CE30 – Discussion 10

Beam Bending

Textbook: 11.1, 11.2, 12.1, 12.2

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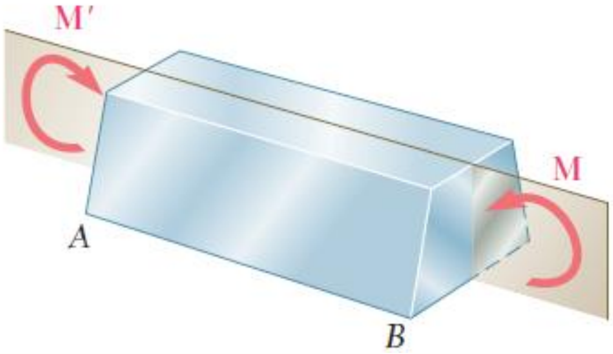
Announcements

- HW10 Problems from the textbook:

  11.3, 11.13, 12.13, 12.16, 12.27, 12.53

Pure Bending

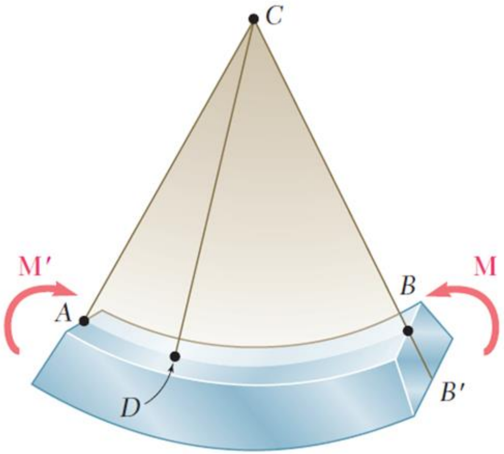
- Members subjected to an equal and opposite couple moment

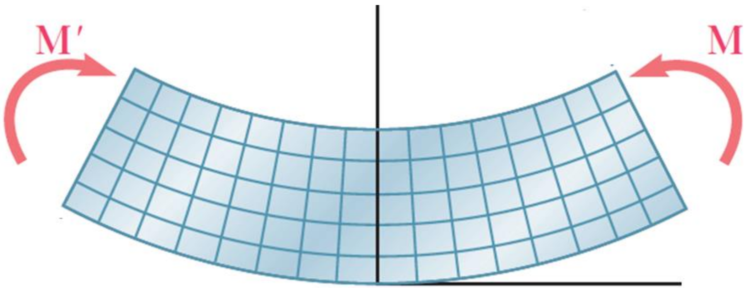


- Analysis of symmetric beams
- Beam: A structural member that carries lateral loads

Symmetric Beam in Pure Bending

- End couple will bend the beam
- We want to analyze the resulting deformations
- Key assumption: **Transverse sections remain plane**



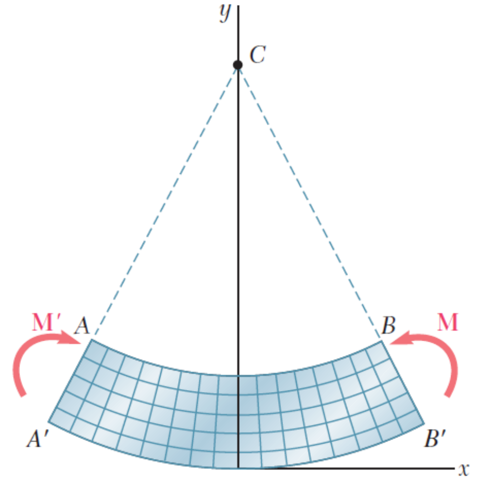


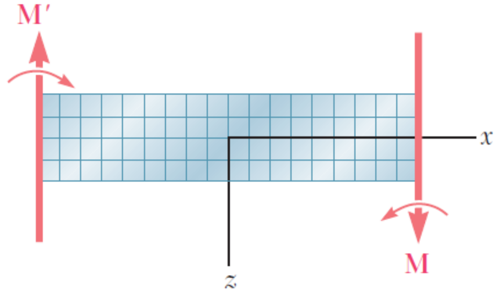
Elastic Beam theory: Euler-Bernoulli Beam

Deformations in Pure Bending

- Due to the symmetry and the plane sections remain plane assumption, most stress and strain components are zero.

