CH5

CH5.1 2P

Step 1 of 9

Given that $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$

We need to compute the Eigen values and eigenvectors of the matrix A

Now $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}1-\lambda & -1\\ 2 & 4-\lambda\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 2 of 9

Then

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & -1\\ 2 & 4-\lambda\end{vmatrix}\\ &= (1-\lambda)(4-\lambda)+2\\ &= 4-\lambda-4\lambda+\lambda^2+2\\ &= \lambda^2-5\lambda+6\end{aligned}$$

Provide feedback (0)

Step 3 of 9

We know that

$$|A-\lambda I|=0$$

$$\begin{aligned}\lambda^2-5\lambda+6 &= \lambda^2-3\lambda-2\lambda+6\\ &= \lambda(\lambda-3)-2(\lambda-3)\\ &= (\lambda-3)(\lambda-2)\end{aligned}$$

$$= (1-\lambda)(4-\lambda)+2$$
$$= 4-\lambda-4\lambda+\lambda^2+2$$
$$= \lambda^2-5\lambda+6$$

Provide feedback (0)

Step 3 of 9

We know that

$$|A-\lambda I| = 0$$
$$\lambda^2-5\lambda+6 = \lambda^2-3\lambda-2\lambda+6$$
$$= \lambda(\lambda-3)-2(\lambda-3)$$
$$= (\lambda-3)(\lambda-2)$$
$$(\lambda-3)(\lambda-2) = 0$$

Now $\lambda = 3, 2$

Hence the Eigen values of A are 3, 2

Provide feedback (0)

Step 4 of 9

Case (1)

Let $\lambda = 3$

Eigen vectors X corresponding to the Eigen value 3 are given by

$(A - 3I)X = 0$

That is

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 9

Add +1 times first row to the second row

$$\begin{bmatrix} -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Step 5 of 9

Add +1 times first row to the second row

$$\begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - x_2 = 0$$

Let $x_2 = k$

Therefore

$$x_1 = -k/2$$

Therefore eigenvectors corresponding to eigenvalue -3 are given by $k\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Step 6 of 9

Case (2):

Let $\lambda = 2$

Eigen vector X corresponding to the Eigen value 2 are given by

$(A - 2I)X = 0$

That is

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add -2 times first row to the second row

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 - x_2 = 0$$

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Step 7 of 9

Step 7 of 9

Let

$x_2 = k(\neq 0)$

Therefore

$x_1 = -k$

Therefore Eigen vectors corresponding to Eigen value 2 are given by $k\begin{bmatrix}-1\\1\end{bmatrix}$

Where k is a non-zero parameter

Provide feedback (0)

Step 8 of 9

The trace of A equals the sum of the Eigen values, and the determinant equals their product.

Verification: The trace of A is $1+4$ (diagonal elements) $=\boxed{5}$ is equals the sum of the Eigen values $3+2=\boxed{5}$

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Step 9 of 9

And the determinant of $A=\begin{vmatrix}1 & -1\\2 & 4\end{vmatrix}=4-(-2)=\boxed{6}$ is equals the product of the Eigen values $3\times 2=\boxed{6}$

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Algebra tutors who can help right now

Scotty G.

Michael B.

CH5.1 1P

Step 1 of 7

Given that $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$. We have to solve the differential equation $\frac{du}{dt}=Au, u(0)=\begin{bmatrix}0\\ 6\end{bmatrix}$ and find the two pure exponential solutions.

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Step 2 of 7

The given differential equation is $\frac{du}{dt}=Au$ and $u(0)=\begin{bmatrix}0\\ 6\end{bmatrix}$

Let pure exponential solution be $u(t)=e^{\lambda t}u(0)$

The number λ is an eigen value of A if and only if

$|A-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}1-\lambda & -1\\ 2 & 4-\lambda\end{vmatrix}=0$$

$$\Rightarrow(1-\lambda)(4-\lambda)-(-1)(2)=0$$

$$\Rightarrow\lambda^2-5\lambda+6=0$$

$$\Rightarrow(\lambda-2)(\lambda-3)=0$$

$$\Rightarrow\lambda=2 \text{ and } \lambda=3$$

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Step 3 of 7

If $\lambda_1=2$ then

$(A-\lambda I)x=0$

$$\Rightarrow\begin{bmatrix}-1 & -1\\ 2 & 2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Rightarrow y+z=0$$

$$\Rightarrow\frac{y}{-1}=\frac{z}{1}$$

CH5.1 3P

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Step 3 of 7

If $\lambda_1 = 2$ then

$(A - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow y + z = 0$

$$\Rightarrow \frac{y}{-1} = \frac{z}{1}$$

Eigen vector for $\lambda_1 = 2$ is $x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 4 of 7

If $\lambda_2 = 3$ and $(A - \lambda_2 I)x$

$$\Rightarrow \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2y + z = 0$

$$\Rightarrow \frac{y}{-1} = \frac{z}{2}$$

Eigen vector for $\lambda_2 = 3$ is $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

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Step 5 of 7

In the differential equation, this produces the special solution $u = e^{\lambda t}.x$.

They are the pure exponential solution to $\frac{du}{dt} = Au$.

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Step 5 of 7

In the differential equation, this produces the special solution $u = e^{\lambda t}.x$.

They are the pure exponential solution to $\frac{du}{dt} = Au$.

Notice e^{3t} and e^{2t}:

$$\begin{aligned} u(t) &= e^{\lambda_1 t} x_1 \\ &= e^{2t}\begin{bmatrix} -1 \\ 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} u(t) &= e^{\lambda_2 t}.x_2 \\ &= e^{3t}\begin{bmatrix} -1 \\ 2 \end{bmatrix} \end{aligned}$$

The complete solution $u(t) = C_1 e^{\lambda_1 t} x_1 + C_2 e^{\lambda_2 t} x_2$

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Step 6 of 7

The initial condition $u(0) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$$\Rightarrow C_1 x_1 + C_2 x_2 = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

The constants are $C_1 = -6$ and $C_2 = 6$.

Hence the solution to the original equation is

$$u(t) = -6.e^{2t}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + 6.e^{3t}\begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

Provide feedback (0)

Step 6 of 7

The initial condition $u(0)=\begin{bmatrix}0\\6\end{bmatrix}$

$$\Rightarrow C_1x_1+C_2x_2=\begin{bmatrix}0\\6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}-1 & -1\\1 & 2\end{bmatrix}\begin{bmatrix}C_1\\C_2\end{bmatrix}=\begin{bmatrix}0\\6\end{bmatrix}$$

The constants are $C_1=-6$ and $C_2=6$.

Hence the solution to the original equation is

$$u(t)=-6.e^{2t}\begin{bmatrix}-1\\1\end{bmatrix}+6.e^{3t}\begin{bmatrix}-1\\2\end{bmatrix}.$$

Provide feedback (0)

Step 7 of 7

Writing the two components separately, we have $v(0)=0, w(0)=6$.

Therefore the solution is

$$\boxed{\begin{aligned}v(t)&=6e^{2t}-6e^{3t}\\w(t)&=-6e^{2t}+12e^{3t}\end{aligned}}$$

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CH5.1 2P

Step 1 of 5

Given that $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$. We have to find the eigen values and eigen vectors of $B=A-7I$ and their relation with those of A.

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Step 2 of 5

Given $B=A-7I$

$$=\begin{bmatrix}-6 & -1\\ 2 & -3\end{bmatrix}$$

The number λ is eigen value of B if and only if

$|B-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}-6-\lambda & -1\\ 2 & -3-\lambda\end{vmatrix}=0$$

$$\Rightarrow(-6-\lambda)(-3-\lambda)-(-1)(2)=0$$

$$\Rightarrow\lambda^2+9\lambda+20=0$$

$$\Rightarrow(\lambda+4)(\lambda+5)=0$$

$$\Rightarrow\lambda=-4 \text{ and } \lambda=-5$$

Provide feedback (0)

Step 3 of 5

If $\lambda=-4$ and $(B-\lambda I)x_1=0$

$$\Rightarrow\begin{bmatrix}-2 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Rightarrow 2y+z=0$$

$$\Rightarrow\frac{y}{-1}=\frac{z}{2}$$

The eigen vector is any non-zero multiple of x_1

CH5.1 4P

Provide feedback (0)

Step 3 of 5

If $\lambda = -4$ and $(B - \lambda I)x_1 = 0$

$$\Rightarrow \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2y + z = 0$$

$$\Rightarrow \frac{y}{-1} = \frac{z}{2}$$

The eigen vector is any non-zero multiple of x_1

Eigen vector for $\lambda_1 = -4$ is $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Provide feedback (0)

Step 4 of 5

If $\lambda_2 = -5$ and $(B - \lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow y + z = 0$$

$$\Rightarrow \frac{y}{-1} = \frac{z}{1}$$

The eigen vector is any non zero multiple of x_2.

Eigen vector for $\lambda_2 = -5$ *is* $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 5 of 5

From the above eigen values of A and B, it is clear that both the eigen values are reduced by 7 to that of A and there is no change in their corresponding eigen vectors.

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Step 4 of 5

If $\lambda_2 = -5$ and $(B - \lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow y + z = 0$$

$$\Rightarrow \frac{y}{-1} = \frac{z}{1}$$

The eigen vector is any non zero multiple of x_2.

Eigen vector for $\lambda_2 = -5$ *is* $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

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Step 5 of 5

From the above eigen values of A and B, it is clear that both the eigen values are reduced by 7 to that of A and there is no change in their corresponding eigen vectors.

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CH5.1 3P

CH5.1 5P

Step 1 of 10

Consider the following projection matrix P:

$$P=\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

Solve the following:

$$\frac{du}{dt}=Pu$$

With

$$u(0)=\begin{bmatrix}5\\3\end{bmatrix}$$

Here, part of $u(0)$ increases exponentially while the null space part stays fixed.

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Step 2 of 10

To solve the following differential equation with $u=u(0)$ at $t=0$.

$$\frac{du}{dt}=Pu$$

Consider the following pure exponential solution:

$$u(t)=e^{at}u(0)$$

Provide feedback (0)

Step 2 of 10

To solve the following differential equation with $u = u(0)$ at $t = 0$.

$$\frac{du}{dt} = Pu$$

Consider the following pure exponential solution:

$$u(t) = e^{at}u(0)$$

Provide feedback (0)

Step 3 of 10

Let eigen value equation be:

$$Px = \lambda x$$

Let eigen value problem be:

$$\frac{1}{2}y + \frac{1}{2}z = \lambda y$$
$$\frac{1}{2}y + \frac{1}{2}z = \lambda z$$

Provide feedback (0)

Step 4 of 10

Do the following calculations:

$$P - \lambda I = \begin{bmatrix} \frac{1}{2} - \lambda & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} - \lambda \end{bmatrix}$$

$$|P - \lambda I| = \left(\frac{1}{2} - \lambda\right)^2 - \frac{1}{2}\left(\frac{1}{2}\right)$$

Provide feedback (0)

Step 4 of 10

Do the following calculations:

$$P-\lambda I=\begin{bmatrix}\frac{1}{2}-\lambda & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}-\lambda\end{bmatrix}$$

$$\begin{aligned}|P-\lambda I|&=\left(\frac{1}{2}-\lambda\right)^2-\frac{1}{2}\left(\frac{1}{2}\right)\\ &=\lambda^2-\lambda\end{aligned}$$

This is the characteristic polynomial. Put it equal to zero to get eigen values-

$$\begin{aligned}\lambda^2-\lambda&=0\\ \lambda(\lambda-1)&=0\end{aligned}$$

So, following are the eigen values.

$$\begin{aligned}\lambda&=0\\ \lambda&=1\end{aligned}$$

Provide feedback (0)

Step 5 of 10

To get the eigen vectors corresponding to the eigen values do the following calculations:

For $\lambda_1=0$, eigen vector x_1 is :

$$\begin{bmatrix}\frac{1}{2}-0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}-0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 5 of 10

To get the eigen vectors corresponding to the eigen values do the following calculations:

For $\lambda_1 = 0$, eigen vector x_1 is :

$$\begin{bmatrix} \frac{1}{2}-0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}-0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, eigen vector will be as follows:

$$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

For $\lambda_2 = 1$, eigen vector x_2 is :

$$\begin{bmatrix} \frac{1}{2}-1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}-1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step 6 of 10

For $\lambda_2 = 1$, eigen vector x_2 is :

$$\begin{bmatrix} \frac{1}{2}-1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}-1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, eigen vector will be as follows:

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 10

Solutions corresponding to these eigen vectors are as follows:

$$\begin{aligned} u_1(t) &= e^{\lambda_1 t} x_1 \\ &= e^{0 \cdot t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

And

$$\begin{aligned} u_2(t) &= e^{\lambda_2 t} x_2 \\ &= e^{1 \cdot t}\begin{bmatrix} 1 \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
u_1(t) &= e^{\lambda_1 t} x_1 \\
&= e^{0 \cdot t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{aligned}
$$

And

$$
\begin{aligned}
u_2(t) &= e^{\lambda_2 t} x_2 \\
&= e^{1 \cdot t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
&= e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

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Step 8 of 10

If $u_1(t)$ and $u_2(t)$ satisfy the linear differential equation $\frac{du}{dt} = Pu$, so will their sum $u_1(t) + u_2(t)$. Therefore, complete solution is as follows:

$$u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$$

Here, c_1 and c_2 are constants and can be chosen in a manner that it satisfies the initial condition $u = u(0)$ at $t = 0$.

Provide feedback (0)

Step 9 of 10

At $t = 0$, $u(t)$ becomes:

Provide feedback (0)

Step 9 of 10

At $t=0$, $u(t)$ becomes:

$$c_1x_1+c_2x_2=u(0)$$
$$\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}5\\3\end{bmatrix}$$

Or

$$c_1+c_2=5$$
$$-c_1+c_2=3$$

On solving these two equations value of constants can be calculated:

$$c_1=1$$
$$c_2=4$$

Provide feedback (0)

Step 10 of 10

Therefore the solution is:

$$\boxed{u(t)=1\cdot\begin{bmatrix}1\\-1\end{bmatrix}+4e^t\begin{bmatrix}1\\1\end{bmatrix}}$$

Provide feedback (0)

Algebra tutors who can help right now

CH5.1 4P

Step 1 of 21

Given that $A=\begin{bmatrix}3&4&2\\0&1&2\\0&0&0\end{bmatrix}$ and $B=\begin{bmatrix}0&0&2\\0&2&0\\2&0&0\end{bmatrix}$

Now $A=\begin{bmatrix}3&4&2\\0&1&2\\0&0&0\end{bmatrix}$

$$A-\lambda I=\begin{bmatrix}3&4&2\\0&1&2\\0&0&0\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}3&4&2\\0&1&2\\0&0&0\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}$$

$$=\begin{bmatrix}3-\lambda&4&2\\0&1-\lambda&2\\0&0&-\lambda\end{bmatrix}$$

Provide feedback (0)

CH5.1 6P

Step 2 of 21

Now

$$|A-\lambda I|=\begin{vmatrix}3-\lambda&4&2\\0&1-\lambda&2\\0&0&-\lambda\end{vmatrix}$$

$$=(3-\lambda)((1-\lambda)\times-\lambda)$$

$$=(3-\lambda)(-\lambda+\lambda^2)$$

$$=-3\lambda+3\lambda^2+\lambda^2-\lambda^3$$

$$=-\lambda^3+4\lambda^2-3\lambda$$

$$= -\lambda^3 + 4\lambda^2 - 3\lambda$$

Provide feedback (1)

Step 3 of 21

Now $|A - \lambda I| = 0$

$$-\lambda^3 + 4\lambda^2 - 3\lambda = 0$$
$$\lambda^3 - 4\lambda^2 + 3\lambda = 0$$
$$\lambda(\lambda^2 - 4\lambda + 3) = 0$$
$$\lambda = 0, \lambda^2 - 4\lambda + 3 = 0$$
$$\lambda^2 - 4\lambda + 3 = 0$$
$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$
$$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$$
$$(\lambda - 3)(\lambda - 1) = 0$$

Provide feedback (0)

Step 4 of 21

Therefore the eigenvalues are 0, 3, and 1

Case (i)

Let $\lambda_1 = 0$

Eigen vectors X corresponding to the Eigen value 0 are given by

$(A - 0I)X = 0$

That is

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + 4x_2 + 2x_3 = 0$ and $x_2 + 2x_3 = 0$

Provide feedback (0)

Step 5 of 21

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3x_1 + 4x_2 + 2x_3 = 0$ and $x_2 + 2x_3 = 0$

