CH4.2

Step 1 of 3

Given matrices are

$$A=\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&0\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}0&0&1&2\\0&3&4&5\\6&7&8&9\\0&0&0&1\end{bmatrix}$$

The big formula is

$$\det A=\sum_{\text{all } p\text{'s}}\left(a_{1\alpha}a_{\beta}\ldots\ldots a_{n}\gamma\right)\det p$$

Provide feedback (0)

Step 2 of 3

Here

$$\det A=\det\begin{vmatrix}0&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&0\end{vmatrix}$$

Term is $(-1)^2 a_{12}a_{21}a_{34}a_{43}$ as there are exactly two interchanges in permutations of 1, 2 and 3, 4.

An the value of $\det A=(-1)^2.1.1.1.1=\boxed{1}$

Provide feedback (0)

Step 3 of 3

And in $\det B=\det\begin{vmatrix}0&0&1&2\\0&3&4&5\\6&7&8&9\\0&0&0&1\end{vmatrix}$, the only non zero term is $(-1)\,b_{13}b_{22}b_{31}b_{44}$ as exactly one

CH4.3 2P

CH4.2

Step 1 of 3

Given matrices are

$$A=\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&0\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}0&0&1&2\\0&3&4&5\\6&7&8&9\\0&0&0&1\end{bmatrix}$$

The big formula is

$$\det A=\sum_{\text{all } p\text{'s}}\left(a_{1\alpha}a_{\beta}\ldots\ldots a_{n}\gamma\right)\det p$$

Provide feedback (0)

Step 2 of 3

Here

$$\det A=\det\begin{vmatrix}0&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&0\end{vmatrix}$$

Term is $(-1)^2 a_{12}a_{21}a_{34}a_{43}$ as there are exactly two interchanges in permutations of 1, 2 and 3, 4.

An the value of $\det A=(-1)^2.1.1.1.1=\boxed{1}$

Provide feedback (0)

Step 3 of 3

And in $\det B=\det\begin{vmatrix}0&0&1&2\\0&3&4&5\\6&7&8&9\\0&0&0&1\end{vmatrix}$, the only non zero term is $(-1)\,b_{13}b_{22}b_{31}b_{44}$ as exactly one

CH4.3 2P

$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$

The big formula is

$$\det A = \sum_{\text{all p's}} \left(a_{1\alpha} a_{\beta} \ldots\ldots a_{n} \gamma\right) \det p$$

Provide feedback (0)

Step 2 of 3

Here

$$\det A = \det \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

Term is $(-1)^2 a_{12} a_{21} a_{34} a_{43}$ as there are exactly two interchanges in permutations of 1, 2 and 3, 4.

An the value of $\det A = (-1)^2 .1.1.1.1 = \boxed{1}$

Provide feedback (0)

Step 3 of 3

And in $\det B = \det \begin{vmatrix} 0 & 0 & 1 & 2 \\ 0 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}$, the only non zero term is $(-1) b_{13} b_{22} b_{31} b_{44}$ as exactly one interchange of 1, 3 is involved in the permutation and the value of

$$\det B = (-1).1.3.6.1$$
$$= \boxed{-18}$$

Provide feedback (0)

CH4.3 1P

Step 1 of 7

Given

$$A=\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&1&0&1\\0&0&1&0\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}0&0&1&2\\0&3&4&5\\6&7&8&9\\0&0&0&1\end{bmatrix}$$

Provide feedback (0)

Step 2 of 7

First let us find the cofactors of 1, the elements in first row of A, that is

$$\begin{aligned}\text{Cofactor of } a_{11}=A_{11}&=\begin{vmatrix}0&1&0\\1&0&1\\0&1&0\end{vmatrix}\\&=0\\\text{Cofactor of } a_{12}=A_{12}&=-\begin{vmatrix}1&1&0\\0&0&1\\0&1&0\end{vmatrix}\\&=-(0-1)\\&=1\end{aligned}$$

Provide feedback (0)

Step 3 of 7

$$\begin{aligned}\text{Cofactor of } a_{13}=A_{13}&=\begin{vmatrix}1&0&0\\0&1&1\\0&0&0\end{vmatrix}\\&=0\\\text{Cofactor of } a_{14}=A_{14}&=-\begin{vmatrix}1&0&1\\0&1&0\end{vmatrix}\end{aligned}$$

CH4.3 3P

Step 3 of 7

$$\text{Cofactor of } a_{13} = A_{13} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$
$$= 0$$
$$\text{Cofactor of } a_{14} = A_{14} = -\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= \boxed{-1}$$

Provide feedback (0)

Step 4 of 7

Now determinant of A is

$$\det A = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} + a_{14}A_{14}$$
$$= 0(0) + 1(1) + 0(0) + 0(-1)$$
$$= \boxed{1}$$

Provide feedback (0)

Step 5 of 7

(B)

The elements in first row of B,

$$\text{Cofactor of } b_{11} = B_{11} = \begin{vmatrix} 3 & 4 & 5 \\ 7 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= 3(8) - 4(7) + 5(0)$$
$$= 24 - 28$$
$$= -4$$

The elements in first row of B,

$$\text{Cofactor of } b_{11} = B_{11} = \begin{vmatrix} 3 & 4 & 5 \\ 7 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= 3(8) - 4(7) + 5(0)$$
$$= 24 - 28$$
$$= -4$$

$$\text{Cofactor of } b_{12} = B_{12} = -\begin{vmatrix} 0 & 4 & 5 \\ 6 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= -\left[0(8-0) - 4(6-0) + 5(0)\right]$$
$$= 24$$

Provide feedback (0)

Step 6 of 7

$$\text{Cofactor of } b_{13} = B_{13} = \begin{vmatrix} 0 & 3 & 5 \\ 6 & 7 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= 0(7-0) - 3(6-0) + 5(0-0)$$
$$= -18$$

$$\text{Cofactor of } b_{14} = B_{14} = -\begin{vmatrix} 0 & 3 & 4 \\ 6 & 7 & 8 \\ 0 & 0 & 0 \end{vmatrix}$$
$$= 0$$

Provide feedback (0)

Step 7 of 7

Now, determinant of B is

$\det B = b_{11}B_{11} + b_{12}B_{12} + b_{13}B_{13} + b_{14}B_{14}$

$$\text{Cofactor of } b_{13} = B_{13} = \begin{vmatrix} 0 & 3 & 5 \\ 6 & 7 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= 0(7-0) - 3(6-0) + 5(0-0)$$
$$= -18$$
$$\text{Cofactor of } b_{14} = B_{14} = -\begin{vmatrix} 0 & 3 & 4 \\ 6 & 7 & 8 \\ 0 & 0 & 0 \end{vmatrix}$$
$$= 0$$

Provide feedback (0)

Step 7 of 7

Now, determinant of B is

$$\det B = b_{11}B_{11} + b_{12}B_{12} + b_{13}B_{13} + b_{14}B_{14}$$
$$= 0(-4) + 0(24) + 1(-18) + 2(0)$$
$$= \boxed{-18}$$

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CH4.3 2P

Step 1 of 4

a) Given statement
"The determinant of $S^{-1}AS$ equals to determinant of A" is **true**.

Reason: $\det\left(S^{-1}AS\right) = \det\left(S^{-1}\right)\det A \det S$

$$= \frac{1}{\det S}.\det A.\det S$$

$$= \det A$$

Is always true for any non singular matrix S
Thus, the determinant of $S^{-1}AS$ is same as the determinant of A

Provide feedback (0)

Step 2 of 4

b) Given statement
"If $\det A = 0$ then atleast on the cofactors must be zero" is **false**

Consider $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ so that $\det A = 0$

But the cofactors of all the elements of A are either 1 or -1
Hence the given statement is false.

Provide feedback (0)

Step 3 of 4

c) Given statement
"A matrix whose entries are os and 1s has determinant $1, 0,$ or -1" Is **false**

Reason: The determinant of a matrix whose entries are os and 1s need not always be $1, 0,$ or -1

Consider $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

CH4.3 4P

b) Given statement
"If $\det A = 0$ then atleast on the cofactors must be zero" is **false**

Consider $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ so that $\det A = 0$

But the cofactors of all the elements of A are either 1 or -1
Hence the given statement is false.

Provide feedback (0)

Step 3 of 4

c) Given statement
"A matrix whose entries are os and 1s has determinant $1, 0,$ or -1" Is **false**

Reason: The determinant of a matrix whose entries are os and 1s need not always be $1, 0,$ or -1

Consider $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

Provide feedback (0)

Step 4 of 4

$$\det A = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$$
$$= -1 - 1$$
$$= -2$$
$$\neq 0, 1, -1$$

Hence the given statement is false.

Provide feedback (0)

CH4.3 3P

Step 1 of 5

A) determinant of 4 by 4 matrix whose entries are a_{ij} = smaller of i and j

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

And

$$A = LU \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The pivots are $\boxed{1,1,1,1}$ and $\det A = \boxed{1}$

Provide feedback (0)

CH4.3 5P

Step 2 of 5

b) Given $n_1 = 2, n_2 = 6, n_3 = 8, n_4 = 10$

a_{ij} = smaller of n_i nad n_j

$$\text{Here } A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 2 & 6 & 8 & 10 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 2 & 6 & 8 & 10 \end{vmatrix}$$

Provide feedback (0)

Provide feedback (0)

Step 3 of 5

Now

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the third row to the fourth}$$

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the second row to the third}$$

Provide feedback (0)

Step 4 of 5

Then

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the first row to the second}$$

$$= (2)(4)(2)(2)$$

$$= \boxed{32}$$

Provide feedback (0)

Step 5 of 5

We can give a general rule for any $n_1 \le n_2 \le n_3 \le n_4$ as $\det A$ for A defined as above I s given by

$\det A = n_1(n_2 - n_1)(n_3 - n_2)(n_4 - n_3)$

$$\begin{vmatrix} 0 & 0 & 0 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the second row to the third}$$

Provide feedback (0)

Step 4 of 5

Then

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the first row to the second}$$

$$= (2)(4)(2)(2)$$

$$= \boxed{32}$$

Provide feedback (0)

Step 5 of 5

We can give a general rule for any $n_1 \le n_2 \le n_3 \le n_4$ as $\det A$ for A defined as above I s given by

$$\det A = n_1(n_2 - n_1)(n_3 - n_2)(n_4 - n_3)$$

Provide feedback (0)

Algebra tutors who can help right now

Michael B. Scotty G.

