CH6.2

CH6.3 2P

Step 1 of 5

Consider the following matrix:

$$A=\begin{pmatrix}1 & 4\\ 2 & 8\end{pmatrix}$$

Then,

$$A^T=\begin{pmatrix}1 & 2\\ 4 & 8\end{pmatrix}$$

Now,

$$\begin{aligned}A^TA&=\begin{pmatrix}1 & 2\\ 4 & 8\end{pmatrix}\begin{pmatrix}1 & 4\\ 2 & 8\end{pmatrix}\\ &=\begin{pmatrix}1+4 & 4+16\\ 4+16 & 16+64\end{pmatrix}\\ &=\boxed{\begin{pmatrix}5 & 20\\ 20 & 80\end{pmatrix}}\end{aligned}$$

Provide feedback (0)

Step 2 of 5

Now the characteristic equation is,

$$\begin{aligned}|A-I\lambda|&=0\\ \begin{vmatrix}5-\lambda & 20\\ 20 & 80-\lambda\end{vmatrix}&=0\\ (5-\lambda)(80-\lambda)-(20)(20)&=0\\ 400-85\lambda+\lambda^2-400&=0\end{aligned}$$

$$\begin{aligned}\lambda^2-85\lambda&=0\\ \lambda(\lambda-85)&=0\end{aligned}$$

Step 2 of 5

Now the characteristic equation is,

$$|A - I\lambda| = 0$$

$$\begin{vmatrix} 5-\lambda & 20 \\ 20 & 80-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(80-\lambda)-(20)(20)=0$$

$$400-85\lambda+\lambda^2-400=0$$

$$\lambda^2-85\lambda=0$$

$$\lambda(\lambda-85)=0$$

$$\lambda = 0 \text{ or } 85$$

Therefore, the Eigen values are 0 and 85.

Now,

$$\sigma_1^2 = 85$$

$$\sigma_1 = \sqrt{85}$$

Therefore, $A^T A = \begin{pmatrix} 5 & 20 \\ 20 & 80 \end{pmatrix}$ has only $\boxed{\sigma_1^2 = 85}$.

Provide feedback (0)

Step 3 of 5

Let $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be Eigen vector corresponding to $\lambda_2 = 85$.

Then,

$$\begin{pmatrix} 5-85 & 20 \\ 20 & 80-85 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -80 & 20 \\ 20 & -5 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Now,

$$\sigma_1^2 = 85$$
$$\sigma_1 = \sqrt{85}$$

Therefore, $A^T A = \begin{pmatrix} 5 & 20 \\ 20 & 80 \end{pmatrix}$ has only $\boxed{\sigma_1^2 = 85}$.

Provide feedback (0)

Step 3 of 5

Let $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be Eigen vector corresponding to $\lambda_2 = 85.$

Then,

$$\begin{pmatrix} 5-85 & 20 \\ 20 & 80-85 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} -80 & 20 \\ 20 & -5 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} -80 & 20 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (\text{Apply } R_2 \to R_1 + 4R_2)$$
$$-4y_1 + y_2 = 0$$

Hence, the Eigen vector is $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

Now the unit Eigen vector corresponding to $\lambda_2 = 85$ is $\boxed{v_2 = \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{pmatrix}}$.

Provide feedback (0)

$$-4y_1 + y_2 = 0$$

Hence, the Eigen vector is $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

Now the unit Eigen vector corresponding to $\lambda_2 = 85$ is $\boxed{v_2 = \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{pmatrix}}$.

Provide feedback (0)

Step 4 of 5

Let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be Eigen vector corresponding to Eigen value $\lambda_1 = 0$.

So,

$$\begin{pmatrix} 5-0 & 20 \\ 20 & 80-0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 20 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (\text{Apply } R_2 \to R_2 - 4R_1)$$

$$5x_1 + 20x_2 = 0$$

$$x_1 + 4x_2 = 0$$

Therefore, the Eigen vector is $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Thus, the unit Eigen vector corresponding to $\lambda_1 = 0$ is $\boxed{v_1 = \begin{pmatrix} \frac{4}{\sqrt{17}} \\ \frac{-1}{\sqrt{17}} \end{pmatrix}}$.

Provide feedback (0)

Provide feedback (0)

Step 4 of 5

Let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be Eigen vector corresponding to Eigen value $\lambda_1 = 0.$

So,

$$\begin{pmatrix} 5-0 & 20 \\ 20 & 80-0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 20 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (\text{Apply } R_2 \to R_2 - 4R_1)$$

$$5x_1 + 20x_2 = 0$$

$$x_1 + 4x_2 = 0$$

Therefore, the Eigen vector is $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Thus, the unit Eigen vector corresponding to $\lambda_1 = 0$ is $\boxed{v_1 = \begin{pmatrix} \frac{4}{\sqrt{17}} \\ \frac{-1}{\sqrt{17}} \end{pmatrix}}$.

Provide feedback (0)

Step 5 of 5

Therefore, $\boxed{A^T A = \begin{pmatrix} 5 & 20 \\ 20 & 80 \end{pmatrix} \text{has only } \sigma_1^2 = 85 \text{ with } v_2 = \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{pmatrix}, \text{so } v_2 = \begin{pmatrix} \frac{4}{\sqrt{17}} \\ \frac{-1}{\sqrt{17}} \end{pmatrix}}$.

Provide feedback (0)

CH6.3 1P

Step 1 of 4

Consider the matrix:

$$A=\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix}$$

Thus,

$$A^T=\begin{bmatrix}1 & 2\\ 4 & 8\end{bmatrix}$$

Therefore, AA^T is given by,

$$\begin{aligned}A^TA&=\begin{bmatrix}1 & 2\\ 4 & 8\end{bmatrix}\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix}\\ &=\begin{bmatrix}1+4 & 4+16\\ 4+16 & 16+64\end{bmatrix}\\ &=\begin{bmatrix}5 & 20\\ 20 & 80\end{bmatrix}\end{aligned}$$

The Eigen values are given by,

$$\begin{aligned}\begin{vmatrix}17-\lambda & 34\\ 34 & 68-\lambda\end{vmatrix}&=0\\ (17-\lambda)(68-\lambda)&=34\times 34\\ 17\times 68-17\lambda-68\lambda+\lambda^2&=34\times 34\\ \lambda^2-85\lambda+17\times 17(4-4)&=0\\ \lambda(\lambda-85)&=0\\ \lambda&=85,0\end{aligned}$$

CH6.3 3P

Provide feedback (0)

$$17\times 68-17\lambda-68\lambda+\lambda^2=34\times 34$$
$$\lambda^2-85\lambda+17\times 17(4-4)=0$$
$$\lambda(\lambda-85)=0$$
$$\lambda=85,0$$

Provide feedback (0)

Step 2 of 4

Therefore, we get,

$$\sigma_1=\sqrt{85}$$
$$\sigma_2=0$$

The unit eigenvectors are given by,

$$v_1=\begin{bmatrix}\frac{1}{\sqrt{17}}\\ \frac{4}{\sqrt{17}}\end{bmatrix}$$
$$v_2=\begin{bmatrix}-\frac{4}{\sqrt{17}}\\ \frac{1}{\sqrt{17}}\end{bmatrix}$$

The unit eigenvectors are given by,

$$u_1=\frac{1}{\sigma_1}Av_1$$
$$=\frac{1}{\sqrt{85}}\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{17}}\\ \frac{4}{\sqrt{17}}\end{bmatrix}$$

$$v_1 = \begin{bmatrix} \frac{4}{\sqrt{17}} \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -\frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{bmatrix}$$

The unit eigenvectors are given by,

$$\begin{aligned} u_1 &= \frac{1}{\sigma_1} A v_1 \\ &= \frac{1}{\sqrt{85}} \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{bmatrix} \\ &= \frac{1}{\sqrt{85}} \begin{bmatrix} \frac{1}{\sqrt{17}} + \frac{16}{\sqrt{17}} \\ \frac{2}{\sqrt{17}} + \frac{32}{\sqrt{17}} \end{bmatrix} \\ &= \frac{1}{\sqrt{85}} \begin{bmatrix} \sqrt{17} \\ 2\sqrt{17} \end{bmatrix} \\ u_1 &= \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 3 of 4

Consider the equation:

Provide feedback (1)

Provide feedback (0)

Step 3 of 4

Consider the equation:

Provide feedback (1)

Step 4 of 4

$$\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{85} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{17}} & -\frac{4}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} & \frac{1}{\sqrt{17}} \end{bmatrix}^T$$

$$= \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{85} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{17}} & \frac{4}{\sqrt{17}} \\ -\frac{4}{\sqrt{17}} & \frac{1}{\sqrt{17}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 4\sqrt{5} \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$$

Thus, $\boxed{A^T A = \begin{bmatrix} 5 & 20 \\ 20 & 80 \end{bmatrix}}$, $\boxed{v_1 = \begin{bmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{bmatrix}, v_2 = \begin{bmatrix} -\frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{bmatrix}}$, and $\boxed{u_1 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}, u_1 = \begin{bmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}}$

Provide feedback (0)

CH6.3 2P

Step 1 of 4

Consider the matrix:

$$A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

Thus,

$$A^T=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

Therefore, A^TA is given by,

$$\begin{aligned}A^TA&=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\\ &=\begin{bmatrix}1+1 & 1+0\\ 1+0 & 1+0\end{bmatrix}\\ &==\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\end{aligned}$$

The Eigen values are given by,

$$\begin{aligned}\begin{vmatrix}2-\lambda & 1\\ 1 & 1-\lambda\end{vmatrix}&=0\\ (2-\lambda)(1-\lambda)&=1\\ 2-2\lambda-\lambda+\lambda^2&=1\\ \lambda^2-3\lambda+1&=0\\ \lambda&=\frac{3\pm\sqrt{9-4}}{2}\\ &=\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2}\end{aligned}$$

CH6.3 4P

$$\begin{bmatrix} 1+0 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

The Eigen values are given by,

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(1-\lambda) = 1$$

$$2 - 2\lambda - \lambda + \lambda^2 = 1$$

$$\lambda^2 - 3\lambda + 1 = 0$$

$$\lambda = \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$$

Provide feedback (0)

