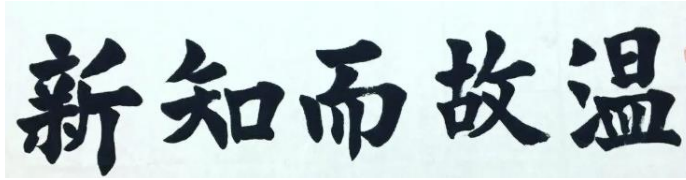
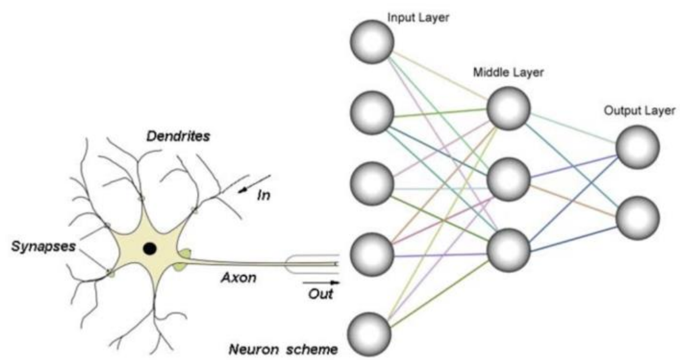
Lecture 2
Review Statics of Particles



To gain new knowledge by reviewing the old first

------Confucius



What is Mechanics?

- Mechanics is the study of body motions (deformation) under the action of forces.

- Categories of Mechanics:
  - Rigid bodies
    - *Statics*
    - Dynamics
  - Deformable solid bodies
  - Fluids
  - Soft matter

It is a STEM class

That is my research area.

- Engineering Mechanics is an applied science - it is not an abstract or pure science but does not have the empiricism found in other engineering sciences.

- Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite for many other subjects.

Fundamental Concepts

- ***Force*** **-** In physics, a **force** is any influence that causes a free body to undergo a change in velocity, including a change in its direction, or a change in shape (and size) **under equilibrium.**
- Force can also be described by intuitive concepts such as a push or pull that can cause an object with mass to change its velocity (which includes to begin moving from a state of rest), i.e., to accelerate, or which can cause a flexible object to deform.deform. A force is characterized by its point of application, magnitude, and direction, i.e., a force is a **vector** quantity.

In Newtonian Mechanics, space, time, and mass are absolute concepts, independent of each other. Force, however, is not independent of the other three. The force acting on a body is related to the mass of the body and the variation of its velocity with time.

Systems of Units

- ***Kinetic Units*****:** length, time, mass, and force.

- The first three of the kinetic units, referred to as *basic units*, may be defined arbitrarily. The fourth unit, referred to as a *derived unit*, must have a definition compatible with Newton's 2nd Law,

$$\vec{F} = m\vec{a}$$

- ***International System of Units*** **(SI):** The basic units are length, time, and mass which are arbitrarily defined as the meter (m), second (s), and kilogram (kg). Force is the derived unit,

$$F = ma$$

$$1\,\mathrm{N} = (1\,\mathrm{kg})\left(1\frac{\mathrm{m}}{\mathrm{s}^2}\right)$$

- ***U.S. Customary Units*****:** The basic units are length, time, and force which are arbitrarily defined as the foot (ft), second (s), and pound (lb). Mass is the derived unit,

$$m = \frac{F}{a}$$

$$1\,\mathrm{slug} = \frac{1\,\mathrm{lb}}{1\,\mathrm{ft}/\mathrm{s}}$$

Table 1.2 Principal SI Units Used in Mechanics

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	. . .	m/s^2
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	. . .	rad/s^2
Angular velocity	Radian per second	. . .	rad/s
Area	Square meter	. . .	m^2
Density	Kilogram per cubic meter	. . .	kg/m^3
Energy	Joule	J	N·m
Force	Newton	N	$kg{\cdot}m/s^2$
Frequency	Hertz	Hz	s^{-1}
Impulse	Newton-second	. . .	kg·m/s
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	. . .	N·m
Power	Watt	W	J/s
Pressure	Pascal	Pa	N/m^2
Stress	Pascal	Pa	N/m^2
Time	Second	s	‡
Velocity	Meter per second	. . .	m/s
Volume			
Solids	Cubic meter	. . .	m^3
Liquids	Liter	L	$10^{-3}\ m^3$
Work	Joule	J	N·m

†Supplementary unit (1 revolution = 2π rad = 360°).

‡Base unit.

Table 1.3 U.S. Customary Units and Their SI Equivalents

Quantity	U.S. Customary Unit	SI Equivalent
Acceleration	ft/s^2	$0.3048\ m/s^2$
	$in./s^2$	$0.0254\ m/s^2$
Area	ft^2	$0.0929\ m^2$
	in^2	$645.2\ mm^2$
Energy	ft·lb	1.356 J
Force	kip	4.448 kN
	lb	4.448 N
	oz	0.2780 N
Impulse	lb·s	4.448 N·s
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	oz mass	28.35 g
	lb mass	0.4536 kg
	slug	14.59 kg
	ton	907.2 kg
Moment of a force	lb·ft	1.356 N·m
	lb·in.	0.1130 N·m
Moment of inertia		
Of an area	in^4	$0.4162 \times 10^6\ mm^4$
Of a mass	$lb \cdot ft \cdot s^2$	$1.356\ kg \cdot m^2$
Momentum	lb·s	4.448 kg·m/s
Power	ft·lb/s	1.356 W
	hp	745.7 W
Pressure or stress	lb/ft^2	47.88 Pa
	lb/in^2 (psi)	6.895 kPa
Velocity	ft/s	0.3048 m/s
	in./s	0.0254 m/s
	mi/h (mph)	0.4470 m/s
	mi/h (mph)	1.609 km/h
Volume	ft^3	$0.02832\ m^3$
	in^3	$16.39\ cm^3$
Liquids	gal	3.785 L
	qt	0.9464 L
Work	ft·lb	1.356 J

Numerical Accuracy

- The accuracy of a solution depends on 1) accuracy of the given data, and 2) the accuracy of the computations performed. The solution cannot be more accurate than the less accurate of these two.

- As a general rule for engineering problems, the data are seldom known with an accuracy greater than 0.2%. Therefore, it is usually appropriate to record parameters beginning with "1" with four digits and with three digits in all other cases, i.e., 40.2 lb and 15.58 lb.

