CH5.5

Step 1 of 2

Given that B is similar to A

$\Rightarrow B = M^{-1}AM$

Also, C is similar to $B \Rightarrow C = N^{-1}BN$

We consider $C = N^{-1}BN$

$$\begin{aligned} &= N^{-1}\left(M^{-1}AM\right)N \\ &= \left(N^{-1}M^{-1}\right)A\left(MN\right) \\ &= \left(MN\right)^{-1}A\left(MN\right) \end{aligned}$$

Provide feedback (0)

CH5.6 2P

Step 2 of 2

Assuming $MN = P$, this equation becomes $C = P^{-1}AP$

Therefore C is similar to A.

Considering any non singular matrix M we can write $I = M^{-1}IM$

Therefore, I is similar to I only.

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CH5.6 1P

CH5.6 3P

Step 1 of 4

Let us consider the given matrix $A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

Let the matrix similar to A is $B=\begin{bmatrix}x & y\\ z & w\end{bmatrix}$

So, there exists a matrix $M=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, $ad-bc\neq 0$ such that $B=M^{-1}AM$

So, $\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}x & y\\ z & w\end{bmatrix}$

$\Rightarrow \frac{1}{ad-bc}\begin{bmatrix}d & b\\ -c & -a\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}x & y\\ z & w\end{bmatrix}$

$\frac{1}{ad-bc}\begin{bmatrix}ad+bc & 2bd\\ -2ac & -(ad+bc)\end{bmatrix}=\begin{bmatrix}x & y\\ z & w\end{bmatrix}$

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Step 2 of 4

So, the general matrix similar to $A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ is $B=\begin{bmatrix}\frac{ad+bc}{ad-bc} & \frac{2bd}{ad-bc}\\ -\frac{2ac}{ad-bc} & -\frac{(ad+bc)}{ad-bc}\end{bmatrix}$ when

$M=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is such that $M^{-1}AM=B$

Provide feedback (0)

Step 3 of 4

We know that $M=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ is having the determinant $1+1=2$ and so, $M^{-1}=\frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

such that $B=M^{-1}AM$

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Step 3 of 4

We know that $M=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ is having the determinant $1+1=2$ and so, $M^{-1}=\frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

such that $B=M^{-1}AM$

$$=\frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

Provide feedback (1)

Step 4 of 4

So, the matrix B similar to $A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ is $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

Also, another matrix B is similar to $A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ is obtained by considering $M=\begin{bmatrix}2 & 1\\ 0 & 1\end{bmatrix}$

and $M^{-1}=\frac{1}{2}\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$

$B=M^{-1}AM$

$$=\frac{1}{2}\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}2 & 1\\ 0 & 1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$$

Provide feedback (1)

CH5.6 2P

Step 1 of 1

Suppose that $B = M^{-1}AM$. Then A and B have the same eigen values. …… (1)

Suppose A has eigen values $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_n$

That is the roots of $|A - \lambda I| = 0$ are $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_n$ …… (2)

Replacing A with $A + I$, we get $|(A+I) - \lambda I| = 0 \Rightarrow |A - (\lambda - 1) I| = 0$

So, its roots are $\lambda_1 - 1, \lambda_2 - 1, \ldots, \lambda_n - 1$ which the eigen values of $A + I$ are. …… (3)

We follow that the eigen values of A and that of $A+I$ are not same.

In view of (1), we confirm that A and $A+I$ are not similar.

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CH5.6 4P

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CH5.6 3P

Step 1 of 4

In case of a diagonal matrix, its inverse is obtained by keeping the off diagonal elements as zeros only and then writing reciprocals of the diagonal elements.

Since D is a diagonal matrix, which has either 1 or –1 along the diagonal, its inverse D^{-1} has to be same as D, since reciprocal of 1 is 1 and reciprocal of –1 is –1 only.

Thus, we want $DAD = B$.

Provide feedback (0)

CH5.6 5P

Step 2 of 4

Let $D = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$, where each of a, b, c, and d is either 1 or –1.

Consider the following:

$$\begin{aligned}
\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} &= \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \\
&= \begin{bmatrix} 2a & a & 0 & 0 \\ b & 2b & b & 0 \\ 0 & c & 2c & c \\ 0 & 0 & d & 2d \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \\
&\begin{bmatrix} 2a^2 & ab & 0 & 0 \end{bmatrix}
\end{aligned}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} 2a & a & 0 & 0 \\ b & 2b & b & 0 \\ 0 & c & 2c & c \\ 0 & 0 & d & 2d \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} 2a^2 & ab & 0 & 0 \\ ab & 2b^2 & bc & 0 \\ 0 & bc & 2c^2 & cd \\ 0 & 0 & cd & 2d^2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Thus, we have $\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2a^2 & ab & 0 & 0 \\ ab & 2b^2 & bc & 0 \\ 0 & bc & 2c^2 & cd \\ 0 & 0 & cd & 2d^2 \end{bmatrix}$.

Therefore, we get the following three equations:

$$ab = -1$$
$$bc = -1$$
$$cd = -1$$

Hence, if $a = 1$, then $b = -1$. This gives $c = 1$ and $d = -1$. Or, if $a = -1$, then $b = 1$. This implies that

Thus, we have $\begin{bmatrix} -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} ab & 2b^2 & bc & 0 \\ 0 & bc & 2c^2 & cd \\ 0 & 0 & cd & 2d^2 \end{bmatrix}$.

Therefore, we get the following three equations:

$$ab = -1$$
$$bc = -1$$
$$cd = -1$$

Hence, if $a = 1$, then $b = -1$. This gives $c = 1$ and $d = -1$. Or, if $a = -1$, then $b = 1$. This implies that $c = -1$ and thus, $d = 1$.

Provide feedback (0)

Step 4 of 4

Thus, we get the two following diagonal matrices so that *A* will be symmetric to *B*:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

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CH5.6 4P

Step 1 of 1

Given B is invertiable

$\Rightarrow B^{-1}$ exists such that $BB^{-1} = B^{-1}B = I$.

Let us consider $AB = I(AB)$

$$= \left(B^{-1}B\right)(AB)$$

$$= B^{-1}(BA)B$$

Therefore BA is similar to AB.

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CH5.6 6P

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CH5.6 5P

Step 1 of 3

a) D is invertible $\Rightarrow DD^{-1} = D^{-1}D = I$ and $CD = -DC$. ..... (1)

Let us consider $C = IC$

$$\begin{aligned} &= \left(D^{-1}D\right)C \\ &= D^{-1}\left(DC\right) && \text{By associativity of multiplication of matrices} \\ &= D^{-1}\left(-CD\right) && \text{By (1)} \\ &= D^{-1}\left(-C\right)D \end{aligned}$$

Therefore C is similar to $-C$.

Provide feedback (0)

Step 2 of 3

b) If two matrices are similar, then their eigen values are equal.

We have C is similar to $-C$.

But we follow that the eigen values are respectively the roots of $|C - \lambda I| = 0, |C + \lambda I| = 0$

So, it follows that if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigen values of C then

$-\lambda_1, -\lambda_2, \ldots, -\lambda_n$ are eigen values of $-C$

Therefore, we confirm that the eigen values of the matrices C and $-C$ are the plus $-$ minus pairs of $\lambda_1, \lambda_2, \ldots \lambda_n$.

Provide feedback (0)

Step 3 of 3

c) Given that $Cx = \lambda x$

Then $C\left(Dx\right) = \left(CD\right)x$

$$\begin{aligned} &= \left(-DC\right)x \\ &= -D\left(Cx\right) \\ &= -D\left(\lambda x\right) \\ &= -\lambda\left(Dx\right) \end{aligned}$$

CH5.6 7P

But we follow that the eigen values are respectively the roots of $|C-\lambda I|=0, |C+\lambda I|=0$

So, it follows that if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigen values of C then $-\lambda_1, -\lambda_2, \ldots, -\lambda_n$ are eigen values of $-C$

Therefore, we confirm that the eigen values of the matrices C and $-C$ are the plus – minus pairs of $\lambda_1, \lambda_2, \ldots \lambda_n$.

Provide feedback (0)

Step 3 of 3

c) Given that $Cx = \lambda x$

Then
$$\begin{aligned} C(Dx) &= (CD)x \\ &= (-DC)x \\ &= -D(Cx) \\ &= -D(\lambda x) \\ &= -\lambda(Dx) \end{aligned}$$

Therefore $C(Dx) = -\lambda(Dx)$

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CH5.6 6P

Step 1 of 1

Given that $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $M=\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So, $M^{-1}=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$M^{-1}AM=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} a\cos\theta+d\sin\theta & b\cos\theta+e\sin\theta & c\cos\theta+f\sin\theta \\ -a\sin\theta+d\cos\theta & -b\sin\theta+e\cos\theta & -c\sin\theta+f\cos\theta \\ g & h & i \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The element (3,1) in this product is $g\cos\theta+h\sin\theta$ is given to be zero.

$\Rightarrow g\cos\theta+h\sin\theta=0$

$\Rightarrow h\sin\theta=-g\cos\theta$

$\Rightarrow \tan\theta=\frac{-g}{h}$

$\Rightarrow \theta=\tan^{-1}\left(\frac{-g}{h}\right)$

CH5.6 8P

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CH5.6 7P

CH5.6 9P

Step 1 of 2

Given that $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

The vector $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = C_1V_1 + C_2V_2$

$$= C_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 + C_2 \\ C_1 + 4C_2 \end{bmatrix}$$

$C_1 + C_2 = 3$

$C_1 + 4C_2 = 9$

Solving these, we get $C_1 = 1, C_2 = 2$

Also, $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = d_1v_1 + d_2v_2$

$$= d_1\begin{bmatrix} 2 \\ 5 \end{bmatrix} + d_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2d_1 + d_2 \\ 3d_1 + 4d_2 \end{bmatrix}$$

$\Rightarrow 2d_1 + d_2 = 3$

$5d_1 + 4d_2 = 9$

Provide feedback (0)

Step 2 of 2

Solving these, we get $d_1 = 1, d_2 = 1$

And column of M come from V_1, V_2 as colwntration

$\sum m_{ij}\, v_i$

$= m_{11}v_1 + v_{31}v_2$

$\begin{bmatrix} 1 \end{bmatrix} = m_{11}\begin{bmatrix} 2 \end{bmatrix} + m_{21}\begin{bmatrix} 1 \end{bmatrix}$

Solving these, we get $C_1 = 1, C_2 = 2$

Also, $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = d_1 v_1 + d_2 v_2$

$$= d_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + d_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2d_1 + d_2 \\ 3d_1 + 4d_2 \end{bmatrix}$$

$\Rightarrow 2d_1 + d_2 = 3$

$5d_1 + 4d_2 = 9$

Provide feedback (0)

Step 2 of 2

Solving these, we get $d_1 = 1, d_2 = 1$

And column of M come from V_1, V_2 as colwntration

$\sum m_{ij} v_i$

$$= m_{11} v_1 + v_{31} v_2$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = m_{11} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{21} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2m_{11} + m_{21} \\ 5m_{11} + 4m_{21} \end{bmatrix}$$

$\Rightarrow 2m_{11} + m_{21} = 1$

$5m_{11} + 4m_{21} = 1$

Solving these we get $m_{11} = 1, m_{21} = -1$

Therefore, $M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ changes the basis.

Provide feedback (0)

CH5.6 8P

Step 1 of 5

Given that $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

The vector $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = C_1V_1 + C_2V_2$

$$= C_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 + C_2 \\ C_1 + 4C_2 \end{bmatrix}$$

$$C_1 + C_2 = 3$$
$$C_1 + 4C_2 = 9$$

Provide feedback (0)

Step 2 of 5

Solving these, we get $C_1 = 1, C_2 = 2$

Also, $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = d_1v_1 + d_2v_2$

$$= d_1\begin{bmatrix} 2 \\ 5 \end{bmatrix} + d_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2d_1 + d_2 \\ 3d_1 + 4d_2 \end{bmatrix}$$

$$\Rightarrow 2d_1 + d_2 = 3$$
$$5d_1 + 4d_2 = 9$$

Provide feedback (0)

Step 3 of 5

Solving these we get $d_1 = 1, d_2 = 1$

CH5.6 10P

$$5d_1 + 4d_2 = 9$$

Provide feedback (0)

Step 3 of 5

Solving these we get $d_1 = 1, d_2 = 1$

The column of M comes from V_1, V_2 as colwntration

$$\sum m_{ij} v_i$$

$$V_1 = m_{11} v_1 + v_{31} v_2$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = m_{11} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{21} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2m_{11} + m_{21} \\ 5m_{11} + 4m_{21} \end{bmatrix}$$

$$\Rightarrow \quad 2m_{11} + m_{21} = 1$$

$$5m_{11} + 4m_{21} = 1$$

Provide feedback (0)

Step 4 of 5

Solving these we get $m_{11} = 1, m_{21} = -1$

$$V_2 = m_{12} v_1 + m_{22} v_2$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = m_{12} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{22} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2m_{12} + m_{22} \\ 5m_{12} + 4m_{22} \end{bmatrix}$$

$$\Rightarrow 2m_{12} + m_{22} = 1$$

$$5m_{12} + 4m_{22} = 4$$

Solving these we get $m_{12} = 0$, $m_{22} = 1$

Provide feedback (0)

Provide feedback (0)

Step 4 of 5

Solving these we get $m_{11} = 1, m_{21} = -1$

$$V_2 = m_{12}v_1 + m_{22}v_2$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = m_{12}\begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{22}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2m_{12} + m_{22} \\ 5m_{12} + 4m_{22} \end{bmatrix}$$

$$\Rightarrow 2m_{12} + m_{22} = 1$$

$$5m_{12} + 4m_{22} = 4$$

Solving these we get $m_{12} = 0$, $m_{22} = 1$

Provide feedback (0)

Step 5 of 5

$$M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$MC = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= d$$

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CH5.6 9P

Step 1 of 5

Given that $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

The vector $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = C_1V_1 + C_2V_2$

$$= C_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 + C_2 \\ C_1 + 4C_2 \end{bmatrix}$$

$C_1 + C_2 = 3$

$C_1 + 4C_2 = 9$

Solving these $C_1 = 1, C_2 = 2$

Provide feedback (0)

Step 2 of 5

Also, $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = d_1v_1 + d_2v_2$

$$= d_1\begin{bmatrix} 2 \\ 5 \end{bmatrix} + d_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2d_1 + d_2 \\ 3d_1 + 4d_2 \end{bmatrix}$$

$\Rightarrow 2d_1 + d_2 = 3$

$5d_1 + 4d_2 = 9$

Provide feedback (0)

Step 3 of 5

Solving these, we get $d_1 = 1, d_2 = 1$

CH5.6 11P

$$= d_1\begin{bmatrix}5\end{bmatrix} + d_2\begin{bmatrix}4\end{bmatrix}$$

$$= \begin{bmatrix}2d_1 + d_2 \\ 3d_1 + 4d_2\end{bmatrix}$$

$$\Rightarrow 2d_1 + d_2 = 3$$

$$5d_1 + 4d_2 = 9$$

Provide feedback (0)

