#### POLITICAL SCIENCE W4210 (FALL 2003)

# RESEARCH TOPICS IN GAME THEORY

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Research topics in game theory will cover the study of dynamic games, games of incomplete information, and evolutionary games, with applications in the fields of voting, bargaining, lobbying and violent conflict. Results from the study of social choice theory and mechanism design will also be treated. The course will concentrate on mathematical techniques for constructing and solving games. Students will be required to develop a topic relating political science and game theory and to write a formal research paper. Prerequisite: W4209 or instructor's permission.

#### **Course Overview**

Political Science W4210 is a continuation of W4209 with the objective of preparing the student to use formal models in research in political science.

- In the first two weeks of the course we state and prove results that are of very broad interest to political philosophy, public economics and positive political theory. These results are all very important in their own rights, but they will also be used to introduce key ideas about model construction to be used later on.
- In weeks 3-7, we introduce the tools you need to construct a formal model and prove results. We consider the choices that modelers need to make and the set of options that they have and we review approaches to constructing models and proving results. In these weeks we aim to provide the tools that you will need to construct a model of your own.
- In weeks 8-14 the course will take on more of a seminar format, engaging in close reading of models that use the techniques we have seen to study political problems

#### Requirements

The readings are typically light in terms of page numbers but are compact and heavy in notation. As one of the aims of the course is to develop skills not just in reading but in developing models, you will be expected to work through the proofs of all propositions and theorems covered in the course. Notes on close readings of these texts follow below. In addition:

1. You will be required to write an original paper presenting a model or theorem. This paper is your key output from this course, ideally it should contribute directly to the writing of your dissertation. The paper should motivate a problem, develop a model and prove ensuing propositions, and identify testable predictions resulting from the model. This research paper will account for 55% of the final grade. You may be asked to present parts of your model in class for discussion by the group. The paper is due on 13 December.

- 2. There will be problem sets and exercises to complete throughout the first part of the course; these are intended to evaluate your understanding of the material and to allow for deeper exploration of models studied, and, especially, to practice model construction and proof writing. These account for 15% of the course grade and typically have to be handed in the week after they are assigned. Late problem sets will not be accepted.
- 3. In one week you will be required to review one of the central models / theorems studied in the course. For this presentation you will be expected to (i) give an overview of the question under study (ii) give a brief presentation of the proof, (iii) evaluate the model's assumptions—are all assumptions necessary? are all assumptions reasonable? (iv) discuss the solution concept employed (v) discuss the generality of field of application of the results (vi) suggest ways in which the results could be pushed further. In some weeks these presentations may be constructed in somewhat of a debate format where rival papers are discussed by different students. This presentation will account for 10% of your grade.
- 4. You will be required to participate in weekly sections where problem sets will be reviewed, and class and research material will be discussed. The final 20% of the grade will be based on participation in these sections.
- 5. All submitted writing, for your research paper or your problem sets, should be typed up on a word processor capable of handling the mathematics and symbols. It is strongly recommended, if you do not already know how, that you learn to use either Scientific Word/Workplace (<a href="http://www.mackichan.com/index.html?products/sw.html~mainFrame">http://www.mackichan.com/index.html?products/sw.html~mainFrame</a>) or LaTeX (<a href="http://www.latex-project.org">http://www.maths.tcd.ie/~dwilkins/LaTeXPrimer</a>) during the course of the term and use these tools to write your papers.

## **Topics**

# Part I: Some Big Results in Social Choice

## Week 1 [9 SEPTEMBER] SOCIAL WELFARE, COLLECTIVE CHOICE AND EFFICIENCY

**THEOREMS AND CONCEPTS**: Arrow's Impossibility Theorem, the Impossibility of a Paretian Liberal and the Coase Theorem.

#### Readings

- Class Notes: 1 & 2
- Sen, Amartya. 1970. "The Impossibility of a Paretian Liberal." The Journal of Political Economy, 78:1, pp. 152-57. <a href="http://www.jstor.org/view/00223808/di950932/95p0121q/0">http://www.jstor.org/view/00223808/di950932/95p0121q/0</a>
- Coase,Ronald. 1960. "The Problem of Social Cost." <u>Journal of Law and Economics</u> (<a href="http://www.sfu.ca/~allen/CoaseJLE1960.pdf">http://www.sfu.ca/~allen/CoaseJLE1960.pdf</a>)
- Geanakoplos, John, 2001. Three Brief Proofs of Arrow's Impossibility Theorem <a href="http://cowles.econ.yale.edu/P/cd/d11a/d1123RRR.pdf">http://cowles.econ.yale.edu/P/cd/d11a/d1123RRR.pdf</a>

