

Step 3 of 4 c) We have to find $\det (A^2)$. We know that $\det (A^n) = (\det A)^n$. Now $\det (A^2) = (\det A)^2$ $= \left(\frac{1}{2}\right)^2$ $= \frac{1}{4}$ Thus, $\det (A^2) = \frac{1}{4}$

Provide feedback (0)

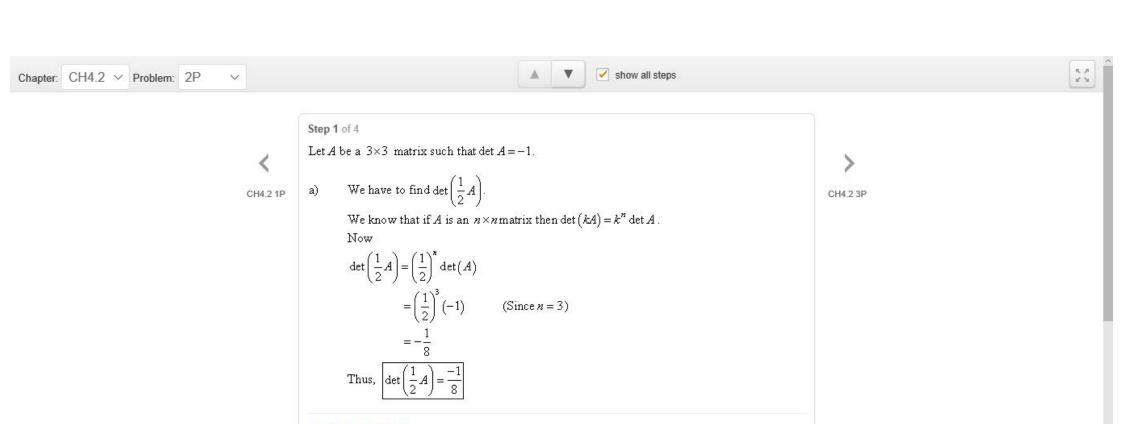
Step 4 of 4

d) We have to find
$$\det \left(A^{-1}\right)$$
.

Now
$$\det \left(A^{-1}\right) = \frac{1}{\det \left(A\right)}$$

$$= \frac{1}{2}$$

$$= 2$$
Thus, $\det \left(A^{-1}\right) = 2$

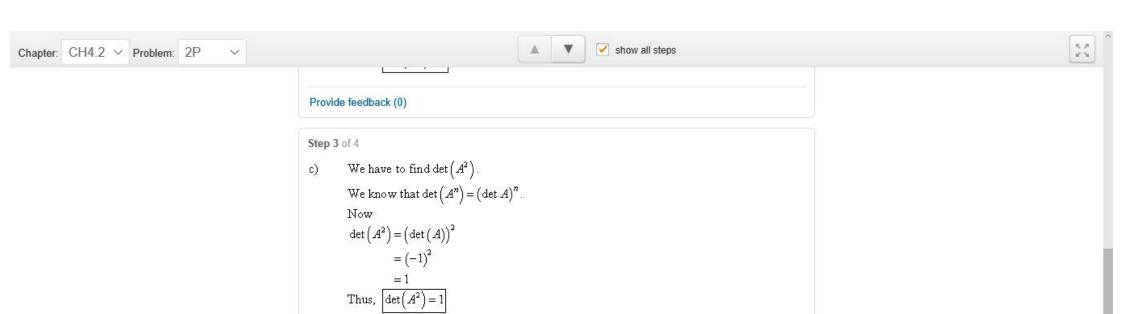


Provide feedback (0)

Step 2 of 4 b) We have to find $\det(-A)$. Now $\det(-A) = -\det(A)$ = -(-1) = 1Thus, $\det(-A) = 1$

Provide feedback (0)

Step 3 of 4

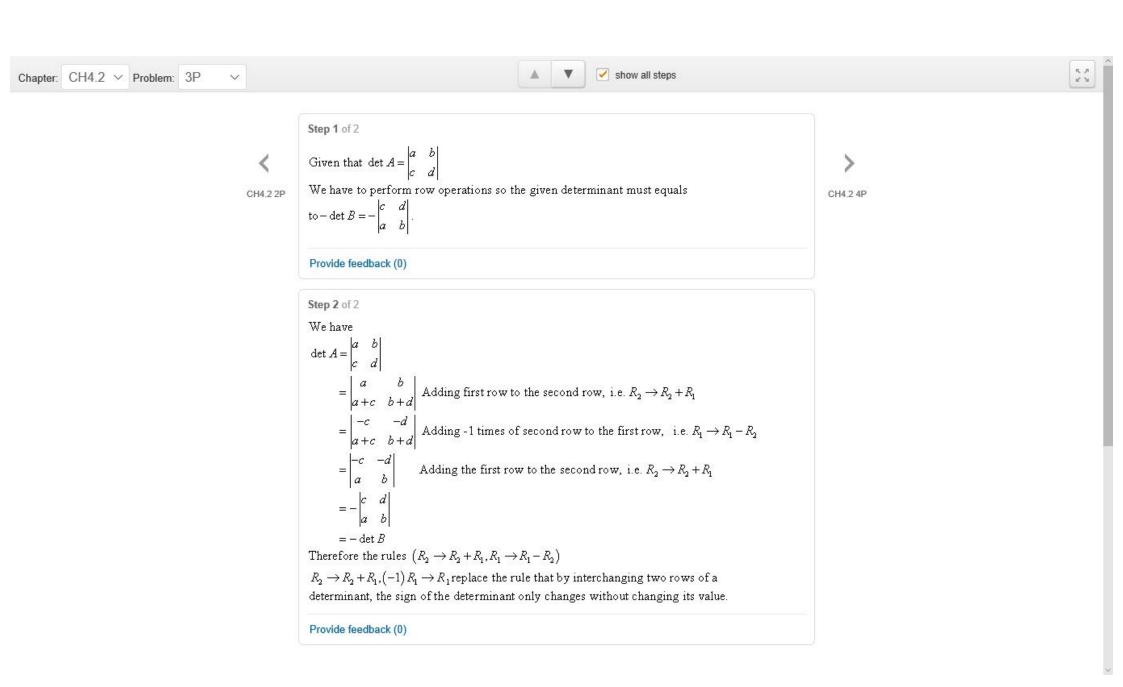


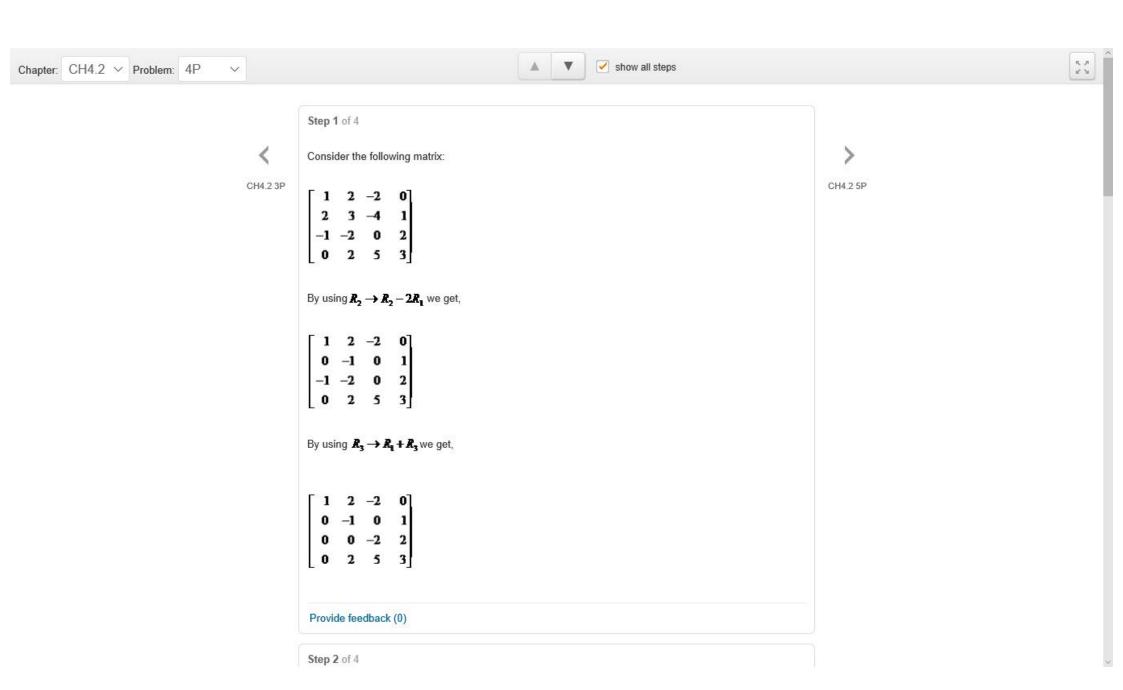
Provide feedback (0)

Step 4 of 4

d) We have to find
$$\det (A^{-1})$$

Now
$$\det (A^{-1}) = \frac{1}{\det (A)}$$
$$= \frac{1}{-1}$$
$$= -1$$
Thus,
$$\det (A^{-1}) = -1$$





By using $R_4 \rightarrow 2R_2 + R_4$, we get,

Provide feedback (0)

Step 3 of 4

By using $R_4 \rightarrow \frac{5}{2} R_3 + R_4$, we get,

We know that if M is triangular, then det M is the product of the diagonal entries.

show all steps

Thus, we have

$$\det \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} = \begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{vmatrix}$$



Step 3 of 4

By using $R_4 \rightarrow \frac{5}{2} R_3 + R_4$, we get,

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

We know that if M is triangular, then det M is the product of the diagonal entries.

Thus, we have

$$\det\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} = \begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{vmatrix}$$
$$= (1)(-1)(-2)(10)$$
$$= \boxed{20}$$

Provide feedback (0)

Step 4 of 4

Consider the following matrix:



show all steps

Step 4 of 4

Consider the following matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

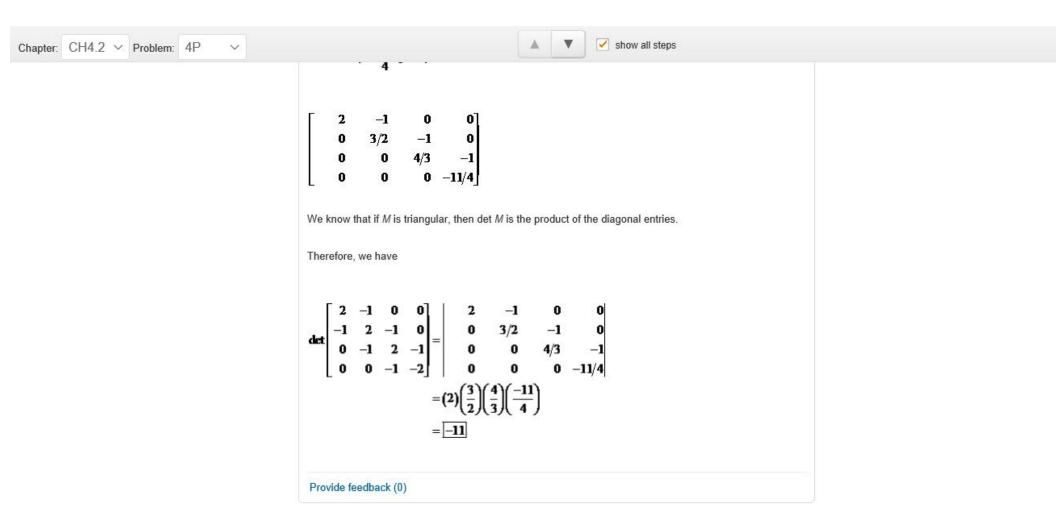
By using $R_2 \rightarrow R_2 + \frac{R_1}{2}$ we get,

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

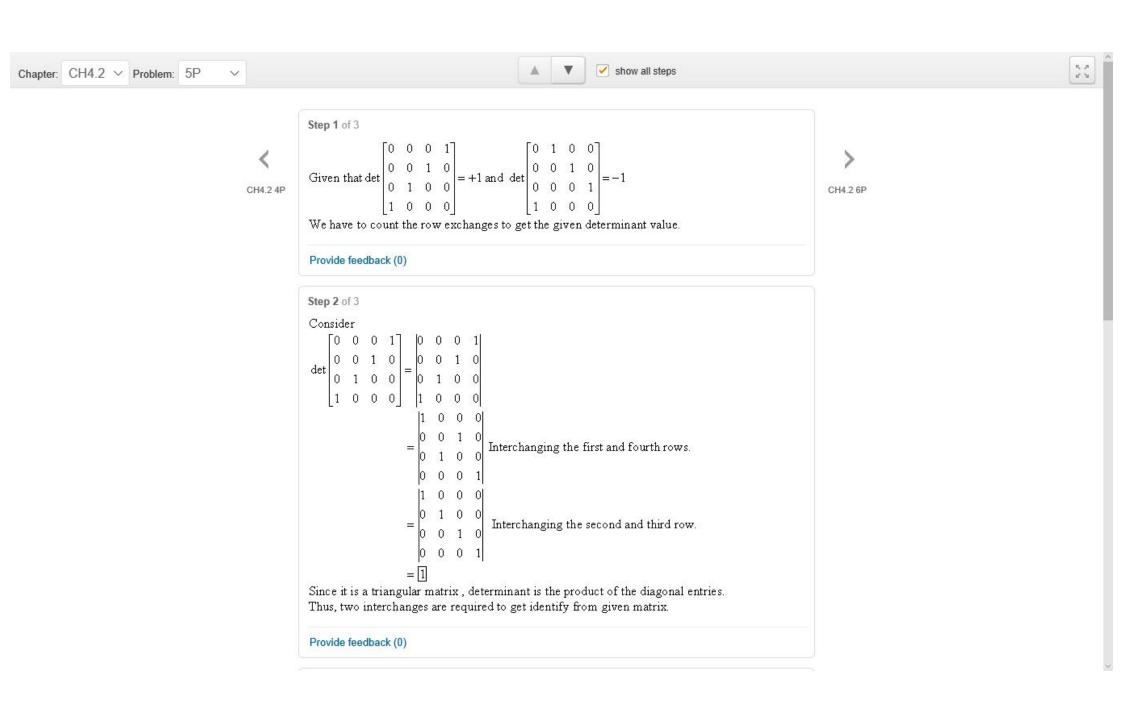
By using $R_3 \rightarrow \frac{2}{3} R_2 + R_3$ we get,

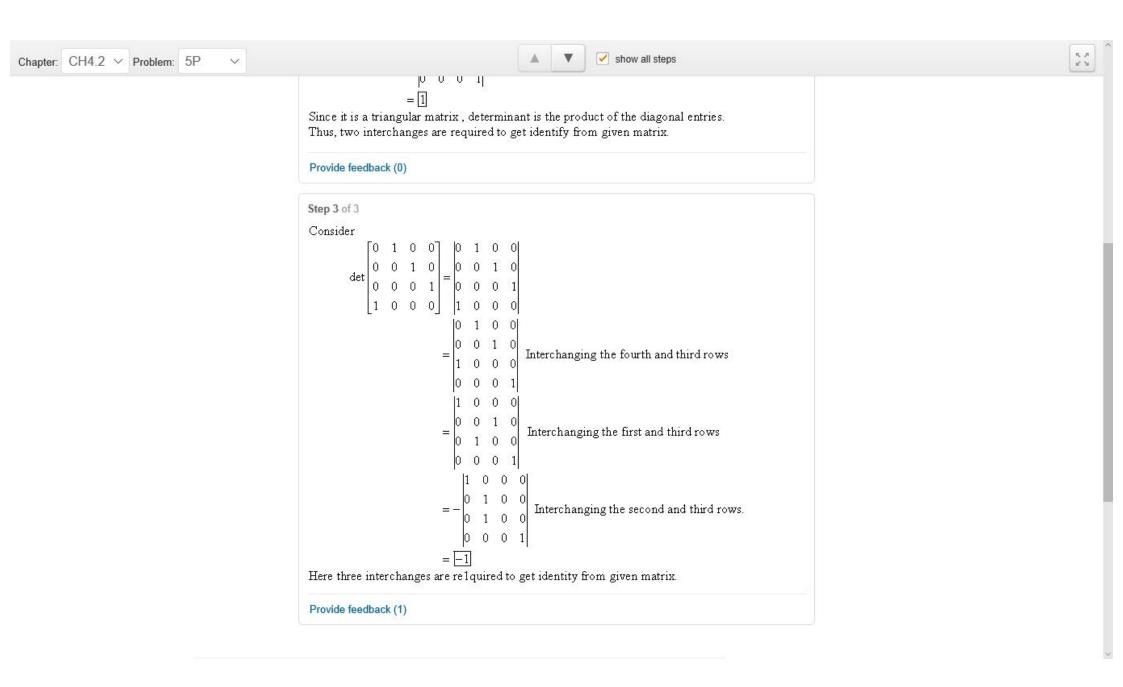
$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

