

**Set Parameters**

$d := .9$        $x1 := 20$      $x2 := 15$        $y1 := 10$      $y2 := 15$        $k := 500$

**Define Utility Functions**

$u1(P1, P2, z1, z2, Y1, Y2) := k - (P1 - x1)^2 - (P2 - x2)^2$      $u2(P1, P2, z1, z2, X1, X2) := k - (P1 - y1)^2 - (P2 - y2)^2$      $u3(P1, P2, z1, z2) := k - (P1 - z1)^2 - (P2 - z2)^2$

**Sample policy:**

$$\begin{pmatrix} P1 \\ P2 \end{pmatrix} := \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

**Define best reply functions**

starting values:

$z1 := 20$

$z2 := 20$

Given  $u3(P1, P2, z1, z2) > d \cdot u3(Y1, Y2, z1, z2)$      $u2(P1, P2, z1, z2, Y1, Y2) > d \cdot u2(Y1, Y2, z1, z2, Y1, Y2)$      $f1(z1, z2, Y1, Y2) := \text{Maximize}(u1, P1, P2)$

Given  $u3(P1, P2, z1, z2) > d \cdot u3(X1, X2, z1, z2)$      $u1(P1, P2, z1, z2, X1, X2) > d \cdot u1(X1, X2, z1, z2, X1, X2)$      $f2(z1, z2, X1, X2) := \text{Maximize}(u2, P1, P2)$

**Example of repeated application of maximization to convergence:**

		$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1, z2, Y1, Y2)$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1, z2, X1, X2)$	$\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 20 \\ 15 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.933 \\ 15.25 \end{pmatrix}$
$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1, z2, Y1, Y2)$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1, z2, X1, X2)$	$\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.603 \\ 14.999 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.576 \\ 15.249 \end{pmatrix}$
$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1, z2, Y1, Y2)$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1, z2, X1, X2)$	$\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.485 \\ 15 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.542 \\ 15.25 \end{pmatrix}$
$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1, z2, Y1, Y2)$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1, z2, X1, X2)$	$\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.475 \\ 15 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.538 \\ 15.25 \end{pmatrix}$
$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1, z2, Y1, Y2)$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1, z2, X1, X2)$	$\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.474 \\ 15 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.538 \\ 15.25 \end{pmatrix}$
$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1, z2, Y1, Y2)$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1, z2, X1, X2)$	$\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.474 \\ 15 \end{pmatrix}$	$\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.538 \\ 15.25 \end{pmatrix}$

**Program to iterate this procedure (ex post check for convergence):**

```

GUESS(z1,z2,startX1,startX2,startY1,startY2,tries) :=
| X ← f1(z1,z2,startY1,startY2)
| X1 ← X0
| X2 ← X1
| Y ← f2(z1,z2,startX1,startX2)
| Y1 ← Y0
| Y2 ← Y1
| for j ∈ 1 .. tries
|   | X ← f1(z1,z2,Y1,Y2)
|   | X1 ← X0
|   | X2 ← X1
|   | Y ← f2(z1,z2,X1,X2)
|   | Y1 ← Y0
|   | Y2 ← Y1
| OUT0 ← X1
| OUT1 ← X2
| OUT2 ← Y1
| OUT3 ← Y2
| OUT

```

Example with 10 and 15 iterations

$$\text{GUESS}(z1,z2,0,0,0,0,10) = \begin{pmatrix} 17.473 \\ 15 \\ 12.538 \\ 15.25 \end{pmatrix} \quad \text{GUESS}(z1,z2,0,0,0,0,15) = \begin{pmatrix} 17.473 \\ 15 \\ 12.538 \\ 15.25 \end{pmatrix}$$

## Utilities at optimum

```

U1(Xgrid, Ygrid, tries) :=
  START ← GUESS(0,0,0,0,0,0,tries-2)
  X1t ← START0
  X2t ← START1
  Y1t ← START2
  Y2t ← START3
  for i ∈ 0 .. Xgrid
    for j ∈ 0 .. Ygrid
      GO ← GUESS(i,j,X1t,X2t,Y1t,Y2t,tries)
      X1t ← GO0
      X2t ← GO1
      Y1t ← GO2
      Y2t ← GO3
      OUTPUTi,j ← u1(X1t,X2t,0,0,0,0)
  OUTPUT
  
```

```

U2(Xgrid, Ygrid, tries) :=
  START ← GUESS(0,0,0,0,0,0,tries-2)
  X1t ← START0
  X2t ← START1
  Y1t ← START2
  Y2t ← START3
  for i ∈ 0 .. Xgrid
    for j ∈ 0 .. Ygrid
      GO ← GUESS(i,j,X1t,X2t,Y1t,Y2t,tries)
      X1t ← GO0
      X2t ← GO1
      Y1t ← GO2
      Y2t ← GO3
      OUTPUT2i,j ← u2(X1t,X2t,0,0,0,0)
  OUTPUT2
  
```

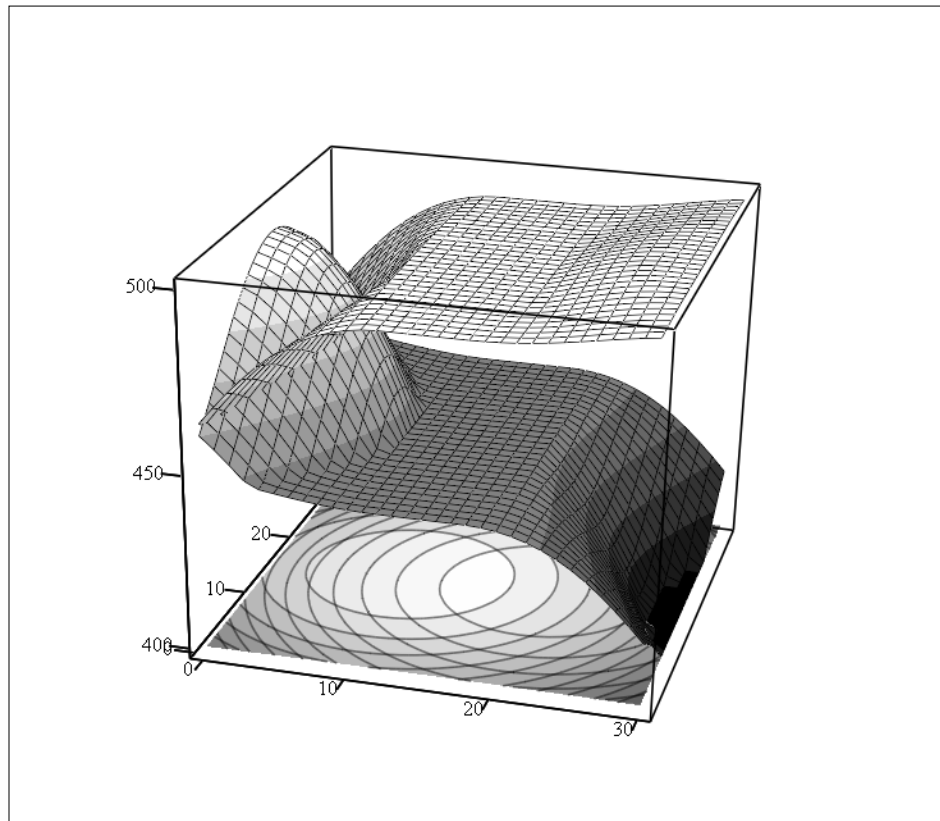
Set grid size                      gr := 30

Estimate utilities on grid:      V1 := U1(gr, gr, 10)      V2 := U2(gr, gr, 10)

Set graph indices                i := 0 .. gr      j := 0 .. gr

Utility graphs for contour plots    Fl1<sub>i,j</sub> := u1(i,j,0,0,0,0)<sup>1</sup>    Fl2<sub>i,j</sub> := u2(i,j,0,0,0,0)<sup>1</sup>

Figure 1



V1, V2, F11, F12