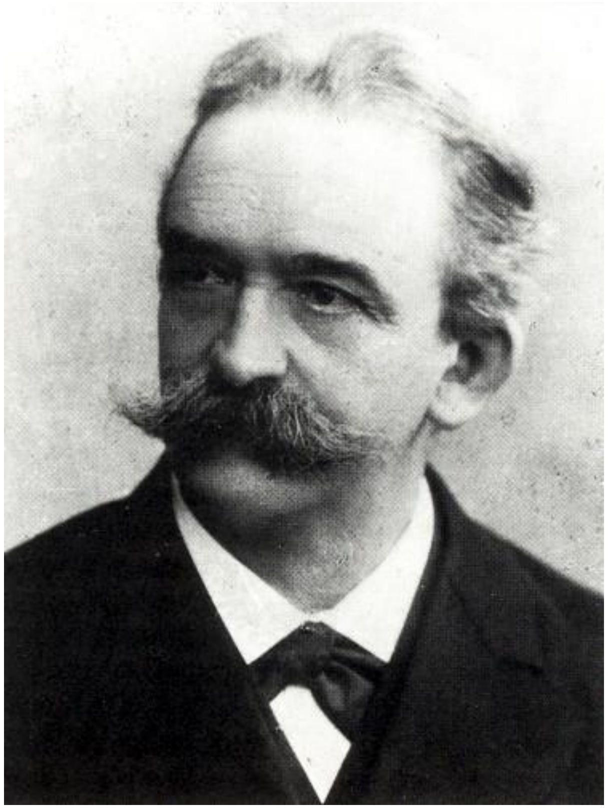
Lecture 35 Mohr's Circle (II)

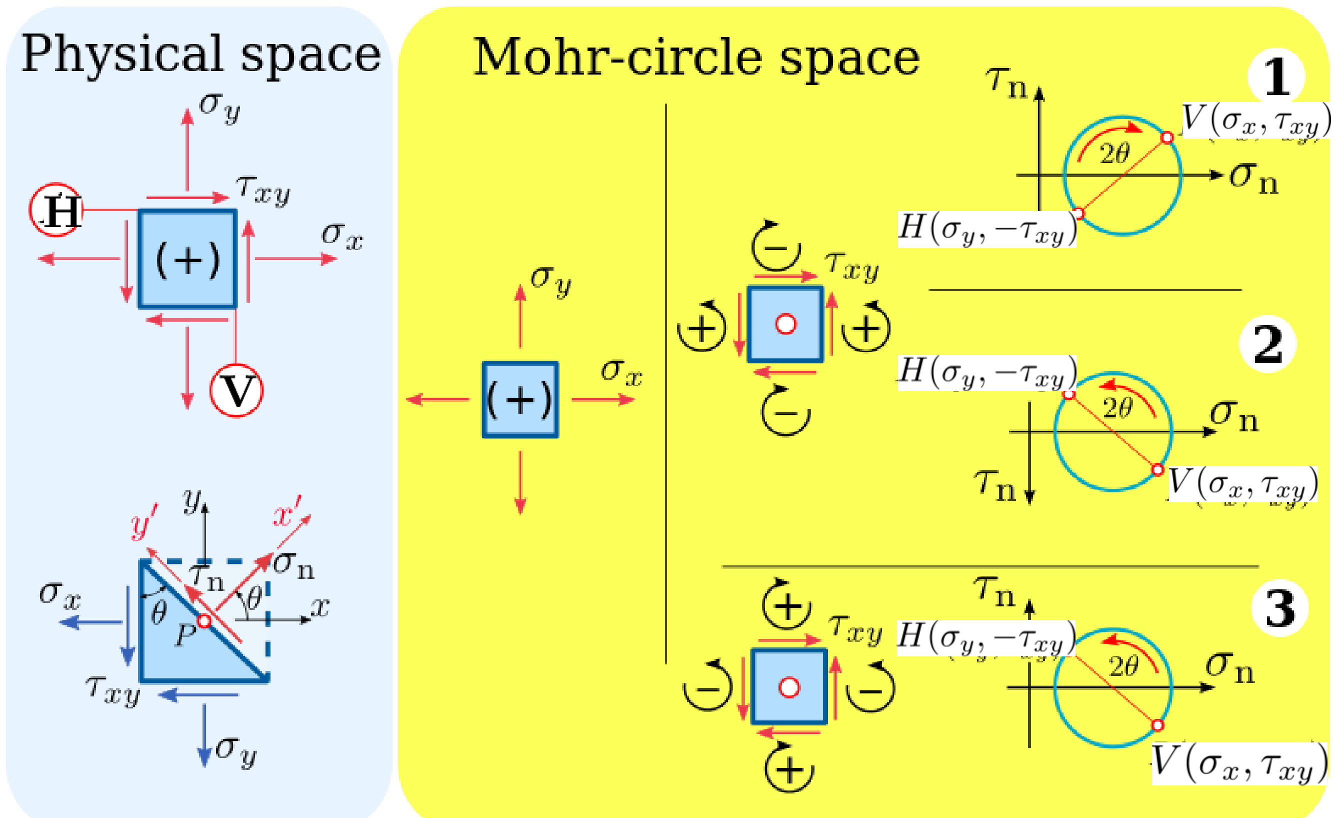
- *Christian Otto Mohr (October 8, 1835 - October 2, 1918) was a German civil engineer, one of the most celebrated of the nineteenth century.*
- *Starting in 1855, his early working life was spent in railroad engineering for the Hanover and Oldenburg state railways, designing some famous bridges and making some of the earliest uses of steel trusses.*



Christian Otto Mohr
(October 8, 1835 – October 2, 1918)

C30/ME85-Mohr-circile-handout.pdf

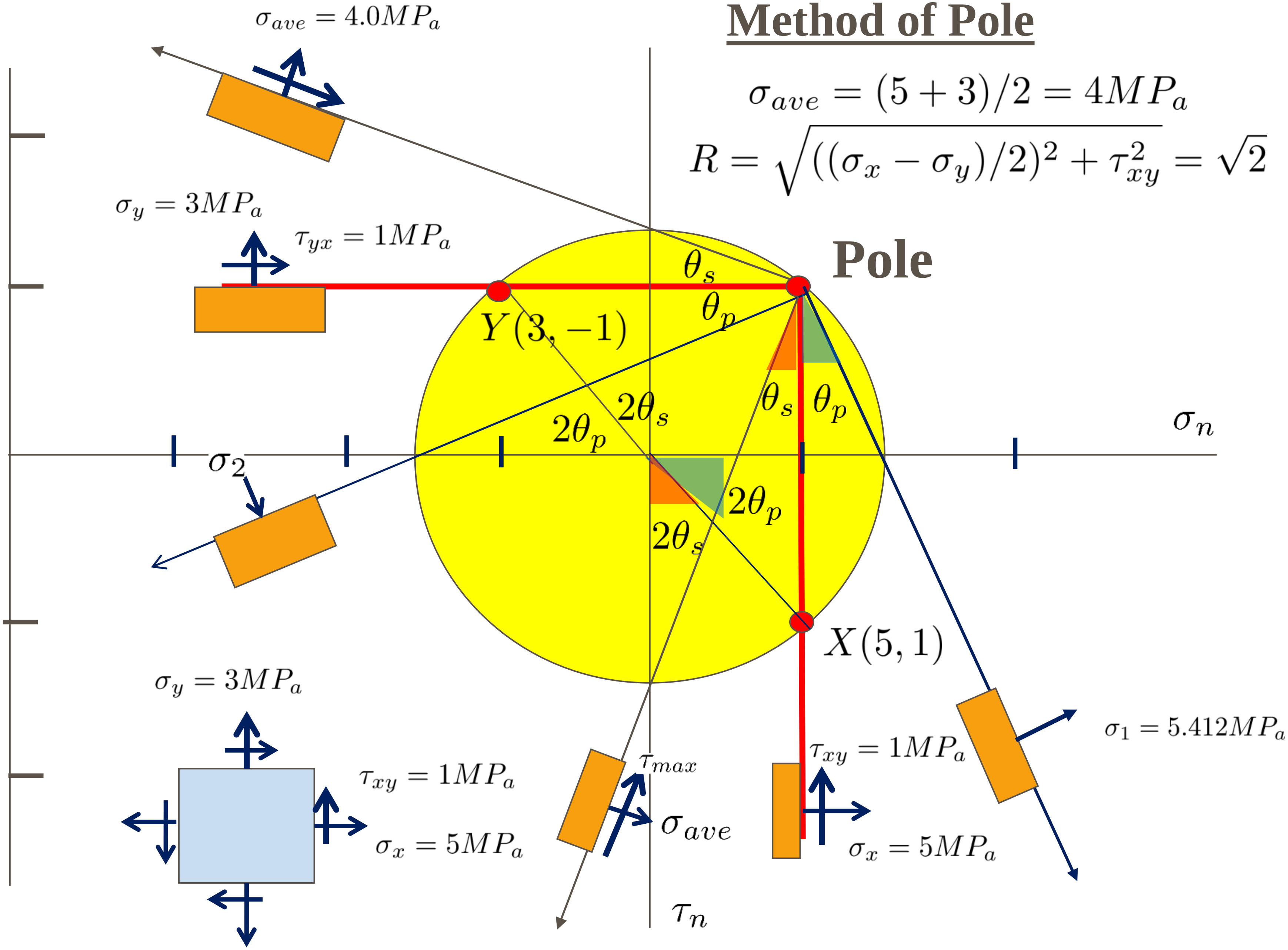
C30M85-Mohr-circle-handout.pdf	Yesterday	Yesterday	Shaofan...	622 KB		
CE30-2023-Spring.pdf	Feb 8, 2023	Feb 8, 2023	Shaofan...	53 KB		



