Exercise Set 2

Lectures 7-10, Weeks 4-5

Dynamic Programming, Spring 2020

by Thomas Jørgensen (original Jeppe Druedahl)

This exercise set contains exercises for solving, simulating and estimating the buffer-stock consumption model. The third (optional) exercise consider the consumption-saving model from lecture 9 with a discrete absorbing retirement choice.

Exercise 1 [L7]: Solving the buffer-stock consumption model with EGM

Consider the canonical buffer-stock consumption model from lecture 7. The exercise will be to add code to *model* such that *Exercise_1* can be run to produce the life-cycle figures from the lecture.

1. Look at *ReadMe.txt* to get an overview of the ex ante code
2. Ensure that you *understand* the following functions:

   model.setup
   model.create_grids
   model.solve

   Hint: *Look at how they are called from Exercise_1*
3. Fill in the missing stuff in the function *model.EGM*
4. Run *Exercise_1* to check that your results are correct
5. (Optional) Could you write a vectorized version of EGM to speed it up?
   (i.e. without no loop over a_t)

Exercise 2 [L9]: Estimating the buffer-stock consumption model with MLE and MSM

Consider the canonical buffer-stock consumption model from lecture 7. The exercise will be to add code to *estimate* such that *Exercise_2* can be run to produce consistent estimates under both MLE and MSM from lecture 9.

1. Ensure that you *understand* the following sections and functions:

   section a) and b) of Exercise_2
   estimate.updatepar
   estimate.maximum_likelihood

2. Fill in the missing stuff in the function *estimate.log_likelihood* and *estimate.maximum_likelihood*

3. Run section c) and d) of *Exercise_2* to check that your results are correct

4. Ensure that you *understand* the following sections and functions:

   section e) of Exercise_2
   estimate.calc_moments
   estimate.method_simulated_moments

5. Fill in the missing stuff in the function *estimate.sum_squared_diff_moments* and *estimate.method_simulated_moments*

6. Run section f) and g) of *Exercise_2* to check that your results are correct

Exercise 3 [L10]: (Optional) Solving Discrete-Continuous Choice Models

Consider the model from lecture 10. The *value function* is given as

$$
\begin{aligned}
v_t(m_t, z_t, \varepsilon_t^0, \varepsilon_t^1) &= \max_{z_{t+1} \in \mathcal{Z}(z_t)} \left\{ \mathcal{V}_t(m_t, z_{t+1}) + \sigma_\varepsilon \varepsilon_t^{L_{t+1}} \right\} \\
& \mathcal{Z}(z_t) = \begin{cases} \{0,1\} & \text{if } z_t = 0 \\ \{1\} & \text{if } z_t = 1 \end{cases}
\end{aligned}
$$

and the *choice-specific value functions* are given by

$$
\begin{aligned}
\mathcal{V}_t(m_t, z_{t+1}) &= \max_{c_t} \frac{c_t^{1-\rho}}{1-\rho} - \alpha \mathbf{1}_{z_{t+1}=0} + \beta \mathbb{E}_t \left[v_{t+1}(\bullet_{t+1}) \right] \\
&\text{s.t.} \\
m_{t+1} &= R(m_t - c_t) + W \xi_{t+1} \mathbf{1}_{z_{t+1}=0} \\
c_t &\leq m_t \\
\log \xi_{t+1} &\sim \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) \\
\varepsilon_{t+1}^0, \varepsilon_{t+1}^1 &\sim \text{Extreme Value Type 1}
\end{aligned}
$$

The exercise will be to add code in *model_dc* such that *Exercise_3* can be run to produce the consumption function figures from the lecture. This cannot be done without understanding *all* the other functions in *model_dc*.

1. Ensure that you understand the function *logsum*
2. Ensure that you understand all functions in *model_dc*
3. Fill in the missing stuff in the function *model_dc.EGM*
4. Run *Exercise_3* to check that your results are correct

Now consider the model extended with permanent income

$$
\begin{aligned}
v_t(m_t, p_t, z_t, \varepsilon_t^0, \varepsilon_t^1) &= \max_{z_{t+1} \in \mathcal{Z}(z_t)} \left\{ \mathcal{V}_t(m_t, p_t, z_{t+1}) + \sigma_\varepsilon \varepsilon_t^{L_{t+1}} \right\} \\
\mathcal{Z}(z_t) &= \begin{cases} \{0,1\} & \text{if } z_t = 0 \\ \{1\} & \text{if } z_t = 1 \end{cases}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{V}_t(m_t, p_t, z_{t+1}) &= \max_{c_t} \frac{c_t^{1-\rho}}{1-\rho} - \alpha \mathbf{1}_{z_{t+1}=0} + \beta \mathbb{E}_t \left[v_{t+1}(\bullet_{t+1}) \right] \\
&\text{s.t.} \\
p_{t+1} &= \begin{cases} p_t & \text{if } z_{t+1} = 1 \\ \xi_{t+1} p_t & \text{if } z_{t+1} = 0 \end{cases} \\
m_{t+1} &= R(m_t - c_t) + W \mathbf{1}_{z_{t+1}=0} p_{t+1} + \kappa \mathbf{1}_{z_{t+1}=1} p_{t+1} \\
c_t &\leq m_t \\
\log \xi_{t+1} &\sim \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) \\
\varepsilon_{t+1}^0, \varepsilon_{t+1}^1 &\sim \text{Extreme Value Type 1}
\end{aligned}
$$

5. Solve the extended model [THIS IS NOT EASY].