Step 1 of 6

First let us observe F_n for $n = 1, 2, 3, 4, 5$

Where

$$F_n = \det\begin{vmatrix} 1 & -1 & & & \\ 1 & 1 & -1 & & \\ & 1 & 1 & -1 & \\ & & \cdot & \cdot & \cdot \\ & & & 1 & 1 \end{vmatrix}$$

$$F_1 = \det[1]$$
$$= 1$$

Provide feedback (0)

Step 2 of 6

Now

$$F_2 = \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$= 1+1$$
$$= 2$$

Provide feedback (0)

Step 3 of 6

Then

$$F_3 = \det\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$= \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + \det\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$
$$= F_2 + \det[1] \text{ Expanding } 2^{\text{nd}} \text{ determinant by } 1^{\text{st}} \text{ column}$$

Step 3 of 6

Then

$$
\begin{aligned}
F_3 &= \det\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \\
&= \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + \det\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \\
&= F_2 + \det[1] \text{ Expanding } 2^{\text{nd}} \text{ determinant by } 1^{\text{st}} \text{ column} \\
&= F_2 + F_1 \\
&= 2+1 \\
&= 3
\end{aligned}
$$

Provide feedback (0)

Step 4 of 6

And

$$
\begin{aligned}
F_4 &= \det\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \\
&= \det\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} + \det\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \\
&= F_3 + \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ Expanding } 2^{\text{nd}} \text{ determinant by } 1^{\text{st}} \text{ column} \\
&= F_3 + F_2 \\
&= 3+2 \\
&= 5
\end{aligned}
$$

Provide feedback (0)

$= 5$

Provide feedback (0)

Step 5 of 6

Then

$F_3 = F_2 + F_1$

$F_4 = F_3 + F_2$

...

...

$F_n = F_{n-1} + F_{n-2}$

This yields the Fibonacci sequence 1, 2, 3, 5, 8, 13...

Provide feedback (0)

Step 6 of 6

In general for any positive integer n , we have

$$F_n = \det \begin{vmatrix} 1 & -1 & & & & \\ 1 & 1 & -1 & & & \\ & 1 & 1 & -1 & & \\ & & \cdot & \cdot & \cdot & \cdot \\ & & & & 1 & 1 \end{vmatrix}$$

$= 1.\text{cofactor of } 1 + (-1)\text{cofactor of } (-1)$

$= 1.\text{minor of } 1 + \text{minor of } (1)$

$= 1 + 12 + 18 - 6 - 4 - 9$

$= 12$

Here rows of A are independent.

Provide feedback (0)

CH4.3 5P

CH4.3 7P

Step 1 of 4

Consider the following tridiagonal matrices,

$$A_1 = [1]$$

$$A_2 = \begin{bmatrix} 1 & \\ 1 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Consider

$$A_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Cofactor of a_{11} is given by,

$$D_4 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

Cofactor of a_{11} is given by,

$$
\begin{aligned}
D_4 &= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \\
&= 0 - 1 \\
&= -1
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
D_3 &= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} \\
&= 0 - 0 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
D_2 &= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} \\
&= 1 - 0 \\
&= 1
\end{aligned}
$$

Therefore, we get,

$$D_4 = D_3 - D_2$$

Thus

Thus,

$$D_n = D_{n-1} - D_{n-2}$$

Provide feedback (0)

Step 2 of 4

Let

$$D_1 = 1$$
$$D_2 = 0$$

By using $D_n = D_{n-1} - D_{n-2}$, we get,

$$\begin{aligned} D_3 &= D_2 - D_1 \\ &= 0 - 1 \\ &= -1 \\ D_4 &= D_3 - D_2 \\ &= -1 - 0 \\ &= -1 \\ D_5 &= D_4 - D_3 \\ &= -1 - (-1) \\ &= 0 \\ D_6 &= D_5 - D_4 \\ &= 0 - (-1) \\ &= 1 \end{aligned}$$

Provide feedback (0)

Step 3 of 4

$$D_7 = D_6 - D_5$$

Provide feedback (0)

Step 3 of 4

$$\begin{aligned} D_7 &= D_6 - D_5 \\ &= 1 - 0 \\ &= 1 \\ D_8 &= D_7 - D_6 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Thus,

$$\begin{aligned} D_2 &= 0 \\ D_3 &= -1 \\ D_4 &= -1 \\ D_5 &= 0 \\ D_6 &= 1 \\ D_7 &= 1 \\ D_8 &= 0 \\ D_{1000} &= -1 \end{aligned}$$

Provide feedback (0)

Step 4 of 4

Thus, $\boxed{D_{1000} = -1}$.

Provide feedback (0)

CH4.3 6P

CH4.3 8P

Step 1 of 4

Given matrix $\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix}$

So, consider

$$\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} - 4\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} + 4\begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Provide feedback (0)

Step 2 of 4

On solving

$$= 4\left[2(2)+1(-1)\right]-4\left[1(2)+(-1)\right]+4\left[(-2)-2(2)+2\right]-4\left[0-1-2(-1)\right]$$

$$= 4(3)-4(1)+4(-4)-4(1)$$

$$= 4(3-1-4-1)$$

$$= 4(-3)$$

$$= \boxed{-12}$$

Provide feedback (0)

Step 3 of 4

b) We recomputed the same determinant by subtracting 1st column from other

On solving

$$= 4\left[2(2)+1(-1)\right]-4\left[1(2)+(-1)\right]+4\left[(-2)-2(2)+2\right]-4\left[0-1-2(-1)\right]$$
$$= 4(3)-4(1)+4(-4)-4(1)$$
$$= 4(3-1-4-1)$$
$$= 4(-3)$$
$$= \boxed{-12}$$

Provide feedback (0)

Step 3 of 4

b) We recomputed the same determinant by subtracting 1st column from other columns.

$$\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 2 & -2 & -1 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix} \text{ subtracting 1st column from remaining columns}$$

$$= 4\begin{vmatrix} 1 & -1 & 0 \\ -2 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

Provide feedback (0)

Step 4 of 4

On solving

$$= 4\begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} \text{expanding by last columns}$$
$$= 4(-1-2)$$
$$= \boxed{-12}$$

Provide feedback (0)

CH4.3 7P

Step 1 of 8

Given

$$A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Then

$$\begin{aligned} \det(A_2) &= 0 - 1 \\ &= -1 \end{aligned}$$

Provide feedback (0)

Step 2 of 8

And

$$A_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Then

$$\begin{aligned} \det(A_3) &= (-1)\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ &= (-1)(-1) + 1(1) \\ &= 2 \end{aligned}$$

Provide feedback (0)

Step 3 of 8

Then

$$A_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \text{ Then}$$

$$\begin{vmatrix} 0 & 1 & 0 & 0 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$$

CH4.3 9P

Step 3 of 8

Then

$$A_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \text{ Then}$$

$$\begin{aligned} \det(A_4) = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{vmatrix} &= -\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} \\ &= -\begin{vmatrix} 1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & -1 & -2 \end{vmatrix} \\ &= -(4-1) \\ &= -3 \end{aligned}$$

Provide feedback (0)

Step 4 of 8

Now

$$\begin{aligned} -3\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} &= -3\left[\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}\right] \\ &= -3[-1+1+1] \\ &= 3 \\ &= -3\left[\det(A_2) + \det(A_3)\right] \end{aligned}$$

Provide feedback (0)

Provide feedback (0)

Step 5 of 8

And

$$A_5 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Expanding by 1st row

$$\det(A_5) = -\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

Provide feedback (0)

Step 6 of 8

Observe that the determinants on R.HS. Are each formed by one interchange of rows from the previous one so we can see that

$$\det(A_5) = -4\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$\left(\begin{array}{l} \text{if we call the determinants on R.H.S as } d_1, d_2, d_3, d_4 \text{ then } d_2 = -d_1, \\ d_3 = d_1, d_4 = -d_1 \end{array}\right)$$

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Step 7 of 8

$$\begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$\left(\begin{array}{l}\text{if we call the determinants on R.H.S as } d_1, d_2, d_3, d_4 \text{ then } d_2 = -d_1, \\ d_3 = d_1, d_4 = -d_1\end{array}\right)$$

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Step 7 of 8

On solving

$$= -4\left[\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}\right] \quad \text{(working on similar lines)}$$

$$= -4\left[\det(A_3) + \det(A_4)\right]$$

$$= -4(2-3)$$

$$= 4$$

So, in general we can predict $\boxed{\det(A_n) = (-1)^{n-1}(n-1)}$

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Step 8 of 8

$$\left(\text{so we get } \det A_2 = -(2-1) = -1\right)$$

$$\det(A_3) = (3-1)$$

$$= 2$$

$$\det(A_4) = -(4-1)$$

$$= -3$$

$$\det(A_5) = 5-1$$

$$= 4 \text{ etc}$$

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CH4.3 8P

Step 1 of 5

a) Big Formula

$$\det A = \sum_{all\,P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$$

The multiplication required in the big formula is n! Each term $(n-1)$

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Step 2 of 5

b) The cofactor formula

$$\det A = a_{i1}C_{i1} + a_{i2}C_{i2} + \ldots + a_{in}C_{in}$$

Also required $(n!+n)$ multiplications because cofactor is calculated with $(n-1)!$ multiplications and then we have to multiply these cofactors of a row with elements of the respective row to obtain determinants

So, we needed

$(n-1)!+1$ Multiplications to find each term of $\det A$ in this way

So, totally we require $n\left((n-1)!+1\right) = n!+n$ multiplications.

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Step 3 of 5

c) When we are reducing a given matrix to upper triangular form use need atleast $(n-1)+(n-2)+\ldots+1$ steps i.e. $\frac{n(n-1)}{2}$ and then we can get determinant by multiplication of pivots.

