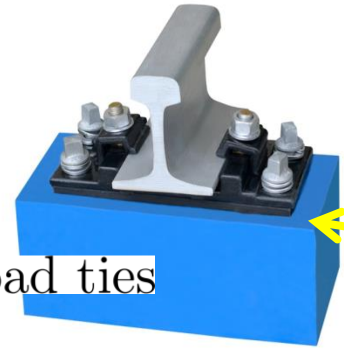
Dmitrii Ivanovich Zhuravskii (1821-1891)

Rail spikes



Railroad ties

$$\tau = \frac{VQ}{It} \ .$$

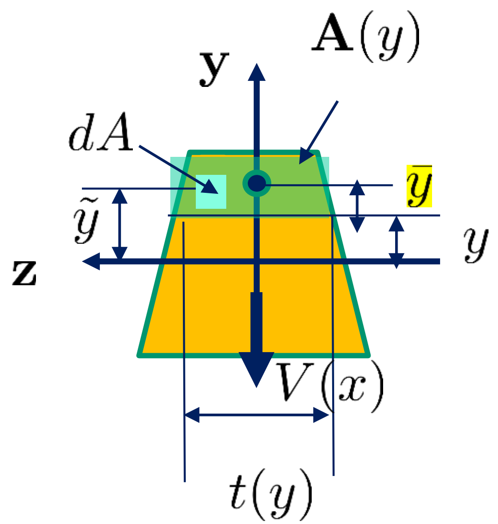
Continuous Welded Rail

Lecture 29 Shear Stress in thin-walled Members

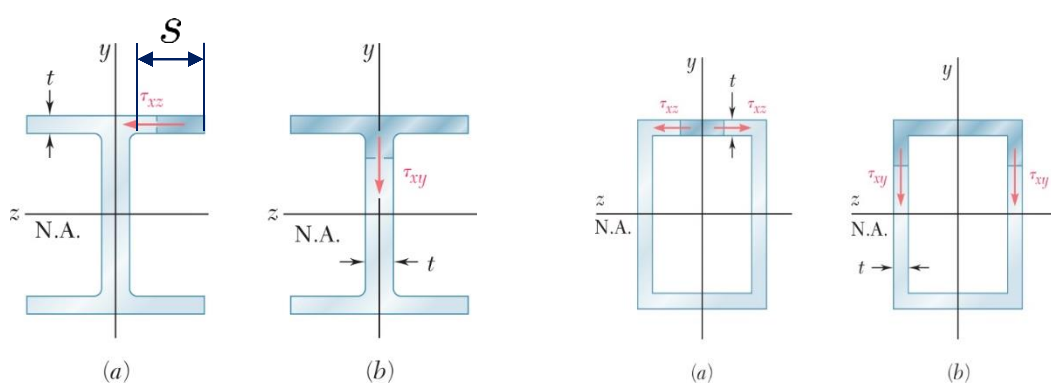
- $$\Delta H = \frac{\Delta M Q}{I_z}$$

  $$q(y) = \frac{\Delta H}{\Delta x} = \frac{VQ(y)}{I_z}$$

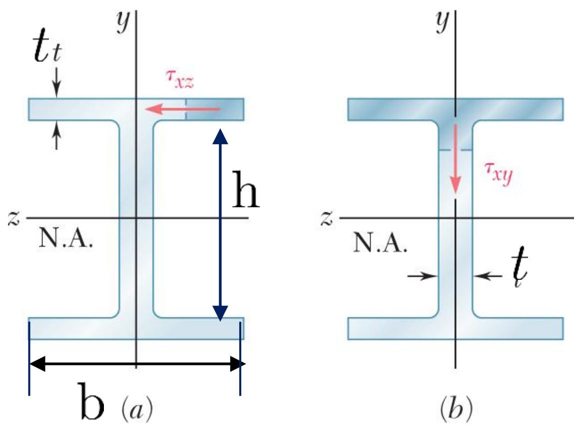
  $$\tau(x,y) = \frac{V(x)Q(y)}{I_z(x)t(y)}$$

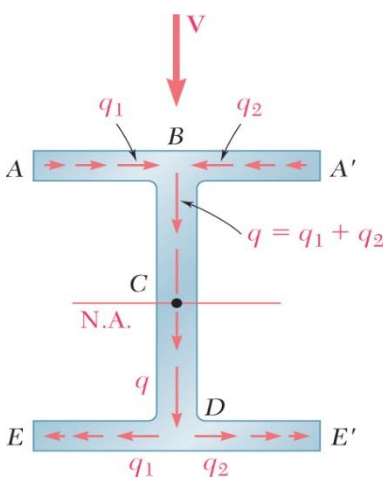


- $$\tau(s) = \frac{VQ(s)}{I_z t} \qquad q = \tau(s)t = \frac{\Delta H}{\Delta x} = \frac{VQ(s)}{I_z}$$