Method of Pole

$$\sigma_{ave} = (5+3)/2 = 4MP_a$$

$$R = \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2} = \sqrt{2}$$



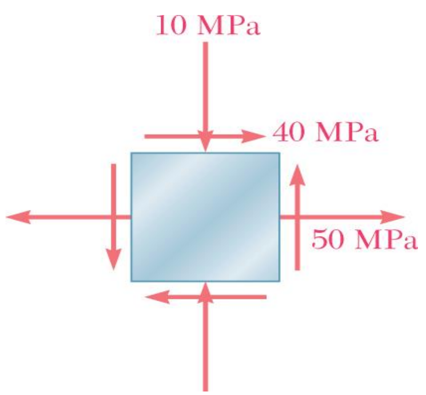
An angle at the circumference of a circle is half the angle at the center **standing on the same arc.**

Example 1

$$V \rightarrow X \text{ and } H \rightarrow Y$$

$$X : (50, 40)$$

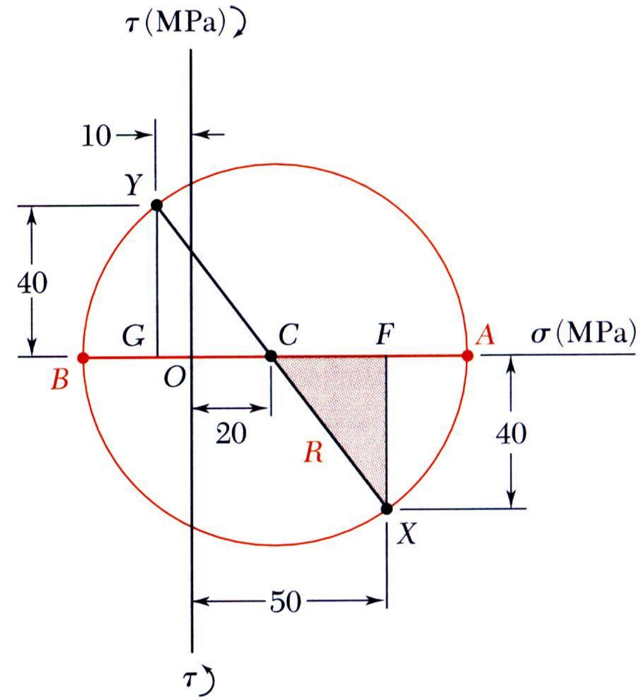
$$Y : (-10, -40)$$



For the state of plane stress shown, **(a) construct Mohr's circle, determine (b) the principal planes, (c) the principal stresses, (d) the maximum shearing stress and the corresponding normal stress.**

$$CF = (\sigma_x - \sigma_y)/2$$

$$FX = \tau_{xy}$$



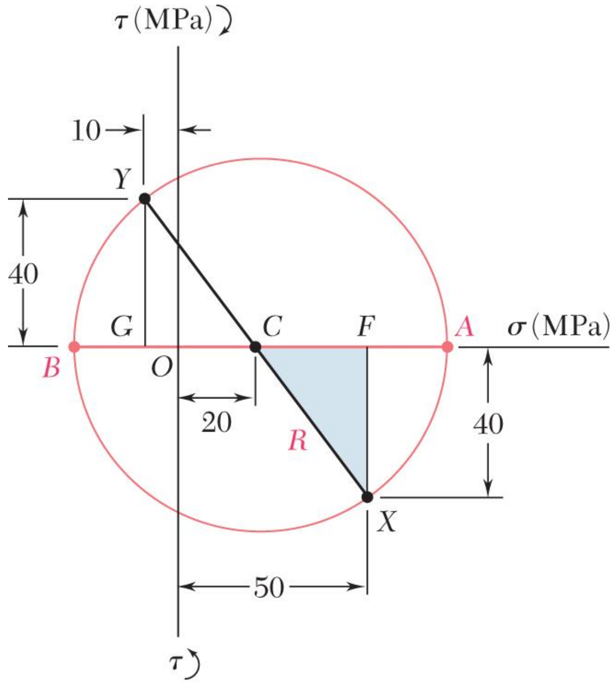
SOLUTION:

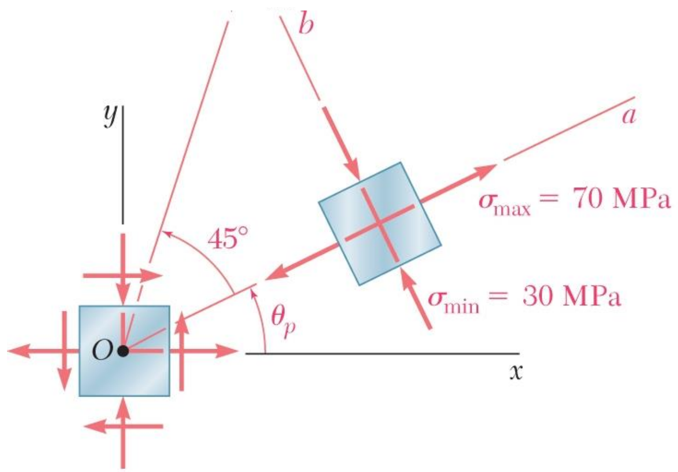
- Construction of Mohr's circle

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{(50)+(-10)}{2} = 20\,\text{MPa}$$

