CH3.1

CH3.2 2P

Step **1** of 2

Schwarz inequality: a, b are any vectors in $\mathbf{R}^n$, then $|a^T b| \le \|a\|\|b\|$

(a) Given that x and y are positive numbers.

$$b = \begin{pmatrix} \sqrt{x} \\ \sqrt{y} \end{pmatrix} \text{ and } a = \begin{pmatrix} \sqrt{y} \\ \sqrt{x} \end{pmatrix}$$

In view of Schwarz inequality, we consider $|a^T b|$

$$\left| \left(\sqrt{y}, \sqrt{x}\right) \begin{pmatrix} \sqrt{x} \\ \sqrt{y} \end{pmatrix} \right| = \left|\sqrt{xy} + \sqrt{xy}\right|$$
$$= 2\left|\sqrt{xy}\right| \qquad \ldots\ldots (1)$$

On the other hand, we consider $\|a\|\|b\| = \left\|\left(\sqrt{y}, \sqrt{x}\right)\right\| \left\|\left(\sqrt{x}, \sqrt{y}\right)\right\|$

$$= \sqrt{\left(\sqrt{y}\right)^2 + \left(\sqrt{x}\right)^2}\sqrt{\left(\sqrt{x}\right)^2 + \left(\sqrt{y}\right)^2}$$
$$= \sqrt{(x+y)^2}$$
$$= x + y \qquad \ldots\ldots (2)$$

Applying Schwarz inequality on (1) and (2), we get $2\left|\sqrt{xy}\right| \le x + y$

Or, $\sqrt{xy} \le \frac{1}{2}(x+y)$

Therefore, geometric mean $\le$ arithmetic mean

Provide feedback (0)

Step **2** of 2

(b) We consider $\|x+y\|^2$

On the other hand, we consider $\|a\|\|b\| = \left\|\left(\sqrt{y}, \sqrt{x}\right)\right\|\left\|\left(\sqrt{x}, \sqrt{y}\right)\right\|$

$$= \sqrt{\left(\sqrt{y}\right)^2 + \left(\sqrt{x}\right)^2}\sqrt{\left(\sqrt{x}\right)^2 + \left(\sqrt{y}\right)^2}$$

$$= \sqrt{(x+y)^2}$$

$$= x+y \qquad \ldots\ldots (2)$$

Applying Schwarz inequality on (1) and (2), we get $2\left|\sqrt{xy}\right| \le x+y$

Or, $\sqrt{xy} \le \frac{1}{2}(x+y)$

Therefore, geometric mean $\le$ arithmetic mean

Provide feedback (0)

Step 2 of 2

(b) We consider $\|x+y\|^2$

By definition, we get
$$= (x+y)^T(x+y)$$
$$= \left(x^T + y^T\right)(x+y)$$
$$= \left(x^T x + x^T y + y^T x + y^T y\right)$$
$$= \|x\|^2 + x^T y + y^T x + \|y\|^2$$
$$= \|x\|^2 + 2\left|x^T y\right| + \|y\|^2 \text{ in } \mathbf{R}^2$$

Using the result in (a) here, we get
$$\le \|x\|^2 + 2\|x\|\|y\| + \|y\|^2$$
$$\le \left(\|x\| + \|y\|\right)^2$$

While norm is a non negative quantity, we apply the square root on both sides, we get

$\|x+y\| \le \|x\| + \|y\|$

This is the required triangular inequality.

Provide feedback (0)

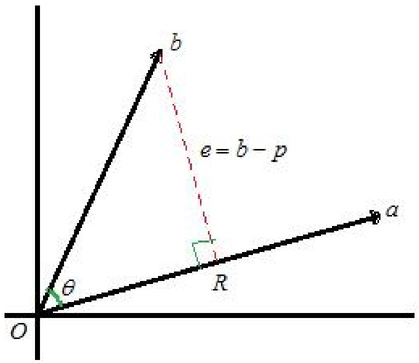
Chapter: CH3.2 Problem: 2P

show all steps

CH3.2 1P

CH3.2 3P

Step 1 of 2



Provide feedback (0)

Step 2 of 2

We see that $OR = p$ is the projection of b on a.

We have $p = \hat{x}a = \frac{a^T b}{a^T a}a = \frac{a^T b}{\|a\|^2}a$ and ...... (1)

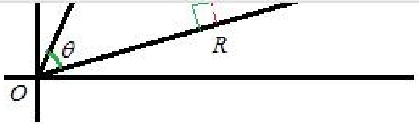
$$\cos\theta = \frac{a^T b}{\|a\|\|b\|} \quad \text{...... (2)}$$

By definition of norm, we have $\|p\|^2 = p^T p$

Using (1), we get $\|p\|^2 = \left(\frac{a^T b}{\|a\|^2}a\right)^T \frac{a^T b}{\|a\|^2}a$

Using (2), we can write this as $= \frac{(\|a\|\|b\|\cos\theta)a^T}{\|a\|^2}\frac{(\|a\|\|b\|\cos\theta)a}{\|a\|^2}$ while norm is a scalar

$$= \frac{(\|a\|\|b\|\cos\theta)^2 a^T a}{\|a\|^2\|a\|^2}$$



Provide feedback (0)

Step 2 of 2

We see that $OR = p$ is the projection of b on a.

We have $p = \hat{x}a = \dfrac{a^T b}{a^T a}a = \dfrac{a^T b}{\|a\|^2}a$ and ...... (1)

$$\cos\theta = \frac{a^T b}{\|a\|\|b\|} \qquad \text{...... (2)}$$

By definition of norm, we have $\|p\|^2 = p^T p$

Using (1), we get $\|p\|^2 = \left(\dfrac{a^T b}{\|a\|^2}a\right)^T \dfrac{a^T b}{\|a\|^2}a$

Using (2), we can write this as $= \dfrac{(\|a\|\|b\|\cos\theta)a^T}{\|a\|^2}\dfrac{(\|a\|\|b\|\cos\theta)a}{\|a\|^2}$ while norm is a scalar

$$= \frac{(\|a\|\|b\|\cos\theta)^2 a^T a}{\|a\|^2\|a\|^2}$$

$$= \frac{(\|a\|\|b\|\cos\theta)^2 \|a\|^2}{\|a\|^2\|a\|^2}$$

$$= \|b\|^2\cos^2\theta$$

Since norm is a non negative quantity, by applying square root on both sides, we get $\|p\| = \|b\|\cos\theta$

Provide feedback (1)

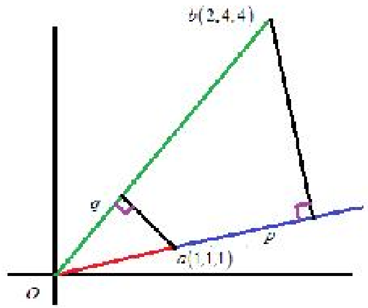
Chapter: CH3.2 Problem: 3P

show all steps

CH3.2 2P

CH3.2 4P

Step 1 of 3



Provide feedback (0)

Step 2 of 3

The vector p is the projection of b upon a.
So, the length of p is the part of $a(1,1,1)$ closest the vector b.

