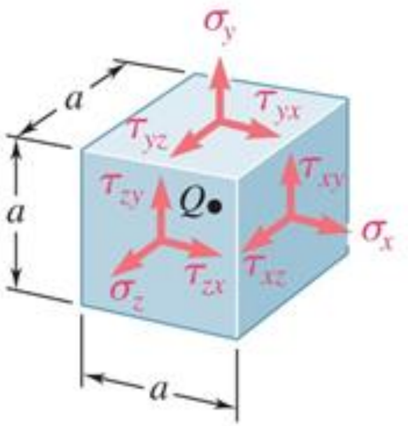
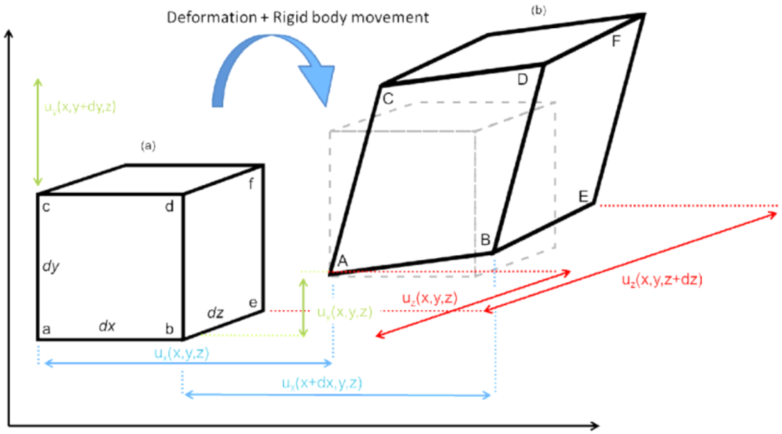
# Last Time: We defined Stress and strain




$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

# Recap: Lecture 17

1. **Stress is the intensity of internal force.**

2. **Displacement is the change of the position of material particle.**

3. **Deformation is the relative position,**

4. **Strain is the intensity of deformation or relative deformation.**

5. **Both stress and strain are symmetric tensors.**

6. **Shear strain is the change of a right angle.**

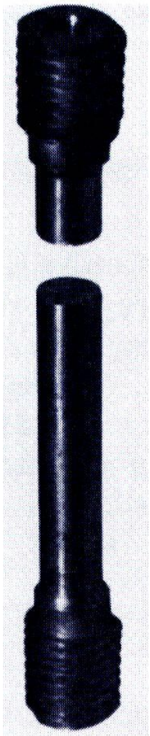
# A Fundamental Question:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

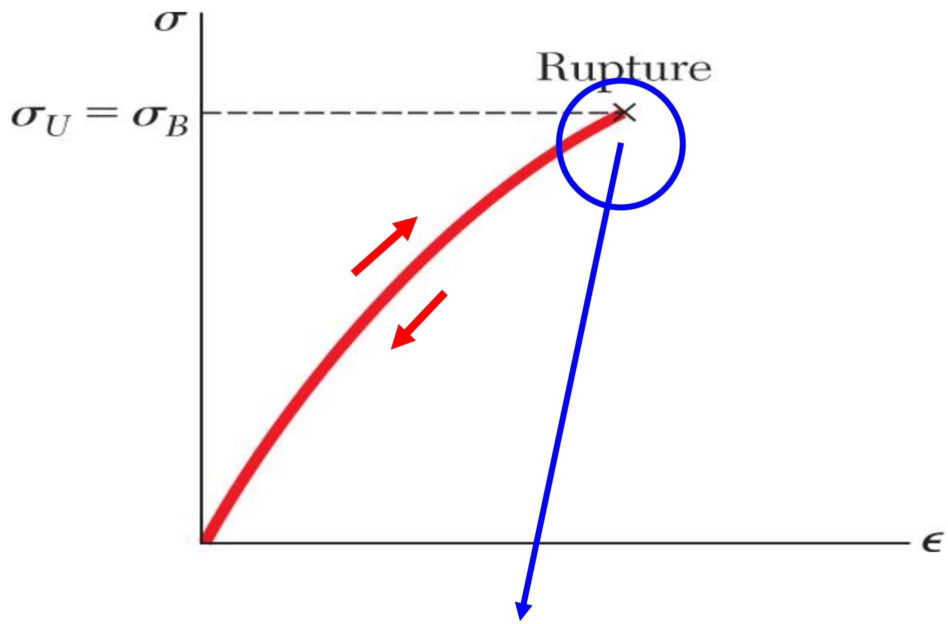
$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{bmatrix}$$

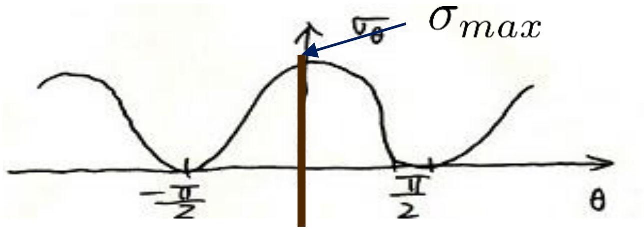
# Stress first or Strain first ?

# Stress-Strain Relation

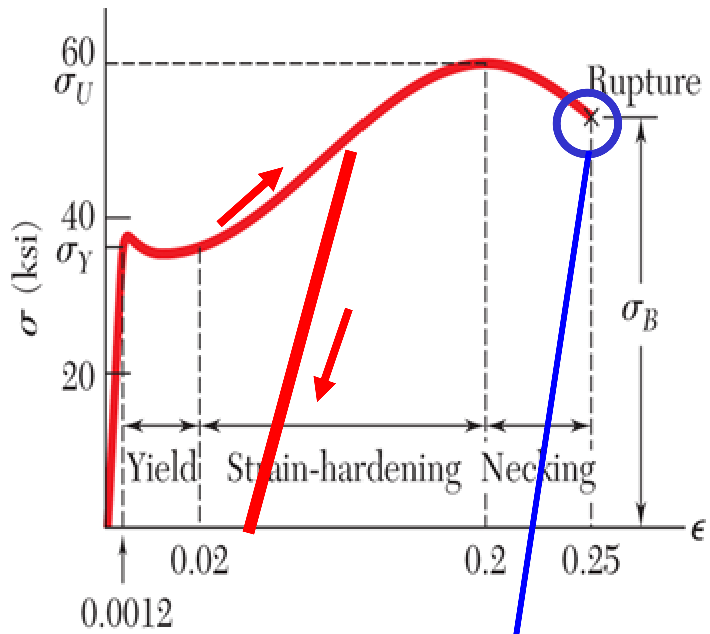


## Brittle Material

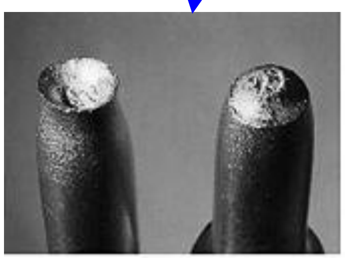


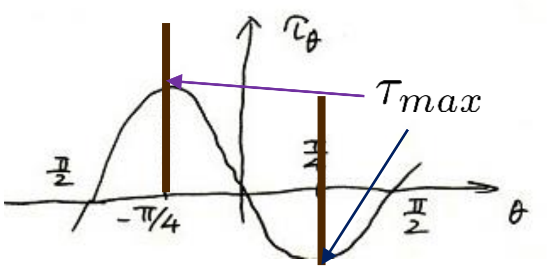
# Stress-Strain Relation:  Ductile Materials



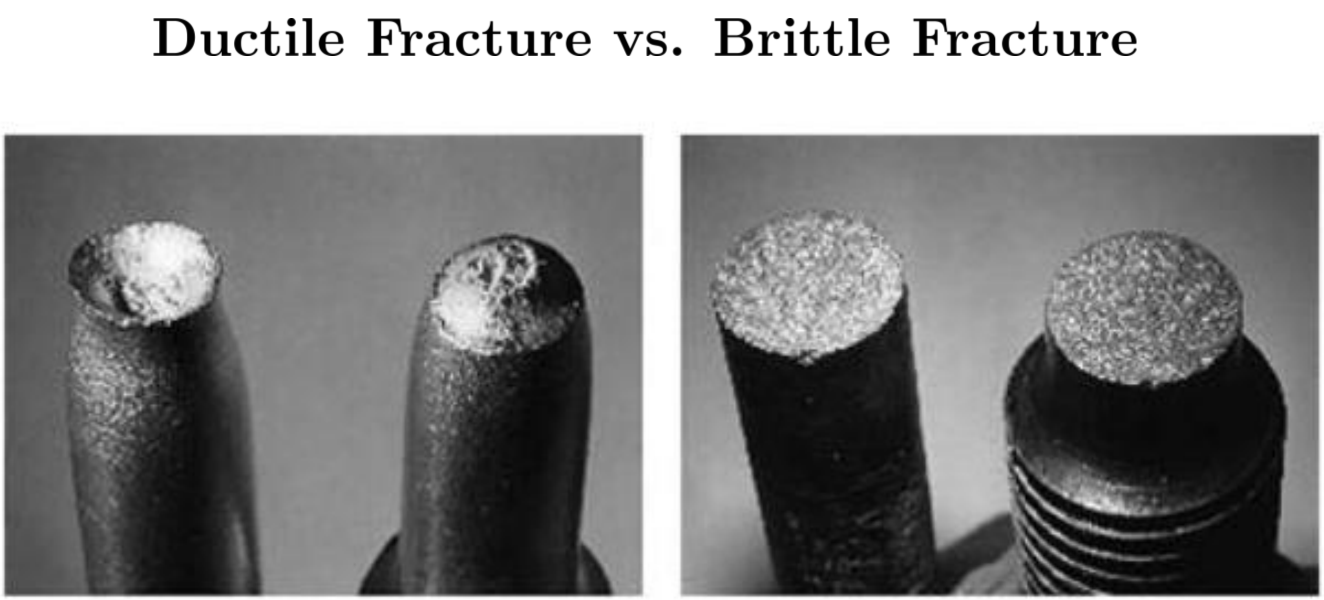

(*a*) Low-carbon steel

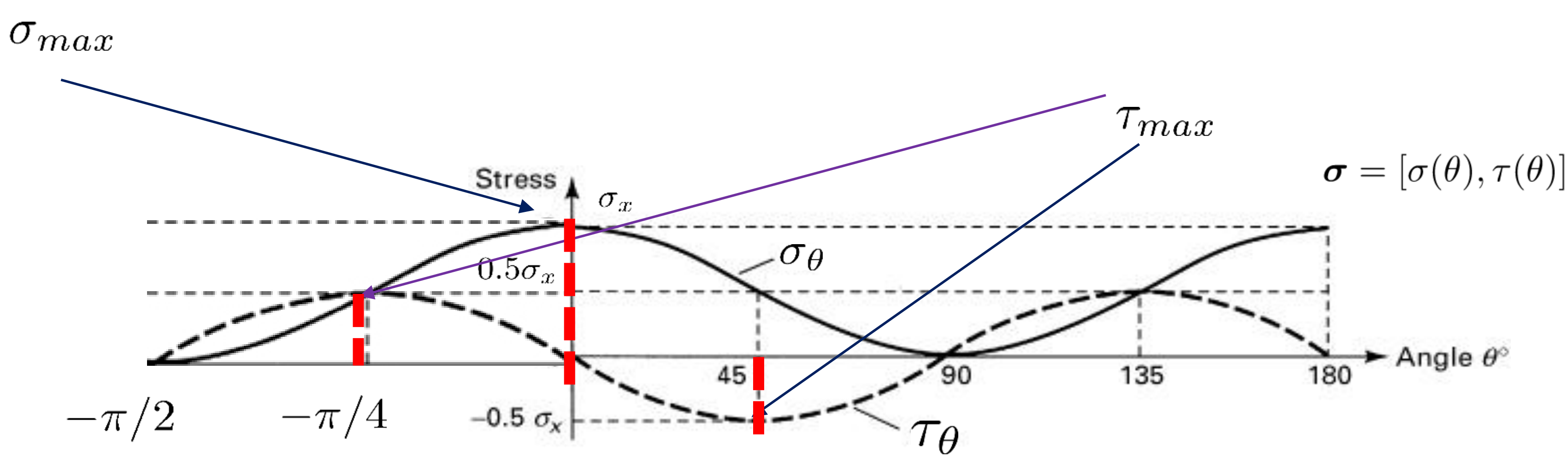


- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.



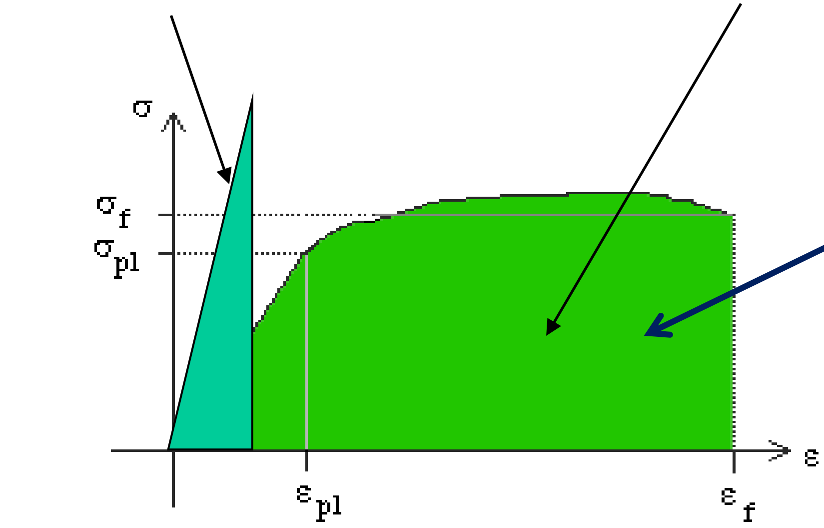
# Brittle Materials via Ductile Materials



# Material toughness

**Toughness:** **refers to the capacity of a material of absorb energy prior to failure. Its value is equal to the entire area under the stress-strain curve. In most cases, the area under the elastic portion of the curve is a very small percentage of the total area and may be ignored in the calculation of the modulus of toughness.**



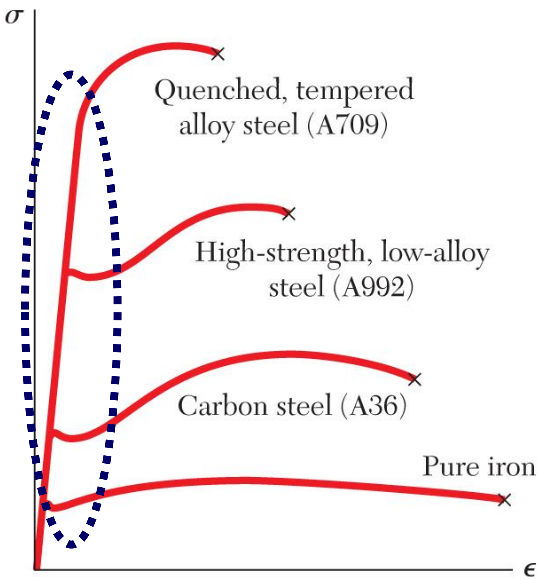

$$
\begin{aligned}
\text{Area} &= \int_0^{\epsilon} \tilde{\sigma}(\tilde{\epsilon})\, d\tilde{\epsilon} \\
&= \int_0^{\Delta L/L} \frac{\tilde{P}}{A} \frac{d\tilde{L}}{L} \\
&= \frac{1}{V} \int_0^{\Delta L} \tilde{P}\, d\tilde{L}
\end{aligned}
$$

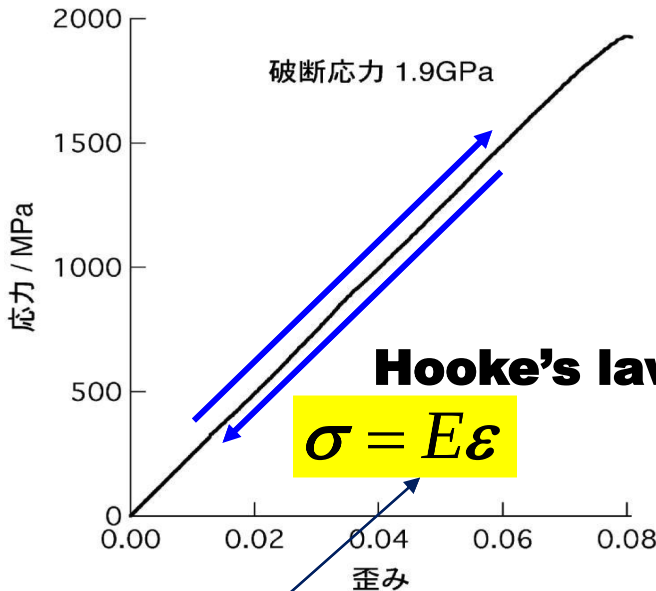
Work per unit volume

# Constitutive Modeling:
# What is elasticity ?

**Neverland ----- Forever Young**

# Constitutive Modeling: Linear Elastic Model




**Hooke's law**

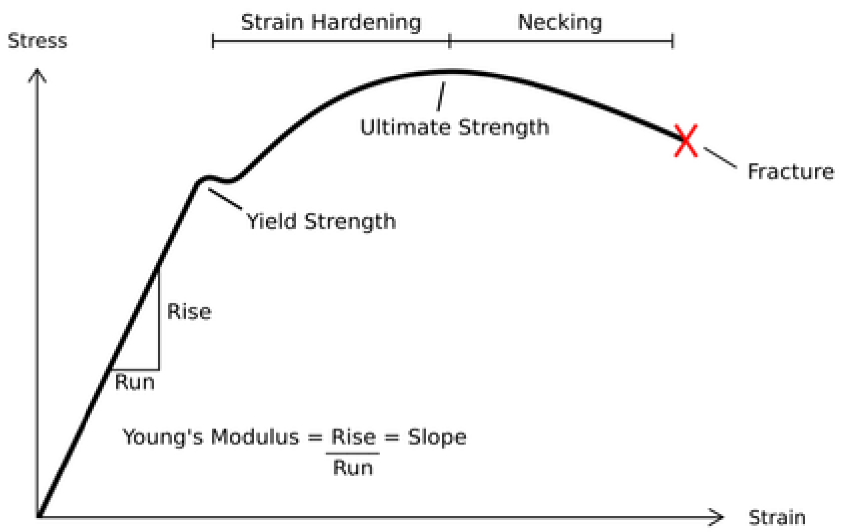
$$\sigma = E\varepsilon$$

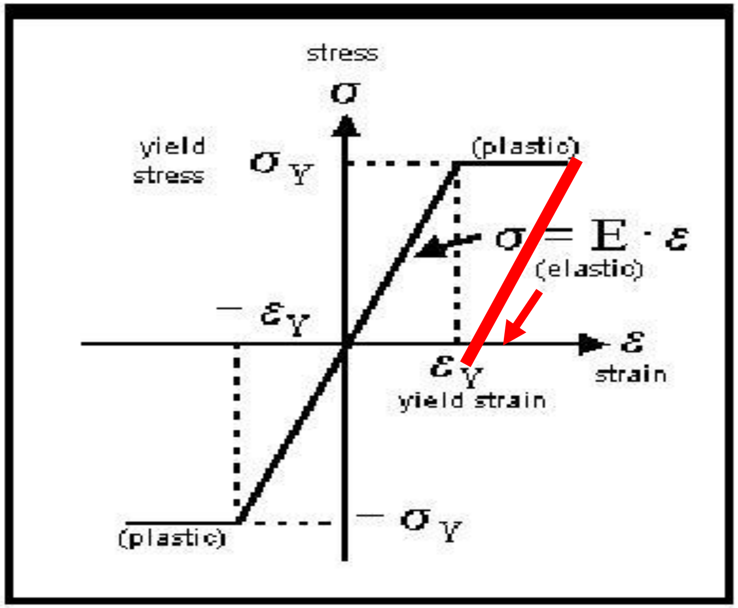
E is Young's modulus.

