CH5.3

Step 1 of 8

Let following be the difference equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 8

Find the Eigen values and Eigen vectors of matrix A:

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (-1-\lambda)(-1-\lambda) - 1 &= 0 \\ \lambda^2 + 2\lambda &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= -2 \end{aligned}$$

Therefore, Eigen values are $\boxed{0, -2}$

Provide feedback (0)

CH5.4 2P

Provide feedback (0)

Step 3 of 8

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$

$$\begin{bmatrix} -1-0 & 1 \\ 1 & -1-0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=0$ is as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 8

Similarly, Eigen vectors corresponding to Eigen value $\lambda=-2$ is as follows:

$$(A-\lambda_2 I)x=0$$

$$\begin{bmatrix} -1+2 & 1 \\ 1 & -1+2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=-2$ is as follows:

$$x_2=\begin{bmatrix} y \\ z \end{bmatrix}$$

Provide feedback (0)

Step 4 of 8

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -2$ is as follows:

$$(A - \lambda_2 I)x = 0$$

$$\begin{bmatrix} -1+2 & 1 \\ 1 & -1+2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda = -2$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

Therefore Eigen values are:

$$x_1 = \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$x_2 = \boxed{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

Provide feedback (0)

Step 6 of 8

Step 6 of 8

Find the value of the exponential matrix e^{At}. Recall the following:

$$e^{At} = Se^{\Lambda t}S^{-1}$$

Here Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

Substitute the values in the above equation and solve.

Provide feedback (0)

Step 7 of 8

Matrix e^{At} can be written as follows:

$$\begin{aligned} e^{At} &= Se^{\Lambda t}S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{0t} \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} e^{-2t} & 1 \\ -e^{-2t} & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} e^{-2t}+1 & -e^{-2t}+1 \\ -e^{-2t}+1 & e^{-2t}+1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 8 of 8

Therefore, value of the exponential e^{At} matrix is:

CH5.4 1P

CH5.4 3P

Step 1 of 9

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Initial condition:

$$u(0) = (3,1)$$

Find the general and specific solution that matches the initial condition. Also, find the steady state when $t \to \infty$.

Provide feedback (0)

Step 2 of 9

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (-1-\lambda)(-1-\lambda) - 1 &= 0 \\ \lambda^2 + 2\lambda &= 0 \end{aligned}$$

After solving following values are obtained:

$$\lambda_1 = 0$$

Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}-1-\lambda & 1\\ 1 & -1-\lambda\end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(-1-\lambda)(-1-\lambda)-1=0$$
$$\lambda^2+2\lambda=0$$

After solving following values are obtained:

$$\lambda_1=0$$
$$\lambda_2=-2$$

Therefore, Eigen values are $0,-2$

Provide feedback (0)

Step 3 of 9

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}-1+2 & 1\\ 1 & -1+2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=-2$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$
$$=\begin{bmatrix}1\\ -1\end{bmatrix}$$

Step 4 of 9

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} -1-0 & 1 \\ 1 & -1-0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda = 0$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 9

Therefore Eigen values are:

$$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 9

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$x_1 = \begin{bmatrix} \\ -1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 9

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

Therefore, the general solution of the differential equation is:

$$u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$$

Or

$$\boxed{u(t) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 8 of 9

So, the solution for differential equation can be written as follows:

$$\begin{aligned}
u(t) &= Se^{\Lambda t}S^{-1}u(0) \\
&= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{0t} \end{bmatrix}\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{-2t} & 1 \\ -e^{-2t} & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{-2t}+1 & -e^{-2t}+1 \\ -e^{-2t}+1 & e^{-2t}+1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} 2e^{-2t}+4 \\ -2e^{-2t}+4 \end{bmatrix} \\
u(t) &= \begin{bmatrix} e^{-2t}+2 \\ -e^{-2t}+2 \end{bmatrix}
\end{aligned}$$

Provide feedback (0)

Step 9 of 9

Therefore, specific solution of the differential equation is:

$$\boxed{u(t) = \begin{bmatrix} e^{-2t}+2 \\ -e^{-2t}+2 \end{bmatrix}}$$

Steady state when $t \to \infty$ will be:

$$= \frac{1}{2}\begin{bmatrix} e^{-2t} & 1 \\ -e^{-2t} & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} e^{-2t}+1 & -e^{-2t}+1 \\ -e^{-2t}+1 & e^{-2t}+1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2e^{-2t}+4 \\ -2e^{-2t}+4 \end{bmatrix}$$

$$u(t) = \begin{bmatrix} e^{-2t}+2 \\ -e^{-2t}+2 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 9

Therefore, specific solution of the differential equation is:

$$\boxed{u(t) = \begin{bmatrix} e^{-2t}+2 \\ -e^{-2t}+2 \end{bmatrix}}$$

Steady state when $t \to \infty$ will be:

$$\boxed{u_\infty = \begin{bmatrix} 2 \\ 2 \end{bmatrix}}$$

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CH5.4 2P

CH5.4 4P

Step 1 of 9

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Initial condition:

$$u(0) = (3,1)$$

Find the specific solution that matches the initial condition. Also, show that it blows up instead of decaying as $t \to \infty$.

Provide feedback (0)

Step 2 of 9

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (1-\lambda)(1-\lambda) - 1 &= 0 \\ \lambda^2 - 2\lambda &= 0 \end{aligned}$$

After solving following values are obtained:

$$\lambda_1 = 2$$

Provide feedback (0)

Step 2 of 9

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}1-\lambda & -1\\ -1 & 1-\lambda\end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(1-\lambda)(1-\lambda)-1=0$$
$$\lambda^2-2\lambda=0$$

After solving following values are obtained:

$$\lambda_1=2$$
$$\lambda_2=0$$

Therefore, Eigen values are $0, 2$

Provide feedback (0)

Step 3 of 9

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}1-2 & -1\\ -1 & 1-2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=2$ are as follows:

$$x_1=\begin{bmatrix}y\end{bmatrix}$$

Provide feedback (0)

Step 3 of 9

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}1-2 & -1\\ -1 & 1-2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=2$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ -1\end{bmatrix}$$

Provide feedback (0)

Step 4 of 9

Similarly, Eigen vectors corresponding to Eigen value $\lambda=0$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}1-0 & -1\\ -1 & 1-0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of y and z corresponding to $\lambda=0$ is as follows:

$$= \begin{bmatrix} \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 9

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-0 & -1 \\ -1 & 1-0 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda = 0$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 9

Therefore Eigen values are:

$$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 9

Therefore Eigen values are:

$$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 9

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

Therefore, the general solution of the differential equation is:

$$\begin{aligned} u(t) &= c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2 \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \end{aligned}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 8 of 9

So, the solution for differential equation can be written as follows:

$$
\begin{aligned}
u(t) &= Se^{\Lambda t}S^{-1}u(0) \\
&= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{0t} \end{bmatrix}\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{2t} & 1 \\ -e^{2t} & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{2t}+1 & -e^{2t}+1 \\ -e^{2t}+1 & e^{2t}+1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} 2e^{2t}+4 \\ -2e^{2t}+4 \end{bmatrix} \\
u(t) &= \begin{bmatrix} e^{2t}+2 \\ -e^{2t}+2 \end{bmatrix}
\end{aligned}
$$

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Step 9 of 9

Therefore, specific solution of the differential equation is:

$$\boxed{u(t) = \begin{bmatrix} e^{2t}+2 \\ -e^{2t}+2 \end{bmatrix}}$$

When $t \to \infty$, e^{2t} becomes ∞. This blows up the solution of u_∞.

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CH5.4 3P

CH5.4 5P

Step 1 of 5

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Pu$$

Here, projection matrix P is defined as follows:

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

To show the following from the infinite series:

$$e^P = I + 1.718P$$

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Step 2 of 5

Recall the following:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \cdots$$

Substitute the following:

$$A = P$$
$$t = 1$$

Thus, e^{At} becomes:

$$e^P = I + P + \frac{(P)^2}{2!} + \cdots$$

$$e^P = I + P + \frac{(P)^2}{2!} + \cdots$$

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Step 3 of 5

A calculation on projection matrix shows the following:

$$P \cdot P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= P$$

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Step 4 of 5

Substitute the above result $P^2 = P$ in e^P:

$$\begin{aligned} e^P &= I + P + \frac{P}{2!} + \frac{P}{3!} \cdots \\ &= I + P\left(1 + \frac{1}{2!} + \frac{1}{3!} \cdots\right) \\ &= I + P\left(e^1 - 1\right) \\ &= I + P(2.718 - 1) \\ e^P &= I + 1.718P \end{aligned}$$

Provide feedback (0)

Provide feedback (0)

Step 4 of 5

Substitute the above result $P^2 = P$ in e^P:

$$\begin{aligned} e^P &= I + P + \frac{P}{2!} + \frac{P}{3!} \cdots \\ &= I + P\left(1 + \frac{1}{2!} + \frac{1}{3!} \cdots\right) \\ &= I + P\left(e^1 - 1\right) \\ &= I + P(2.718 - 1) \\ e^P &= I + 1.718P \end{aligned}$$

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Step 5 of 5

Therefore, $\boxed{e^P = I + 1.718P}$.

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STEP-BY-STEP SOLUTION

Step 1 of 10

Rule 1: $e^{\Lambda(t+T)} = e^{\Lambda t} e^{\Lambda T}$

Every diagonal entry of a diagonalize matrix satisfies the rule 1.

Step 2 of 10

(a) Explain why $e^{A(t+T)} = e^{At} e^{AT}$.

Recall the following;

$$e^{At} = Se^{\Lambda t} S^{-1}$$

Substitute this on the left hand side:

$$\begin{aligned} e^{A(t+T)} &= Se^{\Lambda(t+T)} S^{-1} \\ &= Se^{\Lambda t} e^{\Lambda T} S^{-1} \\ &= Se^{\Lambda t} S^{-1} Se^{\Lambda T} S^{-1} \\ &= e^{At} \cdot e^{AT} \end{aligned}$$

Step 3 of 10

Therefore, $\boxed{e^{A(t+T)} = e^{At} e^{AT}}$.

Step 4 of 10

(b) Let matrix A and B are defined as follows:

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

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Chapter: CH5.4 Problem: 5P

Step 4 of 10

(b) Let matrix A and B are defined as follows:

$$A=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$
$$B=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}$$

To show the following:

$$e^{A+B}\neq e^{A}e^{B}$$

Step 5 of 10

Recall the following:

$$e^{At}=I+At+\frac{(At)^2}{2!}+\cdots$$

Substitute $t=1$:

$$e^{A}=I+A+\frac{(A)^2}{2!}+\cdots$$

Step 6 of 10

The calculation on matrix A and B shows the following:

$$A\cdot A=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$
$$A^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$B\cdot B=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}$$
$$B^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Chapter: CH5.4 Problem: 5P

Step 6 of 10

The calculation on matrix A and B shows the following:

$$A \cdot A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B \cdot B = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 7 of 10

Substitute the above result in e^A:

$$\begin{aligned} e^A &= I + A + \frac{(A)^2}{2!} + \cdots \\ &= I + A \\ &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Similarly,

$$\begin{aligned} e^B &= I + B \\ &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Multiplication of these two matrices is:

$$\begin{aligned} e^A \cdot e^B &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

Step 8 of 10

$$=\begin{bmatrix}1 & -1\\ 1 & 0\end{bmatrix}$$

Chapter: CH5.4 Problem: 5P

Step 8 of 10

Calculate $A+B$:

$$A+B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

Since, in matrix $A+B$, trace is zero and determinant is 1. So, Eigen values are purely imaginary.

$$\det(A+B-\lambda I)=0$$
$$\begin{bmatrix}-\lambda & -1\\ 1 & -\lambda\end{bmatrix}=0$$
$$\lambda^2+1=0$$
$$\lambda=\pm i$$

Step 9 of 10

The Eigen vectors are $(1,i)$ and $(1,-i)$. The solution is:

$$e^{(A+B)t}=\frac{1}{2}e^{it}\begin{bmatrix}1\\ -i\end{bmatrix}+\frac{1}{2}e^{-it}\begin{bmatrix}1\\ i\end{bmatrix}$$

Substitute $\cos t\pm i\sin t$ for e^{it} and e^{-it}. Thus, $e^{(A+B)t}$ can be written as follows:

$$e^{(A+B)t}=\begin{bmatrix}\cos t & -\sin t\\ \sin t & \cos t\end{bmatrix}$$

For $t=1$:

$$e^{(A+B)}=\begin{bmatrix}\cos 1 & -\sin 1\\ \sin 1 & \cos 1\end{bmatrix}$$

This matrix is not equal to the product matrix of e^A and e^B.

Step 9 of 10

Chapter: CH5.4 Problem: 5P

The Eigen vectors are $(1,i)$ and $(1,-i)$. The solution is:

$$e^{(A+B)t} = \frac{1}{2}e^{it}\begin{bmatrix}1\\-i\end{bmatrix} + \frac{1}{2}e^{-it}\begin{bmatrix}1\\i\end{bmatrix}$$

Substitute $\cos t \pm i\sin t$ for e^{it} and e^{-it}. Thus, $e^{(A+B)t}$ can be written as follows:

$$e^{(A+B)t} = \begin{bmatrix}\cos t & -\sin t\\ \sin t & \cos t\end{bmatrix}$$

For $t=1$:

$$e^{(A+B)} = \begin{bmatrix}\cos 1 & -\sin 1\\ \sin 1 & \cos 1\end{bmatrix}$$

This matrix is not equal to the product matrix of e^A and e^B.

Step 10 of 10

Therefore, $\boxed{e^{A+B} \neq e^A e^B}$.

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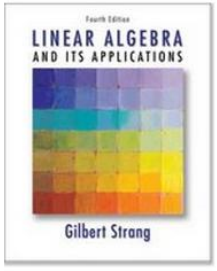

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CH5.4 5P

Step 1 of 5

Consider the higher order equation $y'' + y = 0$.

This equation written as a first order system by introducing the velocity y' as another unknown is given by

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} y' \\ -y \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

$$u' = Au$$

Therefore write this system in the form of $\frac{du}{dt} = Au$, using 2 by 2 matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Hence the matrix A is $\boxed{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}$.

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CH5.4 7P

Step 2 of 5

Consider $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

To find the eigenvalues of A, solve the characteristic equation $\det(A - \lambda I) = 0$.

$$\det\begin{bmatrix} 0-\lambda & 1 \\ -1 & 0-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\lambda = \pm i$$

Therefore, the eigenvalues of matrix A are $\lambda_1 = \boxed{-i}$ and $\lambda_2 = \boxed{i}$.

Provide feedback (0)

Step 2 of 5

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$$\lambda^2=-1$$
$$\lambda=\pm i$$

Therefore, the eigenvalues of matrix A are $\lambda_1=\boxed{-i}$ and $\lambda_2=\boxed{i}$.

Provide feedback (0)

Step 3 of 5

Find the eigenvector for $\lambda_1=-i$.

$$(A-\lambda_1 I)X=0$$
$$\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}-(-i)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$
$$\begin{bmatrix}i & 1\\ -1 & i\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

Add - i times first row to the second row

$$\begin{bmatrix}i & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Equation is $ix_1+x_2=0$

Add - i times first row to the second row

$$\begin{bmatrix} i & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Equation is $ix_1 + x_2 = 0$

Let $x_2 = k$

Therefore $x_1 = -k / i$

$= ik$

The solution (the first eigenvector) is any nonzero multiple of x_1, thus eigenvector for $\lambda_1 = -i$ is $x_1 = \boxed{\begin{bmatrix} 1 \\ i \end{bmatrix}}$.

Provide feedback (0)

Step 4 of 5

Find the eigenvector for $\lambda_1 = i$.

$$(A - \lambda_1 I)X = 0$$

$$\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - (i)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Add i times first row to the second row

$$\begin{bmatrix} -i & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Equation is $-ix_1 + x_2 = 0$

Let $x_2 = k$

Provide feedback (0)

Step 4 of 5

Find the eigenvector for $\lambda_1 = i$.

$$(A - \lambda_1 I)X = 0$$

$$\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - (i)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Add i times first row to the second row

$$\begin{bmatrix} -i & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Equation is $-ix_1 + x_2 = 0$

Let $x_2 = k$

Therefore $x_1 = k / i$

$= -ik$

The solution (the second eigenvector) is any nonzero multiple of x_2, thus eigenvector for $\lambda_2 = i$ is

$x_2 = \boxed{\begin{bmatrix} 1 \\ -i \end{bmatrix}}$.

