CH6.1

Step 1 of 3

Given the matrix $A=\begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}$

If A is positive definite, then $a>0$ and

$$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0$$

$$\Rightarrow a^2-4>0$$

$$\Rightarrow a>2$$

Therefore A is positive definite if $\boxed{a>2}$.

Provide feedback (0)

Step 2 of 3

Given matrix $B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{pmatrix}$

$$|B_{11}|=1>0$$

$$|B_{22}|=\begin{vmatrix} 1 & 2 \\ 2 & b \end{vmatrix}$$

$$\Rightarrow b-4>0$$

$$\Rightarrow b>4$$

$$\Rightarrow b=5(\text{say})$$

Provide feedback (0)

Step 3 of 3

$$B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \\ 4 & 8 & 7 \end{pmatrix}$$

CH6.2 2P

Provide feedback (0)

Step 3 of 3

$$B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \\ 4 & 8 & 7 \end{pmatrix}$$

$$\Rightarrow |B| = 1(35-64) - 2(14-32) + 4(16-20)$$

$$\Rightarrow |B| = -29 + 36 - 16$$

$$\Rightarrow |B| = -9 < 0$$

So the matrix B is $\boxed{\text{never positive definite}}$.

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CH6.2 1P

Step 1 of 4

Given matrix, $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

$$|A_{11}| = 2 > 0$$

$$\begin{aligned} |A_{22}| &= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \\ &= 4-1 \\ &= 3 > 0 \end{aligned}$$

$$\begin{aligned} |A_{33}| &= |A| \\ &= 2(4-1)+1(-2-1)-(1+2) \\ &= 6-3-3 \\ &= 0 \end{aligned}$$

So A is $\boxed{\text{not positive definite}}$.

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CH6.2 3P

Step 2 of 4

Given matrix, $B = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$

$$|B_{11}| = 2 > 0$$

$$\begin{aligned} |B_{22}| &= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \\ &= 4-1 \\ &= 3 > 0 \end{aligned}$$

$$\begin{aligned} |B_{33}| &= |B| \\ &= 2(4-1)+1(-2+1)-(-1+2) \\ &= 6-1-1 \\ &= 4 > 0 \end{aligned}$$

$$= 6-3-3$$
$$= 0$$

So A is $\boxed{\text{not positive definite}}$.

Provide feedback (0)

Step 2 of 4

Given matrix, $B = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$

$$|B_{11}| = 2 > 0$$

$$|B_{22}| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$
$$= 4-1$$
$$= 3 > 0$$

$$|B_{33}| = |B|$$
$$= 2(4-1)+1(-2+1)-(-1+2)$$
$$= 6-1-1$$
$$= 4 > 0$$

Thus $|B_{11}| > 0,\ |B_{22}| > 0,\ |B_{33}| > 0.$

Therefore, B $\boxed{\text{is positive definite}}$.

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Step 3 of 4

Given matrix, $C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}^2$

$$C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

Therefore, B $\boxed{\text{is positive definite}}$

Provide feedback (0)

Step 3 of 4

Given matrix, $C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}^2$

$$C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix}$$

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Step 4 of 4

Now calculate determinant of left sub matrices

$|C_{11}| = 5 > 0$

$$|C_{22}| = \begin{vmatrix} 5 & 2 \\ 2 & 2 \end{vmatrix}$$

$$= 10 - 4$$

$$= 6 > 0$$

$$|C_{33}| = |C|$$

$$= 5(6) - 2(8) + 1(2)$$

$$= 30 - 16 + 2$$

$$= 16 > 0$$

Thus $|C_{11}| > 0$, $|C_{22}| > 0$, $|C_{33}| > 0$.

Therefore C $\boxed{\text{is positive definite}}$.

CH6.2 2P

Step 1 of 2

Given matrix is $A=\begin{pmatrix}1 & b & -b\\ b & 1 & b\\ -b & b & 1\end{pmatrix}$ with $|b|<1$.

$$\begin{aligned}|A| &= 1\left(1-b^2\right)-b\left(b+b^2\right)-b\left(b^2+b\right)\\ &= 1-b^2-b^2-b^3-b^3-b^2\\ &= -2b^3-3b^2+1\end{aligned}$$

Therefore, the determinant of matrix A is $-2b^3-3b^2+1$

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Step 2 of 2

Let $-2b^3-3b^2+1<0$

Solving this using CAS

We get $b>\frac{1}{2}$, so let $b=\frac{2}{3}$.

Substitute this value in the given matrix.

So,

$$A=\begin{pmatrix}1 & \frac{2}{3} & -\frac{2}{3}\\ \frac{2}{3} & 1 & \frac{2}{3}\\ -\frac{2}{3} & \frac{2}{3} & 1\end{pmatrix}$$

Then the required matrix is $\boxed{A=\begin{pmatrix}1 & \frac{2}{3} & -\frac{2}{3}\\ \frac{2}{3} & 1 & \frac{2}{3}\\ -\frac{2}{3} & \frac{2}{3} & 1\end{pmatrix}}$.

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CH6.2 4P

CH6.2 3P

Step 1 of 2

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be Eigen values of A.

A is positive definite.

$\Rightarrow \lambda_1 > 0, \lambda_2 > 0, \ldots, \lambda_n > 0.$

We know that if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are Eigen values of A then

$\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2$ are Eigen values of A^2.

Clearly $\lambda_1^2 > 0, \lambda_2^2 > 0, \ldots, \lambda_n^2 > 0.$

So the Eigen values of A^2 are all positive.

Thus A^2 $\boxed{\text{is also positive definite}}$.

Provide feedback (0)

Step 2 of 2

We know that if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are Eigen values of A, then

$\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots, \frac{1}{\lambda_n}$ are Eigen values of A^{-1}.

Clearly $\frac{1}{\lambda_1} > 0, \frac{1}{\lambda_2} > 0, \ldots, \frac{1}{\lambda_n} > 0.$

So the Eigen values of A^{-1} are also positive.

Hence the matrix A^{-1} $\boxed{\text{is also positive definite}}$.

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CH6.2 5P

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CH6.2 4P

Step 1 of 2

If A is positive definite then,

$\Rightarrow x^T Ax > 0$ for any $x \neq 0$ ..... (1)

If B is positive definite then,

$\Rightarrow x^T Bx > 0$ for any $x \neq 0$ ..... (2)

Provide feedback (0)

Step 2 of 2

Now for $x \neq 0$,

$$x^T (A+B) x = x^T Ax + x^T Bx$$

$$> 0 \qquad \text{(using (1) and (2))}$$

Thus $x^T (A+B) x > 0$ for $x \neq 0$.

Therefore, $A+B$ $\boxed{\text{is also positive definite}}$.

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CH6.2 6P

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Chapter: CH6.2 Problem: 6P

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CH6.2 5P

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CH6.2 7P

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CH6.2 6P

Step 1 of 10

Consider $A = Q\Lambda Q^T$ is symmetric positive definite and then $R = Q\sqrt{\Lambda}Q^T$ is its symmetric positive definite square root.

Since $R = Q\sqrt{\Lambda}Q^T$ is positive definite, then for all $x \neq 0$, we have

$$x^T Rx > 0$$

We have to show that *R* have positive eigenvalues, so consider:

$$\begin{aligned} Rx &= \lambda x \\ x^T Rx &= x^T \lambda x \\ x^T Rx &= \lambda \|x\|^2 \end{aligned}$$

We know that $x^T Rx > 0$, so $\lambda \|x\|^2 > 0$, thus $\lambda > 0$.

Therefore, *R* have positive eigenvalues, $\boxed{\lambda > 0}$.

Provide feedback (0)

CH6.2 8P

Step 2 of 10

Consider $A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$.

Now to find *R*, we have to first find *Q*.

Therefore, R have positive eigenvalues, $\boxed{\lambda > 0}$.

Provide feedback (0)

Step 2 of 10

Consider $A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$.

Now to find R, we have to first find Q.

Using $l_{21} = \frac{6}{10}$ we have

$$\begin{aligned} A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} &\to \begin{bmatrix} 10 & 6 \\ 6 - \left(\frac{6}{10} \cdot 10\right) & 10 - \left(\frac{6}{10} \cdot 6\right) \end{bmatrix} \\ &\to \begin{bmatrix} 10 & 6 \\ 0 & \frac{32}{5} \end{bmatrix} \\ &= U \end{aligned}$$

Provide feedback (0)

Step 3 of 10

Here, $U = \begin{bmatrix} 10 & 6 \\ 0 & \frac{32}{5} \end{bmatrix}$ and $L = \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix}$, so we get

Step 3 of 10

Here, $U = \begin{bmatrix} 10 & 6 \\ 0 & \frac{32}{5} \end{bmatrix}$ and $L = \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix}$, so we get

$$\begin{aligned} LDU &= \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} \begin{bmatrix} 1 & \frac{6}{10} \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix}^T \end{aligned}$$

Provide feedback (0)

Step 4 of 10

Comparing LDU with $Q\sqrt{\Lambda}Q^T$, we have

$$Q = \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix} = L$$

And

$$\Lambda = \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} = D$$

$$\begin{bmatrix} \frac{}{10} & 1 \end{bmatrix}\begin{bmatrix} 0 & \frac{}{5} \end{bmatrix}\begin{bmatrix} \frac{}{10} & 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 10

Comparing LDU with $Q\sqrt{\Lambda}Q^T$, we have

$$Q = \begin{bmatrix} 1 & 0 \\ \frac{6}{10} & 1 \end{bmatrix} = L$$

And

$$\Lambda = \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} = D$$

And

$$Q^T = \begin{bmatrix} 1 & \frac{6}{10} \\ 0 & 1 \end{bmatrix} = L^T$$

We know that $A = Q\Lambda Q^T$ is symmetric positive definite and $R = Q\sqrt{\Lambda}Q^T$ symmetric positive definite square root, so we take $R = \sqrt{\Lambda}Q^T$.

$$\begin{bmatrix} 1 & \frac{6}{} \end{bmatrix} \quad \begin{bmatrix} \sqrt{10} & 0 \end{bmatrix}$$

We know that $A = Q\Lambda Q^T$ is symmetric positive definite and $R = Q\sqrt{\Lambda}Q^T$ symmetric positive definite square root, so we take $R = \sqrt{\Lambda}Q^T$.

To compute *R*, use $Q^T = \begin{bmatrix} 1 & \frac{6}{10} \\ 0 & 1 \end{bmatrix}$ and $\sqrt{\Lambda} = \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{\frac{32}{5}} \end{bmatrix}$.

$$\begin{aligned} R &= \sqrt{\Lambda}Q^T \\ &= \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{\frac{32}{5}} \end{bmatrix} \begin{bmatrix} 1 & \frac{6}{10} \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{10} & \frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix} \end{aligned}$$

Therefore, $R = \boxed{\begin{bmatrix} \sqrt{10} & \frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix}}$.

Provide feedback (0)

Step 5 of 10

We have to verify that $R^2 = A$.

$R^2 = R^T R$

Step 5 of 10

We have to verify that $R^2 = A$.

$$\begin{aligned} R^2 &= R^T R \\ &= \begin{bmatrix} \sqrt{10} & 0 \\ \frac{3}{5}\left(\sqrt{10}\right) & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix} \begin{bmatrix} \sqrt{10} & \frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix} \\ &= \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \\ &= A \end{aligned}$$

Provide feedback (0)

Step 6 of 10

Therefore, $\boxed{R^2 = A}$.

Provide feedback (0)

Step 7 of 10

Consider $A = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}$

Now to find *R*, we have to again find *Q*.

Using $l_{21} = \frac{-6}{10}$ we have

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Step 7 of 10

Consider $A=\begin{bmatrix}10 & -6\\ -6 & 10\end{bmatrix}$

Now to find R, we have to again find Q.

Using $l_{21}=\frac{-6}{10}$ we have

$$A=\begin{bmatrix}10 & -6\\ -6 & 10\end{bmatrix}\to\begin{bmatrix}10 & -6\\ -6-\left(-\frac{6}{10}\cdot 10\right) & 10-\left(-\frac{6}{10}\cdot(-6)\right)\end{bmatrix}$$
$$\to\begin{bmatrix}10 & -6\\ 0 & \frac{32}{5}\end{bmatrix}$$
$$=U$$

Provide feedback (0)

Step 8 of 10

Here, $U=\begin{bmatrix}10 & -6\\ 0 & \frac{32}{5}\end{bmatrix}$ and $L=\begin{bmatrix}1 & 0\\ -\frac{6}{10} & 1\end{bmatrix}$, so we get

$$LDU=\begin{bmatrix}1 & 0\\ 6 & 1\end{bmatrix}\begin{bmatrix}10 & 0\\ 0 & 32\end{bmatrix}\begin{bmatrix}1 & -\frac{6}{10}\end{bmatrix}$$

$$\rightarrow \begin{bmatrix} & \\ 0 & \frac{32}{5} \end{bmatrix}$$
$$= U$$

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Step 8 of 10

Here, $U = \begin{bmatrix} 10 & -6 \\ 0 & \frac{32}{5} \end{bmatrix}$ and $L = \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix}$, so we get

$$\begin{aligned} LDU &= \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} \begin{bmatrix} 1 & -\frac{6}{10} \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix}^T \end{aligned}$$

Now comparing LDU with $Q\sqrt{\Lambda}Q^T$, we have

$$Q = \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix} = L$$

And

$$\Lambda = \begin{bmatrix} 10 & 0 \\ 0 & 32 \end{bmatrix} = D$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} \begin{bmatrix} 1 & -\frac{6}{10} \\ 0 & 1 \end{bmatrix}$$

Now comparing LDU with $Q\sqrt{\Lambda}Q^T$, we have

$$Q = \begin{bmatrix} 1 & 0 \\ -\frac{6}{10} & 1 \end{bmatrix} = L$$

And

$$\Lambda = \begin{bmatrix} 10 & 0 \\ 0 & \frac{32}{5} \end{bmatrix} = D$$

$$Q^T = \begin{bmatrix} 1 & -\frac{6}{10} \\ 0 & 1 \end{bmatrix} = L^T$$

Provide feedback (0)

Step 9 of 10

We know that $A = Q\Lambda Q^T$ is symmetric positive definite and $R = Q\sqrt{\Lambda}Q^T$ symmetric positive definite square root, so we take $R = \sqrt{\Lambda}Q^T$.

To compute R, use $Q^T = \begin{bmatrix} 1 & -\frac{6}{10} \end{bmatrix}$ and $\sqrt{\Lambda} = \begin{bmatrix} \sqrt{10} & 0 \end{bmatrix}$

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Step 9 of 10

We know that $A = Q\Lambda Q^T$ is symmetric positive definite and $R = Q\sqrt{\Lambda}Q^T$ symmetric positive definite square root, so we take $R = \sqrt{\Lambda}Q^T$.

To compute R, use $Q^T = \begin{bmatrix} 1 & -\frac{6}{10} \\ 0 & 1 \end{bmatrix}$ and $\sqrt{\Lambda} = \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{\frac{32}{5}} \end{bmatrix}$.

$$\begin{aligned} R &= \sqrt{\Lambda}Q^T \\ &= \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{\frac{32}{5}} \end{bmatrix} \begin{bmatrix} 1 & -\frac{6}{10} \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{10} & -\frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix} \end{aligned}$$

Therefore, $R = \boxed{\begin{bmatrix} \sqrt{10} & -\frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix}}$.

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Step 10 of 10

$$\begin{bmatrix} 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix}$$

Therefore, $R = \boxed{\begin{bmatrix} \sqrt{10} & -\frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix}}$.

Provide feedback (0)

Step 10 of 10

We have to verify that $R^2 = A$.

$$\begin{aligned} R^2 &= R^T R \\ &= \begin{bmatrix} \sqrt{10} & 0 \\ -\frac{3}{5}\left(\sqrt{10}\right) & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix} \begin{bmatrix} \sqrt{10} & -\frac{3}{5}\left(\sqrt{10}\right) \\ 0 & \frac{4}{5}\left(\sqrt{10}\right) \end{bmatrix} \\ &= \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix} \\ &= A \end{aligned}$$

Therefore, $\boxed{R^2 = A}$.

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CH6.2 7P

Step 1 of 2

Given that A is symmetric which means that

$\Rightarrow A^T = A.$

Given that A is positive definite which means that

$\Rightarrow x^T Ax > 0$ for $x \neq 0.$

So,

$$\begin{aligned} B^T &= \left(C^T AC\right)^T \\ &= C^T A^T \left(C^T\right)^T \qquad \left(\text{since } (AB)^T = B^T A^T\right) \\ &= C^T A^T C \qquad \left(\text{since } \left(C^T\right)^T = C\right) \\ &= B \end{aligned}$$

Therefore, B is symmetric matrix.

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Step 2 of 2

Now we need to prove that B is positive definite.

$$\begin{aligned} x^T Bx &= x^T \left(C^T AC\right) x \\ &= \left(x^T C^T\right) A(Cx) \\ &= (Cx)^T A(Cx) \\ &> 0 \qquad \text{(Since } A \text{ is positive definite)} \end{aligned}$$

Therefore, B is also positive definite.

Therefore, $\boxed{B \text{ is symmetric positive definite}}$.

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CH6.2 9P

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CH6.2 8P

Step 1 of 2

If $A = R^T R$, then we need to prove the generalized Schwarz inequality.

i.e. $\left|x^T Ay\right|^2 \le \left(x^T Ax\right)\left(y^T Ay\right)$.

So,

$$\begin{aligned} x^T Ay &= x^T \left(R^T R\right) y \\ &= \left(x^T R^T\right)\left(Ry\right) \\ &= \left(Rx\right)^T \left(Ry\right) \end{aligned}$$

Also, $\left\|\left(Rx\right)\right\|^T = \left\|\left(Rx\right)\right\|$.

Provide feedback (0)

Step 2 of 2

Therefore,

$$\begin{aligned} \left|x^T Ay\right|^2 &= \left|\left(Rx\right)^T \left(Ry\right)\right|^2 \\ &\le \left\|\left(Rx\right)\right\|^2 \left\|\left(Ry\right)\right\|^2 \quad \text{(by Schwarz inequality)} \\ &= \left(x^T R^T Rx\right)\left(y^T R^T Ry\right) \\ &= \left(x^T Ax\right)\left(y^T Ay\right) \end{aligned}$$

Thus, $\boxed{\left|x^T Ay\right|^2 = \left(x^T Ax\right)\left(y^T Ay\right)}$.

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CH6.2 10P

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CH6.2 9P

Step 1 of 2

Given ellipse is $u^2+4v^2=1$, which corresponds to the matrix $A=\begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}$.

Given ellipse can be re written as $1.(1.u+0.v)^2+4.(0.u+1.v)^2=1$.

So the Eigen values are 1 and 4.

$\begin{pmatrix}1\\0\end{pmatrix}$ is the Eigen vector corresponding to the Eigen value 1 and $\begin{pmatrix}0\\1\end{pmatrix}$ is the Eigen vector corresponding to the Eigen value 4.

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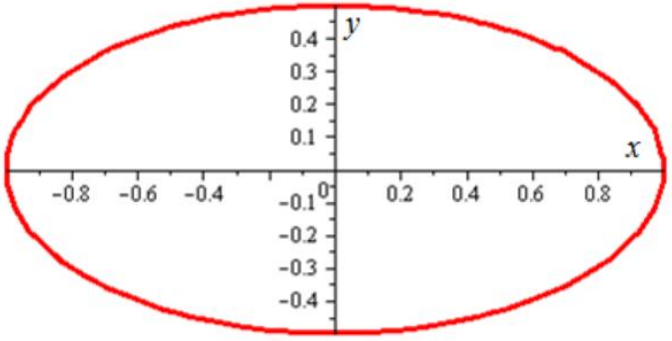
CH6.2 11P

Step 2 of 2

The graph of ellipse is as shown below.



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CH6.2 10P

Step 1 of 6

Given quadric is $3u^2 - 2\sqrt{2}uv + 2v^2 = 1$,

Corresponding matrix A is $A = \begin{pmatrix} 3 & -\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$.

First, we need to find the Eigen values of A.

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & -\sqrt{2} \\ -\sqrt{2} & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)(2-\lambda) - 2 = 0$

$\Rightarrow \lambda^2 - 5\lambda + 4 = 0$

$\Rightarrow \lambda = 1, 4.$

Thus the Eigen values are $\boxed{1 \text{ and } 4}$.

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CH6.2 12P

Step 2 of 6

If $\lambda = 1$, then $A - \lambda I = \begin{pmatrix} 2 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}$

Apply $R_2 \to R_2 + \frac{1}{\sqrt{2}} R_1$,

$$= \begin{pmatrix} 2 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}$$

Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$ be Eigen vector corresponding to $\lambda = 1$.

Then $(A - \lambda I)X = 0$

$$\begin{pmatrix} 2 & -\sqrt{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow 2x - \sqrt{2}y = 0$

Step 2 of 6

If $\lambda = 1$, then $A - \lambda I = \begin{pmatrix} 2 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}$

Apply $R_2 \to R_2 + \frac{1}{\sqrt{2}} R_1$,

$$= \begin{pmatrix} 2 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}$$

Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$ be Eigen vector corresponding to $\lambda = 1$.

Then $(A - \lambda I)X = 0$

$$\begin{pmatrix} 2 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow 2x - \sqrt{2}y = 0$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$$

Therefore, the Eigen vector corresponding to $\lambda = 1$ is $\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}}$.

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Step 3 of 6

If $\lambda = 4$ then $A - \lambda I = \begin{pmatrix} -1 & -\sqrt{2} \\ -\sqrt{2} & -2 \end{pmatrix}$

Apply $R_2 \to R_2 - \sqrt{2}R_1$

$$= \begin{pmatrix} -1 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}$$

Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$ be the Eigen vector corresponding to $\lambda = 4$.

Step 3 of 6

If $\lambda = 4$ then $A - \lambda I = \begin{pmatrix} -1 & -\sqrt{2} \\ -\sqrt{2} & -2 \end{pmatrix}$

Apply $R_2 \to R_2 - \sqrt{2}R_1$

$$= \begin{pmatrix} -1 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}$$

Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$ be the Eigen vector corresponding to $\lambda = 4$.

Then $(A - \lambda I)X = 0$

$$= \begin{pmatrix} -1 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow -x - \sqrt{2}y = 0$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \end{pmatrix}$$

Therefore, the Eigen vector corresponding to $\lambda = 4$ is $\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \end{pmatrix}}$.

Provide feedback (0)

Step 4 of 6

Now the unit Eigen vectors corresponding to $\lambda = 1$ is

$$\Rightarrow \frac{1}{\sqrt{1+2}}\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}$$

$$\begin{pmatrix} \\ -\frac{1}{2} \end{pmatrix}$$

Provide feedback (0)

Step 4 of 6

Now the unit Eigen vectors corresponding to $\lambda = 1$ is

$$\Rightarrow \frac{1}{\sqrt{1+2}}\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}$$

Unit Eigen vector corresponding to $\lambda = 4$ is

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{2}+\frac{1}{4}}}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$$

Now we can write the given to sum of squares as

$$1\left(\frac{1}{\sqrt{3}}u + \frac{\sqrt{2}}{\sqrt{3}}v\right)^2 + 4\left(\frac{\sqrt{2}}{\sqrt{3}}u - \frac{1}{\sqrt{3}}v\right)^2 = 1$$

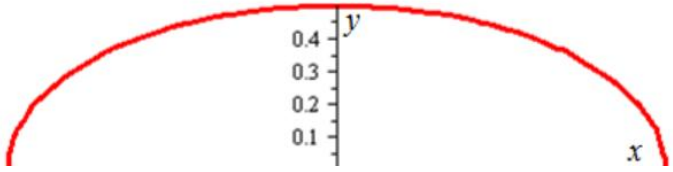
Provide feedback (0)

Step 5 of 6

The graph of the ellipse is as shown below.