Provide feedback (0)

Step 5 of 21

Let

$x_3 = k(\neq 0)$

$x_2 = -2k$

$x_1 = 2k$

Therefore eigenvectors corresponding to eigenvalue 0 are given by

$k\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ Where k is a non-zero parameter

Provide feedback (0)

Step 6 of 21

Case (ii)

Let $\lambda_2 = 3$

Eigen vectors X corresponding to the Eigen value 3 are given by

$$(A - 3I)X = 0$$

That is

$$\begin{bmatrix} 0 & 4 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4x_2 + 2x_3 = 0,\ -2x_2 + 2x_3 = 0 \text{ and } -3x_3 = 0$$

Provide feedback (0)

Step 7 of 21

Step 6 of 21

Case (ii)

Let $\lambda_2 = 3$

Eigen vectors X corresponding to the Eigen value 3 are given by

$$(A - 3I)X = 0$$

That is

$$\begin{bmatrix} 0 & 4 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4x_2 + 2x_3 = 0,\ -2x_2 + 2x_3 = 0 \text{ and } -3x_3 = 0$$

Provide feedback (0)

Step 7 of 21

Therefore $x_3 = 0$ and $x_2 = 0$

Let $x_1 = k(\neq 0)$

Therefore eigenvectors corresponding to Eigen value 3 are given by

Provide feedback (0)

Step 8 of 21

$$k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ Where } k \text{ is a non-zero parameter}$$

Case (iii)

Let $\lambda_3 = 1$

Eigen vectors X corresponding to the Eigen value 1 are given by

$$(A - I)X = 0$$

$$\begin{bmatrix} 2 & 4 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + 4x_2 + 2x_3 = 0,\ 2x_3 = 0$ and $-x_3 = 0$

Therefore eigenvectors corresponding to Eigen value 3 are given by

Provide feedback (0)

Step 8 of 21

$$k\begin{bmatrix}1\\0\\0\end{bmatrix} \text{ Where } k \text{ is a non-zero parameter}$$

Case (iii)

Let $\lambda_3 = 1$

Eigen vectors X corresponding to the Eigen value 1 are given by

$$(A - I)X = 0$$

$$\begin{bmatrix}2 & 4 & 2\\0 & 0 & 2\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$2x_1 + 4x_2 + 2x_3 = 0,\ 2x_2 = 0 \text{ and } -x_3 = 0$$

Provide feedback (3)

Step 9 of 21

Let $x_1 = k$

Therefore eigenvectors corresponding to Eigen value 1 are given by

$$k\begin{bmatrix}1\\0\\0\end{bmatrix} \text{ Where } k \text{ is a non-zero parameter}$$

Provide feedback (5)

Step 10 of 21

$$\text{Now } B = \begin{bmatrix}0 & 0 & 2\\0 & 2 & 0\\ & & \end{bmatrix}$$

Provide feedback (5)

Step 10 of 21

$$\text{Now } B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} -\lambda & 0 & 2 \\ 0 & 2-\lambda & 0 \\ 2 & 0 & -\lambda \end{bmatrix} \end{aligned}$$

Provide feedback (1)

Step 11 of 21

Now

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} -\lambda & 0 & 2 \\ 0 & 2-\lambda & 0 \\ 2 & 0 & -\lambda \end{vmatrix} \\ &= (-\lambda)(2-\lambda)(-\lambda) + 2(-(2-\lambda)2) \\ &= \lambda^2(2-\lambda) + 2(-4+2\lambda) \\ &= 2\lambda^2 - \lambda^3 - 8 + 4\lambda \\ &= -\lambda^3 + 2\lambda^2 + 4\lambda - 8 \end{aligned}$$

Provide feedback (1)

Provide feedback (1)

Step 12 of 21

Then

$$|B-\lambda I|=0$$

$$\lambda^3-2\lambda^2-4\lambda+8=0$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ & 0 & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \\ \hline \end{array}$$

$$(\lambda-2)(\lambda^2-4)=0$$

$$\lambda=2,\lambda^2=4$$

$$\lambda=2,2,-2$$

Therefore the eigenvalues are 2, 2, and -2

Provide feedback (0)

Step 13 of 21

Case (i)

Let $\lambda_1=2$

Eigen vectors X corresponding to the Eigen value 2 are given by

$$(A-2I)X=0$$

$$\begin{bmatrix} -2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1+2x_3=0 \text{ and } 2x_1-2x_3=0$$

Provide feedback (0)

Step 14 of 21

Let $x_3=x_1=k$

And let $x_2=k$

Provide feedback (0)

Step 14 of 21

Let $x_3 = x_1 = k$

And let $x_2 = k$

Therefore eigenvectors corresponding to Eigen value 2 are given by $k\begin{bmatrix}1\\1\\1\end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 15 of 21

Case (ii)

Let $\lambda_2 = 2$

Eigen vectors X corresponding to Eigen value 2 are given by

$(A - 2I)X = 0$

Provide feedback (0)

Step 16 of 21

That is

$$\begin{bmatrix}-2 & 0 & 2\\0 & 0 & 0\\2 & 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$-2x_1 + 2x_3 = 0 \text{ and } 2x_1 - 2x_3 = 0$$

Let $x_3 = x_1 = k$

And let $x_2 = k$

Therefore eigenvectors corresponding to Eigen value 2 are given by $k\begin{bmatrix}1\\1\\1\end{bmatrix}$ where k

Step 16 of 21

That is

$$\begin{bmatrix} -2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2x_3 = 0 \text{ and } 2x_1 - 2x_3 = 0$$

Let $x_3 = x_1 = k$

And let $x_2 = k$

Therefore eigenvectors corresponding to Eigen value 2 are given by $k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Step 17 of 21

Case (iii)

Let $\lambda_3 = -2$

Eigen vectors X corresponding to Eigen value -2 are given by

$$(A + 2I)X = 0$$

$$\begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 2x_3 = 0,\ 4x_2 = 0 \text{ and } 2x_1 + 2x_3 = 0$$

Provide feedback (0)

Step 18 of 21

Put $x_3 = k$

$$2x_1 + 2k = 0$$

$$2x_1 = -2k$$

Provide feedback (0)

Step 18 of 21

Put $x_3 = k$

$$2x_1 + 2k = 0$$
$$2x_1 = -2k$$
$$x_1 = -k, x_2 = 0$$

Therefore eigenvectors corresponding to Eigen value -2 are given by $k\begin{bmatrix}-1\\0\\1\end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 19 of 21

In A matrix the eigenvalues are 0, 3, and 1

$\lambda_1 + \lambda_2 + \lambda_3 = 0+3+1 = 4$ is equal to the trace of A.

That is $A = \begin{bmatrix}3 & 4 & 2\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}$

Provide feedback (0)

Step 20 of 21

If we add the diagonal values of A, $3+1+0 = 4$

$\lambda_1\lambda_2\lambda_3 = 0$ is equal to the determinant of A

That is

$$\begin{vmatrix}3 & 4 & 2\\0 & 1 & 2\\0 & 0 & 0\end{vmatrix} = 3(0-0) - 4(0-0) + 2(0-0)$$
$$= \boxed{0}$$

Provide feedback (0)

Step 20 of 21

If we add the diagonal values of A, $3+1+0=4$
$\lambda_1\lambda_2\lambda_3 = 0$ is equal to the determinant of A
That is

$$\begin{vmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix} = 3(0-0)-4(0-0)+2(0-0)$$
$$= \boxed{0}$$

Provide feedback (0)

Step 21 of 21

In B matrix the eigenvalues are 2, 2, and -2
$\lambda_1+\lambda_2+\lambda_3 = 2+2-2=2$ is equal to the trace of B

That is $B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

If we add the diagonal values of B, $0+2+0=2$
$\lambda_1\lambda_2\lambda_3 = -8$ is equal to the determinant of B
That is

$$\begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{vmatrix} = 2(-2\times 2)$$
$$= \boxed{-8}$$

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CH5.1 5P

Step 1 of 2

Let us consider a 2×2 matrix $A=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$

The characteristic equation of A is $|A-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}2-\lambda & 2\\ 2 & 2-\lambda\end{vmatrix}=0$$

$$\Rightarrow(2-\lambda)^2-4=0$$

$$\Rightarrow 4+\lambda^2-4\lambda-4=0$$

$$\Rightarrow\lambda=0,4$$

Provide feedback (0)

Step 2 of 2

On the other hand, we apply the row transformation $R_2\to R_2-R_1$, we get $A\sim\begin{bmatrix}2 & 2\\ 0 & 0\end{bmatrix}$

Its characteristic equation is $\begin{vmatrix}2-\lambda & 2\\ 0 & 0-\lambda\end{vmatrix}=0$

$$\Rightarrow(2-\lambda)(-\lambda)=0$$

$$\Rightarrow\lambda=0,2$$

Observe that the elementary row operation on the given matrix $A=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ has changed the eigen values from 0, 4 to 0, and 2.
However, the zero eigen value remained in the transformed matrix also.

The reason for this is "if the given matrix has dependent rows, then at least one of the eigen values is 0. Also, when an elementary row or column operation is applied on the given matrix, the dependence of the columns cannot be changed.

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CH5.1 7P

CH5.1 6P

Step 1 of 2

We easily see that if $B = A + k\,I$, λ is the eigen value of A, then $\lambda - k$ is the eigen value of B.

a) Given that λ is an eigen value of A and x is the respective eigen vector, then

$Ax = \lambda x$

$\Rightarrow (A - 7I)x = (\lambda - 7)x$

Let $A - 7I = B$ and $\lambda - 7 = \mu$

Then $Bx = \mu x$

Therefore x is an eigen vector of B

$\Rightarrow x$ is an eigen vector of $A - 7I$.

μ is the eigen value of B.

$\Rightarrow \lambda - 7$ is the eigen value.

That is the eigen values are reduced by 7 to λ with unchanged eigen vectors.

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CH5.1 8P

Step 2 of 2

b) $\lambda \neq 0$

$Ax = \lambda x$

Multiplying both sides with A^{-1}

$(A^{-1}A)x = \lambda A^{-1}x$

$\Rightarrow x = \lambda A^{-1}x$

$\Rightarrow A^{-1}x = \left(\frac{1}{\lambda}x\right)$

We observe that the vector x satisfies $A^{-1}x = \left(\frac{1}{\lambda}x\right)$ and so, the eigen value of A^{-1} is $\frac{1}{\lambda}$ and the eigen vector does not change.

Thus, the eigen value of A^{-1} is the reciprocal of the eigen value of A and the respective eigen vectors are one and the same.

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CH5.1 7P

Step 1 of 1

Given that $\det(A-\lambda I)=(\lambda_1-\lambda)(\lambda_2-\lambda)\ldots\ldots(\lambda_n-\lambda)$ where $\lambda_1,\lambda_2,\ldots,\lambda_n$ are the eigen values of A.

Substituting $\lambda=0$ on either sides, we get

$\det(A-0I)=(\lambda_1-0)(\lambda_2-0)\ldots(\lambda_n-0)$

Or, $\det A=\lambda_1\lambda_2\ldots\lambda_n$

Therefore the determinant of the matrix is equal to the product of the eigen values.

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CH5.1 9P

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CH5.1 8P

Step 1 of 4

To show that trace equals the sum of the eigen values.

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CH5.1 10P

Step 2 of 4

Let $(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n)$ be the eigen values.

Recall that characteristic equation is kept to zero to get the eigen values. This means that determinant value of $A - \lambda I$ must be equal to zero.

$$\det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\ldots(\lambda_n - \lambda)$$

Put the above equation equal to zero to get polynomial equation in different powers of λ. Calculate the coefficient of $(-\lambda^{n-1})$ from the polynomial equation, given as follows:

$$(\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n)$$

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Step 3 of 4

Consider the following matrix:

$$\det(A - \lambda I) = \det\begin{bmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{bmatrix}$$

Find all the terms that involve $(-\lambda^{n-1})$, they will all come from the main diagonal. Following is the coefficient of $(-\lambda^{n-1})$:

Step 3 of 4

Consider the following matrix:

$$\det(A-\lambda I)=\det\begin{bmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{bmatrix}$$

Find all the terms that involve $\left(-\lambda^{n-1}\right)$, they will all come from the main diagonal. Following is the coefficient of $\left(-\lambda^{n-1}\right)$:

$$\left(a_{11}+a_{22}+a_{33}+\cdots+a_{nn}\right)$$

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Step 4 of 4

Compare the coefficients of $\left(-\lambda^{n-1}\right)$ formed by solving determinant value of $A-\lambda I$:

$$\boxed{\left(a_{11}+a_{22}+a_{33}+\cdots+a_{nn}\right)=\left(\lambda_1+\lambda_2+\lambda_3+\cdots+\lambda_n\right)}$$

Therefore, trace of the matrix is equal to the sum of the eigen values.

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Step 1 of 7

(a) We have to construct 2 by 2 matrices such that the eigen values of AB are not the products of the eigen values of A and B, and the eigen values of $A+B$ are not the sums of the individual eigen values.

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Step 2 of 7

Let $A=\begin{bmatrix}1 & -5\\ 2 & -3\end{bmatrix}$

The number λ is an eigen value of A if and only if

$|A-\lambda I|=0$

$\Rightarrow \begin{vmatrix}1-\lambda & -1\\ 2 & 4-\lambda\end{vmatrix}=0$

$\Rightarrow (1-\lambda)(4-\lambda)-(-1)(2)=0$

$\Rightarrow \lambda^2-5\lambda+6=0$

$\Rightarrow (\lambda-2)(\lambda-3)=0$

$\Rightarrow \lambda=2$ and $\lambda=3$

The eigen values are 2 and 3.

Provide feedback (2)

Step 3 of 7

Let $B=\begin{bmatrix}4 & -5\\ 2 & -3\end{bmatrix}$

If λ is an eigen value of B, then

$|B-\lambda I|=0$

$\Rightarrow \begin{vmatrix}4-\lambda & -5\\ 2 & -3-\lambda\end{vmatrix}=0$

$\Rightarrow (4-\lambda)(-3-\lambda)-(-5)(2)=0$

CH5.1 11P

Provide feedback (2)

Step 3 of 7

Let $B = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$

If λ is an eigen value of B, then

$|B - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{vmatrix} = 0$

$\Rightarrow (4-\lambda)(-3-\lambda) - (-5)(2) = 0$

$\Rightarrow \lambda^2 - \lambda - 2 = 0$

$\Rightarrow (\lambda+1)(\lambda-2) = 0$

$\Rightarrow \lambda = -1 \text{ or } 2$

The eigen values are -1 and 2.

Provide feedback (0)

Step 4 of 7

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 16 & -22 \end{bmatrix}$$

If λ is an eigen value of AB, then

$|AB - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & -2 \\ 16 & -22-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(-22-\lambda) - (-2)(16) = 0$

$\Rightarrow \lambda^2 + 20\lambda - 12 = 0$

$\Rightarrow \lambda = -10 \pm 4\sqrt{7}$

Sum of the eigen values is -20 and product is -12.

$\Rightarrow \lambda = -10 \pm 4\sqrt{7}$

Sum of the eigen values is -20 and product is -12.

Provide feedback (0)

Step 5 of 7

$$A+B=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}+\begin{bmatrix}4 & -5\\ 2 & -3\end{bmatrix}$$
$$=\begin{bmatrix}5 & -6\\ 4 & 1\end{bmatrix}$$

If λ is an eigen value of A+B then

$$|(A+B)-\lambda I|=0$$
$$\Rightarrow \begin{vmatrix}5-\lambda & -6\\ 4 & 1-\lambda\end{vmatrix}=0$$
$$\Rightarrow \lambda^2-6\lambda+29=0$$

Here the eigen values are imaginary hence eigen values of AB are not the product of the eigen values of A and B.

Eigen values of $A+B$ are not the sums of the eigen values of A and B.

Provide feedback (0)

Step 6 of 7

(b) The sum of the eigen values of $A+B$ is $\frac{-(-6)}{1}=6$.

The sum of the all the individual eigen values of A and B is $2+3-1+2=6$ which equals to the above sum of the eigen values of $A+B$.

Provide feedback (0)

Step 7 of 7

The product of the eigen values of AB is -12.

The product of the all the individual eigen values of A and B is $(2)(3)(-1)(2)=-12$ which equals to the above product of the eigen values of AB.

$$\begin{vmatrix} 4 & 1-\lambda \end{vmatrix}$$

$\Rightarrow \lambda^2 - 6\lambda + 29 = 0$

Here the eigen values are imaginary hence eigen values of AB are not the product of the eigen values of A and B.