Step 3 of 5

Solving these, we get $d_1 = 1, d_2 = 1$

The column of M comes from V_1, V_2 as colwntration

$$\sum m_{ij} v_i$$

$$V_1 = m_{11}v_1 + v_{21}v_2$$

$$\begin{bmatrix}1\\1\end{bmatrix} = m_{11}\begin{bmatrix}2\\5\end{bmatrix} + m_{21}\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}2m_{11} + m_{21} \\ 5m_{11} + 4m_{21}\end{bmatrix}$$

$$\Rightarrow 2m_{11} + m_{21} = 1$$

$$5m_{11} + 4m_{21} = 1$$

Solving these, we get $m_{11} = 1, m_{21} = -1$

Provide feedback (0)

Step 4 of 5

$$V_2 = m_{12}v_1 + m_{22}v_2$$

$$\begin{bmatrix}1\\4\end{bmatrix} = m_{12}\begin{bmatrix}2\\5\end{bmatrix} + m_{22}\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}2m_{12} + m_{22} \\ 5m_{12} + 4m_{22}\end{bmatrix}$$

$$\Rightarrow 2m_{12} + m_{22} = 1$$

$$5m_{12} + 4m_{22} = 4$$

$$V_2 = m_{12}v_1 + m_{22}v_2$$

$$\begin{bmatrix}1\\4\end{bmatrix} = m_{12}\begin{bmatrix}2\\5\end{bmatrix} + m_{22}\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}2m_{12}+m_{22}\\5m_{12}+4m_{22}\end{bmatrix}$$

$$\Rightarrow 2m_{12}+m_{22}=1$$

$$5m_{12}+4m_{22}=4$$

Solving these, we get $m_{12}=0$, $m_{22}=1$

$$M = \begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix}$$

$$MC = \begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$= \begin{bmatrix}1\\1\end{bmatrix}$$

$$= d$$

Provide feedback (0)

Step 5 of 5

$$m_{21}c_1 + m_{22}c_{22} = (-1)(1)+(1)(2)$$

$$= 1$$

$$= d_2$$

$$C_1V_1 + C_2V_2 = 1\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}3\\9\end{bmatrix}$$

$$d_1v_1 + d_2v_2 = 1\begin{bmatrix}2\\5\end{bmatrix} + 1\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}3\\9\end{bmatrix}$$

$$C_1V_1 + C_2V_2 = d_1v_1 + d_2v_2$$

$$= d_2$$

$$C_1V_1 + C_2V_2 = 1\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}3\\9\end{bmatrix}$$

$$d_1v_1 + d_2v_2 = 1\begin{bmatrix}2\\5\end{bmatrix} + 1\begin{bmatrix}1\\4\end{bmatrix}$$

$$= \begin{bmatrix}3\\9\end{bmatrix}$$

$$C_1V_1 + C_2V_2 = d_1v_1 + d_2v_2$$

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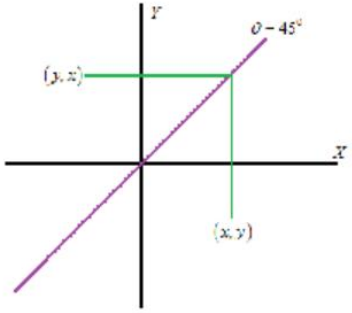
Step 1 of 6

The reflection across the line $\theta = 45^0$ is shown in the following figure.
The point (x,y) is transformed to (y,x) under this reflection.



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Step 2 of 6

So, T is the linear transformation performs this action.
It can be written as $T(x,y) = (y,x)$
Using this, we find the images of the standard basis vectors.
$T(1,0) = (0,1) = 0(1,0) + 1(0,1)$;
$T(0,1) = (1,0) = 1(1,0) + 0(0,1)$
We collect the coefficients of these linear combinations as the columns of the suitable matrix.

So, the matrix of T with respect to standard basis is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. ..... (1)

This action can be written as the product of matrices $(x,y)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (y,x)$

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Step 3 of 6

This action can be written as the product of matrices $(x,y)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}=(y,x)$

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Step 3 of 6

Similarly, we find the images of the other basis.

$T(1,1)=(1,1)$ ..... (2)

$T(1,-1)=(-1,1)$ ..... (3)

Suppose the general vector (a, b) is written as a linear combination of this basis.

i.e., $(a,b)=p(1,1)+q(1,-1)$

$\Rightarrow a=p+q$

$b=p-q$

On solving, we get $p=\dfrac{a+b}{2}$ and $q=\dfrac{a-b}{2}$

So, $(a,b)=\left(\dfrac{a+b}{2}\right)(1,1)+\left(\dfrac{b-a}{2}\right)(1,-1)$ ..... (4)

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Step 4 of 6

We write the images (2) and (3) as in (4).

i.e., $(1,1)=\left(\dfrac{1+1}{2}\right)(1,1)+\left(\dfrac{1-1}{2}\right)(1,-1)$

$=1(1,1)+0(1,-1)$

$(-1,1)=\left(\dfrac{-1+1}{2}\right)(1,1)+\left(\dfrac{1+1}{2}\right)(1,-1)$

$=0(1,1)+1(1,-1)$

We collect the coefficients of these linear combinations as the columns of a matrix.

i.e., $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ is the matrix suitable to the given transformation and the second basis.

..... (5)

For, $\begin{pmatrix} 0 & 1 \end{pmatrix}$ is the matrix suitable to the given transformation and the second such.

$$\ldots\ldots (5)$$

Provide feedback (1)

Step 5 of 6

We now show that (1) and (5) are the similar matrices.

For, we find a matrix M such that $(1) = M^{-1}(5)M$

Suppose $M = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$, then $M^{-1} = \dfrac{1}{xw - yz}\begin{pmatrix} w & -y \\ -z & x \end{pmatrix}$ and

$$\frac{1}{xw - yz}\begin{pmatrix} w & -y \\ -z & x \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \frac{1}{xw - yz}\begin{pmatrix} -y & w \\ x & -z \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \frac{1}{xw - yz}\begin{pmatrix} wz - xy & w^2 - y^2 \\ x^2 - z^2 & xy - wz \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Step 6 of 6

Using $xw - yz \neq 0$, $\dfrac{wz - xy}{xw - yz} = \dfrac{xy - wz}{xw - yz} = 1$, $w^2 - y^2 = 0, x^2 - z^2 = 0$ and $w = \pm y, x = \pm z$

We have $w = y, x = -z$, we get $M = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ such that $M^{-1}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Therefore, (1) and (5) are similar matrices.

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Step 1 of 4

Identity Transformation: When the outputs of linear transformation $T(v_j) = v_j$ are combinations of $\sum_{i=1}^{n} m_{ij} w_i$ then the change of basis matrix is M. Matrix M represents the identity transformation.

Provide feedback (0)

Step 2 of 4

Consider an identity transformation that takes each vector to itself.

$$Tx = x$$

Let the first basis is:

$$v_1 = (1,2)$$
$$v_2 = (3,4)$$

Second basis is:

$$w_1 = (1,0)$$
$$w_2 = (0,1)$$

This basis does not represent an identity matrix. Find the corresponding matrix that represents the identity transformation.

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Step 3 of 4

The input basis are $v_1 = (1,2)$ and $v_2 = (3,4)$. In the identity transformation first input is the output itself. So, output can be expressed as $1w_1 + 2w_2$. Hence, the first column of the matrix M contains 1 and 2.

CH5.6 13P

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Step 3 of 4

The input basis are $v_1 = (1,2)$ and $v_2 = (3,4)$. In the identity transformation first input is the output itself. So, output can be expressed as $1w_1 + 2w_2$. Hence, the first column of the matrix M contains 1 and 2.

Similarly, second column will contain 3 and 4.

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Step 4 of 4

Therefore the matrix M is :

$$\boxed{M = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}}$$

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Step 1 of 4

The derivative of $a+bx+cx^2$ is $b+2cx+0x^2$

a) $$D\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ 2c \\ 0 \end{bmatrix}$$

The 3 by 3 matrix $D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ such that $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ 2c \\ 0 \end{bmatrix}$

So, the required matrix is $D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

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CH5.6 14P

Step 2 of 4

c) $$D^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D^3 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

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$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D^3 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Provide feedback (0)

Step 3 of 4

This means D stands for the derivative and the third derivative of a second degree polynomial is zero.

The first derivative $= b + 2cx$

The second derivative $= 2c$

The derivative $= 0$

The third derivative of $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$

Provide feedback (0)

Step 4 of 4

c) The characteristic equation of $D = |D - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 2 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$= -\lambda(\lambda^2) - 1(0) + 0 = 0$$

$$\Rightarrow \lambda^3 = 0$$

$\Rightarrow \lambda = 0, 0, 0$ are the eigen values of the derivative matrix

Provide feedback (0)

Step 4 of 4

c) The characteristic equation of $D = |D - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 2 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$= -\lambda(\lambda^2) - 1(0) + 0 = 0$$

$$\Rightarrow \lambda^3 = 0$$

$\Rightarrow \lambda = 0, 0, 0$ are the eigen values of the derivative matrix.

To find eigen vector corresponding to $\lambda = 0$, we solve the homogeneous system $(D - 0I)x = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_2 = 0, x_3 = 0$

We further, observe that x_1 is satisfied by any real number $x_1 = k$ and not necessarily zero.

So, the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$

Putting $k = 1$, we get the eigen vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ corresponding to $\lambda = 0$.

Observe that the eigen values are repeated 3 times and the corresponding eigen vector is only one.

Provide feedback (0)

CH5.6 13P

CH5.6 15P

Step 1 of 11

First we have to show that every number is an Eigen value for $Tf(x) = df/dx$.

Suppose $f(x) = e^{ax}$ for a real number *a*, so by the above definition we have:

$$\begin{aligned} Tf(x) &= \frac{df}{dx} \\ &= \frac{d}{dx}\left(e^{ax}\right) \\ &= ae^{ax} \\ &= af(x) \end{aligned}$$

Therefore, $\boxed{\text{every real number } a \text{ is an eigenvalue for } Tf(x) = df/dx}$.

Provide feedback (0)

Step 2 of 11

Now, we have to show that the transformation $Tf(x) = \int_0^x f(t)dt$ has no eigenvalues, here $-\infty < x < \infty$.

Provide feedback (0)

Step 3 of 11

Let us take $Tf(x) = af(x)$, for some real number *a* and function *f*.

Provide feedback (0)

Provide feedback (0)

Step 3 of 11

Let us take $Tf(x) = af(x)$, for some real number *a* and function *f*.

Provide feedback (0)

Step 4 of 11

We know that $Tf(x) = \int_0^x f(t)dt$, so we have

Provide feedback (0)

Step 5 of 11

$$\begin{aligned}\int_0^x f(t)dt &= Tf(x)\\ &= af(x)\end{aligned}$$

Provide feedback (0)

Step 6 of 11

Or we can write $\int_0^x f(t)dt = af(x)$.

Provide feedback (0)

Step 7 of 11

Provide feedback (0)

Step 5 of 11

$$\int_0^x f(t)dt = Tf(x)$$
$$= af(x)$$

Provide feedback (0)

Step 6 of 11

Or we can write $\int_0^x f(t)dt = af(x)$.

Provide feedback (0)

Step 7 of 11

Differentiate both the sides of equation $\int_0^x f(t)dt = af(x)$.

$$f(x) = af'(x)$$

Rewrite the above equation as

$$\frac{f'(x)}{f(x)} = \frac{1}{a}$$

Step 7 of 11

Differentiate both the sides of equation $\int_0^x f(t)dt = af(x)$.

$$f(x) = af'(x)$$

Rewrite the above equation as

$$\frac{f'(x)}{f(x)} = \frac{1}{a}$$

To solve for *f*, first integrate both the sides of equation:

$$\int \frac{f'(x)}{f(x)}dx = \int \frac{1}{a}dx$$

$$\ln|f(x)| = \frac{1}{a}(x) + C$$

$$\ln|f(x)| = \frac{x}{a} + C$$

Provide feedback (0)

Step 8 of 11

Take exponential of both sides of equation $\ln|f(x)| = \frac{x}{a} + C$.

Provide feedback (0)

Step 8 of 11

Take exponential of both sides of equation $\ln|f(x)| = \frac{x}{a} + C$.

$$e^{\ln|f(x)|} = e^{\frac{x}{a}+C}$$

$$|f(x)| = e^{\frac{x}{a}+C}$$

$$|f(x)| = e^{C}e^{\frac{x}{a}}$$

Provide feedback (0)

Step 9 of 11

Substitute *A* for e^{C} (here *A* may be a negative).

Provide feedback (0)

Step 10 of 11

$$f(x) = Ae^{\frac{x}{a}}$$

Provide feedback (0)

Step 11 of 11

Step 10 of 11

$$f(x) = Ae^{\frac{x}{a}}$$

Provide feedback (0)

Step 11 of 11

Use the function $f(x) = Ae^{\frac{x}{a}}$ and definition of *T*, $Tf(x) = \int_0^x f(t)dt$, so

$$Tf(x) = \int_0^x Ae^{\frac{t}{a}} dt$$

By integrating, we have

$$\begin{aligned} Tf(x) &= A\int_0^x e^{\frac{a}{t}} dt \\ &= \left[Aae^{\frac{t}{a}} \right]_0^x \\ &= Aae^{\frac{x}{a}} - Aa \\ &= a\left(Ae^{\frac{x}{a}} - A \right) \end{aligned}$$

Substitute $f(x)$ for $Ae^{\frac{x}{a}}$, so

$$= \left[Aae^{\frac{t}{a}} \right]_0^x$$

$$= Aae^{\frac{x}{a}} - Aa$$

$$= a\left(Ae^{\frac{x}{a}} - A \right)$$

Substitute $f(x)$ for $Ae^{\frac{x}{a}}$, so

$$Tf(x) = a\left(Ae^{\frac{x}{a}} - A \right)$$

$$= a\left(f(x) - A \right)$$

We know that $Tf(x) = af(x)$, so we have

$$af(x) = a\left(f(x) - A \right)$$

$$= af(x) - aA$$

This implies that either $a = 0$ or $A = 0$. So we get $f(x) = 0$ in either conditions,

Therefore, we have concluded that transformation $Tf(x) = \int_0^x f(t)dt$ has $\boxed{\text{no eigenvalues}}$.

Provide feedback (0)

CH5.6 14P

Step 1 of 4

We need to obtain the eigenvalues and eigenmatrices.

Let us suppose $A^{\mathrm{T}} = \lambda A$.

Thus, we want $\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \lambda \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Provide feedback (0)

Step 2 of 4

Therefore, we get the following four equations:

$$\begin{aligned} a &= \lambda a \\ c &= \lambda b \\ b &= \lambda c \\ d &= \lambda d \end{aligned}$$

Provide feedback (0)

Step 3 of 4

Therefore, the eigenvalue λ can be either 1 or –1.

When $\lambda = 1$, we get $b = c$. When $\lambda = -1$, $b = -c$.

$\begin{bmatrix} 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \end{bmatrix}$

CH5.6 16P

Step 3 of 4

Therefore, the eigenvalue λ can be either 1 or –1.

When $\lambda = 1$, we get $b = c$. When $\lambda = -1$, $b = -c$.

Thus, we get the eigenmatrices as follows: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Provide feedback (0)

Step 4 of 4

Thus, the four eigenvalues are 1, 1, 1, and –1 and corresponding to these, the four eigenmatrices are $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

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Step 1 of 9

Given that $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}, \Lambda=\begin{bmatrix}0&0&0\\0&0&0\\0&0&3\end{bmatrix}$

The characteristic equation of A is $|A-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}1-\lambda&1&1\\1&1-\lambda&1\\1&1&1-\lambda\end{vmatrix}=0$$

$$\Rightarrow(1-\lambda)\left((1-\lambda)^2-1\right)-1(1-\lambda-1)+1(1-1+\lambda)=0$$

$$\Rightarrow(1-\lambda)\left(1+\lambda^2-2\lambda-1\right)+\lambda+\lambda=0$$

$$\Rightarrow\lambda^2-2\lambda-\lambda^3+2\lambda^2+2\lambda=0$$

$$\Rightarrow\lambda^3-3\lambda^2=0$$

$$\Rightarrow\lambda^2(\lambda-3)=0$$

$$\Rightarrow\lambda=0,0,3$$

There fore $\Lambda=\begin{bmatrix}0&0&0\\0&0&0\\0&0&3\end{bmatrix}$

Provide feedback (1)

Step 2 of 9

To find eigen vector corresponding to $\lambda=0$, let us consider $(A-\lambda I)x=0$

$$\Rightarrow\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Applying the row operation $R_2\to R_2-R_1, R_3\to R_3-R_1$ upon the coefficient matrix, it reduces to $\begin{bmatrix}1&1&1\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

Step 2 of 9

To find eigen vector corresponding to $\lambda = 0$, let us consider $(A - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying the row operation $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ upon the coefficient matrix, it reduces to $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

The homogeneous from this is $x_1 + x_2 + x_3 = 0$

$\Rightarrow x_1 = -x_2 - x_3$

Using $x_2 = k, x_3 = t$ parameters, the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Putting $k = 1, t = 1$, we get $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are the linearly independent solutions which are the eigen vectors corresponding to $\lambda = 0$.

Provide feedback (0)

Step 3 of 9

We make an orthogonal set using these vectors.

Suppose $u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

We consider $u_2 = v_1 - \frac{v_1^T u_1}{u_1^T u_1} u_1$

$\begin{bmatrix} -1 \end{bmatrix}$

Putting $k = 1, t = 1$, we get $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are the linearly independent solutions which are the eigen vectors corresponding to $\lambda = 0$.

Provide feedback (0)

Step 3 of 9

We make an orthogonal set using these vectors.

Suppose $u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

We consider $u_2 = v_1 - \frac{v_1^T u_1}{u_1^T u_1} u_1$

$$= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \frac{[-1,0,1]\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}{[-1,1,0]\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 9

The eigen vector corresponding to $\lambda = 3$ is obtained by solving $(A - \lambda I)x = 0$

$$\begin{bmatrix} 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 9

The eigen vector corresponding to $\lambda = 3$ is obtained by solving $(A - \lambda I)x = 0$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using $R_2 \to 2R_2 + R_1, R_3 \to 2R_3 + R_1$ on the coefficient matrix, we get

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_2 + R_3, R_2 / -3 \Rightarrow$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, the homogeneous equations from this are

$x_2 - x_3 = 0, -2x_1 + x_2 + x_3 = 0$

$\Rightarrow x_2 = x_3, x_1 = x_3$

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Step 5 of 9

Using $x_3 = m$ a parameter, when $m = 1$, the solution $u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 3$.

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Provide feedback (0)

Step 6 of 9

We consider u_1, u_2, u_3 as the columns of Q such that the pairs of these eigen vectors are orthogonal.

So, the required orthogonal matrix is $Q = \begin{bmatrix} -1 & -1/2 & 1 \\ 1 & -1/2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, and is invertible.

So, $Q^{-1} = \frac{1}{6}\begin{bmatrix} -3 & 3 & 0 \\ -2 & -2 & 4 \\ 2 & 2 & 2 \end{bmatrix}$ such that

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1/2 & 1 \\ 1 & -1/2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}\frac{1}{6}\begin{bmatrix} -3 & 3 & 0 \\ -2 & -2 & 4 \\ 2 & 2 & 2 \end{bmatrix}$$

In other words, $A = Q\Lambda Q^{-1}$

This can also be written as $\Lambda = Q^{-1}AQ$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -3 & 3 & 0 \\ -2 & -2 & 4 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1/2 & 1 \\ 1 & -1/2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

(b) One pair of orthogonal eigen vectors corresponding to $\lambda = 0$ are

$$u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix}$$

So, the ortho normal vectors along these are $X_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, X_2 = \sqrt{\frac{2}{3}}\begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix}$

So, $P = X_1 X_1^T + X_2 X_2^T$

$$\begin{bmatrix}0&0&0\\0&0&0\\0&0&3\end{bmatrix}=\frac{1}{6}\begin{bmatrix}-3&3&0\\-2&-2&4\\2&2&2\end{bmatrix}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}-1&-1/2&1\\1&-1/2&1\\0&1&1\end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

(b) One pair of orthogonal eigen vectors corresponding to $\lambda = 0$ are

$$u_1=\begin{bmatrix}-1\\1\\0\end{bmatrix}, u_2=\begin{bmatrix}-1/2\\-1/2\\1\end{bmatrix}$$

So, the ortho normal vectors along these are $X_1=\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\\0\end{bmatrix}, X_2=\sqrt{\frac{2}{3}}\begin{bmatrix}-1/2\\-1/2\\1\end{bmatrix}$

So, $P = X_1X_1^T + X_2X_2^T$

$$=\frac{1}{2}\begin{bmatrix}-1\\1\\0\end{bmatrix}\begin{bmatrix}-1&1&0\end{bmatrix}+\frac{2}{3}\begin{bmatrix}-1/2\\-1/2\\1\end{bmatrix}\begin{bmatrix}-1/2&-1/2&1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1&-1&0\\-1&1&0\\0&0&0\end{bmatrix}+\frac{2}{3}\begin{bmatrix}1/4&1/4&-1/2\\1/4&1/4&-1/2\\-1/2&-1/2&1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1&-1&0\\-1&1&0\\0&0&0\end{bmatrix}+\frac{1}{3}\begin{bmatrix}1/2&1/2&-1\\1/2&1/2&-1\\-1&-1&2\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}3&-3&0\\-3&3&0\\0&0&0\end{bmatrix}+\frac{1}{6}\begin{bmatrix}1&1&-2\\1&1&-2\\-2&-2&4\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix} \qquad \ldots\ldots (1)$$

Step 8 of 9

On the other hand, we consider the eigen vectors corresponding to $\lambda = 0$

$u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and find the new pair of ortho normal vectors along them.

For, $X_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$,

$$v_2 = u_1 - \frac{v_1^T u_2}{u_2^T u_2} u_2$$

$$= \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} - \frac{[-1\ 1\ 0] \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}{[-1\ 0\ 1] \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1 \\ -1/2 \end{bmatrix}$$

So, $X_2 = \frac{1}{\sqrt{\frac{1}{4} + 1 + \frac{1}{4}}} \begin{bmatrix} -1/2 \\ 1 \\ -1/2 \end{bmatrix}$

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Step 9 of 9

$\sqrt{2} \begin{bmatrix} -1/2 \end{bmatrix}$

$$\sqrt{\frac{1}{4}+1+\frac{1}{4}}\left[-1/2\right]$$

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Step 9 of 9

$$=\sqrt{\frac{2}{3}}\begin{bmatrix}-1/2\\1\\-1/2\end{bmatrix}$$

$$P=X_1X_1^T+X_2X_2^T$$

$$=\frac{1}{2}\begin{bmatrix}-1\\0\\1\end{bmatrix}\begin{bmatrix}-1 & 0 & 1\end{bmatrix}+\frac{2}{3}\begin{bmatrix}-1/2\\1\\-1/2\end{bmatrix}\begin{bmatrix}-1/2 & 1 & -1/2\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1 & 0 & -1\\0 & 0 & 0\\-1 & 0 & 1\end{bmatrix}+\frac{2}{3}\begin{bmatrix}1/4 & -1/2 & 1/4\\-1/2 & 1 & -1/2\\1/4 & -1/2 & 1/4\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}3 & 0 & -3\\0 & 0 & 0\\-3 & 0 & 3\end{bmatrix}+\frac{1}{6}\begin{bmatrix}1 & -2 & 1\\-2 & 4 & -2\\1 & -2 & 1\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}2 & -1 & -1\\-1 & 2 & -1\\-1 & -1 & 2\end{bmatrix} \qquad \ldots\ldots (2)$$

We easily see that (1) and (2) are same as desired.

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CH5.6 16P

Step 1 of 3

A complex matrix with ortho normal columns is called a unitary matrix.
We have the following properties of a unitary matrix U:
The conjugate transpose of the matrix U is denoted by U^H and called U Hermitian.
More precisely, if every entry in U is replaced by its complex conjugate and then the matrix is transposed, we get U^H.

1. $U^H U = I = UU^H, U^H = U^{-1}$
2. Every eigen value of U has absolute value $|\lambda| = 1$
3. The eigen vectors corresponding to different eigen values are orthogonal.

Putting these properties together, we get that any two eigen vectors of U are ortho normal.

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Step 2 of 3

(i) Given that A, U are the unitary matrices.
So, we have $AA^H = I = A^H A, A^{-1} = A^H$ and $UU^H = I = U^H U, U^{-1} = U^H$
Let us consider $T = U^{-1}AU$ and

$$\begin{aligned}
TT^H &= \left(U^{-1}AU\right)\left(U^{-1}AU\right)^H \\
&= \left(U^{-1}AU\right)U^H A^H \left(U^{-1}\right)^H \\
&= \left(U^{-1}A\right)\left(UU^{-1}\right)\left(A^H\left(U^H\right)^H\right) \qquad \because U^{-1} = U^H \\
&= U^{-1}A(I)A^H U \\
&= U^H\left(AA^H\right)U \\
&= U^H(I)U \\
&= U^H U \\
&= I
\end{aligned}$$

So, $TT^H = I$ which shows T is a unitary matrix.

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CH5.6 18P

$$= U^H(I)U$$
$$= U^H U$$
$$= I$$

So, $TT^H = I$ which shows T is a unitary matrix.

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Step 3 of 3

(ii) Suppose T is an upper triangular and unitary matrix.
Then we follow that $TT^H = I$
But when T is the upper triangular matrix, we follow that T^H is the lower triangular matrix with the conjugate entries of T.
The product of upper and lower triangular matrices lead to the identity matrix which is diagonal.
So, the only possibility is T is a diagonal matrix.
i.e., $T = \Lambda$

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CH5.6 17P

CH5.6 19P

Step 1 of 5

Consider the following matrix:

$$N = \begin{bmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{bmatrix}$$

Then

$$N^{H} = \begin{bmatrix} 0 & b & a \\ a & 0 & b \\ b & a & 0 \end{bmatrix}$$

Here, *a* and *b* are any complex numbers. Note that *N* is neither of Hermitian, Skew Hermitian, Unitary, or Diagonal.

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Step 2 of 5

Consider the following:

$$\begin{aligned} N^{H}N &= \begin{bmatrix} 0 & b & a \\ a & 0 & b \\ b & a & 0 \end{bmatrix}\begin{bmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{bmatrix} \\ &= \begin{bmatrix} b^2+a^2 & ab & ba \\ ba & a^2+b^2 & ab \\ ab & ba & b^2+a^2 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} a & b & 0 \end{bmatrix}\begin{bmatrix} b & a & 0 \end{bmatrix}$$
$$= \begin{bmatrix} a^2+b^2 & ba & ab \\ ab & b^2+a^2 & ba \\ ba & ab & a^2+b^2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 5

From the properties of complex numbers, we know that the addition and multiplication of complex numbers are commutative. That is, $z_1 + z_2 = z_2 + z_1$ and $z_1 z_2 = z_2 z_1$, for any $z_1, z_2 \in \square$.

Therefore, $N^H N = NN^H$. Choose $a \neq b$ and we have got the required matrix.

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Step 4 of 5

Every permutation matrix is an orthogonal matrix. Therefore, the transpose of a projection matrix is its inverse.

Therefore, if *P* is a permutation matrix, then $P^{-1} = P^T$. Since, the entries in a permutation matrix are real numbers, we can say that $P^T = P^H$.

Thus, we have $P^{-1} = P^H$.

Provide feedback (0)

Step 5 of 5

Therefore, $N^H N = NN^H$. Choose $a \neq b$ and we have got the required matrix.

Provide feedback (0)

Step 4 of 5

Every permutation matrix is an orthogonal matrix. Therefore, the transpose of a projection matrix is its inverse.

Therefore, if *P* is a permutation matrix, then $P^{-1} = P^T$. Since, the entries in a permutation matrix are real numbers, we can say that $P^T = P^H$.

Thus, we have $P^{-1} = P^H$.

Provide feedback (0)

Step 5 of 5

For any invertible matrix A, we know that $AA^{-1} = A^{-1}A = I$.

Thus, if *P* is any permutation matrix, we get $PP^H = P^H P = I$.

Thus, we have shown that every permutation matrix is normal.

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CH5.6 18P

CH5.6 20P

Step 1 of 3

T is a 3 by 3 upper triangular matrix.
The entries of T are t_{ij}.

