CE30 – Discussion 13

Stress Transformations

Textbook: 14.1, 14.2

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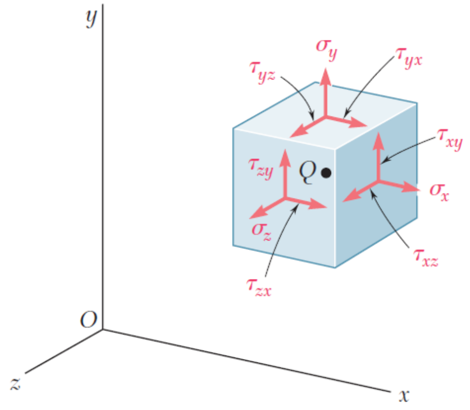
Announcements

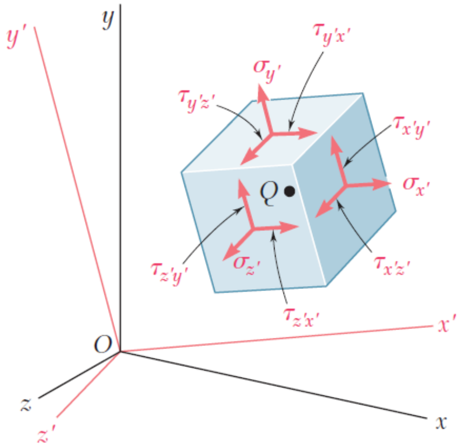
- HW13 Problems from the textbook:

  14.2, 14.7, 14.23, 14.40, 14.43, 14.46

Components of Stress

- In 3D, stress has 6 components (3 normal, 3 shear)
- The stress components are dependent on the **choice of the coordinate system**



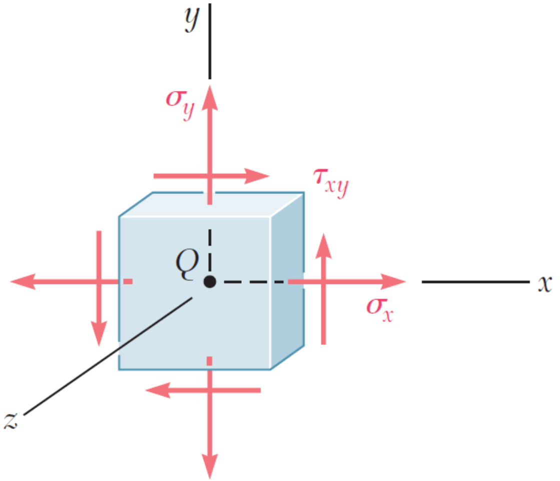


Note: The material point Q experiences same state of stress in both configurations!

Only the way we express the stress components are different.

Stress Transformation: 2D Plane Stress

- Plane stress conditions: Stress in y and z planes are zero!
- Simplifies the analysis for some loading conditions (thin plates, free surfaces, etc.)