Shearing Stresses in I-Beam





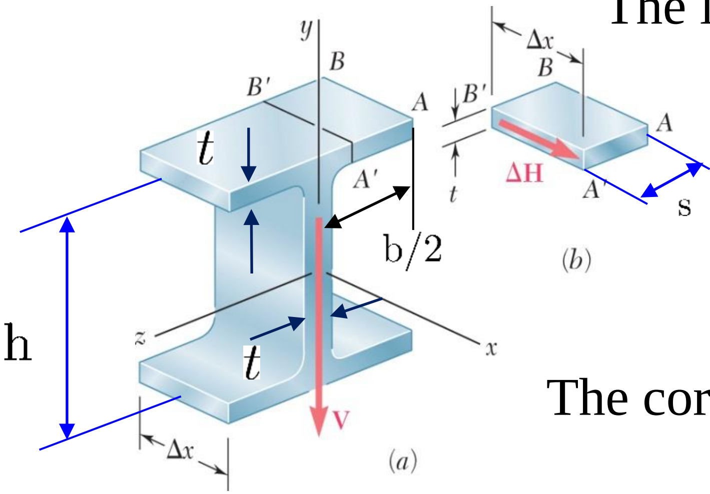
For a wide-flange beam, the shear flow increases symmetrically from zero at *A* and *A'*, reaches a maximum at *C* and then decreases to zero at *E* and *E'*.

The continuity of the variation in *q* and the merging of *q* from section branches suggests an analogy to fluid flow.

$$I = \frac{1}{12} b(h+2t)^3 - \frac{1}{12}\frac{2\times}{2}(b-t)h^3$$

Today's Lecture Attendance Password is: **I-Beam**

Shearing Stresses in I-beam (I. Calculate τ_{xz})

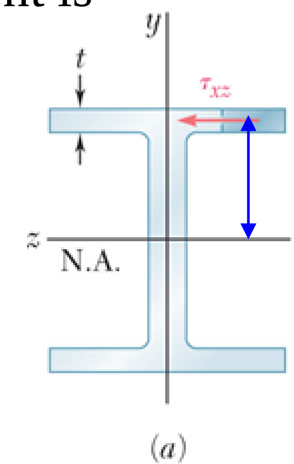


The longitudinal shear force on the element is

$$\Delta H = \frac{\Delta M Q}{I}$$

$$\bar{y} = \frac{h+t}{2}$$

$$A = st$$



The corresponding shear stress is

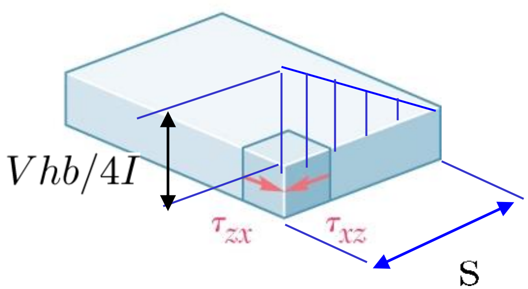
$$\tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t\,\Delta x} = \frac{VQ}{It}$$

$$Q = st \cdot \frac{h+t}{2} \approx st \cdot \frac{h}{2}$$

$$q_h = \frac{VQ}{I} = \frac{Vsth}{2I}$$

When $s = b/2$,

$$\boxed{q_{h,max} = \frac{Vbth}{4I}.}$$



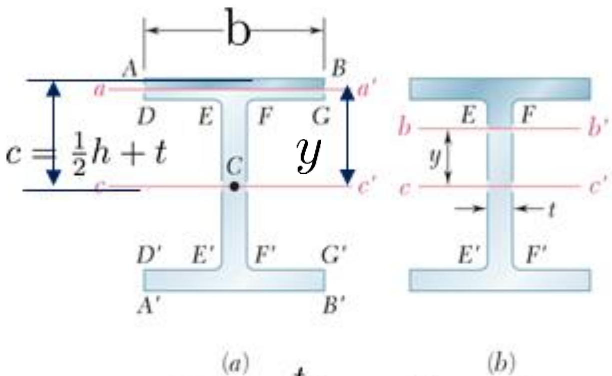
$$\tau_{zx} = \frac{Vhs}{2I}, \quad 0 < s < b/2$$

NOTE: $\tau_{xy} \approx 0$ in the flanges

$\tau_{xz} \approx 0$ in the web

Shearing Stresses in I-Beam: (II) Calculate τ_{xy}

For a rectangular beam, $\bar{y} = y + \frac{1}{2}(c-y) = \frac{1}{2}(c+y)$



$$A = b(c-y)$$

$$Q = \bar{y}A = \frac{b}{2}(c^2 - y^2)$$

$$q_v = \frac{VQ}{I} = \frac{Vb}{2I}(c-y)(c+y)$$

$$Q(y) = \frac{t}{2}(c-y)(c+y)$$

When $y = h/2 \rightarrow Q = \frac{bt(h+t)}{2} \approx \frac{bth}{2}$.