Step 2 of 4

Therefore, we get,

$$\sigma_1^2 = \frac{3+\sqrt{5}}{2}$$

$$\sigma_2^2 = \frac{3-\sqrt{5}}{2}$$

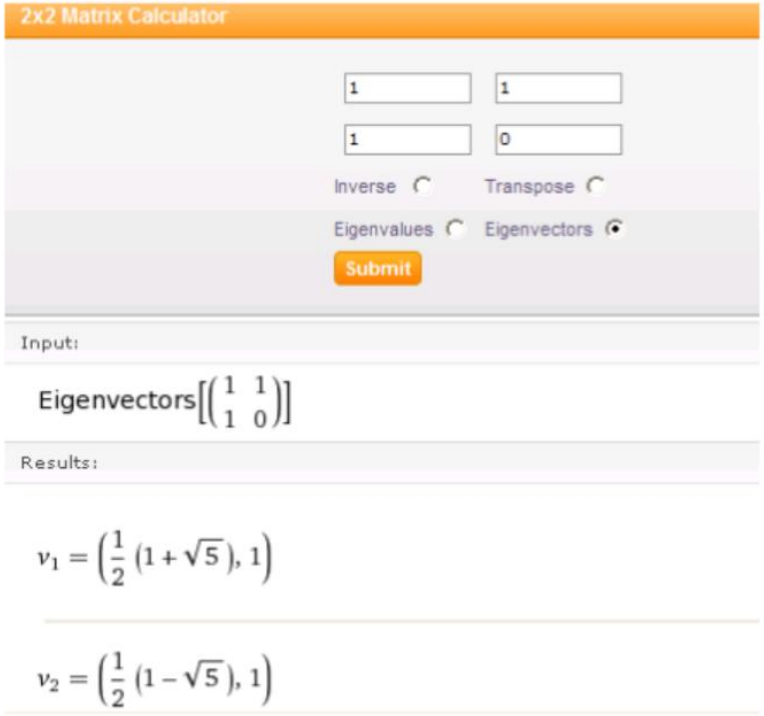
By using matrix calculator (the screenshot is given below), the eigenvectors are given by,

Step 2 of 4

Therefore, we get,

$$\sigma_1^2 = \frac{3+\sqrt{5}}{2}$$
$$\sigma_2^2 = \frac{3-\sqrt{5}}{2}$$

By using matrix calculator (the screenshot is given below), the eigenvectors are given by,



Since $\frac{1}{2}\left(1-\sqrt{5}\right)$ is negative, $\sigma_1 = \frac{1}{2}\left(1+\sqrt{5}\right)$ but $\sigma_2 = -\frac{1}{2}\left(1-\sqrt{5}\right)$.

The unit eigenvectors are given by,

Input:

Eigenvectors$\left[\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\right]$

Results:

$$v_1 = \left(\frac{1}{2}\left(1+\sqrt{5}\right), 1\right)$$

$$v_2 = \left(\frac{1}{2}\left(1-\sqrt{5}\right), 1\right)$$

Since $\frac{1}{2}\left(1-\sqrt{5}\right)$ is negative, $\sigma_1 = \frac{1}{2}\left(1+\sqrt{5}\right)$ but $\sigma_2 = -\frac{1}{2}\left(1-\sqrt{5}\right)$.

The unit eigenvectors are given by,

Provide feedback (1)

Step 3 of 4

$$v_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

$$v_2 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

Step 3 of 4

$$v_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

$$v_2 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

Therefore, we get,

$$u_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

$$u_2 = -\begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

Therefore, we get,

$$u_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

$$u_2 = -\begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

Thus, $\boxed{A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}}$, $\boxed{v_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}, v_2 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}}$, and

$$\begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix} \quad \begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

$$u_2 = -\begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

Thus, $\boxed{A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}}$, $\boxed{v_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}, v_2 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}}$, and

$$\boxed{u_1 = \begin{bmatrix} \dfrac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2+1}} \end{bmatrix}, u_2 = -\begin{bmatrix} \dfrac{\frac{1}{2}\left(1-\sqrt{5}\right)}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \\ \dfrac{1}{\sqrt{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^2+1}} \end{bmatrix}}$$

Provide feedback (0)

Chapter: CH6.3 Problem: 4P

show all steps

CH6.3 3P

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CH6.3 5P

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CH6.3 4P

CH6.3 6P

Step 1 of 8

Consider the matrix:

$$A=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

Thus,

$$A^T=\begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix}$$

Therefore, A^TA is given by,

$$\begin{aligned}A^TA&=\begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\\ &=\begin{bmatrix}1+0 & 1+0 & 0\\ 1+0 & 1+1 & 0+1\\ 0 & 0+1 & 0+1\end{bmatrix}\\ &=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 0 & 1 & 1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 2 of 8

Similarly, AA^T is given by,

$$AA^T=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

CH6.3 4P

CH6.3 6P

Step 1 of 8

Consider the matrix:

$$A=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

Thus,

$$A^T=\begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix}$$

Therefore, A^TA is given by,

$$\begin{aligned}A^TA&=\begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\\ &=\begin{bmatrix}1+0 & 1+0 & 0\\ 1+0 & 1+1 & 0+1\\ 0 & 0+1 & 0+1\end{bmatrix}\\ &=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 0 & 1 & 1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 2 of 8

Similarly, AA^T is given by,

$$AA^T=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

Step 2 of 8

Similarly, AA^T is given by,

$$AA^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+1+0 & 0+1+0 \\ 0+1+0 & 0+1+1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

The eigenvalues of $A^T A$ are given by,

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$
$$(1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0$$
$$(1-\lambda)\left[(2-\lambda)(1-\lambda)-1\right]-(1-\lambda)=0$$
$$(1-\lambda)\left[(2-\lambda)(1-\lambda)-1-1\right]=0$$
$$(1-\lambda)\left[\lambda^2-3\lambda\right]=0$$
$$(1-\lambda)(\lambda-3)\lambda=0$$

Therefore, the eigenvalues are 1, 3, and 0.

Provide feedback (0)

Step 3 of 8

The eigenvalues of AA^T are given by,

Step 3 of 8

The eigenvalues of AA^T are given by,

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$
$$(2-\lambda)(2-\lambda)-1=0$$
$$4-4\lambda+\lambda^2-1=0$$
$$\lambda^2-4\lambda+3=0$$
$$(\lambda-3)(\lambda-1)=0$$
$$\lambda=3,1$$

Therefore, the eigenvalues are 3, and 1.

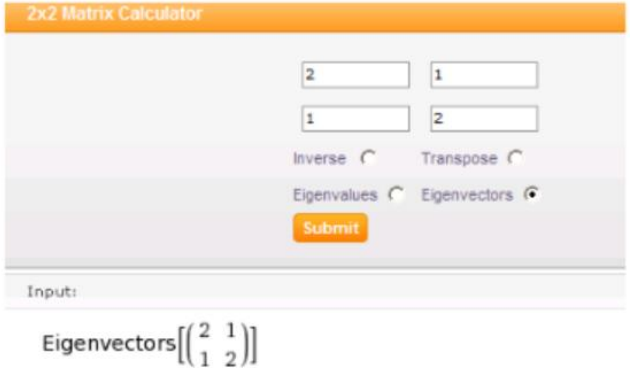
Therefore, we get,

$$\sigma_1^2=3$$
$$\sigma_2^2=1$$

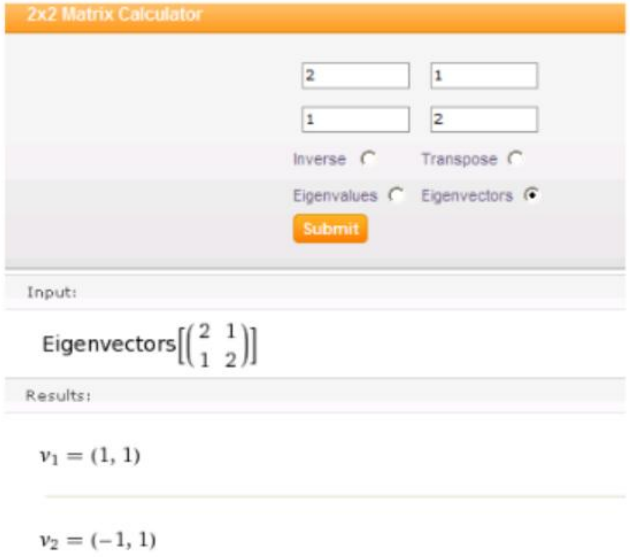
Provide feedback (0)

Step 4 of 8

By using matrix calculator (the screenshot is given below), the eigenvectors of AA^T are given by,



By using matrix calculator (the screenshot is given below), the eigenvectors of AA^T are given by,



The eigenvectors of AA^T are given by,

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The unit eigenvectors of AA^T are given by,

$$u_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$u_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The unit eigenvectors of AA^T are given by,

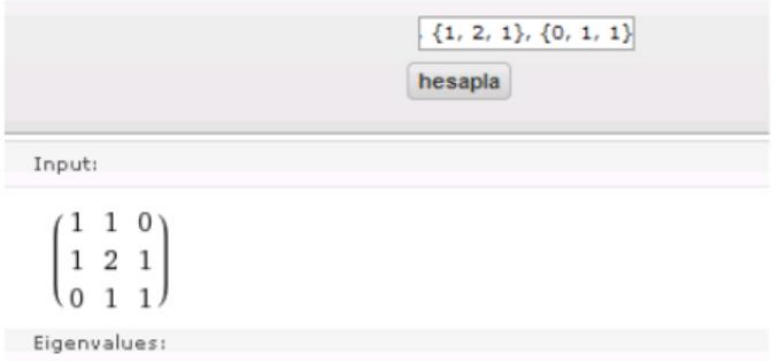
$$u_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$u_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

By using matrix calculator (the screenshot is given below), the eigenvectors of $A^T A$ are given by,

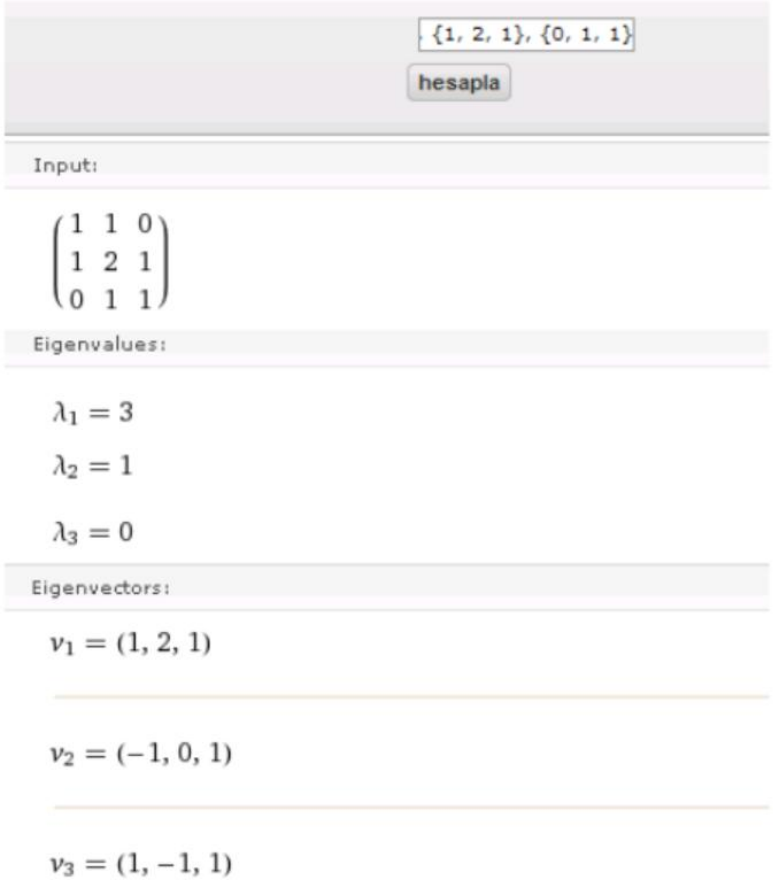


$$u_2 = \begin{bmatrix} \sqrt{2} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

By using matrix calculator (the screenshot is given below), the eigenvectors of $A^T A$ are given by,



$$\begin{pmatrix} \cdot & \cdot & \cdot \\ 0 & 1 & 1 \end{pmatrix}$$

Eigenvalues:

$\lambda_1 = 3$

$\lambda_2 = 1$

$\lambda_3 = 0$

Eigenvectors:

$v_1 = (1, 2, 1)$

$v_2 = (-1, 0, 1)$

$v_3 = (1, -1, 1)$

The eigenvectors of $A^T A$ are given by,

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

$$v_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

The unit eigenvectors of $A^T A$ are given by,

$$u_1 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$u_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$u_3 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

Therefore,

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \end{aligned}$$

$$u_3 = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

Therefore,

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

$$V = \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

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Step 7 of 8

Therefore, we get,

$$A = U\Sigma V^T$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

Step 7 of 8

Therefore, we get,

$$A = U\Sigma V^T$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}^T$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \sqrt{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \times \sqrt{2} & \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \times \sqrt{2} & \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} + \frac{1}{2} & 1 & 1-1 \\ \frac{1}{2} - \frac{1}{2} & 1 & \frac{1}{2} + \frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & 1 & 1-1 \\ \frac{1}{2}-\frac{1}{2} & 1 & \frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 8

Thus, $\boxed{U = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}$, $\boxed{\Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$, and $\boxed{V = \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}}$.

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Chapter: CH6.3 Problem: 6P

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CH6.3 5P

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CH6.3 6P

CH6.3 8P

Step 1 of 4

We know that Singular Value Decomposition for any *m* by *n* matrix *A* is given by

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are *in V* and

$$\begin{aligned} \sigma_i &= \sqrt{\lambda_i\left(A^T A\right)} \\ &= \sqrt{\lambda_i\left(AA^T\right)} \end{aligned}$$

We also know that $Av_j = \sigma_j u_j$.

Here σ_j is the length of eigenvector vector Av_j and u_j is unit eigenvector.

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Step 2 of 4

Since $u = \frac{1}{3}(2,2,1,)$ and $v = \frac{1}{2}(1,1,1,1)$, then matrix *A* must be 3 by 4.

If the *A* has rank-1 then $A^T A$ has also rank-1.

So, only one eigenvalue of $A^T A$ is nonzero.

Provide feedback (0)

Step 2 of 4

Since $u = \frac{1}{3}(2,2,1,)$ and $v = \frac{1}{2}(1,1,1,1)$, then matrix *A* must be 3 by 4.

If the *A* has rank-1 then $A^T A$ has also rank-1.

So, only one eigenvalue of $A^T A$ is nonzero.

The matrix $\sum$ is given by

$$\sum = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there is only one nonzero entry in $\sum$, we get the following equation

$$Av_1 = \sigma_1 u_1$$

So, *u* be the first column of *U* and *v* be the first column of *V*.

Provide feedback (0)

Step 3 of 4

If we find out $A = U \sum V^T$, the only nonzero will come from first column of *U* and first column of *V*.

Provide feedback (0)

Step 3 of 4

If we find out $A = U\sum V^T$, the only nonzero will come from first column of *U* and first column of *V*.

This gives the equation $Av = 12u$.

Therefore, the matrix $\boxed{A = 12uv^T}$ with rank 1 that has $Av = 12u$.

Provide feedback (0)

Step 4 of 4

The length of eigenvector vector Av_j is σ_j and u_j is unit eigenvector.

So, from the equation $A = 12uv^T$, the only one singular value is $\boxed{\sigma_1 = 12}$.

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CH6.3 7P

CH6.3 9P

Step 1 of 2

The matrix A has orthogonal columns $w_1, w_2, w_3, \cdots, w_n$ of lengths $\sigma_1, \sigma_2, \sigma_3, \cdots, \sigma_n$.

Therefore, we get,

$$\Sigma = \begin{bmatrix} \sigma_1 & & & & & & \\ & \sigma_2 & & & & & \\ & & \sigma_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & \sigma_n \end{bmatrix}$$

The singular value decomposition is given by,

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{bmatrix} u_1 & u_2 & u_3 \cdots u_n \end{bmatrix} \begin{bmatrix} \sigma_1 & & & & & & \\ & \sigma_2 & & & & & \\ & & \sigma_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & \sigma_n \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \cdots v_n \end{bmatrix}^T \\ &= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T + \cdots + \sigma_n u_n v_n^T \end{aligned}$$

Provide feedback (0)

Step 2 of 2

Therefore, $\boxed{U\Sigma V^T = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T + \cdots + \sigma_n u_n v_n^T}$.

$$\begin{bmatrix} & & \\ & \ddots & \\ & & \sigma_n \end{bmatrix}$$

$$= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T + \cdots + \sigma_n u_n v_n^T$$

Provide feedback (0)

Step 2 of 2

Therefore, $\boxed{U\Sigma V^T = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T + \cdots + \sigma_n u_n v_n^T}$.

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CH6.3 8P

Step 1 of 3

The singular value decomposition , $U\sum V^T$ expresses *A* as a sum of *r* rank-1 matrices because of the block form of multiplication.

If *m* by *p* matrix *A* has *r* row partitions and ***s*** column partitions and *p* by *n* matrix *B* has ***s*** row partitions and *t* column partitions then the block partitioned matrix product can given by *m* by *n* matrix *C*, which has *r* row partitions and *t* column partitions.

We know that the block form of multiplication is fact, now the key is in singular values in the $\sum$.

We have to show how $\sum$ effect it.

Provide feedback (0)

Step 2 of 3

If the number of columns is more than rows, then by multiplication of $\sum$ reset the rows of the matrix *V* and remove the bottom ones.

Similarly, if the number of rows is more than columns, then by multiplication of $\sum$ reset the columns of *U* and remove the the last ones.

Provide feedback (0)

Step 3 of 3

Using the columns of *U* times rows of $\sum V^T$, we get

$$U\Sigma V^T = \begin{bmatrix} u_1 & \cdots & u_r \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} v_1 & \cdots & v_r \end{bmatrix}^T$$

CH6.3 10P

Provide feedback (0)

Step 2 of 3

If the number of columns is more than rows, then by multiplication of $\sum$ reset the rows of the matrix V and remove the bottom ones.

Similarly, if the number of rows is more than columns, then by multiplication of $\sum$ reset the columns of U and remove the the last ones.

Provide feedback (0)

Step 3 of 3

Using the columns of U times rows of $\sum V^T$, we get

$$\begin{aligned} U\Sigma V^T &= \begin{bmatrix} u_1 & \cdots & u_r \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} v_1 & \cdots & v_r \end{bmatrix}^T \\ &= u_1\sigma_1 v_1^T + \cdots + u_r\sigma_r v_r^T \end{aligned}$$

So, we have found $U\sum V^T$ as a sum of r rank-1 matrices $u_1\sigma_1 v_1^T + \cdots + u_r\sigma_r v_r^T$.

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CH6.3 9P

CH6.3 11P

Step 1 of 2

Let A is a 2 by 2 symmetric matrix with unit eigenvectors u_1 and u_2.

Assume that,

$$A = A^T$$

Therefore, the eigenvectors of $A^T A$ are the same as for A.

The eigenvalues are 3 and -2.

Therefore,

$$\begin{aligned}\Sigma &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{3} & 0 \\ 0 & i\sqrt{2} \end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 2 of 2

Thus, $\boxed{U = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}$, $\boxed{\Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$, and $\boxed{V = \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}}$.

Provide feedback (1)

CH6.3 10P

CH6.3 12P

Step 1 of 2

We know that Singular Value Decomposition for any *m* by *n* matrix *A* can be factor into

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V* .

The *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

Provide feedback (0)

Step 2 of 2

Suppose *A* is a invertible with $\sigma_1 > \sigma_2 > 0$.

$$A = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$$

To make matrix *A* as singular matrix A_0 , we have to do the smallest change in *A* is to set its smallest singular value σ_2 to 0.

Consider the factorization.

Step 2 of 2

Suppose A is a invertible with $\sigma_1 > \sigma_2 > 0$.

$$A = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$$

To make matrix A as singular matrix A_0 , we have to do the smallest change in A is to set its smallest singular value σ_2 to 0.

Consider the factorization.

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T \\ &= u_1\sigma_1 v_1^T + u_2\sigma_2 v_2^T \end{aligned}$$

So, in this factorization $\sigma_2 \to 0$, gives the closest rank-1 matrix as $u_1\sigma_1 v_1^T$.

Therefore, the singular matrix $\boxed{A_0 = u_1\sigma_1 v_1^T}$.

Provide feedback (0)

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CH6.3 11P

CH6.3 13P

Step 1 of 7

(a) Singular Value Decomposition (SVD) for any *m* by *n* matrix *A* is as follows

$$A = U\Sigma V^T$$
$$= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V*.

The *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

Provide feedback (0)

Step 2 of 7

We know that the eigenvalue equation for AA^T is given by

$$AA^T x = \lambda x$$

Multiply both sides of above equation by a constant *c*.

$$cAA^T x = c\lambda x$$

So, if we multiple the above equation by any constant the eigenvectors remain same.

But the eigenvalues changes to c times of λ .

Step 2 of 7

We know that the eigenvalue equation for AA^T is given by

$$AA^T x = \lambda x$$

Multiply both sides of above equation by a constant *c*.

$$cAA^T x = c\lambda x$$

So, if we multiple the above equation by any constant the eigenvectors remain same.

But the eigenvalues changes to c times of λ .

So, if we change *m* by *n* matrix *A* to $4A$, *U* and *V* remain same in the SVD for $4A$.

Provide feedback (0)

Step 3 of 7

We have *m* by *n* matrix as $4A$, so

$$(4A)(4A^T) = 16AA^T$$

The eigenvalue of $16AA^T$ is give by

$$16AA^T x = 16\lambda x$$

Provide feedback (0)

Step 3 of 7

We have *m* by *n* matrix as $4A$, so

$$(4A)(4A^T) = 16AA^T$$

The eigenvalue of $16AA^T$ is give by

$$16AA^T x = 16\lambda x$$

This implies eigenvalue of $16AA^T$ is 16λ .

Provide feedback (0)

Step 4 of 7

We know that the diagonal of $\sum$ are the square roots of the nonzero eigenvalues of AA^T

$$\sigma = \sqrt{\lambda} \cdot$$

Since the eigenvalue of $16AA^T$ is 16λ , so

$$\begin{aligned}\sigma &= \sqrt{16\lambda} \\ &= 4\sqrt{\lambda}\end{aligned}$$

$$16AA^T x = 16\lambda x$$

This implies eigenvalue of $16AA^T$ is 16λ .

Provide feedback (0)

Step 4 of 7

We know that the diagonal of $\sum$ are the square roots of the nonzero eigenvalues of AA^T

$$\sigma = \sqrt{\lambda} \cdot$$

Since the eigenvalue of $16AA^T$ is 16λ , so

$$\begin{aligned}\sigma &= \sqrt{16\lambda} \\ &= 4\sqrt{\lambda}\end{aligned}$$

The diagonal matrix for $16AA^T$ is $4\sum$.

Provide feedback (0)

Step 5 of 7

The SVD for 4*A* is as follows:

$$\begin{aligned}4A &= U(4\sum)V^T \\ &= 4U\sum V^T\end{aligned}$$

Therefore, SVD for 4*A* is $\boxed{4A = 4U\sum V^T}$.

We know that the diagonal of Σ are the square roots of the nonzero eigenvalues of AA^T

$$\sigma = \sqrt{\lambda}.$$

Since the eigenvalue of $16AA^T$ is 16λ, so

$$\begin{aligned}\sigma &= \sqrt{16\lambda}\\ &= 4\sqrt{\lambda}\end{aligned}$$

The diagonal matrix for $16AA^T$ is 4Σ.

Provide feedback (0)

Step 5 of 7

The SVD for 4*A* is as follows:

$$\begin{aligned}4A &= U(4\Sigma)V^T\\ &= 4U\Sigma V^T\end{aligned}$$

Therefore, SVD for 4*A* is $\boxed{4A = 4U\Sigma V^T}$.

Provide feedback (0)

Step 6 of 7

(b) Consider the SVD for *m* by *n* matrix *A*.

Provide feedback (0)

Step 6 of 7

(b) Consider the SVD for *m* by *n* matrix *A*.

$$A = U\Sigma V^T$$

The transpose of *A* is as follows:

$$\begin{aligned} A^T &= \left(U\Sigma V^T\right)^T \\ &= V\Sigma^T U^T \end{aligned}$$

Here Σ^T is *n* by *m* matrix, with *r* nonzero entries σi .

Therefore, SVD for $\boxed{A^T = V\Sigma^T U^T}$.

Provide feedback (0)

Step 7 of 7

If matrix *A* non-singular matrix, then the inverse of *A* is as follows:

$$\begin{aligned} A &= U\Sigma V^T \\ A^+ &= \left(U\Sigma V^T\right)^{-1} \\ &= V\Sigma^{-1}U^{-1} \\ &= V\Sigma^{-1}U^T \end{aligned}$$

The transpose of A is as follows:

$$\begin{aligned} A^T &= \left(U\Sigma V^T\right)^T \\ &= V\Sigma^T U^T \end{aligned}$$

Here Σ^T is n by m matrix, with r nonzero entries σi .

Therefore, SVD for $\boxed{A^T = V\Sigma^T U^T}$.

Provide feedback (0)

Step 7 of 7

If matrix A non-singular matrix, then the inverse of A is as follows:

$$\begin{aligned} A &= U\Sigma V^T \\ A^+ &= \left(U\Sigma V^T\right)^{-1} \\ &= V\Sigma^{-1}U^{-1} \\ &= V\Sigma^{-1}U^T \end{aligned}$$

Here Σ^{-1} is n by m matrix, with r nonzero entries $\frac{1}{\sigma i}$.

If matrix A is square and invertible matrix then $A^+ = A^{-1}$.

Therefore, when A is square and invertible then SVD for $\boxed{A^{-1} = V\Sigma^{-1}U^T}$.

CH6.3 12P

CH6.3 14P

Step 1 of 4

Singular Value Decomposition (SVD) for any *m* by *n* matrix *A* is as follows

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V*.

The *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

Provide feedback (0)

Step 2 of 4

That is $\sigma_r = \sqrt{\lambda_r}$

Provide feedback (0)

Step 3 of 4

Suppose the SVD for $A+I$ involve $\Sigma + I$, then we have

$$\begin{aligned} U(\Sigma+I)V^T &= U\Sigma V^T + UIV^T \\ &= U\Sigma V^T + UV^T \\ &\neq A+I \end{aligned}$$

Step 2 of 4

That is $\sigma_r = \sqrt{\lambda_r}$

Provide feedback (0)

Step 3 of 4

Suppose the SVD for $A+I$ involve $\Sigma+I$, then we have

$$\begin{aligned} U(\Sigma+I)V^T &= U\Sigma V^T + UIV^T \\ &= U\Sigma V^T + UV^T \\ &\neq A+I \end{aligned}$$

Provide feedback (0)

Step 4 of 4

We know that the *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of $A^T A$.

So, singular-values $A+I$ are not $\sigma_r + 1$.

The singular-values $A+I$ come from the eigenvalues of $(A+I)^T(A+I)$.

Therefore, the SVD for $A+I$ just not use $\Sigma+I$.

Provide feedback (0)

CH6.3 13P

CH6.3 15P

Step 1 of 5

Singular Value Decomposition (SVD) for any *m* by *n* matrix *A* is as follows

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V*, and the $\sigma_i = \sqrt{\lambda_i}$.

Provide feedback (0)

Step 2 of 5

Let *A* be an *m* by *n* zero matrix.

So, AA^T be *m* by *m* zero matrix, whose eigenvalues are $\lambda = 0$.

We know that eigenvectors of AA^T are in *U*.

Since eigenvalues are zero, *U* will be *m* by *m* identity matrix.

$$\begin{aligned} U &= [u_1 \cdots u_m] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{m\times m} \end{aligned}$$

We know that eigenvectors of AA^T are in U.

Since eigenvalues are zero, U will be m by m identity matrix.

$$\begin{aligned} U &= [u_1 \cdots u_m] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{m\times m} \end{aligned}$$

Provide feedback (0)

Step 3 of 5

Now $A^T A$ be n by n zero matrix, whose eigenvalues are $\lambda = 0$.

We know that eigenvectors of $A^T A$ are in V.

Since eigenvalues are zero, V will be n by n identity matrix.

$$\begin{aligned} V &= [v_1 \cdots v_n] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{n\times n} \end{aligned}$$

Provide feedback (0)

Step 4 of 5

We know that eigenvectors of $A^T A$ are in V.

Since eigenvalues are zero, V will be n by n identity matrix.

$$\begin{aligned} V &= \left[v_1 \cdots v_n\right] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{n\times n} \end{aligned}$$

Provide feedback (0)

Step 4 of 5

The diagonal of $\sum$ are the square roots of the nonzero eigenvalues of both AA^T.

So, $\sum$ is m by n zero matrix.

$$\Sigma = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

The SVD for m by n zero matrix is as follows:

$$\begin{aligned} A &= U \Sigma V^T \\ &= I_{m\times m} \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma \end{aligned}$$

So, $\sum$ is *m* by *n* zero matrix.

$$\Sigma = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

The SVD for *m* by *n* zero matrix is as follows:

$$\begin{aligned} A &= U \Sigma V^T \\ &= I_{m\times m} \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} \end{aligned}$$

Therefore, SVD for *m* by *n* zero matrix is $\boxed{m \text{ by } n \text{ zero matrix}}$.

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Step 5 of 5

If SVD of *m* by *n* matrix *A* is $A = U \Sigma V^T$, then the pseudoinverse of *A* is

$$A^+ = V \Sigma^+ U^T$$

For *m* by *n* zero matrix, we have

$$= I_{m\times m}\,\Sigma_{m\times n}\,I_{n\times n}$$
$$= \Sigma_{m\times n}\,I_{n\times n}$$
$$= \Sigma_{m\times n}$$

Therefore, SVD for *m* by *n* zero matrix is $\boxed{m \text{ by } n \text{ zero matrix}}$.

Provide feedback (0)

Step 5 of 5

If SVD of *m* by *n* matrix *A* is $A = U\Sigma V^T$, then the pseudoinverse of *A* is

$$A^+ = V\Sigma^+ U^T$$

For *m* by *n* zero matrix, we have

$$U = I_{m\times m}$$
$$V = I_{n\times n}$$

Since Σ is *m* by *n* zero matrix, so Σ^+ is *n* by *m* zero matrix.

Hence, pseudoinverse of *m* by *n* zero matrix *A* is

$$\begin{aligned} A^+ &= V\Sigma^+ U^T \\ &= I_{n\times n}\,\Sigma^+{}_{n\times m}\,I_{m\times m} \\ &= \Sigma^+{}_{n\times m} \\ &= A^T \end{aligned}$$

Step 5 of 5

If SVD of *m* by *n* matrix *A* is $A = U\Sigma V^T$, then the pseudoinverse of *A* is

$$A^+ = V\Sigma^+ U^T$$

For *m* by *n* zero matrix, we have

$$U = I_{m\times m}$$
$$V = I_{n\times n}$$

Since Σ is *m* by *n* zero matrix, so Σ^+ is *n* by *m* zero matrix.

Hence, pseudoinverse of *m* by *n* zero matrix *A* is

$$\begin{aligned} A^+ &= V\Sigma^+ U^T \\ &= I_{n\times n}\Sigma^+{}_{n\times m} I_{m\times m} \\ &= \Sigma^+{}_{n\times m} \\ &= A^T \end{aligned}$$

Therefore, the pseudoinverse of an zero *n* by *m* matrix is $\boxed{\text{equal to its transpose}}$.

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Algebra tutors who can help right now

CH6.3 14P

CH6.3 16P

Step 1 of 18

We know that Singular Value Decomposition for any *m* by *n* matrix *A* is given by

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V* and the diagonal matrix Σ has square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

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Step 2 of 18

Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

So, we have

$$A^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculate AA^T.

Calculate AA^T .

$$AA^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$
$$= [4]$$

Provide feedback (0)

Step 3 of 18

To find the eigenvalues of AA^T , solve the following equation for λ .

$$\det[4-\lambda] = 0$$
$$(\lambda - 4) = 0$$

Therefore, the eigenvalues for AA^T are:

$$\lambda_1 = 4$$

The eigenvectors of AA^T are given by,

$$x_1 = [1]$$

We know that eigenvectors of AA^T are in *U*, so

$\lambda_1 = 4$

The eigenvectors of AA^T are given by,

$x_1 = [1]$

We know that eigenvectors of AA^T are in U, so

$U = [1]$

Provide feedback (0)

Step 4 of 18

Calculate $A^T A$.

$$AA^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

To find the eigenvalues of AA^T , solve the following equation for λ .