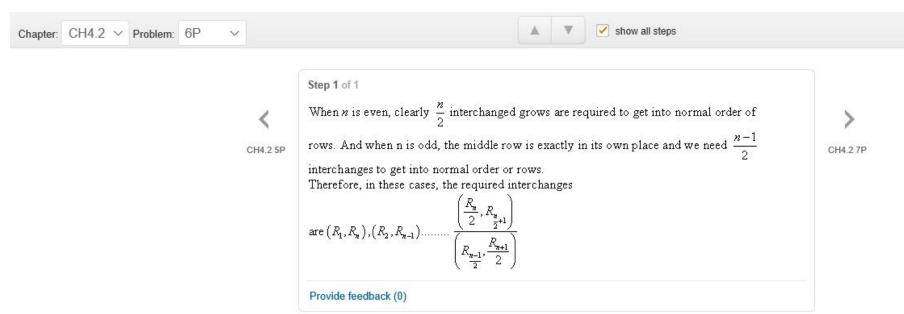
By using $R_4 \rightarrow \frac{3}{4}R_3 + R_4$ we get,



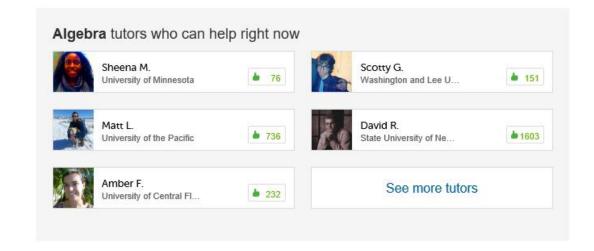


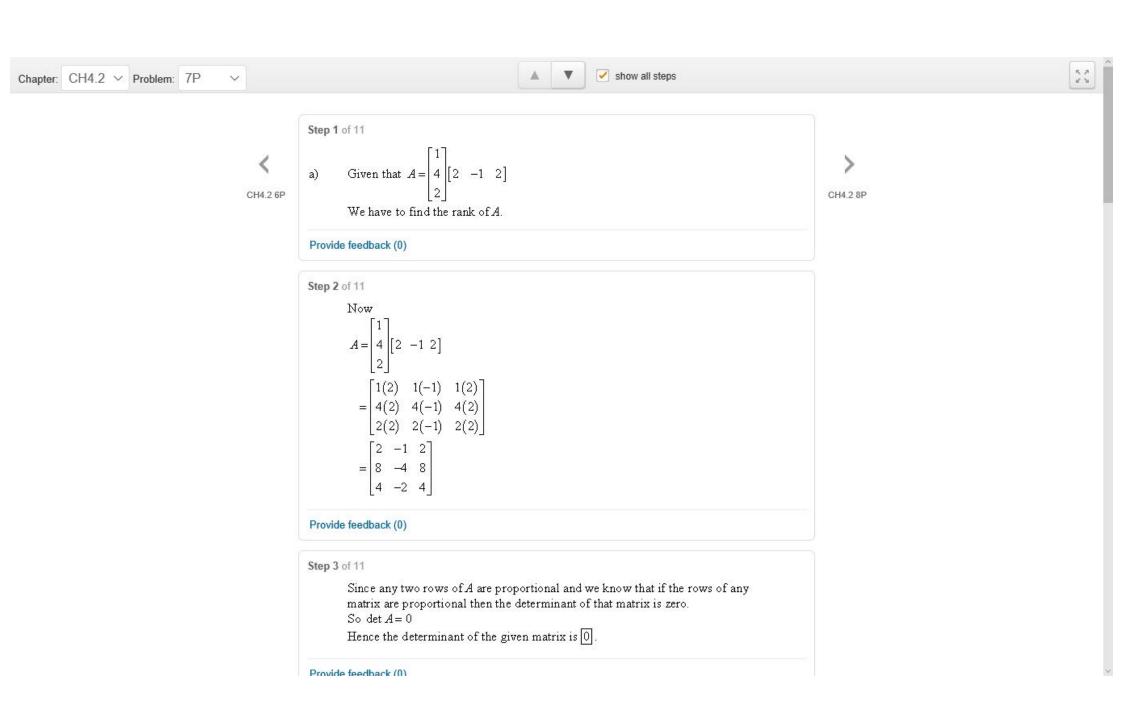


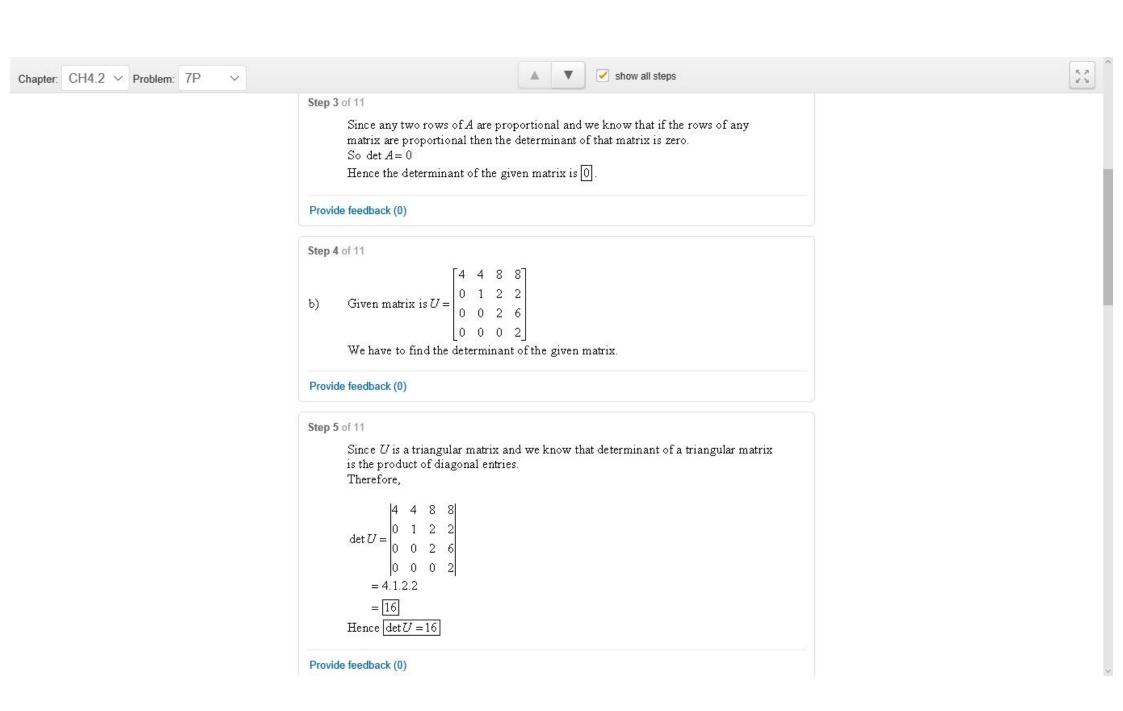


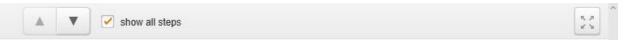


5.7









Step 6 of 11

Chapter: CH4.2 V Problem: 7P

c) Given upper triangular matrix is
$$U = \begin{bmatrix} 4 & 4 & 6 & 6 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Then
$$U^T = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 8 & 2 & 2 & 0 \\ 8 & 2 & 6 & 2 \end{bmatrix}$$
 is a lower triangular matrix.

We have to find the determinant of \boldsymbol{U}^{T}

Provide feedback (0)

Step 7 of 11

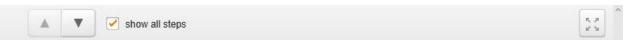
Since U^T is a triangular matrix and we know that determinant of a triangular matrix is the product of diagonal entries. Therefore,

$$\det U^{T} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 8 & 2 & 2 & 0 \\ 8 & 2 & 6 & 2 \end{bmatrix}$$
$$= 4.1.2.2$$
$$= \boxed{16}$$
Hence
$$\det U^{T} = 16$$

Provide feedback (0)

Step 8 of 11

d) We have to find the determinant of the inverse matrix II^{-1}



Step 8 of 11

Chapter: CH4.2 V Problem: 7P

d) We have to find the determinant of the inverse matrix U^{-1} . We know that $\det A^{-1} = \frac{1}{\det A}$, for any matrix A. Therefore,

Provide feedback (0)

Step 9 of 11

$$\det U^{-1} = \frac{1}{\det U}$$

$$= \frac{1}{16}$$
Hence
$$\det U^{-1} = \frac{1}{16}$$

Provide feedback (0)

Step 10 of 11

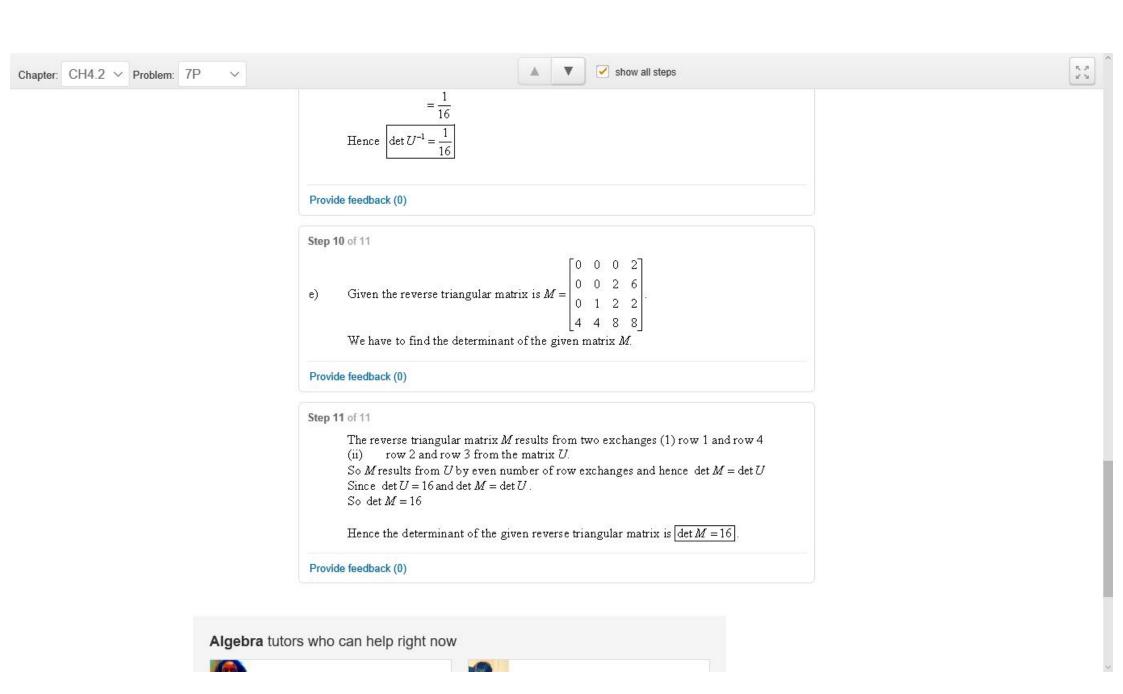
e) Given the reverse triangular matrix is $M = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 1 & 2 & 2 \\ 4 & 4 & 8 & 8 \end{bmatrix}$

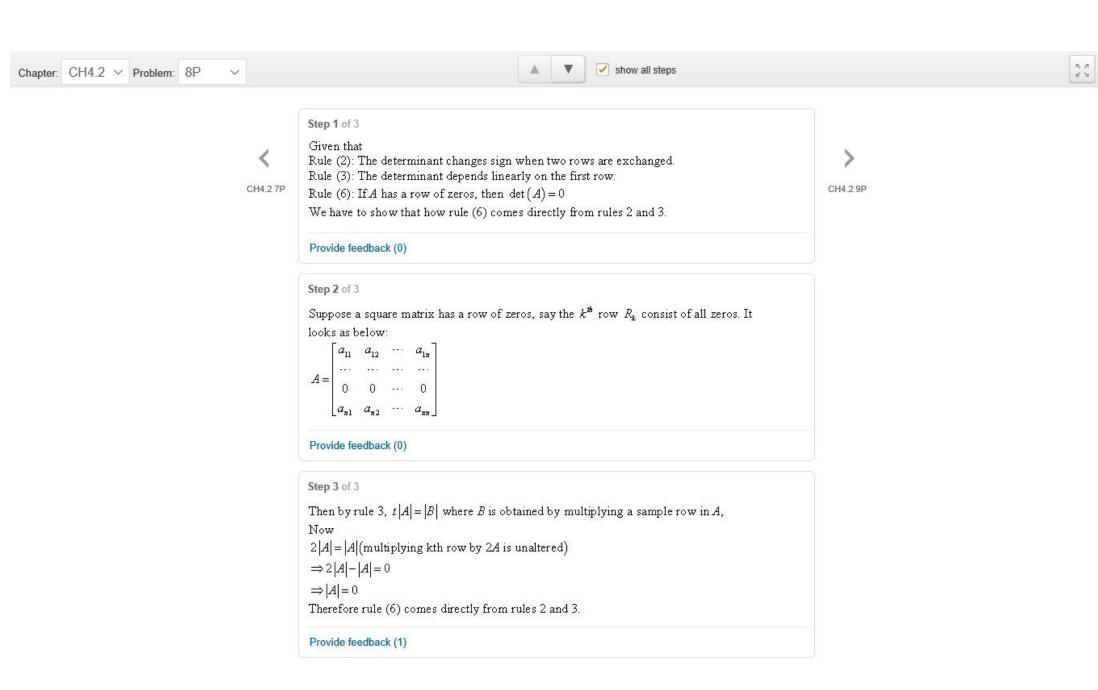
We have to find the determinant of the given matrix M.

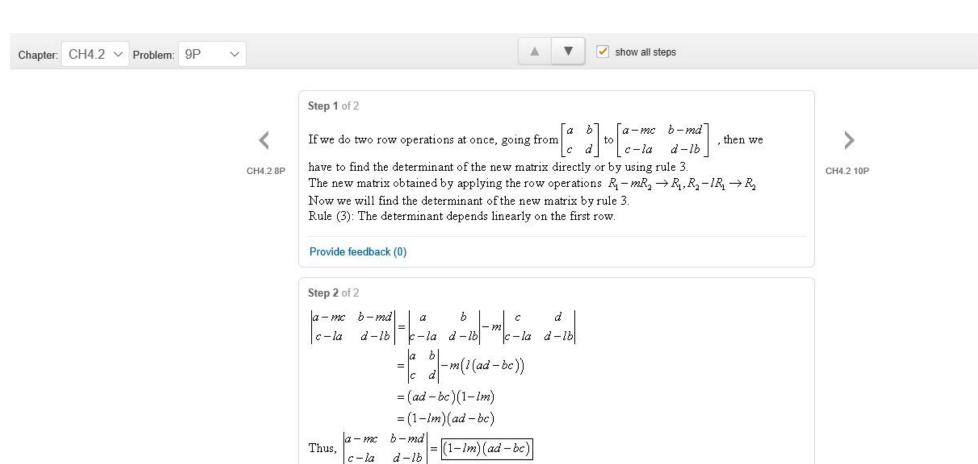
Provide feedback (0)

Step 11 of 11

The reverse triangular matrix M results from two exchanges (1) row 1 and row 4

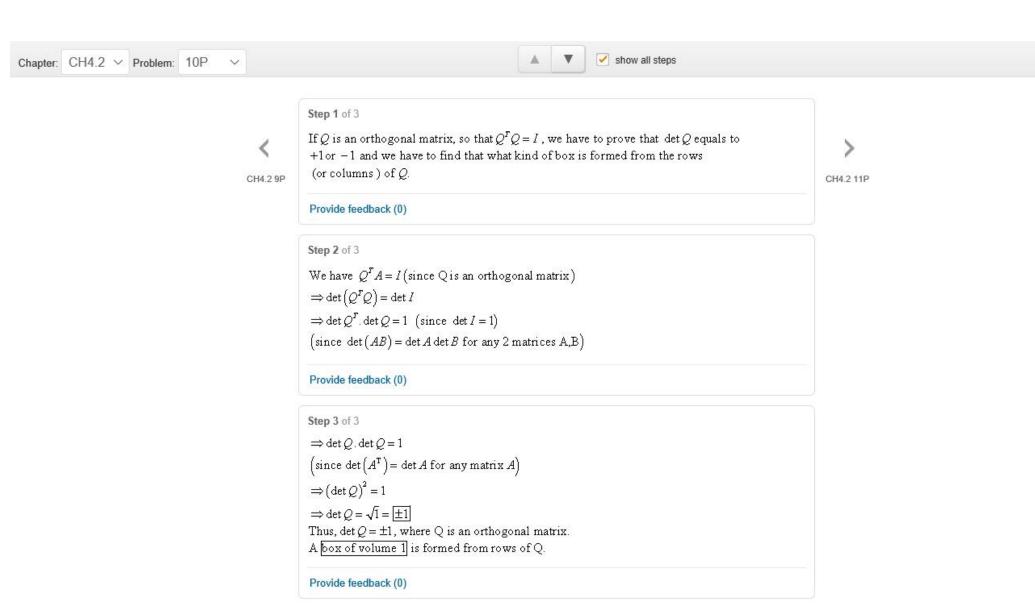






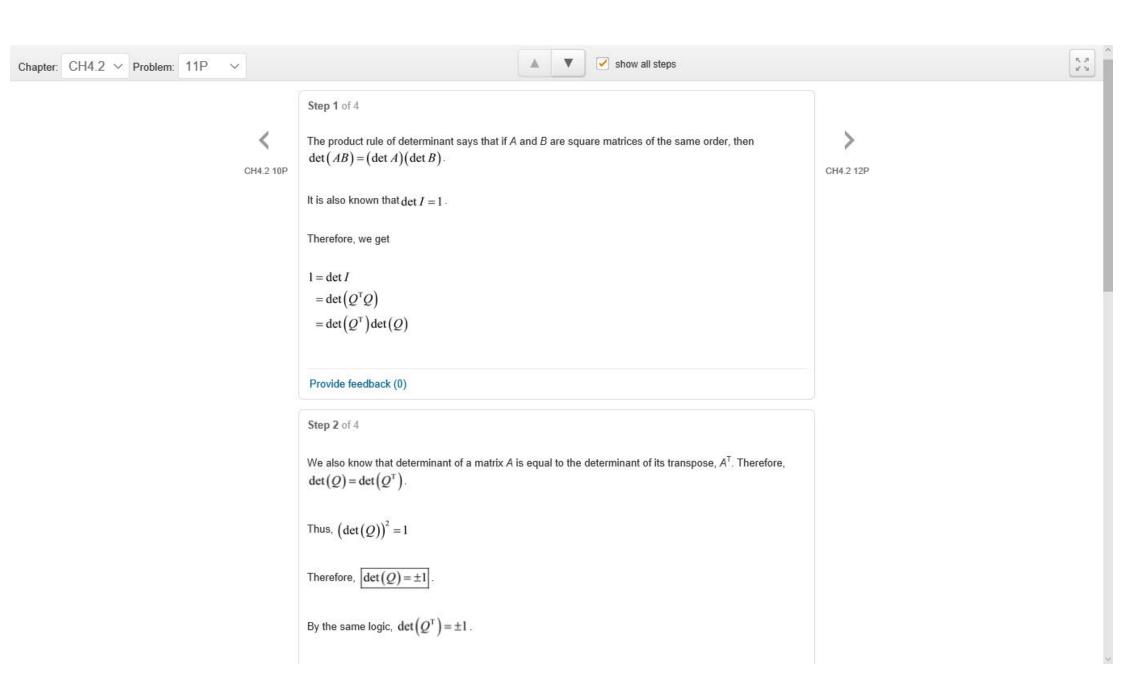
K 2





K 2

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A | W |

I

show all steps

Step 3 of 4

Now we show that Q^2 is also an Orthogonal matrix. Thus, we need to show that $\left(Q^2\right)^{-1} = \left(Q^2\right)^{\mathrm{T}}$.

$$Q^{2}(Q^{2})^{\mathsf{T}} = (Q \cdot Q)(Q^{\mathsf{T}} \cdot Q^{\mathsf{T}})$$

$$= Q(Q \cdot Q^{\mathsf{T}})Q^{\mathsf{T}}$$

$$= Q \cdot Q^{\mathsf{T}}$$

$$= I$$

Thus, Q² is an Orthogonal matrix. Similarly, it can be shown that Qⁿ is also an Orthogonal matrix.

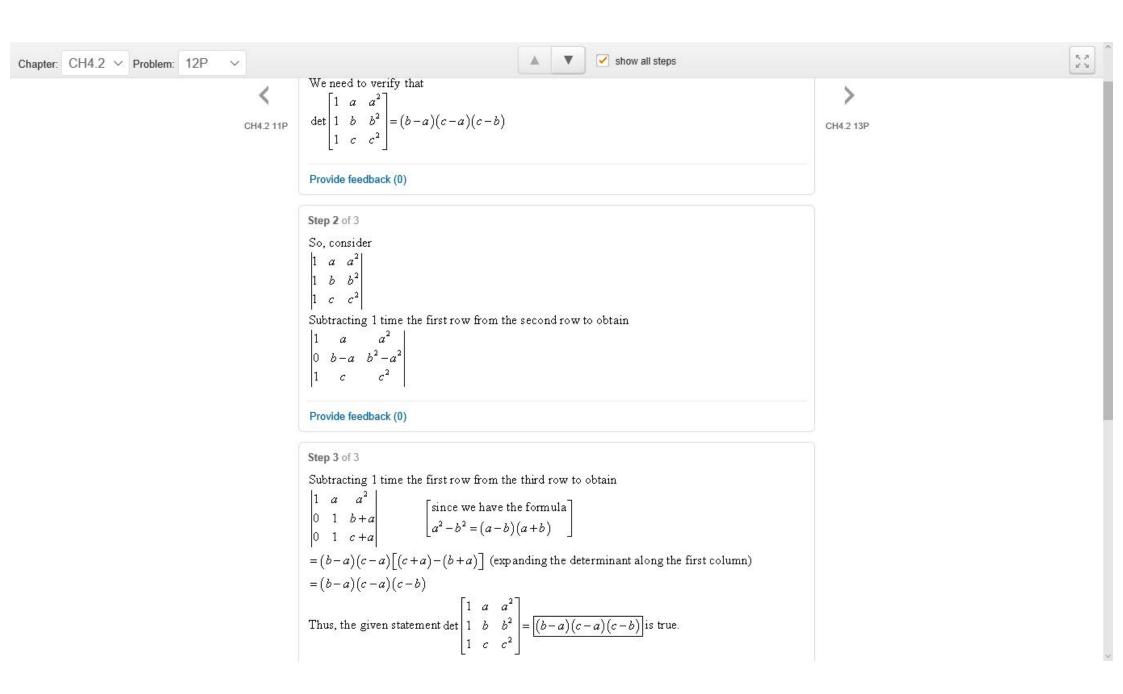
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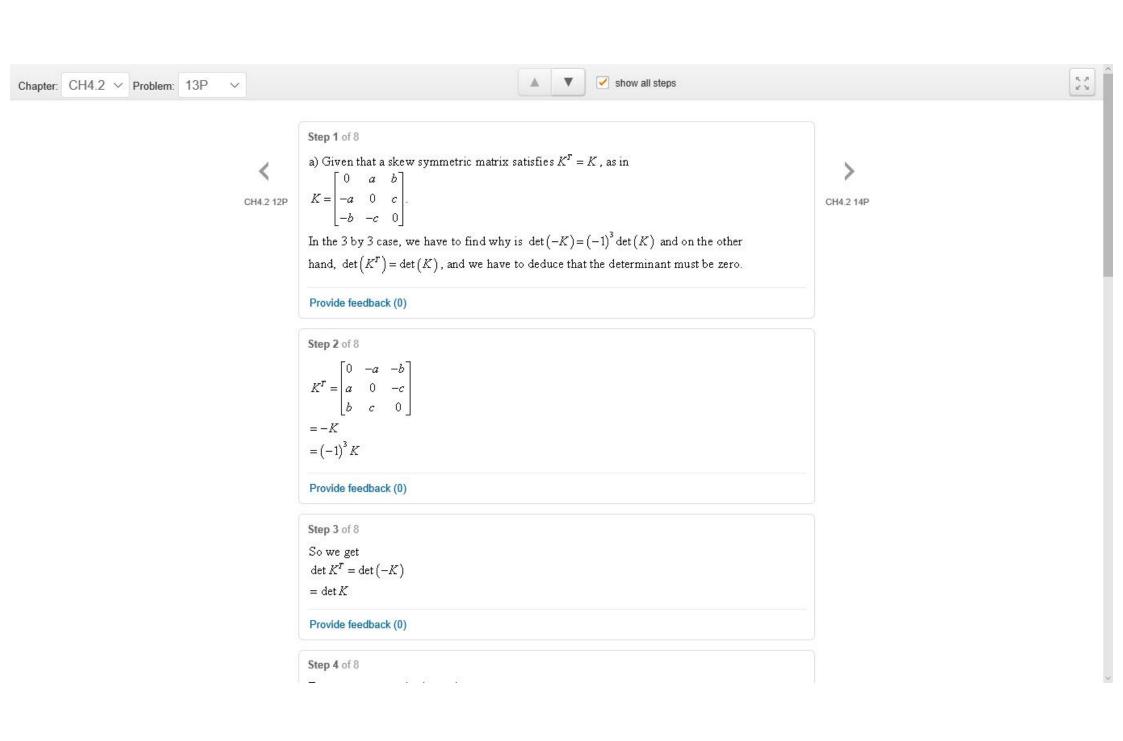
Step 4 of 4

If $\det Q$ is not equal to ± 1 , then $\det \left(Q''\right)$ would either tend to zero or would tend to plus or minus infinity.

But, Qn remains an Orthogonal matrix.

This also shows that $\det Q = \pm 1$





Step 4 of 8

For any $n \times n$ matrix A, we have $\det(tA) = t^n \det A$ So for a 3×3 matrix K we have

 $\det(-K) = (-1)^3 \det K$

 $=-\det K$

Provide feedback (0)

Step 5 of 8

Therefore for skew symmetric matrix K we get $\det K = -\det K$ and hence $2\det K = 0$ Giving that $\det K = 0$

Provide feedback (0)

Step 6 of 8

b) We have to write down a 4 by 4 skew symmetric matrix with det $K \neq 0$ Consider

$$K = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 3 & -1 \\ 0 & -3 & 0 & 2 \\ -2 & 1 & -2 & 0 \end{bmatrix}, \text{ where } K \text{ is a skew symmetric matrix of order } 4 \times 4.$$

Provide feedback (0)

Step 7 of 8

$$\det K = -\begin{vmatrix} -1 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 0 & -3 & 0 \end{vmatrix}$$

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CH4.2 15P



Chapter: CH4.2 V Problem: 14P

CH4.2 13P

Step 1 of 8

We have to verify that the statements (a), (b), (c), (d) and (e) are true or false. a) The given statement is "If A and B are identical except that $b_{11} = 2a_{11}$, then det $B = 2 \det A$ ".

For example

Let
$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$
 where $b_{11} = 2a_{11}$

Provide feedback (0)

Step 2 of 8

Now we have

$$\det A = (1)(1) - (1)(1)$$

$$= 0$$

$$\det B = (2)(1) - (1)(1)$$

$$=1$$

Clearly $\det B \neq 2 \det A$

The given statement is false.

Provide feedback (0)

Step 3 of 8

b) The given statement is "The determinant is the product of pivots" is false.

For example

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Three row exchanges (Row 3, Row 4) and (Row 2, Row 3) result in identity matrix which is upon this probability has no dust of six at $a = (-1)^3$ 1 = -1

Step 3 of 8

b) The given statement is "The determinant is the product of pivots" is false. For example

5.7

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Three row exchanges (Row 3, Row 4) and (Row 2, Row 3) result in identity matrix which is upper triangular here product of pivots =1 but det $A = (-1)^3 \cdot 1 = -1$ The given statement is false

Provide feedback (0)

Step 4 of 8

c) The given statement "If A is invertible B is singular then A+B is invertible". Consider

$$A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Now det A = -2 + 1

$$= -1$$

Provide feedback (0)

Step 5 of 8

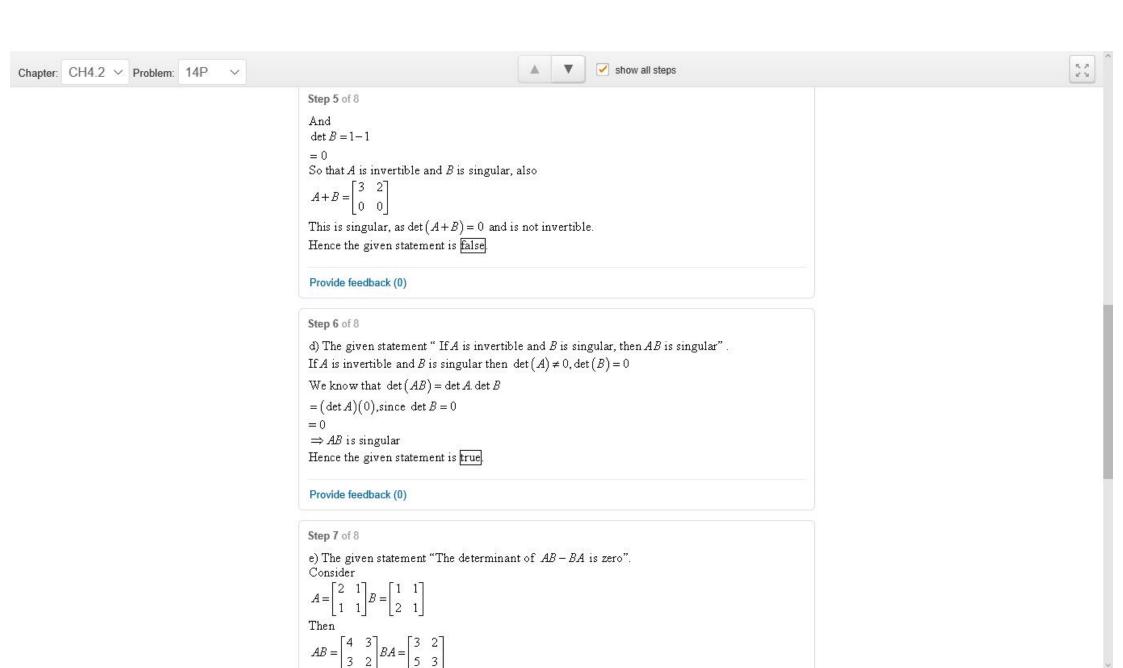
And

 $\det B = 1 - 1$

= 0

So that A is invertible and B is singular, also

$$A+B = \begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}$$





Chapter: CH4.2 V Problem: 14P

If A is invertible and B is singular then $\det(A) \neq 0$, $\det(B) = 0$

We know that
$$\det(AB) = \det A \cdot \det B$$

$$= (\det A)(0)$$
, since $\det B = 0$

= 0

 \Rightarrow AB is singular

Hence the given statement is true.

Provide feedback (0)

Step 7 of 8

e) The given statement "The determinant of AB-BA is zero".

Consider

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Ther

$$AB = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} BA = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 8

Now,

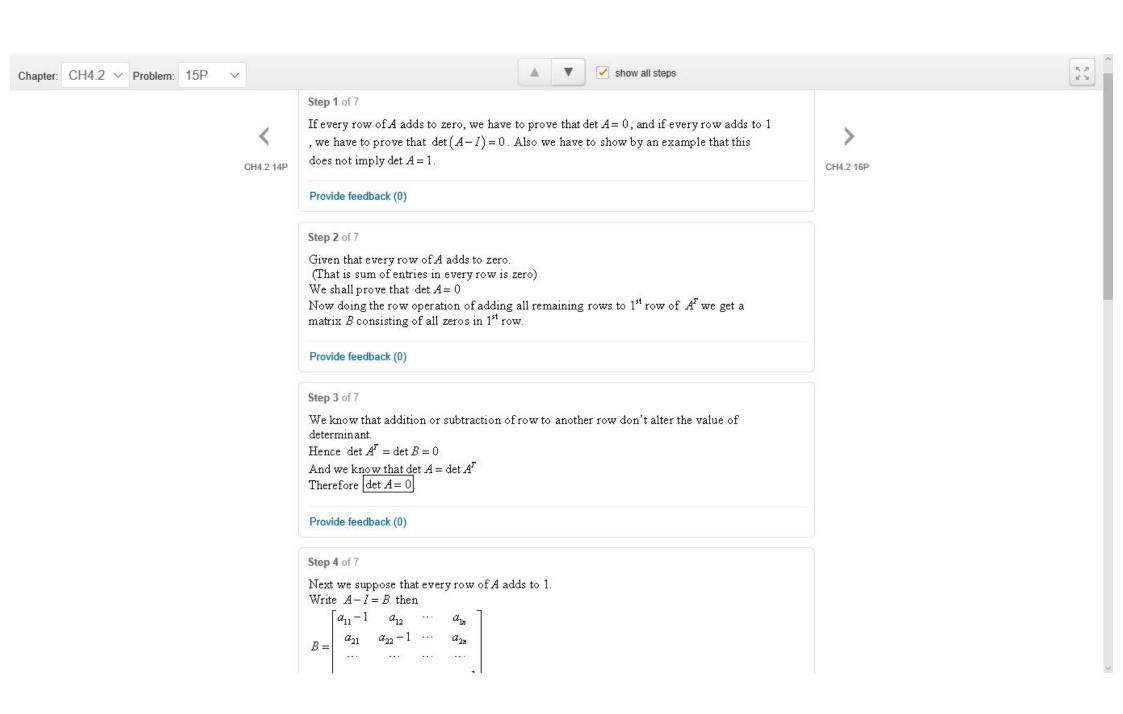
$$AB - BA = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$$

$$\det(AB - BA) = -1 + 2$$

=1

± 1

Hence the given statement is false.





Next we suppose that every row of A adds to 1. Write A-I=B then $\begin{bmatrix} a_{11}-1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-1 & \cdots & a_{2n} \end{bmatrix}$

$$B = \begin{bmatrix} a_{11} - 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - 1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - 1 \end{bmatrix}$$

We can observe that every row of B adds to zero. From the first part, we have $\det B = 0$ Hence $\det (A - I) = 0$

Provide feedback (0)

Step 5 of 7

Now consider

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$
, then

$$A - I = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

Provide feedback (0)

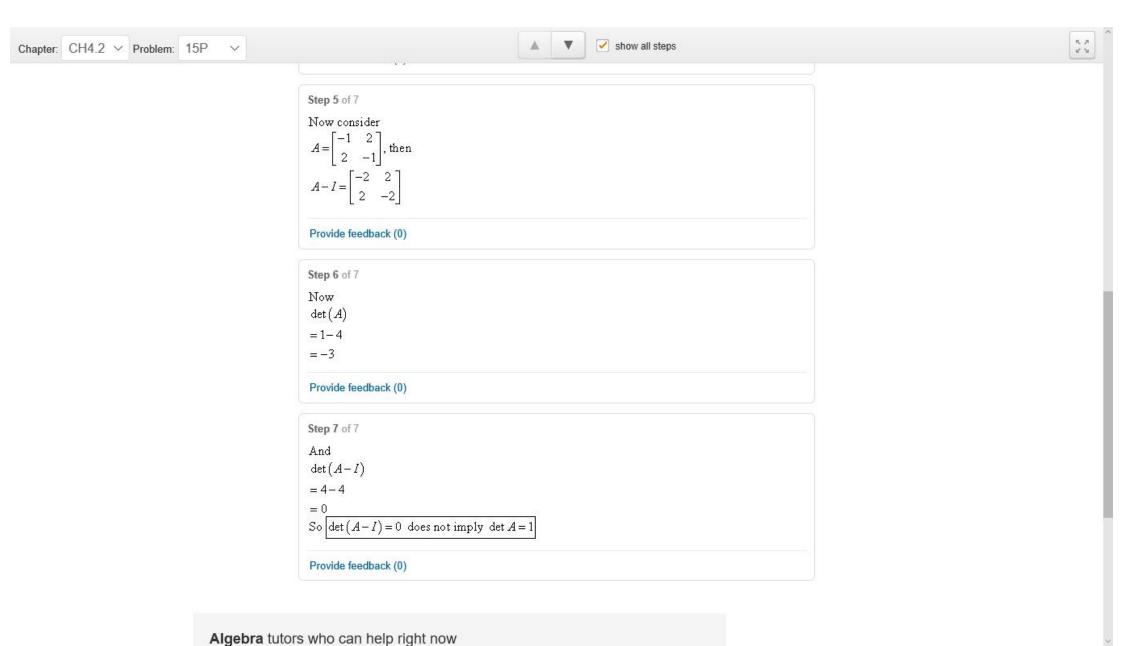
Step 6 of 7

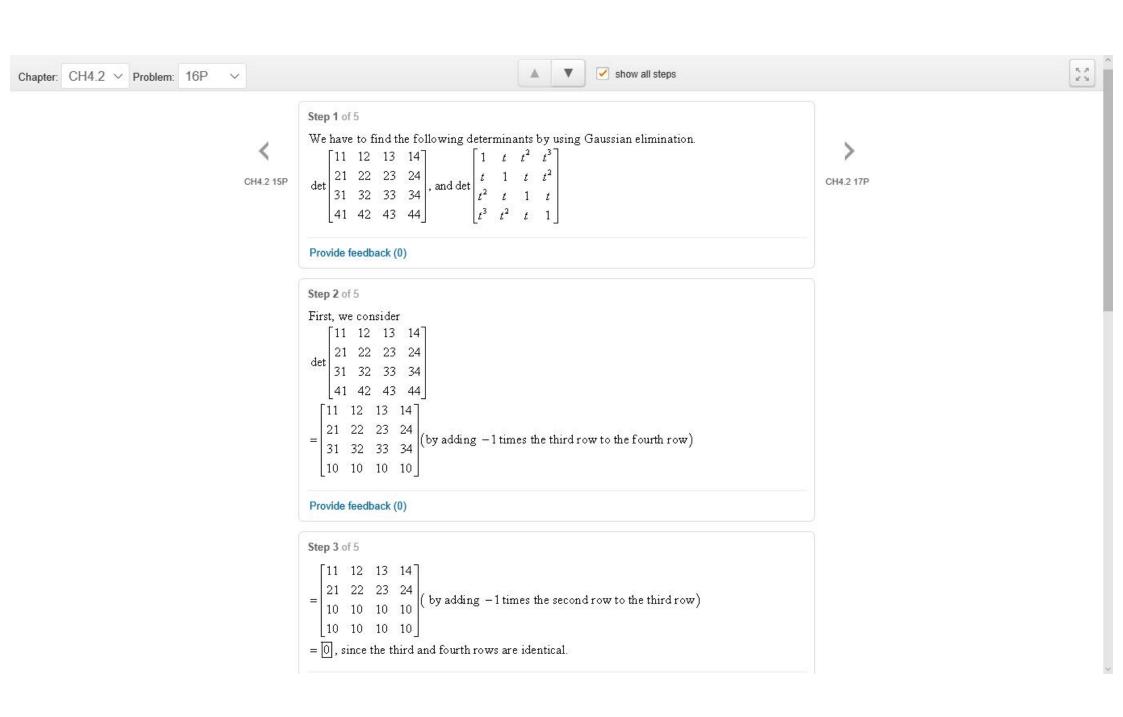
Now

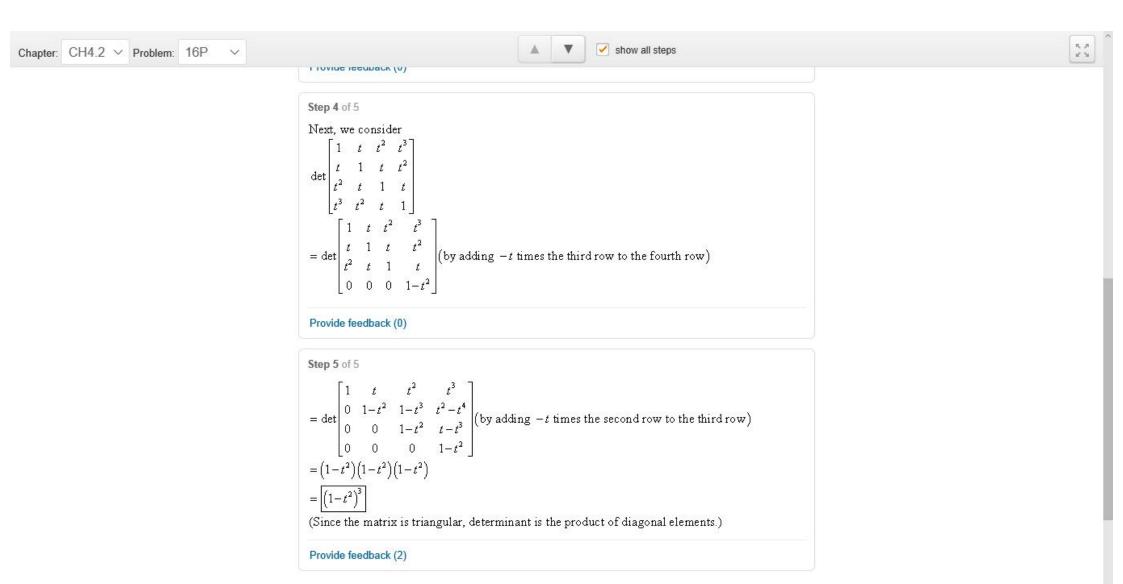
 $\det(A)$

= 1 - 4

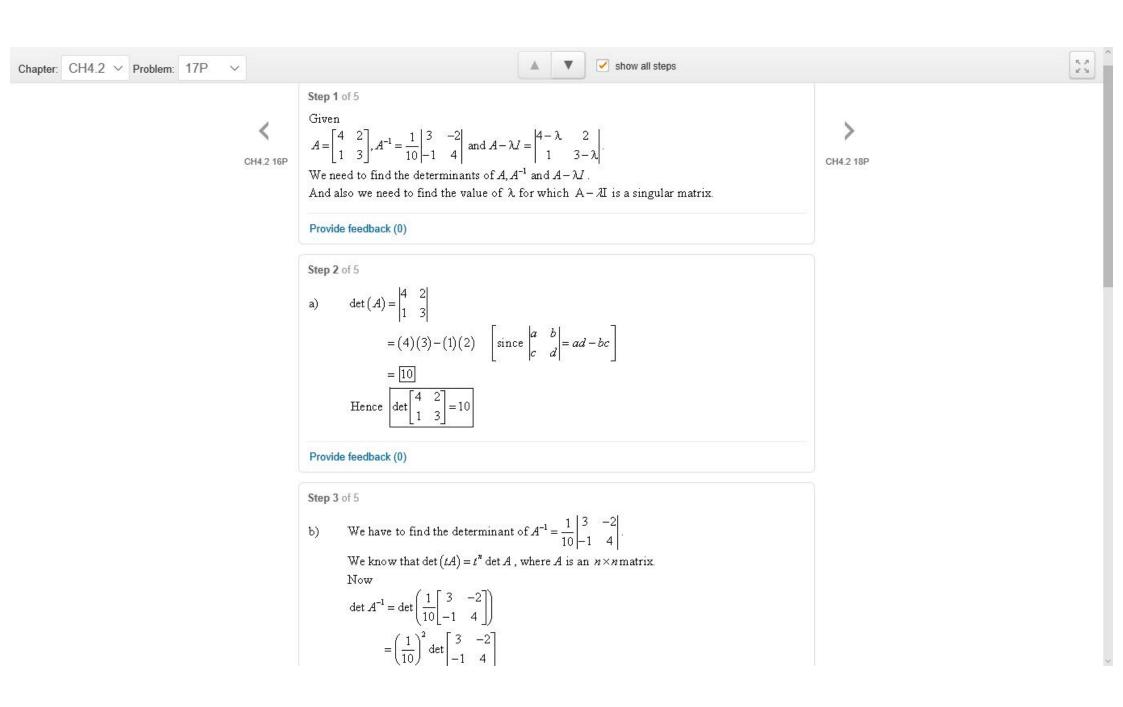
= -3







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Step 3 of 5

Chapter: CH4.2 V Problem: 17P

We have to find the determinant of $A^{-1} = \frac{1}{10} \begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix}$.

We know that $\det (tA) = t^n \det A$, where A is an $n \times n$ matrix.

Now

$$\det A^{-1} = \det \left(\frac{1}{10} \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \right)$$

$$= \left(\frac{1}{10} \right)^2 \det \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$$

$$= \left(\frac{1}{10} \right)^2 \left[(3)(4) - (-1)(-2) \right]$$

$$= \frac{1}{100} (10)$$

$$= \frac{1}{10}$$

Thus, $\det A^{-1} = \frac{1}{10}$

Provide feedback (0)

Step 4 of 5

c) We have to find the determinant of $A - \lambda I = \begin{vmatrix} 4 - \lambda & 2 \\ 1 & 3 - \lambda \end{vmatrix}$.

$$\det(A - \lambda I) = \begin{vmatrix} 4 - \lambda & 2 \\ 1 & 3 - \lambda \end{vmatrix}$$
$$= (4 - \lambda)(3 - \lambda) - (2)(1)$$
$$= 12 - 4\lambda - 3\lambda + \lambda^2 - 2$$

Step 4 of 5

We have to find the determinant of $A - \lambda I = \begin{vmatrix} 4 - \lambda & 2 \\ 1 & 3 - \lambda \end{vmatrix}$.

Now

$$\det(A - \lambda I) = \begin{vmatrix} 4 - \lambda & 2 \\ 1 & 3 - \lambda \end{vmatrix}$$
$$= (4 - \lambda)(3 - \lambda) - (2)(1)$$
$$= 12 - 4\lambda - 3\lambda + \lambda^2 - 2$$
$$= \lambda^2 - 7\lambda + 10$$

Provide feedback (0)

Step 5 of 5

We need to find the values of λ for which $A - \lambda I$ is a singular matrix. We know that the determinant of a singular matrix is 0.