Provide feedback (0)

Step 6 of 6



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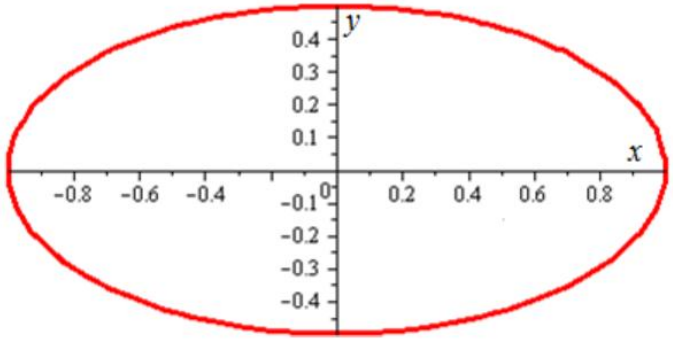
Step 5 of 6

The graph of the ellipse is as shown below.

Provide feedback (0)

Step 6 of 6



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Step 1 of 3

Suppose one or more eigenvalues of λ_1, λ_2, and λ_3 are zero.

Without loss of generality, let $\lambda_3 = 0$ and other eigenvalues be positive.

Then we get $\lambda_1 y_1^2 + \lambda_2 y_2^2 = 1$

This represents an elliptical cylinder in 3 dimensions. If we consider the plane, which contains the vectors y_1 and y_2, then the cross section of this cylinder with the plane is the ellipse, whose equation is given by $\lambda_1 y_1^2 + \lambda_2 y_2^2 = 1$.

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Step 2 of 3

Consider the case of two eigenvalues being zero and the third one being positive. Without loss of generality, let $\lambda_2 = 0$, $\lambda_3 = 0$, and $\lambda_1 > 0$.

This gives, $\lambda_1^2 = 1$.

Therefore, $\lambda_1 = \pm 1$.

This represents pair of two planes, perpendicular to the vector y_1, passing through it at a distance of ± 1 from the origin.

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Step 3 of 3

CH6.2 13P

Provide feedback (0)

Step 2 of 3

Consider the case of two eigenvalues being zero and the third one being positive. Without loss of generality, let $\lambda_2 = 0$, $\lambda_3 = 0$, and $\lambda_1 > 0$.

This gives, $\lambda_1^2 = 1$.

Therefore, $\lambda_1 = \pm 1$.

This represents pair of two planes, perpendicular to the vector y_1, passing through it at a distance of ± 1 from the origin.

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Step 3 of 3

Suppose we consider the case where all the three eigenvalues are zero. Then we get 0 = 1, which is impossible.

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CH6.2 12P

Step 1 of 5

Given that $(-A)$ is positive definite.

(i).

$(-A)$ is positive definite.

$\Rightarrow x^T(-A)x > 0$ for all $x \neq 0$.

$\Rightarrow x^T Ax < 0$ for all $x \neq 0$.

$\therefore$ A is negative definite if $\boxed{x^T Ax < 0}$ for all non zero vectors x.

Provide feedback (0)

Step 2 of 5

(ii).

$(-A)$ is positive definite.

$\Rightarrow$ All the Eigen values of $(-A)$ satisfies $\lambda_i > 0$.

So all the Eigen values of A satisfies $\lambda_i < 0$.

Provide feedback (0)

Step 3 of 5

(iii).

We know that $\det(-A) = (-1)^n \det A$

$(-A)$ is positive definite.

$\Rightarrow$ All the upper left sub matrices of $(-A)$

i.e. A_1, A_2 and A_3 have positive determinants.

That is $\det A_1 > 0, \det A_2 > 0$ and $\det A_3 > 0$.

$\det A_1^1 = (-1)^1 \det A_1 = -\det A_1$

$\Rightarrow \det A_1^1 < 0$ as $\det A_1 > 0$

$\det A_2^1 = (-1)^2 \det A_2 = \det A_2$

$\Rightarrow \det A_2^1 > 0$ as $\det A_2 > 0$

CH6.2 14P

Provide feedback (0)

Step 3 of 5

(iii).

We know that $\det(-A) = (-1)^n \det A$

$(-A)$ is positive definite.

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$\det A_1^1 = (-1)^1 \det A_1 = -\det A_1$

$\Rightarrow \det A_1^1 < 0$ as $\det A_1 > 0$

$\det A_2^1 = (-1)^2 \det A_2 = \det A_2$

$\Rightarrow \det A_2^1 > 0$ as $\det A_2 > 0$

$\det A_3^1 = (-1)^3 \det A_3 = -\det A_3$

$\Rightarrow \det A_3^1 < 0$ as $\det A_3 > 0$

Thus if A is negative definite then

$\det A_1^1 < 0$, $\det A_2^1 > 0$ and $\det A_3^1 < 0$.

Provide feedback (0)

Step 4 of 5

(iv).

$(-A)$ is positive definite.

$\Rightarrow$ All the pivots (without row exchanges) satisfies $d_k > 0$.

So if A is negative definite if all the pivots (without row exchanges) satisfies $d_k < 0$.

Provide feedback (0)

Step 5 of 5

i.e. A_1, A_2 and A_3 have positive determinants.

That is $\det A_1 > 0, \det A_2 > 0$ and $\det A_3 > 0$.

$\det A_1^1 = (-1)^1 \det A_1 = -\det A_1$

$\Rightarrow \det A_1^1 < 0$ as $\det A_1 > 0$

$\det A_2^1 = (-1)^2 \det A_2 = \det A_2$

$\Rightarrow \det A_2^1 > 0$ as $\det A_2 > 0$

$\det A_3^1 = (-1)^3 \det A_3 = -\det A_3$

$\Rightarrow \det A_3^1 < 0$ as $\det A_3 > 0$

Thus if A is negative definite then

$\det A_1^1 < 0$, $\det A_2^1 > 0$ and $\det A_3^1 < 0$.

Provide feedback (0)

Step 4 of 5

(iv).

$(-A)$ is positive definite.

$\Rightarrow$ All the pivots (without row exchanges) satisfies $d_k > 0$.

So if A is negative definite if all the pivots (without row exchanges) satisfies $d_k < 0$.

Provide feedback (0)

Step 5 of 5

(v).

$(-A)$ is positive definite.

$\Rightarrow$ There is a matrix R with independent columns such that $-A = R^T R$.

There is a matrix R with independent columns such that $A = -R^T R$.

Provide feedback (0)

CH6.2 13P

CH6.2 15P

Step 1 of 9

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix}$$

A symmetric matrix A is given by

$$A = A^T$$

The entries of a symmetric matrix are symmetric to the main diagonal

Therefore, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix}$ is a symmetric matrix.

Provide feedback (0)

Step 2 of 9

Using the Gaussian elimination, we have

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix} \xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 3 & 4 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Therefore, $A=\begin{bmatrix}1&2&3\\2&5&4\\3&4&9\end{bmatrix}$ is a symmetric matrix.

Provide feedback (0)

Step 2 of 9

Using the Gaussian elimination, we have

$$\begin{bmatrix}1&2&3\\2&5&4\\3&4&9\end{bmatrix}\xrightarrow{-2R_1+R_2}\begin{bmatrix}1&2&3\\0&1&-2\\3&4&9\end{bmatrix}$$

$$\begin{bmatrix}1&2&3\\0&1&-2\\3&4&9\end{bmatrix}\xrightarrow{-3R_1+R_3}\begin{bmatrix}1&2&3\\0&1&-2\\0&-2&0\end{bmatrix}$$

$$\begin{bmatrix}1&2&3\\0&1&-2\\0&-2&0\end{bmatrix}\xrightarrow{2R_2+R_3}\begin{bmatrix}1&2&3\\0&1&-2\\0&0&-4\end{bmatrix}$$

Here the pivots are $1,1,-4$.

Provide feedback (0)

Step 3 of 9

We know that a symmetric matrix is positive definite, if all pivots (without row exchanges) are positive and it is positive semidefinite, if no pivots are negative.

Since one of the pivots is negative, therefore the symmetric matrix A is $\boxed{\text{indefinite}}$.

Provide feedback (0)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix} \xrightarrow{2R_2 + R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & -4 \end{bmatrix}$$

Here the pivots are $1, 1, -4$.

Provide feedback (0)

Step 3 of 9

We know that a symmetric matrix is positive definite, if all pivots (without row exchanges) are positive and it is positive semidefinite, if no pivots are negative.

Since one of the pivots is negative, therefore the symmetric matrix *A* is $\boxed{\text{indefinite}}$.

Provide feedback (0)

Step 4 of 9

Consider the following matrix:

$$B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

Since entries are symmetric to the main diagonal, so *B* is symmetric matrix.

Provide feedback (0)

Step 5 of 9

Step 4 of 9

Consider the following matrix:

$$B=\begin{bmatrix}1 & 2 & 0 & 0\\ 2 & 6 & -2 & 0\\ 0 & -2 & 5 & -2\\ 0 & 0 & -2 & 3\end{bmatrix}$$

Since entries are symmetric to the main diagonal, so B is symmetric matrix.

Provide feedback (0)

Step 5 of 9

Using the Gaussian elimination, we have

$$\begin{bmatrix}1 & 2 & 0 & 0\\ 2 & 6 & -2 & 0\\ 0 & -2 & 5 & -2\\ 0 & 0 & -2 & 3\end{bmatrix}\xrightarrow{-2R_1+R_2}\begin{bmatrix}1 & 2 & 0 & 0\\ 0 & 2 & -2 & 0\\ 0 & -2 & 5 & -2\\ 0 & 0 & -2 & 3\end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2}\begin{bmatrix}1 & 2 & 0 & 0\\ 0 & 1 & -1 & 0\\ 0 & -2 & 5 & -2\\ 0 & 0 & -2 & 3\end{bmatrix}$$

$$\xrightarrow{2R_2+R_3}\begin{bmatrix}1 & 2 & 0 & 0\\ 0 & 1 & -1 & 0\\ 0 & 0 & 3 & -2\\ 0 & 0 & -2 & 3\end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

$$\xrightarrow{2R_2+R_3} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

Again applying the row operations we have:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \xrightarrow{\frac{1}{3}R_3} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{-2}{3} \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

$$\xrightarrow{2R_3+R_4} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{-2}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix}$$

Here the pivots are $1, 1, 1, \text{and } \frac{5}{3}$.

$$\begin{bmatrix} 0 & 0 & -2 & 3 \end{bmatrix} \qquad \begin{bmatrix} & & & 3 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

$$\xrightarrow{2R_3 + R_4} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{-2}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix}$$

Here the pivots are $1, 1, 1, \text{and } \frac{5}{3}$.

We know that a symmetric matrix is positive definite, if all pivots (without row exchanges) are positive and it is positive semidefinite, if no pivots are negative.

Since all pivots is positive, the symmetric matrix *B* is $\boxed{\text{positive definite}}$.

Provide feedback (0)

Step 6 of 9

Consider the reduced form of *B* as

$$B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{-2}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix}$$

Since $C = -B$, so we have

Since all pivots is positive, the symmetric matrix B is $\boxed{\text{positive definite}}$.

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Step 6 of 9

Consider the reduced form of B as

$$B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{-2}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix}$$

Since $C = -B$, so we have

$$C = \begin{bmatrix} -1 & -2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & \frac{2}{3} \\ 0 & 0 & 0 & -\frac{5}{3} \end{bmatrix}$$

We know that a symmetric matrix is negative definite, if all the pivots (without row exchanges) are negative.

Therefore, matrix C is $\boxed{\text{nagative definite}}$.