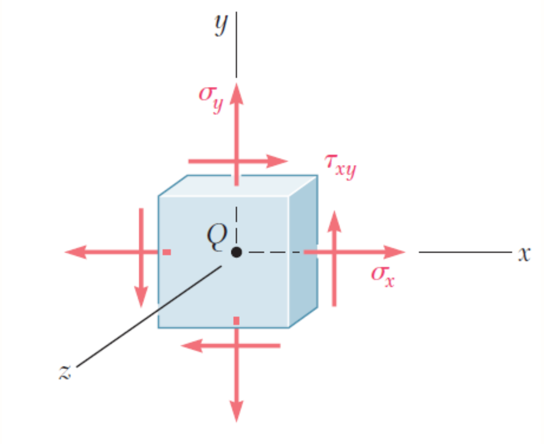


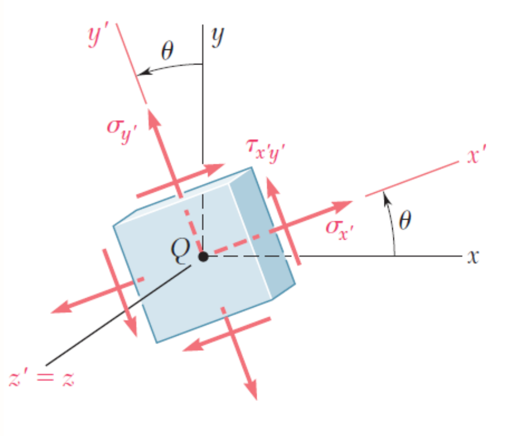
Non-zero stress components in Plane Stress

$\boldsymbol{\sigma_x}$, $\boldsymbol{\sigma_y}$	Normal Stresses
$\boldsymbol{\tau_{xy}}$	Shear Stress

Stress Transformation: 2D Plane Stress

- Find the stress components after the element is rotated by θ ccw





Given: $\boldsymbol{\sigma_x}$, $\boldsymbol{\sigma_y}$, $\boldsymbol{\tau_{xy}}$

Find: $\boldsymbol{\sigma_{x'}}$ $\boldsymbol{\sigma_{y'}}$ $\boldsymbol{\tau_{x'y'}}$

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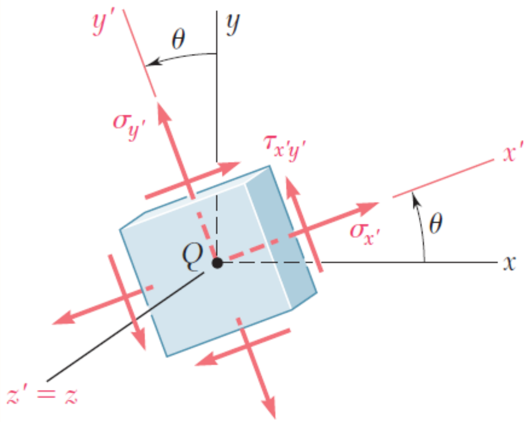
Stress Transformation: 2D Plane Stress



Stresses in the transformed coordinates (x', y')

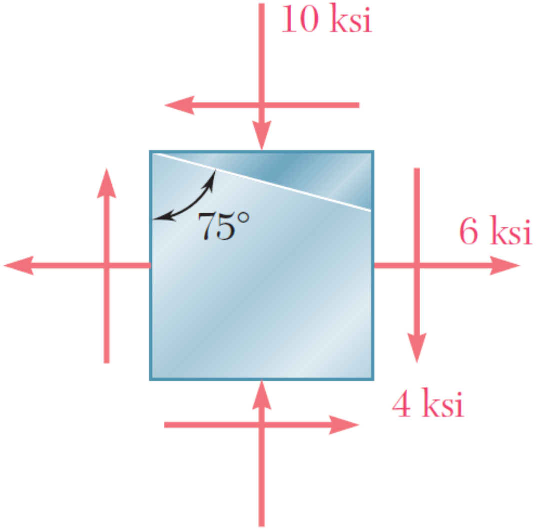
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