Therefore $T = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix}$

Then $T^H = \begin{bmatrix} \bar{t}_{11} & 0 & 0 \\ \bar{t}_{12} & \bar{t}_{22} & 0 \\ \bar{t}_{13} & \bar{t}_{23} & \bar{t}_{33} \end{bmatrix}$

$$TT^H = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix} \begin{bmatrix} \bar{t}_{11} & 0 & 0 \\ \bar{t}_{12} & \bar{t}_{22} & 0 \\ \bar{t}_{13} & \bar{t}_{23} & \bar{t}_{33} \end{bmatrix}$$

$$= \begin{bmatrix} |t_{11}|^2 + |t_{12}|^2 + |t_{13}|^2 & t_{11}\bar{t}_{22} + t_{13}\bar{t}_{23} & t_{13}\bar{t}_{33} \\ t_{22}\bar{t}_{12} + t_{33}\bar{t}_{13} & |t_{22}|^2 + t_{33}\bar{t}_{23} & t_{23}\bar{t}_{33} \\ t_{33}\bar{t}_{13} & t_{33}\bar{t}_{23} & |t_{33}|^2 \end{bmatrix} \qquad \ldots\ldots (1)$$

Also, $T^H T = \begin{bmatrix} \bar{t}_{11} & 0 & 0 \\ \bar{t}_{12} & \bar{t}_{22} & 0 \\ \bar{t}_{13} & \bar{t}_{23} & \bar{t}_{33} \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix}$

$$= \begin{bmatrix} |t_{11}|^2 & \bar{t}_{11}t_{12} & \bar{t}_{11}t_{13} \\ \bar{t}_{12}t_{11} & |t_{12}|^2 + |t_{22}|^2 & \bar{t}_{12}t_{13} + \bar{t}_{22}t_{23} \\ \bar{t}_{13}t_{11} & \bar{t}_{13}t_{12} + \bar{t}_{23}t_{22} & |t_{13}|^2 + |t_{23}|^2 + |t_{33}|^2 \end{bmatrix} \qquad \ldots\ldots (2)$$

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Step 2 of 3

Comparing the ij^{th} entries of (1) and (2) for every i and j, we get
$|t_{11}|^2 + |t_{12}|^2 + |t_{13}|^2 = |t_{11}|^2$

$$= \begin{bmatrix} |t_{11}|^2 & \bar{t}_{11}t_{12} & \bar{t}_{11}t_{13} \\ \bar{t}_{12}t_{11} & |t_{12}|^2 + |t_{22}|^2 & \bar{t}_{12}t_{13} + \bar{t}_{22}t_{23} \\ \bar{t}_{13}t_{11} & \bar{t}_{13}t_{12} + \bar{t}_{23}t_{22} & |t_{13}|^2 + |t_{23}|^2 + |t_{33}|^2 \end{bmatrix} \quad \ldots\ldots (2)$$

Provide feedback (0)

Step 2 of 3

Comparing the ij^{th} entries of (1) and (2) for every i and j, we get

$|t_{11}|^2 + |t_{12}|^2 + |t_{13}|^2 = |t_{11}|^2$

$\Rightarrow t_{12} = t_{13} = 0 \quad \ldots\ldots (3)$

$|t_{22}|^2 + t_{33}\bar{t}_{23} = |t_{12}|^2 + |t_{22}|^2$

From this, we get $t_{33}\bar{t}_{23} = |t_{12}|^2$

(3) $\Rightarrow t_{33}\bar{t}_{23} = 0 \quad \ldots\ldots (4)$

$|t_{33}|^2 = |t_{13}|^2 + |t_{23}|^2 + |t_{33}|^2$

$\Rightarrow t_{13} = t_{23} = 0 \quad \ldots\ldots (5)$

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Step 3 of 3

In view of (3),(4), and (5), we can see that t_{11}, t_{22}, t_{33} entries can take either real or complex values and $t_{12} = t_{13} = t_{23} = 0$ to satisfy the equality of the products (1) and (2).

Thus, $T = \begin{bmatrix} t_{11} & 0 & 0 \\ 0 & t_{22} & 0 \\ 0 & 0 & t_{33} \end{bmatrix}$ which is a diagonal matrix.

Further, such a matrix T is called the normal triangular matrix.
Therefore, every normal triangular matrix T is diagonal.

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CH5.6 19P

Step 1 of 4

We know that a matrix *N* is normal if $NN^H = N^H N$.

Consider *N* is normal matrix, then we have to show that $\|Nx\| = \|N^H x\|$ for every vector *x*.

If *x* is the vector then we know that norm is given by

$$(\|x\|)^2 = \langle x, x \rangle$$

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Step 2 of 4

If we consider the vector $(\|Nx\|)^2$ then we have

$$(\|Nx\|)^2 = \langle Nx, Nx \rangle$$

Multiply the vectors by N^H, we get

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H Nx, N^H Nx \rangle \\ &= \langle N^H Nx, x \rangle\end{aligned}$$

Since $NN^H = N^H N$, so we have

CH5.6 21P

$(\|Nx\|) = \langle Nx, Nx \rangle$

Multiply the vectors by N^H, we get

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H Nx, N^H Nx \rangle \\ &= \langle N^H Nx, x \rangle\end{aligned}$$

Since $NN^H = N^H N$, so we have

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H Nx, x \rangle \\ &= \langle NN^H x, x \rangle\end{aligned}$$

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Step 3 of 4

Now multiply the vectors by N^H, we get

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H NN^H x, N^H x \rangle \\ &= \langle N^H x, N^H x \rangle \\ &= (\|N^H x\|)^2\end{aligned}$$

Therefore, $(\|Nx\|)^2 = (\|N^H x\|)^2$.

Provide feedback (0)

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H N N^H x, N^H x\rangle \\ &= \langle N^H x, N^H x\rangle \\ &= (\|N^H x\|)^2\end{aligned}$$

Therefore, $(\|Nx\|)^2 = (\|N^H x\|)^2$.

Provide feedback (0)

Step 4 of 4

Let the ith column of N be Ne_i , here $e_1,\ldots,e_n$ are the elements of a canonical basis.

Let the ith row of N be $e_i^T N = (N^H e_i)^H$.

We know that the length of a matrix and its transpose are same so we have:

$$\begin{aligned}\|e_i^T N\| &= \|(N^H e_i)^H\| \\ &= \|(N^H e_i)\| \\ &= \|Ne_i\|\end{aligned}$$

Therefore, the length of ith row of N is same as the ith column, $\|e_i^T N\| = \|Ne_i\|$.

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CH5.6 20P

CH5.6 22P

Step 1 of 2

A matrix N is normal if it commutes with N^H.
i.e., if $NN^H = N^H N$, then N is normal.

Suppose N is a matrix with ortho normal eigen vectors.
So, we have $U^{-1}NU = \Lambda$
Or, $N = U\Lambda U^{-1}$
To show N commutes with itself, let us consider $NN^H = \left(U\Lambda U^{-1}\right)\left(U\Lambda U^{-1}\right)^H$

$$\begin{aligned} &= \left(U\Lambda U^{-1}\right)\left(\left(U^{-1}\right)^H \Lambda^H U^H\right) \\ &= U\Lambda\left(\left(U^{-1}\right)\left(U^{-1}\right)^H\right)\Lambda^H U^H \qquad \ldots\ldots (1) \end{aligned}$$

Since U is the unitary matrix, we have $U^H = U^{-1}$ and so,

$$\begin{aligned} \left(U^{-1}\right)\left(U^{-1}\right)^H &= U^{-1}U = I \qquad \ldots\ldots (2) \\ &= UU^{-1} \\ &= \left(U^{-1}\right)^H\left(U^{-1}\right) \qquad \ldots\ldots (3) \end{aligned}$$

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Step 2 of 2

Using (2) in (1), we get $NN^H = U\left(\Lambda\Lambda^H\right)U^H$
Since Λ is a diagonal matrix, we follow that $\Lambda\Lambda^H = \Lambda^H\Lambda$
So, $NN^H = U\left(\Lambda^H I\Lambda\right)U^H$

$$\begin{aligned} &= U\Lambda^H\left(U^{-1}U\right)\Lambda U^H \\ &= \left(\left(U^H\right)^H\Lambda^H U^{-1}\right)\left(U\Lambda U^H\right) \\ &= \left(\left(U^H\right)^H\Lambda^H U^H\right)\left(U\Lambda U^H\right) \\ &= \left(U\Lambda U^H\right)^H\left(U\Lambda U^H\right) \\ &= N^H N \end{aligned}$$

Therefore, N is a normal matrix.

Provide feedback (0)

Step 2 of 2

Using (2) in (1), we get $NN^H = U\left(\Lambda\Lambda^H\right)U^H$

Since Λ is a diagonal matrix, we follow that $\Lambda\Lambda^H = \Lambda^H\Lambda$

So, $NN^H = U\left(\Lambda^H I\Lambda\right)U^H$

$$\begin{aligned}
&= U\Lambda^H\left(U^{-1}U\right)\Lambda U^H \\
&= \left(\left(U^H\right)^H\Lambda^H U^{-1}\right)\left(U\Lambda U^H\right) \\
&= \left(\left(U^H\right)^H\Lambda^H U^H\right)\left(U\Lambda U^H\right) \\
&= \left(U\Lambda U^H\right)^H\left(U\Lambda U^H\right) \\
&= N^H N
\end{aligned}$$

Therefore, N is a normal matrix.

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Step 1 of 14

Consider following matrix:

$$A=\begin{bmatrix}5 & -3\\ 4 & -2\end{bmatrix}$$

We know that the unitary matrix *U* and triangular matrix *T*, such that $U^{-1}AU=T$.

The eigenvalues of matrix *A*, should appear along the diagonal of similar matrix *T*.

First step is to find the Eigen values and Eigen vectors of matrix *A*. To calculate the Eigen values do the following calculations;

$$\det(A-\lambda I)=0$$
$$\det\begin{bmatrix}5 & -3\\ 4 & -2\end{bmatrix}=0$$
$$\lambda^2-3\lambda+2=0$$
$$(\lambda-1)(\lambda-2)=0$$

After solving following values are obtained:

$$\lambda_1=1$$
$$\lambda_2=2$$

Provide feedback (0)

Provide feedback (0)

Step 2 of 14

To calculate eigen vectors x_1 do the following calculations:

$$(A-\lambda_1 I)x_1 = 0$$

$$\begin{bmatrix} 5-1 & -3 \\ 4 & -2-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of *x* and *y* corresponding to $\lambda_1 = 1$ are as follows:

$$x_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 14

To calculate eigen vectors x_2 do the following calculations:

$$(A-\lambda_2 I)x_2 = 0$$

$$\begin{bmatrix} 5-2 & -3 \\ 4 & -2-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -3 \end{bmatrix}\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$(A-\lambda_2 I)x_2 = 0$$

$$\begin{bmatrix} 5-2 & -3 \\ 4 & -2-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -3 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of *x* and *y* corresponding to $\lambda_2 = 2$ are as follows:

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 14

The length of eigenvectors $x_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is

$$\begin{aligned} \|x_1\| &= \sqrt{3^2+4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Next, divide eigenvector by its length we get:

$$\frac{x_1}{\|x_1\|} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$$

Step 4 of 14

The length of eigenvectors $x_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is

$$\begin{aligned} \|x_1\| &= \sqrt{3^2 + 4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Next, divide eigenvector by its length we get:

$$\frac{x_1}{\|x_1\|} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 14

The first column of U is $\begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$, we need the columns of U should be orthonormal.

Therefore, by using these normalized vectors as the columns, U is $\boxed{\begin{bmatrix} 3/5 & -4/5 \\ 4/5 & -3/5 \end{bmatrix}}$.

Provide feedback (1)

Step 6 of 14

$\begin{bmatrix} 3/5 & 4/5 \end{bmatrix}$

$$\begin{aligned}\|x_1\| &= \sqrt{3^2+4^2}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

Next, divide eigenvector by its length we get:

$$\frac{x_1}{\|x_1\|} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 14

The first column of U is $\begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$, we need the columns of U should be orthonormal.

Therefore, by using these normalized vectors as the columns, U is $\boxed{\begin{bmatrix} 3/5 & -4/5 \\ 4/5 & -3/5 \end{bmatrix}}$.

Provide feedback (1)

Step 6 of 14

Since $U = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix}$, so we have

$$U^{-1} = U^T = \begin{bmatrix} 3/5 & 4/5 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 4/5 & -3/5 \end{bmatrix}}$$

Provide feedback (1)

Step 6 of 14

Since $U = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix}$, so we have

$$U^{-1} = U^T = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix}.$$

Using $U^{-1}AU = T$, we will find *T*.

$$\begin{aligned} U^{-1}AU &= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \\ &= \begin{bmatrix} 31/5 & -17/5 \\ 8/5 & -6/5 \end{bmatrix}\begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 7 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

Therefore, the lower triangular matrix is $T = \boxed{\begin{bmatrix} 1 & 7 \\ 0 & 2 \end{bmatrix}}$.

Provide feedback (3)

Step 7 of 14

Step 7 of 14

The length of eigenvectors $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is

$$\begin{aligned} \|x_2\| &= \sqrt{1^2 + 1^2} \\ &= \sqrt{2} \end{aligned}$$

Next, divide eigenvector by its length we get:

$$\frac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 8 of 14

The first column of U is $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, we need the columns of U should be orthonormal.

Therefore, by using these normalized vectors as the columns, U is $\boxed{\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}}$.

Provide feedback (0)

Step 9 of 14

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

Provide feedback (0)

Step 9 of 14

Since $U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$, so we have

$$U^{-1} = U^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}.$$

Using $U^{-1}AU = T$, we will find T.

$$\begin{aligned} U^{-1}AU &= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \\ &= \begin{bmatrix} 9/\sqrt{2} & -5/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \\ &= \begin{bmatrix} 2 & 7 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore, the lower triangular matrix is $T = \boxed{\begin{bmatrix} 2 & 7 \\ 0 & 1 \end{bmatrix}}$.

Provide feedback (0)

Step 10 of 14

Step 10 of 14

Consider following matrix:

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$$

First step is to find the Eigen values and Eigen vectors of matrix *A*. To calculate the Eigen values do the following calculations;

$$\det\left(A-\lambda I\right)=0$$

$$\det\begin{bmatrix}-\lambda & 1 & 0\\ 0 & -\lambda & 0\\ 1 & 0 & -\lambda\end{bmatrix}=0$$

$$\lambda^3=0$$

After solving following values are obtained:

$$\lambda_1=0$$
$$\lambda_2=0$$
$$\lambda_3=0$$

To calculate Eigen vectors do the following calculations:

$$\left(A-\lambda_1 I\right)x_1=0$$

$$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\lambda_1 = 0$$
$$\lambda_2 = 0$$
$$\lambda_3 = 0$$

To calculate Eigen vectors do the following calculations:

$$(A - \lambda_1 I)x_1 = 0$$
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving, values of *x*, *y* and *z* corresponding to $\lambda_1 = 0$ are as follows:

$$x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly, Eigen vectors corresponding to Eigen value λ_2 and λ_3 are as follows:

$$x_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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Step 11 of 14

The lengths of each eigenvectors are 1.

Next, divide each eigenvector by its length we get:

$$\frac{x_1}{\|x_1\|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{x_2}{\|x_2\|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{x_3}{\|x_3\|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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Step 12 of 14

Step 13 of 14

Since $U = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, so we have

$$U^{-1} = U^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Provide feedback (0)

Step 14 of 14

Using $U^{-1}AU = T$, we will find T.

$$\begin{aligned} U^{-1}AU &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= T \end{aligned}$$

Therefore the lower triangular matrix is $T = \boxed{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$.

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$U^{-1} = U^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Provide feedback (0)

Step 14 of 14

Using $U^{-1}AU = T$, we will find T.

$$\begin{aligned} U^{-1}AU &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= T \end{aligned}$$

Therefore the lower triangular matrix is $T = \boxed{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$.

