CE30 – Discussion 12

Deflection of Beams

Textbook: 15.1, 15.2, 15.3

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Announcements

- HW12 Problems from the textbook:

  13. 26, 15.5, 15.19, 15.26, 15.43, 15.52

- MATLAB assignment

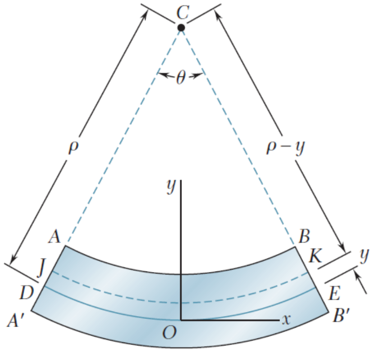
  See Lecture 32 slides for details

Beam Bending

- In Chapter 12 under pure bending, we had $\frac{1}{\rho} = \frac{M}{EI}$
- Internal moment (M) and material properties (EI) can be functions of distance (x)
- Usually, we have the same material and constant cross section, thus

$$\frac{1}{\rho} = \frac{M(x)}{EI}$$

Curvature of beam at point (x)



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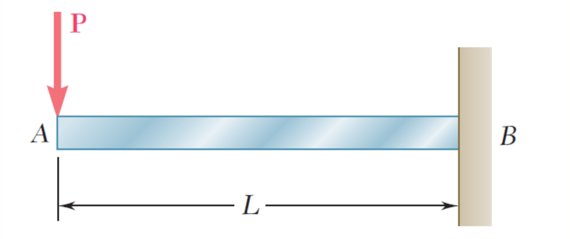
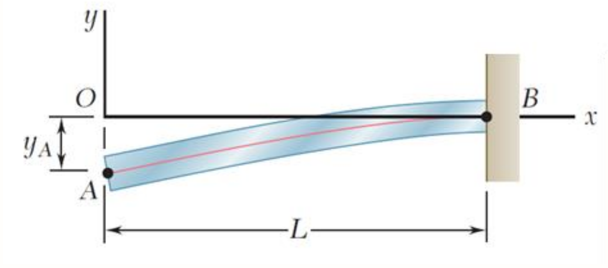
Deflection of Beams

- **Deflection $y(x)$ = Vertical Displacement of Beam**
- Example:



Cantilever beam with end load



Beam moves down in the y-direction

$y(x)$ is the deflection at point (x)

Deflection of Beams: Elastic Curve

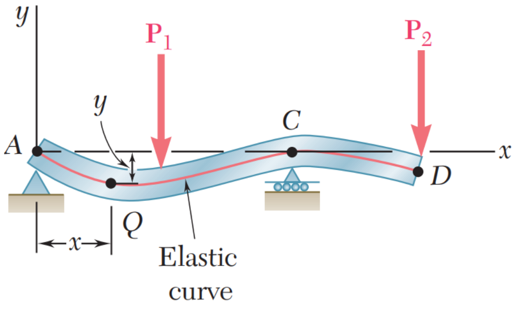
- We want to find the deflection $y(x)$, given loads and section properties.
- Using the curvature formula and some calculus…

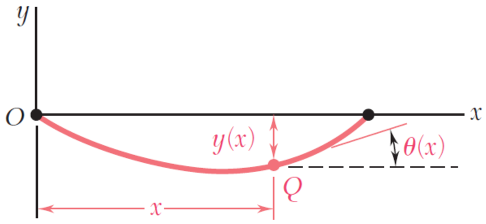
$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

$y(x)$: Deflection at x

$\frac{dy}{dx} = \theta$: Slope at x

The governing equation of the elastic curve





Elastic Curve

Direct integration of this ODE would give us two integraion constants:

Governing ODE: $$EI\,\frac{d^2y}{dx^2} = M(x)$$

Integrate once: $$EI\,\frac{dy}{dx} = \int M(x)\,dx + \boldsymbol{C_1} \xrightarrow{\;\frac{dy}{dx} = \theta\;} EI\,\theta(x) = \int M(x)\,dx + \boldsymbol{C_1}$$

Integrate again: $$EI\,y(x) = \int\int M(x)\,dxdx + \boldsymbol{C_1}x + \boldsymbol{C_2}$$