$\text{The exact number is:} 99.99$

$$99.99 \rightarrow \frac{0.09}{99.9} = 0.0009009 \approx 0.1\% < 0.2\%$$

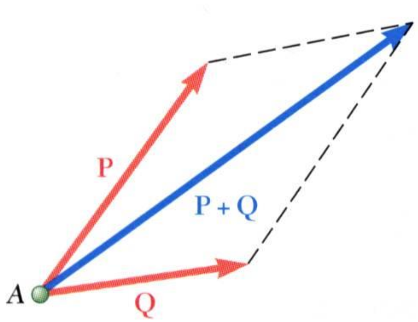
$\text{The exact number is :} \quad 10.09$

$$10.09 \quad \rightarrow \quad \frac{0.09}{10.0} = 0.009 > 0.2\%$$

$$10.009 \quad \rightarrow \quad \frac{0.00}{10.09} = 0.00 < 0.2\%$$

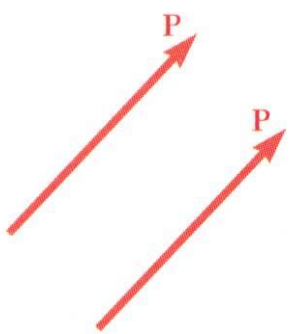
Last Time

Vectors

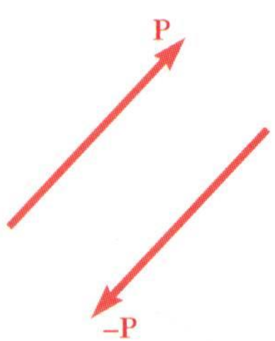


- *Vector*: parameters possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.

- *Scalar*: parameters possessing magnitude but not direction. Examples: mass, volume, temperature



- Vector classifications:
  - ***Fixed or bound vectors*** have well defined points of application that cannot be changed without affecting an analysis.
  - ***Free*** vectors may be freely moved in space without changing their effect on an analysis.
  - ***Sliding*** vectors may be applied anywhere along their line of action without affecting an analysis.

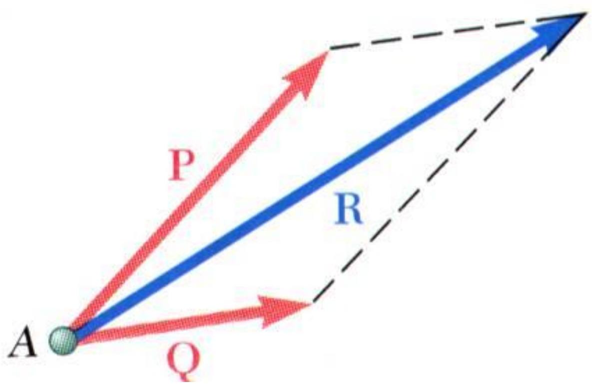


- ***Equal vectors*** have the same magnitude and direction.

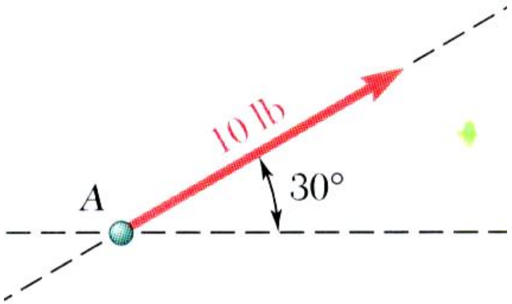
- ***Negative*** vector of a given vector has the same magnitude and the opposite direction.

Resultant of Two Forces

Force is a special vector



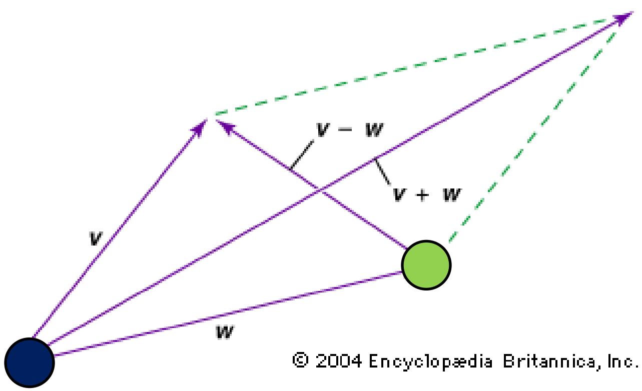
- **force**: action of one body on another; characterized by its ***point of application***, ***magnitude***, ***line of action***, **and *sense***.



Parallelogram Method

- The sum of the two forces may be represented by a single ***resultant* force**.

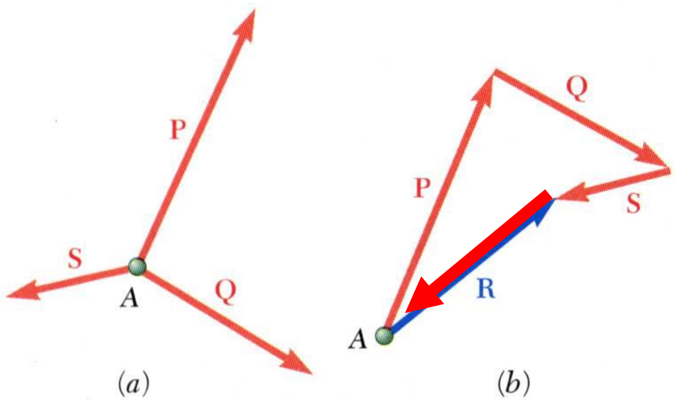
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.



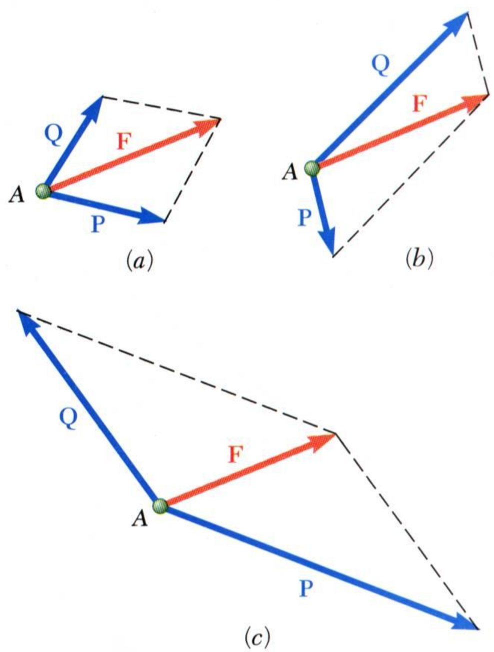
Today's Lecture Attendance Password is:

Resultant

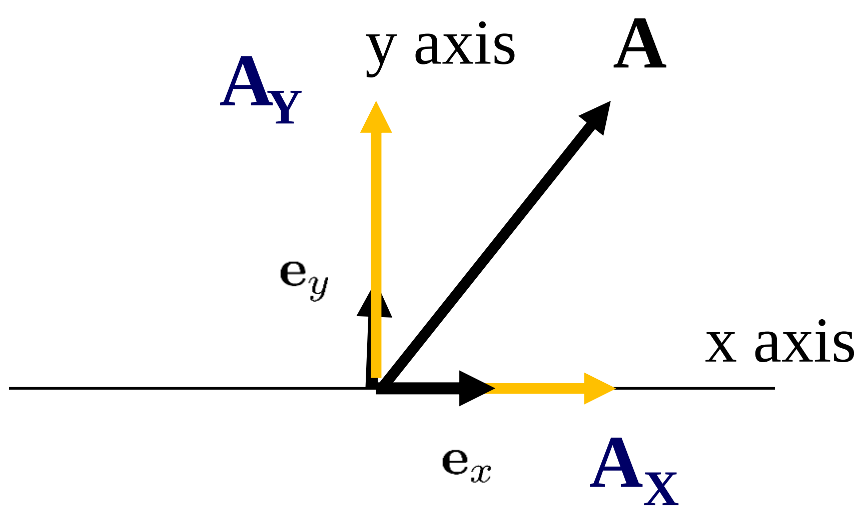
Resultant of Several Concurrent Forces



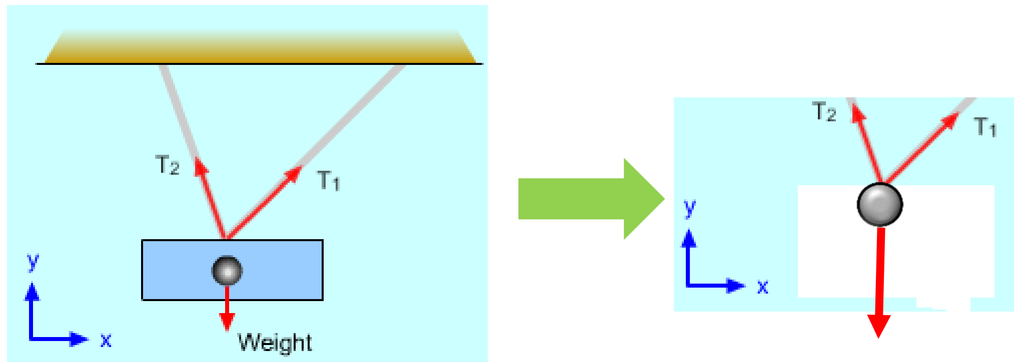
- *Concurrent forces*: set of forces which all pass through the same point.
  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.



- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.



Particle Statics



- The objective for the section is to investigate the effects of forces on particles:
  - replacing multiple forces acting on a particle with a single equivalent or ***resultant*** force,

$$\mathbf{F} = \sum_{i=1}^{n} \mathbf{F}_i$$

  - relations between forces acting on a particle that is in a state of ***equilibrium.***

$$\mathbf{F} = \sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0}$$

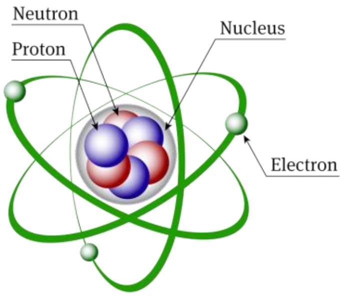
- The focus on *particles* does not imply a restriction to miniscule bodies. Rather, the study is restricted to analyses in which the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.

Rigid Particle is a Mechanics Model

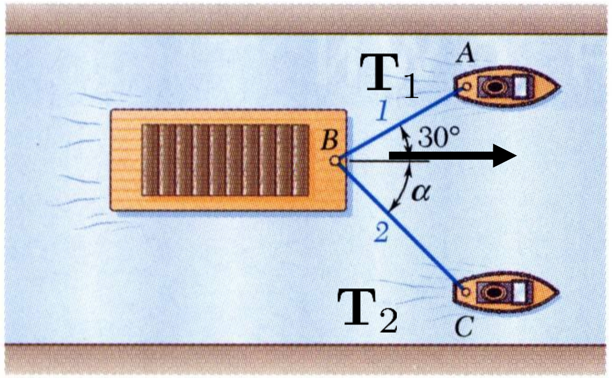
Rigid Particle Model

**This model is a very coarse approximation,
and it may be cruel and humiliating,**

but it is incredibly simple and efficient.



Sample Problem 2.2: Find Resultant Force



A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

a) The tension in each of the ropes for $\alpha = 45^{o}$,

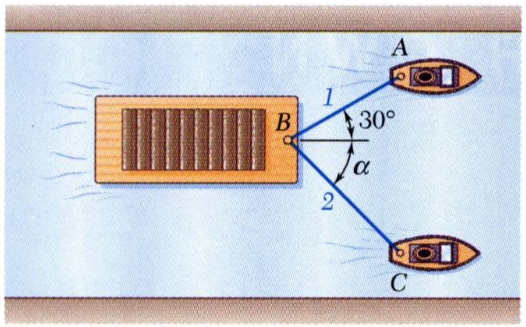
b) The value of α for which the tension in rope 2 is a minimum.

SOLUTION:

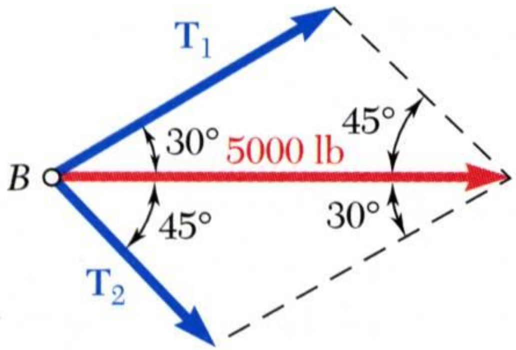
- Find a **graphical solution** by applying the Parallelogram Rule for vector addition.

- Find a **trigonometric solution** by applying the Triangle Rule for vector addition. Apply the Law of Sines to find the rope tensions.

- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.



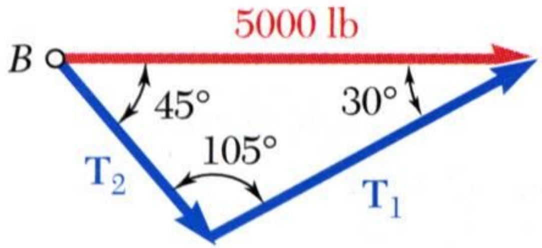
- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.



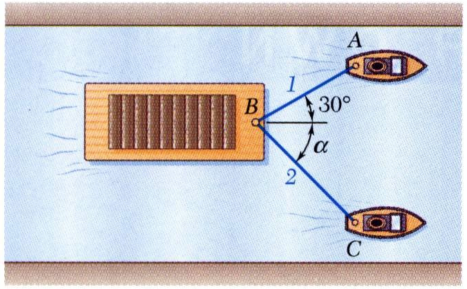
- Trigonometric solution - Triangle Rule with Law of Sines

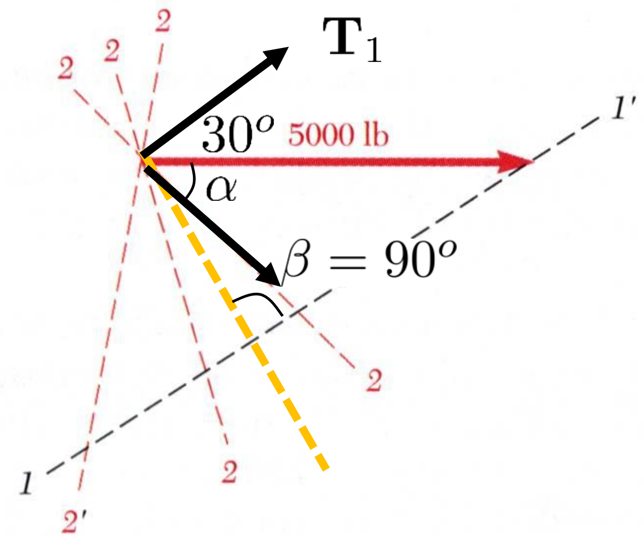
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\,\text{lbf}}{\sin 105°}$$

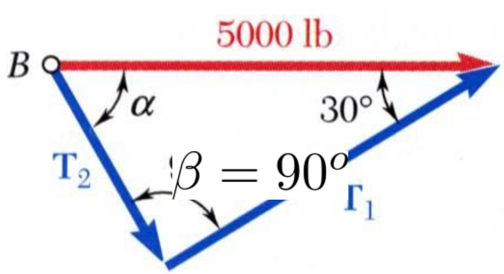
$$\boxed{T_1 = 3660\,\text{lbf} \quad T_2 = 2590\,\text{lbf}}$$



The value of α for which the tension in rope 2 is a minimum.







$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin 30^o} = \frac{5000}{\sin\beta}$$

- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

Under which β, T_2 is the smallest ?

$$T_1 = (5000lbf)\frac{\sin\alpha}{\sin\beta}$$

$$\boxed{T_2 = (5000lbf)\frac{\sin 30^o}{\sin\beta}}$$

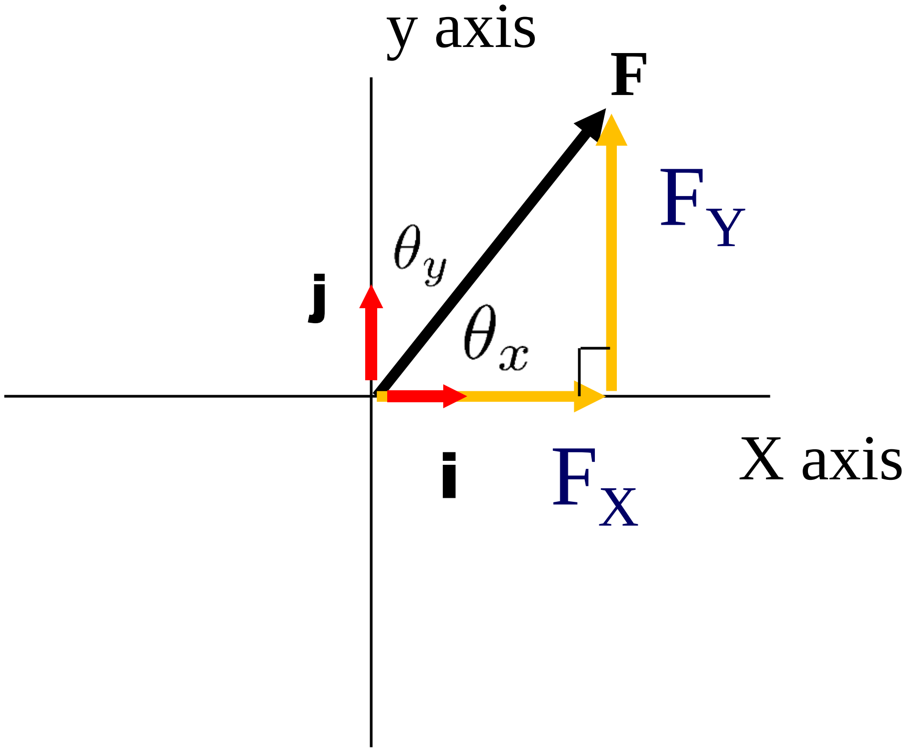
$$\alpha = 90° - 30° \qquad \boxed{\alpha = 60°}$$

$$T_1 = (5000\,\text{lbf})\cos 30° \qquad \boxed{T_1 = 4330\,\text{lbf}}$$

$$T_2 = (5000\,\text{lbf})\sin 30° \qquad \boxed{T_2 = 2500\,\text{lbf}}$$

Coordinate Approach

What are F_x, and F_y ?



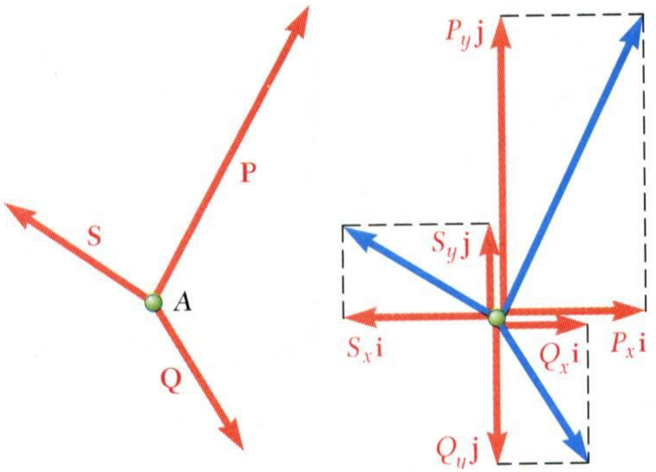
$F_X = F\cos\theta$

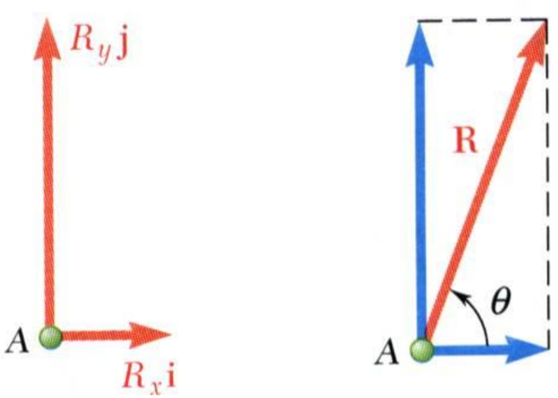
$F_Y = F\sin\theta$

$$\begin{aligned} F_x &= \mathbf{F} \cdot \mathbf{i} = F\cos\theta_x \\ F_y &= \mathbf{F} \cdot \mathbf{j} = F\cos\theta_y \end{aligned}$$

Note that $\cos\theta_y = \sin\theta_x$.

Addition of Forces by Summing Components





- Wish to find the resultant of 3 or more concurrent forces,

  $$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

  $$\begin{aligned} R_x\vec{i} + R_y\vec{j} &= P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j} \\ &= \left(P_x + Q_x + S_x\right)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j} \end{aligned}$$

- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

  $$\begin{aligned} R_x &= P_x + Q_x + S_x \\ &= \sum F_x \end{aligned} \qquad \begin{aligned} R_y &= P_y + Q_y + S_y \\ &= \sum F_y \end{aligned}$$

- To find the resultant magnitude and direction,

  $$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

2.18 and 2.19 Determine the x and y components of each of the forces shown.