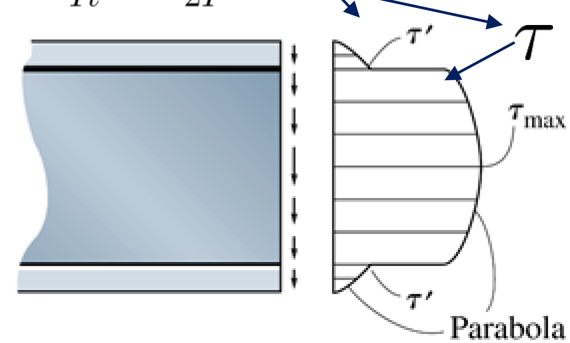
$t(y)$ has a jump from flange to web $(b \rightarrow t)$

$\tau_{xy}(y)$ has a jump from flange to web.

$$\tau_{xy}(t^-) = \tau' = \frac{VQ}{Ib} = \frac{Vth}{2I}$$

$$\tau_{xy}(t^+) = \tau = \frac{VQ}{It} = \frac{Vbh}{2I}$$

$$q = \frac{VQ}{I} \quad \text{and} \quad \tau_{xy} = \frac{VQ}{It(y)}$$



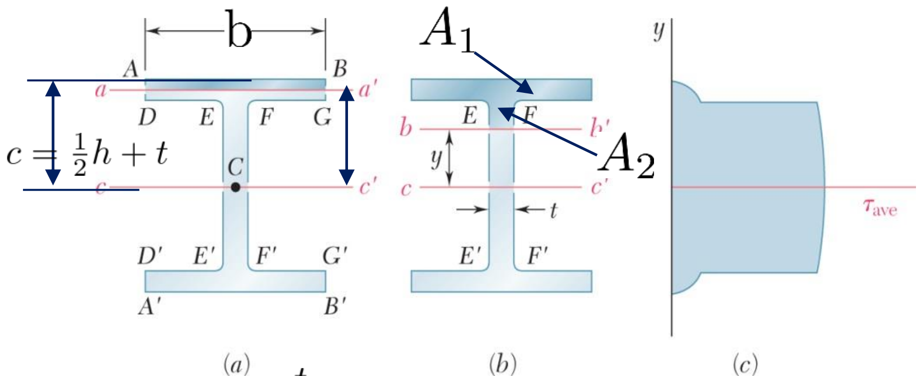
$$q_v = \frac{Vbth}{2I} = 2q_{h,max}$$

Why ?

$$\tau' = \frac{Vth}{2I} \qquad \tau = \frac{Vbh}{2I}$$

Shearing Stresses in I-Beam: (II) Calculate τ_{xy}

$$\tau_{xy} = \frac{VQ}{It(y)}$$



$$Q(y) = \frac{t}{2}(c-y)(c+y)$$

When $y = h/2 \rightarrow Q_1 = \frac{bt_0(h+t)}{2} \approx \frac{bth}{2}.$

$$Q_2 = \bar{y}A = t((h/2) - y)((h/2 - y)/2 + y)$$
$$= \frac{t}{2}((h/2)^2 - y^2)$$

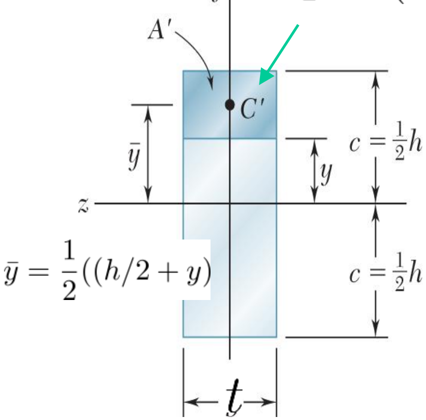
$$\tau = \frac{V}{2I}(bh + ((h/2)^2 - y^2))$$

$$\tau_{max} = \frac{V}{2I}(bh + (h/2)^2)$$

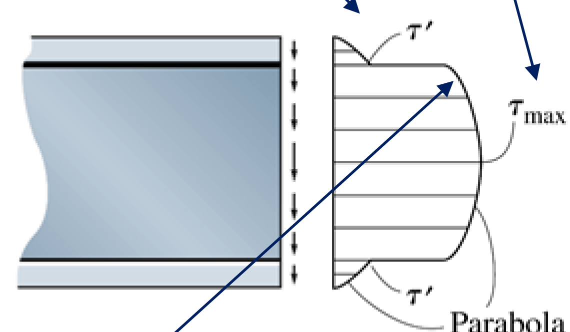
$t(y)$ has a jump from flange to web $(b \rightarrow t)$.

$\tau_{xy}(y)$ has a jump from flange to web.

$$A_2 = t(h/2 - y)$$



$$\tau' = \frac{Vth}{2I}$$



$$\tau = \frac{Vbh}{2I}$$

Summary

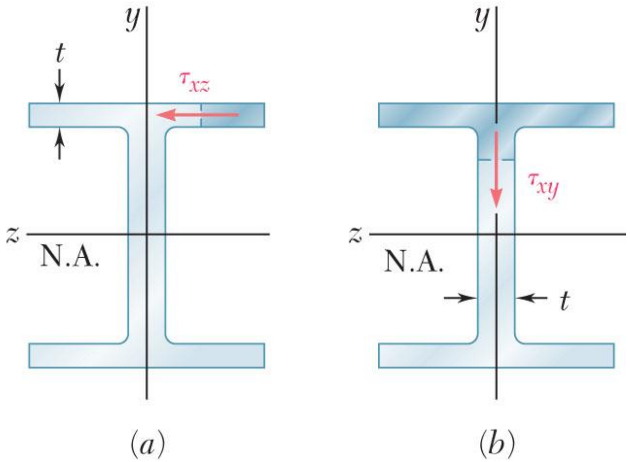


Fig. 13.24 Wide-flange beam showing shearing stress (*a*) in flange (*b*) in web.

$$q_v = \frac{Vbth}{2I} = 2q_{h,max}$$

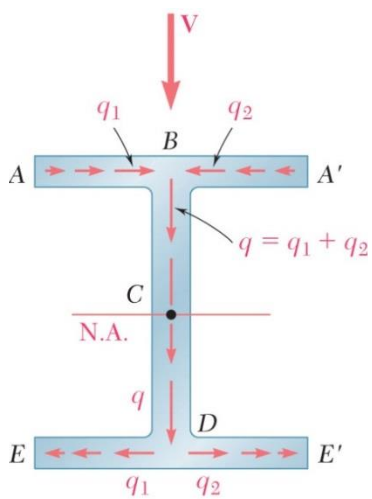
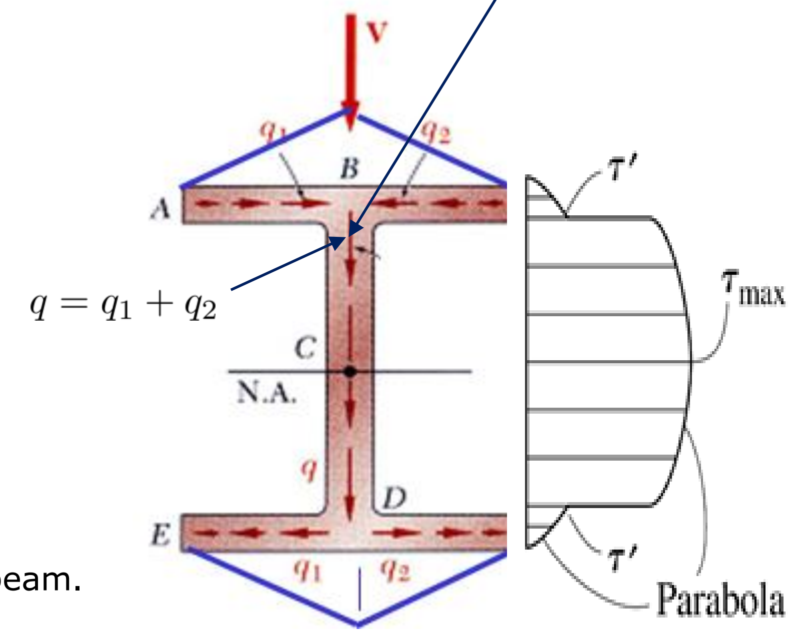


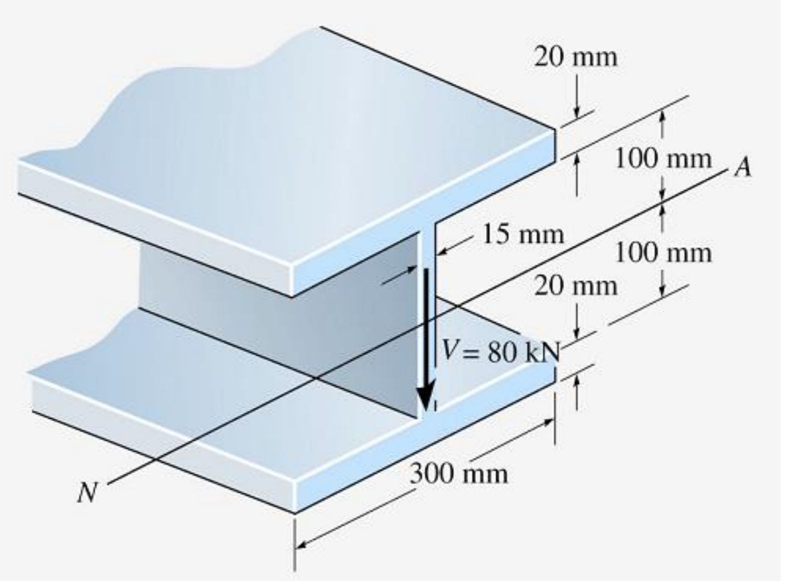
Fig. 13.27 Shear flow, *q*, in a wide-flange beam.