$$CF = 50 - 20 = 30\,\text{MPa} \qquad FX = 40\,\text{MPa}$$

$$R = CX = \sqrt{(30)^2 + (40)^2} = 50\,\text{MPa}$$





$$CA = BC = R$$

- Principal planes and stresses

$$\sigma_{\max} = OA = OC + CA = 20 + 50$$

$$\boxed{\sigma_{\max} = 70\,\text{MPa}}$$

$$\sigma_{\min} = OB = OC - BC = 20 - 50$$

$$\boxed{\sigma_{\min} = -30\,\text{MPa}}$$

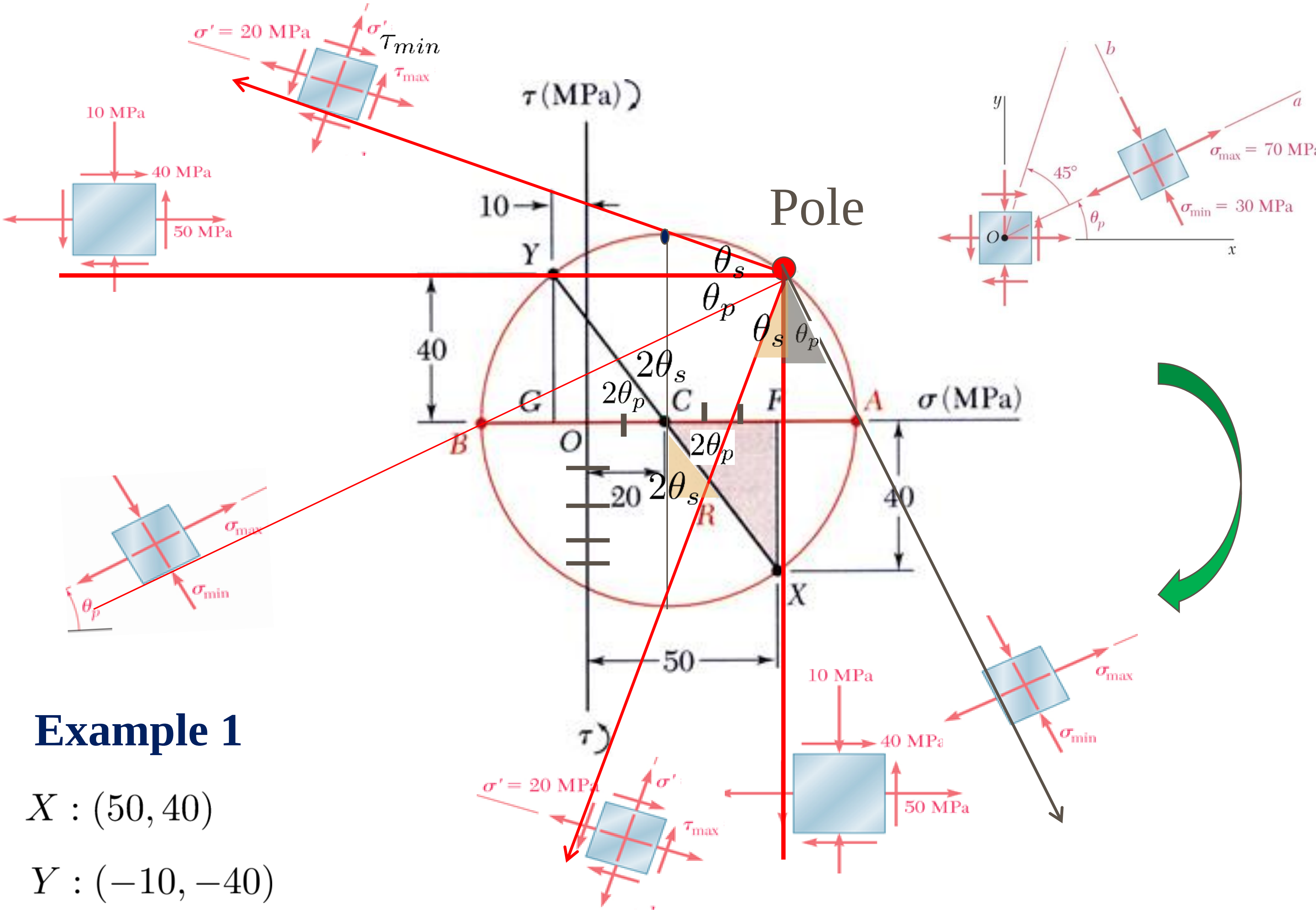
$$\tan 2\theta_p = \frac{FX}{CP} = \frac{40}{30}$$

$$2\theta_p = 53.1°$$

$$\boxed{\theta_p = 26.6°}$$

$$OC = \sigma_{ave}$$

Method of Pole

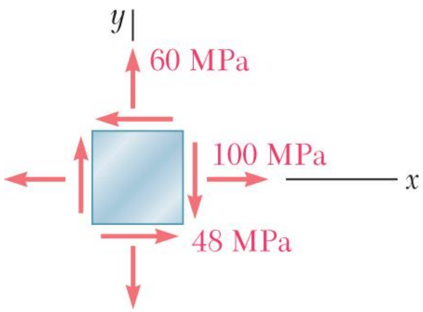


Example 1

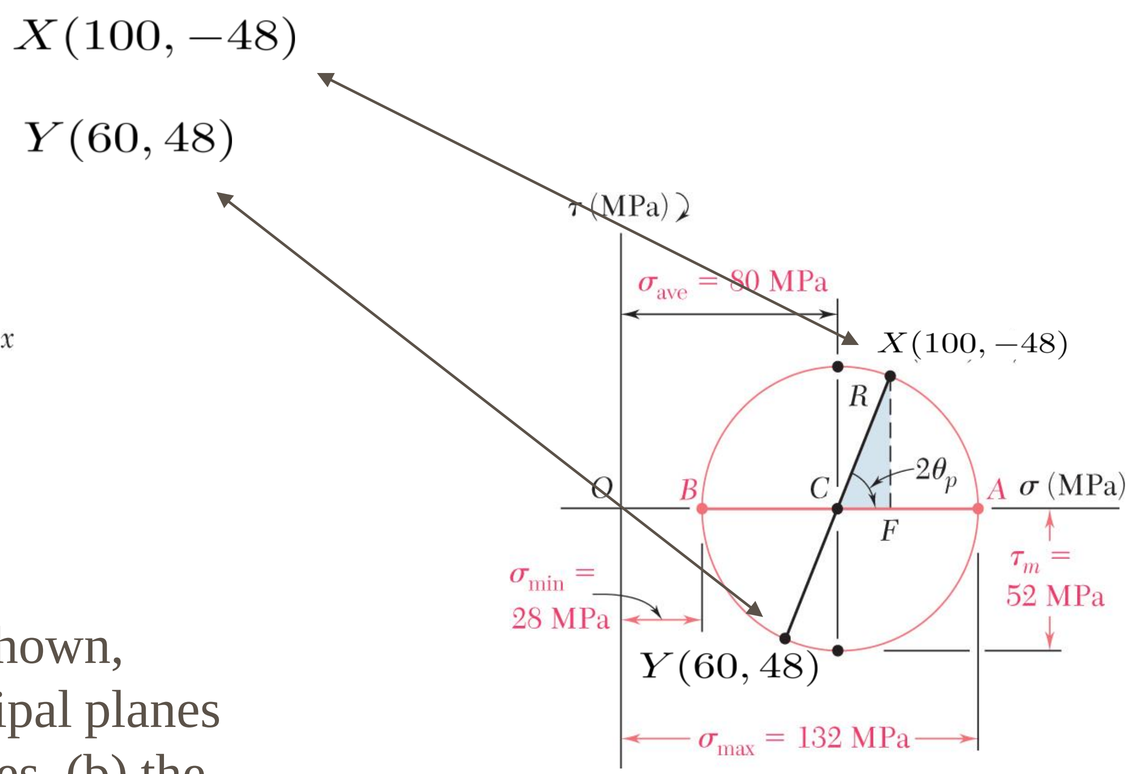
$X : (50, 40)$

$Y : (-10, -40)$

Example 2



For the state of stress shown, determine (a) the principal planes and the principal stresses, (b) the stress components exerted on the element obtained by rotating the given element counterclockwise through 30 degrees.



SOLUTION:

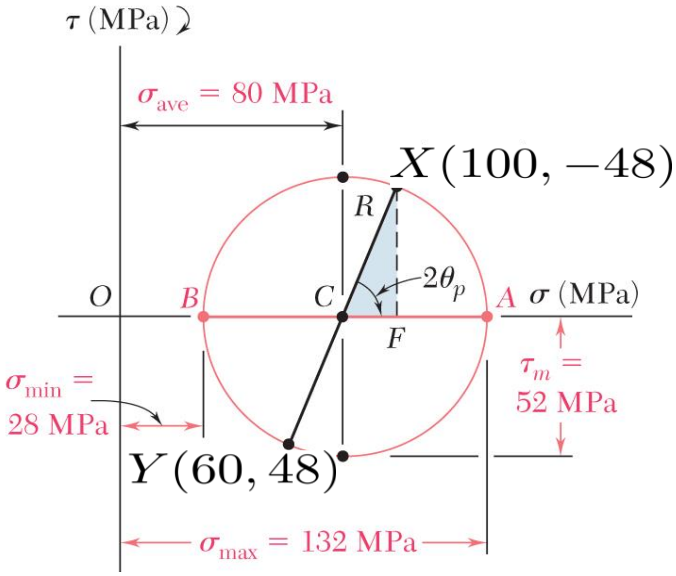
- Construct Mohr's circle

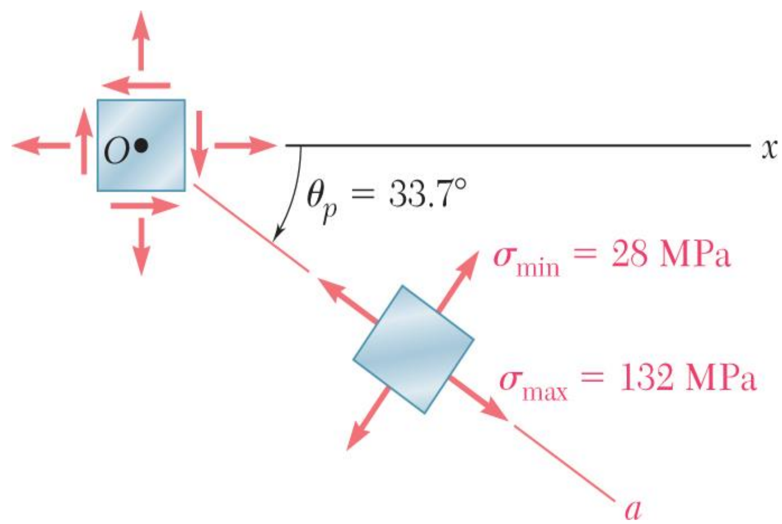
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{100 + 60}{2} = 80\,\text{MPa}$$

$$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(20)^2 + (48)^2} = 52\,\text{MPa}$$

$$XF = \tau_{xy} \qquad CF = (\sigma_x - \sigma_y)/2$$

$$OC = \sigma_{ave} \qquad CA = BC = R$$





- Principal planes and stresses

$$\tan 2\theta_p = \frac{XF}{CF} = -\frac{48}{20} = -2.4$$

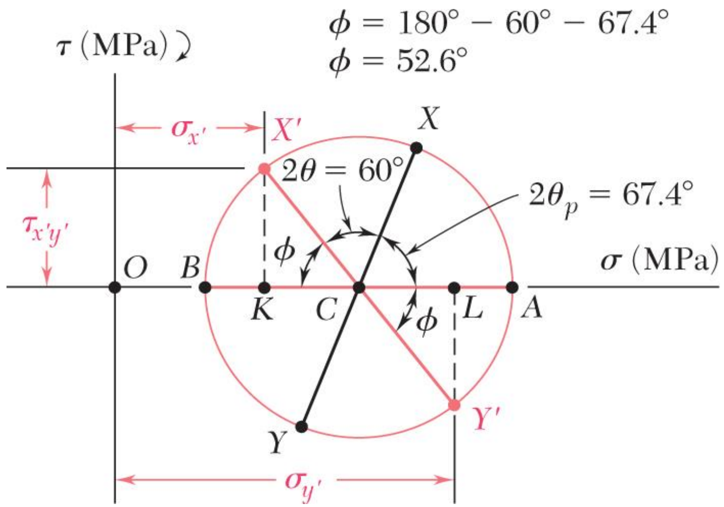
$$2\theta_p = -67.4^\circ$$

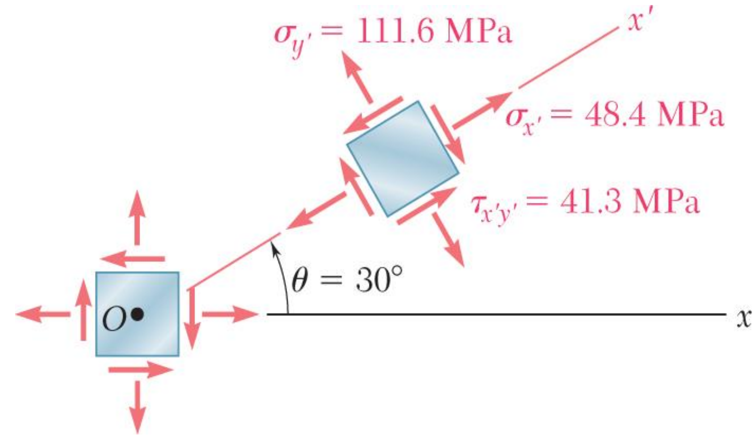
$$\boxed{\theta_p = -33.7^\circ \text{ clockwise}}$$

$$\sigma_{max} = OA = OC + CA$$
$$= 80 + 52$$
$$\boxed{\sigma_{max} = +132\,\text{MPa}}$$

$$\sigma_{min} = OA = OC - BC$$
$$= 80 - 52$$
$$\boxed{\sigma_{min} = +28\,\text{MPa}}$$

$$OC = \sigma_{ave} = 80MP_a \qquad CA = BC = R = 52MP_a$$





- Stress components after rotation by 30º

Points *X'* and *Y'* on Mohr's circle that correspond to stress components on the rotated element are obtained by rotating *XY* counterclockwise through $2\theta = 60°$

$$\phi = 180° - 60° - 67.4° = 52.6°$$

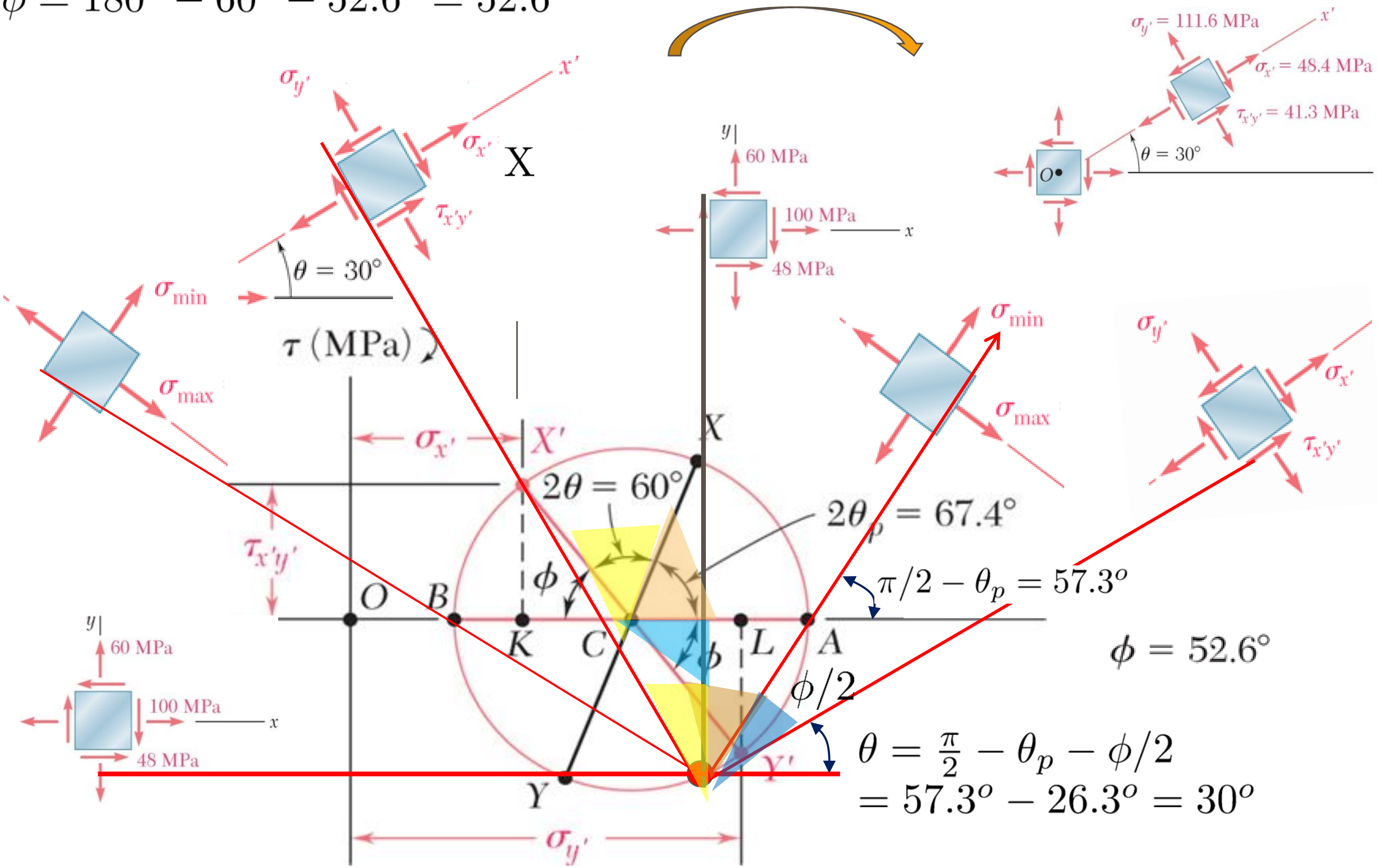
$$\sigma_{x'} = OK = OC - KC = 80 - 52\cos 52.6°$$