We know that the projection p is given by $\frac{a^T b}{a^T a} a$

$$p = \frac{(1,1,1)\begin{pmatrix}2\\4\\4\end{pmatrix}}{(1,1,1)\begin{pmatrix}1\\1\\1\end{pmatrix}}(1,1,1)$$

$$= \frac{2+4+4}{1+1+1}(1,1,1)$$

$$= \frac{10}{3}(1,1,1)$$

So, the projection of b upon a is bigger than the length of a by 10/3 times.

$\boxed{\left(\frac{10}{3}, \frac{10}{3}, \frac{10}{3}\right)}$

$$(1,1,1)\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$=\frac{2+4+4}{1+1+1}(1,1,1)$$

$$=\frac{10}{3}(1,1,1)$$

So, the projection of b upon a is bigger than the length of a by 10/3 times.

The nearest point is $\boxed{\left(\frac{10}{3},\frac{10}{3},\frac{10}{3}\right)}$

Provide feedback (0)

Step 3 of 3

Similarly, q is the projection of a upon b is given by $\frac{b^T a}{b^T b}b$

$$q=\frac{(2,4,4)\begin{pmatrix}1\\1\\1\end{pmatrix}}{(2,4,4)\begin{pmatrix}2\\4\\4\end{pmatrix}}(2,4,4)$$

$$=\frac{2+4+4}{4+16+16}(2,4,4)$$

$$=\frac{5}{18}(2,4,4)$$

So, the nearest part of a to the part of b is $5/18^{\text{th}}$ part of b.

The nearest point is $\boxed{\left(\frac{5}{9},\frac{10}{9},\frac{10}{9}\right)}$

Provide feedback (0)

Chapter: CH3.2 Problem: 4P

show all steps

CH3.2 3P

CH3.2 5P

Step 1 of 3

Since $\cos\theta \le 1$ for any θ, we get the Schwartz's Inequality as follows:

$$|a^T b| \le \|a\|\|b\|$$

Now suppose *a* and *b* are on the same line through the origin. In such case $\theta = 0$ and $\cos\theta = 1$.

Therefore,

$$\begin{aligned} a^T b &= \|a\|\|b\|\cos\theta \\ &= \|a\|\|b\| \end{aligned}$$

Provide feedback (0)

Step 2 of 3

There is no other angle for which $\cos\theta = 1$. Therefore, only when *a* and *b* lie on the same line through the origin, the Schwartz's inequality becomes equality.

Provide feedback (0)

Step 3 of 3

Suppose *a* and *b* lie on the opposite sides of the origin, but on the same line. In this case, the angle between *a* and *b* is $180°$.

Chapter: CH3.2 Problem: 4P

show all steps

$= \|a\|\|b\|$

Provide feedback (0)

Step 2 of 3

There is no other angle for which $\cos\theta = 1$. Therefore, only when *a* and *b* lie on the same line through the origin, the Schwartz's inequality becomes equality.

Provide feedback (0)

Step 3 of 3

Suppose *a* and *b* lie on the opposite sides of the origin, but on the same line. In this case, the angle between *a* and *b* is $180°$.

We know that $\cos 180° = -1$

Therefore, in such case, we get

$$\begin{aligned} a^T b &= \|a\|\|b\|\cos\theta \\ &= \|a\|\|b\|\cos 180° \\ &= \|a\|\|b\|(-1) \\ &= -\|a\|\|b\| \end{aligned}$$

Provide feedback (0)

CH3.2 4P

CH3.2 6P

Step 1 of 6

$a = (1,1,......,1)$ is the given vector in the $n-$ dimensions.

$b = (1,0,......,0)$ is the unit vector along the X_1- axis.

Let θ be the angle between a and b.

Then $\cos\theta = \dfrac{a^T b}{\|a\|\|b\|}$

$$a^T b = (1,1,......,1)\begin{pmatrix}1\\0\\.\\.\\.\\0\end{pmatrix}$$

$$= 1+0+........+0$$

$$= 1$$

Provide feedback (0)

Step 2 of 6

$$\|a\|^2 = a^T a$$

$$= (1,1,......,1)\begin{pmatrix}1\\1\\.\\.\\.\\1\end{pmatrix}$$

$$= 1+1+........+1 (n\ times)$$

$$= n \qquad ...... (1)$$

Step 3 of 6

$$\|b\|^2 = b^T b$$

$$= (1, 0, \ldots\ldots, 0)\begin{pmatrix} 1 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$$

$$= 1 + 0 + \ldots\ldots + 0$$

$$= 1$$

Provide feedback (0)

Step 4 of 6

$$\|a\| = \sqrt{n}, \ \|b\| = 1$$

$$\cos\theta = \frac{1}{\sqrt{n}.1}$$

$$= \frac{1}{\sqrt{n}}$$

Therefore, $\boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{n}}\right)}$

We can easily see that the angle with X_2- axis, ..., X_n-axis is also equal to $\cos^{-1}\left(\frac{1}{\sqrt{n}}\right)$

Provide feedback (0)

Step 5 of 6

We know that the projection p of b upon a is $p = \frac{a^T b}{\ldots}a$ and the projection matrix is

Step 5 of 6

We know that the projection p of b upon a is $p = \frac{a^T b}{a^T a} a$ and the projection matrix is

$$P = \frac{a^T a}{a^T a}$$

$$aa^T = \begin{bmatrix} 1 \\ 1 \\ - \\ - \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & - & - & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & - & - & 1 \\ 1 & 1 & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & 1 \end{bmatrix} \qquad \ldots\ldots (2)$$

Provide feedback (2)

Step 6 of 6

Using (1) and (2), we get $P = \frac{a^T a}{a^T a}$

$$= \frac{1}{n} \begin{bmatrix} 1 & 1 & - & - & 1 \\ 1 & 1 & - & - & 1 \\ - & - & - & - & - \\ - & - & - & - & - \\ 1 & 1 & - & - & 1 \end{bmatrix}$$

Provide feedback (0)

CH3.2 5P

CH3.2 7P

Step 1 of 2

Since $\cos\theta \le 1$ for any θ, we get the Schwartz's Inequality as follows:

$$|a^T b| \le \|a\|\|b\|$$

Note that the Schwartz's inequality can be proved, if *a* and *b* are unit vectors, as follows:

$$\begin{aligned} |a^T b| &= \left|\sum a_i b_i\right| \\ &\le \sum |a_i||b_i| \\ &\le \sum \frac{|a_i|^2 + |b_i|^2}{2} \\ &= \frac{1}{2} + \frac{1}{2} \\ &= 1 \\ &= \|a\|\|b\| \end{aligned}$$

Step 2 of 2

In this case, notice the step $\left|\sum a_i b_i\right| \le \sum |a_i||b_i|$.

