}^{m} M_{zj}^{O} = 0$$

What is REACTION (force)?

Reaction is a type of **passive external forces**, and it usually arises due to the enforcement of a **displacement constraint.**



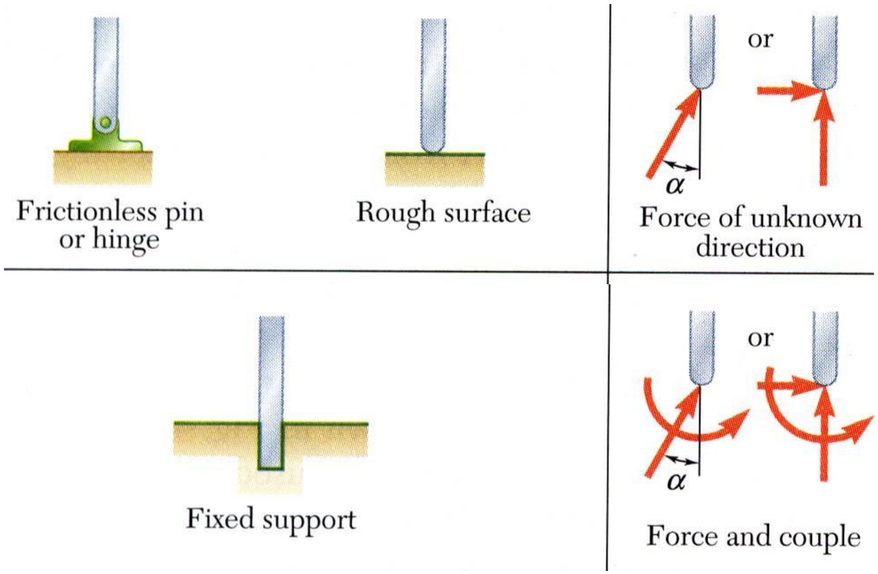
Reactions at Supports and Connections for a 2D Structure



- Reactions equivalent to a force with known line of action.

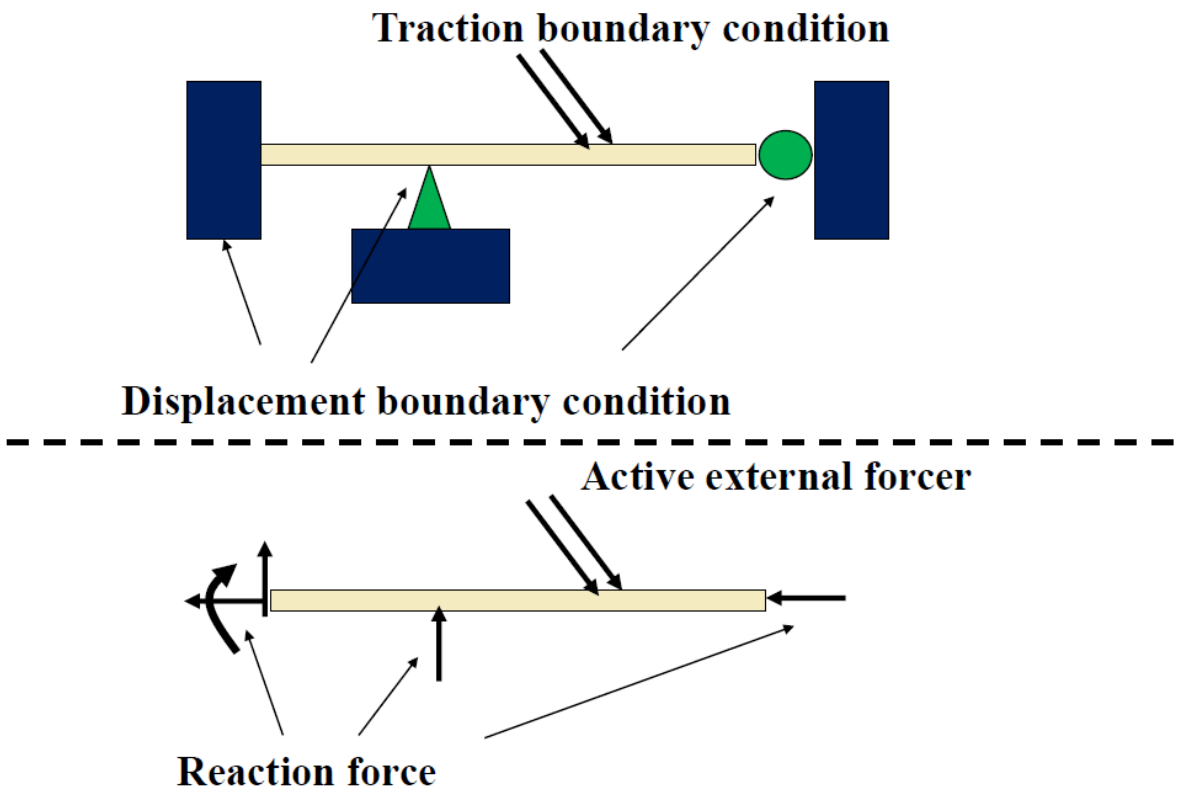
Technical sysmbols for displacement constraints. They specify directions of the reactions,

