CH5.6

Step 1 of 3

Given the quadratic equation f is $f = x^2 + 4xy + 2y^2$
We need to write the given function as a difference of two squares.

Provide feedback (0)

Step 2 of 3

Comparing given quadratic with $f = ax^2 + 2bxy + cy^2$.
So, $a = 1,\ 2b = 4,\ c = 2$.
Now $a = 1 > 0$ and $ac - b^2 = -2 < 0$.
So origin is the saddle point.

Provide feedback (0)

Step 3 of 3

Now,

$$\begin{aligned} f &= x^2 + 4xy + 2y^2 \\ &= x^2 + 2(x)(2y) + 4y^2 - 2y^2 \\ &= (x+2y)^2 - \left(\sqrt{2}y\right)^2 \\ &= (x+2y)^2 - 2y^2 \end{aligned}$$

Therefore, $\boxed{f = (x+2y)^2 - \left(\sqrt{2}y\right)^2}$ (difference of two squares.)

Provide feedback (0)

CH6.1 2P

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CH6.1 1P

Step 1 of 9

(a).

Given matrix is $A=\begin{pmatrix}1 & 3\\ 3 & 5\end{pmatrix}$

Now the corresponding $f=x^T Ax$ is,

$$\begin{aligned} f &= x^T Ax \\ &= \begin{pmatrix} x & y\end{pmatrix}\begin{pmatrix}1 & 3\\ 3 & 5\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \\ &= \begin{pmatrix} x+3y & 3x+5y\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \\ &= (x+3y)x+(3x+5y)y \\ &= x^2+3xy+3xy+5y^2 \\ &= x^2+6xy+5y^2 \end{aligned}$$

Provide feedback (0)

Step 2 of 9

Comparing given quadratic with $f=ax^2+2bxy+cy^2$

So, $a=1,\ 2b=6,\ c=5.$

Now $a>0$ and $ac-b^2=-4<0.$

So $\boxed{f \text{ is not positive definite.}}$

Provide feedback (0)

Step 3 of 9

(b).

Given matrix is, $A=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

Now the corresponding $f=x^T Ax$ is,

$f=x^T Ax$

CH6.1 3P

Step 3 of 9

(b).

Given matrix is, $A=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

Now the corresponding $f=x^T Ax$ is,

$$\begin{aligned} f &= x^T Ax \\ &= \begin{pmatrix} x & y\end{pmatrix}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \\ &= \begin{pmatrix} x-y & -x+y\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \\ &= (x-y)x+(-x+y)y \\ &= x^2-xy-xy+y^2 \\ &= x^2-2xy+y^2 \end{aligned}$$

Provide feedback (0)

Step 4 of 9

Comparing given quadratic with $f=ax^2+2bxy+cy^2$

We get $a=1,\ 2b=-2,\ c=1.$

Now $a>0$ and $ac-b^2=0.$

So $\boxed{f \text{ is not positive definite.}}$

We now write f as

$$\begin{aligned} f &= x^2-2xy+y^2 \\ &= (x-y)^2 \end{aligned}$$

$f=0$

$\Rightarrow y=x.$

Thus along the line $\boxed{x=y, f(x,y)=0.}$

Provide feedback (0)

So $\boxed{f \text{ is not positive definite.}}$

We now write f as

$$f = x^2 - 2xy + y^2$$
$$= (x-y)^2$$
$$f = 0$$
$$\Rightarrow y = x.$$

Thus along the line $\boxed{x = y, f(x,y) = 0.}$

Provide feedback (0)

Step 5 of 9

(c).

Given matrix is $A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$

Now the corresponding $f = x^T Ax$ is,

$$f = x^T Ax$$
$$= \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \begin{pmatrix} 2x+3y & 3x+5y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= (2x+3y)x + (3x+5y)y$$
$$= 2x^2 + 3xy + 3xy + 5y^2$$
$$= 2x^2 + 6xy + 5y^2$$

Provide feedback (0)

Step 6 of 9

Comparing given quadratic with $f = ax^2 + 2bxy + cy^2$

We get $a = 2,\ 2b = 6,\ c = 5.$

Now $a > 0$ and $ac - b^2 = 1 > 0.$

So $\boxed{f \text{ is positive definite}}$

Provide feedback (0)

Step 6 of 9

Comparing given quadratic with $f = ax^2 + 2bxy + cy^2$

We get $a = 2,\ 2b = 6,\ c = 5.$

Now $a > 0$ and $ac - b^2 = 1 > 0.$

So $\boxed{f \text{ is positive definite.}}$

Provide feedback (0)

Step 7 of 9

(d).

Given matrix is $A = \begin{pmatrix} -1 & 2 \\ 2 & -8 \end{pmatrix}$

Now the corresponding $f = x^T Ax$ is

$$f = x^T Ax$$
$$= \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Provide feedback (0)

Step 8 of 9

$$= \begin{pmatrix} -x+2y & 2x-8y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= (-x+2y)x + (2x-8y)y$$
$$= -x^2 + 2xy + 2xy - 8y^2$$
$$= -x^2 + 4xy - 8y^2$$

Provide feedback (0)

Step 9 of 9

$$= \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Provide feedback (0)

Step 8 of 9

$$= \begin{pmatrix} -x+2y & 2x-8y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= (-x+2y)x + (2x-8y)y$$
$$= -x^2 + 2xy + 2xy - 8y^2$$
$$= -x^2 + 4xy - 8y^2$$

Provide feedback (0)

Step 9 of 9

Comparing given quadratic with $f = ax^2 + 2bxy + cy^2$

We get $a = -1,\ 2b = 4,\ c = -8.$

Now $a < 0$ and $ac - b^2 = 4 > 0.$

So $\boxed{f \text{ is negative definite.}}$

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CH6.1 2P

Step 1 of 5

Given symmetric is, $A=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$

Now,

$$A-\lambda I=\begin{pmatrix} a & b \\ b & c \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$=\begin{pmatrix} a-\lambda & b \\ b & c-\lambda \end{pmatrix}$$

Provide feedback (0)

Step 2 of 5

The characteristic equation is,

$$\det(A-\lambda I)=0$$
$$\Rightarrow\begin{vmatrix} a-\lambda & b \\ b & c-\lambda \end{vmatrix}=0$$
$$\Rightarrow(a-\lambda)(c-\lambda)-b^2=0$$
$$\Rightarrow ac-(a+c)\lambda+\lambda^2-b^2=0$$
$$\Rightarrow \lambda^2-(a+c)\lambda+(ac-b^2)=0$$
$$\Rightarrow \lambda=\frac{(a+c)\pm\sqrt{(a+c)^2-4(ac-b^2)}}{2}$$
$$=\frac{(a+c)\pm\sqrt{(a-c)^2+4b^2}}{2}$$

Therefore, the Eigen values

are $\boxed{\lambda_1=\frac{(a+c)+\sqrt{(a-c)^2+4b^2}}{2}}$ and $\boxed{\lambda_2=\frac{(a+c)-\sqrt{(a-c)^2+4b^2}}{2}}$.

Provide feedback (0)

CH6.1 4P

$$\begin{vmatrix} & b & c-\lambda \end{vmatrix}$$

$$\Rightarrow (a-\lambda)(c-\lambda)-b^2=0$$

$$\Rightarrow ac-(a+c)\lambda+\lambda^2-b^2=0$$

$$\Rightarrow \lambda^2-(a+c)\lambda+(ac-b^2)=0$$

$$\Rightarrow \lambda=\frac{(a+c)\pm\sqrt{(a+c)^2-4(ac-b^2)}}{2}$$

$$=\frac{(a+c)\pm\sqrt{(a-c)^2+4b^2}}{2}$$

Therefore, the Eigen values

are $\boxed{\lambda_1=\frac{(a+c)+\sqrt{(a-c)^2+4b^2}}{2}}$ and $\boxed{\lambda_2=\frac{(a+c)-\sqrt{(a-c)^2+4b^2}}{2}}$.

Provide feedback (0)

Step 3 of 5

Let

$$\lambda_1=\frac{(a+c)+\sqrt{(a-c)^2+4b^2}}{2} \quad \text{and} \quad \lambda_2=\frac{(a+c)-\sqrt{(a-c)^2+4b^2}}{2}$$

Given that $a>0,\ ac>b^2 \ \Rightarrow c>0.$

Therefore,

$a+c>0,\ (a-c)^2+4b^2>0$

Provide feedback (0)

Step 4 of 5

Thus $\lambda_1>0$

$$ac-b^2>0$$

$$\Rightarrow 4ac-4b^2>0$$

$$\Rightarrow (a^2+c^2+2ac)>(a^2+c^2-2ac)+4b^2$$

Step 4 of 5

Thus $\lambda_1 > 0$

$$ac - b^2 > 0$$
$$\Rightarrow 4ac - 4b^2 > 0$$
$$\Rightarrow \left(a^2 + c^2 + 2ac\right) > \left(a^2 + c^2 - 2ac\right) + 4b^2$$
$$\Rightarrow (a+c)^2 > (a-c)^2 + 4b^2$$
$$\Rightarrow (a+c) > \sqrt{(a-c)^2 + 4b^2}$$
$$\Rightarrow \lambda_2 > 0$$

Thus both the Eigen value of $\det(A - \lambda I) = 0$ are $\boxed{\text{positive}}$

Provide feedback (0)

Step 5 of 5

Moreover, the product of Eigen values.

$$\lambda_1 \lambda_2 = \frac{ac - b^2}{1}$$
$$= \boxed{ac - b^2}.$$

Therefore, the product of Eigen values is $\boxed{ac - b^2}$.