Eigen values of $A+B$ are not the sums of the eigen values of A and B.

Provide feedback (0)

Step 6 of 7

(b) The sum of the eigen values of $A+B$ is $\frac{-(-6)}{1} = 6$.

The sum of the all the individual eigen values of A and B is $2+3-1+2=6$ which equals to the above sum of the eigen values of $A+B$.

Provide feedback (0)

Step 7 of 7

The product of the eigen values of AB is -12.

The product of the all the individual eigen values of A and B is $(2)(3)(-1)(2) = -12$ which equals to the above product of the eigen values of AB.

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Step 1 of 18

That is true because Transpose $A-\lambda I$: $\det\left(A-\lambda I\right)=\det\left(A-\lambda I\right)^T=\det\left(A^T-\lambda I\right)$

Example:

Suppose $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$

$A^T=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$

Provide feedback (0)

Step 2 of 18

Now $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$

$$\begin{aligned}A-\lambda I&=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &=\begin{bmatrix}1-\lambda & -1\\ 2 & 4-\lambda\end{bmatrix}\end{aligned}$$

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Step 3 of 18

$$\begin{aligned}\left|A-\lambda I\right|&=\begin{vmatrix}1-\lambda & -1\\ 2 & 4-\lambda\end{vmatrix}\\ &=\left(1-\lambda\right)\left(4-\lambda\right)+2\\ &=4-\lambda-4\lambda+\lambda^2+2\\ &=\lambda^2-5\lambda+6\end{aligned}$$

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Step 3 of 18

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix}$$
$$= (1-\lambda)(4-\lambda)+2$$
$$= 4-\lambda-4\lambda+\lambda^2+2$$
$$= \lambda^2-5\lambda+6$$

Provide feedback (0)

Step 4 of 18

we know that $|A-\lambda I| = 0$

$$\lambda^2-5\lambda+6 = \lambda^2-3\lambda-2\lambda+6$$
$$= \lambda(\lambda-3)-2(\lambda-3)$$
$$= (\lambda-3)(\lambda-2)$$

Now $(\lambda-3)(\lambda-2)=0$

$\lambda = 3, 2$

Hence the eigenvalues of A are 3,2

Provide feedback (0)

Step 5 of 18

Case(i) Let $\lambda = 3$

Eigenvectors X corresponding to the eigenvalue 3 are given by

$(A-3I)X = 0$

That is $\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Provide feedback (0)

Step 5 of 18

Case(i) Let $\lambda = 3$

Eigenvectors X corresponding to the eigenvalue 3 are given by

$(A-3I)X = 0$

That is $\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 6 of 18

$By\ R_2 + R_1 = R_2$

$\begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -2x_1 - x_2 = 0$

Let $x_1 = k(say)$

$-2k - x_2 = 0$

$x_2 = -2k$

Provide feedback (0)

Step 7 of 18

Therefore eigenvectors corresponding to eigenvalue 3 are given by $k\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ where k is a non-zero parameter.

Case(ii) Let $\lambda = 2$

Eigenvectors X corresponding to the eigenvalue 2 are given by

$(A-2I)X = 0$

That is $\begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

Provide feedback (0)

Step 7 of 18

Therefore eigenvectors corresponding to eigenvalue 3 are given by $k\begin{bmatrix}1\\-2\end{bmatrix}$ where k is a non-zero parameter.

Case(ii) Let $\lambda = 2$

Eigenvectors X corresponding to the eigenvalue 2 are given by

$(A-2I)X = 0$

That is $\begin{bmatrix}-1 & -1\\2 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

Provide feedback (0)

Step 8 of 18

By $R_2 + 2R_1 = R_2$

$\begin{bmatrix}-1 & -1\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\Rightarrow -x_1 - x_2 = 0$

Provide feedback (0)

Step 9 of 18

Let $x_1 = k(\text{say})$

Therefore $x_2 = -k$

Therefore eigenvectors corresponding to eigenvalue 2 are given by $k\begin{bmatrix}1\\-1\end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Provide feedback (0)

Step 10 of 18

Now $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

Provide feedback (0)

Step 11 of 18

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 1-\lambda & 2 \\ -1 & 4-\lambda \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 12 of 18

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} 1-\lambda & 2 \\ -1 & 4-\lambda \end{vmatrix} \\ &= (1-\lambda)(4-\lambda) + 2 \\ &= 4 - \lambda - 4\lambda + \lambda^2 + 2 \\ &= \lambda^2 - 5\lambda + 6 \end{aligned}$$

Provide feedback (0)

Step 13 of 18

we know that $|A - \lambda I| = 0$

Step 13 of 18

we know that $|A-\lambda I|=0$

$\lambda^2-5\lambda+6=0$

$\lambda^2-3\lambda-2\lambda+6=0$

$\lambda(\lambda-3)-2(\lambda-3)=0$

$(\lambda-3)(\lambda-2)=0$

Now $(\lambda-3)(\lambda-2)=0$

$\lambda=3,2$

Hence the eigenvalues of A^T are 3,2

Provide feedback (0)

Step 14 of 18

Case(i) Let $\lambda=3$

Eigenvectors X corresponding to the eigenvalue 3 are given by

$(A-3I)X=0$

That is $\begin{bmatrix}-2 & 2\\ -1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

Provide feedback (0)

Step 15 of 18

By $2R_2-R_1=R_2$

$\begin{bmatrix}-2 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$\Rightarrow -2x_1+2x_2=0$

Let $x_1=k(\text{say})$

Therefore $x_2=k$

Therefore eigenvectors corresponding to eigenvalue 3 are given by $k\begin{bmatrix}1\\ 1\end{bmatrix}$ where k is a non-zero parameter.

$\begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} x_2 \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix}$

$\Rightarrow -2x_1 + 2x_2 = 0$

Let $x_1 = k(\text{say})$

Therefore $x_2 = k$

Therefore eigenvectors corresponding to eigenvalue 3 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 16 of 18

Case(ii) Let $\lambda = 2$

Eigenvectors X corresponding to the eigenvalue 2 are given by

$(A - 2I)X = 0$

That is $\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 17 of 18

By $R_2 - R_1 = R_2$

$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -x_1 + 2x_2 = 0$

Provide feedback (0)

Step 18 of 18

Let $x_1 = k(\text{say})$

Therefore $x_2 = k/2$

Therefore eigenvectors corresponding to eigenvalue 2 are given by $k\begin{bmatrix} 1 \\ \end{bmatrix}$ where k

$(A-2I)X=0$

That is $\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 17 of 18

By $R_2 - R_1 = R_2$

$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -x_1+2x_2=0$

Provide feedback (0)

Step 18 of 18

Let $x_1 = k(\text{say})$

Therefore $x_2 = k/2$

Therefore eigenvectors corresponding to eigenvalue 2 are given by $k\begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$ where k is a non-zero parameter

The above example shows that the eigenvectors of A and A^T are not the same.

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Step 1 of 7

The number λ is an eigen value of A if and only if $A - \lambda I$ is singular
In other words, $\det(A - \lambda I) = 0$
This is the characteristic equation. Each λ is associated with eigen vector x such that $Ax = \lambda x$.

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Step 2 of 7

In view of this definition, let us consider $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

The characteristic equation of this matrix is $\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 9 - 16 = 0$
$\Rightarrow \lambda_1 = -5, \lambda_2 = 5$ are the roots of the characteristic equation or the eigen values of the given matrix.

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Step 3 of 7

The eigen vector corresponding to the eigen value $\lambda_1 = -5$ satisfies $(A - \lambda_1 I)x_1 = 0$

$$\Rightarrow \begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using the row operations, we reduce this as

$$R_2 \to 2R_2 - R_1, R_1/4 \Rightarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2t_1 + t_2 = 0$
$t_2 = -2t_1$

When $t_1 = 1$, we get $t_2 = -2$ and thus, the solution set is $x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_1 = -5$

CH5.1 13P

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Step 3 of 7

The eigen vector corresponding to the eigen value $\lambda_1 = -5$ satisfies $(A - \lambda_1 I)x_1 = 0$

$$\Rightarrow \begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using the row operations, we reduce this as

$$R_2 \to 2R_2 - R_1, R_1/4 \Rightarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2t_1 + t_2 = 0$$

$$t_2 = -2t_1$$

When $t_1 = 1$, we get $t_2 = -2$ and thus, the solution set is $x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_1 = -5$

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Step 4 of 7

Similarly, using $\lambda_2 = 5$, we get $(A - \lambda_2 I)x_2 = 0$ as $\begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$R_2 \to R_2 + 2R_1, R_1/-2 \Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow t_1 - 2t_2 = 0$$

When $t_2 = 1$, we get $t_1 = 2$ and thus, the solution set is $x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_2 = 5$

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Step 5 of 7

$$R_2 \to R_2 + 2R_1, R_1/-2 \Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow t_1 - 2t_2 = 0$$

When $t_2 = 1$, we get $t_1 = 2$ and thus, the solution set is $x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_2 = 5$

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Step 5 of 7

The characteristic equation of $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ is $\det(A - \lambda I) = \begin{vmatrix} a-\lambda & b \\ b & a-\lambda \end{vmatrix} = 0$

$$\Rightarrow (\lambda - a)^2 - b^2 = 0$$

$$\Rightarrow \lambda^2 - 2a\lambda + (a^2 - b^2) = 0$$

$$\Rightarrow \lambda = \frac{2a + \sqrt{4a^2 - 4(a^2 - b^2)}}{2}, \frac{2a - \sqrt{4a^2 - 4(a^2 - b^2)}}{2}$$

$$\Rightarrow \lambda_1 = a + b, \lambda_2 = a - b$$

Provide feedback (0)

Step 6 of 7

Let the eigen vector corresponding to λ_1 is x_1

Then x_1 satisfies the equation $(A - \lambda_1 I)x_1 = 0$

$$\Rightarrow \begin{bmatrix} -b & b \\ b & -b \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2 + R_1, R_2/-b$, we get $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow t_1 - t_2 = 0$$

Putting $t_1 = 1$, we get $t_2 = 1$ and thus, the respective eigen vector is $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\Rightarrow \lambda = \frac{\quad}{2}, \frac{\quad}{2}$$

$$\Rightarrow \lambda_1 = a+b, \lambda_2 = a-b$$

Provide feedback (0)

Step 6 of 7

Let the eigen vector corresponding to λ_1 is x_1

Then x_1 satisfies the equation $(A-\lambda_1 I)x_1 = 0$

$$\Rightarrow \begin{bmatrix} -b & b \\ b & -b \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2 + R_1, R_2/-b$, we get $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow t_1 - t_2 = 0$$

Putting $t_1 = 1$, we get $t_2 = 1$ and thus, the respective eigen vector is $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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Step 7 of 7

Similarly,

Let the eigen vector corresponding to λ_2 is x_2

Then x_2 satisfies the equation $(A-\lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} b & b \\ b & b \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2 - R_1, R_2/b$, we get $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow t_1 + t_2 = 0$$

Putting $t_1 = 1$, we get $t_2 = -1$ and thus, the respective eigen vector is $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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$\Rightarrow t_1 - t_2 = 0$

Putting $t_1 = 1$, we get $t_2 = 1$ and thus, the respective eigen vector is $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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Step 7 of 7

Similarly,

Let the eigen vector corresponding to λ_2 is x_2

Then x_2 satisfies the equation $(A - \lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} b & b \\ b & b \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2 - R_1, R_2 / b$, we get $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow t_1 + t_2 = 0$

Putting $t_1 = 1$, we get $t_2 = -1$ and thus, the respective eigen vector is $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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CH5.1 12P

Step 1 of 1

The eigen values of either the upper or the lower triangular matrix are nothing but the diagonal entries.

Given matrix is $A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix}$ where B has eigen values 1, 2, 3

C has eigen values 4, 5, 6 and D has eigen values 7, 8, 9.

While this matrix is the upper triangular matrix, we follow that the eigen values of A are nothing but the eigen values of B and of D.

Therefore, the eigen values of A are 1, 2, 3, 7, 8, and 9.

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Step 1 of 9

$$A=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$$

We see that any sub matrix of order 2×2 has the determinant zero.
Using this, we get the determinant of the given matrix is zero and also the determinant of any 3×3 sub matrix is also zero.
Therefore, the rank of the matrix A is < 2.

Since A has non zero entries, rank $A < 2$ gives that rank $A = 1$.

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Step 2 of 9

Also, the characteristic equation of A is $|A-\lambda I|=0$

i.e., $$\begin{vmatrix}1-\lambda&1&1&1\\1&1-\lambda&1&1\\1&1&1-\lambda&1\\1&1&1&1-\lambda\end{vmatrix}=0$$

We use elementary row operations on this to reduce to the echelon form.

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Step 3 of 9

$$R_1\to R_1+R_2+R_3+R_4\Rightarrow\begin{vmatrix}4-\lambda&4-\lambda&4-\lambda&4-\lambda\\1&1-\lambda&1&1\\1&1&1-\lambda&1\\1&1&1&1-\lambda\end{vmatrix}=0$$

$$\begin{vmatrix}1&1&1&1\\1&1-\lambda&1&1\end{vmatrix}$$

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We use elementary row operations on this to reduce to the echelon form.

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Step 3 of 9

$$R_1 \to R_1 + R_2 + R_3 + R_4 \Rightarrow \begin{vmatrix} 4-\lambda & 4-\lambda & 4-\lambda & 4-\lambda \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (4-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

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Step 4 of 9

$$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1 \Rightarrow (4-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0-\lambda & 0 & 0 \\ 0 & 0 & 0-\lambda & 0 \\ 0 & 0 & 0 & 0-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (4-\lambda)\{1(-\lambda)(-\lambda)(-\lambda)\} = 0$$

So, $\lambda = 0, 0, 0, 4$ are the eigen values of the given matrix.

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Step 5 of 9

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Step 5 of 9

$$C=\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&1\\1&0&1&0\end{bmatrix}$$

We know that the elementary operations on C does not change the rank of C.
So, we apply row operations on C to reduce it to echelon form.
Then we confirm the rank of C.

$$\text{For, } R_1 \leftrightarrow R_4, R_2 \leftrightarrow R_3 \Rightarrow \begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{bmatrix}$$

$$R_3 \to R_3 - R_1, R_4 \to R_4 - R_2 \Rightarrow \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

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Step 6 of 9

Clearly, the entries in the principal diagonal are either zero or one.
All the entries below the main diagonal are 0's
So, it is the echelon form.
So, rank C = number of non zero rows in C
$= 2$

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Step 7 of 9

Further, to get the eigen values of C, we consider the characteristic equation of C.

$$|C - \lambda I| = \begin{vmatrix}0-\lambda & 1 & 0 & 1\\1 & 0-\lambda & 1 & 0\end{vmatrix} = 0$$

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Step 7 of 9

Further, to get the eigen values of C, we consider the characteristic equation of C.

i.e., $|C-\lambda I| = \begin{vmatrix} 0-\lambda & 1 & 0 & 1 \\ 1 & 0-\lambda & 1 & 0 \\ 0 & 1 & 0-\lambda & 1 \\ 1 & 0 & 1 & 0-\lambda \end{vmatrix} = 0$

Applying the row operations on this, we see $R_1 \to R_1 + R_2 + R_3 + R_4 \Rightarrow$

$$\begin{vmatrix} 2-\lambda & 2-\lambda & 2-\lambda & 2-\lambda \\ 1 & 0-\lambda & 1 & 0 \\ 0 & 1 & 0-\lambda & 1 \\ 1 & 0 & 1 & 0-\lambda \end{vmatrix} = 0$$

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Step 8 of 9

$$\Rightarrow (2-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 1 \\ 1 & 0 & 1 & -\lambda \end{vmatrix} = 0$$

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Step 9 of 9

$$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1 \Rightarrow (2-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -\lambda-1 & 0 & -1 \\ 0 & 1 & -\lambda & 1 \\ 0 & -1 & 0 & -\lambda-1 \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\begin{vmatrix} -\lambda-1 & 0 & -1 \\ 1 & -\lambda & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & 1 & -\lambda \end{vmatrix}$$

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Step 9 of 9

$$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1 \Rightarrow (2-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -\lambda-1 & 0 & -1 \\ 0 & 1 & -\lambda & 1 \\ 0 & -1 & 0 & -\lambda-1 \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\begin{vmatrix} -\lambda-1 & 0 & -1 \\ 1 & -\lambda & 1 \\ -1 & 0 & -\lambda-1 \end{vmatrix} = 0$$