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CH5.6 22P

Step 1 of 3

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CH5.6 24P

Step 2 of 3

Let us consider $A(A-I)(A-2I)=A^3-3A^2+2A$

In view of (1), we can write this as $A^3-3A^2+2A=Q\Lambda^3Q^{-1}-3Q\Lambda^2Q^{-1}+2Q\Lambda Q^{-1}$

$$=Q\left(\Lambda^3-3\Lambda^2+2\Lambda\right)Q^{-1} \ldots\ldots (2) \text{ where}$$

$$\Lambda=\begin{bmatrix}0&0&0\\0&1&0\\0&0&2\end{bmatrix}$$

So, $\Lambda^2=\begin{bmatrix}0&0&0\\0&1&0\\0&0&4\end{bmatrix}$ and $\Lambda^3=\begin{bmatrix}0&0&0\\0&1&0\\0&0&8\end{bmatrix}$

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Step 3 of 3

Using these in (2) to give

$$A(A-I)(A-2I)=Q\left(\begin{bmatrix}0&0&0\\0&1&0\\0&0&8\end{bmatrix}-3\begin{bmatrix}0&0&0\\0&1&0\\0&0&4\end{bmatrix}+2\begin{bmatrix}0&0&0\\0&1&0\\0&0&2\end{bmatrix}\right)Q^{-1}$$

$$=Q\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}Q^{-1}$$

The diagonal entries of the diagonal matrix are nothing but the eigen values of the matrix on the left hand side.

Therefore, the eigen values of $A(A-I)(A-2I)$ is 0, 0, and 0.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

So, $A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

Provide feedback (0)

Step 3 of 3

Using these in (2) to give

$$A(A-I)(A-2I) = Q\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix} - 3\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} + 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right)Q^{-1}$$

$$= Q\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}Q^{-1}$$

The diagonal entries of the diagonal matrix are nothing but the eigen values of the matrix on the left hand side.

Therefore, the eigen values of $A(A-I)(A-2I)$ is 0, 0, and 0.

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CH5.6 25P

Step 1 of 5

a) Let us consider T is a triangular matrix of order 3 by 3.

$$T=\begin{bmatrix} a & x & y \\ 0 & b & z \\ 0 & 0 & c \end{bmatrix}$$

The characteristic equation $|T-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} a-\lambda & x & y \\ 0 & b-\lambda & z \\ 0 & 0 & c-\lambda \end{vmatrix}=0$$

$$\Rightarrow (a-\lambda)(b-\lambda)(c-\lambda)=0$$

$\Rightarrow \lambda_1=a, \lambda_2=b, \lambda_3=c$ are the eigen values of T.

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Step 2 of 5

We have $T-\lambda_1 I=\begin{bmatrix} 0 & 0 & 0 \\ 0 & b-a & 0 \\ 0 & 0 & c-a \end{bmatrix}$, $T-\lambda_2 I=\begin{bmatrix} a-b & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c-a \end{bmatrix}$ and

$$T-\lambda_3 I=\begin{bmatrix} a-c & 0 & 0 \\ 0 & b-c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Provide feedback (1)

Step 3 of 5

Let us consider $(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & b-a & 0 \\ 0 & 0 & c-a \end{bmatrix}\begin{bmatrix} a-b & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c-b \end{bmatrix}\begin{bmatrix} a-c & 0 & 0 \\ 0 & b-c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 3 of 5

Let us consider $(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & b-a & 0 \\ 0 & 0 & c-a \end{bmatrix} \begin{bmatrix} a-b & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c-b \end{bmatrix} \begin{bmatrix} a-c & 0 & 0 \\ 0 & b-c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (c-a)(c-b) \end{bmatrix} \begin{bmatrix} a-c & 0 & 0 \\ 0 & b-c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Provide feedback (0)

Step 4 of 5

Thus, we have shown that $(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I) = 0 = |T-\lambda I|$

Therefore, the products of the factors directly satisfy the characteristic equation of the triangular matrix T.

Provide feedback (0)

Step 5 of 5

b) Substituting $T = U^{-1}AU$, we get $T^2 = U^{-1}A^2U; T^3 = U^{-1}A^3U$

$$(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I) = 0 =$$
$$= T^3 - (\lambda_1+\lambda_2+\lambda_3)T^2 + (\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)T - \lambda_1\lambda_2\lambda_3 I$$
$$= U^{-1}A^3U - (\lambda_1+\lambda_2+\lambda_3)U^{-1}A^2U + (\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)U^{-1}AU - \lambda_1\lambda_2\lambda_3 I$$
$$= U^{-1}\left(A^3 - (\lambda_1+\lambda_2+\lambda_3)A^2 + (\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)A - \lambda_1\lambda_2\lambda_3 I\right)U$$
$$= U^{-1}(A-\lambda_1 I)(A-\lambda_2 I)(A-\lambda_3 I)U$$

Step 4 of 5

Thus, we have shown that $(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)=0=|T-\lambda I|$

Therefore, the products of the factors directly satisfy the characteristic equation of the triangular matrix T.

Provide feedback (0)

Step 5 of 5

b) Substituting $T=U^{-1}AU$, we get $T^2=U^{-1}A^2U; T^3=U^{-1}A^3U$

$$(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)=0=$$
$$=T^3-(\lambda_1+\lambda_2+\lambda_3)T^2+(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)T-\lambda_1\lambda_2\lambda_3 I$$
$$=U^{-1}A^3U-(\lambda_1+\lambda_2+\lambda_3)U^{-1}A^2U+(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)U^{-1}AU-\lambda_1\lambda_2\lambda_3 I$$
$$=U^{-1}\left(A^3-(\lambda_1+\lambda_2+\lambda_3)A^2+(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)A-\lambda_1\lambda_2\lambda_3 I\right)U$$
$$=U^{-1}(A-\lambda_1 I)(A-\lambda_2 I)(A-\lambda_3 I)U$$

Thus, we have shown that $(A-\lambda_1 I)(A-\lambda_2 I)(A-\lambda_3 I)=0$ while $U\neq 0$

Therefore, A satisfies its characteristic equation.

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Step 1 of 3

Given that $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The characteristic equation is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}=0$$

$$\Rightarrow \lambda^2-(a+d)\lambda+(ad-bc)=0$$

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Step 2 of 3

Cayley – Hamilton theorem states that every square matrix satisfies its characteristic equation.

i.e., when the eigen value λ is replaced by the matrix A in $|A-\lambda I|=0$, it has to get satisfied.

i.e., $A^2-(a+d)A+(ad-bc)I$:

$$A^2=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

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Step 3 of 3

$$A^2-(a+d)A+(ad-bc)I=$$

$$=\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}-(a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}+(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} a^2+bc-a^2-ad+ad-bc & ab+bd-ab-db+0 \\ ac+cd-ac-cd+0 & bc+d^2-ad-d^2+ad-bc \end{bmatrix}$$

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Step 2 of 3

Cayley – Hamilton theorem states that every square matrix satisfies its characteristic equation.

i.e., when the eigen value λ is replaced by the matrix A in $|A-\lambda I|=0$, it has to get satisfied.

i.e., $A^2-(a+d)A+(ad-bc)I$:

$$A^2=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

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Step 3 of 3

$$A^2-(a+d)A+(ad-bc)I=$$

$$=\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}-(a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}+(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} a^2+bc-a^2-ad+ad-bc & ab+bd-ab-db+0 \\ ac+cd-ac-cd+0 & bc+d^2-ad-d^2+ad-bc \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$=0$$

Therefore the matrix A satisfied its characteristic equation.

Thus, Cayley – Hamilton theorem is verified.

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Step 1 of 6

Consider a 4 by 4 matrix in which $a_{ij} = 1$ above the diagonal and $a_{ij} = 0$ elsewhere. Find the Jordan form by finding the Eigen vectors.

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Step 2 of 6

Let the matrix be A defined as follows:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Calculate Eigen values of matrix A:

$$A - \lambda I = \begin{bmatrix} 0-\lambda & 1 & 1 & 1 \\ 0 & 0-\lambda & 1 & 1 \\ 0 & 0 & 0-\lambda & 1 \\ 0 & 0 & 0 & 0-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(-\lambda)(-\lambda^3) = 0$$

$$\lambda^4 = 0$$

On solving above equation Eigen values obtained are $\lambda = (0,0,0,0)$

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Step 3 of 6

Clearly it can be seen that $\lambda = 0$ is the only repeated Eigen value of the matrix A. Eigen vector corresponding to this will be $v_1 = (1,0,0,0)$. Generalised Eigen vectors will be as

On solving above equation Eigen values obtained are $\lambda = (0,0,0,0)$

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Step 3 of 6

Clearly it can be seen that $\lambda = 0$ is the only repeated Eigen value of the matrix A. Eigen vector corresponding to this will be $v_1 = (1,0,0,0)$. Generalised Eigen vectors will be as follows:

$$v_2 = (0,1,0,0)$$
$$v_3 = (0,-1,1,0)$$
$$v_4 = (0,1,-2,1)$$

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Step 4 of 6

Put these vectors in a matrix M. Therefore matrix M and its inverse can be defined as follows:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 6

Recall that Jordan form is given by $M^{-1}AM = J$. Therefore,

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 6

Recall that Jordan form is given by $M^{-1}AM = J$. Therefore,

$$\begin{aligned} M^{-1}AM &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &= J \end{aligned}$$

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Step 6 of 6

As matrix A has only one independent Eigen vector, so Jordan form will have only one block. In Jordan matrix three 1's represents absence of three Eigen vectors. Therefore, Jordan form of matrix A is :

$$= \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= J$$

Provide feedback (0)

Step 6 of 6

As matrix A has only one independent Eigen vector, so Jordan form will have only one block. In Jordan matrix three 1's represents absence of three Eigen vectors. Therefore, Jordan form of matrix A is :

$$\boxed{J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

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Step 1 of 8

Similar matrices: Matrices A and B are similar if $A = M^{-1}BM$ for invertible matrix M. Consider the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Eigen values of A and B matrices are $\lambda = (0,0,0)$.

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Step 2 of 8

One Eigen vector of matrix A is $(1,0,0)$ only. Its Jordan form will be as follows.

$$J_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix A must be similar to J_1.

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Step 3 of 8

Matrix B has additional Eigen vector $(0,1,0)$. Its Jordan form will contain two blocks:

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Matrix A must be similar to J_1.

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Step 3 of 8

Matrix B has additional Eigen vector $(0,1,0)$. Its Jordan form will contain two blocks:

$$J_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix B must be similar to J_2.

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Step 4 of 8

Another three 1 by 1 block Jordan matrix is:

$$J_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The only matrix similar to J_3 is $M^{-1}0M = 0$.

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Step 5 of 8

Show that no two of these Jordan forms are similar.

$$J_1 \neq M^{-1}J_2M$$

Provide feedback (0)

Step 5 of 8

Show that no two of these Jordan forms are similar.

$$J_1 \neq M^{-1}J_2M$$
$$J_1 \neq M^{-1}J_3M$$
$$J_2 \neq M^{-1}J_3M$$

Provide feedback (0)

Step 6 of 8

As mentioned earlier that only matrix similar to J_3 is $M^{-1}0M = 0$. So, J_3 cannot be similar to J_1 and J_2.

$$\boxed{\begin{aligned} J_1 &\neq M^{-1}J_3M \\ J_2 &\neq M^{-1}J_3M \end{aligned}}$$

If matrices J_1 and J_2 are similar than following must be true:

$$\begin{aligned} J_1 &= M^{-1}J_2M \\ MJ_1 &= J_2M \end{aligned}$$

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Step 7 of 8

Consider matrix M containing generalized Eigen vector defined as follows and do the calculations:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 7 of 8

Consider matrix M containing generalized Eigen vector defined as follows and do the calculations:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} MJ_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} J_2M &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

This shows that $MJ_1 \neq J_2M$.

Provide feedback (0)

Step 8 of 8

Therefore, $\boxed{J_1 \neq M^{-1}J_2M}$ matrices J_1 and J_2 are also not similar. So, no two of these Jordan forms are similar.

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Step 1 of 5

Consider the following differential equation:

$$\frac{du}{dt} = Ju$$

Here, J is a 2 by 2 Jordan block defined as below:

$$J = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$$

Whereas,

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Initial value:

$$u(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve by back substitution method.

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Step 2 of 5

Above differential equation can be written as follows:

$$\frac{du_1}{dt} = 5u_1 + u_2$$
$$\frac{du_2}{dt} = 5u_2$$

Provide feedback (0)

Provide feedback (0)

Step 2 of 5

Above differential equation can be written as follows:

$$\frac{du_1}{dt} = 5u_1 + u_2$$
$$\frac{du_2}{dt} = 5u_2$$

Provide feedback (0)

Step 3 of 5

The system is triangular. So, solve the single variable equation and move upwards in further equation by back substitution method:

$$\frac{du_2}{dt} = 5u_2$$

Recall that solution of the differential equation is given as follows:

$$\frac{du}{dt} = au$$

Solution:

$$u(t) = e^{at}u(0)$$

So, the solution of the differential equation starting with initial values is:

$$\begin{aligned} u_2 &= u_2(0)e^{5t} \\ &= 2e^{5t} \end{aligned}$$

Provide feedback (0)

So, the solution of the differential equation starting with initial values is:

$$\begin{aligned} u_2 &= u_2(0)e^{5t} \\ &= 2e^{5t} \end{aligned}$$

Provide feedback (0)

Step 4 of 5

Solve the next equation having variable u_2 and substitute the values:

$$\begin{aligned} \frac{du_1}{dt} &= 5u_1 + u_2 \\ u_1 &= \left(u_1(0) + tu_2(0)\right)e^{5t} \\ &= (1+2t)e^{5t} \end{aligned}$$

Provide feedback (0)

Step 5 of 5

Therefore, the solution is:

$$\boxed{\begin{aligned} u_1 &= (1+2t)e^{5t} \\ u_2 &= 2e^{5t} \end{aligned}}$$

Provide feedback (0)

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Rohan M.
BITS pilani
87

Jonathan M.
North Carolina State U...
61

CH5.6 28P

CH5.6 30P

Step 1 of 5

Consider the following matrix:

$$\begin{aligned} A &= \begin{bmatrix} 14 & 9 \\ -16 & -10 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \\ &= MJM^{-1} \end{aligned}$$

Compute A^{10} and e^{A}.

Provide feedback (0)

Step 2 of 5

Recall the following:

$$\begin{aligned} A^k &= MJ^kM^{-1} \\ e^{At} &= Me^{Jt}M^{-1} \end{aligned}$$

Also,

$$\begin{aligned} (J_i)^k &= \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}^k \\ &= \begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{1}{2}k(k-1)\lambda^{k-2} \\ 0 & \lambda^k & k\lambda^{k-1} \\ 0 & 0 & \lambda^k \end{bmatrix} \\ e^{J_it} &= \begin{bmatrix} e^{\lambda t} & te^{\lambda t} & \frac{1}{2}t^2e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \end{bmatrix} \end{aligned}$$

$$(J_i)^k = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}^k$$

$$= \begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{1}{2}k(k-1)\lambda^{k-2} \\ 0 & \lambda^k & k\lambda^{k-1} \\ 0 & 0 & \lambda^k \end{bmatrix}$$

$$e^{J_i t} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} & \frac{1}{2}t^2 e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{bmatrix}$$

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Step 3 of 5

Compute first J^{10}:

$$J = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$(J)^{10} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^{10}$$

$$= \begin{bmatrix} 2^{10} & 10 \cdot 2^{10-1} \\ 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

Provide feedback (0)

Step 3 of 5

Compute first J^{10}:

$$J = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{aligned} (J)^{10} &= \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^{10} \\ &= \begin{bmatrix} 2^{10} & 10 \cdot 2^{10-1} \\ 0 & 2^{10} \end{bmatrix} \\ &= 2^{10} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

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Step 4 of 5

Therefore, A^{10} is:

$$\begin{aligned} A^{10} &= MJ^{10}M^{-1} \\ &= \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} 2^{10} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \\ &= 2^{10} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \\ &= 2^{10} \begin{bmatrix} 3 & 13 \\ -4 & -17 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \end{aligned}$$

$$\boxed{A^{10} = 2^{10} \begin{bmatrix} 61 & 45 \\ -80 & -59 \end{bmatrix}}$$

Provide feedback (0)

$$A^{10} = MD^{10}M^{-1}$$

$$= \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} 2^{10} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 3 & 13 \\ -4 & -17 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\boxed{A^{10} = 2^{10} \begin{bmatrix} 61 & 45 \\ -80 & -59 \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 5

And e^A is:

$$e^A = Me^J M^{-1}$$

$$= \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} e^2 & e^2 \\ 0 & e^2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} e^2 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= e^2 \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$$

$$\boxed{e^A = e^2 \begin{bmatrix} 13 & 9 \\ -16 & -11 \end{bmatrix}}$$

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CH5.6 29P

Step 1 of 8

We know that *A* and *B* are similar if $B = M^{-1}AM$.

Above equation can be written as follows:

$$MB = MM^{-1}AM$$
$$MB = AM$$

Provide feedback (0)

CH5.6 31P

Step 2 of 8

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

So we have:

$$MB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & a+b \\ 0 & c+d \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

So we have:

$$\begin{aligned} MB &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & a+b \\ 0 & c+d \end{bmatrix} \end{aligned}$$

$$\begin{aligned} AM &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} a & b \\ a & b \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 3 of 8

We know that $MB = AM$, so we can write the following equations:

$$\begin{aligned} a &= 0 \\ a+b &= b \\ c+d &= b \end{aligned}$$

From the last two equations, we have

$$a+b = c+d$$

$$a+b=c+d$$
$$a=-b+c+d$$

As $a=0$, so one of the possible values of *b*, *c*, and *d* are

$$b=0$$
$$c=1$$
$$d=-1$$

Therefore, *A* and *B* are similar, if one of the possible matrix $M=\boxed{\begin{bmatrix}0 & 0\\ 1 & -1\end{bmatrix}}$, such that

$$MB=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=AM .$$

Provide feedback (2)

Step 4 of 8

Consider the following matrices:

$$A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$
$$B=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$
$$M=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

So we have:

Provide feedback (2)

Step 4 of 8

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

So we have:

$$\begin{aligned} MB &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} a-b & -a+b \\ c-d & -c+d \end{bmatrix} \end{aligned}$$

$$\begin{aligned} AM &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix} \end{aligned}$$

We know that $MB = AM$, so we can write the following equations:

$$a-b = a+c$$
$$c-d = a+c$$

$$AM = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix}$$

We know that $MB = AM$, so we can write the following equations:

$$a - b = a + c$$
$$c - d = a + c$$
$$-a + b = b + d$$
$$-c + d = b + d$$

From the above equations we get:

$$-b = c$$
$$-d = a$$
$$-a = d$$
$$-c = b$$

So, one of the possible values of a, *b*, *c*, and *d* are

$$a = 1$$
$$b = 1$$
$$c = -1$$
$$d = -1$$

Provide feedback (0)

Step 5 of 8

Therefore, *A* and *B* are similar, if one of the possible matrix $M = \boxed{\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}}$, such that

$$MB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = AM .$$

Provide feedback (0)

Step 6 of 8

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
$$B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$
$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

So we have:

$$\begin{aligned} MB &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4a+2b & 3a+b \\ 4c+2d & 3c+d \end{bmatrix} \end{aligned}$$

$$AM = \begin{bmatrix} 1 & 2 \\ & \end{bmatrix}\begin{bmatrix} a & b \\ & \end{bmatrix}$$

$$MB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4a+2b & 3a+b \\ 4c+2d & 3c+d \end{bmatrix}$$

$$AM = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

Provide feedback (0)

Step 7 of 8

We know that $MB = AM$, so we can write the following equations

$$4a+2b = a+2c$$
$$4c+2d = 3a+4c$$
$$3a+b = b+2d$$
$$3c+d = 3b+4d$$

From the above equations we get

$$a = \frac{2}{3}c - \frac{2}{3}b$$
$$a = \frac{2}{3}d$$
$$a = \frac{2}{3}d$$
$$d = c - b$$

From the above equations we get

$$a = \frac{2}{3}c - \frac{2}{3}b$$
$$a = \frac{2}{3}d$$
$$a = \frac{2}{3}d$$
$$d = c - b$$

So one of the possible values of a, *b*, *c*, and *d* are

$$a = 0$$
$$b = 1$$
$$c = 1$$
$$d = 0$$

Provide feedback (0)

Step 8 of 8

Therefore, *A* and *B* are similar, if one of the possible matrix $M = \boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$, such that

$$MB = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = AM .$$

Provide feedback (0)

CH5.6 30P

CH5.6 32P

Step 1 of 5

Consider the following matrices:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Determine the matrices which are similar.

Provide feedback (0)

Step 2 of 5

Loading solution. Please wait ...

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Step 3 of 5

Calculate Eigen values of matrix A_2;

$$A_2 - \lambda I = \begin{bmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{bmatrix}$$

Therefore, $\boxed{A_3, A_6}$ are similar matrices.

Provide feedback (0)

Step 3 of 5

Calculate Eigen values of matrix A_2;

$$A_2 - \lambda I = \begin{bmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{bmatrix}$$

$$\det(A_2 - \lambda I) = 0$$

$$(-\lambda)(-\lambda) - 1 = 0$$

$$\lambda^2 - 1 = 0$$

On solving above equation following Eigen values are obtained:

$$\lambda_2 = (1, -1)$$

Provide feedback (0)

Step 4 of 5

Eigen values of matrix A_4 will be:

$$A_4 - \lambda I = \begin{bmatrix} 0-\lambda & 0 \\ 1 & 1-\lambda \end{bmatrix}$$

$$\det(A_4 - \lambda I) = 0$$

$$(-\lambda)(1-\lambda) = 0$$

$$\lambda^2 - \lambda = 0$$

On solving above equation following Eigen values are obtained:

$$\lambda_4 = (1, 0)$$

Similarly, Eigen values of matrix A_5 will be $\lambda_5 = (1, 0)$.

On solving above equation following Eigen values are obtained:

$$\lambda_2 = (1, -1)$$

Provide feedback (0)

Step 4 of 5

Eigen values of matrix A_4 will be:

$$A_4 - \lambda I = \begin{bmatrix} 0-\lambda & 0 \\ 1 & 1-\lambda \end{bmatrix}$$

$$\det(A_4 - \lambda I) = 0$$

$$(-\lambda)(1-\lambda) = 0$$

$$\lambda^2 - \lambda = 0$$

On solving above equation following Eigen values are obtained:

$$\lambda_4 = (1, 0)$$

Similarly, Eigen values of matrix A_5 will be $\lambda_5 = (1, 0)$.

Provide feedback (0)

Step 5 of 5

Therefore, following matrices are similar:

$$\boxed{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}}$$

Matrix A_1 and A_2 are similar to themselves.

Provide feedback (0)

CH5.6 31P

CH5.6 33P

Step 1 of 6

Similar matrices have the same eigenvalues. Consider the following 2 by 2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Its eigenvalues can be obtained by solving $\det(A - \lambda I) = 0$.

Consider

$$\begin{aligned} 0 &= \det(A - \lambda I) \\ &= \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} \\ &= (a-\lambda)(d-\lambda) - bc \\ &= \lambda^2 - (a+d)\lambda + (ad - bc) \end{aligned}$$

Provide feedback (0)

Step 2 of 6

Since, any of *a*, *b*, *c*, and *d* can be either 0 or 1, there are only 3 choices (0 or 1 or 2) possible for *a* + *d*.

Suppose $a + d = 0$. Then $a = 0$ and $d = 0$. Now *bc* can be equal to 1 or 0. Thus, the following two equations are possible:

$$\begin{aligned} \lambda^2 &= 0 \\ \lambda^2 - 1 &= 0 \end{aligned}$$

Step 2 of 6

Since, any of *a*, *b*, *c*, and *d* can be either 0 or 1, there are only 3 choices (0 or 1 or 2) possible for *a* + *d*.

Suppose $a+d=0$. Then $a=0$ and $d=0$. Now *bc* can be equal to 1 or 0. Thus, the following two equations are possible:

$$\lambda^2=0$$
$$\lambda^2-1=0$$

Suppose $a+d=1$. Then one of *a* and *d* is 1 and the other is 0. Now *bc* can be equal to 1 or 0. Thus, the following two equations are possible:

$$\lambda^2-\lambda=0$$
$$\lambda^2-\lambda-1=0$$

Suppose $a+d=2$. Then both *a* and *d* are equal to 1. Now *bc* can be equal to 1 or 0. Thus, the following two equations are possible:

$$\lambda^2-2\lambda=0$$
$$\lambda^2-2\lambda+1=0$$

Provide feedback (0)

Step 3 of 6

Let us obtain the roots of the 6 quadratic equations.

For $\lambda^2=0$, the roots are 0 and 0. For $\lambda^2-1=0$, the roots are 1 and –1. For $\lambda^2-\lambda=0$, the roots are 0 and 1. For $\lambda^2-\lambda-1=0$, the roots are $\frac{1\pm\sqrt{5}}{2}$. For $\lambda^2-2\lambda=0$, the roots are 0 and 2. Finally, for

Step 3 of 6

Let us obtain the roots of the 6 quadratic equations.

For $\lambda^2 = 0$, the roots are 0 and 0. For $\lambda^2 - 1 = 0$, the roots are 1 and –1. For $\lambda^2 - \lambda = 0$, the roots are 0 and 1. For $\lambda^2 - \lambda - 1 = 0$, the roots are $\frac{1 \pm \sqrt{5}}{2}$. For $\lambda^2 - 2\lambda = 0$, the roots are 0 and 2. Finally, for $\lambda^2 - 2\lambda + 1 = 0$, the roots are 1 and 1.

Each of these roots corresponds to some eigenvalue of a 2 by 2 matrix. Thus, 6 different eigenvalues are possible. These are: –1, 0, 1, 2, $\frac{1 \pm \sqrt{5}}{2}$.

We have obtained 6 distinct quadratic equations and each of these have a unique set of 2 eigenvalues.

Thus, it is clear that there are $\boxed{6}$ families of similar matrices.

Provide feedback (0)

Step 4 of 6

Let us enlist all possible matrices in each of the 6 families:

Let the characteristic equation be $\lambda^2 = 0$. Then both *a* and *d* should be zero and either one or both of *b* and *c* should be zero. Thus, we get the following three matrices in this family:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Let the characteristic equation be $\lambda^2 - 1 = 0$. Then both *a* and *d* should be zero and both *b* and *c* should be 1. Thus, we get the following (one and only one) matrix in this family:

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Step 4 of 6

Let us enlist all possible matrices in each of the 6 families:

Let the characteristic equation be $\lambda^2 = 0$. Then both *a* and *d* should be zero and either one or both of *b* and *c* should be zero. Thus, we get the following three matrices in this family:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Let the characteristic equation be $\lambda^2 - 1 = 0$. Then both *a* and *d* should be zero and both *b* and *c* should be 1. Thus, we get the following (one and only one) matrix in this family:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Let the characteristic equation be $\lambda^2 - \lambda = 0$. Then exactly one of *a* and *d* should be equal to one and at least one of *b* and *c* should be equal to zero. Thus, we get the following six matrices in this family:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Let the characteristic equation be $\lambda^2 - \lambda - 1 = 0$. Then exactly one of *a* and *d* should be equal to one and both *b* and *c* should be equal to one. Thus, we get the following two matrices in this family:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 6

Let the characteristic equation be $\lambda^2 - 2\lambda = 0$. Then both *a* and *d* should be equal to one and both *b* and *c* should be equal to one. Thus, we get the following one matrix in this family:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Let the characteristic equation be $\lambda^2 - 2\lambda + 1 = 0$. Then both *a* and *d* should be equal to one and at least one of *b* and *c* should be equal to zero. Thus, we get the following three matrices in this family:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Step 6 of 6

Thus, we have categorised all the 16 matrices in the 6 families.

Provide feedback (0)

CH5.6 32P

Step 1 of 2

a) Given that x is a null space of A

$\Rightarrow Ax = 0$

Let us consider $\left(M^{-1}AM\right)\left(M^{-1}x\right)$

$$\begin{aligned} &= M^{-1}A\left(MM^{-1}\right)x \\ &= M^{-1}\left(AI\right)x \\ &= M^{-1}\left(Ax\right) \\ &= M^{-1}\left(0\right) \\ &= 0 \end{aligned}$$

$\left(M^{-1}AM\right)\left(M^{-1}x\right) = 0$ confirms that $M^{-1}x$ is in the null space of $M^{-1}AM$.

Provide feedback (0)

Step 2 of 2

b) We have that similar matrix $B = M^{-1}AM$ is closely connected to A.
Every linear transformation is represented by a matrix.
The matrix depends on the choice of basis.
If we change the basis to M, we change the matrix A to a similar matrix B.

Similar matrices represent the same transformation T with respect to different bases.
But this confirms that the dimension of B is equal to the dimension of A.
The set of linearly independent vectors in A are reduced to row echelon form by the multiplication of M and its inverse to get B. Thus, the basis of A and that of B are same.

Putting these things together, we get $M^{-1}AM$ and A have the same vectors, bases and dimension.

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CH5.6 34P

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CH5.6 33P

Step 1 of 11

Consider two matrices A and B. If these have same Eigen values and same Eigen vectors then are they considered as equal matrices?

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CH5.6 35P

Step 2 of 11

The answer is no. Consider the following two matrices:

$$A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$
$$B=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

Both matrices are upper triangular matrices. So, Eigen values of both matrices are $\lambda=(1,1)$

Provide feedback (0)

Step 3 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$
$$\begin{bmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix}y\\ \end{bmatrix}$$

Provide feedback (0)

Step 3 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$

$$\begin{bmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$x_2=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 4 of 11

Similarly, Eigen values of other matrix can be calculated. These are given as below:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$x_2=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 3 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$

$$\begin{bmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$x_2=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 4 of 11

Similarly, Eigen values of other matrix can be calculated. These are given as below:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$x_2=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 4 of 11

Similarly, Eigen values of other matrix can be calculated. These are given as below:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 11

Eigen vectors of both matrices are also same. However matrices are not equal.

Therefore, if two matrices A and B have same Eigen values and same Eigen vectors then they can not be considered as equal matrices.

Provide feedback (0)

Step 6 of 11

Find matrices A and B such that $A \neq B$ also Eigen values and Eigen vectors should be as follows:

$$\lambda = (0,0)$$
$$v_1 = (x_1, 0)$$

Provide feedback (0)

Step 7 of 11

Consider the following matrices:

Step 7 of 11

Consider the following matrices:

$$A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$
$$B=\begin{bmatrix}0 & 3\\ 0 & 0\end{bmatrix}$$

Both matrices are upper triangular matrices. So, Eigen values of both matrices are $\lambda=(0,0)$

Provide feedback (0)

Step 8 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$
$$\begin{bmatrix}0-\lambda & 1\\ 0 & 0-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 9 of 11

On solving values of y and z corresponding to $\lambda=0$ are as follows:

$$v_1=\begin{bmatrix}y\\ z\end{bmatrix}$$
$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$
$$\begin{bmatrix}0\end{bmatrix}$$

Provide feedback (0)

Step 9 of 11

On solving values of y and z corresponding to $\lambda = 0$ are as follows:

$$v_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 11

Similarly, Eigen values of other matrix can be calculated. These are given as below:

$$v_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 11

Eigen vectors of both matrices A and B are also same. However matrices are not equal. Therefore,

Similarly, Eigen values of other matrix can be calculated. These are given as below:

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$$v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 11

Eigen vectors of both matrices A and B are also same. However matrices are not equal. Therefore,

$$A = \boxed{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$
$$B = \boxed{\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}}$$

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Step 1 of 2

Given that $J = \begin{bmatrix} c & 1 \\ 0 & c \end{bmatrix}$

$$J^2 = \begin{bmatrix} c & 1 \\ 0 & c \end{bmatrix}\begin{bmatrix} c & 1 \\ 0 & c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c^2 & 2c \\ 0 & c^2 \end{bmatrix}$$

$$J^3 = \begin{bmatrix} c^2 & 2c \\ 0 & c^2 \end{bmatrix}\begin{bmatrix} c & 1 \\ 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} c^3 & 3c^2 \\ 0 & c^3 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 2

Similarly, $J^4 = \begin{bmatrix} c^4 & 4c^{4-1} \\ 0 & c^4 \end{bmatrix}$ and so on.

In view of mathematical induction, we can say $J^k = \begin{bmatrix} c^k & kc^{k-1} \\ 0 & c^k \end{bmatrix}$

Substitute $k = 0$

$$\Rightarrow J^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also, substituting $k = -1$, we get $J^{-1} = \begin{bmatrix} \frac{1}{c} & \frac{-1}{c^2} \\ 0 & \frac{1}{c} \end{bmatrix}$

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CH5.6 36P

CH5.6 35P

CH5.6 37P

Step 1 of 6

Consider the 5 by 5 Jordan block defined as below:

$$J=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$$

Here, Eigen values are zero, $\lambda=0$. Find value of J^2 and its Eigen vectors. Also find its Jordan form in two blocks.

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Step 2 of 6

Jordan block powers can be given as follows:

$$\begin{aligned}(J_i)^k&=\begin{bmatrix}\lambda&1&0\\0&\lambda&1\\0&0&\lambda\end{bmatrix}^k\\&=\begin{bmatrix}\lambda^k&k\lambda^{k-1}&\frac{1}{2}k(k-1)\lambda^{k-2}\\0&\lambda^k&k\lambda^{k-1}\\0&0&\lambda^k\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 3 of 6

To find the value of J^2 do the following calculations:

$$\begin{bmatrix}0&1&0&0&0\end{bmatrix}^2$$

Provide feedback (0)

Step 3 of 6

To find the value of J^2 do the following calculations:

$$J^2 = \begin{bmatrix} \mathbf{0} & 1 & 0 & 0 & 0 \\ 0 & \mathbf{0} & 1 & 0 & 0 \\ 0 & 0 & \mathbf{0} & 1 & 0 \\ 0 & 0 & 0 & \mathbf{0} & 1 \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix}^2$$

$$= \begin{bmatrix} \mathbf{0} & 2\cdot\mathbf{0}^{2-1} & \frac{1}{2}2(2-1)\mathbf{0}^{2-2} & 0 & 0 \\ 0 & \mathbf{0} & 2\cdot\mathbf{0}^{2-1} & \frac{1}{2}2(2-1)\mathbf{0}^{2-2} & 0 \\ 0 & 0 & \mathbf{0} & 2\cdot\mathbf{0}^{2-1} & \frac{1}{2}2(2-1)\mathbf{0}^{2-2} \\ 0 & 0 & 0 & \mathbf{0} & 2\cdot\mathbf{0}^{2-1} \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{0} & 1 & 0 \\ 0 & 0 & \mathbf{0} & \mathbf{0} & 1 \\ 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix}$$

Provide feedback (0)

Step 4 of 6

Therefore,

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{0} & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 6

Therefore,

$$\boxed{J^2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{0} & 1 & 0 \\ 0 & 0 & \mathbf{0} & \mathbf{0} & 1 \\ 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 6

For every missing Eigen vector Jordan form will have 1 in the matrix. Here in J^2 matrix three 1 can be seen. This implies that three Eigen vectors are missing. So, from 5 Eigen vectors corresponding to 5 repeated Eigen values 3 are missing.

Therefore, J^2 have $\boxed{2}$ independent Eigen vectors corresponding to Eigen value $\lambda = 0$.

Provide feedback (0)

Step 6 of 6

Jordan form of J^2 can be written as follows in two blocks:

$$\begin{bmatrix} J_3 & \\ & J_2 \end{bmatrix}$$

Here, both blocks are defined as:

$$J_3 = \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$$

Step 5 of 6

For every missing Eigen vector Jordan form will have 1 in the matrix. Here in J^2 matrix three 1 can be seen. This implies that three Eigen vectors are missing. So, from 5 Eigen vectors corresponding to 5 repeated Eigen values 3 are missing.

Therefore, J^2 have $\boxed{2}$ independent Eigen vectors corresponding to Eigen value $\lambda = 0$.

Provide feedback (0)

Step 6 of 6

Jordan form of J^2 can be written as follows in two blocks:

$$\begin{bmatrix} J_3 & \\ & J_2 \end{bmatrix}$$

Here, both blocks are defined as:

$$J_3 = \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$J_2 = \boxed{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$

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Step 1 of 7

Consider the following differential equation:

$$\frac{du}{dt} = Ju$$

Here, J is a 3 by 3 Jordan block defined as below:

$$J = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

Whereas,

$$u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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Step 2 of 7

Above differential equation can be written as follows:

$$\frac{dx}{dt} = 5x + y$$
$$\frac{dy}{dt} = 5y + z$$
$$\frac{dz}{dt} = 5z$$

Add a fourth equation defined as follows:

$$\frac{dw}{dt} = 5w + x$$

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Add a fourth equation defined as follows:

$$\frac{dw}{dt} = 5w + x$$

Find the value of w.

Provide feedback (0)

Step 3 of 7

Add fourth equation in differential equation:

$$\frac{du}{dt} = Ju$$

Thus,

$$u = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$
$$J = \begin{bmatrix} 5 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 7

Add fourth equation in differential equation:

$$\frac{du}{dt} = Ju$$

Thus,

$$u = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

$$J = \begin{bmatrix} 5 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 7

The system is triangular. So, solve the single variable equation and move upwards in further equation by back substitution method:

$$\frac{dz}{dt} = 5z$$

Recall that solution of the differential equation is given as follows:

$$\frac{du}{dt} = au$$

Solution:

$$\frac{dz}{dt} = 5z$$

Recall that solution of the differential equation is given as follows:

$$\frac{du}{dt} = au$$

Solution:

$$u(t) = e^{at}u(0)$$

So, the solution of the differential equation starting with initial values is:

$$z = z(0)e^{5t}$$

Solve the next equation having variable z and substitute the values:

$$\frac{dy}{dt} = 5y + z$$
$$y = \left(y(0) + tz(0)\right)e^{5t}$$

Provide feedback (0)

Step 5 of 7

Similarly, solutions of other two differential equations are as follows:

$$\frac{dx}{dt} = 5x + y$$
$$x = \left(x(0) + ty(0) + \frac{t^2}{2!}z(0)\right)e^{5t}$$

Provide feedback (0)

Step 6 of 7

Another added equation solution will be:

Provide feedback (0)

Step 5 of 7

Similarly, solutions of other two differential equations are as follows:

$$\frac{dx}{dt} = 5x + y$$

$$x = \left(x(0) + ty(0) + \frac{t^2}{2!} z(0) \right) e^{5t}$$

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Step 6 of 7

Another added equation solution will be:

$$\frac{dw}{dt} = 5w + x$$

$$w = \left(w(0) + tx(0) + \frac{t^2}{2!} y(0) + \frac{t^3}{3!} z(0) \right) e^{5t}$$

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Step 7 of 7

Therefore, the solution of the added equation is:

$$\boxed{w = \left(w(0) + tx(0) + \frac{t^2}{2} y(0) + \frac{t^3}{6} z(0) \right) e^{5t}}$$

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CH5.6 37P

CH5.6 39P

Step 1 of 7

Consider Jordan matrices:

$$J = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ and } K = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

These Jordan matrices have eigenvalues 0, 0, 0, 0 and two eigenvectors.

Here the block sizes are not matching and J is not similar to K.

Provide feedback (0)

Step 2 of 7

We have to find the eigenvectors of Jordan matrices J and K.

We know that the Jordan matrices have all eigenvalues as 0, so we will find vectors in nullspace of these matrices.

Consider the matrix $J = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$, it is in the reduced echelon form.

So, we have $Jx_1 = 0$, and vector x_1 is a linear combination of columns of J

Here the block sizes are not matching and J is not similar to K.

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Step 2 of 7

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We know that the Jordan matrices have all eigenvalues as 0, so we will find vectors in nullspace of these matrices.

Consider the matrix $J = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$, it is in the reduced echelon form.

So, we have $Jx_1 = 0$, and vector x_1 is a linear combination of columns of J

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Therefore, the eigenvectors of Jordan matrix J are $\boxed{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}$.

Therefore, the eigenvectors of Jordan matrix J are $\boxed{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}}$.

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Step 3 of 7

Consider the matrix $K = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$, it is also in the reduced echelon form.

So, we have $Kx_2 = 0$, and vector x_2 is a linear combination of columns of K

$$\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right\}$$

Therefore, the eigenvectors of Jordan matrices K are $\boxed{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}}$.

Provide feedback (0)

Provide feedback (0)

Step 4 of 7

Consider the following matrices

$$J=\left[\begin{array}{cc|cc}0&1&0&0\\0&0&0&0\\\hline 0&0&0&1\\0&0&0&0\end{array}\right],\ K=\left[\begin{array}{ccc|c}0&1&0&0\\0&0&1&0\\0&0&0&0\\\hline 0&0&0&0\end{array}\right]\text{ and }M=\begin{bmatrix}m_{11}&m_{12}&m_{13}&m_{14}\\m_{21}&m_{22}&m_{23}&m_{24}\\m_{31}&m_{32}&m_{33}&m_{34}\\m_{41}&m_{42}&m_{43}&m_{44}\end{bmatrix}$$

such that $JM=MK$.

We have to show, if $JM=MK$ then M is not invertible.

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Step 5 of 7

Find JM .

$$\begin{aligned}JM&=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}\begin{bmatrix}m_{11}&m_{12}&m_{13}&m_{14}\\m_{21}&m_{22}&m_{23}&m_{24}\\m_{31}&m_{32}&m_{33}&m_{34}\\m_{41}&m_{42}&m_{43}&m_{44}\end{bmatrix}\\&=\begin{bmatrix}m_{21}&m_{22}&m_{23}&m_{24}\\0&0&0&0\\m_{41}&m_{42}&m_{43}&m_{44}\\0&0&0&0\end{bmatrix}\end{aligned}$$

$$JM = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

$$= \begin{bmatrix} m_{21} & m_{22} & m_{23} & m_{24} \\ 0 & 0 & 0 & 0 \\ m_{41} & m_{42} & m_{43} & m_{44} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now find MK .

$$JM = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & m_{11} & m_{12} & 0 \\ 0 & m_{21} & m_{22} & 0 \\ 0 & m_{31} & m_{32} & 0 \\ 0 & m_{41} & m_{42} & 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Using the condition $JM = MK$, we have

$$m_{21} = m_{22} = m_{24} = m_{41} = m_{42} = m_{44} = 0$$

Step 6 of 7

Using the condition $JM = MK$, we have

$$m_{21} = m_{22} = m_{24} = m_{41} = m_{42} = m_{44} = 0$$

Substitute these values in the matrix *M*.

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ 0 & 0 & m_{23} & 0 \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & m_{43} & 0 \end{bmatrix}$$

From matrix *M*, it is clear that 2nd row is multiple of 4th row.

So, rank of matrix *M* is not 4, if so then *M* is not invertible.

Provide feedback (0)

Step 7 of 7

If we consider that matrices *J* and *K* are similar, then there exists an invertible matrix, say *M* such that

$$M^{-1}JM = K$$

This implies that $MK = JM$.

We have already shown that *M* is not invertible.

Step 7 of 7

If we consider that matrices *J* and *K* are similar, then there exists an invertible matrix, say *M* such that

$$M^{-1}JM = K$$

This implies that $MK = JM$.

We have already shown that *M* is not invertible.

Therefore, $M^{-1}JM = K$ is impossible, so *J* is not similar to *K*.

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Step 1 of 3

Prove that A^T is always similar to A in three steps:

(a) For matrix A matrix M_i of permutations is calculated so that $M_i^{-1}J_iM_i = J_i^T$.
(b) Matrix M_0 is constructed from blocks so that $M_0^{-1}JM_0 = J^T$
(c) For any matrix A following is true:

$$\begin{aligned} A &= MJM^{-1} \\ A^T &= \left(MJM^{-1}\right)^T \\ &= \left(M^{-1}\right)^T J^T M^T \end{aligned}$$

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Step 2 of 3

From step 2 substitutes $M_0^{-1}JM_0 = J^T$.

$$A^T = \left(M^{-1}\right)^T M_0^{-1}JM_0M^T$$

Substitute $M^{-1}AM = J$. Thus,

$$\begin{aligned} A^T &= \left(M^{-1}\right)^T M_0^{-1}M^{-1}AMM_0M^T \\ &= \left(MM_0M^T\right)^{-1} A\left(MM_0M^T\right) \end{aligned}$$

Provide feedback (0)

Step 3 of 3

Therefore, $\boxed{A^T \text{ is always similar to } A.}$

Provide feedback (0)

CH5.6 39P

CH5.6 41P

Step 1 of 6

Consider the following matrices:

$$A_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$A_2 = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$
$$A_3 = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$
$$A_4 = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$$

Determine the pairs which are similar.

Provide feedback (0)

Step 2 of 6

Recall that similar matrices have same Eigen values. In other words trace of the similar matrices will be same.