Calculate $A^T A$.

$$AA^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

To find the eigenvalues of AA^T , solve the following equation for λ .

$$\det \begin{bmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{bmatrix} = 0$$
$$\lambda^4 - 4\lambda^3 = 0$$

Therefore, the eigenvalues for $A^T A$ are:

$$\lambda_1 = 4$$
$$\lambda_2 = 0$$
$$\lambda_3 = 0$$
$$\lambda_4 = 0$$

The eigenvectors of $A^T A$ are:

$\lambda_1 = 4$
$\lambda_2 = 0$
$\lambda_3 = 0$
$\lambda_4 = 0$

The eigenvectors of $A^T A$ are:

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$
$$x_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
$$x_4 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 18

The unit eigenvectors of $A^T A$ are given by

Provide feedback (0)

Step 5 of 18

The unit eigenvectors of $A^T A$ are given by

$$x_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$x_4 = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

We know that eigenvectors of $A^T A$ are in V, so

$$V = \begin{bmatrix} 1/2 & -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/2 & 0 & 0 & 1/\sqrt{2} \\ 1/2 & 1/\sqrt{2} & 0 & 0 \\ \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$x_4 = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

We know that eigenvectors of $A^T A$ are in V, so

$$V = \begin{bmatrix} 1/2 & -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/2 & 0 & 0 & 1/\sqrt{2} \\ 1/2 & 1/\sqrt{2} & 0 & 0 \\ 1/2 & 0 & 1/\sqrt{2} & 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 18

The diagonal matrix Σ has eigenvalues from BB^T.

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sqrt{4} & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore, we get,

$$B = U \Sigma V^T$$

Provide feedback (0)

Step 6 of 18

The diagonal matrix Σ has eigenvalues from BB^T .

$$\begin{aligned}\Sigma &= \begin{bmatrix}\sqrt{4} & 0 & 0 & 0\end{bmatrix}\\ &= \begin{bmatrix}2 & 0 & 0 & 0\end{bmatrix}\end{aligned}$$

Therefore, we get,

$$\begin{aligned}B &= U\Sigma V^T\\ &= [1]\begin{bmatrix}2 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1/2 & -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2}\\ 1/2 & 0 & 0 & 1/\sqrt{2}\\ 1/2 & 1/\sqrt{2} & 0 & 0\\ 1/2 & 0 & 1/\sqrt{2} & 0\end{bmatrix}^T\end{aligned}$$

We know that if $A = U\Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V\Sigma^+ U^T$$

Therefore, pseudoinverse of *A* is given by

$$\begin{aligned}A^+ &= V\Sigma^+ U^T\\ &= \begin{bmatrix}1/2 & -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2}\\ 1/2 & 0 & 0 & -1/\sqrt{2}\\ 1/2 & 1/\sqrt{2} & 0 & 0\end{bmatrix}\begin{bmatrix}1/2\\ 0\\ 0\end{bmatrix}[1]\end{aligned}$$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, we get,

$$B = U \Sigma V^T$$

$$= [1]\begin{bmatrix} 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/2 & 0 & 0 & 1/\sqrt{2} \\ 1/2 & 1/\sqrt{2} & 0 & 0 \\ 1/2 & 0 & 1/\sqrt{2} & 0 \end{bmatrix}^T$$

We know that if $A = U \Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V \Sigma^+ U^T$$

Therefore, pseudoinverse of A is given by

$$A^+ = V \Sigma^+ U^T$$

$$= \begin{bmatrix} 1/2 & -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/2 & 0 & 0 & -1/\sqrt{2} \\ 1/2 & 1/\sqrt{2} & 0 & 0 \\ 1/2 & 0 & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{bmatrix} [1]$$

$$= \boxed{\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}}$$

Provide feedback (0)

Provide feedback (0)

Step 7 of 18

Consider the matrix

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

So, we have

$$B^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Calculate BB^T .

$$\begin{aligned} BB^T &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 8 of 18

To find the eigenvalues of BB^T , solve the following equation for λ .

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} & \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 18

To find the eigenvalues of BB^T , solve the following equation for λ .

$$\det\begin{bmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(1-\lambda) = 0$$

$$(1-\lambda)^2 = 0$$

Therefore, the eigenvalues for BB^T are:

$$\lambda_1 = 1$$
$$\lambda_2 = 1$$

The eigenvectors of BB^T are given by

$$x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

The eigenvectors of BB^T are given by

$$x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 18

The unit eigenvectors of BB^T are given by

$$u_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$u_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We know that eigenvectors of BB^T are in U, so

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

The unit eigenvectors of BB^T are given by

$$u_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$u_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We know that eigenvectors of BB^T are in U, so

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 10 of 18

Calculate $B^T B$.

$$\begin{aligned} B^T B &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

To find the eigenvalues of $B^T B$, solve the following equation for λ .

Provide feedback (0)

Step 10 of 18

Calculate $B^T B$.

$$B^T B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To find the eigenvalues of $B^T B$, solve the following equation for λ .

$$\det \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 0-\lambda \end{bmatrix} = 0$$
$$(1-\lambda)(1-\lambda)(-\lambda) = 0$$

Therefore, the eigenvalues for $B^T B$ are:

$$\lambda_1 = 1$$
$$\lambda_2 = 1$$
$$\lambda_3 = 0$$

The eigenvectors of $B^T B$ are:

Therefore, the eigenvalues for $B^T B$ are:

$$\lambda_1 = 1$$
$$\lambda_2 = 1$$
$$\lambda_3 = 0$$

The eigenvectors of $B^T B$ are:

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 18

The unit eigenvectors of $B^T B$ are given by

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 18

The unit eigenvectors of $B^T B$ are given by

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We know that eigenvectors of $B^T B$ are in V, so

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 12 of 18

The diagonal matrix Σ has eigenvalue from BB^T.

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 12 of 18

The diagonal matrix Σ has eigenvalue from BB^T.

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

Therefore, we get,

$$\begin{aligned} B &= U \Sigma V^T \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \end{aligned}$$

Provide feedback (0)

Step 13 of 18

We know that if $A = U \Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V \Sigma^+ U^T$$

Step 13 of 18

We know that if $A = U\Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V\Sigma^+ U^T$$

Hencee, pseudoinverse of *B* is given by

$$\begin{aligned} B^+ &= V\Sigma^+ U^T \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore, $B^+ = \boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}}$

Consider the matrix

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

So, we have

$$\begin{bmatrix} 0 & 0 \end{bmatrix}$$

Consider the matrix

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

So, we have

$$C^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Calculate CC^T .

$$\begin{aligned} CC^T &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 14 of 18

To find the eigenvalues of CC^T , solve the following equation for λ .

$$\begin{aligned} \begin{bmatrix} 2-\lambda & 0 \\ 0 & 0-\lambda \end{bmatrix} &= 0 \\ (2-\lambda)(-\lambda) &= 0 \end{aligned}$$

Provide feedback (0)

Step 14 of 18

To find the eigenvalues of CC^T , solve the following equation for λ .

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 0-\lambda \end{bmatrix} = 0$$
$$(2-\lambda)(-\lambda) = 0$$

Therefore, the eigenvalues for CC^T are:

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of CC^T are given by

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 15 of 18

The unit eigenvectors of CC^T are given by,

Provide feedback (0)

Step 14 of 18

To find the eigenvalues of CC^T , solve the following equation for λ .

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 0-\lambda \end{bmatrix} = 0$$
$$(2-\lambda)(-\lambda) = 0$$

Therefore, the eigenvalues for CC^T are:

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of CC^T are given by

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 15 of 18

The unit eigenvectors of CC^T are given by,

Provide feedback (0)

Step 15 of 18

The unit eigenvectors of CC^T are given by,

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

We know that eigenvectors of CC^T are in U, so

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 16 of 18

Calculate $C^T C$.

$$\begin{aligned} C^T C &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Step 16 of 18

Calculate $C^T C$.

$$C^T C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The eigenvalues of $C^T C$ are given by,

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$
$$(1-\lambda)^2 - 1 = 0$$
$$\lambda^2 - 2\lambda = 0$$

Therefore, the eigenvalues for $C^T C$ are:

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of $C^T C$ are given by

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The unit eigenvectors of $C^T C$ are given by

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$
$$v_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

We know that eigenvectors of $C^T C$ are in V, so

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 17 of 18

The diagonal matrix has Σ eigenvalue from CC^T.

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

We know that eigenvectors of $C^T C$ are in V, so

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 17 of 18

The diagonal matrix has Σ eigenvalue from CC^T .

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore, we get,

$$\begin{aligned} C &= U \Sigma V^T \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T \end{aligned}$$

Provide feedback (0)

CH6.3 15P

CH6.3 17P

Step 1 of 5

We know that if *m* by *n* matrix *Q* has orthonormal columns then

$$QQ^T = I_{m\times m}$$
$$Q^TQ = I_{n\times n}$$

Singular Value Decomposition (SVD for any *m* by *n* matrix *Q* is as follows

$$\begin{aligned} Q &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of QQ^T are in *U*, eigenvectors of Q^TQ are in *V*.

The *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both QQ^T and Q^TQ .

Provide feedback (0)

Step 2 of 5

Since Q^TQ be *m* by *m* identity matrix, so eigenvalues are $\lambda = 1$.

We know that eigenvectors of QQ^T are in *U*.

Eigenvectors of QQ^T will make matrix *U* as *m* by *m* identity matrix.

The r singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both QQ^T and Q^TQ.

Provide feedback (0)

Step 2 of 5

Since Q^TQ be m by m identity matrix, so eigenvalues are $\lambda = 1$.

We know that eigenvectors of QQ^T are in U.

Eigenvectors of QQ^T will make matrix U as m by m identity matrix.

$$\begin{aligned} U &= \left[u_1 \cdots u_m\right] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{m\times m} \end{aligned}$$

Provide feedback (0)

Step 3 of 5

Now Q^TQ be n by n identity matrix, so eigenvalues are $\lambda = 1$.

We know that eigenvectors of Q^TQ are in V.

Eigenvectors of Q^TQ will make matrix U as n by n identity matrix.

Provide feedback (0)

Step 3 of 5

Now $Q^T Q$ be *n* by *n* identity matrix, so eigenvalues are $\lambda = 1$.

We know that eigenvectors of $Q^T Q$ are in *V*.

Eigenvectors of $Q^T Q$ will make matrix *U* as *n* by *n* identity matrix.

$$\begin{aligned} V &= \left[v_1 \cdots v_n \right] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{n \times n} \end{aligned}$$

Provide feedback (0)

Step 4 of 5

The diagonal of $\sum$ are the square roots of the nonzero eigenvalues of QQ^T .

When *A* has rank *r*, $\sum$ is *m* by *n* matrix, where

$$\sigma_1, \cdots, \sigma_r = 1$$

And all other entries will be zero.

Step 4 of 5

The diagonal of $\sum$ are the square roots of the nonzero eigenvalues of QQ^T .

When *A* has rank *r*, $\sum$ is *m* by *n* matrix, where

$$\sigma_1, \cdots, \sigma_r = 1$$

And all other entries will be zero.

Let us denote $\sum = I_{m\times n}$.

The SVD for *m* by *n* matrix *Q* is as follows:

$$\begin{aligned} Q &= U\sum V^T \\ &= I_{m\times m} I_{m\times n} \left(I_{n\times n}\right)^T \\ &= I_{m\times m} I_{m\times n} I_{n\times n} \\ &= I_{m\times n} \end{aligned}$$

Therefore, SVD for *m* by *n* matrix *Q* with orthonormal column is $\boxed{m \text{ by } n \text{ identity matrix}}$.

