CH4.3

Step 1 of 10

We have

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

We find $\det(A)$ by cofactor expansion along the first column.

$$\begin{aligned} \det(A) &= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{vmatrix} \\ &= \begin{vmatrix} 4 & 0 \\ 0 & 5 \end{vmatrix} \\ &= (4)(5) - 0 \\ &= 20 \end{aligned}$$

Provide feedback (0)

Step 2 of 10

<u>Second method</u>: since A is a triangular matrix, we find $\det(A)$ by taking the product of the diagonal entries.

$$\begin{aligned} \det(A) = |A| &= (1)(4)(5) \\ &= 20 \end{aligned}$$

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Step 3 of 10

The cofactor of A are

$$C_{11} = \begin{vmatrix} 4 & 0 \\ 0 & 5 \end{vmatrix}$$

CH4.4 2P

Step 3 of 10

The cofactor of A are

$$C_{11} = \begin{vmatrix} 4 & 0 \\ 0 & 5 \end{vmatrix}$$
$$= 20$$
$$C_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & 5 \end{vmatrix}$$
$$= 0$$
$$C_{13} = \begin{vmatrix} 0 & 4 \\ 0 & 0 \end{vmatrix}$$
$$= 0$$

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Step 4 of 10

And

$$C_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$$
$$= -10$$
$$C_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix}$$
$$= 5$$
$$C_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}$$
$$= 0$$

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Step 5 of 10

Then

$$\begin{vmatrix} 2 & 3 \end{vmatrix}$$

Step 5 of 10

Then

$$C_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix}$$
$$= -12$$
$$C_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix}$$
$$= 0$$
$$C_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix}$$
$$= 4$$

Provide feedback (0)

Step 6 of 10

So, all the nine cofactors c_{ij} of A are

$C_{11} = 20,\ \ C_{12} = 0, C_{13} = 0,$

$C_{21} = -10, C_{22} = 5, C_{23} = 0,$

$C_{31} = -12, C_{32} = 0, C_{33} = 4$

The matrix of cofactor is

$$C = \begin{bmatrix} 20 & 0 & 0 \\ -10 & 5 & 0 \\ -12 & 0 & 4 \end{bmatrix}$$

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Step 7 of 10

Now

$$C^T = \begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \end{bmatrix}$$

Step 7 of 10

Now

$$C^T = \begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

We need to verify that $AC^T = (\det A)\, I$

So, consider

$$AC^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 10

$$= \begin{bmatrix} 20+0+0 & -10+10+0 & -12+0+12 \\ 0+0+0 & 0+20+0 & -0+0+0 \\ 0+0+0 & -0+0+0 & -0+0+20 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$= 20 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (\det A)\, I$$

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Step 9 of 10

C^T

Step 9 of 10

Thus, $AC^T = \dfrac{C^T}{\det(A)}$

$$= \frac{1}{20}\begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{1}{2} & \frac{-3}{5} \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$$

Provide feedback (0)

Step 10 of 10

Thus

$$A^{-1} = \boxed{\begin{bmatrix} 1 & -\frac{1}{2} & \frac{-3}{5} \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}}$$

Note: The inverse of an upper triangular matrix is also an upper triangular matrix.

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CH4.4 1P

Step 1 of 15

We have

$$A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & 3\end{bmatrix}$$

The cofactors of A are

$$\begin{aligned} C_{11} &= \begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix}\\ &= 4-1\\ &= 3 \end{aligned}$$

Provide feedback (0)

Step 2 of 15

Now

$$\begin{aligned} C_{12} &= -\begin{vmatrix}-1 & -1\\ 0 & 2\end{vmatrix}\\ &= -(-2-0)\\ &= 2 \end{aligned}$$

$$\begin{aligned} C_{13} &= \begin{vmatrix}-1 & 2\\ 0 & -1\end{vmatrix}\\ &= 1-0\\ &= 1 \end{aligned}$$

Provide feedback (0)

Step 3 of 15

Then

$$C_{21} = -\begin{vmatrix}-1 & 0\end{vmatrix}$$

CH4.4 3P

Step 3 of 15

Then

$$C_{21} = -\begin{vmatrix} -1 & 0 \\ -1 & 2 \end{vmatrix}$$
$$= -(-2-0)$$
$$= 2$$
$$C_{22} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$
$$= 4-0$$
$$= 4$$

Provide feedback (0)

Step 4 of 15

And

$$C_{23} = -\begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix}$$
$$= -(-2-0)$$
$$= 2$$
$$C_{31} = \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix}$$
$$= 1-0$$
$$= 1$$

Provide feedback (0)

Step 5 of 15

Now

$$C_{32} = -\begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix}$$

Step 5 of 15

Now

$$C_{32} = -\begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix}$$
$$= -(-2-0)$$
$$= 2$$
$$C_{33} = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$
$$= 4-1$$
$$= 3$$

Provide feedback (0)

Step 6 of 15

The matrix of cofactors of A is

$$C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Now

$$C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 15

And

$$\det(A) = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix}$$

Step 7 of 15

And

$$\det(A) = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix}$$

$$= 2(3) - 2$$

$$= 4$$

$$A^{-1} = \frac{C^T}{\det(A)}$$

$$= \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 15

The cofactors of B are

$$C_{11} = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix}$$

$$= 6 - 4$$

$$= 2$$

$$C_{12} = -\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= -(3 - 2)$$

$$= -1$$

Provide feedback (0)

Step 9 of 15

Now

Step 9 of 15

Now

$$C_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$
$$= 2 - 2$$
$$= 0$$
$$C_{21} = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$
$$= -(3 - 2)$$
$$= -1$$

Provide feedback (0)

Step 10 of 15

And

$$C_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}$$
$$= 3 - 1$$
$$= 2$$
$$C_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= -(2 - 1)$$
$$= -1$$

Provide feedback (0)

Step 11 of 15

Now

$$C_{31} = \begin{vmatrix} 1 & 1 \end{vmatrix}$$

Step 11 of 15

Now

$$C_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$$
$$= 2 - 2$$
$$= 0$$

$$C_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= -(2-1)$$
$$= -1$$

$$C_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= 2 - 1$$
$$= 1$$

Provide feedback (0)

Step 12 of 15

The matrix of cofactors of B is

$$C = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

And $C^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ and

$$\det(B) = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$
$$= (2) - (1)$$
$$= 1$$

Step 13 of 15

Now

$$B^{-1} = \frac{C^T}{\det(A)}$$

$$= \frac{1}{1}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Provide feedback (1)

Step 14 of 15

Thus, the inverses of the given matrices A and B are

$$A^{-1} = \boxed{\frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}} \text{ And } B^{-1} = \boxed{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}}$$

Provide feedback (0)

Step 15 of 15

CH4.4 2P

Step 1 of 14

a) The system of equations is

$$ax + by = 1$$
$$cx + dy = 0$$

Cramer's rule:

The j Th component of $x = A^{-1}b$ is the ratio

$$x_j = \frac{\det B_j}{\det A}, \text{ where } B_j = \begin{bmatrix} a_{11} & a_{12} & b_1 & a_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & b_n & a_{nn} \end{bmatrix} \text{ has } b \text{ in column } j.$$

Provide feedback (0)

Step 2 of 14

We need to solve the given system by cramer's rule

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad x = \begin{bmatrix} x \\ y \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & b \\ 0 & d \end{bmatrix} \qquad A_2 = \begin{bmatrix} a & 1 \\ c & 0 \end{bmatrix}$$

Now

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$

Provide feedback (0)

Step 3 of 14

And

CH4.4 4P

Step 3 of 14

And

$$\det(A_1) = \begin{vmatrix} 1 & b \\ 0 & d \end{vmatrix}$$
$$= d$$
$$\det(A_2) = \begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}$$
$$= -c$$

Provide feedback (0)

Step 4 of 14

Thus, by crammers rule, we have

$$x = \frac{\det(A_1)}{\det(A)}$$
$$= \frac{\begin{vmatrix} 1 & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$
$$= \frac{d}{ad - bc}$$

Provide feedback (0)

Step 5 of 14

And

$$y = \frac{\det(A_2)}{\det(A)}$$
$$\begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}$$

Step 5 of 14

And

$$y = \frac{\det(A_2)}{\det(A)}$$
$$= \frac{\begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$
$$= \frac{-c}{ad - bc}$$

Provide feedback (0)

Step 6 of 14

Thus, the solution for the given systems

$$\boxed{\begin{aligned} x &= \frac{d}{ad - bc} \\ y &= \frac{-c}{ad - bc} \end{aligned}}$$

Provide feedback (0)

Step 7 of 14

b) The given system is

$$\begin{aligned} x + 4y - z &= 1 \\ x + y + z &= 0 \\ 2x \quad + 3z &= 0 \end{aligned}$$

We need to solve the given system by cramers rule

$$A - \begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Step 7 of 14

b) The given system is

$$x+4y-z=1$$
$$x+y+z=0$$
$$2x \quad +3z=0$$

We need to solve the given system by cramers rule

$$A-\begin{bmatrix}1 & 4 & -1\\ 1 & 1 & 1\\ 2 & 0 & 3\end{bmatrix} X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} b=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 8 of 14

Replacing the first, second and third columns of A with b we get the matrices A_1, A_2 and A_3

$$A_1=\begin{bmatrix}1 & 4 & -1\\ 0 & 1 & 1\\ 0 & 0 & 3\end{bmatrix}, A_2=\begin{bmatrix}1 & 1 & -1\\ 1 & 0 & 1\\ 2 & 0 & 3\end{bmatrix}$$

Provide feedback (0)

Step 9 of 14

$$A_3=\begin{bmatrix}1 & 4 & 1\\ 1 & 1 & 0\\ 2 & 0 & 0\end{bmatrix}$$

Now

$$\det(A)=|A|=\begin{vmatrix}1 & 4 & -1\\ 1 & 1 & 1\\ 2 & 0 & 3\end{vmatrix}$$
$$=\begin{vmatrix}1 & 1\\ \end{vmatrix}-4\begin{vmatrix}1 & 1\\ \end{vmatrix}-\begin{vmatrix}1 & 1\\ \end{vmatrix}$$

Step 9 of 14

$$A_3 = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Now

$$\begin{aligned} \det(A) = |A| &= \begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} - 4\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} \\ &= (3) - 4(1) - (-2) \\ &= 1 \end{aligned}$$

Provide feedback (0)

Step 10 of 14

Now

$$\begin{aligned} \det(A_1) = |A_1| &= \begin{vmatrix} 1 & 4 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} - 4\begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \\ &= (3) - 4(0) - (0) \\ &= 3 \end{aligned}$$

Provide feedback (0)

Step 11 of 14

And

Step 11 of 14

And

$$\det(A_2) = |A_2| = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$
$$= (0) - (1) - (0)$$
$$= -1$$
$$\det(A_3) = |A_3| = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix}$$
$$= 1(0) - 4(0) + 1(-2)$$
$$= -2$$

Provide feedback (0)

Step 12 of 14

Thus, by cramers rule we have

$$x = \frac{\det(A_1)}{\det(A)}$$
$$= \frac{\begin{vmatrix} 1 & 4 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}}$$
$$= \frac{3}{1}$$
$$= 3$$

Provide feedback (0)

Provide feedback (0)

Step 13 of 14

And

$$y=\frac{\det(A_2)}{\det(A)}=\frac{\begin{vmatrix}1&1&-1\\1&0&1\\2&0&3\end{vmatrix}}{\begin{vmatrix}1&4&-1\\1&1&1\\2&0&3\end{vmatrix}}$$
$$=\frac{-1}{1}$$
$$=-1$$