EXAMPLE 7–2 Calculate τ_{xy}

A steel wide-flange beam has the dimensions shown in Fig. 7–11*a*. If it is subjected to a shear of $V = 80$ kN, (*a*) plot the shear-stress distribution acting over the beam's cross-sectional area, and (*b*) determine the shear force resisted by the web.

$$\begin{aligned} I &= \frac{1}{12}(0.30)(0.24)^3 - \frac{1}{12}(0.285)(0.20)^3 \\ &= 155.6 \times 10^{-6} m^4 \end{aligned}$$



For point B', $t_{B'} = 0.300$ m, and A' is the dark shaded area shown

$$Q_{B'} = \bar{y}'A' = [0.110 \text{ m}](0.300 \text{ m})(0.02 \text{ m}) = 0.660(10^{-3}) \text{ m}^3$$

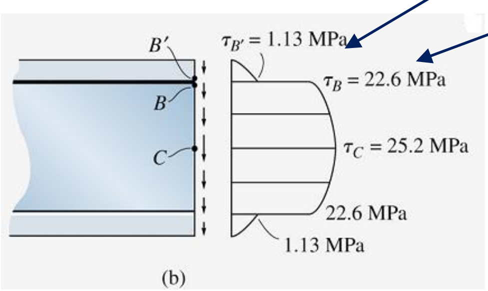
$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80 \text{ kN}(0.660(10^{-3}) \text{ m}^3)}{155.6(10^{-6}) \text{ m}^4 \boxed{(0.300 \text{ m})}} = 1.13 \text{ MPa}$$

$Q_B = Q_{B'}$

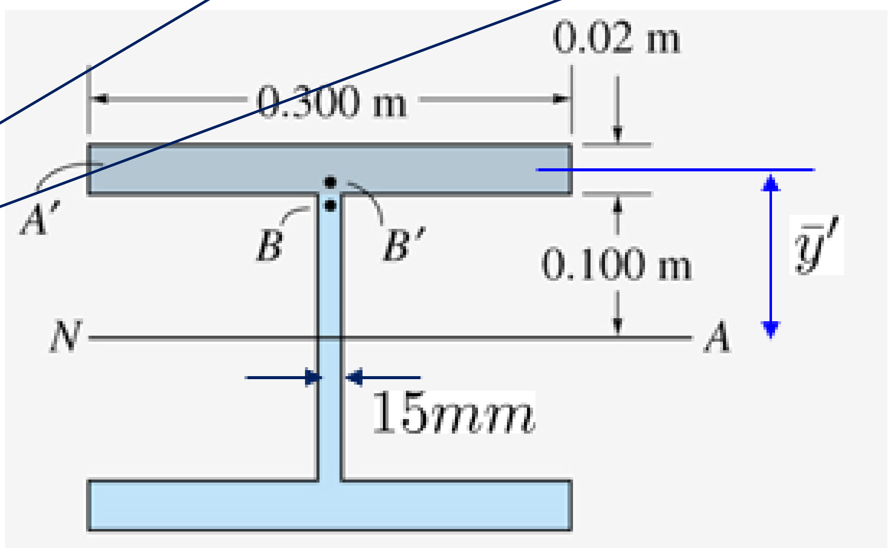
For point B, $t_B = 0.015$ m and $Q_B = Q_{B'}$,

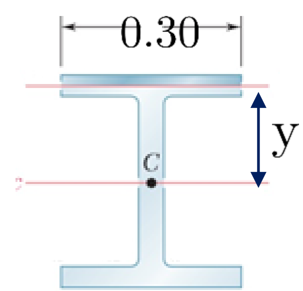
$$\tau_B = \frac{VQ_B}{It_B} = \frac{80 \text{ kN}(0.660(10^{-3}) \text{ m}^3)}{155.6(10^{-6}) \text{ m}^4 \boxed{(0.015 \text{ m})}} = 22.6 \text{ MPa}$$

$t_B \neq t_{B'}$



(b)





results an abrupt change in τ_{xy} !