$$\sigma_{y'} = OL = OC + CL = 80 + 52\cos 52.6°$$

$$\tau_{x'y'} = KX' = 52\sin 52.6°$$

$$\boxed{\begin{aligned}\sigma_{x'} &= +48.4\,\text{MPa}\\ \sigma_{y'} &= +111.6\,\text{MPa}\\ \tau_{x'y'} &= 41.3\,\text{MPa}\end{aligned}}$$

$$\phi = 180^o - 60^o - 52.6^o = 52.6^o$$

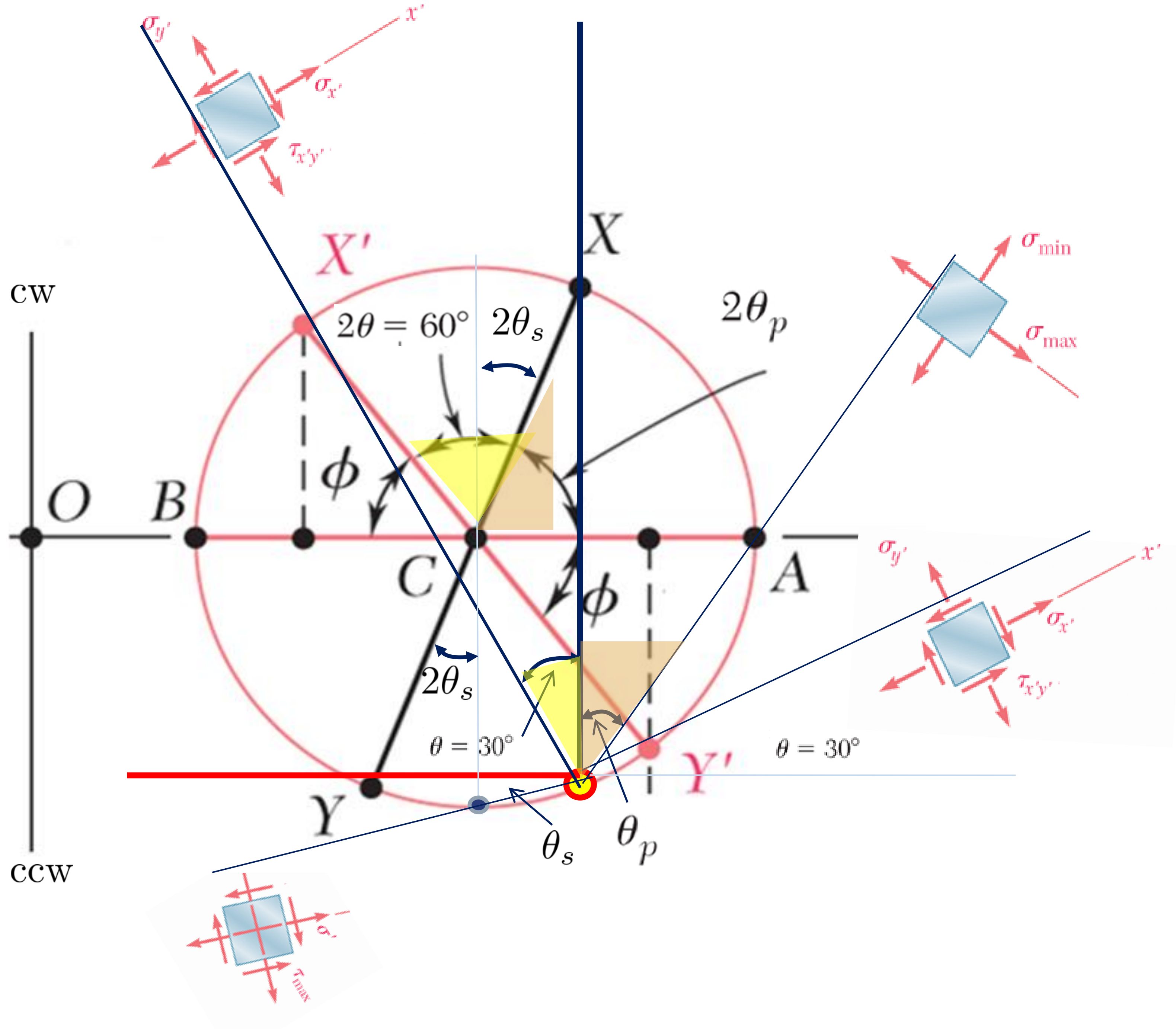


Example 2

Pole

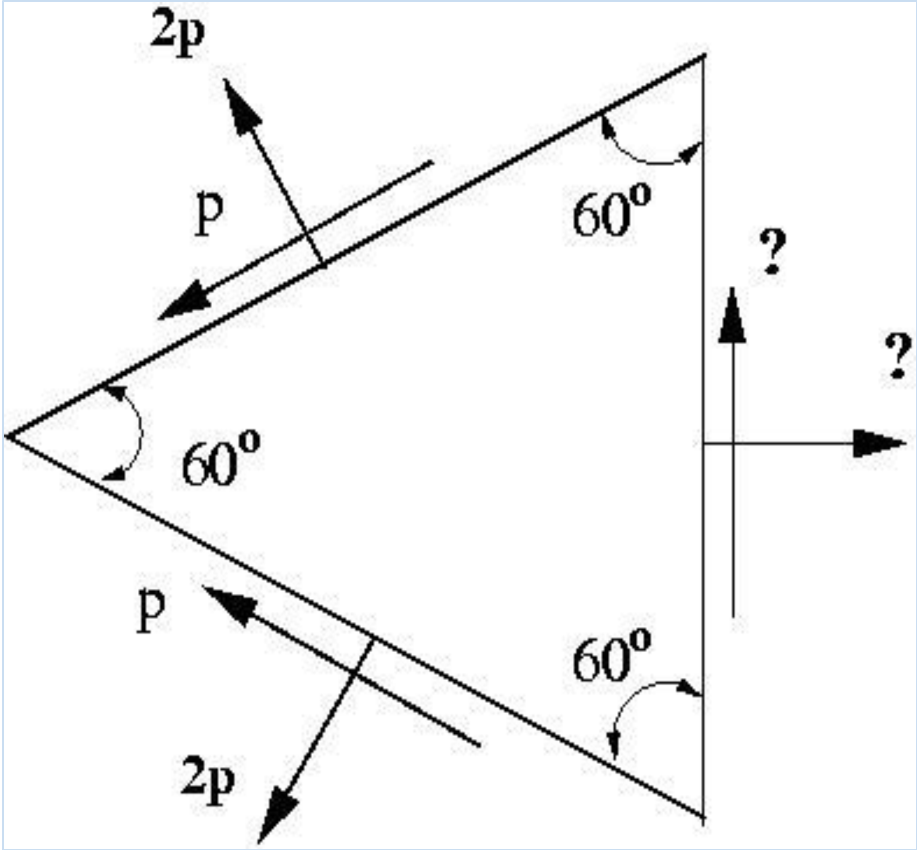
$\theta_p = 33.7°$

Method of Pole



$\sigma_{y'}$
x'
$\sigma_{x'}$
$\tau_{x'y'}$
X
X'
σ_{min}
σ_{max}
cw
$2\theta = 60°$
$2\theta_s$
$2\theta_p$
ϕ
O
B
C
A
ϕ
$\sigma_{y'}$
x'
$\sigma_{x'}$
$\tau_{x'y'}$
$2\theta_s$
$\theta = 30°$
$\theta = 30°$
Y'
Y
θ_s
θ_p
ccw
τ_{max}

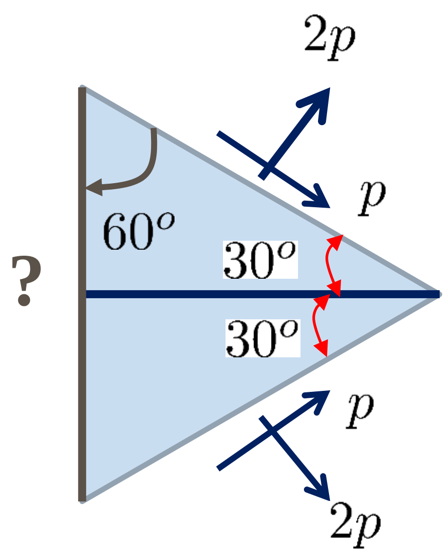
This problem is rated R.