Provide feedback (0)

Step 5 of 5

The eigenvectors are $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and the solution is given by

Provide feedback (0)

Step 5 of 5

The eigenvectors are $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and the solution is given by

$$u(t) = \frac{1}{2}e^{-it}\begin{bmatrix} 1 \\ i \end{bmatrix} + \frac{1}{2}e^{it}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} e^{-it} + e^{it} \\ ie^{-it} - ie^{it} \end{bmatrix}$$

By know that if $\lambda = a + ib$ then

$$e^{\lambda t} = e^{at}(\cos bt + i\sin bt)$$

Hence we have:

$$e^{it} = \cos t + i\sin t$$
$$e^{-it} = \cos t - i\sin t$$

By substituting $\cos t + i\sin t$ for e^{it} and $\cos t - i\sin t$ for e^{-it} we get:

$$u(t) = \frac{1}{2}\begin{bmatrix} e^{-it} + e^{it} \\ ie^{-it} - ie^{it} \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} \cos t - i\sin t + \cos t + i\sin t \\ i(\cos t - i\sin t) - i(\cos t + i\sin t) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$$
$$= \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

Therefore $u(t) = \boxed{\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}}$

$e^{it} = \cos t + i\sin t$

$e^{-it} = \cos t - i\sin t$

By substituting $\cos t + i\sin t$ for e^{it} and $\cos t - i\sin t$ for e^{-it} we get:

$$u(t) = \frac{1}{2}\begin{bmatrix} e^{-it} + e^{it} \\ ie^{-it} - ie^{it} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \cos t - i\sin t + \cos t + i\sin t \\ i(\cos t - i\sin t) - i(\cos t + i\sin t) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$$

$$= \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

Therefore $u(t) = \boxed{\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}}$

If we start form a differential $u(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, the solution $u(t)$ is given by

$$u(t) = \boxed{\begin{bmatrix} \frac{1}{2}\cos t & \sin t \\ -\frac{1}{2}\sin t & \cos t \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}}$$

Provide feedback (0)

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CH5.4 6P

CH5.4 8P

Step 1 of 4

We know that the matrix exponential for *n* by *n* matix is given by the series

$$e^{At} = I + At + \frac{(At)^2}{2!} + \ldots.$$

Consider a first-order system $\frac{du}{dt} = Au$ as:

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

Provide feedback (0)

Step 2 of 4

Since we have 2 by 2 matrix so matrix exponential e^{At} is as follows:

$$e^{At} = I + At$$

Substituting $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for *A* and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for *I* we get:

$$e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} t$$

Provide feedback (0)

Step 2 of 4

Since we have 2 by 2 matrix so matrix exponential e^{At} is as follows:

$$e^{At} = I + At$$

Substituting $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for *A* and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for *I* we get:

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} t \\ &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore, $e^{At} = \boxed{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}$.

Provide feedback (0)

Step 3 of 4

The solution $u(t) = e^{At}u(0)$, starting from $y(0) = 3$ and $y'(0) = 4$ is as follows:

$$\begin{aligned} u(t) &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\ &= \boxed{\begin{bmatrix} 3+4t \\ \end{bmatrix}} \end{aligned}$$

Provide feedback (0)

Step 3 of 4

The solution $u(t)=e^{At}u(0)$, starting from $y(0)=3$ and $y'(0)=4$ is as follows:

$$\begin{aligned} u(t) &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} \\ &= \boxed{\begin{bmatrix} 3+4t \\ 4 \end{bmatrix}} \end{aligned}$$

Provide feedback (0)

Step 4 of 4

Since we have $\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix}=\begin{bmatrix} y' \\ 0 \end{bmatrix}$, so we have

$$\begin{aligned} \frac{dy}{dt} &= y' \\ \frac{d^2y}{dt^2} &= y'' \\ &= 0 \end{aligned}$$

Hence, (y, y'), satisfied $\boxed{y''=0}$.

Provide feedback (0)

CH5.4 7P

CH5.4 9P

Step 1 of 10

(a) Consider the rabbit population *r* and the wolf population *w* is as follows:

$$\frac{dr}{dt} = 4r - 2w$$
$$\frac{dw}{dt} = r + w$$

We have to find whether this system is stable or neutrally stable or unstable.

We know that the differential equation $\frac{du}{dt} = Au$ is

stable, when all $\operatorname{Re}\lambda_i < 0$

neutrally stable, when all $\operatorname{Re}\lambda_i \le 0$ and $\operatorname{Re}\lambda_1 = 0$

unstable and e^{At} is unbounded if any eigenvalue has $\operatorname{Re}\lambda_i > 0$.

Provide feedback (0)

Step 2 of 10

Consider $P(t)$ is the population vector, define as follows:

$$P(t) = \begin{bmatrix} r(t) \\ w(t) \end{bmatrix}$$

Step 2 of 10

Consider $P(t)$ is the population vector, define as follows:

$$P(t)=\begin{bmatrix} r(t) \\ w(t) \end{bmatrix}$$

Using the population vector, we can write the system as

$$P'(t)=\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} P(t)$$

Let $A=\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$, we have to find eigenvalues of matrix A.

To find the eigenvalues of matrix A, we have to solve for $\det(A-\lambda I)=0$.

$$\begin{aligned} \det(A-\lambda I)&=0 \\ \begin{vmatrix} 4-\lambda & -2 \\ 1 & 1-\lambda \end{vmatrix}&=0 \\ (4-\lambda)(1-\lambda)-(-2)&=0 \\ (4-\lambda)(1-\lambda)+2&=0 \end{aligned}$$

By simplifying we get:

$$\begin{aligned} 4-4\lambda-\lambda+\lambda^2+2&=0 \\ 6-5\lambda+\lambda^2&=0 \end{aligned}$$

To find the eigenvalues of matrix *A*, we have to solve for $\det(A-\lambda I)=0$.

$$\begin{aligned}\det(A-\lambda I)&=0\\ \begin{vmatrix}4-\lambda & -2\\ 1 & 1-\lambda\end{vmatrix}&=0\\ (4-\lambda)(1-\lambda)-(-2)&=0\\ (4-\lambda)(1-\lambda)+2&=0\end{aligned}$$

By simplifying we get:

$$\begin{aligned}4-4\lambda-\lambda+\lambda^2+2&=0\\ 6-5\lambda+\lambda^2&=0\\ \lambda^2-5\lambda+6&=0\\ (\lambda-3)(\lambda-2)&=0\end{aligned}$$

This gives $\lambda_1=3$ and $\lambda_2=2$.

Provide feedback (0)

Step 3 of 10

Hence, the eigenvalues of matrix *A* are $\lambda_1=3$ and $\lambda_2=2$.

Since both eigenvalues are greater than 0, so the system is $\boxed{\text{unstable}}$.

Provide feedback (0)

Step 4 of 10

Step 4 of 10

(b) If initially $r = 300$ and $w = 200$, then we have to find the populations at time *t*.

We know that, suppose *A* can be diagonalized, $A = S\Lambda S^{-1}$, then $du/dt = Au$ has a solution

$$\begin{aligned} u(t) &= e^{At}u(0) \\ &= Se^{\Lambda t}S^{-1}u(0) \end{aligned}$$

Provide feedback (0)

Step 5 of 10

We know that the eigenvalues of matrix *A* are $\lambda_1 = 3$ and $\lambda_2 = 2$.

Now, to diagonalize the Matrix *A*, we will find the eigenvectors of *A*.

We know that for each eigenvalue solve the equation $(A - \lambda I)x = 0$. The determinant is zero, so the solutions are $x \neq 0$. Those are the eigenvectors.

Provide feedback (0)

Step 6 of 10

The nullspace contains a whole line of eigenvectors, so eigenvector for $\lambda_1 = 3$ is given by

$$(A - 3I)x = \begin{bmatrix} 1 & -2 \\ \end{bmatrix}\begin{bmatrix} y \\ \end{bmatrix}$$

Step 6 of 10

The nullspace contains a whole line of eigenvectors, so eigenvector for $\lambda_1 = 3$ is given by

$$\begin{aligned}(A-3I)x &= \begin{bmatrix}1 & -2\\ 1 & -2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}\\ &= \begin{bmatrix}0\\ 0\end{bmatrix}\end{aligned}$$

The solution (the first eigenvector) is any nonzero multiple of $\boldsymbol{x}_1$, thus eigenvector for $\lambda_1 = 3$ is $\boldsymbol{x}_1 = \begin{bmatrix}2\\ 1\end{bmatrix}$.

The nullspace contains a whole line of eigenvectors, so eigenvector for $\lambda_2 = 2$ is given by

$$\begin{aligned}(A-2I)x &= \begin{bmatrix}2 & -2\\ 1 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}\\ &= \begin{bmatrix}0\\ 0\end{bmatrix}\end{aligned}$$

The solution (the second eigenvector) is any nonzero multiple of $\boldsymbol{x}_1$, thus eigenvector for $\lambda_2 = 2$ is $\boldsymbol{x}_2 = \begin{bmatrix}1\\ 1\end{bmatrix}$.

Provide feedback (0)

Step 7 of 10

The eigenvector matrix of A is given by

The solution (the second eigenvector) is any nonzero multiple of $\boldsymbol{x}_1$, thus eigenvector for $\lambda_2 = 2$ is $\boldsymbol{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 7 of 10

The eigenvector matrix of *A* is given by

$S = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ And we have

$$S^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

The Eigen values matrix of *A* is given by

$$\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 10

Using the above matices we have to find $A = S\Lambda S^{-1}$.

$$\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 10

Using the above matices we have to find $A = S\Lambda S^{-1}$.

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{aligned} e^{At} &= Se^{\Lambda t}S^{-1} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{3t} & 0 \\ 0 & e^{2t} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 9 of 10

By using the equation $u(t) = e^{At}u(0)$, we have

$$\begin{aligned} P(t) &= e^{At}P(0) \\ &= \begin{bmatrix} 2 & 1 \\ \end{bmatrix}\begin{bmatrix} e^{3t} & 0 \\ \end{bmatrix}\begin{bmatrix} 1 & -1 \\ \end{bmatrix}\begin{bmatrix} 300 \\ \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 9 of 10

By using the equation $u(t)=e^{At}u(0)$, we have

$$\begin{aligned}P(t)&=e^{At}P(0)\\&=\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}e^{3t}&0\\0&e^{2t}\end{bmatrix}\begin{bmatrix}1&-1\\-1&2\end{bmatrix}\begin{bmatrix}300\\200\end{bmatrix}\\&=\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}e^{3t}&0\\0&e^{2t}\end{bmatrix}\begin{bmatrix}100\\100\end{bmatrix}\\&=\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}100e^{3t}\\100e^{2t}\end{bmatrix}\end{aligned}$$

$$P(t)=\begin{bmatrix}200e^{3t}+100e^{2t}\\100e^{3t}+100e^{2t}\end{bmatrix}$$

Therefore, when $r=300$ and $w=300$, the rabbit population *r* at time *t* is

$\boxed{r(t)=200e^{3t}+100e^{2t}}$ and the wolf population *w* at time *t* is $\boxed{w(t)=100e^{3t}+100e^{2t}}$.

Provide feedback (1)

Step 10 of 10

(c) If time is very long the value of e^{3t} is greater than the value of e^{2t} , so we can only consider the lager value.

Provide feedback (1)

Step 10 of 10

(c) If time is very long the value of e^{3t} is greater than the value of e^{2t}, so we can only consider the lager value.

Hence, the equation $P(t)=\begin{bmatrix}200e^{3t}+100e^{2t}\\100e^{3t}+100e^{2t}\end{bmatrix}$ can be written as

$$P(t)=\begin{bmatrix}200e^{3t}\\100e^{3t}\end{bmatrix}$$

So, for a long time, the proportion vector is approximately

$$\begin{aligned}P(t)&\approx\begin{bmatrix}200e^{3t}\\100e^{3t}\end{bmatrix}\\&\approx e^{3t}\begin{bmatrix}200\\100\end{bmatrix}\end{aligned}$$

Thus , when time is long, the rabbit population *r* at time *t* is $\boxed{r(t)=200e^{3t}}$ and the wolf population *w* at time *t* is $\boxed{w(t)=100e^{3t}}$.

Therefore, the population of rabbits is $\boxed{\text{double}}$ than the population of wolves.

Provide feedback (0)

CH5.4 8P

Step 1 of 10

We know that the differential equation $\frac{du}{dt} = Au$ is

stable, when all $\operatorname{Re}\lambda_i < 0$

neutrally stable, when all $\operatorname{Re}\lambda_i \le 0$ and $\operatorname{Re}\lambda_1 = 0$

unstable and e^{At} is unbounded if any Eigen values has $\operatorname{Re}\lambda_i > 0$.

Provide feedback (0)

Step 2 of 10

Consider $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, we have to find Eigen values of matrix *A*.

To find the Eigen values of matrix *A*, we have to solve for $\det(A - \lambda I) = 0$.

$$\begin{aligned} \det(A - \lambda I) &= 0 \\ \begin{vmatrix} 2-\lambda & 3 \\ 4 & 5-\lambda \end{vmatrix} &= 0 \\ (2-\lambda)(5-\lambda) - 12 &= 0 \\ \lambda^2 - 7\lambda - 2 &= 0 \end{aligned}$$

Provide feedback (0)

CH5.4 10P

Provide feedback (0)

Step 3 of 10

Solve the equation $\lambda^2 - 7\lambda - 2 = 0$, using the following formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{49+8}}{2}$$

$$= \frac{7 \pm \sqrt{57}}{2}$$

Solving the above equation, we get λ_1 and λ_2 as

$$\lambda_1 = \frac{7+\sqrt{57}}{2}$$

$$\lambda_2 = \frac{7-\sqrt{57}}{2}$$

Provide feedback (0)

Step 4 of 10

Hence, the Eigen values of matrix *A* are $\lambda_1 = \frac{7+\sqrt{57}}{2}$ and $\lambda_2 = \frac{7-\sqrt{57}}{2}$.

Provide feedback (0)

Step 4 of 10

Hence, the Eigen values of matrix *A* are $\lambda_1 = \frac{7+\sqrt{57}}{2}$ and $\lambda_2 = \frac{7-\sqrt{57}}{2}$.

Since both Eigen values are greater than 0, that is $\operatorname{Re}\lambda_i > 0$, so the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{unstable}}$.

Provide feedback (1)

Step 5 of 10

Consider $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, we have to find Eigen values of matrix *A*.

To find the Eigen values of matrix *A*, we have to solve for $\det(A - \lambda I) = 0$.

$$\begin{aligned} \begin{vmatrix} 1-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} &= 0 \\ (1-\lambda)(-1-\lambda) - 6 &= 0 \\ -1-\lambda+\lambda+\lambda^2-6 &= 0 \\ \lambda^2 &= 7 \end{aligned}$$

Provide feedback (0)

Step 6 of 10

Step 6 of 10

Taking square root of both side of equation $\lambda^2 = 7$, we get.

$$\lambda = \pm\sqrt{7}$$

Hence, the Eigen values of matrix *A* are $\lambda_1 = \sqrt{7}$ and $\lambda_2 = -\sqrt{7}$.

Since Eigen values $\lambda_1 = \sqrt{7}$ is greater than 0, that is $\operatorname{Re}\lambda_i > 0$, so the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{unstable}}$.

Consider $A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$, we have to find Eigen values of matrix *A*.

To find the Eigen values of matrix *A*, we have to solve for $\det(A - \lambda I) = 0$.

$$\begin{aligned} \det(A - \lambda I) &= 0 \\ \begin{bmatrix} 1-\lambda & 1 \\ 1 & -2-\lambda \end{bmatrix} &= 0 \\ (1-\lambda)(-2-\lambda) - 1 &= 0 \\ \lambda^2 + \lambda - 3 &= 0 \end{aligned}$$

Solve the equation $\lambda^2 + \lambda - 3 = 0$, using the following formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\lambda^2 + \lambda - 3 = 0$

Solve the equation $\lambda^2 + \lambda - 3 = 0$, using the following formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\lambda = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{13}}{2}$$

Solving the above equation, we get λ_1 and λ_2 as

$$\lambda_1 = \frac{-1 + \sqrt{13}}{2}$$
$$\lambda_2 = \frac{-1 - \sqrt{13}}{2}$$

Provide feedback (0)

Step 7 of 10

Hence, the Eigen values of matrix *A* are $\lambda_1 = \frac{-1 + \sqrt{13}}{2}$ and $\lambda_2 = \frac{-1 - \sqrt{13}}{2}$.