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CH6.1 3P

Step 1 of 4

(a).

Given function is $F(x,y)=-1+4(e^x-x)-5x\sin y+6y^2$ and given stationary point is $(x,y)=(0,0)$.

We need to decide between a minimum, maximum, or saddle point.

We need to find the first and second derivatives now.

$F_x=4e^x-4-5\sin y.$

$F_{xx}=4e^x$

$\Rightarrow (F_{xx})_{(0,0)}$

$\Rightarrow 4>0$

$F_y=-5x\cos y+12y$

$F_{yy}=5x\sin y+12$

$F_{xy}=-5\cos y$

Provide feedback (0)

CH6.1 5P

Step 2 of 4

Given the stationary point is $(x,y)=(0,0)$.

Now $(F_{xy})_{(0,0)}=-5$

And $(F_{yy})_{(0,0)}=12$

And,

$$(F_{xx})(F_{yy})=4(12)$$
$$=48$$

$(F_{xy})^2=25$

Clearly $(F_{xx})(F_{yy})>(F_{xy})^2$

So $F(x,y)$ has minimum at $(0,0)$.

Therefore, $\boxed{F(x,y) \text{ has minimum at } (0,0).}$

Step 2 of 4

Given the stationary point is $(x,y)=(0,0)$.

Now $(F_{xy})_{(0,0)}=-5$

And $(F_{yy})_{(0,0)}=12$

And,

$$(F_{xx})(F_{yy})=4(12)$$
$$=48$$
$$(F_{xy})^2=25$$

Clearly $(F_{xx})(F_{yy})>(F_{xy})^2$

So $F(x,y)$ has minimum at $(0,0)$.

Therefore, $\boxed{F(x,y) \text{ has minimum at } (0,0).}$

Provide feedback (0)

Step 3 of 4

(b).

Given function is $F(x,y)=(x^2-2x)\cos y$.

We need to decide between a minimum, maximum, or saddle point.
We need to find the first and second derivatives now.

$$F_x=(2x-2)\cos y$$
$$F_{xx}=2\cos y$$
$$F_y=-(x^2-2x)\sin y$$
$$F_{yy}=-(x^2-2x)\cos y$$
$$F_{xy}=-(2x-2)\sin y$$

Provide feedback (0)

Step 4 of 4

We need to find the first and second derivatives now.

$F_x = (2x-2)\cos y$

$F_{xx} = 2\cos y$

$F_y = -(x^2-2x)\sin y$

$F_{yy} = -(x^2-2x)\cos y$

$F_{xy} = -(2x-2)\sin y$

Provide feedback (0)

Step 4 of 4

Given stationary point is $(x,y)=(1,\pi)$.

Now,

$$(F_{xx})_{(1,\pi)} = 2\cos\pi$$
$$= -2 < 0$$

And,

$$(F_{xx})(F_{yy}) = (-2)(-1)$$
$$= 2$$
$$(F_{xy})^2 = 0$$

Clearly $(F_{xx})(F_{yy}) > (F_{xy})^2$

So $F(x,y)$ has maximum at $(1,\pi)$.

Therefore, $\boxed{F(x,y) \text{ has maximum at } (1,\pi).}$

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CH6.1 4P

Step 1 of 9

(a).

Given matrix is, $A=\begin{pmatrix}1 & b\\ b & 9\end{pmatrix}$

We know that the matrix $\begin{pmatrix}a & b\\ b & c\end{pmatrix}$ is positive definite if and only if $ac-b^2>0$ and $a>0$.

Therefore A is positive definite

Provide feedback (0)

Step 2 of 9

If $(1)(9)-b^2>0$

$\Rightarrow (b+3)(3-b)<0$ ..... (1)

If $b^2-9<0$

$(b+3)(b-3)<0$ ..... (2)

From the both equations, the range is $-3<b<3$

Therefore, $A$is $\boxed{\text{positive definite when } -3<b<3}$.

Provide feedback (0)

Step 3 of 9

(b).

$A=\begin{pmatrix}1 & b\\ b & 9\end{pmatrix}$

So, $L=\begin{pmatrix}1 & 0\\ 6 & 1\end{pmatrix}$

$A=\begin{pmatrix}1 & b\\ b & 9\end{pmatrix}$

$=\begin{pmatrix}1 & 0\\ b & 1\end{pmatrix}\begin{pmatrix}1 & b\\ 0 & 9-b^2\end{pmatrix}$

CH6.1 6P

Step 3 of 9

(b).

$$A=\begin{pmatrix}1 & b\\ b & 9\end{pmatrix}$$

So, $L=\begin{pmatrix}1 & 0\\ 6 & 1\end{pmatrix}$

$$\begin{aligned}A&=\begin{pmatrix}1 & b\\ b & 9\end{pmatrix}\\ &=\begin{pmatrix}1 & 0\\ b & 1\end{pmatrix}\begin{pmatrix}1 & b\\ 0 & 9-b^2\end{pmatrix}\\ &=\begin{pmatrix}1 & 0\\ b & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 9-b^2\end{pmatrix}\begin{pmatrix}1 & b\\ 0 & 1\end{pmatrix}\\ &=LDL^T\end{aligned}$$

Where L have is on the diagonal and D is the diagonal matrix of points.

Here b is in the range of positive definiteness.

Therefore, $\boxed{A=\begin{pmatrix}1 & 0\\ b & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 9-b^2\end{pmatrix}\begin{pmatrix}1 & b\\ 0 & 1\end{pmatrix}}$

Provide feedback (1)

Step 4 of 9

(c).

Let $F(x,y)=\frac{1}{2}\left(x^2+2bxy+9y^2\right)-y$

The first and second derivatives with respect to x are,

$$\begin{aligned}F_x&=\frac{1}{2}(2x+2by)\\ &=x+by\end{aligned}$$

$F_{xx}=1>0$

$a+c>2b,\ ac<b^2$

$$\begin{aligned}(a-c)^2&=(a+c)^2-4ab\\ &=4b^2-4b^2\end{aligned}$$

Step 4 of 9

(c).

Let $F(x,y)=\frac{1}{2}\left(x^2+2bxy+9y^2\right)-y$

The first and second derivatives with respect to x are,

$$F_x=\frac{1}{2}(2x+2by)$$
$$=x+by$$
$$F_{xx}=1>0$$
$$a+c>2b,\ ac<b^2$$
$$(a-c)^2=(a+c)^2-4ab$$
$$=4b^2-4b^2$$

Provide feedback (0)

Step 5 of 9

Now,

The first and second derivatives are,

$$F_y=\frac{1}{2}(2bx+18y)-1$$
$$F_{yy}=9$$
$$F_{xy}=b$$

Now,

$$(F_{xx})(F_{yy})=(1)(9)$$
$$=9$$
$$(F_{xy})^2=b^2$$
$$\Rightarrow(F_{xx})(F_{yy})-(F_{xy})^2>0$$
$$\Rightarrow 9>b^2$$
$$\Rightarrow b^2-9<0$$
$$\Rightarrow -3<b<3$$

Thus $F_{xx}>0$ and $(F_{xx})(F_{yy})-(F_{xy})^2>0$

Provide feedback (0)

Step 5 of 9

Now,
The first and second derivatives are,

$$F_y = \frac{1}{2}(2bx+18y)-1$$
$$F_{yy} = 9$$
$$F_{xy} = b$$

Now,

$$(F_{xx})(F_{yy}) = (1)(9)$$
$$= 9$$
$$(F_{xy})^2 = b^2$$
$$\Rightarrow (F_{xx})(F_{yy})-(F_{xy})^2 > 0$$
$$\Rightarrow 9 > b^2$$
$$\Rightarrow b^2 - 9 < 0$$
$$\Rightarrow -3 < b < 3$$

Thus $F_{xx} > 0$ and $(F_{xx})(F_{yy})-(F_{xy})^2 > 0$

Provide feedback (0)

Step 6 of 9

For b in this range.
Stationary points are given by,

$$F_x = 0 \text{ and } F_y = 0$$
$$\Rightarrow x+by = 0 \text{ and } bx+9y-1=0$$
$$\Rightarrow x = -by \text{ and } bx+9y-1=0$$
$$\Rightarrow b(-by)+9y-1=0$$
$$\Rightarrow (9-b^2)y = 1$$
$$\Rightarrow y = \frac{1}{(9-b^2)}$$

Therefore, $x = \frac{-b}{(9-b^2)}$ and $y = \frac{1}{(9-b^2)}$

$F_y = \frac{1}{2}(2bx+18y)-1$

$F_{yy} = 9$

$F_{xy} = b$

Now,

$(F_{xx})(F_{yy}) = (1)(9)$

$= 9$

$(F_{xy})^2 = b^2$

$\Rightarrow (F_{xx})(F_{yy}) - (F_{xy})^2 > 0$

$\Rightarrow 9 > b^2$

$\Rightarrow b^2 - 9 < 0$

$\Rightarrow -3 < b < 3$

Thus $F_{xx} > 0$ and $(F_{xx})(F_{yy}) - (F_{xy})^2 > 0$

Provide feedback (0)

Step 6 of 9

For b in this range.

Stationary points are given by,

$F_x = 0$ and $F_y = 0$

$\Rightarrow x + by = 0$ and $bx + 9y - 1 = 0$

$\Rightarrow x = -by$ and $bx + 9y - 1 = 0$

$\Rightarrow b(-by) + 9y - 1 = 0$

$\Rightarrow (9 - b^2)y = 1$

$\Rightarrow y = \frac{1}{(9-b^2)}$

Therefore, $x = \frac{-b}{(9-b^2)}$ and $y = \frac{1}{(9-b^2)}$

Provide feedback (0)

Thus $F_{xx} > 0$ and $(F_{xx})(F_{yy}) - (F_{xy})^2 > 0$

Provide feedback (0)

Step 6 of 9

For b in this range.
Stationary points are given by,

$F_x = 0$ and $F_y = 0$

$\Rightarrow x + by = 0$ and $bx + 9y - 1 = 0$

$\Rightarrow x = -by$ and $bx + 9y - 1 = 0$

$\Rightarrow b(-by) + 9y - 1 = 0$

$\Rightarrow (9 - b^2)y = 1$

$\Rightarrow y = \dfrac{1}{(9 - b^2)}$

Therefore, $x = \dfrac{-b}{(9 - b^2)}$ and $y = \dfrac{1}{(9 - b^2)}$

Provide feedback (0)

Step 7 of 9

So the minimum value of F is

$$= \frac{1}{2}\left(\frac{b^2}{(9-b^2)^2} + 2b\left(\frac{-b}{9-b^2}\right)\left(\frac{1}{9-b^2}\right) + 9\left(\frac{1}{9-b^2}\right)^2\right) - \frac{1}{9-b^2}$$

$$= \frac{1}{2}\left(\frac{b^2+9}{(9-b^2)^2}\right) - \frac{1}{9-b^2}$$

$$= -\frac{1}{2(9-b^2)}$$

Thus the stationary point is $= \left(\dfrac{-b}{9-b^2}, \dfrac{1}{9-b^2}\right)$.

Therefore, the minimum value of F is

$\Rightarrow y = \frac{1}{(9-b^2)}$

Therefore, $x = \frac{-b}{(9-b^2)}$ and $y = \frac{1}{(9-b^2)}$

Provide feedback (0)

Step 7 of 9

So the minimum value of F is

$$= \frac{1}{2}\left(\frac{b^2}{(9-b^2)^2} + 2b\left(\frac{-b}{9-b^2}\right)\left(\frac{1}{9-b^2}\right) + 9\left(\frac{1}{9-b^2}\right)^2\right) - \frac{1}{9-b^2}$$

$$= \frac{1}{2}\left(\frac{b^2+9}{(9-b^2)^2}\right) - \frac{1}{9-b^2}$$

$$= -\frac{1}{2(9-b^2)}$$

Thus the stationary point is $= \left(\frac{-b}{9-b^2}, \frac{1}{9-b^2}\right)$.

Therefore, the minimum value of F is

$$\boxed{-\frac{1}{2(9-b^2)} \text{ when } \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ which is } \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(9-b^2)}\begin{bmatrix} -b \\ 1 \end{bmatrix}}.$$

Provide feedback (0)

Step 8 of 9

(d).

Given $b = 3$.

Then minimum value is $-\frac{1}{0} = -\infty$.

Thus there is no minimum when $b = 3$.

Thus the stationary point is $=\left(\frac{-b}{9-b^2},\frac{1}{9-b^2}\right)$.

Therefore, the minimum value of F is

$$\boxed{-\frac{1}{2\left(9-b^2\right)}\text{ when }\begin{bmatrix}1 & b\\ b & 9\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 1\end{bmatrix}\text{ which is }\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{\left(9-b^2\right)}\begin{bmatrix}-b\\ 1\end{bmatrix}}.$$

Provide feedback (0)

Step 8 of 9

(d).

Given $b=3$.

Then minimum value is $-\frac{1}{0}=-\infty$.

Thus there is no minimum when $b=3$.

Provide feedback (0)

Step 9 of 9

$x+by=0$

$\Rightarrow x+3y=0$

$\Rightarrow x=-3y$

Let $y\to\infty$, $x=-3y$, then $x-y\to-\infty$.

Therefore, $\boxed{\text{No minimum. let } y\to\infty,\ x=-3y,\ \text{then } x-y\to-\infty}$.

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CH6.1 5P

Step 1 of 2

Consider $F(x,y)=x^2+10xy+10y^2$.

Compare this with $a^2+2bxy+cy^2$,

So, $a=1,\ 2b=10,\ c=10$

So that $a+c=10+1=11$ and $b=5$

Clearly $a+c>2b$ and $ac=10,\ b^2=25$

Provide feedback (0)

Step 2 of 2

Thus the required matrix is $\begin{pmatrix} a & b \\ b & c \end{pmatrix}=\begin{pmatrix} 1 & 5 \\ 5 & 10 \end{pmatrix}$

So,

$ac-b^2<0$

$\Rightarrow 10-25<0$

$\Rightarrow -15<0$

The matrix is $\boxed{\text{not positive definite.}}$

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CH6.1 7P

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CH6.1 6P

Step 1 of 4

(a).

Given that $f_1 = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$

The symmetric matrix corresponding to it is

$$A_1 = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 2 of 4

The matrix corresponding to the quadratic $ax_1^2 + bx_2^2 + cx_3^2 + 2hx_1x_2 + 2gx_1x_3 + 2fx_2x_3$ is

$$\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}.$$

Now the matrix A_2 corresponding to f_2 is

$$A_2 = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & -2 \\ -1 & -2 & 11 \end{pmatrix}.$$

Therefore, $\boxed{A_1 = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \text{ and } A_2 = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & -2 \\ -1 & -2 & 11 \end{pmatrix}}$.

Provide feedback (0)

Step 3 of 4

(b).

Given that $f_1 = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$

$$= (x_1 - x_2 - x_3)^2$$

CH6.1 8P

Provide feedback (0)

Step 3 of 4

(b).

Given that $f_1 = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$

$$= (x_1 - x_2 - x_3)^2$$

For $x_1 = 2,\ x_2 = x_3 = 1,$

$f_1(x_1, x_2, x_3) = 0$

Therefore f_1 is not positive definite.

Therefore, $\boxed{f_1 = (x_1 - x_2 - x_3)^2 = 0 \text{when } x_1 - x_2 - x_3 = 0}$.

Provide feedback (0)

Step 4 of 4

(c).

Now,

$$\begin{aligned} f_2 &= x_1^2 + 2x_2^2 + 11x_3^2 - 2x_1x_2 - 2x_1x_3 - 4x_2x_3 \\ &= x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3 - 2x_1x_3 + x_2^2 + 10x_3^2 - 6x_2x_3 \\ &= (x_1 - x_2 - x_3)^2 + x_2^2 + 9x_3^2 - 6x_2x_3 + x_3^2 \\ &= (x_1 - x_2 - x_3)^2 + (x_1 - 3x_3)^2 + x_3^2. \end{aligned}$$

Now we can write L as

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -3 & 1 \end{pmatrix}$$

So that,

$$\begin{aligned} A &= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \\ &= LL^T. \end{aligned}$$

Step 4 of 4

(c).

Now,

$$\begin{aligned} f_2 &= x_1^2 + 2x_2^2 + 11x_3^2 - 2x_1x_2 - 2x_1x_3 - 4x_2x_3 \\ &= x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3 - 2x_1x_3 + x_2^2 + 10x_3^2 - 6x_2x_3 \\ &= (x_1 - x_2 - x_3)^2 + x_2^2 + 9x_3^2 - 6x_2x_3 + x_3^2 \\ &= (x_1 - x_2 - x_3)^2 + (x_1 - 3x_3)^2 + x_3^2. \end{aligned}$$

Now we can write L as

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -3 & 1 \end{pmatrix}$$

So that,

$$\begin{aligned} A &= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \\ &= LL^T. \end{aligned}$$

Therefore, $\boxed{f_2 = (x_1 - x_2 - x_3)^2 + (x_1 - 3x_3)^2 + x_3^2;\ L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -3 & 1 \end{pmatrix}}$.

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CH6.1 7P

Step 1 of 3

Given that the matrix $A=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is positive definite

We need to test $A^{-1}=\begin{bmatrix} p & q \\ q & r \end{bmatrix}$ for positive definiteness.

So that $a>0$ and $ac-b^2>0$.
Also it implies that $c>0$.

