Feature transformation

The process of modifying the original dataset to create new features that capture the variability and structures from the original data. Enhance model performance.

- Principal Component Analysis
- Factor Analysis

You an then reduce dimensionality while preserving as much information as possible.

Feature transformation methods need numeric data only.

```
% Script to fetch and combine closing prices for multiple stock indices with
interpolation

% List of stock index symbols
symbols = {'^GSPC','^GSPTSE','^FCHI','^GDAXI','FTSEMIB.MI','^N225',
'^FTSE','^HSI','IMOEX.ME'};


% Define the date range and parameters
startDate = '05-Mar-2013';
endDate = '30-May-2023';
interval = '1d';         % Daily data
fields = {'close'};      % Only closing prices

% Number of symbols
numSymbols = length(symbols);

% Cell arrays to store data
stockDataCell = cell(numSymbols, 1);
timetablesCell = cell(numSymbols, 1);

% Fetch data for each symbol and convert to timetable
for k = 1:numSymbols
    symbol = symbols{k};
    fprintf('Fetching data for %s...\n', symbol);

    try
        % Fetch data using the custom function
        stockData = fetchYahooFinanceData(symbol, startDate, endDate, interval,
fields);

        % Remove time component and set dates to midnight
```

```
        stockData.Date = dateshift(stockData.Date, 'start', 'day');

        % Rename the 'Close' column to the symbol name
        varName = matlab.lang.makeValidName(symbol);
        stockData.Properties.VariableNames{'Close'} = varName;

        % Convert to timetable
        tt = table2timetable(stockData, 'RowTimes', 'Date');

        % Store the timetable
        timetablesCell{k} = tt(:, varName);  % Only include the price column
    catch ME
        warning('Failed to fetch data for %s: %s', symbol, ME.message);
        timetablesCell{k} = timetable();  % Empty timetable
    end
end
```

```
Fetching data for ^GSPC...
Fetching data for ^GSPTSE...
Fetching data for ^FCHI...
Fetching data for ^GDAXI...
Fetching data for FTSEMIB.MI...
Fetching data for ^N225...
Fetching data for ^FTSE...
Fetching data for ^HSI...
Fetching data for IMOEX.ME...
```

```

% Remove empty timetables
nonEmpty = ~cellfun(@isempty, timetablesCell);
timetablesCell = timetablesCell(nonEmpty);
symbols = symbols(nonEmpty);

% Collect all unique dates from all indices
allDates = [];
for k = 1:length(timetablesCell)
    allDates = [allDates; timetablesCell{k}.Date];
end

% Get unique sorted dates
commonDates = unique(allDates);

% Sort dates
commonDates = sort(commonDates);

% Resample all timetables to the common dates with interpolation
for k = 1:length(timetablesCell)
    % Synchronize to common dates with linear interpolation
    timetablesCell{k} = retime(timetablesCell{k}, commonDates, 'linear');
end

% Combine all timetables into one
combinedTimetable = timetablesCell{1};
```

```
for k = 2:length(timetablesCell)
    combinedTimetable = synchronize(combinedTimetable, timetablesCell{k}, 'union',
'linear');
end

% Convert timetable to table
combinedData = timetable2table(combinedTimetable);

% Rename the time variable to 'Date'
combinedData.Properties.VariableNames{1} = 'Date';

% Display the first few rows
disp('Combined Data (First 10 Rows):');
```

```
Combined Data (First 10 Rows):
```

```
disp(head(combinedData, 10));
```

Date	x_GSPC	x_GSPTSE	x_FCHI	x_GDAXI	FTSEMIB_MI	x_N225	x_FTSE	x_HSI	IMOEX_ME
05-Mar-2013	1539.8	12736	3787.2	7870.3	15974	11683	6432	22560	1486.4
06-Mar-2013	1541.5	12832	3773.8	7919.3	15900	11932	6427.6	22778	1491.5
07-Mar-2013	1544.3	12826	3793.8	7939.8	15947	11968	6439.2	22771	1491.3
08-Mar-2013	1551.2	12836	3840.1	7986.5	16204	12284	6483.6	23092	1495
11-Mar-2013	1556.2	12858	3836.3	7984.3	16092	12349	6503.6	23091	1505.9
12-Mar-2013	1552.5	12879	3840	7966.1	16024	12315	6510.6	22891	1499.7
13-Mar-2013	1554.5	12744	3836	7970.9	15745	12240	6481.5	22557	1495.9
14-Mar-2013	1563.2	12800	3871.6	8058.4	16131	12381	6529.4	22619	1502.5
15-Mar-2013	1560.7	12830	3844	8042.9	16061	12561	6489.7	22533	1496.6
18-Mar-2013	1552.1	12782	3825.5	8010.7	15924	12221	6457.9	22083	1462.6

```
% Write the combined data to an Excel file
writetable(combinedData, 'Data.xlsx');
fprintf('Data with interpolated missing values has been written to
StockIndicesData_Interpolated.xlsx\n');
```

```
Data with interpolated missing values has been written to StockIndicesData_Interpolated.xlsx
```