**Robert Hooke (1635-1703)**

Thomas Young (1773-1829)

# Constitutive Modeling




**Data**

**Model**

<u>Linear elastic-perfectly plastic</u>

SH Chicken first or egg first

**From ChatGPT**

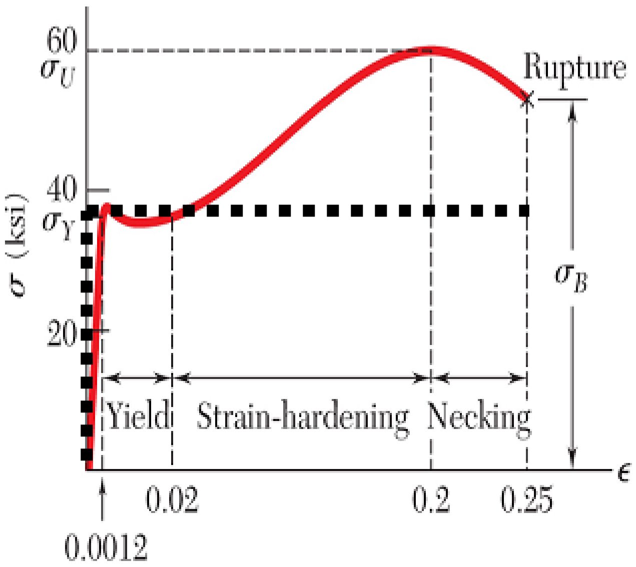
The question of whether the chicken or the egg came first is a classic philosophical and scientific puzzle that has been debated for centuries.

From a biological perspective, the answer is that the egg came first. This is because birds, including chickens, evolved from reptiles, and over a long period of time, there were gradual changes in the genetic makeup of these reptiles that eventually gave rise to the first bird-like creatures. These early birds would have laid eggs with genetic mutations that eventually led to the first chicken.

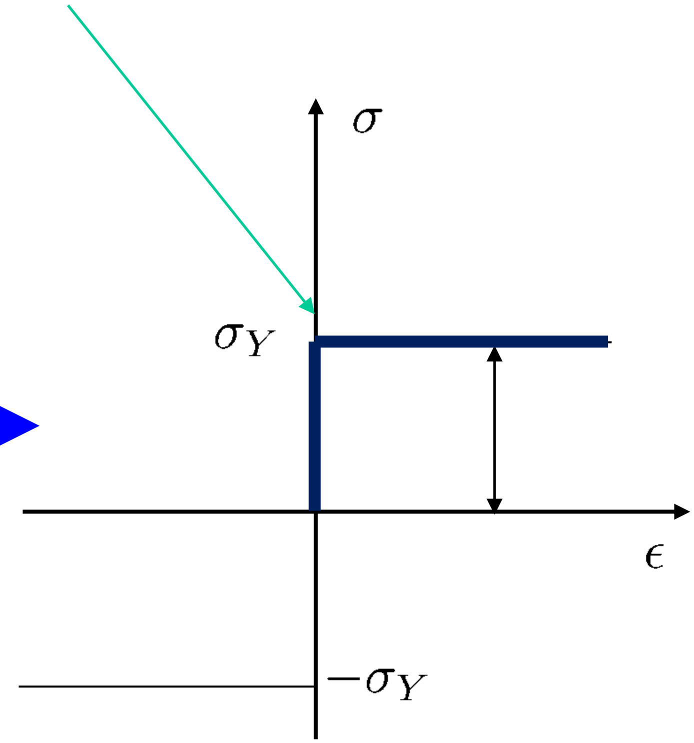
In other words, the first chicken would have hatched from an egg, and that egg would have been laid by a bird that was not quite a chicken but very closely related. So, scientifically speaking, the egg came first.

However, from a philosophical or metaphysical perspective, this question can be seen as more of a paradox or a thought experiment, and there is no definitive answer. It raises questions about causality and the origins of species, and it can be a fun topic for discussion and contemplation.

**Since a rigid body can transfer internal force, stress comes first!**




(a) Low-carbon steel




<u>perfectly or rigid plastic</u>

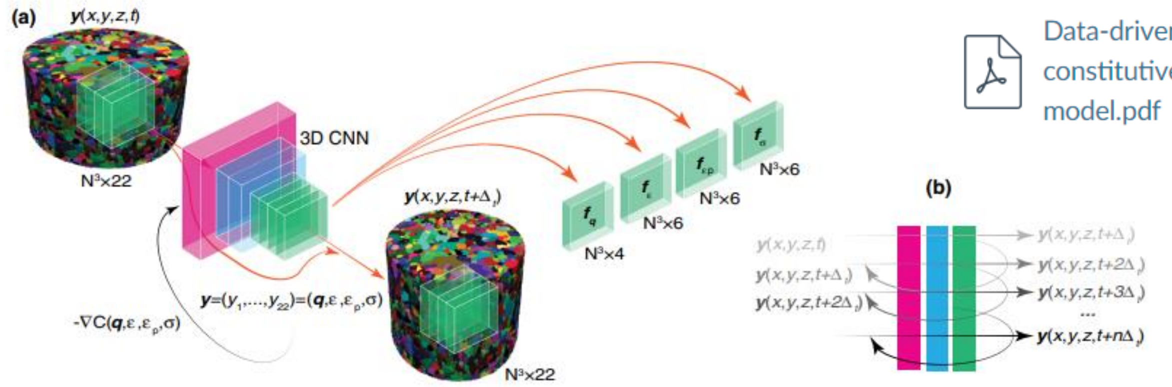



Fig. 1.. The 3D convolutional neural network architecture (a) with recursive inputs/outputs (b). A multilayer feedforward 3D convolutional network is employed, where each material state creates input data for the next material state through nested transformations.

Data-driven-constitutive-model.pdf 7:45pm 7:45pm 23.2 MB

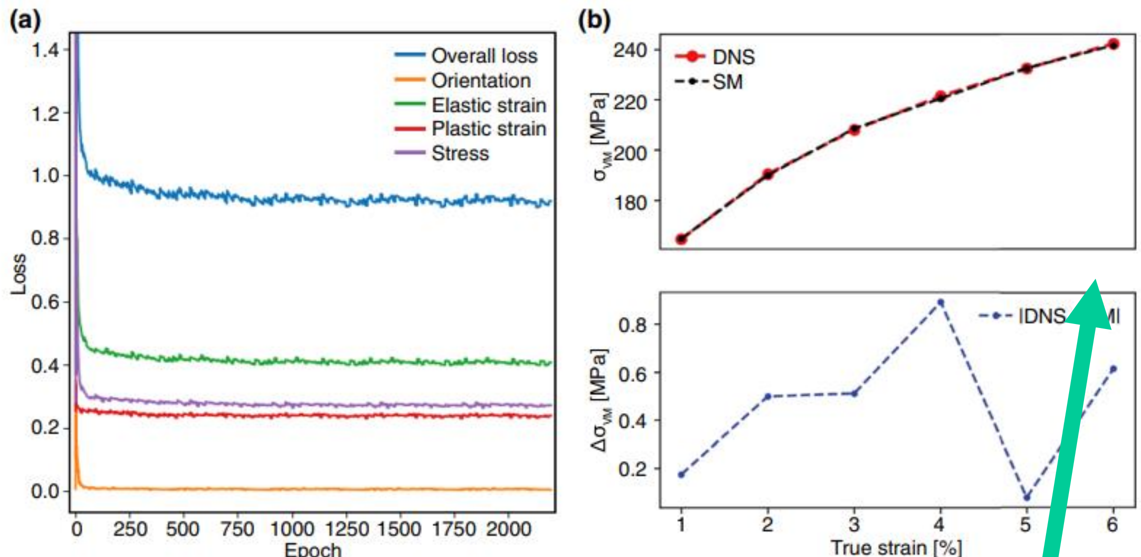
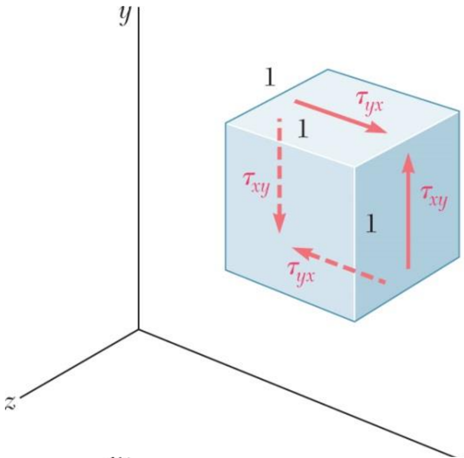



Fig. 2. (a) Training history. (b) Average von Mises stress evolution with macroscopic strain.

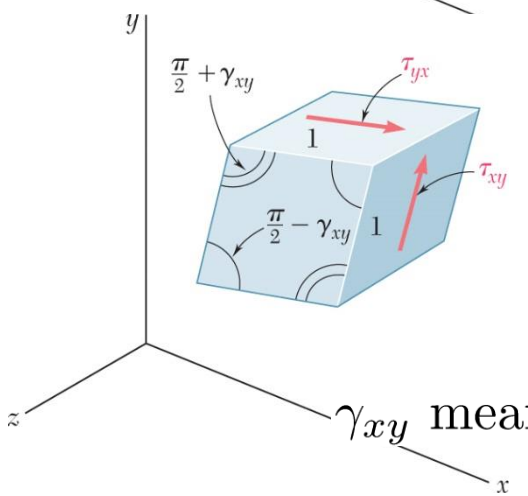
## Physics-Informed Data-Driven Surrogate Model for Polycrystalline Material

# Hooke's Law in Shear

Last time: Hooke's Law in tension $\sigma = E\epsilon$.




- A cubic element subjected to a shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,




- Hooke's law in shear

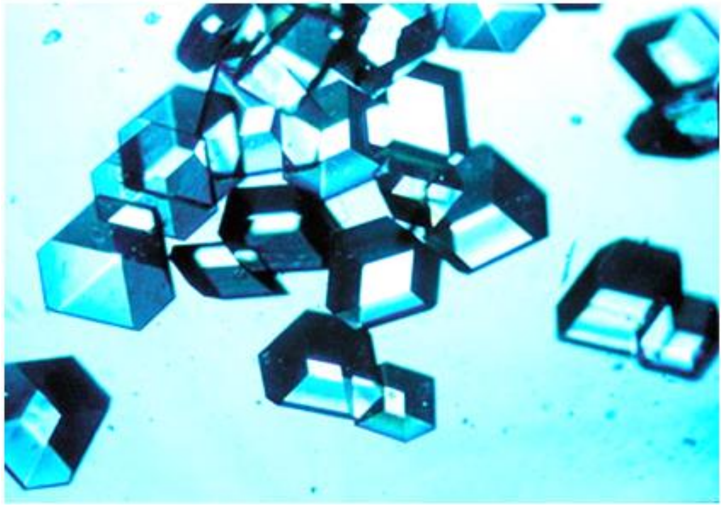
$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where $G$ is the shear modulus.

$\gamma_{xy}$ means the change of angle between x-axis and y-axis.

Obviously, $\gamma_{xy} = \gamma_{yx}$.

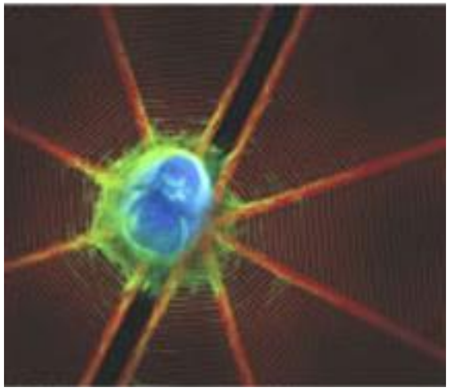
# Solids vs. Liquids



$$\mathbf{G \neq 0}$$

$$\mathbf{G = 0}$$

Soft Matter



$$\mathbf{G = G(X, t)}$$

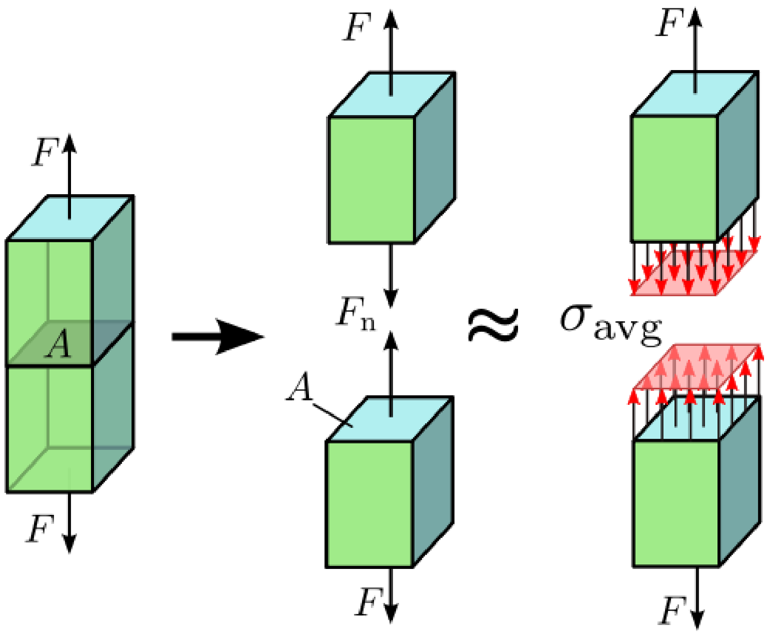
# Lecture 18 Axial Deformable Bars

- The material is linear elastic.
- The bar only deforms axially, i.e.