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 10 = 0$$

$$\Rightarrow (\lambda - 2)(\lambda - 5) = 0$$

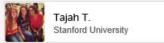
$$\Rightarrow \lambda = 2 \text{ or } 5$$

Thus, $A - \lambda I$ is a singular matrix if $\lambda = 2$ or 5

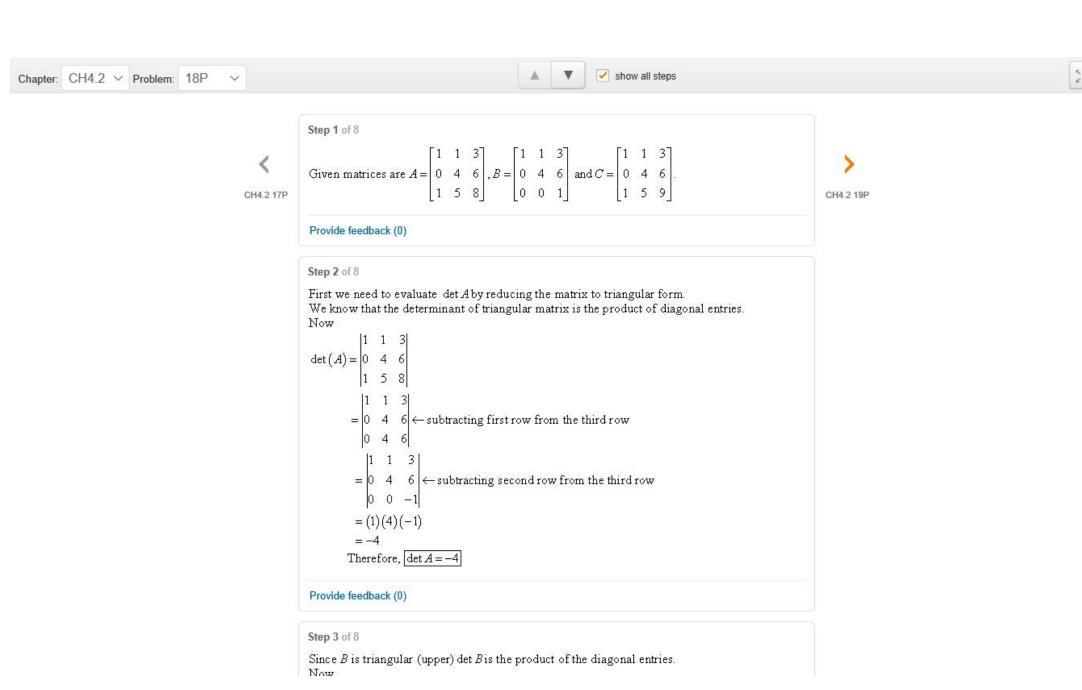
Provide feedback (0)

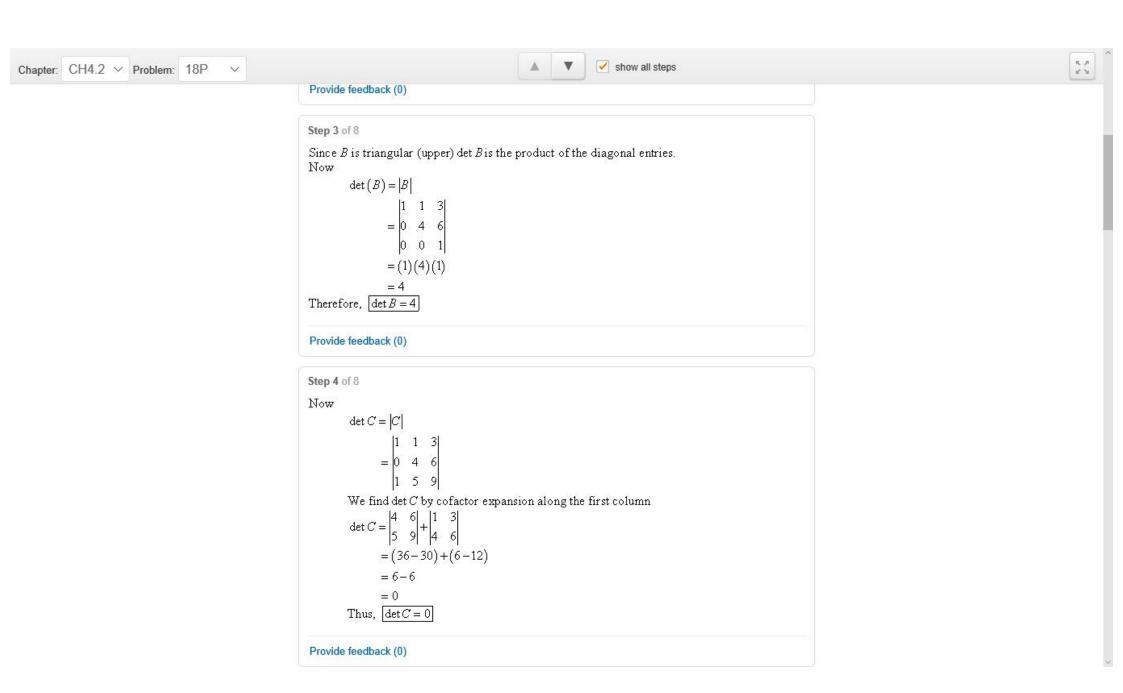
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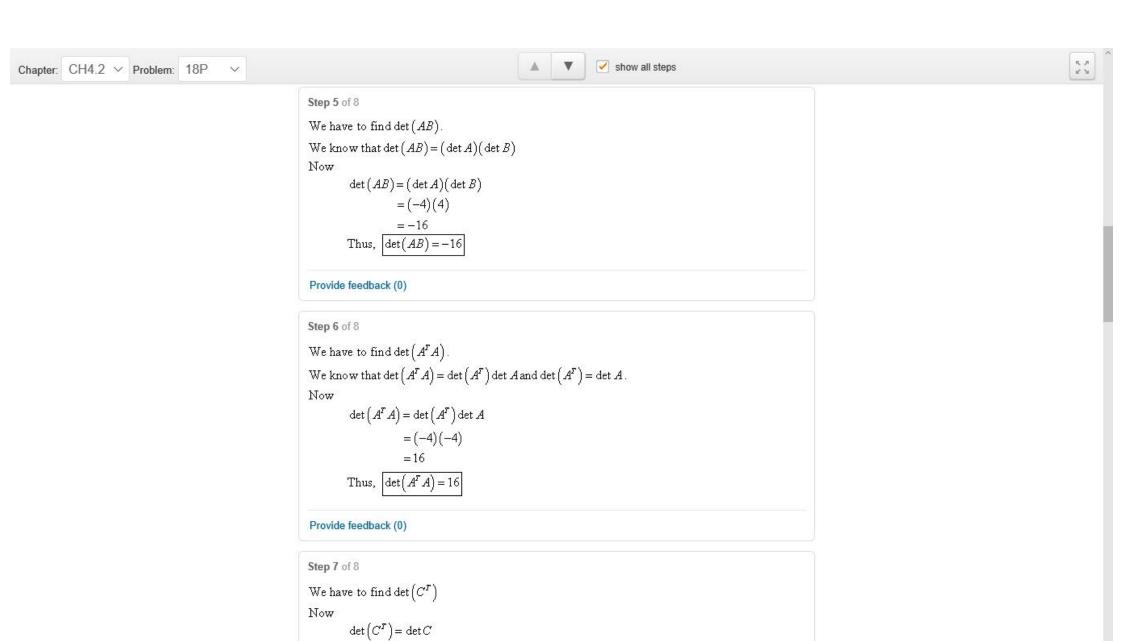




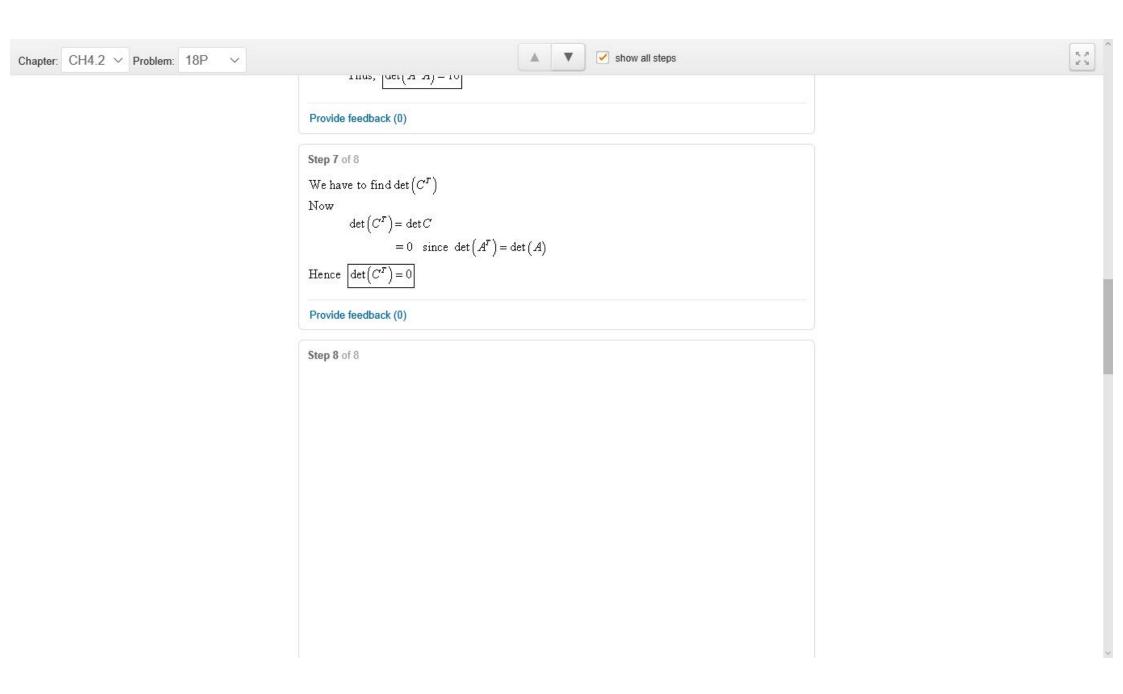
b 0

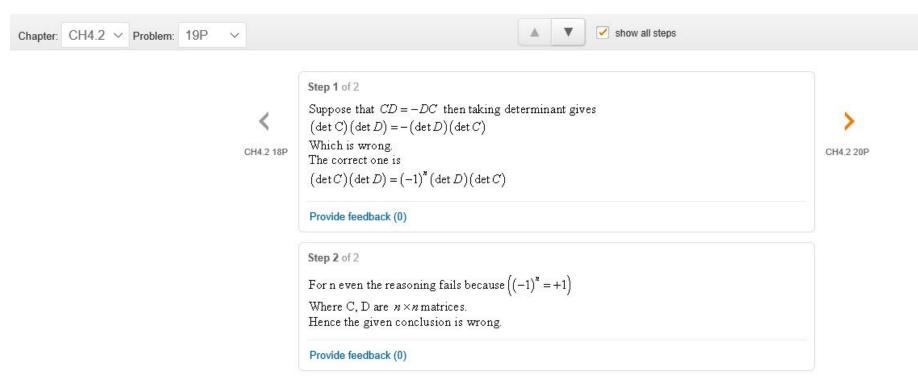




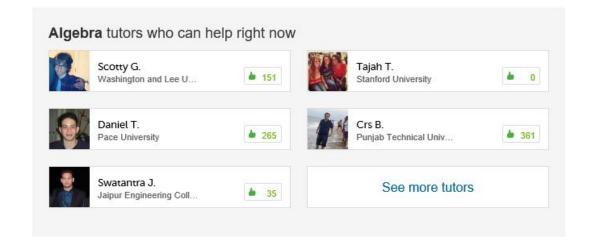


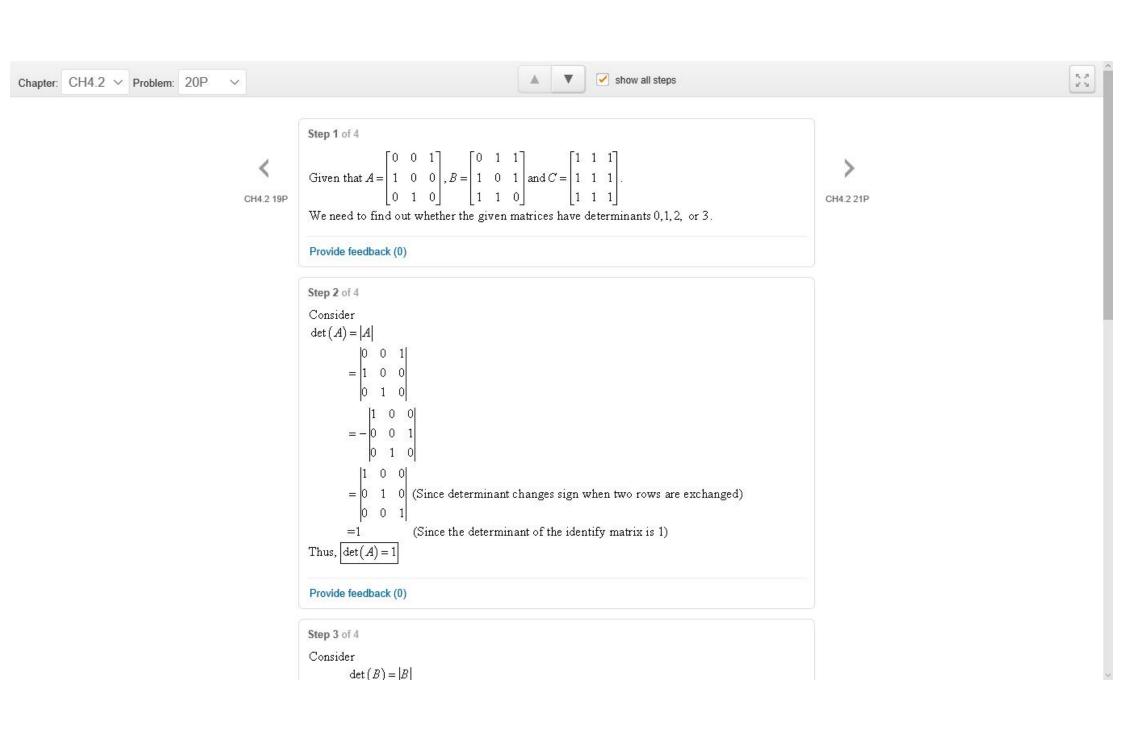
=0 since $\det(A^T)=\det(A)$





5.7









▲ ▼ show all steps

Step 3 of 4

Consider

$$\det(B) = |B|$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

We find det(B) by cofactor expansion along the first row.

Therefore,

$$\det B = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$
$$= (-1) - (1)$$
$$= -2$$
Thus,
$$\det B = -2$$

Provide feedback (1)

Step 4 of 4

Consider

$$\det C = |C|$$

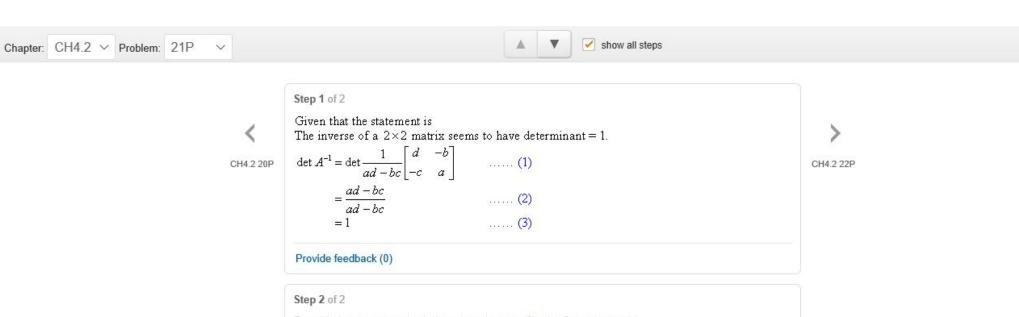
$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 0 \qquad \text{(Since the rows of the matrix C are identical)}$$

$$Thus, \det C = 0$$

Thus, out of the 3 matrices A, B and C, only A and C have the determinants 1 and 0.

Provide feedback (1)



5.7

But this is a wrong calculation since in step (2), the formula used is
$$\det(tA) \det(tA) = t \cdot \det(A)$$
 which is false.