Provide feedback (0)

Step 14 of 14

And

$$z=\frac{\det(A_3)}{\det(A)}=\frac{\begin{vmatrix}1&4&1\\1&1&0\\2&0&0\end{vmatrix}}{\begin{vmatrix}1&4&-1\\1&1&1\\2&0&3\end{vmatrix}}$$
$$=\frac{-2}{1}$$
$$=-2$$

Thus, the solution for the given system is

$$\boxed{x=3,\ y=-1,\ z=-2}$$

Provide feedback (0)

CH4.4 3P

CH4.4 5P

Step 1 of 5

a) $$M = \begin{bmatrix} 1 & 0 & x_1 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \\ - & - & - & - & - \\ 0 & 0 & x_n & - & 1 \end{bmatrix}$$

Determinant is M is $$1\begin{vmatrix} 1 & 0 & x_2 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \\ - & - & - & - & - \\ 0 & 0 & x_n & - & 1 \end{vmatrix} + (-1)^{1+j} x_1 \begin{vmatrix} 0 & 1 & - & - & 0 \\ 0 & 0 & 1 & - & 0 \\ 0 & 0 & - & - & 0 \\ - & - & - & - & 1 \\ 0 & 0 & 0 & - & 0 \end{vmatrix} \quad \ldots\ldots (1)$$

$$= 1\begin{vmatrix} 1 & 0 & x_2 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \\ - & - & - & - & - \\ 0 & 0 & x_n & - & 1 \end{vmatrix} + (-1)^{1+j} x_1 (0)$$

Provide feedback (0)

Step 2 of 5

$$= 1\begin{vmatrix} 1 & 0 & x_3 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \\ - & - & - & - & - \\ 0 & 0 & x_n & - & 1 \end{vmatrix} + (-1)^{2+j} x_2 \begin{vmatrix} 0 & 1 & - & - & 0 \\ 0 & 0 & 1 & - & 0 \\ 0 & 0 & - & - & 0 \\ - & - & - & - & 1 \\ 0 & 0 & 0 & - & 0 \end{vmatrix}$$

Continuing this, we get the determinant M is $(-1)^{j+j} x_j = x_j$

Provide feedback (0)

Step 3 of 5

Further, if $x_j = 0$, it directly follows from (1) that the j th column determinant on the left which is the multiple of 1 becomes zero and so, the determinant is zero.

Provide feedback (0)

Step 4 of 5

b) Suppose the non homogeneous system of m linear equations in n variables is

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned}$$

It can be written as the product of matrices $Ax = b$ shown to be

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ . \\ . \\ b_m \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix}_{m\times n} \begin{bmatrix} x_1 \\ x_2 \\ - \\ - \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ - \\ - \\ b_m \end{bmatrix}_{m\times 1}$$

We consider $AM = \begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \end{bmatrix} \begin{bmatrix} 1 & 0 & x_1 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \end{bmatrix}$ ..... (2)

We consider $AM = \begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix} \begin{bmatrix} 1 & 0 & x_1 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \\ - & - & - & - & - \\ 0 & 0 & x_n & - & 1 \end{bmatrix}$ ..... (2)

$$= \begin{bmatrix} a_{11} & a_{12} & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n & - & a_{1n} \\ a_{21} & a_{22} & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n & - & a_{mn} \end{bmatrix}$$

Observe that the jth column is nothing but the given system on the left side of the equations.
So, they can be replaced by the right side values of the system as

$$= \begin{bmatrix} a_{11} & a_{12} & b_1 & - & a_{1n} \\ a_{21} & a_{22} & b_2 & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & b_m & - & a_{mn} \end{bmatrix} \quad \ldots\ldots (3)$$

When the jth column of the coefficient matrix is changed by the column matrix b, the resultant is denoted by B_j.

Provide feedback (0)

Step 5 of 5

(c) Considering the determinant on both sides of (2) and (3), we get

Provide feedback (0)

Step 5 of 5

(c) Considering the determinant on both sides of (2) and (3), we get

$$\text{Det}\left\{\begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix}\begin{bmatrix} 1 & 0 & x_1 & - & 0 \\ 0 & 1 & - & - & 0 \\ 0 & 0 & x_j & - & 0 \\ - & - & - & - & - \\ 0 & 0 & x_n & - & 1 \end{bmatrix}\right\} = \det\begin{bmatrix} a_{11} & a_{12} & b_1 & - & a_{1n} \\ a_{21} & a_{22} & b_2 & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & b_m & - & a_{mn} \end{bmatrix}$$

Using the result in (a), we follow that

$$\text{Det}\begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix} \times x_j = \det\begin{bmatrix} a_{11} & a_{12} & b_1 & - & a_{1n} \\ a_{21} & a_{22} & b_2 & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & b_m & - & a_{mn} \end{bmatrix}$$

$$\text{Therefore, } x_j = \frac{\det\begin{bmatrix} a_{11} & a_{12} & b_1 & - & a_{1n} \\ a_{21} & a_{22} & b_2 & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & b_m & - & a_{mn} \end{bmatrix}}{\det\begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix}}$$

This is nothing but the Cramer's rule.

CH4.4 4P

CH4.4 6P

Step 1 of 10

a) We first find the area of the parallelogram.

Suppose $OP = (x_1, y_1)$, $OQ = (x_2, y_2)$

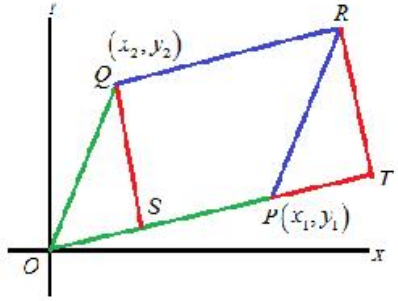
Extend the line PR such that it is parallel to OQ and is of length OQ.

Join QR to complete the parallelogram.

Suppose S is the point on OP such that QS is perpendicular to OP.

So, OQS form a right triangle.

From this, QS is the height and OS is the base.



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Step 2 of 10

Then the projection of OQ on OP is $OS = \dfrac{OQ \cdot OP}{|OP|} = \dfrac{(x_2, y_2)\cdot(x_1, y_1)}{\sqrt{x_1^2 + y_1^2}}$

$$= \frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2 + y_1^2}}$$

So, height of the right triangle is $QS = \sqrt{hypotenuse^2 - base^2}$

$$= \sqrt{\left(x_2^2 + x_1^2\right) - \left(\frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2 + y_1^2}}\right)^2}$$

Step 2 of 10

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$$= \sqrt{\left(x_2^2 + x_1^2\right) - \left(\frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2 + y_1^2}}\right)^2}$$

$$= \sqrt{\frac{\left(x_1^2x_2^2 + y_1^2y_2^2 + x_1^2y_2^2 + x_2^2y_1^2\right) - \left(x_1^2x_2^2 + y_1^2y_2^2 - 2x_1x_2y_1y_2\right)}{x_1^2 + y_1^2}}$$

$$= \sqrt{\frac{(x_1y_2 - x_2y_1)^2}{x_1^2 + y_1^2}}$$

$$= \frac{x_1y_2 - x_2y_1}{|OP|} \qquad \ldots\ldots (1)$$

Provide feedback (0)

Step 3 of 10

Further, we extend the vector OP to PT such that $QSTR$ is the rectangle.
Obviously, the right triangle OQS is identical to triangle PRT.
Consequently, the base of rectangle $= |ST| = |OP| \qquad \ldots\ldots (2)$
Area of the rectangle is base $\times$ height $= (1) \times (2) = x_1y_2 - x_2y_1$

$$= \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

We easily see that by removing the triangle PRT from the rectangle $QSRT$ and adding the triangle OQS, we get the parallelogram.
While the triangles PRT and OQS are identical, it follows that area of the rectangle $OSRT$ is equal to the area of the parallelogram $OPQR$.

Step 3 of 10

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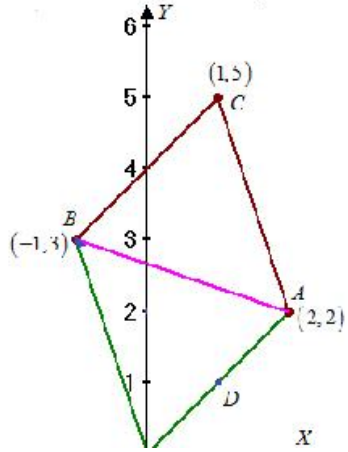
Thus, the area of the parallelogram $= \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$ with the adjoining sides $OP = (x_1, y_1)$, and $OQ = (x_2, y_2)$.

Provide feedback (0)

Step 4 of 10

Using the above discussion, we follow that OAB triangle is half $OABC$ parallelogram.



Step 4 of 10

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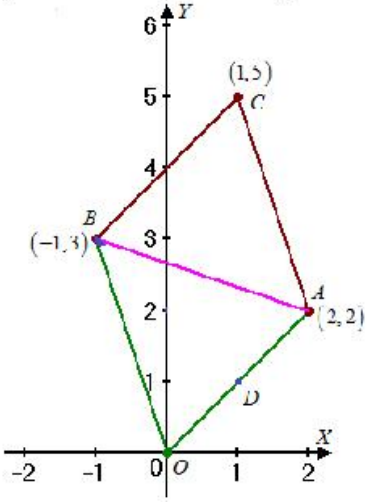


Provide feedback (0)

Step 5 of 10

The area of parallelogram $OABC = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}$

So, area of the triangle OAB is $\frac{1}{2}\begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}$

Provide feedback (0)

Step 6 of 10

(b) Suppose $N(x_3, y_3), P(x_1, y_1), Q(x_2, y_2)$ are three vertices

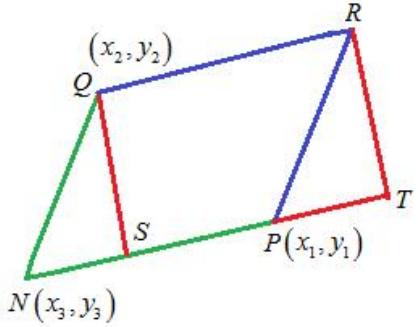
Step 7 of 10

Then $NP=(x_1-x_3,y_1-y_3)$, $NQ=(x_2-x_3,y_2-y_3)$ are the adjoining sides of the parallelogram and proceeding as in the above case, we get NS is the projection of NQ upon NP given by $NS=\dfrac{NQ\cdot NP}{|NP|}$

$$=\frac{(x_1-x_3,y_1-y_3)\cdot(x_2-x_3,y_2-y_3)}{\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}} \quad \ldots\ldots (3)$$

Provide feedback (0)

Step 8 of 10



Provide feedback (0)

Step 9 of 10

Consequently, the height of the right triangle NQS is QS

$$=\sqrt{\left\{(x_1-x_3)^2+(y_1-y_3)^2\right\}-\left\{\frac{(x_1-x_3,y_1-y_3)\cdot(x_2-x_3,y_2-y_3)}{\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}}\right\}^2}$$

$$(x_2-x_3)(y_1-y_3)-(x_1-x_3)(y_2-y_3)$$

Step 9 of 10

Consequently, the height of the right triangle NQS is QS

$$= \sqrt{\left\{(x_1-x_3)^2+(y_1-y_3)^2\right\}-\left\{\frac{(x_1-x_3,y_1-y_3)\cdot(x_2-x_3,y_2-y_3)}{\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}}\right\}^2}$$

$$= \frac{(x_2-x_3)(y_1-y_3)-(x_1-x_3)(y_2-y_3)}{|NP|} \qquad \ldots\ldots (4)$$

Proceeding to construct the rectangle in the above case, the base is NP and so, the area of the rectangle is $|NP|\times\frac{(x_2-x_3)(y_1-y_3)-(x_1-x_3)(y_2-y_3)}{|NP|}$

$$=(x_2-x_3)(y_1-y_3)-(x_1-x_3)(y_2-y_3)$$

Provide feedback (0)

Step 10 of 10

So, the area of the parallelogram NPQR is $(x_2-x_3)(y_1-y_3)-(x_1-x_3)(y_2-y_3)$

$$= x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Therefore, the area of the triangle $NPQ = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

In our case, replacing NPQ by $C(1,-4), A(2,2), B(-1,3)$, the area of the required triangle $= \boxed{\frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & -4 & 1 \end{vmatrix}}$

CH4.4 5P

Step 1 of 1

Consider volume of a box of length l, breadth b, height h is given by lbh.

