

Statistical Methods of Machine Learning

Assignment 1

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I.1.1.1

Given

$$a = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \quad b = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

Then $a^T b = 9$

I.1.1.2

The l_2 -norm or *Euclidean norm* $\|a\| = \sqrt{1^2 + 2^2 + 2^2} = 3$

I.1.1.3

The outer product

$$ab^T = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 4 & 2 \\ 6 & 4 & 2 \end{bmatrix}$$

I.1.1.4

The inverse matrix of M is

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

I.1.1.5

The matrix-vector product $Ma = \begin{pmatrix} 1 \\ 8 \\ 4 \end{pmatrix}$

I.1.1.6

$$A = ab^T = \begin{bmatrix} 3 & 6 & 6 \\ 2 & 4 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

I.1.1.7

The rank of $A = 1$, because the rows are linearly dependent. We can verify this by observing that the third row can produce the first and second rows with a multiple, e.g. the first row (3 6 6) is the same as the third row (1 2 2) x 3.

I.1.1.8

As A is not full rank, it is not invertible.

I.1.2.1

The derivative of $f(w) = (wx + b)^2$ with respect to w is

$$(wx + b)^2 = w^2x^2 + 2wxb + b^2 = 2x^2w + 2xb = 2x(xw + b)$$

I.1.2.2

In general

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

Therefore, differentiating for w we get:

$$\begin{aligned}
 f(x) &= 1 \\
 f'(x) &= 0 \\
 g(x) &= (wx + b)^2 \\
 g'(x) &= 2x(wx + b) \\
 \left(\frac{f}{g}\right)'(w) &= \frac{0 \cdot (wx + b)^2 - 1 \cdot 2x(wx + b)}{((wx + b)^2)^2} \\
 &= \frac{-1 \cdot 2x(wx + b)}{(wx + b)^4} \\
 &= \frac{-2x}{(wx + b)^3}
 \end{aligned}$$

I.1.2.3

In general

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Therefore, differentiating for x we get:

$$\begin{aligned}
 f(x) &= x \\
 g(x) &= e^x \\
 (f \cdot g)'(x) &= 1e^x + xe^x
 \end{aligned}$$

I.2.1

The plots with gaussian distributions for (sigma, nu) pairs (-1,1), (0,2) and (2,3) can be seen in Figure 1. The code for generating the plots can be found in `gauss_run.m`, and the code for our gaussian distribution function can be found in `gauss.m`.

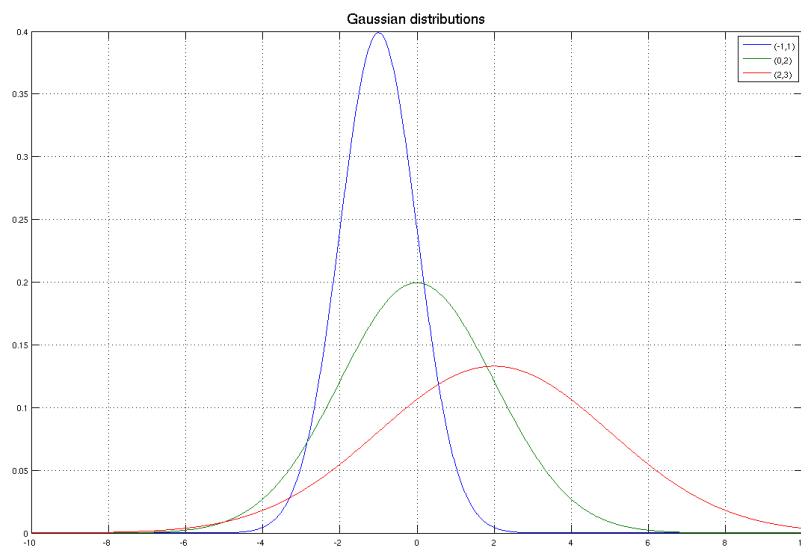


Figure 1: Gaussian distributions plotted with different values for (σ, μ) .