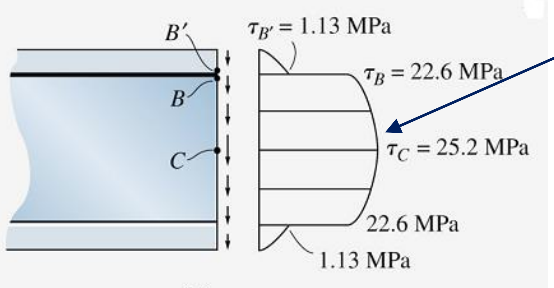
An abrupt change in t at B

For point C, $t_C = 0.015$ m and A' is the dark shaded area shown

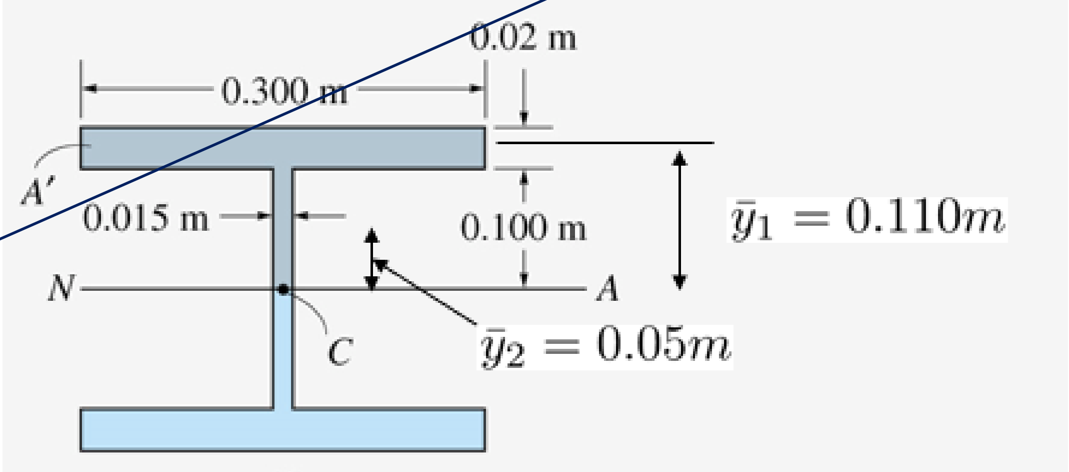
Considering this area to be composed of two rectangles,

$$Q_C = \Sigma\, \bar{y}'A' = [0.110 \text{ m}](0.300 \text{ m})(0.02 \text{ m}) \leftarrow Q_1$$
$$+ [0.05 \text{ m}](0.015 \text{ m})(0.100 \text{ m}) \leftarrow Q_2$$
$$= 0.735(10^{-3}) \text{ m}^3$$

$$\tau_C = \tau_{\max} = \frac{VQ_C}{It_C} = \frac{80 \text{ kN}[0.735(10^{-3}) \text{ m}^3]}{155.6(10^{-6}) \text{ m}^4(0.015 \text{ m})} = 25.2 \text{ MPa}$$



(b)



(d)

For $0 \le y \le 0.1$:

$$Q_2 = \bar{y}A = t((h/2) - y)(h/2 - y)/2 + y)$$
$$= \frac{t}{2}((h/2)^2 - y^2)$$

Part (b). The shear force in the web will be determined by first formulating the shear stress at the *arbitrary* location *y* within the web, Fig. 7–11*e*. Using units of meters, we have

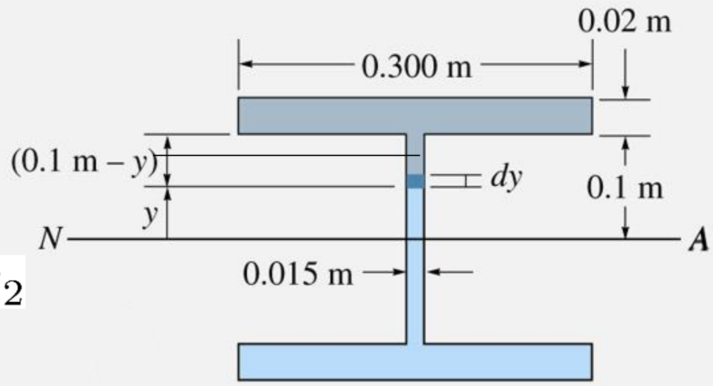
$$I = 155.6(10^{-6})\ \mathrm{m}^4$$
$$t = 0.015\ \mathrm{m}$$
$$A' = (0.300\ \mathrm{m})(0.02\ \mathrm{m}) + (0.015\ \mathrm{m})(0.1\ \mathrm{m} - y)$$

$$Q = \Sigma \bar{y}'A' = (0.11\ \mathrm{m})(0.300\ \mathrm{m})(0.02\ \mathrm{m}) \leftarrow Q_1$$

$$+[y + \tfrac{1}{2}(0.1\ \mathrm{m} - y)](0.015\ \mathrm{m})(0.1\ \mathrm{m} - y) \leftarrow Q_2$$

$$= (0.735 - 7.50\,y^2)(10^{-3})\ \mathrm{m}^3$$

(d)



(e)

$$Q = (0.735 - 7.50y^2)(10^{-3})m^3$$

$Q = (0.735 - 7.50y^2)(10^{-3})m^3$

so that

$$\tau = \frac{VQ}{It} = \frac{80 \text{ kN}(0.735 - 7.50\ y^2)(10^{-3})\text{ m}^3}{(155.6(10^{-6})\text{ m}^4)(0.015\text{ m})}$$

$$= (25.192 - 257.07\ y^2)\text{ MPa}$$

This stress acts on the area strip $dA = 0.015\ dy$

and therefore the shear force resisted by the web is

$$V_w = \int_{A_w} \tau\, dA = \int_{-0.1\text{ m}}^{0.1\text{ m}} (25.192 - 257.07\ y^2)(10^6)\boxed{(0.015\text{ m})\ dy}$$