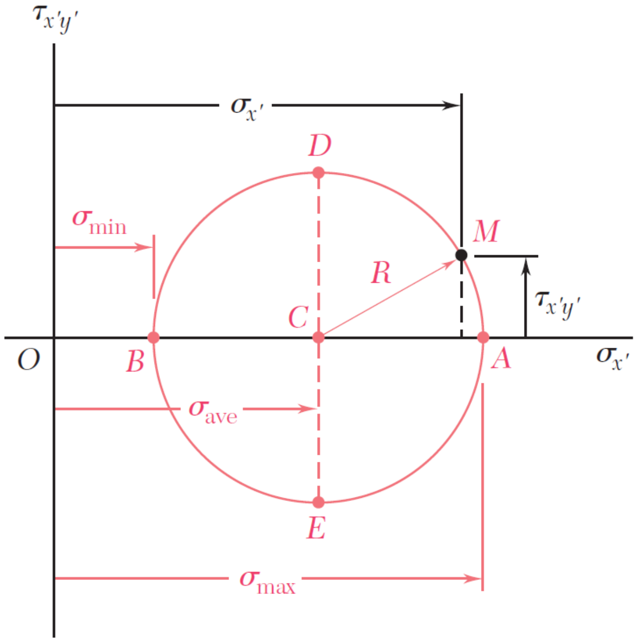
Practice – Similar to HW P14.2

Find the normal and shear stresses exerted on the oblique face of the shaded triangular element.



Principal Stresses

- Using the stress transformation equations, we can find a relationship between the normal and shear stresses. This relationship represents the equation of a circle.



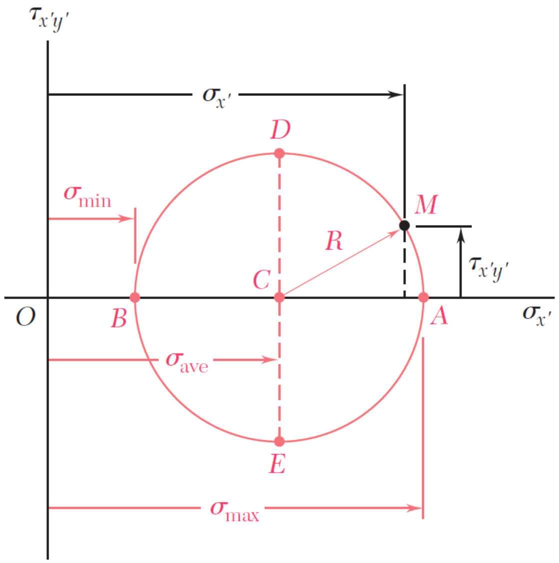
Center: $$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Radius: $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

x-axis: Normal stress

y-axis: Shear stress

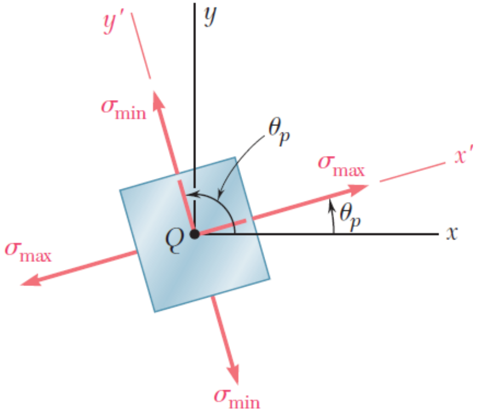
Principal Stresses



- At point A, normal stress is maximum and shear stress is zero.
- At point B, normal stress is minimum and shear stress is zero.
- These orientations are known as the ***principal planes***

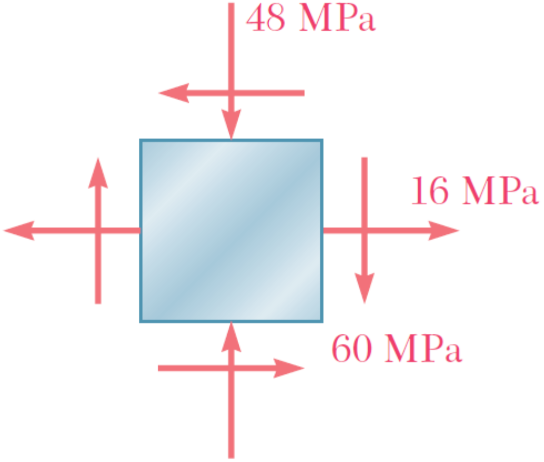
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- **Orientation of the principal plane** = θ_p
- **Principal stresses** = $(\sigma_{max}, \ \sigma_{min})$
- Two principal planes are 90° apart