Suppose if each side is made 't' times, then the volume is clearly $tl.tb.th = t^3lbh$

Now det A is volume of a box of n sides in an n - dimensional space. When we multiply A by 3, each row vector is made 3 times,

So, every side of the given box A is made 3 times its original length.

While the case is of 3 dimensions, we get

Volume of the new box = det 3A

$= 3^n.\det A$

Provide feedback (1)

CH4.4 7P

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CH4.4 6P

CH4.4 8P

Step 1 of 5

In order to guess the pivots, we proceed as follows:

$$|2| = 2$$

$$\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} = 6$$

$$\begin{vmatrix} 2 & 1 & 2 \\ 4 & 5 & 0 \\ 2 & 7 & 0 \end{vmatrix} = 36$$

Thus, the pivot entries must be $\boxed{2, 3, \text{ and } 6}$.

Provide feedback (0)

Step 2 of 5

Let us obtain the pivot entries by elimination:

Consider $\begin{bmatrix} 2 & 1 & 2 \\ 4 & 5 & 0 \\ 2 & 7 & 0 \end{bmatrix}$

Row 3 – Row 1 and Row 2 – 2Row 1 gives

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

Step 2 of 5

Let us obtain the pivot entries by elimination:

Consider $\begin{bmatrix} 2 & 1 & 2 \\ 4 & 5 & 0 \\ 2 & 7 & 0 \end{bmatrix}$

Row 3 – Row 1 and Row 2 – 2Row 1 gives

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & -4 \\ 0 & 6 & -2 \end{bmatrix}$$

Row 3 – 2Row 2 gives

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & -4 \\ 0 & 0 & 6 \end{bmatrix}$$

Thus, we confirm that the pivot elements are 2, 3, and 6.

Provide feedback (0)

Step 3 of 5

Consider the following determinant:

Step 3 of 5

Consider the following determinant:

$$B = \begin{vmatrix} 2 & 1 & 2 \\ 4 & 5 & 3 \\ 2 & 7 & 0 \end{vmatrix}$$

Provide feedback (0)

Step 4 of 5

In order to guess the pivots, we proceed as follows:

$$\begin{vmatrix} 2 \end{vmatrix} = 2$$

$$\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} = 6$$

$$\begin{vmatrix} 2 & 1 & 2 \\ 4 & 5 & 3 \\ 2 & 7 & 0 \end{vmatrix} = 0$$

Thus, the pivot entries must be $\boxed{2, 3, \text{ and } 0}$.

Provide feedback (0)

Step 5 of 5

Let us obtain the pivot entries by elimination:

Step 5 of 5

Let us obtain the pivot entries by elimination:

Consider $\begin{bmatrix} 2 & 1 & 2 \\ 4 & 5 & 3 \\ 2 & 7 & 0 \end{bmatrix}$

Row 3 – Row 1 and Row 2 – 2Row 1 gives

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 0 & 6 & -2 \end{bmatrix}$$

Row 3 – 2Row 2 gives

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, we confirm that the pivot elements are $\boxed{2, 3, \text{ and } 0}$.

Provide feedback (0)

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CH4.4 7P

Step 1 of 8

We need to find all the odd permutations of $\{1,2,3,4\}$

The odd permutations on four symbols are $\frac{1}{2}(4!)=12$ in number

They are

One inter change $(1,2,4,3),(1,3,4,2)(1,4,3,2),(2,1,3,4),(3,2,1,4),(4,2,3,1)$

Three interchanges $(2,3,4,1),(2,4,1,3),(3,1,4,2),(3,4,2,1),(4,1,2,3),(4,3,1,2)$

Provide feedback (0)

Step 2 of 8

The permutations that lead to $\det P=-1$ are odd permutations.
The row interchanges are shown by 1's in the respective places of the 4×4 matrix.
If only one interchange has taken place, then $\det P=(-1)^1=-1$ and so, P is an odd permutation.

1st and 2nd rows interchange $\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

3rd and 4th rows interchange $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$

Provide feedback (0)

Step 3 of 8

$$\begin{bmatrix}1&0&0&0\end{bmatrix}$$

CH4.4 9P

Step 3 of 8

$$2^{nd} \text{ and } 4^{th} \text{ rows interchange } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1^{st} \text{ and } 4^{th} \text{ rows interchange } \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 8

$$2^{nd} \text{ and } 3^{rd} \text{ rows interchange } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$1^{st} \text{ and } 3^{rd} \text{ rows interchange } \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

Three row interchange:

$$1234 \to 1243 \to 1423 \to 4123 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Step 5 of 8

Three row interchange:

$$1234 \to 1243 \to 1423 \to 4123 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$1234 \to 2134 \to 2143 \to 2413 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

$$1234 \to 1324 \to 3142 \to 3412 \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1234 \to 2134 \to 2314 \to 2134 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 8

$$1234 \to 1243 \to 1423 \to 1432 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 8

$$1234 \to 1243 \to 1423 \to 1432 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1234 \to 1324 \to 1342 \to 3142 \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

We easily see that $\det P = (-1)^{\text{number of row interchanges}}$

Provide feedback (0)

Step 8 of 8

$= (-1)^3$

$= -1$

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Step 1 of 3

A permutation is nothing but a bijective function.

So, we follow that $P^2 = P \circ P$ which is the composition of functions.

Given that P takes (1,2,3,4,5) to (5,4,1,2,3)

This can be interpreted as $P = \begin{bmatrix} & & & & 1 \\ & & & 1 & \\ 1 & & & & \\ & 1 & & & \\ & & 1 & & \end{bmatrix}$

That means $1 \to 5, 2 \to 4, 3 \to 1, 4 \to 2, 5 \to 3$

Provide feedback (0)

Step 2 of 3

When composition with P itself is considered, the resultant can be seen as

$1 \to 5 \to 3, 2 \to 4 \to 2, 3 \to 1 \to 5, 4 \to 2 \to 4, 5 \to 3 \to 1$

Or, this can simply be understood as $1 \to 3, 2 \to 2, 3 \to 5, 4 \to 4, 5 \to 1$

In other words, P^2 takes (1,2,3,4,5) to (3,2,5,4,1).

$$P^2 = \begin{bmatrix} & & 1 & & \\ & 1 & & & \\ & & & & 1 \\ & & & 1 & \\ 1 & & & & \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

Similarly, considering P as $1 \to 5, 2 \to 4, 3 \to 1, 4 \to 2, 5 \to 3$, we can reverse them as

$1 \to 3, 2 \to 4, 3 \to 5, 4 \to 2, 5 \to 1$

CH4.4 10P

This can be interpreted as $P = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & & \end{bmatrix}$

That means $1 \to 5, 2 \to 4, 3 \to 1, 4 \to 2, 5 \to 3$

Provide feedback (0)

Step 2 of 3

When composition with P itself is considered, the resultant can be seen as
$1 \to 5 \to 3, 2 \to 4 \to 2, 3 \to 1 \to 5, 4 \to 2 \to 4, 5 \to 3 \to 1$

Or, this can simply be understood as $1 \to 3, 2 \to 2, 3 \to 5, 4 \to 4, 5 \to 1$

In other words, P^2 takes (1,2,3,4,5) to (3,2,5,4,1).

$$P^2 = \begin{bmatrix} & & 1 & & \\ & 1 & & & \\ & & & & 1 \\ & & & 1 & \\ 1 & & & & \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

Similarly, considering P as $1 \to 5, 2 \to 4, 3 \to 1, 4 \to 2, 5 \to 3$, we can reverse them as
$1 \to 3, 2 \to 4, 3 \to 5, 4 \to 2, 5 \to 1$

This is nothing but P^{-1}.

So, P^{-1} takes (1,2,3,4,5) to (3,4,5,2,1).

Provide feedback (0)

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CH4.4 9P

Step 1 of 2

An odd permutation is nothing but the number of interchanges of the immediate entries in that permutation raised to the power of -1.

i.e., $\det P = (-1)^j$ where j is the number of interchanges odd and so, the permutation is odd.

$P^2 = P \circ P$ is nothing but the composition of functions while a permutation is a bijective function.

If there are j interchanges performed in the permutation P, then there will be $j + j = 2j$ interchanges performed in $P \circ P$

While $2j$ is even, we see that the number of interchanges in P^2 is even and so, $(-1)^{2j} = 1$ which confirms that P^2 is an even permutation.

Provide feedback (0)

Step 2 of 2

On the other hand, if the permutation P requires j interchanges, then we follow that to reverse these interchanges, we require j interchanges.

So, if j is odd, then we follow that both P and P^{-1} are odd permutations.

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Step 1 of 3

Note the following;

$$\begin{aligned} P(PA) &= P^2 A \\ P(P(PA)) &= P^3 A \\ &\vdots \\ P(P(\ldots(PA))) &= P^n A \end{aligned}$$

When a Permutation Matrix P is multiplied by itself, the resultant matrix is also a Permutation Matrix.

Thus, P, P^2, P^3, ..., P^n are all Permutation Matrices.

Provide feedback (0)

Step 2 of 3

When we consider n by n Permutation Matrices, we know that there are only $n!$ distinct Permutation Matrices.

Thus, if we go on multiplying by Permutation Matrices, there are bound to be some P^k and P^r such that $P^k = P^r$. Without loss of generality, let $k > r$. Thus, we have

$$\begin{aligned} P^r &= P^k \\ &= P^{r+k-r} \\ &= P^r P^{k-r} \end{aligned}$$

This clearly indicates that $P^{k-r} = I$.

Provide feedback (0)

Step 2 of 3

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This clearly indicates that $P^{k-r} = I$.

Provide feedback (0)

Step 3 of 3

Thus, when we multiply A by P, as many as $k-r$ times, we get

$$\begin{aligned} P^{k-r} A &= IA \\ &= A \end{aligned}$$

Thus, the first row of the matrix $P^{k-r} A$ must be same as that of the matrix A.

Provide feedback (0)

CH4.4 11P

Step 1 of 6

If A is 5 by 5 matrix with all $|a_{ij}| \le 1$, then we have to find $\det A \le$ ___ .

Suppose $\vec{v_i}$ be the ith column of A. Then we have

$$|\vec{v_i}| = \sqrt{a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 + a_{5i}^2}$$

We know that $|a_{ij}| \le 1$ for all i and j, so $a_{ij}^2 \le 1$ is true.

Provide feedback (0)

Step 2 of 6

Since $a_{ij}^2 \le 1$, so

$$\begin{aligned}\sqrt{a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 + a_{5i}^2} &\le \sqrt{1+1+1+1+1} \\ &\le \sqrt{5}\end{aligned}$$

Now consider $|\vec{v_i}| = \sqrt{a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 + a_{5i}^2}$, then we can write

$$|\vec{v_i}| \le \sqrt{5}$$

CH4.4 13P

Step 3 of 6

Now we have $|v_i| \le \sqrt{5}$, hence the edges of a 5-dimensional box spanned by the columns of A is less than $\sqrt{5}$.

The volume of such box, V, can be calculate as follows

$$\begin{aligned} V &\le \left(\sqrt{5}\right)^5 \\ &\le 25\sqrt{5} \\ &\le 55.091 \end{aligned}$$

Provide feedback (0)

Step 4 of 6

We know that $|\det A|$ is equivalent to the volume of 5-dimensional box spanned by the columns of A.

So, by using this fact we get

$$|\det A| \le \boxed{55.091}$$

Provide feedback (0)

Step 5 of 6

Provide feedback (0)

Step 5 of 6

Now, to find an upper bound on the determinant, we use the Big formula for the determinant

$$\det A = \sum_{all p's} \left(a_{1\alpha} a_{2\beta} \ldots a_{nv}\right) \det P$$

The sum determinant of the whole matrix is the sum of $n!$.

Provide feedback (0)

Step 6 of 6

We have considered A is 5 by 5 matrix with all $|a_{ij}| \le 1$, so the number of terms in the sum is

$$5! = 120$$

Since $|a_{ij}| \le 1$, so that all of the 120 terms in the sum is smaller than 1 .

Therefore, $|\det A| \le \boxed{120}$.

Provide feedback (0)

CH4.4 12P

Step 1 of 10

a) The given system of equations is

$$2x_1 + 5x_2 = 1$$
$$x_1 + 4x_2 = 2$$

We need to solve the given system by cramer's rule

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 10

Replacing the first and second columns of A with b, we get the matrices B_1 and B_2

$$B_1 = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}, B_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Now

$$\begin{aligned} \det(A) = |A| &= \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix} \\ &= (2)(4) - (5)(1) \\ &= 3 \end{aligned}$$

Provide feedback (0)

Step 3 of 10

And

$$\begin{aligned} \det(B_1) = |A_1| &= \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix} \\ &= (1)(4) - (2)(5) \\ &= -6 \end{aligned}$$

CH4.4 14P

Step 3 of 10

And

$$\det(B_1) = |A_1| = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}$$
$$= (1)(4) - (2)(5)$$
$$= -6$$
$$\det(B_2) = |A_2| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= (2)(2) - (1)(1)$$
$$= 3$$

Provide feedback (0)

Step 4 of 10

Thus, by cramers rule we have

$$x_1 = \frac{\det(B_1)}{\det(A)}$$
$$= \frac{-6}{3}$$
$$= -2$$
$$x_2 = \frac{\det(B_2)}{\det(A)}$$
$$= \frac{3}{3}$$
$$= 1$$

Thus, the solution for the given system is $\boxed{x_1 = -2 \text{ and } x_2 = 1}$

Provide feedback (0)

Step 5 of 10

Step 5 of 10

b) The given system of equations is

$$2x_1 + x_2 = 1$$
$$x_1 + 2x_2 + x_3 = 0$$
$$x_2 + 2x_3 = 0$$

We need to solve the given system by cramer's rule

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

Replacing the first, second and third columns of A with b we get the matrices B_1, B_2 and B_3

$$B_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} B_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \text{ And } B_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Now

$$\det(A) = |A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$
$$= 2(3) - (2)$$
$$= 4$$

Provide feedback (0)

Step 7 of 10

And

Step 7 of 10

And

$$\det(B_1) = |B_1| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$
$$= (3) - (0)$$
$$= 3$$
$$\det(B_2) = |B_2|$$

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Step 8 of 10

$$= \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{vmatrix}$$
$$= 2(0) - (2)$$
$$= -2$$
$$\det(B_3) = |B_3|$$
$$= \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$
$$= 2(0) - (0) + (1)$$
$$= 1$$

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Step 9 of 10

Thus, by cramers rule we have

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Step 9 of 10

Thus, by cramers rule we have

$$x_1 = \frac{\det(B_1)}{\det(A)}$$
$$= \frac{3}{4}$$
$$x_2 = \frac{\det(B_2)}{\det(A)}$$
$$= \frac{-2}{4}$$
$$= \frac{-1}{2}$$
$$x_3 = \frac{\det(B_3)}{\det(A)}$$
$$= \frac{1}{4}$$

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Step 10 of 10

Thus, the solution for the given system is

$$\boxed{x_1 = \frac{3}{4}, x_2 = -\frac{1}{2}, x_3 = \frac{1}{4}}$$

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CH4.4 13P

Step 1 of 7

a) The given system of equation is

$ax + by = 1$

$cx + dy = 0$

We need to solve for y by cramers rule

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 7

Replacing the first and second columns of A with b we get the matrices B_1 and B_2

$$B_1 = \begin{bmatrix} 1 & b \\ 0 & d \end{bmatrix} B_2 = \begin{bmatrix} a & 1 \\ c & 0 \end{bmatrix}$$

Now

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$

Provide feedback (0)

Step 3 of 7

We need not compute $\det(B_1)$ because we need to solve for y only

$$\det(B_2) = |B_2| = \begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}$$
$$= -c$$

Thus, by cramers rule

$$y = \frac{\det(B_2)}{\det(A)} = \frac{-c}{ad - bc}$$

CH4.4 15P

Step 3 of 7

We need not compute $\det(B_1)$ because we need to solve for y only

$$\det(B_2) = |B_2| = \begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}$$
$$= -c$$

Thus, by cramers rule

$$y = \frac{\det(B_2)}{\det(A)} = \frac{-c}{ad-bc}$$

Hence $\boxed{y = \frac{-c}{ad-bc}}$

Provide feedback (0)

Step 4 of 7

b) The given system of equations is

$$ax + by + cz = 1$$
$$dx + ey + fz = 0$$
$$gx + hy + iz = 0$$

We need to solve for y by cramers rule

Let us name the coefficient matrix with D

$$D = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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Step 5 of 7

Replacing the first, second and third columns of D with b

We get the matrices B_1, B_2 and B_3

$$\begin{bmatrix} a & b & 1 \end{bmatrix} \quad \begin{bmatrix} a & 1 & c \end{bmatrix}$$

Step 5 of 7

Replacing the first, second and third columns of D with b
We get the matrices B_1, B_2 and B_3

$$B_1 = \begin{bmatrix} a & b & 1 \\ d & e & 0 \\ g & h & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} a & 1 & c \\ d & 0 & f \\ g & 0 & i \end{bmatrix}$$

$$B_3 = \begin{bmatrix} a & b & 1 \\ d & e & 0 \\ g & h & 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Now

$$\begin{aligned} \det(D) &= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \\ &= a(ei - fh) - b(di - gf) + c(dh - eg) \\ &= aei - afh - bdi + bgf + cdh - ceg \end{aligned}$$

$$\begin{aligned} \det(B_2) &= |B_2| \\ &= \begin{vmatrix} a & 1 & c \\ d & 0 & f \\ g & 0 & i \end{vmatrix} \\ &= a(0) - (di - gf) + c(0) \\ &= gf - di \end{aligned}$$

Provide feedback (0)

Step 6 of 7

Now

$$\det(D) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a(ei - fh) - b(di - gf) + c(dh - eg)$$

$$= aei - afh - bdi + bgf + cdh - ceg$$

$$\det(B_2) = |B_2|$$

$$= \begin{vmatrix} a & 1 & c \\ d & 0 & f \\ g & 0 & i \end{vmatrix}$$

$$= a(0) - (di - gf) + c(0)$$

$$= gf - di$$

Provide feedback (0)

Step 7 of 7

Thus, by cramers rule we have

$$y = \frac{\det(B_2)}{\det(A)}$$

$$= \boxed{\frac{gf - di}{aei - afh - bdi + bgf + cdh - ceg}}$$

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CH4.4 14P

CH4.4 16P

Step 1 of 3

a) $2x_1 + 3x_2 = 1$

$4x_1 + 6x_2 = 1$

$$Ax = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = b$$

$|A| = 12 - 12 = 0$

So, Cramer's rule cannot confirm the solution of the given system.
But by multiplying the 1st equation with 2 and if 2nd is subtracted from it, we get $0 = 1$.
This is an absurdity.
In other words, the given system is called inconsistent.
Or, there is not solution to it.

Provide feedback (0)

Step 2 of 3

b) As in the case (a), $\det A = 0$ and so, Cramer's rule is not useful to solve the system.

$2x_1 + 3x_2 = 1$

$4x_1 + 6x_2 = 2$

On the other hand, we can easily see that the 2nd equation is a multiple of the 1st equation and so, the 2nd equation can be neglected and thus, we are left with $2x_1 + 3x_2 = 1$ only.

Provide feedback (0)

Step 3 of 3

It can be written as $x_1 = \dfrac{1 - 3x_2}{2}$

In other words, we can write $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

But by multiplying the 1st equation with 2 and if 2nd is subtracted from it, we get $0 = 1$.
This is an absurdity.
In other words, the given system is called inconsistent.
Or, there is not solution to it.

Provide feedback (0)

Step 2 of 3

b) As in the case (a), $\det A = 0$ and so, Cramer's rule is not useful to solve the system.

$$2x_1 + 3x_2 = 1$$
$$4x_1 + 6x_2 = 2$$

On the other hand, we can easily see that the 2nd equation is a multiple of the 1st equation and so, the 2nd equation can be neglected and thus, we are left with $2x_1 + 3x_2 = 1$ only.

Provide feedback (0)

Step 3 of 3

It can be written as $x_1 = \dfrac{1-3x_2}{2}$

In other words, we can write $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

Putting $x_2 = k$ a parameter, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + k \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

For infinite real values of k, there will be infinite solutions to $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Provide feedback (0)

CH4.4 15P

CH4.4 17P

Step 1 of 3

We know that the determinant is defined only for a square matrix.
So, from the given details, we follow that $Ax = b$ is a non homogeneous system of three linear equations in three variables seen as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

This can otherwise be written as $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \mathbf{b}$ where $\mathbf{a}_i$ is the ith column of the coefficient matrix A.

This equation can simply be written as $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$

Provide feedback (0)

Step 2 of 3

Let us consider $|\mathbf{b}\ \mathbf{a}_2\ \mathbf{a}_3| = |\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$

$= |\mathbf{a}_1 x_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + |\mathbf{a}_2 x_2\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + |\mathbf{a}_3 x_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$ By the properties of determinants.

$$\begin{aligned} &= x_1 |\mathbf{a}_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + x_2 |\mathbf{a}_2\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + x_3 |\mathbf{a}_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3| \\ &= x_1 |A| + x_2 (0) + x_3 (0) \\ &= x_1 |A| \end{aligned}$$

Consequently, $x_1 = \dfrac{|\mathbf{b}\ \ \mathbf{a}_2\ \ \mathbf{a}_3|}{|\mathbf{a}_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3|} = \dfrac{B_1}{|A|}$

Provide feedback (0)

Step 3 of 3

(b) Proceeding as above by using $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$ in the middle place

This can otherwise be written as $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = \mathbf{b}$ where $\mathbf{a}_i$ is the ith column of the coefficient matrix A.

This equation can simply be written as $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$

Provide feedback (0)

Step 2 of 3

Let us consider $|\mathbf{b}\ \mathbf{a}_2\ \mathbf{a}_3| = |\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$

$= |\mathbf{a}_1 x_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + |\mathbf{a}_2 x_2\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + |\mathbf{a}_3 x_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$ By the properties of determinants.

$$= x_1 |\mathbf{a}_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + x_2 |\mathbf{a}_2\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + x_3 |\mathbf{a}_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$$
$$= x_1 |A| + x_2 (0) + x_3 (0)$$
$$= x_1 |A|$$

Consequently, $x_1 = \dfrac{|\mathbf{b}\ \ \mathbf{a}_2\ \ \mathbf{a}_3|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|} = \dfrac{B_1}{|A|}$

Provide feedback (0)

Step 3 of 3

(b) Proceeding as above by using $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$ in the middle place of $[\mathbf{a}_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3]$, we get $x_2 = \dfrac{|\mathbf{a}_1\ \ \mathbf{b}\ \ \mathbf{a}_3|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|} = \dfrac{B_2}{|A|}$,

Similarly, $x_3 = \dfrac{|\mathbf{a}_1\ \ \mathbf{a}_2\ \ \mathbf{b}|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|} = \dfrac{B_3}{|A|}$

Provide feedback (0)

CH4.4 16P

Step 1 of 2

Suppose the given system is $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

It can be written as $Ax = b$.

We can write this as $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \mathbf{b}$ where $\mathbf{a}_i$ is the column of A. $\mathbf{b}$ is the column vector on the right of the given system.

If $\mathbf{a}_1x_1 + \mathbf{a}_2x_2 + \mathbf{a}_3x_3 = \mathbf{b}$ goes into the 3rd column to produce B_3, then we can write

$$\begin{aligned}
|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{b}| &= |\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_1x_1 + \mathbf{a}_2x_2 + \mathbf{a}_3x_3| \\
&= |\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_1x_1| + |\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_2x_2| + |\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3x_3| \quad \text{By properties of determinants} \\
&= x_1|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_1| + x_2|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_2| + x_3|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3| \quad \text{By the properties of determinants} \\
&= x_1(0) + x_2(0) + x_3|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3| \quad \text{If two columns are identical, then the determinant is zero.} \\
&= x_3|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|
\end{aligned}$$

Provide feedback (0)

CH4.4 18P

Step 2 of 2

So, $|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{b}| = x_3|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|$

Or, $x_3 = \dfrac{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{b}|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|}$

Similarly, $x_2 = \dfrac{|\mathbf{a}_1\ \mathbf{b}\ \mathbf{a}_2|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|}$

Provide feedback (0)

CH4.4 17P

Step 1 of 4

$$A=\begin{bmatrix}1 & 2 & 0\\ 0 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$$

To find the cofactor matrix or the adjoint of A, we follow the steps

$$1)\text{ Co factors of the entries of } A=\begin{bmatrix}\begin{vmatrix}3 & 0\\ 4 & 1\end{vmatrix} & \begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix} & \begin{vmatrix}0 & 3\\ 0 & 4\end{vmatrix}\\ \begin{vmatrix}2 & 0\\ 4 & 0\end{vmatrix} & \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} & \begin{vmatrix}1 & 2\\ 0 & 4\end{vmatrix}\\ \begin{vmatrix}2 & 0\\ 3 & 0\end{vmatrix} & \begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix} & \begin{vmatrix}1 & 2\\ 0 & 3\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}3 & 0 & 0\\ 0 & 1 & 4\\ 0 & 0 & 3\end{bmatrix}$$

Provide feedback (1)

CH4.4 19P

Step 2 of 4

2) Multiplying each entry with the sign of the position:

$$=\begin{bmatrix}(-1)^{1+1}3 & (-1)^{1+2}0 & (-1)^{1+3}0\\ (-1)^{2+1}0 & (-1)^{2+2}1 & (-1)^{2+3}4\\ (-1)^{3+1}0 & (-1)^{3+2}0 & (-1)^{3+3}3\end{bmatrix}$$

$$C=\begin{bmatrix}3 & 0 & 0\\ 0 & 1 & -4\\ 0 & 0 & 3\end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

3) Transposing this matrix, we get C^T.

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 3 \end{bmatrix}$$

Also, the determinant of the given matrix is $|A| = 1\begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix} - 0\begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix} + 0\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix}$

$$= 3$$

$$A^{-1} = \frac{C^T}{\det A}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

b) $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Clearly, $\det A = 3 + 1$

$$= 4$$

(A is tridiagonal matrix $-1, 2, -1$)

$$C = \begin{bmatrix} 3 & -(-2) & (1-0) \\ -(-2) & 4 & -(-2) \\ 1-0 & -(-2) & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

b) $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Clearly, $\det A = 3+1$
$= 4$

(A is tridiagonal matrix $-1, 2, -1$)

$$C = \begin{bmatrix} 3 & -(-2) & (1-0) \\ -(-2) & 4 & -(-2) \\ 1-0 & -(-2) & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

While C is symmetric, its transpose is also equal to C and thus, $A^{-1} = \frac{C^T}{|A|}$

$$= \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Provide feedback (0)

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CH4.4 18P

Step 1 of 2

Suppose $A = \begin{bmatrix} a_{11} & a_{12} & - & a_{1n} \\ a_{21} & - & - & a_{2n} \\ - & - & - & - \\ a_{n1} & a_{n2} & - & a_{nn} \end{bmatrix}$

The cofactor of each entry a_{ij} is $A_{ij} = 0$

So, the determinant of A is $(-1)^{1+1} a_{11}A_{11} + (-1)^{1+2} a_{12}A_{12} + \ldots + (-1)^{1+n} a_{1n}A_{1n}$

$$= \sum_{j=1}^{n} (-1)^{1+j} a_{1j} (0)$$

$$= 0$$

Provide feedback (0)

Step 2 of 2

Now, the inverse of A is given by $A^{-1} = \dfrac{C^T}{|A|}$

$$= \frac{C^T}{0} \text{ does not exist.}$$

Provide feedback (0)

CH4.4 20P

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CH4.4 19P

CH4.4 21P

Step 1 of 3

We have $A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$

To find the cofactor matrix and the transpose, we follow

(1) We replace each entry of A by its co – factor:

$$\begin{bmatrix} \begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & 0 \\ -3 & 1 & 1 \\ -6 & -2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 3

(2) Multiplying each entry with the sign of its position

$$= \begin{bmatrix} (-1)^{1+1} 6 & (-1)^{1+2} 3 & (-1)^{1+3} 0 \\ (-1)^{2+1}(-3) & (-1)^{2+2} 1 & (-1)^{2+3} 1 \\ (-1)^{3+1}(-6) & (-1)^{3+2}(-2) & (-1)^{3+3} 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 3

(2) Multiplying each entry with the sign of its position

$$= \begin{bmatrix} (-1)^{1+1}6 & (-1)^{1+2}3 & (-1)^{1+3}0 \\ (-1)^{2+1}(-3) & (-1)^{2+2}1 & (-1)^{2+3}1 \\ (-1)^{3+1}(-6) & (-1)^{3+2}(-2) & (-1)^{3+3}1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

(3) We transpose this matrix to get $C^T = \begin{bmatrix} 6 & 3 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$

So, $AC^T = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 3 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= 3I$$

$$= \det A.I$$

$$\Rightarrow \det A = \boxed{3}$$

Provide feedback (0)

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CH4.4 20P

Step 1 of 2

If A is a $n \times n$ matrix, then $A^{-1} = \dfrac{C^T}{|A|}$ ..... (1)

We know that every cofactor A_{ij} of the entries of A.

So, multiplying each of them with their respective sign $(-1)^{i+j}$ and then transposing the matrix, we get C^T

(1) can be written as $|A|A^{-1} = C^T$

Or, using $\det A = 1$, we get $A^{-1} = C^T$ ..... (2)

Consequently, $|A^{-1}| = |C^T|$

$= |A^{n-1}|$

$= |A|^{n-1}$ By the properties of determinants.

$= 1$ ..... (3)

Provide feedback (0)

CH4.4 22P

Step 2 of 2

While the cofactor matrix of A is equal to A^{-1}, we follow that the cofactor matrix of A^{-1} is nothing but A.

But the cofactor matrix of C^T is C itself.

Therefore, $\boxed{A = C}$.

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CH4.4 21P

Step 1 of 2

We have $A^{-1}A = I$

$$\Rightarrow \frac{C^T}{|A|}A = I$$

$$\Rightarrow C^T A = |A| I$$

$$= |A| \begin{bmatrix} 1 & 0 & - & 0 \\ 0 & 1 & - & 0 \\ - & - & - & - \\ 0 & 0 & - & 1 \end{bmatrix}$$

$$= \begin{bmatrix} |A| & 0 & - & 0 \\ 0 & |A| & - & 0 \\ - & - & - & - \\ 0 & 0 & - & |A| \end{bmatrix}$$

Provide feedback (0)

Step 2 of 2

Applying determinant on both sides, we get $|C^T A| = |A|^n$

By the properties of determinants, we get $|C^T||A| = |A|^n$

$$|C^T| = |A|^{n-1}$$

$\therefore |C| = |A|^{n-1}$

Provide feedback (0)

CH4.4 23P

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We know that,

$$\det A = (\det C)^{\frac{1}{n-1}}$$

In this case $n = 4$.

Therefore, we get,

$$A^{-1} = \frac{C^T}{\det A}$$

The matrix A can be obtained by inverting A^{-1}.

Consider the matrix A

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

Therefore, we get,

$$C = \begin{bmatrix} \begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & 1 \\ -6 & 2 & 1 \end{bmatrix}$$

C^T

By using the formula $A^{-1} = \frac{C^T}{\det A}$, we get,

$$
\begin{aligned}
A^{-1} &= \frac{C^T}{\det A} \\
&= \frac{\begin{bmatrix} 6 & -3 & 0 \\ 6 & 1 & 0 \\ -6 & -2 & 1 \end{bmatrix}^T}{\det \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}} \\
&= \frac{\begin{bmatrix} 6 & 6 & -6 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}{\begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}} \\
&= \frac{\begin{bmatrix} 6 & 6 & -6 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}{6-3+0} \\
&= \frac{1}{3}\begin{bmatrix} 6 & 6 & -6 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

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Step 2 of 2

Thus, we can A^{-1} by using the formula $A^{-1} = \frac{C^T}{\det A}$ and by inverting A^{-1}

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Step 1 of 2

If A is a square matrix with integer entries and determinant A is not zero, then

$$A^{-1} = \frac{C^T}{\det A}$$

If $\det A$ is -1 or 1, then $A^{-1} = \frac{C^T}{(-1)}$ or $A^{-1} = \frac{C^T}{1}$

In either case, we have $A^{-1} = C^T$ or $A^{-1} = -C^T$

We know that the cofactor of each entry in A with integer entries is also an integer and so, $A^{-1} = C^T$ or $A^{-1} = -C^T$ are matrices with integer entries.

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Step 2 of 2

$$A = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix}$$

$$\begin{aligned} \det A &= 6 - 5 \\ &= 1 \end{aligned}$$

the cofactors of A are $C_{11} = 2, C_{12} = -1, C_{21} = -5, C_{22} = 3$ which are all integers

$$\begin{aligned} A^{-1} &= \frac{1}{\det A}.C^T \\ &= C^T \\ &= \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \end{aligned}$$

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CH4.4 25P

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CH4.4 24P

CH4.4 26P

Step 1 of 2

The determinant of a lower triangular matrix is nothing but the product of diagonal entries.

Given that the determinant of $L = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$ is not zero.