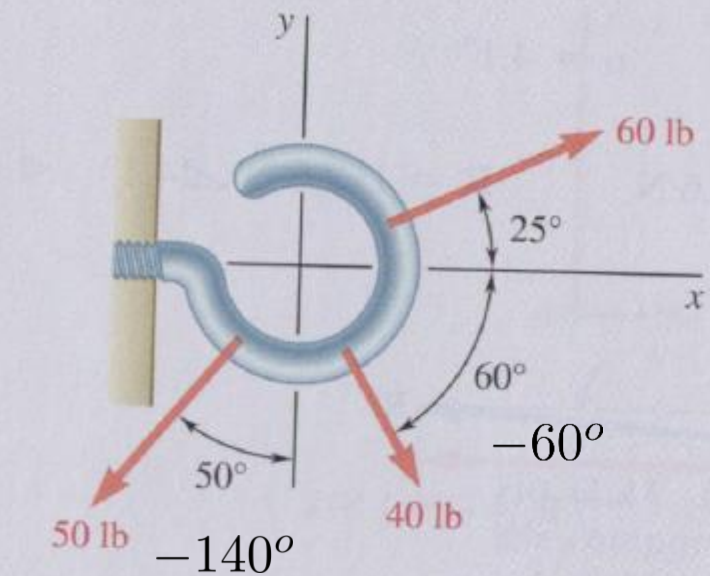


Fig. P2.18

$$F_x = \sum_{i=1}^{3} F_{ix} = \sum_{i=1}^{3} F_i \cos\theta_i$$

$$F_y = \sum_{i=1}^{3} F_{iy} = \sum_{i=1}^{3} F_i \sin\theta_i$$

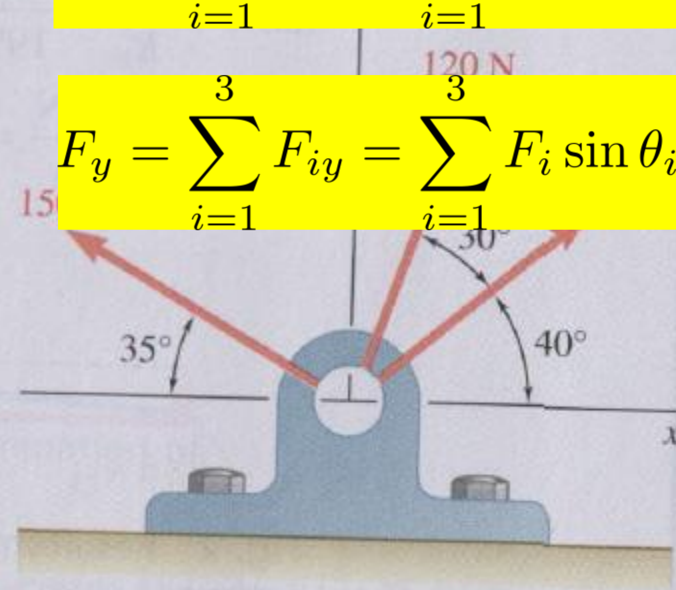


Fig. P2.19

$$F_x = 60\cos 25^o + 40\cos(-60^o) + 50\cos(-140^o) = ?$$

$$F_y = 60\sin 25^o + 40\sin(-60^o) + 50\sin(-140^o) = ?$$

Statics of Particles: Equilibrium Equations

Equilibrium of a particle:

Newton's first Law

$$\sum_i \mathbf{F}_i = \mathbf{R} = 0 \ , \qquad (m\ddot{\mathbf{r}} = 0)$$

In 2D,

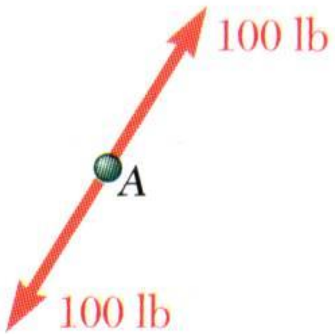
$$\sum_i \mathbf{F}_{ix} = 0, \quad \text{and} \quad \sum_i \mathbf{F}_{iy} = 0;$$

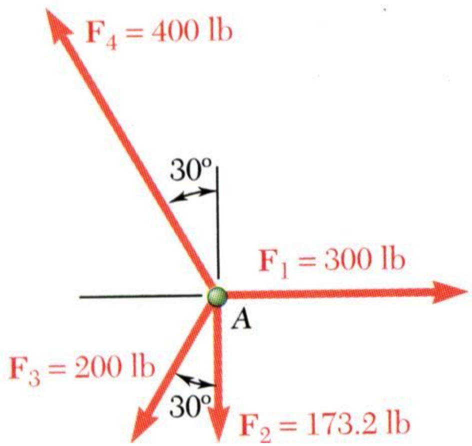
In 3D

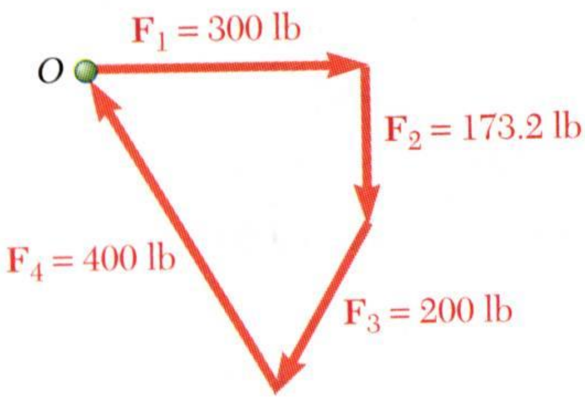
$$\sum_i \mathbf{F}_{ix} = 0, \quad \sum_i \mathbf{F}_{iy} = 0, \ \text{and} \ \sum_i \mathbf{F}_{iz} = 0 \ .$$

Equilibrium of a Particle: A tug-of-war

- When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.
- ***Newton's First Law***: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.