Provide feedback (0)

Provide feedback (0)

Step 7 of 9

Consider the reduced form of A as

$$A=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -2\\ 0 & 0 & -4\end{bmatrix}$$

Sinc $D=A^{-1}$, so we have

$$\begin{aligned}D&=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -2\\ 0 & 0 & -4\end{bmatrix}^{-1}\\ &=\begin{bmatrix}1 & -2 & \frac{7}{2}\\ 0 & 1 & -\frac{1}{2}\\ 0 & 0 & -\frac{1}{4}\end{bmatrix}\end{aligned}$$

Here the pivots are $1,1,-\frac{1}{4}$.

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Step 8 of 9

We know that a symmetric matrix is positive definite, if all pivots (without row exchanges) are positive and it

Here the pivots are $1, 1, -\frac{1}{4}$.

Provide feedback (0)

Step 8 of 9

We know that a symmetric matrix is positive definite, if all pivots (without row exchanges) are positive and it is positive semidefinite, if no pivots are negative.

Since one of the pivots is negative, therefore the symmetric matrix D is $\boxed{\text{indefinite}}$.

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Step 9 of 9

We have

$$\begin{aligned} -X^T AX &= -\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ &= -x^2 - 5y^2 - 9z^2 - 4xy - 6xz - 8yz \end{aligned}$$

This is the real solution of $-X^T AX = 1$.

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CH6.2 14P

Step 1 of 2

Let A is symmetric positive definite and Q is an orthogonal matrix.

Since Q is an orthogonal matrix, therefore, Q must contain eigenvectors of A.
Hence, $Q^T AQ$ is not a diagonal matrix.

The eigenvalues of A are equal to eigenvalues of $Q^T AQ$.

The matrix Q is an orthogonal, therefore, we get,

$$Q^T = Q^{-1}$$

Therefore, we get,

$$Q^T AQ = A$$
$$Q^{-1} AQ = A$$

Therefore,

(a)False
(b)True
(c)True
(d)True

Provide feedback (0)

Step 2 of 2

Thus,

$$\boxed{(a)\text{False}(b)\text{True}(c)\text{True}(d)\text{True}}.$$

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CH6.2 16P

CH6.2 15P

CH6.2 17P

Step 1 of 3

Suppose A is an n by n matrix, which is positive definite.

By using cofactors, we can write the following:

$$|A| = a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$$

Since, A is positive definite, its determinant should be positive. Thus we get,

$$a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > 0$$

Therefore,

$$a_{11}A_{11} > -\left(a_{12}A_{12} + \ldots + a_{1n}A_{1n}\right)$$

Provide feedback (0)

Step 2 of 3

Suppose, the value of a_{11} is increased to b_{11} .

Thus, $a_{11} < b_{11}$.

Let if possible, $a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > b_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$

This gives, $a_{11}A_{11} > b_{11}A_{11}$.

Thus, $a_{11} < b_{11}$.

Let if possible, $a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > b_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$

This gives, $a_{11}A_{11} > b_{11}A_{11}$.

But since $a_{11} < b_{11}$, we can get $a_{11}A_{11} > b_{11}A_{11}$ only if $A_{11} < 0$.

Provide feedback (0)

Step 3 of 3

But the matrix *A* is positive definite. Therefore, it is automatically positive semidefinite. Thus, none of its principal submatrices has negative determinant.

This contradicts with $A_{11} < 0$.

Therefore, our assumption that $a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > b_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$ is wrong.

Therefore, as a_{11} is increased, the determinant of *A* must increase.

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Step 1 of 3

From the relation $A = R^{\mathrm{T}} R$, we can write down the following:

$$\begin{aligned}\det(A) &= \det\left(R^{\mathrm{T}} R\right) \\ &= \det\left(R^{\mathrm{T}}\right)\det(R) \\ &= \det(R)\det(R) \\ &= \left(\det(R)\right)^2\end{aligned}$$

Provide feedback (0)

Step 2 of 3

Now $\left(\det(R)\right)^2$ is equal to the square of the *R* parallelepiped.

Note that, a_{jj} is equal to the product of the jth row of R^{T} and the jth column of *R*. Therefore, a_{jj} is equal to the length squared of the jth column of *R*.

It is obvious that the volume of the *R* parallelepiped cannot be greater than the product of the length squared columns of *R*.

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Step 3 of 3

Therefore, volume of the *R* parallelepiped is less than or equal to $a_{11}a_{22}\cdots a_{nn}$.

CH6.2 18P

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Step 2 of 3

Now $\left(\det(R)\right)^2$ is equal to the square of the *R* parallelepiped.

Note that, a_{jj} is equal to the product of the jth row of R^T and the jth column of *R*. Therefore, a_{jj} is equal to the length squared of the jth column of *R*.

It is obvious that the volume of the *R* parallelepiped cannot be greater than the product of the length squared columns of *R*.

Provide feedback (0)

Step 3 of 3

Therefore, volume of the *R* parallelepiped is less than or equal to $a_{11}a_{22}\cdots a_{nn}$.

Therefore, $\boxed{\det A \le a_{11}a_{22}\cdots a_{nn}}$.

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Step 1 of 3

Thus, we get $2\lambda x^{H} Ax = -x^{H} x$.

Note that $x^{H} x$ is a positive real number, since *x* is a non zero vector. Thus, in the above equation, the right hand side is negative. If *x* is a real vector, then $x^{H} Ax$ too will be a positive real number. Therefore, the real part of the eigenvalue ? will be negative.

Provide feedback (0)

Step 2 of 3

Let us show that the real part of the eigenvalue ? is less than zero. Consider the following:

$$\begin{aligned} AM + M^{H} A &= -I \\ x^{H}\left(AM + M^{H} A\right) &= -x^{H} I \\ x^{H}\left(AM + M^{H} A\right)x &= -x^{H} Ix \\ x^{H} AMx + x^{H} M^{H} Ax &= -x^{H} x \end{aligned}$$

Provide feedback (0)

Step 3 of 3

We have $Mx = \lambda x$. Also, we can write $x^{H} M^{H} = \left(Mx\right)^{H}$. Therefore, we get

$$\begin{aligned} x^{H} AMx + x^{H} M^{H} Ax &= -x^{H} x \\ x^{H} A\left(\lambda x\right) + \left(Mx\right)^{H} Ax &= -x^{H} x \\ \lambda x^{H} Ax + \left(\lambda x\right)^{H} Ax &= -x^{H} x \\ \lambda x^{H} Ax + \lambda x^{H} Ax &= -x^{H} x \end{aligned}$$

Provide feedback (0)

CH6.2 19P

CH6.2 18P

Step 1 of 4

(a). Given quadratic is,

$$f = 2\left(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3\right)$$

$$= 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$$

$$= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

Provide feedback (0)

CH6.2 20P

Step 2 of 4

Now the upper left determinants are,

$$A_1 = 2 > 0$$

$$A_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 > 0$$

$$A_3 = |A|$$

$$= 2(4-1) + 1(-2) + 0$$

$$= 4 > 0$$

The pivots are $\frac{A_1}{1}, \frac{A_2}{A_1}$ and $\frac{A_3}{A_2}$.

i.e. $\frac{2}{1}, \frac{3}{2}$ and $\frac{4}{3}$

Therefore, $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ have pivots $\boxed{2, \frac{3}{2} \text{ and } \frac{4}{3}}$.

Therefore, f is positive definite.

i.e. $\frac{2}{1}, \frac{3}{2}$ and $\frac{4}{3}$

Therefore, $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ have pivots $\boxed{2, \frac{3}{2} \text{ and } \frac{4}{3}}$.

Therefore, f is positive definite.

Provide feedback (0)

Step 3 of 4

(b). Given quadratic is,

$$\begin{aligned} f &= 2\left(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_1x_3 - x_2x_3\right) \\ &= 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3 \\ &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \end{aligned}$$

$$\Rightarrow A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

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Step 4 of 4

Now the upper last determinants are

$$A_1 = 2 > 0$$

$$A_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 > 0$$

$$\begin{aligned} A_3 &= |A| \\ &= 2(4-1) + 1(-2-1) - 1(1+2) \\ &= 6 - 3 - 3 \\ &= 0 \end{aligned}$$

$$\begin{pmatrix} 2 & -1 & -1 \end{pmatrix}$$

$$A_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 > 0$$

$$\begin{aligned} A_3 &= |A| \\ &= 2(4-1)+1(-2-1)-1(1+2) \\ &= 6-3-3 \\ &= 0 \end{aligned}$$

Therefore, $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ is singular.

Therefore, $A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

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Step 1 of 3

Given matrix is $A=\begin{pmatrix}2&2&0\\2&5&3\\0&3&8\end{pmatrix}$.

Now the upper left determinants are

$A_1=2,$

$A_2=\begin{vmatrix}2&2\\2&5\end{vmatrix}=6,$

$A_3=|A|$

$=2(40-9)-2(16)+0$

$=30$

Therefore, the upper left determinants are $\boxed{A_1=2, A_2=6 \text{ and } A_3=30}$.

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Step 2 of 3

Now we determine the pivots of A.

$A=\begin{pmatrix}2&2&0\\2&5&3\\0&3&8\end{pmatrix}$

Apply $R_2\to R_2-R_1$

$=\begin{pmatrix}2&2&0\\0&3&3\\0&3&8\end{pmatrix}$

Apply $R_3\to R_3-R_2$

$=\begin{pmatrix}2&2&0\\0&3&3\\0&0&5\end{pmatrix}$

So the pivots of A are $\boxed{2,3 \text{ and } 5}$.

CH6.2 21P

Therefore, the upper left determinants are $\boxed{A_1 = 2, A_2 = 6 \text{ and } A_3 = 30}$.

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Step 2 of 3

Now we determine the pivots of A.

$$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 3 \\ 0 & 3 & 8 \end{pmatrix}$$

Apply $R_2 \to R_2 - R_1$

$$= \begin{pmatrix} 2 & 2 & 0 \\ 0 & 3 & 3 \\ 0 & 3 & 8 \end{pmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$= \begin{pmatrix} 2 & 2 & 0 \\ 0 & 3 & 3 \\ 0 & 0 & 5 \end{pmatrix}$$

So the pivots of A are $\boxed{2, 3 \text{ and } 5}$.

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Step 3 of 3

Verification,

The pivots are $\frac{A_1}{1}, \frac{A_2}{A_1}$ and $\frac{A_3}{A_2}$.

i.e. $\frac{2}{1}, \frac{6}{2}, \frac{30}{6}$

That is $2, 3, 5$.

Thus pivots are verified.

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CH6.2 20P

Step 1 of 1

We need to prove that A is not positive definite.

Given matrix is $A=\begin{pmatrix}4 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 5\end{pmatrix}$.

Now,

$$x^T Ax=\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}\begin{pmatrix}4 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

$$=4x_1^2+5x_3^2+2x_1x_2+2x_1x_3+4x_2x_3$$

Now for $(x_1,x_2,x_3)=(0,1,0)$

$x^T Ax=0+0+0+0+0=0$

Therefore, $\boxed{A \text{ is not positive definite when } (x_1,x_2,x_3)=(0,1,0)}$, because positive definite matrix cannot have a zero on its diagonal.

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Step 1 of 1

We know that a positive definite matrix cannot have a zero on its diagonal.
So a diagonal entry a_{jj} of a symmetric matrix cannot be smaller than all the λ's.
If it were, then $A - a_{jj}I$ would have $\boxed{\text{positive}}$ Eigen values and would be positive definite. But $A - a_{jj}I$ has a $\boxed{\text{zero}}$ on the main diagonal.

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Step 1 of 4

(a).
Given statement is "Every positive definite matrix is invertible."
We know that for a positive definite matrix determinant is always positive and hence determinant is non-zero. So the matrix is non singular.
Hence every positive definite matrix is invertible.
Hence the given statement is true.

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Step 2 of 4

(b).
Given statement is "the only positive definite projection matrix is $P = I$".
We know that all projection matrices except identity matrix I are singular.
Also we know the positive definite matrix is non-singular.
So the only positive definite projection matrix is $P = I$.
Hence the given statement is true.

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Step 3 of 4

(c).
Given statement is "A diagonal matrix with positive diagonal entries is positive definite."
We know that diagonal entries of diagonal matrix are the Eigen values of the matrix.
So, all the Eigen values of the matrix are positive. So the given diagonal matrix with positive definite diagonal entries is positive definite.
Hence, the given statement is true.

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Step 4 of 4

(d).
Given statement is "A symmetric matrix with a positive determinant might not be positive definite."

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Step 2 of 4

(b).
Given statement is "the only positive definite projection matrix is $P = I$".
We know that all projection matrices except identity matrix I are singular.
Also we know the positive definite matrix is non-singular.
So the only positive definite projection matrix is $P = I$.
Hence the given statement is true.

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Step 3 of 4

(c).
Given statement is "A diagonal matrix with positive diagonal entries is positive definite."
We know that diagonal entries of diagonal matrix are the Eigen values of the matrix.
So, all the Eigen values of the matrix are positive. So the given diagonal matrix with positive definite diagonal entries is positive definite.
Hence, the given statement is true.

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Step 4 of 4

(d).
Given statement is "A symmetric matrix with a positive determinant might not be positive definite."
We know that if A is a square matrix of order 'n' then
$\det(A) = (-1)^n \det A$.
So the negative definite matrix $-I$ has determinant $+1$ when n is even.
Thus negative definite matrix has positive determinant.
Thus a symmetric matrix with a positive determinant might not be positive definite.
Hence the givens statement is true.

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CH6.2 23P

Step 1 of 3

Given matrices are $A=\begin{pmatrix} s & -4 & -4 \\ -4 & s & -4 \\ -4 & -4 & s \end{pmatrix}$ and $B=\begin{pmatrix} t & 3 & 0 \\ 3 & t & 4 \\ 0 & 4 & t \end{pmatrix}$.

We have to found the values of s and t for which the matrices A and B are positive definite.

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Step 2 of 3

We need to find the determinants of A and B.

$$\begin{aligned} \det A &= s\left(s^2-16\right)+4(-4s-16)-4(16+4s) \\ &= s^3-16s-16s-64-64-16s \\ &= s^3-48s-128 \end{aligned}$$

Now for A to be positive definite we must have

$s>0, s^2-16>0$ and $s^3-48s-128>0$.

So if $s>8$ all the above three conditions are satisfied and hence A is positive definite.

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Step 3 of 3

$$\begin{aligned} \det B &= t\left(t^2-16\right)-3(3t)+0 \\ &= t^3-25t = t\left(t^2-25\right) \end{aligned}$$

Now if $t^3-25t>0$ then given matrix is positive definite.

So if $t>5$, then $t>0$, $t^2-9>0$ and $t^3-25t>0$.

So the matrix B is positive definite if $t>5$.

Therefore, the matrices $\boxed{A \text{ is positive definite if } s>8 \text{ and } B \text{ is positive definite if } t>5}$.

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CH6.2 25P

CH6.2 24P

Step 1 of 2

Equation of given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

We can written this equation as $\lambda_1 x^2+\lambda_2 y^2=1$.

Compare these two equations,

So, $\lambda_1=\frac{1}{a^2}, \lambda_2=\frac{1}{b^2}$

$\Rightarrow a=\frac{1}{\sqrt{\lambda_1}}, b=\frac{1}{\sqrt{\lambda_2}}$

Thus half lengths of axes are $\boxed{a=\frac{1}{\sqrt{\lambda_1}}, b=\frac{1}{\sqrt{\lambda_2}}}$.

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Step 2 of 2

Given ellipse equation is $9x^2+16y^2=1$.

$\Rightarrow \frac{x^2}{\left(\frac{1}{3}\right)^2}+\frac{y^2}{\left(\frac{1}{4}\right)^2}=1$

$\Rightarrow a=\frac{1}{3},\ b=\frac{1}{4}$.

Thus half lengths of axes are $\boxed{a=\frac{1}{3},\ b=\frac{1}{4}}$.

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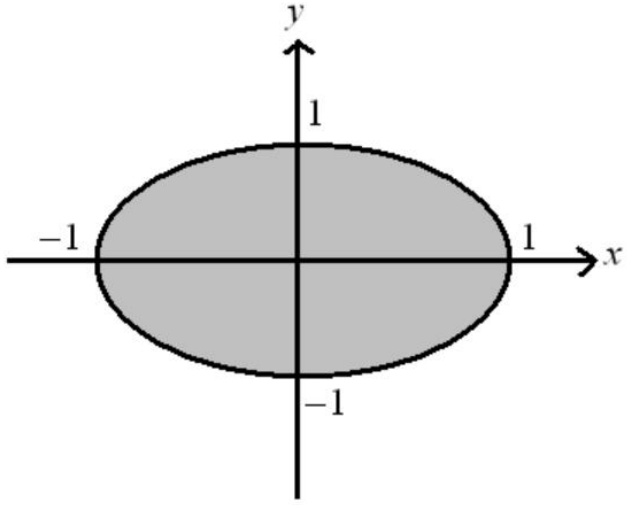
Step 1 of 3

The graph of tilted ellipse $x^2 + xy + y^2 = 1$ is as shown below.



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CH6.2 27P

Step 2 of 3

The corresponding matrix A is $A = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$

Eigen values of A are,

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & \frac{1}{2} \\ \frac{1}{2} & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)^2 - \frac{1}{4} = 0$$

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Step 2 of 3

The corresponding matrix A is $A=\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$

Eigen values of A are,

$|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & \frac{1}{2} \\ \frac{1}{2} & 1-\lambda \end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)^2-\frac{1}{4}=0$$

$$\Rightarrow \left(1-\lambda+\frac{1}{2}\right)\left(1-\lambda-\frac{1}{2}\right)=0$$

$$\Rightarrow \lambda_1=\frac{1}{2},\ \ \lambda_1=\frac{3}{2}.$$

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Step 3 of 3

We know that the half lengths of its axes are $a=\frac{1}{\sqrt{\lambda_1}}$ and $b=\frac{1}{\sqrt{\lambda_2}}$.

$$\Rightarrow \lambda_1=\frac{1}{2} \text{ and } \lambda_1=\frac{3}{2}.$$

$$\Rightarrow a=\sqrt{2} \text{ and } b=\sqrt{\frac{2}{3}}$$

Therefore $\boxed{a=\sqrt{2},\ \ b=\sqrt{\frac{2}{3}}}$

CH6.2 26P

Step 1 of 3

Given that the matrix is $C=\begin{pmatrix}3 & 0\\ 1 & 2\end{pmatrix}$,

We need to find the matrix A.

We can determine A by using Cholesky factorization. i.e. $A=CC^T$.

So,

$$A=\begin{pmatrix}3 & 0\\ 1 & 2\end{pmatrix}\begin{pmatrix}3 & 1\\ 0 & 2\end{pmatrix}$$

$$=\begin{pmatrix}9 & 3\\ 3 & 5\end{pmatrix}$$

Therefore, the matrix $\boxed{A=\begin{pmatrix}9 & 3\\ 3 & 5\end{pmatrix}}$.

Provide feedback (0)

Step 2 of 3

Given that $A=\begin{pmatrix}4 & 8\\ 8 & 25\end{pmatrix}$,

We need to find C.

We know that if $A=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$ then a and $\dfrac{ac-b^2}{a}$ are the pivots.

So,

$$\begin{pmatrix}a & b\\ b & c\end{pmatrix}=LDL^T$$

$$=\begin{pmatrix}1 & 0\\ \frac{b}{a} & 1\end{pmatrix}\begin{pmatrix}a & 0\\ 0 & \frac{ac-b^2}{a}\end{pmatrix}\begin{pmatrix}1 & \frac{b}{a}\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 9\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$

CH6.2 28P

Step 2 of 3

Given that $A=\begin{pmatrix}4 & 8\\ 8 & 25\end{pmatrix}$,

We need to find C.

We know that if $A=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$ then a and $\frac{ac-b^2}{a}$ are the pivots.

So,

$$\begin{aligned}\begin{pmatrix}a & b\\ b & c\end{pmatrix} &= LDL^T\\ &= \begin{pmatrix}1 & 0\\ \frac{b}{a} & 1\end{pmatrix}\begin{pmatrix}a & 0\\ 0 & \frac{ac-b^2}{a}\end{pmatrix}\begin{pmatrix}1 & \frac{b}{a}\\ 0 & 1\end{pmatrix}\\ &= \begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 9\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}\end{aligned}$$

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Step 3 of 3

Here $a=4$, $b=8$ and $c=25$.

Now $L=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$, $D=\begin{pmatrix}4 & 0\\ 0 & 9\end{pmatrix}$

Now,

$$\begin{aligned}C &= L\sqrt{D}\\ C &= \begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}\\ &= \begin{pmatrix}2 & 0\\ 4 & 3\end{pmatrix}\end{aligned}$$

Thus, the matrix $\boxed{C=\begin{pmatrix}2 & 0\\ 4 & 3\end{pmatrix}}$.

Therefore, the matrices are $\boxed{A=\begin{pmatrix}9 & 3\\ 3 & 5\end{pmatrix} \text{ and } C=\begin{pmatrix}2 & 0\\ 4 & 3\end{pmatrix}}$.

CH6.2 27P

Step 1 of 7

Given that $A=\begin{pmatrix}9&0&0\\0&1&2\\0&2&8\end{pmatrix}$.

In the Cholesky factorization, $A=CC^T$, with $C=L\sqrt{D}$, the square roots of the pivots are on the diagonal of C.

$$A=\begin{pmatrix}9&0&0\\0&1&2\\0&2&8\end{pmatrix}$$

Apply $R_3\to R_3+(-)R_2$

$$=\begin{pmatrix}9&0&0\\0&1&2\\0&0&4\end{pmatrix}$$

Thus, the pivots are $\boxed{9,1,4}$.

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CH6.2 29P

Step 2 of 7

Now,

$$A=LDL^T$$
$$=\left(L\sqrt{D}\right)\left(\sqrt{D}L^T\right)$$
$$=\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}\begin{pmatrix}9&0&0\\0&1&0\\0&0&4\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&2\\0&0&1\end{pmatrix}$$

Here,

$$L=\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix},\ D=\begin{pmatrix}9&0&0\\0&1&0\\0&0&4\end{pmatrix}\text{ and }\sqrt{D}=\begin{pmatrix}3&0&0\\0&1&0\\0&0&2\end{pmatrix}$$

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Step 2 of 7

Now,

$$A = LDL^T$$
$$= \left(L\sqrt{D}\right)\left(\sqrt{D}L^T\right)$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 9 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

Here,

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix},\ D = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \text{ and } \sqrt{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

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Step 3 of 7

So,

$$C = L\sqrt{D}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

Therefore, the matrix $\boxed{C = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}}$

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Step 4 of 7

Given matrix $A=\begin{pmatrix}1&1&1\\1&2&2\\1&2&7\end{pmatrix}$

Apply operations $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$

$$=\begin{pmatrix}1&1&1\\0&1&1\\0&1&6\end{pmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$=\begin{pmatrix}1&1&1\\0&1&1\\0&0&5\end{pmatrix}$$

Thus, the pivots are $\boxed{1,1,5}$.

