CH4

CH4.2 2P

Step 1 of 4

Let A be a 4 by 4 matrix with $\det A = \frac{1}{2}$.

a) We have to find $\det(2A)$

We know that if A is an $n \times n$ matrix then $\det(kA) = k^n \det A$.

Now

$$\begin{aligned}
\det(2A) &= 2^n \det(A) \quad (\text{Since } n = 4) \\
&= 2^4\left(\frac{1}{2}\right) \\
&= (16)\left(\frac{1}{2}\right) \\
&= 8
\end{aligned}$$

Thus, $\boxed{\det(2A) = 8}$.

Step 2 of 4

b) We have to find $\det(-A)$.

Now

$$\begin{aligned}
\det(-A) &= -\det(A) \\
&= -\frac{1}{2}
\end{aligned}$$

Thus, $\boxed{\det(-A) = \frac{-1}{2}}$

Step 3 of 4

c) We have to find $\det(A^2)$.

Step 3 of 4

c) We have to find $\det\left(A^2\right)$.

We know that $\det\left(A^n\right) = \left(\det A\right)^n$.

Now

$$\det\left(A^2\right) = \left(\det A\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Thus, $\boxed{\det\left(A^2\right) = \frac{1}{4}}$

Provide feedback (0)

Step 4 of 4

d) We have to find $\det\left(A^{-1}\right)$.

Now

$$\det\left(A^{-1}\right) = \frac{1}{\det\left(A\right)} = \frac{1}{\frac{1}{2}} = 2$$

Thus, $\boxed{\det\left(A^{-1}\right) = 2}$

Provide feedback (0)

CH4.2 1P

CH4.2 3P

Step 1 of 4

Let A be a 3×3 matrix such that $\det A = -1$.

a) We have to find $\det\left(\frac{1}{2}A\right)$.

We know that if A is an $n\times n$ matrix then $\det(kA) = k^n \det A$.

Now

$$\det\left(\frac{1}{2}A\right) = \left(\frac{1}{2}\right)^n \det(A)$$
$$= \left(\frac{1}{2}\right)^3 (-1) \qquad (\text{Since } n=3)$$
$$= -\frac{1}{8}$$

Thus, $\boxed{\det\left(\frac{1}{2}A\right) = \frac{-1}{8}}$

Provide feedback (0)

Step 2 of 4

b) We have to find $\det(-A)$.

Now

$$\det(-A) = -\det(A)$$
$$= -(-1)$$
$$= 1$$

Thus, $\boxed{\det(-A) = 1}$

Provide feedback (0)

Step 3 of 4

Step 3 of 4

c) We have to find $\det\left(A^2\right)$.

We know that $\det\left(A^n\right)=\left(\det A\right)^n$.

Now

$$\begin{aligned}\det\left(A^2\right)&=\left(\det\left(A\right)\right)^2\\&=\left(-1\right)^2\\&=1\end{aligned}$$

Thus, $\boxed{\det\left(A^2\right)=1}$

Step 4 of 4

d) We have to find $\det\left(A^{-1}\right)$.

Now

$$\begin{aligned}\det\left(A^{-1}\right)&=\frac{1}{\det\left(A\right)}\\&=\frac{1}{-1}\\&=-1\end{aligned}$$

Thus, $\boxed{\det\left(A^{-1}\right)=-1}$

CH4.2 2P

CH4.2 4P

Step 1 of 2

Given that $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

We have to perform row operations so the given determinant must equals to $-\det B = -\begin{vmatrix} c & d \\ a & b \end{vmatrix}$.

Provide feedback (0)

Step 2 of 2

We have

$$\begin{aligned} \det A &= \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\ &= \begin{vmatrix} a & b \\ a+c & b+d \end{vmatrix} \quad \text{Adding first row to the second row, i.e. } R_2 \to R_2 + R_1 \\ &= \begin{vmatrix} -c & -d \\ a+c & b+d \end{vmatrix} \quad \text{Adding -1 times of second row to the first row, i.e. } R_1 \to R_1 - R_2 \\ &= \begin{vmatrix} -c & -d \\ a & b \end{vmatrix} \quad \text{Adding the first row to the second row, i.e. } R_2 \to R_2 + R_1 \\ &= -\begin{vmatrix} c & d \\ a & b \end{vmatrix} \\ &= -\det B \end{aligned}$$

Therefore the rules $(R_2 \to R_2 + R_1, R_1 \to R_1 - R_2)$ $R_2 \to R_2 + R_1, (-1)R_1 \to R_1$ replace the rule that by interchanging two rows of a determinant, the sign of the determinant only changes without changing its value.

Provide feedback (0)

CH4.2 3P

CH4.2 5P

Step 1 of 4

Consider the following matrix:

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}$$

By using $R_2 \to R_2 - 2R_1$ we get,

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}$$

By using $R_3 \to R_1 + R_3$ we get,

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix}$$

Step 2 of 4

Step 2 of 4

By using $R_4 \to 2R_2 + R_4$, we get,

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

By using $R_4 \to \frac{5}{2}R_3 + R_4$, we get,

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

We know that if M is triangular, then det M is the product of the diagonal entries.

Thus, we have

$$\det\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} = \begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{vmatrix}$$

Provide feedback (0)

Step 3 of 4

By using $R_4 \to \frac{5}{2}R_3 + R_4$, we get,

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

We know that if M is triangular, then det M is the product of the diagonal entries.

Thus, we have

$$\det\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} = \begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{vmatrix}$$
$$= (1)(-1)(-2)(10)$$
$$= \boxed{20}$$

Provide feedback (0)

Step 4 of 4

Consider the following matrix:

Step 4 of 4

Consider the following matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

By using $R_2 \to R_2 + \frac{R_1}{2}$ we get,

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

By using $R_3 \to \frac{2}{3}R_2 + R_3$ we get,

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

By using $R_4 \to \frac{3}{4}R_3 + R_4$ we get,

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & -11/4 \end{bmatrix}$$

We know that if *M* is triangular, then det *M* is the product of the diagonal entries.

Therefore, we have

$$\det\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix} = \begin{vmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & -11/4 \end{vmatrix}$$

$$= (2)\left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{-11}{4}\right)$$

$$= \boxed{-11}$$

CH4.2 4P

CH4.2 6P

Step 1 of 3

Given that $\det\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix}=+1$ and $\det\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}=-1$

We have to count the row exchanges to get the given determinant value.

Provide feedback (0)

Step 2 of 3

Consider

$$\det\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix}=\begin{vmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{vmatrix}$$

$$=\begin{vmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{vmatrix}\text{ Interchanging the first and fourth rows.}$$

$$=\begin{vmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{vmatrix}\text{ Interchanging the second and third row.}$$

$$=\boxed{1}$$

Since it is a triangular matrix , determinant is the product of the diagonal entries.
Thus, two interchanges are required to get identify from given matrix.

Provide feedback (0)

$$\begin{vmatrix} 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= \boxed{1}$$

Since it is a triangular matrix , determinant is the product of the diagonal entries.
Thus, two interchanges are required to get identify from given matrix.

Provide feedback (0)

Step 3 of 3

Consider

$$\det\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \text{ Interchanging the fourth and third rows}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \text{ Interchanging the first and third rows}$$

$$= -\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \text{ Interchanging the second and third rows.}$$

$$= \boxed{-1}$$

Here three interchanges are re1quired to get identity from given matrix.