Each elimination step while reducing first row require n multiplications.

So, total $n(n-1)$ multiplications and for second row we need $(n-1)(n-2)$ steps and finally in last step we need 2 multiplications.

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CH4.3 10P

Each elimination step while reducing first row require n multiplications.

So, total $n(n-1)$ multiplications and for second row we need $(n-1)(n-2)$ steps and finally in last step we need 2 multiplications.

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Step 4 of 5

So, total steps required $\sum_{k=2}^{n} k(k-1)$

$$= \sum_{k=2}^{n} (k^2 - k)$$

$$= \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k$$

$(\text{for } k = 1 \text{, we have } k^2 = 1)$

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Step 5 of 5

On solving

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(\frac{2n+1}{3} - 1\right)$$

$$= \frac{n(n+1)(2n-2)}{6}$$

$$= \boxed{\frac{n(n^2-1)}{3}}$$

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CH4.3 9P

Step 1 of 1

Given

$p = (5,4,3,2,1)$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$ is an even permutation

Hence $a_{15}a_{24}a_{33}a_{42}a_{51}$ goes with a +ve sign in determinant of 5×5 matrix (a_{ij})

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CH4.3 11P

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CH4.3 10P

CH4.3 12P

Step 1 of 3

Consider matrices A and B, here A is m by n and B is n by m.

We have to show that $\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} = \det(AB)$.

We know that $\det\begin{bmatrix} I & 0 \\ B & I \end{bmatrix} = 1$.

We will compute the left hand side first

$$\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} \cdot 1 = \det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} \det\begin{bmatrix} I & 0 \\ B & I \end{bmatrix}$$

We know that the determinant of AB is the product of det A times det B, so we have:

$$\begin{aligned} \det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} \cdot 1 &= \det\left(\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix}\begin{bmatrix} I & 0 \\ B & I \end{bmatrix}\right) \\ &= \det\begin{bmatrix} AB & A \\ 0 & I \end{bmatrix} \\ &= \det(AB) \end{aligned}$$

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Step 2 of 3

Step 2 of 3

Therefore, $\boxed{\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} = \det(AB)}$.

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Step 3 of 3

Let us take $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

So we get

$$\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} = 5 = \det(AB)$$

And if we take $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$.

Then we get

$$\begin{aligned} \det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} &= 0 \\ &= \det(AB) \end{aligned}$$

CH4.3 11P

CH4.3 13P

Step 1 of 4

Let A be a 2×2 matrix.

Let $A=\begin{bmatrix} a_{11} & b \\ c & d \end{bmatrix}$, where $b\neq 0$, $c\neq 0$ and $d=0$.

We have,

$$\begin{aligned}\det(A) &= (a_{11}d)-(bc)\\ &= 0-bc\\ &= -bc\\ &\neq 0\end{aligned}$$

Therefore, in such case, whatever is the value of a_{11}, determinant of A cannot be zero.

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Step 2 of 4

Let A be a 2×2 matrix.

Let $A=\begin{bmatrix} a_{11} & b \\ c & d \end{bmatrix}$, where $b=0$ and $d=0$.

We have,

Step 2 of 4

Let A be a 2×2 matrix.

Let $A=\begin{bmatrix} a_{11} & b \\ c & d \end{bmatrix}$, where $b=0$ and $d=0$.

We have,

$$\begin{aligned}\det(A) &= (a_{11}d)-(bc)\\ &= 0-0\\ &= 0\end{aligned}$$

Therefore, in such case, whatever is the value of a_{11}, determinant of A is always zero.

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Step 3 of 4

Otherwise, the determinant of A can be found out from the cofactor expression as follows:

$$\det A = a_{11}C_{11}+a_{12}C_{12}+\ldots+a_{1n}C_{1n}$$

Now suppose $\det A=0$. This gives,

$$a_{11}C_{11}+a_{12}C_{12}+\ldots+a_{1n}C_{1n}=0$$

$$a_{11}C_{11}=-(a_{12}C_{12}+\ldots+a_{1n}C_{1n})$$

Provide feedback (0)

Step 3 of 4

Otherwise, the determinant of A can be found out from the cofactor expression as follows:

$$\det A = a_{11}C_{11} + a_{12}C_{12} + \ldots + a_{1n}C_{1n}$$

Now suppose $\det A = 0$. This gives,

$$\begin{aligned} a_{11}C_{11} + a_{12}C_{12} + \ldots + a_{1n}C_{1n} &= 0 \\ a_{11}C_{11} &= -\left(a_{12}C_{12} + \ldots + a_{1n}C_{1n}\right) \\ a_{11} &= \frac{-\left(a_{12}C_{12} + \ldots + a_{1n}C_{1n}\right)}{C_{11}} \end{aligned}$$

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Step 4 of 4

Since, the matrix A is fixed, the value of $\frac{-\left(a_{12}C_{12} + \ldots + a_{1n}C_{1n}\right)}{C_{11}}$ is also fixed. Therefore, only when $\boxed{a_{11} = \frac{-\left(a_{12}C_{12} + \ldots + a_{1n}C_{1n}\right)}{C_{11}}}$, we get $\det A = 0$.

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CH4.3 12P

Step 1 of 5

We have exactly 6 permutations of 1, 2, 3 to 1, 2, and 3
They are

$$e=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix} f=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} g=\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$$

$$h=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix} \phi=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} \psi=\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$$

And we have f, g, h are odd and e, ϕ, ψ are even.

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Step 2 of 5

Big Formula

$$\det A=\sum_{all\,P's}\left(a_{1\alpha}a_{2\beta}\ldots a_{n\upsilon}\right)\det P$$

So $\det\left(a_{ij}\right)_{3\times3}=a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}-a_{13}a_{22}a_{31}$

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Step 3 of 5

Considering

I) $$A=\begin{bmatrix}1&2&3\\3&1&2\\3&2&1\end{bmatrix}$$

$$\begin{aligned}\det A&=1.1.1+2.2.3+3.2.3-2.3.1-1.2.2-3.1.3\\&=1+12+18-6-4-9\\&=\boxed{12}\end{aligned}$$

Here rows of A are independent

Provide feedback (0)

CH4.3 14P

$$= 1+12+18-6-4-9$$
$$= \boxed{12}$$

Here rows of A are independent

Provide feedback (0)

Step 4 of 5

ii) Given $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$

$$\det B = 1.4.7 + 2.4.5 + 3.4.6 - 2.4.7 - 1.4.6 - 3.4.5$$
$$= 28 + 40 + 72 + 56 - 24 - 60$$
$$= \boxed{0}$$

Note that rows of B are dependent since $row1 + row2 = row3$

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Step 5 of 5

iii) Given $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\det C = 1.1.0 + 1.0.1 + 1.1.0 - 1.0.0 - 1.0.1 - 1.1.1$$
$$= -1$$
$$\neq 0$$

Rows of C are independent.

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CH4.3 13P

Step 1 of 4

Big Formula

$$\det A = \sum_{all\,P's} \left(a_{1\alpha}a_{2\beta}\ldots a_{n\upsilon}\right)\det P$$

By using big formula, we get

$$\det\left(a_{ij}\right)_{3\times3} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31}$$

Provide feedback (0)

Step 2 of 4

i) Considering $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$\begin{aligned} \det A &= 1.0.1 + 1.1.0 + 0.1.1 - 1.1.1 - 1.1.1 - 0.0.0 \\ &= -1 - 1 \\ &= \boxed{-2} \\ &\neq 0 \end{aligned}$$

So, columns of A are independent

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Step 3 of 4

ii) Given $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Computing det using big formula, we have

$$\begin{aligned} \det B &= 1.5.9 + 2.6.7 + 3.4.8 - 1.6.8 - 2.4.9 - 3.7.5 \\ &= 45 + 84 + 96 - 48 - 72 - 105 \\ &= \boxed{0} \end{aligned}$$

Columns of B are dependent. Observe that second column is average of remaining columns.

CH4.3 15P

Provide feedback (0)

Step 3 of 4

ii) Given $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Computing det using big formula, we have

$$\begin{aligned} \det B &= 1.5.9 + 2.6.7 + 3.4.8 - 1.6.8 - 2.4.9 - 3.7.5 \\ &= 45 + 84 + 96 - 48 - 72 - 105 \\ &= \boxed{0} \end{aligned}$$

Columns of B are dependent. Observe that second column is average of remaining columns.

So, we get $\det B = 0$

Provide feedback (0)

Step 4 of 4

iii) Given $C = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 0 & 7 & 8 & 9 \end{bmatrix}$$

As columns of B are independent we get that columns of C are also dependent since 5th column is average of 4th and 6th columns of C.

Here we get $\det C = \boxed{0}$

Provide feedback (0)

CH4.3 14P

Step 1 of 2

We have

$$A=\begin{bmatrix} x & x & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}$$

$$A=\begin{bmatrix} a & b & c \\ 0 & 0 & d \\ 0 & 0 & e \end{bmatrix}$$ Where a,b,c,d,e are non $-$zero values marked in the place of x

Provide feedback (0)

Step 2 of 2

Then using row operation of row 3 $-\left(\text{Row 2 multiplied by } ed^{-1}\right)$ on A

We get a zero row so that

$$\det A=\begin{vmatrix} a & b & c \\ 0 & 0 & d \\ 0 & 0 & 0 \end{vmatrix}$$

$$=\boxed{0}$$ Since it contains a zero row

And clearly rank of $A=\boxed{2}$

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CH4.3 15P

CH4.3 17P

Step 1 of 4

Given

$$A=\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \end{bmatrix}$$

$$A=\begin{bmatrix} a & b & c & d & e \\ p & q & r & s & t \\ 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & z & f \\ 0 & 0 & 0 & g & h \end{bmatrix}$$

$a,b,c,d,e,p,q,r,s,t,x,y,z,f,g,h$ Can be any real numbers

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Step 2 of 4

a) The last three rows of A are linearly dependent.
Since if any of x,z,g is non zero
We can make remaining two places zero by suitable subtraction of a constant multiple of the vector from remaining vectors.