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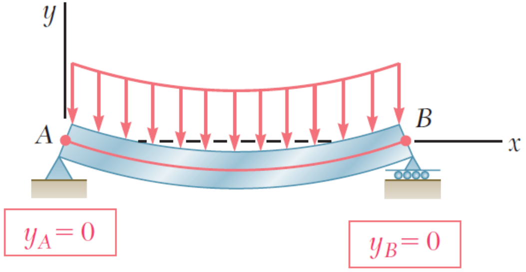
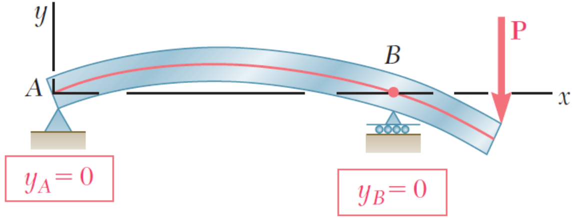
Elastic Curve

$$EI\, y(x) = \int \int M(x)\, dxdx + C_1 x + C_2$$

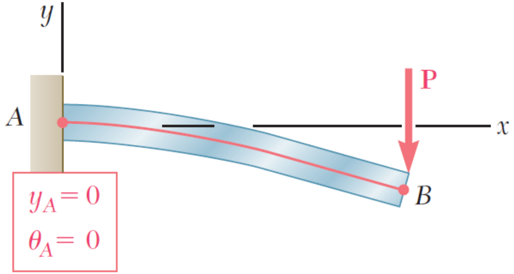
- Constants C_1 and C_2 can be found using the *boundary conditions*
- Depends on support conditions:



(a) Simply supported beam

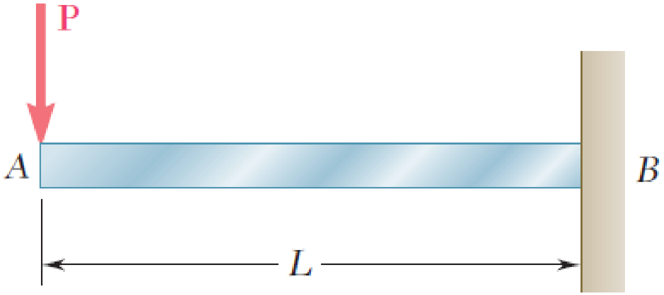


(b) Overhanging beam



(c) Cantilever beam

Example: Cantilever Beam



$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

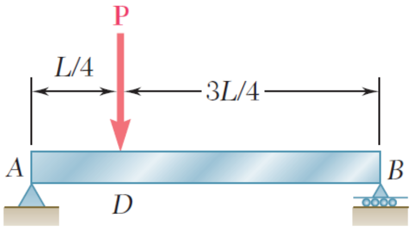
BC (1): No deflection at point B $\quad y(x = L) = 0$

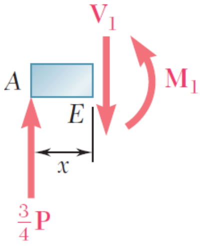
BC (2): No rotation at point B $\quad \theta(x = L) = 0$

$$y(x) = ?$$

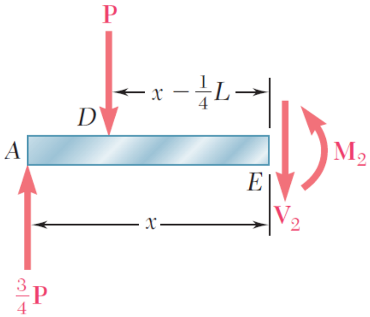
Elastic Curve: Other considerations

- M(x) might have different expression along the length



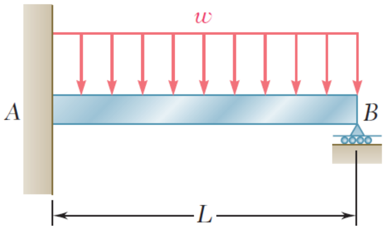


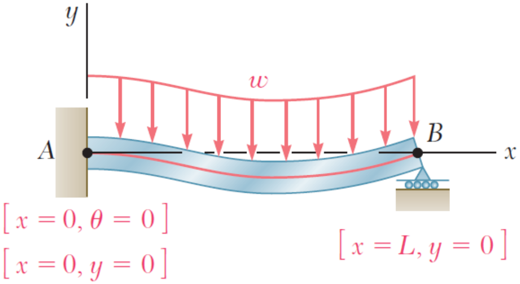
Between A-D



Between D-B

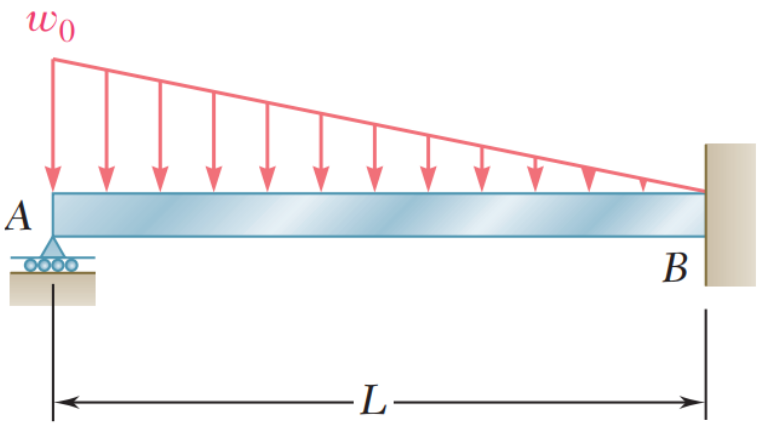
- Problem might be statically indeterminate