Considering the middle column for the determinant, we get

$$(2-\lambda)(-\lambda)\left\{(-\lambda-1)^2 - 1\right\} = 0$$

$$\Rightarrow (2-\lambda)(-\lambda)\lambda(\lambda+2) = 0$$

Therefore, the eigen values of C are 0, 0, -2, and 2

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Step 1 of 11

Let $A = \begin{bmatrix} 1 & 1 & - & - & 1 \\ 1 & 1 & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & 1 \end{bmatrix}_{n\times n}$

Applying elementary row operations $R_i \to R_i - R_1, 2 \le i \le n$, this matrix becomes

$$\sim \begin{bmatrix} 1 & 1 & - & - & 1 \\ 0 & 0 & - & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & - & - & 0 \end{bmatrix}_{n\times n}$$

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Step 2 of 11

Again applying the column operations $C_i \to C_i - C_1, 2 \le i \le n$, it will become

$$\sim \begin{bmatrix} 1 & 0 & - & - & 0 \\ 0 & 0 & - & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & - & - & 0 \end{bmatrix}_{n\times n}$$

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Step 3 of 11

We observe that the entries in the principal diagonal are 1's or 0's and the entries below the principal diagonal are 0's.
So, this is the echelon form.
The rank of the matrix is nothing but the number of non zero rows or non zero columns in

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Step 4 of 11

Further, the characteristic equation of A can be written as

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & - & - & 1 \\ 1 & 1-\lambda & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & 1-\lambda \end{vmatrix}_{n\times n} = 0$$

Applying elementary row operations on this, we can make it

$$R_1 \to \sum_{i=1}^{n} R_i \Rightarrow \begin{vmatrix} n-\lambda & n-\lambda & - & - & n-\lambda \\ 1 & 1-\lambda & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & n-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (n-\lambda)\begin{vmatrix} 1 & 1 & - & - & 1 \\ 1 & 1-\lambda & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & n-\lambda \end{vmatrix} = 0$$

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Step 5 of 11

Applying $R_i \to R_i - R_1, 2 \le i \le n$, we get $(n-\lambda)\begin{vmatrix} 1 & 1 & - & - & 1 \\ 0 & -\lambda & - & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & - & - & -\lambda \end{vmatrix} = 0$

We observe that the entries below the principal diagonal are zero and so, the determinant is the product of diagonal entries.

$$\Rightarrow (n-\lambda)\begin{vmatrix} 1 & 1-\lambda & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & n-\lambda \end{vmatrix} = 0$$

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Step 5 of 11

Applying $R_i \to R_i - R_1, 2 \le i \le n$, we get $(n-\lambda)\begin{vmatrix} 1 & 1 & - & - & 1 \\ 0 & -\lambda & - & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & - & - & -\lambda \end{vmatrix} = 0$

We observe that the entries below the principal diagonal are zero and so, the determinant is the product of diagonal entries.

i.e., $|A-\lambda I| = (n-\lambda)(-\lambda)^{n-1} = 0$

Thus, the eigen values of A are n, 0, 0,…, 0

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Step 6 of 11

$C = \begin{bmatrix} 1 & 0 & 1 & - & 1 \\ 0 & 1 & 0 & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 1 & - & - & 0 \end{bmatrix}_{n\times n}$ The entries of last row and last column depend on n is even or odd.

If n is even, then the last row entries are 0, 1, 0, 1,…, 1.

If n is odd, then the last row and last column are identical to the first row and first column respectively.

To find the rank of this matrix, we apply the elementary row operations and reduce it to the echelon form.

$$C=\begin{bmatrix}1&0&1&-&1\\0&1&0&-&0\\-&-&-&-&-\\-&-&-&-&-\\0&1&-&-&0\end{bmatrix}_{n\times n}$$ The entries of last row and last column depend on n is even or odd.

If n is even, then the last row entries are 0, 1, 0, 1,..., 1.
If n is odd, then the last row and last column are identical to the first row and first column respectively.

To find the rank of this matrix, we apply the elementary row operations and reduce it to the echelon form.

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Step 7 of 11

Applying $R_{2i-1}\to R_{2i-1}-R_1, R_{2i}\to R_{2i}-R_2, 2\le i\le\left\lfloor\frac{n}{2}\right\rfloor$, we get

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Step 8 of 11

$$C\sim\begin{bmatrix}1&0&1&-&1\\0&1&0&-&0\\0&0&0&-&0\\-&-&-&-&-\\0&0&0&-&0\end{bmatrix}_{n\times n}$$

The entries below the principal diagonal are zero and so, it is the echelon form.
The rank = number of non zero rows in the echelon form
$=2$

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$$C \sim \begin{bmatrix} 1 & 0 & 1 & - & 1 \\ 0 & 1 & 0 & - & 0 \\ 0 & 0 & 0 & - & 0 \\ - & - & - & - & - \\ 0 & 0 & 0 & - & 0 \end{bmatrix}_{n\times n}$$

The entries below the principal diagonal are zero and so, it is the echelon form.
The rank = number of non zero rows in the echelon form
$= 2$

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Step 9 of 11

The characteristic equation of C is $|C-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 & - & - \\ 0 & 1-\lambda & 0 & - & - \\ 1 & 0 & - & - & - \\ - & 1 & 0 & - & 0 \\ 1 & 0 & 1 & - & 1-\lambda \end{vmatrix}$

Applying the row operation $R_1 \to \sum_{i=1}^{n} R_i$, we get $\begin{vmatrix} \left\lfloor \frac{n}{2} \right\rfloor - \lambda & \left\lfloor \frac{n}{2} \right\rfloor - \lambda & - & - & \left\lfloor \frac{n}{2} \right\rfloor - \lambda \\ 0 & 1-\lambda & 0 & - & - \\ 1 & 0 & - & - & - \\ - & 1 & 0 & - & 0 \\ 1 & 0 & 1 & - & 1-\lambda \end{vmatrix}$

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Step 10 of 11

It can be written as $\left(\left\lfloor \frac{n}{2} \right\rfloor - \lambda\right) \begin{vmatrix} 1 & 1 & - & - & 1 \\ 0 & 1-\lambda & 0 & - & - \\ 1 & 0 & - & - & - \\ - & 1 & 0 & - & 0 \end{vmatrix} = 0$

Step 10 of 11

It can be written as $\left(\left\lfloor\frac{n}{2}\right\rfloor-\lambda\right)\begin{vmatrix}1 & 1 & - & - & 1\\ 0 & 1-\lambda & 0 & - & -\\ 1 & 0 & - & - & -\\ - & 1 & 0 & - & 0\\ 1 & 0 & 1 & - & 1-\lambda\end{vmatrix}=0$

Applying the row operation $R_{2i-1}\to R_{2i-1}-R_1, 2\le i\le\left\lfloor\frac{n}{2}\right\rfloor$, we get

$$\left(\left\lfloor\frac{n}{2}\right\rfloor-\lambda\right)\begin{vmatrix}1 & 1 & - & - & 1\\ 0 & -\lambda & 0 & - & -\\ 0 & -1 & - & - & -\\ - & 0 & 0 & - & 0\\ 0 & -1 & 0 & - & 0-\lambda\end{vmatrix}=0$$

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Step 11 of 11

The other $\left\lfloor\frac{n}{2}\right\rfloor$ operations provide the echelon form and the entries of the principal diagonal are $\left(\left\lfloor\frac{n}{2}\right\rfloor+\lambda\right)(-\lambda)^{n-2}$

Thus, the determinant is $\left(\left\lfloor\frac{n}{2}\right\rfloor-\lambda\right)\left(\left\lfloor\frac{n}{2}\right\rfloor+\lambda\right)(-\lambda)^{n-2}=0$

Therefore, the eigen values of C are $0, 0, \ldots, 0, \frac{n}{2}, -\frac{n}{2}$

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Step 1 of 6

A is the 4×4 matrix of ones

$$A=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$$

$$A-I=B=\begin{bmatrix}0&1&1&1\\1&0&1&1\\1&1&0&1\\1&1&1&0\end{bmatrix}$$

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Step 2 of 6

The characteristic equation of $A-I=B$ is $|B-\lambda I|=0$

i.e., $$\begin{vmatrix}-\lambda&1&1&1\\1&-\lambda&1&1\\1&1&-\lambda&1\\1&1&1&-\lambda\end{vmatrix}=0$$

we apply the elementary operations to reduce this matrix to the echelon form and find the determinant.

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Step 3 of 6

$$R_1\to R_1+R_2+R_3+R_4\Rightarrow\begin{vmatrix}3-\lambda&3-\lambda&3-\lambda&3-\lambda\\1&-\lambda&1&1\\1&1&-\lambda&1\\1&1&1&-\lambda\end{vmatrix}=0$$

$$\begin{vmatrix}1&1&1&1\end{vmatrix}$$

Step 3 of 6

$$R_1 \to R_1 + R_2 + R_3 + R_4 \Rightarrow \begin{vmatrix} 3-\lambda & 3-\lambda & 3-\lambda & 3-\lambda \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

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Step 4 of 6

$$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1 \Rightarrow (3-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -\lambda-1 & 1 & 1 \\ 0 & 0 & -\lambda-1 & 0 \\ 0 & 0 & 0 & -\lambda-1 \end{vmatrix} = 0$$

While this is the determinant of the upper triangular matrix, it is equal to the product of the diagonal entries.

$= (3-\lambda)(-1-\lambda)^3 = 0$

Therefore, the eigen values are -1, -1, -1, and 3.

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Step 5 of 6

Also, the determinant of $A - I$ is $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3 + R_4$ we get $\begin{vmatrix} 3 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 \end{vmatrix}$

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Step 5 of 6

Also, the determinant of $A-I$ is $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3 + R_4$, we get $\begin{vmatrix} 3 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

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Step 6 of 6

$$= 3\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1 \Rightarrow 3\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= 3(-1)^3$$

$$= \boxed{-3}$$

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Step 1 of 2

Let us suppose $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ a & b & c\end{bmatrix}$

$\det(A-\lambda I)=\begin{vmatrix}-\lambda & 1 & 0\\ 0 & -\lambda & 1\\ a & b & c-\lambda\end{vmatrix}=0$ is the characteristic equation

It can be simplified as $-\lambda\begin{vmatrix}-\lambda & 1\\ b & c-\lambda\end{vmatrix}-1\begin{vmatrix}0 & 1\\ a & c-\lambda\end{vmatrix}=0$

$$\Rightarrow -\lambda\left\{\lambda^2-c\lambda-b\right\}+a=0$$

$$\Rightarrow -\lambda^3+c\lambda^2+b\lambda+a=0$$

Provide feedback (0)

Step 2 of 2

On the other hand, the given characteristic equation is $-\lambda^3+4\lambda^2+5\lambda+6=0$
Comparing both the equations, we get $\boxed{c=4,\ b=5,\ a=6}$.

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CH5.1 17P

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Step 1 of 4

(a) The nullspace of *A* consists of all those vectors *x* such that $Ax = 0$. Since, *u*, *v*, and *w* are independent vectors, any vector *x* can be expressed as a linear combination of these three vectors.

Consider

$$\begin{aligned} 0 &= Ax \\ &= A(au + bv + cw) \\ &= a(Au) + b(Av) + c(Aw) \\ &= 3bv + 5cw \end{aligned}$$

Therefore, $Ax = 0$ implies that $b = 0$, $c = 0$, and there is no restriction on *a*.

Thus, the basis of nullspace is .

Provide feedback (0)

Step 2 of 4

Therefore, the basis of row space is.

This indicates that the column space is spanned by.

Since, row rank = column rank, the basis of column space is.

Provide feedback (0)

Step 3 of 4

Step 2 of 4

Therefore, the basis of row space is $\{v, w\}$.

This indicates that the column space is spanned by $\{Av, Aw\}$.

Since, row rank = column rank, the basis of column space is $\boxed{\{Av, Aw\}}$.

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Step 3 of 4

(b) Consider the equation $Ax = v + w$. We need to find a particular solution for the same.

We have the following:

$$Av = 3v$$
$$Aw = 5w$$

Let us consider $x = \frac{1}{3}v + \frac{1}{5}w$

Then

$$\begin{aligned} Ax &= A\left(\frac{1}{3}v + \frac{1}{5}w\right) \\ &= \frac{1}{3}Av + \frac{1}{5}Aw \\ &= \frac{1}{3}(3v) + \frac{1}{5}(5w) \\ &= v + w \end{aligned}$$

$$= \frac{1}{3}(3v) + \frac{1}{5}(5w)$$
$$= v + w$$

Thus, a particular solution of $Ax = v + w$ is $\boxed{x = \frac{1}{3}v + \frac{1}{5}w}$.

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Step 4 of 4

Since, *u*, *v*, and *w* are independent vectors, any vector *x* can be expressed as a linear combination of these three vectors.

Let $x = au + bv + cw$. Then we get

$$\begin{aligned} v + w &= Ax \\ &= A(au + bv + cw) \\ &= a(Au) + b(Av) + c(Aw) \\ &= 3bv + 5cw \end{aligned}$$

Therefore, if $Ax = v + w$, there is no restriction on *a*. Also $b = \frac{1}{3}$ and $c = \frac{1}{5}$. Therefore, the general solution of $Ax = v + w$ is given by $\boxed{\left\{\left(a, \frac{1}{3}, \frac{1}{5}\right) \mid a \in \square\right\}}$.

(c) Consider the equation $Ax = u$. Since, *u*, *v*, and *w* are independent vectors, any vector *x* can be expressed as a linear combination of these three vectors.

Let $x = au + bv + cw$. Then we get

Therefore, if $Ax = v + w$, there is no restriction on *a*. Also $b = \frac{1}{3}$ and $c = \frac{1}{5}$. Therefore, the general solution of $Ax = v + w$ is given by $\boxed{\left\{\left(a, \frac{1}{3}, \frac{1}{5}\right) \mid a \in \square\right\}}$.

(c) Consider the equation $Ax = u$. Since, *u*, *v*, and *w* are independent vectors, any vector *x* can be expressed as a linear combination of these three vectors.

Let $x = au + bv + cw$. Then we get

$$\begin{aligned} u &= Ax \\ &= A(au + bv + cw) \\ &= a(Au) + b(Av) + c(Aw) \\ &= 3bv + 5cw \end{aligned}$$

Since, *u*, *v*, and *w* are independent vectors, none of these three can be expressed as a linear combination of the others. Therefore, $u = 3bv + 5cw$ is impossible.

Therefore, the equation $Ax = u$ has no solution. It may be noted that if this equation had some solution, then *u* would be in the column space of *A*.

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CH5.1 18P

Step 1 of 8

$$A=\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$$

To find the eigen values of A, we consider $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} \frac{8}{10}-\lambda & \frac{3}{10} \\ \frac{2}{10} & \frac{7}{10}-\lambda \end{vmatrix}=0$$

$$\Rightarrow (.8-\lambda)(.7-\lambda)-(.3)(.2)=0$$

$$\Rightarrow 0.56-1.5\lambda+\lambda^2-0.6=0$$

$$\Rightarrow \lambda^2-1.5\lambda+0.5=0$$

$$\Rightarrow \lambda^2-\lambda-0.5\lambda+0.5=0$$

$$\Rightarrow \lambda(\lambda-1)-0.5(\lambda-1)=0$$

$$\Rightarrow (\lambda-1)(\lambda-0.5)=0$$

$\Rightarrow \lambda=1, \lambda=0.5$ are the eigen values of the given matrix

Provide feedback (0)

Step 2 of 8

To find the eigen vector corresponding to $\lambda=1$, we consider $|A-\lambda I|x=0$

$$\begin{bmatrix} .8-1 & .3 \\ .2 & .7-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -.2 & .3 \\ .2 & -.3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2+R_1$, we get $\begin{bmatrix} -0.2 & 0.3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix.