- **Only non-zero stress component is σ_x**
- **Non-zero strain component ϵ_x**
- Uniaxial stress state





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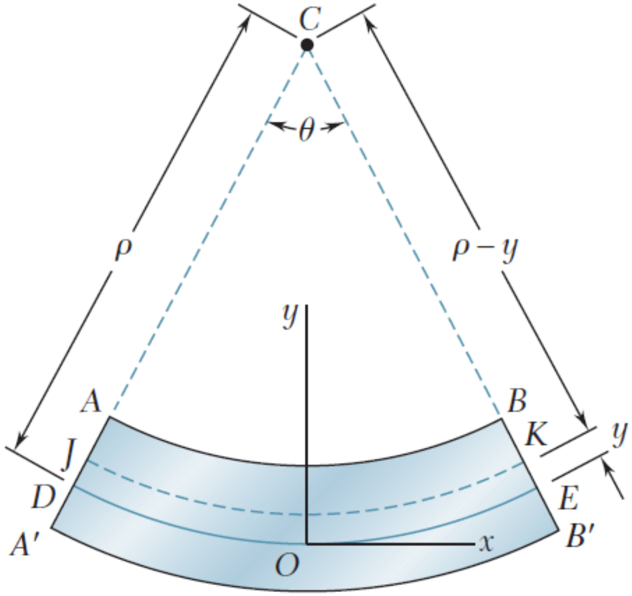
Deformations in Pure Bending



<u>**Line (AB) gets shorter**</u>

- Upper portion of the member is in compression
- $\sigma_x < 0$ and $\epsilon_x < 0$

<u>**Line (A′B′) gets longer**</u>

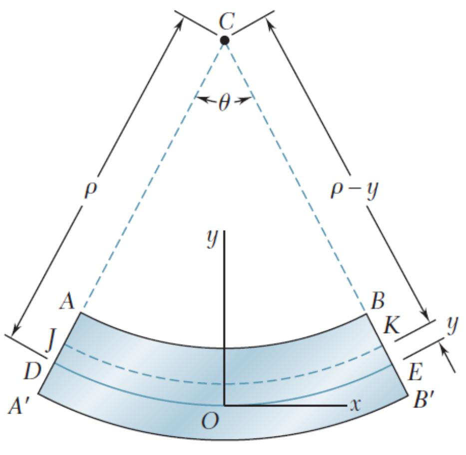
- Lower portion of the member is in tension
- $\sigma_x > 0$ and $\epsilon_x > 0$

There exists an axis in between where there is zero longitudinal strain/stress (line DE)

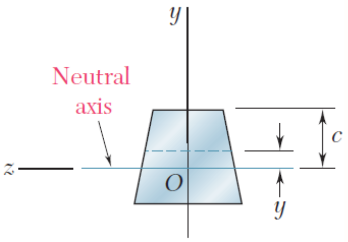
This is axis is called **neutral axis**

Neutral axis coincides with the centroid of the section

Deformations in Pure Bending



(*a*) Longitudinal, vertical section (plane of symmetry)



(*b*) Transverse section

- Constant curvature along the beam
- ρ : Radius of curvature
- From the geometry, we obtain:

$$\epsilon_x = -\frac{y}{\rho}$$

Longitudinal strain (ϵ_x) varies linearly with distance from the neutral axis (y)

Deformations in Pure Bending

- Strain varies linearly from the neutral axis

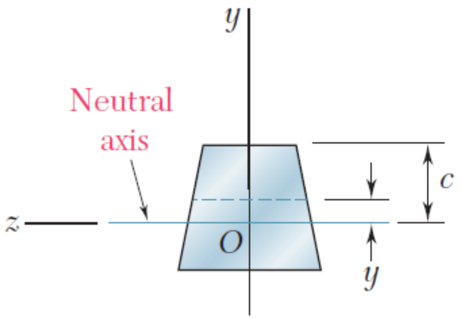
$$\epsilon_x = -\frac{y}{\rho}$$

- Maximum strains happen at the surfaces

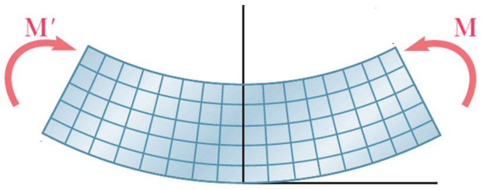
$$\epsilon_m = -\frac{c}{\rho}$$

- Negative sign indicates top surface compression

  (Standard sign convention in bending)



Cross-sectional view



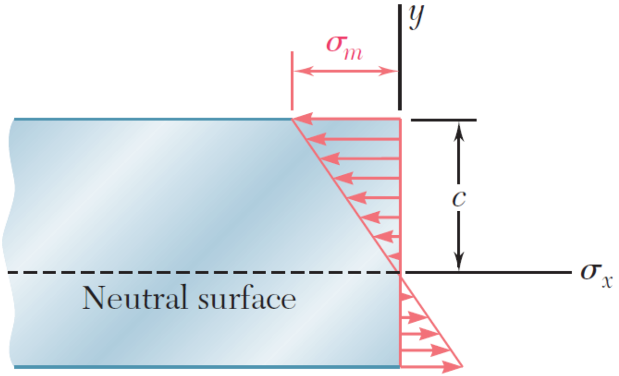
Positive sign convention

Stress in Pure Bending

- From the Hooke's law: $\sigma_x = E\,\epsilon_x$
- Using the strain relations, we get

$$\boldsymbol{\sigma_x = -\frac{y}{\rho}\sigma_m}$$

Stress (σ_x) varies linearly with distance from the neutral axis (y)

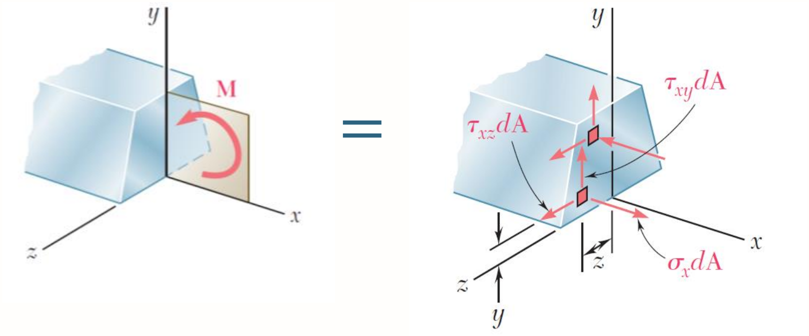


Negative stress (compressive) above

Positive stress (tensile) below

Stress in Pure Bending

- Static equilibrium in cross section:



1. Force equilibrium in x-direction

$$\int \sigma_x \, dA = 0$$

2. Moment equilibrium around z-axis

$$\int (-y\sigma_x \, dA) = M$$

- Equation 1 results in (neutral axis = centroid)
- Equation 2 gives relation between stress and applied moment

Stress in Pure Bending

- Moment-Stress relations (flexural formulas):

$$\boxed{\boldsymbol{\sigma_m = \frac{Mc}{I} \qquad\qquad \sigma_x = -\frac{My}{I}}}$$

- The stress σ_x is also known as the ***flexural stress***
- The ratio (I/c) is called the section modulus (S)