#### Further reading:

• ! Aivazian, V.A. and Jeffrey L. Callen. 1981 "The Coase Theorem and the Empty Core." <u>Journal of Law and Economics</u> v.24: 175 -181. See also Coase's response, same journal. *Not on-line*.

## Part II: Tools for Constructing and Solving Games

# Week 2 [16 SEPTEMBER] HOW TO PROVE IT: STRATEGIES OF PROOF

**THEOREMS AND CONCEPTS:** Direct Method of Proof, Proof by Contradiction, by Induction, Truth Tables, WLOG, Existence, Uniqueness, Monotonicity, Weierstrass Theorem, Intermediate Value Theorem, Fixed Point Theorems: (Brouwer, Kakutani, Borzak-Ulam, Gale-Nikaido, Hairy Ball). Proof of Nash's theorem.

#### Required Readings

- Class Notes: 3 & 4
- Velleman, Daniel J. 1994. <u>How to Prove It: A Structured Approach</u>. Cambridge: Cambridge University Press. Chapters 1-3.
- Osborne and Rubinstein, Section 2.4.

# Recommended Readings:

- The rest of Velleman
- For an older political science article that uses a fixed point theorem read Herbert A. Simon's short 1954 piece: "Bandwagon and Underdog Effects and the Possibility of Election Predictions" Public Opinion Quarterly, 18:3. pp. 245-253. http://links.jstor.org/sici?sici=0033-362X%28195423%2918%3A3%3C245%3ABAUEAT%3E2.0.CO%3B2-Y
- How to Write Mathematics (from the webpages of the Mathematics Society of Trent) <a href="http://xaravve.trentu.ca/mascot/handbook/SEC">http://xaravve.trentu.ca/mascot/handbook/SEC</a> write.pdf
- For more on the mathematical results see the appendices in Mas-Colell, Whinston, and Green or
  in Rasmusen's <u>Games and Information</u>, or in topology texts such as Berge's, <u>Topological Spaces</u>.

# Week 3 [23 September\*] What to Prove: Defining Games, Identifying Propositions, Representing Preferences

**THEOREMS AND CONCEPTS:** Defining Games Formally. Generality of Propositions and Strength of Assumptions, Choices (Discrete v. Continuous Action Spaces, Discrete v. Continuous Time, Atomic v. Continuous Distributions, Finite v. Infinite Horizons), Genericity and Robustness. Choices for representing preferences: Representing attitudes to risk, single and multidimensional problems, intertemporal utility. The Expected utility theorem.

### Required Reading

- Class Notes 5, 6 & 7
- Varian, Hal How to Build an Economic Model in Your Spare Time http://www.sims.berkeley.edu/%7Ehal/Papers/how.pdf
- Starmer, Chris. 2000. "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk" <u>Journal of Economic Literature</u> Vol. XXXVIII 332–382 <a href="http://www.nottingham.ac.uk/~lezcs/pdf">http://www.nottingham.ac.uk/~lezcs/pdf</a> files/STARMER JEL.PDF
- ! Davies, Philip et al. "The Creation of New Mathematics: An Application of the Lakatos Heuristic," Ch 44 in Rasmusen, <u>Readings</u> (Call Number: QA269 .R42 2001).

## **Further Reading**

- Harry Roberts and Roman Weil, 1970. "Starting Research Early," University of Chicago, Graduate School of Business, <a href="http://pacioli.bus.indiana.edu/erasmuse/GL\_reader/05c.roberts.htm">http://pacioli.bus.indiana.edu/erasmuse/GL\_reader/05c.roberts.htm</a>
- Recommended to help your writing: MIT's guidelines: <u>Writing a Math Phase Two Paper</u>, <u>http://www.mit.edu/afs/athena.mit.edu/course/other/mathp2/www/piil.html</u>.
- Of historical interest: John Von Neumann and Oscar Morgenstern, 1990. <u>Theory of Games and Economic Behavior</u>, Princeton University Press, Chapter 3: "The Notion of Utility."

\*Note: Clash with ECPR meetings. This class may have to be held on 16 or 17 September.

# Week 4 [30 SEPTEMBER] SOLUTION CONCEPTS FOR NORMAL FORM GAMES AND EVOLUTIONARY GAMES

**THEOREMS AND CONCEPTS:** Elimination of Dominated Strategies, Rationalizability, Nash Equilibrium, Resistance, Risk Dominance, Viscosity, Strong Equilibrium, Coalition-Proof Equilibrium, Correlated Equilibrium. Evolutionarily Stable Strategies, Weak Evolutionarily Stable Strategies, Neutrally Stable Strategies, Limit Evolutionarily Stable Strategies, Evolutionarily Stable Sets, Proper Equilibrium, Asymptotically Stable Equilibria, Stochastically Stable Equilibrium.

#### **Required Readings**

- Class Notes: 8
- Osborne, Martin J, and Ariel Rubinstein. 1994. <u>A Course in Game Theory</u>, MIT University Press, Chapters 2, 3, 4, and 13.
- Samuelson, Larry. 1998. <u>Evolutionary Games and Equilibrium Selection</u> Cambridge: MIT Press. Chapters 2, 3 and 4.

#### **Recommended Readings:**

- H. Peyton Young. 1998. <u>Individual Strategy and Social Structure: An Evolutionary Theory of Institutions</u>. Princeton: Princeton University Press.
- Brian Skyrms, 1996. Evolution of the Social Contract. Cambridge: Cambridge University Press.

# Week 5 [7 October] Solution Concepts and Tools for Extensive Form Games I

**CONCEPTS AND THEOREMS**: SPNE, Fuzzy SPNE, Refinements, Zermelo's Theorem. The One Stage Deviation Principle, The Bellman Equation, Existence and Uniqueness of Equilibrium in Stahl-Rubinstein Game, Folk-theoretic results.

## Required Readings:

- Class Notes 9 & 10
- Muthoo, Abhinay. 1999. <u>Bargaining Theory with Applications</u>, Cambridge: Cambridge University Press. 3.1-.2.

- Osborne and Rubinstein, Sections 8.1-5.
- ! Ferejohn, John. 1986. "Incumbent Performance and Electoral Control" <u>Public Choice</u> 50: 5-26
   or Chapter 1 in Perrson and Tabellini. 1995. <u>Monetary and Fiscal Policy 2: Politics</u>, Cambridge,
   MIT University Press.

# Recommended Readings:

- For a more general presentation of the Rubinstein model, see Osborne and Rubinstein, Sections 7.2-3 or the rest of Muthoo Chapter 3
- For a richer but tougher model than Ferejohn's try: Jeffrey S. Banks and Rangarajan Sundaram.
   "Adverse Selection and Moral Hazard in a Repeated Elections Model," in <u>Political Economy: Institutions, Information, Competition, and Representation</u>, (W. Barnett, et al, Eds.), Cambridge University Press, Cambridge and New York, 1993.
- A recent application of folk theoretic ideas to the study of ethnic politics is found in Fearon,
  James D. and David D. Laitin. 1996. "Explaining Interethnic Cooperation," <u>APSR</u>, 90:715-735.
  <a href="http://links.jstor.org/sici?sici=0003-0554%28199612%2990%3A4%3C715%3AEIC%3E2.0.CO%3B2-8">http://links.jstor.org/sici?sici=0003-0554%28199612%2990%3A4%3C715%3AEIC%3E2.0.CO%3B2-8</a>

! DUE!: \*\*\* HAND I

\*\*\* HAND IN MODEL OUTLINE \*\*\*

# Week 6 [14 October] Solution Concepts and Tools for Repeated Games II (WITH MORE UNCERTAINTY)

**CONCEPTS AND THEOREMS**: Using Bayes' Rule, using equilibrium refinements, Perfect Bayesian Equilibria, Sequential equilibrium, Trembling Hand Perfection, Forward Induction, The Intuitive Criterion, Signaling and Screening, The Single-Crossing Property.

#### Required Readings:

- Class Notes 11
- Nalebuff, Barry (1991), "Rational Deterrence in an Imperfect World" World Politics, 43:3, pp. 313-335.
- Mas-Colell, Andreu, Michael Whinston, and Jerry Green, 1995. <u>Microeconomic Theory</u>. Oxford, Oxford University Press. See BUSINESS: HB172 .M6247 1995. Chapter 13

## **Further Reading**

- Osborne and Rubinstein, Chapter 12.
- To see some of the refinements "in action" see Banks, Jeffrey, Colin Camerer, and David Porter, David., 1996. "An Experimental Analysis of Nash Refinements in Signaling Games," Games and Economic Behavior. 6: 1-31.

Week 7 [21 OCTOBER] BREATHER

**CLASS PRESENTATIONS OF MODEL DESIGNS** 

#### Week 8 [28 NOVEMBER] SOLUTION CONCEPTS FOR COOPERATIVE GAMES

CONCEPTS AND THEOREMS: THE NEGOTIATION SET, THE NASH BARGAINING SOLUTION, THE CORE, THE NUCLEOLIS, THE SHAPLEY VALUE, PLOTT'S THEOREM, THE UNCOVERED SET. MIXING COOPERATIVE AND NON-COOPERATICE GAME THEORY.

# Required Readings:

- Class Notes 12
- Mas-Colell, Andreu, Michael Whinston, and Jerry Green, 1995. <u>Microeconomic Theory</u>. Oxford, Oxford University Press. See BUSINESS: <u>HB172</u>. <u>M6247</u> 1995. Chapter 13
- Diermeier, Daniel and Keith Krehbiel, 2003. "<u>Institutionalism as a Methodology</u>" <u>Journal of Theoretical Politics</u> 15: 2. <a href="http://ejournals.ebsco.com/Article.asp?ContributionID=4476343">http://ejournals.ebsco.com/Article.asp?ContributionID=4476343</a>
- Austen-Smith, David and Jeffrey S. Banks. 1998. "Social Choice Theory, Game Theory and Positive Political Theory" in N.W. Polsby [Eds.] <u>Annual Review of Political Science</u>, Palo Alto: Annual Reviews. <a href="http://arjournals.annualreviews.org/doi/pdf/10.1146/annurev.polisci.1.1.259">http://arjournals.annualreviews.org/doi/pdf/10.1146/annurev.polisci.1.1.259</a>
- Aumann, Robert. 1998. "On the State of the Art in Game Theory," Games and Economic Behavior. 24: 181-210. http://sv5.vwl.tuwien.ac.at/literatur/GEB/Vol24/0612a.pdf

#### **Further Reading**

- Osborne and Rubinstein, Chapter 12.
- To see some of the refinements "in action" see Banks, Jeffrey, Colin Camerer, and David Porter, David., 1996. "An Experimental Analysis of Nash Refinements in Signaling Games," Games and Economic Behavior. 6: 1-31.

# Week 9 [4 NOVEMBER] TURNING GAME THEORY ON ITS HEAD: MECHANISM DESIGN, AUCTION THEORY

**THEOREMS AND CONCEPTS:** The Revelation Principle, The Gibbard-Satterthwaite Theorem, the Revenue Equivalence Theorem. The Myerson-Satterthwaite Theorem.

#### Required Reading

- Class Notes 13
- Osborne and Rubinstein, Chapter 10.
- Klemperer, Paul. 2000. <u>Auction Theory: A Guide to the Literature</u> in <u>The Economic Theory of Auctions</u>, Klemperer (ed.); on-line: <a href="http://www.nuff.ox.ac.uk/users/klemperer/Survey.pdf">http://www.nuff.ox.ac.uk/users/klemperer/Survey.pdf</a>

#### **Further Reading**

- Reny, 2000. "Arrow's Theorem and the Gibbard Satterthwaite Theorem: A Unified Approach," Working Paper. <a href="http://www.src.uchicago.edu/users/preny/arrow-gibbard-satterthwaite.pdf">http://www.src.uchicago.edu/users/preny/arrow-gibbard-satterthwaite.pdf</a>
- / Myerson, Roger. B., and Satterthwaite, M. A. 1983. "Efficient mechanisms for bilateral trading."
   Journal of Economic Theory 28:265--281. (Not on-line)
- Richard D. McKelvey and Talbot Page. 2002. "Status Quo Bias in Bargaining" <u>Journal of Economic Theory</u> 107, 336–355. <a href="http://www.sciencedirect.com/science/journal/00220531">http://www.sciencedirect.com/science/journal/00220531</a>

#### Part III: Game Theory Applied to Politics

## Week 10 [11 NOVEMBER] NEW WORK IN JUDGING, LOBBYING AND BARGAINING

# Required Readings:

- Cameron, Charles, Jeffrey Segal and Donald Songer. 2000. "Strategic Auditing in a Political hierarchy: An Informational model of the Supreme Court's Certorari Decisions." <u>American Political Science Review</u>. 95 (1)
- Gene Grossman and Elhanan Helpman. <u>Special Interest Politics</u>. Cambridge: MIT University Press. Chapters 7 and 8.
- Slantchev, Branislav. 2004. The Principle of Convergence in Wartime Negotiations
   <u>American Political Science Review</u>. 98 or Slantchev, Branislav. 2003 The Power to Hurt: Costly
   Conflict with Completely Informed States. <u>American Political Science Review</u>. 97 (1)

### Further Reading:

Banks, Jeffrey and John Duggan. 2000. "A Bargaining Model of Collective Choice." <u>American Political Science Review</u> 94: 73–88. <a href="http://www.columbia.edu/cgi-bin/cul/resolve?AUX2407.001">http://www.columbia.edu/cgi-bin/cul/resolve?AUX2407.001</a>

# Week 11 [18 NOVEMBER] NO CLASSES (THANKSGIVING)

# Week 12 [25 NOVEMBER] NEW WORK IN GENERALIZATION

#### Required Reading

- Robert Powell. 2004. The Inefficient Use of Power: Costly Conflict with Complete Information. American Political Science Review. 98 (2)
- Jonathan Bendor and Adam Meirowitz. 2004. "Spatial Models of Delegation" <u>American Political Science Review</u>. 98 (2)

#### Week 13 [2 DECEMBER] VIOLENT GAMES

#### Required Readings:

- Hirshleifer, Jack. 1995. "Theorizing about Conflict." in <u>Handbook of Defense Economics</u>, ed. Hartley, K. and Todd Sandler, 165-89. Vol.1, Amsterdam, Elsevier Science. In Library. See also: <a href="http://www.econ.ucla.edu/workingpapers/wp727.pdf">http://www.econ.ucla.edu/workingpapers/wp727.pdf</a>
- Dickson, Eric and Ethan Buena de Mesquita. 2004. "The Propaganda of the Deed: Terror as a Tool for Mobilization." Paper presented at APSA meetings, Chicago. [To be posted]

#### **Recommended Reading**

 Bates, Robert H., Avner Greif, and Smita Singh. 2002. "Organizing Violence." Journal of Conflict Resolution 46:5. http://www.columbia.edu/cgi-bin/cul/resolve?clio3326743.004 • Grossman, Herschel. I. 1991. "A General Equilibrium Model of Insurrections." <u>American Economic Review</u>. 81, pp. 912-21. <a href="http://links.jstor.org/sici?sici=0002-8282%28199109%2981%3A4%3C912%3AAGEMOI%3E2.0.CO%3B2-F">http://links.jstor.org/sici?sici=0002-8282%28199109%2981%3A4%3C912%3AAGEMOI%3E2.0.CO%3B2-F</a>

# Week 14 [9 DECEMBER] TAKING GAMES TO DATA

- Charles Cameron and Rebecca Morton. 2002. "Formal Theory Meets Data" in Katznelson and Milner. <u>Political Science: State of the Discipline</u>. Washington. Norton.
- Jeffrey B. Lewis and Kenneth A. Schultz 2003. Revealing Preferences: Empirical Estimation of a Crisis Bargaining Game with Incomplete Information. *Political Analysis*, 11:4. http://pan.oupjournals.org/cgi/reprint/11/4/345
- Curtis Signorino. 2003. "Structure and uncertainty in Discrete Choice Models." Political Analysis 11: 316 344.