(There is no transverse deformation).

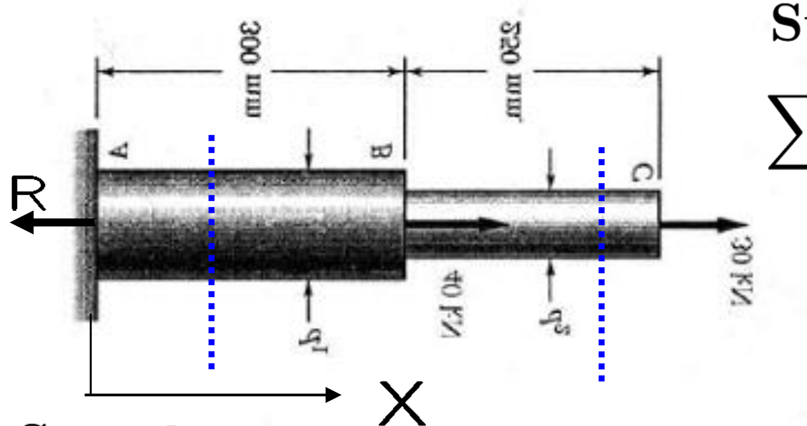
- There is no shear stress/strain.

$$\sigma_x \sim \epsilon_x \sim u_x(x)$$




**The first structural mechanics model that we shall learn.**

# 1. How to draw internal force diagram ?

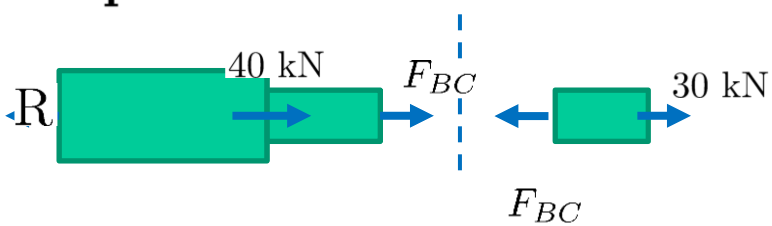



**Step 1**

$$\sum F_x = 0 \ \rightarrow \ -R + 40(kN) + 30(kN) = 0$$

$$R = 70kN.$$

**Step 2**

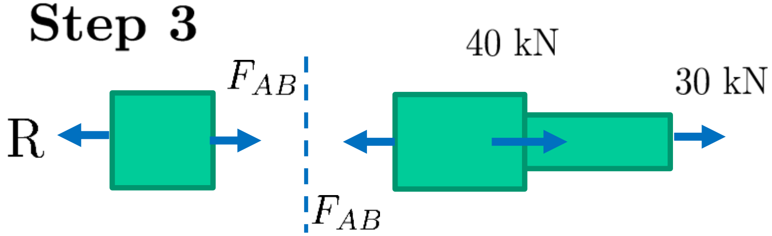



$$\sum F_x = 0, \ \rightarrow \ -F_{BC} + 30kN = 0$$

$$F_{BC} = 30kN$$

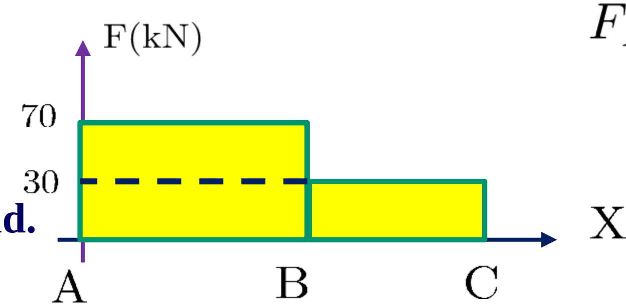
**The internal force has an evil twin.**

**Step 3**




$$\sum F_x = 0, \ \rightarrow \ -R + F_{AB} = 0$$

$$F_{AB} = R = 70kN$$

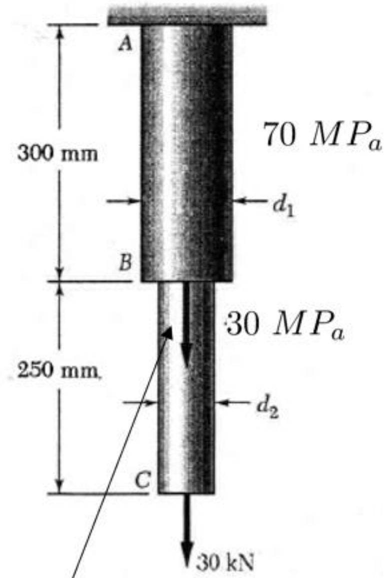



**Internal force is also a distribution field.**

Problem 8.2

**8.2** Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 140 MPa in either rod, determine the smallest allowable values of $d_1$ and $d_2$.

$\sigma_{allow}$




Rod AB.

$$P = 40 + 30 = 70\text{ kN} = 70\times10^3\text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4}d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi\sigma_{AB}}} = \sqrt{\frac{(4)(70\times10^3)}{\pi(140\times10^6)}} = 25.2\times10^{-3}\text{ m}$$

$$d_1 = 25.2\text{ mm}$$

Rod BC

$$P = 30\text{ kN} = 30\times10^3\text{ N}$$

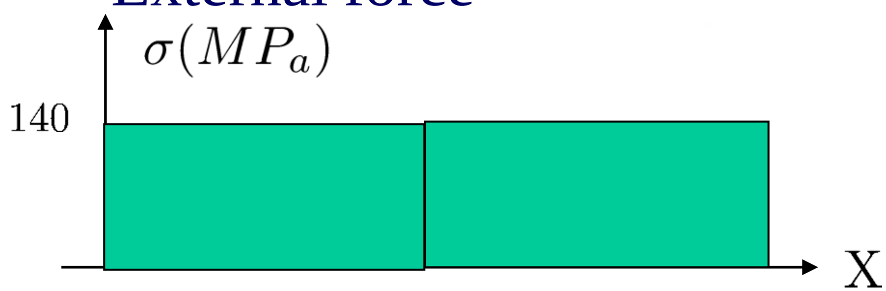
Internal forces

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4}d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi\sigma_{BC}}} = \sqrt{\frac{(4)(30\times10^3)}{\pi(140\times10^6)}} = 16.52\times10^{-3}\text{ m}$$