So, back substituting the equations, we get $x_1=1.5x_2$

Putting $x_2=2$, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is the eigen vector corresponding to $\lambda=1$

CH5.1 20P

Provide feedback (0)

Step 2 of 8

To find the eigen vector corresponding to $\lambda = 1$, we consider $|A - \lambda I|x = 0$

$$\begin{bmatrix} .8-1 & .3 \\ .2 & .7-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -.2 & .3 \\ .2 & -.3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2 + R_1$, we get $\begin{bmatrix} -0.2 & 0.3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix.
So, back substituting the equations, we get $x_1 = 1.5x_2$

Putting $x_2 = 2$, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 1$

Provide feedback (0)

Step 3 of 8

Similarly, to find the eigen vector corresponding to $\lambda = 0.5$, we consider $|A - \lambda I|x = 0$

$$\begin{bmatrix} .8-0.5 & .3 \\ .2 & .7-0.5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} .3 & .3 \\ .2 & .2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying row operation $R_2 \to R_2 - \frac{2}{3}R_1$, we get $\begin{bmatrix} 0.3 & 0.3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix.
So, back substituting the equations, we get $x_1 = -x_2$

Putting $x_1 = 1$, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 0.5$

Applying row operation $R_2 \to R_2 - \frac{2}{3}R_1$, we get $\begin{bmatrix} 0.3 & 0.3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix.

So, back substituting the equations, we get $x_1 = -x_2$

Putting $x_1 = 1$, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 0.5$

Provide feedback (0)

Step 4 of 8

The matrix whose columns are the eigen vectors of A is $P = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$

$$P^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$$

So, by the diagonalizability, we have $P^{-1}AP = D$ where D is the diagonal matrix.

$$\begin{aligned} \frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} &= \frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & .5 \\ 2 & -.5 \end{bmatrix} \\ &= \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 0 & 2.5 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \end{aligned}$$

This is the diagonal matrix whose diagonal entries are nothing but the eigen values of A.

Provide feedback (0)

Step 5 of 8

The above equation can otherwise be written as $A = PDP^{-1}$

Raising this equation to the power n, we easily see

$$\begin{aligned} A^n &= \left(PDP^{-1}\right)\left(PDP^{-1}\right)\ldots\left(PDP^{-1}\right) \\ &= PD^nP^{-1} \end{aligned}$$

So, $A^2 = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1^2 & 0 \\ 0 & -3 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$ ..... (1)

This is the diagonal matrix whose diagonal entries are nothing but the eigen values of A.

Provide feedback (0)

Step 5 of 8

The above equation can otherwise be written as $A = PDP^{-1}$
Raising this equation to the power n, we easily see

$$A^n = \left(PDP^{-1}\right)\left(PDP^{-1}\right)\ldots\left(PDP^{-1}\right)$$
$$= PD^nP^{-1}$$

$$\text{So, } A^2 = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1^2 & 0 \\ 0 & .5^2 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix} \qquad \ldots\ldots (1)$$
$$= \frac{1}{5}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0.5 & -0.75 \end{bmatrix}$$
$$= \frac{1}{5}\begin{bmatrix} 3.5 & 2.25 \\ 1.5 & 2.75 \end{bmatrix}$$
$$= \begin{bmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

$$\lim_{n\to\infty} A^n = P\left\{\lim_{n\to\infty} D^n\right\}P^{-1}$$
$$= \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\left\{\lim_{n\to\infty}\begin{bmatrix} 1^n & 0 \\ 0 & 0.5^n \end{bmatrix}\right\}\frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 8

$$= \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$$

$$5\begin{bmatrix}2 & -1\end{bmatrix}\begin{bmatrix}0.5 & -0.75\end{bmatrix}$$
$$= \frac{1}{5}\begin{bmatrix}3.5 & 2.25\\ 1.5 & 2.75\end{bmatrix}$$
$$= \begin{bmatrix}0.7 & 0.45\\ 0.3 & 0.55\end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

$$\lim_{n\to\infty} A^n = P\left\{\lim_{n\to\infty} D^n\right\}P^{-1}$$
$$= \begin{bmatrix}3 & 1\\ 2 & -1\end{bmatrix}\left\{\lim_{n\to\infty}\begin{bmatrix}1^n & 0\\ 0 & 0.5^n\end{bmatrix}\right\}\frac{1}{5}\begin{bmatrix}1 & 1\\ 2 & -3\end{bmatrix}$$

Provide feedback (0)

Step 7 of 8

$$= \begin{bmatrix}3 & 1\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\frac{1}{5}\begin{bmatrix}1 & 1\\ 2 & -3\end{bmatrix}$$
$$= \begin{bmatrix}3 & 0\\ 2 & 0\end{bmatrix}\frac{1}{5}\begin{bmatrix}1 & 1\\ 2 & -3\end{bmatrix}$$
$$= \frac{1}{5}\begin{bmatrix}3 & 3\\ 2 & 2\end{bmatrix}$$
$$= \begin{bmatrix}0.6 & 0.6\\ 0.4 & 0.4\end{bmatrix} \qquad \ldots\ldots (2)$$

Provide feedback (0)

Step 8 of 8

We have the equation $A^n = PD^nP^{-1}$

$$= \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 2 & 0 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \qquad \ldots\ldots (2)$$

Provide feedback (0)

Step 8 of 8

We have the equation $A^n = PD^nP^{-1}$

So, the eigen values of A^n are nothing but the nth powers of the diagonal matrix of A.
That is $1^n, 0.5^n$

We know that $1^2 = 1, \lim_{n\to\infty} 1^n = 1$

The average of $1, \lim_{n\to\infty} 1^n$ is 1^2 $\qquad \ldots\ldots (3)$

But $0.5^2 = \frac{1}{2}(0.5), \lim_{n\to\infty} 0.5^n = 0$

The average of 0.5 and 0 is 0.5^2 $\qquad \ldots\ldots (4)$

Putting (3) and (4) together, we can say that the eigen values of A^2 are nothing but the averages of the eigen values of A, and $\lim_{n\to\infty} A^n$.

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CH5.1 19P

Step 1 of 19

Given that $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and $A+I=\begin{bmatrix}2 & 4\\ 2 & 4\end{bmatrix}$

We need to compute the eigenvalues and the eigenvectors of the above two matrices.

Provide feedback (0)

Step 2 of 19

Now $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}1-\lambda & 4\\ 2 & 3-\lambda\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 3 of 19

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & 4\\ 2 & 3-\lambda\end{vmatrix}\\ &= (1-\lambda)(3-\lambda)-8\\ &= 3-\lambda-3\lambda+\lambda^2-8\\ &= \lambda^2-4\lambda-5\end{aligned}$$

Provide feedback (0)

Step 4 of 19

CH5.1 21P

$$= 3 - \lambda - 3\lambda + \lambda^2 - 8$$
$$= \lambda^2 - 4\lambda - 5$$

Provide feedback (0)

Step 4 of 19

we know that $|A - \lambda I| = 0$

$$\lambda^2 - 4\lambda - 5 = 0$$
$$\lambda^2 - 5\lambda + \lambda - 5 = 0$$
$$\lambda(\lambda - 5) + 1(\lambda - 5) = 0$$
$$(\lambda - 5)(\lambda + 1) = 0$$

Provide feedback (0)

Step 5 of 19

$$\lambda = 5, -1$$
$$\lambda = 5, -1$$

Hence the eigenvalues of A are 5,-1

Provide feedback (0)

Step 6 of 19

Case(i) Let $\lambda = 5$

Eigenvectors X corresponding to the eigenvalue 5 are given by

$$(A - 5I)X = 0$$

That is $\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 7 of 19

Step 6 of 19

Case(i) Let $\lambda = 5$

Eigenvectors X corresponding to the eigenvalue 5 are given by

$(A-5I)X = 0$

That is $\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 7 of 19

$By\ 2R_2 + R_1 = R_2$

$\begin{bmatrix} -4 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -4x_1 + 4x_2 = 0$

Let $x_1 = k(\text{say})$

Therefore $x_2 = k$

Therefore eigenvectors corresponding to eigenvalue 5 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 8 of 19

Case(ii) Let $\lambda = -1$

Eigenvectors X corresponding to the eigenvalue -1 are given by

$(A+I)X = 0$

That is $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Provide feedback (0)

Step 8 of 19

Case(ii) Let $\lambda = -1$

Eigenvectors X corresponding to the eigenvalue -1 are given by

$(A+I)X = 0$

That is $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 9 of 19

$By\ R_2 - R_1 = R_2$

$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_1 + 4x_2 = 0$

$\Rightarrow x_1 + 2x_2 = 0$

Provide feedback (0)

Step 10 of 19

Let $x_1 = k(\text{say})$

Therefore $2x_2 = -k$

$x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -1 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Step 11 of 19

Let $x_1 = k(\text{say})$

Therefore $2x_2 = -k$

$x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -1 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Step 11 of 19

Now $A+I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

Provide feedback (0)

Step 12 of 19

$$\begin{aligned}(A+I)-\lambda I &= \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 2-\lambda & 4 \\ 2 & 4-\lambda \end{bmatrix} \\ &= (2-\lambda)(4-\lambda)-8 \\ &= 8-2\lambda-4\lambda+\lambda^2-8\end{aligned}$$

Provide feedback (0)

Step 13 of 19

we know that $|A-\lambda I| = 0$

$$\lambda^2 - 6\lambda = 0$$

Provide feedback (0)

Step 13 of 19

we know that $|A-\lambda I|=0$

$$\lambda^2-6\lambda=0$$
$$\lambda(\lambda-6)=0$$
$$\lambda=6,0$$

Provide feedback (0)

Step 14 of 19

Hence the eigenvalues of $A+I$ are 6,0

Case(i) Let $\lambda=6$

Eigenvectors X corresponding to the eigenvalue 6 are given by

$$\left((A+I)-6I\right)X=0$$

That is $\begin{bmatrix}-4 & 4\\ 2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

Provide feedback (0)

Step 15 of 19

$By\ 2R_2+R_1=R_2$

$$\begin{bmatrix}-4 & 4\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Rightarrow -4x_1+4x_2=0$$

$$\Rightarrow x_1-x_2=0$$

Provide feedback (0)

Step 16 of 19

Step 15 of 19

$By\ 2R_2 + R_1 = R_2$

$$\begin{bmatrix} -4 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -4x_1 + 4x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

Provide feedback (0)

Step 16 of 19

Let $x_1 = k(\text{say})$

Therefore $x_2 = k$

Therefore eigenvectors corresponding to eigenvalue 6 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Step 17 of 19

Case(ii) Let $\lambda = 0$

Eigenvectors X corresponding to the eigenvalue 0 are given by

$((A+I) - 0I)X = 0$

That is $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 18 of 19

$By\ R_2 - R_1 = R_2$

$\begin{bmatrix} 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

Eigenvectors X corresponding to the eigenvalue 0 are given by

$$((A+I)-0I)X=0$$

That is $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 18 of 19

$By\ R_2 - R_1 = R_2$

$$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2x_1+4x_2=0$$

$$\Rightarrow x_1+2x_2=0$$

Provide feedback (0)

Step 19 of 19

Let $x_1 = k(say)$

Therefore $x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue 0 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter.

$A+I$ has the same eigenvectors as A. Its eigenvalues are different by 1.

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CH5.1 20P

Step 1 of 19

Given that $A=\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}-3/4 & 1/2\\ 1/2 & 0\end{bmatrix}$

We need to compute the eigenvalues and the eigenvectors of the above two matrices.

Provide feedback (0)

CH5.1 22P

Step 2 of 19

Now $A=\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}-\lambda & 2\\ 2 & 3-\lambda\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 3 of 19

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}-\lambda & 2\\ 2 & 3-\lambda\end{vmatrix}\\ &= (-\lambda)(3-\lambda)-4\\ &= -3\lambda+\lambda^2-4\\ &= \lambda^2-3\lambda-4\end{aligned}$$

Provide feedback (0)

Step 4 of 19

$$= -3\lambda + \lambda^2 - 4$$
$$= \lambda^2 - 3\lambda - 4$$

Provide feedback (0)

Step 4 of 19

we know that $|A - \lambda I| = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\lambda = 4, -1$

$\lambda = 4, -1$

Hence the eigenvalues of A are 4,-1

Provide feedback (0)

Step 5 of 19

Case(i) Let $\lambda = 4$

Eigenvectors X corresponding to the eigenvalue 4 are given by

$(A - 4I)X = 0$

That is $\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 6 of 19

$By\ 2R_2 + R_1 = R_2$

$\begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -4x_1 + 2x_2 = 0$

Step 6 of 19

$By\ 2R_2 + R_1 = R_2$

$$\begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -4x_1 + 2x_2 = 0$

Let $x_1 = k(\text{say})$

Therefore $x_2 = 2k$

Therefore eigenvectors corresponding to eigenvalue 4 are given by $k\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 7 of 19

Case(ii) Let $\lambda = -1$

Eigenvectors X corresponding to the eigenvalue -1 are given by

$(A+I)X = 0$

That is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 8 of 19

$By\ R_2 - 2R_1 = R_2$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + 2x_2 = 0$

Provide feedback (0)

Provide feedback (0)

Step 8 of 19

$By\ R_2 - 2R_1 = R_2$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + 2x_2 = 0$

Provide feedback (0)

Step 9 of 19

Let $x_1 = k(\text{say})$

Therefore $2x_2 = -k$

$x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -1 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter

Now $\boxed{A^{-1} = \begin{bmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{bmatrix}}$

$$\left(A^{-1}\right) - \lambda I = \begin{bmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 19

$$= \begin{bmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -3/4 - \lambda & 1/2 \\ 1/2 & -\lambda \end{bmatrix}$$

Step 10 of 19

$$= \begin{bmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -3/4-\lambda & 1/2 \\ 1/2 & -\lambda \end{bmatrix}$$

$$= (-3/4-\lambda)(-\lambda) - 1/4$$

$$= \frac{3}{4}\lambda + \lambda^2 - \frac{1}{4}$$

$$= \lambda^2 + \frac{3}{4}\lambda - \frac{1}{4}$$

Provide feedback (0)

Step 11 of 19

We know that $|A - \lambda I| = 0$

$$\lambda^2 + \frac{3}{4}\lambda - \frac{1}{4} = 0$$

$$4\lambda^2 + 3\lambda - 1 = 0$$

Provide feedback (0)

Step 12 of 19

Compare this equation with $ax^2 + bx + c = 0$

$$\lambda = \frac{-3 \pm \sqrt{9+16}}{8}$$

$$= \frac{-3 \pm \sqrt{25}}{8}$$

$$= \frac{-3 \pm 5}{8}$$

$$= \frac{-3+5}{8}, \frac{-3-5}{8}$$

Step 12 of 19

Compare this equation with $ax^2+bx+c=0$

$$\lambda=\frac{-3\pm\sqrt{9+16}}{8}$$
$$=\frac{-3\pm\sqrt{25}}{8}$$
$$=\frac{-3\pm5}{8}$$
$$=\frac{-3+5}{8},\frac{-3-5}{8}$$
$$=\frac{1}{4},-1$$

Provide feedback (0)

Step 13 of 19

Case(i) Let $\lambda=\frac{1}{4}$

Eigenvectors X corresponding to the eigenvalue $\frac{1}{4}$ are given by

$$\left(\left(A^{-1}\right)-\frac{1}{4}I\right)X=0$$

That is $\begin{bmatrix}-1 & \frac{1}{2}\\ \frac{1}{2} & -\frac{1}{4}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

Provide feedback (0)

Step 14 of 19

$By\ R_2+\frac{R_1}{2}=R_2$

$\lfloor 2 \quad 4 \rfloor$

Provide feedback (0)

Step 14 of 19

$By\ R_2 + \frac{R_1}{2} = R_2$

$$\begin{bmatrix} -1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x_1 + \frac{1}{2} x_2 = 0$$

Provide feedback (0)

Step 15 of 19

Let $x_1 = k(\text{say})$

$-k + \frac{1}{2} x_2 = 0$

Therefore $x_2 = 2k$

Therefore eigenvectors corresponding to eigenvalue $\frac{1}{4}$ are given by $k \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ where k is a non-zero parameter

Provide feedback (0)

Step 16 of 19

Case(ii) Let $\lambda = -1$

Eigenvectors X corresponding to the eigenvalue -1 are given by

$\left(\left(A^{-1} \right) + I \right) X = 0$

That is $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \end{bmatrix} \begin{bmatrix} x_1 \\ \end{bmatrix} = \begin{bmatrix} 0 \\ \end{bmatrix}$

Step 16 of 19

Case(ii) Let $\lambda = -1$

Eigenvectors X corresponding to the eigenvalue -1 are given by

$$\left(\left(A^{-1}\right)+I\right)X=0$$

That is $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 17 of 19

$By\ R_2/2-R_1=R_2$

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{4}x_1+\frac{1}{2}x_2=0$$

$$\Rightarrow x_1+2x_2=0$$

Provide feedback (0)

Step 18 of 19

Let $x_1 = k(\text{say})$

Therefore $x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -1 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 19 of 19

By $x_2/2 = x_1 = x_2$

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{4}x_1 + \frac{1}{2}x_2 = 0$$

$$\Rightarrow x_1 + 2x_2 = 0$$

Provide feedback (0)

Step 18 of 19

Let $x_1 = k(\text{say})$

Therefore $x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -1 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 19 of 19

A^{-1} has the same eigenvectors as A. When A has eigenvalues λ_1 and λ_2, its inverse has eigenvalues $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$.

