CH5.2

Step 1 of 3

Fibonacci sequence: Fibonacci sequence is given as follows:

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots$$

To prove that every third Fibonacci number in Fibonacci sequence is an even number.

Provide feedback (0)

Step 2 of 3

Fibonacci number follows the following rule.

$$F_{k+2} = F_{k+1} + F_k$$

This produces the following sequence of numbers:

Even, odd, odd, even, odd, odd, even…..

Here, every even number comes at third place.

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Step 3 of 3

Therefore, every third Fibonacci number in Fibonacci sequence is an even number.

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CH5.3 2P

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CH5.3 1P

Step 1 of 7

Consider the beetle matrix:

$$A = \begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}$$

Let x_k represents the number of beetles that are *k* years old.

So, we get the following equation from the matrix *A*.

$$x_1 = 6x_3$$
$$x_2 = \frac{1}{2}x_1$$
$$x_3 = \frac{1}{3}x_2$$

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CH5.3 3P

Step 2 of 7

To find the distribution of 3,000 beetles for six years, we have to compute

$$A^k \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

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Step 2 of 7

To find the distribution of 3,000 beetles for six years, we have to compute

$$A^k \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

For $k = 1, 2, 3, 4, 5,$ and 6.

For $k = 1$, we have

$$\begin{aligned} A^1 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} \\ &= \begin{bmatrix} 18,000 \\ 1,500 \\ 1,000 \end{bmatrix} \end{aligned}$$

Provide feedback (1)

Step 3 of 7

Step 3 of 7

Therefore, the population of the beetles for first year is $\boxed{\begin{bmatrix} 18,000 \\ 1,500 \\ 1,000 \end{bmatrix}}$.

Provide feedback (0)

Step 4 of 7

For $k = 2$, we have

$$A^2 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}^2 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ \frac{1}{6} & 0 & 0 \end{bmatrix} \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

$$= \begin{bmatrix} 6,000 \\ 9,000 \\ 500 \end{bmatrix}$$

Therefore, the population of the beetles for second year is $\boxed{\begin{bmatrix} 6,000 \\ 9,000 \\ 500 \end{bmatrix}}$.

$$\begin{bmatrix} 500 \end{bmatrix}$$

Therefore, the population of the beetles for second year is $\boxed{\begin{bmatrix} 6,000 \\ 9,000 \\ 500 \end{bmatrix}}$.

For $k = 3$, we have

$$\begin{aligned} A^3 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}^3 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} \\ &= \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} \end{aligned}$$

Therefore, the population of the beetles for third year is $\boxed{\begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}}$.

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Step 5 of 7

Step 5 of 7

For $k=4$, we have

$$A^4\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}=\begin{bmatrix}0&0&6\\\frac{1}{2}&0&0\\0&\frac{1}{3}&0\end{bmatrix}^4\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$

$$=\begin{bmatrix}0&0&6\\\frac{1}{2}&0&0\\0&\frac{1}{3}&0\end{bmatrix}\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$

$$=\begin{bmatrix}18,000\\1,500\\1,000\end{bmatrix}$$

Therefore, the population of the beetles for fourth year is $\boxed{\begin{bmatrix}18,000\\1,500\\1,000\end{bmatrix}}$.

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Step 6 of 7

For $k=5$, we have

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Step 6 of 7

For $k=5$, we have

$$A^5\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}=\begin{bmatrix}0&0&6\\\frac{1}{2}&0&0\\0&\frac{1}{3}&0\end{bmatrix}^5\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$

$$=\begin{bmatrix}0&2&0\\0&0&3\\\frac{1}{6}&0&0\end{bmatrix}\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$

$$=\begin{bmatrix}6,000\\9,000\\500\end{bmatrix}$$

Therefore, the population of the beetles for fifth year is $\boxed{\begin{bmatrix}6,000\\9,000\\500\end{bmatrix}}$.

For $k=6$, we have

$$\begin{bmatrix}3,000\end{bmatrix}\begin{bmatrix}0&0&6\\1\end{bmatrix}^6\begin{bmatrix}3,000\end{bmatrix}$$

For $k=6$, we have

$$A^6\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}=\begin{bmatrix}0&0&6\\\frac{1}{2}&0&0\\0&\frac{1}{3}&0\end{bmatrix}^6\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$
$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$
$$=\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$

Therefore, the population of the beetles for sixth year is $\boxed{\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}}$.

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Step 7 of 7

By observing the above distributions, we have seen that after every 3 years we got the initial population.

$$A^3\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}=\begin{bmatrix}3,000\\3,000\\3,000\end{bmatrix}$$

$$= \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

Therefore, the population of the beetles for sixth year is $\boxed{\begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}}$.

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Step 7 of 7

By observing the above distributions, we have seen that after every 3 years we got the initial population.

$$A^3 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} = \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

$$A^6 \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix} = \begin{bmatrix} 3,000 \\ 3,000 \\ 3,000 \end{bmatrix}$$

This only possible if $A^3 = I$.

Thus, $\boxed{A^3 = I}$.

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CH5.3 2P

CH5.3 4P

Step 1 of 5

Fibonacci sequence: Fibonacci sequence is given as follows:

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots$$

Let the Fibonacci matrix be as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

To compute A^2, A^3 and A^4. Also, calculate F_{20} using the text and calculator.

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Step 2 of 5

To compute A^2, A^3 and A^4 do the following calculations:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$A^3 \cdot A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$A^3 \cdot A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

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Step 3 of 5

Therefore, values of A^2, A^3 and A^4 are as follows:

$$A^2 = \boxed{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}}$$

$$A^3 = \boxed{\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}}$$

$$A^4 = \boxed{\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}}$$

Provide feedback (0)

Step 4 of 5

To calculate Fibonacci 20th number put $n = 20$ in the following:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$$A^4 = \boxed{\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}}$$

Provide feedback (0)

Step 4 of 5

To calculate Fibonacci 20th number put $n = 20$ in the following:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

And solve it using calculator:

$$\begin{aligned} F_{20} &= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{20} - \left(\frac{1-\sqrt{5}}{2}\right)^{20}\right] \\ &= 6764.99 \\ &\approx 6765 \end{aligned}$$

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Step 5 of 5

Therefore, $\boxed{F_{20} = 6765}$.

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CH5.3 3P

Step 1 of 12

Gibonacci number: Let G_{k+2} be the Gibonacci number which is defined by the average of two previous number G_{k+1}, G_k.

$$G_{k+2} = \frac{G_{k+1} + G_k}{2}$$

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Step 2 of 12

Let following be the difference equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrices u_{k+1}, A, u_k are defined as follows:

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

$$u_{k+1} = \begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix}$$

$$u_k = \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}$$

Provide feedback (2)

Step 3 of 12

(a) Find the Eigen values and Eigen vectors of matrix A:

To calculate the Eigen values do the following calculations;

$$\det(A - \lambda I) = 0$$

CH5.3 5P

$$u_k = \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}$$

Provide feedback (2)

Step 3 of 12

(a) Find the Eigen values and Eigen vectors of matrix A:

To calculate the Eigen values do the following calculations;

$$\det(A - \lambda I) = 0$$
$$\left(\frac{1}{2} - \lambda\right)(-\lambda) - \frac{1}{2} = 0$$
$$2\lambda^2 - \lambda - 1 = 0$$

After solving following values are obtained:

$$\lambda_1 = 1$$
$$\lambda_2 = \frac{-1}{2}$$

Therefore, Eigen values are $\boxed{1, -1/2}$

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Step 4 of 12

To calculate Eigen vectors do the following calculations:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} \frac{1}{2} - 1 & \frac{1}{2} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 12

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} \frac{1}{2}-1 & \frac{1}{2} \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=1$ is as follows:

$$\begin{aligned} x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 5 of 12

Similarly, Eigen vectors corresponding to Eigen value $\lambda=-\frac{1}{2}$ is as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -2 \end{bmatrix} \end{aligned}$$

Therefore Eigen values are:

Step 5 of 12

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -\frac{1}{2}$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Therefore Eigen values are:

$$x_1 = \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$
$$x_2 = \boxed{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}$$

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Step 6 of 12

(b) Find the limit of the following matrices when $n \to \infty$.

$$A^n = S\Lambda^n S^{-1}$$

Provide feedback (0)

Step 7 of 12

Matrix A can be written as follows:

$$A = S\Lambda S^{-1}$$
$$= \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{-1}{2} \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}$$

$$A^n = S\Lambda^n S^{-1}$$

Provide feedback (0)

Step 7 of 12

Matrix A can be written as follows:

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{-1}{2} \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \end{aligned}$$

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Step 8 of 12

Power matrix:

$$\begin{aligned} A^n &= S\Lambda^n S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1^n & 0 \\ 0 & \left(\frac{-1}{2}\right)^n \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \end{aligned}$$

Take the limit $n \to \infty$. Value of $\left(\frac{-1}{2}\right)^n$ becomes very small, so neglect it and do the above calculations.

$$\begin{aligned} A^n &= \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1^n & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \\ &= \begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix} \end{aligned}$$

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Provide feedback (0)

Step 10 of 12

(c) Consider the following values:

$$G_0 = 0$$
$$G_1 = 1$$

To show that Gibonacci numbers approaches to $\frac{2}{3}$.

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Step 11 of 12

Difference equation can be written as follows:

$$G_{k+1} = A^k G_0$$

Substitute the values to get the value of G_{k+1}

$$G_{k+1} = \begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$$

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Step 12 of 12

Therefore, above result shows that Gibonacci numbers approaches to $\boxed{\frac{2}{3}}$.

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Step 1 of 4

To complete the diagonalization of Fibonacci matrix by completing S^{-1}:

$$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}\lambda_1 & \lambda_2\\1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1 & 0\\0 & \lambda_2\end{bmatrix}\begin{bmatrix}\quad & \quad\\\quad & \quad\end{bmatrix}$$

Also, find the second component of the matrix by multiplying $S\Lambda^k S^{-1}\begin{bmatrix}1\\0\end{bmatrix}$.

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Step 2 of 4

From the following matrices, matrix S can be given as follows:

$$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}=\begin{bmatrix}\lambda_1 & \lambda_2\\1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1 & 0\\0 & \lambda_2\end{bmatrix}\begin{bmatrix}\quad & \quad\\\quad & \quad\end{bmatrix}$$

Thus, matrix S is :

$$S=\begin{bmatrix}\lambda_1 & \lambda_2\\1 & 1\end{bmatrix}$$

Inverse of S is given as follows:

$$S^{-1}=\frac{1}{\lambda_1-\lambda_2}\begin{bmatrix}1 & -\lambda_2\\-1 & \lambda_1\end{bmatrix}$$

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Step 3 of 4

Multiplying $S\Lambda^k S^{-1}\begin{bmatrix}1\\0\end{bmatrix}$ gives the following:

CH5.3 6P

Step 3 of 4

Multiplying $S\Lambda^k S^{-1}\begin{bmatrix}1\\0\end{bmatrix}$ gives the following:

$$S\Lambda^k S^{-1}\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{\lambda_1-\lambda_2}\begin{bmatrix}\lambda_1 & \lambda_2\\1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1^k & 0\\0 & \lambda_2^k\end{bmatrix}\begin{bmatrix}1 & -\lambda_2\\-1 & \lambda_1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$
$$=\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}\lambda_1^{k+1}-\lambda_2^{k+1}\\\lambda_1^k-\lambda_2^k\end{pmatrix}$$

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Step 4 of 4

Therefore, the second component is $\boxed{\frac{\lambda_1^k-\lambda_2^k}{\lambda_1-\lambda_2}}$. This is the kth Fibonacci number.

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CH5.3 5P

Step 1 of 4

Fibonacci rule is given by: $F_{k+2} = F_{k+1} + F_k$.

The numbers λ_1^k and λ_2^k satisfy the Fibonacci rule:

$$\lambda_1^{k+2} = \lambda_1^{k+1} + \lambda_1^k$$
$$\lambda_2^{k+2} = \lambda_2^{k+1} + \lambda_2^k$$

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Step 2 of 4

Let the Fibonacci matrix be as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

To find the Eigen values determinant can be written as follows:

$$\det(A - \lambda I) = \begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix}$$
$$= \lambda^2 - \lambda - 1$$

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Step 3 of 4

Put the determinant equal to zero.

$$\lambda^2 - \lambda - 1 = 0$$
$$\lambda^2 = \lambda + 1$$

Multiply it by λ_1^k and λ_2^k.

$$\lambda^{k+2} = \lambda^{k+1} + \lambda^k$$

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Step 3 of 4

Put the determinant equal to zero.

$$\lambda^2 - \lambda - 1 = 0$$
$$\lambda^2 = \lambda + 1$$

Multiply it by λ_1^k and λ_2^k.

$$\lambda_1^{k+2} = \lambda_1^{k+1} + \lambda_1^k$$
$$\lambda_2^{k+2} = \lambda_2^{k+1} + \lambda_2^k$$

Therefore, this shows that the numbers λ_1^k and λ_2^k satisfy the Fibonacci rule:

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Step 4 of 4

Any combination of λ_1^k and λ_2^k satisfies the rule. One of the combinations is given as follows:

$$F_k = \frac{\left(\lambda_1^k - \lambda_2^k\right)}{\lambda_1 - \lambda_2}$$

By putting different values of k, different value of F_k can be calculated.

Put $k = 0$ we get value of F_0.

$$F_0 = 0$$

Put $k = 1$ we get value of F_0.

$$F_1 = 1$$

Put the determinant equal to zero.

$$\lambda^2 - \lambda - 1 = 0$$
$$\lambda^2 = \lambda + 1$$

Multiply it by λ_1^k and λ_2^k.

$$\lambda_1^{k+2} = \lambda_1^{k+1} + \lambda_1^k$$
$$\lambda_2^{k+2} = \lambda_2^{k+1} + \lambda_2^k$$

Therefore, this shows that the numbers λ_1^k and λ_2^k satisfy the Fibonacci rule:

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Step 4 of 4

Any combination of λ_1^k and λ_2^k satisfies the rule. One of the combinations is given as follows:

$$F_k = \frac{\left(\lambda_1^k - \lambda_2^k\right)}{\lambda_1 - \lambda_2}$$

By putting different values of k, different value of F_k can be calculated.

Put $k = 0$ we get value of F_0.

$$F_0 = 0$$

Put $k = 1$ we get value of F_0.

$$F_1 = 1$$

Similarly, different values of k can give different Fibonacci numbers.

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CH5.3 6P

Step 1 of 6

Fibonacci rule is given by: $F_{k+2} = F_{k+1} + F_k$.
Let the Fibonacci matrix be as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

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Step 2 of 6

Lucas numbers starts with L_0, L_1 is given as follows:

$$L_0 = 2$$
$$L_1 = 1$$

Lucas rule is the same as Fibonacci rule.

$$L_{k+2} = L_{k+1} + L_k$$

Evaluate Lucas numbers L_{10} and λ_1^{10}.

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Step 3 of 6

First ten Lucas number is as follows:

$$\begin{aligned} L_2 &= L_1 + L_0 \\ &= 1 + 2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} L_3 &= L_2 + L_1 \\ &= 3 + 1 \end{aligned}$$

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Step 3 of 6

First ten Lucas number is as follows:

$$\begin{aligned} L_2 &= L_1 + L_0 \\ &= 1+2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} L_3 &= L_2 + L_1 \\ &= 3+1 \\ &= 4 \end{aligned}$$

Similarly, other calculated Lucas numbers are as follows:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123.

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Step 4 of 6

Therefore, $\boxed{L_{10} = 123}$

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Step 5 of 6

Eigen value of Lucas number will be:

$$\lambda_1 = \frac{1}{2}\left(1+\sqrt{5}\right)$$
$$\lambda_2 = \frac{1}{2}\left(1-\sqrt{5}\right)$$

So,

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Step 5 of 6

Eigen value of Lucas number will be:

$$\lambda_1 = \frac{1}{2}\left(1+\sqrt{5}\right)$$
$$\lambda_2 = \frac{1}{2}\left(1-\sqrt{5}\right)$$

So,

$$\begin{aligned}\lambda_1^{10} &= \left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{10}\\ &= 122.99\\ &\approx 123\end{aligned}$$

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Step 6 of 6

Therefore, $\boxed{\lambda_1^{10} = 123}$.

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Step 1 of 4

Stability status: Stability depends on the Eigen values. The difference equation $u_{k+1} = Au_k$ will have following stability status corresponding to the Eigen values:

Stable: If all Eigen values are $|\lambda_i| < 1$.

Neutrally stable: If some $|\lambda_i| < 1$ and some are $|\lambda_i| = 1$

Unstable: If atleast one Eigen value has $|\lambda_i| > 1$.

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Step 2 of 4

Find the steady state for the following equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrices u_{k+1}, A, u_k are defined as follows:

$$A = \begin{bmatrix} 1 & \frac{1}{4} & 0 \\ 0 & \frac{3}{4} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$u_{k+1} = \begin{bmatrix} d_{k+1} \\ s_{k+1} \\ w_{k+1} \end{bmatrix}$$

$$u_k = \begin{bmatrix} d_k \\ s_k \\ w_k \end{bmatrix}$$

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$$\begin{bmatrix} & 4 & 2 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$u_{k+1} = \begin{bmatrix} d_{k+1} \\ s_{k+1} \\ w_{k+1} \end{bmatrix}$$

$$u_k = \begin{bmatrix} d_k \\ s_k \\ w_k \end{bmatrix}$$

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Step 3 of 4

Matrix A is a triangular matrix, so main diagonal gives the Eigen values. Thus, Eigen values are as follows:

$$\lambda_1 = 1$$
$$\lambda_2 = \frac{3}{4}$$
$$\lambda_3 = \frac{1}{2}$$

From the above values see that $\lambda_1 = 1$ and all other Eigen values are $|\lambda_j| < 1$. This shows that difference equation is neutrally stable,

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Step 4 of 4

Therefore, $\boxed{u_{k+1} = Au_k \text{ is neutrally stable}}$.

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CH5.3 10P

Step 1 of 3

We know that in a markov transition matrix, each column adds to 1 and it has no negative entries.

Since in every week the chemistry course taught in section A is $\frac{1}{4}$ course and in section B $\frac{1}{3}$ course and $\frac{1}{6}$ of each section transfer to the other section.

In the first column we take two entries as $\frac{1}{4}$ and $\frac{1}{6}$.

To find the third entry in the first column, we do the following calculation:

$$\begin{aligned} 1-\left(\frac{1}{4}+\frac{1}{6}\right) &= 1-\left(\frac{2+3}{12}\right) \\ &= \frac{12-5}{12} \\ &= \frac{7}{12} \end{aligned}$$

Provide feedback (0)

Step 2 of 3

Now in the second column we take two entries as $\frac{1}{3}$ and $\frac{1}{6}$.

To find the third entry in the second column, we do the following calculation:

Step 2 of 3

Now in the second column we take two entries as $\frac{1}{3}$ and $\frac{1}{6}$.

To find the third entry in the second column, we do the following calculation:

$$\begin{aligned} 1-\left(\frac{1}{3}+\frac{1}{6}\right) &= 1-\left(\frac{2+1}{6}\right) \\ &= \frac{6-3}{6} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

Provide feedback (0)

Step 3 of 3

And in the third column we take entries as $0, 0, 1$.

Therefore, the transition matrix for a chemistry course that is taught in section A and Section B is

$$\boxed{\begin{bmatrix} \frac{7}{12} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{3} & 1 \end{bmatrix}}.$$

Provide feedback (0)

CH5.3 9P

CH5.3 11P

Step 1 of 7

Let matrices u_{k+1}, A, and u_k be defined as follows:

$$A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$$

$$u_{k+1} = \begin{bmatrix} y_{k+1} \\ z_{k+1} \end{bmatrix}$$

$$u_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

Let following be the difference equation of matrices:

$$u_{k+1} = Au_k$$

$$\begin{bmatrix} y_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

Provide feedback (0)

Step 2 of 7

Find the Eigen values and Eigen vectors of matrix *A*:

To calculate the Eigen values do the following calculations;

$$\det(A - \lambda I) = 0$$

$$(0.8 - \lambda)(0.7 - \lambda) - (0.3 \times 0.2) = 0$$

Provide feedback (0)

Step 2 of 7

Find the Eigen values and Eigen vectors of matrix *A*:

To calculate the Eigen values do the following calculations;

$$\det(A-\lambda I)=0$$
$$(0.8-\lambda)(0.7-\lambda)-(0.3\times 0.2)=0$$
$$\lambda^2-1.5\lambda+0.5=0$$

After solving following values are obtained:

$$\lambda_1=0.5$$
$$\lambda_2=1$$

Therefore, Eigen values are 0.5 and 1.

Provide feedback (0)

Step 3 of 7

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}0.8-\lambda & 0.3\\ 0.2 & 0.7-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Step 3 of 7

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 0.8-\lambda & 0.3 \\ 0.2 & 0.7-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_1 = 0.5$ is as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda_2 = 1$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Step 4 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda_2 = 1$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 7

Find the limit of the following matrices when $n \to \infty$.

$$A^n = S\Lambda^n S^{-1}$$

Matrix A can be written as follows:

$$A = S\Lambda S^{-1}$$
$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix}$$

Power matrix:

$$A^k = S\Lambda^k S^{-1}$$
$$\begin{bmatrix} -1 & 3 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \end{bmatrix}$$

Step 5 of 7

Find the limit of the following matrices when $n \to \infty$.

$$A^n = S\Lambda^n S^{-1}$$

Matrix A can be written as follows:

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix} \end{aligned}$$

Power matrix:

$$\begin{aligned} A^k &= S\Lambda^k S^{-1} \\ &= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 6 of 7

By using the initial condition $\begin{bmatrix} y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$, we have

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = A^k \begin{bmatrix} y_0 \\ z_0 \end{bmatrix}$$

Step 6 of 7

By using the initial condition $\begin{bmatrix} y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$, we have

$$\begin{aligned}
\begin{bmatrix} y_k \\ z_k \end{bmatrix} &= A^k \begin{bmatrix} y_0 \\ z_0 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3\times 0.5^k \\ 1^k \end{bmatrix}
\end{aligned}$$

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -3\times 0.5^k + 3\times 1^k \\ 3\times 0.5^k + 2\times 1^k \end{bmatrix}$$

Therefore, the formulas are $y_k = \boxed{-3\times 0.5^k + 3\times 1^k}$ and $z_k = \boxed{3\times 0.5^k + 2\times 1^k}$.

Provide feedback (0)

Step 7 of 7

Take the limit $k \to \infty$. Value of $(0.5)^k$ becomes very small, so neglect it, so we get

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -3\times 0.5^k + 3\times 1^k \\ 3\times 0.5^k + 2\times 1^k \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3\times 0.5^k \\ 1^k \end{bmatrix}$$

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -3\times 0.5^k + 3\times 1^k \\ 3\times 0.5^k + 2\times 1^k \end{bmatrix}$$

Therefore, the formulas are $y_k = \boxed{-3\times 0.5^k + 3\times 1^k}$ and $z_k = \boxed{3\times 0.5^k + 2\times 1^k}$.

Provide feedback (0)

Step 7 of 7

Take the limit $k \to \infty$. Value of $(0.5)^k$ becomes very small, so neglect it, so we get

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -3\times 0.5^k + 3\times 1^k \\ 3\times 0.5^k + 2\times 1^k \end{bmatrix}$$

$$\begin{bmatrix} y_\infty \\ z_\infty \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Therefore, when $k \to \infty$, the limiting values of $y_k = \boxed{3}$ and $z_k = \boxed{2}$.

Provide feedback (0)

CH5.3 10P

CH5.3 12P

Step 1 of 12

Consider the following matrix:

$$A=\begin{bmatrix}0.2 & 0.4 & 0.3\\ 0.4 & 0.2 & 0.3\\ 0.4 & 0.4 & 0.4\end{bmatrix}$$

Here, columns are linearly dependent as column 1 + column 2 = 2(column 3).

Provide feedback (0)

Step 2 of 12

(a) Find one Eigen value and Eigen vectors of matrix A.

To find the Eigen values calculate the following:

$$\det(A-\lambda I)=0$$

$$\begin{bmatrix}\frac{1}{5}-\lambda & \frac{2}{5} & \frac{3}{10}\\ \frac{2}{5} & \frac{1}{5}-\lambda & \frac{3}{10}\\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5}-\lambda\end{bmatrix}=0$$

$$-5\lambda^3+4\lambda^2+\lambda=0$$

$$\lambda(5\lambda+1)(-\lambda+1)=0$$

After solving following values are obtained:

$$\lambda=0$$

$$\lambda=1$$

$$\lambda=\frac{-1}{5}$$

$$\begin{bmatrix} \frac{2}{5} & \frac{2}{5} & \frac{2}{5}-\lambda \end{bmatrix}$$

$$-5\lambda^3+4\lambda^2+\lambda=0$$

$$\lambda(5\lambda+1)(-\lambda+1)=0$$

After solving following values are obtained:

$$\lambda=0$$

$$\lambda=1$$

$$\lambda=\frac{-1}{5}$$

Provide feedback (0)

Step 3 of 12

Lets take one Eigen value to calculate Eigen vectors:

$$\lambda=0$$

$$(A-0I)x=0$$

$$\begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 12

Solving it using Gaussians method:

$$\left(\begin{array}{ccc:c} \frac{1}{5} & \frac{2}{5} & \frac{3}{10} & 0 \\ 2 & 1 & 3 & \end{array}\right) \left(\begin{array}{ccc:c} 1 & 2 & \frac{3}{2} & 0 \\ & -3 & -3 & \end{array}\right)$$

Step 4 of 12

Solving it using Gaussians method:

$$\left(\begin{array}{ccc:c} \frac{1}{5} & \frac{2}{5} & \frac{3}{10} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{3}{10} & 0 \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & 0 \end{array}\right) \lrcorner \left(\begin{array}{ccc:c} 1 & 2 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{5} & \frac{-3}{10} & 0 \\ 0 & \frac{-2}{5} & \frac{-1}{5} & 0 \end{array}\right)$$

$$\urcorner \left(\begin{array}{ccc:c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Provide feedback (0)

Step 5 of 12

Thus, Eigen vectors are as follows:

$$x = \begin{bmatrix} -\frac{1}{2}c_1 \\ -\frac{1}{2}c_1 \\ c_1 \end{bmatrix}$$

Lets take the value of constant c_1 as follows and then solve:

$$c_1 = -2$$

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Step 5 of 12

Thus, Eigen vectors are as follows:

$$x = \begin{bmatrix} -\frac{1}{2}c_1 \\ -\frac{1}{2}c_1 \\ c_1 \end{bmatrix}$$

Lets take the value of constant c_1 as follows and then solve:

$$c_1 = -2$$

$$x = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 12

Therefore, Eigen vectors corresponding to Eigen value $\lambda = 0$ are as follows:

$$x = \boxed{\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}}$$

Provide feedback (0)

Step 7 of 12

(b) Find the other Eigen values of matrix A.

As solved in part (a) other Eigen values are as follows:

Provide feedback (0)

Step 7 of 12

(b) Find the other Eigen values of matrix A.

As solved in part (a) other Eigen values are as follows:

$$\lambda = \boxed{1}$$
$$\lambda = \boxed{\frac{-1}{5}}$$

Provide feedback (0)

Step 8 of 12

(c) Find the limit of $A^k u_0$ when $k \to \infty$ for the following value:

$$u_0 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 12

Difference equation can be written as follows:

$$\begin{aligned} u_{k+1} &= A^k u_0 \\ &= S\Lambda^k S^{-1} \cdot u_0 \end{aligned}$$

Following is the Eigen vector matrix.

$$\begin{bmatrix} 1 & 3 & -1 \end{bmatrix}$$

$$u_0 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 12

Difference equation can be written as follows:

$$\begin{aligned} u_{k+1} &= A^k u_0 \\ &= S\Lambda^k S^{-1} \cdot u_0 \end{aligned}$$

Following is the Eigen vector matrix.

$$S = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 3 & 1 \\ -2 & 4 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 12

Now, calculate the following:

$$\begin{aligned} u_{k+1} &= S\Lambda^k S^{-1} \cdot u_0 \\ &= \begin{bmatrix} 1 & 3 & -1 \\ 1 & 3 & 1 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1^k & 0 \\ 0 & 0 & \left(-\frac{1}{5}\right)^k \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-3}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{-1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Provide feedback (0)

Step 11 of 12

Taking the limit $k \to \infty$ makes the element $(-1/5)^k$ very small, so neglect it.

$$u_\infty = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 3 & 1 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-3}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{-1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-3}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{-1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$$

Provide feedback (0)

Step 12 of 12

Therefore, the limit of $A^k u_0$ is as follows:

$$u_\infty = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{}{10} & \frac{}{10} & \frac{}{10} \\ \frac{-1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-3}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{-1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$$

Provide feedback (0)

Step 12 of 12

Therefore, the limit of $A^k u_0$ is as follows:

$$\boxed{\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}}$$

Provide feedback (0)

CH5.3 11P

CH5.3 13P

Step 1 of 6

Let us consider the numbers of trucks in Los Angeles, Boston, and Chicago, are $a, b,$ and c respectively.

Since every month $\frac{1}{2}$ of those in Boston and in Los Angeles go to Chicago, the other $\frac{1}{2}$ stay where they are and the trucks in Chicago are split equally between Boston and Los Angeles.

So after one month, the new numbers of trucks in Los Angeles and Boston are given by

$$A = a/2 + c/2$$
$$B = b/2 + c/2$$
$$C = a/2 + b/2$$

The system of equation can be written as

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Provide feedback (0)

Step 2 of 6

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ & & \end{bmatrix}$$

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Step 2 of 6

Consider $A = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$.

Find one eigenvalue and eigenvector of matrix *A*.

To find the eigenvalue calculate the following:

$$\det(A - \lambda I) = 0$$

$$\begin{bmatrix} \frac{1}{2} - \lambda & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} - \lambda & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 - \lambda \end{bmatrix} = 0$$

$$\lambda^3 - \lambda^2 - 0.25\lambda + 0.25 = 0$$

After solving following values are obtained:

$$\lambda = 1$$

$$\lambda = \frac{1}{2}$$

$$\lambda = -\frac{1}{2}$$

Step 3 of 6

Let us take eigenvalue $\lambda = 1$ to calculate eigenvector:

$$(A-\lambda I)x = 0$$

$$\begin{bmatrix} \frac{1}{2}-1 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2}-1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0-1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 6

The augmented matrix is given by

$$\begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{bmatrix} \square \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 6

The augmented matrix is given by

$$\begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{bmatrix} \square \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

So, we have

$$\begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving the above matrix equation we get:

$$-\frac{a}{2} + \frac{c}{2} = 0$$
$$-\frac{b}{2} + \frac{c}{2} = 0$$

This implies

$$a = b = c$$

$a = b = c$

Provide feedback (0)

Step 5 of 6

Therefore, eigenvector corresponding to eigenvalue $\lambda = 1$ is as follows:

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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Step 6 of 6

The initial number of trucks are in Los Angeles, Boston, and Chicago are as follows

$$a = 1$$
$$b = 1$$
$$c = 1$$

After one month the new numbers of trucks in Los Angeles and Boston are given by

$$\begin{aligned} A &= 1/2 + 1/2 \\ &= 1 \\ B &= 1/2 + 1/2 \end{aligned}$$

$a = 1$
$b = 1$
$c = 1$

After one month the new numbers of trucks in Los Angeles and Boston are given by

$$\begin{aligned} A &= 1/2 + 1/2 \\ &= 1 \\ B &= 1/2 + 1/2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} C &= 1/2 + 1/2 \\ &= 1 \end{aligned}$$

Suppose we have the equal numbers of trucks in the three major centers and after every month the numbers of trucks will be same, so this defines a steady state.

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CH5.3 12P

CH5.3 14P

Step 1 of 10

Consider the equation

$$\begin{aligned} u_{k+1} &= Au_k \\ &= \begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix} u_k \end{aligned}$$

We know that according to Markov process, matrix has no negative entries and each column of the Markov matrix has summation equal to 1.

Thus, the range of *a* and *b* are $\boxed{0 \le a \le 1}$ and $\boxed{0 \le b \le 1}$.

Provide feedback (0)

Step 2 of 10

To compute $u_k = S\Lambda S^{-1}u_0$, we will find the eigenvalues and eigenvector of matrix *A*.

$$\begin{aligned} \det(A-\lambda I) &= 0 \\ \det\begin{bmatrix} a-\lambda & b \\ 1-a & 1-b-\lambda \end{bmatrix} &= 0 \\ (a-\lambda)(1-b-\lambda)-b(1-a) &= 0 \\ \lambda^2-\lambda(a-b+1)+(a-b) &= 0 \end{aligned}$$

Provide feedback (0)

Provide feedback (0)

Step 3 of 10

Solve the $\lambda^2 - \lambda(a-b+1) + (a-b) = 0$, using the following formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{(a-b+1) \pm \sqrt{(a-b+1)^2 - 4(1)(a-b)}}{2a}$$

$$= \frac{(a-b+1) \pm (a-b+1)}{2a}$$

Solving the above equation, we get λ_1 and λ_2 as

$$\lambda_1 = 1$$
$$\lambda_2 = a-b$$

Provide feedback (0)

Step 4 of 10

Now, the eigenvector for $\lambda_1 = 1$ is given by

$$(A - \lambda_1 I)x_1 = \begin{bmatrix} a-1 & b \\ 1-a & -b \end{bmatrix} x_1$$
$$= 0$$

Provide feedback (0)

Step 4 of 10

Now, the eigenvector for $\lambda_1 = 1$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix} a-1 & b \\ 1-a & -b \end{bmatrix} x_1 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_1 = 1$ is given by $\begin{bmatrix} b/(1-a) \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 5 of 10

Now, the eigenvector for $\lambda_2 = a-b$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} a-(a-b) & b \\ 1-a & (1-b)-(a-b) \end{bmatrix} x_2 \\ &= \begin{bmatrix} b & b \\ 1-a & (1-a) \end{bmatrix} x_2 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_2 = a-b$ $\lambda_1 = 1$ is given by $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Provide feedback (0)

Provide feedback (0)

Step 6 of 10

So, we have

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & a-b \end{bmatrix} \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \end{aligned}$$

To find A^k, and the distribution after k , we have to change $S\Lambda S^{-1}$ to $S\Lambda^k S^{-1}$:

$$\begin{aligned} A^k &= S\Lambda S^{-1} \\ &= \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \end{aligned}$$

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Step 7 of 10

Now $u_k = S\Lambda S^{-1}u_0$ is given by

$$\begin{aligned} u_k &= S\Lambda S^{-1}u_0 \\ &= \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} u_0 \end{aligned}$$

Consider $u_0 = \begin{bmatrix} 1 \\ \end{bmatrix}$, we have

Provide feedback (0)

Step 7 of 10

Now $u_k = S\Lambda S^{-1}u_0$ is given by

$$\begin{aligned} u_k &= S\Lambda S^{-1}u_0 \\ &= \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} u_0 \end{aligned}$$

Consider $u_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, we have

$$\begin{aligned} u_k &= \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} b/(1-a) & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{2b}{b-a+1} - \dfrac{1-a-b}{b-a+1}(a-b)^k \\ \dfrac{2(1-a)}{b-a+1} - \dfrac{1-a-b}{b-a+1}(a-b)^k \end{bmatrix} \end{aligned}$$

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Step 8 of 10

Therefore, $u_k = \boxed{\begin{bmatrix} \dfrac{2b}{b-a+1} - \dfrac{1-a-b}{b-a+1}(a-b)^k \\ \dfrac{2(1-a)}{b-a+1} - \dfrac{1-a-b}{b-a+1}(a-b)^k \end{bmatrix}}$.

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Step 9 of 10

Consider $u_k = \begin{bmatrix} \frac{2b}{b-a+1} - \frac{1-a-b}{b-a+1}(a-b)^k \\ \frac{2(1-a)}{b-a+1} - \frac{1-a-b}{b-a+1}(a-b)^k \end{bmatrix}$, if $|a-b|<1$, then u_k approach a finite limit

as $k \to \infty$., that is

$$u_k \to \begin{bmatrix} \frac{2b}{b-a+1} \\ \frac{2(1-a)}{b-a+1} \end{bmatrix}$$

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Step 10 of 10

We know that according to Markov process, matrix has no negative entries and each column of the Markov matrix has summation equal to 1.

Thus, the limit of *a* and *b* should be $\boxed{a = \frac{1}{3}}$ and $\boxed{b = -\frac{1}{3}}$.

With these values of *a* and *b*, we get

$$A = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

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Step 10 of 10

We know that according to Markov process, matrix has no negative entries and each column of the Markov matrix has summation equal to 1.

Thus, the limit of a and b should be $\boxed{a=\frac{1}{3}}$ and $\boxed{b=-\frac{1}{3}}$.

With these values of a and b, we get

$$A=\begin{bmatrix}\frac{1}{3} & -\frac{1}{3}\\ 1-\frac{1}{3} & 1-\left(-\frac{1}{3}\right)\end{bmatrix}$$
$$=\begin{bmatrix}\frac{1}{3} & -\frac{1}{3}\\ \frac{2}{3} & \frac{4}{3}\end{bmatrix}$$

Since matrix A has negative entry, A $\boxed{\text{is not a markov matrix}}$.

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CH5.3 14P

CH5.3 16P

Step 1 of 3

We know that Markov matrix has no negative entries, $a_{ij} \geq 0$ and each column of the matrix adding to 1.

Consider A is a Markov matrix and let $y = Ax$, here x is a vector.

We have to show that the sum of the components of y is equal to the sum of the components of vector x.

We know that for a vector x, the sum of the component of x is given by

$$\begin{aligned} x_1 + x_2 + x_3 + \cdots + x_n &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \\ &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} x \end{aligned}$$

We have considered A is Markov matrix, so we have

$$\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} A = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$$

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Step 2 of 3

The sum of the components of y is given by

$$y_1 + y_2 + y_3 + \cdots + y_n = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y \\ \vdots \end{bmatrix}$$

Step 2 of 3

The sum of the components of y is given by

$$\begin{aligned} y_1 + y_2 + y_3 + \cdots + y_n &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y \\ \vdots \\ y_n \end{bmatrix} \\ &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} y \end{aligned}$$

Since $y = Ax$ and $\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} A = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$, so we have

$$\begin{aligned} y_1 + y_2 + y_3 + \cdots + y_n &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} y \\ &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} Ax \\ &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} x \\ &= x_1 + x_2 + x_3 + \cdots + x_n \end{aligned}$$

Therefore, the sum of the components of y is $\boxed{\text{equal}}$ to the sum of the components of vector x.

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Step 3 of 3

We know that each column of the Markov matrix adding to 1, so the component of Ax is given by

$$x_1 + x_2 + x_3 + \cdots + x_n$$

And the component of λx is given by

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Step 3 of 3

We know that each column of the Markov matrix adding to 1, so the component of Ax is given by

$$x_1 + x_2 + x_3 + \cdots + x_n$$

And the component of λx is given by

$$\lambda\left(x_1 + x_2 + x_3 + \cdots + x_n\right)$$

If $Ax = \lambda x$ then we have

$$x_1 + x_2 + x_3 + \cdots + x_n = \lambda\left(x_1 + x_2 + x_3 + \cdots + x_n\right).$$

Since $\lambda \neq 1$ then $x_1 + x_2 + x_3 + \cdots + x_n$ must be $\boxed{\text{zero}}$.

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CH5.3 15P

CH5.3 17P

Step 1 of 18

Consider the solution for differential equation

$$\begin{aligned}\frac{du}{dt} &= Au \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} u\end{aligned}$$

goes around in a circle $u = (\cos t, \sin t)$.

Matrix $I + A$ is given by

$$\begin{aligned} I + A &= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 2 of 18

To find the eigenvalues of matrix $I + A$, solve the equation $\det((I + A) - \lambda I) = 0$, for λ.

$$\det \begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

Provide feedback (0)

Step 2 of 18

To find the eigenvalues of matrix $I+A$, solve the equation $\det\left(\left(I+A\right)-\lambda I\right)=0$, for λ.

$$\det\begin{bmatrix}1-\lambda & -1\\ 1 & 1-\lambda\end{bmatrix}=0$$
$$\left(\lambda-1\right)^2=-1$$
$$\lambda-1=\pm i$$

Therefore, the eigenvalues of matrix $I+A$ are $\lambda_1=\boxed{1-i}$ and $\lambda_2=\boxed{1+i}$.

Provide feedback (0)

Step 3 of 18

To calculate eigenvector x_1 do the following calculation:

$$\det\left(\left(I+A\right)-\lambda_1 I\right)x_1=0$$
$$\begin{bmatrix}1-\left(1-i\right) & -1\\ 1 & 1-\left(1-i\right)\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_1=1-i$ is as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

Step 3 of 18

To calculate eigenvector x_1 do the following calculation:

$$\det\left((I+A)-\lambda_1 I\right)x_1 = 0$$

$$\begin{bmatrix} 1-(1-i) & -1 \\ 1 & 1-(1-i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_1 = 1-i$ is as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ i \end{bmatrix}$$

Provide feedback (0)

Step 4 of 18

To calculate eigenvector x_2 do the following calculation:

$$\det\left((I+A)-\lambda_2 I\right)x_2 = 0$$

$$\begin{bmatrix} 1-(1+i) & -1 \\ 1 & 1-(1+i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_2 = 1+i$ is as follows:

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Step 4 of 18

To calculate eigenvector x_2 do the following calculation:

$$\det\left(\left(I+A\right)-\lambda_2 I\right)x_2 = 0$$

$$\begin{bmatrix} 1-(1+i) & -1 \\ 1 & 1-(1+i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_2 = 1+i$ is as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -i \end{bmatrix} \end{aligned}$$

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Step 5 of 18

Therefore, the eigenvectors for the matrix $I+A$ are $x_1 = \boxed{\begin{bmatrix} 1 \\ i \end{bmatrix}}$ and $x_2 = \boxed{\begin{bmatrix} 1 \\ -i \end{bmatrix}}$.

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Step 6 of 18

Step 6 of 18

Matrix $(I-A)^{-1}$ is given by

$$\begin{aligned}(I-A)^{-1} &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}\end{aligned}$$

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Step 7 of 18

To find the eigenvalues of matrix $(I-A)^{-1}$, solve the equation $\det\left((I-A)^{-1}-\lambda I\right)=0$, for λ.

$$\begin{aligned}\det\begin{bmatrix} 1/2-\lambda & -1/2 \\ 1/2 & 1/2-\lambda \end{bmatrix} &= 0 \\ \left(\frac{1}{2}-\lambda\right)^2+\frac{1}{4} &= 0\end{aligned}$$

Therefore, the eigenvalues of matrix $(I-A)^{-1}$ are $\lambda_1=\boxed{\frac{1}{2}-\frac{1}{2}i}$ and $\lambda_2=\boxed{\frac{1}{2}+\frac{1}{2}i}$.

To calculate eigenvector x_1 do the following calculation:

Step 7 of 18

To find the eigenvalues of matrix $(I-A)^{-1}$, solve the equation $\det\left((I-A)^{-1}-\lambda I\right)=0$, for λ.

$$\det\begin{bmatrix} 1/2-\lambda & -1/2 \\ 1/2 & 1/2-\lambda \end{bmatrix}=0$$

$$\left(\frac{1}{2}-\lambda\right)^2+\frac{1}{4}=0$$

Therefore, the eigenvalues of matrix $(I-A)^{-1}$ are $\lambda_1=\boxed{\frac{1}{2}-\frac{1}{2}i}$ and $\lambda_2=\boxed{\frac{1}{2}+\frac{1}{2}i}$.

To calculate eigenvector x_1 do the following calculation:

$$\det\left((I-A)^{-1}-\lambda_1 I\right)x_1=0$$

$$\begin{bmatrix} 1/2-(1/2-1/2i) & -1/2 \\ 1/2 & 1/2-(1/2-1/2i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_1=(1/2-1/2i)$ is as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ i \end{bmatrix}$$

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Step 8 of 18

To calculate eigenvector x_2 do the following calculation:

$$\det\left((I-A)^{-1} - \lambda_2 I\right) x_2 = 0$$

$$\begin{bmatrix} 1/2-(1/2+1/2i) & -1/2 \\ 1/2 & 1/2-(1/2+1/2i) \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_2 = (1/2+1/2i)$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

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Step 9 of 18

Therefore, the eigenvectors for the matrix $(I-A)^{-1}$ are $x_1 = \boxed{\begin{bmatrix} 1 \\ i \end{bmatrix}}$ and $x_2 = \boxed{\begin{bmatrix} 1 \\ -i \end{bmatrix}}$.

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$$= \begin{bmatrix} \cdot \\ -i \end{bmatrix}$$

Provide feedback (0)

Step 9 of 18

Therefore, the eigenvectors for the matrix $(I-A)^{-1}$ are $x_1 = \boxed{\begin{bmatrix} 1 \\ i \end{bmatrix}}$ and $x_2 = \boxed{\begin{bmatrix} 1 \\ -i \end{bmatrix}}$.

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Step 10 of 18

Consider matrix $B = \left(I - \frac{1}{2}A\right)^{-1}\left(I + \frac{1}{2}A\right)$ is given by

$$\begin{aligned} B &= \begin{bmatrix} 1 & 1/2 \\ -1/2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1/2 \\ 1/2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4/5 & -2/5 \\ 2/5 & 4/5 \end{bmatrix} \begin{bmatrix} 1 & -1/2 \\ 1/2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix} \end{aligned}$$

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Step 11 of 18

To find the eigenvalues of matrix B, solve the equation $\det(B - \lambda I) = 0$, for λ.

To find the eigenvalues of matrix B, solve the equation $\det(B-\lambda I)=0$, for λ.

$$\det\begin{bmatrix} 3/5-\lambda & -4/5 \\ 4/5 & 3/5-\lambda \end{bmatrix}=0$$
$$\left(\frac{3}{5}-\lambda\right)^2+\frac{16}{25}=0$$

Therefore, the eigenvalues of matrix B are $\lambda_1=\boxed{\frac{3}{5}-\frac{4}{5}i}$ and $\lambda_2=\boxed{\frac{3}{5}+\frac{4}{5}i}$.

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Step 12 of 18

To calculate eigenvector x_1 do the following calculation:

$$\det(B-\lambda_1 I)x_1=0$$
$$\begin{bmatrix} 3/5-(3/5-4/5i) & -4/5 \\ 4/5 & 3/5-(3/5-4/5i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda_1=(3/5-4/5i)$ is as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$
$$=\begin{bmatrix} 1 \\ i \end{bmatrix}$$

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Step 12 of 18

To calculate eigenvector x_1 do the following calculation:

$$\det\left(B-\lambda_1 I\right)x_1=0$$

$$\begin{bmatrix} 3/5-(3/5-4/5i) & -4/5 \\ 4/5 & 3/5-(3/5-4/5i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_1=(3/5-4/5i)$ is as follows:

$$\begin{aligned} x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ i \end{bmatrix} \end{aligned}$$

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Step 13 of 18

To calculate eigenvector x_2 do the following calculation:

$$\det\left(B-\lambda_2 I\right)x_2=0$$

$$\begin{bmatrix} 3/5-(3/5+4/5i) & -4/5 \\ 4/5 & 3/5-(3/5+4/5i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Step 13 of 18

To calculate eigenvector x_2 do the following calculation:

$$\det\left(B-\lambda_2 I\right)x_2=0$$

$$\begin{bmatrix} 3/5-(3/5+4/5i) & -4/5 \\ 4/5 & 3/5-(3/5+4/5i) \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda_2=(3/5+4/5i)$ is as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -i \end{bmatrix} \end{aligned}$$

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Step 14 of 18

Therefore, the eigenvectors for the matrix $(I-A)^{-1}$ are $x_1=\boxed{\begin{bmatrix} 1 \\ i \end{bmatrix}}$ and $x_2=\boxed{\begin{bmatrix} 1 \\ -i \end{bmatrix}}$.

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Step 15 of 18

Consider (x_1, x_2) lies on unit circle then $\sqrt{(x_1)^2+(x_2)^2}=1$, that is

Step 15 of 18

Consider (x_1, x_2) lies on unit circle then $\sqrt{(x_1)^2+(x_2)^2}=1$, that is

$$(x_1)^2+(x_2)^2=1.$$

Take the forward difference (**F**):

$$u_{n+1}=(I+A)u_n$$

So, solving the following equation, we have

$$\begin{aligned}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= \begin{bmatrix} x_1-x_2 \\ x_1+x_2 \end{bmatrix}\end{aligned}$$

If (y_1, y_2) lies on unit circle then $y_1^2+y_2^2=1$.

Solve the following equation:

$$\begin{aligned} y_1^2+y_2^2 &= (x_1-x_2)^2+(x_1+x_2)^2 \\ &= 2\left[(x_1)^2+(x_2)^2\right] \\ &= 2(1) \\ &\neq 1 \end{aligned}$$

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Provide feedback (0)

Step 16 of 18

Therefore, for the differential equation $u_{n+1} = (I + A)u_n$, solution does not stay on a circle.

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Step 17 of 18

Consider the forward difference (**B**):

$$u_{n+1} = (I - A)^{-1} u_n$$

So, solving the following equation, we have

$$\begin{aligned}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &= \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= \begin{bmatrix} 1/2\,x_1 - 1/2\,x_2 \\ 1/2\,x_1 + 1/2\,x_2 \end{bmatrix}\end{aligned}$$

If (z_1, z_2) lies on unit circle then $z_1^2 + z_2^2 = 1$. For this solve the following equation:

$$\begin{aligned} z_1^2 + z_2^2 &= \left(\frac{1}{2}x_1 - \frac{1}{2}x_2\right)^2 + \left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right)^2 \\ &= \frac{1}{4}\left[(x_1)^2 + (x_2)^2\right] \\ &= \frac{1}{4}(1) \end{aligned}$$

$$u_{n+1} = (I - A)^{-1} u_n$$

So, solving the following equation, we have

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= \begin{bmatrix} 1/2\, x_1 - 1/2\, x_2 \\ 1/2\, x_1 + 1/2\, x_2 \end{bmatrix}$$

If (z_1, z_2) lies on unit circle then $z_1^2 + z_2^2 = 1$. For this solve the following equation:

$$\begin{aligned} z_1^2 + z_2^2 &= \left(\frac{1}{2}x_1 - \frac{1}{2}x_2\right)^2 + \left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right)^2 \\ &= \frac{1}{4}\left[(x_1)^2 + (x_2)^2\right] \\ &= \frac{1}{4}(1) \\ &\neq 1 \end{aligned}$$

Therefore, for the differential equation $u_{n+1} = (I - A)^{-1} u_n$, solution does not stay on a circle.

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Step 18 of 18

Consider the forward difference (**C**):

$$u_{n+1} = \left(I - \frac{1}{2}A\right)^{-1} \left(I + \frac{1}{2}A\right) u_n$$

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Step 18 of 18

Consider the forward difference (**C**):

$$u_{n+1} = \left(I - \frac{1}{2}A\right)^{-1}\left(I + \frac{1}{2}A\right)u_n$$

So, solving the following equation, we have

$$\begin{aligned}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} &= \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= \begin{bmatrix} 3/5\,x_1 - 4/5\,x_2 \\ 4/5\,x_1 + 3/5\,x_2 \end{bmatrix}\end{aligned}$$

If $(w_1,\ w_2)$ lies on unit circle then $w_1^2 + w_2^2 = 1$. For this solve the following equation:

$$\begin{aligned} w_1^2 + w_2^2 &= \left(\frac{3}{5}x_1 - \frac{4}{5}x_2\right)^2 + \left(\frac{4}{5}x_1 + \frac{3}{5}x_2\right)^2 \\ &= \frac{25}{25}\left[(x_1)^2 + (x_2)^2\right] \\ &= 1(1) \\ &= 1 \end{aligned}$$

Therefore, for the differential equation $u_{n+1} = \left(I - \frac{1}{2}A\right)^{-1}\left(I + \frac{1}{2}A\right)u_n$, solution does stay on a circle.

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CH5.3 16P

CH5.3 18P

Step 1 of 6

Stability status: Stability depends on the Eigen values. The difference equation $u_{k+1} = Au_k$ will have following stability status corresponding to the Eigen values:

Stable: If all Eigen values are $|\lambda_i| < 1$.

Neutrally stable: If some $|\lambda_i| < 1$ and some are $|\lambda_i| = 1$

Unstable: If atleast one Eigen value has $|\lambda_i| > 1$.

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Step 2 of 6

To find the value of α which produces instability in the following:

$$v_{n+1} = \alpha v_n + \alpha w_n$$
$$w_{n+1} = \alpha v_n + \alpha w_n$$

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Step 3 of 6

Above defined equations can be converted into the following equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrices u_{k+1}, A, u_k are defined as follows:

$$A = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix}$$
$$u_{k+1} = \begin{bmatrix} v_{n+1} \\ w_{n+1} \end{bmatrix}$$
$$u_k = \begin{bmatrix} v_n \\ w_n \end{bmatrix}$$

Above defined equations can be converted into the following equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrices u_{k+1}, A, u_k are defined as follows:

$$A = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix}$$

$$u_{k+1} = \begin{bmatrix} v_{n+1} \\ w_{n+1} \end{bmatrix}$$

$$u_k = \begin{bmatrix} v_n \\ w_n \end{bmatrix}$$

Provide feedback (0)

Step 4 of 6

Calculate the Eigen values of matrix A by doing following calculations:

$$\det(A - \lambda I) = 0$$

$$\alpha^2 + \lambda^2 - 2\alpha\lambda - \alpha^2 = 0$$

$$\lambda(\lambda - 2\alpha) = 0$$

$$\lambda = 0$$

$$\text{Or } \lambda = 2\alpha$$

Provide feedback (0)

Step 5 of 6

Difference equation $u_{k+1} = Au_k$ will have following stability status:

Stable: If $|\lambda_i| < 1$ or

$$|2\alpha| < 1$$

$$|\alpha| < \frac{1}{}$$

Or $\lambda = 2\alpha$

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Step 5 of 6

Difference equation $u_{k+1} = Au_k$ will have following stability status:

Stable: If $|\lambda_i| < 1$ or

$$|2\alpha| < 1$$
$$|\alpha| < \frac{1}{2}$$

Unstable: If $|\lambda_i| > 1$ or

$$|2\alpha| > 1$$
$$|\alpha| > \frac{1}{2}$$

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Step 6 of 6

Therefore, value of $\boxed{|\alpha| > \frac{1}{2}}$ gives the instability in $u_{k+1} = Au_k$.

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CH5.3 17P

Step 1 of 10

Consider the following matrix:

$$A=\begin{bmatrix} a & -0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(A-\lambda I)=0$. This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} a-\lambda & -0.8 \\ 0.8 & 0.2-\lambda \end{vmatrix} \\ &= (a-\lambda)(0.2-\lambda)+0.64 \\ &= \lambda^2-(0.2+a)\lambda+0.2a+0.64 \end{aligned}$$

Provide feedback (0)

CH5.3 19P

Step 2 of 10

Consider the equation, $\lambda^2-(0.2+a)\lambda+0.2a+0.64=0$.

Thus,

$$\begin{aligned} \lambda &= \frac{(0.2+a)\pm\sqrt{(0.2+a)^2-4(0.2a+0.64)}}{2} \\ &= \frac{(0.2+a)\pm\sqrt{(0.04+0.4a+a^2)-(0.8a+2.56)}}{2} \\ &= \frac{(0.2+a)\pm\sqrt{a^2-0.4a-2.52}}{} \end{aligned}$$

Provide feedback (0)

Step 2 of 10

Consider the equation, $\lambda^2-(0.2+a)\lambda+0.2a+0.64=0$.

Thus,

$$\begin{aligned}\lambda &= \frac{(0.2+a)\pm\sqrt{(0.2+a)^2-4(0.2a+0.64)}}{2}\\ &= \frac{(0.2+a)\pm\sqrt{(0.04+0.4a+a^2)-(0.8a+2.56)}}{2}\\ &= \frac{(0.2+a)\pm\sqrt{a^2-0.4a-2.52}}{2}\end{aligned}$$

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Step 3 of 10

Since $|\lambda_i|<1$, we get the following:

$$\begin{aligned}-2<(0.2+a)\pm\sqrt{a^2-0.4a-2.52}&<2\\ -2.2<a\pm\sqrt{a^2-0.4a-2.52}&<1.98\\ -2.2-a<\pm\sqrt{a^2-0.4a-2.52}&<1.98-a\\ (-2.2-a)^2<a^2-0.4a-2.52&<(1.98-a)^2\end{aligned}$$

Simplifying, we get

Provide feedback (0)

Step 3 of 10

Since $|\lambda_i| < 1$, we get the following:

$$\begin{aligned}
-2 < (0.2+a) \pm \sqrt{a^2 - 0.4a - 2.52} < 2 \\
-2.2 < a \pm \sqrt{a^2 - 0.4a - 2.52} < 1.98 \\
-2.2 - a < \pm\sqrt{a^2 - 0.4a - 2.52} < 1.98 - a \\
(-2.2-a)^2 < a^2 - 0.4a - 2.52 < (1.98-a)^2
\end{aligned}$$

Simplifying, we get

$$\begin{aligned}
a^2 + 4.4a + 4.84 < a^2 - 0.4a - 2.52 < a^2 - 3.96a + 3.9204 \\
4.4a + 4.84 < -0.4a - 2.52 < -3.96a + 3.9204 \\
4.4a + 7.36 < -0.4a < -3.96a + 6.4404 \\
-11a - 18.4 > a > 9.9a - 16.101
\end{aligned}$$

Provide feedback (0)

Step 4 of 10

Consider the two inequalities separately.

$$\begin{aligned}
-11a - 18.4 > a \\
-18.4 > 12a \\
\frac{-18.4}{12} > a \\
-1.533 > a
\end{aligned}$$

Simplifying, we get

$$\begin{aligned} a^2+4.4a+4.84 &< a^2-0.4a-2.52 < a^2-3.96a+3.9204 \\ 4.4a+4.84 &< -0.4a-2.52 < -3.96a+3.9204 \\ 4.4a+7.36 &< -0.4a < -3.96a+6.4404 \\ -11a-18.4 &> a > 9.9a-16.101 \end{aligned}$$

Provide feedback (0)

Step 4 of 10

Consider the two inequalities separately.

$$\begin{aligned} -11a-18.4 &> a \\ -18.4 &> 12a \\ \frac{-18.4}{12} &> a \\ -1.533 &> a \end{aligned}$$

$$\begin{aligned} a &> 9.9a-16.101 \\ -8.9a &> -16.101 \\ a &< \frac{16.101}{8.9} \\ a &< 1.809 \end{aligned}$$

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Step 5 of 10

Thus, if matrix A has to be stable, $\boxed{a < -1.533}$.

Step 5 of 10

Thus, if matrix *A* has to be stable, $\boxed{a < -1.533}$.

If the matrix A has to be neutrally stable, some of its eigenvalues satisfy $|\lambda_i| = 1$ and other eigenvalues satisfy $|\lambda_i| < 1$.

This gives, $\boxed{a = 1.809}$ or $\boxed{a = -1.533}$.

Consider the following matrix:

$$B = \begin{bmatrix} b & 0.8 \\ 0 & 0.2 \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(B - \lambda I) = 0$. This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} b-\lambda & 0.8 \\ 0 & 0.2-\lambda \end{vmatrix} \\ &= (b-\lambda)(0.2-\lambda) + 0 \\ &= \lambda^2 - (0.2+b)\lambda + 0.2b \end{aligned}$$

Provide feedback (0)

Step 6 of 10

Consider the equation $\lambda^2 - (0.2+b)\lambda + 0.2b = 0$.

Thus,

Step 6 of 10

Consider the equation $\lambda^2-(0.2+b)\lambda+0.2b=0$.

Thus,

$$\begin{aligned}\lambda &= \frac{(0.2+b)\pm\sqrt{(0.2+b)^2-0.8b}}{2}\\ &= \frac{(0.2+b)\pm\sqrt{b^2+0.4b+0.04-0.8b}}{2}\\ &= \frac{(0.2+b)\pm\sqrt{b^2-0.4b+0.04}}{2}\\ &= \frac{(0.2+b)\pm\sqrt{(b-0.2)^2}}{2}\\ &= \frac{(0.2+b)\pm(b-0.2)}{2}\\ &= b \text{ or } 0.2\end{aligned}$$

Provide feedback (0)

Step 7 of 10

For the matrix *B* to be stable, we want $|\lambda_i|<1$. Therefore, $\boxed{b<1}$.

For the matrix *B* to be neutrally stable, we want some eigenvalue λ such that $|\lambda_i|=1$ and other λ such that $|\lambda_i|<1$. Therefore, $\boxed{b=1}$.

For the matrix B to be stable, we want $|\lambda_i| < 1$. Therefore, $\boxed{b < 1}$.

For the matrix B to be neutrally stable, we want some eigenvalue λ such that $|\lambda_i| = 1$ and other λ such that $|\lambda_i| < 1$. Therefore, $\boxed{b = 1}$.

Provide feedback (0)

Step 8 of 10

Consider the following matrix:

$$C = \begin{bmatrix} c & 0.8 \\ 0.2 & c \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(C - \lambda I) = 0$. This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} c-\lambda & 0.8 \\ 0.2 & c-\lambda \end{vmatrix} \\ &= (c-\lambda)^2 - 0.16 \\ &= \lambda^2 - 2c\lambda + (c^2 - 0.16) \end{aligned}$$

Provide feedback (0)

Step 9 of 10

Consider the equation $\lambda^2 - 2c\lambda + (c^2 - 0.16) = 0$.

$$= (c-\lambda)^2 - 0.16$$
$$= \lambda^2 - 2c\lambda + (c^2 - 0.16)$$

Provide feedback (0)

Step 9 of 10

Consider the equation $\lambda^2 - 2c\lambda + (c^2 - 0.16) = 0$.

Thus,

$$\lambda = \frac{2c \pm \sqrt{4c^2 - 4(c^2 - 0.16)}}{2}$$
$$= \frac{2c \pm \sqrt{0.64}}{2}$$
$$= \frac{2c \pm 0.8}{2}$$
$$= c \pm 0.4$$

Provide feedback (0)

Step 10 of 10

For the matrix *C* to be stable, we want $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 < c < 0.6}$.

For the matrix *C* to be neutrally stable, we want some eigenvalue λ such that $|\lambda_i| = 1$ and other λ such that $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 \le c \le 0.6}$.

$$= \frac{2c \pm 0.8}{2}$$
$$= c \pm 0.4$$

Provide feedback (0)

Step 10 of 10

For the matrix C to be stable, we want $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 < c < 0.6}$.

For the matrix C to be neutrally stable, we want some eigenvalue λ such that $|\lambda_i| = 1$ and other λ such that $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 \le c \le 0.6}$.

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CH5.3 18P

Step 1 of 5

This series $\left(I+A+A^2+\ldots\right)$ represent $\left(I-A\right)^{-1}$.

Check the following:

$$\left(I-A\right)\left(I+A+A^2+\ldots\right)=I$$

Solve the left hand side.

$$\begin{aligned}\left(I-A\right)\left(I+A+A^2+\ldots\right)&=I\left(I+A+A^2+\ldots\right)-A\left(I+A+A^2+\ldots\right)\\&=\left(I+A+A^2+\ldots\right)-\left(A+A^2+A^3+\ldots\right)\\&=I\end{aligned}$$

Therefore, left hand side comes out to be equal to right hand side.

Provide feedback (0)

Step 2 of 5

Consider the following matrix which has $\lambda_{\max}=0$.

$$A=\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix}$$

Provide feedback (0)

Step 3 of 5

Find the powers of matrix A as follows:

$$A\cdot A=\begin{bmatrix}0&1&1\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}0&1&1\\0&0&1\end{bmatrix}$$

CH5.3 20P

Step 3 of 5

Find the powers of matrix A as follows:

$$A \cdot A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, A^3 results into zero matrix. so other higher powers of matrix A will also be zero.

Provide feedback (0)

Step 4 of 5

Substitute the values into the following series:

$$\begin{aligned}(I-A)^{-1} &= \left(I + A + A^2\right) \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

Here, A^3 results into zero matrix. so other higher powers of matrix A will also be zero.

Provide feedback (0)

Step 4 of 5

Substitute the values into the following series:

$$\begin{aligned}(I-A)^{-1} &= \left(I+A+A^2\right)\\ &= \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix}+\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\\ &= \begin{bmatrix}1&1&2\\0&1&1\\0&0&1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 5 of 5

Therefore, above result shows the following:

$$\boxed{(I-A)^{-1} = \begin{bmatrix}1&1&2\\0&1&1\\0&0&1\end{bmatrix}}$$

Provide feedback (0)

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CH5.3 19P

CH5.3 21P

Step 1 of 6

This series $(I+A+A^2+\ldots)$ represent $(I-A)^{-1}$.

Consider the following matrix which has $\lambda_{max}=0$.

$$A=\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

Find the powers of matrix A and show that it equals to $(I-A)^{-1}$.

Provide feedback (0)

Step 2 of 6

Find the powers of matrix A as follows:

$$A\cdot A=\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}\cdot\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

$$A^2=\begin{bmatrix}0 & 0.1\\ 0 & 0.25\end{bmatrix}$$

$$A^2\cdot A=\begin{bmatrix}0 & 0.1\\ 0 & 0.25\end{bmatrix}\cdot\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

$$A^3=\begin{bmatrix}0 & 0.05\\ 0 & 0.125\end{bmatrix}$$

Provide feedback (0)

Step 3 of 6

Similarly,

$$A^3\cdot A=\begin{bmatrix}0 & 0.05\\ 0 & 0.125\end{bmatrix}\cdot\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

$$\begin{bmatrix}0 & 0.25\end{bmatrix}\begin{bmatrix}0 & 0.5\end{bmatrix}$$

$$A^3 = \begin{bmatrix}0 & 0.05\\ 0 & 0.125\end{bmatrix}$$

Provide feedback (0)

Step 3 of 6

Similarly,

$$A^3 \cdot A = \begin{bmatrix}0 & 0.05\\ 0 & 0.125\end{bmatrix} \cdot \begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

$$A^4 = \begin{bmatrix}0 & 0.025\\ 0 & 0.0625\end{bmatrix}$$

$$A^4 \cdot A = \begin{bmatrix}0 & 0.025\\ 0 & 0.0625\end{bmatrix} \cdot \begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

$$A^5 = \begin{bmatrix}0 & 0.0125\\ 0 & 0.03125\end{bmatrix}$$

$$A^5 \cdot A = \begin{bmatrix}0 & 0.0125\\ 0 & 0.03125\end{bmatrix} \cdot \begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}$$

$$A^6 = \begin{bmatrix}0 & 0.00625\\ 0 & 0.015625\end{bmatrix}$$

Here, A^6 matrix elements are very small and reduce further in other higher powers of matrix A.

Provide feedback (0)

Step 4 of 6

Substitute the values into the following series:

$$(I-A)^{-1} = \left(I + A + A^2 + A^3 + A^4 + A^5 + A^6\right)$$

$$= \begin{bmatrix}1 & 0\end{bmatrix} + \begin{bmatrix}0 & 0.2\end{bmatrix} + \begin{bmatrix}0 & 0.1\end{bmatrix} + \begin{bmatrix}0 & 0.05\end{bmatrix}$$

Here, A^6 matrix elements are very small and reduce further in other higher powers of matrix A.

Provide feedback (0)

Step 4 of 6

Substitute the values into the following series:

$$\begin{aligned}(I-A)^{-1} &= \left(I+A+A^2+A^3+A^4+A^5+A^6\right)\\ &= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}+\begin{bmatrix}0 & 0.1\\ 0 & 0.25\end{bmatrix}+\begin{bmatrix}0 & 0.05\\ 0 & 0.125\end{bmatrix}\\ &\quad+\begin{bmatrix}0 & 0.025\\ 0 & 0.0625\end{bmatrix}+\begin{bmatrix}0 & 0.0125\\ 0 & 0.03125\end{bmatrix}+\begin{bmatrix}0 & 0.00625\\ 0 & 0.015625\end{bmatrix}\\ &= \begin{bmatrix}1 & 0.39375\\ 0 & 1.98437\end{bmatrix}\\ &\approx \begin{bmatrix}1 & 0.4\\ 0 & 2\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 5 of 6

Now, calculate the following:

$$\begin{aligned}(I-A)^{-1} &= \left\{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 0.2\\ 0 & 0.5\end{bmatrix}\right\}^{-1}\\ &= \begin{bmatrix}1 & 0.4\\ 0 & 2\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 6 of 6

$$
\begin{aligned}
(I-A)^{-1} &= \left(I+A+A^2+A^3+A^4+A^5+A^6\right) \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix} + \begin{bmatrix} 0 & 0.1 \\ 0 & 0.25 \end{bmatrix} + \begin{bmatrix} 0 & 0.05 \\ 0 & 0.125 \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 & 0.025 \\ 0 & 0.0625 \end{bmatrix} + \begin{bmatrix} 0 & 0.0125 \\ 0 & 0.03125 \end{bmatrix} + \begin{bmatrix} 0 & 0.00625 \\ 0 & 0.015625 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0.39375 \\ 0 & 1.98437 \end{bmatrix} \\
&\approx \begin{bmatrix} 1 & 0.4 \\ 0 & 2 \end{bmatrix}
\end{aligned}
$$

Provide feedback (0)

Step 5 of 6

Now, calculate the following:

$$
\begin{aligned}
(I-A)^{-1} &= \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix} \right\}^{-1} \\
&= \begin{bmatrix} 1 & 0.4 \\ 0 & 2 \end{bmatrix}
\end{aligned}
$$

Provide feedback (0)

Step 6 of 6

Therefore, above result shows the following:

$$
\boxed{(I-A)^{-1} = \left(I+A+A^2+\ldots\right)}
$$

Provide feedback (0)

Chapter: CH5.3 Problem: 21P

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CH5.3 20P

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CH5.3 21P

CH5.3 23P

Step 1 of 13

To find the limits as $k \to \infty$ for the following matrices:

(a) Consider the following matrix:

$$A = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$$

$$u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Then to find the limit for $A^k u_0$.

Provide feedback (0)

Step 2 of 13

To find the Eigen values calculate the following:

$$\det(A - \lambda I) = 0$$

$$\begin{bmatrix} 0.4-\lambda & 0.2 \\ 0.6 & 0.8-\lambda \end{bmatrix} = 0$$

$$5\lambda^2 - 6\lambda + 1 = 0$$

$$(5\lambda - 1)(\lambda - 1) = 0$$

After solving following values are obtained:

$$\lambda = 1$$

$$\lambda = \frac{1}{5}$$

Provide feedback (0)

Step 3 of 13

Step 3 of 13

Calculate Eigen vectors for $\lambda = 1$:

$$(A - 1I)x = 0$$

$$\begin{bmatrix} 0.4-1 & 0.2 \\ 0.6 & 0.8-1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{-3}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{-1}{5} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 13

Similarly, Eigen vectors for $\lambda = \frac{1}{5}$ are as follows:

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Therefore, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix}$$

Step 4 of 13

Similarly, Eigen vectors for $\lambda = \frac{1}{5}$ are as follows:

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Therefore, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix}$$

Provide feedback (0)

Step 5 of 13

Difference equation can be written as follows:

$$\begin{aligned} u_{k+1} &= A^k u_0 \\ &= S\Lambda^k S^{-1} \cdot u_0 \end{aligned}$$

Now, calculate the following:

$$\begin{aligned} u_{k+1} &= S\Lambda^k S^{-1} \cdot u_0 \\ &= \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & \left(\frac{1}{5}\right)^k \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 5 of 13

Difference equation can be written as follows:

$$\begin{aligned} u_{k+1} &= A^k u_0 \\ &= S\Lambda^k S^{-1} \cdot u_0 \end{aligned}$$

Now, calculate the following:

$$\begin{aligned} u_{k+1} &= S\Lambda^k S^{-1} \cdot u_0 \\ &= \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & \left(\frac{1}{5}\right)^k \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 6 of 13

After taking the limit $k \to \infty$ makes the element $(1/5)^k$ very small, so neglect it.

$$\begin{aligned} u_\infty &= \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &\begin{bmatrix} \frac{1}{4} \end{bmatrix} \end{aligned}$$

After taking the limit $k \to \infty$ makes the element $(1/5)^k$ very small, so neglect it.

$$u_\infty = \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$

Provide feedback (0)

Step 7 of 13

Therefore, the limit of $A^k u_0$ is as follows:

$$\boxed{\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}}$$

Provide feedback (0)

Step 8 of 13

Step 8 of 13

(b) Consider the following matrix:

$$A = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$$

$$u_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Then to find the limit for $A^k u_0$.

Provide feedback (0)

Step 9 of 13

As the matrix A is same as in part (a) every step will be same only last step will change as value of u_0 is change. So, calculate the last step:

$$\begin{aligned} u_\infty &= \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix} \end{aligned}$$

Step 8 of 13

(b) Consider the following matrix:

$$A = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$$

$$u_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Then to find the limit for $A^k u_0$.

Provide feedback (0)

Step 9 of 13

As the matrix A is same as in part (a) every step will be same only last step will change as value of u_0 is change. So, calculate the last step:

$$\begin{aligned} u_\infty &= \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} \frac{3}{4} \end{bmatrix}$$

Provide feedback (0)

Step 10 of 13

Therefore, the limit of $A^k u_0$ is as follows:

$$\boxed{\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}}$$

Provide feedback (0)

Step 11 of 13

(c) Consider the following matrix:

$$A = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$$

Then to find the limit for A^k.

Provide feedback (0)

Step 12 of 13

As the matrix A is same as in part (a) every step will be same only the last step will change. So, calculate the last step:

$$u_\infty = \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix}$$

Provide feedback (0)

Step 12 of 13

As the matrix A is same as in part (a) every step will be same only the last step will change. So, calculate the last step:

$$u_\infty = \begin{bmatrix} \frac{1}{3} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

Provide feedback (0)

Step 13 of 13

Therefore, the limit of A^k is as follows:

$$\boxed{\begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix}}$$

Provide feedback (0)

CH5.3 22P

Step 1 of 8

Let A be the following matrix:

$$A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$$

Diagonalize A and compute $S\Lambda^k S^{-1}$ to get following formula for A^k:

$$A^k=\frac{1}{2}\begin{bmatrix}5^k+1 & 5^k-1\\ 5^k-1 & 5^k+1\end{bmatrix}$$

Provide feedback (0)

Step 2 of 8

To diagonalize the matrix A follow the following steps:

$$A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$$

$$A-\lambda I=\begin{bmatrix}3-\lambda & 2\\ 2 & 3-\lambda\end{bmatrix}$$

$$\begin{aligned}\det(A-\lambda I)&=(3-\lambda)^2-4\\ &=\lambda^2-6\lambda+5\end{aligned}$$

Put the determinant value equal to zero, to get following roots as Eigen values:

$$\lambda_1=5$$
$$\lambda_2=1$$

Provide feedback (0)

Step 3 of 8

CH5.3 24P

Step 3 of 8

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1 = 5$

$$(A-\lambda I)x_1 = 0$$
$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 8

For $\lambda_2 = 1$

$$(A-\lambda_2 I)x_2 = 0$$
$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

Thus, Eigen vector matrix is as follows:

$$\begin{bmatrix} -1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

Therefore, diagonalisation of matrix A is as follows:

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \boxed{\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} \end{aligned}$$

Provide feedback (0)

Step 7 of 8

Now, do the following calculations to get A^k:

$$\begin{aligned} A^k &= S\Lambda^k S^{-1} \\ &= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5^k & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 5^k+1 & 5^k-1 \\ 5^k-1 & 5^k+1 \end{bmatrix} \end{aligned}$$

$$A = S\Lambda S^{-1}$$
$$= \boxed{\frac{1}{2}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}5 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}}$$

Provide feedback (0)

Step 7 of 8

Now, do the following calculations to get A^k:

$$A^k = S\Lambda^k S^{-1}$$
$$= \frac{1}{2}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}5^k & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix}5^k+1 & 5^k-1\\5^k-1 & 5^k+1\end{bmatrix}$$

Provide feedback (0)

Step 8 of 8

Therefore,

$$\boxed{A^k = \frac{1}{2}\begin{bmatrix}5^k+1 & 5^k-1\\5^k-1 & 5^k+1\end{bmatrix}}$$

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Step 1 of 8

Let B be the following matrix:

$$B=\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$$

Diagonalize B and compute $S\Lambda^k S^{-1}$ to get following formula for B^k:

$$B^k=\frac{1}{2}\begin{bmatrix}3^k & 3^k-2^k\\ 0 & 2^k\end{bmatrix}$$

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Step 2 of 8

To diagonalize the matrix B follow the following steps:

$$\begin{aligned}B&=\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\\ B-\lambda I&=\begin{bmatrix}3-\lambda & 1\\ 0 & 2-\lambda\end{bmatrix}\\ \det(B-\lambda I)&=(3-\lambda)(2-\lambda)\\ &=\lambda^2-5\lambda+6\end{aligned}$$

Put the determinant value equal to zero, to get following roots as Eigen values:

$$\begin{aligned}\lambda_1&=2\\ \lambda_2&=3\end{aligned}$$

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CH5.3 25P

Step 3 of 8

Eigen vectors corresponding to the Eigen values are calculated as follows:

Step 3 of 8

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1 = 2$

$$(B-\lambda_1 I)x_1 = 0$$
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

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Step 4 of 8

For $\lambda_2 = 3$

$$(B-\lambda_2 I)x_2 = 0$$
$$\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

Thus, Eigen vector matrix is as follows:

Step 5 of 8

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$
$$S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

Therefore, diagonalisation of matrix B is as follows:

$$\begin{aligned} B &= S\Lambda S^{-1} \\ &= \boxed{\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}} \end{aligned}$$

Provide feedback (0)

Step 7 of 8

Now, do the following calculations to get B^k:

$$\begin{aligned} B^k &= S\Lambda^k S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3^k & 0 \\ 0 & 2^k \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix} \end{aligned}$$

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Step 8 of 8

Therefore

$$B = S\Lambda S^{-1}$$
$$= \boxed{\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}}$$

Provide feedback (0)

Step 7 of 8

Now, do the following calculations to get B^k:

$$B^k = S\Lambda^k S^{-1}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3^k & 0 \\ 0 & 2^k \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$$

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Step 8 of 8

Therefore,

$$\boxed{B^k = \frac{1}{2}\begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}}$$

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Step 1 of 10

Let A and B be two matrices defined as follows:

$$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$
$$B = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 10

Consider the following Eigen values of matrix A and B given as follows:

For matrix A:

$$\lambda_1 = 1$$
$$\lambda_2 = 9$$

For matrix B:

$$\lambda_1 = -1$$
$$\lambda_2 = 9$$

Let $R = S\Lambda^{1/2}S^{-1}$, then to find the matrix square root of A from R. Also explain that why there is no real matrix square root of B.

Provide feedback (0)

Step 3 of 10

Let A be the following matrix:

$$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

CH5.3 26P

$$\lambda_2 = 9$$

Let $R = S\Lambda^{1/2}S^{-1}$, then to find the matrix square root of A from R. Also explain that why there is no real matrix square root of B.

Provide feedback (0)

Step 3 of 10

Let A be the following matrix:

$$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Compute $S\Lambda^{1/2}S^{-1}$ to get the desired result.

Provide feedback (0)

Step 4 of 10

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1 = 1$

$$\begin{aligned} (A - \lambda_1 I)x_1 &= 0 \\ \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 5 of 10

Step 5 of 10

For $\lambda_2 = 9$

$$(A-\lambda_2 I)x_2 = 0$$
$$\begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$
$$S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 10

Now, do the following calculations to get R:

$$R = S\Lambda^{1/2}S^{-1}$$
$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \end{bmatrix}$$

Step 7 of 10

Now, do the following calculations to get R:

$$\begin{aligned} R &= S\Lambda^{1/2}S^{-1} \\ &= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{aligned}$$

Next, compute the following:

$$\begin{aligned} R^2 &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \\ &= A \end{aligned}$$

Provide feedback (0)

Step 8 of 10

Therefore, $\boxed{A = R^2}$. This shows that R is a matrix square root of A.

Provide feedback (0)

Step 9 of 10

Next, to find matrix square root of B, first calculate the following:

$$\Lambda^{1/2} = \begin{bmatrix} \sqrt{-1} & 0 \\ 0 & 3 \end{bmatrix}$$

This shows that one of the Eigen values of matrix square root of B is imaginary. In other words trace is not real.

Step 8 of 10

Therefore, $\boxed{A = R^2}$. This shows that R is a matrix square root of A.

Provide feedback (0)

Step 9 of 10

Next, to find matrix square root of B, first calculate the following:

$$\Lambda^{1/2} = \begin{bmatrix} \sqrt{-1} & 0 \\ 0 & 3 \end{bmatrix}$$

This shows that one of the Eigen values of matrix square root of B is imaginary. In other words trace is not real.

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Step 10 of 10

Therefore, $\boxed{\text{there can not be any real matrix square root of } B}$.

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Step 1 of 3

Fill up the blank:

Let A and B be two matrices. Consider that they have the same Eigen values with full set of independent Eigen vectors. If they are factorized into same …….. . Then $A = B$

Provide feedback (0)

Step 2 of 3

To get diagonalize any matrix is factorize into $S\Lambda S^{-1}$. So, if matrix A and B have same, $S\Lambda S^{-1}$, factorization value, they are said to be equal.

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Step 3 of 3

Therefore, to get equality matrix A and B should be factorized into same $\boxed{S\Lambda S^{-1}}$.

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Step 1 of 4

Let A and B be two matrices. Consider that they have the same full set of Eigen vectors, such that:

$$A = S\Lambda_1 S^{-1}$$
$$B = S\Lambda_2 S^{-1}$$

Prove that $AB = BA$.

Provide feedback (0)

Step 2 of 4

Lets calculate first AB

$$\begin{aligned} AB &= \left(S\Lambda_1 S^{-1}\right)\left(S\Lambda_2 S^{-1}\right) \\ &= S\Lambda_1 S^{-1} S\Lambda_2 S^{-1} \\ &= S\Lambda_1 I \Lambda_2 S^{-1} \\ &= S\Lambda_1 \Lambda_2 S^{-1} \end{aligned}$$

Diagonal matrix always gives the following result:

$$\Lambda_1 \Lambda_2 = \Lambda_2 \Lambda_1$$

Provide feedback (0)

Step 3 of 4

So,

$$\begin{aligned} AB &= S\Lambda_1 \Lambda_2 S^{-1} \\ &= S\Lambda_2 \Lambda_1 S^{-1} \\ &= S\Lambda_2 I \Lambda_1 S^{-1} \\ &= S\Lambda_2 S^{-1} S\Lambda_1 S^{-1} \end{aligned}$$

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$$\Lambda_1\Lambda_2 = \Lambda_2\Lambda_1$$

Provide feedback (0)

Step 3 of 4

So,

$$\begin{aligned} AB &= S\Lambda_1\Lambda_2S^{-1} \\ &= S\Lambda_2\Lambda_1S^{-1} \\ &= S\Lambda_2I\Lambda_1S^{-1} \\ &= S\Lambda_2S^{-1}S\Lambda_1S^{-1} \\ &= \left(S\Lambda_2S^{-1}\right)\left(S\Lambda_1S^{-1}\right) \\ AB &= BA \end{aligned}$$

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Step 4 of 4

Therefore, when A and B have the same full set of Eigen vectors then $\boxed{AB = BA}$.

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Step 1 of 2

Consider a matrix A with Eigen values $\lambda = 0$ and $\lambda \neq 0$. Let $N(A)$ be the null space and $C(A)$ be the column space.

(a) Determine when Eigen vectors for $\lambda = 0$ span the subspace $N(A)$.

Eigen vectors for $\lambda = 0$ always span the null space of matrix A.

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Step 2 of 2

(b) Determine when all the Eigen vectors for $\lambda \neq 0$ span the column space $C(A)$.

Eigen vectors $x_1, x_2, \ldots, x_j$ that correspond to different nonzero Eigen values are linearly independent. Let j be the independent vectors, then Eigen vectors for $\lambda_j \neq 0$ spans the column space of matrix A.

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Step 1 of 5

Consider a matrix A with Eigen values $|\lambda_i|>1$ and $|\lambda_i|<1$. The powers A^k approach zero if all $|\lambda_i|<1$, and they blow up if any of the $|\lambda_i|>1$.

Let B and C be two matrices defined as follows:

$$B=\begin{bmatrix}3 & 2\\ -5 & -3\end{bmatrix}$$
$$C=\begin{bmatrix}5 & 7\\ -3 & -4\end{bmatrix}$$

Determine the Eigen values of B and C and show the following:

$$B^4=I$$
$$C^3=-I$$

Provide feedback (0)

Step 2 of 5

To find the Eigen values of B do the following calculations:

$$\begin{aligned}\det(B-\lambda I)&=0\\ (3-\lambda)(3-\lambda)+10&=0\\ \lambda^2+1&=0\\ \lambda&=\pm i\end{aligned}$$

Therefore, Eigen values of matrix B is $\boxed{\lambda=\pm i}$.

Provide feedback (0)

Step 3 of 5

CH5.4

$$\det(B-\lambda I)=0$$
$$(3-\lambda)(3-\lambda)+10=0$$
$$\lambda^2+1=0$$
$$\lambda=\pm i$$

Therefore, Eigen values of matrix B is $\boxed{\lambda=\pm i}$.

Provide feedback (0)

Step 3 of 5

Eigen values of matrix B^4 will be λ^4.

$$\lambda^4=1$$

This shows that $\boxed{B^4=I}$.

Provide feedback (0)

Step 4 of 5

To find the Eigen values of C do the following calculations:

$$\det(C-\lambda I)=0$$
$$(5-\lambda)(-4-\lambda)+21=0$$
$$\lambda^2-\lambda+1=0$$
$$\lambda=e^{\pm i\pi/3}$$

Therefore, Eigen values of matrix C is $\boxed{\lambda=e^{\pm i\pi/3}}$.

Provide feedback (0)

Step 5 of 5

$$\lambda^2 = 1$$

This shows that $\boxed{B^4 = I}$.

Provide feedback (0)

Step 4 of 5

To find the Eigen values of C do the following calculations:

$$\begin{aligned} \det(C - \lambda I) &= 0 \\ (5-\lambda)(-4-\lambda)+21 &= 0 \\ \lambda^2 - \lambda + 1 &= 0 \\ \lambda &= e^{\pm i\pi/3} \end{aligned}$$

Therefore, Eigen values of matrix C is $\boxed{\lambda = e^{\pm i\pi/3}}$.

Provide feedback (0)

Step 5 of 5

Eigen values of matrix C^3 will be λ^3.

$$\begin{aligned} \lambda^3 &= \left(e^{\pm i\pi/3}\right)^3 \\ &= e^{\pm i\pi} \\ \lambda^3 &= -1 \end{aligned}$$

This shows that $\boxed{C^3 = -I}$.

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