Network Analysis and Mining
5. Community detection

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Community detection

Goal: Identify automatically relevant groups of nodes.

Applications:

- Understand the structure of a network
- Detect specific communities (web pages, proteins, ...)
- Help visualization
- Improvement information retrieval (search engines, recommendation, ...)

Challenges:

- Unknown number of communities
- Unknown sizes of communities
- Scalability

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What is a community?

Set of nodes that share something:

- Affiliation (friends, colleagues, club, ...)
- Similar interests (tagging systems, ...)
- Similar contents (movies, books, products, web pages, ...)
- . . .

What is the connexion with the network structure?

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What is the connexion with the network structure?

More densely connected inside than outside

Outline

Structural approaches: cohesive subgraphs

clique: complete subgraph

***k*-plex**: maximal subgraph H such that each node is connected to at least $|H| - k$ other nodes (if $k = 1$: clique)

α-set: maximal subgraph such that any node as α times more internal than external links

Exercise: suggest other relevant definitions.

Optimization approach: quality function

Quality function: quantitatively evaluate the quality of a set of nodes as a community.

Local optimum of $f(n, m, s, l_2, l_1)$, with

- s: number of nodes in the set
- l_2: number of links with both end nodes in the set
- l_1: number of links with exactly one node in the set

Examples:

- $\frac{l_2}{l_1+l_2}$
- edit distance: $\frac{s(s-1)}{2} - l_2 + l_1$

Exercise: suggest other relevant quality functions.

Optimization approach: quality function

Local optimum of $f(n, m, t, s, l_2, l_1, t_1, t_2, t_3)$, with

- t: number of triangles in the graph
- t_3: number of triangles with three nodes in the set
- t_2: number of triangles with exactly two nodes in the set
- t_1: number of triangles with exactly one node in the set

Examples:

- triangle density: $\frac{t_3}{s}$
- $C = \frac{t_3}{\binom{s}{3}} \times \frac{t_3}{t_3+t_2}$ Triangles to Capture Social Cohesion - *Friggeri et al.*

Exercise: suggest other relevant quality functions.

Optimization approach: Optimization

Use a simple greedy or stochastic approach:

- **Initialization:** start from a set containing only one node
- **Optimization:** at each iteration, add a randomly chosen node, neighbor of the set, that increases the quality f
- **Stop:** when the quality function can no longer be increased

Can be done efficiently by updating s, l_1 and l_2 locally:
`https://github.com/maxdan94/mocda`

Outline

A simple and fast algorithm: Label propagation

Near linear time algorithm to detect community structures in large-scale networks - *Raghavan et al.*

- **Step 1:** give a unique label to each node in the network
- **Step 2:** Arrange the nodes in the network in a random order
- **Step 3:** for each node in the network (in this random order) set its label to a label occurring with the highest frequency among its neighbours
- **Step 4:** go to 2 as long as there exists a node with a label that does not have the highest frequency among its neighbours.

To shuffle in a clean way:
`https://en.wikipedia.org/wiki/Fisher-Yates_shuffle`

A simple and fast algorithm: Label propagation

Exercise: why such an algorithm should lead to relevant groups?

Exercise: Which data structure should be used to implement this algorithm efficiently?

A simple and fast algorithm: Label propagation

Exercise: why such an algorithm should lead to relevant groups?

- Densely connected groups should reach a common label.
- When such a consensus group is created it should expand until being stopped by other equivalent consensus groups.

Exercise: Which data structure should be used to implement this algorithm efficiently?

Community structure

Structural definitions

- A **community** is a set of nodes that are more connected among themselves than to the rest of the network
- **Modularity** is a measure to evaluate the quality of a community partioning of a graph **(one among others)**

What is modularity (intuitively)?

The difference between:

- the **number of links in a group**
- and the **expected number of links in the same group** of a comparable random graph

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Modularity definition: preliminary remarks

Graph G : m links, n nodes

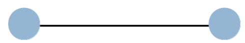
Group S : sum of degree d_s , number of internal links m_s

In a random graph with fixed degree distribution:

probability for one end of a link to be in S:

$\Rightarrow$ probability for a link to be in S:

$\Rightarrow$ expected number of links in S:



Modularity definition: preliminary remarks

Graph G : m links, n nodes

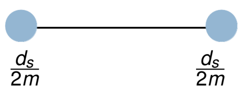
Group S : sum of degree d_s , number of internal links m_s

In a random graph with fixed degree distribution:

probability for one end of a link to be in S: $\frac{d_s}{2m}$

$\Rightarrow$ probability for a link to be in S: $\frac{d_s}{2m} \cdot \frac{d_s}{2m}$

$\Rightarrow$ expected number of links in S: $m.\frac{d_s}{2m} \cdot \frac{d_s}{2m} = \frac{d_s^2}{4m}$



Modularity definition

$$Q = \frac{1}{m} \sum_{s=1}^{K} \left(m_s - \frac{d_s^2}{4m} \right)$$

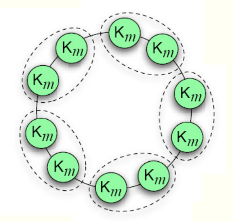
- m_s: number of internal links in group s
- m: number of links in the graph

Known problem: resolution limit

Ring of cliques: α cliques of size β

$$Q_{single} = 1 - \frac{2}{\beta(\beta-1)+2} - \frac{1}{\alpha}$$

$$Q_{pairs} = 1 - \frac{1}{\beta(\beta-1)+2} - \frac{2}{\alpha}$$



Known problem: resolution limit

Ring of cliques: α cliques of size β

$$Q_{single} > Q_{pairs} \iff \beta(\beta-1)+2 > \alpha$$

Suppose 30 cliques of size 5 then:

- $\alpha = 30$ and $\beta(\beta-1)+2 = 22 \Rightarrow Q_{single} < Q_{pairs}$
- $Q_{single} = 0.876$, $Q_{pairs} = 0.888$

counter-intuitive
Tendency to favour large communities...
... may appear at any length scale

Greedy and efficient optimization of Modularity

- **Step 1.** Initialization: node = community
- **Step 2.** Remove node *u* from its community
- **Step 3.** Insert node *u* in a neighboring community that maximizes Q
- **Step 4.** Iterate from step 1 until the partition does not evolve

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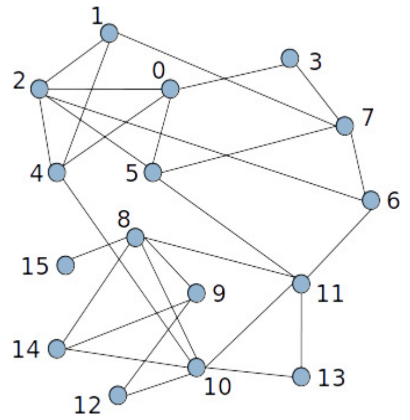
Can be trapped in bad local minima

The Louvain algorithm

- **Step 1.** Initialization: node = community
- **Step 2.** Remove node u from its community
- **Step 3.** Insert node u in a neighboring community that maximizes Q
- **Step 4.** Iterate from step 1 until the partition does not evolve
- **Step 5.** Transform the communities into (hyper-)nodes and go back to step 1 with the new graph

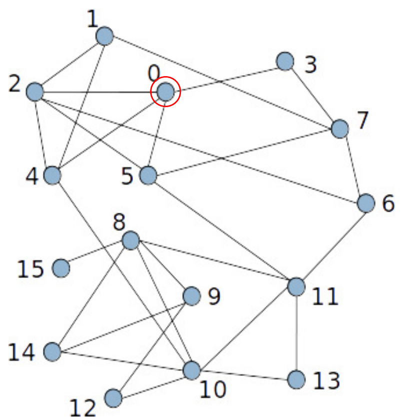
Leads to better local optima

Example



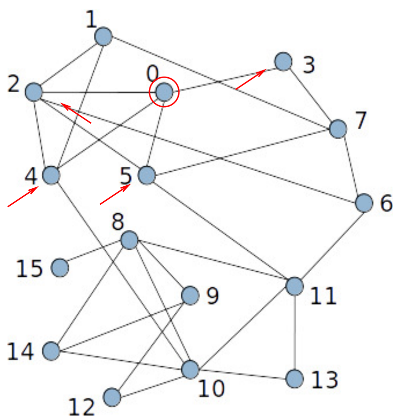
First passage, first iteration: isolated nodes

Example



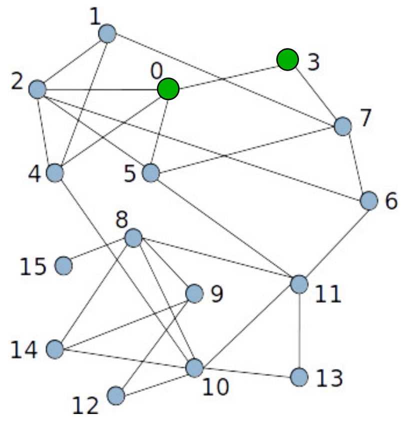
considering 0...

Example



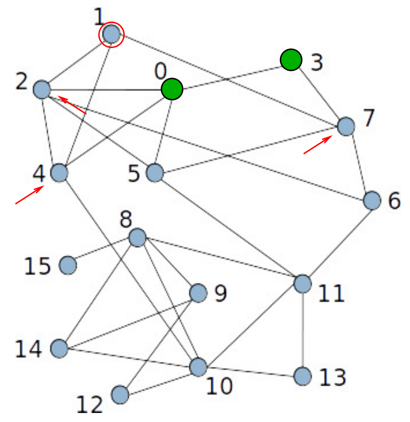
its neighboring communities are...

Example



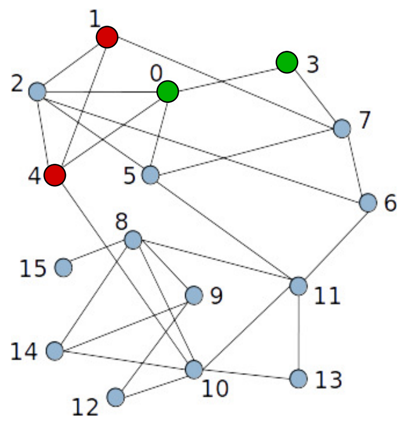
0 is put in C(3), best Q increase

Example



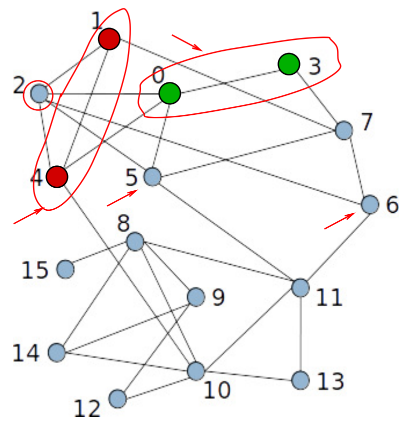
considering 1, its neighboring communities are...

Example



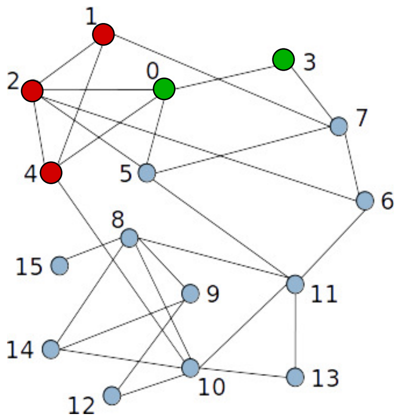
1 is put in C(4), best Q increase

Example



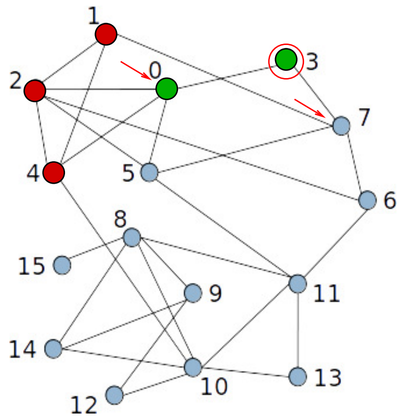
considering 2, its neighboring communities are...