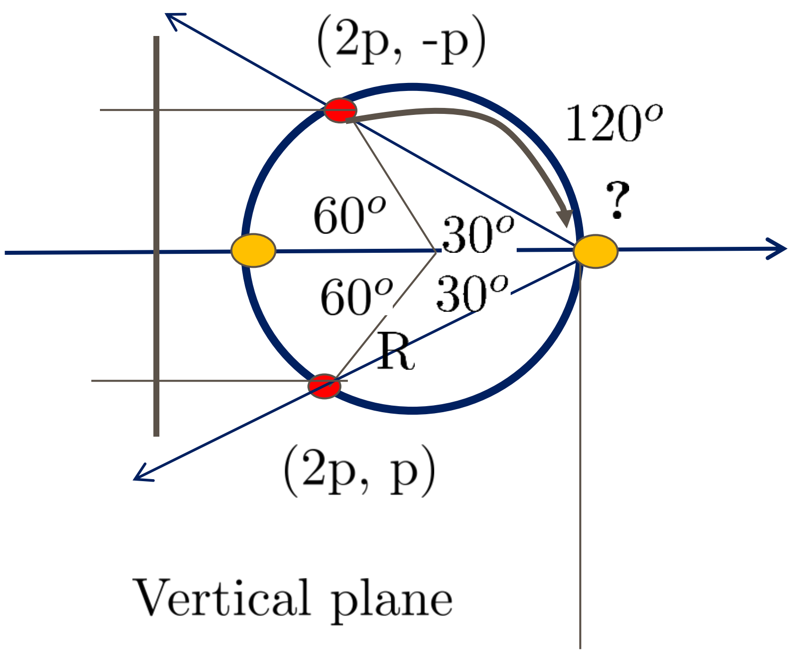


Today's Lecture Passcode is: Method of Pole

The magnitude and direction of stresses components on two inclined planes are shown in the figure.
Find σ_x, σ_y and τ_{xy}.

This problem is rated R !





$$R \sin 60 = p \quad \rightarrow \quad R = \frac{2\sqrt{3}}{3} p$$

$$\sigma_{ave} = 2p + R\cos 60 = (2 + \frac{\sqrt{3}}{3})p$$

Vertical plane

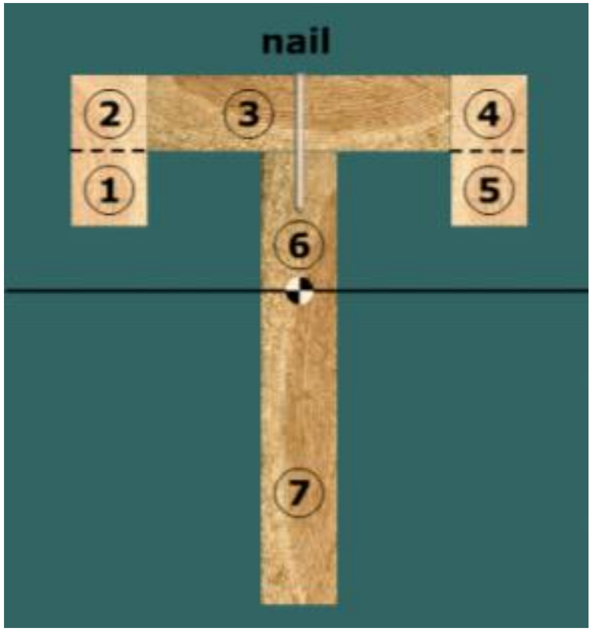
$$\sigma_x = \sigma_{ave} + R = \sigma_1, \;\; \tau_{xy} = 0;$$

Horizontal plane

$$\sigma_y = \sigma_{ave} - R = \sigma_2, \;\; \tau_{yx} = 0;$$

We move the rated-R problem to Friday

Q1. In calculating the shear flow associated with the nail shown, which areas should be included in the calculation of Q ?



(**A**) Areas (1) and (5) ;

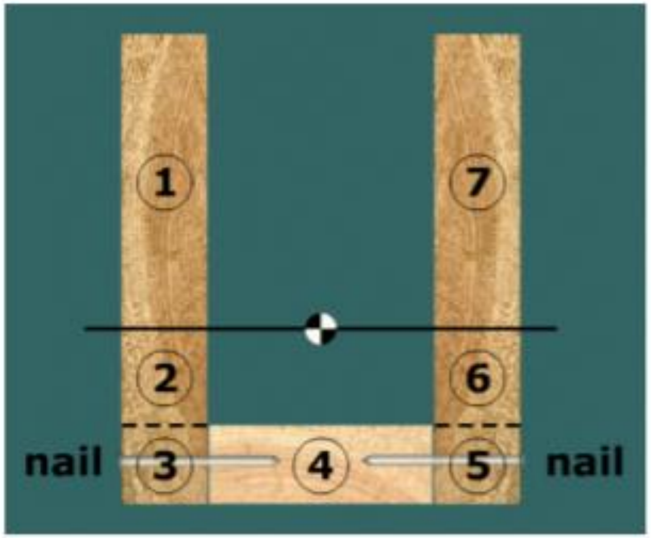
(**B**) Areas (1) through (5) ;

(**C**) Areas (2), (3) and (4) ;

(**D**) Areas (1), (2), (4), and (5)

Ans: (B)

Q2. In calculating the shear flow associated with the two nails shown, which areas should be included in the calculation of Q? ?



(**A**) Areas (2) and (6) ;

(**B**) Areas (2),(3), (5) through (6) ;

(**C**) Area (4) ;

(**D**) Areas (3), (4) and (5)

Ans: (C)

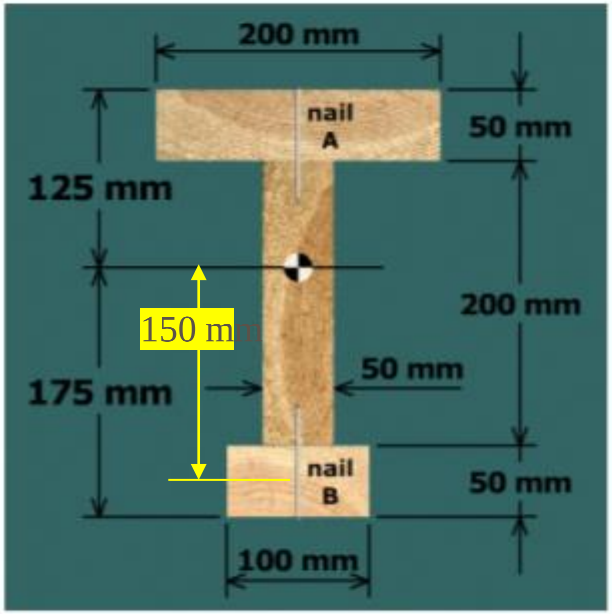
Q3. What is the value of Q needed to determine the shear force acting on nail B?