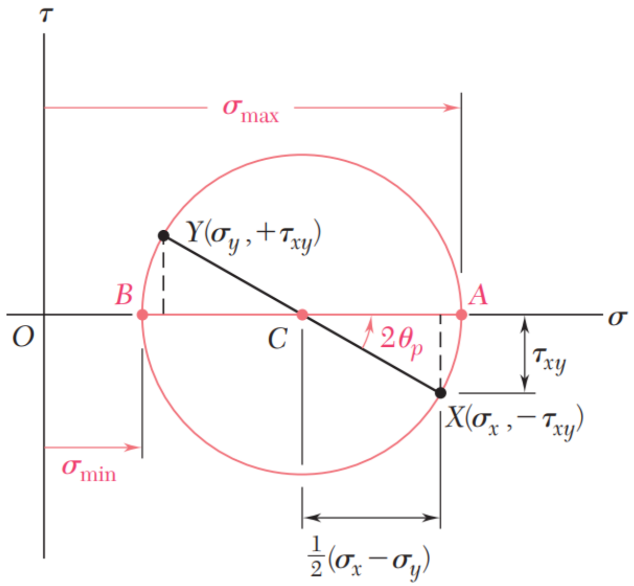
Practice – Similar to HW P14.7

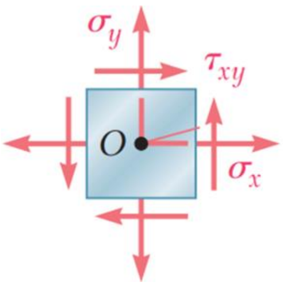
For the givens stress state, find (a) principal planes, (b) principal stresses



Mohr's Circle

- The circle we drew before is known as the ***Mohr's Circle***
- We can construct it with only the geometry.



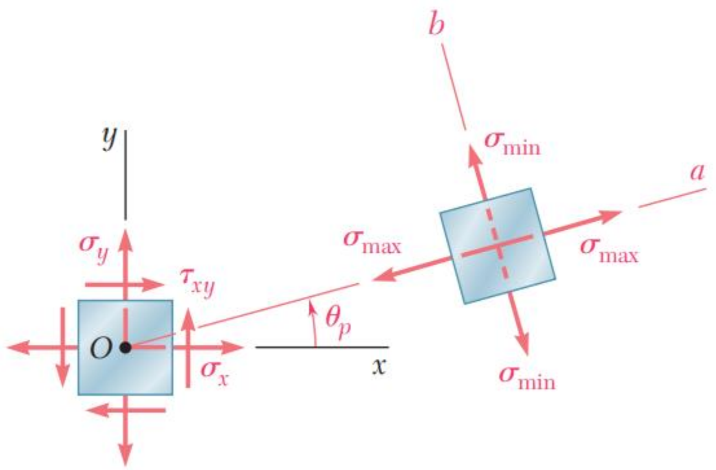


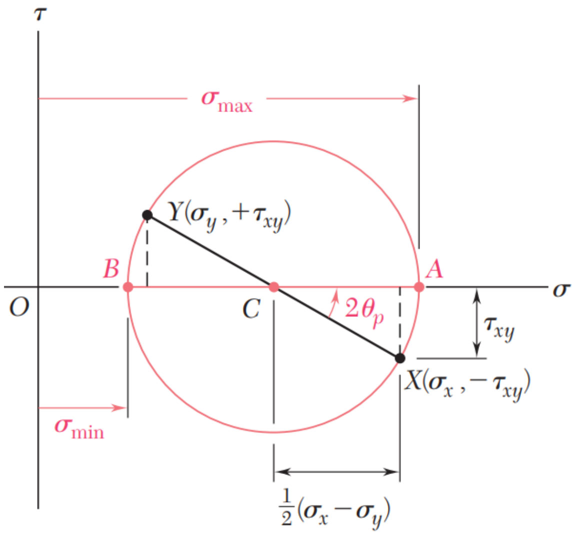
For any given stress state:

1. **Plot point X** $= \ (\boldsymbol{\sigma_x}, -\boldsymbol{\tau_{xy}})$
2. **Plot point Y** $= \ (\boldsymbol{\sigma_y}, +\boldsymbol{\tau_{xy}})$
3. **Connect (X,Y), where the intersection is** $C = \frac{\sigma_x + \sigma_y}{2}$

Mohr’s Circle

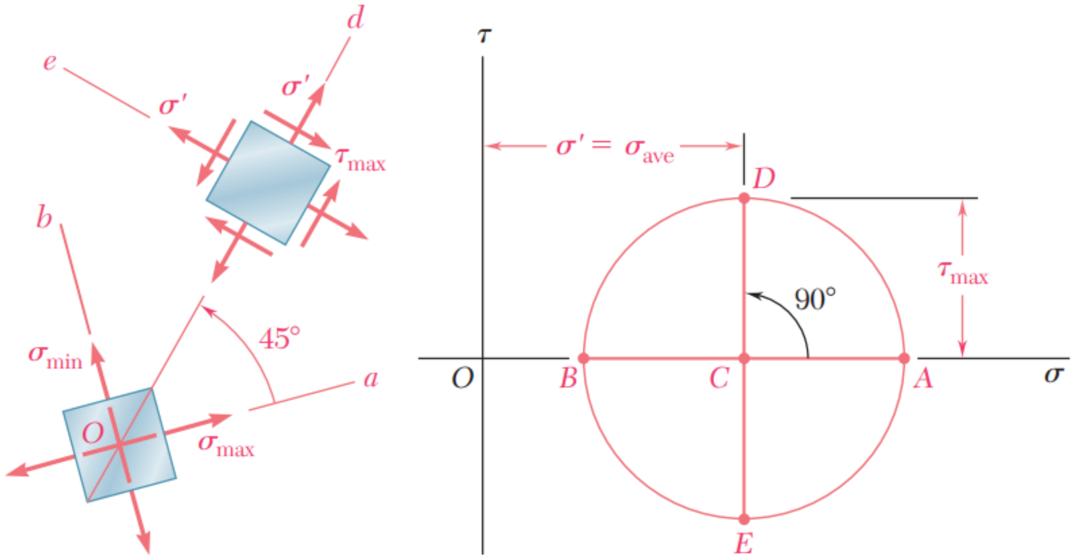
- Principal stresses/planes and maximum shear can be found
- θ rotation in the stress element corresponds to 2θ rotation in Mohr’s circle





Mohr’s Circle

- θ rotation in the stress element corresponds to 2θ rotation in Mohr’s circle
- Maximum shear happens at 45 from the principal planes



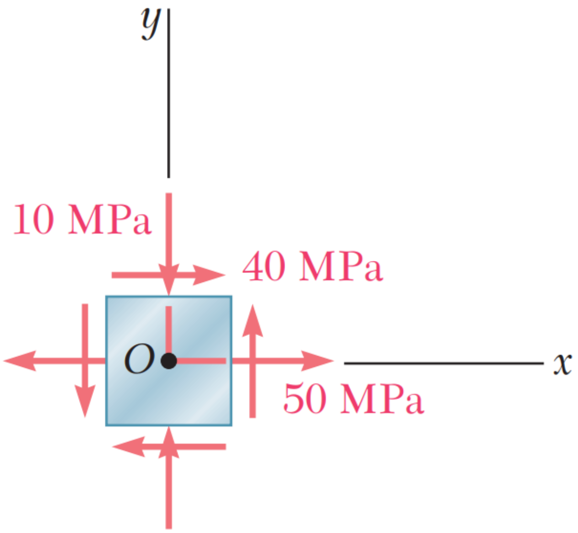
Example: Mohr's Circle

For the given state of plane stress :

(a) Draw the Mohr's Circle

(b) Find the principal stress and its directions

(c) Find the maximum shear stress



Practice – Similar to HW P14.46

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown