

&lt; CH1

## Step 1 of 2

Consider the following equations:

$$x + y = 4$$

$$2x - 2y = 4$$

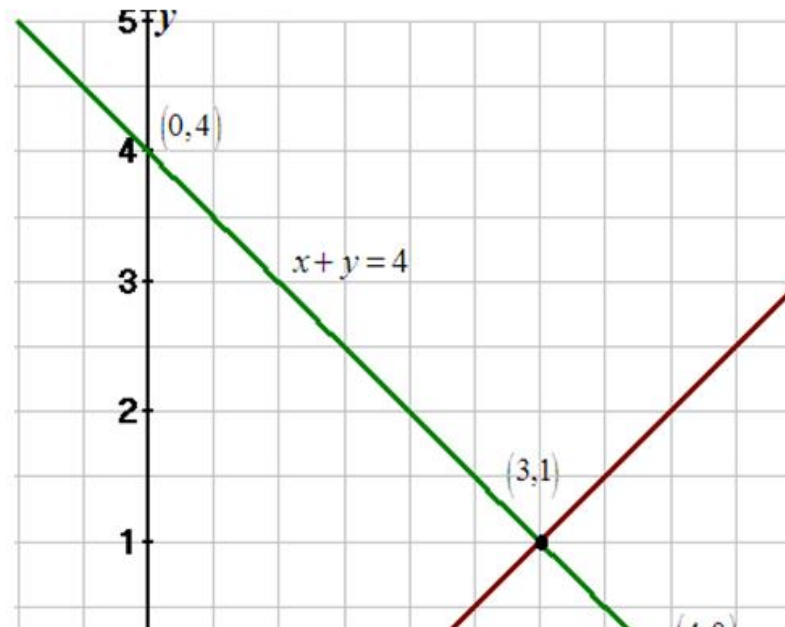
The equation  $x + y = 4$  represented by a straight line in the  $x$ - $y$  plane. The line goes through the points  $x = 2, y = 2$  and  $x = 4, y = 0$ .

The second equation  $2x - 2y = 4$  is also represented by a straight line in the  $x$ - $y$  plane.

This line goes through the points  $x = 0, y = -2$  and it crosses the first line at the solution.

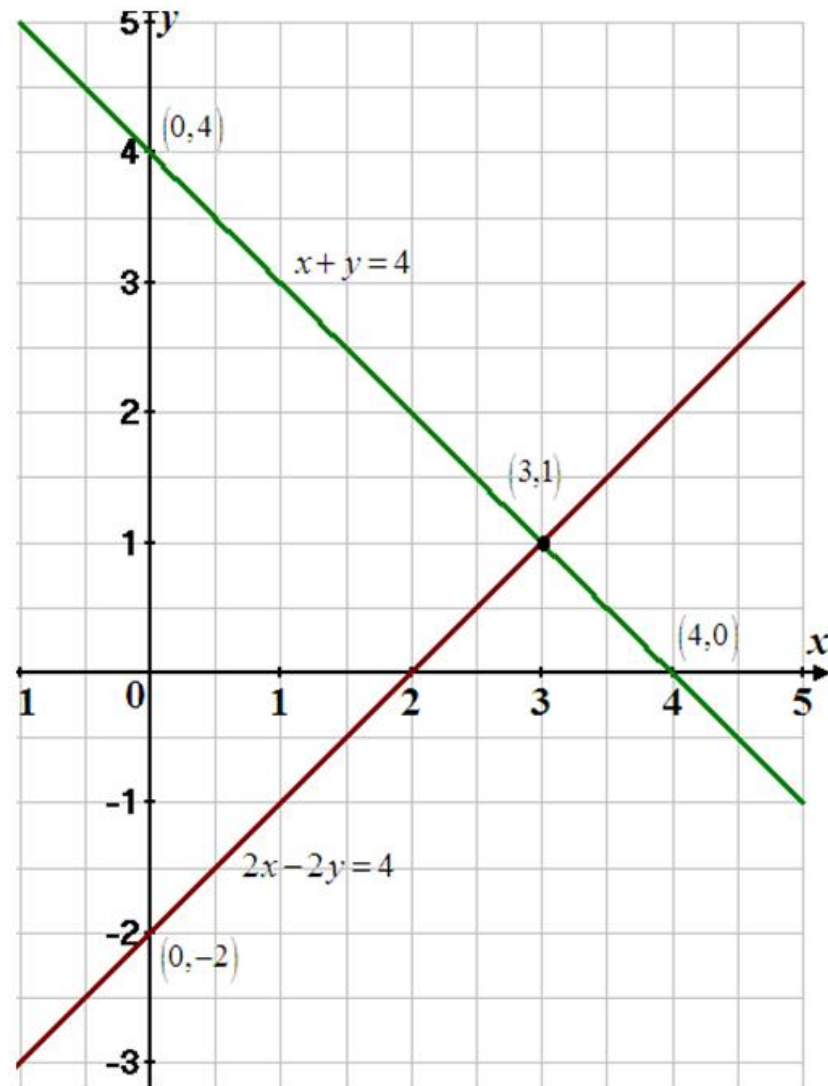
Draw the row picture and the column picture for the equations  $x + y = 4, 2x - 2y = 4$

The required diagram is shown as follows:



CH1.2 2P &gt;

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The two lines intersect at the point  $(3, 1)$ .

## Step 2 of 2

Column form of the given equations as follows

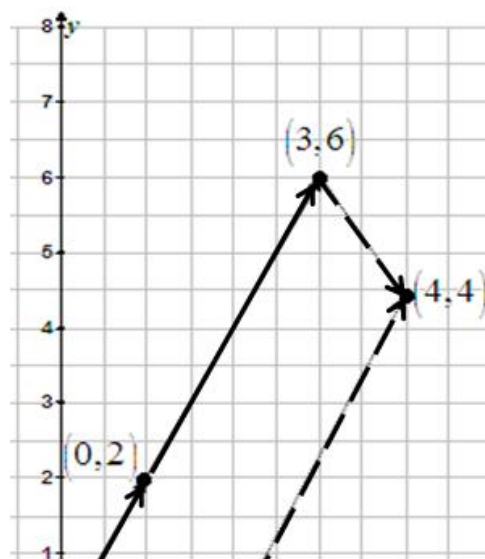
$$x \begin{pmatrix} 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Now consider

$$\begin{aligned} 3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} &= \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 3+1 \\ 6-2 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \end{aligned}$$

Therefore  $3(\text{column1}) + 1(\text{column2}) = (4, 4)$

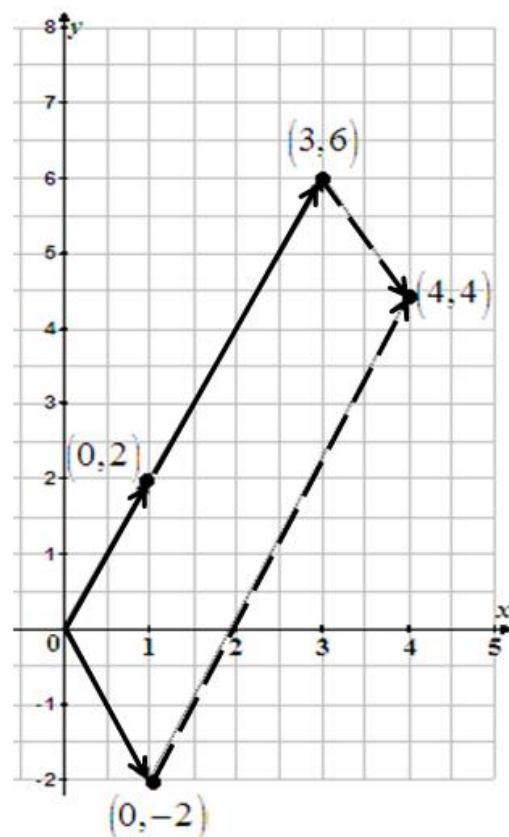
Sketch the column picture as shown below.



$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Therefore  $3(\text{column1}) + 1(\text{column2}) = (4, 4)$

Sketch the column picture as shown below.



[Provide feedback \(0\)](#)

 CH1.2 1P

## Step 1 of 1

Consider the triangular system

$$\left. \begin{array}{l} u - v - w = b_1 \\ v + w = b_2 \\ w = b_3 \end{array} \right\} \dots (1)$$

As the system is in triangular form use back substitution to solve for the elements of  $b$ .From the last equation,  $w = b_3$ Substitute  $w = b_3$  in second equation  $v + w = b_2$ 

$$\begin{aligned} v + b_3 &= b_2 \\ v &= b_2 - b_3 \end{aligned}$$

Substitute  $w = b_3, v = b_2 - b_3$  in first equation to find  $u$ .

$$\begin{aligned} u - (b_2 - b_3) - b_3 &= b_1 \\ u - b_2 + b_3 - b_3 &= b_1 \\ u - b_2 &= b_1 \\ u &= b_1 + b_2 \end{aligned}$$

Column form of system (1) can be written as

$$u \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + v \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Substitute  $u, v$  and  $w$  in the column form that gives the linear combination of the vectors of  $b$ .Hence the column combination as in  $b$  is

$$\left[ (b_1 + b_2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (b_2 - b_3) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + (b_3) \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \right]$$

CH1.2 3P


CH1.2 1P

$$\left. \begin{aligned} u - v - w &= b_1 \\ v + w &= b_2 \\ w &= b_3 \end{aligned} \right\} \dots\dots (1)$$

As the system is in triangular form use back substitution to solve for the elements of  $b$ .

From the last equation,  $w = b_3$

Substitute  $w = b_3$  in second equation  $v + w = b_2$

$$\begin{aligned} v + b_3 &= b_2 \\ v &= b_2 - b_3 \end{aligned}$$

Substitute  $w = b_3, v = b_2 - b_3$  in first equation to find  $u$ .

$$\begin{aligned} u - (b_2 - b_3) - b_3 &= b_1 \\ u - b_2 + b_3 - b_3 &= b_1 \\ u - b_2 &= b_1 \\ u &= b_1 + b_2 \end{aligned}$$

Column form of system (1) can be written as

$$u \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + v \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Substitute  $u, v$  and  $w$  in the column form that gives the linear combination of the vectors of  $b$ .

Hence the column combination as in  $b$  is

$$(b_1 + b_2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (b_2 - b_3) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + (b_3) \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

[Provide feedback \(0\)](#)

CH1.2 3P

 CH1.2 2P

## Step 1 of 3

Solve the following equations for  $u$ ,  $v$ ,  $w$  and  $z$ .

$$u + v + w + z = 6$$

$$u + w + z = 4$$

$$u + w = 2$$

$$\left. \begin{array}{l} u + w + z = 4 \\ u + w = 2 \end{array} \right\} \Rightarrow z = 2$$

$$\left. \begin{array}{l} u + v + w + z = 6 \\ u + w = 2 \\ z = 2 \end{array} \right\} \Rightarrow v = 2$$

Hence the obtained equations are

$$u + w = 2$$

$$z = 2$$

$$v = 2$$

These equations contain one dependent variable, one independent variable and two constants, so these three equations represent a line in four-dimensional space.

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 CH1.2 4P

## Step 2 of 3

Include the equation  $u = -1$  in the given set of equations and solve them for  $u$ ,  $v$ ,  $w$  and  $z$ .

$$u + v + w + z = 6$$

$$u + w + z = 4$$

$$u + w = 2$$

$$u = -1$$

$$u + w + z = 2$$



$$u + w = 2$$

$$z = 2$$

$$v = 2$$

These equations contain one dependent variable, one independent variable and two constants, so these three equations represent a line in four-dimensional space.

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### Step 2 of 3

Include the equation  $u = -1$  in the given set of equations and solve them for  $u$ ,  $v$ ,  $w$  and  $z$ .

$$u + v + w + z = 6$$

$$u + w + z = 4$$

$$u + w = 2$$

$$u = -1$$

$$\left. \begin{array}{l} u + w = 2 \\ u = -1 \end{array} \right\} \Rightarrow w = 3$$

$$\left. \begin{array}{l} u + w + z = 4 \\ w = 3 \\ u = -1 \end{array} \right\} \Rightarrow z = 2$$

$$\left. \begin{array}{l} u + v + w + z = 6 \\ z = 2 \\ w = 3 \\ u = -1 \end{array} \right\} \Rightarrow v = 2$$

Hence the solution to the set of equations is  $(-1, 2, 3, 2)$  which represents a point in a four-dimensional space.

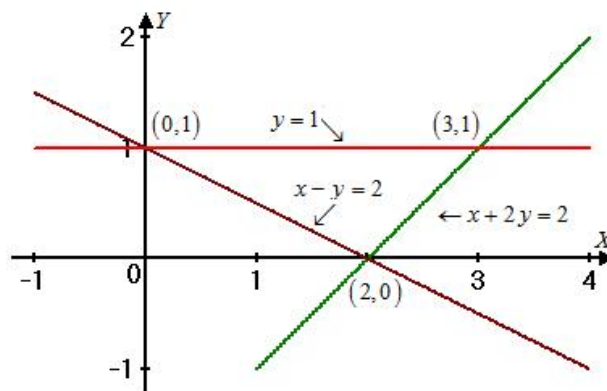
Therefore the intersection of the given four planes is the point  $(-1, 2, 3, 2)$ .



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CH1.2 3P**Step 1 of 3**

The system of equations is solvable if they have at least one point in common.

we sketch the equations  $x + 2y = 2$ ;  $x - y = 2$ ;  $y = 1$  along with the points of intersection.

[Provide feedback \(1\)](#)>  
CH1.2 5P**Step 2 of 3**

From the figure above, we note that every pair of lines have a point of intersection. But three lines put together have no common point. Therefore, the given equations have no solution.

[Provide feedback \(0\)](#)**Step 3 of 3**

Now, writing the given equations as  $x + 2y = 0$ ;  $x - y = 0$ ;  $y = 0$ , we observe that all these lines pass through origin. In other words, the origin  $(0, 0)$  is the solution for the given system when the right hand

## Step 1 of 2

We have to find two points on the line of intersection of the three planes  $t = 0$ ,  $z = 0$ , and  $x + y + z + t = 1$  in four dimensional space.

[Provide feedback \(0\)](#)

## Step 2 of 2

Substituting  $t = 0, z = 0$  in  $x + y + z + t = 1$ , we get

$$x + y = 1$$

By letting  $y = 0$  in  $x + y = 1$  gives  $x = 1$  then the solution becomes  $(1, 0, 0, 0)$

By letting  $x = 0$  in  $x + y = 1$  gives  $y = 1$  then the solution becomes  $(0, 1, 0, 0)$

Therefore the two solutions are  $(1, 0, 0, 0)$  and  $(0, 1, 0, 0)$

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## Step 1 of 1

We have to find a solution  $(u, v, w)$  for the following system

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix} = b$$

From this, we can observe that  $4(\text{column1}) + (-1)(\text{column2}) + (-1)(\text{column3}) = b$

So a solution for the equations is  $(u, v, w) = (4, -1, -1)$

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CH1.2 7P

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CH1.2 6P

## Step 1 of 5

Given system is

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = b$$

We have to give two examples for  $b$  in which the system is solvable, and the system is not solvable.

[Provide feedback \(0\)](#)>  
CH1.2 8P

## Step 2 of 5

Letting  $b = (3, 5, 8)$  then the column picture for the given equation is

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 8 \end{pmatrix}$$

In the above equation, the second row can be obtained by subtracting the first row from the third row, so the system has infinite solution; hence in this case the system is solvable when  $b = (3, 5, 8)$

[Provide feedback \(0\)](#)

## Step 3 of 5

Letting  $b = (1, 2, 3)$  then the column picture for the given equation is

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

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CH1.2 6P

**Step 1 of 5**

Given system is

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = b$$

We have to give two examples for  $b$  in which the system is solvable, and the system is not solvable.

[Provide feedback \(0\)](#)

>  
CH1.2 8P

**Step 2 of 5**

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**Step 3 of 5**

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$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

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In the above equation, the second row can be obtained by subtracting the first row from the third row, so the system has infinite solution; hence in this case the system is solvable when  $b = (1, 2, 3)$

[Provide feedback \(0\)](#)

#### Step 4 of 5

Letting  $b = (3, 5, 7)$ , then the column picture is

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$$

In the above equation, the left-side of the second row can be obtained by subtracting the first row from the third row, but not on the right-side (since  $7 - 3 \neq 5$ ) so the system has no solution; hence in this case the system is not solvable when  $b = (3, 5, 7)$

[Provide feedback \(0\)](#)

#### Step 5 of 5

Letting  $b = (1, 2, 2)$ , then the column picture is

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

In the above equation, the left-side of the second row can be obtained by subtracting the first row from the third row, but not on the right-side (since  $2 - 1 \neq 2$ ) so the system has no solution; hence in this case the system is not solvable when  $b = (1, 2, 2)$


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CH1.2 7P

## Step 1 of 2

Given equations are

$$u + v + w = 2 \quad \dots (1)$$

$$u + 2v + 3w = 1 \quad \dots (2)$$

$$v + 2w = 0 \quad \dots (3)$$

$$(1) + (3) - (2) \Rightarrow 0 = 1$$

an absurdity  $0 = 1$  occurred

So, the system is singular or inconsistent.

[Provide feedback \(0\)](#)

 >  
CH1.2 9P

## Step 2 of 2

To make the system non singular or possess a solution, we change (3) as  $v + 2w = -1$   
Then  $(1) + (3) - (2) \Rightarrow 0 = 0$  and so, there will be infinitely many solutions to the system.

Now, the system can be written in the matrix notation as

$$u \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + v \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Replacing  $u = 4$ ,  $v = -3$  and  $w = 1$ , we see that this matrix equation is satisfied.

$$4 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (-3) \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

therefore the solution is  $\boxed{b = (4, -3, 1)}$

[Provide feedback \(2\)](#)


  
CH1.2 8P

Given system is

$$u \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + v \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = b$$

We have to show that the three columns on the left lie on the same plane by expressing the third column as a combination of the first two, and we have to find the all solutions  $(u, v, w)$  if  $b = (0, 0, 0)$

[Provide feedback \(0\)](#)

  
CH1.2 10P
**Step 2 of 3**

By performing, 2 times (column 2) - column 1,

$$2 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

= 3rd column

Hence the three columns lie in the same plane.

[Provide feedback \(1\)](#)
**Step 3 of 3**If  $b = (0, 0, 0)$  then

$$u \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + v \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Then by letting  $(u, v, w) = (k, -2k, k)$ , then it become the solution for the system.





CH1.2 9P

## Step 1 of 2

Given points are  $(0, y_1), (1, y_2), (2, y_3)$ , we have to find that under what conditions on  $y_1, y_2, y_3$  do the given points lie on a straight line.

Three points are said to lie on the same line if the area of the triangle formed with these three points is zero.

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CH1.2 11P

## Step 2 of 2

The area of the triangle from the analytical geometry gives,

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$$\Rightarrow \frac{1}{2} |0(y_2 - y_3) + 1(y_3 - y_1) + 2(y_1 - y_2)| = 0$$

$$\Rightarrow |0 + y_3 - y_1 + 2y_1 - 2y_2| = 0$$

$$\Rightarrow y_3 + y_1 - 2y_2 = 0$$

$$\Rightarrow 2y_2 = y_1 + y_3$$

So the relation between  $y_1, y_2, y_3$  is  $2y_2 = y_1 + y_3$

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CH1.2 10P

## Step 1 of 2

Given equations are

$$ax + 2y = 0$$

$$2x + ay = 0$$

We have to find that which values of  $a$ , there is a whole line of solutions.[Provide feedback \(0\)](#)>  
CH1.2 12P

## Step 2 of 2

For  $a = 2$ , from the given equations we have only one equation  $x + y = 0$ , this is a straight line passing through the origin. And for  $a = -2$ , from the given equations we have only one equation  $x - y = 0$ , this is also a straight line passing through the origin.

Therefore for  $a = 2, -2$  the given system has a whole line of solutions.For all other values of  $a$ , the given system has the only solution  $x = y = 0$ [Provide feedback \(0\)](#)

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CH1.2 11P

## Step 1 of 3

Given that an equation of a line is  $x+4y=7$ , we have to find the equation for the parallel line through  $x=0, y=0$ , also we have to find the equation of the another line that meets the first at  $x=3, y=1$

[Provide feedback \(0\)](#)>  
CH1.2 13P

## Step 2 of 3

Let the equation of the line parallel to the line  $x+4y=7$  be  $x+4y=k$ , but it passing through the point  $(0,0)$ .

So  $k=0$ , so the required parallel line is  $x+4y=0$

[Provide feedback \(0\)](#)

## Step 3 of 3

And given that another line meet the  $x$ -axis at  $x=3$  and meet the  $y$ -axis at  $y=1$  and which intersects the line  $x+4y=7$

So the required line is

$$\frac{x}{3} + \frac{y}{1} = 1$$

$$\Rightarrow x+3y=3$$

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CH1.2 12P

## Step 1 of 3

We have to draw the row and column pictures for the equations:

$$x - 2y = 0 \quad \dots (1)$$

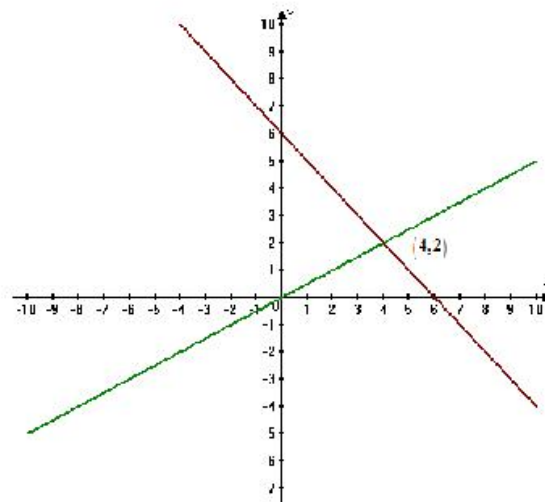
$$x + y = 6 \quad \dots (2)$$

By solving (1) and (2), we will get the point of intersection as  $(4, 2)$

[Provide feedback \(0\)](#)>  
CH1.2 14P

## Step 2 of 3

Row picture of the equations in the plane as shown as follows:

[Provide feedback \(0\)](#)

## Step 3 of 3

The column picture for the equations as follows:

CH1.2 12P

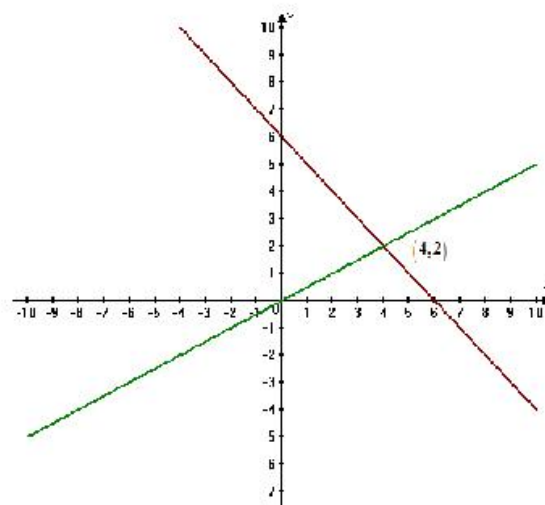
By solving (1) and (2), we will get the point of intersection as  $(4, 2)$

[Provide feedback \(0\)](#)

CH1.2 14P

**Step 2 of 3**

Row picture of the equations in the plane as shown as follows:

[Provide feedback \(0\)](#)**Step 3 of 3**

The column picture for the equations as follows:

$$x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

By performing  $4(\text{first column}) + 2(\text{second column}) = (0, 6)$ , we will get the solution as

$$(x, y) = (4, 2)$$

<  
CH1.2 13P

## Step 1 of 3

We have to find that whether the solutions lie on a line or a plane for two linear equations in three unknowns  $x, y, z$ .

For example, consider the system

$$x + y + z = 2$$

$$x - 2z = 3$$

[Provide feedback \(0\)](#)>  
CH1.2 15P

## Step 2 of 3

For the solution, put  $z = k$

$$\Rightarrow x = 3 + 2k$$

And

$$y = 2 - x - z$$

$$= 2 - 3 - 2k - k$$

$$= -1 - 3k$$

[Provide feedback \(0\)](#)

## Step 3 of 3

Therefore the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 + 2k \\ -1 - 3k \\ k \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} + k \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

This is a linear solution, so it lies on a straight line.

[Provide feedback \(0\)](#)

&lt; CH1.2 14P

We have to fill in the blanks for the following question:  
 For four linear equations in two unknowns  $x$  and  $y$ , the row picture shows four \_\_\_\_\_.  
 The column picture is in \_\_\_\_\_ dimensional space. The equations have no solution unless the vector on the right-hand side is a combination of \_\_\_\_\_.

[Provide feedback \(0\)](#)

&gt; CH1.2 16P

#### Step 2 of 4

Let us take the required system as

$$ax + by = k_1$$

$$cx + dy = k_2$$

$$ex + fy = k_3$$

$$gx + hy = k_4$$

From this, it is clear that the row picture shows four lines.

[Provide feedback \(0\)](#)

#### Step 3 of 4

The column picture for the system is as follows:

$$\begin{bmatrix} a \\ c \\ e \\ g \end{bmatrix} x + \begin{bmatrix} b \\ d \\ f \\ h \end{bmatrix} y = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}$$

From this, four dimensional vectors form the column space.

[Provide feedback \(0\)](#)

#### Step 4 of 4

And from the above equation it is clear that, the equations have no solution unless the vector on the right-hand side is a combination of the two columns.

<  
CH1.2 15P**Step 1 of 6**

Given that the equations of two planes are  $x + y + 3z = 6$  and  $x - y + z = 4$

We have to find a point with  $z = 2$  on the intersection line of the given planes; we have to find a point with  $z = 0$  on the intersection line of the given planes, and we have to find a third point halfway between these two.

[Provide feedback \(0\)](#)>  
CH1.2 17P**Step 2 of 6**

When  $z = 2$  the equations becomes,

$$x + y = 0$$

$$x - y = 2$$

The column vector of the above equations is

$$x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

[Provide feedback \(0\)](#)**Step 3 of 6**

The linear combination for the vectors is  $1(\text{first column}) + (-1)(\text{second column})$  that gives right hand side vector and for taking  $z = 2$ , the required point is  $(1, -1, 2)$

[Provide feedback \(0\)](#)**Step 4 of 6**

When  $z = 0$  the equations becomes,

$$x + y = 6$$

$$x - y = 4$$

The column vector of the above equations is



[Provide feedback \(0\)](#)**Step 3 of 6**

The linear combination for the vectors is  $1(\text{first column}) + (-1)(\text{second column})$  that gives right hand side vector and for taking  $z = 2$ , the required point is  $(1, -1, 2)$

[Provide feedback \(0\)](#)**Step 4 of 6**

When  $z = 0$  the equations becomes,

$$x + y = 6$$

$$x - y = 4$$

The column vector of the above equations is

$$x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

[Provide feedback \(0\)](#)**Step 5 of 6**

The linear combination for the vectors is  $5(\text{first column}) + 1(\text{second column})$  that gives right hand side vector and by taking  $z = 0$ , the required point is  $(5, 1, 0)$

[Provide feedback \(0\)](#)**Step 6 of 6**

The point that is half way between  $(1, -1, 2)$  and  $(5, 1, 0)$  is  $(3, 0, 1)$

Since it is the mid point of the points  $(1, -1, 2)$  and  $(5, 1, 0)$



CH1.2 16P

**Step 1 of 6**

Given that the first of these equations plus the second equals to the third:

$$x + y + z = 2$$

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 5$$

And given that the first two planes meet along a line. If  $x, y, z$  satisfy the first two equations then we have to find that the plane in which they lie. And we have to find the three solutions of the infinite number of solutions of the given system.

[Provide feedback \(0\)](#)

CH1.2 18P

**Step 2 of 6**

Form the given planes; the third plane occurs by the sum of the first two planes, so if  $x, y, z$  satisfy the first two equations then they will lie on the third plane.

[Provide feedback \(0\)](#)

**Step 3 of 6**

The given three planes can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Apply  $R_3 \rightarrow R_3 - (R_1 + R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

[Provide feedback \(0\)](#)

[Provide feedback \(0\)](#)**Step 2 of 6**

Form the given planes; the third plane occurs by the sum of the first two planes, so if  $x, y, z$  satisfy the first two equations then they will lie on the third plane.

[Provide feedback \(0\)](#)**Step 3 of 6**

The given three planes can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Apply  $R_3 \rightarrow R_3 - (R_1 + R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

[Provide feedback \(0\)](#)**Step 4 of 6**

Apply  $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Apply  $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

Apply  $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

[Provide feedback \(0\)](#)

#### Step 5 of 6

$$\Rightarrow x + z = 1$$

$$y = 1$$

Put  $z = k$

$$\Rightarrow x = 1 - k$$

[Provide feedback \(0\)](#)

#### Step 6 of 6

Therefore the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-k \\ 1 \\ k \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

For  $k = 1$ ,  $(x, y, z) = (0, 1, 1)$

Therefore the three solution of the given equations are  $(1, 1, 0), (-1, 0, 1), (0, 1, 1)$

[Provide feedback \(1\)](#)

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CH1.2 17P

## Step 1 of 2

Given that the equations of three planes:

$$x + y + z = 2$$

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 9$$

And given that the first two planes meet along a line.

We have to explain why the three equations have no solution.

[Provide feedback \(0\)](#)>  
CH1.2 19P

## Step 2 of 2

From the equations, we observe that the left hand side of the third equation occurs by adding the left hand side of the first two equations where as the right hand side is not. So the given equations have no solution, and hence third plane does not intersect the intersection line  $L$  of the first two planes.

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CH1.2 18P

## Step 1 of 7

Given that the equations of three planes:

$$x + y + z = 2$$

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 5$$

We have to find that why the three columns form singular case, and we have to find two combinations of the columns that give  $b = (2, 3, 5)$ , and finally we have to find the value of  $c$  such that  $b = (4, 6, c)$

[Provide feedback \(0\)](#)

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CH1.2 20P

## Step 2 of 7

The three columns of the above equations are

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

The first and third columns are equal, so on column operation  $C_3 \rightarrow C_3 - C_1$ , we will get a zero column, and hence three columns will form the singular case.

[Provide feedback \(0\)](#)

## Step 3 of 7

The given three planes can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Apply  $R_3 \rightarrow R_3 - (R_1 + R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

## Step 3 of 7

The given three planes can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Apply  $R_3 \rightarrow R_3 - (R_1 + R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

[Provide feedback \(0\)](#)

## Step 4 of 7

Apply  $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Apply  $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

[Provide feedback \(0\)](#)

## Step 5 of 7

$$\Rightarrow x + z = 1$$

$$y = 1$$

$$\text{Put } z = k$$

$$\Rightarrow x = 1 - k$$

## Step 5 of 7

$$\Rightarrow x + z = 1$$

$$y = 1$$

$$\text{Put } z = k$$

$$\Rightarrow x = 1 - k$$

[Provide feedback \(0\)](#)

## Step 6 of 7

Therefore the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-k \\ 1 \\ k \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

From the above, the two combinations of the columns that give  $b = (2, 3, 5)$  are

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } k \text{ is any real number}$$

[Provide feedback \(0\)](#)

## Step 7 of 7

If we multiply  $b = (2, 3, 5)$  with 2, then we get  $(4, 6, 10)$ , compare this vector with

$$b = (4, 6, c), \text{ we get } \boxed{c = 10}$$

[Provide feedback \(0\)](#)





CH1.2 19P

**Step 1 of 3**

We have to find that at which the four planes in four dimensional space meet. We have to find that what combination of  $(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0), (1, 1, 1, 1)$  produce  $b = (3, 3, 3, 2)$  and also we have to find that what four equations for  $x, y, z, t$  are we solving.

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CH1.2 21P

**Step 2 of 3**

Let

$$x \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 2 \end{pmatrix}$$

From the system, we can observe that

$0(\text{column1}) + 0(\text{column1}) + 1(\text{column3}) + 2(\text{column4})$  gives the right-hand side vector.

[Provide feedback \(0\)](#)**Step 3 of 3**

So the solution of the above planes is  $(0, 0, 1, 2)$ , so the four planes meet at there.

And from the above equation, we can observe that the equations for solving this is

$$\begin{array}{l} x + y + z + t = 3 \\ y + z + t = 3 \\ z + t = 3 \\ t = 2 \end{array}$$

[Provide feedback \(0\)](#)


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CH1.2 20P

## Step 1 of 6

When equation 1 is added to the equation 2, then we have to find that which of the following are changed.  
The planes in the row picture, the column picture, the coefficient matrix and the solution.

[Provide feedback \(0\)](#)

 >  
CH1.2 22P

## Step 2 of 6

For example, a system of four planes is

$$x + y + z + t = 3$$

$$y + z + t = 3$$

$$z + t = 3$$

$$t = 2$$

The column picture for this system is

$$x \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 2 \end{pmatrix}$$

[Provide feedback \(0\)](#)

## Step 3 of 6

The coefficient matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the system, we can observe that

$0(\text{column1}) + 0(\text{column1}) + 1(\text{column3}) + 2(\text{column4})$  gives the right-hand side vector.

So the solution of the above planes is  $(0, 0, 1, 2)$

## Step 4 of 6

When equation 1 is added to equation 2 gives, we have the following system

$$x + 2y + 2z + 2t = 6$$

$$z + t = 3$$

$$t = 2$$

Column picture is

$$x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

[Provide feedback \(0\)](#)

## Step 5 of 6

The coefficient matrix is  $\begin{pmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

From the column picture, we observe that

$0(\text{column } 1) + 0(\text{column } 2) + 1(\text{column } 3) + 2(\text{column } 4)$  gives the right-hand side vector.

The solution matrix is  $(0, 0, 1, 2)$ .

[Provide feedback \(0\)](#)

## Step 6 of 6

In the first system we have 4 planes where as in the second system we have 3 planes, so the planes in the row picture changed.

In the first system we have 4 four dimensional columns where as in the second system we have 4 three dimensional columns, so the column picture changed.

The coefficient matrix for the two system is changed.

And the solution  $(0, 0, 1, 2)$  not changed.

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CH1.2 21P

## Step 1 of 3

If  $(a, b)$  is a multiple of  $(c, d)$  with  $abcd \neq 0$ , then we have to show that  $(a, c)$  is a multiple of  $(b, d)$ .

If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  has dependent rows, then we have to show that it has dependent columns.

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CH1.2 23P

## Step 2 of 3

Let  $(a, b) = (2, 4)$  and  $(c, d) = (1, 2)$

Then it is clear that  $(a, b)$  is a multiple of  $(c, d)$ , since 2 times of  $(c, d)$  is  $(a, b)$

And  $(a, c) = (2, 1), (b, d) = (4, 2)$

From this, it is clear that  $(a, c) = \frac{1}{2}(b, d)$

So  $(a, c)$  is a multiple of  $(b, d)$ .

[Provide feedback \(0\)](#)

## Step 3 of 3

Now the matrix  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$

Here the first row is the two times of the second row, so the rows are dependent and, and from this we have the second column is the two times of the first column.

So we conclude that if  $A$  has dependent rows then it has dependent columns also.

[Provide feedback \(0\)](#)

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CH1.2 22P

## Step 1 of 2

Given system of equations is

$$6u + 7v + 8w = 8$$

$$4u + 5v + 9w = 9$$

$$2u - 2v + 7w = 7$$

We have to find the solution for this system in the column form.

[Provide feedback \(0\)](#)>  
CH1.3

## Step 2 of 2

The column picture for this system is as follows:

$$u \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} + v \begin{bmatrix} 7 \\ 5 \\ 2 \end{bmatrix} + w \begin{bmatrix} 8 \\ 9 \\ 7 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 7 \end{bmatrix} = b$$

We can observe that the coefficient of  $w$  and  $b$  are the same, so we have

$$0(\text{column1}) + 0(\text{column2}) + 1(\text{column3}) = b$$

So the solution for the given system is  $\boxed{(0, 0, 1)}$ [Provide feedback \(0\)](#)

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