Example



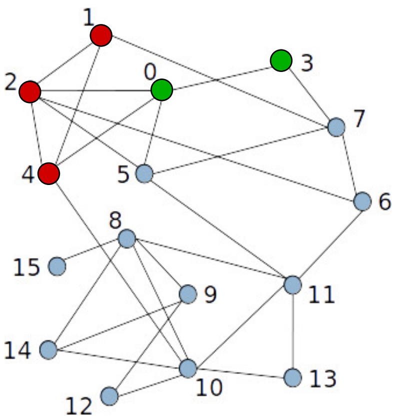
2 is put in C(1,4), best Q increase

Example



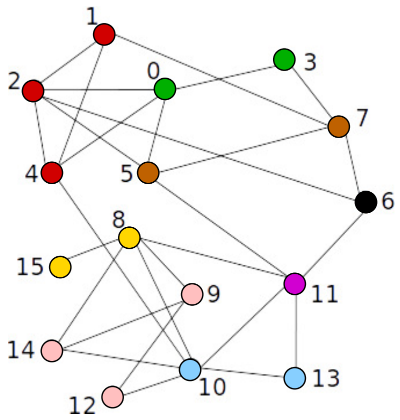
considering 3, its neighboring communities are...

Example



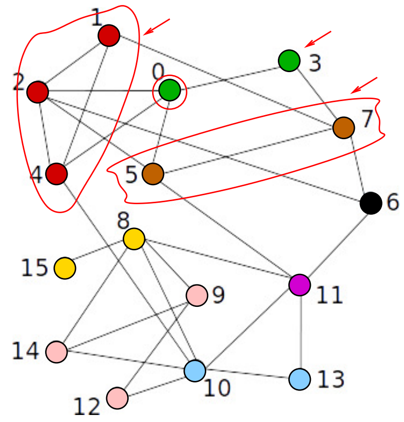
3 stays in the same community C(0,3), otherwise Q decreases

Example



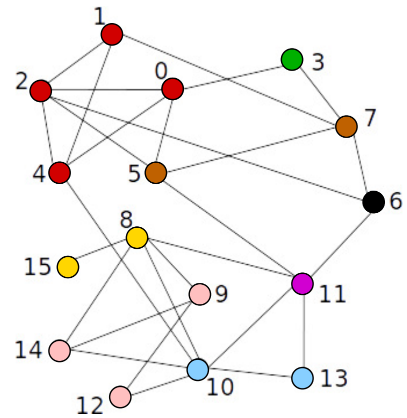
and so on...

Example



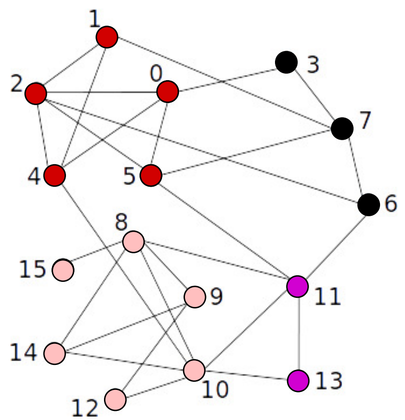
First passage, second iteration: considering 0...

Example



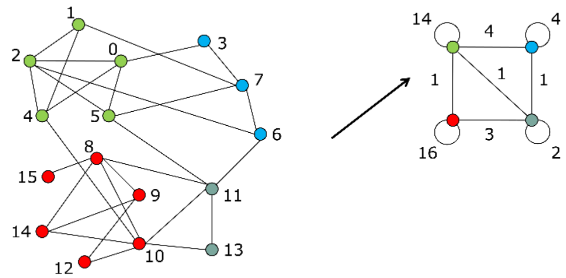
0 is put in C(1,2,4), best Q increase

Example



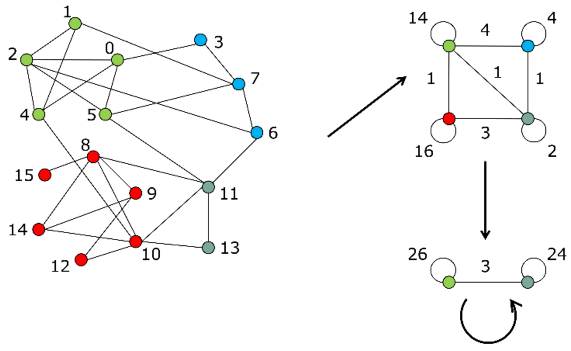
after 4 iterations, no change anymore

Example



Second passage

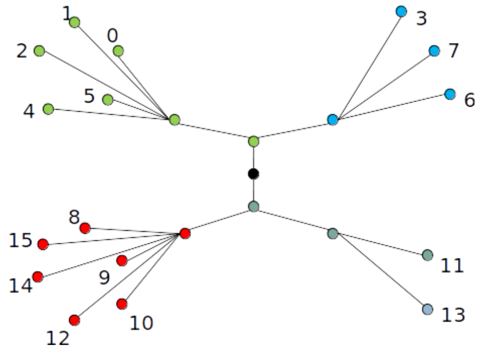
Example



Third passage

Example

Outcome: non-binary dendrogram



Other experimental observations

- Graphs on the highest levels of the dendrogram are small, **the first passages are the most expensive** practically, the first passage is more than 90% of the time
- **Few iterations per passage** (less than < 33 for all the networks tested)
- **Processing a node is simple** (cheap)

Modularity

$$Q = \frac{1}{m}\sum_s m_s - \frac{d_s^2}{4m} = \sum_s \frac{m_s}{m} - \left(\frac{d_s}{2m}\right)^2$$

m_s: links $\in S$
d_s: sum of the degrees of nodes in S

Note that the contribution of an isolated node is then:

$$Q(i) = -\left(\frac{k_i}{2m}\right)^2$$

with k_i: degree of i
$\Rightarrow$ always merged with a neighboring community

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The cost of moving one node

An isolated node i may be moved to S with a gain of:

$$\Delta Q(S,i) = \left[\frac{m_s}{m} + \frac{k_{i,s}}{m} - \left(\frac{d_s + k_i}{2m}\right)^2\right] - \left[\frac{m_s}{m} - \left(\frac{d_s}{2m}\right)^2 - \left(\frac{k_i}{2m}\right)^2\right]$$

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Only depends on S and i, linear complexity with k_i

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Data structures

We have to keep in memory:

- the adjacency lists: size $(2m + n)$
- vectors m_s, d_s and *node2comm* (stores $k_{i,s}$): size n each

A total of $2m + 4n$, meaning a few GB for a billion links graph

Conclusion

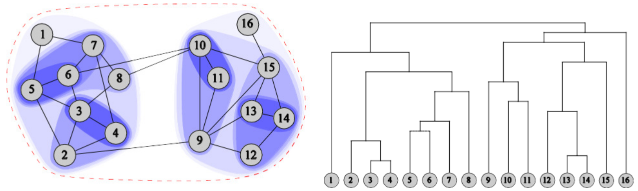
Fast unfolding of communities in large networks - *Blondel et al, 2008*

- What kind of approach is it?
  - **greedy local** approach
  - **modularity-based**
- **Good results** in terms of modularity
- **Quasi linear** complexity
  $\Rightarrow$ allow to process very large graphs
- **Non-deterministic** algorithm

Divisive and agglomerative approaches

Many other algorithms can be found in the literature:
Community detection in graphs - *Fortunato, 2010*

A large amount of them can be seen as divisive or agglomerative approaches.



Agglomerative approach

- **Step 1:** Each node is in a community (initialization)
- **Step 2:** Compute the similarity between each pair of communities
- **Step 3:** Merge the two closest communities
- **Step 4:** Iterate from step 1

Exercise: Suggest a relevant similarity metric between two communities

Divisive approaches

- **Step 1:** All nodes are in a unique community (initialization)
- **Step 2:** Compute a strength score for each link
- **Step 3:** Delete the weakest link
- **Step 4:** Iterate from step 1

Exercise: Suggest a relevant strength score for a link

Outline

Many algorithms

Again there is a plethora of algorithms for finding overlapping communities:

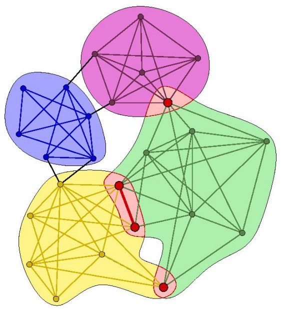
Overlapping Community Detection in Networks: The State-of-the-art and Comparative Study - *Xie at al. 2013*

We just show one which was among the first to do the job.

k-clique percolation method

Definition: Two k-cliques are considered adjacent if they share $k-1$ nodes.

Definition: A community is defined as the maximal union of k-cliques that can be reached from each other through a series of adjacent k-cliques.



Exercise: how can we find all “communities” efficiently for $k=3$?