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Step 5 of 7

Now,

$A = LDL^T$

$$=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&5\end{pmatrix}\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$$

Here,

$$L=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}, D=\begin{pmatrix}1&0&0\\0&1&0\\0&0&5\end{pmatrix} \text{ and } \sqrt{D}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&\sqrt{5}\end{pmatrix}$$

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$$\begin{pmatrix}1 & 1 & 1\end{pmatrix} \quad \begin{pmatrix}0 & 0 & 5\end{pmatrix} \quad \begin{pmatrix}0 & 0 & \sqrt{5}\end{pmatrix}$$

Provide feedback (0)

Step 6 of 7

Now,

$$C = L\sqrt{D}$$

$$= \begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 1 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \sqrt{5}\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 1 & \sqrt{5}\end{pmatrix}.$$

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Step 7 of 7

Therefore, the matrix $\boxed{C = \begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 1 & \sqrt{5}\end{pmatrix}}$

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Step 1 of 3

The symmetric factorization $A = LDL^T$ means that $x^T Ax = x^T LDL^T x$

$$\Rightarrow \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & \frac{ac-b^2}{a} \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow ax^2 + 2bxy + cy^2$$

$$\Rightarrow \begin{pmatrix} x + \frac{b}{a}y & y \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & \frac{ac-b^2}{a} \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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Step 2 of 3

So,

$$\Rightarrow \begin{pmatrix} a\left(x + \frac{b}{a}y\right) & \left(\frac{ac-b^2}{a}\right)y \end{pmatrix} \begin{pmatrix} \left(x + \frac{b}{a}y\right) \\ y \end{pmatrix}$$

$$\Rightarrow a\left(x + \frac{b}{a}y\right)^2 + \left(\frac{ac-b^2}{a}\right)y^2$$

Therefore, $\boxed{ax^2 + 2bxy + cy^2 = a\left(x + \frac{b}{a}y\right)^2 + \left(\frac{ac-b^2}{a}\right)y^2}$

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Step 3 of 3

Now taking $a = 2,\ b = 4,\ c = 10.$

Therefore,

$2x^2 + 8xy + 10y^2$

Provide feedback (0)

Step 2 of 3

So,

$$\Rightarrow \begin{pmatrix} a\left(x+\frac{b}{a}y\right) & \left(\frac{ac-b^2}{a}\right)y \end{pmatrix} \begin{pmatrix} \left(x+\frac{b}{a}y\right) \\ y \end{pmatrix}$$

$$\Rightarrow a\left(x+\frac{b}{a}y\right)^2+\left(\frac{ac-b^2}{a}\right)y^2$$

Therefore, $\boxed{ax^2+2bxy+cy^2=a\left(x+\frac{b}{a}y\right)^2+\left(\frac{ac-b^2}{a}\right)y^2}$.

Provide feedback (0)

Step 3 of 3

Now taking $a=2,\ b=4,\ c=10$.

Therefore,

$2x^2+8xy+10y^2$

$$\Rightarrow a\left(x+\frac{b}{a}y\right)^2+\left(\frac{ac-b^2}{a}\right)y^2$$

$$\Rightarrow 2\left(x+\frac{4}{2}y\right)^2+\left(\frac{(2)(10)-(4)^2}{2}\right)y^2$$

$$\Rightarrow 2(x+2y)^2+2y^2.$$

Therefore, $\boxed{2x^2+8xy+10y^2=2(x+2y)^2+2y^2}$.

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Step 1 of 4

Given $A=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 5\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$

Now without multiplying we have to determine the information of A.

(a).

The determinant of the matrix is,

$$\det A=(2)(5)$$
$$=10.$$

Therefore, the determinant of the matrix is $\boxed{10}$.

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Step 2 of 4

(b).

Clearly, the Eigen values of A are the diagonal elements

i.e. 2 and 5.

Therefore, the Eigen values of A are $\boxed{2\text{ and }5}$.

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Step 3 of 4

(c).

The Eigen vectors of A are $\boxed{\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}^T \text{ and } \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}^T}$.

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Step 4 of 4

(d).

As the Eigen values of A are both positive, A is positive definite.

Therefore $\boxed{A\text{ is positive definite}}$

CH6.2 31P

Therefore, the Eigen values of A are $\boxed{2 \text{ and } 5}$.

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Step 3 of 4

(c).

The Eigen vectors of A are $\boxed{\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}^T \text{ and } \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}^T}$.

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Step 4 of 4

(d).

As the Eigen values of A are both positive, A is positive definite.

Therefore, $\boxed{A \text{ is positive definite}}$.

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Step 1 of 3

Given matrix is $A=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}$.

So,

$$x^T Ax=\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}\begin{pmatrix}x\\ x_2\\ x_3\end{pmatrix}$$

$$=2x_1^2+2x_2^2+2x_3^2-2x_1x_2-2x_2x_3-2x_3x_1$$

$$=2\left(x_1^2+x_2^2+x_3^2-x_1x_2-x_2x_3-x_3x_1\right)$$

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Step 2 of 3

$$A=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}.$$

Apply $R_2\to R_2+\frac{1}{2}R_1$ and $R_3\to R_3+\frac{1}{2}R_1$

$$=\begin{pmatrix}2 & -1 & -1\\ 0 & \frac{3}{2} & -\frac{3}{2}\\ 0 & -\frac{3}{2} & \frac{3}{2}\end{pmatrix}$$

Apply $R_3\to R_3+R_2$

$$=\begin{pmatrix}2 & -1 & -1\\ 0 & \frac{3}{2} & -\frac{3}{2}\\ 0 & 0 & 0\end{pmatrix}$$

Provide feedback (0)

Step 2 of 3

$$A=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}.$$

Apply $R_2 \to R_2+\frac{1}{2}R_1$ and $R_3 \to R_3+\frac{1}{2}R_1$

$$=\begin{pmatrix}2 & -1 & -1\\ 0 & \frac{3}{2} & -\frac{3}{2}\\ 0 & -\frac{3}{2} & \frac{3}{2}\end{pmatrix}$$

Apply $R_3 \to R_3+R_2$

$$=\begin{pmatrix}2 & -1 & -1\\ 0 & \frac{3}{2} & -\frac{3}{2}\\ 0 & 0 & 0\end{pmatrix}$$

So we can write it as $x^T Ax = 2\left(x_1-\frac{x_2}{2}-\frac{x_3}{2}\right)^2+\frac{3}{2}(x_2-x_3)^2$

Therefore, $\boxed{x^T Ax = 2\left(x_1-\frac{x_2}{2}-\frac{x_3}{2}\right)^2+\frac{3}{2}(x_2-x_3)^2}$

Thus, $x^T Ax$ is sum of two squares and A has two pivots.

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Step 3 of 3

$$B=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

So we can write it as $x^T Ax = 2\left(x_1 - \frac{x_2}{2} - \frac{x_3}{2}\right)^2 + \frac{3}{2}(x_2 - x_3)^2$

Therefore, $\boxed{x^T Ax = 2\left(x_1 - \frac{x_2}{2} - \frac{x_3}{2}\right)^2 + \frac{3}{2}(x_2 - x_3)^2}$.

Thus, $x^T Ax$ is sum of two squares and A has two pivots.

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Step 3 of 3

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} x^T Bx &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_1 \\ &= (x_1 + x_2 + x_3)^2 \end{aligned}$$

Therefore, $\boxed{x^T Bx = (x_1 + x_2 + x_3)^2}$.

Thus, $x^T Bx$ can be written as one square and B has one pivot.

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Step 1 of 8

Given matrix is $A=\begin{pmatrix}1&1&1\\1&1&1\\1&1&0\end{pmatrix}$.

(i).

$$x^TAx=\begin{pmatrix}x&y&z\end{pmatrix}\begin{pmatrix}1&1&1\\1&1&1\\1&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$=x^2+y^2+2xy+2yz+2zx$$

$$=(x+y)^2+2z(x+y)$$

Clearly $x^TAx\geq 0$ for some $x,y\,\&\,z$.

So given matrix is $\boxed{\text{indefinite.}}$

Provide feedback (0)

Step 2 of 8

(ii).

Characteristic equations of A is

$$\begin{vmatrix}1-\lambda&1-\lambda&1-\lambda\\1-\lambda&1-\lambda&1-\lambda\\1-\lambda&1-\lambda&-\lambda\end{vmatrix}=0$$

$$\Rightarrow(1-\lambda)\left[-\lambda(1-\lambda)-1\right]-1\left[-\lambda-1\right]+1\left[1-(1-\lambda)\right]=0$$

$$\Rightarrow\lambda^2-\lambda-1-\lambda^3+\lambda^2+\lambda+2\lambda+1=0$$

$$\Rightarrow-\lambda^3+2\lambda^2+2\lambda=0$$

$$\Rightarrow-\lambda\left(\lambda^2-2\lambda-2\right)=0$$

$$\Rightarrow\lambda=0,\ \lambda=1+\sqrt{3},1-\sqrt{3}$$

Therefore the Eigen values of A are $\boxed{\text{positive and zero.}}$

Provide feedback (0)

Step 2 of 8

(ii).

Characteristic equations of A is

$$\begin{vmatrix} 1-\lambda & 1-\lambda & 1-\lambda \\ 1-\lambda & 1-\lambda & 1-\lambda \\ 1-\lambda & 1-\lambda & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left[-\lambda(1-\lambda)-1\right]-1\left[-\lambda-1\right]+1\left[1-(1-\lambda)\right]=0$$

$$\Rightarrow \lambda^2-\lambda-1-\lambda^3+\lambda^2+\lambda+2\lambda+1=0$$

$$\Rightarrow -\lambda^3+2\lambda^2+2\lambda=0$$

$$\Rightarrow -\lambda\left(\lambda^2-2\lambda-2\right)=0$$

$$\Rightarrow \lambda=0,\ \lambda=1+\sqrt{3},1-\sqrt{3}$$

Therefore the Eigen values of A are $\boxed{\text{positive and zero.}}$

Provide feedback (0)

Step 3 of 8

(iii).

$$A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Apply $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$=\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Apply $R_3 \leftrightarrow R_2$

$$=\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus the pivots are $\boxed{1,-1}$.

So the given matrix is $\boxed{\text{indefinite.}}$.

(iii).

$$A=\begin{pmatrix}1&1&1\\1&1&1\\1&1&0\end{pmatrix}$$

Apply $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$=\begin{pmatrix}1&1&1\\0&0&0\\0&0&-1\end{pmatrix}$$

Apply $R_3 \leftrightarrow R_2$

$$=\begin{pmatrix}1&1&1\\0&0&-1\\0&0&0\end{pmatrix}$$

Thus the pivots are $\boxed{1,-1}$.

So the given matrix is $\boxed{\text{indefinite.}}$.

Provide feedback (0)

Step 4 of 8

Given second matrix is $A=\begin{pmatrix}2&1&2\\1&1&1\\2&1&2\end{pmatrix}$

(i).

Characteristic equation is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}2-\lambda&1&2\\1&1-\lambda&1\\2&1&2-\lambda\end{vmatrix}=0$$

$$\Rightarrow (2-\lambda)\left[(1-\lambda)(2-\lambda)-1\right]-1\left[2-\lambda-2\right]+2\left[1-2(2-\lambda)\right]=0$$

$$\Rightarrow (2-\lambda)\left[\lambda^3-3\lambda+1\right]+\lambda-2+4\lambda=0$$

$$\Rightarrow 2\lambda^2-6\lambda+2-\lambda^3+3\lambda^2-\lambda+\lambda-2+4\lambda=0$$

$$\begin{pmatrix} 2 & 1 & 2 \end{pmatrix}$$

(i).

Characteristic equation is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 2 \\ 1 & 1-\lambda & 1 \\ 2 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\left[(1-\lambda)(2-\lambda)-1\right]-1\left[2-\lambda-2\right]+2\left[1-2(2-\lambda)\right]=0$$

$$\Rightarrow (2-\lambda)\left[\lambda^3-3\lambda+1\right]+\lambda-2+4\lambda=0$$

$$\Rightarrow 2\lambda^2-6\lambda+2-\lambda^3+3\lambda^2-\lambda+\lambda-2+4\lambda=0$$

Provide feedback (0)

Step 5 of 8

So,

$$\Rightarrow -\lambda^3+5\lambda^2-2\lambda=0$$

$$\Rightarrow -\lambda\left(\lambda^2-5\lambda+2\right)=0$$

$$\Rightarrow \lambda=0,\ \lambda=\frac{5\pm\sqrt{25-8}}{2}$$

$$\Rightarrow \lambda=0,\ \frac{5}{2}+\frac{\sqrt{17}}{2},\frac{5}{2}-\frac{\sqrt{17}}{2}>0$$

Thus the given matrix is $\boxed{\text{positive semi definite.}}$

Provide feedback (0)

Step 6 of 8

(ii).

$$A=\begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

Apply $R_2 \to R_2-\frac{1}{2}R_1$ and $R_3 \to R_3-R_1$

Thus the given matrix is $\boxed{\text{positive semi definite.}}$

Provide feedback (0)

Step 6 of 8

(ii).