Provide feedback (1)

Chapter: CH4.2 Problem: 6P

show all steps

CH4.2 5P

Step 1 of 1

When n is even, clearly $\frac{n}{2}$ interchanged grows are required to get into normal order of rows. And when n is odd, the middle row is exactly in its own place and we need $\frac{n-1}{2}$ interchanges to get into normal order or rows.

Therefore, in these cases, the required interchanges are $(R_1, R_n), (R_2, R_{n-1}) \ldots\ldots\ldots \dfrac{\left(\frac{R_n}{2}, R_{\frac{n}{2}+1}\right)}{\left(R_{\frac{n-1}{2}}, \frac{R_{n+1}}{2}\right)}$

Provide feedback (0)

CH4.2 7P

CH4.2 6P

CH4.2 8P

Step 1 of 11

a) Given that $A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$

We have to find the rank of A.

Provide feedback (0)

Step 2 of 11

Now

$$A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2) & 1(-1) & 1(2) \\ 4(2) & 4(-1) & 4(2) \\ 2(2) & 2(-1) & 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 2 \\ 8 & -4 & 8 \\ 4 & -2 & 4 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 11

Since any two rows of A are proportional and we know that if the rows of any matrix are proportional then the determinant of that matrix is zero.

So $\det A = 0$

Hence the determinant of the given matrix is $\boxed{0}$.

Provide feedback (0)

Step 3 of 11

Since any two rows of A are proportional and we know that if the rows of any matrix are proportional then the determinant of that matrix is zero.
So $\det A = 0$
Hence the determinant of the given matrix is $\boxed{0}$.

Provide feedback (0)

Step 4 of 11

b) Given matrix is $U = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

We have to find the determinant of the given matrix.

Provide feedback (0)

Step 5 of 11

Since U is a triangular matrix and we know that determinant of a triangular matrix is the product of diagonal entries.
Therefore,

$$\det U = \begin{vmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$
$$= 4.1.2.2$$
$$= \boxed{16}$$

Hence $\boxed{\det U = 16}$

Provide feedback (0)

Step 6 of 11

c) Given upper triangular matrix is $U = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Then $U^T = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 8 & 2 & 2 & 0 \\ 8 & 2 & 6 & 2 \end{bmatrix}$ is a lower triangular matrix.

We have to find the determinant of U^T

Provide feedback (0)

Step 7 of 11

Since U^T is a triangular matrix and we know that determinant of a triangular matrix is the product of diagonal entries.
Therefore,

$$\det U^T = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 8 & 2 & 2 & 0 \\ 8 & 2 & 6 & 2 \end{bmatrix}$$
$$= 4.1.2.2$$
$$= \boxed{16}$$

Hence $\boxed{\det U^T = 16}$

Provide feedback (0)

Step 8 of 11

d) We have to find the determinant of the inverse matrix U^{-1}

Step 8 of 11

d) We have to find the determinant of the inverse matrix U^{-1}.

We know that $\det A^{-1} = \dfrac{1}{\det A}$, for any matrix A.

Therefore,

Provide feedback (0)

Step 9 of 11

$$\det U^{-1} = \frac{1}{\det U} = \frac{1}{16}$$

Hence $\boxed{\det U^{-1} = \frac{1}{16}}$

Provide feedback (0)

Step 10 of 11

e) Given the reverse triangular matrix is $M = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 1 & 2 & 2 \\ 4 & 4 & 8 & 8 \end{bmatrix}$.

We have to find the determinant of the given matrix M.

Provide feedback (0)

Step 11 of 11

The reverse triangular matrix M results from two exchanges (1) row 1 and row 4

$$= \frac{1}{16}$$

Hence $\boxed{\det U^{-1} = \frac{1}{16}}$

Step 10 of 11

e) Given the reverse triangular matrix is $M = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 1 & 2 & 2 \\ 4 & 4 & 8 & 8 \end{bmatrix}$.

We have to find the determinant of the given matrix M.

Step 11 of 11

The reverse triangular matrix M results from two exchanges (1) row 1 and row 4
(ii) row 2 and row 3 from the matrix U.
So M results from U by even number of row exchanges and hence $\det M = \det U$
Since $\det U = 16$ and $\det M = \det U$.
So $\det M = 16$

Hence the determinant of the given reverse triangular matrix is $\boxed{\det M = 16}$.

CH4.2 7P

CH4.2 9P

Step 1 of 3

Given that
Rule (2): The determinant changes sign when two rows are exchanged.
Rule (3): The determinant depends linearly on the first row.
Rule (6): If A has a row of zeros, then $\det(A)=0$
We have to show that how rule (6) comes directly from rules 2 and 3.

Provide feedback (0)

Step 2 of 3

Suppose a square matrix has a row of zeros, say the k^{th} row R_k consist of all zeros. It looks as below:

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

Then by rule 3, $t|A|=|B|$ where B is obtained by multiplying a sample row in A,
Now
$2|A|=|A|$(multiplying kth row by $2A$ is unaltered)
$\Rightarrow 2|A|-|A|=0$
$\Rightarrow |A|=0$
Therefore rule (6) comes directly from rules 2 and 3.

Provide feedback (1)

CH4.2 8P

CH4.2 10P

Step 1 of 2

If we do two row operations at once, going from $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to $\begin{bmatrix} a-mc & b-md \\ c-la & d-lb \end{bmatrix}$, then we have to find the determinant of the new matrix directly or by using rule 3.

The new matrix obtained by applying the row operations $R_1 - mR_2 \to R_1, R_2 - lR_1 \to R_2$

Now we will find the determinant of the new matrix by rule 3.

Rule (3): The determinant depends linearly on the first row.

Step 2 of 2

$$\begin{aligned}
\begin{vmatrix} a-mc & b-md \\ c-la & d-lb \end{vmatrix} &= \begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} - m\begin{vmatrix} c & d \\ c-la & d-lb \end{vmatrix} \\
&= \begin{vmatrix} a & b \\ c & d \end{vmatrix} - m\left(l\left(ad-bc\right)\right) \\
&= \left(ad-bc\right)\left(1-lm\right) \\
&= \left(1-lm\right)\left(ad-bc\right)
\end{aligned}$$

Thus, $\begin{vmatrix} a-mc & b-md \\ c-la & d-lb \end{vmatrix} = \boxed{(1-lm)(ad-bc)}$

Chapter: CH4.2 Problem: 10P

show all steps

CH4.2 9P

CH4.2 11P

Step 1 of 3

If Q is an orthogonal matrix, so that $Q^T Q = I$, we have to prove that $\det Q$ equals to $+1$ or -1 and we have to find that what kind of box is formed from the rows (or columns) of Q.

Provide feedback (0)

Step 2 of 3

We have $Q^T A = I$ (since Q is an orthogonal matrix)

$\Rightarrow \det\left(Q^T Q\right) = \det I$

$\Rightarrow \det Q^T . \det Q = 1 \quad (\text{since } \det I = 1)$

$\left(\text{since } \det(AB) = \det A \det B \text{ for any 2 matrices A,B}\right)$

Provide feedback (0)

Step 3 of 3

$\Rightarrow \det Q . \det Q = 1$

$\left(\text{since } \det\left(A^T\right) = \det A \text{ for any matrix } A\right)$

$\Rightarrow (\det Q)^2 = 1$

$\Rightarrow \det Q = \sqrt{1} = \boxed{\pm 1}$

Thus, $\det Q = \pm 1$, where Q is an orthogonal matrix.

A $\boxed{\text{box of volume 1}}$ is formed from rows of Q.

Provide feedback (0)

CH4.2 10P

CH4.2 12P

Step 1 of 4

The product rule of determinant says that if *A* and *B* are square matrices of the same order, then $\det(AB) = (\det A)(\det B)$.

It is also known that $\det I = 1$.

Therefore, we get

$$\begin{aligned}1 &= \det I \\ &= \det\left(Q^{T}Q\right) \\ &= \det\left(Q^{T}\right)\det(Q)\end{aligned}$$

Step 2 of 4

We also know that determinant of a matrix *A* is equal to the determinant of its transpose, A^{T}. Therefore, $\det(Q) = \det\left(Q^{T}\right)$.

Thus, $\left(\det(Q)\right)^{2} = 1$

Therefore, $\boxed{\det(Q) = \pm 1}$.

By the same logic, $\det\left(Q^{T}\right) = \pm 1$.

Step 3 of 4

Now we show that Q^2 is also an Orthogonal matrix. Thus, we need to show that $\left(Q^2\right)^{-1} = \left(Q^2\right)^T$.

$$\begin{aligned} Q^2\left(Q^2\right)^T &= (Q \cdot Q)\left(Q^T \cdot Q^T\right) \\ &= Q\left(Q \cdot Q^T\right)Q^T \\ &= Q \cdot Q^T \\ &= I \end{aligned}$$

Thus, Q^2 is an Orthogonal matrix. Similarly, it can be shown that Q^n is also an Orthogonal matrix.

Provide feedback (0)

Step 4 of 4

If $\det Q$ is not equal to ± 1, then $\det\left(Q^n\right)$ would either tend to zero or would tend to plus or minus infinity.

But, Q^n remains an Orthogonal matrix.

This also shows that $\boxed{\det Q = \pm 1}$.

Provide feedback (0)

CH4.2 11P

CH4.2 13P

We need to verify that

$$\det\begin{bmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{bmatrix}=(b-a)(c-a)(c-b)$$

Provide feedback (0)

Step 2 of 3

So, consider

$$\begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix}$$

Subtracting 1 time the first row from the second row to obtain

$$\begin{vmatrix}1 & a & a^2\\0 & b-a & b^2-a^2\\1 & c & c^2\end{vmatrix}$$

Provide feedback (0)

Step 3 of 3

Subtracting 1 time the first row from the third row to obtain

$$\begin{vmatrix}1 & a & a^2\\0 & 1 & b+a\\0 & 1 & c+a\end{vmatrix} \qquad \begin{bmatrix}\text{since we have the formula}\\ a^2-b^2=(a-b)(a+b)\end{bmatrix}$$

$=(b-a)(c-a)\left[(c+a)-(b+a)\right]$ (expanding the determinant along the first column)

$=(b-a)(c-a)(c-b)$

Thus, the given statement $\det\begin{bmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{bmatrix}=\boxed{(b-a)(c-a)(c-b)}$ is true.

CH4.2 12P

CH4.2 14P

Step 1 of 8

a) Given that a skew symmetric matrix satisfies $K^T = K$, as in

$$K = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}.$$

In the 3 by 3 case, we have to find why is $\det(-K) = (-1)^3 \det(K)$ and on the other hand, $\det(K^T) = \det(K)$, and we have to deduce that the determinant must be zero.

Provide feedback (0)

Step 2 of 8

$$K^T = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= -K$$

$$= (-1)^3 K$$

Provide feedback (0)

Step 3 of 8

So we get

$$\det K^T = \det(-K)$$

$$= \det K$$

Provide feedback (0)

Step 4 of 8

Step 4 of 8

For any $n \times n$ matrix A , we have
$\det(tA) = t^n \det A$
So for a 3×3 matrix K we have
$\det(-K) = (-1)^3 \det K$
$= -\det K$

Provide feedback (0)

Step 5 of 8

Therefore for skew symmetric matrix K we get
$\det K = -\det K$ and hence $2\det K = 0$
Giving that $\det K = 0$

Provide feedback (0)

Step 6 of 8

b) We have to write down a 4 by 4 skew symmetric matrix with $\det K \neq 0$
Consider

$$K = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 3 & -1 \\ 0 & -3 & 0 & 2 \\ -2 & 1 & -2 & 0 \end{bmatrix}$$, where K is a skew symmetric matrix of order 4×4.

Provide feedback (0)

Step 7 of 8

$$\det K = -\begin{vmatrix} -1 & 3 & -1 \\ 0 & 0 & 2 \\ \end{vmatrix} - 2\begin{vmatrix} -1 & 0 & 3 \\ 0 & -3 & 0 \\ \end{vmatrix}$$

b) We have to write down a 4 by 4 skew symmetric matrix with $\det K \neq 0$
Consider

$$K=\begin{bmatrix}0 & 1 & 0 & 2\\ -1 & 0 & 3 & -1\\ 0 & -3 & 0 & 2\\ -2 & 1 & -2 & 0\end{bmatrix}$$, where K is a skew symmetric matrix of order 4×4.

Provide feedback (0)

Step 7 of 8

$$\det K=-\begin{vmatrix}-1 & 3 & -1\\ 0 & 0 & 2\\ -2 & -2 & 0\end{vmatrix}-2\begin{vmatrix}-1 & 0 & 3\\ 0 & -3 & 0\\ -2 & 1 & -2\end{vmatrix}$$
$$=-\left[-2(2+6)\right]-2\left[(-3)(2+6)\right]$$
$$=64$$
$$\neq 0$$

Provide feedback (0)

Step 8 of 8

Thus $k=\begin{bmatrix}0 & 1 & 0 & 2\\ -1 & 0 & 3 & -1\\ 0 & -3 & 0 & 2\\ -2 & 1 & -2 & 0\end{bmatrix}$ is an example of a 4 by skew symmetric matrix with $\det K \neq 0$.

Provide feedback (0)

CH4.2 13P

CH4.2 15P

Step 1 of 8

We have to verify that the statements (a), (b), (c), (d) and (e) are true or false.

a) The given statement is "If A and B are identical except that $b_{11} = 2a_{11}$, then $\det B = 2\det A$".

For example

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ where $b_{11} = 2a_{11}$

Provide feedback (0)

Step 2 of 8

Now we have

$\det A = (1)(1) - (1)(1)$

$= 0$

$\det B = (2)(1) - (1)(1)$

$= 1$

Clearly $\det B \neq 2\det A$

The given statement is $\boxed{\text{false}}$.

Provide feedback (0)

Step 3 of 8

b) The given statement is "The determinant is the product of pivots'' is false.

For example

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Three row exchanges (Row 3, Row 4) and (Row 2 , Row3) result in identity matrix

which is upper triangular here product of pivots =1 but $\det A = (-1)^3 \cdot 1 = -1$

Step 3 of 8

b) The given statement is "The determinant is the product of pivots'' is false.
For example

$$A=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$$

Three row exchanges (Row 3, Row 4) and $(\text{Row } 2\text{ , Row}3)$ result in identity matrix

which is upper triangular here product of pivots =1 but $\det A=(-1)^3.1=-1$

The given statement is $\boxed{\text{false}}$

Provide feedback (0)

Step 4 of 8

c) The given statement "If A is invertible B is singular then $A+B$ is invertible'' .
Consider

$$A=\begin{bmatrix}2&1\\-1&-1\end{bmatrix} \text{ and } B=\begin{bmatrix}1&1\\1&1\end{bmatrix}$$

Now $\det A=-2+1$
$=-1$
$\neq 0$

Provide feedback (0)

Step 5 of 8

And
$\det B=1-1$
$=0$
So that A is invertible and B is singular, also

$$A+B=\begin{bmatrix}3&2\\0&0\end{bmatrix}$$

Step 5 of 8

And

$\det B = 1 - 1$

$= 0$

So that A is invertible and B is singular, also

$$A + B = \begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}$$

This is singular, as $\det(A+B) = 0$ and is not invertible.

Hence the given statement is $\boxed{\text{false}}$.

Provide feedback (0)

Step 6 of 8

d) The given statement " If A is invertible and B is singular, then AB is singular" .

If A is invertible and B is singular then $\det(A) \neq 0, \det(B) = 0$

We know that $\det(AB) = \det A . \det B$

$= (\det A)(0)$, since $\det B = 0$

$= 0$

$\Rightarrow AB$ is singular

Hence the given statement is $\boxed{\text{true}}$.

Provide feedback (0)

Step 7 of 8

e) The given statement "The determinant of $AB - BA$ is zero".

Consider

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Then

$$AB = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} BA = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

If A is invertible and B is singular then $\det(A) \neq 0, \det(B) = 0$

We know that $\det(AB) = \det A . \det B$

$= (\det A)(0)$, since $\det B = 0$

$= 0$

$\Rightarrow AB$ is singular

Hence the given statement is $\boxed{\text{true}}$.

Provide feedback (0)

Step 7 of 8

e) The given statement "The determinant of $AB - BA$ is zero".

Consider

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Then

$$AB = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} BA = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 8

Now,

$$AB - BA = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$$

$\det(AB - BA) = -1 + 2$

$= 1$

$\neq 0$

Hence the given statement is $\boxed{\text{false}}$.

Provide feedback (0)

CH4.2 14P

CH4.2 16P

Step 1 of 7

If every row of A adds to zero, we have to prove that $\det A = 0$, and if every row adds to 1, we have to prove that $\det(A-I)=0$. Also we have to show by an example that this does not imply $\det A = 1$.

Provide feedback (0)

Step 2 of 7

Given that every row of A adds to zero.
(That is sum of entries in every row is zero)
We shall prove that $\det A = 0$
Now doing the row operation of adding all remaining rows to 1^{st} row of A^T we get a matrix B consisting of all zeros in 1^{st} row.

Provide feedback (0)

Step 3 of 7

We know that addition or subtraction of row to another row don't alter the value of determinant.
Hence $\det A^T = \det B = 0$
And we know that $\det A = \det A^T$
Therefore $\boxed{\det A = 0}$.

Provide feedback (0)

Step 4 of 7

Next we suppose that every row of A adds to 1.
Write $A - I = B$ then

$$B = \begin{bmatrix} a_{11}-1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Step 4 of 7

Next we suppose that every row of A adds to 1.
Write $A-I=B$ then

$$B=\begin{bmatrix} a_{11}-1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-1 \end{bmatrix}$$

We can observe that every row of B adds to zero. From the first part, we have $\det B=0$

Hence $\boxed{\det(A-I)=0}$

Provide feedback (0)

Step 5 of 7

Now consider

$A=\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, then

$$A-I=\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Now

$\det(A)$

$=1-4$

$=-3$

Provide feedback (0)

Step 7 of 7

Step 5 of 7

Now consider

$$A=\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}, \text{ then}$$

$$A-I=\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}$$

Step 6 of 7

Now

$\det(A)$

$=1-4$

$=-3$

Step 7 of 7

And

$\det(A-I)$

$=4-4$

$=0$

So $\boxed{\det(A-I)=0 \text{ does not imply } \det A=1}$

CH4.2 15P

CH4.2 17P

Step 1 of 5

We have to find the following determinants by using Gaussian elimination.

$$\det\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}, \text{ and } \det\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 5

First, we consider

$$\det\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 10 & 10 & 10 & 10 \end{bmatrix} \text{(by adding } -1 \text{ times the third row to the fourth row)}$$

Provide feedback (0)

Step 3 of 5

$$= \begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix} \text{(by adding } -1 \text{ times the second row to the third row)}$$

$= \boxed{0}$, since the third and fourth rows are identical.

Step 4 of 5

Next, we consider

$$\det\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$$

$$= \det\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ 0 & 0 & 0 & 1-t^2 \end{bmatrix} \text{(by adding } -t \text{ times the third row to the fourth row)}$$

Step 5 of 5

$$= \det\begin{bmatrix} 1 & t & t^2 & t^3 \\ 0 & 1-t^2 & 1-t^3 & t^2-t^4 \\ 0 & 0 & 1-t^2 & t-t^3 \\ 0 & 0 & 0 & 1-t^2 \end{bmatrix} \text{(by adding } -t \text{ times the second row to the third row)}$$

$$= (1-t^2)(1-t^2)(1-t^2)$$

$$= \boxed{(1-t^2)^3}$$

(Since the matrix is triangular, determinant is the product of diagonal elements.)

CH4.2 16P

CH4.2 18P

Step 1 of 5

Given

$A=\begin{bmatrix}4 & 2\\1 & 3\end{bmatrix}, A^{-1}=\frac{1}{10}\begin{vmatrix}3 & -2\\-1 & 4\end{vmatrix}$ and $A-\lambda I=\begin{vmatrix}4-\lambda & 2\\1 & 3-\lambda\end{vmatrix}$.

We need to find the determinants of A, A^{-1} and $A-\lambda I$.

And also we need to find the value of λ for which $A-\lambda I$ is a singular matrix.

Provide feedback (0)

Step 2 of 5

a) $\det(A)=\begin{vmatrix}4 & 2\\1 & 3\end{vmatrix}$

$=(4)(3)-(1)(2) \quad \left[\text{since } \begin{vmatrix}a & b\\c & d\end{vmatrix}=ad-bc\right]$

$=\boxed{10}$

Hence $\boxed{\det\begin{bmatrix}4 & 2\\1 & 3\end{bmatrix}=10}$

Provide feedback (0)

Step 3 of 5

b) We have to find the determinant of $A^{-1}=\frac{1}{10}\begin{vmatrix}3 & -2\\-1 & 4\end{vmatrix}$.

We know that $\det(tA)=t^n \det A$, where A is an $n\times n$ matrix.

Now

$\det A^{-1}=\det\left(\frac{1}{10}\begin{bmatrix}3 & -2\\-1 & 4\end{bmatrix}\right)$

$=\left(\frac{1}{10}\right)^2 \det\begin{bmatrix}3 & -2\\-1 & 4\end{bmatrix}$

Step 3 of 5

b) We have to find the determinant of $A^{-1} = \frac{1}{10}\begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix}$.

We know that $\det(tA) = t^n \det A$, where A is an $n \times n$ matrix.

Now

$$\det A^{-1} = \det\left(\frac{1}{10}\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\right)$$
$$= \left(\frac{1}{10}\right)^2 \det\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$$
$$= \left(\frac{1}{10}\right)^2 \left[(3)(4) - (-1)(-2)\right]$$
$$= \frac{1}{100}(10)$$
$$\boxed{= \frac{1}{10}}$$

Thus, $\boxed{\det A^{-1} = \frac{1}{10}}$

Provide feedback (0)

Step 4 of 5

c) We have to find the determinant of $A - \lambda I = \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix}$.

Now

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix}$$
$$= (4-\lambda)(3-\lambda) - (2)(1)$$
$$= 12 - 4\lambda - 3\lambda + \lambda^2 - 2$$

Step 4 of 5

c) We have to find the determinant of $A-\lambda I=\begin{vmatrix}4-\lambda & 2\\ 1 & 3-\lambda\end{vmatrix}$.

Now

$$\begin{aligned}\det(A-\lambda I)&=\begin{vmatrix}4-\lambda & 2\\ 1 & 3-\lambda\end{vmatrix}\\ &=(4-\lambda)(3-\lambda)-(2)(1)\\ &=12-4\lambda-3\lambda+\lambda^2-2\\ &=\lambda^2-7\lambda+10\end{aligned}$$

Step 5 of 5

We need to find the values of λ for which $A-\lambda I$ is a singular matrix.
We know that the determinant of a singular matrix is 0.

$$\det(A-\lambda I)=0$$
$$\Rightarrow \lambda^2-7\lambda+10=0$$
$$\Rightarrow (\lambda-2)(\lambda-5)=0$$
$$\Rightarrow \lambda=2 \text{ or } 5$$

Thus, $A-\lambda I$ is a singular matrix if $\boxed{\lambda=2 \text{ or } 5}$

CH4.2 17P

CH4.2 19P

Step 1 of 8

Given matrices are $A=\begin{bmatrix}1&1&3\\0&4&6\\1&5&8\end{bmatrix}$, $B=\begin{bmatrix}1&1&3\\0&4&6\\0&0&1\end{bmatrix}$ and $C=\begin{bmatrix}1&1&3\\0&4&6\\1&5&9\end{bmatrix}$.

Provide feedback (0)

Step 2 of 8

First we need to evaluate $\det A$ by reducing the matrix to triangular form.
We know that the determinant of triangular matrix is the product of diagonal entries.
Now

$$\det(A)=\begin{vmatrix}1&1&3\\0&4&6\\1&5&8\end{vmatrix}$$

$$=\begin{vmatrix}1&1&3\\0&4&6\\0&4&6\end{vmatrix}\leftarrow\text{subtracting first row from the third row}$$

$$=\begin{vmatrix}1&1&3\\0&4&6\\0&0&-1\end{vmatrix}\leftarrow\text{subtracting second row from the third row}$$

$$=(1)(4)(-1)$$

$$=-4$$

Therefore, $\boxed{\det A=-4}$

Provide feedback (0)

Step 3 of 8

Since B is triangular (upper) $\det B$ is the product of the diagonal entries.
Now

Provide feedback (0)

Step 3 of 8

Since B is triangular (upper) det B is the product of the diagonal entries.
Now

$$\begin{aligned} \det(B) &= |B| \\ &= \begin{vmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{vmatrix} \\ &= (1)(4)(1) \\ &= 4 \end{aligned}$$

Therefore, $\boxed{\det B = 4}$

Provide feedback (0)

Step 4 of 8

Now

$$\begin{aligned} \det C &= |C| \\ &= \begin{vmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 9 \end{vmatrix} \end{aligned}$$

We find det C by cofactor expansion along the first column

$$\begin{aligned} \det C &= \begin{vmatrix} 4 & 6 \\ 5 & 9 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} \\ &= (36-30)+(6-12) \\ &= 6-6 \\ &= 0 \end{aligned}$$

Thus, $\boxed{\det C = 0}$

Provide feedback (0)

Step 5 of 8

We have to find $\det(AB)$.

We know that $\det(AB) = (\det A)(\det B)$

Now

$$\begin{aligned}\det(AB) &= (\det A)(\det B)\\ &= (-4)(4)\\ &= -16\end{aligned}$$

Thus, $\boxed{\det(AB) = -16}$

Provide feedback (0)

Step 6 of 8

We have to find $\det(A^T A)$.

We know that $\det(A^T A) = \det(A^T)\det A$ and $\det(A^T) = \det A$.

Now

$$\begin{aligned}\det(A^T A) &= \det(A^T)\det A\\ &= (-4)(-4)\\ &= 16\end{aligned}$$

Thus, $\boxed{\det(A^T A) = 16}$

Provide feedback (0)

Step 7 of 8

We have to find $\det(C^T)$

Now

$$\begin{aligned}\det(C^T) &= \det C\\ &= 0 \quad \text{since } \det(A^T) = \det(A)\end{aligned}$$

Chapter: CH4.2 Problem: 18P

show all steps

Thus, $\boxed{\det\left(A^T A\right) = 10}$

Provide feedback (0)

Step 7 of 8

We have to find $\det\left(C^T\right)$

Now

$$\begin{aligned}\det\left(C^T\right) &= \det C \\ &= 0 \quad \text{since } \det\left(A^T\right) = \det\left(A\right)\end{aligned}$$

Hence $\boxed{\det\left(C^T\right) = 0}$

Provide feedback (0)

Step 8 of 8

Chapter: CH4.2 Problem: 19P

show all steps

CH4.2 18P

CH4.2 20P

Step 1 of 2

Suppose that $CD = -DC$ then taking determinant gives

$(\det C)(\det D) = -(\det D)(\det C)$

Which is wrong.

The correct one is

$(\det C)(\det D) = (-1)^n (\det D)(\det C)$

Provide feedback (0)

Step 2 of 2

For n even the reasoning fails because $\left((-1)^n = +1\right)$

Where C, D are $n \times n$ matrices.

Hence the given conclusion is wrong.

Provide feedback (0)

CH4.2 19P

CH4.2 21P

Step 1 of 4

Given that $A=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}, B=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$ and $C=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$.

We need to find out whether the given matrices have determinants $0,1,2,$ or 3.

Provide feedback (0)

Step 2 of 4

Consider

$$\begin{aligned}\det(A)&=|A|\\&=\begin{vmatrix}0&0&1\\1&0&0\\0&1&0\end{vmatrix}\\&=-\begin{vmatrix}1&0&0\\0&0&1\\0&1&0\end{vmatrix}\\&=\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}\quad\text{(Since determinant changes sign when two rows are exchanged)}\\&=1\quad\text{(Since the determinant of the identify matrix is 1)}\end{aligned}$$

Thus, $\boxed{\det(A)=1}$

Provide feedback (0)

Step 3 of 4

Consider

$\det(B)=|B|$

Step 3 of 4

Consider

$$\det(B) = |B|$$
$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

We find $\det(B)$ by cofactor expansion along the first row.

Therefore,

$$\det B = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$
$$= (-1) - (1)$$
$$= -2$$

Thus, $\boxed{\det B = -2}$

Provide feedback (1)

Step 4 of 4

Consider

$$\det C = |C|$$
$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$
$$= 0 \quad \text{(Since the rows of the matrix } C \text{ are identical)}$$

Thus, $\boxed{\det C = 0}$

Thus, out of the 3 matrices A, B and C, only A and C have the determinants 1 and 0.

Provide feedback (1)

CH4.2 20P

CH4.2 22P

Step 1 of 2

Given that the statement is
The inverse of a 2×2 matrix seems to have determinant $= 1$.

$$\det A^{-1} = \det \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \ldots\ldots (1)$$

$$= \frac{ad-bc}{ad-bc} \quad \ldots\ldots (2)$$

$$= 1 \quad \ldots\ldots (3)$$

Provide feedback (0)

Step 2 of 2

But this is a wrong calculation since in step (2), the formula used is
$\det(tA)$ $\det(tA) = t.\det(A)$ which is false.

We need to find correct $\det A^{-1}$
Now

$$\det A^{-1} = \det \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \left(\frac{1}{ad-bc}\right)^2 (ad-bc) \quad \text{(Here } n = 2\text{, since the size of the matrix is 2 by 2)}$$

$$= \frac{1}{ad-bc} \qquad \left(\text{since } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc\right)$$

Hence the correct calculation is $\boxed{\det A^{-1} = \frac{1}{ad-bc}}$.

Provide feedback (0)

CH4.2 21P

CH4.2 23P

Step 1 of 4

Properties of the determinant

1. The determinant of the identity matrix is 1
2. The determinant changes sign when two rows are exchanged
3. The determinant depends linearly on the first row.
4. If two rows of A are equal, then $\det A = 0$
5. Subtracting a multiple of one row from another row leaves the same determinant.
6. If A has a row of zeros, then $\det A = 0$
7. If A is triangular, then $\det A$ is the product $a_{11}a_{22}a_{33}...a_{nn}$ of the diagonal entries.
8. If A is singular, then $\det A = 0$. If A is invertible, then $\det A \neq 0$.
9. The determinant of AB is product of $\det A$ times $\det B$
10. The transpose of A has the same determinant as A itself; $\det A^T = \det A$

Provide feedback (0)

Step 2 of 4

D)

Given matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Subtract second row from third row

Subtract first row from second row

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = U$$

Therefore

$$\det A = \det U = \text{product of pivots}$$

$$=1.1.1$$

$$=\boxed{1}$$

Provide feedback (0)

Provide feedback (0)

Step 3 of 4

ii) Given matrix

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 2 & 3\\ 3 & 3 & 3\end{bmatrix}$$

Subtract sum of first row and second row from third row

Add -2 times first row to the second row

$$\sim\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & -3\\ 0 & -1 & -3\end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

Subtract second row from the third row

$$\sim\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & -3\\ 0 & 0 & 0\end{bmatrix}=U$$

Therefore

$$\begin{aligned}\det A = \det U &= \text{product of pivots}\\ &= 1(-1)(0)\\ &= \boxed{0}\end{aligned}$$

Provide feedback (1)

CH4.2 22P

CH4.2 24P

Step 1 of 9

We need to apply row operations to produce an upper triangular U to compute the determinants of the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \text{and} \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Step 2 of 9

(a) So, first consider

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{subtractign 2 times the first row from the second row}$$

Step 3 of 9

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{Adding first row to the third}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \end{bmatrix}$$

Step 3 of 9

$$=\begin{bmatrix}1&2&3&0\\0&2&0&1\\0&2&3&3\\0&2&0&7\end{bmatrix}\leftarrow \text{Adding first row to the third}$$

$$=\begin{bmatrix}1&2&3&0\\0&2&0&1\\0&0&3&2\\0&2&0&7\end{bmatrix}\leftarrow \text{Substracting the second row from the third row}$$

Provide feedback (0)

Step 4 of 9

$$=\begin{bmatrix}1&2&3&0\\0&2&0&1\\0&0&3&2\\0&0&0&6\end{bmatrix}\leftarrow \text{Substracting the second row from the fourth row}$$

$$=(1)(2)(3)(6)$$
$$=36$$

Note: if A is triangular, then det A is the product $a_{11}a_{22}...a_{nn}$ of the diagonal entries.

Provide feedback (0)

Step 5 of 9

(b) Now consider

$$\begin{vmatrix}2&1&1&1\\1&2&1&1\\1&1&2&1\\1&1&1&2\end{vmatrix}$$

Step 5 of 9

(b) Now consider

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 2 \end{vmatrix} \leftarrow \text{substracting the fourth row from the second and third rows}$$

Provide feedback (0)

Step 6 of 9

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{2} \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the first row to the fourth.}$$

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & \frac{1}{2} & 2 \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the second row to the fourth.}$$

Provide feedback (0)

Step 7 of 9

$$\begin{vmatrix} 2 & 1 & 1 & 1 \end{vmatrix}$$

Step 7 of 9

$$=\begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & \frac{5}{2} \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the third row to the fourth.}$$

$$=(2)(1)(1)\left(\frac{5}{2}\right)$$

$$=5$$

Provide feedback (0)

Step 8 of 9

Note: if A is triangular, then det A is the product $a_{11}, a_{22}.....a_{nn}$ of the diagonal entries.

Provide feedback (0)

Step 9 of 9

Thus, $\boxed{\det\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} = 36}$

And

$$\boxed{\det\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = 5}$$

Provide feedback (0)

CH4.2 23P

CH4.2 25P

Step 1 of 8

We need to apply row operations to simplify and compute the determinants of the following matrices:

$$\begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 8

(a) So, first consider

$$\begin{vmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{vmatrix}$$

$$= \begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} \leftarrow \text{substracting the first row from the second and third rows}$$

Provide feedback (0)

Step 3 of 8

$$= \begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} \leftarrow \text{substracting 2 times the second row from the third}$$

$$= 0$$

Note: If A has a row of zeros, then det A=0.

Provide feedback (0)

Chapter: CH4.2 Problem: 24P

show all steps

Step 4 of 8

(b) Now, consider

$$\begin{vmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & t & t^2 \\ t-1 & 1-t & t-t^2 \\ t^2-1 & 0 & 1-t^2 \end{vmatrix} \leftarrow \text{substracting the first row from the second and third rows}$$

Provide feedback (0)

Step 5 of 8

$$= \begin{vmatrix} 1 & t & t^2 \\ -(1-t) & 1-t & t(1-t) \\ -(1-t^2) & 0 & 1-t^2 \end{vmatrix}$$

$$= (1-t)(1-t^2) \begin{vmatrix} 1 & t & t^2 \\ -1 & 1 & t \\ -1 & 0 & 1 \end{vmatrix}$$

$\leftarrow$ taking $(1-t)\,\&\,(1-t^2)$ common from the second and third rows

Provide feedback (0)

Step 6 of 8

$$= (1-t)(1-t^2) \begin{vmatrix} 1 & t & t^2 \\ 0 & 1+t & t+t^2 \\ 0 & t & 1+t^2 \end{vmatrix}$$

Row evaluating the det by cofactor expansion along the first column

Step 6 of 8

$$=(1-t)(1-t^2)\begin{vmatrix}1 & t & t^2\\ 0 & 1+t & t+t^2\\ 0 & t & 1+t^2\end{vmatrix}$$

Row evaluating the det by cofactor expansion along the first column

$$=(1-t)(1-t^2)\begin{vmatrix}1+t & t+t^2\\ t & 1+t^2\end{vmatrix}$$

Provide feedback (0)

Step 7 of 8

$$=(1-t)(1-t^2)\left[(1+t)(1+t^2)-t(t+t^2)\right]$$
$$=(1-t)(1-t^2)\left[1+t^2+t+t^3-t^2-t^3\right]$$
$$=(1-t)(1-t^2)(1+t)$$
$$=(1-t^2)(1-t^2)$$
$$=(1-t^2)^2$$

Provide feedback (0)

Step 8 of 8

Thus,

$$\boxed{\det\begin{bmatrix}101 & 201 & 301\\ 102 & 202 & 303\\ 103 & 203 & 303\end{bmatrix}=0} \text{ and } \boxed{\det\begin{bmatrix}1 & t & t^2\\ t & 1 & t\\ t^2 & t & 1\end{bmatrix}=(1-t^2)^2}$$

Provide feedback (0)