This is justified by the simple fact that modulus of sum is less than or equal to the sum of the moduli.

$$\leq \sum \frac{|a_i|^2 + |b_i|^2}{2}$$

$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$
$$= \|a\|\|b\|$$

Provide feedback (0)

Step **2** of 2

In this case, notice the step $\left|\sum a_i b_i\right| \leq \sum |a_i||b_i|$.

This is justified by the simple fact that modulus of sum is less than or equal to the sum of the moduli.

Next, there is one more inequality $\sum |a_i||b_i| \leq \sum \frac{|a_i|^2 + |b_i|^2}{2}$.

This is same as $\sum \sqrt{|a_i|^2 |b_i|^2} \leq \sum \frac{|a_i|^2 + |b_i|^2}{2}$. This step is justified by the fact that the geometric mean of positive numbers is always less than or equal to the arithmetic mean of the same numbers.

Provide feedback (0)

CH3.2 6P

CH3.2 8P

Step 1 of 7

By choosing the correct vector in b in the Schwarz inequality, we have to prove that $(a_1+a_2+\cdots+a_n)^2 \le n(a_1^2+a_2^2+\cdots+a_n^2)$, and also we have to find that, when does the equality holds.

Provide feedback (0)

Step 2 of 7

Let $a=\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

Choose $b=(1,1,\dots,1)$

Schwarz inequality is $|a^Tb| \le \|a\|\,\|b\|$ ...... (1)

Provide feedback (0)

Step 3 of 7

$$a^Tb=(a_1,a_2,\dots,a_n)\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$= a_1+a_2+\cdots+a_n$$

$$\Rightarrow |a^Tb| = |a_1+a_2+\cdots+a_n|$$

$$\Rightarrow |a^Tb|^2 = (a_1+a_2+\cdots+a_n)^2$$

Provide feedback (0)

Provide feedback (0)

Step 4 of 7

And $\|a\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$

$\|b\| = \sqrt{1^2 + 1^2 + \cdots + 1^2 \,(n \text{ times})}$

$= \sqrt{n.1}$

Provide feedback (0)

Step 5 of 7

$\Rightarrow \|a\|^2 = a_1^2 + a_2^2 + \cdots + a_n^2$ and $\|b\|^2 = n$

By (1), $|a^T b|^2 \le \|a\|^2 \|b\|^2$

Hence $\boxed{(a_1 + a_2 + \cdots + a_n)^2 \le n\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)}$

Provide feedback (0)

Step 6 of 7

If $a_1 = a_2 = \cdots = a_n$ then

$(a_1 + a_2 + \cdots + a_n)^2$

$= (na_1)^2$

$= n^2 a_1^2$

Provide feedback (0)

Step 7 of 7

And

$n\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)$

Provide feedback (0)

Step 5 of 7

$\Rightarrow \|a\|^2 = a_1^2 + a_2^2 + \cdots + a_n^2$ and $\|b\|^2 = n$

By (1), $|a^T b|^2 \le \|a\|^2 \|b\|^2$

Hence $\boxed{(a_1 + a_2 + \cdots + a_n)^2 \le n(a_1^2 + a_2^2 + \cdots + a_n^2)}$

Provide feedback (0)

Step 6 of 7

If $a_1 = a_2 = \cdots = a_n$ then

$(a_1 + a_2 + \cdots + a_n)^2$

$= (na_1)^2$

$= n^2 a_1^2$

Provide feedback (0)

Step 7 of 7

And

$n(a_1^2 + a_2^2 + \cdots + a_n^2)$

$= n(na_1^2)$

$= n^2 a_1^2$

Therefore $(a_1 + a_2 + \cdots + a_n)^2 = n(a_1^2 + a_2^2 + \cdots + a_n^2)$

Hence the equality holds if $\boxed{a_1 = a_2 = \cdots = a_n}$

Provide feedback (0)

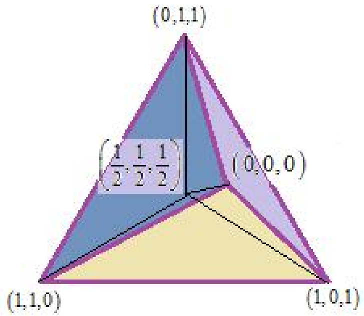
Chapter: CH3.2 Problem: 8P

show all steps

CH3.2 7P

CH3.2 9P

Step 1 of 5



Provide feedback (0)

Step 2 of 5

Let O = (0, 0, 0), B = (1, 1, 0), C= (1, 0, 1), D = (0, 1, 1) are the vertices and

$E=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ is center of the regular tetrahedron.

We determine the cosine of the angle θ between the rays from E to any to of above four vertices

Suppose $u=EA, v=EB$

$$\begin{aligned}u&=EA\\&=OA-OE\\&=(1,1,0)-\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\\&=\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)\end{aligned}$$

Provide feedback (2)

Step 3 of 5

Step 3 of 5

$$v = EB$$
$$= (1,0,1) - \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$$
$$= \left(\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)$$

Angle between u and v is given by $\cos\theta = \frac{u^T v}{\|u\|\|v\|}$ ...... (1)

$$u^T v = \left(\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)\begin{pmatrix}1/2\\-1/2\\1/2\end{pmatrix}$$
$$= \frac{1}{4}-\frac{1}{4}-\frac{1}{4}$$
$$= -\frac{1}{4}$$

Provide feedback (0)

Step 4 of 5

$$\|u\| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2}$$
$$= \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$
$$= \sqrt{\frac{3}{4}}$$
$$\|v\| = \sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$$
$$= \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$

$$= \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$\|v\| = \sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

Provide feedback (0)

Step 5 of 5

Using all these results in (1), we get $\cos\theta = \dfrac{-1/4}{\sqrt{\frac{3}{4}}\sqrt{\frac{3}{4}}} = \dfrac{\left(-\frac{1}{4}\right)}{\frac{3}{4}} = \dfrac{-1}{3}$

Similarly, the angle between any two rays at E is $\boxed{\cos^{-1}\left(\pm\frac{1}{3}\right)}$

Provide feedback (0)

CH3.2 8P

Step **1** of 1

We have the projection matrix P upon to a line a is $P = \frac{a^T a}{a^T a}$

$$
\begin{aligned}
P^2 &= \left(\frac{aa^T}{a^T a}\right)\left(\frac{aa^T}{a^T a}\right) \\
&= \frac{\left(aa^T\right)\left(aa^T\right)}{\left(a^T a\right)\left(a^T a\right)} \\
&= \frac{a\left(a^T a\right)a^T}{\left(a^T a\right)\left(a^T a\right)} \\
&= \frac{aa^T}{a^T a} \qquad \text{Since } a^T a \text{ is a scalar, we cancelled it.} \\
&= P
\end{aligned}
$$

Therefore, $\boxed{P^2 = P}$ when P is a projection matrix.

Provide feedback (0)

CH3.2 10P

Chapter: CH3.2 Problem: 10P

show all steps

CH3.2 9P

CH3.2 11P

Step 1 of 3

We have to verify that is the projection matrix P invertible.

