

- The assignment is due at Gradescope on 5/17/24.
- A LaTeX template will be provided for each homework. You can either type your homework into this template or scan your handwritten work. If you writing by hand, please fill in the solutions in this template, inserting additional sheets as necessary. This will help facilitate the grading.
- You are permitted to discuss the problems with up to 2 other students in the class (per problem); however, you must write up your own solutions, in your own words. Do not submit anything you cannot explain. If you do collaborate with any of the other students on any problem, please list all your collaborators in the appropriate spaces.
- Similarly, please list any other source you have used for each problem, including other textbooks or websites.
- Show your work. Answers without justification will be given little credit.
- Algorithms discussed during lecture may be used as black boxes.
- Unfortunately, due to the time constraints of the quarter, there are no resubmittable problems.
- Congrats on making it to the final homework assignment! You've done great :)

Problem 1 (Clique Reductions) We define a k -Clique as follows:

Definition 1 (k -Clique) A k -clique is a set of k vertices $\{v_1, v_2, \dots, v_k\}$ in $G = (V, E)$ such that for every $1 \leq i \leq j \leq k$, there is exists an edge $(v_i, v_j) \in E$. In other words, the subgraph formed by the vertices in C form a complete graph.

Now, consider the following problems:

Definition 2 (Clique) Given a graph $G = (V, E)$ and a value k , find a k -clique in G .

Definition 3 (Half-Clique) Given a graph $G = (V, E)$, find a $(|V|/2)$ -clique in G .

- (a) Show that the Half-Clique problem reduces to the k -clique problem.
- (b) Show that the k -clique problem reduces to the half-clique problem.
- (c) Using only the result from part (b), if half-clique is NP-hard, what guarantees (if any) can we make about the hardness of k -clique?

Collaborators:

Solution: