CH1.4

Step 1 of 1

We have to find that when is an upper triangular matrix nonsingular.
The upper triangular matrix U becomes non singular when no entry on the main diagonal is zero.

For instance $U = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ is an upper triangular matrix and non singular matrix.

Because the lower triangle below the main diagonal is 0 and $\det U = 12 \neq 0$

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CH1.5 2P

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CH1.5 1P

Step 1 of 3

We have to find that what multiple l_{32} of row 2 of A will subtract from row 3 of A. And by use the following factored form we have to find the all pivots.

$$A=\begin{pmatrix}1&0&0\\2&1&0\\1&4&1\end{pmatrix}\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

Form the above, $\boxed{l_{32}=\frac{1}{2}}$ eliminate the second pivot in A , we subtract $\frac{1}{2}$ times of row2 from row3.

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Step 2 of 3

Then

$$A=\begin{pmatrix}1&0&0\\2&1&0\\0&\frac{7}{2}&1\end{pmatrix}\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

Subtracting 2 times of row 1 from row 2 gives

$$A=\begin{pmatrix}1&0&0\\0&1&0\\0&\frac{7}{2}&1\end{pmatrix}\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

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Step 3 of 3

Subtracting 7/2 times of row 2 from row 3 gives

CH1.5 3P

from row3.

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Step 2 of 3

Then

$$A=\begin{pmatrix}1&0&0\\2&1&0\\0&\frac{7}{2}&1\end{pmatrix}\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

Subtracting 2 times of row 1 from row 2 gives

$$A=\begin{pmatrix}1&0&0\\0&1&0\\0&\frac{7}{2}&1\end{pmatrix}\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

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Step 3 of 3

Subtracting 7/2 times of row 2 from row 3 gives

$$A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

The left hand side is identity matrix, so

$$A=\begin{pmatrix}5&7&8\\0&2&3\\0&0&6\end{pmatrix}$$

$\boxed{\text{Hence the pivots are } 5, 2, 6}$

CH1.5 2P

Step 1 of 7

We have to multiply the matrices $E^{-1}F^{-1}G^{-1}$ and GFE, and then we have to multiply the following matrices:

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

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Step 2 of 7

Now

$$\begin{aligned}
&\left(E^{-1}F^{-1}G^{-1}\right)(GFE) = E^{-1}F^{-1}\left(G^{-1}G\right)FE \\
&= E^{-1}F^{-1}(I)FE \\
&= E^{-1}F^{-1}(IF)E \\
&= E^{-1}\left(F^{-1}F\right)E
\end{aligned}$$

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Step 3 of 7

$$\begin{aligned}
&= E^{-1}(I)E \\
&= E^{-1}(IE) \\
&= E^{-1}E \\
&= I
\end{aligned}$$

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Step 4 of 7

CH1.5 4P

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Step 4 of 7

Since I is identity matrices and by associative law, similarly

$$(GFE)\left(E^{-1}F^{-1}G^{-1}\right) = GF\left(E^{-1}E\right)F^{-1}G^{-1}$$

$$= GF(I)F^{-1}G^{-1}$$

$$= G(FI)F^{-1}G^{-1}$$

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Step 5 of 7

$$= G(F)F^{-1}G^{-1}$$

$$= G\left(FF^{-1}\right)G^{-1}$$

$$= GG^{-1}$$

$$= I$$

Therefore the matrices $E^{-1}F^{-1}G^{-1}$ and GFE are inverses to each other.

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Step 6 of 7

And

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= I$$

$= GG^{-1}$

$= I$

Therefore the matrices $E^{-1}F^{-1}G^{-1}$ and GFE are inverses to each other.

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Step 6 of 7

And

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= I$

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Step 7 of 7

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= I$

Therefore the matrices in the above are inverse to each other.

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CH1.5 3P

Step 1 of 7

We apply the elementary row operations to reduce the given matrix to the echelon form which is nothing but the upper triangular matrix.
We know that each elementary row operation is identical the multiplication of the given matrix with an elementary row matrix.
Further, the product of elementary matrices form a non singular matrix and is a lower triangular matrix.
Therefore, we write A as the product of elementary row matrices and the matrix in the echelon form which looks like LU.
Suppose $L_k \ldots L_2 L_1 A = U$ where each L_i is an elementary row matrix, then

$$\begin{aligned} A &= L_1^{-1} L_2^{-1} \ldots L_k^{-1} U \\ &= LU \end{aligned}$$

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CH1.5 5P

Step 2 of 7

$$A = \begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix}$$

$$R_2 \to R_2 - 4R_1 \Rightarrow \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$$

This matrix is the upper triangular matrix U.

We use the inverse elementary row operation on the identity matrix to get L_1^{-1}

i.e., $R_2 \to R_2 + 4R_1$ is applied on $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ to give $L_1^{-1} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$

Therefore, $A = L_1^{-1} U$

$$\begin{aligned} \Rightarrow \begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} \\ &= LU \text{ where } L = L_1^{-1} \end{aligned}$$

Step 3 of 7

$$A=\begin{pmatrix}3&1&1\\1&3&1\\1&1&3\end{pmatrix}$$

$$R_2\to 3R_2-R_1, R_3\to 3R_3-R_1 \Rightarrow \begin{pmatrix}3&1&1\\0&8&2\\0&2&8\end{pmatrix} \qquad \ldots\ldots (1)$$

So, we get the elementary matrix E by applying the same row operation($R_2\to 3R_2-R_1$) on the identity matrix of same order.

i.e., $E=\begin{bmatrix}1&0&0\\-1&3&0\\0&0&1\end{bmatrix}$

$$E^{-1}:\begin{bmatrix}1&0&0\\\frac{1}{3}&\frac{1}{3}&0\\0&0&1\end{bmatrix}$$

Similarly, F is the elementary matrix obtained by applying $R_3\to 3R_3-R_1$

i.e., $F=\begin{bmatrix}1&0&0\\0&1&0\\-1&0&3\end{bmatrix}$

$$F^{-1}:\begin{bmatrix}1&0&0\\0&1&0\\\frac{1}{3}&0&\frac{1}{3}\end{bmatrix}$$

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Step 4 of 7

$$F^{-1}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix}$$

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Step 4 of 7

At the next level, we use $R_3 \to 4R_3 - R_2$ on (1) to give the upper triangular matrix $U =$

$$\begin{pmatrix} 3 & 1 & 1 \\ 0 & 8 & 2 \\ 0 & 0 & 30 \end{pmatrix} \quad \ldots\ldots (2)$$

So, the elementary matrix G is obtained by $R_3 \to 4R_3 - R_2$ on the identity.

i.e., $G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 4 \end{bmatrix}$ and $G^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

we noted above that $GFEA = U$ using (2).

so, we can write $A = E^{-1}F^{-1}G^{-1}U$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{pmatrix} 3 & 1 & 1 \\ 0 & 8 & 2 \\ 0 & 0 & 30 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} \begin{pmatrix} 3 & 1 & 1 \\ 0 & 8 & 2 \\ 0 & 0 & 30 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

$LU = A$

This is the product of lower and upper triangular matrices.

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Step 6 of 7

$$A=\begin{pmatrix}1&1&1\\1&4&4\\1&4&8\end{pmatrix}$$

$$R_2\to R_2-R_1\Rightarrow\begin{pmatrix}1&1&1\\0&3&3\\1&4&8\end{pmatrix}\qquad\ldots\ldots(1)$$

$$\text{So, } E=\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}$$

$$E^{-1}=\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}$$

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Step 7 of 7

$$\text{Applying } R_3\to R_3-R_1 \text{ on (1) we get } \begin{pmatrix}1&1&1\\0&3&3\\0&3&7\end{pmatrix}\qquad\ldots\ldots(2)$$

$$F=\begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix}$$

$$F^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

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Step 7 of 7

Applying $R_3 \to R_3 - R_1$ on (1) we get $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 3 & 7 \end{pmatrix}$ ..... (2)

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$F^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Applying $R_3 \to R_3 - R_2$ on (2), we get $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{pmatrix} = U$

Consequently, $G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$$G^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Now, $A = E^{-1}F^{-1}G^{-1}U$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{pmatrix} = LU$$

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CH1.5 4P

Step 1 of 4

We have to factor A into LU, and we have to write down the upper triangular system $Ux = c$ after elimination for the system:

$$Ax = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

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Step 2 of 4

Here the matrix $A = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix}$

Subtracting 3 times row 1 from row 3 gives

$$U = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix}$$

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Step 3 of 4

Applying the reverse rule that is adding three times of row 1 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$LU = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix}$$

CH1.5 6P

Step 3 of 4

Applying the reverse rule that is adding three times of row 1 to row 3 gives

$$L=\begin{pmatrix}1&0&0\\0&1&0\\3&0&1\end{pmatrix}$$

$$LU=\begin{pmatrix}1&0&0\\0&1&0\\3&0&1\end{pmatrix}\begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}$$

$$=\begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix}$$

$$\Rightarrow \boxed{A=LU=\begin{pmatrix}1&0&0\\0&1&0\\3&0&1\end{pmatrix}\begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}}$$

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Step 4 of 4

The given system is

$$Ax=\begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}2\\2\\5\end{pmatrix}$$

After elimination that is subtracting 3 times row 1 from row 3 both sides of $Ax=b$ gives

$$\boxed{Ux=\begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}2\\2\\-1\end{pmatrix}}$$

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CH1.5 5P

Step 1 of 2

Given that $E = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$

$$\begin{aligned} E^2 &= E.E \\ &= \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 6+6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 2(6) & 1 \end{pmatrix} \end{aligned}$$

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Step 2 of 2

Similarly $E^8 = \begin{pmatrix} 1 & 0 \\ 8(6) & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 48 & 1 \end{pmatrix}$$

$$E^{-1} = \frac{1}{1-0}\begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$$

$$E^{-1} = \frac{1}{1-0}\begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$$

$$\left(\because A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \right)$$

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CH1.5 7P

CH1.5 6P

Step 1 of 3

We have to find the products FGH and HGF if

$$F=\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},G=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&2&1&0\\0&0&0&1\end{pmatrix},\text{and}$$

$$H=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}$$

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Step 2 of 3

$$FGH=\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\left\{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&2&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\right\}$$

$$=\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&2&1&0\\0&0&2&1\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&2&1&0\\0&0&2&1\end{pmatrix}}$$

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CH1.5 8P

$$\begin{pmatrix}0&0&1&0\\0&0&0&1\end{pmatrix}\left[\begin{pmatrix}0&2&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&2&1\end{pmatrix}\right]$$

$$=\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&2&1&0\\0&0&2&1\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&2&1&0\\0&0&2&1\end{pmatrix}}$$

Provide feedback (0)

Step 3 of 3

And

$$HGF=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\left\{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&2&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\2&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\right\}$$

$$=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&2&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\2&1&0&0\\4&2&1&0\\0&0&2&1\end{pmatrix}$$

$$=\boxed{\begin{pmatrix}1&0&0&0\\2&1&0&0\\4&2&1&0\\8&4&2&1\end{pmatrix}}$$

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CH1.5 7P

Step 1 of 3

Matrix A can be factorized into product of lower and upper triangular matrices.

$$A = LU$$

Here, matrix L is a lower triangular matrix with 1 at the diagonal position and matrix U is the upper triangular matrix with pivots at the diagonal position.

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Step 2 of 3

(a) The third row of matrix U is the third pivot that comes from the third row of matrix A by subtracting multiplies of row 1 and 2 of matrix U.

Row 3 of matrix U = row 3 of A $-l_{31}$ (row 1 of matrix U) $-l_{32}$ (row 2 of matrix U).

Here, rows of matrix U is subtracted and not rows of matrix A. Because by the time a pivot row is used, original rows of matrix A are changed by elimination.

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Step 3 of 3

(b) Above equation can be written as follows:

CH1.5 9P

Row 3 of matrix U = row 3 of A $-l_{31}$ (row 1 of matrix U) $-l_{32}$ (row 2 of matrix U).

Here, rows of matrix U is subtracted and not rows of matrix A. Because by the time a pivot row is used, original rows of matrix A are changed by elimination.

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Step 3 of 3

(b) Above equation can be written as follows:

$$\text{Row 3 of matrix } A = (\text{row 1 of matrix } U) + l_{32}(\text{row 2 of matrix } U)(\text{row 3 of } U)$$

Here, multiplication seems to be done row wise, rows of matrix L is multiplied by rows of matrix U, however row 3 of matrix A is obtained by row times column multiplication rule. Actual multiplication after solving will give the same result as the following equation:

$$\begin{aligned}\text{Row 3 of matrix } A = {} & (\text{row 3 of matrix } L)\text{-}(\text{column 1 of } U), \\ & (\text{row 3 of matrix } L)\text{-}(\text{column 2 of matrix } U), \\ & (\text{row 3 of matrix } L)\text{-}(\text{column 3 of } U).\end{aligned}$$

Therefore, matrix multiplication rule followed is row times column.

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CH1.5 8P

Step 1 of 6

(a) We have to find that under what conditions is the following product nonsingular:

$$A=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}d_1&&\\&d_2&\\&&d_3\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

The matrix A is nonsingular if $\det(A)\neq 0$. If any one of d_1,d_2,d_3 is 0 then $\det(A)=0$

So the given matrices product becomes nonsingular when $\boxed{d_1d_2d_3\neq 0}$

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Step 2 of 6

(b) We have to solve the system $Ax=b$ starting with $Lc=b$:

$$\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}=b$$

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Step 3 of 6

$$A=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}d_1&0&0\\0&d_2&0\\0&0&d_3\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}d_1&0&0\\-d_1&d_2&0\\0&-d_2&d_3\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}d_1&-d_1&0\\-d_1&d_1+d_2&-d_2\\0&-d_2&d_2+d_3\end{bmatrix}$$

CH1.5 10P

Step 4 of 6

Therefore $Ax = b$

$$\Rightarrow \begin{bmatrix} d_1 & -d_1 & 0 \\ -d_1 & d_1+d_2 & -d_2 \\ 0 & -d_2 & d_2+d_3 \end{bmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\xrightarrow[R_2 \to R_2 + R_1]{R_3 \to R_1 + R_2 + R_3,} \begin{bmatrix} d_1 & -d_1 & 0 \\ 0 & d_2 & -d_2 \\ 0 & 0 & d_3 \end{bmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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Step 5 of 6

$\Rightarrow d_3 w = 1$

$\Rightarrow w = \dfrac{1}{d_3}$

And $d_1 u - d_1 v = 0$

$\Rightarrow d_1 (u - v) = 0$

$\Rightarrow u - v = 0, d_1 \neq 0$

$\Rightarrow u = v$

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Step 6 of 6

Similarly $v = w$

Therefore $u = v = w = \dfrac{1}{d_3}$

Hence the solution of the system is $\boxed{x = \begin{bmatrix} 1/d_3 \\ 1/d_3 \\ 1/d_3 \end{bmatrix}}$

CH1.5 9P

Step 1 of 2

Loading solution. Please wait ...

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CH1.5 11P

Step 2 of 2

(b) We have to find that how many steps need for elimination use in solving 10 systems with the same 60 by 60 coefficient matrix A.

Here $n = 10$, because there are n unknowns are there, so it needs $\boxed{\frac{n^2}{2} = \frac{10^2}{2} = 50}$, steps to solve the equations.

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Step 1 of 2

(a) We have to explain that why does it takes approximately $\frac{n^2}{2}$ multiplication-subtraction steps to solve each $Lc = b$ and $Ux = c$

Because there are n unknowns are in both the systems $Lc = b$ and $Ux = c$, and both L and U of order n, so it takes $\frac{n^2}{2}$ steps to solve.

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Step 2 of 2

(b) We have to find that how many steps need for elimination use in solving 10 systems with the same 60 by 60 coefficient matrix A.

Here $n = 10$, because there are n unknowns are there, so it needs $\boxed{\frac{n^2}{2} = \frac{10^2}{2} = 50}$, steps to solve the equations.

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Step 1 of 5

We have to solve as two triangular systems, without multiplying LU to find A:

$$LUx = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

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Step 2 of 5

Taking the $Lc = b$ systems first we have

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

Going down ward elimination gives that is subtracting row 1 from row 2 and subtracting row 1 from row 3 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$\Rightarrow c = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 5

Taking $Ux = c$ gives

$$\begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

CH1.5 12P

Step 3 of 5

Taking $Ux = c$ gives

$$\begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

Going upward elimination gives, that is subtracting 2 times of row 2 from row1

$$\begin{pmatrix} 2 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$

Provide feedback (2)

Step 4 of 5

Again subtracting 2 times of row 3 from row 2 gives

$$\begin{pmatrix} 2 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$

Again subtracting 2 times of row 2 from row 1 gives

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix}$$

Provide feedback (0)

Step 5 of 5

Dividing the row 1 by 2 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

Going upward elimination gives, that is subtracting 2 times of row 2 from row1

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Provide feedback (2)

Step 4 of 5

Again subtracting 2 times of row 3 from row 2 gives

$$\begin{pmatrix} 2 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$

Again subtracting 2 times of row 2 from row 1 gives

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix}$$

Provide feedback (0)

Step 5 of 5

Dividing the row 1 by 2 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$

Then the solution x gives $\boxed{x = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}}$

CH1.5 11P

Step 1 of 3

Suppose A is the given matrix.
We apply elementary row operations on A to reduce it to the echelon form.
The echelon form is nothing but the upper triangular matrix.
On the other hand, each elementary row operation on A is nothing but pre multiplying A with an elementary row matrix.
So, the number of row operations on A to change it to the upper triangular matrix are the number of elementary row matrices pre multiplied with A.
Each elementary row matrix is a lower triangular matrix.
An elementary row matrix is obtained by applying the same elementary row operation that was applied on A is applied on the identity matrix.
The inverse of the elementary matrix is also elementary and a lower triangular matrix.
Product of lower triangular matrices is also a lower triangular matrix.
Thus, the given matrix has become the product of lower and upper triangular matrices.
The above procedure is exhibited as

(1) $E_k E_{k-1} \ldots E_2 E_1 A = U_1$ where E_i's are elementary row matrices and U_1 is the echelon form.

(2) $A = E_1^{-1} E_2^{-1} \ldots E_k^{-1} U_1$
$= L_1 U_1$

Provide feedback (0)

CH1.5 13P

Step 2 of 3

We are now interested to write the given matrix A as the product of upper and lower triangular matrices.
So, we start to make zero the entries above the principal diagonal.
For this procedure, we use the elementary column operations result in elementary column matrices.
Applying an elementary column operation on A is nothing but post multiplying A with an elementary column matrix.
Each elementary column matrix is an upper triangular matrix.
Inverse of an elementary column matrix is also an elementary column matrix.
Product of upper triangular matrices is also an upper triangular matrix.

This above procedure is summarized as

(1) $E_k E_{k-1} \ldots E_2 E_1 A = U_1$ where E_i's are elementary row matrices and U_1 is the echelon form.

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Provide feedback (0)

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This way we write the given matrix as the product of upper and lower triangular matrices.

The above procedure is performed as

(1) $AF_1 F_2 \ldots F_s = L_2$ where each F_i is the post multiplied elementary column matrix with A and ultimately, it is reduced to the lower triangular matrix.

(2) $A = F_s^{-1} F_{s-1}^{-1} \ldots F_1^{-1} L_2$

$= U_2 L_2$

Provide feedback (0)

Step 3 of 3

Observe that the entire procedure used in this case and the previous case are different. So, the triangularisations are also different.
i.e., $U_1 \neq U_2$ and $L_1 \neq L_2$ but $L_1 U_1 = L_2 U_2$

form.

(2) $A = E_1^{-1}E_2^{-1}\ldots E_k^{-1}U_1$

$= L_1U_1$

Provide feedback (0)

Step 2 of 3

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(1) $AF_1F_2\ldots F_s = L_2$ where each F_i is the post multiplied elementary column matrix with A and ultimately, it is reduced to the lower triangular matrix.

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Provide feedback (0)

Step 3 of 3

Observe that the entire procedure used in this case and the previous case are different. So, the triangularisations are also different.

i.e., $U_1 \neq U_2$ and $L_1 \neq L_2$ but $L_1U_1 = L_2U_2$

Provide feedback (0)

CH1.5 12P

Step 1 of 8

We have to solve by elimination, exchanging row if necessary:

$u+4v+2w=-2$ $\quad v+w=0$

$-2u-8v+3w=32$, and $u+v=0$

$v+w=1$ $\quad u+v+w=1$

And we have to find that which permutation matrices are required.

Provide feedback (0)

Step 2 of 8

Converting into matrix form gives

$$\begin{pmatrix} 1 & 4 & 2 \\ -2 & -8 & 3 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 32 \\ 1 \end{pmatrix}$$

Here the first pivot is 1 so adding 2 times of row1 to row2 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 0 & 7 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 28 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 8

Dividing row 2 by 7 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$$

It requires row changing so by changing row 2 and row 3 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}=\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

CH1.5 14P

Step 3 of 8

Dividing row 2 by 7 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$$

It requires row changing so by changing row 2 and row 3 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 8

Now here the pivot is 1, subtracting 4 times row2 from row 1 gives

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix}$$

Subtracting row 3 from row 2 and adding 2 times row 3 to row 1 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

Provide feedback (0)

Step 5 of 8

Hence the solution is $\boxed{x = (2, -3, 4)}$

The permutation matrix has same rows as in identity matrix (in some order)

So here the required permutation matrix is $\boxed{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$

Step 4 of 8

Now here the pivot is 1, subtracting 4 times row2 from row 1 gives

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Provide feedback (0)

Step 5 of 8

Hence the solution is $\boxed{x = (2, -3, 4)}$

The permutation matrix has same rows as in identity matrix (in some order)

So here the required permutation matrix is $\boxed{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}}$

Provide feedback (0)

Step 6 of 8

Now consider the second system.
Converting into matrix form gives

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Here the first row first element is 0 so it needs row exchange so exchanging row 1 and row 2 gives

Step 6 of 8

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$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

Subtracting row 1 from row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Now here the pivot is 1, subtracting row2 from row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 8 of 8

Subtracting row 2 from row 1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

row 2 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

Subtracting row 1 from row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Now here the pivot is 1, subtracting row2 from row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 8 of 8

Subtracting row 2 from row 1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Hence the solution is $\boxed{x = (1,-1,1)}$

Here the permutation matrix is $\boxed{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$

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CH1.5 13P

Step 1 of 4

We have to write down all six 3 by 3 permutation matrices including $P = I$, and we have to identify that which of their inverses are also permutation matrices.
The six permutation matrices of 3 by 3 are

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Step 2 of 4

$$P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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CH1.5 15P

$$P_5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 4

We know that P^{-1} is same as P^T it also true for I

For the permutation matrix P_1,

$$P_1^{-1} = P_1^T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 4

Since changing of rows and columns

$$P_1^{-1}P_1^T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= I$$

Similarly we can verify that remaining permutation matrices are also follows the same property $P^{-1}P = I$.

Provide feedback (0)

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CH1.5 14P

Step 1 of 11

Consider the matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

Find $PA = LDU$ factorizations.

In the matrix A, the pivot is zero, 0.

So, to change the pivot entry, exchange the first and second rows so that the entry 1 in the second row moves into the pivot.

Provide feedback (0)

Step 2 of 11

For this row exchange, use the permutation matrix P which is the same as an identity matrix.

So, the PA matrix will be as follows:

$$\begin{aligned} PA &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \end{aligned}$$

Here, the permutation matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Provide feedback (0)

CH1.5 16P

Step 3 of 11

Find the lower and upper (*L* and *U*) triangular matrices for the matrix PA.

Find the upper triangular matrix *U* for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$.

Step 1: Subtract two times the first row from the third row.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \quad R_3 \to R_3 - 2R_1 \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{bmatrix}$$

Step 2: Subtract three times the second row from the third row.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{bmatrix} \quad R_3 \to R_3 - 3R_2 \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Therefore, the upper triangular matrix *U* is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$.

Provide feedback (0)

Step 4 of 11

Find the lower triangular matrix *L* for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$.

To find the matrix *L*, do the same row operations reversely on the identity matrix *I*.

Step 1: Add two times the first row from the third row.

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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_3 \to R_3 + 2R_1 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Step 2: Add three times the second row from the third row.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad R_3 \to R_3 + 3R_2 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

Therefore, the lower triangular matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$.

Provide feedback (0)

Step 5 of 11

Find D:

Use the fact that D is the diagonal matrix of pivots, and the pivots are the diagonal entries of the upper triangular matrix U.

Here, the pivots are $1, 1, -1$ because, $1, 1, -1$ are the diagonal entries of the upper triangular matrix U.

So, the matrix D is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

The LU factorization is the following:

$$PA = LU$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$PA = LU$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

To make the diagonal entries all 1 in the matrix *U*, one needs to split *LU* into *LDU* such that $DU' = U$ as follows:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, *L* and *U* are the lower and upper triangular matrices which contain 1 on diagonal

D is the diagonal matrices which consists pivots 1, 1,-1 on the diagonal.

Therefore, write the $PA = LDU$ factorizations as follows:

$$\boxed{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}.$$

Provide feedback (0)

Step 6 of 11

Check:

Find LDU:

$$LDU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 6 of 11

Check:

Find LDU:

$$\begin{aligned} LDU &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \\ &= PA \end{aligned}$$

Therefore, by row column multiplication of the above two sides we get the same matrices.

Provide feedback (0)

Step 7 of 11

Consider the matrix.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find $PA = LDU$ factorizations.

In the matrix A, the pivot is 4.

So, to change the pivot entry, exchange the second and third rows, so that the entry 1 in the third row moves into the pivot.

So, PA matrix will be the following:

So, PA matrix will be the following:

$$PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix}$$

Here, the permutation matrix $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Provide feedback (0)

Step 8 of 11

Find the lower and upper (*L* and *U*) triangular matrices for the matrix PA .

Find the upper triangular matrix *U* for the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix}$.

Step 1: Subtract one the first row from the second row.

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix} \quad R_2 \to R_2 - R_1 \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 2 & 4 & 2 \end{bmatrix}$$

Step 2: Subtract two times the first row from the third row.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 2 & 4 & 2 \end{bmatrix} \quad R_3 \to R_3 - 2R_1 \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, the upper triangular matrix U is the following:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find the lower triangular matrix L for the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix}$:

To find the matrix L, do the same row operations reversely on the identity matrix I.

Step 1: Add one the first row to the second row.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \to R_2 + R_1 \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2: Add two times the first row to the third row.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_3 \to R_3 + 2R_1 \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Therefore, the lower triangular matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

Provide feedback (0)

Step 9 of 11

Find D.

Use the fact that D is the diagonal matrix of pivots, and the pivots are the diagonal entries of the upper

Step 9 of 11

Find D.

Use the fact that D is the diagonal matrix of pivots, and the pivots are the diagonal entries of the upper triangular matrix U.

Here, the pivots are $1, -1, 0$ because, $1, -1, 0$ are the diagonal entries of the upper triangular matrix U.

So, the matrix D is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Provide feedback (0)

Step 10 of 11

The LU factorization is as follows:

$$PA = LU$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To make the diagonal entries all 1 in the matrix U, one needs to split LU into LDU such that $DU' = U$ as follows:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, L and U are the lower and upper triangular matrices, which contain 1 on diagonal

$$= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, *L* and *U* are the lower and upper triangular matrices, which contain 1 on diagonal

D is the diagonal matrices which consists pivots 1,-1, 0 on the diagonal.

Therefore, $PA = LDU$ factorizations is as follows:

$$\boxed{\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}.$$

Provide feedback (0)

Step 11 of 11

Check:

Find LDU:

$$\begin{aligned} LDU &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix} \\ &= PA \end{aligned}$$

Therefore, by row column multiplication of the above two sides we get the same matrices.

Provide feedback (0)

CH1.5 15P

Step 1 of 1

A permutation matrix is a square matrix that has exactly one 1 in each row or column and the rest of the entries are zero.

We write a 4 by 4 permutation matrix that becomes the identity matrix with just three row exchanges.

Let us consider $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

1st exchange: $R_4 \leftrightarrow R_2 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

2nd exchange: $R_1 \leftrightarrow R_3 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

3rd exchange: $R_3 \leftrightarrow R_4 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$

CH1.5 17P

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CH1.5 16P

Step 1 of 2

Given that the less familiar form $A = LPU$ exchanges rows only at the end:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix} \to L^{-1}A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 3 & 6 \end{bmatrix}$$

$$= PU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

We have to find L in this case.

Provide feedback (0)

Step 2 of 2

It is seem to be that $L^{-1}A$ be obtained by subtracting row 1 from row 2 and subtracting 2 times row 2 from row 3 of A. Now L is the lower triangular matrix that obtain by adding row 1 to row 2 and adding 2 times row 1 to row 3 on the identity matrix so

$$\boxed{L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}}$$

Provide feedback (1)

CH1.5 18P

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CH1.5 17P

Step 1 of 11

We have to decide whether the following systems are singular or nonsingular, and whether they have no solution, one solution or infinitely many solutions:

$v-w=2 \quad v-w=0 \qquad v+w=1$

$u-v=2$, $u-v=0$, and $u+v=1$

$u-w=2 \quad u-w=0 \qquad u+w=1$

Provide feedback (0)

Step 2 of 11

Consider the system

$v-w=2$

$u-v=2$

$u-w=2$

Converting the given equations into $Ax=b$ form gives

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 11

First coefficient is zero, so it needs row exchange hence exchange row 1 and row 2, then

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

Subtracting row 1 from row 3, we have

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

CH1.5 19P

Subtracting row 2 from row 3 gives

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

From the above the last equation is 0 = -2,

So the system has no solution, and hence it is singular.

Provide feedback (0)

Step 5 of 11

Consider the second system

$v - w = 0$

$u - v = 0$

$u - w = 0$

Converting the given equations into $Ax = b$ form gives

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Provide feedback (0)

Step 6 of 11

First coefficient is zero, so it needs row exchange hence exchange row 1 and row 2, then

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Subtracting row 1 from row 3 gives

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Step 7 of 11

Subtracting row 2 from row 3 gives

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Now from the system, we obtained the last equation as $0 = 0$ and no further elimination is possible, so we have two equations in three variables.

$\boxed{\text{Hence the system has infinitely many solutions, and hence it is nonsingular}}$.

Provide feedback (0)

Step 8 of 11

Consider the third system

$v + w = 1$

$u + v = 1$

$u + w = 1$

Converting the given equations into $Ax = b$ form gives

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 9 of 11

First coefficient is zero, so it needs row exchange hence exchange row 1 and row 2

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Provide feedback (0)

Step 10 of 11

Subtracting row 1 from row 3, we have

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Adding row 2 to row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 11 of 11

Now the pivot is 2, so subtracting $\frac{1}{2}$ times row 3 from row 2 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1/2 \\ 1 \end{pmatrix}$$

Subtracting row 2 from row 1 and divide row 3 by 2 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

Hence the system has unique solution which is $\boxed{u = v = w = \frac{1}{2}}$

$\boxed{\text{Hence it is a nonsingular system}}$.

Provide feedback (0)

CH1.5 18P

Step 1 of 5

For the following matrices we have to find that which numbers a, b, c lead to row exchange, and which make the matrix singular.

$$A = \begin{pmatrix} 1 & 2 & 0 \\ a & 8 & 3 \\ 0 & b & 5 \end{pmatrix}, \text{ and } A = \begin{pmatrix} c & 2 \\ 6 & 4 \end{pmatrix}$$

Provide feedback (0)

Step 2 of 5

Fist we consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ a & 8 & 3 \\ 0 & b & 5 \end{pmatrix}$$

If $a = 4$ then the matrix is

$$\begin{pmatrix} 1 & 2 & 0 \\ 4 & 8 & 3 \\ 0 & b & 5 \end{pmatrix}$$

And subtracting 4 times row 1 from row 2 gives

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & b & 5 \end{pmatrix}$$

There is 0 in the position of second pivot.

$\boxed{\text{So the matrix needs row exchange when } a = 4}$

Provide feedback (1)

Step 3 of 5

The matrix is singular if and only if the determinant is zero so

CH1.5 20P

Provide feedback (1)

Step 3 of 5

The matrix is singular if and only if the determinant is zero so
By finding the determinant A, we have
$40-3b-10a=0$
$\Rightarrow 10a+3b=40$
$\boxed{\text{So the matrix is singular if } 10a+3b=40}$

Provide feedback (0)

Step 4 of 5

Now consider the second matrix
$$A=\begin{pmatrix} c & 2 \\ 6 & 4 \end{pmatrix}$$
If $c=0$ then it leads to row exchange because if $c=0$ then the matrix is
$$\begin{pmatrix} 0 & 2 \\ 6 & 4 \end{pmatrix}$$
Here the first coefficient is zero so it needs row exchange.
$\boxed{\text{Hence } A \text{ needs row exchange if } c=0}$

Provide feedback (0)

Step 5 of 5

The matrix is singular when $c=3$
Since if $c=3$, its determinant becomes zero so it is singular.
$\boxed{\text{Hence the matrix } A \text{ is singular if } c=3}$

Provide feedback (0)

CH1.5 19P

Step 1 of 4

l_{21} = One time row 1 is subtracted from row 2 to make $\begin{bmatrix} 1 & 1 & 5 \\ 1 & 2 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \end{bmatrix}$

Provide feedback (0)

Step 2 of 4

The reverse step is adding l_{21}= one time row 1 to row 2 to make $L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ in an identity matrix.

Provide feedback (0)

Step 3 of 4

$Ax = b \Rightarrow LUx = b$

So,

$A = LU$

Provide feedback (0)

Step 4 of 4

$A = LDU$

Provide feedback (0)

CH1.5 21P

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Scotty G

Mareeka N

CH1.5 20P

Step 1 of 1

We write the augmented matrix for the system

$$\begin{aligned} x+\ \ y+\ z&=5\\ x+2y+3z&=7\\ x+3y+6z&=11 \end{aligned}$$

$$[A|b]=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 1 & 2 & 3 & 7\\ 1 & 3 & 6 & 11 \end{array}\right]$$

l_{21}: one time row 1 subtracted from row 2 to give $\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & 1 & 2 & 2\\ 1 & 3 & 6 & 11 \end{array}\right]$

l_{31}: one time row 1 subtracted from row 3 to give $\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & 1 & 2 & 2\\ 0 & 2 & 5 & 6 \end{array}\right]$

l_{32}: two times row 2 is subtracted from row 3, we get $\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & 1 & 2 & 2\\ 0 & 0 & 1 & 2 \end{array}\right]$

Thus, the system is reduced as

$$\begin{aligned} x+y+z\ &=5\\ y+2z&=2\\ z&=2 \end{aligned}$$

This can simply be written as $Ux=c$

CH1.5 22P

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CH1.5 21P

Step 1 of 4

We have the system $Ax = b$ is $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 11 \end{pmatrix}$

Applying the forward elimination method, we get $Ux = c$ is

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$$

Provide feedback (0)

Step 2 of 4

To find the matrix L, we apply the reverse process to identity matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Adding row 1 to row 2 and adding row 1 to row 3

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

Provide feedback (1)

Step 3 of 4

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 5 \end{pmatrix} \quad \begin{pmatrix} 5 \end{pmatrix}$$

CH1.5 23P

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

Provide feedback (1)

Step 3 of 4

$$\text{Now } Lc = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 21 \end{pmatrix}$$

Since by matrix multiplication

Lc = b solves for (5, 7, 21)

Provide feedback (0)

Step 4 of 4

Further, $Ux = c = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$ can be written as the system of equations from below as

$$\begin{aligned} z &= 2 \\ y + 2z &= 2 \\ x + y + z &= 5 \end{aligned}$$

Consequently, $z = 2, y = -2$ and $x = 5$

So, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 2 \end{pmatrix}$ satisfies the equation $Ux = c$.

Provide feedback (0)

CH1.5 22P

Step 1 of 8

Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$

We have to find that the two elimination matrices E_{21}, E_{32} to put A into upper triangular form $E_{32}E_{21}A = U$ and we have to multiply by E_{32}^{-1}, E_{21}^{-1} to factor A into $LU = E_{21}^{-1}E_{32}^{-1}U$

Provide feedback (0)

Step 2 of 8

Subtracting 2 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix}$$

Subtracting 2 times row 2 from row 3 gives

$$U = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 8

To find L reversing the operations on identity matrix;
Adding 2 times row 1 to row 2 and adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

CH1.5 24P

CH1.5 22P

Step 1 of 8

Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$

We have to find that the two elimination matrices E_{21}, E_{32} to put A into upper triangular form $E_{32}E_{21}A = U$ and we have to multiply by E_{32}^{-1}, E_{21}^{-1} to factor A into $LU = E_{21}^{-1}E_{32}^{-1}U$

Provide feedback (0)

Step 2 of 8

Subtracting 2 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix}$$

Subtracting 2 times row 2 from row 3 gives

$$U = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 8

To find L reversing the operations on identity matrix;
Adding 2 times row 1 to row 2 and adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

CH1.5 24P

Step 3 of 8

To find L reversing the operations on identity matrix;
Adding 2 times row 1 to row 2 and adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$LU = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 8

Applying elementary operation on identity matrix that is
Subtracting 2 times of row 1 from row 2 gets E_{21}

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Subtracting 2 times of row 2 from row 3 gets E_{32}

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 5 of 8

Step 5 of 8

Now

$$E_{32}E_{21}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix} = U$$

This is the required upper triangular form.

Provide feedback (0)

Step 6 of 8

By reversing the operations held on E_{21} gets E_{21}^{-1}

$$E_{32}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

By reversing the operations held on E_{32} gets E_{32}^{-1}

$$E_{21}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

Therefore

By reversing the operations held on E_{32} gets E_{32}

$$E_{21}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

Therefore

$$E_{21}^{-1}E_{32}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 8 of 8

$$\Rightarrow E_{21}^{-1}E_{32}^{-1}U = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix}$$

$$= LU$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$$

$$= A$$

So this is the required factor form A.

Provide feedback (0)

CH1.5 23P

Step 1 of 3

We consider $A=\begin{pmatrix}1&0&1\\2&2&2\\3&4&5\end{pmatrix}$

E_{21}: Subtracting 2 times row 1 from row 2

$$A\sim\begin{pmatrix}1&0&1\\0&2&0\\3&4&5\end{pmatrix}$$

E_{31}: Subtracting 3 times row 1 from row 3

$$A\sim\begin{pmatrix}1&0&1\\0&2&0\\0&4&2\end{pmatrix}$$

E_{32}: Subtracting 2 times row 2 from row 3

$$A\sim\begin{pmatrix}1&0&1\\0&2&0\\0&0&2\end{pmatrix}=U$$

Provide feedback (0)

CH1.5 25P

Step 2 of 3

$E_{32}E_{31}E_{21}A=U$

Now, we write $A=E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U$

E_{21}^{-1}: applying the reverse operations for the above operations:

Adding two times row 1 to row 2:

$$E_{21}^{-1}:\begin{bmatrix}1&0&0\\2&1&0\\0&0&1\end{bmatrix}$$

E_{31}^{-1}: Adding 3 times row 1 to row 3

$E_{32}E_{31}E_{21}A = U$

Now, we write $A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U$

E_{21}^{-1}: applying the reverse operations for the above operations:

Adding two times row 1 to row 2:

$$E_{21}^{-1}: \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E_{31}^{-1}: Adding 3 times row 1 to row 3

$$: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

E_{32}^{-1}: adding 2 times row 2 to row 3

$$: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

Provide feedback (1)

Step 3 of 3

$$\text{So,} \quad A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= LU$$

CH1.5 24P

Givent that when zero appears in a pivot position, $A = LU$ is not possible(we need non-zero pivots d, f, I in U). We have to sow directly that why the following are impossible.

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ l & 1 \end{pmatrix}\begin{pmatrix} d & e \\ 0 & f \end{pmatrix}, \text{ and } \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ m & n & 1 \end{pmatrix}\begin{pmatrix} d & e & g \\ 0 & f & h \\ 0 & 0 & i \end{pmatrix}$$

Provide feedback (0)

CH1.5 26P

Step 2 of 3

Given $A = LU$ system is

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ l & 1 \end{pmatrix}\begin{pmatrix} d & e \\ 0 & f \end{pmatrix}$$

In right hand side by matrices multiplication

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} d & e \\ dl & le \end{pmatrix}$$

But d is not zero but here by comparing $d = 0$ which not allowed so $A = LU$ is not allowed.

Provide feedback (0)

Step 3 of 3

Given $A = LU$ system is

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ m & n & 1 \end{pmatrix}\begin{pmatrix} d & e & g \\ 0 & f & h \\ 0 & 0 & i \end{pmatrix}$$

In right hand side by matrices multiplication

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} d & e & g \\ ld & le+f & lg+h \\ md & me+nf & mg+nh+i \end{pmatrix}$$

But g is not zero but here by comparing $g = 0$ which is not allowed so $A = LU$ is not allowed.

CH1.5 25P

Step 1 of 5

Given that $A = \begin{pmatrix} 1 & c & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$

We have to find that which number leads to zero in the second pivot position. And we have to find that which c produces 0 in third pivot position.

Provide feedback (0)

Step 2 of 5

If c is 2 then the second pivot becomes zero.
Since when $c = 2$ then the matrix is

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 5

By subtracting 2 times row 1 from row 2 gives

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

$\boxed{\text{So } c = 2 \text{ leads to zero in the second pivot position}}$.

Provide feedback (0)

Step 4 of 5

If c is $\frac{5}{3}$ then the third pivot becomes zero.

CH1.5 27P

Step 4 of 5

If c is $\frac{5}{3}$ then the third pivot becomes zero.

Since when $c = \frac{5}{3}$ then the matrix is

$$\begin{pmatrix} 1 & \frac{5}{3} & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

Provide feedback (1)

Step 5 of 5

By subtracting 3 times row 1 from row 3gives

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Here the third pivot (third diagonal element) is zero.

$\boxed{\text{So } c = \frac{5}{3} \text{ leads to zero in the third pivot position}}$

Provide feedback (3)

CH1.5 26P

Step 1 of 4

Given $A=\begin{pmatrix}2&4&8\\0&3&9\\0&0&7\end{pmatrix}$

We have to find L and D for this matrix and we have to find U in $A=LU$, and then we have to find new U in $A=LDU$

Provide feedback (0)

Step 2 of 4

The matrix A is an upper triangular matrix and 2, 3, 7 are pivots, so here A can be written as

$$A=\begin{pmatrix}2&4&8\\0&3&9\\0&0&7\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}2&4&8\\0&3&9\\0&0&7\end{pmatrix}$$

This is form the form $A=LU$

Therefore U in $A=LU$ is

$$\boxed{U=A=\begin{pmatrix}2&4&8\\0&3&9\\0&0&7\end{pmatrix}}$$

Provide feedback (0)

Step 3 of 4

To factorize $A=LDU$, U should be an upper triangular with 1's on the diagonal.

Dividing Row 1 with 2, row 2 with 3, row 3 with 7 in U from $A=LU$ gives

$$A=\begin{pmatrix}2&4&8\\0&3&9\\0&0&7\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}(2)(3)(7)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\frac{1}{(2)(3)(7)}\begin{pmatrix}2&4&8\\0&3&9\\0&0&7\end{pmatrix}$$

$$\begin{pmatrix}2&4&8\end{pmatrix}\begin{pmatrix}1&0&0\end{pmatrix}\begin{pmatrix}2&0&0\end{pmatrix}\begin{pmatrix}1&2&4\end{pmatrix}$$

CH1.5 28P

$$\boxed{U = A = \begin{pmatrix} 0 & 3 & 9 \\ 0 & 0 & 7 \end{pmatrix}}$$

Provide feedback (0)

Step 3 of 4

To factorize $A = LDU$, U should be an upper triangular with 1's on the diagonal.
Dividing Row 1 with 2, row 2 with 3, row 3 with 7 in U from $A = LU$ gives

$$A = \begin{pmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}(2)(3)(7)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\frac{1}{(2)(3)(7)}\begin{pmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{pmatrix}\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 4

This is of the form $A = LDU$
So here

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

So U in $A = LDU$ is

$$\boxed{U = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}}$$

Provide feedback (0)

CH1.5 27P

Step 1 of 8

Given symmetric matrices are

$$A=\begin{pmatrix}2 & 4\\ 4 & 11\end{pmatrix}, \text{ and } B=\begin{pmatrix}1 & 4 & 0\\ 4 & 12 & 4\\ 0 & 4 & 0\end{pmatrix}$$

We have to find their triple factorizations LDU and we have to say that how U and L are related.

Provide feedback (0)

Step 2 of 8

Given $A=\begin{pmatrix}2 & 4\\ 4 & 11\end{pmatrix}$

Subtracting 2 times row 1 from row 2

$$U=\begin{pmatrix}2 & 4\\ 0 & 3\end{pmatrix}$$

Dividing row 1 with 2 and row 2 with 3 gets

$$U=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$

Provide feedback (0)

Step 3 of 8

To get L, we have to do reverse operations on the identity matrix I_2 which are held on A;

Adding 2 times row 1 to row 2 gives

$$L=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$$

D is the diagonal matrix with pivots 2, 3 on the diagonal so

$$D=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$$

CH1.5 29P

Step 3 of 8

To get L, we have to do reverse operations on the identity matrix I_2 which are held on A;
Adding 2 times row 1 to row 2 gives

$$L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

D is the diagonal matrix with pivots 2, 3 on the diagonal so

$$D = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 8

Now $A = LDU$ is $\boxed{\begin{pmatrix} 2 & 4 \\ 4 & 11 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}}$

And we can observe that $\boxed{L^T = U \text{ and } U^T = L}$.

Provide feedback (0)

Step 5 of 8

Given $B = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix}$

Subtracting 4 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 4 & 0 \\ 0 & -4 & 4 \\ 0 & 4 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 6 of 8

Adding row 2 to row 3 gets

$$=\begin{pmatrix}1 & 4 & 0\\ 0 & -4 & 4\\ 0 & 0 & 4\end{pmatrix}$$

Dividing row 2 with -4 and row 3 with 4 gets U

$$U=\begin{pmatrix}1 & 4 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

To get L, we have to do reverse operations on the identity matrix I_3 which are held on A;
Adding 4 times row 1 to row 2 and subtracting row 2 from row 3 gives

$$L=\begin{pmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 0 & -1 & 1\end{pmatrix}$$

D is the diagonal matrix with pivots 1,-4, 4 on the diagonal so

$$D=\begin{pmatrix}1 & 0 & 0\\ 0 & -4 & 0\\ 0 & 0 & 4\end{pmatrix}$$

Provide feedback (0)

Step 8 of 8

The factorization $A = LDU$ is

$$\begin{pmatrix}1 & 4 & 0\\ 4 & 12 & 4\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 4 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & -4 & 0\end{pmatrix}\begin{pmatrix}1 & 4 & 0\\ 0 & 1 & -1\end{pmatrix}$$

$$= \begin{pmatrix} 0 & -4 & 4 \\ 0 & 0 & 4 \end{pmatrix}$$

Dividing row 2 with -4 and row 3 with 4 gets U

$$U = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

To get L, we have to do reverse operations on the identity matrix I_3 which are held on A;
Adding 4 times row 1 to row 2 and subtracting row 2 from row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

D is the diagonal matrix with pivots 1,-4, 4 on the diagonal so

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Provide feedback (0)

Step 8 of 8

The factorization $A = LDU$ is

$$\begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 4 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

And we can observe that $\boxed{L^T = U \text{ and } U^T = L}$.

Provide feedback (0)

CH1.5 28P

Step 1 of 6

We have to compute L and U for the symmetric matrix:

$$A=\begin{pmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{pmatrix}$$

Also we have to find four conditions on a,b,c,d to get $A=LU$ with four pivots.

Provide feedback (0)

Step 2 of 6

Subtracting row 1 from row 2 and row 3 and row 4 gives

$$A\sim\begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{pmatrix}$$

Subtracting row 2 from row 3 and row 4 gives

$$A\sim\begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{pmatrix}$$

Provide feedback (0)

Step 3 of 6

Subtracting row 3 from row 4 gives

$$A\sim\begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \end{pmatrix}=U$$

CH1.5 30P

Step 3 of 6

Subtracting row 3 from row 4 gives

$$A \sim \begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{pmatrix} = U$$

Provide feedback (0)

Step 4 of 6

The lower triangular matrix is

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix}$$

Fix $l_{21} = 1$for the row operation, subtracting row 1 from row 2, so from the above row operations we get $l_{21} = l_{31} = l_{41} = 1, l_{32} = l_{42} = 1, l_{43} = 1$

Provide feedback (0)

Step 5 of 6

Therefore

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

We can observe that

$$A = \begin{pmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \end{pmatrix}$$

$$L=\begin{bmatrix}1&0&0&0\\l_{21}&1&0&0\\l_{31}&l_{32}&1&0\\l_{41}&l_{42}&l_{43}&1\end{bmatrix}$$

Fix $l_{21}=1$for the row operation, subtracting row 1 from row 2, so from the above row operations we get $l_{21}=l_{31}=l_{41}=1,\ l_{32}=l_{42}=1, l_{43}=1$

Provide feedback (0)

Step 5 of 6

Therefore

$$L=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}$$

We can observe that

$$A=\begin{pmatrix}a&a&a&a\\a&b&b&b\\a&b&c&c\\a&b&c&d\end{pmatrix}=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}\begin{pmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&0&d-c\end{pmatrix}$$

Provide feedback (0)

Step 6 of 6

The four pivots are $a, b-a, c-b, d-c$ (the diagonal elements of U) and the factorization is possible if $a\neq 0, b\neq a, c\neq b, d\neq c$

Provide feedback (0)

CH1.5 29P

Step 1 of 6

We have to find L and U for the non-symmetric matrix

$$A=\begin{pmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{pmatrix}$$

And also we have to find the four conditions on a,b,c,d,r,s,t to get $A=LU$ with four pivots.

Provide feedback (0)

Step 2 of 6

Subtracting row 1 from row 2 and row 3 and row 4 gives

$$\sim\begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & b-r & c-r & t-r \\ 0 & b-r & c-r & d-r \end{pmatrix}$$

Subtracting row 2 from row 3 and row 4 gives

$$\sim\begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & c-s & d-s \end{pmatrix}$$

Provide feedback (0)

Step 3 of 6

Subtracting row 3 from row 4 gives

$$\sim\begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \end{pmatrix}=U$$

CH1.5 31P

Step 3 of 6

Subtracting row 3 from row 4 gives

$$\sim \begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & 0 & d-t \end{pmatrix} = U$$

Provide feedback (0)

Step 4 of 6

The lower triangular matrix is

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix}$$

Fix $l_{21} = 1$for the row operation, subtracting row 1 from row 2, so from the above row operations we get $l_{21} = l_{31} = l_{41} = 1, l_{32} = l_{42} = 1, l_{43} = 1$

Provide feedback (0)

Step 5 of 6

Therefore

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

We can observe that

$$A = \begin{pmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \end{pmatrix}$$

The lower triangular matrix is

$$L=\begin{bmatrix}1&0&0&0\\l_{21}&1&0&0\\l_{31}&l_{32}&1&0\\l_{41}&l_{42}&l_{43}&1\end{bmatrix}$$

Fix $l_{21}=1$for the row operation, subtracting row 1 from row 2, so from the above row operations we get $l_{21}=l_{31}=l_{41}=1,\ l_{32}=l_{42}=1, l_{43}=1$

Provide feedback (0)

Step 5 of 6

Therefore

$$L=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}$$

We can observe that

$$A=\begin{pmatrix}a&r&r&r\\a&b&s&s\\a&b&c&t\\a&b&c&d\end{pmatrix}=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}\begin{pmatrix}a&r&r&r\\0&b-r&s-r&s-r\\0&0&c-s&t-s\\0&0&0&d-t\end{pmatrix}$$

Provide feedback (0)

Step 6 of 6

The four pivots are $a, b-r, c-s, d-t$ (the diagonal elements of U) and the factorization is possible if $a\neq 0, b\neq r, c\neq s, d\neq t$

Provide feedback (0)

CH1.5 30P

Step 1 of 8

We have to factor the following matrices into $A = LU$ and $A = LDV$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \text{ and } A = \begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}$$

Provide feedback (0)

Step 2 of 8

Given

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Subtracting 1 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Subtracting row 2 from row 3, we gets

$$U = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 8

To get L, we have to do reverse operations on the identity matrix which are held on A;
Adding 1 times row 1 to row 2 and adding row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

CH1.5 32P

Step 3 of 8

To get L, we have to do reverse operations on the identity matrix which are held on A;
Adding 1 times row 1 to row 2 and adding row 2 to row 3 gives

$$L=\begin{pmatrix}1&0&0\\1&1&0\\0&1&1\end{pmatrix}$$

$$\boxed{\text{Therefore } A=LU \text{ factorization is } \begin{pmatrix}1&1&0\\1&2&1\\0&1&2\end{pmatrix}=\begin{pmatrix}1&0&0\\1&1&0\\0&1&1\end{pmatrix}\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}}$$

Provide feedback (0)

Step 4 of 8

The diagonal elements of D are the pivots of U and V is obtained by the each row of U, by the pivots of U.

$$\boxed{\text{The factorization } A=LDV \text{ is } \begin{pmatrix}1&1&0\\1&2&1\\0&1&2\end{pmatrix}=\begin{pmatrix}1&0&0\\1&1&0\\0&1&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}}$$

Provide feedback (0)

Step 5 of 8

$$\text{Given } A=\begin{pmatrix}a&a&0\\a&a+b&b\\0&b&b+c\end{pmatrix}$$

Subtracting row 1 from row 2 gives

$$=\begin{pmatrix}a&a&0\\0&b&b\\0&b&b+c\end{pmatrix}$$

Step 6 of 8

Subtracting row 2 from row 3 gives

$$U=\begin{pmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{pmatrix}$$

To get L, we have to do reverse operations on the identity matrix which are held on A;
Adding 1 times row 1 to row 2 and adding row 2 to row 3 gives

$$L=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 8

Therefore $A=LU$ factorization is

$$\boxed{\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{pmatrix}}$$

Provide feedback (0)

Step 8 of 8

The diagonal elements of D are the pivots of U and V is obtained by the each row of U, by the pivots of U.
The factorization $A=LDV$ is

$$\boxed{\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}$$

Provide feedback (0)

CH1.5 31P

Step 1 of 4

To solve $Lc=b$, we write $\begin{pmatrix}1 & 0\\4 & 1\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}2\\11\end{pmatrix}$

Subtracting 4 times row 1 from row 2 gives

$$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}$$

From this the matrices $c=\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}$

Provide feedback (0)

Step 2 of 4

To solve $Ux=c$, we consider $\begin{pmatrix}2 & 4\\0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-5\\3\end{pmatrix}$$

Provide feedback (0)

Step 3 of 4

$A=LU$ implies

$$A=\begin{pmatrix}1 & 0\\4 & 1\end{pmatrix}\begin{pmatrix}2 & 4\\0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}2 & 4\\8 & 17\end{pmatrix}$$

Provide feedback (0)

CH1.5 33P

$$\begin{pmatrix} x_2 \end{pmatrix} \quad \begin{pmatrix} 3 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 4

$A = LU$ implies

$$A = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 8 & 17 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 4

On the other hand, to solve $Ax=b$ the system is $\begin{pmatrix} 2 & 4 \\ 8 & 17 \end{pmatrix} x = \begin{pmatrix} 2 \\ 11 \end{pmatrix}$

Subtracting 4 times row 1 from row 2

$$\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Divide row 2 with 2 gives

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Subtracting 2times row 2 from row 1 gives

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

Here also $x = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

Provide feedback (0)

CH1.5 32P

CH1.5 34P

Step 1 of 6

Given

$$L=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix},\ U=\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix},\text{ and } b=\begin{pmatrix}4\\5\\6\end{pmatrix}$$

We have to solve $Lc=b$ to find c, we have to solve $Ux=c$ to find x, and finally we have to find A

Provide feedback (0)

Step 2 of 6

To solve $Lc=b$, the system is

$$\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}c=\begin{pmatrix}4\\5\\6\end{pmatrix}$$

Subtracting row 1 from row 2 and row 3 gives

$$\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}c=\begin{pmatrix}4\\1\\2\end{pmatrix}$$

Provide feedback (0)

Step 3 of 6

Subtracting row 2 from row 3 gives

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}c=\begin{pmatrix}4\\1\\1\end{pmatrix}$$

From this system, $\boxed{c=\begin{pmatrix}4\\1\\1\end{pmatrix}}$

Step 4 of 6

To solve $Ux = c$, the system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

Subtracting 1 times row 3 from row 1 and row 2 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

Provide feedback (0)

Step 5 of 6

Subtracting 1 times row 2 from row 1 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

From this system, $\boxed{x = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}}$

Provide feedback (0)

Step 6 of 6

$$A = LU$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 \end{pmatrix}$$

Provide feedback (0)

Step 5 of 6

Subtracting 1 times row 2 from row 1 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

From this system, $\boxed{x = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}}$

Provide feedback (0)

Step 6 of 6

$$A = LU$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\Rightarrow \boxed{A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}}$$

Provide feedback (2)

CH1.5 33P

Step 1 of 4

Matrix A can be factorized into product of lower and upper triangular matrices.

$$A = LU$$

Here, matrix L is a lower triangular matrix with 1 at the diagonal position and matrix U is the upper triangular matrix with pivots at the diagonal position.

Provide feedback (0)

CH1.5 35P

Step 2 of 4

Consider the following matrices:

$$A = \begin{bmatrix} x & x & x & x \\ x & x & x & 0 \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$

$$B = \begin{bmatrix} x & x & x & 0 \\ x & x & 0 & x \\ x & 0 & x & x \\ 0 & x & x & x \end{bmatrix}$$

Here, nonzero positions are marked by x. Determine which zeros will be still zero in their factors L and U.

Provide feedback (0)

Step 3 of 4

First take matrix *A*.

$$A=\begin{bmatrix} x & x & x & x \\ x & x & x & 0 \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$

It can be seen that a part (3×3) of the (4×4) matrix is already in lower triangular form. Now recall that if a row of matrix *A* starts with zeros, so does that row of matrix *L* and if a column starts with zero so does that column of matrix *U*.

Therefore, matrix *L* will contain three zeros at the position $\boxed{a_{31}, a_{41}, a_{42}}$ however matrix *U* may not contain zero at the position $\boxed{a_{24}}$.

Provide feedback (0)

Step 4 of 4

Similarly, in the case of matrix *B*, bottom left side zero will be in matrix *L* and top right side zero will be in matrix *U*. Rest zeros may be filled in by the non zeros.

Therefore, matrix *L* will contain one zero at the position $\boxed{a_{41}}$ and matrix *U* will contain zero at the position $\boxed{a_{14}}$.

CH1.5 34P

Step 1 of 2

If A has pivots 2, 7, 6 with no row exchanges, we have to say that what are the pivots for the upper left 2 by 2 sub-matrix B (without row 3 and column 3)

Provide feedback (0)

CH1.5 36P

Step 2 of 2

From the given data, we have

$$A = \begin{bmatrix} 2 & & \\ & 7 & \\ & & 6 \end{bmatrix}$$

Now the elimination on A starts in the upper left corner with elimination on B.

$$\Rightarrow B = \begin{bmatrix} 2 & \\ & 7 \end{bmatrix}$$

Therefore the 2 by 2 upper sub matrix B has the first two $\boxed{\text{pivots } 2, 7}$.

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CH1.5 35P

Step 1 of 3

We have to produce M by starting from a 3 by 3 matrix A with pivots 2, 7, 6, and by adding a fourth row and column to A, and we have to find the first three pivots for M, and we have to find the fourth row and the fourth column to produce 9 as a fourth pivot.

Provide feedback (0)

CH1.5 37P

Step 2 of 3

Given

$$A = \begin{bmatrix} 2 & & \\ & 7 & \\ & & 6 \end{bmatrix}$$

If the 3 by 3 matrix starts with pivots 2, 7, 6 and by adding the column and row with 0,0,0,1 then there is no effect on elimination since the fourth pivot there is 1.

Provide feedback (0)

Step 3 of 3

That is

$$\boxed{M = \begin{bmatrix} 2 & & & 0 \\ & 7 & & 0 \\ & & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

By multiplying with 9 the fourth pivot becomes 9.

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CH1.5 36P

Step 1 of 1

The Pascal 5 pattern matrix is $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{pmatrix}$

Pascal 5 triangle in L and U by using MATLAB's LU code is

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

CH1.5 38P

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CH1.5 37P

Step 1 of 2

We have to find that for which numbers c is $A = LU$ impossible with three pivots.

Given matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & c & 1 \\ 0 & 1 & 1 \end{pmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 2 & 0 \\ 0 & c-6 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Provide feedback (0)

CH1.5 39P

Step 2 of 2

Now if $c = 6$, then the second pivot becomes 0.
We know that if any element in the pivot position is zero then LU factorization is not possible, so if $\boxed{c = 6}$ then LU is impossible.

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CH1.5 39P

Step 1 of 2

Given that there are 12 even permutations of $(1,2,3,4)$, with an even number of exchanges. And two of them are $(1,2,3,4)$ with no exchanges and $(4,3,2,1)$ with two exchanges. We have to list the other ten and instead of writing 4 by 4 matrix, we have use the numbers 4, 3, 2, 1 to give the position of the 1 in each row.

Provide feedback (0)

Step 2 of 2

The other ten permutations of (1, 2, 3, 4):
With one exchange the permutations are $\boxed{(2,1,3,4),(1,3,2,4),(1,2,4,3)}$
With two exchanges are $\boxed{(2,1,4,3),(3,1,2,4),(1,3,4,2)}$
The other exchange permutations are $\boxed{(3,1,4,2),(4,1,3,2),(4,3,1,2),(3,4,1,2)}$

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CH1.5 41P

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CH1.5 40P

Step 1 of 5

We have to find the number of exchanges will permute $(5,4,3,2,1)$ back to $(1,2,3,4,5)$

And we have to find the number of exchanges to change $(6,5,4,3,2,1)$ back to $(1,2,3,4,5,6)$, one is even and the other is odd.

For $(n,\ldots,1)$ to $(1,\ldots,n)$, we have to show that $n=100$ and 101 are even, $n=102$ and 103 are odd.

Provide feedback (0)

Step 2 of 5

First we exchange 1 and 5 then it becomes $(1,4,3,2,5)$, then we exchange 4 and 2 then it becomes $(1,2,3,4,5)$.

It requires 2 exchanges to permute $(5,4,3,2,1)$ back to $(1,2,3,4,5)$

Provide feedback (0)

Step 3 of 5

First we exchange 1and 6 then 2 and 5 then 3 and 4.

Provide feedback (0)

Step 4 of 5

Therefore the number of exchanges to be perform that (6,5,4,3,2,1) back to (1,2,3,4,5,6)

Provide feedback (0)

Step 5 of 5

is 3.

For $n=100$ there are 50 exchanges and then 51 exchanges for $n=102$ requires.

CH1.5 42P

CH1.5 41P

Step 1 of 9

Given that the permutation matrices are the matrices obtained by exchange of rows in the identity matrix

We have to give examples of permutation matrices with $P_1P_2 \neq P_2P_1$ and $P_3P_4 = P_4P_3$

Provide feedback (0)

CH1.5 43P

Step 2 of 9

Let the two permutation matrices of order 3 be

$$P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 9

Now

$$P_1P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 9

And

$$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$$

Step 4 of 9

And

$$P_2P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

P_1P_2 is also a permutation matrix.

And $P_1P_2 \neq P_2P_1$

Provide feedback (0)

Step 5 of 9

So the permutation matrices such that $P_1P_2 \neq P_2P_1$ are

$$\boxed{P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}$$

Provide feedback (0)

Step 6 of 9

Let the two permutation matrices of order 3 be

$$P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } P_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

Then

$$P_3P_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 9

And

$$P_4P_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 9

Therefore $P_3P_4 = P_4P_3$

Hence the permutation matrices such that $P_3P_4 = P_4P_3$ are

$$\boxed{P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } P_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

CH1.5 42P

Step 1 of 7

Given

$$A=\begin{pmatrix}0&0&6\\1&2&3\\0&4&5\end{pmatrix}$$

We have to find that which permutation makes PA upper triangular, and which permutations make P_1AP_2 lower triangular, and we have to verify that whether P_2 gives column exchange or row exchange of A.

Provide feedback (0)

Step 2 of 7

If we perform R_{12}, R_{23} on A then we have

$$A\sim\begin{pmatrix}1&2&3\\0&4&5\\0&0&6\end{pmatrix}$$

This is upper triangular.

Provide feedback (0)

Step 3 of 7

So for getting permutation matrix P, we have to perform the same operations R_{12}, R_{23} on I_3, then we get

$$P=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$$

And we have

$$PA=\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}0&0&6\\1&2&3\end{pmatrix}=\begin{pmatrix}1&2&3\\0&4&5\end{pmatrix}$$

CH1.5 44P

Step 3 of 7

So for getting permutation matrix P, we have to perform the same operations R_{12}, R_{23} on I_3, then we get

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

And we have

$$PA = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 7

So the required matrix P is

$$\boxed{P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}$$

Provide feedback (0)

Step 5 of 7

To convert P_1AP_2 into lower triangular, we can write A as

$I_3AI_3 = A$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{pmatrix}$$

Now we perform the row operations on the first identity matrix and the corresponding column operations on the second identity matrix.

Provide feedback (0)

Step 6 of 7

Now we perform R_{23}, C_{13} on A on the right-hand side of the above equation, and R_{23} on first identity matrix, and C_{13} on second identity matrix, then we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 7 of 7

This is of the form

$P_1 A P_2 = \text{lower triangular matrix}$

$$\Rightarrow \boxed{P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \text{and } P_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}}$$

$\boxed{\text{And from this, we can observe that } P_2 \text{ gives column exchange } C_{13} \text{ of } A}$

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CH1.5 43P

Step 1 of 7

We have to find a 3 by 3 permutation matrix with $P^3 = I$ (but not $P = I$), and we have to find a 4 by 4 permutation P with $P^4 \neq I$

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CH1.5 45P

Step 2 of 7

Let a 3 by 3 permutation matrix P be

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ then}$$

$$P^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 7

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= I$$

Provide feedback (0)

Step 4 of 7

Hence it is clear that $P^3 = I$

So the required matrix is

$$\boxed{P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}$$

Provide feedback (0)

Step 5 of 7

Let a 4 by 4 permutation matrix P be

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 6 of 7

Then

$$P^4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence it is clear that $P^4 \neq I$

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Provide feedback (0)

Step 6 of 7

Then

$$P^4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence it is clear that $P^4 \neq I$

Provide feedback (1)

Step 7 of 7

So the required matrix is

$$\boxed{P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

Provide feedback (0)

CH1.5 44P

Step 1 of 5

If we take powers of a permutation, then we have to explain that why is some P^k eventually equal to I, and we have to find a 5 by 5 permutation P so that the smallest power to equal I is P^6

Provide feedback (0)

Step 2 of 5

If the order of matrix is n then, there exist $n!$ permutation matrices
Eventually two powers of permutation matrix P must be the same.
If $P^r = P^s$ then $P^{r-s} = I$
$\Rightarrow r - s \le n!$

Provide feedback (0)

Step 3 of 5

Let $P = \begin{pmatrix} P_2 & 0 \\ 0 & P_3 \end{pmatrix}$ in 5 by 5 matrices with

$$P_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ P_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 5

Then

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

CH1.5 46P

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

Provide feedback (0)

Step 4 of 5

Then

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Then by using CAS, we get $P^6 = I$

Provide feedback (0)

Step 5 of 5

So the required P is

$$\boxed{P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}}$$

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Permutation matrix: Matrix *P* has single 1 in every row and every column. It has the rows of identity matrix *I* in any order.

Provide feedback (0)

Step 2 of 4

Let matrix *P* is a rotation matrix which multiplies to a vector (x, y, z) to give (z, y, x). Determine *P* and P^3.

Matrix *P* if multiplied to the vector (x, y, z) puts last column first by shifting other columns. This gives the permutation matrix P defined as follows. Therefore,

$$\boxed{P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$$

Provide feedback (0)

Step 3 of 4

Now do the following calculations:

$$P \cdot P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Now do the following calculations:

$$P \cdot P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P^2 \cdot P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^3 = I$$

Therefore, $\boxed{P^3 = I}$.

Provide feedback (0)

Step 4 of 4

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^3 = I$$

Therefore, $\boxed{P^3 = I}$.

Provide feedback (0)

Step 4 of 4

Next consider the rotation axis $a = (1,1,1)$ that doesn't move but equals to *Pa*. determine the angle of rotation from $v = (2,3,-5)$ to $Pv = (-5,2,3)$.

Matrix *P* if multiplied to the vector $v = (2,3,-5)$ puts last column first by shifting other columns. This gives the permutation matrix P as defined above.

Also, above calculation, $P^3 = I$, shows that after taking three rotations matrix equals to identity matrix. This implies that three rotations equals to $360°$, so one rotation will make an angle of $360°/3 = 120°$.

Therefore, angle of rotation from *v* to $P \cdot v$ around $a = (1,1,1)$ is $\boxed{120°}$.

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CH1.5 46P

Step 1 of 2

If P is any permutation matrix, we have to find a nonzero matrix x such that $(I-P)x=0$

Given that $(I-P)x=0$

$\Rightarrow Ix-Px=0$

$\Rightarrow Ix=Px$

$\Rightarrow x=Px$

Since by matrices multiplication and because I is identity so $Ix=x$

$\boxed{\text{This will happens if } x=(1,1,\ldots,1)}$

Provide feedback (0)

Step 2 of 2

Because for instance let $x=(1,1,1)$ and P be 3 by 3 permutation matrix equals to I then

$$Px=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$=\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$=x$$

Provide feedback (0)

CH1.5 48P

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Permutation matrix: Matrix P has single 1 in every row and every column. It has the rows of identity matrix I in any order.

Provide feedback (0)

Step 2 of 3

Consider P has 1s on the anti-diagonal from $(1,n)$ to $(n,1)$ and let A be any matrix then describe PAP.

Matrix P with 1s on the anti-diagonal from $(1,n)$ to $(n,1)$ will be:

$$P=\begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Here, $P=P^T$. Recall that $P^T=P^{-1}$, so $P=P^{-1}$.

Provide feedback (0)

Step 3 of 3

Let A be any matrix. Multiplication with matrix P, (PA) destroys the symmetry of matrix A. To recover the symmetry it is multiplied by a permutation matrix Q (PAQ). This matrix Q is none other than P^T. So, PAP^T guarantees to be symmetric.

Therefore, PAP becomes a symmetric matrix.