We need to find correct $\det A^{-1}$

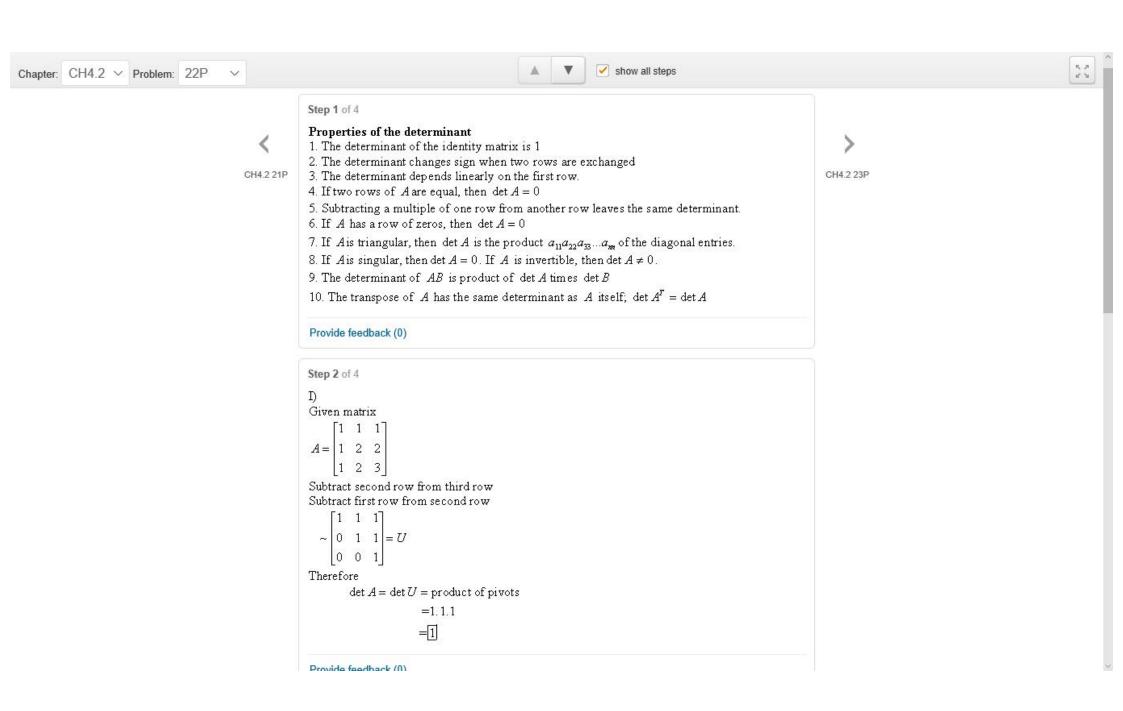
Now
$$\det A^{-1} = \det \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

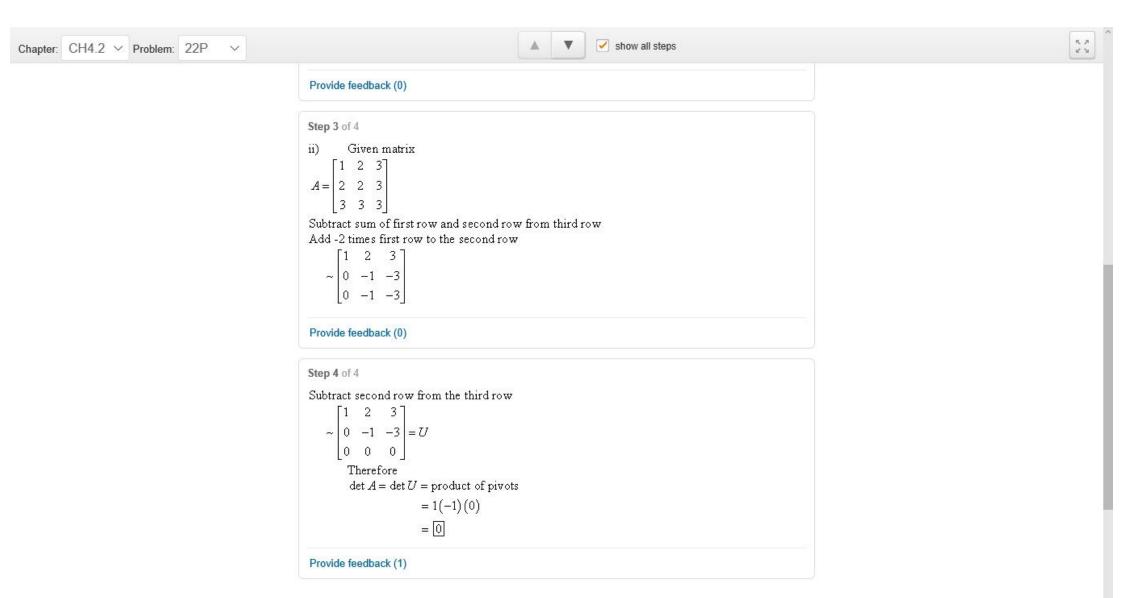
$$= \left(\frac{1}{ad - bc}\right)^2 (ad - bc) \quad \text{(Here } n = 2 \text{, since the size of the matrix is 2 by 2)}$$

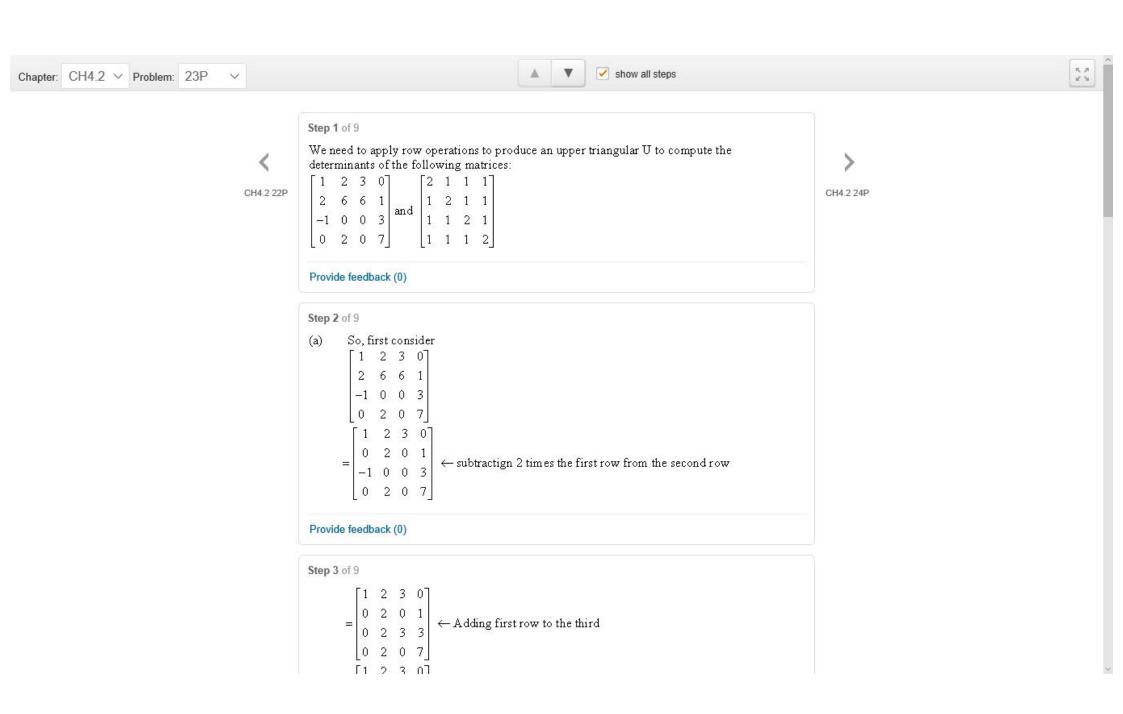
$$= \frac{1}{ad - bc} \qquad \left(\text{since } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \right)$$

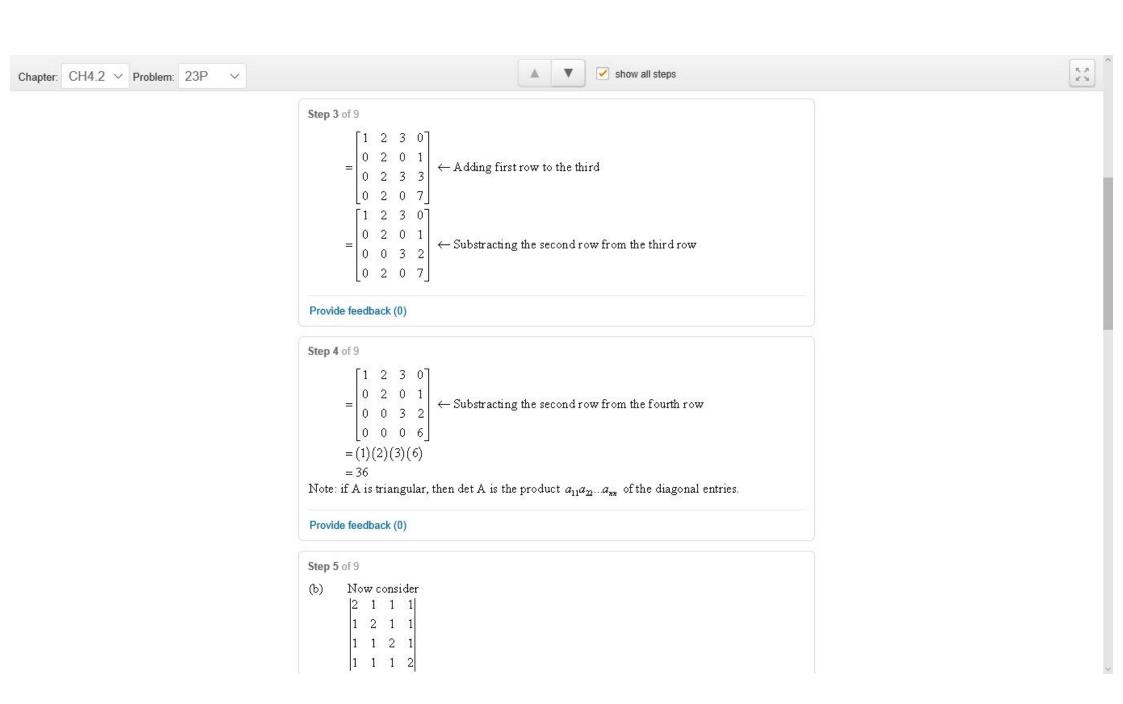
Hence the correct calculation is $\det A^{-1} = \frac{1}{ad - bc}$.

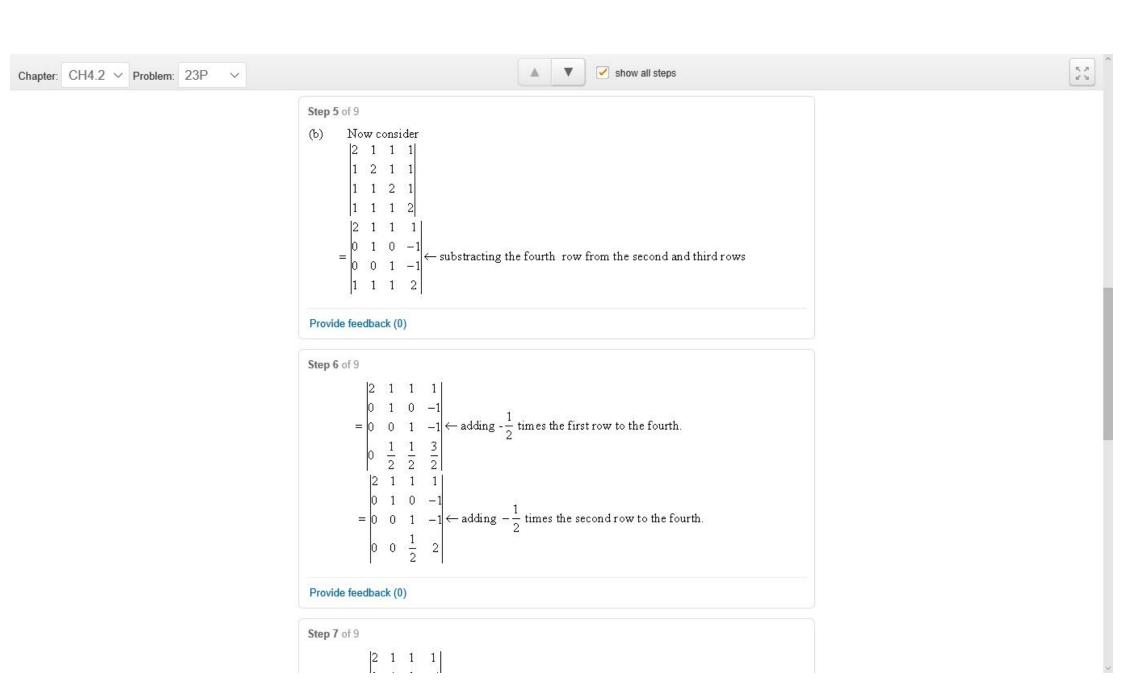
Provide feedback (0)

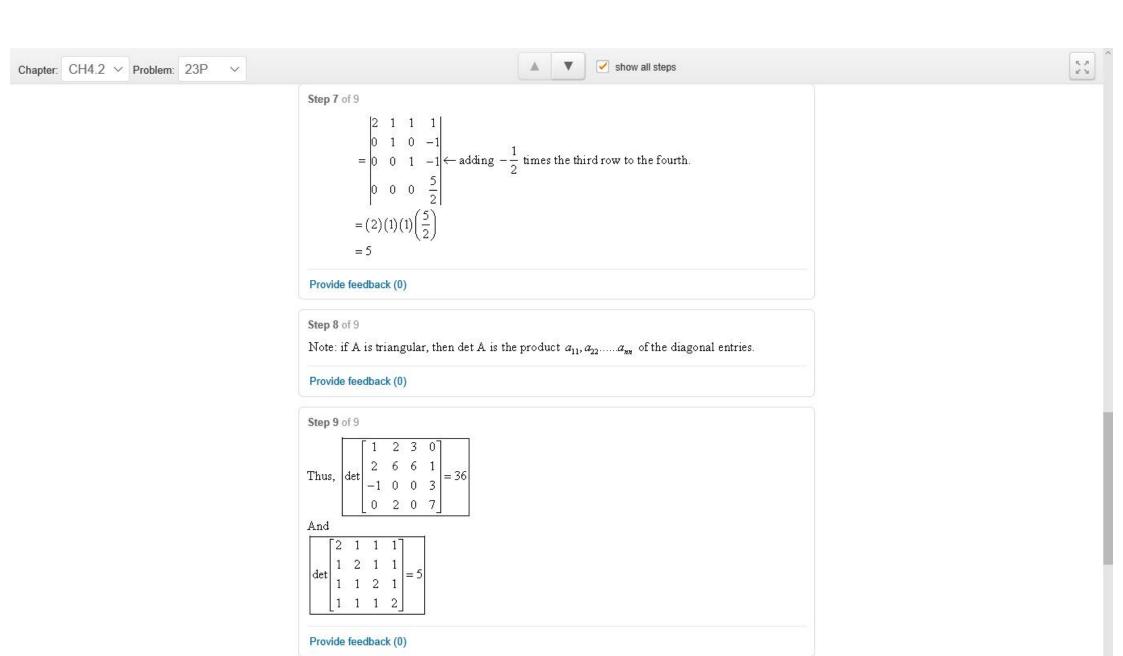


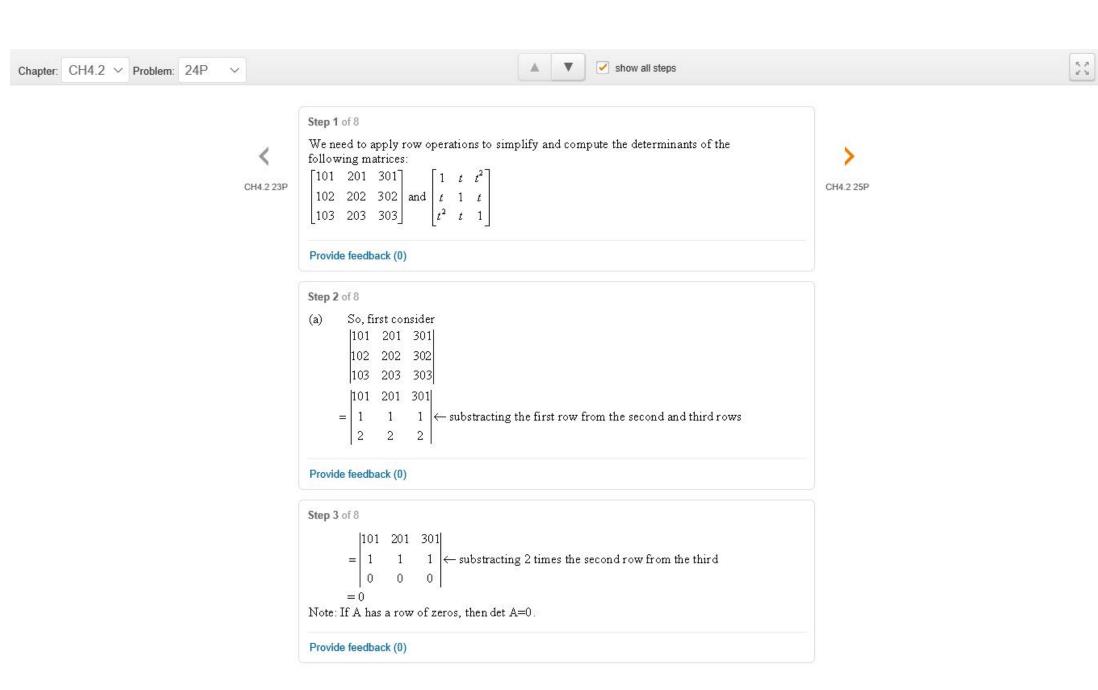












Step 4 of 8

Chapter: CH4.2 V Problem: 24P

(b) Now, consider $\begin{vmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{vmatrix}$ $= \begin{vmatrix} 1 & t & t^2 \\ t-1 & 1-t & t-t^2 \\ t^2-1 & 0 & 1-t^2 \end{vmatrix} \leftarrow \text{substracting the first row from the second and third rows}$