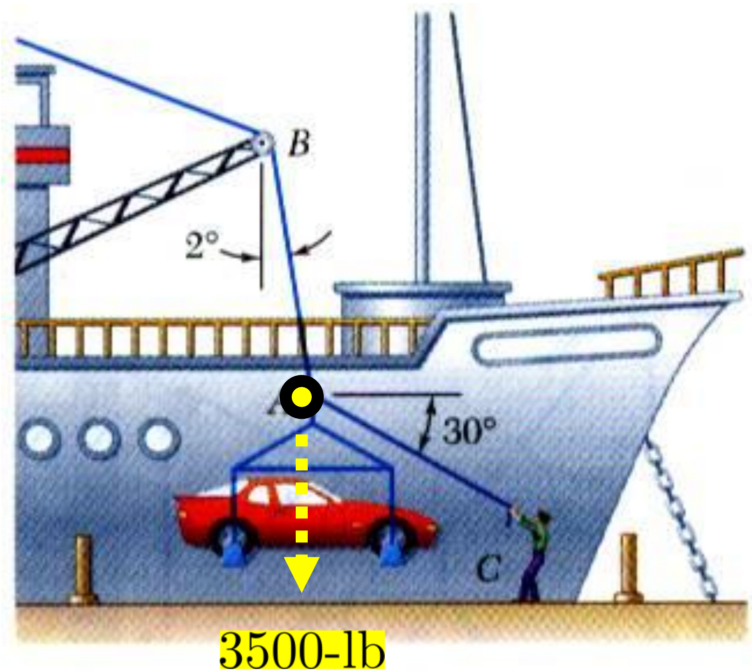
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense
- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$ Vector algebraic equation

$$\sum F_x = 0 \qquad \sum F_y = 0$$

scalar algebraic equations

Sample Problem 2.4

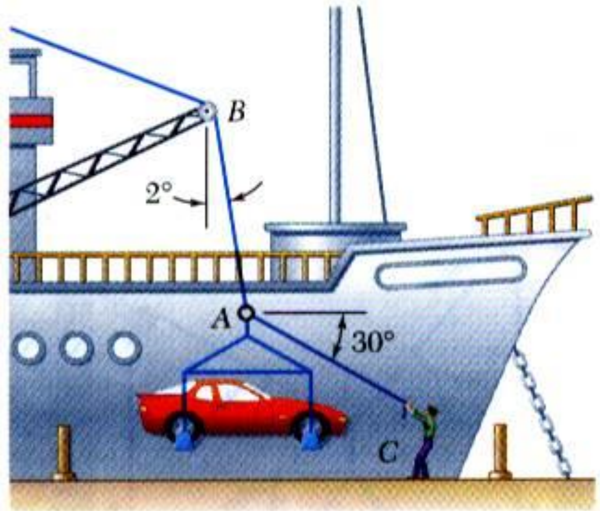


In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

- **Construct a free-body diagram**
- for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

Sample Problem 2.4



SOLUTION:

- Construct a free-body diagram for the particle at *A*.

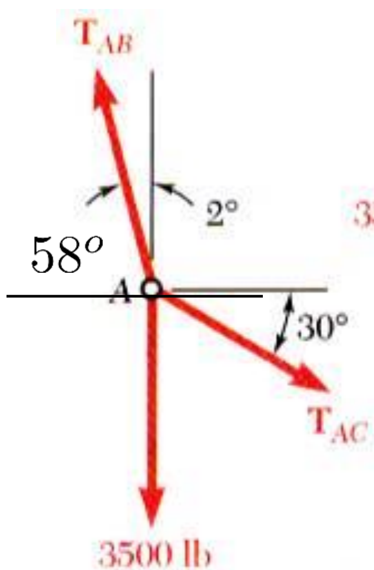
- Apply the conditions for equilibrium.

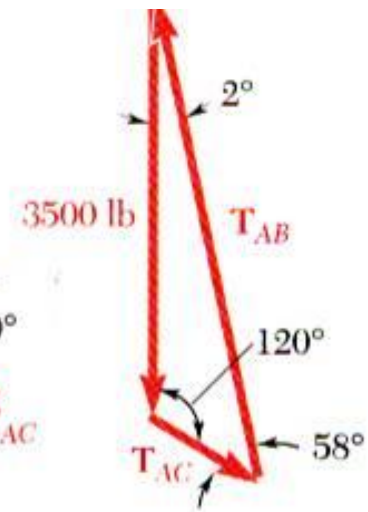
- Solve for the unknown force magnitudes.