CH4.2 24P

CH4.2 26P

Step 1 of 6

Properties of the determinant

1. The determinant of the identity matrix is 1
2. The determinant changes sign when two rows are exchanged
3. The determinant depends linearly on the first row.
4. If two rows of A are equal, then $\det A = 0$
5. Subtracting a multiple of one row from another row leaves the same determinant.
6. If A has a row of zeros, then $\det A = 0$
7. If A is triangular, then $\det A$ is the product $a_{11}a_{22}a_{33}\ldots a_{nn}$ of the diagonal entries.
8. If A is singular, then $\det A = 0$. If A is invertible, then $\det A \neq 0$.
9. The determinant of AB is product of $\det A$ times $\det B$
10. The transpose of A has the same determinant as A itself; $\det A^T = \det A$

Provide feedback (0)

Step 2 of 6

We have

$$A=\begin{bmatrix}3&3&4\\6&8&7\\-3&5&-9\end{bmatrix}=LU=\begin{bmatrix}1&0&0\\2&1&0\\-1&4&1\end{bmatrix}\begin{bmatrix}3&3&4\\0&2&-1\\0&0&-1\end{bmatrix}$$

Here

$$L=\begin{bmatrix}1&0&0\\2&1&0\\-1&4&1\end{bmatrix}\quad\text{And } U=\begin{bmatrix}3&3&4\\0&2&-1\\0&0&-1\end{bmatrix}$$

Provide feedback (0)

Step 3 of 6

Both L and U are triangular matrices

$$\det L=(1)(1)(1)$$
$$=1$$

Step 3 of 6

Both L and U are triangular matrices

$$\det L = (1)(1)(1)$$
$$= 1$$
$$\det U = (3)(2)(-1)$$
$$= -6$$

Provide feedback (0)

Step 4 of 6

Therefore

$$\det A = \det L \times \det U$$
$$= (1)(-6)$$
$$= -6$$

Provide feedback (0)

Step 5 of 6

And

$$A^{-1} = (LU)^{-1}$$
$$= U^{-1}L^{-1}$$
$$\det U^{-1}L^{-1} = \det A^{-1}$$
$$= \frac{1}{\det A}$$
$$= \frac{-1}{6}$$

Provide feedback (0)

Step 6 of 6

Step 5 of 6

And

$$A^{-1} = (LU)^{-1}$$
$$= U^{-1}L^{-1}$$
$$\det U^{-1}L^{-1} = \det A^{-1}$$
$$= \frac{1}{\det A}$$
$$= \frac{-1}{6}$$

Step 6 of 6

Further

$$U^{-1}L^{-1}A = A^{-1}A = I$$
$$\det\left(U^{-1}L^{-1}A\right) = \det I = 1$$

Thus

$$\boxed{\begin{aligned} \det L &= 1 \\ \det U &= -6 \\ \det A &= -6 \\ \det U^{-1}L^{-1} &= \frac{-1}{6} \\ \det\left(U^{-1}L^{-1}A\right) &= 1 \end{aligned}}$$

Chapter: CH4.2 Problem: 26P

show all steps

CH4.2 25P

CH4.2 27P

Step 1 of 3

Let $A = (a_{ij})_{n\times n}$ where

$a_{ij} = ij$ for $1 \le i, j \le n$

Then

$A_1 = (1)$ so $\det A_1 = 1$(exceptional case)

$A_2 = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ so that $\det A_2 = 0$

$A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ so that

Provide feedback (0)

Step 2 of 3

$$\begin{aligned}\det A_3 &= \begin{vmatrix} 4 & 6 \\ 6 & 9 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 3 & 9 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} \\ &= 0 - 2(0) + 3(0) \\ &= 0\end{aligned}$$

Provide feedback (0)

Step 3 of 3

For any $n \ge 2$ we can note that

$A_n = \begin{pmatrix} 1 & 2 & 3 \dots n \\ 2 & 4 & 6 \dots 2n \\ n & 2n & 3n \dots n^2 \end{pmatrix}$

And clearly any two rows of A_n are proportional and hence $\det A_n = 0$

Thus if a_{ij} is I times j, then $\det A = 0$

(exception when A=[1])

$$\det A_3 = \begin{vmatrix}4 & 6\\6 & 9\end{vmatrix} - 2\begin{vmatrix}2 & 6\\3 & 9\end{vmatrix} + 3\begin{vmatrix}2 & 4\\3 & 6\end{vmatrix}$$
$$= 0 - 2(0) + 3(0)$$
$$= 0$$

Provide feedback (0)

Step 3 of 3

For any $n \geq 2$ we can note that

$$A_n = \begin{pmatrix}1 & 2 & 3\ldots n\\2 & 4 & 6\ldots 2n\\n & 2n & 3n\ldots n^2\end{pmatrix}$$

And clearly any two rows of A_n are proportional and hence $\det A_n = 0$

Thus if a_{ij} is I times j, then $\det A = 0$

(exception when A=[1])

Provide feedback (0)

Chapter: CH4.2 Problem: 27P

show all steps

CH4.2 26P

CH4.2 28P

Step 1 of 5

Let $A_n = \left(a_{ij}\right)_{n\times n}$ where

$$A_1 = (1+1)$$
$$= (2)$$
$$\Rightarrow \det A_1 = 2$$

Provide feedback (0)

Step 2 of 5

$$A_2 = \begin{pmatrix} 1+1 & 1+2 \\ 2+1 & 2+2 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$$
$$\Rightarrow \det A_2 = 8-9$$
$$= -1$$

Provide feedback (0)

Step 3 of 5

$$A_3 = \begin{pmatrix} 1+1 & 1+2 & 1+3 \\ 2+1 & 2+2 & 2+3 \\ 3+1 & 3+2 & 3+3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 5

$$= \begin{pmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 5

$$\Rightarrow \det A_3 = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$ Adding -1 time the second row to the third and -1 time the first row to the second.

=0 (since two rows are qual.)

Provide feedback (0)

Step 5 of 5

For any $n \geq 3$

$$A_n = \begin{pmatrix} 1 & 3 & 4 & \dots & n+1 \\ 3 & 4 & 5 & \dots & n+2 \\ 4 & 5 & 6 & \dots & n+3 \\ n+1 & n+2 & n+3 & \dots & 2n \end{pmatrix}$$

Cleary subtracting 1^{st} row from 2^{nd} row and 2^{nd} row from third two result in a matrix of two identical rows containing all entries equal to 1 and hence $\det A_n = 0$ for $n \geq 3$

Thus, if a_{ij} is $i+j$, we have $\det A = 0$ (exception when $n = 1$ or 2)

Provide feedback (0)

CH4.2 27P

CH4.2 29P

Step 1 of 7

Properties of the determinant

1. The determinant of the identity matrix is 1
2. The determinant changes sign when two rows are exchanged
3. The determinant depends linearly on the first row.
4. If two rows of A are equal, then $\det A = 0$
5. Subtracting a multiple of one row from another row leaves the same determinant.
6. If A has a row of zeros, then $\det A = 0$
7. If A is triangular, then $\det A$ is the product $a_{11}a_{22}a_{33}...a_{nn}$ of the diagonal entries.
8. If A is singular, then $\det A = 0$. If A is invertible, then $\det A \neq 0$.
9. The determinant of AB is product of $\det A$ times $\det B$
10. The transpose of A has the same determinant as A itself; $\det A^T = \det A$

Provide feedback (0)

Step 2 of 7

a) We have

$$A = \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{vmatrix}$$

$$= -\begin{vmatrix} 0 & 0 & b \\ 0 & a & 0 \\ c & 0 & 0 \end{vmatrix} \text{ interchaning first and second rows}$$

$$= -\begin{vmatrix} c & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{vmatrix} \text{ interchaning first and third rows}$$

Provide feedback (0)

Step 3 of 7

Therefore

$$|A| = \begin{vmatrix} c & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{vmatrix}$$
$$= \boxed{abc}$$