Provide feedback (0)

Step 5 of 8

$$= \begin{vmatrix} 1 & t & t^2 \\ -(1-t) & 1-t & t(1-t) \\ -(1-t^2) & 0 & 1-t^2 \end{vmatrix}$$

$$= (1-t)(1-t^2) \begin{vmatrix} 1 & t & t^2 \\ -1 & 1 & t \\ -1 & 0 & 1 \end{vmatrix}$$

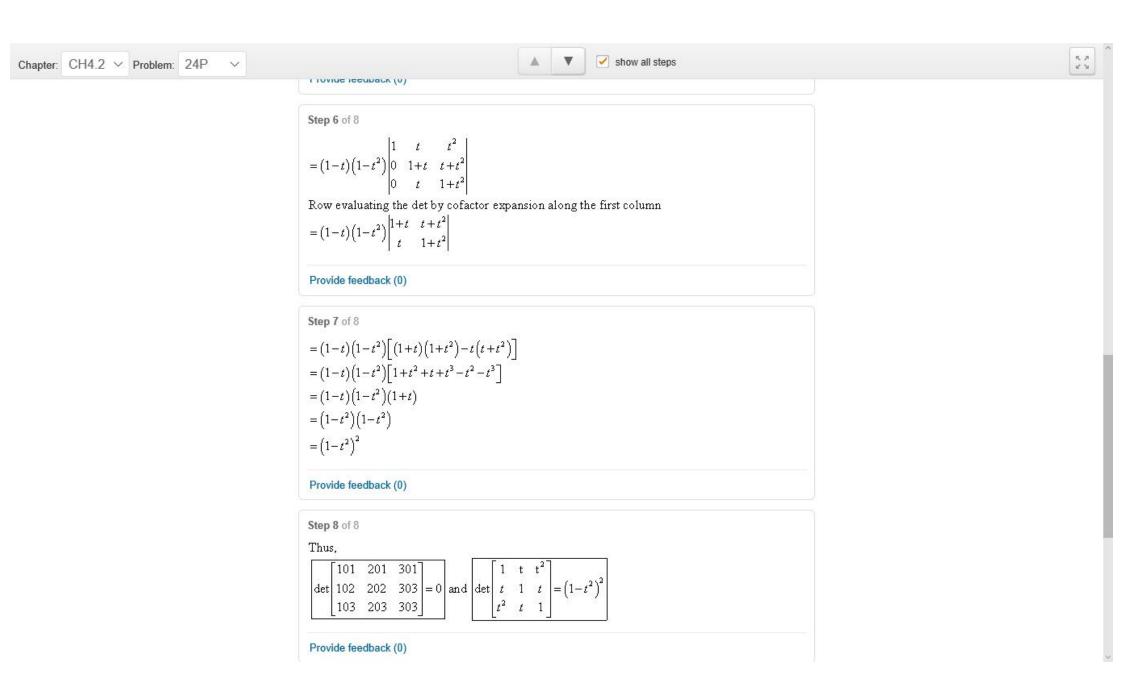
$$\leftarrow \text{ taking } (1-t) & (1-t^2) \text{ common from the second and third rows}$$

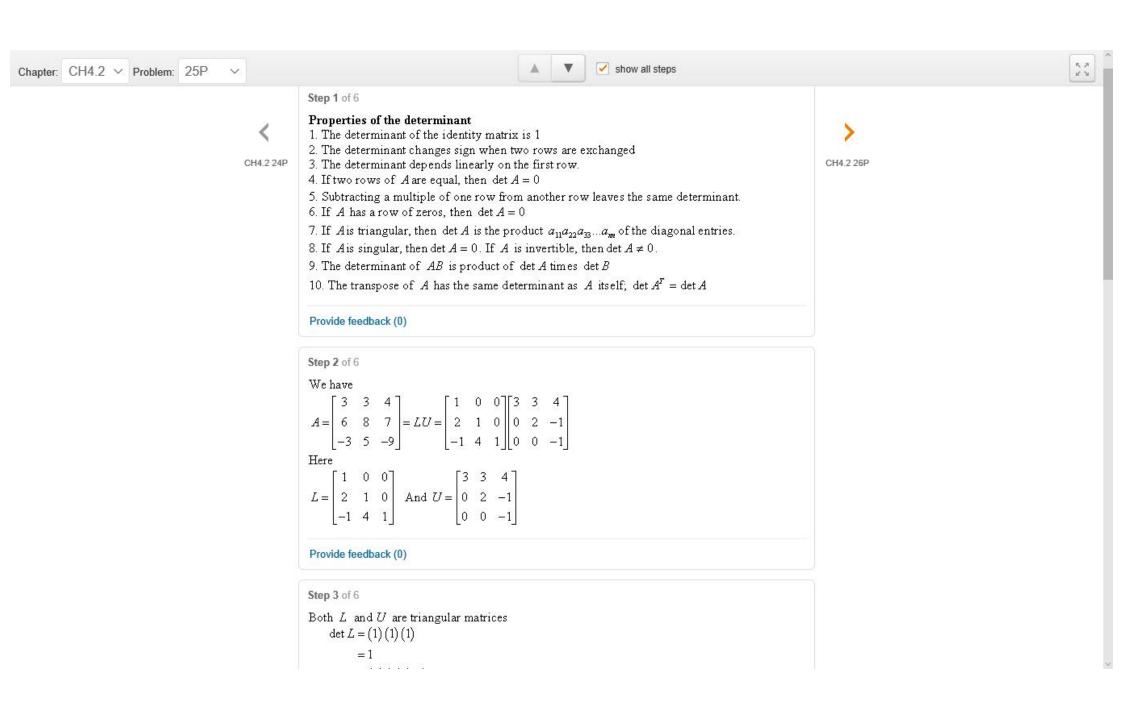
Provide feedback (0)

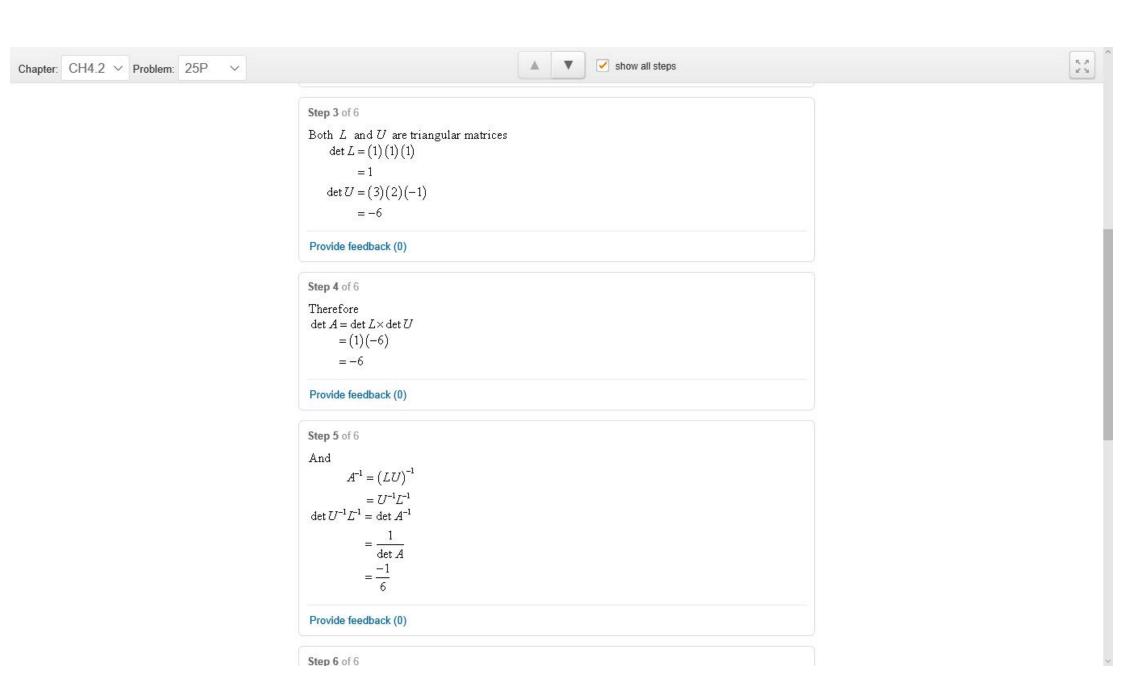
Step 6 of 8

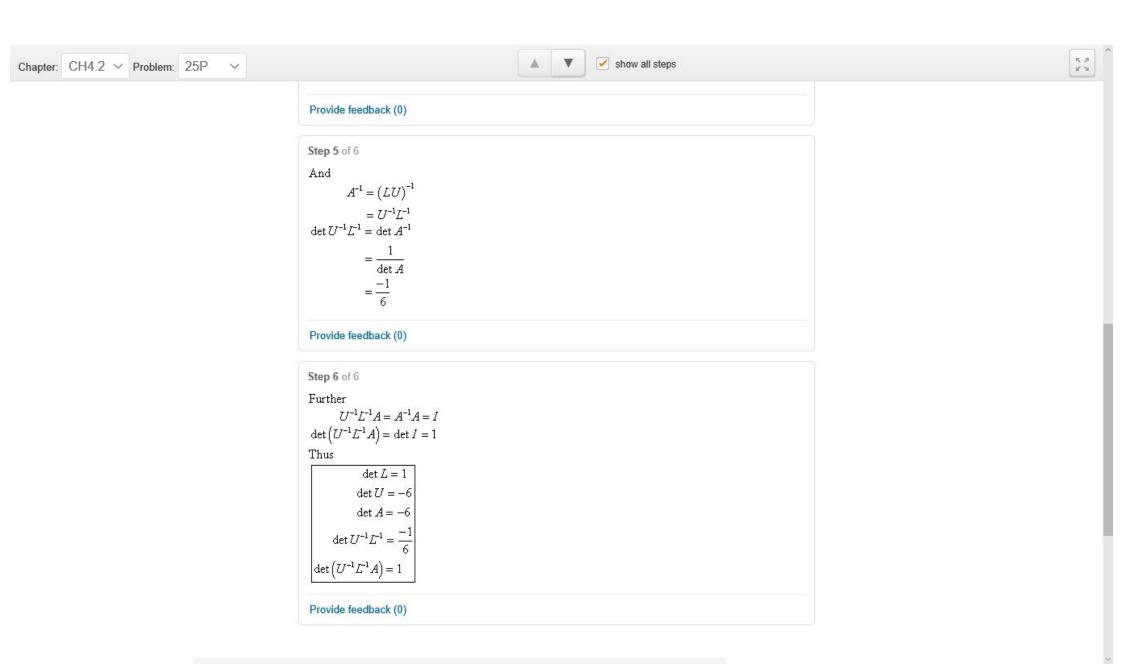
$$= (1-t)(1-t^2)\begin{vmatrix} 1 & t & t^2 \\ 0 & 1+t & t+t \\ 0 & t & 1+t \end{vmatrix}$$

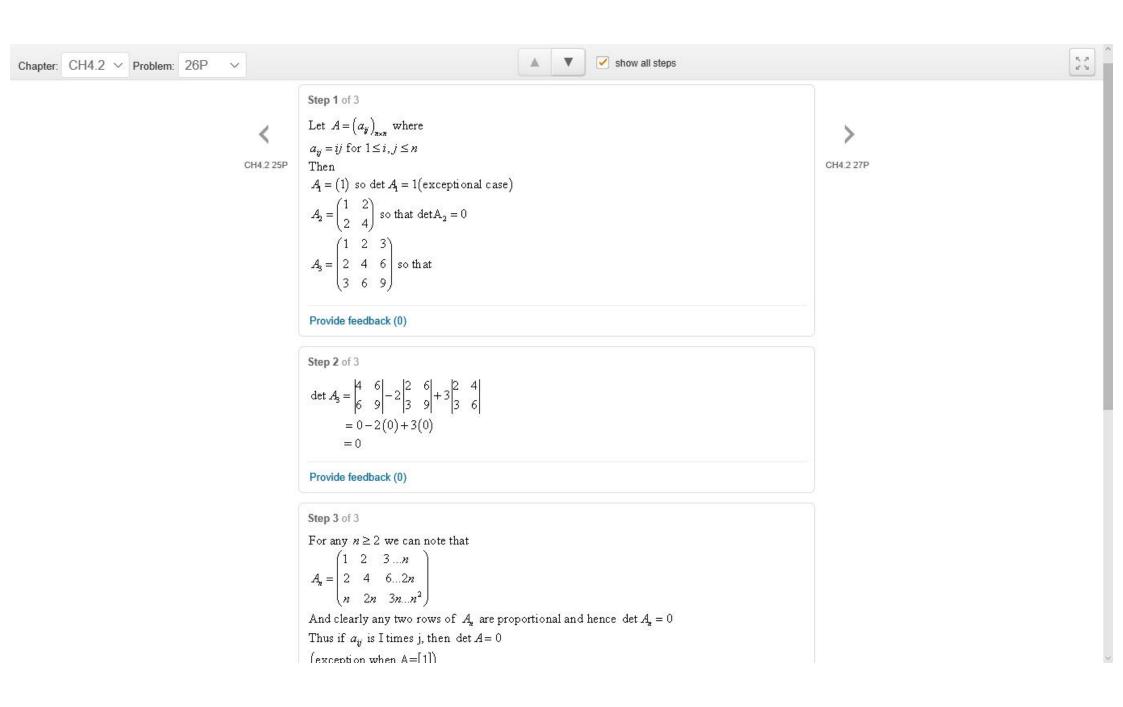
Row evaluating the det by cofactor expansion along the first column



