CE30 – Discussion 6

Distributed Loads & Moment of Interia

Textbook: 4.4, 5.1, 5.2, 5.3, 7.1, 7.2

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Announcements

- HW6 Problems from the textbook:

  4.95, 5.13, 5.53, 5.81, 7.12, 7.25, 7.47

- GSI Qijun Chen will cover the discussions next week 4/3-6/3

Friction Forces

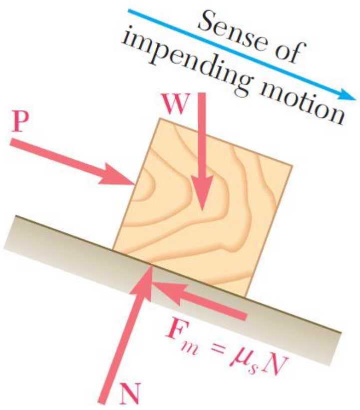
- Acts in the opposite way of motion

- Maximum friction force:

$$F_m = \mu_s N$$

μ_s: Coefficient of (static)friction

N: Normal force



Centroids of Areas

$$\bar{x}A = \int x\, dA \qquad \bar{y}A = \int y\, dA$$

- The integral here is known as the ***first moment of the area***.
- Coordinates of the centroid ($\bar{x}$, $\bar{y}$) can be found using the first moments, i.e.:

Compute the first moment

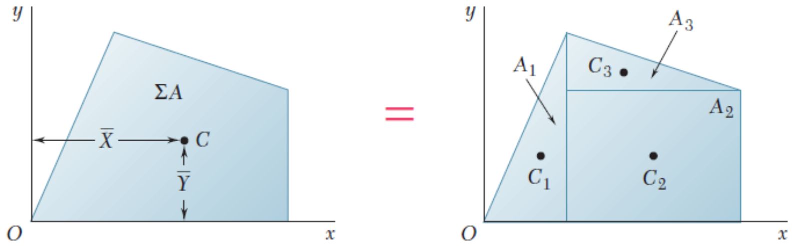
$$Q_x = \int y\, dA$$

Use it to find the centroid

$$\bar{x} = \frac{Q_x}{A}$$

Composite Areas

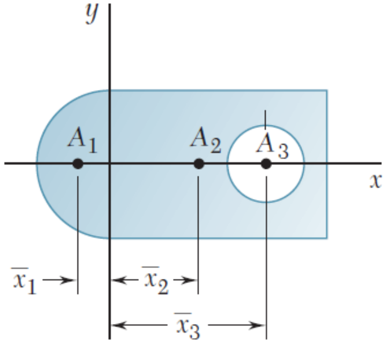
Decompose complex areas into simpler shapes and use the principle of superposition



$$Q_y = \bar{X}\,\Sigma A = \Sigma\,\bar{x}\,A$$

$$Q_x = \bar{Y}\,\Sigma A = \Sigma\,\bar{y}\,A$$

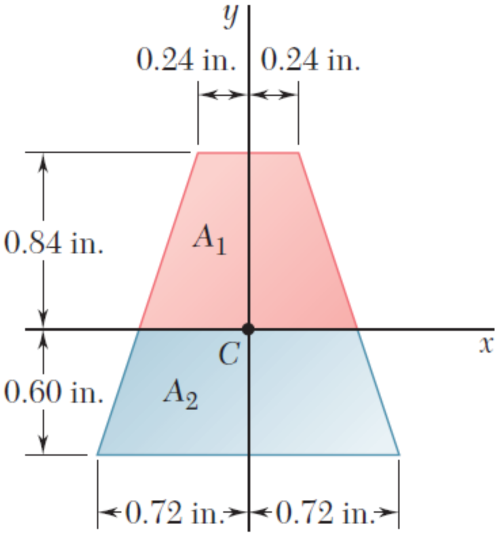
Also useful if there are holes



	$\bar{x}$	A	$\bar{x}A$
A_1 Semicircle	–	+	–
A_2 Full rectangle	+	+	+
A_3 Circular hole	+	–	–

Practice – Similar to HW P5.13

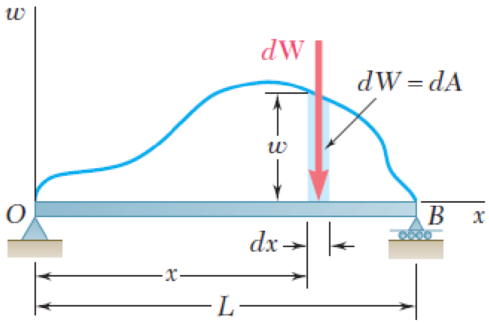
The horizontal x axis is drawn through the centroid C of the area shown and divides it into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x axis and explain the results obtained.