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Step 3 of 4

For example if $x \neq 0$ by making row operations row 4 goes to $row4+\left(-x^{-1}z\right)row3$ and row 5 goes to $row4+\left(-x^{-1}z\right)$row 3 and row 5 goes to $row5+\left(-x^{-1}g\right)$row 3 given a matrix of the form

$$\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \end{bmatrix}$$

We can make remaining two places zero by suitable subtraction of a constant multiple of the vector from remaining vectors.

Provide feedback (0)

Step 3 of 4

For example if $x \neq 0$ by making row operations row 4 goes to $row4+\left(-x^{-1}z\right)row3$ and row 5 goes to $row4+\left(-x^{-1}z\right)$row 3 and row 5 goes to $row5+\left(-x^{-1}g\right)$row 3 given a matrix of the form

$$B=\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & \otimes \\ 0 & 0 & 0 & 0 & \otimes \end{bmatrix}$$

Once again if at least one of places marked as $\otimes$ is non zero, by another row operation we get a matrix of zero rows
Hence $\det A$ is always zero for any values inplaces marked as letters.

Provide feedback (0)

Step 4 of 4

b) Big Formula

$$\det A=\sum_{all\,P's}\left(a_{1\alpha}a_{2\beta}\ldots a_{n\upsilon}\right)\det P$$

In the big formula each term is forward taking exactly one number from each column and each row, now if we take non zero entries from $4,5^{th}$ rows we must compulsorily choose zero from 3rd row and hence the product of three terms (I, e any term in big formula) is invariably zero.

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CH4.3 16P

Step 1 of 3

When $A=\begin{bmatrix}1&0&0&1\\0&1&1&1\\1&1&0&1\\1&0&0&1\end{bmatrix}$ there are exactly two none zero terms in the big formula

$-a_{11}a_{23}a_{32}a_{44}+a_{14}a_{23}a_{32}a_{41}$

And $\det A=-1+1$

$=\boxed{0}$

Also note that last column of A is sum of 1st and 3rd columns. Of A and 1st row, last row of A are identical.

Provide feedback (0)

Step 2 of 3

Big Formula

$$\det A=\sum_{all\,P's}\left(a_{1\alpha}a_{2\beta}\ldots a_{n\upsilon}\right)\det P$$

And

$$B=\begin{bmatrix}1&0&0&2\\0&3&4&5\\5&4&0&3\\2&0&0&1\end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

B Also have zero in exactly same places as A and hence exactly two none zero terms occur in the big formula.

They are $-b_{11}b_{23}b_{32}b_{44}+b_{44}b_{23}b_{32}b_{41}$

And hence

$\det B=-1.4.4.1+2.4.4.2$

CH4.3 18P

And $\det A = -1+1$

$= \boxed{0}$

Also note that last column of A is sum of 1st and 3rd columns. Of A and 1st row, last row of A are identical.

Provide feedback (0)

Step 2 of 3

Big Formula

$$\det A = \sum_{all\,P's} \left(a_{1\alpha}a_{2\beta}\ldots a_{n\upsilon}\right)\det P$$

And

$$B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

B Also have zero in exactly same places as A and hence exactly two none zero terms occur in the big formula.

They are $-b_{11}b_{23}b_{32}b_{44} + b_{44}b_{23}b_{32}b_{41}$

And hence

$\det B = -1.4.4.1 + 2.4.4.2$

$= -16 + 64$

$= \boxed{48}$

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CH4.3 17P

Step 1 of 3

We need not out any zero to get 4×4 matrix A such that $\det A = 0$

For example consider

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 11 \end{bmatrix}$$

Here $\det A = 0$ since first two rows of A are dependent.

Provide feedback (0)

Step 2 of 3

Consider

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \det A &= \begin{vmatrix} 0 & 3 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \\ &= -3\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} \\ &= -12 \\ &\neq 0 \end{aligned}$$

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Step 3 of 3

And a diagonal matrix or matrixes that can be got into such form by row interchanges are

CH4.3 19P

Step 2 of 3

Consider

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \det A &= \begin{vmatrix} 0 & 3 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \\ &= -3\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} \\ &= -12 \\ &\neq 0 \end{aligned}$$

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Step 3 of 3

And a diagonal matrix or matrixes that can be got into such form by row interchanges are the only non singular matrices with maximum number of zero entries.

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Step 1 of 3

Big Formula

$$\det A = \sum_{all\,P's} \left(a_{1\alpha}a_{2\beta}\ldots a_{n\nu}\right)\det P$$

By using big formula, we get

$$\det\left(a_{ij}\right)_{3\times3} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31}$$

Provide feedback (0)

Step 2 of 3

a) If $a_{11} = a_{22} = a_{33} = 0$, four terms in the big formula are zeros.

They are $a_{11}a_{22}a_{33}, -a_{11}a_{23}a_{32}, -a_{12}a_{21}a_{33}, -a_{13}a_{22}a_{31}$

Provide feedback (0)

Step 3 of 3

b) If $a_{11} = a_{22} = a_{33} = a_{44} = 0$, then zero terms in the big formula are

$a_{11}a_{22}a_{33}a_{44}, a_{12}a_{21}a_{33}a_{44}, a_{11}a_{24}a_{33}a_{42}$

$a_{11}a_{22}a_{34}a_{43}, a_{11}a_{23}a_{32}a_{44}, a_{14}a_{22}a_{33}a_{41},$

$a_{13}a_{22}a_{31}a_{44}, a_{11}a_{23}a_{34}a_{42}, a_{11}a_{24}a_{32}a_{43},$

$a_{13}a_{22}a_{34}a_{41}, a_{14}a_{22}a_{31}a_{43}, a_{12}a_{24}a_{33}a_{41},$

$a_{14}a_{21}a_{33}a_{42}, a_{12}a_{23}a_{31}a_{44}, a_{13}a_{21}a_{32}a_{44}$

So, a total 15 terms vanish in the big formula.

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CH4.3 19P

Step 1 of 3

In $5! = 5\times4\times3\times2\times1 = 120$ permutations of 5 elements exactly 60 are even permutations so in $\det P = +1$ for 60 terms from example

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Needs 4 exchange's of rows then (row 1 and row 2)

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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CH4.3 21P

Step 2 of 3

(row 2 and row 3), (row 3 and row 4), (row 4 and row 5)

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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Step 2 of 3

$(\text{row 2 and row 3}),(\text{row 3 and row 4}),(\text{row 4 and row 5})$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

Now

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Exchanges bring identity matrix.

This P is associated with the term $a_{12}a_{23}a_{34}a_{45}a_{51}(2\ 3\ 4\ \ 5\ 1)$

Permutation corresponds to this term.

Provide feedback (0)

CH4.3 20P

Step 1 of 3

Since with every ordering of the rows of A leaves at least one zero on the diagonal, there must be a column of all zeros in the original square matrix A.

Let i^{th} column be the column of zeros in the matrix A.

Consider again, $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$

Provide feedback (0)

Step 2 of 3

While carrying out the summation in the right hand side of $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$, every term contains one and only one term from the ith column.

That is, every term in the summation must be equal to zero, since zero multiplied by any number is zero.

Therefore, $\sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P = 0$

Provide feedback (0)

Step 3 of 3

This implies that $\det A = 0$. But this is a contradiction, since we have assumed that $\det A \neq 0$.

CH4.3 22P

$\text{all } P's$

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Step 2 of 3

While carrying out the summation in the right hand side of $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$, every term contains one and only one term from the ith column.

That is, every term in the summation must be equal to zero, since zero multiplied by any number is zero.

Therefore, $\sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P = 0$

Provide feedback (0)

Step 3 of 3

This implies that $\det A = 0$. But this is a contradiction, since we have assumed that $\det A \neq 0$.

Therefore, our assumption that every ordering of A leaves at least one zero on the diagonal is wrong.

Thus, there exists some ordering of A, which leaves no zero on the diagonal.

Provide feedback (0)

CH4.3 21P

CH4.3 23P

Step 1 of 3

All possible 2×2 matrices with entries $1,-1$ are

$$\begin{bmatrix}1&1\\1&1\end{bmatrix},\begin{bmatrix}-1&1\\1&1\end{bmatrix},\begin{bmatrix}1&-1\\1&1\end{bmatrix},\begin{bmatrix}1&1\\-1&1\end{bmatrix},\begin{bmatrix}1&1\\1&-1\end{bmatrix},\begin{bmatrix}-1&-1\\1&1\end{bmatrix},$$

$$\begin{bmatrix}-1&1\\1&-1\end{bmatrix},\begin{bmatrix}-1&1\\-1&1\end{bmatrix},\begin{bmatrix}1&-1\\-1&1\end{bmatrix},\begin{bmatrix}1&-1\\1&-1\end{bmatrix},\begin{bmatrix}1&1\\-1&-1\end{bmatrix},$$

$$\begin{bmatrix}-1&-1\\-1&-1\end{bmatrix},\begin{bmatrix}-1&-1\\-1&1\end{bmatrix},\begin{bmatrix}-1&1\\-1&-1\end{bmatrix},\begin{bmatrix}1&-1\\-1&-1\end{bmatrix},\begin{bmatrix}-1&-1\\1&-1\end{bmatrix},$$

We can observe that the values of determinants of all these matrices are zero $-2,2$ only.

Provide feedback (0)

Step 2 of 3

So that maximum values of determinant in 2×2 case is 2.
Now getting to 3×3 case, we should compute $a_{22}c_{11}+a_{12}c_{12}+a_{13}c_{13}$ to get $\det A$ where c_{ij} cofactors are a_{ij}
Exploring possibilities suppose $a_{11}=1, a_{12}=1\ a_{13}=1$
The other two rows should not be proportional to this row and proportional to each other so that determinant is non zero so they can be

$$\begin{bmatrix}1&-1&1\\-1&1&1\end{bmatrix} or \begin{bmatrix}-1&1&1\\1&-1&1\end{bmatrix} or \begin{bmatrix}1&1&-1\\-1&1&1\end{bmatrix} or$$

$$\begin{bmatrix}-1&1&1\\1&1&-1\end{bmatrix} or \begin{bmatrix}1&-1&1\\1&1&-1\end{bmatrix} or \begin{bmatrix}1&1&-1\\1&-1&1\end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

The remaining cases are negatives of rows of these versions.
Clearly at least one factor vanishes.
So, the determinant value can not exceed 4

We can observe that the values of determinants of all these matrices are zero $-2, 2$ only.