That means $acf \neq 0$

So, $a \neq 0, c \neq 0, f \neq 0$

We can simply see that the cofactors of b, d, e are $-\begin{vmatrix} 0 & 0 \\ e & f \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ c & a \end{vmatrix}, -\begin{vmatrix} a & 0 \\ b & 0 \end{vmatrix}$ respectively.

$L^{-1} = \dfrac{C^T}{\det L} = \dfrac{1}{acf}\begin{bmatrix} cf & 0 & 0 \\ -bf & af & 0 \\ be-cd & -ae & ac \end{bmatrix}$ is also a lower triangular matrix.

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Step 2 of 2

b) $S = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$ is a symmetric matrix

The cofactor matrix of S is $C^T = \begin{bmatrix} (-1)^{1+1}\begin{vmatrix} c & e \\ e & f \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} b & e \\ d & f \end{vmatrix} & (-1)^{1+3}\begin{vmatrix} b & c \\ d & e \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} b & d \\ e & f \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} a & d \\ d & f \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} a & b \\ d & e \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} b & d \\ c & e \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} a & d \\ b & e \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} a & b \\ b & c \end{vmatrix} \end{bmatrix}$

$$= \begin{bmatrix} cf-e^2 & de-bf & be-cd \\ de-bf & af-d^2 & bd-ae \end{bmatrix}$$

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Step 2 of 2

b) $S=\begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$ is a symmetric matrix

The cofactor matrix of S is $C^T=\begin{bmatrix} (-1)^{1+1}\begin{vmatrix} c & e \\ e & f \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} b & e \\ d & f \end{vmatrix} & (-1)^{1+3}\begin{vmatrix} b & c \\ d & e \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} b & d \\ e & f \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} a & d \\ d & f \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} a & b \\ d & e \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} b & d \\ c & e \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} a & d \\ b & e \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} a & b \\ b & c \end{vmatrix} \end{bmatrix}$

$$=\begin{bmatrix} cf-e^2 & de-bf & be-cd \\ de-bf & af-d^2 & bd-ae \\ be-cd & bd-ae & ac-b^2 \end{bmatrix}$$

Observe that the ij^{th}, and ji^{th} entries of this matrix are same for every $i \neq j$. So, this matrix is symmetric.

Further, the determinant of S is $a\left(cf-e^2\right)+b\left(de-b^2\right)+d\left(be-cd\right)$

$$=acf+2bde-ae^2-d^2\left(b+c\right)$$

Consequently, $S^{-1}=\dfrac{1}{\det S}C^T$

$$=\frac{1}{acf+2bde-ae^2-d^2\left(b+c\right)}\begin{bmatrix} cf-e^2 & de-bf & be-cd \\ de-bf & af-d^2 & bd-ae \\ be-cd & bd-ae & ac-b^2 \end{bmatrix}$$ is also symmetric.

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CH4.4 25P

Step 1 of 5

We consider a 5×5matrix by name A.
A cofactor of each entry a_{ij} of A is nothing but the determinant B_{ij} of 4×4 sub matrix obtained by leaving the ith row and j^{th} column in which a_{ij} exists.

Also, B_{ij} is to be multiplied with $(-1)^{i+j}$ and the entire matrix is to be transposed to get the adjoint of the given matrix A.
While there are 25 entries in A, we see that there are 25 cofactors in the adjoint matrix.

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Step 2 of 5

Again, each B_{ij} is a 4×4 matrix and so, it has 16 entries.
Leaving each entry B_{ij} and the row and column in which it is standing, we get a sub matrix C_{ij} of order 3×3 whose determinant is the cofactor of B_{ij}.

Also, we have to multiply each cofactor with $(-1)^{i+j}$ where i is the number of row and j is the number of column in which the entry standing.

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Step 3 of 5

The above discussion repeats and each 3×3 sub matrix has nine 2×2 matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ whose determinant is $ad-bc$ is the cofactor of each entry in the 3×3 matrix.

Each entry has to be multiplied with $(-1)^{i+j}$ as discussed above.

We observe that $(-1)^{i+j}(ad-bc)$ requires one additive operation and three multiplications.
One column or row of any 3×3 sub matrix is enough to calculate the determinant of the sub matrix of size 3×3

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multiplications.

One column or row of any 3×3 sub matrix is enough to calculate the determinant of the sub matrix of size 3×3

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Step 4 of 5

In a column, there will be 3 entries and so, 3 additions and 9 multiplications are required.

In a 4×4 matrix, there will be 4 entries in a column or a row and so, 4 times the above process is required. So, the number of operations is 4×3 additions and 4×9 multiplication.

In a 5×5 matrix, there will be 5 entries in a column or a row and so, $5\times4\times3$ additions and $5\times4\times9$ multiplications are required

So, the total number of calculations required to find the determinant of the 5×5 matrix is 240.

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Step 5 of 5

On the other hand, each 4×4 sub matrix of the given matrix requires $4\times3=12$ additions and $4\times9=36$ multiplications.

So, all the 25 entries require $25(12+36)=1200$ calculations.

Transposing requires 10 operations.

Putting all the number of calculations and operations together we require 1440 additive and multiplicative operations and 10 entry replacing operations to find the inverse of the 5×5 matrix.

So, when compared to the Gauss – Jordan method to find the inverse, this method requires considerably more operations.

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Step 1 of 4

a) Area of parallelogram with edges $v=(3,2)$ and $w=(1,4)$ is

$$\begin{vmatrix} u \\ v \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$
$$= 12-2$$
$$= 10 \text{ sq.units}$$

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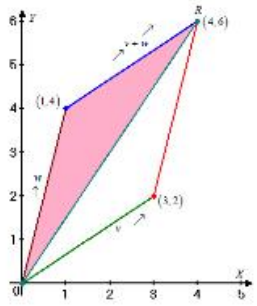
Step 2 of 4

(b) The area of the triangle with sides $v, w, v+w$ is half the area of the parallelogram with the adjacent edges v, w

$$= \frac{10}{2}$$
$$= 5 \text{ sq.units}$$

We draw the vectors $v = (3, 2)$ and $w = (1, 4)$ starting from the origin $(0, 0)$.
We extend a line from v parallel to w whose length is equal to that of w in the positive direction of w with the end point R.
We join OR to get the vector $v + w$.



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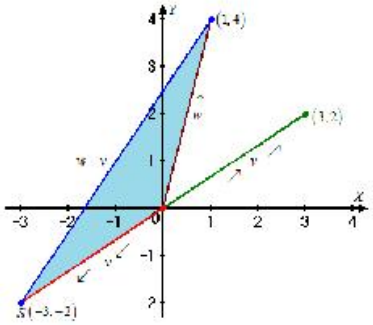
Step 3 of 4

c) Observe that $-v$ is the vector in the negative direction of v and with the same magnitude that of v.
Also, the vector $w-v$ is a vector that joins OS where S is the end point of the vector drawn from w in the negative direction of v and is parallel to v of magnitude v.



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Step 4 of 4

The area of the triangle with sides $v, w, w-v$ is also equal to half the area of the parallelogram with adjacent edges v and w.
Therefore, the required area is 5 sq.units

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Step 1 of 3

The volume of the box with adjacent edges meet at O (0,0,0) and the end vertices

(x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3) is $\det\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

Given that the edges of the box are from $(0,0,0)$ to $(3,1,1), (1,3,1), (1,1,3)$ respectively.

$$\begin{aligned} \text{So, its volume} &= \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix} \\ &= 3(9-1)-(3-1)+(1-3) \\ &= 20 \text{ cubic units} \end{aligned}$$

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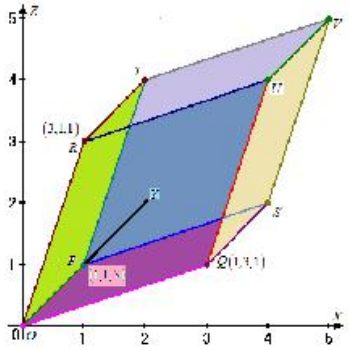
Step 2 of 3

A box with three adjacent sides meeting at the origin will have 6 faces.



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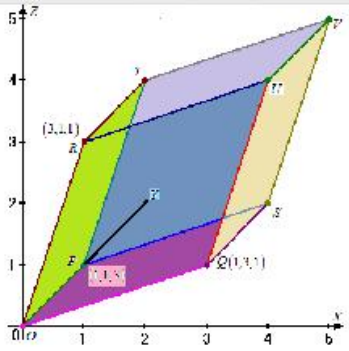

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Step 3 of 3

We see that OPQS, RTUV, OPRT, QSVU, ORUQ and PTVS are the six faces of the box.
We easily see that the opposite sides of this box are parallel and so have the same area.

On the other hand, the length of each given vectors is equal to $\sqrt{1^2+3^2+1^2}=\sqrt{11}$
So, all the parallelograms are having equal areas.
Area of OPQS =

$$
\begin{aligned}
&= \left|(3\mathbf{i}+\mathbf{j}+\mathbf{k})\times(\mathbf{i}+3\mathbf{j}+\mathbf{k})\right| \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} \\
&= \left|-2\vec{i}-2\vec{j}+8\vec{k}\right| \\
&= \sqrt{4+4+64} \\
&= \sqrt{72}\ \text{sq.units}
\end{aligned}
$$

Therefore, each side of the box is of $6\sqrt{2}$ square units area.

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Step 1 of 3

(a) The corners of a triangle are $(2,1),(3,4)$ and $(0,5)$

the area of triangle

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & 4 & 1 \\ 0 & 5 & 1 \end{vmatrix}$$

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Step 2 of 3

$$A = \frac{1}{2}\begin{vmatrix} 2 & -4 & 0 \\ 3 & -3 & 0 \\ 0 & 5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(-3)+4(3)\right]$$

$$= \frac{1}{2}\left|-6+12\right|$$

$$= \frac{1}{2}(6)$$

$$= 3.\ \text{sq.units}$$

Provide feedback (3)

Step 3 of 3

(b) By adjoining the vertex $(-1,0)$,

Area lopsided =3+new triangle area

$$_1\begin{vmatrix} 2 & 1 & 1 \end{vmatrix}$$

Provide feedback (0)

Step 2 of 3

$$A=\frac{1}{2}\begin{vmatrix}2 & -4 & 0\\ 3 & -3 & 0\\ 0 & 5 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[2(-3)+4(3)\right]$$

$$=\frac{1}{2}\left|-6+12\right|$$

$$=\frac{1}{2}(6)$$

$$=3.\ \text{sq.units}$$

Provide feedback (3)

Step 3 of 3

(b) By adjoining the vertex $(-1,0)$,

Area lopsided =3+new triangle area

$$=3+\frac{1}{2}\begin{vmatrix}2 & 1 & 1\\ 0 & 5 & 1\\ -1 & 0 & 1\end{vmatrix}$$

$$=3+\frac{1}{2}\left[2(5)-1(1)+1(5)\right]$$

$$=3+\frac{1}{2}[14]$$

$$=3+7$$

$$\boxed{\text{Area lopsided}=10\ \text{sq.units}}$$

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CH4.4 29P

Step 1 of 4

The area of parallelogram with edges $(2,1),(2,3)$ $= \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}$

$= 6-2$

$= 4$ sq.units

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Step 2 of 4

The area of parallelogram with edges $(2,2),(1,3)$

$= \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= 6-2$

$= 4$ sq.units

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Step 3 of 4

Observe that the edges of 2nd parallelogram are nothing but columns of 1st parallelogram the areas are equal since determinant of transpose of a matrix is equal to determinant of the matrix. These parallelogram look as below in figure.