Step 4 of 7

b) Given $B = \begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ d & 0 & 0 & 0 \end{bmatrix}$

Now

$$\det B = \begin{vmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ d & 0 & 0 & 0 \end{vmatrix}$$
$$= -\begin{vmatrix} 0 & 0 & b & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & c \\ d & 0 & 0 & 0 \end{vmatrix} \text{interchanging first and second rows}$$

Step 5 of 7

On solving

$$\begin{vmatrix} 0 & 0 & b & 0 \end{vmatrix}$$

On solving

$$= \begin{vmatrix} 0 & 0 & b & 0 \\ 0 & a & 0 & 0 \\ d & 0 & 0 & 0 \\ 0 & 0 & 0 & c \end{vmatrix} \text{interchanging fourth and third rows}$$

$$= -\begin{vmatrix} d & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{vmatrix} \text{interchanging first and third rows}$$

$$\det B = -(d)(a)(b)(c)$$

$$= \boxed{-abcd}$$

Provide feedback (0)

Step 6 of 7

iii) We have

$$c = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

$$\det c = \begin{vmatrix} a & a & a \\ a & b & b \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a & a \\ a & b & b \\ 0 & 0 & c-b \end{vmatrix} \text{Adding } -1 \text{ time second row to the thired row}$$

Provide feedback (0)

Step 7 of 7

$$\det B = -(d)(a)(b)(c)$$
$$= \boxed{-abcd}$$

Provide feedback (0)

Step 6 of 7

iii) We have

$$c = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

$$\det c = \begin{vmatrix} a & a & a \\ a & b & b \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a & a \\ a & b & b \\ 0 & 0 & c-b \end{vmatrix} \text{Adding } -1 \text{ time second row to the thired row}$$

Provide feedback (0)

Step 7 of 7

$$= \begin{vmatrix} a & a & a \\ a & b-a & b-a \\ 0 & 0 & c-b \end{vmatrix} \text{Adding } -1 \text{ time the first row to the second row}$$
$$= U$$

$$\det C = \det U$$
$$= a(b-a)(c-b)$$
$$= \boxed{a(a-b)(b-c)}$$

Provide feedback (1)

Step 1 of 1

CH4.2 28P

CH4.2 30P

Although this proof seems to be correct, the matrix A may be rectangular. In such case A^T is also rectangular.

We define determinant of a square matrix only. Determinant of a rectangular matrix is nor defined.

Thus, the expression $|A|\frac{1}{|A^T||A|}|A^T|$ is meaningless.

This is the mistake in the above proof.

Provide feedback (0)

CH4.2 29P

CH4.2 31P

Step 1 of 3

Given that

$f(a,b,c,d) = \ln(ad - bc)$

$\Rightarrow \dfrac{\partial f}{\partial a} = \dfrac{d}{ad - bc}$

$\dfrac{\partial f}{\partial b} = \dfrac{1}{ad - bc}(-c)$

Provide feedback (0)

Step 2 of 3

$\dfrac{\partial f}{\partial c} = \dfrac{-b}{ad - bc}$

$\dfrac{\partial f}{\partial d} = \dfrac{a}{ad - bc}$

Provide feedback (0)

Step 3 of 3

Therefore $\begin{bmatrix} \dfrac{\partial f}{\partial a} & \dfrac{\partial f}{\partial c} \\ \dfrac{\partial f}{\partial b} & \dfrac{\partial f}{\partial d} \end{bmatrix} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \boxed{A^{-1}}$

Provide feedback (0)

CH4.2 33P

CH4.2 35P

Step 1 of 4

We have

$\det A = 6$ and

$$\det A = \begin{vmatrix} \text{row } 1 \\ \text{row } 2 \\ \text{row } 3 \end{vmatrix} = \begin{vmatrix} \text{row } 1 + \text{row } 2 \\ \text{row } 2 + \text{row } 3 \\ \text{row } 3 \end{vmatrix}$$

Provide feedback (0)

Step 2 of 4

$$\text{Therefore} \quad \det B = \begin{vmatrix} \text{row } 1 + \text{row } 2 \\ \text{row } 2 + \text{row } 3 \\ \text{row } 3 + \text{row } 1 \end{vmatrix}$$

$$= \det A + \begin{vmatrix} \text{row } 1 + \text{row } 2 \\ \text{row2+row } 3 \\ \text{row } 1 \end{vmatrix}$$

Provide feedback (0)

Step 3 of 4

$$\det B = \det A + \begin{vmatrix} \text{row } 2 \\ \text{row } 2 + \text{row } 3 \\ \text{row } 1 \end{vmatrix}$$

$$= \det A + \begin{vmatrix} \text{row } 2 \\ \text{row } 3 \\ \text{row } 1 \end{vmatrix}$$

Provide feedback (0)

Step 3 of 4

$$\det B = \det A + \begin{vmatrix} \text{row } 2 \\ \text{row } 2 + \text{row } 3 \\ \text{row } 1 \end{vmatrix}$$

$$= \det A + \begin{vmatrix} \text{row } 2 \\ \text{row } 3 \\ \text{row } 1 \end{vmatrix}$$

Step 4 of 4

$$\det B = \det A - \begin{vmatrix} \text{row } 2 \\ \text{row } 1 \\ \text{row } 3 \end{vmatrix}$$

$$= \det A + \begin{vmatrix} \text{row } 1 \\ \text{row } 2 \\ \text{row } 3 \end{vmatrix}$$

$$= \det A + \det A$$
$$= 6 + 6$$
$$= \boxed{12}$$

CH4.2 34P

CH4.3

Step 1 of 3

Given that

$$\det(I+M)=\begin{vmatrix}1+a & b & c & d\\ a & 1+b & c & d\\ a & b & 1+c & d\\ a & b & c & 1+d\end{vmatrix}$$

$$=\begin{vmatrix}1+a+b+c+d & b & c & d\\ 1+a+b+c+d & 1+b & c & d\\ 1+a+b+c+d & b & 1+c & d\\ 1+a+b+c+d & b & c & 1+d\end{vmatrix}$$

(Adding second, third, fourth columns to the first column)

Provide feedback (0)

Step 2 of 3

$$=(1+a+b+c+d)\begin{vmatrix}1 & b & c & d\\ 1 & 1+b & c & d\\ 1 & b & 1+c & d\\ 1 & b & c & 1+d\end{vmatrix}$$

$$=(1+a+b+c+d)\begin{vmatrix}1 & b & c & d\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}$$

Provide feedback (0)

Step 3 of 3

(Adding -1 time the first row to the second, third and fourth rows)

$$= \begin{vmatrix} 1+a+b+c+d & 1+b & c & d \\ 1+a+b+c+d & b & 1+c & d \\ 1+a+b+c+d & b & c & 1+d \end{vmatrix}$$

(Adding second, third, fourth columns to the first column)

Provide feedback (0)

Step 2 of 3

$$= (1+a+b+c+d) \begin{vmatrix} 1 & b & c & d \\ 1 & 1+b & c & d \\ 1 & b & 1+c & d \\ 1 & b & c & 1+d \end{vmatrix}$$

$$= (1+a+b+c+d) \begin{vmatrix} 1 & b & c & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Provide feedback (0)

Step 3 of 3

(Adding -1 time the first row to the second, third and fourth rows)

$= (1a + a + b + c + d)$

When $a = b = c = d = 1$,

$\det(1+M) = 1+1+1+1+1$

$= \boxed{5}$

Provide feedback (0)