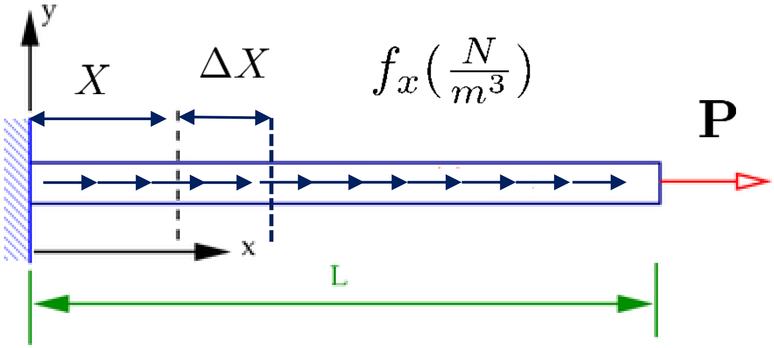
$$d_2 = 16.52\text{ mm}$$

External force

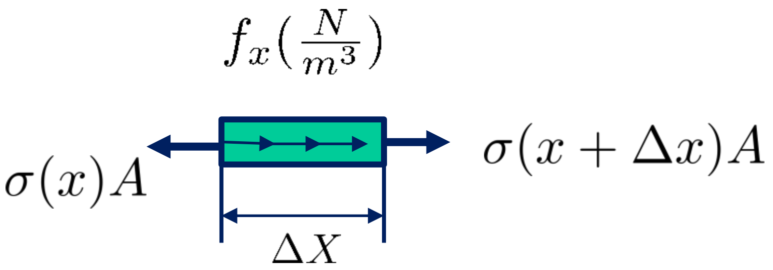



← Stress distribution

# Axial Deformable Bar (Differential Equation Approach)




Neglect transverse load and shear deformation



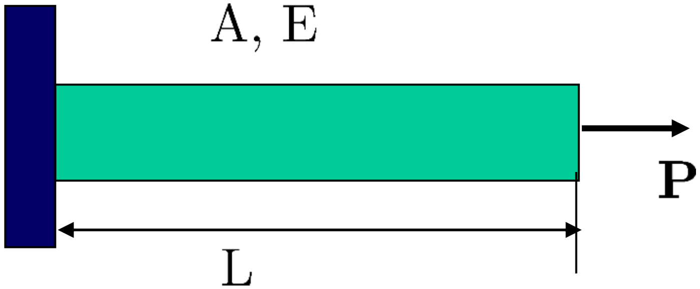

$$\sum F_x = 0, \quad \rightarrow \quad \sigma(X+\Delta X)A - \sigma(X)A + f_x \Delta X A = 0$$

$$\frac{\sigma(X+\Delta X) - \sigma(X)}{\Delta X} + f_x = 0, \quad \Rightarrow \quad \frac{d\sigma}{dx} + f_x = 0 \ .$$

$$(1) \qquad \epsilon = \frac{du}{dx}; \quad (2)\ \sigma = E\epsilon = E\frac{du}{dx};$$

$$(3) \qquad \frac{d\sigma}{dx} + f_x = 0; \quad \boxed{(4)\ \frac{d}{dx}E\left(\frac{du}{dx}\right) + f_x = 0 \ .}$$

## Example II. Special Case: Tow-force member (b(x) = 0, P=const.)




$$du = \epsilon dx = \frac{\sigma}{E}dx = \frac{P(x)}{EA}dx$$

The total elongation of the bar is,

$$\delta \;=\; u(L) - u(0) = \int_0^L \frac{P}{EA}dx \quad \rightarrow \quad \delta = \frac{PL}{EA}$$

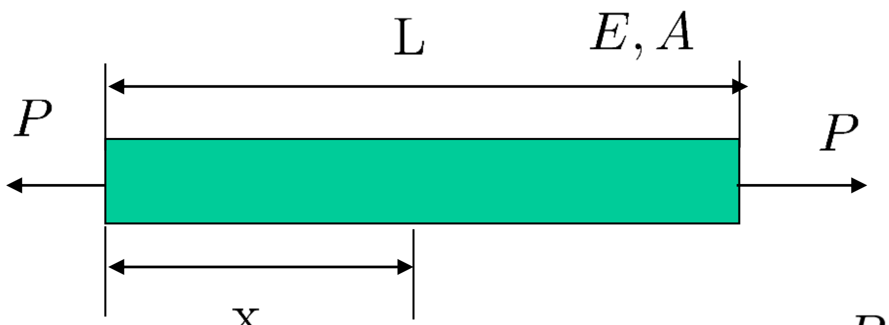
**elongation** **internal foce**

$$\delta = \left(\frac{L}{EA}\right)P \qquad\qquad P = \left(\frac{EA}{L}\right)\delta$$

$$f := \frac{L}{EA} \text{ Flexibility} \qquad\qquad k := \frac{EA}{L} \text{ Stiffness}$$

