Fractal Computation

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Abstract

The objective of the lab is to compute the basin of attractions of Newton's method to compute the cubic roots of unity. The resulting set is an example of a Julia set (fractal).

Write a CUDA code to compute the Julia set associated with the Newton iteration for the complex function $f(z) = z^3 - 1$. In other words, consider the complex function $f(z) = z - g(z)/g'(z)$ where $g(z) = z^3 - 1$. If has 3 attractors: the cubit roots of unity. For each point z_0 in the complex plane, compute the sequence $z_1 = f(z_0)$, $z_{n+1} = f(z_n)$. That sequence converges to one of the attractors. Color your image with red for 1, green for $e^{2i\pi/3}$ and blue for $e^{-2i\pi/3}$.

Compute a 1024×768 matrix storing the image corresponding to the complex plane with the range of the real part between -2.2 and 2.2. The range of the imaginary part should be such that the basis is orthonormal i.e. 1 unit of distance on the x-axis corresponds to 1 unit of distance on the y axis.

Submit:

1. a serial code computing the matrix with values of 1 for red, 2 for blur, and 3 for green.
2. a CUDA code doing the same computation using 1 thread per pixel.
3. a CUDA code computing the matrix and then plotting the image.
4. the image you obtained in png format.

To check your answer, your image should be similar to `http://upload.wikimedia.org/wikipedia/commons/6/6a/Julia-set_N_z3-1.png` except that boundaries should be sharps instead of smooth.

P.S. Many fancy pictures can be generated from that code when using $f(z) = z^2 + c$ for different values of c. See `http://en.wikipedia.org/wiki/Julia_set`.