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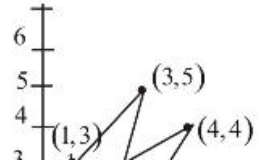
Step 4 of 4



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$= 4$ sq.units

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Step 3 of 4

Observe that the edges of 2nd parallelogram are nothing but columns of 1st parallelogram the areas are equal since determinant of transpose of a matrix is equal to determinant of the matrix. These parallelogram look as below in figure.

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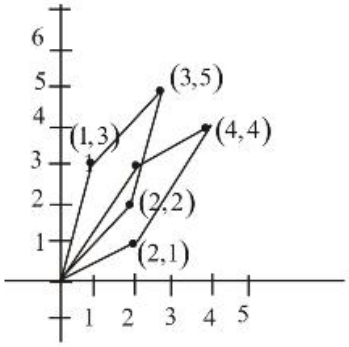
Step 4 of 4



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Step 1 of 4

Given Hadamard matrix H has orthogonal rows

$$\det H = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 \\ 0 & -2 & -2 & 0 \\ 0 & -2 & 0 & -2 \end{vmatrix} \text{ adding } -1 \text{ time the first row to the second, third and fourth}$$

rows.

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Step 2 of 4

$$\det H = \begin{vmatrix} 0 & -2 & -2 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{vmatrix} \text{ expanding the determinant by 1 column.}$$

$$= (-2)^3 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

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Step 3 of 4

$$\det H = -8 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= 8 \begin{vmatrix} 1 & 1 \end{vmatrix}$$

$$= (-2)^3 \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

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Step 3 of 4

$$\det H = -8 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= 8 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= 8(2)$$

$$= 16$$

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Step 4 of 4

The edges of the hypercube have length $\sqrt{1+1+1+1} = 2$.
Volume of a hypercube in

$\mathbb{R}^4 = 2^4$

$= 16$

$H/2$ has orthonormal columns. Then $\det(H/2) = 1$ leads again to $\det H = 16$.

Thus, the volume of a hypercube in $\mathbb{R}^4$ is 16 cubn.units.

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Step 1 of 2

If the columns of a 4 by 4 matrix have lengths L_1, L_2, L_3, L_4 the largest possible value of the determinant, is the largest possible volume when the lengths of its edges are given. The largest possible volume is obtained only when the edges are mutually orthogonal and hence we note that the largest possible value of determinant 4×4 whose column lengths are $L_1, L_2, L_3, L_4 = L_1 . L_2 . L_3 . L_4$.

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Step 2 of 2

When all the entries are 1 or -1, each length is equal to $\sqrt{1+1+1+1} = 2$ and hence maximum determinant is $2^4 = 16$ units.

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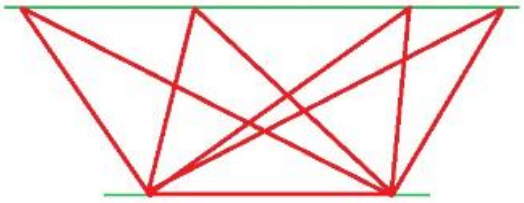
Step 1 of 4

Also, we know that the areas of the triangles, which have common base and same height are same.



The various triangles, shown above, have same base and the same height and therefore, their areas are equal.

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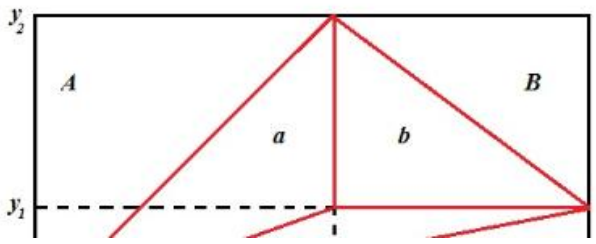
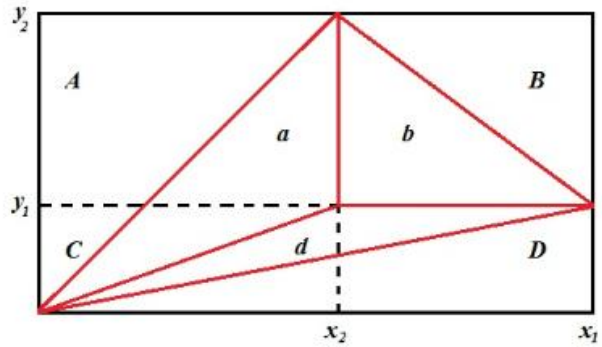
Step 2 of 4

Let $x_1 > x_2$ and $y_2 > y_1$. Consider the figure, drawn below. We have removed the area of a smaller rectangle $(x_2 y_1)$ from a bigger rectangle $(x_1 y_2)$. After the removal, the remaining part is divided into three parts, as shown:



Step 2 of 4

Let $x_1 > x_2$ and $y_2 > y_1$. Consider the figure, drawn below. We have removed the area of a smaller rectangle $(x_2 y_1)$ from a bigger rectangle $(x_1 y_2)$. After the removal, the remaining part is divided into three parts, as shown:



Here C is the removed rectangle. From the properties of the triangle, which we discussed above, it is clear that

$$A(\square A) = 2A(\Delta a)$$
$$A(\square B) = 2A(\Delta b)$$
$$A(\square D) = 2A(\Delta d)$$

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Step 3 of 4

Step 3 of 4

This gives,

$$2A(\Delta a)+2A(\Delta b)+2A(\Delta d)=A(\square A)+A(\square B)+A(\square D)$$
$$2\left(A(\Delta a)+A(\Delta b)+A(\Delta d)\right)=A(\square A)+A(\square B)+A(\square D)$$
$$\begin{aligned}A(\Delta a)+A(\Delta b)+A(\Delta d)&=\frac{1}{2}\left(A(\square A)+A(\square B)+A(\square D)\right)\\&=\frac{1}{2}(x_1y_2-x_2y_1)\end{aligned}$$

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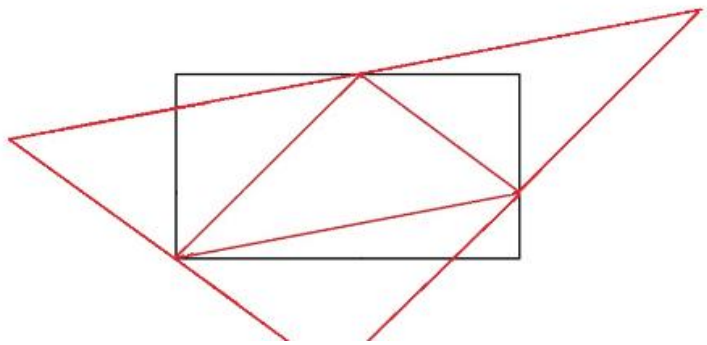
Step 4 of 4

Thus, the area of the biggest triangle in the figure is equal to half the area of the bigger rectangle minus the removed the rectangle. Now to obtain, the parallelogram, we need to attach another congruent triangle to the biggest triangle. This will form a parallelogram. Observe the figure, drawn below. There are three parallelograms. Each has the same area, equal to $x_1y_2-x_2y_1$.



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Step 1 of 4

When the edge vectors a, b, c are perpendicular, the volume of the box is $\|a\|$ time $\|b\|$ times $\|c\|$ .and

When $A = \left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is 3 by 3 matrix.

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Step 2 of 4

With mutually perpendicular vectors $\vec{a}, \vec{b}, \vec{c}$ we have

$$A^T A = \begin{bmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{bmatrix} \left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

$$= \begin{bmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{bmatrix}$$

$$= \begin{bmatrix} \|a\|^2 & 0 & 0 \\ 0 & \|b\|^2 & 0 \\ 0 & 0 & \|c\|^2 \end{bmatrix}$$

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Step 3 of 4

And hence

$$\det\left(A^T A\right) = \det A^T . \det A$$

$$= \det A . \det A$$

CH4.4 35P

$$= \begin{bmatrix} 0 & \|b\| & 0 \\ 0 & 0 & \|c\|^2 \end{bmatrix}$$

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Step 3 of 4

And hence

$$\det\left(A^T A\right) = \det A^T . \det A$$
$$= \det A . \det A$$
$$\det\left(A^T A\right) = \left(\det A\right)^2$$

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Step 4 of 4

$$\det\left(A^T A\right) = \|a\|^2 \|b\|^2 \|c\|^2$$
$$\Rightarrow \det A = \|a\|\|b\|\|c\|$$

Thus,

$$\boxed{\det A^T A = \|a\|^2 \|b\|^2 \|c\|^2}$$
$$\boxed{\det A = \|a\|\|b\|\|c\|}$$

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Step 1 of 2

In 2 dimensional cubes we have 4 corners and 3 edges and one face.
In 3 dimensional cube we have 8 corners and 12 edges and 6 faces.
In n dimensional cube we have 2^n corners and $n.2^{n-1}$edges (since each corner is adjoined $n-1$ other corners to give n edges).

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Step 2 of 2

At each corner $n-1$ edges give rise to a face of $n-1$ dimensions so that n faces are joined at a corner.
Each face is counted 2^{n-1} times. So, the number of faces is $2n$.
The n cube whose edges are the rows of 21 has volume equal to 2^n since 1 has mutually orthogonal columns.

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Step 1 of 4

The triangle with corners $(0,0),(1,0),(0,1)$ has area $=\frac{1}{2}\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}$

$$=\frac{1}{2}(1-0)$$

$$=\frac{1}{2}$$

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Step 2 of 4

The pyramid with four corners $(0,0,0),(1,0,0),(0,1,0),(0,0,1)$

$$\text{Volume of the pyramid}=\frac{1}{6}\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$$

$$=\frac{1}{6}\left[1(1-0)\right]$$

$$=\frac{1}{6}$$

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Step 3 of 4

The pyramid with five corners in $\mathbb{R}^4$ i.e. the pyramid in $\mathbb{R}^4$ with five corners at $(0,0,0,0)$ and the rows of 1 has volume $\frac{1}{8}$ cubic units.

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$$= \frac{1}{6}\left[1(1-0)\right]$$
$$= \frac{1}{6}$$

Provide feedback (0)

Step 3 of 4

The pyramid with five corners in $\mathbb{R}^4$ i.e. the pyramid in $\mathbb{R}^4$ with five corners at $(0,0,0,0)$ and the rows of 1 has volume $\frac{1}{8}$ cubic units.