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

Apply $R_2 \to R_2 - \frac{1}{2}R_1$ and $R_3 \to R_3 - R_1$

Provide feedback (0)

Step 7 of 8

$$= \begin{pmatrix} 2 & 1 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus the pivots are $\boxed{2, \frac{1}{2}}$ and one pivot is missing.

Therefore given matrix is $\boxed{\text{positive semi definite.}}$

Provide feedback (0)

Step 8 of 8

(iii).

Now the upper half determinants

$A_1 = 2 > 0$

$$A_2 = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 > 0$$

Step 7 of 8

$$= \begin{pmatrix} 2 & 1 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus the pivots are $\boxed{2, \frac{1}{2}}$ and one pivot is missing.

Therefore given matrix is $\boxed{\text{positive semi definite.}}$

Provide feedback (0)

Step 8 of 8

(iii).

Now the upper half determinants

$A_1 = 2 > 0$

$A_2 = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 > 0$

$A_3 = |A| = 0$

Thus the given matrix is $\boxed{\text{positive semi definite.}}$

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Step 1 of 6

Thus, we have

$$\begin{aligned} 0 &= \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} \\ &= (1-\lambda)^2 - 1 \\ &= \lambda^2 - 2\lambda \end{aligned}$$

Thus, one eigenvalue of A is positive and the other is zero.

Provide feedback (0)

Step 2 of 6

Consider the matrix $C^{\mathrm{T}}AC$.

We have

$$\begin{aligned} C^{\mathrm{T}}AC &= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} \end{aligned}$$

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$$= \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 6

To obtain the eigenvalues of $C^{\mathrm{T}}AC$, we solve $\det\left(C^{\mathrm{T}}AC - \lambda I\right) = 0$. This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} 4-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} \\ &= (4-\lambda)(1-\lambda) - 4 \\ &= \lambda^2 - 5\lambda + 4 - 4 \\ &= \lambda^2 - 5\lambda \end{aligned}$$

Thus, here also one eigenvalue of $C^{\mathrm{T}}AC$ is positive and the other is zero.

Provide feedback (0)

Step 4 of 6

Let $Q = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Consider the following chain of matrices:

$$\begin{aligned} C(t) &= tQ + (1-t)C \\ &= t\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + (1-t)\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Step 4 of 6

Let $Q=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$.

Consider the following chain of matrices:

$$\begin{aligned}C(t)&=tQ+(1-t)C\\&=t\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}+(1-t)\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix}\\&=\begin{bmatrix}t & 0\\ 0 & -t\end{bmatrix}+\begin{bmatrix}2(1-t) & 0\\ 0 & t-1\end{bmatrix}\\&=\begin{bmatrix}2-t & 0\\ 0 & -1\end{bmatrix}\end{aligned}$$

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Step 5 of 6

Note that

$$\begin{aligned}C(0)&=\begin{bmatrix}2-0 & 0\\ 0 & -1\end{bmatrix}\\&=\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix}\\C(1)&=\begin{bmatrix}2-1 & 0\\ 0 & -1\end{bmatrix}\\&=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 5 of 6

Note that

$$C(0)=\begin{bmatrix}2-0 & 0\\ 0 & -1\end{bmatrix}$$
$$=\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix}$$
$$C(1)=\begin{bmatrix}2-1 & 0\\ 0 & -1\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

Provide feedback (0)

Step 6 of 6

Thus, $\boxed{C(t)=tQ+(1-t)C}$ is the required chain of non-singular matrices.

Since C has one positive and one zero eigenvalue and the identity matrix I has both positive eigenvalues, it is impossible to have such a chain of matrices from C to I.

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Step 1 of 5

Let $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

Then we know that the pivots of A are 2, $\frac{3}{2}$, $\frac{4}{3}$.

Thus, every pivot of A is greater than 1.

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Step 2 of 5

To obtain the eigenvalues of A, solve $\det(A - \lambda I) = 0$.

This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \\ &= (2-\lambda)^3 - (2-\lambda) - (2-\lambda) \\ &= \left(8 - 12\lambda + 6\lambda^2 - \lambda^3\right) - 4 + 2\lambda \\ &= -\lambda^3 + 6\lambda^2 - 10\lambda + 4 \end{aligned}$$

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$$
\begin{aligned}
0 &= \begin{vmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \\
&= (2-\lambda)^3 - (2-\lambda) - (2-\lambda) \\
&= \left(8 - 12\lambda + 6\lambda^2 - \lambda^3\right) - 4 + 2\lambda \\
&= -\lambda^3 + 6\lambda^2 - 10\lambda + 4
\end{aligned}
$$

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Step 3 of 5

The roots of the equation $\lambda^3 - 6\lambda^2 + 10\lambda - 4 = 0$ are calculated by using online eigenvalue calculator. See the screenshot below:

Provide feedback (0)

Step 4 of 5

The three roots are 2, 3.414, and 0.585.

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Step 5 of 5

Thus, the three eigenvalues of A are 2, 3.414, and 0.585. Thus, one of the eigenvalues is less than 1.

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Step 1 of 1

First we need to find pivots of $A-\frac{1}{2}I$.

$$A-\frac{1}{2}I=\begin{pmatrix}2.5 & 3 & 0\\ 3 & 5.9 & 7\\ 0 & 7 & 7.5\end{pmatrix}$$

Apply $R_2 \to R_2-\frac{3}{2.5}R_1$

$$\Rightarrow A-\frac{1}{2}I=\begin{pmatrix}2.5 & 3 & 0\\ 0 & 5.9 & 7\\ 0 & 7 & 7.5\end{pmatrix}$$

Apply $R_3 \to R_3-\frac{7}{5.9}R_2$

$$\Rightarrow A-\frac{1}{2}I=\begin{pmatrix}2.5 & 3 & 0\\ 0 & 5.9 & 7\\ 0 & 0 & -0.805\end{pmatrix}$$

Thus the pivots are $\boxed{2.5, 5.9 \text{ and } -0.81}$..

So, $\boxed{\text{one Eigen value of } A-\frac{1}{2}I \text{ is negative}}$.

$A-\frac{1}{2}I$ has negative pivot, so that A has Eigen value smaller than $\frac{1}{2}$.

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Step 1 of 6

(a) Let us show that the $p+q$ number of vectors $x_1, x_2, ..., x_p$ and $y_1, y_2, ..., y_q$ are linearly independent. We will show this by contradiction.

Suppose, if possible, there exists a non zero linear combination of these vectors, which produces the zero vector.

Therefore, we can write the following:

$$\begin{aligned} z &= a_1x_1 + ... + a_px_p \\ &= b_1C_1y + ... + b_qCy_q \end{aligned}$$

Provide feedback (0)

Step 2 of 6

Now consider the following:

$$\begin{aligned} z^T Az &= \left(a_1x_1 + ... + a_px_p\right) A \begin{bmatrix} a_1x_1 \\ \vdots \\ a_px_p \end{bmatrix} \\ &= \left(a_1x_1 + ... + a_px_p\right) \begin{bmatrix} a_1Ax_1 \\ \vdots \\ a_pAx_p \end{bmatrix} \\ &= \left(a_1x_1 + ... + a_px_p\right) \begin{bmatrix} a_1\lambda_1x_1 \\ \vdots \\ a_p\lambda_px_p \end{bmatrix} \\ &= \lambda_1a_1^2 + ... + \lambda_pa_p^2 \end{aligned}$$

The last equality is true because the vectors $x_1, x_2, ..., x_p$ are orthonormal vectors.

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Step 3 of 6

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$$= \left(b_1 C' x_1 + \ldots + b_p C' x_p\right) \begin{bmatrix} \vdots \\ b_p A C x_p \end{bmatrix}$$

$$= \mu_1 b_1^2 + \ldots + \mu_p b_p^2$$

Provide feedback (0)

Step 4 of 6

Now the eigenvalues $?_i$ are assumed to be positive and the eigenvalues μ_i are assumed to be negative. Therefore, we get,

$$\boxed{\begin{aligned} \lambda_1 a_1^2 + \ldots + \lambda_p a_p^2 \geq 0 \\ \mu_1 b_1^2 + \ldots + \mu_p b_p^2 \leq 0 \end{aligned}}$$

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Step 5 of 6

(b) But, $\lambda_1 a_1^2 + \ldots + \lambda_p a_p^2 = \mu_1 b_1^2 + \ldots + \mu_p b_p^2$. Therefore, each must be equal to be zero. This is true only if all *a*'s are zero and all *b*'s are zero.

Therefore, the vectors $x_1, x_2, \ldots, x_p$ and $y_1, y_2, \ldots, y_q$ are linearly independent. If *A* is assumed to be an *n* by *n* matrix, then the number of its eigenvectors cannot exceed *n*. Therefore, we have

$$\boxed{p + q \leq n}$$

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Step 6 of 6

(c) Under the assumption that there is no zero eigenvalue, we can say that *A* has $n - p$ negative eigenvalues and $C^T AC$ has $n - q$ positive eigenvalues.

Arguing as above, we can say that $(n-p)+(n-q) \leq n$. This gives the following:

$$\begin{aligned} (n-p)+(n-q) &\leq n \\ 2n - p - q &\leq n \end{aligned}$$

$\boxed{p+q=n}$

Provide feedback (0)

Step 6 of 6

(c) Under the assumption that there is no zero eigenvalue, we can say that *A* has $n-p$ negative eigenvalues and $C^{\mathrm{T}}AC$ has $n-q$ positive eigenvalues.

Arguing as above, we can say that $(n-p)+(n-q)\le n$. This gives the following:

$$\begin{aligned}(n-p)+(n-q)&\le n\\ 2n-p-q&\le n\\ n&\le p+q\end{aligned}$$

Thus, we have $p+q\le n$ and $p+q\ge n$. This shows that $p+q=n$. This further gives $p=n-q$.

Recall that we have assumed that *A* has *p* positive eigenvalues and $C^{\mathrm{T}}AC$ has *q* negative eigenvalues. Also, we have assumed that there are no negative eigenvalues. Therefore, $C^{\mathrm{T}}AC$ has $n-q$ positive eigenvalues. Now we have shown that $p=n-q$. Therefore, the number of positive eigenvalues of *A* is equal to the number of positive eigenvalue of $C^{\mathrm{T}}AC$. By the same argument, the number of negative eigenvalues of *A* is equal to the number of negative eigenvalue of $C^{\mathrm{T}}AC$.

Thus, law of inertia is proved algebraically.

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Step 1 of 3

Given that C is non-singular.
We have to prove that A and $C^T AC$ have the same rank.
Here obviously C is square and the sizes of A and $C^T AC$ are same.
We know that

$$\operatorname{rank}\left(C^T AC\right) \le \operatorname{rank}(A) \qquad \ldots\ldots (1)$$

Since C is non-singular and C^{-1} exists, it is also non-singular.

Provide feedback (0)

Step 2 of 3

Now applying equation (1) to the matrix $C^T AC$ and the non-singular matrix C^{-1},

$$\begin{aligned}\operatorname{rank}\left(C^T AC\right) &\ge \operatorname{rank}\left(\left(C^{-1}\right)^T C^T AC\left(C^{-1}\right)\right)\\ &= \operatorname{rank}\left(\left(C^T\right)^{-1} C^T AC\left(C^{-1}\right)\right)\\ &= \operatorname{rank}(IAI)\\ &= \operatorname{rank}(A)\end{aligned}$$

Thus $\operatorname{rank}\left(C^T AC\right) \ge \operatorname{rank}(A) \qquad \ldots\ldots (2)$

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Step 3 of 3

From (1) and (2),
$\operatorname{rank}(A) = \operatorname{rank}\left(C^T AC\right)$.