Reactions at Supports and Connections for a 2D Structure

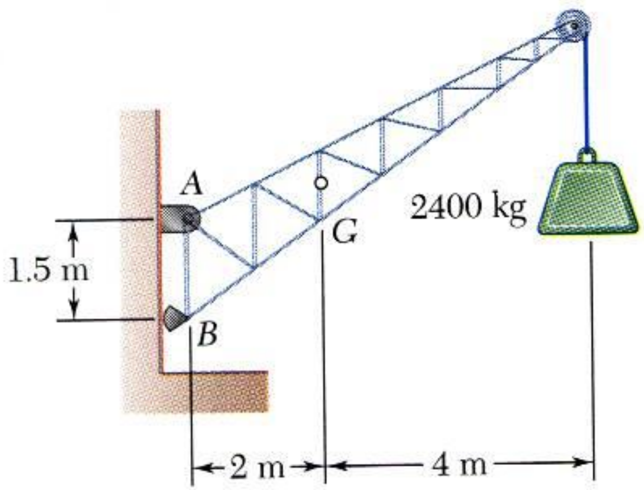


- Reactions equivalent to a force of unknown direction and magnitude.

- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude



Free-body Diagram



A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the reactions at *A* and *B*.

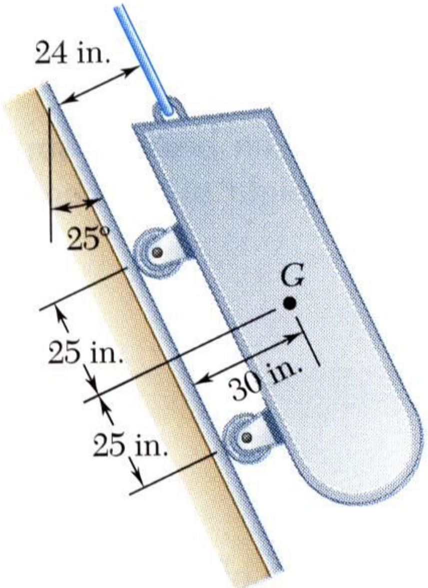
SOLUTION:

- Create a free-body diagram for the crane.

- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.

- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.

Sample Problem 4.3



A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at *G*. The cart is held in position by the cable.