(**A**) 390,625 mm^3 ;

(**B**) 1,140,625 mm^3 ;

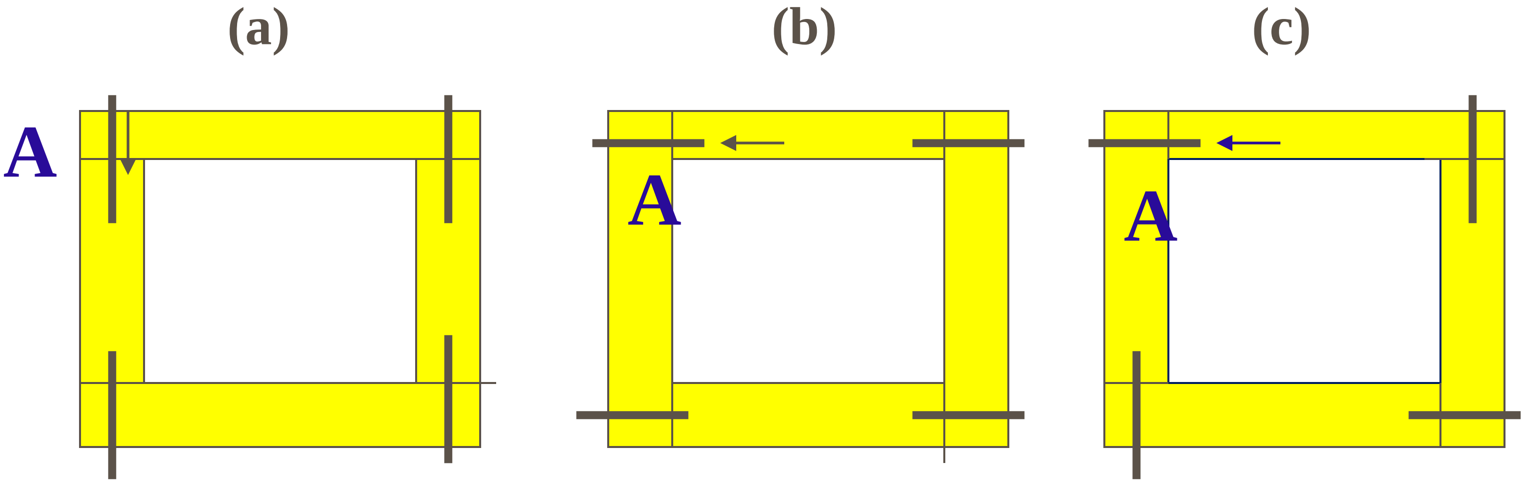
(**C**) 875,000 mm^3 ;

(**D**) 750,000 mm^3



Ans: (D)

Question 4



We have three box-beams with the same cross section and the same nail spacing.
Q: What is the shear stress component in the top blank near nail A ?

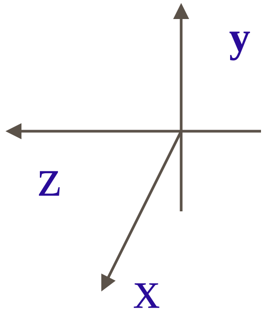
$(A)\quad (a)\ \tau_{xz},\ (b)\ \tau_{xy},\ \text{and}\ (c)\ \tau_{xz};$

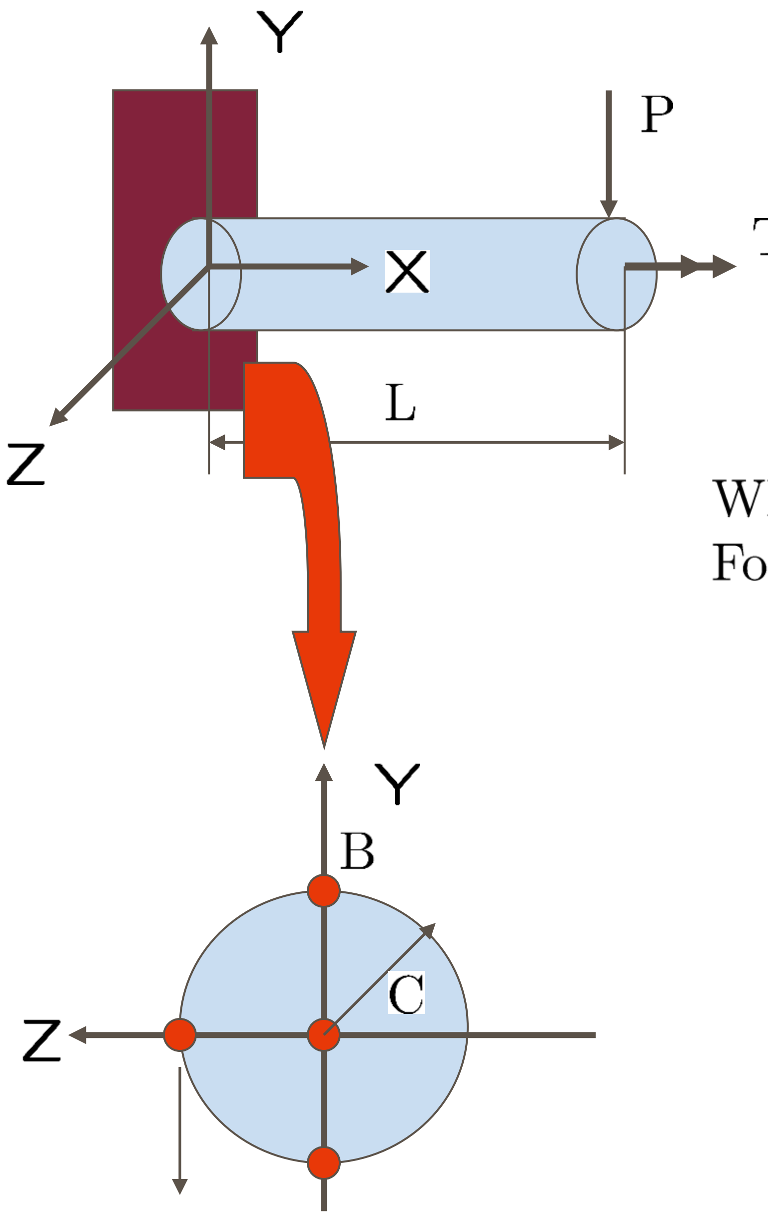
$(B)\quad (a)\ \tau_{xy},\ (b)\ \tau_{xz},\ \text{and}\ (c)\ \tau_{xy};$

$(C)\quad (a)\ \tau_{xy}\ ,(b)\ \tau_{xz}\ ,\ \text{and}\ (c)\ \tau_{xz};$

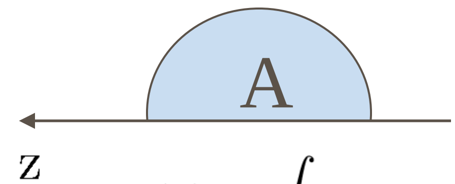
$(D)\quad (a)\ \tau_{xy},\ (b)\ \tau_{xy}\ ,\ \text{and}(c)\ \tau_{xy};$

Ans: (c)





Question 5



$$Q(0) = \int_A y dA$$

Which the following statements are correct:
For the shear stress at point B:

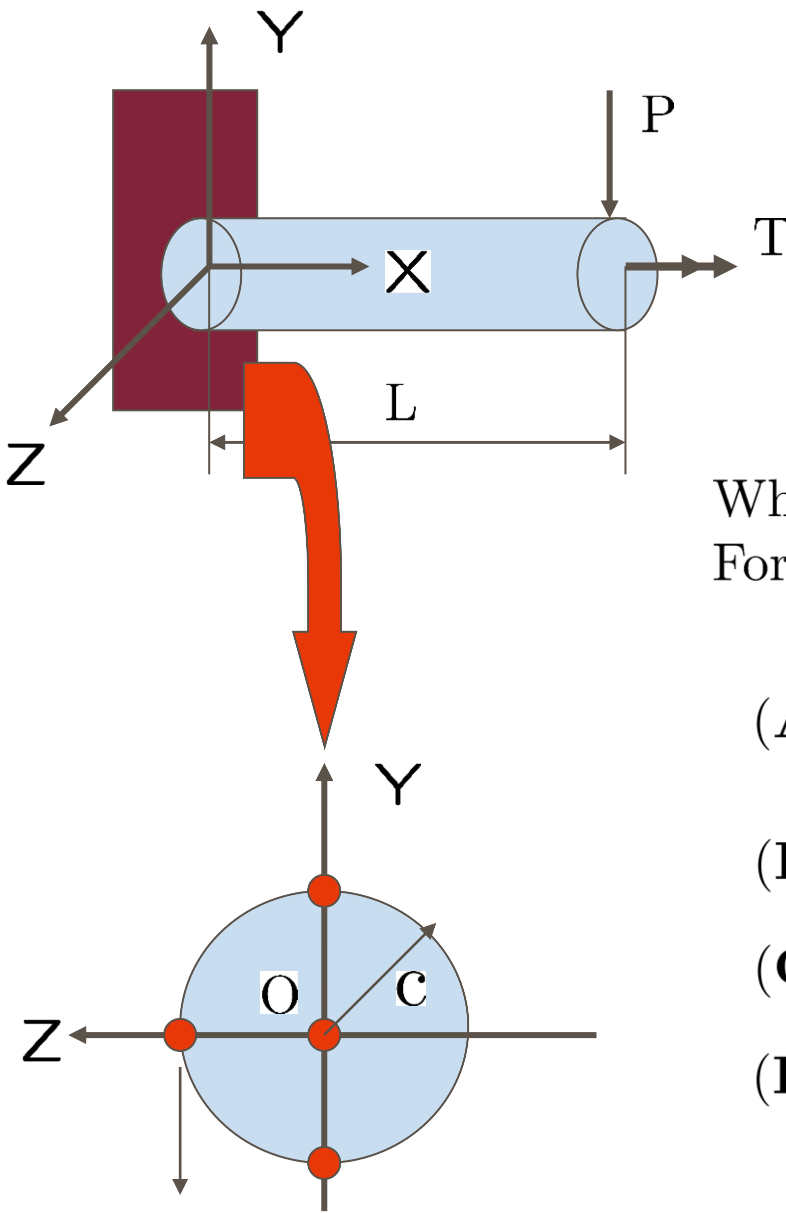
$$(\mathbf{A}) \quad \tau_{xz} = \frac{Tc}{J}$$

$$(\mathbf{B}) \quad \tau_{xy} = \frac{PQ(0)}{2cI_z}$$

$$(\mathbf{C}) \quad \tau_{xz} = 0;$$

$$(\mathbf{D}) \quad \tau_{xz} = -\frac{Tc}{J};$$

Ans: (a)



Question 6

Which the following statements are correct:
For the shear stress at point O:

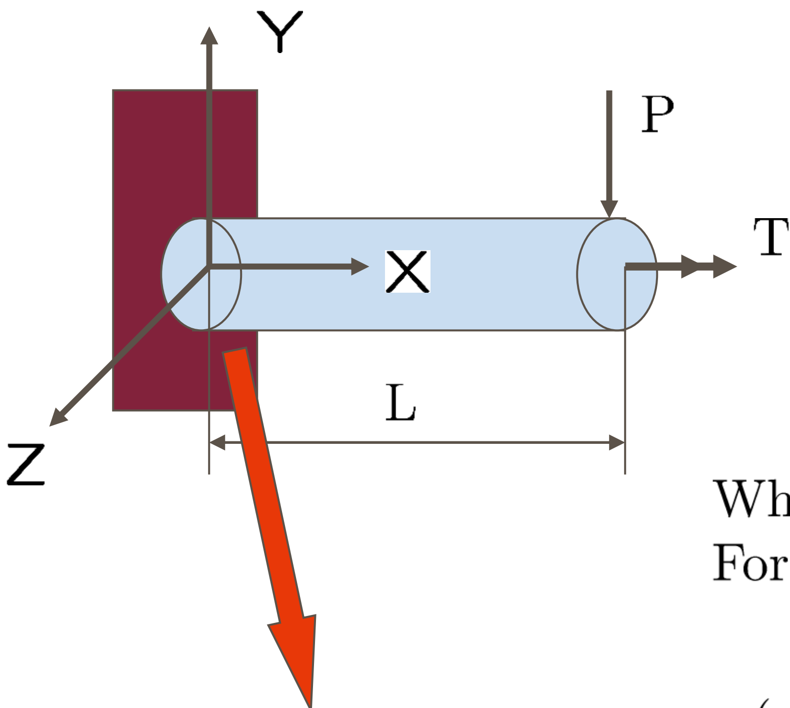
$$(\mathbf{A}) \quad \tau = \frac{Tc}{J} + \frac{PQ(0)}{2cI_z}$$

$$(\mathbf{B}) \quad \tau = \frac{PQ(0)}{2cI_z}$$

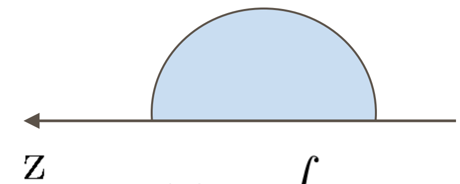
$$(\mathbf{C}) \quad \tau_{xy} = \tau;$$

$$(\mathbf{D}) \quad \tau_{xz} = -\frac{Tc}{J};$$

Ans: (b)

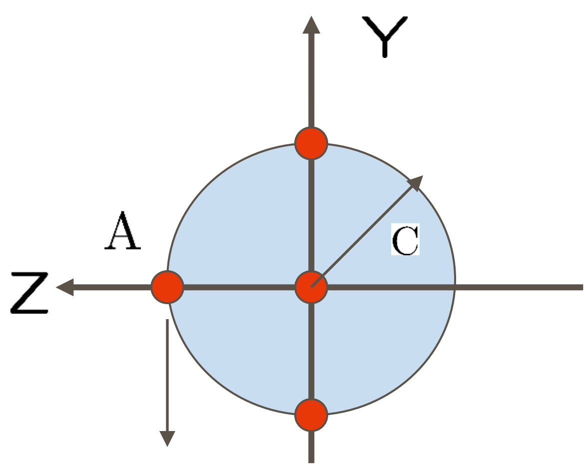


Question 7



$$Q(0) = \int_A y dA$$

Which the following statements are correct:
For the shear stress at point A:



(A) $\tau = \dfrac{Tc}{J}$

(B) $\tau = \dfrac{Tc}{J} - \dfrac{PQ(0)}{2cI_z}$

(C) $\tau = \dfrac{Tc}{J} + \dfrac{PQ(0)}{2cI_z}$

(D) $\tau \ is \ \tau_{xy};$

(E) $\tau \ is \ \tau_{xz};$

Ans: (c)

Question 8

A principal plane is a plane of

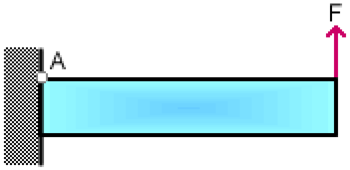
(a) Zero tensile stress;
(b) Zero compressive stress;
(c) Zero shear stress;
(d) None of above.

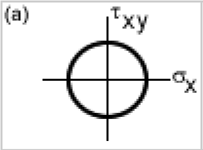
Ans: (C)

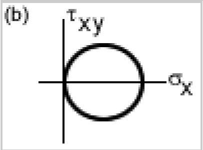
Question 9

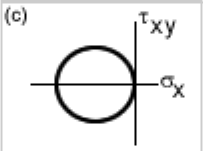
4

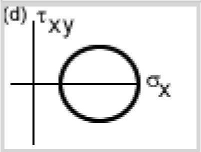
A cantilever beam is loaded as shown here. Which one of the Mohr's circle shown here represents the stress element at point A?



A) 

B) 

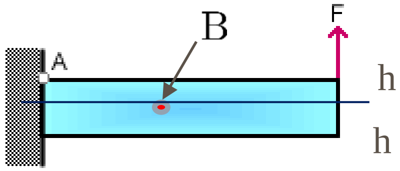
C) 

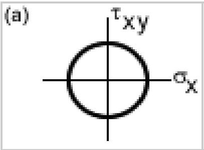
D) 

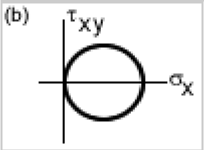
Ans: (c)

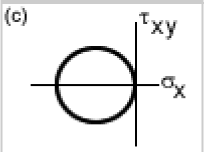
Question 10

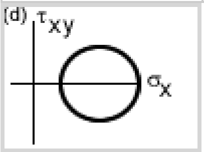
A cantilever beam is loaded as shown here. Which one of Mohr's circles shown below represents the stress state at point B?



A) 

B) 

C) 

D) 

Ans: (A)