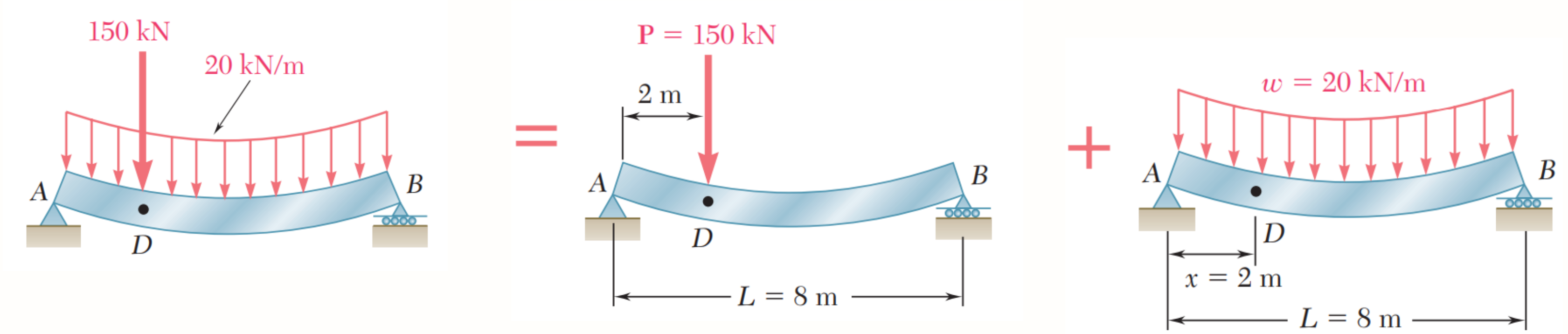
Practice – Similar to HW P15.19

For the beam and loading shown, determine the reaction at the roller support.



Method of Superposition

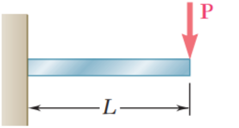
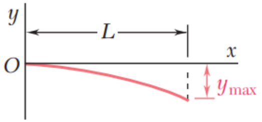
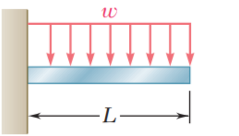
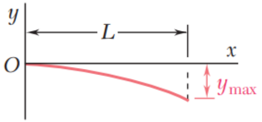
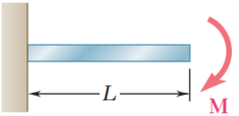
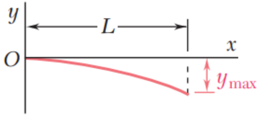
- Deflections/slopes can be computed for each individual load
- We can get their combined effect by the *principle of superposition*



Method of Superposition

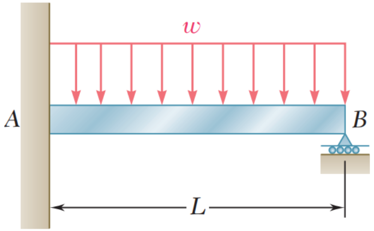
- Deflection/slope formulas are given in **Appendix C** for typical loads and supports.

APPENDIX C **Beam Deflections and Slopes**

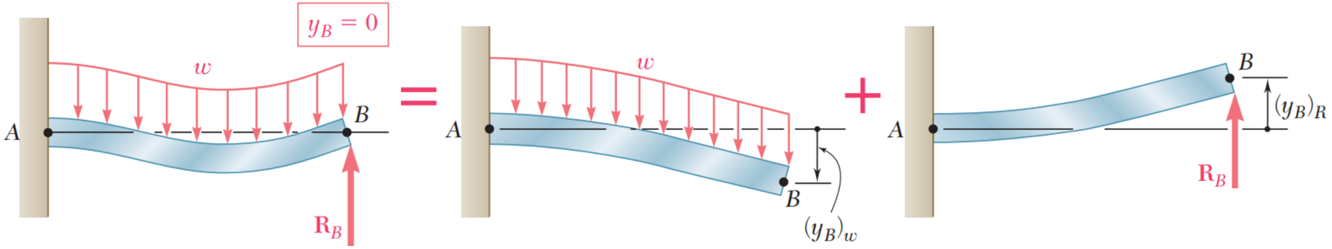
Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$	$y = \dfrac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$	$y = -\dfrac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$	$y = -\dfrac{M}{2EI}x^2$

Method of Superposition: Indeterminate

- For indeterminate beams, choose redundant reactions and apply superposition
- Consider the indeterminate beam:



- The reaction at (B) is chosen as redundant:



$$y = y_{(Load\ 1)} + y_{(Load\ 2)}$$

Practice – Similar to HW P15.43

For the statically indeterminate beam below, **find the reactions at B**