Since Eigen values $\lambda_1 = \frac{-1 + \sqrt{13}}{2}$ is greater than 0, that is $\operatorname{Re} \lambda_i > 0$, so the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{unstable}}$.

$$\lambda_2 = \frac{-1-\sqrt{13}}{2}$$

Provide feedback (0)

Step 7 of 10

Hence, the Eigen values of matrix A are $\lambda_1 = \frac{-1+\sqrt{13}}{2}$ and $\lambda_2 = \frac{-1-\sqrt{13}}{2}$.

Since Eigen values $\lambda_1 = \frac{-1+\sqrt{13}}{2}$ is greater than 0, that is $\operatorname{Re}\lambda_i > 0$, so the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{unstable}}$.

Provide feedback (0)

Step 8 of 10

Consider $A = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$, we have to find Eigen values of matrix A.

To find the Eigen values of matrix A, we have to solve for $\det(A - \lambda I) = 0$.

$$\begin{aligned} \begin{bmatrix} -1-\lambda & -1 \\ -1 & -1-\lambda \end{bmatrix} &= 0 \\ (-1-\lambda)(-1-\lambda)-1 &= 0 \\ (-1)(1+\lambda)(-1)(1+\lambda)-1 &= 0 \\ (1+\lambda)^2-1 &= 0 \end{aligned}$$

Step 8 of 10

Consider $A=\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}$, we have to find Eigen values of matrix *A*.

To find the Eigen values of matrix *A*, we have to solve for $\det(A-\lambda I)=0$.

$$\begin{aligned}\begin{bmatrix}-1-\lambda & -1\\ -1 & -1-\lambda\end{bmatrix}&=0\\ (-1-\lambda)(-1-\lambda)-1&=0\\ (-1)(1+\lambda)(-1)(1+\lambda)-1&=0\\ (1+\lambda)^2-1&=0\end{aligned}$$

Provide feedback (0)

Step 9 of 10

Solve the equation $(1+\lambda)^2-1=0$, to find the value of λ.

$$\begin{aligned}(1+\lambda)^2-1&=0\\ (1+\lambda)^2&=1\\ 1+\lambda&=\pm\sqrt{1}\end{aligned}$$

Solving the above equation, we get λ_1 and λ_2 as

$$\lambda_1=0$$

Step 9 of 10

Solve the equation $(1+\lambda)^2 - 1 = 0$, to find the value of λ.

$$(1+\lambda)^2 - 1 = 0$$
$$(1+\lambda)^2 = 1$$
$$1+\lambda = \pm\sqrt{1}$$

Solving the above equation, we get λ_1 and λ_2 as

$$\lambda_1 = 0$$
$$\lambda_2 = -2$$

Provide feedback (0)

Step 10 of 10

Hence, the Eigen values of matrix *A* are $\lambda_1 = 0$ and $\lambda_2 = -2$.

Since Eigen values $\lambda_1 = 0$ and $\lambda_2 = -2$, that is $\operatorname{Re}\lambda_i \le 0$ and $\operatorname{Re}\lambda_1 = 0$, so the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{neutrally stable}}$.

Provide feedback (0)

CH5.4 9P

Step 1 of 8

Consider the differential equations $\frac{dv}{dt} = w$ and $\frac{dw}{dt} = v$.

Let $u = \begin{bmatrix} u \\ v \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, so we can write

$$\frac{du}{dt} = Au$$

$$\frac{d}{dt}\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}$$

Provide feedback (1)

Step 2 of 8

To find the eigen values for matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, use the equation $\det(A - \lambda I) = 0$.

$$\det\begin{bmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

So, we get λ_1 and λ_2 as follows:

CH5.4 11P

$$\lambda_1 = 1$$
$$\lambda_2 = -1$$

Provide feedback (0)

Step 3 of 8

The eigenvector for $\lambda_1 = 1$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix} 0-1 & 1 \\ 1 & 0-1 \end{bmatrix} x_1 \\ &= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} x_1 \\ &= 0\end{aligned}$$

So, the eigenvector for $\lambda_1 = 1$ is $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 4 of 8

The eigenvector for $\lambda_2 = -1$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} 0+1 & 1 \\ 1 & 0+1 \end{bmatrix} x_2 \\ &= \begin{bmatrix} 1 & 1 \end{bmatrix} x\end{aligned}$$

Provide feedback (0)

Step 4 of 8

The eigenvector for $\lambda_2 = -1$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} 0+1 & 1 \\ 1 & 0+1 \end{bmatrix} x_2 \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x_2 \\ &= 0\end{aligned}$$

So, the eigenvector for $\lambda_2 = -1$ is $x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 5 of 8

We know that the differential equation $\frac{du}{dt} = Au$ is

stable and $e^{At} \to 0$, when all $\operatorname{Re}\lambda_i < 0$

neutrally stable, when all $\operatorname{Re}\lambda_i \le 0$ and $\operatorname{Re}\lambda_1 = 0$

unstable and e^{At} is unbounded if any eigenvalue has $\operatorname{Re}\lambda_i > 0$.

Provide feedback (0)

We know that the differential equation $\frac{du}{dt} = Au$ is

stable and $e^{At} \to 0$, when all $\operatorname{Re}\lambda_i < 0$

neutrally stable, when all $\operatorname{Re}\lambda_i \le 0$ and $\operatorname{Re}\lambda_1 = 0$

unstable and e^{At} is unbounded if any eigenvalue has $\operatorname{Re}\lambda_i > 0$.

Provide feedback (0)

Step 6 of 8

The eigenvalue $\lambda_1 = 1$ is greater than 0, that is $\operatorname{Re}\lambda_i > 0$.

So, the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{unstable}}$ and $e^{\lambda_1 t} \to \infty$.

The eigenvalue $\lambda_2 = -1$ is less than 0, that is $\operatorname{Re}\lambda_i < 0$.

So, the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{stable}}$ and $e^{\lambda_2 t} \to 0$.

Provide feedback (0)

Step 7 of 8

The eigenvector matrix of *A* is given by

$$S = \begin{bmatrix} 1 & -1 \\ \cdot & \cdot \end{bmatrix}$$

The eigenvalue $\lambda_2 = -1$ is less than 0, that is $\operatorname{Re}\lambda_i < 0$.

So, the differential equation $\frac{du}{dt} = Au$ is $\boxed{\text{stable}}$ and $e^{\lambda_2 t} \to 0$.

Provide feedback (0)

Step 7 of 8

The eigenvector matrix of *A* is given by

$$S = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

And we have

$$S^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The Eigen values matrix of *A* is given by

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Using the above matices we have to find $A = S\Lambda S^{-1}$.

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Using the above matices we have to find $A = S\Lambda S^{-1}$.

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{aligned} e^{At} &= Se^{\Lambda t}S^{-1} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{3t} & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 8 of 8

We know that any specific solution $u(t)$ is some combination of

$$u(t) = Se^{\Lambda t}S^{-1}u(0)$$

$$\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}0 & e^{t}\end{bmatrix}\begin{bmatrix}-\frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

Provide feedback (0)

Step 8 of 8

We know that any specific solution $u(t)$ is some combination of

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= c_1 e^{\lambda_1 t}x_1 + \cdots + c_n e^{\lambda_n t}x_n \end{aligned}$$

If one of them blows up, then except for very special starting conditions the solution will blow up.

So, the solution for this system $\boxed{\text{blows up}}$, unless $u(0) = x_2$.

Decaying to zero is given by $\boxed{u(t) = x_2 e^{-t}}$.

Provide feedback (0)

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CH5.4 10P

CH5.4 12P

Step 1 of 10

Stability: Real parts of the Eigen values govern the stability. The differential equation $du/dt = Au$ is:

Stable: If $\mathrm{Re}(\lambda_i) < 0$

Neutrally stable: If all $\mathrm{Re}(\lambda_i) \le 0$ and $\lambda_1 = 0$.

Unstable: If any Eigen value has $\mathrm{Re}(\lambda_i) > 0$.

Provide feedback (0)

Step 2 of 10

A matrix is stable if it passes two tests:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Tests:

The trace: $T = a + d$ must be negative.
The determinant: $D = ad - bc$ must be positive.

If Eigen values are complex numbers then determinant will be always positive and trace will be negative if real part of complex number is negative. So, negative trace means that the real part is negative and the matrix is stable.

Provide feedback (0)

Step 3 of 10

Evaluate the value of t for which the following matrix is stable and instable. Stability can be with real and complex Eigen values. Use the concept of trace and determinant of the matrix.

Let matrix A_1 is defined as follows:

If Eigen values are complex numbers then determinant will be always positive and trace will be negative if real part of complex number is negative. So, negative trace means that the real part is negative and the matrix is stable.

Provide feedback (0)

Step 3 of 10

Evaluate the value of t for which the following matrix is stable and instable. Stability can be with real and complex Eigen values. Use the concept of trace and determinant of the matrix.

Let matrix A_1 is defined as follows:

$$A_1 = \begin{bmatrix} 1 & -1 \\ t & -1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 10

For stability, trace and determinant of matrix is as follows:

Trace: $1-1=0$
Determinant: $-1+t>0$ or $t>1$.

As trace is zero so matrix will be neutrally stable.

Therefore, A_1 will be neutrally stable if $\boxed{t \geq 1}$ and unstable if $\boxed{t<1}$.

Provide feedback (0)

Step 5 of 10

Let another matrix A_2 is defined as follows:

$$A_2 = \begin{bmatrix} 0 & 4-t \\ 1 & -2 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 10

Let another matrix A_2 is defined as follows:

$$A_2 = \begin{bmatrix} 0 & 4-t \\ 1 & -2 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

For stability, trace and determinant of matrix is as follows:

Trace:

$$\begin{aligned} 0-2 &= -2 \\ &< 0 \end{aligned}$$

Determinant: $-4+t>0$ or $t>4$.

As trace is negative so matrix is stable.

Provide feedback (0)

Step 7 of 10

Therefore, A_2 will be neutrally stable if $\boxed{t=4}$ and unstable if $\boxed{t<4}$. Also, it will be stable if $\boxed{t>4}$.

Provide feedback (0)

Step 8 of 10

Let another matrix A_3 is defined as follows:

$$\begin{bmatrix} t & -1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 10

Let another matrix A_3 is defined as follows:

$$A_3 = \begin{bmatrix} t & -1 \\ 1 & t \end{bmatrix}$$

Provide feedback (0)

Step 9 of 10

For stability, trace and determinant of matrix is as follows:

Trace:

$$\begin{aligned} t + t &= 2t \\ &< 0 \end{aligned}$$

Determinant: $t^2 + 1 > 0$.

As determinant is positive so matrix can be stable.

Provide feedback (0)

Step 10 of 10

Therefore, A_3 will be unstable if $\boxed{t > 0}$ and it will be stable if $\boxed{t < 0}$.

Provide feedback (0)

Algebra tutors who can help right now

CH5.4 11P

Step 1 of 12

Orthogonal Matrix: If matrix A is skew- symmetric then e^{At} is an orthogonal matrix.

Skew symmetric: If transpose of matrix A $\left(A^T\right)$ is equal to negative of matrix A, then matrix A is skew-symmetric.

In a conservative system following are observed:

$$A^T = -A$$
$$\left(e^{At}\right)^T = e^{-At}$$
$$\left\|e^{At}u(0)\right\| = \left\|u(0)\right\|$$

Provide feedback (0)

Step 2 of 12

Compute the Eigen values and Eigen vectors of the following matrix:

$$A = \begin{bmatrix} 0 & -3 & 0 \\ 3 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$$

Without any computation comment on the orthogonality of e^{At} and $\left\|u(t)\right\|^2 = u_1^2 + u_2^2 + u_3^2$ will be constant.

Provide feedback (0)

Step 3 of 12

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{bmatrix} 0-\lambda & -3 & 0 \end{bmatrix}$$

CH5.4 13P

Provide feedback (0)

Step 3 of 12

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}0-\lambda & -3 & 0\\ 3 & 0-\lambda & -4\\ 0 & 4 & 0-\lambda\end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(-\lambda)^3+25(-\lambda)=0$$

$$\lambda(\lambda^2+25)=0$$

After solving following values are obtained:

$$\lambda_1=0$$
$$\lambda_2=5i$$
$$\lambda_3=-5i$$

Therefore, Eigen values are $\boxed{0,5i,-5i}$.

Provide feedback (0)

Step 4 of 12

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}0-\lambda & -3 & 0\\ 3 & 0-\lambda & -4\\ 0 & 4 & 0-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & -3 & 0\\ 3 & 0 & -4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 4 of 12

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 0-\lambda & -3 & 0 \\ 3 & 0-\lambda & -4 \\ 0 & 4 & 0-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 & 0 \\ 3 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving, values of x, y and z corresponding to $\lambda=0$ are as follows:

$$x_1=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 12

Similarly, Eigen vectors corresponding to Eigen value $\lambda=5i$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 0-\lambda & -3 & 0 \\ 3 & 0-\lambda & -4 \\ 0 & 4 & 0-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0-5i & -3 & 0 \end{bmatrix}\begin{bmatrix} x \end{bmatrix}\begin{bmatrix} 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 12

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 5i$ is as follows:

$$(A-\lambda I)x = 0$$

$$\begin{bmatrix} 0-\lambda & -3 & 0 \\ 3 & 0-\lambda & -4 \\ 0 & 4 & 0-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0-5i & -3 & 0 \\ 3 & 0-5i & -4 \\ 0 & 4 & 0-5i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 12

On solving values of x, y and z are as follows:

$$x_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -3/4 \\ 5i/4 \\ 1 \end{bmatrix}$$

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -5i$ is as follows:

$$(A-\lambda I)x = 0$$

$$\begin{bmatrix} 0-\lambda & -3 & 0 \\ 3 & 0-\lambda & -4 \\ 0 & 4 & 0-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0+5i & -3 & 0 \end{bmatrix}\begin{bmatrix} x \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3/4 \\ 5i/4 \\ 1 \end{bmatrix}$$

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -5i$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 0-\lambda & -3 & 0 \\ 3 & 0-\lambda & -4 \\ 0 & 4 & 0-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0+5i & -3 & 0 \\ 3 & 0+5i & -4 \\ 0 & 4 & 0+5i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving values of x, y and z are as follows:

$$x_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -3/4 \\ -5i/4 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 12

Therefore Eigen values are:

$$x_1 = \boxed{\begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 12

Therefore Eigen values are:

$$x_1 = \boxed{\begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}}$$

$$x_2 = \boxed{\begin{bmatrix} -3/4 \\ 5i/4 \\ 1 \end{bmatrix}}$$

$$x_3 = \boxed{\begin{bmatrix} -3/4 \\ -5i/4 \\ 1 \end{bmatrix}}$$

Provide feedback (0)

Step 8 of 12

Next:

$$\begin{aligned} A^T &= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 4 \\ 0 & -4 & 0 \end{bmatrix} \\ &= -A \end{aligned}$$

Matrix A is skew-symmetric so, e^{At} will be an orthogonal matrix.

Provide feedback (0)

Provide feedback (0)

Step 9 of 12

Now,

$$\|u(t)\|^2 = u_1^2 + u_2^2 + u_3^2$$

Differentiate the following:

$$\frac{d\|u(t)\|^2}{dt} = 2u_1u_1' + 2u_2u_2' + 2u_3u_3'$$

Provide feedback (0)

Step 10 of 12

Do the following calculations for right hand side as given below:

$$\begin{aligned}\frac{du}{dt} &= Au \\ &= \begin{bmatrix} 0 & -3 & 0 \\ 3 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\end{aligned}$$

Calculate the following:

$$\begin{aligned} u_1' &= -3u_2 \\ u_2' &= 3u_1 - 4u_3 \\ u_3' &= 4u_2 \end{aligned}$$

Provide feedback (0)

Step 11 of 12

$$u_2' = 3u_1 - 4u_3$$
$$u_3' = 4u_2$$

Provide feedback (0)

Step 11 of 12

Now,

$$u_1u_1' = -3u_2u_1$$
$$u_2u_2' = 3u_1u_2 - 4u_3u_2$$
$$u_3u_3' = 4u_2u_2$$
$$u_1u_1' + u_2u_2' + u_3u_3' = 0$$

Substitute the above result, to get derivate of $\|u(t)\|^2$ equal to zero. This shows that $\|u(t)\|^2$ is constant.