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CH5.1 21P

Step 1 of 14

Given that $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$ and $A^2=\begin{bmatrix}7 & -3\\ -2 & 6\end{bmatrix}$

Now $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}-1-\lambda & 3\\ 2 & -\lambda\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 2 of 14

Now

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}-1-\lambda & 3\\ 2 & -\lambda\end{vmatrix}\\ &= (-1-\lambda)(-\lambda)-6\\ &= \lambda+\lambda^2-6\\ &= \lambda^2+\lambda-6\end{aligned}$$

Provide feedback (0)

Step 3 of 14

We know that

$$\begin{aligned}|A-\lambda I| &= 0\\ \lambda^2+\lambda-6 &= \lambda^2+3\lambda-2\lambda-6\\ &= \lambda(\lambda+3)-2(\lambda+3)\\ &= (\lambda+3)(\lambda-2)\end{aligned}$$

CH5.1 23P

Step 3 of 14

We know that

$$|A-\lambda I|=0$$

$$\begin{aligned}\lambda^2+\lambda-6&=\lambda^2+3\lambda-2\lambda-6\\&=\lambda(\lambda+3)-2(\lambda+3)\\&=(\lambda+3)(\lambda-2)\end{aligned}$$

$$(\lambda+3)(\lambda-2)=0$$

$$\lambda=-3,2$$

Hence the Eigen values of A are -3, 2

Provide feedback (0)

Step 4 of 14

Case (i) Let $\lambda=-3$

Eigen vectors X corresponding to Eigen value -3 are given by

$$(A-(-3)I)X=0$$

$$\begin{bmatrix}2&3\\2&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

Add -1 times second row to the first row to the second row

$$\begin{bmatrix}2&3\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$2x_1+3x_2=0$$

Provide feedback (0)

Step 5 of 14

Let $x_2=k$

Therefore

$$x_1=-3k/2$$

Therefore eigenvectors corresponding to eigenvalue -3 are given by $k\begin{bmatrix}-3/2\\1\end{bmatrix}$ where k

$$2x_1 + 3x_2 = 0$$

Provide feedback (0)

Step 5 of 14

Let $x_2 = k$

Therefore

$$x_1 = -3k/2$$

Therefore eigenvectors corresponding to eigenvalue -3 are given by $k\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 6 of 14

Case (ii) Let $\lambda = 2$

Eigen vectors X corresponding to the Eigen value 2 are given by

$$(A - 2I)X = 0$$

$$\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add +1 times first row to the second row

$$\begin{bmatrix} -3 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 14

Add 3 times second row and -1 times first row to second row

$$\begin{bmatrix} -3 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + 3x_2 = 0$$

Step 7 of 14

Add 3 times second row and -1 times first row to second row

$$\begin{bmatrix} -3 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + 3x_2 = 0$$

Let $x_2 = k(\neq 0)$

Therefore $x_1 = k$

Therefore Eigen vectors corresponding to Eigen value 2 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Where k is a non-zero parameter

Provide feedback (0)

Step 8 of 14

$$\text{Now } A^2 = \begin{bmatrix} 7 & -3 \\ -2 & 6 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 7 & -3 \\ -2 & 6 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7-\lambda & -3 \\ -2 & 6-\lambda \end{bmatrix}$$

Provide feedback (0)

Step 9 of 14

$$|A - \lambda I| = \begin{vmatrix} 7-\lambda & -3 \\ -2 & 6-\lambda \end{vmatrix}$$

$$= (7-\lambda)(6-\lambda) - 6$$

$$= 42 - 7\lambda - 6\lambda + \lambda^2 - 6$$

$$= \lambda^2 - 13\lambda + 36$$

We know that

$$\begin{bmatrix} -2 & 6-\lambda \end{bmatrix}$$

Provide feedback (0)

Step 9 of 14

$$\begin{aligned} |A-\lambda I| &= \begin{vmatrix} 7-\lambda & -3 \\ -2 & 6-\lambda \end{vmatrix} \\ &= (7-\lambda)(6-\lambda)-6 \\ &= 42-7\lambda-6\lambda+\lambda^2-6 \\ &= \lambda^2-13\lambda+36 \end{aligned}$$

We know that

$$|A-\lambda I| = 0$$

$$\lambda^2-13\lambda+36=0$$

Provide feedback (0)

Step 10 of 14

We have a formula $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\begin{aligned} &= \frac{13\pm\sqrt{(13)^2-4\times1\times36}}{2\times1} \\ &= \frac{13\pm\sqrt{169-144}}{2} \\ &= \frac{13\pm\sqrt{25}}{2} \\ &= \frac{13\pm5}{2} \\ &= \frac{18}{2},\frac{8}{2} \\ &= 9,4 \\ \lambda &= 9,4 \end{aligned}$$

Hence the Eigen value of A^2 are 9, 4

$$\lambda = 9, 4$$

Hence the Eigen value of A^2 are 9, 4

Provide feedback (0)

Step 11 of 14

Case (i) Let $\lambda = 9$

Eigen vectors X corresponding to Eigen value 9 are given by

$$(A - 9I)X = 0$$

$$\begin{bmatrix} -2 & -3 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add -1 times second row and +1 times the first row to the second row

$$\begin{bmatrix} -2 & -3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - 3x_2 = 0$$

Provide feedback (0)

Step 12 of 14

Let $x_2 = k$

Therefore $x_1 = -3k/2$

Therefore eigenvectors corresponding to eigenvalue 9 are given by $k\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ where k is a non-zero parameter.

Provide feedback (0)

Step 13 of 14

Case (ii) Let $\lambda = 4$

Eigen vectors X corresponding to Eigen value 4 are given by

$$(A - 4I)X = 0$$

$$\begin{bmatrix} 3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix}$$

Provide feedback (0)

Step 13 of 14

Case (ii) Let $\lambda = 4$

Eigen vectors X corresponding to Eigen value 4 are given by

$$(A-4I)X = 0$$

$$\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add +1 times the first row to the second row

$$\begin{bmatrix} 3 & -3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 14 of 14

Add 3 times second row and -1 times first row to second row

$$\begin{bmatrix} 3 & -3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x_1 - 3x_2 = 0$$

Let $x_2 = k$

Therefore $x_1 = k$

Therefore Eigen vectors corresponding to Eigen value 4 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Where k is a non-zero parameter

A^2 has the same square as A

When A has an Eigen value λ_1, λ_2 then A^2 has Eigen values λ_1^2, λ_2^2

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CH5.1 22P

Step 1 of 2

(a) If you know x is an Eigen vector, then the way to find λ is
Multiply Ax to see λx which reveals λ

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Step 2 of 2

(b) If you know λ is an Eigen value, then the way to find x is
Solve $(A-\lambda I)x=0$ to find x

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CH5.1 24P

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CH5.1 23P

Step 1 of 3

Given that $Ax = \lambda x$

a) Multiplying A on both sides, we get $A(Ax) = A(\lambda x)$

But λ is a scalar and so, it commutes with the matrix A.

So, the above equation becomes $A(Ax) = \lambda(Ax)$

$$\Rightarrow A^2 x = \lambda(\lambda x)$$
$$= \lambda^2 x$$

Therefore λ^2 is the eigen value of A^2.

Provide feedback (0)

Step 2 of 3

b) $Ax = \lambda x$

Multiplying both sides with A^{-1}, we get

$(A^{-1}A)x = \lambda(A^{-1}x)$ by the commutativity of λ

$$Ix = \lambda(A^{-1}x)$$
$$\Rightarrow A^{-1}x = \frac{1}{\lambda}x$$

Therefore $\frac{1}{\lambda}$ is the eigen value of A^{-1}.

Provide feedback (0)

Step 3 of 3

c) $Ax = \lambda x$

We know that when x is a column of size $n \times 1$, then it can be written as $x = Ix$ where I is the identity matrix of size $n \times n$.

So, adding $Ix = x$ on the respective sides of the above equation, we get

$$(A+I)x = (\lambda+1)x$$

Therefore $\lambda+1$ is the eigen value of $A+I$

CH5.1 25P

Step 2 of 3

b) $Ax = \lambda x$

Multiplying both sides with A^{-1}, we get

$(A^{-1}A)x = \lambda(A^{-1}x)$ by the commutativity of λ

$$Ix = \lambda(A^{-1}x)$$

$$\Rightarrow A^{-1}x = \frac{1}{\lambda}x$$

Therefore $\frac{1}{\lambda}$ is the eigen value of A^{-1}.

Provide feedback (0)

Step 3 of 3

c) $Ax = \lambda x$

We know that when x is a column of size $n \times 1$, then it can be written as $x = Ix$ where I is the identity matrix of size $n \times n$.

So, adding $Ix = x$ on the respective sides of the above equation, we get

$$(A+I)x = (\lambda+1)x$$

Therefore $\lambda+1$ is the eigen value of $A+I$

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CH5.1 24P

Step 1 of 6

Given unit vector is $u=\left(\frac{1}{6},\frac{1}{6},\frac{3}{6},\frac{5}{6}\right)$

P is the projection matrix given by $P=uu^T$

$$=\begin{bmatrix}\frac{1}{6} & \frac{1}{6} & \frac{3}{6} & \frac{5}{6}\end{bmatrix}\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\\ \frac{3}{6}\\ \frac{5}{6}\end{bmatrix}$$

$$=\left[\frac{1}{36}+\frac{1}{36}+\frac{9}{36}+\frac{25}{36}\right]$$

$$=[1]$$

Clearly, this is a matrix of size 1×1, with determinant 1

So, the rank of this matrix is 1.

Provide feedback (0)

Step 2 of 6

a) $Pu=\left(uu^T\right)u$

$=u\left(u^Tu\right)$ by the associative property

$=u\cdot I$ while u is the unit vector

$=u$

We know that if λ is the eigen value of a matrix A and x is the corresponding eigen vector, then it follows that $Ax=\lambda x$.

In view of this, we can write $Pu=u$ as $Pu=1u$ and thus, $\lambda=1$ is the corresponding eigen value of u.

Provide feedback (0)

CH5.1 26P

So, the rank of this matrix is 1.

Provide feedback (0)

Step 2 of 6

a) $Pu = \left(uu^T\right)u$

$= u\left(u^T u\right)$ by the associative property

$= u \cdot I$ while u is the unit vector

$= u$

We know that if λ is the eigen value of a matrix A and x is the corresponding eigen vector, then it follows that $Ax = \lambda x$.

In view of this, we can write $Pu = u$ as $Pu = 1u$ and thus, $\lambda = 1$ is the corresponding eigen value of u.

Provide feedback (0)

Step 3 of 6

b) Suppose v is a perpendicular vector to u.

Then $u^T v = 0$

$Pv = \left(uu^T\right)v$ while $P = uu^T$

$= u\left(u^T v\right)$ by associate

$= u(0)$ since $u^T v = 0$

$= 0$

This can be written as $Pv = \lambda v$ where $\lambda = 0$

Therefore, the eigen value of P corresponding to eigen vector v is 0.

Provide feedback (0)

Step 4 of 6

c) We consider

$x_1 = (-1,1,0,0)$

$x_2 = (-3,0,1,0)$

$x_3 = (-5,0,0,1)$

Provide feedback (0)

Step 4 of 6

c) We consider

$$x_1 = (-1,1,0,0)$$
$$x_2 = (-3,0,1,0)$$
$$x_3 = (-5,0,0,1)$$

Then

$$u^T x_1 = \begin{bmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{3}{6} \\ \frac{5}{6} \end{bmatrix} (-1,1,0,0)$$
$$= \left[\frac{-1}{6} + \frac{1}{6} + 0 + 0 \right]$$
$$= 0$$

Provide feedback (0)

Step 5 of 6

$$u^T x_2 = \begin{bmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{3}{6} \\ \frac{5}{6} \end{bmatrix} [-3,0,1,0]$$
$$= 0$$
$$\begin{bmatrix} \frac{1}{6} \end{bmatrix}$$

Step 5 of 6

$$u^T x_2 = \begin{bmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{3}{6} \\ \frac{5}{6} \end{bmatrix} [-3, 0, 1, 0]$$

$$= 0$$

$$u^T x_3 = \begin{bmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{3}{6} \\ \frac{5}{6} \end{bmatrix} [-5 \quad 0 \quad 0 \quad 1]$$

$$= \left[\frac{-5}{6} + 0 + 0 + \frac{5}{6} \right]$$

$$= 0$$

Therefore x_1, x_2, x_3 are orthogonal to u.

Provide feedback (0)

Step 6 of 6

So, in view of the result (b), we confirm that the eigen value of P with respect to the eigen vectors x_1, x_2, x_3 is $\lambda = 0$.

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CH5.1 25P

Step 1 of 4

Given that $Q=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

The characteristic equation of this matrix is $\det(Q-\lambda I)=0$

$$\Rightarrow\begin{vmatrix}\cos\theta-\lambda & -\sin\theta\\ \sin\theta & \cos\theta-\lambda\end{vmatrix}=0$$

$$\Rightarrow(\cos-\lambda)^2+\sin^2\theta=0$$

$$\Rightarrow\cos^2\theta+\lambda^2-2\lambda\cos\theta+\sin^2\theta=0$$

$$\Rightarrow\lambda^2-2\lambda\cos\theta+1=0$$

$$\Rightarrow\lambda=\frac{2\cos\theta\pm\sqrt{4\cos^2\theta-4}}{2}$$

$$=\frac{2\left(\cos\theta\pm\sqrt{1-\cos^2\theta}\right)}{2}$$

$$\Rightarrow\cos\theta\pm\sqrt{\sin^2\theta}$$

$\Rightarrow\cos\theta+i\sin\theta,\cos\theta-i\sin\theta$ are the eigen values of the given matrix that rotates the xy – plane through an angle θ.