**Structure constants**

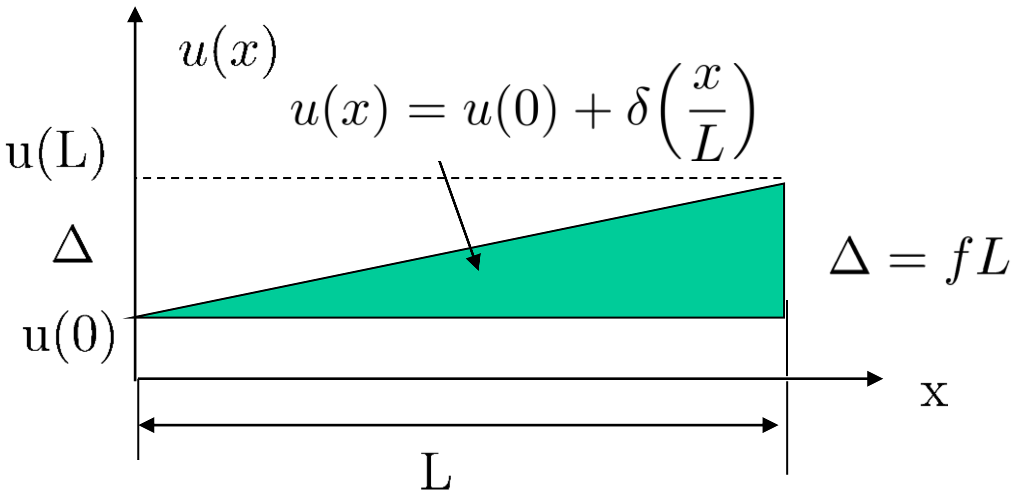
# Two-force Member Under Axial Loading



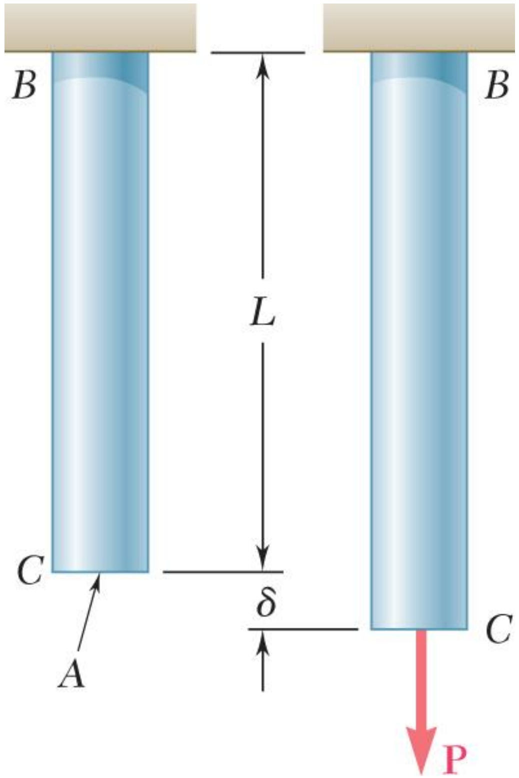

$$\sigma = \frac{P}{A}, \quad \epsilon = \frac{P}{EA} \quad \rightarrow \quad du = \epsilon dx = \frac{P}{EA} dx$$

$$u(x) - u(0) = \frac{Px}{EA} = \delta\left(\frac{x}{L}\right)$$

$$\delta = \frac{PL}{EA}$$



## Two-force Member Under Axial Loading




- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

- Equating and solving for the deformation,

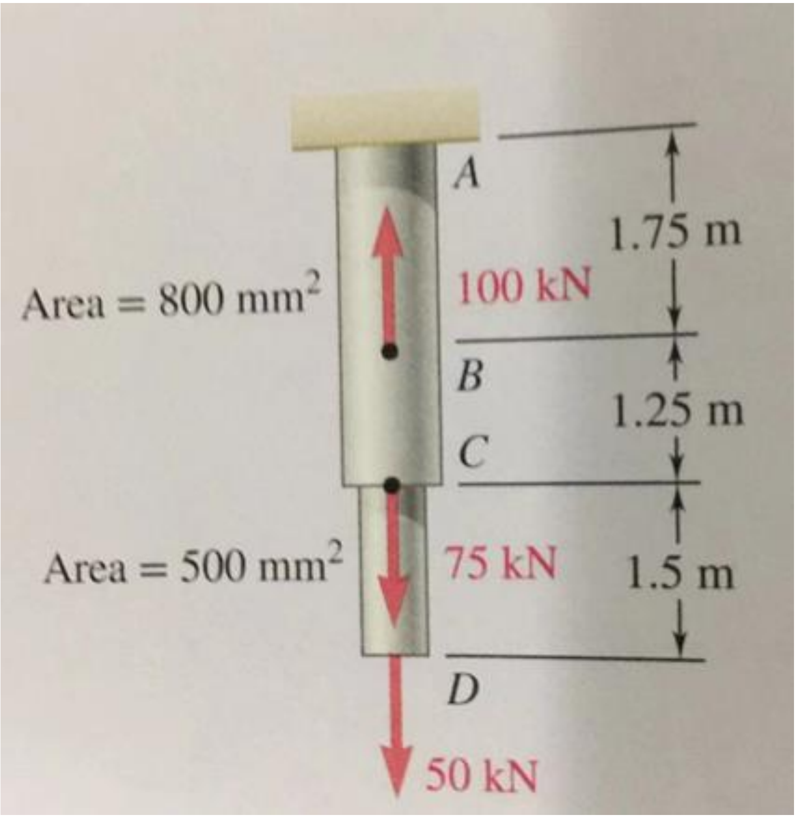
$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

## PROBLEM 9.14

The rod *ABCD* is made of an aluminum for which $E = 70$ GPa. For the loading shown, determine the deflection of (*a*) point *B*, (*b*) point *D*.



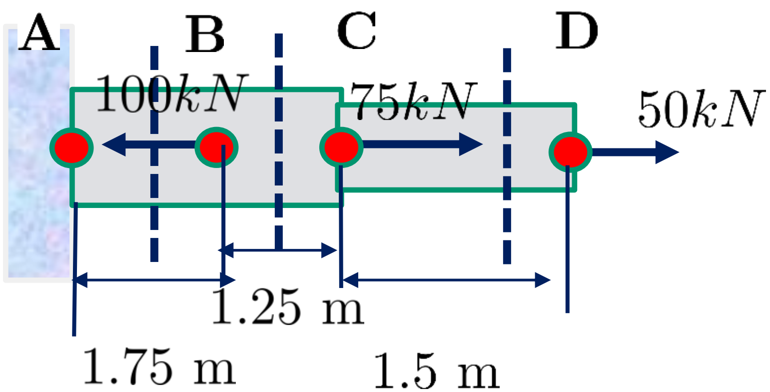

SOLUTION:

- Divide the rod into components of multiple two-force members
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

How many two-force member bars do we have here ?

SOLUTION:

- Divide the rod into three two-force members

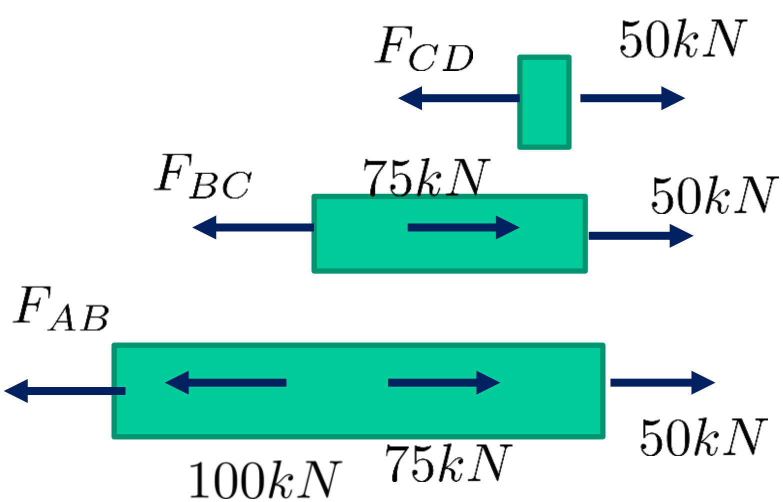



- Apply free-body analysis to each component to determine internal forces,

$$A_{AB} = 800mm^2, \quad L_{AB} = 1.75m$$

$$A_{BC} = 800mm^2, \quad L_{AB} = 1.25m$$

$$A_{CD} = 500mm^2, \quad L_{AB} = 1.5m$$




$$\sum F_x = 0 \rightarrow$$

$$-F_{CD} + 50 = 0 \rightarrow F_{CD} = 50kN$$

$$-F_{BC} + 75 + 50 = 0 \rightarrow F_{BC} = 125kN$$

$$-F_{AB} - 100 + 75 + 50 = 0 \rightarrow$$

$$F_{AB} = 25kN$$