Recall that $u(t) = e^{At}u(0)$, then

$$\begin{aligned}\|u(t)\|^2 &= \|e^{At}u(0)\|^2 \\ &= \|u(0)\|^2 \\ &= \text{constant}\end{aligned}$$

Provide feedback (0)

Step 12 of 12

Therefore, $\boxed{\|u(t)\|^2 = \|u(0)\|^2}$

Provide feedback (0)

CH5.4 12P

Step 1 of 9

Orthogonal Matrix: If matrix A is skew- symmetric then e^{At} is an orthogonal matrix.

Skew symmetric: If transpose of matrix A $\left(A^T\right)$ is equal to negative of matrix A, then matrix A is skew-symmetric.

In a conservative system following are observed:

$$A^T = -A$$
$$\left(e^{At}\right)^T = e^{-At}$$
$$\left\|e^{At}u(0)\right\| = \left\|u(0)\right\|$$

Provide feedback (0)

Step 2 of 9

Consider the skew-symmetric equation:

$$\frac{du}{dt} = Au$$
$$= \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 9

(a) Compute u_1', u_2', u_3' and confirm the following:

$$u_1u_1' + u_2u_2' + u_3u_3' = 0$$

Calculate the following from the skew-symmetric equation

CH5.4 14P

Step 3 of 9

(a) Compute u_1', u_2', u_3' and confirm the following:

$$u_1u_1' + u_2u_2' + u_3u_3' = 0$$

Calculate the following from the skew-symmetric equation

$$u_1' = cu_2 - bu_3$$
$$u_2' = -cu_1 + au_3$$
$$u_3' = bu_1 - au_2$$

Provide feedback (0)

Step 4 of 9

Now,

$$u_1u_1' = cu_2u_1 - bu_3u_1$$
$$u_2u_2' = -cu_1u_2 + au_3u_2$$
$$u_3u_3' = bu_1u_3 - au_2u_3$$
$$u_1u_1' + u_2u_2' + u_3u_3' = 0$$

Therefore, $\boxed{u_1u_1' + u_2u_2' + u_3u_3' = 0}$.

Provide feedback (0)

Step 5 of 9

(b) Evaluate the length of $u_1^2 + u_2^2 + u_3^2$.

Length of $u_1^2 + u_2^2 + u_3^2$ can be written as $\|u(t)\|^2$. Now,

Step 4 of 9

Now,

$$u_1u_1' = cu_2u_1 - bu_3u_1$$
$$u_2u_2' = -cu_1u_2 + au_3u_2$$
$$u_3u_3' = bu_1u_3 - au_2u_3$$
$$u_1u_1' + u_2u_2' + u_3u_3' = 0$$

Therefore, $\boxed{u_1u_1' + u_2u_2' + u_3u_3' = 0}$.

Provide feedback (0)

Step 5 of 9

(b) Evaluate the length of $u_1^2 + u_2^2 + u_3^2$.

Length of $u_1^2 + u_2^2 + u_3^2$ can be written as $\|u(t)\|^2$. Now,

$$\|u(t)\|^2 = u_1^2 + u_2^2 + u_3^2$$

Differentiate the following:

$$\frac{d\|u(t)\|^2}{dt} = 2u_1u_1' + 2u_2u_2' + 2u_3u_3'$$

Substitute the above result, to get derivate of $\|u(t)\|^2$ equal to zero. This shows that $\|u(t)\|^2$ is constant.

Provide feedback (0)

Provide feedback (0)

Step 6 of 9

Recall that $u(t)=e^{At}u(0)$, then

$$\begin{aligned}\|u(t)\|^2 &= \|e^{At}u(0)\|^2\\ &= \|u(0)\|^2\\ &= \text{constant}\end{aligned}$$

Therefore, $\boxed{\|u(t)\|^2=\|u(0)\|^2}$.

Provide feedback (0)

Step 7 of 9

(c) Find the Eigen values of matrix A.

To calculate the Eigen values do the following calculations;

Provide feedback (0)

Step 8 of 9

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}0-\lambda & c & -b\\ -c & 0-\lambda & a\\ b & -a & 0-\lambda\end{bmatrix}\\ \det(A-\lambda I)&=0\\ (-\lambda)^3-\lambda\left(a^2+b^2+c^2\right)&=0\\ -\lambda\left(\lambda^2+\left(a^2+b^2+c^2\right)\right)&=0\end{aligned}$$

After solving following values are obtained:

Provide feedback (0)

Step 7 of 9

(c) Find the Eigen values of matrix A.

To calculate the Eigen values do the following calculations;

Provide feedback (0)

Step 8 of 9

$$A-\lambda I=\begin{bmatrix}0-\lambda & c & -b\\ -c & 0-\lambda & a\\ b & -a & 0-\lambda\end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(-\lambda)^3-\lambda\left(a^2+b^2+c^2\right)=0$$

$$-\lambda\left(\lambda^2+\left(a^2+b^2+c^2\right)\right)=0$$

After solving following values are obtained:

$$\lambda_1=0$$

$$\lambda_2=\sqrt{\left(a^2+b^2+c^2\right)}$$

$$\lambda_3=-\sqrt{\left(a^2+b^2+c^2\right)}$$

Provide feedback (0)

Step 9 of 9

Therefore, Eigen values are $\boxed{0,\pm\sqrt{\left(a^2+b^2+c^2\right)}}$.

Provide feedback (0)

CH5.4 13P

CH5.4 15P

Step 1 of 7

Consider the second-order equation

$$\frac{d^2u}{dt^2}=\begin{bmatrix}-5 & 4\\ 4 & -5\end{bmatrix}u$$

We know that the if A has negative eigenvalues $\lambda_1,\ldots,\lambda_n$ and if $\omega_j=\sqrt{-\lambda_j}$, then general solution of $\frac{d^2u}{dt^2}=Au$ is given by

$$u(t)=(a_1\cos\omega_1 t+b_1\sin\omega_1 t)x_1+\cdots+(a_n\cos\omega_n t+b_n\sin\omega_n t)x_n$$

Provide feedback (0)

Step 2 of 7

To find the eigenvalues for matrix $A=\begin{bmatrix}-5 & 4\\ 4 & -5\end{bmatrix}$, by using the equation

$\det(A-\lambda I)=0$.

$$\begin{aligned}\det(A-\lambda I)&=0\\ \det\begin{bmatrix}-5-\lambda & 4\\ 4 & -5-\lambda\end{bmatrix}&=0\\ (-5-\lambda)(-5-\lambda)-(16)&=0\end{aligned}$$

Provide feedback (0)

Step 2 of 7

To find the eigenvalues for matrix $A=\begin{bmatrix}-5 & 4\\ 4 & -5\end{bmatrix}$, by using the equation

$\det(A-\lambda I)=0$.

$$\begin{aligned}\det(A-\lambda I)&=0\\ \det\begin{bmatrix}-5-\lambda & 4\\ 4 & -5-\lambda\end{bmatrix}&=0\\ (-5-\lambda)(-5-\lambda)-(16)&=0\\ (-5-\lambda)^2-16&=0\end{aligned}$$

Provide feedback (0)

Step 3 of 7

Solve the $(-5-\lambda)^2-16=0$, to find the value of λ.

$$\begin{aligned}(-5-\lambda)^2&=16\\ -5-\lambda&=\pm\sqrt{16}\\ -5-\lambda&=\pm 4\\ -5-\lambda&=-4 \text{ or } -5-\lambda=4\end{aligned}$$

So, we get λ_1 and λ_2 as

Solve the $(-5-\lambda)^2-16=0$, to find the value of λ.

$$(-5-\lambda)^2=16$$
$$-5-\lambda=\pm\sqrt{16}$$
$$-5-\lambda=\pm 4$$
$$-5-\lambda=-4 \text{ or } -5-\lambda=4$$

So, we get λ_1 and λ_2 as

$$\lambda_1=-1$$
$$\lambda_2=-3$$

Provide feedback (2)

Step 4 of 7

Therefore, the eigenvalues for matrix A are $\lambda_1=\boxed{-1}$ and $\lambda_2=\boxed{-3}$.

Provide feedback (1)

Step 5 of 7

The eigenvector for $\lambda_1=-1$ is given by

$$(A-\lambda_1 I)x_1=\begin{bmatrix}-5+1 & -1\\ -1 & -5+1\end{bmatrix}x_1$$
$$\begin{bmatrix}-4 & 4\end{bmatrix}x_1$$

Provide feedback (1)

Step 5 of 7

The eigenvector for $\lambda_1 = -1$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix} -5+1 & -1 \\ -1 & -5+1 \end{bmatrix} x_1 \\ &\begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix} x_1 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_1 = -1$ is given by $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 6 of 7

The eigenvector for $\lambda_2 = -9$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} -5+9 & 4 \\ 4 & -5+9 \end{bmatrix} x_2 \\ &\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} x_2 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_2 = -9$ is given by $x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Step 7 of 7

We know that the frequencies $\omega_j = \sqrt{-\lambda_j}$.

Substituting -1 for λ_1 in equation $\omega_1 = \sqrt{-\lambda_1}$, we get

$$\begin{aligned}\omega_1 &= \sqrt{-(-1)}\\ &= 1\end{aligned}$$

Similarly substituting -9 for λ_2 in equation $\omega_2 = \sqrt{-\lambda_2}$, we get

$$\begin{aligned}\omega_2 &= \sqrt{-(-9)}\\ &= \sqrt{9}\\ &= 3\end{aligned}$$

Therefore, the frequencies are $\omega_1 = \boxed{1}$ and $\omega_2 = \boxed{3}$.

Since we have

$$\begin{aligned}x_1 &= \begin{bmatrix}1\\1\end{bmatrix}\\ x_2 &= \begin{bmatrix}-1\\1\end{bmatrix}\\ \omega_1 &= 1\\ \omega_2 &= 3\end{aligned}$$

$= 3$

Therefore, the frequencies are $\omega_1 = \boxed{1}$ and $\omega_2 = \boxed{3}$.

Since we have

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$\omega_1 = 1$$
$$\omega_2 = 3$$

Therefore, the general solution of $\frac{d^2u}{dt^2} = \begin{bmatrix} -5 & 4 \\ 4 & -5 \end{bmatrix} u$ is

$$u(t) = \boxed{(a_1 \cos t + b_1 \sin t)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (a_2 \cos 3t + b_2 \sin 3t)\begin{bmatrix} -1 \\ 1 \end{bmatrix}}.$$

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Step 1 of 9

Consider the second-order equation

$$\frac{d^2u}{dt^2}=\begin{bmatrix}-5 & -1\\ -1 & -5\end{bmatrix}u$$

With the following conditions

$$u(0)=\begin{bmatrix}1\\0\end{bmatrix}$$
$$u'(0)=\begin{bmatrix}0\\0\end{bmatrix}$$

We know that the if A has negative eigenvalues $\lambda_1,\ldots,\lambda_n$ and if $\omega_j=\sqrt{-\lambda_j}$, then solution of $\frac{d^2u}{dt^2}=Au$ is given by

$$u(t)=(a_1\cos\omega_1 t+b_1\sin\omega_1 t)x_1+\cdots+(a_n\cos\omega_n t+b_n\sin\omega_n t)x_n$$

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Step 2 of 9

To find the eigenvalues for matrix $A=\begin{bmatrix}-5 & -1\\ -1 & -5\end{bmatrix}$, by using the equation

Step 2 of 9

To find the eigenvalues for matrix $A=\begin{bmatrix}-5 & -1\\ -1 & -5\end{bmatrix}$, by using the equation

$\det(A-\lambda I)=0$.

$$\begin{aligned}\det(A-\lambda I)&=0\\ \det\begin{bmatrix}-5-\lambda & -1\\ -1 & -5-\lambda\end{bmatrix}&=0\\ (-5-\lambda)(-5-\lambda)-(1)&=0\\ (-5-\lambda)^2-1&=0\end{aligned}$$

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Step 3 of 9

Solve the $(-5-\lambda)^2-1=0$, to find the value of λ.

$$\begin{aligned}(-5-\lambda)^2&=1\\ -5-\lambda&=\pm\sqrt{1}\\ -5-\lambda&=\pm 1\\ -5-\lambda&=-1 \text{ or } -5-\lambda=1\end{aligned}$$

So, we get λ_1 and λ_2 as

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Step 3 of 9

Solve the $(-5-\lambda)^2 - 1 = 0$, to find the value of λ.

$$\begin{aligned}(-5-\lambda)^2 &= 1\\ -5-\lambda &= \pm\sqrt{1}\\ -5-\lambda &= \pm 1\\ -5-\lambda &= -1 \text{ or } -5-\lambda = 1\end{aligned}$$

So, we get λ_1 and λ_2 as

$$\begin{aligned}\lambda_1 &= -4\\ \lambda_2 &= -6\end{aligned}$$

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Step 4 of 9

The eigenvector for $\lambda_1 = -4$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix}-5+4 & -1\\ -1 & -5+4\end{bmatrix}x_1\\ &\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}x_1\\ &= 0\end{aligned}$$

$\lambda_2 = -6$

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Step 4 of 9

The eigenvector for $\lambda_1 = -4$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix} -5+4 & -1 \\ -1 & -5+4 \end{bmatrix} x_1 \\ &\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} x_1 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_1 = -4$ is given by $x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

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Step 5 of 9

The eigenvector for $\lambda_2 = -6$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} -5+6 & -1 \\ -1 & -5+6 \end{bmatrix} x_2 \\ &\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} x_2 \\ &= 0\end{aligned}$$

$$= 0$$

The eigenvector for $\lambda_2 = -6$ is given by $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

We know that $a = S^{-1}u(0)$, here

$$S = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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Step 6 of 9

So, we get

$$\begin{aligned} a &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \end{aligned}$$

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Step 7 of 9

Step 7 of 9

Thus, $a_1 = \frac{1}{2}$ and $a_2 = \frac{1}{2}$.

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Step 8 of 9

We know that $\omega_j = \sqrt{-\lambda_j}$.

Substituting -4 for λ_1 in equation $\omega_1 = \sqrt{-\lambda_1}$, we get

$$\begin{aligned}\omega_1 &= \sqrt{-(-4)}\\ &= 2\end{aligned}$$

Similarly substituting -6 for λ_2 in equation $\omega_2 = \sqrt{-\lambda_2}$, we get

$$\begin{aligned}\omega_2 &= \sqrt{-(-6)}\\ &= \sqrt{6}\end{aligned}$$

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Step 9 of 9

Since $u'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, so all the terms $b_n \sin \omega_n t = 0$, thus we have the following solution:

Similarly substituting -6 for λ_2 in equation $\omega_2 = \sqrt{-\lambda_2}$, we get

$$\begin{aligned}\omega_2 &= \sqrt{-(-6)} \\ &= \sqrt{6}\end{aligned}$$

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Step 9 of 9

Since $u'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, so all the terms $b_n \sin \omega_n t = 0$, thus we have the following solution:

$$u(t) = (a_1 \cos \omega_1 t) x_1 + \cdots + (a_n \cos \omega_n t) x_n$$

Therefore, the solution form $u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is $u(t) = \boxed{\left(\frac{1}{2}\cos 2t\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \left(\frac{1}{2}\cos \sqrt{6}t\right)\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$.

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Step 1 of 4

Consider a pure exponential equation:

$$u = xe^{i\omega t}$$

Substitute this equation in the following equation and find a generalised Eigen value problem for the frequency ω.

$$Mu'' + Ku = 0$$

Here, M is a mass matrix.