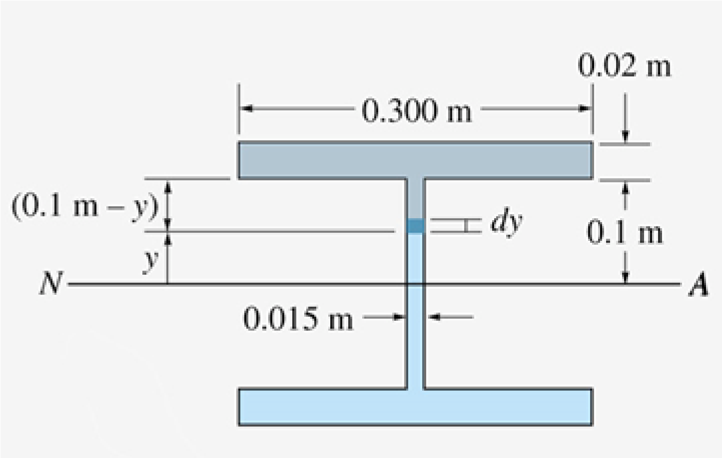
$$\boxed{V_w = 73.0\text{ kN}}$$

V= 80 kN

Note that by comparison, the web supports $\boxed{91\%}$ of the total shear (80 kN), whereas the flanges support the remaining 9%. Try solving this

Recall in the lecture before Spring Break:
How much bending moment does the web sustain ?

$$\sigma_x = -\frac{My}{I_z}$$



$$\begin{aligned} M_w &= -\int_{A_w} \sigma_x y dA = \frac{M}{I_z}\int_{A_w} y^2 dA \\ &= M\frac{I_{A_w}}{I_z} \end{aligned}$$

What are the takeaways ?

$$I_{A_w} = \tfrac{1}{12}(0.015)(0.2)^3$$
$$= 4.444 \times 10^{-6} m^4$$

1. Web sustains main part of shear force, while
2. Flange resists main part of bending moment.

$$I = 155.6 \times 10^{-6} m^4$$

$$\frac{M_w}{M} = \frac{I_{A_w}}{I_z} = \frac{4.4444}{155.6} = 2.856\%$$

12.27 Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint for Prob. 12.25.)

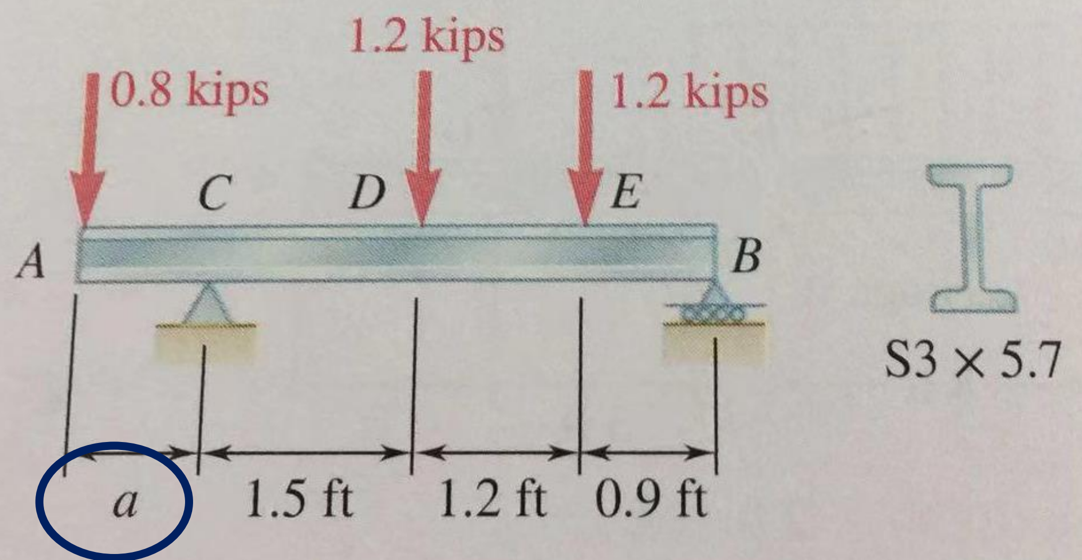
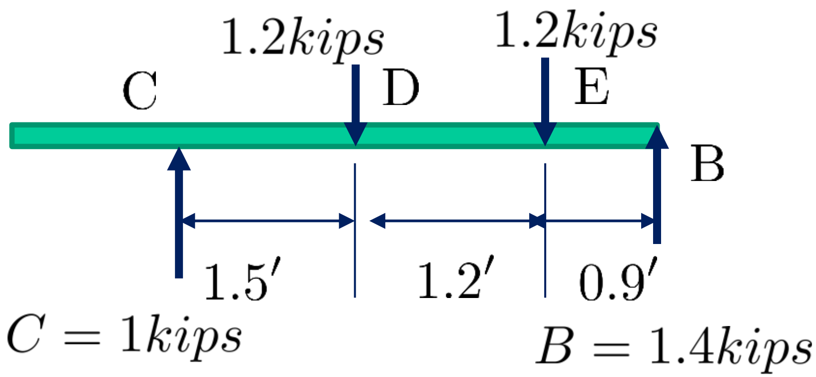
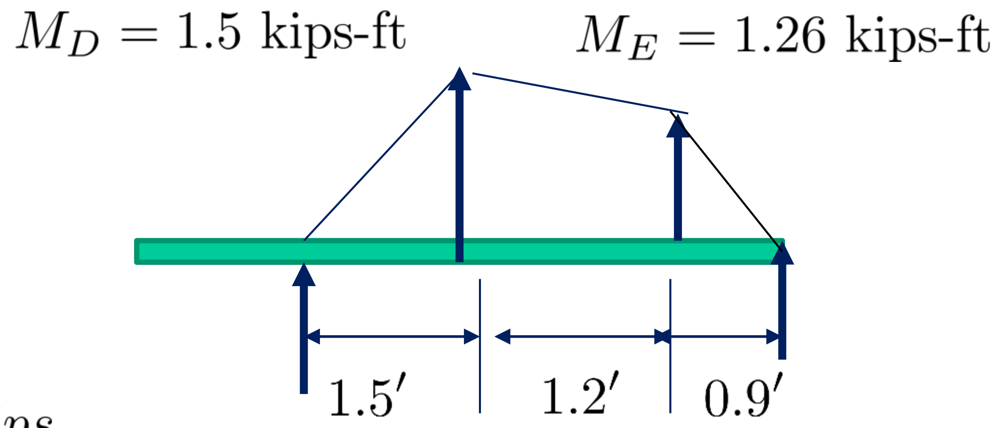