Summary: Beam Bending Formulas

Maximum Strain

$$\epsilon_m = \frac{c}{\rho}$$

Strain

$$\epsilon_x = -\frac{y}{\rho} = -\frac{y}{c}\epsilon_m$$

Maximum Stress

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

Stress

$$\sigma_x = -\frac{My}{I}$$

Curvature

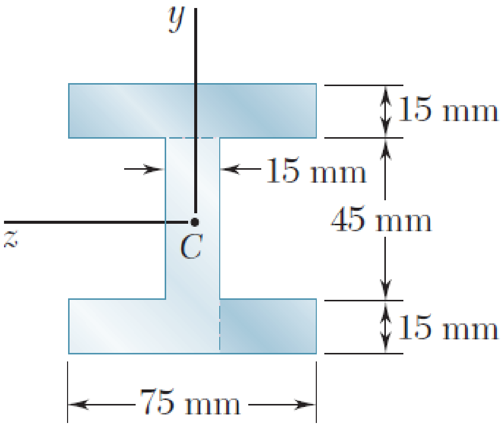
$$\frac{1}{\rho} = \frac{M}{EI}$$

Practice – Similar to HW P11.13

Beam is bent about the horizontal axis with a bending moment 4 kN.m, determine

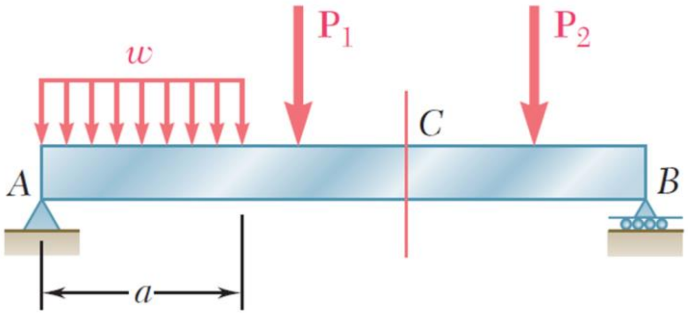
a) Total force acting on the top flange

b) Total force acting on the shaded portion of the lower flange



Analysis of Beams

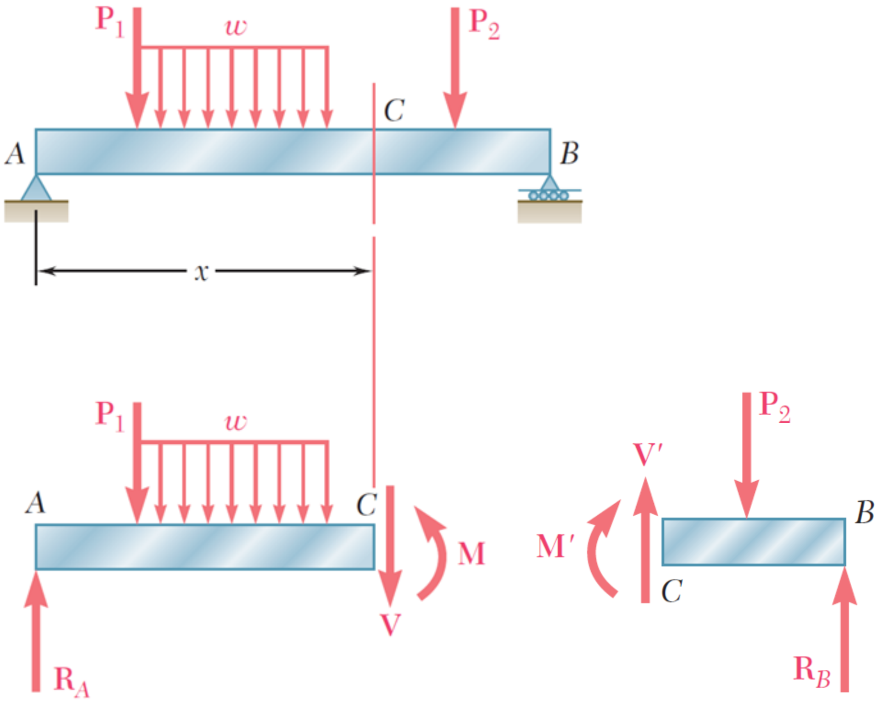
- Beams are usually subjected to a combination of point/distributed loads