Provide feedback (0)

Step 2 of 3

Now,

$$A^{-1}=\frac{1}{ac-b^2}\begin{pmatrix} c & -b \\ -b & a \end{pmatrix}$$
$$=\begin{pmatrix} \frac{c}{ac-b^2} & \frac{-b}{ac-b^2} \\ \frac{-b}{ac-b^2} & \frac{a}{ac-b^2} \end{pmatrix}$$
$$=\begin{bmatrix} p & q \\ q & r \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

As $c>0$ and $ac-b^2>0$,

So, $A=\frac{c}{ac-b^2}>0$,

And $AC-B^2=\left(\frac{c}{ac-b^2}\right)\left(\frac{a}{ac-b^2}\right)-\left(\frac{-b}{ac-b^2}\right)^2$

$$=\frac{ac}{(ac-b^2)^2}-\frac{b^2}{(ac-b^2)^2}$$

CH6.1 9P

Provide feedback (0)

Step 3 of 3

As $c > 0$ and $ac - b^2 > 0$,

So, $A = \dfrac{c}{ac - b^2} > 0$,

$$\begin{aligned}
\text{And } AC - B^2 &= \left(\frac{c}{ac-b^2}\right)\left(\frac{a}{ac-b^2}\right) - \left(\frac{-b}{ac-b^2}\right)^2 \\
&= \frac{ac}{(ac-b^2)^2} - \frac{b^2}{(ac-b^2)^2} \\
&= \frac{1}{ac-b^2} \\
&> 0 \qquad (\text{since } ac - b^2 > 0)
\end{aligned}$$

Therefore, $\boxed{\text{the matrix } A^{-1} = \begin{bmatrix} p & q \\ q & r \end{bmatrix} \text{ is also positive definite.}}$

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CH6.1 8P

Step 1 of 2

Given quadratic is $f(x_1, x_2) = 3(x_1 + 2x_2)^2 + 4x_2^2$

Positive definite is,

$$f(x_1, x_2) = 3x_1^2 + 16x_2^2 + 12x_1x_2$$
$$= 3x_1^2 + 6x_1x_2 + 6x_1x_2 + 16x_2^2$$

So the corresponding matrix is $A = \begin{pmatrix} 3 & 6 \\ 6 & 16 \end{pmatrix}$.

Provide feedback (0)

Step 2 of 2

Now we have to write A as LDL^T.

$$f(x_1, x_2) = 3(x_1 + 2x_2)^2 + 4(0.x_1 + 1.x_2)^2$$

Now the coefficients of the squares are the pivots in D and the coefficients inside the squares are columns of L.

Therefore,

$$A = \begin{pmatrix} 3 & 6 \\ 6 & 16 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$
$$= LDL^T$$

Therefore, $\boxed{A = \begin{pmatrix} 3 & 6 \\ 6 & 16 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}}$; the coefficients of the squares are the pivots in D and the coefficients inside the squares are columns of L.

Provide feedback (0)

CH6.1 10P

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CH6.1 9P

Step 1 of 2

Given the $R=\begin{pmatrix} p & q \\ r & s \end{pmatrix}$.

We need to write R^2 and need to check that it is positive definite unless R is singular.

$$R^2=\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix}$$
$$=\begin{pmatrix} p^2+q^2 & pq+qr \\ pq+qr & q^2+r^2 \end{pmatrix}$$

If R is singular, then $\det R=0$,

That is, $pq-r^2=0$.

Provide feedback (0)

Step 2 of 2

Clearly $p^2+q^2>0$,

And $\left(p^2+q^2\right)\left(q^2+r^2\right)-\left(pq+qr\right)^2$

$$\Rightarrow p^2q^2+p^2r^2+q^4+q^2r^2-q^2\left(p+q\right)^2$$
$$\Rightarrow p^2q^2+p^2r^2+q^4+q^2r^2-q^2p^2-q^2r^2-2prq^2$$
$$\Rightarrow p^2r^2-prq^2-prq^2+q^4$$
$$\Rightarrow pr\left(pr-q^2\right)-q^2\left(pr-q^2\right)$$
$$\Rightarrow \left(pr-q^2\right)\left(pr-q^2\right)$$
$$\Rightarrow \left(pr-q^2\right)^2$$

Therefore, $\boxed{\text{both the conditions of positive definite are satisfied unless } R \text{ is singular.}}$

Provide feedback (0)

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CH6.1 10P

CH6.1 12P

Step 1 of 8

Given that $A=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is Hermitian (complex b)

(a) We need to find the pivots and determinant of the Hermitian.

$$A=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

Apply this row operation $R_2 \to R_2 - \frac{b}{a}R_1$

$$\begin{pmatrix} a & b \\ 0 & c-\frac{|b|^2}{a} \end{pmatrix}$$

So the pivots of A are a and $c-\frac{|b|^2}{a}$

Provide feedback (0)

Step 2 of 8

The determinant is,

$$\det A = a\left(c-\frac{|b|^2}{a}\right)$$
$$= ac-|b|^2$$

Therefore, the pivots of A are $\boxed{a \text{ and } c-\frac{|b|^2}{a}}$ and the determinant is $\boxed{ac-|b|^2}$.

Provide feedback (0)

Step 3 of 8

(b).

We need to complete the square for $x^H Ax$.

$$= ac - |b|^2$$

Therefore, the pivots of A are $\boxed{a \text{ and } c - \frac{|b|^2}{a}}$ and the determinant is $\boxed{ac - |b|^2}$.

Provide feedback (0)

Step 3 of 8

(b).

We need to complete the square for $x^H Ax$.

Given $x^H = \begin{pmatrix} \bar{x}_1 & \bar{x}_2 \end{pmatrix}$ can be complex.

So,

$$x^H Ax = \begin{pmatrix} \bar{x}_1 & \bar{x}_2 \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} a\bar{x}_1 + b\bar{x}_2 & b\bar{x}_1 + c\bar{x}_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \left(a\bar{x}_1 + b\bar{x}_2\right) x_1 + \left(b\bar{x}_1 + c\bar{x}_2\right) x_2$$

$$= a\bar{x}_1 x_1 + b\bar{x}_2 x_1 + b\bar{x}_1 x_2 + c\bar{x}_2 x_2$$

$$= a\bar{x}_1 x_1 + b\left(\bar{x}_2 x_1 + \bar{x}_1 x_2\right) + c\bar{x}_2 x_2$$

$$= a|\bar{x}_1|^2 + b.2\operatorname{Re}\bar{x}_1 x_2 + c|\bar{x}_2|^2$$

Therefore, the value of $x^H Ax$ is $\boxed{a|\bar{x}_1|^2 + b.2\operatorname{Re}\bar{x}_1 x_2 + c|\bar{x}_2|^2}$.

Provide feedback (0)

Step 4 of 8

Now we complete the square by using pivots a and $c - \frac{|b|^2}{a}$.

Therefore

$$a|\bar{x}_1|^2 + 2\operatorname{Re} b\bar{x}_1 x_2 + c|\bar{x}_2|^2$$

Therefore, the value of $x^H Ax$ is $\boxed{a|\bar{x}_1|^2 + b.2\operatorname{Re}\bar{x}_1 x_2 + c|\bar{x}_2|^2}$.

Provide feedback (0)

Step 4 of 8

Now we complete the square by using pivots a and $c - \frac{|b|^2}{a}$.

Therefore

$$a|\bar{x}_1|^2 + 2\operatorname{Re} b\bar{x}_1 x_2 + c|\bar{x}_2|^2$$

$$= a\left|x_1 + \left(\frac{b}{a}\right)x_2\right|^2 + \left(c - \frac{|b|^2}{a}\right)|\bar{x}_2|^2.$$

Therefore, $a|\bar{x}_1|^2 + b.2\operatorname{Re}\bar{x}_1 x_2 + c|\bar{x}_2|^2 == a\left|x_1 + \left(\frac{b}{a}\right)x_2\right|^2 + \boxed{\left(c - \frac{|b|^2}{a}\right)}|\bar{x}_2|^2$.

Provide feedback (0)

Step 5 of 8

(c).

As $x^H Ax$ is a sum of squares, so the pivots are greater than zero.

$a > 0$ and $c - \frac{|b|^2}{a} > 0$

$\Rightarrow a > 0$ and $ac - |b|^2 > 0$.

$\Rightarrow a > 0$ and $ac > |b|^2$.

Now these facts ensure that $\boxed{A \text{ is positive definite}}$.

Provide feedback (0)

Step 6 of 8

(d).

$a > 0$ and $c - \frac{|b|^2}{a} > 0$

$\Rightarrow a > 0$ and $ac - |b|^2 > 0$.

$\Rightarrow a > 0$ and $ac > |b|^2$.

Now these facts ensure that $\boxed{A \text{ is positive definite}}$.

Provide feedback (0)

Step 6 of 8

(d).

Let $B = \begin{pmatrix} 1 & 1+i \\ 1-i & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 4+i \\ 4-i & 6 \end{pmatrix}$.

$$\begin{aligned} \det B &= 2-(1-i)(1+i) \\ &= 2-(1+1) \\ &= 2-2 \\ &= 0 \end{aligned}$$

Therefore, $\boxed{B \text{ is positive semi definite.}}$

Provide feedback (0)

Step 7 of 8

$$\begin{aligned} \det C &= 18-(4-i)(4+i) \\ &= 18-(16+1) \\ &= 18-17 \\ &= 1 \end{aligned}$$

Provide feedback (0)

Step 8 of 8

Therefore, $\boxed{C \text{ is positive in definite.}}$

.

$$\begin{aligned}\det B &= 2-(1-i)(1+i)\\ &= 2-(1+1)\\ &= 2-2\\ &= 0\end{aligned}$$

Therefore, $\boxed{B \text{ is positive semi definite.}}$

Provide feedback (0)

Step 7 of 8

$$\begin{aligned}\det C &= 18-(4-i)(4+i)\\ &= 18-(16+1)\\ &= 18-17\\ &= 1\end{aligned}$$

Provide feedback (0)

Step 8 of 8

Therefore, $\boxed{C \text{ is positive in definite.}}$

.

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CH6.1 11P

Step 1 of 5

Given the function $F = x^2y^2 - 2x - 2y$

So the derivatives are,

$F_x = 2xy^2 - 2$

$F_{xx} = 2y^2$

$F_y = 2x^2y - 2$

$F_{yy} = 2x^2$

$F_{xy} = 4xy$

Provide feedback (0)

Step 2 of 5

We need to find the critical points,

In order to find the critical points, make the first derivatives equals to zero.

$F_x = 0 \Rightarrow 2xy^2 - 2 = 0$

$\Rightarrow xy^2 - 1 = 0$ ..... (1)

$F_y = 0 \Rightarrow 2x^2y - 2 = 0$

$\Rightarrow x^2y - 1 = 0$ ..... (2)

Provide feedback (0)

Step 3 of 5

Equation (1) − equation (2) = 0

$\Rightarrow (xy^2 - 1) - (x^2y - 1) = 0$

$\Rightarrow xy^2 - x^2y = 0$

$\Rightarrow xy(y - x) = 0$

$\Rightarrow x = y$ or $xy = 0$

Provide feedback (0)

CH6.1 13P

$\Rightarrow (xy^2 - 1) - (x^2y - 1) = 0$

$\Rightarrow xy^2 - x^2y = 0$

$\Rightarrow xy(y - x) = 0$

$\Rightarrow x = y$ or $xy = 0$

Provide feedback (0)

Step 4 of 5

Using $x = y$, plug this in equation (1),

$xy^2 - 1 = 0$

$\Rightarrow x^3 = 1$

$\Rightarrow x = 1$

So the values of x and y are 1 and 1.

Provide feedback (0)

Step 5 of 5

Now

$$(F_{xx})(F_{yy})_{(1,1)} = (2)(2)$$
$$= 4$$
$$(F_{xy})_{(1,1)} = 4(1)(1)$$
$$= 4$$
$$(F_{xy})^2_{(1,1)} = 16.$$

Thus the condition $(F_{xx})(F_{yy}) > (F_{xy})^2$ is not satisfied at $(1,1)$.

So F has no maximum at $(1,1)$.

Therefore, F has $\boxed{\text{no maximum at } (1,\ 1)}$.

Provide feedback (1)

CH6.1 12P

Step 1 of 2

Given $ax^2+2bxy+cy^2>x^2+y^2$.

$\Rightarrow (a-1)x^2+2bxy+(c-1)y^2>0\ \forall x,y.$

Provide feedback (0)

CH6.1 14P

Step 2 of 2

Let $F=(a-1)x^2+2bxy+(c-1)y^2$

We have to find conditions on a,b,c for which F is positive definite.

We know that $ax^2+2bxy+cy^2$ is positive definite if $a>0$ and $ac>b^2$.

So the condition on a,b,c for which F is positive definite are

$(a-1)>0$ and $(a-1)(c-1)>b^2$.

Therefore, the condition on a,b,c for which F is positive definite are

$\boxed{a>1 \text{ and } (a-1)(c-1)>b^2}$.

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Step 1 of 6

Consider the matrix A_1 as shown below:

$$A_1 = \begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix}$$

In this matrix,

$a = 5$
$b = 6$
$c = 7$

Since $a > 0$ but $ac < b^2$, the matrix A_1 has one positive and one negative eigenvalue.

Provide feedback (0)

Step 2 of 6

Consider the matrix A_2 as shown below:

In this matrix,

Since and, the matrix has negative eigenvalues.

Provide feedback (0)

Step 3 of 6

Consider the matrixas shown below:

In this matrix,

CH6.1 15P

Consider the matrix A_2 as shown below:

$$A_2 = \begin{bmatrix} -1 & -2 \\ -2 & -5 \end{bmatrix}$$

In this matrix,

$a = -1$
$b = -2$
$c = -5$

Since $a < 0$ and $ac > b^2$, the matrix A_2 has negative eigenvalues.

Provide feedback (0)

Step 3 of 6

Consider the matrix A_3 as shown below:

$$A_3 = \begin{bmatrix} 1 & 10 \\ 10 & 100 \end{bmatrix}$$

In this matrix,

$a = 1$
$b = 10$
$c = 100$

Since $a > 0$ and $ac = b^2$, the matrix A_3 has one positive and one zero eigenvalue.

Provide feedback (0)

Step 4 of 6

Consider the matrix A_4 as shown below:

$$A_4 = \begin{bmatrix} 1 & 10 \\ 10 & 101 \end{bmatrix}$$

Step 3 of 6

Consider the matrix A_3 as shown below:

$$A_3 = \begin{bmatrix} 1 & 10 \\ 10 & 100 \end{bmatrix}$$

In this matrix,

$$a = 1$$
$$b = 10$$
$$c = 100$$

Since $a > 0$ and $ac = b^2$, the matrix A_3 has one positive and one zero eigenvalue.

Provide feedback (0)

Step 4 of 6

Consider the matrix A_4 as shown below:

$$A_4 = \begin{bmatrix} 1 & 10 \\ 10 & 101 \end{bmatrix}$$

In this matrix,

$$a = 1$$
$$b = 10$$
$$c = 101$$

Since $a > 0$ and $ac > b^2$, the matrix A_4 has positive eigenvalues.

Provide feedback (0)

Step 5 of 6

We want a vector *x*, such that $x^{\mathrm{T}} A_1 x < 0$. Let $x = (x_1, x_2)^{\mathrm{T}}$.

Therefore, we get

Step 4 of 6

Consider the matrix A_4 as shown below:

$$A_4 = \begin{bmatrix} 1 & 10 \\ 10 & 101 \end{bmatrix}$$

In this matrix,

$$a = 1$$
$$b = 10$$
$$c = 101$$

Since $a > 0$ and $ac > b^2$, the matrix A_4 has positive eigenvalues.

Provide feedback (0)

Step 5 of 6

We want a vector x, such that $x^{\mathrm{T}} A_1 x < 0$. Let $x = (x_1, x_2)^{\mathrm{T}}$.

Therefore, we get

$$\begin{aligned} x^{\mathrm{T}} A_1 x &< 0 \\ (x_1, x_2)\begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &< 0 \\ (x_1, x_2)\begin{bmatrix} 5x_1 + 6x_2 \\ 6x_1 + 7x_2 \end{bmatrix} &< 0 \\ 5x_1^2 + 6x_1x_2 + 6x_1x_2 + 7x_2^2 &< 0 \end{aligned}$$

This gives $5x_1^2 + 12x_1x_2 + 7x_2^2 < 0$. Note that the quantities $5x_1^2$ and $7x_2^2$ cannot be negative. Therefore, in order to satisfy this inequality, the necessary condition is that exactly one of the quantities x_1 and x_2 should be negative and the other should be positive.

Fix $x_1 = a$ and let $x_2 = x$. We will vary x so that $5a^2 + 12ax + 7x^2 < 0$.

Observe the following:

Provide feedback (0)

Step 5 of 6

We want a vector x, such that $x^{\mathrm{T}}A_1x<0$. Let $x=(x_1,x_2)^{\mathrm{T}}$.

Therefore, we get

$$\begin{aligned}
x^{\mathrm{T}}A_1x&<0\\
(x_1,x_2)\begin{bmatrix}5&6\\6&7\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}&<0\\
(x_1,x_2)\begin{bmatrix}5x_1+6x_2\\6x_1+7x_2\end{bmatrix}&<0\\
5x_1^2+6x_1x_2+6x_1x_2+7x_2^2&<0
\end{aligned}$$

This gives $5x_1^2+12x_1x_2+7x_2^2<0$. Note that the quantities $5x_1^2$ and $7x_2^2$ cannot be negative. Therefore, in order to satisfy this inequality, the necessary condition is that exactly one of the quantities x_1 and x_2 should be negative and the other should be positive.

Fix $x_1=a$ and let $x_2=x$. We will vary x so that $5a^2+12ax+7x^2<0$.

Observe the following:

$$\begin{aligned}
5a^2+12ax+7x^2&<0\\
7x^2+12ax+5a^2&<0\\
\left(\sqrt{7}x\right)^2+2\left(\sqrt{7}\right)\left(\frac{6}{\sqrt{7}}\right)ax+\frac{36a^2}{7}-\frac{a^2}{7}&<0\\
\left(\sqrt{7}x\right)^2+2\left(\sqrt{7}\right)\left(\frac{6}{\sqrt{7}}\right)ax+\frac{36a^2}{7}&<\frac{a^2}{7}\\
\left(x\sqrt{7}+\frac{6a}{\sqrt{7}}\right)^2&<\left(\frac{a}{\sqrt{7}}\right)^2
\end{aligned}$$

Therefore, whenever $x\sqrt{7}+\frac{6a}{\sqrt{7}}<\frac{a}{\sqrt{7}}$, we are through. That is, $x\sqrt{7}<-\frac{5a}{\sqrt{7}}$. This is same as $x<-\frac{5a}{}$.

$$(x_1, x_2)\begin{bmatrix} & \\ 6 & 7 \end{bmatrix}\begin{bmatrix} \\ x_2 \end{bmatrix} < 0$$

$$(x_1, x_2)\begin{bmatrix} 5x_1 + 6x_2 \\ 6x_1 + 7x_2 \end{bmatrix} < 0$$

$$5x_1^2 + 6x_1x_2 + 6x_1x_2 + 7x_2^2 < 0$$

This gives $5x_1^2 + 12x_1x_2 + 7x_2^2 < 0$. Note that the quantities $5x_1^2$ and $7x_2^2$ cannot be negative. Therefore, in order to satisfy this inequality, the necessary condition is that exactly one of the quantities x_1 and x_2 should be negative and the other should be positive.

Fix $x_1 = a$ and let $x_2 = x$. We will vary x so that $5a^2 + 12ax + 7x^2 < 0$.

Observe the following:

$$5a^2 + 12ax + 7x^2 < 0$$

$$7x^2 + 12ax + 5a^2 < 0$$

$$\left(\sqrt{7}x\right)^2 + 2\left(\sqrt{7}\right)\left(\frac{6}{\sqrt{7}}\right)ax + \frac{36a^2}{7} - \frac{a^2}{7} < 0$$

$$\left(\sqrt{7}x\right)^2 + 2\left(\sqrt{7}\right)\left(\frac{6}{\sqrt{7}}\right)ax + \frac{36a^2}{7} < \frac{a^2}{7}$$

$$\left(x\sqrt{7} + \frac{6a}{\sqrt{7}}\right)^2 < \left(\frac{a}{\sqrt{7}}\right)^2$$

Therefore, whenever $x\sqrt{7} + \frac{6a}{\sqrt{7}} < \frac{a}{\sqrt{7}}$, we are through. That is, $x\sqrt{7} < -\frac{5a}{\sqrt{7}}$. This is same as $x < -\frac{5a}{7}$.

Provide feedback (0)

Step 6 of 6

Therefore, if $x_1 = a$, x_2 should be less than $-\frac{5a}{7}$. Let $x_1 = 7$ and $x_2 = -6$. Now observe the following:

$$x^T A_1 x = (7, -6)\begin{bmatrix} 5 & 6 \end{bmatrix}\begin{bmatrix} 7 \end{bmatrix}$$

Fix $x_1 = a$ and let $x_2 = x$. We will vary x so that $5a^2 + 12ax + 7x^2 < 0$.

Observe the following:

$$\begin{aligned} 5a^2 + 12ax + 7x^2 &< 0 \\ 7x^2 + 12ax + 5a^2 &< 0 \\ \left(\sqrt{7}x\right)^2 + 2\left(\sqrt{7}\right)\left(\frac{6}{\sqrt{7}}\right)ax + \frac{36a^2}{7} - \frac{a^2}{7} &< 0 \\ \left(\sqrt{7}x\right)^2 + 2\left(\sqrt{7}\right)\left(\frac{6}{\sqrt{7}}\right)ax + \frac{36a^2}{7} &< \frac{a^2}{7} \\ \left(x\sqrt{7} + \frac{6a}{\sqrt{7}}\right)^2 &< \left(\frac{a}{\sqrt{7}}\right)^2 \end{aligned}$$

Therefore, whenever $x\sqrt{7} + \frac{6a}{\sqrt{7}} < \frac{a}{\sqrt{7}}$, we are through. That is, $x\sqrt{7} < -\frac{5a}{\sqrt{7}}$. This is same as $x < -\frac{5a}{7}$.

Provide feedback (0)

Step 6 of 6

Therefore, if $x_1 = a$, x_2 should be less than $-\frac{5a}{7}$. Let $x_1 = 7$ and $x_2 = -6$. Now observe the following:

$$\begin{aligned} x^{\mathrm{T}} A_1 x &= (7, -6)\begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -6 \end{bmatrix} \\ &= (7, -6)\begin{bmatrix} -1 \\ 0 \end{bmatrix} \\ &= -7 \end{aligned}$$

Thus, we have obtained $\boxed{x^{\mathrm{T}} A_1 x < 0}$.

Provide feedback (0)

CH6.1 14P

Step 1 of 4

Consider the given matrix $A=\begin{pmatrix}1 & 2\\ 2 & 9\end{pmatrix}$.

Let $X^T=\begin{pmatrix}x & y\end{pmatrix}$ so that $X=\begin{pmatrix}x\\ y\end{pmatrix}$.

Now X^TAX

$$=\begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}1 & 2\\ 2 & 9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

$$=\begin{pmatrix}x+2y & 2x+9y\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

$$=(x+2y)x+(2x+9y)y$$

$$=x^2+4xy+9y^2$$

Provide feedback (0)

CH6.1 16P

Step 2 of 4

Thus the quadratic for the given metro A is $x^2+4xy+9y^2$.

Now,

$$\begin{aligned} f &= x^2+4xy+9y^2\\ &= x^2+2(x)(2y)+4y^2-4y^2+9y^2\\ &= (x+2y)^2+5y^2\\ &= d_1(x+2y)^2+d_2y^2 \end{aligned}$$

So that $d_1=1$ and $d_2=5$.

Therefore, the quadratic $f\boxed{=x^2+4xy+9y^2=(x+2y)^2+5y^2}$.

Provide feedback (0)

Step 3 of 4

$(1 \quad 3)$

Step 2 of 4

Thus the quadratic for the given metro A is $x^2+4xy+9y^2$.
Now,

$$\begin{aligned} f &= x^2+4xy+9y^2 \\ &= x^2+2(x)(2y)+4y^2-4y^2+9y^2 \\ &= (x+2y)^2+5y^2 \\ &= d_1(x+2y)^2+d_2y^2 \end{aligned}$$

So that $d_1=1$ and $d_2=5$.

Therefore, the quadratic f $\boxed{= x^2+4xy+9y^2=(x+2y)^2+5y^2}$.

Provide feedback (0)

Step 3 of 4

Now consider the second matrix, $A=\begin{pmatrix}1 & 3\\ 3 & 9\end{pmatrix}$.

$$\begin{aligned} X^TAX &= \begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}1 & 3\\ 3 & 9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \\ &= \begin{pmatrix}x+3y & 3x+9y\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} \\ &= (x+3y)x(3x+9y)y \\ &= x^2+6xy+9y^2 \end{aligned}$$

Provide feedback (0)

Step 4 of 4

Thus the quadratic for the given matrix A is, $x^2+6xy+9y^2$.
Now,

$$\begin{aligned} f &= x^2+6xy+9y^2 \\ &= x^2+2(x)(2y)+(3y)^2 \\ &= (x+3y)^2 \end{aligned}$$

$$= x^2 + 2(x)(2y) + 4y^2 - 4y^2 + 9y^2$$
$$= (x+2y)^2 + 5y^2$$
$$= d_1(x+2y)^2 + d_2y^2$$

So that $d_1 = 1$ and $d_2 = 5$.

Therefore, the quadratic $f \boxed{= x^2 + 4xy + 9y^2 = (x+2y)^2 + 5y^2}$.

Provide feedback (0)

Step 3 of 4

Now consider the second matrix, $A = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$.

$$X^T AX = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \begin{pmatrix} x+3y & 3x+9y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= (x+3y)x(3x+9y)y$$
$$= x^2 + 6xy + 9y^2$$

Provide feedback (0)

Step 4 of 4

Thus the quadratic for the given matrix A is, $x^2 + 6xy + 9y^2$.

Now,

$$f = x^2 + 6xy + 9y^2$$
$$= x^2 + 2(x)(2y) + (3y)^2$$
$$= (x+3y)^2$$

Therefore, the quadratic $f \boxed{= x^2 + 6xy + 9y^2 = (x+3y)^2}$.

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CH6.1 15P

Step 1 of 4

Given the quadratic form $f(x,y)=x^2+4xy+3y^2$.

We need to show that f does not have a minimum at $(0,0)$ even though it has positive coefficients, need to write f as a difference of squares and find a point (x,y) where f is negative.

Provide feedback (0)

Step 2 of 4

The matrix of the quadratic is,

$$A=\begin{pmatrix}1 & 2\\ 2 & 3\end{pmatrix}$$
$$=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$$

Comparing above two matrices,

So, $a=1$, $b=2$, and $c=3$.

Now,

$ac-b^2=-1<0$.