Provide feedback (0)

Step 2 of 3

So that maximum values of determinant in 2×2 case is 2.
Now getting to 3×3 case, we should compute $a_{22}c_{11}+a_{12}c_{12}+a_{13}c_{13}$ to get $\det A$ where c_{ij} cofactors are a_{ij}
Exploring possibilities suppose $a_{11}=1, a_{12}=1\, a_{13}=1$
The other two rows should not be proportional to this row and proportional to each other so that determinant is non zero so they can be

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} or \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} or \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} or$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} or \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} or \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

The remaining cases are negatives of rows of these versions.
Clearly at least one factor vanishes.
So, the determinant value can not exceed 4
Similarly we can dispose all cases where exactly one entry of 1st row is -1 and exactly two entries of 1st row are -1's etc.
In any case we note that atleast one cofactor vanishes and hence determinant of 3×3 matrix of entries 1 or -1 cannot exceed 4.

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CH4.3 22P

CH4.3 24P

Step 1 of 3

Identity matrix has $\frac{4!}{2} = 12$ even exchange matrices P whose value is equal to 1

This is to say that there are 12 even permutations on 4 symbols $(1,2,3,4)$

They are given by

$(1,2,3,4),(2,1,4,3),(4,3,2,1),(3,4,1,2),(2,3,1,4)$

$(3,1,2,4),(1,3,4,2),,(1,4,2,3),(3,2,4,1),(4,2,1,3)$

$(2,4,3,1),(4,1,3,2)$

Provide feedback (0)

Step 2 of 3

P Matrices that arise from 4×4 matrix identity I by exchanges of rows that have value 1 are given by I

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

And

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \end{bmatrix}$$

Step 2 of 3

P Matrices that arise from 4×4 matrix identity I by exchanges of rows that have value 1 are given by I

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix},\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix},\begin{bmatrix}0&0&1&0\\1&0&0&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

And

$$\begin{bmatrix}0&1&0&0\\0&0&1&0\\1&0&0&0\\0&0&0&1\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&1&0&0\\0&0&1&0\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\\0&1&0&0\end{bmatrix},\begin{bmatrix}0&0&0&1\\0&1&0&0\\1&0&0&0\\0&0&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&0&1&0\\0&1&0&0\\0&0&0&1\\1&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&0&1&0\\0&1&0&0\end{bmatrix},\begin{bmatrix}0&1&0&0\\0&0&0&1\\0&0&1&0\\1&0&0&0\end{bmatrix}$$

$\det\left(I+P_{even}\right)=16$ or 4 or 0

$(16$ comes from $I+I)$

Provide feedback (0)

CH4.3 23P

Step 1 of 6

i) Given matrix

$$A=\begin{bmatrix}2 & 1\\ 3 & 6\end{bmatrix}$$

Given

$$C_{ij}=(-1)^{i+j}\det M_{ij}$$

Then $C_A=\begin{bmatrix}6 & -3\\ -1 & 2\end{bmatrix}$ and $C_A^T=\begin{bmatrix}6 & -1\\ -3 & 2\end{bmatrix}$

Provide feedback (0)

Step 2 of 6

Now

$$\begin{aligned}AC_A^T&=\begin{bmatrix}2 & 1\\ 3 & 6\end{bmatrix}\begin{bmatrix}6 & -1\\ -3 & 2\end{bmatrix}\\ &=\begin{bmatrix}2(6)+1(-3) & 2(-1)+1(2)\\ 3(6)+6(-3) & 3(-1)+6(2)\end{bmatrix}\\ &=\begin{bmatrix}12-3 & -2+2\\ 18-18 & -3+12\end{bmatrix}\\ &=\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}\end{aligned}$$

Note that

$$\begin{aligned}\det A&=12-3\\ &=9\end{aligned}$$

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Step 3 of 6

So, we get

CH4.3 25P

Provide feedback (0)

Step 3 of 6

So, we get

$$AC_A^T = \det A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \det A \cdot I$$

Provide feedback (0)

Step 4 of 6

ii) $$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix}$$

Then

$$C_B = \begin{bmatrix} 0 & 42 & -35 \\ 0 & -21 & 14 \\ -3 & 6 & -3 \end{bmatrix}$$

$$C_B^T = \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 6

Now

$$B.C_B^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 84-105 & -42+42 & -3+12-9 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 6 & -3 \end{bmatrix}$$

$$C_B^T = \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 6

Now

$$B.C_B^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 84-105 & -42+42 & -3+12-9 \\ 210-210 & -105+84 & -12+30-18 \\ 0 & 0 & -21 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 6

And

$$= \begin{bmatrix} -21 & 0 & 0 \\ 0 & -21 & 0 \\ 0 & 0 & -21 \end{bmatrix}$$

$$= -21 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= -21I$$

And note that $\det B = \boxed{-21}$

Provide feedback (0)

CH4.3 24P

Step 1 of 9

We have

$$A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$$

The cofactors of the elements of A are:

$$\begin{aligned}C_{11}&=\begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix}\\ &=4-1\\ &=3\end{aligned}$$

Provide feedback (0)

Step 2 of 9

And

$$\begin{aligned}C_{12}&=-\begin{vmatrix}-1 & -1\\ 0 & 2\end{vmatrix}\\ &=-(-2-0)\\ &=2\end{aligned}$$

$$\begin{aligned}C_{13}&=\begin{vmatrix}-1 & 2\\ 0 & -1\end{vmatrix}\\ &=1-0\\ &=1\end{aligned}$$

Provide feedback (0)

Step 3 of 9

Then

$$C_{21}=-\begin{vmatrix}-1 & 0\\ -1 & 2\end{vmatrix}$$

CH4.3 26P

Step 3 of 9

Then

$$C_{21} = -\begin{vmatrix} -1 & 0 \\ -1 & 2 \end{vmatrix}$$
$$= -(-2-0)$$
$$= 2$$
$$C_{22} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$
$$= 4-0$$
$$= 4$$

Provide feedback (0)

Step 4 of 9

And

$$C_{23} = -\begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix}$$
$$= -(-2-0)$$
$$= 2$$
$$C_{31} = \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix}$$
$$= 1-0$$
$$= 1$$

Provide feedback (0)

Step 5 of 9

Now

$$C_{32} = -\begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix}$$

Step 5 of 9

Now

$$C_{32} = -\begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix}$$
$$= -(-2-0)$$
$$= 2$$
$$C_{33} = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$
$$= 4-1$$
$$= 3$$

Provide feedback (0)

Step 6 of 9

So, the matrix of cofactors is $C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Then $C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Provide feedback (0)

Step 7 of 9

And

$$AC^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Step 7 of 9

And

$$AC^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-2+0 & 4-4+0 & 2-2+0 \\ -3+4-1 & -2+8-2 & -1+4-3 \\ 0-2+2 & 0-4+4 & 0-2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= 4I$$

Provide feedback (0)

Step 8 of 9

We can also see $\det A$

$= 2.\text{cofactor of } 2 + (-1)\text{cofactor of } (-1)(\text{cofactor expansion along first row})$

$= 2(3) + (-1)2$

$= 6 - 2$

$= 4$

Provide feedback (0)

Step 9 of 9

So, we observe that $\frac{1}{\det A}.C^T = A^{-1}$

Hence

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$$

$$AC^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-2+0 & 4-4+0 & 2-2+0 \\ -3+4-1 & -2+8-2 & -1+4-3 \\ 0-2+2 & 0-4+4 & 0-2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= 4I$$

Provide feedback (0)

Step 8 of 9

We can also see $\det A$

$= 2.\text{cofactor of } 2 + (-1)\text{cofactor of } (-1)(\text{cofactor expansion along first row})$

$= 2(3) + (-1)2$

$= 6 - 2$

$= 4$

Provide feedback (0)

Step 9 of 9

So, we observe that $\frac{1}{\det A}.C^T = A^{-1}$

Hence

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

CH4.3 25P

Step 1 of 4

Given

The matrix B_n is the $-1,2,-1$ matrix A_n except that $b_{11}=1$ instead of $a_{11}=2$

So that

$$B_2=\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix},\ B_3=\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$$

$$B_4=\begin{bmatrix}1 & -1 & 0 & 0\\ -1 & 2 & -1 & 0\\ 0 & -1 & 2 & -1\\ 0 & 0 & -1 & 2\end{bmatrix}$$

Provide feedback (0)

Step 2 of 4

Then

$$\begin{aligned}\det(B_2)&=2-1\\ &=2\end{aligned}$$

$$\begin{aligned}\det(B_3)&=(4-1)+(-2)=2\det(B_2)+\begin{vmatrix}1 & 0\\ -1 & -1\end{vmatrix}\\ &=2\det(B_3)-\det(B_2)\ \text{(Expanding determinant by least columns)}\\ &=3-2\\ &=1\end{aligned}$$

Provide feedback (1)

Step 3 of 4

Expanding any B_n by last column use get

$$\begin{bmatrix}1 & -1 & 0 & 0\ldots\ldots\ldots 0\\ -1 & 2 & -1 & 0\end{bmatrix}$$

CH4.3 27P

$$\det(B_3) = (4-1)+(-2) = 2\det(B_2) + \begin{vmatrix} & \\ -1 & -1 \end{vmatrix}$$

$$= 2\det(B_3) - \det(B_2) \text{ (Expanding determinant by least columns)}$$

$$= 3-2$$

$$= 1$$

Provide feedback (1)

Step 3 of 4

Expanding any B_n by last column use get

$$\det(B_n) = \begin{bmatrix} 1 & -1 & 0 & 0.....0 \\ -1 & 2 & -1 & 0..... \\ 0 & -1 & 2 & -1..... \\ 0 & 0 & 0 & .....-1 \end{bmatrix} + 2\det(B_{n-1})$$

$$= -\det(B_{n-2}) + 2\det(B_{n-1})$$

(expanding the earlier determinant. 1 term in R.H.S by last column)