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500\,\text{lb}}{\sin 58°}$$

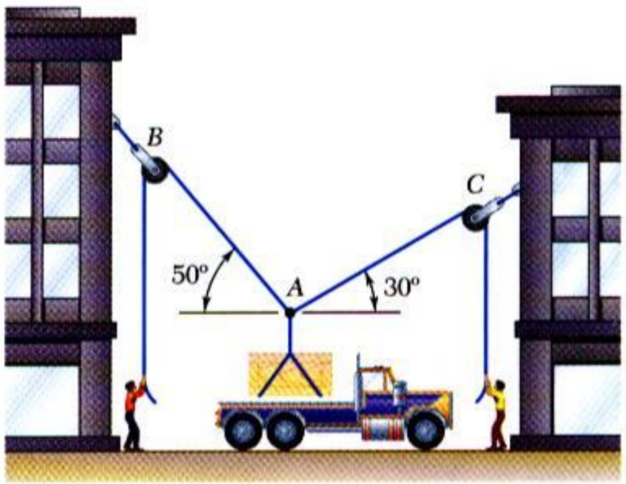
$$T_{AB} = 3570\,\text{lb}$$

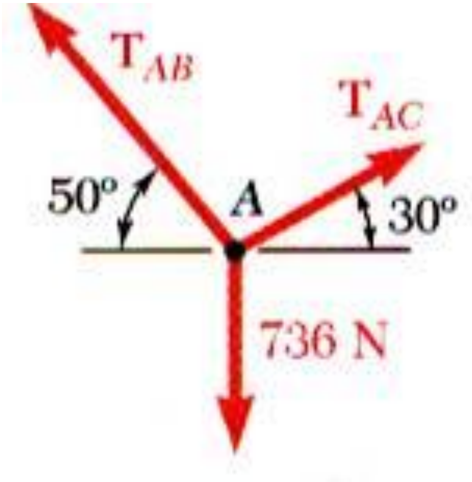
$$\boxed{T_{AC} = 144\,\text{lb}}$$





How to draw Free-Body Diagrams



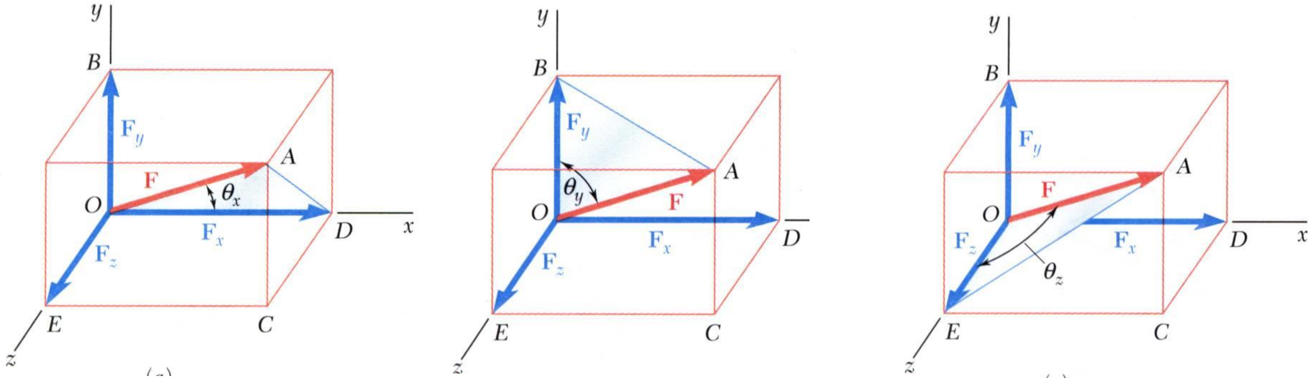


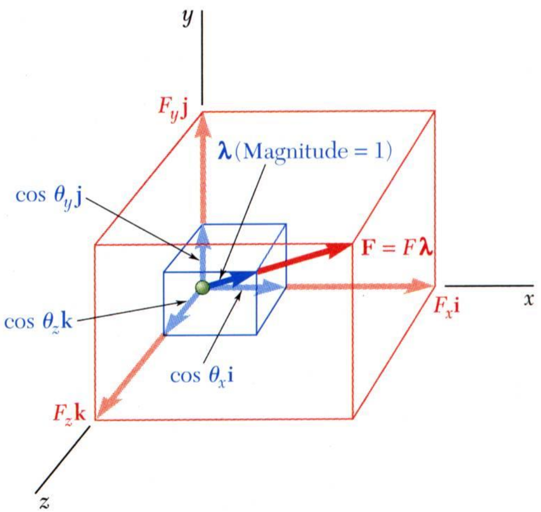
Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle.

1. **Identify the particle;**
2. **Separate the particle with its environment;**
3. **Draw all the forces acting on it**
4. **Mark the direction and magnitude of the forces .**

Coordinate Approach: Rectangular Components in 3D Space





- With the angles between $\vec{F}$ and the axes,

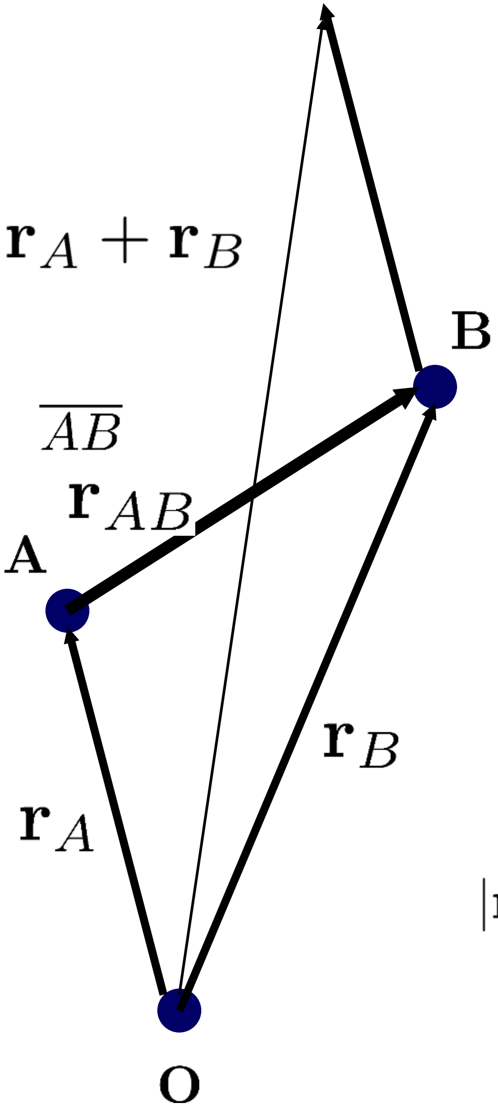
$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\begin{aligned}\vec{F} &= F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \\ &= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right) \\ &= F\vec{\lambda}\end{aligned}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

1. Position vector and relative position vector



$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A =: \mathbf{r}_{B/A}$$

Note $\mathbf{r}_A = \mathbf{r}_{OA}$

Recall Triangle Law (or Parallelogram Law)

$$\mathbf{r}_{AB} = (X_B - X_A)\mathbf{i} + (Y_B - Y_A)\mathbf{j} + (Z_B - Z_A)\mathbf{k}$$

Length:

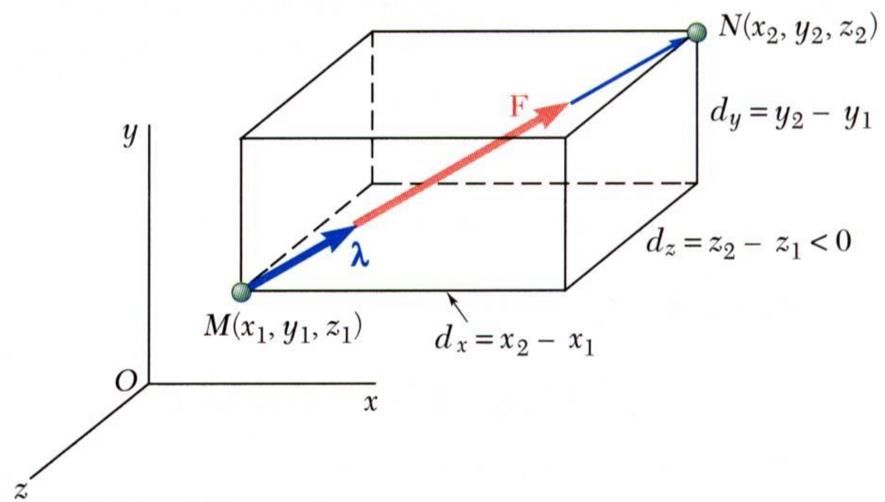
$$|\mathbf{r}_{AB}| = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2}$$

Unit vector along direction $\overline{AB}$:

2. Unit direction vector

$$\boldsymbol{\lambda} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|};$$

Find the direction unit vector



Direction of the force is defined by the location of two points,
$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$

Directional Cosine

$$\cos\theta_x = \frac{d_x}{d}$$
$$\cos\theta_y = \frac{d_y}{d}$$
$$\cos\theta_z = \frac{d_z}{d}$$

and

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$\vec{d} = \text{vector joining } M \text{ and } N$$
$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$

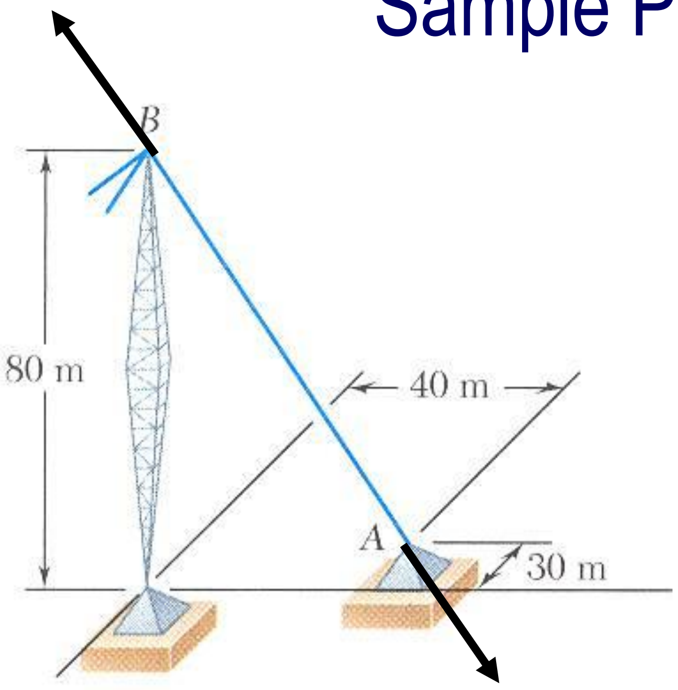
$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$\vec{F} = F\vec{\lambda}$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