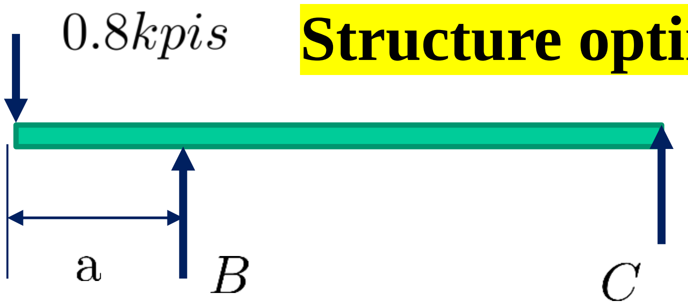
Fig. P12.27

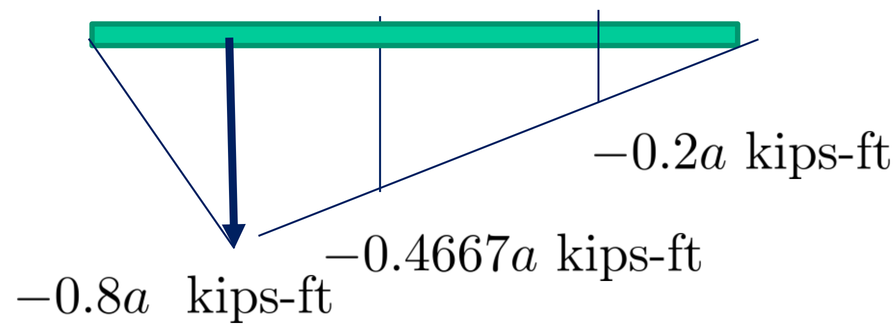
Hint: Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.

Structure optimization via linear programming





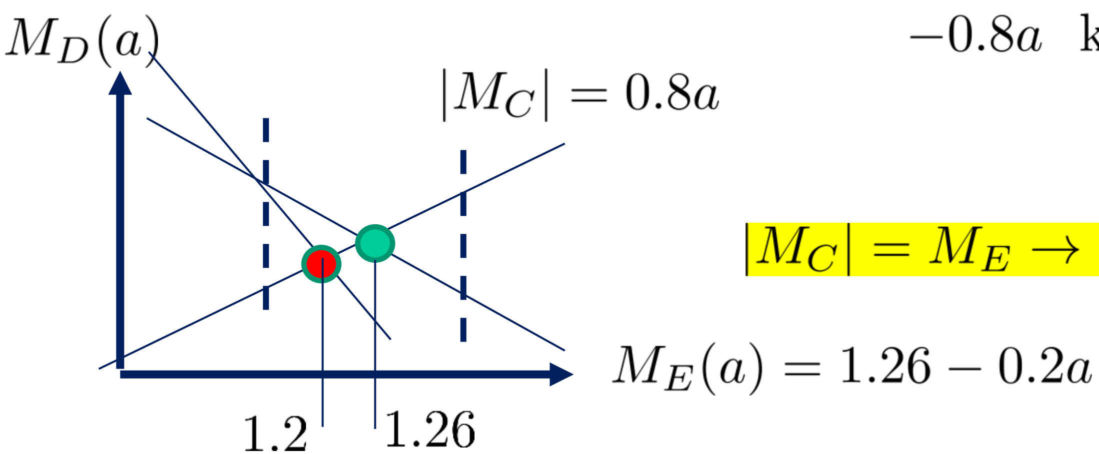




$$M_E = 1.26 - 0.2a$$

$$M_D = 1.5 - 0.4667a$$

$$|M_C| = M_E \rightarrow 1.26 - 0.2a = 0.8a \rightarrow a = 1.26ft$$



$$|M_C| = M_D \rightarrow 1.5 - 0.4667a = 0.8a \rightarrow a = 1.20ft$$