Provide feedback (0)

Step 4 of 4

So, the recursion formula is as of A_n 's

Here we have

$|B_1| = |B_2| = |B_3| = 1$

And we notice that all pivots are equal to 1

Provide feedback (0)

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CH4.3 26P

Step 1 of 2

Given

B_n is still the same as A_n except for $b_{11} = 1$

Using linearity in 1st row of determinants we get

$$|B_n| = \begin{vmatrix} 1 & -1 & 0 \\ -1 & & \\ & & A_{n-1} \\ 0 & & \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 \\ -1 & & \\ & & A_{n-1} \\ 0 & & \end{vmatrix} - \begin{vmatrix} 1 & 0 & 0 \\ -1 & & \\ & & A_{n-1} \\ 0 & & \end{vmatrix}$$

Provide feedback (0)

CH4.3 28P

Step 2 of 2

On solving

$= |A_n| - |A_{n-1}|$ expanding 2nd determinant by 1st row

$= (n+1) - n$

$= 1$ For all $n \in N$

Thus

$|B_n| = \boxed{1}$

Provide feedback (0)

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CH4.3 27P

Step 1 of 8

Given

$$C_1 = |0|$$

$$C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

Provide feedback (0)

Step 2 of 8

(a)

Then determinants are

$$\begin{aligned} C_1 &= |0| \\ &= \boxed{0} \\ C_2 &= \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ &= 0 - 1 \\ &= \boxed{-1} \end{aligned}$$

Provide feedback (0)

Step 3 of 8

CH4.3 29P

Step 3 of 8

Then

$$C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= 0\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 0 - 0$$

$$= \boxed{0}$$

Provide feedback (0)

Step 4 of 8

And

$$C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \boxed{1}$$

Provide feedback (0)

Step 5 of 8

Provide feedback (0)

Step 5 of 8

(b)
Now

$$C_5 = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= -\left[\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}\right]$$

$$= \boxed{0}$$

Provide feedback (0)

Step 6 of 8

From the above calculations, we observed that

$$\begin{aligned} C_5 = 0 &= -C_{(5-2)} \\ &= -C_3 \\ &= 0 \\ C_4 = 1 &= -C_{(4-2)} \\ &= -C_2 \\ &= -(-1) \\ &= 1 \end{aligned}$$

$$= -C_3$$
$$= 0$$
$$C_4 = 1 = -C_{(4-2)}$$
$$= -C_2$$
$$= -(-1)$$
$$= 1$$

Provide feedback (0)

Step 7 of 8

$$C_3 = 0 = -(C_{3-2})$$
$$= -C_1$$
$$= 0$$

In general
We can observe

$$C_n = -C_{n-2}$$

Provide feedback (0)

Step 8 of 8

So

$$C_{10} = -C_8$$
$$= C_6$$
$$= -C_4$$
$$= \boxed{-1}$$

Thus

$$C_{10} = \boxed{-1}$$

Provide feedback (0)

CH4.3 28P

CH4.3 30P

Step 1 of 5

If we let C_n to denote the n by n matrices, with the same properties, then it is obviously clear that no exchange of rows can produce 1's along the diagonal of C_1.

Consider $C_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Exchanging first and second row gives all 1's along the main diagonal.

Consider $C_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. In order to have 1 in the first row and first column, we need to exchange first row and second row. This gives $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Thus, we cannot have a 1 in the third row and third column.

Thus, in case of C_3 also, no exchange of rows can produce only 1's along the main diagonal.

Consider $C_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Exchange row 1 and row 2. Then exchange row 3 and row 4. This will make all 1's along the main diagonal.

Provide feedback (0)

Step 2 of 5

Let us show that if n is even, it is always possible to exchange rows to obtain all 1's along the main diagonal. Exchange first row and second row. Then exchange third row and fourth row. Then exchange fifth

Provide feedback (0)

Step 2 of 5

Let us show that if n is even, it is always possible to exchange rows to obtain all 1's along the main diagonal. Exchange first row and second row. Then exchange third row and fourth row. Then exchange fifth row and sixth row. Continue this process and finally exchange $n-1^{st}$ row and n^{th} row. This makes all 1's along the main diagonal.

Provide feedback (0)

Step 3 of 5

When n is even, either it is divisible by 4 or it is not divisible by 4. Thus, we can write $n = 4k$ or $n = 4k + 2$, where k is some integer.

Consider the following:

$$\begin{aligned} n &= 4k \\ \frac{n}{2} &= \frac{4k}{2} \\ &= 2k \end{aligned}$$

Thus, in such case, row exchange is done $2k$ number of times. That is, row exchange is done even number of times. Every row exchange changes the sign of the determinant. When row exchange is done even number of times, this is same as applying proper permutation to C_n. Since the value of the determinant will remain the same along with the sign (due to even number of exchanges), this is same as even permutation.

Provide feedback (0)

Step 4 of 5

remain the same along with the sign (due to even number of exchanges), this is same as even permutation.

Provide feedback (0)

Step 4 of 5

Consider the following:

$$n = 4k + 2$$
$$\frac{n}{2} = \frac{4k+2}{2}$$
$$= 2k + 1$$

Thus, in such case, row exchange is done $2k+1$ number of times. That is, row exchange is done odd number of times. Every row exchange changes the sign of the determinant. When row exchange is done odd number of times, this is same as applying proper permutation to C_n. Since the value of the determinant will remain the same but the sign will change (due to odd number of exchanges), this is same as odd permutation.

Provide feedback (0)

Step 5 of 5

Thus, for n = 4, 8, 12,... we get even permutation and when n = 2, 6, 10,... we get odd permutation.

It may be noted that when n is an odd number, there is no exchange of rows possible, due to which we can get all 1's along the main diagonal.

Provide feedback (0)

CH4.3 29P

Step 1 of 7

We have

$$V_4 = \det\begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & x & x^2 & x^3 \end{bmatrix}$$

Is designed to the determinant is zero at $x = a, x = b,$ and $x = c$. So, this cannot contain a higher power of x than x^3.

Provide feedback (0)

Step 2 of 7

By subtracting last row from each of 1, 2, 3 row we get

$$V_4 = \det\begin{bmatrix} 0 & a-x & a^2-x^2 & a^3-x^3 \\ 0 & b-x & b^2-x^2 & b^3-x^3 \\ 0 & c-x & c^2-x^2 & c^3-x^3 \\ 0 & x & x^2 & x^3 \end{bmatrix}$$

$$= \begin{bmatrix} a-x & a^2-x^2 & a^3-x^3 \\ b-x & b^2-x^2 & b^3-x^3 \\ c-x & c^2-x^2 & c^3-x^3 \end{bmatrix} \text{ Expanding by } 1^{st} \text{ column}$$

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Step 3 of 7

$$V_4 = (a-x)(b-x)(c-x)\begin{bmatrix} 1 & a+x & a^2+ax+x^2 \\ 1 & b+x & b^2+bx+x^2 \\ 1 & c+x & c^2+cx+x^2 \end{bmatrix}$$

(taking $(a\text{-}x), (b-x) \forall (c-x)$ common from 1st, II and III rows respectively)

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Step 3 of 7

$$V_4=(a-x)(b-x)(c-x)\begin{bmatrix}1 & a+x & a^2+ax+x^2\\ 1 & b+x & b^2+bx+x^2\\ 1 & c+x & c^2+cx+x^2\end{bmatrix}$$

(taking $(a\text{-}x),(b-x)\,\forall\,(c-x)$ common from 1st, II and III rows respectively)

$$=-(x-a)(x-b)(x-c)\begin{bmatrix}1 & a+x & a^2+ax+x^2\\ 0 & b-a & b^2-a^2+x(b-a)\\ 0 & c-b & c^2-b^2+x(c-b)\end{bmatrix}$$

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Step 4 of 7

(Subtracting 2nd row from the thired row and 1st row from the 2nd row)

$$V_4=-(x-a)(x-b)(x-c)\begin{vmatrix}1 & a+x & a^2+ax+x^2\\ 0 & b-a & (b-a)(b+a+x)\\ 0 & c-b & (c-b)(c+b+x)\end{vmatrix}$$

$$=-(x-a)(x-b)(x-c)(b-a)(c-b)\begin{vmatrix}1 & a+x & a^2+ax+x^2\\ 0 & 1 & b+a+x\\ 0 & 1 & c+b+x\end{vmatrix}$$

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Step 5 of 7

$$V_4=-(x-a)(x-b)(x-c)(b-a)(c-b)\left[(c+b+x)-(a+b+x)\right]$$
$$=(x-a)(x-b)(x-c)(a-b)(b-c)(c-a)$$
$$=(x-a)(x-b)(x-c)V_3$$

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Step 6 of 7

Where $V_3 = \text{cofactor of } x^3 \text{ in } V_4$

$$= \det\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

$$= (a-b)(b-c)(c-a)$$

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Step 7 of 7

Can be obtained as below.

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-b & c^2-b^2 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a+b \\ 0 & 1 & b+c \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

Thus $\boxed{V_4 = (x-a)(x-b)(x-c)V_3}$

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Step 1 of 6

Given

$$S_1 = |3|, \quad S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}, \quad S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

We need to compute the determinants of S_1, S_2, S_3 of these 1, 3, 1 tridiagonal matrices.