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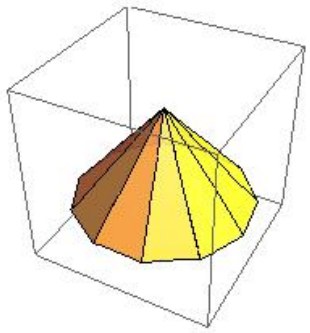
Step 4 of 4

Visual representation of pyramid



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Step 1 of 3

Given that
Polar coordinates satisfy $x = r\cos\theta$ and $y = r\sin\theta$. Polar area $Jdrd\theta$ includes J:

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

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Step 2 of 3

$$\frac{\partial x}{\partial r} = \cos\theta,\ \frac{\partial x}{\partial \theta} = -r\sin\theta,\ \frac{\partial y}{\partial r} = \sin\theta,\ \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

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Step 3 of 3

The two columns are orthogonal their lengths are $\boxed{\sqrt{\cos^2\theta + \sin^2\theta} = 1}$

And $\sqrt{r^2\left(\cos^2\theta + \sin^2\theta\right)} = \sqrt{r^2}$
$= r$

Thus $\boxed{J = r}$

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Step 1 of 5

Spherical coordinates P, ϕ, θ given

$x = P\sin\phi\cos\theta, y = P\sin\phi\sin\theta, z = P\cos\phi$

Now

$x = P\sin\phi\cos\theta$

$$\Rightarrow \frac{\partial x}{\partial P} = \sin\phi\cos\theta$$

$$\frac{\partial x}{\partial \phi} = P\cos\phi\cos\theta$$

$$\frac{\partial x}{\partial \theta} = -P\sin\phi\sin\theta$$

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Step 2 of 5

$y = P\sin\phi\sin\theta$

$$\Rightarrow \frac{\partial y}{\partial P} = \sin\phi\sin\theta$$

$$\frac{\partial y}{\partial \theta} = P\sin\phi\cos\theta$$

$$\frac{\partial y}{\partial \phi} = P\cos\phi\sin\theta$$

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Step 3 of 5

And

$Z = P\cos\phi$

$$\Rightarrow \frac{\partial z}{\partial P} = \cos\phi$$

$$\frac{\partial z}{\partial \phi} = -P\sin\phi$$

CH4.4 39P

$$\frac{\partial z}{\partial \phi} = -P\sin\phi$$

$$\frac{\partial z}{\partial \theta} = 0$$

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Step 4 of 5

So, the Jacobian matrix of partial derivatives is

$$\det J = \begin{vmatrix} \frac{\partial x}{\partial P} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial P} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial Z}{\partial \phi} & \frac{\partial Z}{\partial \phi} & \frac{\partial Z}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} \sin\phi\cos\theta & P\cos\phi\cos\theta & -P\sin\phi\sin\theta \\ \sin\phi\sin\theta & P\cos\phi\sin\phi & P\sin\phi\cos\theta \\ \cos\phi & -P\sin\phi & 0 \end{vmatrix}$$

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Step 5 of 5

The columns are mutually orthogonal and length of columns are given by

$$\sqrt{\sin^2\phi\cos^2\theta + \sin^2\phi\sin^2\theta + \cos^2\theta} = 1$$

$$\sqrt{P\left(\cos^2\phi\cos^2\theta + \cos^2\phi\sin^2\theta + \sin^2\phi\right)} = P$$

$$\sqrt{P^2\sin^2\phi\sin^2\phi + P^2\sin^2\phi\cos^2\phi} = P\sin\phi$$

$$\therefore \det J = 1.P.P.\sin\phi$$

$$= P^2\sin\phi$$

And hence $\boxed{dV = P^2\sin\phi dP d\phi d\theta}$

CH4.4 38P

Step 1 of 5

The matrix that connects r, θ to x, y where polar coordinates satisfy $x = r\cos\theta$ and $y = r\sin\theta$. Polar are $J\,drd\theta$ includes J:

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial x}{\partial r} = \cos\theta, \quad \frac{\partial x}{\partial \theta} = -r\sin\theta, \quad \frac{\partial y}{\partial r} = \sin\theta, \quad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

Provide feedback (0)

Step 2 of 5

And inverting the matrix we get

$$J^{-1} = \begin{vmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & \sin\theta \\ -\frac{\sin\theta}{r} & \frac{\cos\theta}{r} \end{vmatrix}$$

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Step 3 of 5

So that

$$\frac{\partial r}{\partial x} = \cos\theta, \frac{\partial r}{\partial y} = \sin\theta$$

CH4.4 40P

Step 3 of 5

So that

$$\frac{\partial r}{\partial x}=\cos\theta,\ \frac{\partial r}{\partial y}=\sin\theta$$

$$\frac{\partial \theta}{\partial x}=\frac{-\sin\theta}{r},\ \frac{\partial \theta}{\partial y}=\frac{\cos\theta}{r}$$

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Step 4 of 5

Therefore,

$$\frac{\partial x}{\partial r}\cdot\frac{\partial r}{\partial x}+\frac{\partial x}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}$$

$$=(\cos\theta)(\cos\theta)+(-r\sin\theta)\left(\frac{-\sin\theta}{r}\right)$$

$$=\cos^2\theta+\sin^2\theta$$

$$=1$$

$$=\frac{\partial x}{\partial x}$$

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Step 5 of 5

Thus, the product $JJ^{-1}=J$ gives the chain rule

$$\frac{\partial x}{\partial x}=\frac{\partial x}{\partial r}\cdot\frac{\partial r}{\partial x}+\frac{\partial x}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}$$

$$=1$$

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CH4.4 39P

Step 1 of 5

The area of triangle with corners $(0,0),(6,0)$ and $(1,4)$ is

$$= \frac{1}{2}\begin{vmatrix} 6 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= \frac{1}{2}\left[6(4)-0\right]$$

$$= \frac{1}{2}(24)$$

$$= 12 \text{ sq.units}$$

Provide feedback (0)

Step 2 of 5

When the axes are rotated by $\theta = 60°$
The rotation matrix has determinant=1

$$\text{Since } \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4}+\frac{3}{4}$$

$$=1$$

Provide feedback (0)

Step 3 of 5

The new coordinates of the corners of the triangle are
$(6\cos 60°, -6\sin 60°)$, $(\cos 60° + 4\sin 60°, -\sin 60° + 4\cos 60°)$

CH4.4 41P

Step 3 of 5

The new coordinates of the corners of the triangle are

$$(6\cos 60°, -6\sin 60°),\ (\cos 60° + 4\sin 60°, -\sin 60° + 4\cos 60°)$$

$$= \left(6.\frac{1}{2}, -6.\frac{\sqrt{3}}{2}\right), \left(\frac{1}{2} + 4\left(\frac{\sqrt{3}}{2}\right), -\frac{\sqrt{3}}{2} + 4\left(\frac{1}{2}\right)\right)$$

$$= \left(3, -3\sqrt{3}\right), \left(\frac{1+4\sqrt{3}}{2}, \frac{4-\sqrt{3}}{2}\right)$$

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Step 4 of 5

Area of triangle with new coordinates is

$$A = \frac{1}{2}\begin{vmatrix} 3 & -3\sqrt{3} \\ \frac{1+4\sqrt{3}}{2} & \frac{4-\sqrt{3}}{2} \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{12 - 3\sqrt{3} - \left(-3\sqrt{3} - 36\right)}{2}\right]$$

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Step 5 of 5

$$A = \frac{1}{2}\left[\frac{12+36}{2}\right]$$

$$= \frac{1}{2}(24)$$

$$\boxed{= 12 \text{ sq.units}}$$

So, by rotating the axes by $60°$ the area of the triangle is unaltered.

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CH4.4 40P

Step 1 of 4

Let $P=(1,0,-1), Q=(1,1,1), R=(2,2,1)$ and

Let $S=(x,y,z)$ be the fourth vertex so that PQRS is a parallelogram. Then we have

$$\left.\begin{aligned}1+2&=x+1\\0+2&=y+1\\-1+1&=2+1\end{aligned}\right\}\Rightarrow(x,y,z)=(2,1,-1)\text{ So,}$$

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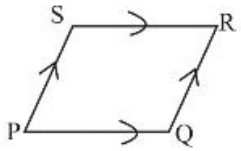
Step 2 of 4

Coordinates of $S=(2,1,-1)$



$\overrightarrow{PQ}=(0,1,2),\ \overrightarrow{PS}=(1,1,0)$

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Step 3 of 4

$$\text{Area of parallelogram PQRS} = \text{length of vector}\begin{vmatrix} i & j & k\\ 0 & 1 & 2\\ 1 & 1 & 0\end{vmatrix}$$

$=\left|-2i+2j-k\right|$

$=\sqrt{4+4+1}$

$=3$ sq.units

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CH4.4 42P

$\overrightarrow{PQ} = (0,1,2),\ \overrightarrow{PS} = (1,1,0)$

Provide feedback (0)

Step 3 of 4

Area of parallelogram PQRS = length of vector $\begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$

$= |-2i + 2j - k|$

$= \sqrt{4+4+1}$

$= 3$ sq.units

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Step 4 of 4

We can choose

$T = (0,1,2), U = (1,1,0), V = (1,2,2)$ to get a parallelepiped OPQRSTUV and its volume is given by

$$\left[\overrightarrow{OP}\ \overrightarrow{OT}\ \overrightarrow{OU}\right] = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$= |1(0-2) - 1(0-1)|$

$= |-2+1|$

$= 1$

$\boxed{= 1 \text{ cubic units}}$

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CH4.4 41P

Step 1 of 2

We have

$(x,y,z),(1,1,0)$ and $(1,2,1)$ lie on a plane through the origin.i.e

If $P(x,y,z),Q(1,1,0),R(1,2,1)$ and $O(0,0,0)$ are coplanar.

Then we get $\overrightarrow{OP},\overrightarrow{OQ},\overrightarrow{OR}$ are coplanar vectors and hence $\begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$

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Step 2 of 2

And this gives the plane equation as

$x(1-0)-y(1-0)+z(2-1)=0$

$\Rightarrow x-y+z=0$

Thus, the plane equation is $\boxed{x-y+z=0}$

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Step 1 of 2

(x,y,z) is a linear combination of $(2,3,1),(1,2,3)$ so that the vectors $(x,y,z),(2,3,1),(1,2,3)$ are linearly dependent and hence we get

$$\begin{vmatrix} x & y & z \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

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Step 2 of 2

Equation of the plane is

$x(9-2)-y(6-1)+z(4-3)=0$

$\Rightarrow 7x-5y+z=0$

All linear combinations (x,y,z) of $(2,3,1),(1,2,3)$ satisfy the equation $\boxed{7x-5y+z=0}$

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Step 1 of 3

If $Ax = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, by cramers rule

$$B_1 = \begin{bmatrix} 1 & a_{12}, \ldots, a_{1n} \\ 0 & a_{22}, \ldots, a_{2n} \\ \vdots & \vdots \quad \vdots \\ 0 & a_{n2}, \ldots, a_{nn} \end{bmatrix}$$

$$= C_{11}$$

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Step 2 of 3

$$\Rightarrow x_1 = \frac{C_{11}}{\det A}$$

$$B_2 = \begin{bmatrix} a_{11} & 1 & a_{13}, \ldots, a_{1n} \\ a_{21} & 0 & a_{23}, \ldots, a_{2n} \\ \vdots & \vdots & \vdots \quad \vdots \\ a_{n1} & 0 & a_{n3}, \ldots, a_{nn} \end{bmatrix}$$

$$= C_{12}$$

$$\Rightarrow x_2 = \frac{C_{12}}{\det A}$$

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Step 3 of 3

CH4.4 45P

Step 2 of 3

$$\Rightarrow x_1 = \frac{C_{11}}{\det A}$$

$$B_2 = \begin{bmatrix} a_{11} & 1 & a_{13},\ldots,a_{1n} \\ a_{21} & 0 & a_{23},\ldots,a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & 0 & a_{n3},\ldots,a_{nn} \end{bmatrix}$$

$$= C_{12}$$

$$\Rightarrow x_2 = \frac{C_{12}}{\det A}$$

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Step 3 of 3

Containing this process we get

$B_n = C_{1n} \Rightarrow x_n = \frac{C_{1n}}{\det A}$ here C_{ij} denotes the cofactor the cofactor of i jth element of a_{ij} of A.

Thus,

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \frac{1}{\det A}\begin{bmatrix} c_{11} \\ c_{12} \\ \vdots \\ c_{1n} \end{bmatrix}$$ is the first column of A^{-1}.

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Step 1 of 2

We have 5 – 2 = 3, an odd number, as expected!

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CH5

Step 2 of 2

In *AVIS*, we find the following pairs of letters, which are reversed in the alphabetical order.

VI

VS

Thus, there are 2 pairs of letters in the reversed alphabetical order.

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