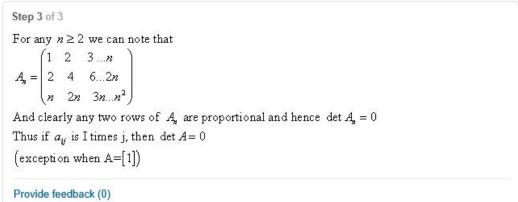


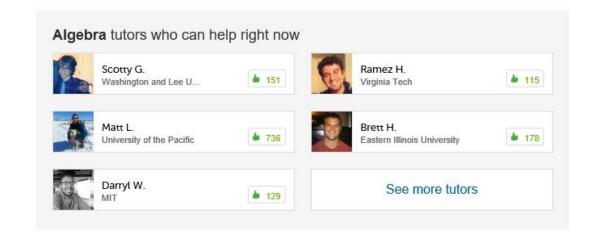


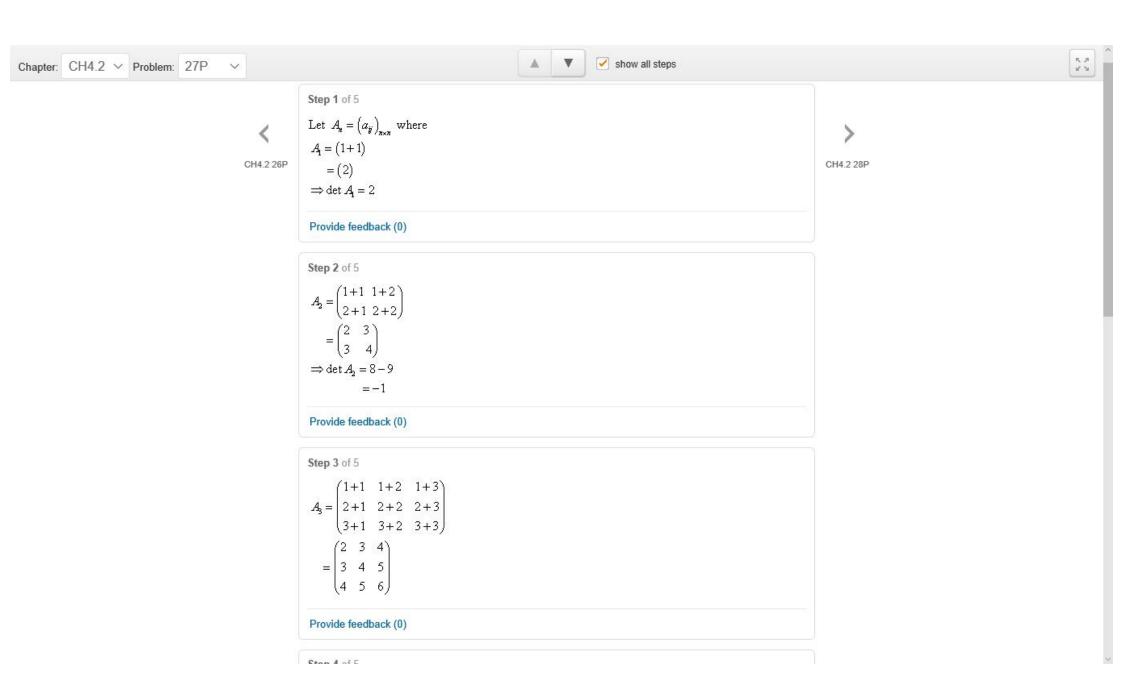


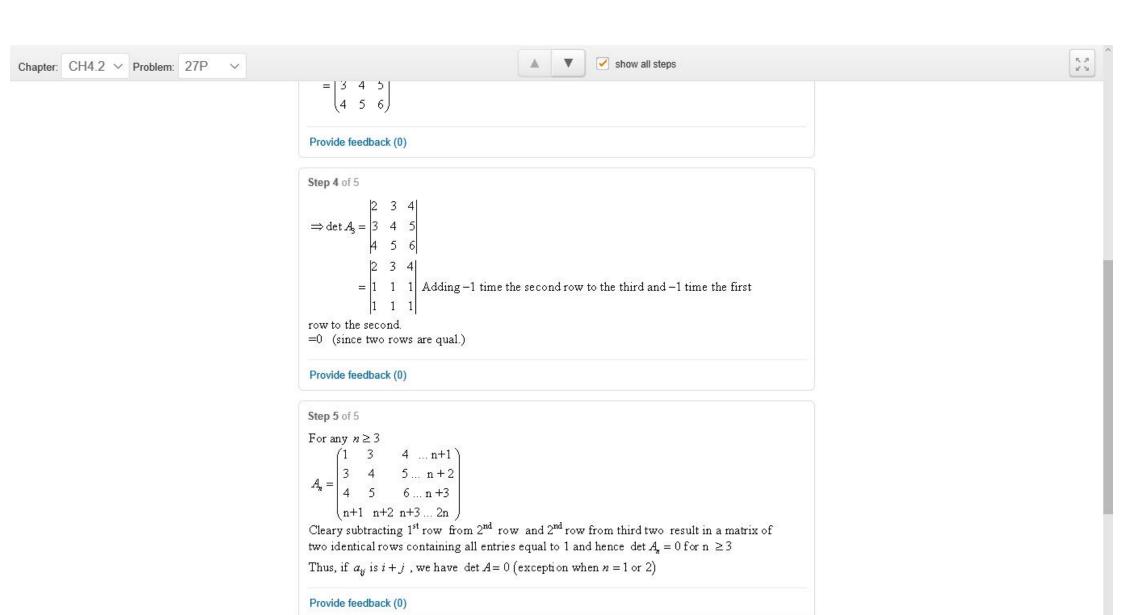


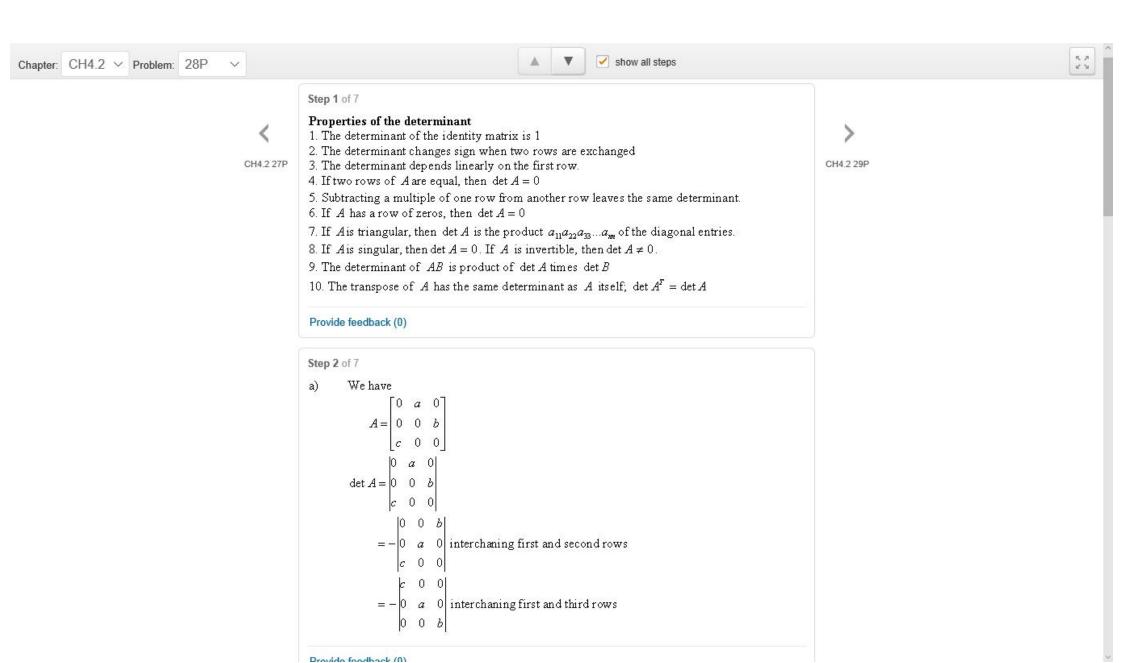


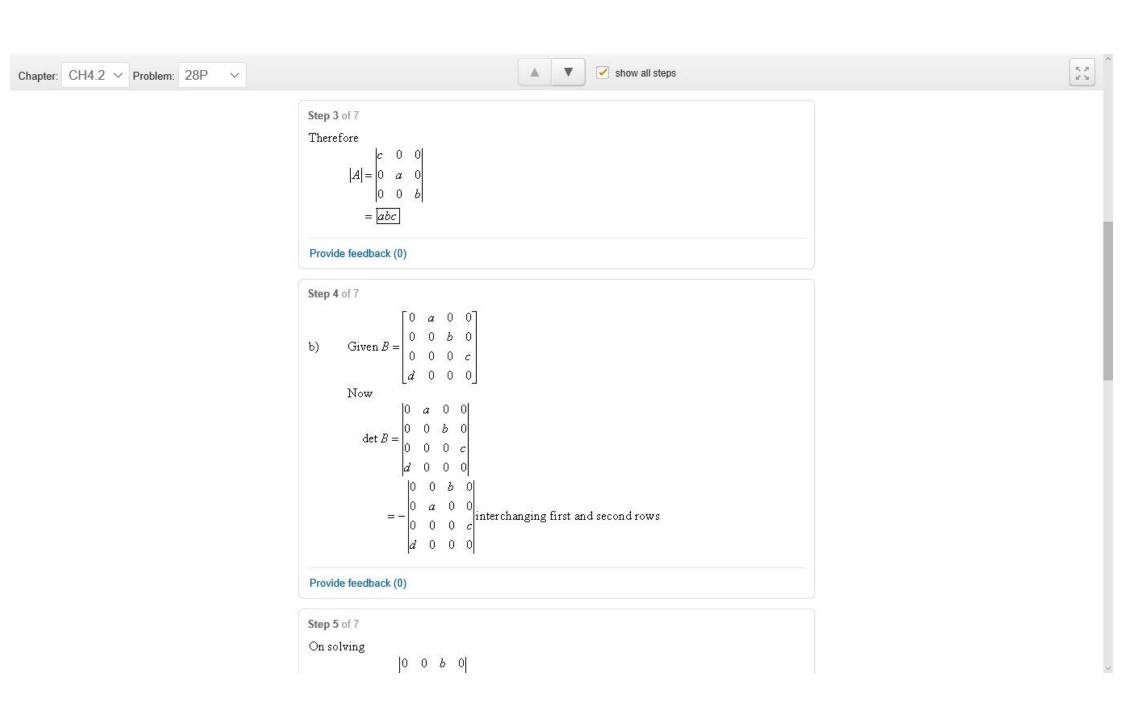


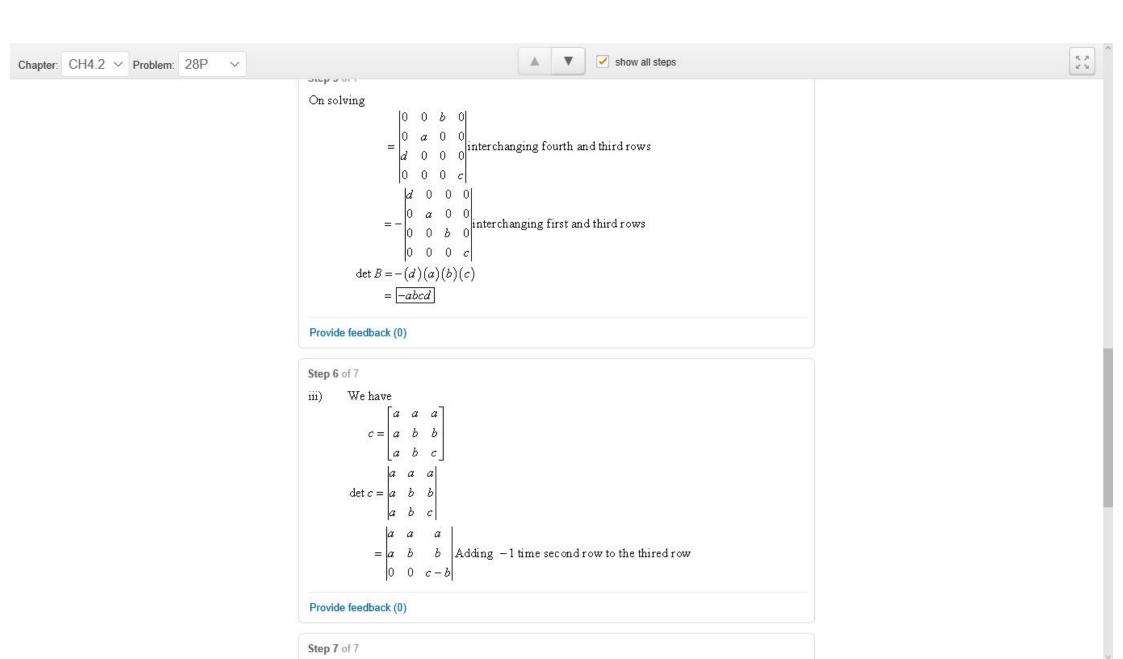


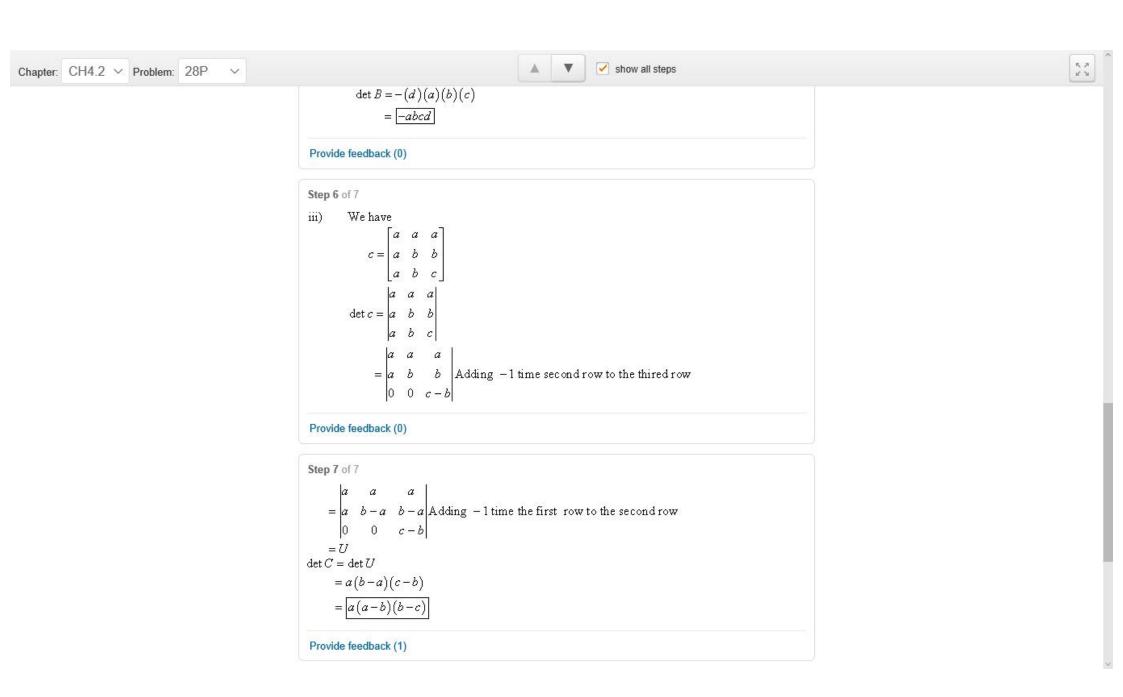


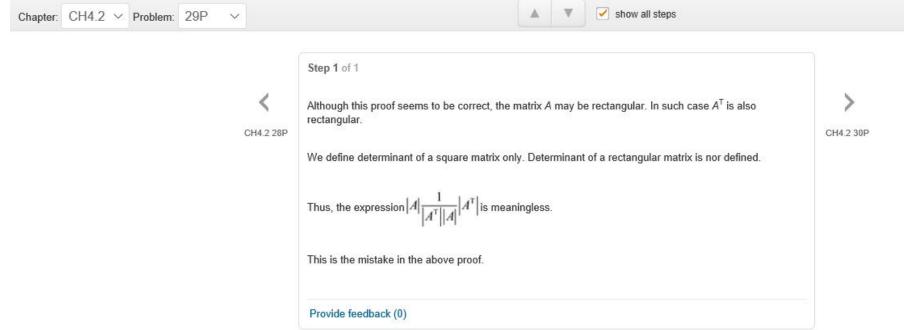




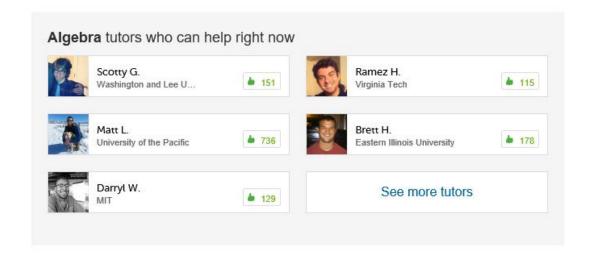


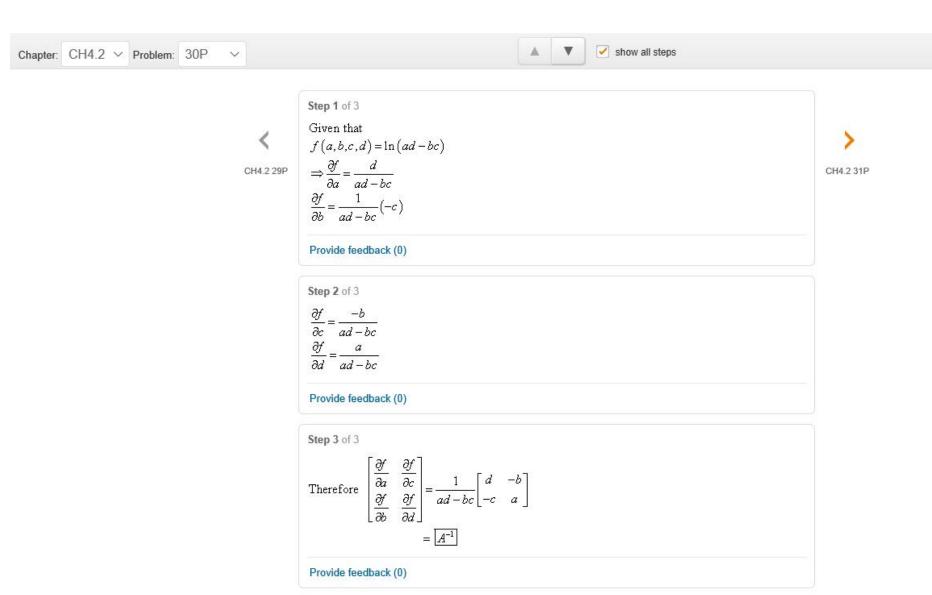






5.7





22

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