Therefore, $\boxed{f(x,y)\text{ is not positive definite,}}$ so f $\boxed{\text{does not have a minimum at }(0,0)}$ even though it has positive coefficients,

Provide feedback (0)

Step 3 of 4

$$\begin{aligned}f(x,y)&=x^2+4xy+3y^2\\&=x^2+2.x.(2y)+4y^2-y^2\\&=(x+2y)^2-y^2\end{aligned}$$

Therefore, $\boxed{f(x,y)\text{ is a difference of squares.}}$

CH6.1 17P

$$A = \begin{pmatrix} & \\ 2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

Comparing above two matrices,
So, $a = 1$, $b = 2$, and $c = 3$.
Now,
$ac - b^2 = -1 < 0$.
Therefore, $\boxed{f(x,y) \text{ is not positive definite,}}$ so f $\boxed{\text{does not have a minimum at } (0,0)}$ even though it has positive coefficients,

Provide feedback (0)

Step 3 of 4

$$\begin{aligned} f(x,y) &= x^2 + 4xy + 3y^2 \\ &= x^2 + 2.x.(2y) + 4y^2 - y^2 \\ &= (x+2y)^2 - y^2 \end{aligned}$$

Therefore, $\boxed{f(x,y) \text{ is a difference of squares.}}$

Provide feedback (0)

Step 4 of 4

For $(x,y) = (2,-1)$,

$$\begin{aligned} f(x,y) &= f(2,-1) \\ &= 0 - 1 \\ &= -1 < 0 \end{aligned}$$

Thus at $(2,-1)$, $f(x,y)$ is negative.

Therefore, the point is $\boxed{(2,-1)}$. $f(x,y)$ is negative.

Provide feedback (0)

CH6.1 16P

Step 1 of 2

Given that if A has independent columns, and then $A^T A$ is square and symmetric and invertible.

So,

$$x^T A^T A x = (Ax)^T (Ax)$$

$$= \text{length squared}$$

So, $x^T A^T A x = 0$ only if $Ax = 0$

Provide feedback (0)

Step 2 of 2

Given that A have independent columns

So, $Ax = 0$ only when $x = 0$

Therefore, $A^T A$ is positive definite.

Provide feedback (0)

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Step 1 of 7

Given matrix $A=\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$.

Now $A^T=\begin{pmatrix}1 & 0\\ 2 & 3\end{pmatrix}$

$$A^TA=\begin{pmatrix}1 & 0\\ 2 & 3\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$$

$$=\begin{pmatrix}1 & 2\\ 2 & 13\end{pmatrix}$$

Provide feedback (0)

Step 2 of 7

Compare this with $\begin{pmatrix}a & b\\ b & c\end{pmatrix}$,

So, $a=1,\ b=2,\ c=13$.

Clearly $a=1>0$ and $ac-b^2=13-4=9>0$

Thus the matrix $\boxed{A^TA \text{ is positive definite.}}$

Provide feedback (0)

Step 3 of 7

Given second matrix is $A=\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 1\end{pmatrix}$

$$A^TA=\begin{pmatrix}1 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 1\end{pmatrix}$$

$$=\begin{pmatrix}6 & 5\end{pmatrix}$$

CH6.1 19P

Provide feedback (0)

Step 3 of 7

Given second matrix is $A=\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 1\end{pmatrix}$

$$A^TA=\begin{pmatrix}1 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 1\end{pmatrix}$$

$$=\begin{pmatrix}6 & 5\\ 5 & 6\end{pmatrix}$$

Provide feedback (0)

Step 4 of 7

Compare this with $\begin{pmatrix}a & b\\ b & c\end{pmatrix}$,

So, $a=6,\ b=5,\ c=6.$

Clearly $a=6>0$ and $ac-b^2=6(6)-25=11>0$

Thus the matrix $\boxed{A^TA \text{ is positive definite.}}$

Provide feedback (0)

Step 5 of 7

Given third matrix is $A=\begin{pmatrix}1 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}$

$$A^TA=\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 1\end{pmatrix}\begin{pmatrix}1 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}$$

$$\begin{pmatrix}2 & 3 & 3\end{pmatrix}$$

Compare this with $\begin{pmatrix} b & c \end{pmatrix}$,

So, $a = 6,\ b = 5,\ c = 6.$

Clearly $a = 6 > 0$ and $ac - b^2 = 6(6) - 25 = 11 > 0$

Thus the matrix $\boxed{A^T A \text{ is positive definite.}}$

Provide feedback (0)

Step 5 of 7

Given third matrix is $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$

$$A^T A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 & 3 \\ 3 & 5 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

Provide feedback (0)

Step 6 of 7

Let $X^T = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$, then

$$X^T A X = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 3 & 5 & 4 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= 2x_1^2 + 5x_2^2 + 5x_3^2 + 6x_1x_2 + 6x_1x_3 + 8x_2x_3$$

Provide feedback (0)

Step 7 of 7

$\begin{pmatrix} 1 & 1 & 2 \end{pmatrix}$

$$\begin{pmatrix} 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 & 3 \\ 3 & 5 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

Provide feedback (0)

Step 6 of 7

Let $X^T = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$, then

$$X^T AX = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}\begin{pmatrix} 2 & 3 & 3 \\ 3 & 5 & 4 \\ 3 & 4 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= 2x_1^2 + 5x_2^2 + 5x_3^2 + 6x_1x_2 + 6x_1x_3 + 8x_2x_3$$

Provide feedback (0)

Step 7 of 7

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

Apply this operation $R_2 \to R_2 - R_1$

$$= \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$$

Clearly A have independent columns.

So $A^T A$ is square and symmetric and invertible.

Therefore, $\boxed{A^T A \text{is positive definite.}}$

Provide feedback (1)

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Step 1 of 3

Given that $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 4\left(x_1 - x_2 + 2x_3\right)^2$

So,

$$= 4\left(x_1^2 + x_2^2 + 4x_3^2 - 2x_1x_2 - 4x_2x_3 + 4x_1x_3\right)$$
$$= 4x_1^2 + 4x_2^2 + 16x_3^2 - 8x_1x_2 - 8x_2x_3 + 16x_1x_3$$
$$= Ax_1^2 + Bx_2^2 + Cx_3^2 + 2Hx_1x_2 + 2Gx_2x_3 + 2Fx_1x_3$$

Provide feedback (0)

Step 2 of 3

Now the corresponding matrix A is,

$$A = \begin{pmatrix} A & H & G \\ H & B & F \\ G & F & C \end{pmatrix}$$
$$= \begin{pmatrix} 4 & -4 & 8 \\ -4 & 4 & -8 \\ 8 & -8 & 16 \end{pmatrix}$$

Apply row operations,

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + (-2)R_1$

$$= \begin{pmatrix} 4 & -4 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So A has only one point $= 4$

and $Rank(A) =$ number of non-zero rows

$= 1$.

Also $\det A = 0$ (since A has zero rows)

CH6.1 20P

Step 2 of 3

Now the corresponding matrix A is,

$$A=\begin{pmatrix} A & H & G \\ H & B & F \\ G & F & C \end{pmatrix}$$

$$=\begin{pmatrix} 4 & -4 & 8 \\ -4 & 4 & -8 \\ 8 & -8 & 16 \end{pmatrix}$$

Apply row operations,

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + (-2) R_1$

$$=\begin{pmatrix} 4 & -4 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So A has only one point $=4$

and $Rank(A) =$ number of non-zero rows

$= 1.$

Also $\det A = 0$ (since A has zero rows)

Provide feedback (1)

Step 3 of 3

Eigen values of A are,

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 4-\lambda & -4 & 8 \\ -4 & 4-\lambda & -8 \\ 8 & -8 & 16-\lambda \end{vmatrix} = 0$$

Now applying the transformation

$R_1 \to R_1 + R_2$

$$\Rightarrow \begin{vmatrix} -\lambda & -\lambda & 0 \\ -4 & 4-\lambda & -8 \\ 8 & -8 & 16-\lambda \end{vmatrix} = 0$$

Step 3 of 3

Eigen values of A are,

$|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 4-\lambda & -4 & 8 \\ -4 & 4-\lambda & -8 \\ 8 & -8 & 16-\lambda \end{vmatrix}=0$$

Now applying the transformation

$R_1 \to R_1+R_2$

$$\Rightarrow \begin{vmatrix} -\lambda & -\lambda & 0 \\ -4 & 4-\lambda & -8 \\ 8 & -8 & 16-\lambda \end{vmatrix}=0$$

$R_3 \to R_3+2R_2$

$$\Rightarrow \begin{vmatrix} -\lambda & -\lambda & 0 \\ 0-4 & 4-\lambda & -8 \\ 8 & -2\lambda & -\lambda \end{vmatrix}=0$$

$$\Rightarrow (-\lambda)(-\lambda)\begin{vmatrix} 1 & 1 & 0 \\ -4 & 4-\lambda & -8 \\ 8 & -2 & 1 \end{vmatrix}=0$$

$\Rightarrow \lambda^2\left((4-\lambda)+16+1(+4)+0\right)=0$

$\Rightarrow \lambda^2(24-\lambda)=0$

$\Rightarrow \lambda=0,0,24.$

Thus the Eigen values are $24,0,0.$

Therefore,

$\boxed{A \text{ has only one point, rank } = 1, \text{ determinant is } 0 \text{ and Eigen values are } 24,0,0.}$