Since the ranks of A and $C^T AC$ are same, then have the same number of zero Eigen values.

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CH6.2 39P

Step 1 of 7

Consider the following matrix:

$$A = \begin{bmatrix} I & B \\ B^{\mathrm{T}} & 0 \end{bmatrix}$$

Here B is a matrix of order $\frac{n}{2}$, such that B is a non singular matrix. Therefore, it is clear that n should be even.

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Step 2 of 7

The diagonal elements of A are 1, 1,..., 1, 0, 0,..., 0. The I block is symmetric and 0 block of zero matrix is also symmetric. Although B may or may not be symmetric, we are writing B in the upper right triangle and B^{T} in the lower left triangle. Therefore, it should be clear that A is a symmetric matrix.

Thus, the signs of the pivots of A agree with the signs of the eigenvalues of A.

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Step 3 of 7

Now while obtaining pivots of A, to obtain zeros below the I block, we will be multiplying the initial rows by some non zero constant and we will be subtracting them from the below rows.

This will create negative numbers in the diagonal along the 0 block.

Step 3 of 7

Now while obtaining pivots of *A*, to obtain zeros below the *I* block, we will be multiplying the initial rows by some non zero constant and we will be subtracting them from the below rows.

This will create negative numbers in the diagonal along the 0 block.

Thus, there should be $\frac{n}{2}$ positive pivots and $\frac{n}{2}$ negative pivots.

Provide feedback (0)

Step 4 of 7

Let $A=\begin{bmatrix}1 & 3\\ 3 & 0\end{bmatrix}$. Then, $\det(A-\lambda I)=0$ gives the following,

$$\begin{aligned}0&=\begin{vmatrix}1-\lambda & 3\\ 3 & -\lambda\end{vmatrix}\\ &=(1-\lambda)(-\lambda)-9\\ &=\lambda^2-\lambda-9\end{aligned}$$

Provide feedback (0)

Step 5 of 7

Therefore, we have

$$\lambda=\frac{1\pm\sqrt{1+36}}{2}$$

Provide feedback (0)

Step 5 of 7

Therefore, we have

$$\lambda = \frac{1 \pm \sqrt{1+36}}{2}$$

Thus, there is one positive eigenvalue and one negative eigenvalue.

Provide feedback (0)

Step 6 of 7

Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 7 \\ 2 & 4 & 0 & 0 \\ 3 & 7 & 0 & 0 \end{bmatrix}$

We obtain its eigenvalues by online eigenvalue calculator. See the screenshot below:

Calculator for Eigenvalues and Eigenvectors

Input the numbers of the matrix:

Thus, there are 2 positive eigenvalues and 2 negative eigenvalues.

Provide feedback (0)

Step 6 of 7

Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 7 \\ 2 & 4 & 0 & 0 \\ 3 & 7 & 0 & 0 \end{bmatrix}$

We obtain its eigenvalues by online eigenvalue calculator. See the screenshot below:

Calculator for Eigenvalues and Eigenvectors

Input the numbers of the matrix:

Thus, there are 2 positive eigenvalues and 2 negative eigenvalues.

Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 & 5 \\ 0 & 1 & 0 & -1 & -1 & 3 \\ 0 & 0 & 1 & 8 & 7 & 2 \\ 2 & -1 & 8 & 0 & 0 & 0 \\ 3 & -1 & 7 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 0 \end{bmatrix}$

We obtain its eigenvalues by online eigenvalue calculator. See the screenshot below:

Calculator for Eigenvalues and Eigenvectors

Thus, there are 2 positive eigenvalues and 2 negative eigenvalues.

Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 & 5 \\ 0 & 1 & 0 & -1 & -1 & 3 \\ 0 & 0 & 1 & 8 & 7 & 2 \\ 2 & -1 & 8 & 0 & 0 & 0 \\ 3 & -1 & 7 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 0 \end{bmatrix}$

We obtain its eigenvalues by online eigenvalue calculator. See the screenshot below:

Thus, here we find that there are 3 positive eigenvalues and 3 negative eigenvalues.

Provide feedback (0)

Step 7 of 7

Thus, for an n by n matrix of the form $\begin{bmatrix} I & B \\ B^{\mathrm{T}} & 0 \end{bmatrix}$, we find that it has $\frac{n}{2}$ positive and $\frac{n}{2}$ negative eigenvalues.

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Step 1 of 2

In order to satisfy law of inertia both the matrices A and $C^T AC$ have same size.
If A is not a square matrix then the size of matrices A and $C^T AC$ are not the same

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Step 2 of 2

For example,
If C is a 2×3 matrix and A is 2×2 matrix, then the size of the matrix $C^T AC$ is 3×3.
Thus the size of matrix A and matrix $C^T AC$ are not same.
Hence, if C is $\boxed{\text{Not square}}$, then A and $C^T AC$ $\boxed{\text{do not}}$ satisfy the law of inversion.

Therefore, the answer is $\boxed{\text{No}}$.

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Step 1 of 1

Consider the equation:

$$Ax = \lambda Mx$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} x = \lambda \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} x$$

$$\left\{ \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda m_1 & 0 \\ 0 & \lambda m_2 \end{bmatrix} \right\} x = 0$$

$$\begin{bmatrix} 2-\lambda m_1 & -1 \\ -1 & 2-\lambda m_2 \end{bmatrix} x = 0$$

By substituting $m_1 = 1$ and $m_2 = 2$, we get,

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-2\lambda \end{bmatrix} x = 0$$

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-2\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$(2-\lambda)x_1 - x_2 = 0$$

$$-x_1 + x_2(2-2\lambda) = 0$$

Therefore, the normal modes are M-orthogonal.

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Step 1 of 3

Given that $Ax = \lambda Mx$

$$\Rightarrow \begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix} x = \frac{\lambda}{18}\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} x$$

$$\Rightarrow \begin{pmatrix} 6-\frac{4\lambda}{18} & -3-\frac{\lambda}{18} \\ -3-\frac{\lambda}{18} & 6-\frac{4\lambda}{18} \end{pmatrix} x = 0$$

For Eigen values,

$$\begin{vmatrix} 6-\frac{4\lambda}{18} & -3-\frac{\lambda}{18} \\ -3-\frac{\lambda}{18} & 6-\frac{4\lambda}{18} \end{vmatrix} = 0$$

$$\Rightarrow \left(6-\frac{4\lambda}{18}-3-\frac{\lambda}{18}\right)\left(6-\frac{4\lambda}{18}+3+\frac{\lambda}{18}\right) = 0$$

$$\Rightarrow \left(3-\frac{5\lambda}{18}\right)\left(9-\frac{3\lambda}{18}\right) = 0$$

$$\Rightarrow \lambda_1 = \frac{54}{5},\ \lambda_2 = 54$$

Therefore, the Eigen values are $\boxed{\lambda_1 = \frac{54}{5} \text{ and } \lambda_2 = 54}$.

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Step 2 of 3

Let $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1 = \frac{54}{5}$.

Now,

$$\Rightarrow \begin{pmatrix} 6-\frac{4(54)}{18} & -3-\frac{54}{18} \\ -3-\frac{54}{18} & 6-\frac{4(54)}{18} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Let $x=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1=\frac{54}{5}$.

Now,

$$\Rightarrow \begin{pmatrix} 6-\frac{4(54)}{18} & -3-\frac{54}{18} \\ -3-\frac{54}{18} & 6-\frac{4(54)}{18} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -6 & -6 \\ -6 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1+x_2=0$$

Thus $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$ is the Eigen vector corresponding to $\lambda_2=54$.

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Step 3 of 3

Let $x=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1=\frac{54}{5}$.

$$\Rightarrow \begin{pmatrix} 6-\frac{4}{18}\cdot\frac{54}{5} & -3-\frac{1}{18}\cdot\frac{54}{5} \\ -3-\frac{1}{18}\cdot\frac{54}{5} & 6-\frac{4}{18}\cdot\frac{54}{5} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{18}{5} & -\frac{18}{5} \\ -\frac{18}{5} & \frac{18}{5} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1-x_2=0$$

Thus $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ is the Eigen vector corresponding to $\lambda_2=\frac{54}{5}$.

Therefore, the Eigen vectors are $\boxed{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$.

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Step 3 of 3

Let $x=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1=\frac{54}{5}$.

$$\Rightarrow \begin{pmatrix} 6-\frac{4}{18}\cdot\frac{54}{5} & -3-\frac{1}{18}\cdot\frac{54}{5} \\ -3-\frac{1}{18}\cdot\frac{54}{5} & 6-\frac{4}{18}\cdot\frac{54}{5} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{18}{5} & -\frac{18}{5} \\ -\frac{18}{5} & \frac{18}{5} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1-x_2=0$$

Thus $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ is the Eigen vector corresponding to $\lambda_2=\frac{54}{5}$.

Therefore, the Eigen vectors are $\boxed{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$.

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Step 1 of 3

Consider the equation $Ax = \lambda Mx$. We need to show that if the matrices A and M are symmetric indefinite, then the eigenvalues might not be real.

Consider the following:

$$\begin{aligned}
0 &= \det(A - \lambda M) \\
&= \det\left(\begin{bmatrix} a & b \\ b & d \end{bmatrix} - \lambda \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\right) \\
&= \det\left(\begin{bmatrix} a - \lambda m_1 & b \\ b & d - \lambda m_2 \end{bmatrix}\right) \\
&= (a - \lambda m_1)(d - \lambda m_2) - b^2 \\
&= ad - (am_2 + dm_1)\lambda + m_1 m_2 \lambda^2 - b^2 \\
&= m_1 m_2 \lambda^2 - (am_2 + dm_1)\lambda + (ad - b^2)
\end{aligned}$$

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Step 2 of 3

Consider the quadratic equation $m_1 m_2 \lambda^2 - (am_2 + dm_1)\lambda + (ad - b^2) = 0$. Since, we want complex eigenvalues, the roots of this equation should be complex numbers. We know that a quadratic equation $ax^2 + bx + c = 0$ doesn't have real roots if $b^2 - 4ac < 0$, that is, if $b^2 < 4ac$.

Consider the following:

$$\begin{aligned}
(am_2 + dm_1)^2 &< 4m_1 m_2 (ad - b^2) \\
a^2 m_2^2 + 2adm_1 m_2 + d^2 m_1^2 &< 4m_1 m_2 ad - 4m_1 m_2 b^2 \\
a^2 m_2^2 - 2adm_1 m_2 + d^2 m_1^2 &< -4m_1 m_2 b^2 \\
(am_2 - dm_1)^2 &< -4m_1 m_2 b^2
\end{aligned}$$

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$$a^2m_2^2+2adm_1m_2+d^2m_1^2<4m_1m_2ad-4m_1m_2b^2$$
$$a^2m_2^2-2adm_1m_2+d^2m_1^2<-4m_1m_2b^2$$
$$(am_2-dm_1)^2<-4m_1m_2b^2$$

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Step 3 of 3

Therefore, if we have $A=\begin{bmatrix} a & b \\ b & d \end{bmatrix}$ and $M=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ such that $(am_2-dm_1)^2<-4m_1m_2b^2$, then the equation $Ax=\lambda Mx$ will not have real eigenvalues. Consider the following example $A=\begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$, $M=\begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$.

Here we get

$$\begin{aligned}(am_2-dm_1)^2&=(3\times 2-(-3)\times(-2))^2\\&=(6-6)^2\\&=0\end{aligned}$$

$$\begin{aligned}-4m_1m_2b^2&=-4(-3)(2)(1)^2\\&=24\end{aligned}$$

Note that $(am_2-dm_1)^2<-4m_1m_2b^2$.

Thus, a required example is $\boxed{A=\begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}}$ and $\boxed{M=\begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}}$. Of course, we can create any number of such examples.

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