Compute returns

```
ret=diff(log(combinedData(:,2:end)))
```

ret = 2663×9 table

...

	x_GSPC	x_GSPTSE	x_FCHI	x_GDAXI	FTSEMIB_MI	x_N225	x_FTSE
1	0.0011	0.0075	-0.0036	0.0062	-0.0046	0.0211	-6.8430e-04
2	0.0018	-4.2871e-04	0.0053	0.0026	0.0030	0.0030	0.0018

	x_GSPC	x_GSPTSE	x_FCHI	x_GDAXI	FTSEMIB_MI	x_N225	x_FTSE
3	0.0045	7.0919e-04	0.0121	0.0059	0.0160	0.0260	0.0069
4	0.0032	0.0018	-0.0010	-2.7302e-04	-0.0069	0.0053	0.0031
5	-0.0024	0.0016	9.6400e-04	-0.0023	-0.0042	-0.0028	0.0011
6	0.0013	-0.0105	-0.0010	6.0112e-04	-0.0176	-0.0061	-0.0045
7	0.0056	0.0044	0.0092	0.0109	0.0242	0.0115	0.0074
8	-0.0016	0.0023	-0.0071	-0.0019	-0.0043	0.0144	-0.0061
9	-0.0055	-0.0038	-0.0048	-0.0040	-0.0086	-0.0275	-0.0049
10	-0.0024	-6.1821e-04	-0.0131	-0.0079	-0.0160	0.0201	-0.0026
11	0.0067	0.0041	0.0142	0.0068	0.0218	0.0067	-0.0013
12	-0.0083	-0.0062	-0.0144	-0.0087	-0.0050	0.0066	-0.0069
13	0.0071	7.4494e-04	-0.0012	-0.0027	0.0069	-0.0238	6.5716e-04
14	-0.0033	-0.0060	-0.0113	-0.0051	-0.0254	0.0167	-0.0023
15	0.0078	0.0020	0.0055	0.0011	-0.0095	-0.0060	0.0033
16	-5.8852e-04	-5.2745e-04	-0.0099	-0.0116	-0.0092	0.0018	-0.0018
17	0.0040	0.0039	0.0053	7.9826e-04	-9.7742e-04	-0.0127	0.0038
18	-0.0011	-0.0011	0.0040	0.0038	0.0028	0.0050	0.0025
19	-0.0034	-0.0032	0.0118	0.0113	0.0084	-0.0214	0.0073
20	0.0052	-0.0010	0.0039	0.0037	0.0028	-0.0109	0.0024
21	-0.0106	-0.0207	-0.0133	-0.0087	-0.0231	0.0295	-0.0109
22	0.0040	-0.0048	-0.0077	-0.0073	-0.0030	0.0218	-0.0119
23	-0.0043	-0.0025	-0.0170	-0.0205	0.0063	0.0156	-0.0150
24	0.0063	0.0010	9.0039e-04	5.0780e-04	-4.5912e-04	0.0276	0.0043
25	0.0035	0.0112	0.0011	-0.0033	0.0126	-1.8210e-05	0.0058
26	0.0121	0.0041	0.0197	0.0224	0.0314	0.0072	0.0117
27	0.0035	-0.0043	0.0085	0.0078	0.0058	0.0195	0.0045
28	-0.0028	-0.0116	-0.0124	-0.0162	-0.0152	-0.0047	-0.0050
29	-0.0232	-0.0273	-0.0051	-0.0042	-0.0096	-0.0157	-0.0064
30	0.0142	0.0095	-0.0067	-0.0039	-0.0062	-0.0041	-0.0062
31	-0.0144	-0.0143	-0.0238	-0.0236	-0.0096	0.0121	-0.0096
32	-0.0067	0.0041	3.6153e-05	-0.0039	0.0063	-0.0122	-8.0078e-05
33	0.0088	0.0058	0.0145	-0.0018	0.0179	0.0073	0.0068
34	0.0047	0.0021	4.6528e-05	0.0024	0.0164	0.0187	-9.5487e-04
35	0.0104	1.6558e-05	0.0352	0.0238	0.0289	-0.0029	0.0198

	x_GSPC	x_GSPTSE	x_FCHI	x_GDAXI	FTSEMIB_MI	x_N225	x_FTSE
36	6.3402e-06	0.0147	0.0157	0.0131	0.0044	0.0229	0.0040
37	0.0040	0.0048	-6.4294e-04	0.0095	0.0052	0.0060	0.0017
38	-0.0018	-0.0089	-0.0080	-0.0023	-0.0051	-0.0030	-0.0025
39	0.0072	0.0075	0.0153	0.0075	0.0218	-0.0013	0.0049
40	0.0025	0.0116	-0.0031	0.0051	-0.0096	-4.1961e-04	-0.0043
41	-0.0094	-0.0109	2.6055e-04	0.0030	-5.9655e-04	-0.0044	0.0033
42	0.0094	0.0047	2.6048e-04	0.0030	-5.9691e-04	-0.0077	0.0015
43	0.0105	0.0047	0.0139	0.0200	0.0103	0.0071	0.0094
44	0.0019	0.0013	-0.0015	-0.0013	-0.0035	0.0209	0.0041
45	0.0052	8.1862e-04	0.0036	0.0086	0.0152	0.0069	0.0014
46	0.0041	0.0097	0.0089	0.0083	0.0078	0.0074	0.0040
47	-0.0037	-0.0033	-0.0070	0.0016	-0.0096	-0.0066	0.0014
48	0.0043	0.0036	0.0064	0.0019	0.0112	0.0289	0.0049
49	4.2888e-05	-0.0047	-0.0022	8.4575e-05	-0.0065	0.0119	0.0010
50	0.0101	0.0038	0.0053	0.0072	0.0083	-0.0016	0.0082
51	0.0051	-0.0082	0.0041	0.0028	0.0102	0.0226	0.0011
52	-0.0050	0.0027	-7.9382e-04	8.9052e-04	0.0029	-0.0039	-8.6692e-04
53	0.0102	0.0084	0.0056	0.0034	0.0035	0.0067	0.0053
54	-7.0787e-04	0.0077	0.0054	0.0069	-0.0056	0.0146	0.0048
55	0.0017	0.0025	0.0033	0.0019	-0.0046	0.0013	0.0071
56	-0.0083	7.9228e-04	0.0037	0.0069	0.0067	0.0159	0.0053
57	-0.0029	-0.0074	-0.0209	-0.0212	-0.0311	-0.0760	-0.0212
58	-5.5152e-04	7.1870e-04	-0.0026	-0.0056	-0.0065	0.0088	-0.0064
59	0.0047	0.0023	0.0097	0.0093	0.0154	-0.0327	0.0121
60	0.0016	0.0043	0.0138	0.0116	0.0208	0.0119	0.0040
61	-0.0071	-0.0014	-0.0191	-0.0172	-0.0163	0.0010	-0.0201
62	0.0037	0.0011	0.0056	0.0076	0.0066	-0.0528	0.0045
63	-0.0144	-0.0076	-0.0120	-0.0061	-0.0079	0.0136	-0.0112
64	0.0059	-0.0032	-0.0071	-0.0076	-0.0091	-0.0379	-0.0088
65	-0.0055	-0.0013	0.0013	0.0012	0.0046	0.0203	0.0051
66	-0.0139	-0.0120	-0.0189	-0.0121	-0.0097	-0.0391	-0.0215
67	0.0085	-0.0028	-0.0100	-0.0120	-0.0266	-0.0086	-0.0130
68	0.0127	-0.0029	0.0152	0.0191	0.0100	-0.0021	0.0119

	x_GSPC	x_GSPTSE	x_FCHI	x_GDAXI	FTSEMIB_MI	x_N225	x_FTSE
69	-3.4687e-04	7.5944e-04	-0.0021	0.0064	-0.0081	0.0483	-0.0018
70	-0.0102	-0.0129	-0.0140	-0.0103	-0.0164	-0.0147	-0.0095
71	-0.0084	-0.0093	-0.0044	-0.0097	-0.0163	-0.0021	-0.0064
72	0.0147	0.0137	0.0011	-0.0059	0.0057	-0.0656	8.0928e-04
73	-0.0059	-0.0073	0.0019	0.0040	0.0023	0.0192	5.8665e-04
74	0.0075	0.0083	0.0153	0.0107	0.0025	0.0270	0.0035
75	0.0078	0.0064	-8.0522e-04	0.0017	2.4697e-04	-0.0020	0.0069
76	-0.0139	-0.0081	-0.0055	-0.0039	-0.0094	0.0181	-0.0040
77	-0.0253	-0.0247	-0.0373	-0.0333	-0.0315	-0.0176	-0.0303
78	0.0027	0.0023	-0.0111	-0.0177	-0.0191	0.0164	-0.0071
79	-0.0122	-0.0133	-0.0172	-0.0125	-0.0094	-0.0127	-0.0143
80	0.0095	0.0141	0.0150	0.0153	-0.0036	-0.0072	0.0120
81	0.0095	-0.0045	0.0207	0.0165	0.0201	-0.0105	0.0104
82	0.0062	0.0045	0.0097	0.0062	0.0044	0.0291	0.0126
83	-0.0043	0.0102	-0.0062	-0.0040	-0.0125	0.0345	-0.0045
84	0.0054	0.0030	0.0076	0.0031	0.0144	0.0127	0.0147
85	-5.4506e-04	0.0010	-0.0066	-0.0092	-0.0061	0.0176	-6.1846e-04
86	8.2371e-04	-0.0027	-0.0109	-0.0103	-0.0054	-0.0031	-0.0118
87	0.0051	0.0017	0.0286	0.0209	0.0338	-0.0026	0.0303
88	0.0051	-0.0026	-0.0147	-0.0238	-0.0175	0.0205	-0.0072
89	0.0052	0.0061	0.0185	0.0206	0.0170	-0.0141	0.0116
90	0.0072	0.0072	0.0051	0.0111	-5.6978e-04	0.0254	0.0097
91	1.8158e-04	7.9668e-04	-7.8865e-04	0.0011	-0.0072	-0.0039	-0.0012
92	0.0135	0.0150	0.0074	0.0114	0	0.0039	0.0059
93	0.0031	-0.0025	-0.0036	0.0066	-0.0158	0.0023	2.2921e-04
94	0.0014	0.0053	0.0061	0.0027	0.0107	0.0048	0.0063
95	-0.0037	-9.1031e-04	-0.0071	-0.0041	-0.0043	0.0016	-0.0045
96	0.0028	0.0041	0.0054	0.0065	0.0106	0.0011	0.0024
97	0.0050	0.0048	0.0143	0.0099	0.0226	0.0132	0.0095
98	0.0016	0.0044	-6.2904e-04	-6.6226e-04	0.0044	-0.0149	-5.5781e-04
99	0.0020	0.0058	0.0037	-6.1304e-05	0.0068	0.0047	-0.0011
100	-0.0019	-0.0010	-0.0043	-0.0020	3.0795e-04	0.0082	-0.0039

⋮

```
ret=table2array(ret)
```

```
ret = 2663×9
    0.0011    0.0075   -0.0036    0.0062   -0.0046    0.0211   -0.0007    0.0096 ···
    0.0018   -0.0004    0.0053    0.0026    0.0030    0.0030    0.0018   -0.0003
    0.0045    0.0007    0.0121    0.0059    0.0160    0.0260    0.0069    0.0140
    0.0032    0.0018   -0.0010   -0.0003   -0.0069    0.0053    0.0031   -0.0000
   -0.0024    0.0016    0.0010   -0.0023   -0.0042   -0.0028    0.0011   -0.0087
    0.0013   -0.0105   -0.0010    0.0006   -0.0176   -0.0061   -0.0045   -0.0147
    0.0056    0.0044    0.0092    0.0109    0.0242    0.0115    0.0074    0.0028
   -0.0016    0.0023   -0.0071   -0.0019   -0.0043    0.0144   -0.0061   -0.0038
   -0.0055   -0.0038   -0.0048   -0.0040   -0.0086   -0.0275   -0.0049   -0.0202
   -0.0024   -0.0006   -0.0131   -0.0079   -0.0160    0.0201   -0.0026   -0.0019
    0.0067    0.0041    0.0142    0.0068    0.0218    0.0067   -0.0013    0.0097
   -0.0083   -0.0062   -0.0144   -0.0087   -0.0050    0.0066   -0.0069   -0.0014
    0.0071    0.0007   -0.0012   -0.0027    0.0069   -0.0238    0.0007   -0.0050
   -0.0033   -0.0060   -0.0113   -0.0051   -0.0254    0.0167   -0.0023    0.0061
    0.0078    0.0020    0.0055    0.0011   -0.0095   -0.0060    0.0033    0.0027
      ⋮
```

To perform feature transformation, one need to normalize the data so that all variables have the same mean and standard deviation.

That's because methods that rely on distance or covariance (like PCA) can be dominated by features with large numeric ranges unless you scale them.

One can normalize your data using the `zscore` function.

```
[retZ, muR, sdR] = zscore(ret);   % retZ is standardized returns
```

.

PCA is essentially a transformation of the data into the **space spanned by the eigenvectors of its covariance (or correlation) matrix**.

Principal Component Analysis

Principal component analysis (PCA) transforms an *n*-dimensional feature space into a new *n*-dimensional space of orthogonal components. The components are ordered by the variation explained in the data.

```
[pcs, scrs, ~, ~, pctExp] = pca(retZ, ...
    'Algorithm','eig', ...        % eigen-decomposition (appropriate here)
    'Centered', true, ...         % retZ already centered; true is fine
    'NumComponents', size(retZ,2) ...
);


% Compute eigenvalues (variances explained by each component)
lambda = var(scrs, 0, 1)';  % column vector
```

The `pca` function transforms data into its principal components. The transformed data are pcs.

The second and fifth outputs, here scrs and `pctExp`, are the transformed data and the percent variance explained by each principal component.

One can name other outputs to return additional information about the transformation or use ~ to skip these outputs. For more information, see the `pca` documentation.

The transformed data `scrs` and the percentage variance explained `pexp` are ordered in descending order according to the percentage variance explained.

pcs - Principal components coefficients. The contribution of each predictor to the *n* principal components.

(*n*-by-*n* matrix)

scrs - Principal component scores. The representations of the *m* observations in `X` in the principal component space.

(*m*-by-*n* matrix)

pctExp - Percent of the variance in the data explained by each principal component.

(1-by-*n* vector)

The transformed variables contain the same amount of information as the original data. However, assuming that the data contains some amount of noise, the components that contain the last few percent of explained variance are likely to represent noise more than information.

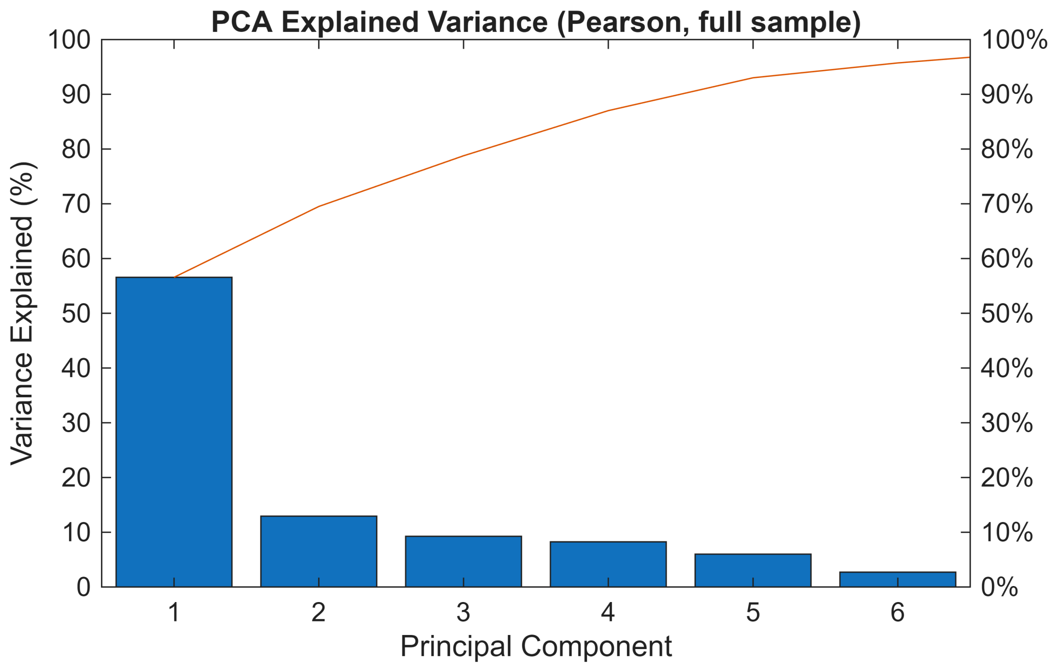
(Equivalent) PCA by diagonalizing Pearson correlation

```
% -----------------------------
R = corrcoef(ret, 'Rows','pairwise');    % Pearson correlation of raw returns
R = (R + R')/2;                          % symmetrize for numerical stability
[Vec, Lam] = eig(R);
[lambda, ord] = sort(diag(Lam), 'descend');
V = Vec(:, ord);                         % eigenvectors (loadings)
pcs_R   = V;
pctExp_R = lambda / sum(lambda) * 100;   % same as pctExp (percent)
```

D) DIAGNOSTICS & PLOTS

You can use the **`pareto` function** (explained variance) to visualize the variance explained by the principle components.

```
figure; pareto(pctExp);
title('PCA Explained Variance (Pearson, full sample)');...
    xlabel('Principal Component'); ylabel('Variance Explained (%)');
```



A Pareto function describes a situation where a small number of causes explain a large part of the effect — often called the 80/20 rule.

For example:

20% of customers might generate 80% of sales.

20% of bugs might cause 80% of software crashes.

Mathematically, the Pareto function (or Pareto distribution) models this type of imbalance.

It's a power-law distribution where large values are rare but have a big impact.

Its probability density function (PDF) is given by:

$$f(x) = \begin{cases} \dfrac{\alpha x_m^{\alpha}}{x^{\alpha+1}}, & x \geq x_m, \\ 0, & x < x_m, \end{cases}$$

where:

- $x_m > 0$ is the scale parameter (the minimum possible value),
- $\alpha > 0$ is the shape parameter, which determines the heaviness of the tail.

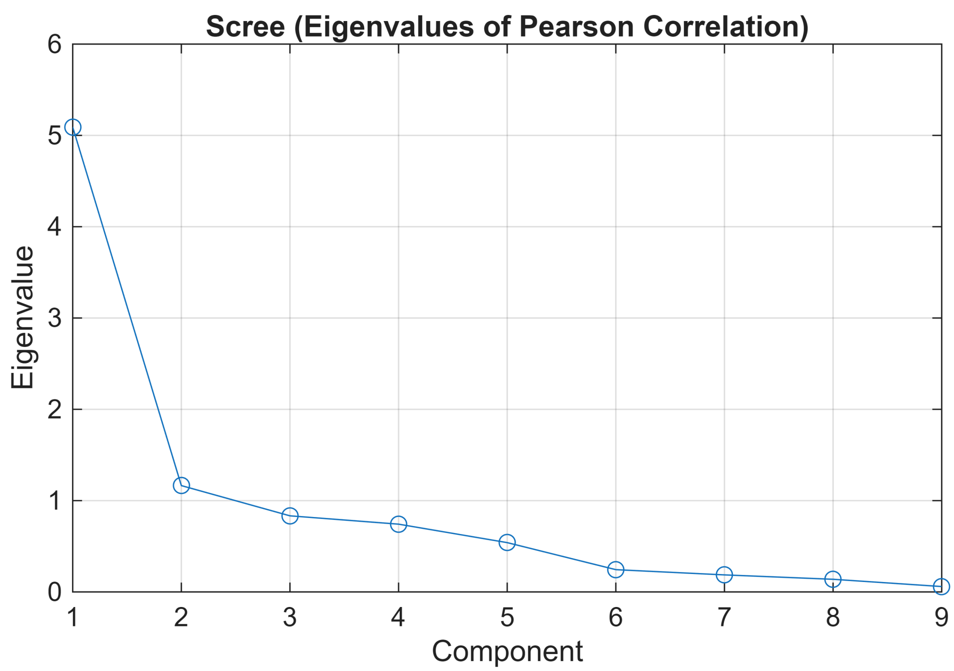
You can use the `cumsum` and `find` functions to calculate the cumulative sum and find the first element that is greater than a threshold.

```
% Cumulative variance and k for thresholds (e.g., 90% and 95%)
cumExp = cumsum(pctExp);
k90 = find(cumExp >= 90, 1);
k95 = find(cumExp >= 95, 1);
fprintf('k (>=90%%): %d   |  k (>=95%%): %d\n', k90, k95);
```

```
k (>=90%): 5   |  k (>=95%): 6
```

Scree plot

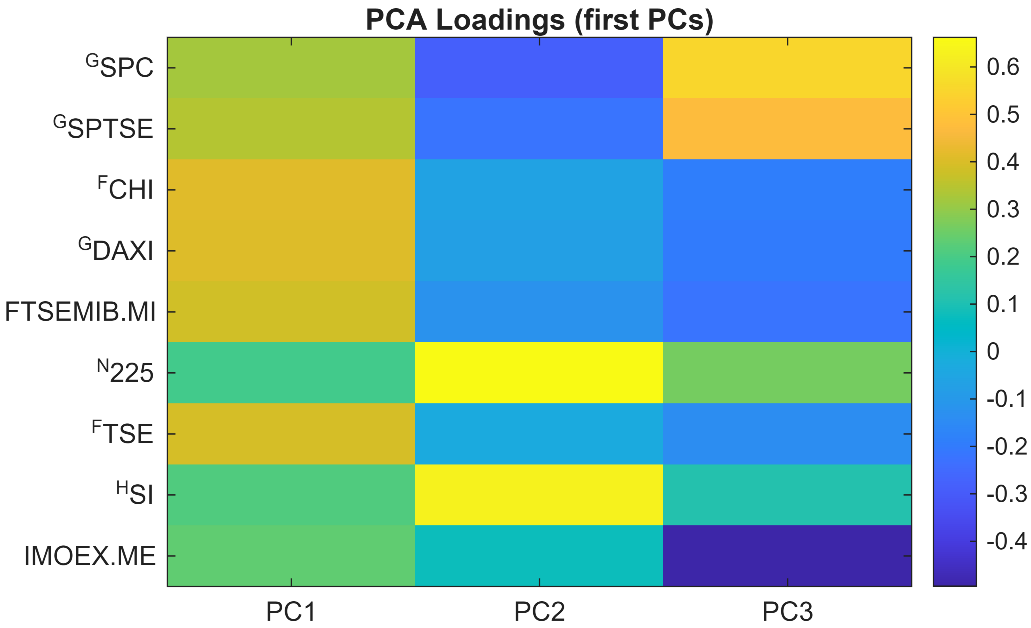
```
figure; plot(lambda, 'o-'); grid on;
title('Scree (Eigenvalues of Pearson Correlation)');...
    xlabel('Component'); ylabel('Eigenvalue');
```



Loadings heatmap for first few PCs (e.g., first 3)

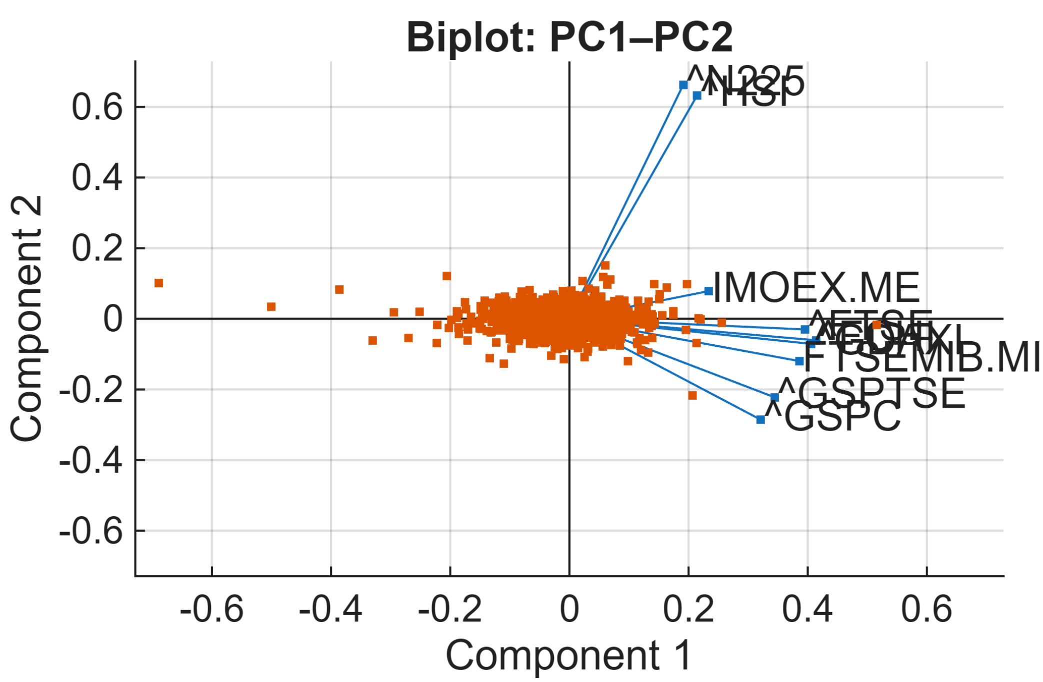
```
varNames=symbols;

kShow = min(3, size(pcs,2));
figure; imagesc(pcs(:,1:kShow)); colorbar;
set(gca,'XTick',1:kShow); xticklabels(compose('PC%d',1:kShow));
```

```
set(gca,'YTick',1:numel(varNames), 'YTickLabel', varNames);
title('PCA Loadings (first PCs)');
```



Biplot for first two PCs (optional, interpret which assets drive PC1/PC2)

```
figure; biplot(pcs(:,1:2), 'Scores', scrs(:,1:2), 'VarLabels', varNames);
title('Biplot: PC1–PC2');
```



SELECT k COMPONENTS & FORM TRANSFORMED DATA

```
k = k95;                           % for example, keep enough for 95% variance
PCA_scores   = scrs(:,1:k);        % transformed predictors (PC time series)
PCA_loadings = pcs(:,1:k);         % loadings (assets -> PCs)
```

If you want "PCA features" per date in a table:

```
PCA_tbl = [ table(combinedData.Date(2:end), 'VariableNames', {'Date'}) , ...
            array2table(PCA_scores, 'VariableNames', compose('PC%d',1:k)) ];
```

QUICK INTERPRETATION HELPERS

Market mode strength (PC1 share)

```
pc1_share = pctExp(1);
fprintf('PC1 share of variance: %.2f%%\n', pc1_share);
```

```
PC1 share of variance: 56.57%
```

Contribution of each asset to PC1 (loading magnitudes)

```
[~, topIdx] = sort(abs(pcs(:,1)), 'descend');
disp('Top contributors to PC1 (by absolute loading):');
```

```
Top contributors to PC1 (by absolute loading):
```

```
disp(varNames(topIdx(1:min(5,end))));
```

```
    {'^FCHI'}    {'^GDAXI'}    {'^FTSE'}    {'FTSEMIB.MI'}    {'^GSPTSE'}
```

Reconstruct correlation using first k PCs (sanity check)

```
Rk = pcs(:,1:k) * diag(lambda(1:k)) * pcs(:,1:k)';   % if you used eig on R
% Or, from pca() route, approximate covariance with kept components:
% covApprox = pcs(:,1:k) * diag(var(scrs(:,1:k))) * pcs(:,1:k)';  % since retZ has
unit variances
```

The Pareto and scree analyses indicate that the first principal component accounts for 56.6 % of total return variance, confirming a single dominant global market mode.

The steep spectral decay (Figure X) suggests a highly integrated system, while the heatmap and biplot (Figures Y–Z) reveal regional clusters — Europe and North America forming the core, Asia and Russia representing orthogonal directions.

Principal Component Analysis as an Eigenvector Transformation

Let $\mathbf{X} \in \mathbb{R}^{n\times p}$ be a data matrix with n observations and p variables.

Each row of $\mathbf{X}$ represents an observation, and each column a variable.

1. Data centering and standardization

First, center the data by subtracting the mean of each variable:

$$\mathbf{X}_c = \mathbf{X} - \mathbf{1}_n \mu^\top,$$

where μ is the vector of column means and $\mathbf{1}_n$ is an n-dimensional column vector of ones.

Optionally, standardize each column to have unit variance using the z-score:

$$\mathbf{Z} = \frac{\mathbf{X}_c}{\sigma}$$

so that each variable has mean 0 and standard deviation 1.

2. Covariance matrix

Compute the empirical covariance matrix:

$$\Sigma = \frac{1}{n-1}\mathbf{X}_c^\top \mathbf{X}_c.$$

If the data were standardized, Σ becomes the correlation matrix.

3. Eigen-decomposition

Solve the eigenvalue problem:

$\Sigma \mathbf{v}_i = \lambda_i \mathbf{v}_i,$

where $\mathbf{v}_i$ is the i-th eigenvector and λ_i the corresponding eigenvalue.

The eigenvectors $\mathbf{v}_i$ form an orthonormal basis for the feature space, and the eigenvalues represent the variance explained by each principal direction.

4. Transformation to principal component space

Arrange the eigenvectors in descending order of their eigenvalues:

$\mathbf{V} = [\,\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\,], \qquad \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p.$

The projection of the centered data onto this new orthogonal basis is:

$\mathbf{Y} = \mathbf{X}_c \mathbf{V}.$

Each column of $\mathbf{Y}$ corresponds to a \emph{principal component}, and the data have now been expressed in the \emph{principal component space}, whose axes are given by the eigenvectors of the covariance matrix.

5. Geometric interpretation

PCA can thus be interpreted as a rotation of the coordinate system in the feature space:

- The first principal component $\mathbf{v}_1$ points in the direction of maximum variance. It represents the axis along which the observations are most spread out, capturing the dominant pattern of co-movement across all variables.

- The second component $\mathbf{v}_2$ is orthogonal to $\mathbf{v}_1$ and and captures the largest remaining variance not explained by the first component. It describes the next most significant, linearly independent mode of variability in the system.
- Each subsequent component $\mathbf{v}_i$ $(i = 3, \ldots, p)$ is orthogonal to all preceding ones and accounts for progressively smaller portions of the total variance.

Geometrically, this transformation corresponds to a rotation and re-scaling of the original coordinate axes so that the new axes (the principal components) align with the directions of maximum spread in the data cloud.

In financial terms, the first principal component corresponds to the **global market mode,** capturing synchronized movements among all indices, while higher-order components describe \textbf{regional or idiosyncratic deviations} orthogonal to that global trend.