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Step 1 of 5

Consider the function,

$$F_1(x,y)=\frac{1}{4}x^4+x^2y+y^2$$

By taking the partial derivatives, we get,

$$\frac{\partial F_1(x,y)}{\partial x}=x^3+2xy$$
$$\frac{\partial^2 F_1(x,y)}{\partial x^2}=3x^2+2y$$
$$\frac{\partial^2 F_1(x,y)}{\partial x\partial y}=2x$$

By taking the partial derivatives, we get,

$$\frac{\partial F_1(x,y)}{\partial y}=x^2+2y$$
$$\frac{\partial^2 F_1(x,y)}{\partial y^2}=2$$
$$\frac{\partial^2 F_1(x,y)}{\partial y\partial x}=2x$$

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Step 2 of 5

The second derivative matrix A_1 is given by,

$$A_1=\begin{bmatrix}\frac{\partial^2 F_1(x,y)}{\partial x^2} & \frac{\partial^2 F_1(x,y)}{\partial x\partial y}\\ \end{bmatrix}$$

Step 2 of 5

The second derivative matrix A_1 is given by,

$$A_1 = \begin{bmatrix} \dfrac{\partial^2 F_1(x,y)}{\partial x^2} & \dfrac{\partial^2 F_1(x,y)}{\partial x \partial y} \\ \dfrac{\partial^2 F_1(x,y)}{\partial y \partial x} & \dfrac{\partial^2 F_1(x,y)}{\partial y^2} \end{bmatrix}$$

On substitution, we get,

$$A_1 = \begin{bmatrix} 3x^2 + 2y & 2x \\ 2x & 2 \end{bmatrix}$$

By equating $\dfrac{\partial F_1(x,y)}{\partial x}$ to zero, we get,

$$\begin{aligned} \frac{\partial F_1(x,y)}{\partial x} &= 0 \\ x^3 + 2xy &= 0 \\ x\left(x^2 + 2y\right) &= 0 \\ x &= 0 \\ y &= 0 \end{aligned}$$

Provide feedback (0)

Step 3 of 5

Consider the function,

$$F_2(x,y) = x^3 + xy - x$$

$$x(x^2+2y)=0$$
$$x=0$$
$$y=0$$

Provide feedback (0)

Step 3 of 5

Consider the function,

$$F_2(x,y)=x^3+xy-x$$

By taking the partial derivatives, we get,

$$\frac{\partial F_1(x,y)}{\partial x}=3x^2+y-1$$
$$\frac{\partial^2 F_1(x,y)}{\partial x^2}=6x$$
$$\frac{\partial^2 F_1(x,y)}{\partial x\partial y}=0$$

By taking the partial derivatives, we get,

$$\frac{\partial F_2(x,y)}{\partial y}=x$$
$$\frac{\partial^2 F_1(x,y)}{\partial y^2}=0$$
$$\frac{\partial^2 F_1(x,y)}{\partial y\partial x}=1$$

The second derivative matrix A_2 is given by,

$$A_2=\begin{bmatrix}\frac{\partial^2 F_2(x,y)}{\partial x^2} & \frac{\partial^2 F_2(x,y)}{\partial x\partial y}\end{bmatrix}$$

$$\frac{\partial F_1(x,y)}{\partial y \partial x} = 1$$

The second derivative matrix A_2 is given by,

$$A_2 = \begin{bmatrix} \frac{\partial^2 F_2(x,y)}{\partial x^2} & \frac{\partial^2 F_2(x,y)}{\partial x \partial y} \\ \frac{\partial^2 F_2(x,y)}{\partial y \partial x} & \frac{\partial^2 F_2(x,y)}{\partial y^2} \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

On substitution, we get,

$$A_2 = \begin{bmatrix} 6x & 0 \\ 1 & 0 \end{bmatrix}$$

By equating $\frac{\partial F_1(x,y)}{\partial x}$ to zero, we get,

$$\begin{aligned} \frac{\partial F_1(x,y)}{\partial x} &= 0 \\ x^3 + 2xy &= 0 \\ x\left(x^2 + 2y\right) &= 0 \\ x &= 0 \\ y &= 0 \end{aligned}$$

Provide feedback (0)

Step 5 of 5

$$\partial x$$

$$x^3 + 2xy = 0$$

$$x\left(x^2 + 2y\right) = 0$$

$$x = 0$$

$$y = 0$$

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Step 5 of 5

Thus, the minimum point of F_1 is $\boxed{(0,0)}$ and the saddle point of F_2 is $\boxed{(0,0)}$.

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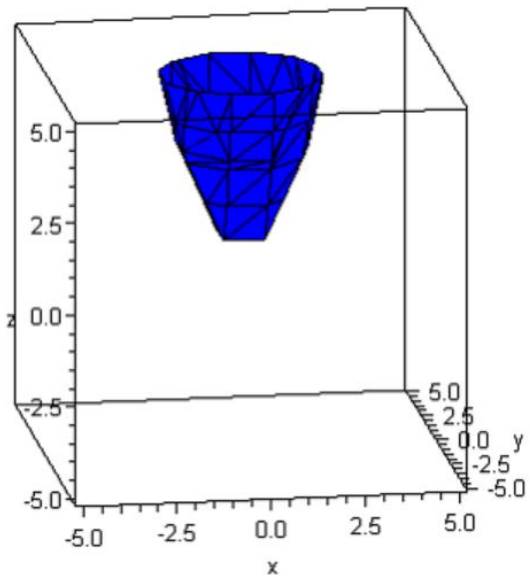
Step 1 of 4

The graph of $z = x^2 + y^2$ is a bowl opening upward as given below.



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Step 2 of 4

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Step 3 of 4

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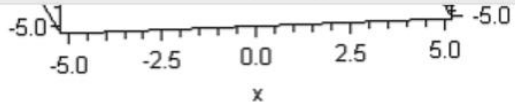

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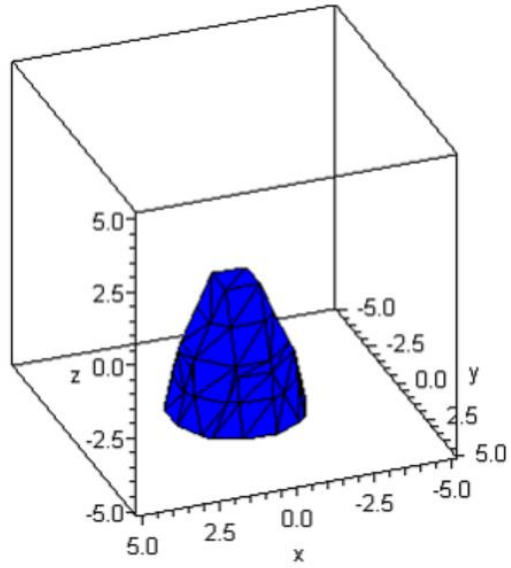
Step 2 of 4

The graph of $z = -x^2 - y^2$ is a bowl opening down ward as given below:



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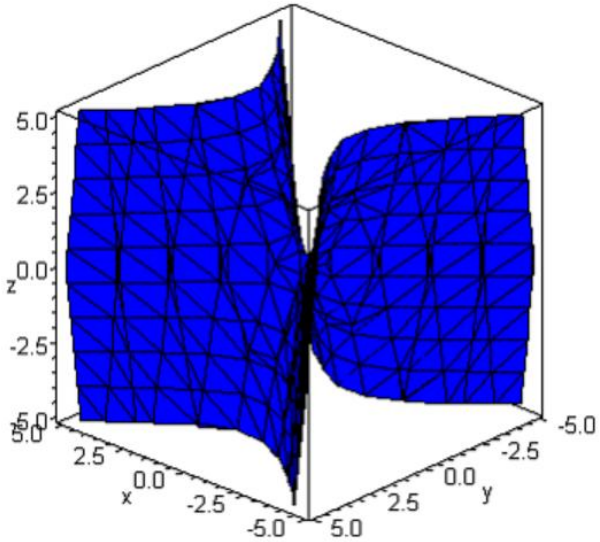
Step 3 of 4

The graph of $z = x^2 - y^2$ is a saddle as given below:



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Step 4 of 4

$ax^2+2bxy+cy^2$ has a saddle point at $(0,0)$ if $ac<b^2$ and the matrix is indefinite, that is, the Eigen values of the matrix are both positive and negative.

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Step 1 of 2

Given quadratic is $z = 4x^2 + 12xy + cy^2$.

Comparing with $ax^2 + 2bxy + cy^2$,

So, $a = 4,\ 2b = 12,\ c = c$.

For saddle point the condition is,

$ac - b^2 < 0$

$\Rightarrow (4)(c) - (6)^2 < 0$

$\Rightarrow 4c - 36 < 0$

$\Rightarrow c < 9$

Thus if $\boxed{c > 9}$, $4x^2 + 12xy + cy^2$ is positive definite and hence graph of z is a bowl.

Provide feedback (0)

Step 2 of 2

If $c < 9$,

$$z = 4x^2 + 12xy + cy^2$$
$$= (2x + 3y)^2$$

Thus if $\boxed{c = 9}$, then the graph of z is a trough staying at zero on this line $2x + 3y = 0$.

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CH6.2

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