Distributed Loads

- Structures often carry distributed loads (snow on roof, self weight…)
- Described in terms of force per length (N/m, lb/ft …)
- We can replace the distributed load with a concentrated load,

  that would result in the **same support reactions**



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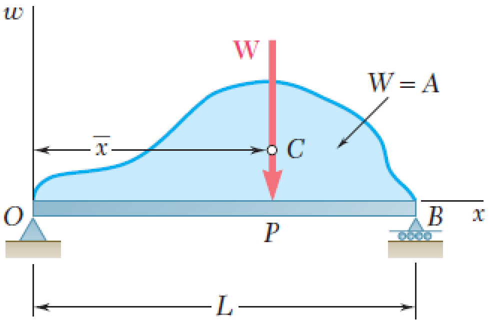
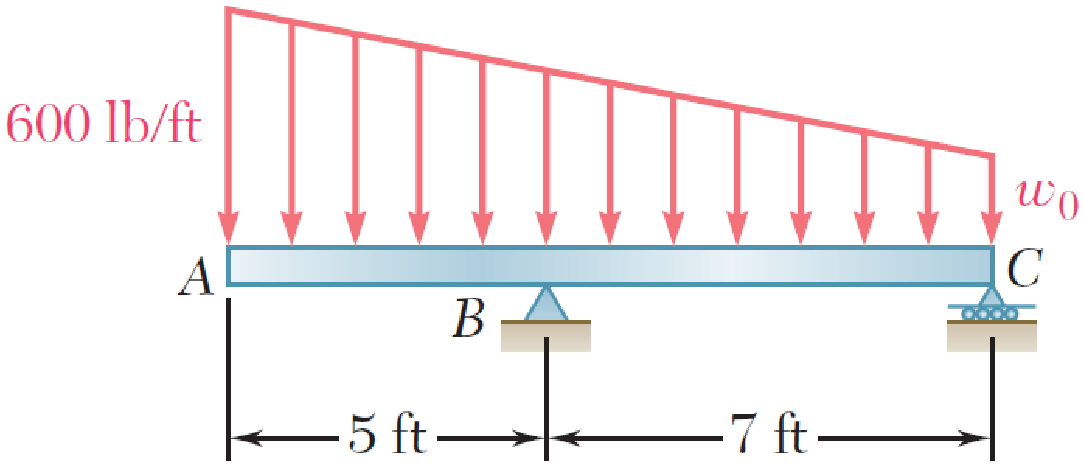


From the Moment around *O*

$$P\,\bar{x} = \int x\, dW$$

Practice – Similar to HW P5.53 – P.5.81

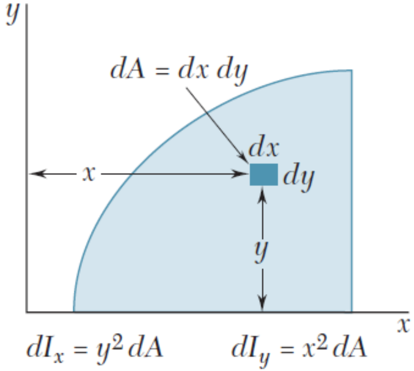
Determine the reactions at the beam supports for the given loading when $w_0 = 450$ lb/ft.

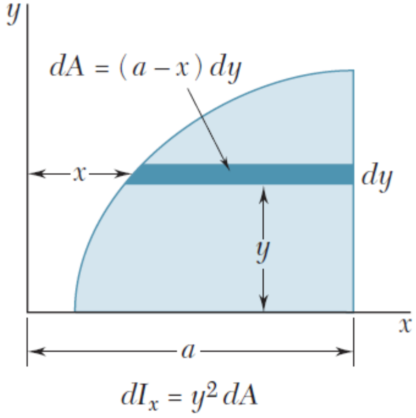


Moment of Inertia

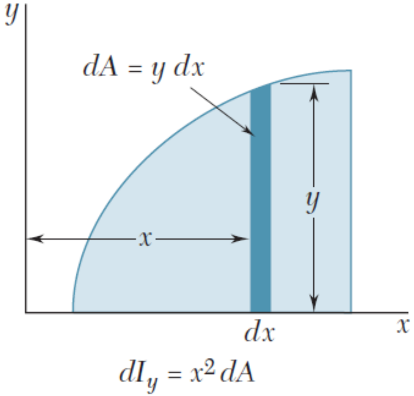
- An important geometric property in beam bending problems,
- Efficiency of a cross-section to “resist” bending
- **Second moment of the area**

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$





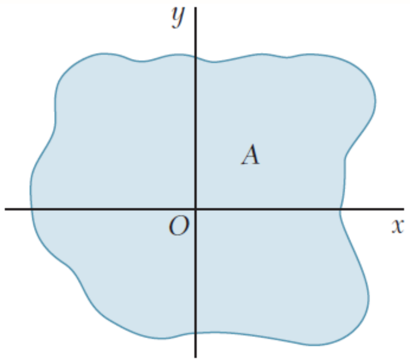
w.r.t. x-axis

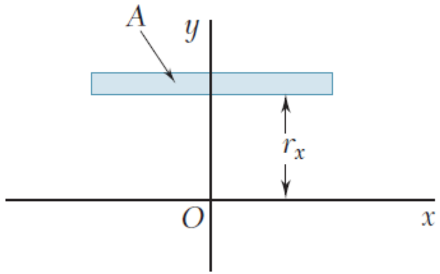


w.r.t y-axis

Radius of Gyration

Distance to an axis where the concentrated area would have the same moment of inertia