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Step 2 of 4

Do the following calculations:

$$\begin{aligned} u &= xe^{i\omega t} \\ u' &= i\omega xe^{i\omega t} \\ u'' &= -\omega^2 xe^{i\omega t} \end{aligned}$$

Substitute these values in the following equation and solve:

$$\begin{aligned} Mu'' + Ku &= 0 \\ -\omega^2 xe^{i\omega t} M + Kxe^{i\omega t} &= 0 \\ -\omega^2 xM + Kx &= 0 \\ \omega^2 x &= KM^{-1}x \end{aligned}$$

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Step 3 of 4

Assume the following:

$$Mu'' + Ku = 0$$
$$-\omega^2 x e^{i\omega t} M + K x e^{i\omega t} = 0$$
$$-\omega^2 x M + K x = 0$$
$$\omega^2 x = K M^{-1} x$$

Provide feedback (0)

Step 3 of 4

Assume the following:

$$KM^{-1} = A$$

Above equation can be written as follows:

$$Ax = \omega^2 x$$
$$\left(A - \omega^2 I\right) x = 0$$

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Step 4 of 4

Therefore, generalised Eigen value problem that must be solved for the frequency ω and the vector x is:

$$\boxed{Ax = \omega^2 x}$$

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Step 1 of 3

Consider a pure exponential equation:

$$u = xe^{\lambda t}$$

Substitute this equation in the following equation and find a quadratic Eigen value problem for λ.

$$u'' + Fu' - Au = 0$$

Here, F is a friction matrix.

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Step 2 of 3

Loading solution. Please wait ...

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Step 3 of 3

Therefore, quadratic Eigen value problem is:

$$\boxed{Ax = \left(\lambda^2 I + F\lambda\right)x}$$

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Step 2 of 3

Do the following calculations:

$$u = xe^{\lambda t}$$
$$u' = \lambda xe^{\lambda t}$$
$$u'' = \lambda^2 xe^{\lambda t}$$

Substitute these values in the following equation and solve:

$$u'' + Fu' - Au = 0$$
$$\lambda^2 xe^{\lambda t} + F\lambda xe^{\lambda t} - Axe^{\lambda t} = 0$$
$$\lambda^2 x + F\lambda x - Ax = 0$$

Above equation can be written as follows:

$$Ax = \lambda^2 x + F\lambda x$$
$$Ax = \left(\lambda^2 I + F\lambda\right)x$$
$$\left(A - \left(\lambda^2 I + F\lambda\right)\right)x = 0$$

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Step 3 of 3

Therefore, quadratic Eigen value problem is:

$$\boxed{Ax = \left(\lambda^2 I + F\lambda\right)x}$$

Provide feedback (0)

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Step 1 of 10

Consider the second-order equation

$$\frac{d^2u}{dt^2}=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}u$$

With the following conditions

$$u(0)=\begin{bmatrix}0\\0\end{bmatrix}$$
$$u'(0)=\begin{bmatrix}1\\0\end{bmatrix}$$

We know that the if A has negative eigenvalues $\lambda_1,\ldots,\lambda_n$ and if $\omega_j=\sqrt{-\lambda_j}$, then solution of $\frac{d^2u}{dt^2}=Au$ is given by

$$u(t)=(a_1\cos\omega_1 t+b_1\sin\omega_1 t)x_1+\cdots+(a_n\cos\omega_n t+b_n\sin\omega_n t)x_n \quad \ldots\ldots(1)$$

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Step 2 of 10

To find the eigenvalues for matrix $A=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}$, by using the equation

Step 2 of 10

To find the eigenvalues for matrix $A=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}$, by using the equation

$$\det(A-\lambda I)=0 .$$

$$\det\begin{bmatrix}-2-\lambda & 1\\ 1 & -2-\lambda\end{bmatrix}=0$$
$$(-2-\lambda)(-2-\lambda)-(1)=0$$
$$(-2-\lambda)^2-1=0$$

Provide feedback (0)

Step 3 of 10

Solve the $(-2-\lambda)^2-1$, to find the value of λ .

$$(-2-\lambda)^2=1$$
$$-2-\lambda=\pm\sqrt{1}$$
$$-2-\lambda=\pm 1$$
$$-2-\lambda=-1 \text{ or } -2-\lambda=1$$

So, we get λ_1 and λ_2 as

$$\lambda_1=-1$$

Step 3 of 10

Solve the $(-2-\lambda)^2-1$, to find the value of λ .

$$\begin{aligned}(-2-\lambda)^2 &= 1\\ -2-\lambda &= \pm\sqrt{1}\\ -2-\lambda &= \pm 1\\ -2-\lambda &= -1 \text{ or } -2-\lambda = 1\end{aligned}$$

So, we get λ_1 and λ_2 as

$$\begin{aligned}\lambda_1 &= -1\\ \lambda_2 &= -3\end{aligned}$$

Provide feedback (0)

Step 4 of 10

The eigenvector for $\lambda_1 = -1$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix}-2+1 & 1\\ 1 & -2+1\end{bmatrix}x_1\\ &\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}x_1\\ &= 0\end{aligned}$$

The eigenvector for $\lambda_1 = -1$ is given by $x_1 = \begin{bmatrix}1\\ 1\end{bmatrix}$.

$$\lambda_1 = -1$$
$$\lambda_2 = -3$$

Provide feedback (0)

Step 4 of 10

The eigenvector for $\lambda_1 = -1$ is given by

$$\begin{aligned}(A-\lambda_1 I)x_1 &= \begin{bmatrix} -2+1 & 1 \\ 1 & -2+1 \end{bmatrix} x_1 \\ &\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} x_1 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_1 = -1$ is given by $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

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Step 5 of 10

The eigenvector for $\lambda_2 = -3$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} -2+3 & 1 \\ 1 & -2+3 \end{bmatrix} x_2 \\ &\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x_2\end{aligned}$$

Step 5 of 10

The eigenvector for $\lambda_2 = -3$ is given by

$$\begin{aligned}(A-\lambda_2 I)x_2 &= \begin{bmatrix} -2+3 & 1 \\ 1 & -2+3 \end{bmatrix} x_2 \\ &\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x_2 \\ &= 0\end{aligned}$$

The eigenvector for $\lambda_2 = -3$ is given by $x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

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Step 6 of 10

We know that $\omega_j = \sqrt{-\lambda_j}$.

Substituting -1 for λ_1 in equation $\omega_1 = \sqrt{-\lambda_1}$, we get

$$\begin{aligned}\omega_1 &= \sqrt{-(-1)} \\ &= 1\end{aligned}$$

Similarly substituting -3 for λ_2 in equation $\omega_2 = \sqrt{-\lambda_2}$, we get

Step 6 of 10

We know that $\omega_j = \sqrt{-\lambda_j}$.

Substituting -1 for λ_1 in equation $\omega_1 = \sqrt{-\lambda_1}$, we get

$$\begin{aligned}\omega_1 &= \sqrt{-(-1)}\\ &= 1\end{aligned}$$

Similarly substituting -3 for λ_2 in equation $\omega_2 = \sqrt{-\lambda_2}$, we get

$$\begin{aligned}\omega_2 &= \sqrt{-(-3)}\\ &= \sqrt{3}\end{aligned}$$

To find the values of *a*'s, substitute $t = 0$ into the equation (1).

$$u(0) = \left(a_1 \cos(0) + b_1 \sin(0)\right)x_1 + \cdots + \left(a_n \cos(0) + b_n \sin(0)\right)x_n$$

so all $b_n \sin(0) = 0$ and $a_n \cos(0) = 1$, this gives:

$$a = S^{-1}u(0)$$

We have:

$$\begin{bmatrix}0\end{bmatrix}$$

$$u(0)=\begin{bmatrix}0\\0\end{bmatrix}$$

and

$$S=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$$

So, the values of *a*'s are:

$$\begin{aligned}a&=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}^{-1}\begin{bmatrix}0\\0\end{bmatrix}\\&=\begin{bmatrix}0\\0\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 7 of 10

Thus, $a_1=0$ and $a_2=0$.

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Step 8 of 10

To find the values of *b*'s, first differentiating $u(t)$ and then substituting $t=0$ we get:

Step 7 of 10

Thus, $a_1 = 0$ and $a_2 = 0$.

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Step 8 of 10

To find the values of *b*'s, first differentiating $u(t)$ and then substituting $t = 0$ we get:

$$u'(t) = b_1\omega_1 x_1 + \cdots + b_n\omega_n x_n$$

We have:

$$u'(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

And

$$\omega_1 = 1$$
$$\omega_2 = \sqrt{3}$$

So, the values of *b*'s are:

$$\omega_1 = 1$$
$$\omega_2 = \sqrt{3}$$

So, the values of *b*'s are:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = b_1 \times 1 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b_2 \times \sqrt{3} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} b_1 - \sqrt{3}b_2 \\ b_1 + \sqrt{3}b_2 \end{bmatrix}$$

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Step 9 of 10

Solving the following system of equation, we get the values of *b*'s.

$$b_1 - \sqrt{3}b_2 = 1$$
$$b_1 + \sqrt{3}b_2 = 0$$

Thus, $b_1 = \frac{1}{2}$ and $b_2 = -\frac{1}{2\sqrt{3}}$.

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Step 10 of 10

Substituting the values of *a*'s and *b*'s into the equation (1), we get

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Step 10 of 10

Substituting the values of *a*'s and *b*'s into the equation (1), we get

$$u(t)=\left(0\times\cos(1)t+\frac{1}{2}\sin(1)t\right)\begin{bmatrix}1\\1\end{bmatrix}+\left(0\times\cos\sqrt{3}t+\left(\frac{1}{-2\sqrt{3}}\right)\sin\sqrt{3}t\right)\begin{bmatrix}-1\\1\end{bmatrix}$$
$$=\frac{1}{2}\sin t\begin{bmatrix}1\\1\end{bmatrix}-\frac{1}{2\sqrt{3}}\sin\sqrt{3}t\begin{bmatrix}-1\\1\end{bmatrix}$$

Therefore, the motion $\boxed{u(t)=\frac{1}{2}\sin t\begin{bmatrix}1\\1\end{bmatrix}-\frac{1}{2\sqrt{3}}\sin\sqrt{3}t\begin{bmatrix}-1\\1\end{bmatrix}}$.

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Step 1 of 4

Note: Real parts of the Eigen values govern the stability. The differential equation $du/dt = Au$ is:

Stable: If $\mathrm{Re}(\lambda_i) < 0$

Neutrally stable: If all $\mathrm{Re}(\lambda_i) \le 0$ and $\lambda_1 = 0$.

Unstable: If any Eigen value has $\mathrm{Re}(\lambda_i) > 0$.

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Step 2 of 4

(a) Consider the following matrix.

$$A = \begin{bmatrix} a & b+c \\ b-c & -a \end{bmatrix}$$

Here, trace of matrix A is zero. To show that Eigen values are real exactly when $a^2 + b^2 \ge c^2$.

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Step 3 of 4

First step is to find the Eigen values of matrix A. Do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} a-\lambda & b+c \\ b-c & -a-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (a-\lambda)(-a-\lambda) - (b^2 - c^2) &= 0 \\ \lambda^2 - a^2 &= (b^2 - c^2) \end{aligned}$$

To get the real Eigen values following must be true:

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Step 3 of 4

First step is to find the Eigen values of matrix A. Do the following calculations;

$$A-\lambda I=\begin{bmatrix} a-\lambda & b+c \\ b-c & -a-\lambda \end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(a-\lambda)(-a-\lambda)-(b^2-c^2)=0$$

$$\lambda^2-a^2=(b^2-c^2)$$

To get the real Eigen values following must be true:

$$\lambda^2>0$$

$$a^2+b^2-c^2>0$$

$$a^2+b^2>c^2$$

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Step 4 of 4

Therefore, Eigen values are real exactly when $\boxed{a^2+b^2\geq c^2}$.

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Step 1 of 7

Compute the Eigen values and Eigen vectors of the following matrix:

$$A=\begin{bmatrix}1 & 2 & 1\\ 0 & 3 & 6\\ 0 & 0 & 4\end{bmatrix}$$

Solve the following:

$$\frac{du}{dt}=Au$$

Initial conditions are as follows:

$$u(0)=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

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Step 2 of 7

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}1-\lambda & 2 & 1\\ 0 & 3-\lambda & 6\\ 0 & 0 & 4-\lambda\end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(1-\lambda)(3-\lambda)(4-\lambda)=0$$

After solving following values are obtained:

$$\lambda_1=4$$

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Step 2 of 7

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}1-\lambda & 2 & 1\\ 0 & 3-\lambda & 6\\ 0 & 0 & 4-\lambda\end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(1-\lambda)(3-\lambda)(4-\lambda)=0$$

After solving following values are obtained:

$$\lambda_1=4$$
$$\lambda_2=3$$
$$\lambda_3=1$$

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Step 3 of 7

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}1-4 & 2 & 1\\ 0 & 3-4 & 6\\ 0 & 0 & 4-4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-3 & 2 & 1\\ 0 & -1 & 6\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

On solving, values of x, y and z corresponding to $\lambda=4$ are as follows:

$\lambda_3 = 4$

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Step 3 of 7

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 1-4 & 2 & 1 \\ 0 & 3-4 & 6 \\ 0 & 0 & 4-4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 & 1 \\ 0 & -1 & 6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving, values of x, y and z corresponding to $\lambda=4$ are as follows:

$$x_1=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 13/3 \\ 6 \\ 1 \end{bmatrix}$$

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Step 4 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda=3$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 1-3 & 2 & 1 \\ 0 & 3-3 & 6 \\ 0 & 0 & 4-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 4 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 3$ is as follows:

$$(A-\lambda I)x = 0$$

$$\begin{bmatrix} 1-3 & 2 & 1 \\ 0 & 3-3 & 6 \\ 0 & 0 & 4-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 1 \\ 0 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving values of x, y and z are as follows:

$$x_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

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Step 5 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 1$ is as follows:

$$(A-\lambda I)x = 0$$

$$\begin{bmatrix} 1-1 & 2 & 1 \\ 0 & 3-1 & 6 \\ 0 & 0 & 4-1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 1$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 1-1 & 2 & 1 \\ 0 & 3-1 & 6 \\ 0 & 0 & 4-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving values of x, y and z are as follows:

$$x_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$u(t) = e^{At}u(0)$$

$$= Se^{\Lambda t}S^{-1}u(0)$$

$$x_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$\begin{aligned} u(t) &= e^{At}u(0) \\ &= Se^{\Lambda t}S^{-1}u(0) \\ &= \begin{bmatrix} 1 & 1 & 13/3 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{t} & 0 & 0 \\ 0 & e^{3t} & 0 \\ 0 & 0 & e^{4t} \end{bmatrix} \begin{bmatrix} 1 & -1 & 5/3 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} e^{t} & e^{3t} & (13/3)e^{4t} \\ 0 & e^{3t} & 6e^{4t} \\ 0 & 0 & e^{4t} \end{bmatrix} \begin{bmatrix} (8/3) \\ -6 \\ 1 \end{bmatrix} \end{aligned}$$

$$u(t) = \begin{bmatrix} (8/3)e^{t} - 6e^{3t} + (13/3)e^{4t} \\ -6e^{3t} + 6e^{4t} \\ e^{4t} \end{bmatrix}$$

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Step 7 of 7

Therefore, solution is as follows:

$$\boxed{\begin{bmatrix} (8/3)e^{t} - 6e^{3t} + (13/3)e^{4t} \end{bmatrix}}$$

$$
\begin{aligned}
u(t) &= e^{At}u(0) \\
&= Se^{\Lambda t}S^{-1}u(0) \\
&= \begin{bmatrix} 1 & 1 & 13/3 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{t} & 0 & 0 \\ 0 & e^{3t} & 0 \\ 0 & 0 & e^{4t} \end{bmatrix} \begin{bmatrix} 1 & -1 & 5/3 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} e^{t} & e^{3t} & (13/3)e^{4t} \\ 0 & e^{3t} & 6e^{4t} \\ 0 & 0 & e^{4t} \end{bmatrix} \begin{bmatrix} (8/3) \\ -6 \\ 1 \end{bmatrix} \\
u(t) &= \begin{bmatrix} (8/3)e^{t} - 6e^{3t} + (13/3)e^{4t} \\ -6e^{3t} + 6e^{4t} \\ e^{4t} \end{bmatrix}
\end{aligned}
$$

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Step 7 of 7

Therefore, solution is as follows:

$$
\boxed{u(t) = \begin{bmatrix} (8/3)e^{t} - 6e^{3t} + (13/3)e^{4t} \\ -6e^{3t} + 6e^{4t} \\ e^{4t} \end{bmatrix}}
$$

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Step 1 of 10

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 10

Initial condition:

$$u(0) = (5, -2)$$

Find the Eigen values and Eigen vectors, so that $u = e^{\lambda t}x$ solves the differential equation. Also, compute the general solution that starts from the initial value.