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Step 2 of 6

Now

$$\begin{aligned} S_1 &= |3| \\ &= 3 \\ S_2 &= \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

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Step 3 of 6

Then

$$\begin{aligned} S_3 &= \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} \\ &= 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \text{ expanding I row} \\ &= 3S_2 - 3 \\ &= 3S_2 - S_1 \\ &= 24 - 3 \end{aligned}$$

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Step 3 of 6

Then

$$S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \text{ expanding I row}$$

$$= 3S_2 - 3$$

$$= 3S_2 - S_1$$

$$= 24 - 3$$

$$= 21$$

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Step 4 of 6

So, we can guess that

$$S_4 = 3S_3 - S_2$$

$$= 3(21) - 8$$

$$= 63 - 8$$

$$= \boxed{55}$$

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Step 5 of 6

Calculating S_4 directly we get

$$S_4 = \begin{vmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}$$

Provide feedback (0)

Step 5 of 6

Calculating S_4 directly we get

$$S_4 = \begin{vmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

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Step 6 of 6

On solving

$$= 3.S_3 - \left[\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix}\right]$$

$$= 3(21) - [9 - 1]$$

$$= 63 - 8$$

$$= \boxed{55}$$

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Step 1 of 7

Given

$$S_1 = |3|, \quad S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}, \quad S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

Provide feedback (0)

Step 2 of 7

Now

$$\begin{aligned} S_1 &= |3| \\ &= 3 \\ S_2 &= \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

Provide feedback (0)

Step 3 of 7

Then

$$\begin{aligned} S_3 &= \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} \\ &= 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \text{ expanding I row} \\ &= 3S_2 - 3 \\ &= 3S_2 - S_1 \\ &= 24 - 3 \\ &= 21 \end{aligned}$$

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Step 3 of 7

Then

$$S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \text{ expanding I row}$$

$$= 3S_2 - 3$$

$$= 3S_2 - S_1$$

$$= 24 - 3$$

$$= 21$$

Provide feedback (0)

Step 4 of 7

Cofactors of 1, 3, 1 tridiagonal matrices gives the relation

$S_n = 3S_{n-1} - S_{n-2}$

We have Fibonacci numbers

$1, 1, 2, 3, 5, 8, 13, 21$

$\quad\;\; | \qquad\quad |$

$\quad 4th \qquad 8th$

Are given by the rule $F_K = F_{K-1} + F_{K-2}$

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Step 5 of 7

Therefore

$$F_{2n+2} = F_{2n+2-1} + F_{2n+2-2}$$

$$= F_{2n+1} + F_{2n}$$

$$= F_{2n} + F_{2n-1} + F_{2n}$$

$$= 2F_{2n} + (F_{2n} - F_{2n-1})$$

Step 5 of 7

Therefore

$$\begin{aligned} F_{2n+2} &= F_{2n+2-1} + F_{2n+2-2} \\ &= F_{2n+1} + F_{2n} \\ &= F_{2n} + F_{2n-1} + F_{2n} \\ &= 2F_{2n} + (F_{2n} - F_{2n-2}) \\ &(\because\ F_{2n} = F_{2n-1} + F_{2n-2}) \\ F_{2n+2} &= 3F_{2n} - F_{2n-2} \qquad \ldots(i) \end{aligned}$$

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Step 6 of 7

Observe that

$$\begin{aligned} S_1 &= 3 \\ &= F_{2+2} \\ &= 3 \\ S_2 &= F_{4+2} = F_6 \\ &= 8 \\ S_3 &= F_{6+2} = F_8 \\ &= 21 \end{aligned}$$

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Step 7 of 7

Therefore, by induction

$$\begin{aligned} S_n &= 3S_{n-1} - S_{n-2} \\ &= 3.F_{2(n-1)+2} - F_{2(n-2)+2} \\ &= 3F_{2n} - F_{2n-2} \\ &= F_{2n+2} \qquad (from\,(i)) \end{aligned}$$

$F_{2n+2} = 3F_{2n} - F_{2n-2} \qquad \ldots(i)$

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Step 6 of 7

Observe that

$$\begin{aligned} S_1 &= 3 \\ &= F_{2+2} \\ &= 3 \\ S_2 &= F_{4+2} = F_6 \\ &= 8 \\ S_3 &= F_{6+2} = F_8 \\ &= 21 \end{aligned}$$

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Step 7 of 7

Therefore, by induction

$$\begin{aligned} S_n &= 3S_{n-1} - S_{n-2} \\ &= 3.F_{2(n-1)+2} - F_{2(n-2)+2} \\ &= 3F_{2n} - F_{2n-2} \\ &= F_{2n+2} \qquad (from\,(i)) \end{aligned}$$

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Step 1 of 5

Given

$$S_1 = |3|,\ S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix},\ S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

Now changing 3 to 2 in the upper left corner of these matrices, we get

$$A_1 = |2|$$

$$\begin{aligned} A_2 &= \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ &= 6 - 1 \\ &= \boxed{5} \end{aligned}$$

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Step 2 of 5

Now

$$\begin{aligned} A_3 &= \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} \\ &= 2\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \\ &= 2\left[\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix}\right] - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \\ &= 2(5) + 2(3) - 3 \\ &= 16 - 3 \\ &= \boxed{13} \end{aligned}$$

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Step 3 of 5

In general

$$S_n = \begin{vmatrix} 2 & 1 & 0 & \ldots\ldots 0 \\ 1 & S_{n-1} & & \\ \vdots & & & \end{vmatrix} + \begin{vmatrix} 1 & 0 & \ldots\ldots 0 \\ 1 & S_{n-1} & \\ \vdots & & \end{vmatrix}$$

(by linearity on row 1)

$$|S_1| = |A_n| + |S_{n-1}|$$

$$|A_n| = |S_n| - |S_{n-1}|$$

$$|A_n| = F_{2n+2} - F_{2n}$$

$$= F_{2n+1}$$

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Step 4 of 5

We have in Fibonacci series

$$F_k = F_{k-1} + F_{2n-1}$$

$$= F_{2n} + F_{2n-1}$$

So

$$F_{2n-1} = F_{2n} + F_{2n-1}$$

$$= F_{2n} + F_{2n} - F_{2n-2}$$

$$= 2F_{2n} - F_{2n-2}$$

Provide feedback (0)

Step 5 of 5

As in earlier problem we can note that

$A_n = F_{2n+1}$ For each n

So when problem 31 gives all even terms

(Starting from 4th term) of Fibonacci sequence $(1,1,2,3,5,8,13,21\ldots\ldots)$ gives all odd terms (starting from 3rd term) of Fibonacci sequence $(1,1,2,3,5,8,13,21\ldots\ldots)$

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Step 4 of 5

We have in Fibonacci series

$$F_k = F_{k-1} + F_{2n-1}$$
$$= F_{2n} + F_{2n-1}$$

So

$$F_{2n-1} = F_{2n} + F_{2n-1}$$
$$= F_{2n} + F_{2n} - F_{2n-2}$$
$$= 2F_{2n} - F_{2n-2}$$

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Step 5 of 5

As in earlier problem we can note that

$A_n = F_{2n+1}$ For each n

So when problem 31 gives all even terms

(Starting from 4th term) of Fibonacci sequence $(1,1,2,3,5,8,13,21.....)$ gives all odd terms (starting from 3rd term) of Fibonacci sequence $(1,1,2,3,5,8,13,21,.....)$

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Step 1 of 11

(a) We have to show the first statement $\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A||D|$ is true, Somehow B doesn't enter.

Consider A is n by n matrix and I is m by m identity matrix and B is n by m matrix.

First we will show that $\begin{vmatrix} A & B \\ 0 & I \end{vmatrix} = |A|$.

Now, we use induction and expansion along the last row.

If $m = 1$, then for matrix A that is n by n, expansion along the last row is given by

$$\begin{aligned} \begin{vmatrix} A & \begin{matrix} b_1 \\ \vdots \\ b_n \end{matrix} \\ 0 \cdots 0 & 1 \end{vmatrix} &= (-1)^{(n+1)+(n+1)} |A| \\ &= |A| \end{aligned}$$

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Step 2 of 11

If $m \geq 1$, then for matrix A that is n by n, expansion along the last row is given by

Step 2 of 11

If $m \geq 1$, then for matrix A that is n by n, expansion along the last row is given by

$$\begin{aligned}\begin{vmatrix} A & B \\ 0 & I_{m+1} \end{vmatrix} &= \begin{vmatrix} A & B_1 & B_2 \\ 0 & I_m & 0 \\ 0 & 0 & 1 \end{vmatrix} \\ &= (-1)^{(n+m+1)+(n+m+1)} \begin{vmatrix} A & B_1 \\ 0 & I_m \end{vmatrix} \\ &= |A|\end{aligned}$$

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Step 3 of 11

Thus, we have proved that, for all $m \geq 1$, $\begin{vmatrix} A & B \\ 0 & I \end{vmatrix} = |A|$.

Similarly, we can prove that $\begin{vmatrix} I & B \\ 0 & D \end{vmatrix} = |D|$.

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Step 4 of 11

Consider A is n by n matrix and I is m by m identity matrix and B is n by m matrix.

Step 4 of 11

Consider A is n by n matrix and I is m by m identity matrix and B is n by m matrix.

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Step 5 of 11

Let $B = AX + YD$

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Step 6 of 11

Here X is n by m and Y is m by n.

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Step 7 of 11

So, we have

Provide feedback (0)

Step 8 of 11

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = \begin{vmatrix} A & AX+YD \\ D & I \end{vmatrix}$$
$$= \begin{vmatrix} A & Y \end{vmatrix} \begin{vmatrix} I_n & X \end{vmatrix}$$

Step 8 of 11

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = \begin{vmatrix} A & AX+YD \\ D & I \end{vmatrix}$$
$$= \begin{vmatrix} A & Y \\ 0 & I_m \end{vmatrix} \begin{vmatrix} I_n & X \\ 0 & D \end{vmatrix}$$
$$= |A||D|$$

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Step 9 of 11

Therefore, the first statement $\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A||D|$ is $\boxed{\text{true}}$.

Provide feedback (0)

Step 10 of 11

(b) When we enter C in the equality $\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A||D|$, this equality fails .

Consider the following matrix.

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \left|\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right|$$

Step 10 of 11

(b) When we enter C in the equality $\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A||D|$, this equality fails .

Consider the following matrix.

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \left|\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right|$$
$$= 1$$

And

$$A = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$
$$B = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$C = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$
$$D = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

So, we have $|A||D| = 0$.

We know that $1 \neq 0$, therefore $\boxed{\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq |A||D|}$, when C enters.