Sample Problem 2.7



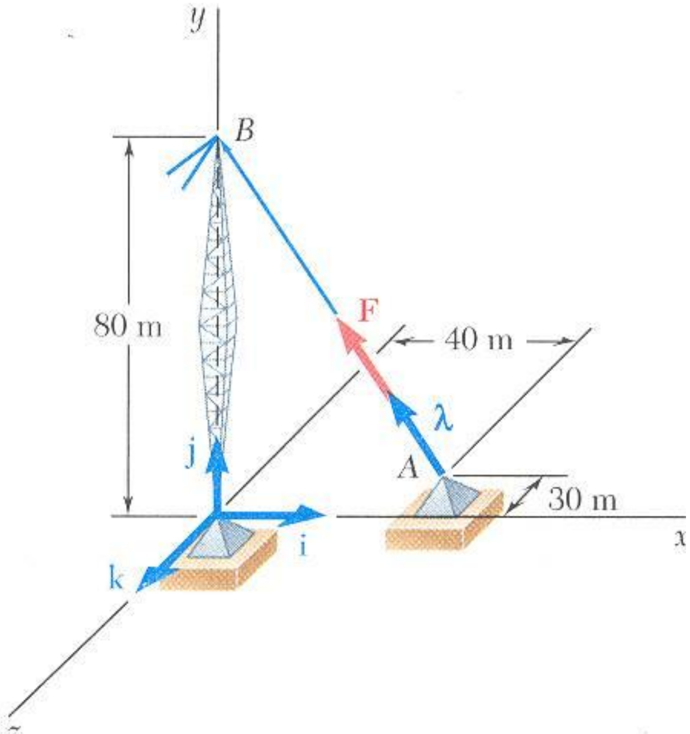
The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles θ_x, θ_y, θ_z defining the direction of the force

SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force acting on *A*.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.



$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A$$

$$\mathbf{r}_B = (0, 80, 0)$$
$$\mathbf{r}_A = (40, 0, -30)$$

SOLUTION: $F = 2500N$

- Determine the unit vector pointing from *A* towards *B*.

$$\overrightarrow{AB} = (-40\,\text{m})\vec{i} + (80\,\text{m})\vec{j} + (30\,\text{m})\vec{k}$$

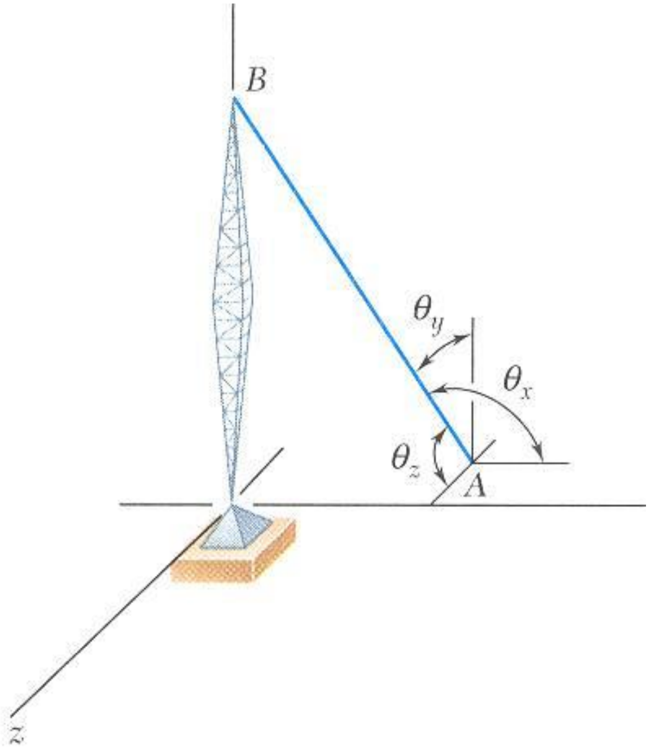
$$AB = \sqrt{(-40\,\text{m})^2 + (80\,\text{m})^2 + (30\,\text{m})^2}$$
$$= 94.3\,\text{m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

- Determine the components of the force.

$$\vec{F} = F\vec{\lambda}$$
$$= (2500\,\text{N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$
$$= (-1060\,\text{N})\vec{i} + (2120\,\text{N})\vec{j} + (795\,\text{N})\vec{k}$$

$F = 2500N$



- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ \end{aligned}}$$

$$\mathbf{F} = 2500\,\overrightarrow{\lambda}$$

Summary

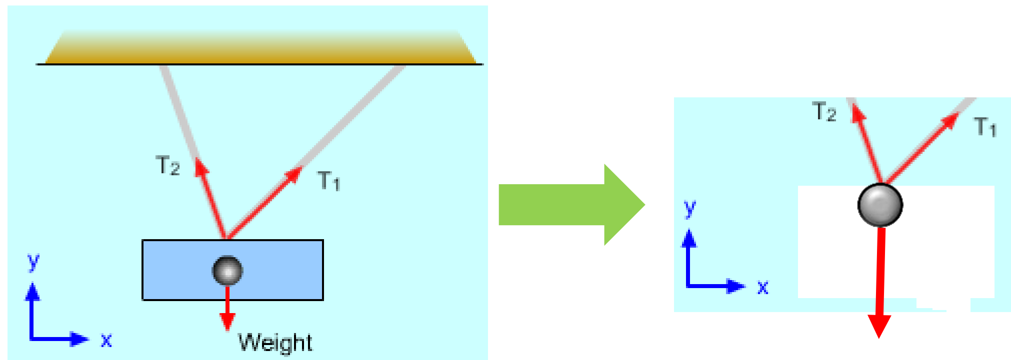
The approach to particle statics is systematic; there is no drama and no twist.

1. **Draw Coordinate System;**
2. **Draw Free Body Diagam;**
3. **Set up Equilibrium Equations;**
4. **Identify Unknow Forces;**
5. **Solve Equilibrium Equations .**

Summary

Basic Concepts in Rigid Particle Statics Model

1. Draw Free-body diagram



2.

- replacing multiple forces acting on a particle with a single equivalent or ***resultant*** force,

$$\mathbf{F} = \sum_{i=1}^{n} \mathbf{F}_i$$

3.

- relations between forces acting on a particle that is in a state of ***equilibrium.***

$$\mathbf{F} = \sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0}$$

Have a good weekend!