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Step 5 of 5

If SVD of *m* by *n* matrix *A* is $A = U\sum V^T$, then the pseudoinverse of *A* is

$$Q = U \Sigma V$$
$$= I_{m\times m} I_{m\times n} \left(I_{n\times n}\right)^T$$
$$= I_{m\times m} I_{m\times n} I_{n\times n}$$
$$= I_{m\times n}$$

Therefore, SVD for *m* by *n* matrix *Q* with orthonormal column is $\boxed{m \text{ by } n \text{ identity matrix}}$.

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Step 5 of 5

If SVD of *m* by *n* matrix *A* is $A = U \Sigma V^T$, then the pseudoinverse of *A* is

$$A^+ = V \Sigma^+ U^T$$

For *m* by *n* matrix *Q* with orthonormal columns, we have

$$U = I_{m\times m}$$
$$U^T = I_{m\times m}$$
$$V = I_{n\times n}$$

When *A* has rank *r*, Σ^+ is *n* by *m* matrix, where

$$\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_r} = 1$$

And all other entries will be zero.

CH6.3 16P

CH6.3 18P

Step 1 of 7

Consider the matrix:

$$A = \frac{1}{\sqrt{10}}\begin{bmatrix} 10 & 6 \\ 0 & 8 \end{bmatrix}$$

Thus,

$$A^T = \frac{1}{\sqrt{10}}\begin{bmatrix} 10 & 0 \\ 6 & 8 \end{bmatrix}$$

Therefore, $A^T A$ is given by,

$$\begin{aligned} A^T A &= \frac{1}{\sqrt{10}}\begin{bmatrix} 10 & 0 \\ 6 & 8 \end{bmatrix}\frac{1}{\sqrt{10}}\begin{bmatrix} 10 & 6 \\ 0 & 8 \end{bmatrix} \\ &= \frac{1}{10}\begin{bmatrix} 100 & 60 \\ 60 & 100 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \end{aligned}$$

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Step 2 of 7

The eigenvalues of $A^T A$ are given by,

$$\begin{aligned} &\begin{vmatrix} 10-\lambda & 6 \\ 6 & 10-\lambda \end{vmatrix} = 0 \\ &(10-\lambda)(10-\lambda) = 36 \\ &100 - 20\lambda + \lambda^2 = 36 \\ &\lambda^2 - 20\lambda + 64 = 0 \\ &(\lambda-16)(\lambda-4) = 0 \end{aligned}$$

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Step 2 of 7

The eigenvalues of $A^T A$ are given by,

$$\begin{vmatrix} 10-\lambda & 6 \\ 6 & 10-\lambda \end{vmatrix} = 0$$

$$(10-\lambda)(10-\lambda) = 36$$

$$100 - 20\lambda + \lambda^2 = 36$$

$$\lambda^2 - 20\lambda + 64 = 0$$

$$(\lambda - 16)(\lambda - 4) = 0$$

$$\lambda = 16, 4$$

Therefore, the eigenvalues are 16 and 4.

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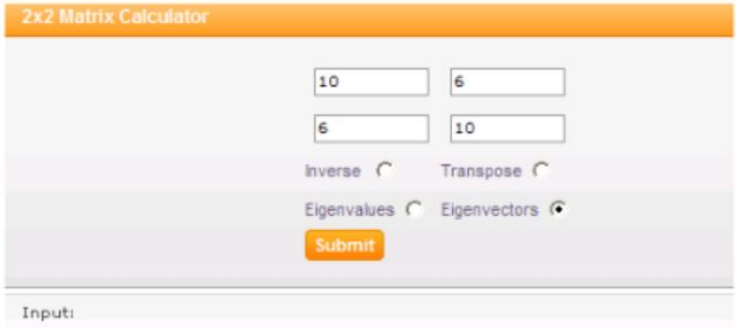
Step 3 of 7

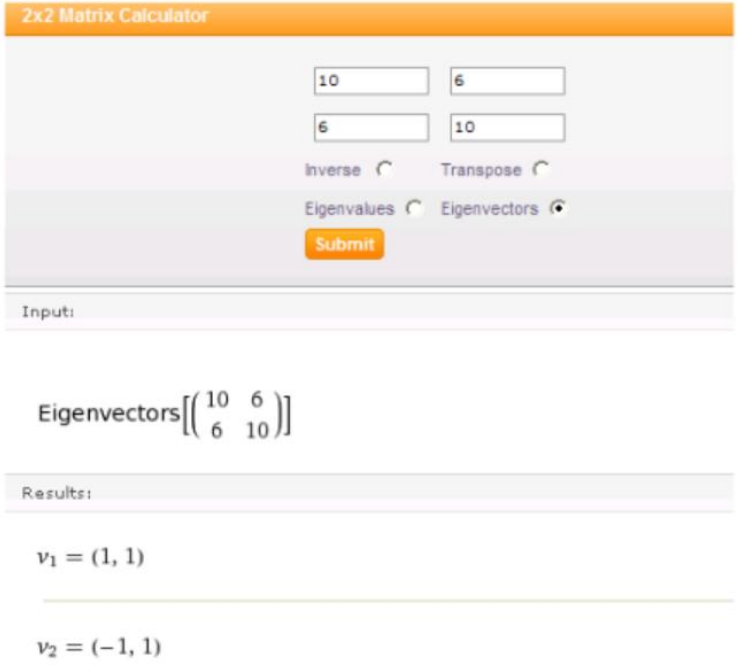
By using matrix calculator (the screenshot is given below), the eigenvectors of AA^T are given by,



$$\text{Eigenvectors}\left[\begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}\right]$$

Step 3 of 7

By using matrix calculator (the screenshot is given below), the eigenvectors of AA^T are given by,



The eigenvectors of AA^T are given by,

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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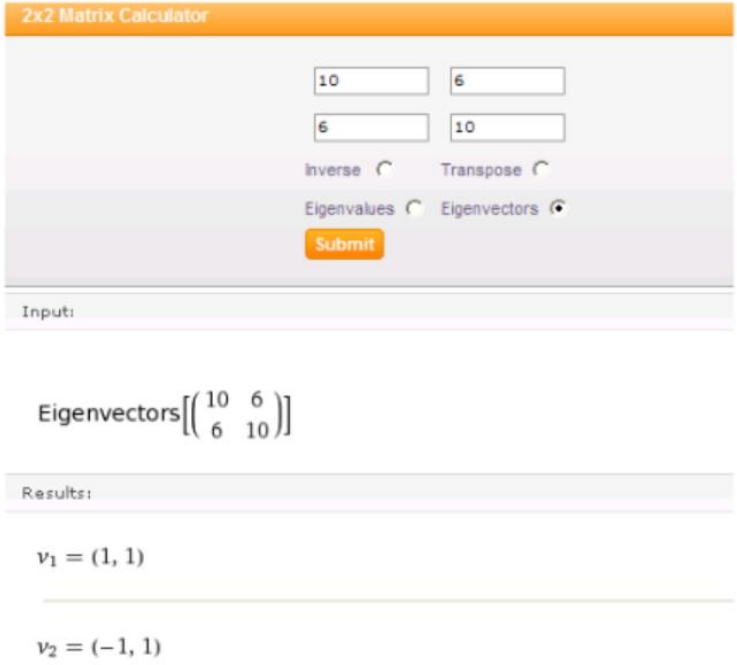
Step 4 of 7

Therefore,

Step 3 of 7

By using matrix calculator (the screenshot is given below), the eigenvectors of AA^T are given by,



The eigenvectors of AA^T are given by,

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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Step 4 of 7

Therefore,

Step 4 of 7

Therefore,

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$
$$= \begin{bmatrix} 16 & 0 \\ 0 & 4 \end{bmatrix}$$
$$V = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

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Step 5 of 7

The positive definite square root $S = V\Sigma^{\frac{1}{2}}V^T$ is given by,

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Step 6 of 7

$$S = V\Sigma^{\frac{1}{2}}V^T$$
$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 16 \end{bmatrix}^{\frac{1}{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^T$$
$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

The inverse of S is given by,

$$S^{-1} = \begin{bmatrix} \frac{3}{8} & \frac{1}{8} \end{bmatrix}$$

Step 6 of 7

$$
\begin{aligned}
S &= V\Sigma^{\frac{1}{2}}V^T \\
&= \frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}4 & 0\\ 0 & 16\end{bmatrix}^{\frac{1}{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}^T \\
&= \frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix} \\
&= \begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}
\end{aligned}
$$

The inverse of S is given by,

$$
S^{-1} = \begin{bmatrix}\frac{3}{8} & \frac{1}{8}\\ -\frac{1}{8} & \frac{3}{8}\end{bmatrix}
$$

Therefore, we get,

$$
\begin{aligned}
Q &= AS^{-1} \\
&= \frac{1}{\sqrt{10}}\begin{bmatrix}10 & 6\\ 0 & 8\end{bmatrix}\begin{bmatrix}\frac{3}{8} & \frac{1}{8}\\ -\frac{1}{8} & \frac{3}{8}\end{bmatrix} \\
&= \frac{1}{\sqrt{10}}\begin{bmatrix}3 & 1\\ -1 & 3\end{bmatrix}
\end{aligned}
$$

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Step 7 of 7

Therefore, we get,

$$
\begin{aligned}
Q &= AS^{-1} \\
&= \frac{1}{\sqrt{10}}\begin{bmatrix} 10 & 6 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} \frac{3}{8} & \frac{1}{8} \\ -\frac{1}{8} & \frac{3}{8} \end{bmatrix} \\
&= \frac{1}{\sqrt{10}}\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}
\end{aligned}
$$

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Step 7 of 7

Thus, $\boxed{S = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}}$ and $\boxed{Q = \frac{1}{\sqrt{10}}\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}}$.

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CH6.3 17P

Step 1 of 4

We will first obtain the general solution to $A^{\mathrm{T}}A\hat{x} = A^{\mathrm{T}}b$.

Obtain $A^{\mathrm{T}}A$ as follows:

$$A^{\mathrm{T}}A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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Step 2 of 4

Let $\hat{x} = (x_1, x_2, x_3)$. Thus, we have

$$A^{\mathrm{T}}A\hat{x} = A^{\mathrm{T}}b$$
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$
$$\begin{bmatrix} 3x_1 + x_2 + x_3 \\ x_1 + x_2 + x_3 \\ x_1 + x_2 + x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$$

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Step 3 of 4

Consider the following three equations:

$$3x_1 + x_2 + x_3 = 4$$

CH6.3 19P

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_3 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \end{bmatrix}$$

$$\begin{bmatrix} 3x_1 + x_2 + x_3 \\ x_1 + x_2 + x_3 \\ x_1 + x_2 + x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Consider the following three equations:

$$\begin{aligned} 3x_1 + x_2 + x_3 &= 4 \\ x_1 + x_2 + x_3 &= 2 \\ x_1 + x_2 + x_3 &= 2 \end{aligned}$$

Subtracting second (or third) equation from the first equation, we get $2x_1 = 2$. This gives $x_1 = 1$. Substituting this value in the second (or third) equation, we get $x_2 + x_3 = 1$. That is, $x_3 = 1 - x_2$.