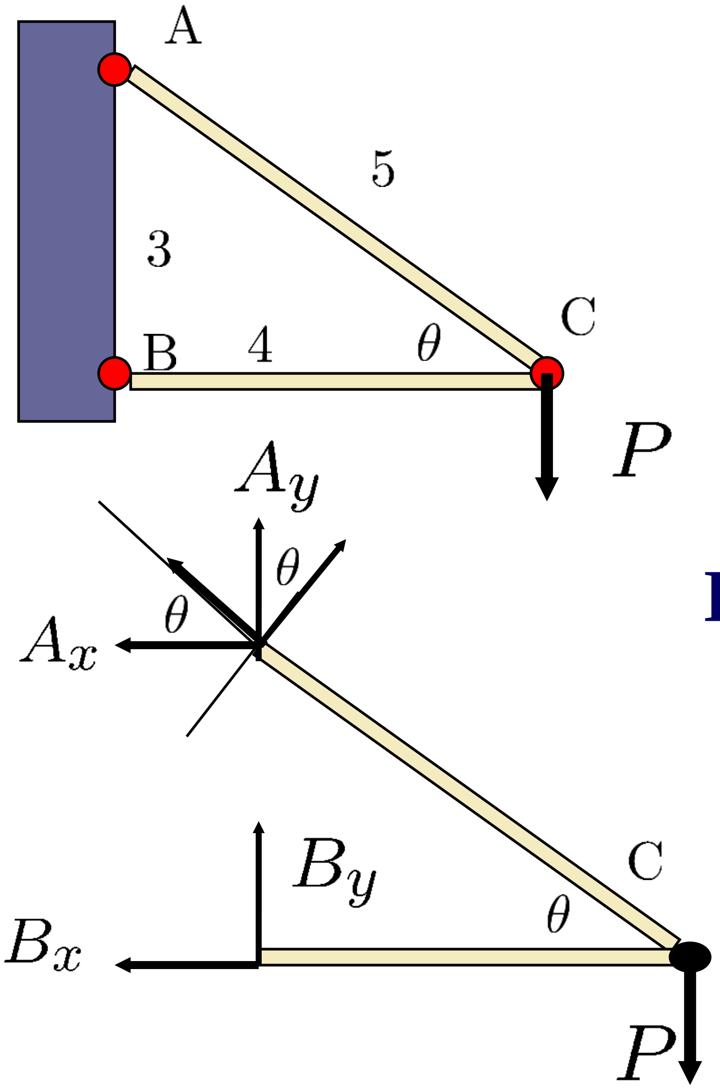
Determine the tension in the cable and the reaction at each pair of wheels.

SOLUTION:

- Create a free-body diagram for the car with the coordinate system aligned with the track.

- Determine the reactions at the wheels by solving equations for the sum of moments about points above each axle.

- Determine the cable tension by solving the equation for the sum of force components parallel to the track.

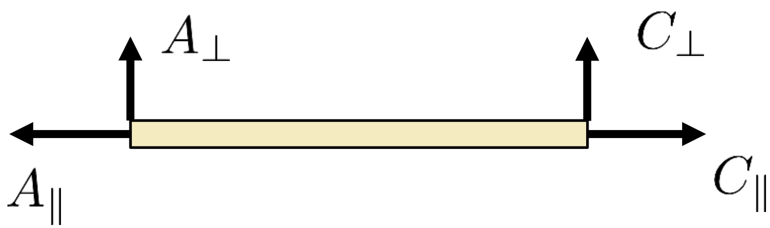


How to draw free-body diagram ?

resultant forces

For both rods, there are only two forces acting on each rod, and these two forces have the same magnitude, the same line of action with opposite direction.

Is this a coincident ? **No ! This is a general principle !**



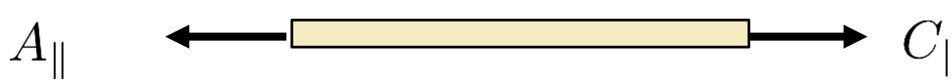
One can always resolve forces components at the point A into components $A_\perp$ and $A_\parallel$.

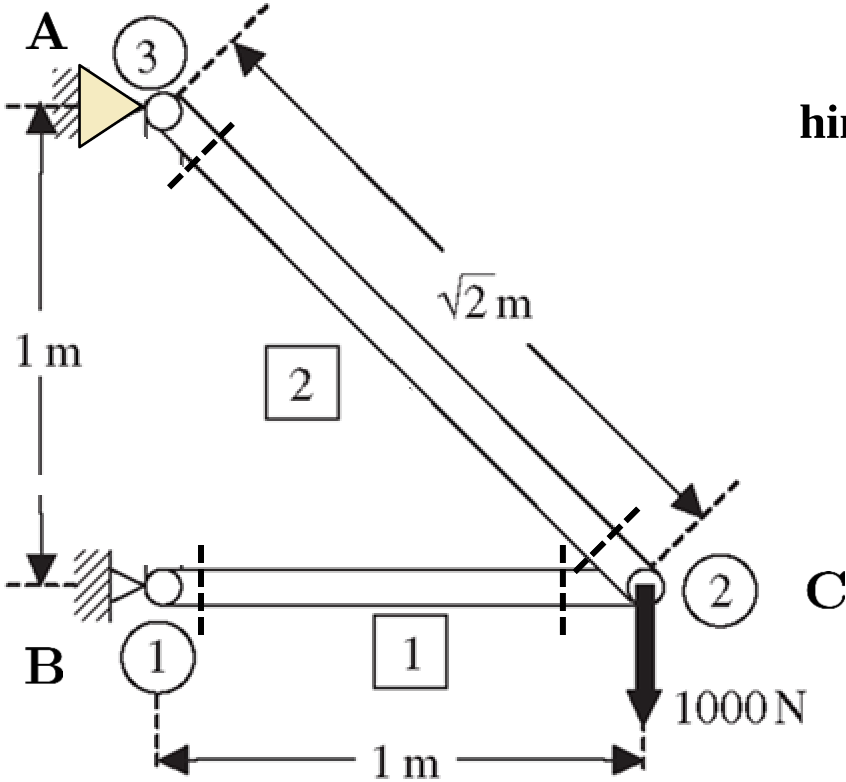
$$\text{Then} \quad \sum M_c = 0 \;\; \rightarrow \;\; -5A_\perp = 0 \;\rightarrow\; A_\perp = 0$$

The same is true that

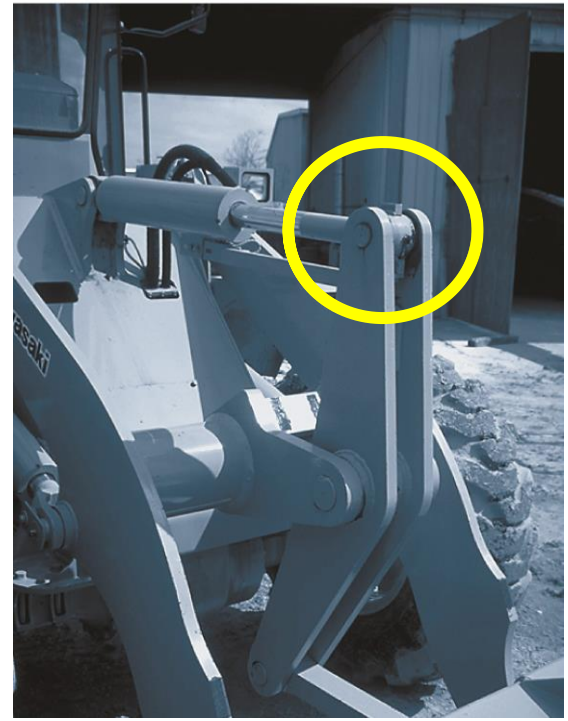
$$\sum M_A = 0 \;\; \rightarrow \;\; -5C_\perp = 0 \;\rightarrow\; C_\perp = 0 \; .$$