We know that the projection matrix $P = \frac{aa^T}{a^T a}$

Now

$$PP^{-1} = \frac{aa^T}{a^T a}\left(\frac{aa^T}{a^T a}\right)^{-1}$$

Provide feedback (1)

Step 2 of 3

Since $(kA)^{-1} = \frac{1}{k}A^{-1}$, k is a scalar, here $a^T a$ is a scalar, we have

$$PP^{-1} = \frac{aa^T}{a^T a} a^T a \left(a^T\right)^{-1} a^{-1}$$

Provide feedback (1)

Step 3 of 3

Therefore

$$PP^{-1} = aa^T \left(a^T\right)^{-1} a^{-1}$$

$$= a\left(a^T\left(a^T\right)^{-1}\right)a^{-1}$$

$$= (aI)\, a^{-1}\left(\text{since } a^T \left(a^T\right)^{-1} = I\right)$$

$$= aa^{-1}$$

$$= I$$

Hence $\boxed{P \text{ is invertible}}$.

Provide feedback (3)

CH3.2 10P

CH3.2 12P

Step 1 of 4

(a) The projection matrix $P_1 = \frac{aa^T}{a^Ta}$

$$aa^T = \begin{bmatrix}1\\3\end{bmatrix}\begin{bmatrix}1 & 3\end{bmatrix} = \begin{bmatrix}1 & 3\\3 & 9\end{bmatrix}$$

And $a^Ta = \begin{bmatrix}1 & 3\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix} = 1+9 = 10$

Therefore, $P_1 = \frac{1}{10}\begin{bmatrix}1 & 3\\3 & 9\end{bmatrix} = \begin{bmatrix}\frac{1}{10} & \frac{3}{10}\\ \frac{3}{10} & \frac{9}{10}\end{bmatrix}$

Provide feedback (0)

Step 2 of 4

While P_1 is the projection onto a subspace, we follow that $I - P_1 = P_2$ is the matrix related to the subspace perpendicular to the projection of P_1

$$P_2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}\frac{1}{10} & \frac{3}{10}\\ \frac{3}{10} & \frac{9}{10}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{9}{10} & \frac{-3}{10}\\ \frac{-3}{10} & \frac{1}{10}\end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Step 3 of 4

(b) while P_1 and P_2 are the orthogonal projection matrices, their sum must be the identity matrix.

$$P_1+P_2=\begin{bmatrix}\frac{1}{10} & \frac{3}{10}\\ \frac{3}{10} & \frac{9}{10}\end{bmatrix}+\begin{bmatrix}\frac{9}{10} & -\frac{3}{10}\\ -\frac{3}{10} & \frac{1}{10}\end{bmatrix}$$
$$=\begin{bmatrix}\frac{1}{10}+\frac{9}{10} & \frac{3}{10}-\frac{3}{10}\\ \frac{3}{10}-\frac{3}{10} & \frac{9}{10}+\frac{1}{10}\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

The product of the matrices related to the orthogonal projections must be the null matrix.

$$P_1P_2=\begin{bmatrix}\frac{1}{10} & \frac{3}{10}\\ \frac{3}{10} & \frac{9}{10}\end{bmatrix}\begin{bmatrix}\frac{9}{10} & -\frac{3}{10}\\ -\frac{3}{10} & \frac{1}{10}\end{bmatrix}$$
$$=\begin{bmatrix}\frac{9}{100}-\frac{9}{100} & \frac{-3}{100}+\frac{3}{100}\\ \frac{27}{100}-\frac{27}{100} & \frac{-9}{100}+\frac{9}{100}\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Provide feedback (0)

CH3.2 11P

CH3.2 13P

Step 1 of 3

We have to find the matrix that projects every point in the plane onto the line $x+2y=0$.

Given line is $x+2y=0$

Put

$y=k$

$\Rightarrow x=-2k$

Provide feedback (0)

Step 2 of 3

Therefore any point on the line is $a=(-2k,k)$

$$aa^T=\begin{bmatrix}-2k\\k\end{bmatrix}\begin{bmatrix}-2k & k\end{bmatrix}$$

$$=\begin{bmatrix}4k^2 & -2k^2\\-2k^2 & k^2\end{bmatrix}$$

$$a^Ta=\begin{bmatrix}-2k & k\end{bmatrix}\begin{bmatrix}-2k\\k\end{bmatrix}$$

$$=5k^2$$

Provide feedback (0)

Step 3 of 3

The projection matrix

$$P=\frac{aa^T}{a^Ta}$$

$$=\frac{1}{5k^2}\begin{bmatrix}4k^2 & -2k^2\\-2k^2 & k^2\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}4 & -2\\-2 & 1\end{bmatrix}$$

Provide feedback (0)

Step 2 of 3

Therefore any point on the line is $a = (-2k, k)$

$$aa^T = \begin{bmatrix} -2k \\ k \end{bmatrix} \begin{bmatrix} -2k & k \end{bmatrix}$$

$$= \begin{bmatrix} 4k^2 & -2k^2 \\ -2k^2 & k^2 \end{bmatrix}$$

$$a^T a = \begin{bmatrix} -2k & k \end{bmatrix} \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$$= 5k^2$$

Provide feedback (0)

Step 3 of 3

The projection matrix

$$P = \frac{aa^T}{a^T a}$$

$$= \frac{1}{5k^2} \begin{bmatrix} 4k^2 & -2k^2 \\ -2k^2 & k^2 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

So the required matrix is $\boxed{\frac{1}{5} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}}$

Provide feedback (0)

CH3.2 12P
CH3.2 14P

Step 1 of 4

We have to prove that the trace of $P = \frac{aa^T}{a^Ta}$ is always 1.

Let $a = (a_1, a_2, \dots, a_n)$

$$aa^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$$

$$= \begin{bmatrix} a_1^2 & a_1a_2 & \cdots & a_1a_n^2 \\ a_2a_1 & a_2^2 & \cdots & a_2a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_na_1 & a_na_1 & \cdots & a_n^2 \end{bmatrix}$$

Step 2 of 4

$$a^Ta = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$= a_1^2 + a_2^2 + \cdots + a_n^2$$
$$= \sum a_1^2$$

Step 3 of 4

The projection matrix

aa^T

Step 3 of 4

The projection matrix

$$P = \frac{aa^T}{a^Ta}$$

$$= \frac{1}{\sum a_1^2}\begin{bmatrix} a_1{}^2 & a_1a_2 & \cdots & a_1a_n{}^2 \\ a_2a_1 & a_2{}^2 & \cdots & a_2a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_na_1 & a_na_1 & \cdots & a_n{}^2 \end{bmatrix}$$

Step 4 of 4

Trace $P =$ Sum of its diagonal elements

$$= \frac{a_1a_1}{\sum a_1^2} + \frac{a_2a_2}{\sum a_1^2} + \cdots + \frac{a_na_n}{\sum a_1^2}$$

$$= \frac{a_1^2 + a_2^2 + \cdots + a_n^2}{\sum a_1^2}$$

$$= \frac{\sum a_1^2}{\sum a_1^2}$$

$$= 1$$

Hence trace of P is $\boxed{1}$

CH3.2 13P

CH3.2 15P

Step 1 of 3

Suppose $x = (x, y, t)$ is any point in $\mathbf{R}^3$
We find the orthogonal projection p of a onto the line of intersection of the planes
$x + y + t = 0$ and $x - t = \mathbf{0}$
The orthogonal projection is nothing but the null space of the matrix A whose rows are the coefficients of the planes such that
The matrix form of above equations is $Ax = 0$

So, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ t \end{bmatrix}$

Provide feedback (0)

Step 2 of 3

Applying $R_2 \to R_2 - R_1$ upon this, we get $\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$

$R_2(-1) \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

This is the row reduced form and so, we rewrite the equations from this.

$y + 2t = 0$

$x + y + t = 0$

1^{st} equation gives $y = -2t$ and so, $x = t$

Therefore, $\begin{bmatrix} x \\ y \\ t \end{bmatrix} = k \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ where k $= t$ is the parameter.

Putting $k = 1$, we get $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is the required orthogonal projection p

$y + 2t = 0$

$x + y + t = 0$

1st equation gives $y = -2t$ and so, $x = t$

Therefore, $\begin{bmatrix} x \\ y \\ t \end{bmatrix} = k\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ where $k = t$ is the parameter.

Putting $k = 1$, we get $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is the required orthogonal projection p

Provide feedback (0)

Step 3 of 3

The required projection matrix is $P = \dfrac{pp^T}{p^T p}$

$$= \frac{\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}(1,-2,1)}{(1,-2,1)\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}$$

$$= \boxed{\frac{1}{6}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}}$$

Provide feedback (0)

CH3.2 14P

Step 1 of 1

Suppose $AA^T = A^T A$ ...... (1)

The length of Ax is $\|Ax\|$ where the norm is a non negative quantity. ...... (2)

By definition, we have
$$\begin{aligned}\|Ax\|^2 &= (Ax)^T(Ax)\\ &= x^T(A^T A)x\\ &= x^T AA^T x \qquad \text{By (1)}\\ &= (A^T x)^T(A^T x)\\ &= \|A^T x\|^2\end{aligned}$$

In view of (2), we can write this equation as $\|Ax\| = \|A^T x\|$

In other words, the length of Ax = length of $A^T x$ when $AA^T = A^T A$

CH3.2 16P

CH3.2 15P

CH3.2 17P

Step 1 of 20

Given that, P is the projection matrix onto the line through a
(a) We have to find that, why is the inner product of x with Py equal to the inner product of Px with y.
The inner product of of x with Py

$= x^T(Py)$

$= (x^T P)y$

Provide feedback (0)

Step 2 of 20

$= (x^T P^T)y$ (since the projection P is symmetric, $P^T = P$)

$= (Px)^T y$

$=$ inner product of Px with y.

Suppose $x^T(Py) = (Px)^T y$ ...... (1)

That is, the inner product of x with Py equal to the inner product of Px with y.

Provide feedback (0)

Step 3 of 20

(b) We have to verify that whether the two angles are equal, and we have to find their cosines if $a = (1,1,-1)$, $x = (2,0,1)$, $y = (2,1,2)$

Let θ be the angle between x and Py,

then $\cos\theta = \dfrac{x^T Py}{\|x\|\|Py\|}$ ...... (2)

Provide feedback (0)

Step 4 of 20

Let ϕ be the angle between Px with y,

then $\cos\phi = \dfrac{(Px)^T y}{\|Px\|\|y\|}$ ...... (3)

As $\|x\|\|Py\| \neq \|Px\|\|y\|$, the $\boxed{\text{two angles are not same}}$

Provide feedback (0)

Step 5 of 20

Given $a = (1,1,-1)$

$P =$ the projection matrix onto the line through a

$= \dfrac{aa^T}{a^T a}$

$$aa^T = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 20

$$a^T a = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$= 1+1+1$

$= 3$

Provide feedback (0)

$$P = \frac{aa^T}{a^T a}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 20

$$Py = \frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2+1-2 \\ 2+1-2 \\ -2-1+2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 20

$$x^T(Py) = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{3}(2+0-1)$$

$$= \frac{1}{3}$$

Step 10 of 20

$$\|x\| = \sqrt{4+0+1}$$
$$= \sqrt{5}$$
$$\|Py\| = \frac{1}{3}\sqrt{1+1+1}$$
$$= \frac{\sqrt{3}}{3}$$

Provide feedback (0)

Step 11 of 20

By (2),

$$\cos\theta = \frac{1/3}{\sqrt{15}/3}$$
$$= \frac{1}{\sqrt{15}}$$

Provide feedback (0)

Step 12 of 20

$$Px = \frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$
$$= \frac{1}{3}\begin{bmatrix} 2+0-1 \\ 2+0-1 \\ -2-0+1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

$$(Px)^T y = \frac{1}{3}\begin{bmatrix}1 & 1 & -1\end{bmatrix}\begin{bmatrix}2\\1\\2\end{bmatrix}$$

$$= \frac{1}{3}(2+1-2)$$

$$= \frac{1}{3}$$

Provide feedback (0)

Step 14 of 20

$$\|y\| = \sqrt{4+1+4}$$

$$= 3$$

$$\|Px\| = \frac{1}{3}\sqrt{1+1+1}$$

$$= \frac{\sqrt{3}}{3}$$

Provide feedback (0)

Step 15 of 20

By (3),

$$\cos\phi = \frac{1/3}{\sqrt{3}}$$

$$= \frac{1}{3\sqrt{3}}$$

$$\boxed{\text{Hence required cosine angles are } \frac{1}{\sqrt{15}} \text{ and } \frac{1}{3\sqrt{3}}}$$

Provide feedback (0)

Step 16 of 20

(c) We have to verify that why the inner product of Px with Py is same the inner product of x with Py and the inner product of Px with y, and we have to find that what is the angle between those two.