Thus, the general solution is $\hat{x} = (1, x_2, 1 - x_2)$.

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Step 4 of 4

Now suppose $\hat{x} = (1, x_2, 1 - x_2)$ lies in the row space of the matrix *A*. Therefore, $\hat{x}$ can be expressed as a linear combination of the rows of *A*.

Let us consider the following:

$$\begin{aligned} (1, x_2, 1 - x_2) &= a(1,0,0) + b(1,0,0) + c(1,1,1) \\ &= (a + b + c, c, c) \end{aligned}$$

Thus, $x_2 = 1 - x_2$.

Therefore, $x_2 = \frac{1}{2}$.

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Step 4 of 4

Now suppose $\hat{x} = (1, x_2, 1 - x_2)$ lies in the row space of the matrix *A*. Therefore, $\hat{x}$ can be expressed as a linear combination of the rows of *A*.

Let us consider the following:

$$\begin{aligned}(1, x_2, 1 - x_2) &= a(1,0,0) + b(1,0,0) + c(1,1,1) \\ &= (a + b + c, c, c)\end{aligned}$$

Thus, $x_2 = 1 - x_2$.

Therefore, $x_2 = \frac{1}{2}$.

This gives, $\boxed{\hat{x} = \left(1, \frac{1}{2}, \frac{1}{2}\right)}$.

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Step 1 of 4

Consider the following:

$$\begin{aligned}\left(A^{T}A\right)x^{+} &= \left(A^{T}A\right)A^{+}b \\ &= \left(A^{T}A\right)\left(A^{T}A\right)^{-1}A^{T}b \\ &= A^{T}AA^{-1}\left(A^{T}\right)^{-1}A^{T}b \\ &= A^{T}b\end{aligned}$$

Thus, we have shown that $A^{T}Ax^{+} = A^{T}b$.

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Step 2 of 4

(b) Let A be square matrix, whose rows are independent. Therefore, the rows of A span the entire vector space $\mathbf{R}^{n}$.

In this case, the right inverse of A is given by

$$A^{+} = A^{T}\left(AA^{T}\right)^{-1}$$

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Step 3 of 4

Let $x^{+} = A^{+}b$

Therefore, $x^{+} = A^{T}\left(AA^{T}\right)^{-1}b$

CH6.3 20P

In this case, the right inverse of A is given by

$$A^{+} = A^{\mathrm{T}}\left(AA^{\mathrm{T}}\right)^{-1}$$

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Step 3 of 4

Let $x^{+} = A^{+}b$

Therefore, $x^{+} = A^{\mathrm{T}}\left(AA^{\mathrm{T}}\right)^{-1}b$.

Obviously, x^{+} is in the row space, because A^{T} multiplied by any vector has to be in the row space.

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Step 4 of 4

Consider the following:

$$\begin{aligned} A^{\mathrm{T}}Ax^{+} &= A^{\mathrm{T}}A\left(A^{\mathrm{T}}\left(AA^{\mathrm{T}}\right)^{-1}b\right) \\ &= A^{\mathrm{T}}A\left(A^{\mathrm{T}}\left(\left(A^{\mathrm{T}}\right)^{-1}A^{-1}\right)\right)b \\ &= A^{\mathrm{T}}AA^{-1}b \\ &= A^{\mathrm{T}}b \end{aligned}$$

Thus, we have shown that $A^{\mathrm{T}}Ax^{+} = A^{\mathrm{T}}b$.

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Step 4 of 4

Consider the following:

$$\begin{aligned} A^{\mathrm{T}}Ax^{+} &= A^{\mathrm{T}}A\left(A^{\mathrm{T}}\left(AA^{\mathrm{T}}\right)^{-1}b\right) \\ &= A^{\mathrm{T}}A\left(A^{\mathrm{T}}\left(\left(A^{\mathrm{T}}\right)^{-1}A^{-1}\right)\right)b \\ &= A^{\mathrm{T}}AA^{-1}b \\ &= A^{\mathrm{T}}b \end{aligned}$$

Thus, we have shown that $A^{\mathrm{T}}Ax^{+} = A^{\mathrm{T}}b$.

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Step 1 of 3

If matrix A is invertible and the matrix factorization for A is as follows

$$A = U\Sigma V^T$$

Consider $U^T U = I$, so we can write

$$\begin{aligned} A &= U\Sigma U^T U V^T \\ &= \left(U\Sigma U^T\right)\left(UV^T\right) \end{aligned}$$

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Step 2 of 3

Consider $Q = UV^T$, since

$$\begin{aligned} Q^T Q &= \left(VU^T\right)\left(UV^T\right) \\ &= VU^T U V^T \\ &= I \end{aligned}$$

The factor $Q = UV^T$ is orthogonal.

Consider $S' = U\Sigma U^T$, since

$$\begin{aligned} S' &= QSQ^T \\ &= \sqrt{AA^T} \\ &= U\Sigma U^T \end{aligned}$$

The factor $S' = U\Sigma U^T$ is the symmetric and positive semidefinite.

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Step 3 of 3

Therefore, we can split $A = U\Sigma V^T$ in the reverse polar decomposition as $\boxed{A = QS'}$, here

CH6.3 21P

$$= (U\Sigma U^T)(UV^T)$$

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Step 2 of 3

Consider $Q = UV^T$, since

$$\begin{aligned} Q^T Q &= (VU^T)(UV^T) \\ &= VU^T UV^T \\ &= I \end{aligned}$$

The factor $Q = UV^T$ is orthogonal.

Consider $S' = U\Sigma U^T$, since

$$\begin{aligned} S' &= QSQ^T \\ &= \sqrt{AA^T} \\ &= U\Sigma U^T \end{aligned}$$

The factor $S' = U\Sigma U^T$ is the symmetric and positive semidefinite.

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Step 3 of 3

Therefore, we can split $A = U\Sigma V^T$ in the reverse polar decomposition as $\boxed{A = QS'}$, here

Q is orthogonal and S' is symmetric positive semidefinite.

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CH6.3 20P

Step 1 of 15

We know that Singular Value Decomposition for any *m* by *n* matrix *A* is given by

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V* and the diagonal matrix Σ has square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

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CH6.3 22P

Step 2 of 15

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Then we get

$$\begin{aligned} AB &= C \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

So, we have

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Then we get

$$\begin{aligned} AB &= C \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

So, we have

$$C^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Calculate CC^T .

$$\begin{aligned} CC^T &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 3 of 15

To find the eigenvalues of CC^T , solve the following equation for λ .

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 0-\lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 15

To find the eigenvalues of CC^T , solve the following equation for λ .

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 0-\lambda \end{bmatrix} = 0$$
$$(2-\lambda)(-\lambda) = 0$$

Therefore, the eigenvalues for CC^T are:

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of CC^T are given by

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 15

The unit eigenvectors of CC^T are given by

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 15

The unit eigenvectors of CC^T are given by,

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

We know that eigenvectors of CC^T are in U, so

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 5 of 15

Calculate $C^T C$.

$$U = \begin{bmatrix} u_1 & u_2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 15

Calculate $C^T C$.

$$C^T C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The eigenvalues of $C^T C$ are given by

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$
$$(1-\lambda)^2 - 1 = 0$$
$$\lambda^2 - 2\lambda = 0$$

Provide feedback (0)

Step 6 of 15

Therefore, the eigenvalues for $C^T C$ are:

Provide feedback (0)

Step 6 of 15

Therefore, the eigenvalues for $C^T C$ are:

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of $C^T C$ are given by

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The unit eigenvectors of $C^T C$ are given by

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$
$$v_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

We know that eigenvectors of $C^T C$ are in V, so

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

The unit eigenvectors of $C^T C$ are given by

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$
$$v_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

We know that eigenvectors of $C^T C$ are in V, so

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 7 of 15

The diagonal matrix has Σ eigenvalue from CC^T.

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, we get,

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 7 of 15

The diagonal matrix has Σ eigenvalue from CC^T .

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore, we get,

$$\begin{aligned} C &= U \Sigma V^T \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T \end{aligned}$$

Provide feedback (0)

Step 8 of 15

We know that if $A = U \Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V \Sigma^+ U^T$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 8 of 15

We know that if $A = U\Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V\Sigma^+ U^T$$

Hence pseudoinverse of C is given by

$$\begin{aligned} C^+ &= V\Sigma^+ U^T \\ &= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \end{aligned}$$

Therefore,

$$\begin{aligned} C^+ &= (AB)^+ \\ &= \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 9 of 15

$$C^{+} = (AB)^{+}$$
$$= \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 15

Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

Similarly we can find the pseudoinverse of A as

$$A^{+} = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$$

Consider the matrix $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$.

So, we have

$$B^{T} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Calculate BB^{T}.

$$BB^{T} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

Calculate BB^T .

$$BB^T = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 15

To find the eigenvalues of BB^T , solve the following equation for λ .

$$\begin{bmatrix} 0-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} = 0$$
$$(2-\lambda)(-\lambda) = 0$$

Therefore, the eigenvalues for BB^T are:

$$\lambda_1 = 0$$
$$\lambda_2 = 2$$

The eigenvectors of CC^T are given by

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lambda_1 = 0$$
$$\lambda_2 = 2$$

The eigenvectors of CC^T are given by

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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Step 11 of 15

The unit eigenvectors of BB^T are given by,

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

We know that eigenvectors of BB^T are in U, so

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

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Step 11 of 15

The unit eigenvectors of BB^T are given by,

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

We know that eigenvectors of BB^T are in U, so

$$\begin{aligned} U &= \begin{bmatrix} u_1 & u_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

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Step 12 of 15

Calculate $B^T B$.

$$\begin{aligned} B^T B &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

The eigenvalues of $B^T B$ are given by

Step 12 of 15

Calculate $B^T B$.

$$B^T B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The eigenvalues of $B^T B$ are given by

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$
$$(1-\lambda)^2 - 1 = 0$$
$$\lambda^2 - 2\lambda = 0$$

Therefore, the eigenvalues for $B^T B$ are:

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of $B^T B$ are given by

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\lambda_1 = 2$$
$$\lambda_2 = 0$$

The eigenvectors of $B^T B$ are given by

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The unit eigenvectors of $B^T B$ are given by

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$
$$v_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

We know that eigenvectors of $C^T C$ are in V, so

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

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Step 13 of 15

The diagonal matrix has Σ eigenvalue from BB^T

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

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Step 13 of 15

The diagonal matrix has Σ eigenvalue from BB^T .

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore, we get,

$$\begin{aligned} B &= U \Sigma V^T \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T \end{aligned}$$

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Step 14 of 15

We know that if $A = U \Sigma V^T$, then its pseudoinverse is give by

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Step 14 of 15

We know that if $A = U\Sigma V^T$, then its pseudoinverse is give by

$$A^+ = V\Sigma^+ U^T$$

Hence pseudoinverse of *B* is given by

$$\begin{aligned} B^+ &= V\Sigma^+ U^T \\ &= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \end{aligned}$$

Therefore,

$$B^+ = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$$

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Step 15 of 15

We have

$$B^{+}=\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}$$

Provide feedback (0)

Step 15 of 15

We have

$$A^{+}=\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}$$
$$B^{+}=\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}$$

Calculate $B^{+}A^{+}$.

$$\begin{aligned}B^{+}A^{+}&=\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}\\ &=\begin{bmatrix}1/4 & 0\\ 1/4 & 0\end{bmatrix}\end{aligned}$$

Since $(AB)^{+}=\begin{bmatrix}1/2 & 0\\ 1/2 & 0\end{bmatrix}$ and $B^{+}A^{+}=\begin{bmatrix}1/4 & 0\\ 1/4 & 0\end{bmatrix}$, so

$$\boxed{(AB)^{+}\neq B^{+}A^{+}}$$

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Step 1 of 3

Now consider the following:

$$\begin{aligned}
A^{+}b &= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\left(\underline{L}^{\mathrm{T}}\underline{L}\right)^{-1}\underline{L}^{\mathrm{T}}b \\
&= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}b \\
&= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}\right)b \\
&= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}b \\
&= \underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\underline{U}^{-1}\underline{L}^{-1}b \\
&= \left(\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\right)\underline{U}^{-1}\underline{L}^{-1}b \\
&= \underline{U}^{-1}\underline{L}^{-1}b \\
&= \left(\underline{L}\underline{U}\right)^{-1}b \\
&= A^{-1}b
\end{aligned}$$

Therefore, it is clear that $A^{+}b$ is in the row space.

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Step 2 of 3

Consider next:

$$\begin{aligned}
A^{T}AA^{+}b &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\left(\underline{L}^{\mathrm{T}}\underline{L}\right)^{-1}\underline{L}^{\mathrm{T}}b \\
&= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}b \\
&= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}\right)b \\
&= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}b
\end{aligned}$$

$$= \left(\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\right)\underline{U}^{-1}\underline{L}^{-1}b$$

$$= \underline{U}^{-1}\underline{L}^{-1}b$$
$$= \left(\underline{LU}\right)^{-1}b$$
$$= A^{-1}b$$

Therefore, it is clear that $A^{+}b$ is in the row space.

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Step 2 of 3

Consider next:

$$A^{T}AA^{+}b = A^{T}\left(\underline{LU}\right)\underline{U}^{\mathrm{T}}\left(\underline{UU}^{\mathrm{T}}\right)^{-1}\left(\underline{L}^{\mathrm{T}}\underline{L}\right)^{-1}\underline{L}^{\mathrm{T}}b$$
$$= A^{T}\left(\underline{LU}\right)\underline{U}^{\mathrm{T}}\left(\underline{UU}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}b$$
$$= A^{T}\left(\underline{LU}\right)\underline{U}^{\mathrm{T}}\left(\underline{UU}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}\right)b$$

$$= A^{T}\left(\underline{LU}\right)\underline{U}^{\mathrm{T}}\left(\underline{UU}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}b$$
$$= A^{T}\left(\underline{LU}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\underline{U}^{-1}\underline{L}^{-1}b$$
$$= A^{T}\left(\underline{LU}\right)\left(\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\right)\underline{U}^{-1}\underline{L}^{-1}b$$

$$= A^{T}\left(\underline{LU}\right)\underline{U}^{-1}\underline{L}^{-1}b$$
$$= A^{T}\underline{L}\left(\underline{UU}^{-1}\right)\underline{L}^{-1}b$$
$$= A^{T}\underline{LL}^{-1}b$$
$$= A^{T}b$$

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Step 3 of 3

Step 2 of 3

Consider next:

$$
\begin{aligned}
A^T AA^+ b &= A^T (\underline{L}\underline{U})\underline{U}^{\mathrm{T}} \left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1} \left(\underline{L}^{\mathrm{T}}\underline{L}\right)^{-1} \underline{L}^{\mathrm{T}} b \\
&= A^T (\underline{L}\underline{U})\underline{U}^{\mathrm{T}} \left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1} \underline{L}^{-1} \left(\underline{L}^{\mathrm{T}}\right)^{-1} \underline{L}^{\mathrm{T}} b \\
&= A^T (\underline{L}\underline{U})\underline{U}^{\mathrm{T}} \left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1} \underline{L}^{-1} \left(\left(\underline{L}^{\mathrm{T}}\right)^{-1} \underline{L}^{\mathrm{T}}\right) b \\
&= A^T (\underline{L}\underline{U})\underline{U}^{\mathrm{T}} \left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1} \underline{L}^{-1} b \\
&= A^T (\underline{L}\underline{U})\underline{U}^{\mathrm{T}} \left(\underline{U}^{\mathrm{T}}\right)^{-1} \underline{U}^{-1} \underline{L}^{-1} b \\
&= A^T (\underline{L}\underline{U})\left(\underline{U}^{\mathrm{T}} \left(\underline{U}^{\mathrm{T}}\right)^{-1}\right) \underline{U}^{-1} \underline{L}^{-1} b \\
&= A^T (\underline{L}\underline{U})\underline{U}^{-1} \underline{L}^{-1} b \\
&= A^T \underline{L}\left(\underline{U}\underline{U}^{-1}\right) \underline{L}^{-1} b \\
&= A^T \underline{L}\underline{L}^{-1} b \\
&= A^T b
\end{aligned}
$$

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Step 3 of 3

Therefore, we have shown that $\boxed{A^T AA^+ b = A^T b}$.

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Ryan C.

CH6.3 22P

Step 1 of 6

Any m by n matrix A can be factor into

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in U, eigenvectors of $A^T A$ are in V.

The r singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

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Step 2 of 6

To show AA^+ is projection matrix, we have to show

$$\left(AA^+\right)^2 = AA^+$$

Consider the matrix factorization for m by n matrix A as

$$A = Q_1 \Sigma Q_2^T$$

So, its pseudoinverse is given by

$$A^+ = Q_2 \Sigma^+ Q_1^T$$

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Step 3 of 6

Find AA^+ as follows:

$$\begin{aligned} AA^+ &= \left(Q_1 \Sigma Q_2^T\right)\left(Q_2 \Sigma^+ Q_1^T\right) \\ &= Q_1 \Sigma \Sigma^+ Q_1^T \end{aligned}$$

CH6.4

The r singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

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Step 2 of 6

To show AA^+ is projection matrix, we have to show

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Step 3 of 6

Find AA^+ as follows:

$$\begin{aligned} AA^+ &= \left(Q_1 \Sigma Q_2^T\right)\left(Q_2 \Sigma^+ Q_1^T\right) \\ &= Q_1 \Sigma \Sigma^+ Q_1^T \end{aligned}$$

Squaring both sides of above equation we get

$$\begin{aligned} \left(AA^+\right)^2 &= \left(Q_1 \Sigma \Sigma^+ Q_1^T\right)^2 \\ &= Q_1 \Sigma \Sigma^+ Q_1^T Q_1 \Sigma \Sigma^+ Q_1^T \\ &= Q_1 \Sigma \Sigma^+ \Sigma \Sigma^+ Q_1^T \\ &= Q_1 \Sigma \Sigma^+ Q_1^T \end{aligned}$$

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So, its pseudoinverse is given by

$$A^+ = Q_2 \Sigma^+ Q_1^T$$

Provide feedback (0)

Step 3 of 6

Find AA^+ as follows:

$$\begin{aligned} AA^+ &= \left(Q_1 \Sigma Q_2^T\right)\left(Q_2 \Sigma^+ Q_1^T\right) \\ &= Q_1 \Sigma \Sigma^+ Q_1^T \end{aligned}$$

Squaring both sides of above equation we get

$$\begin{aligned} \left(AA^+\right)^2 &= \left(Q_1 \Sigma \Sigma^+ Q_1^T\right)^2 \\ &= Q_1 \Sigma \Sigma^+ Q_1^T Q_1 \Sigma \Sigma^+ Q_1^T \\ &= Q_1 \Sigma \Sigma^+ \Sigma \Sigma^+ Q_1^T \\ &= Q_1 \Sigma \Sigma^+ Q_1^T \end{aligned}$$

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Step 4 of 6

So, we have

$$\begin{aligned} \left(AA^+\right)^2 &= Q_1 \Sigma \Sigma^+ Q_1^T \\ &= AA^+ \end{aligned}$$

Therefore, $\boxed{AA^+ \text{ is projection matrix}}$.

We know that every projection matrix is symmetric.

Therefore, $\boxed{AA^+ \text{ is symmetric matrix}}$.

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Step 4 of 6

So, we have

$$\begin{aligned}(AA^+)^2 &= Q_1\Sigma\Sigma^+Q_1^T\\ &= AA^+\end{aligned}$$

Therefore, $\boxed{AA^+ \text{ is projection matrix}}$.

We know that every projection matrix is symmetric.

Therefore, $\boxed{AA^+ \text{ is symmetric matrix}}$.

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Step 5 of 6

Now find A^+A as follows:

$$\begin{aligned}A^+A &= (Q_2\Sigma^+Q_1^T)(Q_1\Sigma Q_2^T)\\ &= Q_2\Sigma^+\Sigma Q_2^T\end{aligned}$$

Squaring both sides of above equation we get

$$\begin{aligned}(A^+A)^2 &= (Q_2\Sigma^+\Sigma Q_2^T)^2\\ &= Q_2\Sigma^+\Sigma Q_2^TQ_2\Sigma^+\Sigma Q_2^T\\ &= Q_2\Sigma^+\Sigma\Sigma^+\Sigma Q_2^T\\ &= Q_2\Sigma^+\Sigma Q_2^T\end{aligned}$$

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Step 6 of 6

So, we have

$$= Q_2 \Sigma^+ \Sigma Q_2^T$$

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Step 6 of 6

So, we have

$$\left(A^+A\right)^2 = Q_2 \Sigma^+ \Sigma Q_2^T$$
$$= A^+A$$

Therefore, $\boxed{A^+A \text{ is projection matrix}}$.

We know that every projection matrix is symmetric.

Therefore, $\boxed{A^+A \text{ is symmetric matrix}}$.

We know that for $A = U\Sigma V^T$, U and V gives orthonormal bases for all four fundamental subspaces as follows:

First r columns of U: column space of A

Last $m-r$ columns of U: left null space of A

First r columns of V: row space of A

Last $n-r$ columns of V: null space of A

We have

$$AA^+ = Q_1 \Sigma \Sigma^+ Q_1^T$$
$$A^+A = Q_2 \Sigma^+ \Sigma Q_2^T$$

So, AA^+ and A^+A project onto the $\boxed{\text{column space and row space}}$ of A.

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