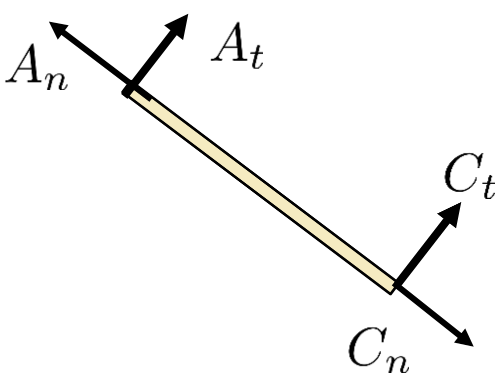
Therefore,

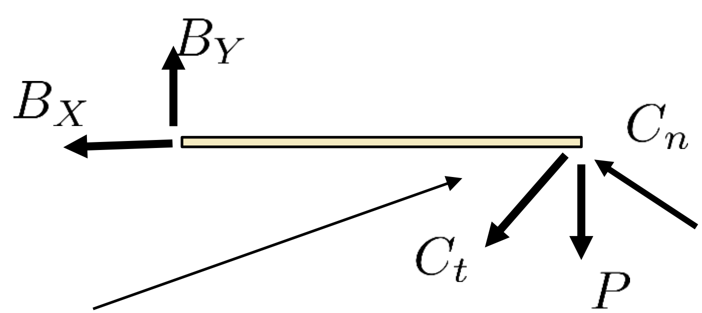




hinge





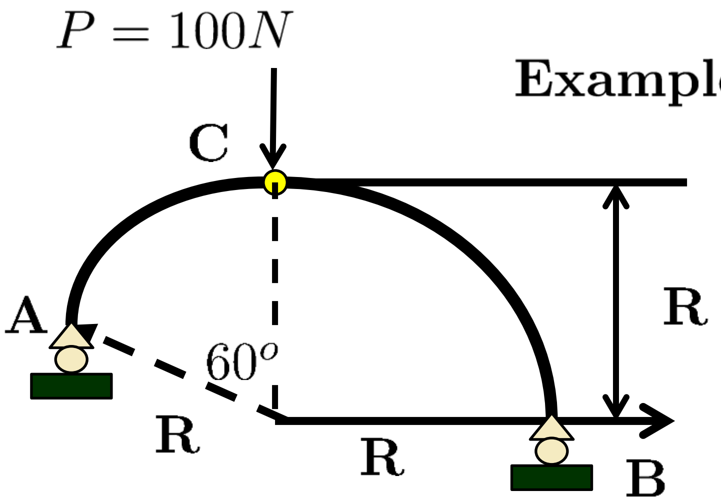


C_n **and** C_t **are internal forces !**

Definition:
A two-force member is a structural element that is only acted upon by two forces.

Two force member principle:
If two forces act on a body in equilibrium, they must be equal in magnitude, co-linear and opposite in sense. The direction of the forces must be along the line that connects the two points where the forces act upon.

Today's Lecture Attendance Password is: Two-force member

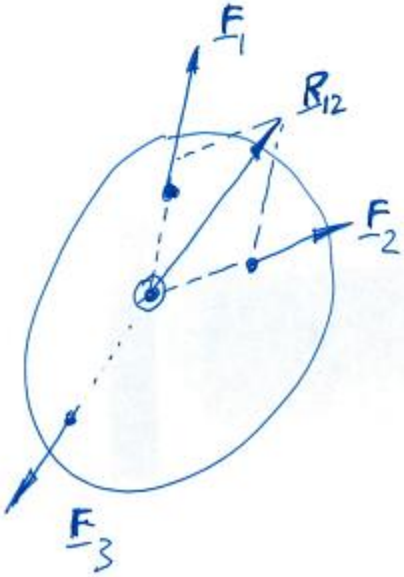


Example : Find the reactions at points A and B ?