Provide feedback (0)

Step 3 of 10

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 3 \\ 0 & 1-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(4-\lambda)(1-\lambda) = 0$$

After solving following values are obtained:

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Step 3 of 10

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}4-\lambda & 3\\ 0 & 1-\lambda\end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(4-\lambda)(1-\lambda)=0$$

After solving following values are obtained:

$$\lambda_1=4$$
$$\lambda_2=1$$

Therefore, Eigen values are $\boxed{4,1}$

Provide feedback (0)

Step 4 of 10

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}4-\lambda & 3\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}4-4 & 3\\ 0 & 1-4\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}0 & 3\\ 0 & -3\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=4$ are as follows:

Provide feedback (0)

Step 4 of 10

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}4-\lambda & 3\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}4-4 & 3\\ 0 & 1-4\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 3\\ 0 & -3\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=4$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 5 of 10

Similarly, Eigen vectors corresponding to Eigen value $\lambda=1$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}4-\lambda & 3\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}4-1 & 3\\ 0 & 1-1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}3 & 3\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Step 5 of 10

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 1$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 4-\lambda & 3 \\ 0 & 1-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4-1 & 3 \\ 0 & 1-1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

Therefore Eigen values are:

$$\boxed{\begin{aligned} x_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ x_2 &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}}$$

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Step 7 of 10

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

Therefore Eigen values are:

$$\boxed{\begin{aligned} x_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ x_2 &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}}$$

Provide feedback (0)

Step 7 of 10

Therefore, values of $u = e^{\lambda t} x$ corresponding to both Eigen values are as follows:

$$\boxed{\begin{aligned} u_1 &= e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ u_2 &= e^{t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}}$$

Provide feedback (0)

Step 8 of 10

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

Therefore Eigen values are:

$$\boxed{\begin{aligned} x_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ x_2 &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}}$$

Provide feedback (0)

Step 7 of 10

Therefore, values of $u = e^{\lambda t} x$ corresponding to both Eigen values are as follows:

$$\boxed{\begin{aligned} u_1 &= e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ u_2 &= e^{t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}}$$

Provide feedback (0)

Step 8 of 10

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Step 7 of 10

Therefore, values of $u = e^{\lambda t}x$ corresponding to both Eigen values are as follows:

$$\boxed{\begin{aligned} u_1 &= e^{4t}\begin{bmatrix}1\\0\end{bmatrix} \\ u_2 &= e^{t}\begin{bmatrix}1\\-1\end{bmatrix} \end{aligned}}$$

Provide feedback (0)

Step 8 of 10

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix}4 & 0\\0 & 1\end{bmatrix}$$

Therefore, the general solution of the differential equation is:

$$\begin{aligned} u(t) &= c_1 e^{\lambda_1 t}x_1 + c_2 e^{\lambda_2 t}x_2 \\ &= \begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}\begin{bmatrix}e^{4t} & 0\\0 & e^{t}\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} \end{aligned}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

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Step 9 of 10

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 9 of 10

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} e^{4t} & 0 \\ 0 & e^{t} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} e^{4t} & 0 \\ 0 & e^{t} \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} \end{aligned}$$

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Step 10 of 10

Therefore, general solution of the differential equation is:

$$\boxed{u(t) = 3e^{4t}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2e^{t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

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Step 1 of 3

This tells us that *k* must be equal to 1. There is a condition $z(0)=-2$.

Thus,

$$\begin{aligned} z &= e^{kt}+C_1 \\ -2 &= e^{k(0)}+C_1 \\ &= 1+C_1 \\ C_1 &= -3 \end{aligned}$$

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Step 2 of 3

Now consider the differential equation $\frac{dy}{dt}=4y+3z$, where $y(0)=-5$.

Let $y=e^{mt}+e^{kt}+A$ be the solution of the above differential equation.

Then,

$$\begin{aligned} 4\left(e^{mt}+e^{kt}+A\right)+3\left(e^{kt}+C_1\right) &= \frac{d}{dt}\left(e^{mt}+e^{kt}+A\right) \\ 4e^{mt}+7e^{kt}+4A+3C_1 &= me^{mt}+ke^{kt} \end{aligned}$$

Therefore, $m=4$.

We have $y(0)=-5$. Also, we have $C_1=-3$. This gives $A=1$.

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Step 2 of 3

Now consider the differential equation $\frac{dy}{dt} = 4y + 3z$, where $y(0) = -5$.

Let $y = e^{mt} + e^{kt} + A$ be the solution of the above differential equation.

Then,

$$4\left(e^{mt} + e^{kt} + A\right) + 3\left(e^{kt} + C_1\right) = \frac{d}{dt}\left(e^{mt} + e^{kt} + A\right)$$
$$4e^{mt} + 7e^{kt} + 4A + 3C_1 = me^{mt} + ke^{kt}$$

Therefore, $m = 4$.

We have $y(0) = -5$. Also, we have $C_1 = -3$. This gives $A = 1$.

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Step 3 of 3

Thus, the solution of the differential equation is $\boxed{y = e^{4t} + e^{t}}$.

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Step 1 of 7

The second order differential equation $y'' = 5y' + 4y$ can be written into a vector equation for $u(t) = (y(t), y'(t))$ and as a first order system by introducing the velocity y'.

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} y' \\ y'' \end{bmatrix}$$

Provide feedback (0)

Step 2 of 7

Above differential equation can also be written as follows:

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$\boxed{A = \begin{bmatrix} 0 & 1 \\ 4 & 5 \end{bmatrix}}$$

Provide feedback (0)

Step 3 of 7

Find the Eigen values of matrix A. Substitute $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$ to get the Eigen values again.

Provide feedback (0)

Provide feedback (0)

Step 3 of 7

Find the Eigen values of matrix A. Substitute $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$ to get the Eigen values again.

Provide feedback (0)

Step 4 of 7

First step is to find the Eigen values of matrix A. Do the following calculations:

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} -\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (-\lambda)(5-\lambda) - 4 &= 0 \\ \lambda^2 - 5\lambda - 4 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\lambda_1 = \frac{5+\sqrt{41}}{2}$$
$$\lambda_2 = \frac{5-\sqrt{41}}{2}$$

Therefore, Eigen values are $\boxed{\frac{5 \pm \sqrt{41}}{2}}$

Provide feedback (0)

Step 5 of 7

Eigen values can also be obtained by substituting $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$. For the same do the following calculations:

$$\lambda_2 = \frac{\quad}{2}$$

Therefore, Eigen values are $\boxed{\frac{5 \pm \sqrt{41}}{2}}$

Provide feedback (0)

Step 5 of 7

Eigen values can also be obtained by substituting $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$. For the same do the following calculations:

$$y = e^{\lambda t}$$
$$y' = \lambda e^{\lambda t}$$
$$y'' = \lambda^2 e^{\lambda t}$$

Provide feedback (0)

Step 6 of 7

Substitute the values in the following equation:

$$y'' = 5y' + 4y$$
$$\lambda^2 e^{\lambda t} = 5\lambda e^{\lambda t} + 4e^{\lambda t}$$
$$\lambda^2 - 5\lambda - 4 = 0$$

After solving value of λ is:

$$\lambda = \frac{5 \pm \sqrt{41}}{2}$$

Provide feedback (0)

Step 7 of 7

Therefore, Eigen values are as follows:

Eigen values can also be obtained by substituting $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$. For the same do the following calculations:

$$y = e^{\lambda t}$$
$$y' = \lambda e^{\lambda t}$$
$$y'' = \lambda^2 e^{\lambda t}$$

Provide feedback (0)

Step 6 of 7

Substitute the values in the following equation:

$$y'' = 5y' + 4y$$
$$\lambda^2 e^{\lambda t} = 5\lambda e^{\lambda t} + 4e^{\lambda t}$$
$$\lambda^2 - 5\lambda - 4 = 0$$

After solving value of λ is:

$$\lambda = \frac{5 \pm \sqrt{41}}{2}$$

Provide feedback (0)

Step 7 of 7

Therefore, Eigen values are as follows:

$$\boxed{\lambda = \frac{5 \pm \sqrt{41}}{2}}$$

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Step 1 of 11

Consider the following first order differential equations:

$$\frac{dv}{dt} = w - v$$
$$\frac{dw}{dt} = v - w$$

To show that $v + w$ is constant. Also, write the equations in the following form:

$$\frac{du}{dt} = Au$$

Find the matrix A its Eigen values and Eigen vectors.

Provide feedback (0)

Step 2 of 11

Initial condition:

$$v(0) = 30$$
$$w(0) = 10$$

Find the values of v and w at $t = 1$.

Provide feedback (0)

Step 3 of 11

To show that $v + w$ is constant, do the following calculations:

$$\begin{aligned}\frac{d(v+w)}{dt} &= \frac{dv}{dt} + \frac{dw}{dt} \\ &= w - v + v - w\end{aligned}$$

Find the values of v and w at $t=1$.

Provide feedback (0)

Step 3 of 11

To show that $v+w$ is constant, do the following calculations:

$$\begin{aligned}\frac{d(v+w)}{dt} &= \frac{dv}{dt}+\frac{dw}{dt}\\ &= w-v+v-w\\ &= 0\end{aligned}$$

This shows that $v+w$ is constant and its value is:

$$\begin{aligned}v+w &= 30+10\\ &= \boxed{40}\end{aligned}$$

Provide feedback (0)

Step 4 of 11

Above differential equations can be written as follows:

$$\frac{d}{dt}\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Matrix $u(t)$ is: $u(t)=(v,w)$

Therefore, A is defined as follows:

Provide feedback (0)

Step 4 of 11

Above differential equations can be written as follows:

$$\frac{d}{dt}\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Matrix $u(t)$ is: $u(t) = (v, w)$

Therefore, A is defined as follows:

$$\boxed{A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 11

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (-1-\lambda)(-1-\lambda) - 1 &= 0 \\ \lambda^2 + 2\lambda &= 0 \end{aligned}$$

After solving following values are obtained:

Therefore, A is defined as follows:

$$\boxed{A=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}}$$

Provide feedback (0)

Step 5 of 11

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{aligned} A-\lambda I &= \begin{bmatrix}-1-\lambda & 1\\ 1 & -1-\lambda\end{bmatrix} \\ \det(A-\lambda I) &= 0 \\ (-1-\lambda)(-1-\lambda)-1 &= 0 \\ \lambda^2+2\lambda &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= -2 \\ \lambda_2 &= 0 \end{aligned}$$

Therefore, Eigen values are $\boxed{-2,0}$

Provide feedback (0)

Step 6 of 11

To calculate Eigen vectors do the following calculations:

$$\begin{aligned} (A-\lambda I)x &= 0 \\ \begin{bmatrix}-1-\lambda & 1\\ 1 & -1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix} &= \begin{bmatrix}0\\ 0\end{bmatrix} \end{aligned}$$

Step 6 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}-1-\lambda & 1\\ 1 & -1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-1+2 & 1\\ 1 & -1+2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 7 of 11

On solving, values of y and z corresponding to $\lambda=-2$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ -1\end{bmatrix}$$

Provide feedback (0)

Step 8 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda=0$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}-1-\lambda & 1\\ 1 & -1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-1-0 & 1\\ \end{bmatrix}\begin{bmatrix}y\\ \end{bmatrix}=\begin{bmatrix}0\\ \end{bmatrix}$$

Provide feedback (0)

Step 8 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(A-\lambda I)x = 0$$

$$\begin{bmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1-0 & 1 \\ 1 & -1-0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore Eigen values are:

$$\boxed{\begin{aligned} x_1 &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ x_2 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}}$$

Provide feedback (0)

Step 9 of 11

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

$$x_2 = \begin{bmatrix} \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 11

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, the general solution of the differential equation is:

$$\begin{aligned} u(t) &= c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2 \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \end{aligned}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 10 of 11

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 30 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} e^{-2t} & 1 \\ -e^{-2t} & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 10 of 11

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{-2t} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 30 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} e^{-2t} & 1 \\ -e^{-2t} & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 20 \end{bmatrix} \\ \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} &= \begin{bmatrix} 10e^{-2t}+20 \\ -10e^{-2t}+20 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 11 of 11

Therefore, specific solution of the differential equation is:

$$\begin{aligned} v(t) &= 10e^{-2t}+20 \\ w(t) &= -10e^{-2t}+20 \end{aligned}$$

The values of v and w at $t=1$.

$$\boxed{\begin{aligned} v(1) &= 10e^{-2}+20 \\ w(1) &= -10e^{-2}+20 \end{aligned}}.$$

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Step 1 of 11

Consider the following first order differential equations:

$$\frac{dv}{dt} = w - v$$
$$\frac{dw}{dt} = v - w$$

Here, $v + w$ is constant. These differential equations can be written in the following form:

$$\frac{du}{dt} = Au$$

Provide feedback (0)

Step 2 of 11

Now reverse the above system to the following:

$$\frac{du}{dt} = -Au$$
$$\frac{dv}{dt} = -(w - v)$$
$$\frac{dw}{dt} = -(v - w)$$

Here, also $v + w$ is constant. Find the matrix $-A$ and its Eigen values.

Provide feedback (0)

Step 3 of 11

Initial condition:

$$v(0) = 30$$

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$$\frac{dw}{dt} = -(v - w)$$

Here, also $v + w$ is constant. Find the matrix -A and its Eigen values.

Provide feedback (0)

Step 3 of 11

Initial condition:

$$v(0) = 30$$
$$w(0) = 10$$

Find the value of v when $t \to \infty$.

Provide feedback (0)

Step 4 of 11

Changed differential equations can be written as follows:

$$\frac{d}{dt}\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Bu$$
$$B = -A$$

Matrix $u(t)$ is: $u(t) = (v, w)$

Therefore, matrix B is defined as follows:

$$\boxed{B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

$$B = -A$$

Matrix $u(t)$ is: $u(t) = (v, w)$

Therefore, matrix B is defined as follows:

$$\boxed{B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 11

First step is to find the Eigen values and Eigen vectors of matrix B. To calculate the Eigen values do the following calculations;

$$\begin{aligned} B - \lambda I &= \begin{bmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix} \\ \det(B - \lambda I) &= 0 \\ (1-\lambda)(1-\lambda) - 1 &= 0 \\ \lambda^2 - 2\lambda &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= 2 \\ \lambda_2 &= 0 \end{aligned}$$

Therefore, Eigen values are $\boxed{2, 0}$

Provide feedback (0)

Step 6 of 11

To calculate Eigen vectors do the following calculations:

$$(B - \lambda I)x = 0$$

Provide feedback (0)

Step 6 of 11

To calculate Eigen vectors do the following calculations:

$$(B-\lambda I)x=0$$

$$\begin{bmatrix}1-\lambda & -1\\ -1 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}1-2 & -1\\ -1 & 1-2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 7 of 11

On solving, values of y and z corresponding to $\lambda=2$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ -1\end{bmatrix}$$

Provide feedback (0)

Step 8 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda=0$ is as follows:

$$(B-\lambda I)x=0$$

$$\begin{bmatrix}1-\lambda & -1\end{bmatrix}\begin{bmatrix}y\end{bmatrix}=\begin{bmatrix}0\end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(B - \lambda I)x = 0$$

$$\begin{bmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-0 & -1 \\ -1 & 1-0 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 11

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 11

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, the general solution of the differential equation is:

$$\begin{aligned} u(t) &= c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2 \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \end{aligned}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 10 of 11

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 30 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} e^{2t} & 1 \\ -e^{2t} & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix} \end{aligned}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 10 of 11

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{2t} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 30 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} e^{2t} & 1 \\ -e^{2t} & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 20 \end{bmatrix} \\ \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} &= \begin{bmatrix} 10e^{2t} + 20 \\ -10e^{2t} + 20 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 11 of 11

Therefore, specific solution of the differential equation is:

$$\begin{aligned} v(t) &= 10e^{2t} + 20 \\ w(t) &= -10e^{2t} + 20 \end{aligned}$$

The value of v when $t \to \infty$ is:

$$\boxed{\begin{aligned} v(\infty) &= 10e^{2} + 20 \\ &\to \infty \end{aligned}}$$

Provide feedback (0)

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Step 1 of 7

The second order differential equation $y'' = 0$ can be written as a first order differential equation system by introducing the velocity y' as unknown.

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

The solution to $y'' = 0$ is a straight line $y = C + Dt$.

Provide feedback (0)

Step 2 of 7

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 7

Matrix A cannot be diagonalize so, find A^2 and compute e^{At} from the series and write the solution of $e^{At}u(0)$.