Provide feedback (0)

Step 17 of 20

Inner product of Px with Py is

$=(Px)^T Py$

$=(x^T P^T)Py$

$=x^T PPy$

$=x^T P^2 y = x^T (Py) \qquad \ldots\ldots (4)$

Therefore from (1) and (4), $(Px)^T Py = (Px)^T y = x^T (Py)$

Hence the inner product of Px with Py is same the inner product of x with Py and the inner product of Px with y

Provide feedback (0)

Step 18 of 20

Let θ be the angle between Px with Py

Then $\cos\theta = \dfrac{(Px)^T Py}{\|Px\|\|Py\|}$

Here

$Px = \dfrac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and

$Py = \dfrac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$Py = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 19 of 20

$$(Px)^T Py = \frac{1}{3}\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{9}(1+1+1)$$

$$= \frac{1}{3}$$

Provide feedback (0)

Step 20 of 20

$\|Px\| = \frac{\sqrt{3}}{3}$, and

$$\|Py\| = \frac{\sqrt{3}}{3}$$

$$\cos\theta = \frac{1/3}{3/9}$$

$$= 1$$

Hence $\boxed{\theta = 0}$

Provide feedback (0)

CH3.2 16P

CH3.2 18P

Step 1 of 4

(a) Given vectors are $b = \begin{bmatrix}1\\2\\3\end{bmatrix}$ and $a = \begin{bmatrix}1\\1\\1\end{bmatrix}$

The projection of b onto a = $\hat{x} = \left(\dfrac{a^T b}{a^T a}\right) a$

$$\begin{aligned} a^T b &= \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix} \\ &= 1+2+2 \\ &= 5 \end{aligned}$$

Also,

$$\begin{aligned} a^T a &= \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} \\ &= 1+1+1 \\ &= 3 \end{aligned}$$

Step 2 of 4

So, $\hat{x} = \dfrac{5}{3}$

Therefore $P = \hat{x}\, a = \dfrac{5}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix}$

$e = b - P = \begin{bmatrix}1\\2\\ \end{bmatrix} - \begin{bmatrix}5/3\\5/3\\ \end{bmatrix}$

Step 2 of 4

So, $\hat{x} = \frac{5}{3}$

Therefore $P = \hat{x}\, a = \frac{5}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix}$

$$e = b - P = \begin{bmatrix}1\\2\\2\end{bmatrix} - \begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix}$$
$$= \begin{bmatrix}-2/3\\1/3\\1/3\end{bmatrix}$$

We verify e is perpendicular to a or not.

$$e^T = (-2/3 \quad 1/3 \quad 1/3)$$

$$e^T a = (-2/3 \quad 1/3 \quad 1/3)\begin{bmatrix}1\\1\\1\end{bmatrix}$$
$$= \frac{-2}{3} + \frac{1}{3} + \frac{1}{3}$$
$$= 0$$

Therefore, e is perpendicular to a.

Provide feedback (0)

Step 3 of 4

(b) Given vectors are $b = \begin{bmatrix}1\\3\\1\end{bmatrix}$ and $a = \begin{bmatrix}-1\\-3\\-1\end{bmatrix}$

(b) Given vectors are $b=\begin{bmatrix}1\\3\\1\end{bmatrix}$ and $a=\begin{bmatrix}-1\\-3\\-1\end{bmatrix}$

The projection of b onto a = $\hat{x}=\dfrac{a^Tb}{a^Ta}$

$$\begin{aligned}a^Tb&=\begin{bmatrix}-1 & -3 & -1\end{bmatrix}\begin{bmatrix}1\\3\\1\end{bmatrix}\\&=-1-9-1\\&=-11\end{aligned}$$

$$\begin{aligned}a^Ta&=\begin{bmatrix}-1 & -3 & -1\end{bmatrix}\begin{bmatrix}-1\\-3\\-1\end{bmatrix}\\&=1+9+1\\&=11\end{aligned}$$

Provide feedback (0)

Step 4 of 4

$$\begin{aligned}\hat{x}&=\frac{-11}{11}\\&=-1\end{aligned}$$

Therefore $P=\hat{x}\,a=-1\begin{bmatrix}-1\\-3\\-1\end{bmatrix}=\begin{bmatrix}1\\3\\1\end{bmatrix}=b$

$$e=b-P=\begin{bmatrix}1\\3\\1\end{bmatrix}-\begin{bmatrix}1\\3\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

We easily see that the zero vector is perpendicular to every vector.
So, $e^Ta=0$ verifies the projection of b upon a.

CH3.2 17P

CH3.2 19P

Step 1 of 4

(a) Given vectors are $b = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

The projection of b onto a $= \hat{x}a = \dfrac{a^T b}{a^T a} a$ ...... (1)

$$a^T b = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$
$$= \cos\theta$$
$$a^T a = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= 1 + 0$$
$$= 1$$

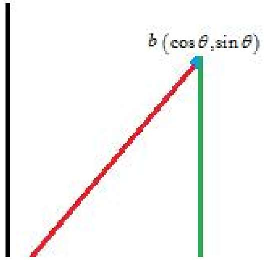
So, $\hat{x} = \dfrac{\cos\theta}{1} = \cos\theta$ ...... (2)

Provide feedback (0)

Step 2 of 4

Using (2) in (1), we get $P = \hat{x}\, a = \cos\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Therefore, the required projection matrix is $P = \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$



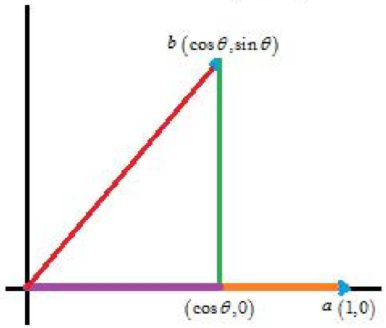
So, $\hat{x} = \dfrac{\cos\theta}{1} = \cos\theta$ ...... (2)

Provide feedback (0)

Step 2 of 4

Using (2) in (1), we get $P = \hat{x}\,a = \cos\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Therefore, the required projection matrix is $P = \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$



Provide feedback (0)

Step 3 of 4

(b) Given vectors are $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The projection of b on to a = $\hat{x}a = \dfrac{a^T b}{a^T a} a$

$$a^T b = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 - 1 = 0$$

And $a^T a = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 + 1 = 2$

Step 3 of 4

(b) Given vectors are $b=\begin{bmatrix}1\\1\end{bmatrix}$ and $a=\begin{bmatrix}1\\-1\end{bmatrix}$

The projection of b on to a $=\hat{x}a=\frac{a^Tb}{a^Ta}a$

$$a^Tb=\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=1-1=0$$

And $a^Ta=\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}=1+1=2$

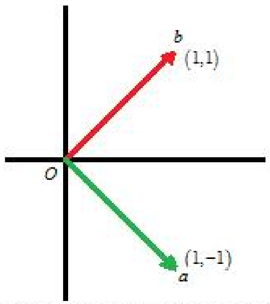
So $\hat{x}=\frac{0}{2}=0$

Therefore $P=\hat{x}a=0\begin{bmatrix}1\\-1\end{bmatrix}$

Hence $P=\begin{bmatrix}0\\0\end{bmatrix}$

Provide feedback (0)

Step 4 of 4



Observe that the vectors (1, 1) and (1, -1) are perpendicular which meet at the origin O and so, the projection of b upon a is the footsteps of b nothing but the origin (0, 0).

CH3.2 18P

CH3.2 20P

Step 1 of 6

(a) The projection matrix $P_1 = \frac{aa^T}{a^Ta}$

$$aa^T = \begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$
$$= \begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$

Also,

$$a^Ta = \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$
$$= 1+1+1$$
$$= 3$$

Provide feedback (0)

Step 2 of 6

$$P_1 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$
$$P_1^2 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix}3 & 3 & 3\\3 & 3 & 3\\3 & 3 & 3\end{bmatrix}$$

$$P_1^2 = \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix}3&3&3\\3&3&3\\3&3&3\end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

Therefore $P_1^2 = P$

Provide feedback (1)

Step 3 of 6

$$P_1 b = \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix}1+2+2\\1+2+2\\1+2+2\end{bmatrix}$$

Therefore $P_1 b = \frac{1}{3}\begin{bmatrix}5\\5\\5\end{bmatrix}$

Provide feedback (0)

Step 4 of 6

(b) The projection matrix $P_2 = \dfrac{aa^T}{a^T a}$

$$aa^T = \begin{bmatrix}-1\\-3\\ \ldots\end{bmatrix}\begin{bmatrix}-1 & -3 & -1\end{bmatrix}$$

Step 4 of 6

(b) The projection matrix $P_2 = \frac{aa^T}{a^T a}$

$$aa^T = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

$$a^T a = \begin{bmatrix} -1 & -3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$$

$$= 1 + 9 + 1$$

$$= 11$$

Provide feedback (0)

Step 5 of 6

$$P_2 = \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

$$P_2^2 = \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix} \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

$$= \frac{1}{121} \begin{bmatrix} 11 & 33 & 11 \\ 33 & 99 & 33 \\ 11 & 33 & 11 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \end{bmatrix}$$

$$P_2^2 = \frac{1}{11}\begin{bmatrix} 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}\frac{1}{11}\begin{bmatrix} 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

$$= \frac{1}{121}\begin{bmatrix} 11 & 33 & 11 \\ 33 & 99 & 33 \\ 11 & 33 & 11 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

Therefore $P_2^2 = P$

Provide feedback (0)

Step 6 of 6

$$P_2 b = \frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 11 \\ 33 \\ 11 \end{bmatrix}$$

Therefore $P_2 b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

Provide feedback (0)

CH3.2 19P

CH3.2 21P

Step 1 of 5

Given vectors are $b = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

The projection of b onto a $= \hat{x}a = \dfrac{a^T b}{a^T a} a$ ...... (1)

$$a^T b = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$
$$= \cos\theta$$
$$a^T a = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= 1 + 0$$
$$= 1$$

So, $\hat{x} = \dfrac{\cos\theta}{1} = \cos\theta$ ...... (2)

Provide feedback (0)

Step 2 of 5

Using (2) in (1), we get $p_1 = \hat{x}\, a$

$$= \cos\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$$

The projection matrix suitable is $P_1 = \dfrac{aa^T}{a^T a}$

$$= \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}}{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

Step 2 of 5

Using (2) in (1), we get $p_1 = \hat{x}\,a$

$$= \cos\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix}$$

The projection matrix suitable is $P_1 = \dfrac{aa^T}{a^Ta}$

$$= \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}}{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \ldots\ldots (3)$$

Step 3 of 5

Given vectors are $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The projection of b on to a = $\hat{x}\,a = \dfrac{a^Tb}{a^Ta}a$

$$a^Tb = \begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 - 1 = 0$$

And $a^Ta = \begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ \end{bmatrix} = 1 + 1 = 2$

And $a^T a = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1+1 = 2$

So $\hat{x} = \frac{0}{2} = 0$

Therefore $p_2 = \hat{x}a$

$$= 0\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

The matrix suitable to this projection is $P_1 = \frac{aa^T}{a^T a}$

$$= \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix}}{\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}{2}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \ldots\ldots (4)$$

Provide feedback (0)

Step 5 of 5

Using (3) and (4), we get $P_1 + P_2 = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

Step 4 of 5

The matrix suitable to this projection is $P_1 = \frac{aa^T}{a^T a}$

$$= \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix}}{\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}{2}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \ldots\ldots (4)$$

Provide feedback (0)

Step 5 of 5

Using (3) and (4), we get $P_1 + P_2 = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

$$P_1 P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$(P_1 + P_2)^2 = \frac{1}{2}\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$P_1^2 + P_2^2 = \frac{1}{2}\begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix}$$

From the above observations, we follow that $(P_1 + P_2)^2 \neq P_1 + P_2$ while $P_1 P_2 \neq 0$

Provide feedback (0)

CH3.2 20P

CH3.2 22P

Step 1 of 3

$P_1 =$ The projection matrix onto the line through $a_1 = \frac{a_1 a_1^T}{a_1^T a_1}$

$$a_1 a_1^T = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} (-1, 2, 2)$$

$$= \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$$

$$a_1^T a_1 = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$= 1 + 4 + 4$$

$$= 9$$

$$P_1 = \frac{1}{9} \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 3

$P_2 =$ The projection matrix onto the line through $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} (2, 2, -1)$$

Chapter: CH3.2 Problem: 21P ▲ ▼ show all steps

Step 2 of 3

$P_2 =$ The projection matrix onto the line through $a_2 = \dfrac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}(2,2,-1)$$

$$= \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$a_2^T a_2 = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$= 4+4+1$$

$$= 9$$

$$P_2 = \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

$$P_1 P_2 = \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}\frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} 4-8+4 & 4-8+4 & -2+4-2 \\ -8+16-8 & -8+16-8 & 4-8+4 \\ -8+16-8 & -8+16-8 & 4-8+4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$= 4+4+1$$
$$= 9$$

$$P_2 = \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

$$P_1P_2 = \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}\frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$
$$= \frac{1}{81}\begin{bmatrix} 4-8+4 & 4-8+4 & -2+4-2 \\ -8+16-8 & -8+16-8 & 4-8+4 \\ -8+16-8 & -8+16-8 & 4-8+4 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore P_1P_2 is a zero matrix because $a_1 \perp a_2$

This result is verified with $a_1^T a_2 = (-1, 2, 2)\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$
$$= -2+4-2$$
$$= 0$$

Provide feedback (0)

CH3.2 21P

CH3.2 23P

Step 1 of 4

Let p_1 = the projection of b onto the line through $a_1 = \frac{a_1^T b}{a_1^T a_1} a_1$

$$a_1^T b = (-1,2,2)\begin{bmatrix}1\\0\\0\end{bmatrix}$$
$$= -1+0+0$$
$$= -1$$

$$a_1^T a_1 = \begin{bmatrix}-1 & 2 & 2\end{bmatrix}\begin{bmatrix}-1\\2\\2\end{bmatrix}$$
$$= 1+4+4$$
$$= 9$$

$$p_1 = \frac{-1}{9}\begin{pmatrix}-1\\2\\2\end{pmatrix}$$
$$= \begin{pmatrix}1/9\\-2/9\\-2/9\end{pmatrix}$$

Provide feedback (0)

Step 2 of 4

p_2 = The projection of b onto the line through $a_2 = \frac{a_2^T b}{a_2^T a_2} a_2$

$$a_2^T b = (2,2,-1)\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$= 2+0+0$$
$$= 2$$

$$a_2^T a_2 = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$
$$= 4+4+1$$
$$= 9$$

$$p_2 = \frac{2}{9} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$
$$= \begin{bmatrix} 4/9 \\ 4/9 \\ -2/9 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

p_3 = The projection of b onto the line through $a_3 = \dfrac{a_3^T b}{a_3^T a_3} a_3$

$$a_3^T b = (2,-1,2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$
$$= 2+0+0$$
$$= 2$$

$$a_3^T a_3 = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$
$$= 4+1+4$$
$$= 9$$

$$= 2+0+0$$
$$= 2$$

$$a_3^T a_3 = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$
$$= 4+1+4$$
$$= 9$$

$$p_3 = \frac{2}{9} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4/9 \\ -2/9 \\ 4/9 \end{bmatrix}$$

Step 4 of 4

$$P_1 + P_2 + P_3 = \begin{bmatrix} 1/9 \\ -2/9 \\ -2/9 \end{bmatrix} + \begin{bmatrix} 4/9 \\ 4/9 \\ -2/9 \end{bmatrix} + \begin{bmatrix} 4/9 \\ -2/9 \\ 4/9 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Therefore $P_1 + P_2 + P_3 = b$

Step 1 of 6

Let P_1 = the projection matrix onto the line through $a_1 = \frac{a_1 a_1^T}{a_1^T a_1}$ ...... (1)

$$a_1 a_1^T = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} (-1, 2, 2)$$

$$= \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$$

$$a^T a = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$= 1 + 4 + 4$$

$$= 9$$

In view of (1), we get $P_1 = \frac{1}{9} \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$

Provide feedback (0)

Step 2 of 6

Let P_2 = the projection matrix onto the line through $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} (2, 2, -1)$$

$$= \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \end{bmatrix}$$

Step 2 of 6

Let $P_2 =$ the projection matrix onto the line through $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} (2, 2, -1)$$
$$= \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$
$$a_2^T a_2 = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$
$$= 4 + 4 + 1$$
$$= 9$$
$$P_2 = \frac{1}{9} \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 6

Let $P_3 =$ the projection matrix onto the line through $a_3 = \frac{a_3 a_3^T}{a_3^T a_3}$

$$a_3 a_3^T = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} (2, -1, 2)$$
$$= \begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$$

$$a_3^T a_3 = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$
$$= 4+1+4$$
$$= 9$$

$$P_3 = \frac{1}{9}\begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 6

$$P_1 + P_2 + P_3 = \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & -2+4-2 & -2-2+4 \\ -2+4-2 & 4+4+1 & 4-2-2 \\ -2-2+4 & 4-2-2 & 4+1+4 \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 6

Chapter: CH3.2 Problem: 23P show all steps

Provide feedback (0)

Step 5 of 6

We easily see that $a_1^T a_2 = (-1,2,2)\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

Provide feedback (0)

Step 6 of 6

$$= -2+4-2$$
$$= 0$$

$$a_2^T a_3 = (2,2,-1)\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$
$$= 4-2-2$$
$$= 0$$

$$a_3^T a_1 = (2,-1,2)\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$
$$= 4-2-2$$
$$= 0$$

That means $\{a_1, a_2, a_3\}$ is an orthogonal set of vectors and so, are linearly independent,
the dimension of each vector is 3 and thus forms a basis to $\mathbf{R}^3$

Provide feedback (0)

CH3.2 23P

CH3.2 25P

Step 1 of 3

Let P_1 = the projection of b onto the line through $a_1 = \frac{a_1^T b}{a_1^T a_1} a_1$

$$a_1^T b = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= 1+0$$
$$= 1$$

$$a_1^T a_1 = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= 1+0$$
$$= 1$$

$$P_1 = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Step 2 of 3

Let P_2 = the projection of b onto the line through $a_2 = \frac{a_2^T b}{a_2^T a_2} a_2$

$$a_2^T b = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= 1+2$$
$$= 3$$

$$a_2^T a_2 = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
$$= 1+4$$

$$= 5$$

$$P_2 = \frac{3}{5}\begin{pmatrix}1\\2\end{pmatrix}$$

$$= \begin{pmatrix}3/5\\6/5\end{pmatrix}$$

Provide feedback (0)

Step 3 of 3

$$P_1 + P_2 = \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}3/5\\6/5\end{pmatrix}$$

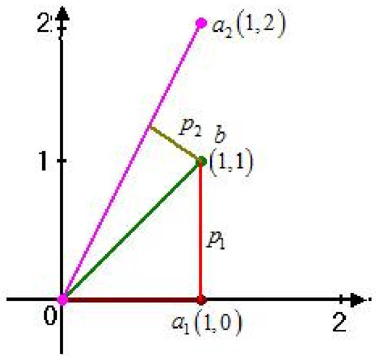
$$= \begin{pmatrix}8/5\\6/5\end{pmatrix}$$

$P_1 + P_2 \neq b$ because a_1, a_2 are not orthogonal

This is established with the help of $a_1^T a_2 = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}$

$$= 1 + 0$$

$$= 1 \neq 0$$



CH3.2 24P

CH3.2 26P

Step 1 of 2

Let $a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Writing $A = [a_1 \quad a_2]$

$= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, the required projection matrix is $P = A\left(A^T A\right)^{-1} A^T$

$A^T A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$

$\left(A^T A\right)^{-1} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix}$

Provide feedback (0)

Step 2 of 2

$A\left(A^T A\right)^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ -2 & 2 \end{bmatrix}$

$A\left(A^T A\right)^{-1} A^T = \frac{1}{4}\begin{bmatrix} 4 & 0 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

Therefore, $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Provide feedback (0)

CH3.2 25P

CH3.3

Step 1 of 3

Let P_1 = the projection matrix onto $a_1 = \frac{a_1 a_1^T}{a_1^T a_1}$

$$a_1 a_1^T = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$$

$$a_1^T a_1 = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 1+0 = 1$$

Therefore, $P_1 = \frac{1}{1}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

$$= \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$$

Provide feedback (0)

Step 2 of 3

Let P_2 = the projection matrix onto $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix} = \begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$$

$$a_2^T a_2 = \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} = 1+4 = 5$$

$$P_2 = \frac{1}{5}\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

$$P = P_1 P_2$$

$$= \begin{bmatrix}1 & 0\\ \end{bmatrix}\frac{1}{\ }\begin{bmatrix}1 & 2\\ \end{bmatrix}$$

Step 2 of 3

Let P_2 = the projection matrix onto $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{pmatrix}1\\2\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix} = \begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$$

$$a_2^T a_2 = \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} = 1+4 = 5$$

$$P_2 = \frac{1}{5}\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

$$\begin{aligned}P &= P_1 P_2\\ &= \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\frac{1}{5}\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}\\ &= \frac{1}{5}\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\end{aligned}$$

$$\begin{aligned}P^2 &= \frac{1}{25}\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\\ &= \frac{1}{25}\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\\ &\neq P\end{aligned}$$

Therefore, PP is not a projection matrix.

Provide feedback (0)