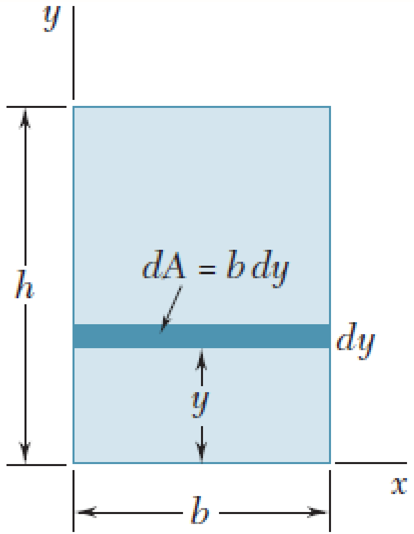




$$r_x = \sqrt{\frac{I_x}{A}}$$

Moment of Inertia: Rectangular Sections

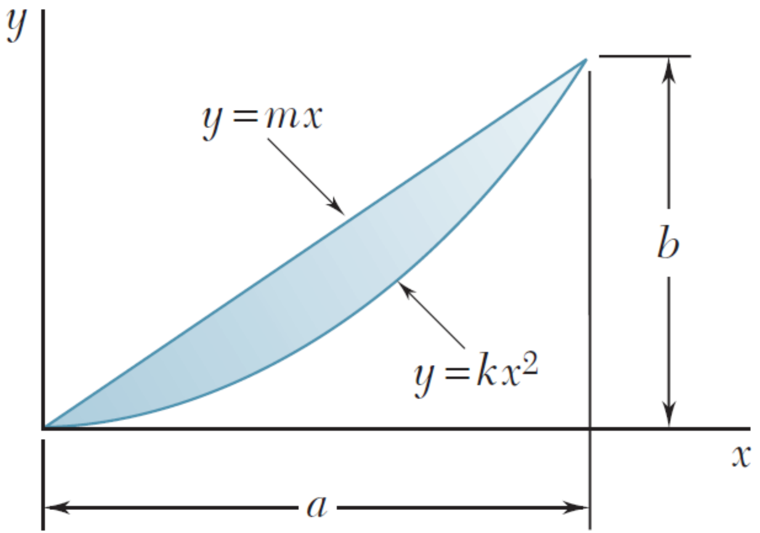
- Simple formula exists for rectangular sections



$$I_x = \int_0^h b y^2 \, dy = \boxed{\tfrac{1}{3} b h^3}$$

Practice – Similar to HW P7.12

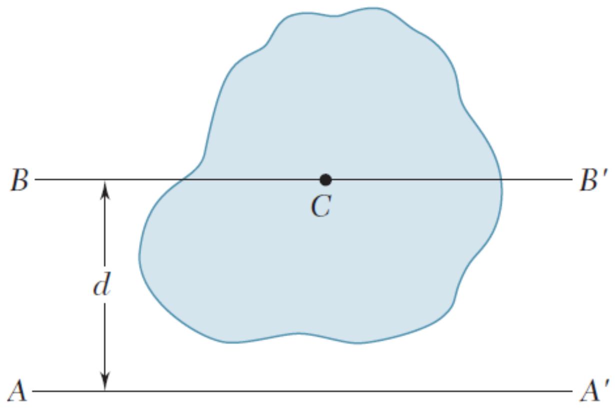
Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.



Parallel Axis Theorem

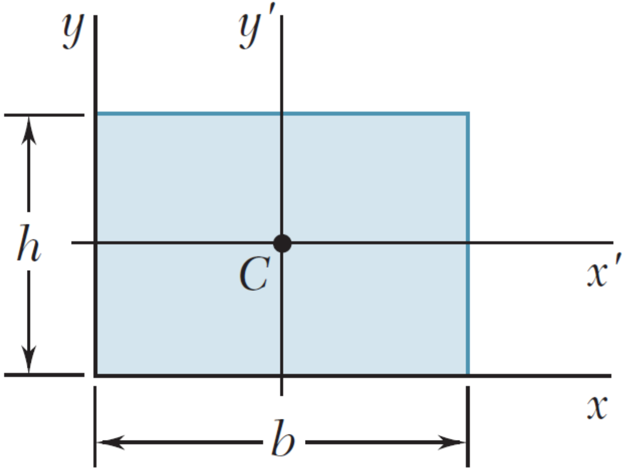
- Find the moment of inertia with respect to any axis

$$I_{AA'} = \bar{I}_{BB'} + Ad^2$$



Parallel Axis Theorem

Using $I_x = \frac{1}{3}bh^3$ and the parallel axis theorem, we can show



$$I'_x = \frac{1}{12}bh^3$$

Practice – Similar to HW P7.25

Determine the moment of inertia and radius of gyration with respect to the x-axis