Provide feedback (0)

Step 4 of 7

Here, matrix A contains only one Eigen value and corresponding to it one Eigen vector

CH5.4 27P

Matrix A cannot be diagonalize so, find A^2 and compute e^{At} from the series and write the solution of $e^{At}u(0)$.

Provide feedback (0)

Step 4 of 7

Here, matrix A contains only one Eigen value and corresponding to it one Eigen vector, which is not sufficient to diagonalize A. So, use the following series to calculate e^{At}.

$$e^{At} = I + At + \frac{(At)^2}{2!} + \cdots$$

Next, compute the following:

$$A \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore:

$$\boxed{A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 7

Thus,

$$\begin{aligned} e^{At} &= I + At \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix} + 0 \\ &= \begin{bmatrix} 1 & t \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 5 of 7

Thus,

$$\begin{aligned} e^{At} &= I + At \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix} + 0 \\ &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore, value of e^{At} is:

$$\boxed{e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}$$

Provide feedback (0)

Step 6 of 7

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= e^{At} u(0) \\ &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y(0) \\ y'(0) \end{bmatrix} \\ \begin{bmatrix} y(t) \\ y'(t) \end{bmatrix} &= \begin{bmatrix} y'(0)t + y(0) \\ y'(0) \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 7 of 7

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} + 0$$

$$= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

Therefore, value of e^{At} is:

$$\boxed{e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}$$

Provide feedback (0)

Step 6 of 7

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= e^{At}u(0) \\ &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y(0) \\ y'(0) \end{bmatrix} \\ \begin{bmatrix} y(t) \\ y'(t) \end{bmatrix} &= \begin{bmatrix} y'(0)t + y(0) \\ y'(0) \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 7 of 7

Therefore, solution of the differential equation is:

$$\boxed{y(t) = y(0) + y'(0)t}$$

This represents a straight line.

Provide feedback (0)

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Step 1 of 9

The second order differential equation $y'' = 6y' - 9y$ can be written into a vector equation for $u(t) = (y(t), y'(t))$ and as a first order differential equation system by introducing y'.

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} y' \\ y'' \end{bmatrix}$$

Provide feedback (0)

Step 2 of 9

Above second order differential equation can also be written as follows:

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 9

Find the Eigen values and Eigen vector of matrix A. Substitute $y = e^{\lambda t}$ into the equation $y'' = 6y' - 9y$ to get the Eigen values again. Also, show that second solution is $y = te^{3t}$.

Provide feedback (0)

Step 3 of 9

Find the Eigen values and Eigen vector of matrix A. Substitute $y = e^{\lambda t}$ into the equation $y'' = 6y' - 9y$ to get the Eigen values again. Also, show that second solution is $y = te^{3t}$.

Provide feedback (0)

Step 4 of 9

First step is to find the Eigen values of matrix A. Do the following calculations:

$$A - \lambda I = \begin{bmatrix} 0-\lambda & 1 \\ -9 & 6-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(-\lambda)(6-\lambda) + 9 = 0$$

$$\lambda^2 - 6\lambda + 9 = 0$$

After solving following values are obtained:

$$\lambda_1 = 3$$
$$\lambda_2 = 3$$

Therefore, Eigen values are $\boxed{3,3}$

Provide feedback (0)

Step 5 of 9

To calculate Eigen vectors do the following calculations:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 0-\lambda & 1 \\ -9 & 6-\lambda \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0-3 & 1 \end{bmatrix} \begin{bmatrix} y \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

Therefore, Eigen values are $\boxed{3,3}$

Provide feedback (0)

Step 5 of 9

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 0-\lambda & 1 \\ -9 & 6-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0-3 & 1 \\ -9 & 6-3 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ -9 & 3 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=3$ are as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Therefore, Eigen vectors are as follows:

$$\boxed{x=\begin{bmatrix} 1 \\ 3 \end{bmatrix}}$$

Provide feedback (0)

Step 6 of 9

Eigen values can also be obtained by substituting $y=e^{\lambda t}$ into the equation $y''=6y'-9y$.
For the same do the following calculations:

Provide feedback (0)

Step 6 of 9

Eigen values can also be obtained by substituting $y = e^{\lambda t}$ into the equation $y'' = 6y' - 9y$. For the same do the following calculations:

$$y = e^{\lambda t}$$
$$y' = \lambda e^{\lambda t}$$
$$y'' = \lambda^2 e^{\lambda t}$$

Provide feedback (0)

Step 7 of 9

Substitute the values in the following equation:

$$y'' = 6y' - 9y$$
$$\lambda^2 e^{\lambda t} = 6\lambda e^{\lambda t} - 9e^{\lambda t}$$
$$\lambda^2 - 6\lambda + 9 = 0$$

Provide feedback (0)

Step 8 of 9

Solve it further to get value of λ as $\boxed{3, 3}$. These are the repeated roots.

To show that second solution is $y = te^{3t}$, do the following calculations:

$$y = te^{3t}$$
$$y' = 3te^{3t} + e^{3t}$$
$$y'' = 9te^{3t} + 3e^{3t} + 3e^{3t}$$
$$= 9te^{3t} + 6e^{3t}$$

Step 7 of 9

Substitute the values in the following equation:

$$\begin{aligned} y'' &= 6y' - 9y \\ \lambda^2 e^{\lambda t} &= 6\lambda e^{\lambda t} - 9e^{\lambda t} \\ \lambda^2 - 6\lambda + 9 &= 0 \end{aligned}$$

Provide feedback (0)

Step 8 of 9

Solve it further to get value of λ as $\boxed{3, 3}$. These are the repeated roots.

To show that second solution is $y = te^{3t}$, do the following calculations:

$$\begin{aligned} y &= te^{3t} \\ y' &= 3te^{3t} + e^{3t} \\ y'' &= 9te^{3t} + 3e^{3t} + 3e^{3t} \\ &= 9te^{3t} + 6e^{3t} \end{aligned}$$

Provide feedback (0)

Step 9 of 9

Substitute the values in the following equation:

$$\begin{aligned} y'' &= 6y' - 9y \\ &= 6\left(3te^{3t} + e^{3t}\right) - 9\left(te^{3t}\right) \\ &= 18te^{3t} + 6e^{3t} - 9te^{3t} \\ &= 9te^{3t} + 6e^{3t} \end{aligned}$$

Solution $y = te^{3t}$ satisfies the equation $y'' = 6y' - 9y$. Therefore, second solution is $y = te^{3t}$.

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Step 8 of 9

Solve it further to get value of λ as $\boxed{3, 3}$. These are the repeated roots.

To show that second solution is $y = te^{3t}$, do the following calculations:

$$\begin{aligned} y &= te^{3t} \\ y' &= 3te^{3t} + e^{3t} \\ y'' &= 9te^{3t} + 3e^{3t} + 3e^{3t} \\ &= 9te^{3t} + 6e^{3t} \end{aligned}$$

Provide feedback (0)

Step 9 of 9

Substitute the values in the following equation:

$$\begin{aligned} y'' &= 6y' - 9y \\ &= 6\left(3te^{3t} + e^{3t}\right) - 9\left(te^{3t}\right) \\ &= 18te^{3t} + 6e^{3t} - 9te^{3t} \\ &= 9te^{3t} + 6e^{3t} \end{aligned}$$

Solution $y = te^{3t}$ satisfies the equation $y'' = 6y' - 9y$. Therefore, second solution is $y = te^{3t}$.

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CH5.4 27P

Step 1 of 3

Write down second order differential equation $my'' + by' + ky = 0$ in the following way:

$$Mu' = Au$$

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Step 2 of 3

Second order differential equation can be written as follows:

$$my'' + by' + ky = 0$$

$$m\frac{d^2y}{dt^2} = -b\frac{dy}{dt} - ky$$

$$m\frac{d^2y}{dt^2} + \frac{dy}{dt} = -b\frac{dy}{dt} - ky + \frac{dy}{dt}$$

In matrix form, it can be written as:

$$\begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix} \cdot \frac{d}{dt}\begin{bmatrix} y' \\ y \end{bmatrix} = \begin{bmatrix} -b & -k \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y' \\ y \end{bmatrix}$$

$$\begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y' \\ y \end{bmatrix}' = \begin{bmatrix} -b & -k \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y' \\ y \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

Therefore, $my'' + by' + ky = 0$ is written in matrix form as:

$$\boxed{Mu' = Au}.$$

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Step 1 of 11

(a) Consider the following differential equation:

$$y'' = -y$$

Find two familiar functions that solve the equation. Also find the function among them which satisfies the following:

$$y(0) = 1$$
$$y'(0) = 0$$

Provide feedback (0)

Step 2 of 11

Let the two familiar functions be $(\cos t, \sin t)$. Next step is to check that these functions solve the differential equation, for this do the following calculations:

$$\begin{aligned} y(t) &= \cos t \\ y'(t) &= -\sin t \\ y''(t) &= -\cos t \\ &= -y \end{aligned}$$

Similarly,

$$\begin{aligned} y(t) &= \sin t \\ y'(t) &= \cos t \\ y''(t) &= -\sin t \\ &= -y \end{aligned}$$

These calculation shows that both functions satisfies the differential equation. Therefore the familiar functions are $\boxed{(\cos t, \sin t)}$.

Among these functions following function satisfy the initial condition

$$y'(t) = \cos t$$
$$y''(t) = -\sin t$$
$$= -y$$

These calculation shows that both functions satisfies the differential equation. Therefore the familiar functions are $\boxed{(\cos t, \sin t)}$.

Among these functions following function satisfy the initial condition.

$$\boxed{y(t) = \cos t}$$

Provide feedback (0)

Step 3 of 11

(b) The second order differential equation $y'' + y = 0$ can be written as a first order system by introducing the velocity y' as unknown factor.

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 11

Initial condition:

$$\begin{bmatrix} -1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 11

Initial condition:

$$u(0)=(1,0)$$

Compute the solution that starts from the initial value.

Provide feedback (0)

Step 5 of 11

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{aligned} A-\lambda I &= \begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix} \\ \det(A-\lambda I) &= 0 \\ (-\lambda)(-\lambda)+1 &= 0 \\ \lambda^2+1 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= i \\ \lambda_2 &= -i \end{aligned}$$

Provide feedback (0)

Step 6 of 11

To calculate Eigen vectors do the following calculations:

Provide feedback (0)

Step 6 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}-\lambda & 1\\ -1 & -\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-i & 1\\ -1 & -i\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 7 of 11

On solving, values of y and z corresponding to $\lambda=i$ are as follows:

$$\begin{aligned}x_1&=\begin{bmatrix}y\\ z\end{bmatrix}\\ &=\begin{bmatrix}1\\ i\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 8 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda=-i$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}-\lambda & 1\\ -1 & -\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}i & 1\end{bmatrix}\begin{bmatrix}y\end{bmatrix}\begin{bmatrix}0\end{bmatrix}$$

Provide feedback (0)

Step 8 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -i$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Provide feedback (0)

Step 9 of 11

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

Therefore, the general solution of the differential equation is:

$$u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$$

Step 9 of 11

Recall that: $e^{At} = Se^{\Lambda t}S^{-1}$

Here, Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

Therefore, the general solution of the differential equation is:

$$\begin{aligned} u(t) &= c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2 \\ &= \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \end{aligned}$$

Here, c_1 and c_2 are constants. Their values are determined by the following values:

$$c = S^{-1}u(0)$$

Provide feedback (0)

Step 10 of 11

So, the solution for differential equation can be written as follows:

$$\begin{aligned} u(t) &= Se^{\Lambda t}S^{-1}u(0) \\ &= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} e^{it} & e^{-it} \\ ie^{it} & -ie^{-it} \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} e^{it} + e^{-it} & -ie^{it} + ie^{-it} \\ ie^{it} - ie^{-it} & e^{it} + e^{-it} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} e^{it} + e^{-it} \\ i\left(e^{it} - e^{-it}\right) \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
u(t) &= Se^{\Lambda t}S^{-1}u(0) \\
&= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}\begin{bmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{bmatrix}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{it} & e^{-it} \\ ie^{it} & -ie^{-it} \end{bmatrix}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{it}+e^{-it} & -ie^{it}+ie^{-it} \\ ie^{it}-ie^{-it} & e^{it}+e^{-it} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} e^{it}+e^{-it} \\ i\left(e^{it}-e^{-it}\right) \end{bmatrix} \\
u(t) &= \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}
\end{aligned}
$$

Provide feedback (0)

Step 11 of 11

Therefore, specific solution of the differential equation is:

$$\boxed{u(t) = \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}}$$

This solution can also be obtained by putting $y(t) = \cos t$ from part (a) in the solution $u(t) = (y(t), y'(t))$.

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Step 1 of 13

If matrix A is invertible, then particular solution to following equation is $u_p = A^{-1}B$.

$$du/dt = Au - b$$

Let u_n be the general solution to the following differential equation:

$$du/dt = Au$$

Then the complete solution is $u_n + u_p$.

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Step 2 of 13

(a) Find the complete solution to the following:

$$\frac{du}{dt} = 2u - 8$$

The above solution can be written as follows:

$$\frac{du}{dt} = Au - b$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

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Step 3 of 13

Particular solution will be:

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$$A=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 13

Particular solution will be:

$$\begin{aligned} u_p &= A^{-1}B \\ &= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot 8 \\ &= \frac{1}{2}\cdot 8 \\ &= 4 \end{aligned}$$

Therefore, $u_p = 4$.

Provide feedback (0)

Step 4 of 13

General solution u_n can be calculated as follows:

$$u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$$

Calculate Eigen values of matrix A:

$$\begin{aligned} A-\lambda I &= \begin{bmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} \\ \det(A-\lambda I) &= 0 \\ (2-\lambda)(2-\lambda) &= 0 \\ (2-\lambda)^2 &= 0 \end{aligned}$$

Therefore, repeated Eigen value is $\lambda = 2$.

Step 5 of 13

Let x be the Eigen vectors corresponding to Eigen value $\lambda = 2$. Then general solution u_n will be:

$$u_n = ce^{2t}x$$

Here, c is any constant.

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Step 6 of 13

Therefore, the complete solution is $u_n + u_p$.

$$\boxed{u_n + u_p = ce^{2t}x + 4}$$

Provide feedback (0)

Step 7 of 13

(b) Find the complete solution to the following:

$$\frac{du}{dt} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} u - \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

The above solution can be written as follows:

$$\frac{du}{dt} = Au - B$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

Step 7 of 13

(b) Find the complete solution to the following:

$$\frac{du}{dt}=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}u-\begin{bmatrix}8\\ 6\end{bmatrix}$$

The above solution can be written as follows:

$$\frac{du}{dt}=Au-B$$

Here, matrix A is defined as follows:

$$A=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$$

Provide feedback (0)

Step 8 of 13

Particular solution will be:

$$\begin{aligned}u_p &= A^{-1}B\\ &=\begin{bmatrix}1/2 & 0\\ 0 & 1/3\end{bmatrix}\cdot\begin{bmatrix}8\\ 6\end{bmatrix}\\ &=\begin{bmatrix}4\\ 2\end{bmatrix}\end{aligned}$$

Therefore, particular solution is:

$$u_p=\begin{bmatrix}4\\ 2\end{bmatrix}$$

Provide feedback (0)

Therefore, particular solution is:

$$u_p = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 13

General solution u_n can be calculated as follows:

$$u(t) = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$$

Calculate Eigen values of matrix A:

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(2-\lambda)(3-\lambda) = 0$$

Therefore, Eigen value is $\lambda = 2, 3$.

Provide feedback (0)

Step 10 of 13

To calculate Eigen vectors do the following calculations:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 2-2 & 0 \\ 0 & 3-2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 13

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 2-2 & 0 \\ 0 & 3-2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=2$ are as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 13

To calculate another Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 2-3 & 0 \\ 0 & 3-3 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=3$ are as follows:

$$x_2=\begin{bmatrix} y \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3-3 \end{bmatrix}\begin{bmatrix} z \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda = 3$ are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 12 of 13

Therefore, general solution is:

$$u_n = c_1 e^{2t}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 13 of 13

Therefore, the complete solution is $u_n + u_p$.

$$\boxed{u_n + u_p = c_1 e^{2t}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}}.$$

Provide feedback (0)

Algebra tutors who can help right now

CH5.4 31P

Step 1 of 10

Stability: Real parts of the Eigen values govern the stability. The differential equation $du/dt = Au$ is:

Stable: If $\mathrm{Re}(\lambda_i) < 0$

Neutrally stable: If all $\mathrm{Re}(\lambda_i) \le 0$ and $\lambda_1 = 0$.