Both arches AC and BC are two-force members, so the free-body diagram is as follows:

Three-force member

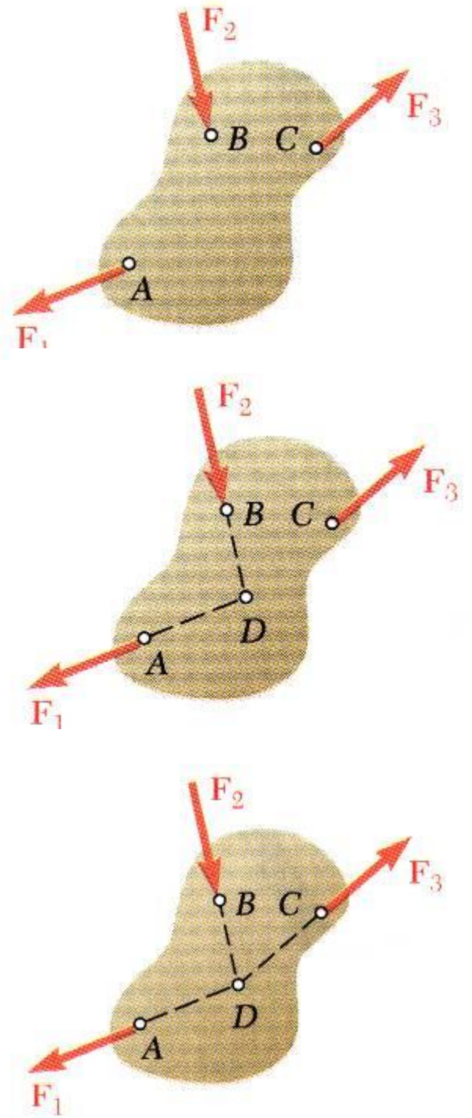
For three non-parallel forces acting on a rigid body, the line of action of the three forces must interect at on common point.



Why ?

because the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$, $\mathbf{R}_{12}$, will form a two-force system with the remaining force $\mathbf{F}_3$.

Equilibrium of a Three-Force Body



- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of ***F**$_1$* and ***F**$_2$* about the point of intersection represented by *D* is zero.
- Since the rigid body is in equilibrium, the sum of the moments of ***F**$_1$*, ***F**$_2$*, and ***F**$_3$* about any axis must be zero. It follows that the moment of ***F**$_3$* about *D* must be zero as well and that the line of action of ***F**$_3$* must pass through *D*.
- The lines of action of the three forces must be **concurrent** or parallel.

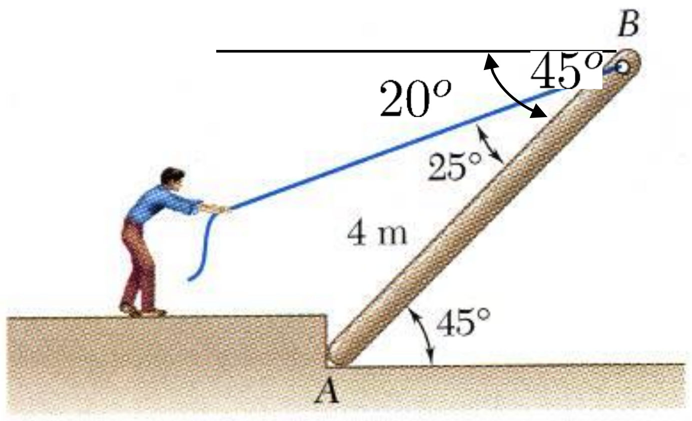
Definition:

If three **non-parallel** forces act on a rigid body in equilibrium, it is known as a three-force member.

Three-force member principle

If a three-force member is in equilibrium, the line of action of all three forces must intersect at a common point; and the total resultant is zero. In other words, any single force is the equilibrant of the two other forces.

Sample Problem 4.6



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at *A*.

SOLUTION:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Main Takeaways

1. Start solving statics problems by drawing the freebody diagram;

2. Identify boundary support conditions;

3. Inspect for two-force body or three-force body, and

4. Judiciously select the point for moment equations.