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Step 11 of 11

(c) We have to show that $\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq |A||D| - |C||B|$, by the example.

Consider the following matrix.

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \left|\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right|$$
$$= 1$$

And

$$A = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$
$$B = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$\begin{vmatrix} 0 & 0 \end{vmatrix}$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \left|\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right|$$
$$= 1$$

And

$$A = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$
$$B = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$C = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$
$$D = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

So, we have $|A||D| - |C||B| = 0$.

We know that $1 \neq 0$, therefore $\boxed{\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq |A||D| - |C||B|}$.

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Step 1 of 7

When we consider the block multiplication, suppose $A = LU$ has $A_k = L_k U_k$ in the upper left corner, as follows:

$$A = \begin{bmatrix} A_k & * \\ * & * \end{bmatrix}$$
$$= \begin{bmatrix} L_k & 0 \\ * & * \end{bmatrix} \begin{bmatrix} U_k & * \\ 0 & * \end{bmatrix}$$

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Step 2 of 7

(a) Suppose the first three pivots of A are 2, 3, and –1. The lower triangular matrices have 1's along the diagonals and therefore, the determinant of L_K is equal to 1, for any K.

Thus, we have

$$\boxed{L_1 = 1, L_2 = 1, L_3 = 1}$$

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Step 3 of 7

It is clear that the determinant of U_K is equal to the determinant of A_K, for each K.

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Step 3 of 7

It is clear that the determinant of U_K is equal to the determinant of A_K, for each K.

The pivot entries of A are 2, 3, and –1.

Therefore, the determinant of A_1 is 2.

The determinant of A_2 is $2 \times 3 = 6$.

The determinant of A_3 is $2 \times 3 \times -1 = -6$.

Provide feedback (0)

Step 4 of 7

Thus, we have the following: $\boxed{A_1 = 2}$, $\boxed{U_1 = 2}$, $\boxed{A_2 = 6}$, $\boxed{U_2 = 6}$, $\boxed{A_3 = -6}$, $\boxed{U_3 = -6}$.

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Step 5 of 7

(b) Let us suppose that the determinants of A_1, A_2, and A_3 are 5, 6, and 7 respectively. We need to find the pivot entries of A.

Since, determinant of A_1 is equal to 5, the first pivot of A must be $\boxed{5}$.

Step 5 of 7

(b) Let us suppose that the determinants of A_1, A_2, and A_3 are 5, 6, and 7 respectively. We need to find the pivot entries of A.

Since, determinant of A_1 is equal to 5, the first pivot of A must be $\boxed{5}$.

Provide feedback (0)

Step 6 of 7

Now, the determinant of A_2 is 6.

Therefore, the second pivot of A must be such that the product its first pivot and second pivot must be 6.

The first pivot is 5.

Therefore, the second pivot must be $\boxed{\frac{6}{5}}$.

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Step 7 of 7

Now, the determinant of A_3 is 7.

Therefore, the third pivot of A must be such that the product its three pivots must be 7.

Therefore, the second pivot must be $\boxed{\frac{6}{5}}$.

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Step 7 of 7

Now, the determinant of A_3 is 7.

Therefore, the third pivot of A must be such that the product its three pivots must be 7.

The first pivot is 5. The second pivot is $\frac{6}{5}$.

Therefore, the third pivot must be $\frac{7}{6}$, since $5 \times \frac{6}{5} \times \frac{7}{6} = 7$.

Therefore, the third pivot must be $\boxed{\frac{7}{6}}$.

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Step 1 of 3

Thus, we have

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

The elements along the diagonal are $(1,5,9)$, $(2,6,7)$, and $(3,4,8)$. The elements along the anti diagonal are $(2,4,9)$, $(1,6,8)$, and $(3,5,7)$.

Therefore, $\det(A) = (1\times5\times9+2\times6\times7+3\times4\times8)-(2\times4\times9+1\times6\times8+3\times5\times7)$.

Provide feedback (0)

Step 2 of 3

Consider the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

Observe that $\boxed{2\times\text{Row 2} - \text{Row 3} = \text{Row 1}}$.

Therefore, the rows of the above matrix are not linearly independent. Therefore, this matrix is not invertible.

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Consider the matrix $\begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

Observe that $\boxed{2\times\text{Row 2} - \text{Row 3} = \text{Row 1}}$.

Therefore, the rows of the above matrix are not linearly independent. Therefore, this matrix is not invertible.

Provide feedback (0)

Step 3 of 3

Also, note the following:

$$\begin{aligned}\det(A) &= (1\times5\times9+2\times6\times7+3\times4\times8)-(2\times4\times9+1\times6\times8+3\times5\times7)\\ &= (45+84+96)-(72+48+105)\\ &= 225-225\\ &= \boxed{0}\end{aligned}$$

This also tells us that the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is not invertible.

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Step 1 of 3

Suppose A_n is the n by n tridiagonal matrix with 1s on the three diagonals:

$$A_1 = [1], \quad A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \ldots$$

Let D_n be the determinant of A_n.

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Step 2 of 3

$$A_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$, the five terms are non zero in det A_4 are given by

$$a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{11}a_{22}a_{34}a_{43} + a_{12}a_{21}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44}$$

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Step 3 of 3

Since $a_{11} = 1 = a_{12}, a_{21} = a_{22} = a_{23} = 1$

$a_{32} = a_{33} = a_{34} = 1, a_{43} = a_{44} = 1$

And remaining entries are zero.

Hence

$\det A_4 = 1 - 1 - 1 + 1 - 1$

$\det A_4 \boxed{= -1}$

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CH4.3 38P

Step 1 of 2

Given that
4×4 Tridiagoanl matrix with entries $-1,2,-1$, is given by

$$B_4 = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

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Step 2 of 2

Here also the five terms that are non zero in big formula for $\det B_4$ are given by

$$a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} + a_{12}a_{21}a_{34}a_{43}$$
$$= (2)(2)(2)(2) - (-1)(-1)(2)(2) - (2)(2)(-1)(-1) - 2(-1)(-1)(2) + (-1)(-1)(-1)(-1)$$
$$= 16 - 4 - 4 - 4 + 1$$
$$\boxed{= 5}$$

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Step 1 of 4

We have

$$P=\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$$

$$\det P=(-1)\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}$$

$$=-1 \quad \text{(expanding by first row with help of cofactors)}$$

to obtain identify from P.

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Step 2 of 4

We have

$$P=\xrightarrow[\text{interchange row3, row4 interchange}]{Row1,Row2}\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix}\xrightarrow[\text{interchange}]{Row2,Row4}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

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Step 3 of 4

We needed 3 exchanges of rows to get identity from P so that

$$\det P=(-1)^2\det I=(-1).1$$

$$=-1$$

$$P^2=\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\xrightarrow[\text{interchange Row2, Row4 interchange}]{Row3,Row1}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}=I$$

$$P = \xrightarrow[\text{interchange row3, row4 interchange}]{Row1, Row2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow[\text{interchange}]{Row2, Row4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Step 3 of 4

We needed 3 exchanges of rows to get identity from P so that

$$\det P = (-1)^2 \det I = (-1).1$$
$$= -1$$

$$P^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow[\text{interchange Row2, Row4 interchange}]{Row3, Row1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

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Step 4 of 4

So P^2 needed by only two interchanges to reach identify hence

$$\boxed{\det P^2 = \det I = 1}$$

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CH4.4

Step 1 of 4

If we consider a Pascal matrix, its determinant is always 1.

Consider

$$\begin{aligned}\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix} &= 2-1\\ &= 1\end{aligned}$$

$$\begin{aligned}\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 6\end{vmatrix} &= 1\begin{vmatrix}2 & 3\\ 3 & 6\end{vmatrix} - 1\begin{vmatrix}1 & 3\\ 1 & 6\end{vmatrix} + 1\begin{vmatrix}1 & 2\\ 1 & 3\end{vmatrix}\\ &= 1(12-9)-1(6-3)+1(3-2)\\ &= 3-3+1\\ &= 1\end{aligned}$$

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Step 2 of 4

In any $n\times n$ Pascal matrix, the largest entry is a_{nn}. If we remove the nth column and n^{th} row from an $n\times n$ Pascal matrix, the remaining matrix is also Pascal matrix of order $n-1\times n-1$. Therefore, the determinant of the matrix, obtained by removing the nth row and column from the $n\times n$ Pascal matrix is also 1.

Let the element in the last row be denoted by a_{n1}, a_{n2}, ..., a_{nn}. The cofactors of these elements be denoted by A_{n1}, A_{n2}, ..., A_{nn}.

Step 2 of 4

In any $n \times n$ Pascal matrix, the largest entry is a_{nn}. If we remove the nth column and n^{th} row from an $n \times n$ Pascal matrix, the remaining matrix is also Pascal matrix of order $n-1 \times n-1$. Therefore, the determinant of the matrix, obtained by removing the nth row and column from the $n \times n$ Pascal matrix is also 1.

Let the element in the last row be denoted by $a_{n1}, a_{n2}, \ldots, a_{nn}$. The cofactors of these elements be denoted by $A_{n1}, A_{n2}, \ldots, A_{nn}$.

Note that $\det(A_{nn}) = 1$.

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Step 3 of 4

If A is the Pascal matrix of order n by n, we get

$$\begin{aligned}
1 &= \det(A) \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + a_{nn}A_{nn} \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1 + 1)A_{nn} \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + A_{nn}
\end{aligned}$$

But then $A_{nn} = 1$. Therefore,

$$\begin{aligned}
1 &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + A_{nn} \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + 1 \\
0 &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn}
\end{aligned}$$

$$
\begin{aligned}
1 &= \det(A) \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + a_{nn}A_{nn} \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1 + 1)A_{nn} \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + A_{nn}
\end{aligned}
$$

But then $A_{nn} = 1$. Therefore,

$$
\begin{aligned}
1 &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + A_{nn} \\
&= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + 1 \\
0 &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn}
\end{aligned}
$$

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Step 4 of 4

Thus, if we make decrease the largest entry of the Pascal matrix by 1, the resultant matrix always has the determinant zero.

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