- First step in the stress analysis is to draw **shear-moment diagrams**

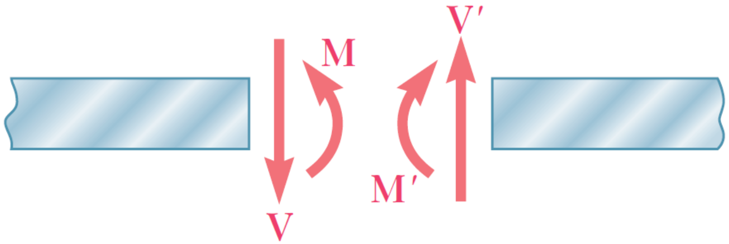
Shear and Moment Diagrams

- Find the internal shear force (V) and bending moment (M) across the length

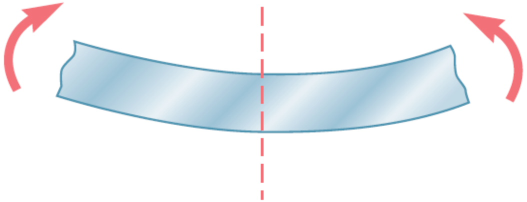


Shear and Moment Diagrams

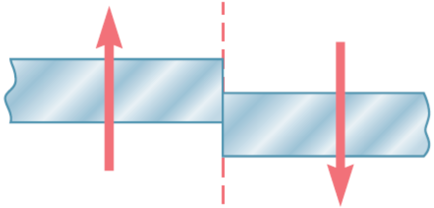
Positive sign convention for internal shear and moment



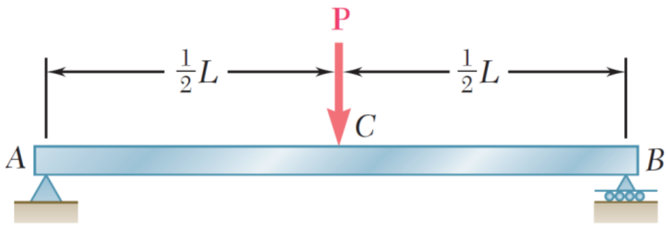
Effect of positive moment

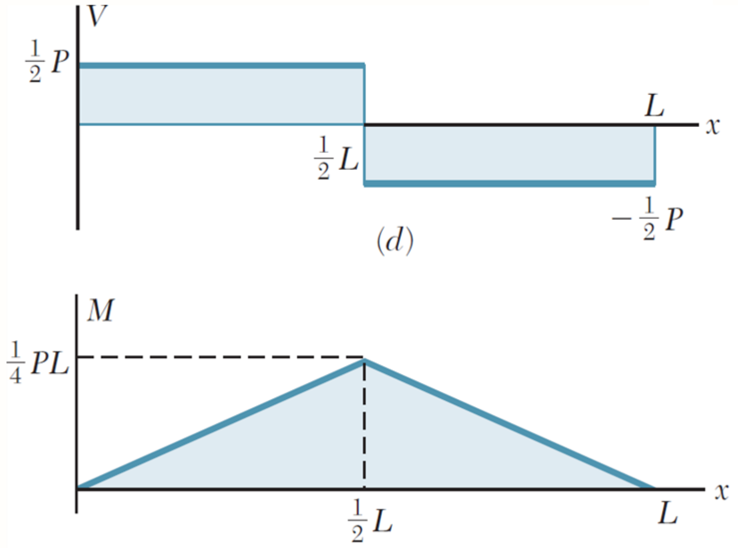


Effect of positive shear



Shear and Moment Diagrams: Example





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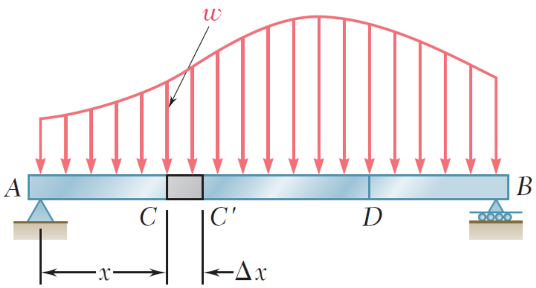
Relations between load, shear and moment



Load-Shear $\dfrac{d\boldsymbol{V}}{d\boldsymbol{x}} = -\boldsymbol{w}$ $V_D - V_C = -\int_C^D w = -$(area under load curve between C and D)

Shear-Moment $\dfrac{d\boldsymbol{M}}{d\boldsymbol{x}} = \boldsymbol{V}$ $M_D - M_C = \int_C^D V = -$(area under shear curve between C and D)

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Practice – Similar to HW P12.53

Draw the shear-moment diagram, find the maximum normal stress due to bending