Provide feedback (0)

Step 2 of 4

We now find the eigen vectors of Q for the eigen value $\lambda=\cos\theta+i\sin\theta$

For, we solve $(Q-\lambda I)x=0$ where x is the required eigen vector.

$$\Rightarrow\begin{bmatrix}-i\sin\theta & -\sin\theta\\ \sin\theta & -i\sin\theta\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

Using the row operation $R_2\to iR_2+R_1$, we get

$$\begin{bmatrix}-i\sin\theta & -\sin\theta\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

This is the row reduced matrix and so, writing the homogeneous equations, we get

$ix_1+x_2=0$

CH5.1 27P

Step 2 of 4

We now find the eigen vectors of Q for the eigen value $\lambda = \cos\theta + i\sin\theta$

For, we solve $(Q - \lambda I)x = 0$ where x is the required eigen vector.

$$\Rightarrow \begin{bmatrix} -i\sin\theta & -\sin\theta \\ \sin\theta & -i\sin\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Using the row operation $R_2 \to iR_2 + R_1$, we get

$$\begin{bmatrix} -i\sin\theta & -\sin\theta \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the row reduced matrix and so, writing the homogeneous equations, we get

$ix_1 + x_2 = 0$

Putting $x_1 = 1$, we get the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$ which is the eigen vector corresponding to $\lambda = \cos\theta + i\sin\theta$

Provide feedback (0)

Step 3 of 4

Similarly, the eigen vector corresponding to the eigen value $\lambda = \cos\theta - i\sin\theta$ is obtained by $\begin{bmatrix} i\sin\theta & -\sin\theta \\ \sin\theta & i\sin\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

Applying $R_2 \to iR_2 - R_1$, we get $\begin{bmatrix} i\sin\theta & -\sin\theta \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow ix_1 - x_2 = 0$

Putting $x_1 = 1$, we get the solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ the eigen vector corresponding to $\lambda = \cos\theta - i\sin\theta$

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Step 4 of 4

Thus, the eigen values of the rotation matrix are $\lambda = \cos\theta + i\sin\theta$, $\lambda = \cos\theta - i\sin\theta$ and

to $\lambda = \cos\theta + i\sin\theta$

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Step 3 of 4

Similarly, the eigen vector corresponding to the eigen value $\lambda = \cos\theta - i\sin\theta$ is obtained by $\begin{bmatrix} i\sin\theta & -\sin\theta \\ \sin\theta & i\sin\theta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

Applying $R_2 \to iR_2 - R_1$, we get $\begin{bmatrix} i\sin\theta & -\sin\theta \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow ix_1 - x_2 = 0$

Putting $x_1 = 1$, we get the solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ the eigen vector corresponding to $\lambda = \cos\theta - i\sin\theta$

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Step 4 of 4

Thus, the eigen values of the rotation matrix are $\lambda = \cos\theta + i\sin\theta$, $\lambda = \cos\theta - i\sin\theta$ and the respective eigen vectors are $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ i \end{bmatrix}$

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CH5.1 26P

CH5.1 28P

Step 1 of 2

If P is a matrix, λ is the eigen value and x is the respective eigen vector, then we have $Px = \lambda x$

In view of this, we consider (1) $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

We consider the characteristic equation of this matrix $|P - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda\left(\lambda^2 - 0\right) - 1(-1) + 0$$

$$\Rightarrow -\lambda^3 + 1 = 0$$

$$\Rightarrow (\lambda - 1)\left(\lambda^2 + \lambda + 1\right) = 0$$

$$\Rightarrow \lambda = 1 \text{ and } \lambda^2 + \lambda + 1 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$$

$$\lambda = 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$$

The other eigen values are $\lambda = \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$

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Step 2 of 2

Similarly, we consider $(P - \lambda I) = 0$ for the matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\lambda = 1, \frac{\quad}{2}, \frac{\quad}{2}$

The other eigen values are $\lambda = \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

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Step 2 of 2

Similarly, we consider $(P-\lambda I)=0$ for the matrix $P=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$

$$\Rightarrow \begin{vmatrix}-\lambda & 0 & 1\\ 0 & 1-\lambda & 0\\ 1 & 0 & -\lambda\end{vmatrix}=0$$

$$\Rightarrow -\lambda(-\lambda(1-\lambda))-0+1(-1-\lambda)=0$$

$$\Rightarrow -\lambda(-\lambda+\lambda^2)-1+\lambda=0$$

$$\Rightarrow \lambda^2-\lambda^3-1+\lambda=0$$

$$\Rightarrow \lambda^3-\lambda^2+1=0$$

$$\Rightarrow \lambda^2(\lambda-1)-1(\lambda-1)=0$$

$$\Rightarrow (\lambda^2-1)(\lambda-1)=0$$

$$\Rightarrow \lambda=1, \lambda^2-1=0$$

$$\Rightarrow \lambda=\pm 1$$

$$\Rightarrow \lambda=1,1,-1$$

The remaining λ values are $-1,1$.

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Step 1 of 2

Suppose $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det(A-\lambda I)=\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}=0$$

$$=(a-\lambda)(d-\lambda)-bc$$

$$=\lambda^2-(a+d)\lambda+(ad-bc)$$

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Step 2 of 2

Given that $|A-\lambda I|=\lambda^2-9\lambda+20$ and $a+d=9, ad-bc=20$

We get, $a+d=9, ad-bc=20$

There can be infinitely many integers which satisfy these equations.

Let us consider the ordered pairs of values of a and d

(1, 8), (2, 7), (3, 6),… which are positive and integer choices.

When $a=1$, $d=8$, we have the choices for b and c obtained by $-bc=12$

This gives $b=-6$, $c=2$ or $b=-3$, $c=4$ or $b=-4$, $c=3$ or $b=6$, $c=-2$

So, some possible choices of matrices are $\begin{bmatrix} 1 & -4 \\ 3 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -3 & 8 \end{bmatrix}, \begin{bmatrix} 1 & -6 \\ 2 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 6 \\ -2 & 8 \end{bmatrix}$

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Step 1 of 2

Suppose $A=\begin{bmatrix}0 & 1\\ a & b\end{bmatrix}$

The characteristic equation of this matrix is $\det(A-\lambda I)=0$

$$\Rightarrow \begin{vmatrix}-\lambda & 1\\ a & b-\lambda\end{vmatrix}=0$$

$$\Rightarrow \lambda^2-b\lambda-a=0$$

$$\Rightarrow \lambda_1=\frac{b+\sqrt{b^2+4a}}{2},\ \lambda_2=\frac{b-\sqrt{b^2+4a}}{2}$$ are the eigen values of A.

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Step 2 of 2

But given eigen values are 4 and 7

So, we follow that $\frac{b+\sqrt{b^2+4a}}{2}=7,\ \frac{b-\sqrt{b^2+4a}}{2}=4$

Adding the respective sides of these equations, we get $b=11$

Further, substation gives $11+\sqrt{121+4a}=14, 11-\sqrt{121+4a}=8$

$$\Rightarrow \sqrt{121+4a}=3,$$

$$\Rightarrow 121+4a=9$$

$$\Rightarrow 4a=-112$$

$$\Rightarrow a=-28$$

Therefore, the required matrix is $\begin{bmatrix}0 & 1\\ -28 & 11\end{bmatrix}$

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CH5.1 30P

Step 1 of 2

Given that $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ a & b & c\end{bmatrix}$ and the eigen values are -3, 0, and 3

The characteristic equation of A is $\det(A-\lambda I)=\begin{vmatrix}-\lambda & 1 & 0\\ 0 & -\lambda & 1\\ a & b & c-\lambda\end{vmatrix}=0$

$$\Rightarrow -\lambda\begin{vmatrix}-\lambda & 1\\ b & c-\lambda\end{vmatrix}-1\begin{vmatrix}0 & 1\\ a & c-\lambda\end{vmatrix}=0$$

$$\Rightarrow -\lambda\left(\lambda^2-c\lambda-b\right)+a=0$$

$$\Rightarrow -\lambda^3+c\lambda^2+b\lambda+a=0$$

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CH5.1 32P

Step 2 of 2

On the other hand, the given characteristic equation is $9\lambda-\lambda^3$
Comparing both the above equations, we follow that $a=0,\ b=9$ and $c=0$

Therefore $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 9 & 0\end{bmatrix}$

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CH5.1 31P

Step 1 of 5

Let $e=(1,1,1)$.

Consider the following:

$$M^{\mathrm{T}}=\begin{bmatrix}\frac{1}{2} & \frac{1}{6} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{6} & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6}\end{bmatrix}$$

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CH5.1 33P

Step 2 of 5

Therefore,

$$\begin{aligned}M^{\mathrm{T}}e&=\begin{bmatrix}\frac{1}{2} & \frac{1}{6} & \frac{1}{3}\\ \frac{1}{3} & \frac{1}{6} & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6}\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}\\&=\begin{bmatrix}\frac{1}{2}+\frac{1}{6}+\frac{1}{3}\\ \frac{1}{3}+\frac{1}{6}+\frac{1}{2}\\ \frac{1}{3}+\frac{1}{2}+\frac{1}{6}\end{bmatrix}\end{aligned}$$

Step 2 of 5

Therefore,

$$
\begin{aligned}
M^{\mathrm{T}}e &= \begin{bmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{2}+\frac{1}{6}+\frac{1}{3} \\ \frac{1}{3}+\frac{1}{6}+\frac{1}{2} \\ \frac{1}{3}+\frac{1}{2}+\frac{1}{6} \end{bmatrix} \\
&= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
&= e
\end{aligned}
$$

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Step 3 of 5

Since, $M^{\mathrm{T}}e = e$, 1 is an eigenvalue of M^{T} .

But the eigenvalues of a matrix are same as the eigenvalues of its transpose.

Therefore, 1 is an eigenvalue of any Markov Matrix *M*.

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Step 3 of 5

Since, $M^{\mathrm{T}}e = e$, 1 is an eigenvalue of M^{T} .

But the eigenvalues of a matrix are same as the eigenvalues of its transpose.

Therefore, 1 is an eigenvalue of any Markov Matrix *M*.

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Step 4 of 5

Consider a 3 by 3 Markov Matrix, which satisfies the following conditions:

1. The matrix is singular

2. The trace of the matrix is $\frac{1}{2}$

Note that the matrix is a Markov Matrix and therefore, one of its eigenvalues should be 1.

Since, the matrix is singular, its determinant must be zero. Now the product of the eigenvalues is always equal to the determinant of the matrix. Thus, at least one of the eigenvalues must be zero.

Finally, the addition of the eigenvalues is equal to the trace of the matrix. Let $\lambda_1 = 0$, $\lambda_2 = 1$. Let the third eigenvalue be λ_3 . Thus, we have $0 + 1 + \lambda_3 = \frac{1}{2}$. This implies that $\lambda_3 = -\frac{1}{2}$.

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Step 4 of 5

Consider a 3 by 3 Markov Matrix, which satisfies the following conditions:

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Finally, the addition of the eigenvalues is equal to the trace of the matrix. Let $\lambda_1 = 0$, $\lambda_2 = 1$. Let the third eigenvalue be λ_3 . Thus, we have $0 + 1 + \lambda_3 = \frac{1}{2}$. This implies that $\lambda_3 = -\frac{1}{2}$.

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Step 5 of 5

Therefore, the three eigenvalues of the Markov matrix are $\boxed{0, 1, \text{and} -\frac{1}{2}}$.

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CH5.1 32P

CH5.1 34P

Step 1 of 5

Let $A=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, C=\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$

$$A^2=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$B^2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$C^2=\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Therefore $A^2=B^2=C^2=0$

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Step 2 of 5

For the eigen values of these matrices, we consider $|A-\lambda I|=0$

$$\begin{vmatrix}-\lambda & 0\\ 1 & -\lambda\end{vmatrix}=0$$
$$\Rightarrow \lambda^2=0$$
$$\Rightarrow \boxed{\lambda_A=0,0}$$

Similarly, we get the eigen values of B are 0, 0.

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$$\begin{vmatrix} 1 & -\lambda \end{vmatrix}$$

$$\Rightarrow \lambda^2 = 0$$

$$\Rightarrow \boxed{\lambda_A = 0, 0}$$

Similarly, we get the eigen values of B are 0, 0.

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Step 3 of 5

We consider $|C - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -1-\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow -(1+\lambda)(1-\lambda)+1 = 0$$

$$\Rightarrow -1-\lambda^2+1 = 0$$

$$\Rightarrow \lambda^2 = 0$$

$$\Rightarrow \boxed{\lambda_C = 0, 0}$$

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Step 4 of 5

Trace of A = sum of the diagonal entries = 0+ 0 =0
Trace of B = 0+0=0
Trace of C = $-1+1=0$

Also,
Determinant of A = Product of eigen value
$= (0)(0) = 0$
Determinant of B = Product of eigen value
$= (0)(0) = 0$
Determinant of C = Product of eigen value
$= (0, 0)$
$= 0$

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Step 4 of 5

Trace of A = sum of the diagonal entries = 0+ 0 =0
Trace of B = 0+0=0
Trace of C = −1+1 = 0

Also,
Determinant of A = Product of eigen value
$= (0)(0) = 0$
Determinant of B = Product of eigen value
$= (0)(0) = 0$
Determinant of C = Product of eigen value
$= (0,0)$
$= 0$

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Step 5 of 5

Thus, $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$ satisfy all the required conditions.

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CH5.1 35P

Step 1 of 5

Given that

$$A=\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 1 & 2\end{bmatrix}=\begin{bmatrix}2 & 1 & 2\\4 & 2 & 4\\2 & 1 & 2\end{bmatrix}$$

$$\det A = 2(4-4)-1(8-8)+2(4-4)$$
$$= 0$$

Therefore A is singular matrix.
And else all 2×2 sub matrices of A having determinant is 0.
Therefore the rank of A = 1.

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Step 2 of 5

For finding the eigen values λ, we consider $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}2-\lambda & 1 & 2\\4 & 2-\lambda & 4\\2 & 1 & 2-\lambda\end{vmatrix}$$

$$\Rightarrow (2-\lambda)\left\{(2-\lambda)^2-4\right\}-1\left\{4(2-\lambda)-8\right\}+2\left\{(4-2(2-\lambda))=0\right\}$$

$$\Rightarrow (2-\lambda)\left\{4+\lambda^2-4\lambda-4\right\}-\left\{8-4\lambda-8\right\}+2\left\{4-4+2\lambda\right\}=0$$

$$\Rightarrow 8+2\lambda^2-8\lambda-8-4\lambda-\lambda^3+4\lambda^2+4\lambda+4\lambda+4\lambda=0$$

$$\Rightarrow -\lambda^3+6\lambda^2=0$$

$$\lambda^3-6\lambda^2=0$$

$$\lambda^2(\lambda-6)=0$$

$\lambda = 0,0,6$ are the eigen values of the given matrix A.

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Step 3 of 5

Step 2 of 5

For finding the eigen values λ, we consider $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 2 \\ 4 & 2-\lambda & 4 \\ 2 & 1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow (2-\lambda)\left\{(2-\lambda)^2-4\right\}-1\{4(2-\lambda)-8\}+2\{(4-2(2-\lambda))=0\}$$

$$\Rightarrow (2-\lambda)\{4+\lambda^2-4\lambda-4\}-\{8-4\lambda-8\}+2\{4-4+2\lambda\}=0$$

$$\Rightarrow 8+2\lambda^2-8\lambda-8-4\lambda-\lambda^3+4\lambda^2+4\lambda+4\lambda+4\lambda=0$$

$$\Rightarrow -\lambda^3+6\lambda^2=0$$

$$\lambda^3-6\lambda^2=0$$

$$\lambda^2(\lambda-6)=0$$

$\lambda=0,0,6$ are the eigen values of the given matrix A.

Provide feedback (0)

Step 3 of 5

To find eigen vector for $\lambda=0$, we consider $(A-\lambda I)x=0$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix,

$R_2 \to R_2-2R_1, R_3 \to R_3-R_1$, we get $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

This is the reduced matrix and so, we rewrite the homogeneous equations from this.

$$2x_1+x_2+2x_3=0$$

Allowing x_1, x_3 to be free variables by putting $x_1=k_1, x_3=k_2$

$$2k_1+x_2+2k_2=0$$

$$x_2=-2k_1-2k_2$$

Step 3 of 5

To find eigen vector for $\lambda = 0$, we consider $(A - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix,

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$, we get $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

This is the reduced matrix and so, we rewrite the homogeneous equations from this.

$2x_1 + x_2 + 2x_3 = 0$

Allowing x_1, x_3 to be free variables by putting $x_1 = k_1, x_3 = k_2$

$2k_1 + x_2 + 2k_2 = 0$

$x_2 = -2k_1 - 2k_2$

So, the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

Therefore, the eigen vectors corresponding to $\lambda = 0$ are $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

Provide feedback (0)

Step 4 of 5

Similarly, to get the eigen vector corresponding to $\lambda = 6$, we solve $(A - \lambda I)x = 0$

i.e., $\begin{bmatrix} 2-6 & 1 & 2 \\ 4 & 2-6 & 4 \\ 2 & 1 & 2-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} -4 & 1 & 2 \\ 4 & -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

Similarly, to get the eigen vector corresponding to $\lambda = 6$, we solve $(A - \lambda I)x = 0$

i.e., $\begin{bmatrix} 2-6 & 1 & 2 \\ 4 & 2-6 & 4 \\ 2 & 1 & 2-6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -4 & 1 & 2 \\ 4 & -4 & 4 \\ 2 & 1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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Step 5 of 5

Applying the row operations on the coefficient matrix, $R_2 \to R_2 + R_1, R_3 \to 2R_3 + R_1$, we get $\begin{bmatrix} -4 & 1 & 2 \\ 0 & -3 & 6 \\ 0 & 3 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_3 \to R_3 + R_2, \frac{R_2}{-3} \Rightarrow \begin{bmatrix} -4 & 1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

This is the reduced matrix.

So, we rewrite the homogeneous equations from this.

$x_2 - 2x_3 = 0$

$-4x_1 + x_2 + 2x_3 = 0$

Using $x_2 = 2x_3$ in the 2nd equation, we get $x_1 = x_3$

So, putting $x_3 = 1$, the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 6$

CH5.1 34P

Step 1 of 2

Given

A And B have the same Eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ with the same independent Eigen vectors $x_1, x_2, \ldots, x_n$.

And any vector x is a combination $c_1x_1 + c_2x_2 + \ldots + c_nx_n$

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Step 2 of 2

Then

$Ax = c_1\lambda_1x_1 + \ldots\ldots + c_n\lambda_nx_n$ Equals $Bx = c_1\lambda_1x_1 + \ldots\ldots + c_n\lambda_nx_n$ for all x

So $\boxed{A = B}$

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CH5.1 37P

Step 1 of 9

Given $A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$ $B=\begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}$ and $C=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}$

Now
$$A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$$
$$\begin{aligned}A-\lambda I&=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\\&=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\\&=\begin{bmatrix}1-\lambda&2&3\\0&4-\lambda&5\\0&0&6-\lambda\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 2 of 9

Then
$$A-\lambda I=\begin{vmatrix}1-\lambda&2&3\\0&4-\lambda&5\\0&0&6-\lambda\end{vmatrix}$$
$$\begin{aligned}(1-\lambda)[(4-\lambda)(6-\lambda)]&=(1-\lambda)[24-4\lambda-6\lambda+\lambda^2]\\&=(1-\lambda)[\lambda^2-10\lambda+24]\\&=\lambda^2-10\lambda+24-\lambda^3+10\lambda^2-24\lambda\\&=-\lambda^3+11\lambda^2-34\lambda+24\end{aligned}$$

Step 2 of 9

Then

$$A-\lambda I=\begin{vmatrix}1-\lambda & 2 & 3\\ 0 & 4-\lambda & 5\\ 0 & 0 & 6-\lambda\end{vmatrix}$$

$$\begin{aligned}(1-\lambda)[(4-\lambda)(6-\lambda)] &= (1-\lambda)[24-4\lambda-6\lambda+\lambda^2]\\ &= (1-\lambda)[\lambda^2-10\lambda+24]\\ &= \lambda^2-10\lambda+24-\lambda^3+10\lambda^2-24\lambda\\ &= -\lambda^3+11\lambda^2-34\lambda+24\end{aligned}$$

Provide feedback (0)

Step 3 of 9

Now $A-\lambda I=0$

$\lambda^3-11\lambda^2+34\lambda-24=0$

$$\begin{array}{c|cccc} 1 & 1 & -11 & 34 & -24\\ & 0 & 1 & -10 & 24\\ \hline 4 & 1 & -10 & 24 & 0\\ & 0 & 4 & -24 & 0\\ \hline & 1 & -6 & 0 & \end{array}$$

The eigenvalues are

$(\lambda-1)(\lambda-4)(\lambda-6)=0$

Therefore the eigenvalues of A is 1, 4, and 6

Provide feedback (0)

Step 4 of 9

$$\text{Now } B=\begin{bmatrix}0 & 0 & 1\\ 0 & 2 & 0\\ 3 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 1\end{bmatrix} \quad \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

Step 4 of 9

$$\text{Now } B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} B - \lambda I &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} -\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 3 & 0 & -\lambda \end{bmatrix} \end{aligned}$$

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Step 5 of 9

$$\begin{aligned} |B - \lambda I| &= \begin{vmatrix} -\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 3 & 0 & -\lambda \end{vmatrix} \\ &= (-\lambda)(2-\lambda)(-\lambda) + 1(-(2-\lambda)3) \\ &= \lambda^2(2-\lambda) - 6 + 3\lambda \\ &= 2\lambda^2 - \lambda^3 - 6 + 3\lambda \\ &= -\lambda^3 + 2\lambda^2 + 3\lambda - 6 \end{aligned}$$

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Step 6 of 9

$$\text{Now} \qquad B - \lambda I = 0$$

Step 6 of 9

Now $B - \lambda I = 0$

$-\lambda^3 + 2\lambda^2 + 3\lambda - 6 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & 0 & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \\ \hline \end{array}$$

$(\lambda - 2)(\lambda^2 - 3) = 0$

The Eigen values are $\boxed{2, \pm\sqrt{3}}$

Therefore the eigenvalues of B is $2, \pm\sqrt{3}$

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Step 7 of 9

Now $C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

$$C - \lambda I = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{bmatrix}$$

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Step 8 of 9

Now

Step 8 of 9

Now

$$C-\lambda I=\begin{vmatrix}2-\lambda & 2 & 2\\ 2 & 2-\lambda & 2\\ 2 & 2 & 2-\lambda\end{vmatrix}$$

$$\begin{aligned}&=(2-\lambda)[(2-\lambda)(2-\lambda)-4]-2[2(2-\lambda)-4]+2[4-2(2-\lambda)]\\&=(2-\lambda)[4-2\lambda-2\lambda+\lambda^2-4]-2[4-2\lambda-4]+2[4-4+2\lambda]\\&=(2-\lambda)[\lambda^2-4\lambda]-2[-2\lambda]+2[2\lambda]\\&=2\lambda^2-8\lambda-\lambda^3+4\lambda^2+4\lambda+4\lambda\\&=-\lambda^3+6\lambda^2\end{aligned}$$

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Step 9 of 9

Then

$$\lambda^3-6\lambda^2=0$$

$$\lambda(\lambda^2-6\lambda)=0$$

$$\lambda=0,\lambda^2-6\lambda=0$$

$$\lambda(\lambda-6)=0$$

$$\lambda=0,6$$

The Eigen values are 0, 0, and 6

Therefore the eigenvalues of C are 0, 0, and 6

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Step 1 of 4

Given that $a+b=c+d$

$Ax=\lambda x$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} a+b \\ c+d \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} a+b \\ a+b \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow(a+b)\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore $\lambda=a+b$ is one eigen value.

Trace of $A=\lambda_1+\lambda_2$

$a+d=(a+b)+\lambda_2$

$\lambda_2=a+d-a-b$

$\quad=d-b$

Therefore the eigen values are $\lambda_1=a+b, \lambda_2=d-b$

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Step 2 of 4

To find eigen vector for $\lambda_1=a+b$, we solve $(A-\lambda_1 I)x=0$

$$\begin{bmatrix} a-(a+b) & b \\ c & d-(a+b) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -b & b \\ c & d-(c+d) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ by replacing $a+b$ with $c+d$.

$$\begin{bmatrix} -b & b \\ c & -c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

Step 2 of 4

To find eigen vector for $\lambda_1 = a+b$, we solve $(A-\lambda_1 I)x = 0$

$$\begin{bmatrix} a-(a+b) & b \\ c & d-(a+b) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -b & b \\ c & d-(c+d) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ by replacing $a+b$ with $c+d$.

$$\begin{bmatrix} -b & b \\ c & -c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

$$R_2 \to bR_2 + cR_1, R_1/-b \Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$x_1 - x_2 = 0$

Putting $x_1 = 1$, the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_1 = a+b$

Provide feedback (0)

Step 3 of 4

Similarly, To find eigen vector for $\lambda_2 = d-b$, we solve $(A-\lambda_2 I)x = 0$

$$\begin{bmatrix} a-(d-b) & b \\ c & d-(d-b) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (a+b)-d & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (c+d)-d & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ by replacing $c+d$ with $a+b$.

$$\begin{bmatrix} c & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we

Provide feedback (0)

Step 3 of 4

Similarly, To find eigen vector for $\lambda_2 = d - b$, we solve $(A - \lambda_2 I)x = 0$

$$\begin{bmatrix} a-(d-b) & b \\ c & d-(d-b) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (a+b)-d & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (c+d)-d & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ by replacing $c+d$ with $a+b$.

$$\begin{bmatrix} c & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

$$R_2 \to R_2 - R_1 \Rightarrow \begin{bmatrix} c & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$cx_1 + bx_2 = 0$

Putting $x_1 = 1$, the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -b/c \end{bmatrix}$ is the eigen vector corresponding to $\lambda_2 = d - b$

Provide feedback (0)

Step 4 of 4

Thus the eigen values are $\lambda_1 = a + b$ and $\lambda_2 = d - b$ while the respective eigen vectors are $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -b/c \end{bmatrix}$

Provide feedback (0)

CH5.1 37P

Step 1 of 5

The eigen values of $A=\begin{bmatrix}1&2&1\\3&6&3\\4&8&4\end{bmatrix}$ are $|A-\lambda I|=0$

$$\begin{vmatrix}1-\lambda&2&1\\3&6-\lambda&3\\1&8&4-\lambda\end{vmatrix}=0$$

$$\Rightarrow -\lambda^3+11\lambda^2=0$$

$$\Rightarrow \lambda^2(-\lambda+11)=0$$

$$\Rightarrow \lambda=0,0,11$$

Provide feedback (0)

Step 2 of 5

If exchange row1 and 2 and columns1 and 2 of A, then we get

$$PAP=\begin{bmatrix}6&3&3\\2&1&1\\8&4&4\end{bmatrix}$$

The eigen values are

$$\begin{vmatrix}6-\lambda&3&3\\2&1-\lambda&1\\8&4&4-\lambda\end{vmatrix}=0$$

$$\Rightarrow \lambda^3-11\lambda^2=0$$

$$\Rightarrow \lambda^2(\lambda-11)=0$$

$$\Rightarrow \lambda=0,0,11$$

Provide feedback (0)

Step 3 of 5

CH5.1 39P

$\Rightarrow \lambda^2(\lambda-11)=0$

$\Rightarrow \lambda = 0,0,11$

Provide feedback (0)

Step 3 of 5

Therefore when P exchange rows 1 and 2, columns 1 and 2, the eigen vector of A for $\lambda = 11$

Taking $(A-\lambda I)x=0$,

$$\begin{bmatrix} 1-11 & 2 & 1 \\ 3 & 6-11 & 3 \\ 4 & 8 & 4-11 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -10 & 2 & 1 \\ 3 & -5 & 3 \\ 4 & 8 & -7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

Applying the row operations on this,

$$R_2 \to 10R_2+3R_1, R_3 \to 10R_3+4R_1 \Rightarrow \begin{bmatrix} -10 & 2 & 1 \\ 0 & -44 & 33 \\ 0 & 88 & -66 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \to R_3+2R_2, R_2/-11 \Rightarrow \begin{bmatrix} -10 & 2 & 1 \\ 0 & 4 & -3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewriting the homogeneous equations, we get

$4x_2 - 3x_3 = 0$

$10x_1 - 2x_2 - x_3 = 0$

$x_2 = \frac{3}{4}x_3$ from the first equation

Step 4 of 5

Applying the row operations on this,

$$R_2 \to 10R_2 + 3R_1, R_3 \to 10R_3 + 4R_1 \Rightarrow \begin{bmatrix} -10 & 2 & 1 \\ 0 & -44 & 33 \\ 0 & 88 & -66 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \to R_3 + 2R_2, R_2 / -11 \Rightarrow \begin{bmatrix} -10 & 2 & 1 \\ 0 & 4 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewriting the homogeneous equations, we get

$4x_2 - 3x_3 = 0$

$10x_1 - 2x_2 - x_3 = 0$

$x_2 = \frac{3}{4}x_3$ from the first equation

Using this in the second equation, we get $x_1 = \frac{1}{4}x_3$

Putting $x_3 = 4$, the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ is the eigen vector corresponding to the eigen value 11.

Provide feedback (0)

Step 5 of 5

Similarly, the eigen vector corresponding to the eigen value 11 for the matrix PAP is also

$$\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

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CH5.1 38P

Step 1 of 2

Yes, there exist a real 2×2 matrix other than I satisfies the given conditions.

Let us take $A=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}$ is a real 2×2 matrix.

$$A^2=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1\\ 1 & -1\end{bmatrix}$$

$$A^3=\begin{bmatrix}0 & -1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}=I$$

Therefore $A^3=I$

To find the eigen values let us take

$$|A-\lambda I|=0$$

$$\Rightarrow\begin{vmatrix}-1-\lambda & 1\\ -1 & -\lambda\end{vmatrix}=0$$

$$\Rightarrow\left(-(1-\lambda)\right)(-\lambda)+1=0$$

$$\Rightarrow\lambda+\lambda^2+1=0$$

$$\Rightarrow\lambda^2+\lambda+1=0$$

$$\Rightarrow\lambda=\frac{-1\pm\sqrt{1-4}}{2}$$

$$=\frac{-1\pm i\sqrt{3}}{2}$$

These are cube root of unity therefore

$$\lambda^3=1$$

$$\lambda_1=\frac{-1+i\sqrt{3}}{2},\lambda_2=\frac{-1-i\sqrt{3}}{2}$$

$$=e^{\frac{2\pi i}{3}},e^{\frac{-2\pi i}{3}}$$

CH5.1 40P

$$\Rightarrow \lambda^2+\lambda+1=0$$

$$\Rightarrow \lambda=\frac{-1\pm\sqrt{1-4}}{2}$$

$$=\frac{-1\pm i\sqrt{3}}{2}$$

These are cube root of unity therefore

$$\lambda^3=1$$

$$\lambda_1=\frac{-1+i\sqrt{3}}{2},\lambda_2=\frac{-1-i\sqrt{3}}{2}$$

$$=e^{\frac{2\pi i}{3}},e^{\frac{-2\pi i}{3}}$$

Provide feedback (0)

Step 2 of 2

The trace of $A=\lambda_1+\lambda_2$

$$=e^{\frac{2\pi i}{3}}+e^{\frac{-2\pi i}{3}}$$

$$=\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)+\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$$

$$=2\cos\frac{2\pi}{3}$$

$$=2\left(\frac{-1}{2}\right)$$

$$=-1$$

The determinant of $A=\lambda_1\lambda_2$

$$=e^{\frac{2\pi i}{3}}.e^{\frac{-2\pi i}{3}}$$

$$=e^0$$

$$=\boxed{1}$$

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CH5.1 39P

Step 1 of 6

Trace: Sum along the main diagonal is called the trace of the matrix.

Let following be the permutation matrix P:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

There can be six 3 by 3 permutation matrices of P. Let $(\lambda_1, \lambda_2, \lambda_3)$ be the eigen values.

Provide feedback (0)

Step 2 of 6

Loading solution. Please wait ...

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Step 3 of 6

The determinants of P will be product of the eigen values:

$$\begin{aligned} \lambda_1\lambda_2\lambda_3 &= 1 \cdot e^{\frac{2\pi i}{3}} \cdot e^{\frac{-2\pi i}{3}} \\ &= 1 \end{aligned}$$

Therefore, determinant of P is $\boxed{1}$.

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Step 4 of 6

Recall that starting element of a matrix to do certain calculations is called as pivot element

CH5.2

Step 2 of 6

Recall that characteristic equation is always put equal to zero to get the eigen values. This means that determinant value of $P-\lambda I$ must be equal to zero.

$$\det(P-\lambda I)=0$$

This gives the equation:

$$\lambda^3=1$$

The solutions of this equation are as follows:

$$\lambda_1=1$$
$$\lambda_2=e^{\frac{2\pi i}{3}}$$
$$\lambda_3=e^{\frac{-2\pi i}{3}}$$

Here, only one value is real and others are complex conjugates.

Provide feedback (0)

Step 3 of 6

The determinants of P will be product of the eigen values:

$$\begin{aligned}\lambda_1\lambda_2\lambda_3&=1\cdot e^{\frac{2\pi i}{3}}\cdot e^{\frac{-2\pi i}{3}}\\&=1\end{aligned}$$

Therefore, determinant of P is $\boxed{1}$.

Provide feedback (0)

Step 4 of 6

Recall that starting element of a matrix to do certain calculations is called as pivot

Therefore, determinant of P is $\boxed{1}$.

Provide feedback (0)

Step 4 of 6

Recall that starting element of a matrix to do certain calculations is called as pivot element.

Therefore, pivot element of P will be $\boxed{1}$.

Provide feedback (0)

Step 5 of 6

Trace of P will be the sum of all the eigen values:

$$\begin{aligned}(\lambda_1+\lambda_2+\lambda_3) &= 1+e^{\frac{2\pi i}{3}}+e^{\frac{-2\pi i}{3}} \\ &= 1+\cos\left(\frac{2\pi i}{3}\right)+i\sin\left(\frac{2\pi i}{3}\right)+\cos\left(\frac{2\pi i}{3}\right)-i\sin\left(\frac{2\pi i}{3}\right) \\ &= 0\end{aligned}$$

Therefore, trace of P is $\boxed{0}$.

Provide feedback (0)

Step 6 of 6

Three numbers that can be eigen values of P will be as follows:

$$\begin{aligned}\lambda_1 &= 1 \\ \lambda_2 &= e^{\frac{2\pi i}{3}} \\ \lambda_3 &= e^{\frac{-2\pi i}{3}}\end{aligned}$$

Provide feedback (0)

Step 6 of 6

Three numbers that can be eigen values of P will be as follows:

$$\lambda_1 = 1$$
$$\lambda_2 = e^{\frac{2\pi i}{3}}$$
$$\lambda_3 = e^{\frac{-2\pi i}{3}}$$

Therefore, eigen values will be $\boxed{\left(1, e^{\frac{2\pi i}{3}}, e^{\frac{-2\pi i}{3}}\right)}$.

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