So, calculate the Eigen values of all the matrices. Eigen values of matrix A_1 will be:

$$\begin{aligned} A_1 - \lambda I &= \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} \\ \det(A_1 - \lambda I) &= 0 \\ (a-\lambda)(d-\lambda) - bc &= 0 \\ \lambda^2 - \lambda(a+d) + (ad-bc) &= 0 \end{aligned}$$

On solving above equation following Eigen values are obtained:

$$\lambda_1 = \frac{1}{2}\left((a+d) + \sqrt{a^2 + d^2 - 2ad + 4bc}\right)$$

$$(a-\lambda)(d-\lambda)-bc=0$$
$$\lambda^2-\lambda(a+d)+(ad-bc)=0$$

On solving above equation following Eigen values are obtained:

$$\lambda_1=\frac{1}{2}\left((a+d)+\sqrt{a^2+d^2-2ad+4bc}\right)$$
$$\lambda_2=\frac{1}{2}\left((a+d)-\sqrt{a^2+d^2-2ad+4bc}\right)$$

Provide feedback (0)

Step 3 of 6

Therefore, trace will be as follows:

$$\begin{aligned}\lambda_1+\lambda_2&=\frac{1}{2}\left((a+d)+\sqrt{a^2+d^2-2ad+4bc}\right)+\frac{1}{2}\left((a+d)-\sqrt{a^2+d^2-2ad+4bc}\right)\\&=\frac{1}{2}(a+d)+\frac{1}{2}(a+d)\\&=a+d\end{aligned}$$

Trace of matrix A_1 is: $a+d$.

Provide feedback (0)

Step 4 of 6

Eigen values of matrix A_2 will be:

$$A_2-\lambda I=\begin{bmatrix}b-\lambda & a\\ d & c-\lambda\end{bmatrix}$$
$$\det(A_2-\lambda I)=0$$
$$(b-\lambda)(c-\lambda)-ad=0$$
$$\lambda^2-\lambda(c+b)+(bc-ad)=0$$

Provide feedback (0)

Step 4 of 6

Eigen values of matrix A_2 will be:

$$A_2 - \lambda I = \begin{bmatrix} b-\lambda & a \\ d & c-\lambda \end{bmatrix}$$
$$\det(A_2 - \lambda I) = 0$$
$$(b-\lambda)(c-\lambda) - ad = 0$$
$$\lambda^2 - \lambda(c+b) + (bc - ad) = 0$$

On solving above equation following Eigen values are obtained:

$$\lambda_1 = \frac{1}{2}\left((b+c) + \sqrt{c^2 + b^2 - 2bc + 4ad}\right)$$
$$\lambda_2 = \frac{1}{2}\left((b+c) - \sqrt{c^2 + b^2 - 2bc + 4ad}\right)$$

Provide feedback (0)

Step 5 of 6

Therefore, trace will be as follows:

$$\begin{aligned}\lambda_1 + \lambda_2 &= \frac{1}{2}\left((b+c) + \sqrt{c^2 + b^2 - 2bc + 4ad}\right) + \frac{1}{2}\left((b+c) - \sqrt{c^2 + b^2 - 2bc + 4ad}\right) \\ &= \frac{1}{2}(b+c) + \frac{1}{2}(b+c) \\ &= (b+c)\end{aligned}$$

Trace of matrix A_2 is: $b+c$.

Provide feedback (0)

$$\lambda_1 = \frac{1}{2}\left((b+c)+\sqrt{c^2+b^2-2bc+4ad}\right)$$
$$\lambda_2 = \frac{1}{2}\left((b+c)-\sqrt{c^2+b^2-2bc+4ad}\right)$$

Provide feedback (0)

Step 5 of 6

Therefore, trace will be as follows:

$$\begin{aligned}\lambda_1+\lambda_2 &= \frac{1}{2}\left((b+c)+\sqrt{c^2+b^2-2bc+4ad}\right)+\frac{1}{2}\left((b+c)-\sqrt{c^2+b^2-2bc+4ad}\right)\\ &= \frac{1}{2}(b+c)+\frac{1}{2}(b+c)\\ &= (b+c)\end{aligned}$$

Trace of matrix A_2 is: $b+c$.

Provide feedback (0)

Step 6 of 6

Similarly, trace of matrix A_3 is $b+c$ and of matrix A_4 is $a+d$. Here, trace of matrix A_1 and A_4 are equal. Hence, they form a pair. Also trace of matrix A_2 and A_3 are equal, so they form another pair.

Therefore, following pairs are similar:

$$\boxed{\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} d & c \\ b & a \end{bmatrix}} \text{ and } \boxed{\begin{bmatrix} b & a \\ d & c \end{bmatrix}, \begin{bmatrix} c & d \\ a & b \end{bmatrix}}$$

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Step 1 of 8

Similar matrices: Matrices A and B are similar if $A = M^{-1}BM$ for some invertible matrix M.

State the reason for following true or false statements:

(a) An invertible matrix can not be similar to a singular matrix.

Singular matrices have determinant value zero. This means that only Eigen value that can satisfy $Ax = 0$ is $\lambda = 0$.

However invertible matrix cannot have zero, $\lambda \neq 0$, as an Eigen value.

Therefore, the statement- an invertible matrix can not be similar to a singular matrix is $\boxed{\text{true}}$.

Provide feedback (0)

Step 2 of 8

(b) A symmetric matrix can't be similar to a non-symmetric matrix.

Symmetric matrix uses transpose instead of inverse for Eigen vector matrix during diagonalisation. Non-symmetric matrix on diagonalisation can give symmetric Eigen value matrix.

Provide feedback (0)

Step 3 of 8

Consider the following non-symmetric matrix:

$$A = \begin{bmatrix} 0 & -i \\ 0 & 1 \end{bmatrix}$$
$$\neq A^H$$

value matrix.

Provide feedback (0)

Step 3 of 8

Consider the following non-symmetric matrix:

$$A = \begin{bmatrix} 0 & -i \\ 0 & 1 \end{bmatrix}$$
$$\neq A^H$$

Now, do the following calculations:

$$\begin{aligned} M^{-1}AM &= \begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &= J \end{aligned}$$

In the above calculation it is clear that non symmetric matrix is similar to symmetric matrix.

Provide feedback (0)

Step 4 of 8

Therefore, a symmetric matrix can be similar to a non-symmetric matrix.

Provide feedback (0)

Step 5 of 8

(c) Matrix A can't be similar to matrix $-A$, unless $A = 0$.

Above statement is false. As matrix A can be similar to matrix $-A$. For this consider an example:

In the above calculation it is clear that non symmetric matrix is similar to symmetric matrix.

Provide feedback (0)

Step 4 of 8

Therefore, a symmetric matrix can be similar to a non-symmetric matrix.

Provide feedback (0)

Step 5 of 8

(c) Matrix A can't be similar to matrix $-A$, unless $A=0$.

Above statement is false. As matrix A can be similar to matrix $-A$. For this consider an example:

$$A=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$$
$$B=\begin{bmatrix}0 & -i\\ -i & 0\end{bmatrix}$$
$$=-A$$

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Step 6 of 8

Now, do the following calculations:

$$M^{-1}AM=\frac{1}{2}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}$$
$$=\begin{bmatrix}-i & 0\\ 0 & i\end{bmatrix}$$
$$=J_1$$

$$A = \begin{bmatrix} & \\ i & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

$$= -A$$

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Step 6 of 8

Now, do the following calculations:

$$M^{-1}AM = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$$

$$= J_1$$

$$M^{-1}BM = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$$

$$= J_2$$

$$= J_1$$

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Step 7 of 8

Therefore, matrix A can be similar to matrix $-A$ without being a zero matrix.

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Step 8 of 8

(d) Matrix $A - I$ can't be similar to $A + I$.

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Step 7 of 8

Therefore, matrix A can be similar to matrix $-A$ without being a zero matrix.

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Step 8 of 8

(d) Matrix $A-I$ can't be similar to $A+I$.

This statement is true. As all Eigen values of $A+I$ is increased by 1 and all Eigen values of $A-I$ are decreased by 1. Different Eigen values are formed. Therefore, Matrix $A-I$ can't be similar to $A+I$ statement is true.

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Step 1 of 2

AB can be expresses as

$$\begin{aligned} AB &= I(AB) \\ &= \left(B^{-1}B\right)(AB) \\ &= B^{-1}(BA)B \end{aligned}$$

Therefore AB is similar to BA.
Similar matrices have the same eigen values.

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Step 2 of 2

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Step 1 of 3

Consider a matrix A of order 6 by 4. Another matrix B is of order 4 by 6. Matrix AB (6×6) and BA (4×4) are of different sizes. Then consider the matrix G formed as below:

$$\begin{aligned} G &= \begin{bmatrix} I & -A \\ 0 & I \end{bmatrix}\begin{bmatrix} AB & 0 \\ B & 0 \end{bmatrix}\begin{bmatrix} I & A \\ 0 & I \end{bmatrix} \\ &= M^{-1}FM \\ &= \begin{bmatrix} 0 & 0 \\ B & BA \end{bmatrix} \end{aligned}$$

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Step 2 of 3

(a) Sizes of blocks are same in each matrix. Determine the sizes of blocks in G.

Following equation shows that matrices F and G are similar matrices.

$$G = M^{-1}FM$$

They have the same 10 Eigen values. Matrix G has one block of BA (4×4) on main diagonal corresponding to 4 Eigen values. Matrix F has one block of AB (6×6) starting from main diagonal corresponding to 6 Eigen values. Total Eigen values are 10 in number and F and G are similar matrices, this implies that another block in matrix G is of (6×6).

Therefore, sizes of blocks in matrix G are: $\boxed{(6\times6)}$ and $\boxed{(4\times4)}$.

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Step 3 of 3

(b) Matrix G and F have the same 10 Eigen values Matrix F has the Eigen values of AB

Step 2 of 3

(a) Sizes of blocks are same in each matrix. Determine the sizes of blocks in G.

Following equation shows that matrices F and G are similar matrices.

$$G = M^{-1}FM$$

They have the same 10 Eigen values. Matrix G has one block of BA (4×4) on main diagonal corresponding to 4 Eigen values. Matrix F has one block of AB (6×6) starting from main diagonal corresponding to 6 Eigen values. Total Eigen values are 10 in number and F and G are similar matrices, this implies that another block in matrix G is of (6×6).

Therefore, sizes of blocks in matrix G are: $\boxed{(6\times6)}$ and $\boxed{(4\times4)}$.

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Step 3 of 3

(b) Matrix G and F have the same 10 Eigen values Matrix F has the Eigen values of AB plus 4 zeros and matrix G has the Eigen values of matrix BA plus 6 zeros.

Matrix F has 4 zeros so remaining $(10-4)$ Eigen values must be Eigen values of matrix AB. Therefore, matrix AB has 6 Eigen values.

Similarly, matrix G has 6 zeros so remaining $(10-6)$ Eigen values must be Eigen values of matrix BA. Therefore, matrix BA has 4 Eigen values.

Therefore, matrix AB has the same Eigen values as matrix BA plus $\boxed{6-4=2}$ zeros.

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CH6.1

Step 1 of 10

Similar matrices: Matrices A and B are similar if $A = M^{-1}BM$ for some invertible matrix M.

State the reason for following true statements:
(a) If matrix A is similar to B, then matrix A^2 is similar to B^2.
If matrix A is similar to matrix B then following must be true:

$$\begin{aligned} A &= M^{-1}BM \\ A \cdot A &= \left(M^{-1}BM\right) \cdot \left(M^{-1}BM\right) \\ &= \left(M^{-1}B\right) I \left(BM\right) \\ &= M^{-1}B^2M \end{aligned}$$

Therefore, $\boxed{A^2 = M^{-1}B^2M}$ shows that the statement, matrix A^2 is similar to B^2 when matrix A is similar to B, is true.

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Step 2 of 10

(b) Matrix A^2 can be similar to B^2 even when matrix A is not similar to B.
For this consider two matrices A and B.

$$\begin{aligned} A &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\ B &= \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} \\ &= -A \end{aligned}$$

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Step 3 of 10

$$B = \begin{bmatrix} & \\ 0 & -1 \end{bmatrix}$$
$$= -A$$

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Step 3 of 10

If these matrices are similar then they must belong to same family. For this do the following calculations:

$$M^{-1}AM = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= J_1$$
$$M^{-1}BM = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$
$$= J_2$$
$$\neq J_1$$

Above calculations shows that matrix A is not similar to matrix B.

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Step 4 of 10

Now, check for matrices A^2 and B^2.

$$M^{-1}A^2M = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Above calculations shows that matrix A is not similar to matrix B.

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Step 4 of 10

Now, check for matrices A^2 and B^2.

$$\begin{aligned} M^{-1}A^2M &= \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ &= J_1 \\ M^{-1}B^2M &= \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ &= J_1 \end{aligned}$$

As matrices are equal so they will belong to same family.

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Step 5 of 10

Therefore, this could be possible that matrix A^2 can be similar to B^2 even when matrix A is not similar to B.

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Step 6 of 10

(c) Matrix A is similar to matrix B defined below:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

Step 5 of 10

Therefore, this could be possible that matrix A^2 can be similar to B^2 even when matrix A is not similar to B.

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Step 6 of 10

(c) Matrix A is similar to matrix B defined below:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$
$$B = \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$$

It can be seen that matrix B is upper triangular matrix with different Eigen values $\lambda = (3,4)$. This implies that it can be diagonalize into a matrix Λ. Matrix S will contain Eigen vectors of matrix B.

$$\begin{aligned} S^{-1}BS &= \Lambda \\ &= A \end{aligned}$$

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Step 7 of 10

Therefore, $\boxed{S^{-1}BS = A}$ implies that matrix A is similar to matrix B.
(d) Matrix A is not similar to matrix B defined below:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$B = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

It can be seen that matrix B is upper triangular matrix with repeated Eigen values

Step 7 of 10

Therefore, $\boxed{S^{-1}BS = A}$ implies that matrix A is similar to matrix B.
(d) Matrix A is not similar to matrix B defined below:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$B = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

It can be seen that matrix B is upper triangular matrix with repeated Eigen values $\lambda = (3,3)$. This implies that it can not be diagonalize into a matrix Λ. However matrix A is equal to Eigen value matrix of B.

$$\begin{aligned} S^{-1}BS &\neq \Lambda \\ &\neq A \end{aligned}$$

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Step 8 of 10

Therefore, $\boxed{S^{-1}BS \neq A}$ implies that matrix A is not similar to matrix B.

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Step 9 of 10

(e) If we exchange rows 1 and 2 of matrix A, and then exchange columns 1 and 2, the Eigen values stay the same.

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Therefore, $\boxed{S^{-1}BS \neq A}$ implies that matrix A is not similar to matrix B.

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Step 9 of 10

(e) If we exchange rows 1 and 2 of matrix A, and then exchange columns 1 and 2, the Eigen values stay the same.

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Now exchange rows 1 and 2.

$$A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

Exchange columns 1 and 2.

$$A_2 = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

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Step 10 of 10

It can be seen that the position of the Eigen values in matrix A_2 are interchanged however, Eigen values remain the same as in matrix A. Therefore, exchanging rows 1 and 2 and then exchanging columns 1 and 2 makes $\boxed{\text{no change}}$ in the Eigen values.

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