Unstable: If any Eigen value has $\mathrm{Re}(\lambda_i) > 0$.

Provide feedback (0)

Step 2 of 10

Define the matrix A to illustrate the following unstable region.

(a) When Eigen values are:

$$\lambda_1 < 0 \text{ and } \lambda_2 > 0$$

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 10

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1-\lambda & 2 \\ 3 & -1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (1-\lambda)(-1-\lambda) - 6 &= 0 \\ \lambda^2 - 7 &= 0 \end{aligned}$$

CH5.4 33P

Provide feedback (0)

Step 3 of 10

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A-\lambda I &= \begin{bmatrix} 1-\lambda & 2 \\ 3 & -1-\lambda \end{bmatrix} \\ \det(A-\lambda I) &= 0 \\ (1-\lambda)(-1-\lambda)-6 &= 0 \\ \lambda^2-7 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= -\sqrt{7} \\ &< 0 \\ \lambda_2 &= \sqrt{7} \\ &> 0 \end{aligned}$$

Provide feedback (0)

Step 4 of 10

Therefore, following matrix satisfies $\lambda_1 < 0$ and $\lambda_2 > 0$

$$\boxed{A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 10

(b) When Eigen values are:

$\lambda_1 > 0$ and $\lambda_2 > 0$

Therefore, following matrix satisfies $\lambda_1 < 0$ and $\lambda_2 > 0$

$$\boxed{A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}}$$

Provide feedback (0)

Step 5 of 10

(b) When Eigen values are:

$$\lambda_1 > 0 \text{ and } \lambda_2 > 0$$

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (1-\lambda)(3-\lambda) &= 0 \\ \lambda^2 - 4\lambda + 3 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= 1 \\ &> 0 \\ \lambda_2 &= 3 \end{aligned}$$

$$\lambda^2 - 4\lambda + 3 = 0$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= 1 \\ &> 0 \\ \lambda_2 &= 3 \\ &> 0 \end{aligned}$$

Provide feedback (0)

Step 7 of 10

Therefore, following matrix satisfies $\lambda_1 > 0$ and $\lambda_2 > 0$

$$\boxed{A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}}$$

Provide feedback (0)

Step 8 of 10

(c) When Eigen values complex λ's real part is greater than zero.

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 9 of 10

To calculate the Eigen values do the following calculations;

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \end{bmatrix}$$

Consider the following matrix:

$$A=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

Provide feedback (0)

Step 9 of 10

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A-\lambda I &= \begin{bmatrix}1-\lambda & 1\\ -1 & 1-\lambda\end{bmatrix} \\ \det(A-\lambda I) &= 0 \\ (1-\lambda)(1-\lambda)+1 &= 0 \\ \lambda^2-2\lambda+2 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= 1+i \\ \lambda_2 &= 1-i \end{aligned}$$

Provide feedback (0)

Step 10 of 10

Therefore, following matrix satisfies the condition that real part is greater than zero.

$$\boxed{A=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}}$$

Provide feedback (0)

CH5.4 32P

Step 1 of 3

To expand and write atleast five terms of the infinite series for e^{At}. After taking derivative of each term with respect to t, show that there are four terms of Ae^{At}.

Provide feedback (0)

CH5.4 34P

Step 2 of 3

Expansion of infinite series for e^{At} is as follows:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots$$

Differentiate each term of infinite series with respect to t.

$$\begin{aligned}
\frac{d}{dt}e^{At} &= A + \frac{2(A^2t)}{2!} + \frac{3(A^3t^2)}{3!} + \frac{4(A^4t^3)}{4!} + \ldots \\
&= A + A^2t + \frac{(A^3t^2)}{2!} + \frac{(A^4t^3)}{3!} + \ldots \\
&= A\left(I + At + \frac{(A^2t^2)}{2!} + \frac{(A^3t^3)}{3!} + \ldots\right) \\
&= Ae^{At}
\end{aligned}$$

Provide feedback (0)

Step 3 of 3

Therefore, after taking derivative of each five terms of e^{At}, $\boxed{\text{four terms}}$ of Ae^{At} results.

Provide feedback (0)

CH5.4 33P

CH5.4 35P

Step 1 of 4

Consider the following matrix:

$$B=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}$$

$$\begin{aligned}B\cdot B&=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\\ &=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\end{aligned}$$

Find e^{Bt} and then check the derivative is Be^{Bt}.

Provide feedback (0)

Step 2 of 4

Substitute B in the expansion of e^{Bt}.

$$\begin{aligned}e^{Bt}&=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -t\\ 0 & 0\end{bmatrix}\\ &=\begin{bmatrix}1 & -t\\ 0 & 1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 3 of 4

Calculate the following:

$$\begin{aligned}B\cdot e^{Bt}&=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & -t\\ 0 & 1\end{bmatrix}\\ &=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\end{aligned}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Calculate the following:

$$\begin{aligned} B \cdot e^{Bt} &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -t \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ &= B \end{aligned}$$

Now, calculate the derivative of e^{Bt}

$$\begin{aligned} \frac{d}{dt} e^{Bt} &= \begin{bmatrix} 1 & -t \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ &= B \\ &= B \cdot e^{Bt} \end{aligned}$$

Provide feedback (0)

Step 4 of 4

Therefore, derivative of e^{Bt} is $\boxed{Be^{Bt}}$

Provide feedback (0)

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CH5.4 34P

Step 1 of 2

Consider the differential equation:

$$\frac{du}{dT} = Au$$

Recall that following is the solution of the above differential equation, starting from $u(0)$.

$$u(T) = e^{AT}u(0)$$

Let an additional time t is taken to reach the following:

$$e^{At}\left(e^{AT}u(0)\right)$$

Write down the solution at time $T+t$.

Provide feedback (0)

Step 2 of 2

Taken additional time solution can be written as follows:

$$e^{At}e^{AT}u(0) = e^{A(T+t)}u(0)$$

Therefore, $\boxed{e^{A(T+t)}u(0)}$ is the solution at time $T+t$. This concludes that e^{At} times e^{AT} is equal to $\boxed{e^{A(T+t)}}$.

Provide feedback (0)

CH5.4 36P

Algebra tutors who can help right now

CH5.4 35P

Step 1 of 8

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 8

Find the Eigen values and Eigen vectors and write the solution in the form of $S\Lambda S^{-1}$. Then find e^{At} from $Se^{\Lambda t}S^{-1}$.

Provide feedback (0)

Step 3 of 8

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ 0 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(1-\lambda)(-\lambda) = 0$$

$$\lambda^2 - \lambda = 0$$

After solving following values are obtained:

$$\lambda_1 = 1$$

CH5.4 37P

Provide feedback (0)

Step 3 of 8

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}1-\lambda & 1\\ 0 & -\lambda\end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(1-\lambda)(-\lambda)=0$$
$$\lambda^2-\lambda=0$$

After solving following values are obtained:

$$\lambda_1=1$$
$$\lambda_2=0$$

Provide feedback (0)

Step 4 of 8

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}1-1 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

Provide feedback (0)

Step 4 of 8

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}1-1 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

Provide feedback (0)

Step 5 of 8

Similarly, Eigen vectors corresponding to Eigen value $\lambda=0$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}1-0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2=\begin{bmatrix}y\\ z\end{bmatrix}$$

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}1-0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ -1\end{bmatrix}$$

Therefore Eigen values are:

$$x_1=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$x_2=\begin{bmatrix}1\\ -1\end{bmatrix}$$

Provide feedback (1)

Step 6 of 8

Recall that: $A=S\Lambda S^{-1}$. Therefore,

$$A=\boxed{\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}}$$

Provide feedback (0)

Step 7 of 8

Step 6 of 8

Recall that: $A = S\Lambda S^{-1}$. Therefore,

$$A = \boxed{\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}}$$

Provide feedback (0)

Step 7 of 8

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} e^t & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & e^t - 1 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 8 of 8

Therefore,

$$e^{At} = \boxed{\begin{bmatrix} e^t & e^t - 1 \\ 0 & 1 \end{bmatrix}}.$$

Provide feedback (0)

CH5.4 36P

CH5.4 38P

Step 1 of 6

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 6

To show that the infinite series produces the following, when $A^2 = A$:

$$e^{At} = I + \left(e^t - 1\right)A$$

Use the above ration to find e^{At}.

Provide feedback (0)

Step 3 of 6

Infinite series for e^{At} is given as follows:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots$$

To get the relation put $A^2 = A$ and do the following calculations:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots$$

Step 3 of 6

Infinite series for e^{At} is given as follows:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots$$

To get the relation put $A^2 = A$ and do the following calculations:

$$\begin{aligned} e^{At} &= I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots \\ &= I + At + \frac{A(t)^2}{2!} + \frac{A(t)^3}{3!} + \frac{A(t)^4}{4!} + \ldots \\ &= I + A\left(t + \frac{(t)^2}{2!} + \frac{(t)^3}{3!} + \frac{(t)^4}{4!} + \ldots\right) \\ &= I + A\left(e^t - 1\right) \end{aligned}$$

Provide feedback (0)

Step 4 of 6

Therefore, $\boxed{e^{At} = I + \left(e^t - 1\right)A}$.

Provide feedback (0)

Step 5 of 6

Now, find e^{At} with the help of above relation and matrix A.

$$\begin{aligned} e^{At} &= I + \left(e^t - 1\right)A \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(e^t - 1\right)\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 4 of 6

Therefore, $\boxed{e^{At} = I + \left(e^{t} - 1\right)A}$.

Provide feedback (0)

Step 5 of 6

Now, find e^{At} with the help of above relation and matrix A.

$$\begin{aligned} e^{At} &= I + \left(e^{t} - 1\right)A \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(e^{t} - 1\right)\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \left(e^{t} - 1\right) & \left(e^{t} - 1\right) \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} e^{t} & \left(e^{t} - 1\right) \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 6 of 6

Therefore,

$$e^{At} = \boxed{\begin{bmatrix} e^{t} & \left(e^{t} - 1\right) \\ 0 & 1 \end{bmatrix}}.$$

Provide feedback (0)

CH5.4 37P

CH5.4 39P

Step 1 of 14

Consider the following matrix:

$$A=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$$
$$B=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}$$

Provide feedback (0)

Step 2 of 14

Show that $e^A e^B$ is different from $e^B e^A$ and they both are different from e^{A+B}.

Provide feedback (0)

Step 3 of 14

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}1-\lambda & 1\\ 0 & -\lambda\end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(1-\lambda)(-\lambda)=0$$
$$\lambda^2-\lambda=0$$

After solving following values are obtained:

$$\lambda_1=1$$
$$\lambda_2=0$$

Provide feedback (0)

After solving following values are obtained:

$$\lambda_1 = 1$$
$$\lambda_2 = 0$$

Provide feedback (0)

Step 4 of 14

To calculate Eigen vectors do the following calculations:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 1-1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda = 1$ are as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 14

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 1-0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} y \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda = 1$ are as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 14

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 0$ is as follows:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 1-0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 14

Therefore Eigen values are:

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 1 \\ \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 14

Therefore Eigen values are:

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 14

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$e^{At} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} e^t & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} e^t & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} e^t & e^t - 1 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 14

Step 7 of 14

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} e^t & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & e^t - 1 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 8 of 14

For $t = 1$:

$$e^A = \begin{bmatrix} e & e-1 \\ 0 & 1 \end{bmatrix}$$

Consider the following matrix

$$\begin{aligned} B &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ B \cdot B &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 9 of 14

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Provide feedback (0)

Step 9 of 14

Substitute B in the expansion of e^{Bt}.

$$\begin{aligned}e^{Bt} &= I + Bt\\ &= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & -t\\0 & 0\end{bmatrix}\\ &= \begin{bmatrix}1 & -t\\0 & 1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 10 of 14

For $t=1$:

$$e^{B}=\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$$

Provide feedback (0)

Step 11 of 14

Now calculate the following:

$$\begin{aligned}e^{A}e^{B} &= \begin{bmatrix}e & e-1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\\ &= \begin{bmatrix}e & -1\\0 & 1\end{bmatrix}\end{aligned}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 14

Now calculate the following:

$$\begin{aligned} e^A e^B &= \begin{bmatrix} e & e-1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} e & -1 \\ 0 & 1 \end{bmatrix} \\ e^B e^A &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e & e-1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} e & e-2 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore, $\boxed{e^A e^B \neq e^B e^A}$.

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Step 12 of 14

Consider the following matrix:

$$\begin{aligned} A+B &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ (A+B)\cdot(A+B) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Consider the following matrix:

$$\begin{aligned} A+B &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ (A+B)\cdot(A+B) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Thus, $(A+B)^2 = (A+B)$.

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Step 13 of 14

Substitute $(A+B)$ in the expansion of $e^{(A+B)t}$.

$$\begin{aligned} e^{(A+B)t} &= I + (A+B)t + \frac{(A+B)^2 t^2}{2!} + \ldots \\ &= I + (A+B)t + \frac{(A+B)t^2}{2!} + \ldots \\ &= I + (A+B)\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right) \\ &= I + (A+B)(e^t - 1) \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^t - 1)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1+(e^t-1) & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} e^t & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$= I + (A+B)t + \frac{\quad}{2!} + \ldots$$

$$= I + (A+B)\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\right)$$

$$= I + (A+B)(e^t - 1)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^t - 1)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (e^t - 1) & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^t & 0 \\ 0 & 1 \end{bmatrix}$$

$$e^{(A+B)} = \begin{bmatrix} e & 0 \\ 0 & 1 \end{bmatrix}$$

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Step 14 of 14

Therefore, $\boxed{e^A e^B \neq e^B e^A}$ and $e^A e^B, e^B e^A$ are different from $\boxed{e^{A+B}}$.

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CH5.4 38P

CH5.4 40P

Step 1 of 9

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

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Step 2 of 9

Find the Eigen values and Eigen vectors and write the solution in the form of $S\Lambda S^{-1}$. Then find e^{At} from $Se^{\Lambda t}S^{-1}$. Check e^{At} when $t = 0$.

Provide feedback (0)

Step 3 of 9

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (1-\lambda)(3-\lambda) &= 0 \\ \lambda^2 - 4\lambda + 3 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\lambda_1 = 3$$

$$\det(A-\lambda I)=0$$
$$(1-\lambda)(3-\lambda)=0$$
$$\lambda^2-4\lambda+3=0$$

After solving following values are obtained:

$$\lambda_1=3$$
$$\lambda_2=1$$

Provide feedback (0)

Step 4 of 9

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}1-3 & 1\\ 0 & 3-3\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}-2 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=3$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$
$$=\begin{bmatrix}1\\ 2\end{bmatrix}$$

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Step 5 of 9

Similarly, Eigen vectors corresponding to Eigen value $\lambda=1$ is as follows:

$$(A-\lambda I)x=0$$

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 9

Similarly, Eigen vectors corresponding to Eigen value $\lambda = 1$ is as follows:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 1-1 & 1 \\ 0 & 3-1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Therefore Eigen values are:

$$x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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Step 6 of 9

Step 6 of 9

Recall that: $A = S\Lambda S^{-1}$. Therefore,

$$A = \boxed{\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1/2 \\ 1 & -1/2 \end{bmatrix}}$$

Provide feedback (0)

Step 7 of 9

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} e^{3t} & 0 \\ 0 & e^{t} \end{bmatrix}\begin{bmatrix} 0 & 1/2 \\ 1 & -1/2 \end{bmatrix} \\ &= \begin{bmatrix} e^{3t} & e^{t} \\ 2e^{3t} & 0 \end{bmatrix}\begin{bmatrix} 0 & 1/2 \\ 1 & -1/2 \end{bmatrix} \\ &= \begin{bmatrix} e^{t} & \frac{1}{2}e^{3t} - \frac{1}{2}e^{t} \\ 0 & e^{3t} \end{bmatrix} \end{aligned}$$

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Step 8 of 9

Therefore,

$$e^{At} = \boxed{\begin{bmatrix} e^{t} & \frac{1}{2}e^{3t} - \frac{1}{2}e^{t} \\ 0 & e^{3t} \end{bmatrix}}$$

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$$= \begin{bmatrix} e^t & \frac{1}{2}e^{3t} - \frac{1}{2}e^t \\ 0 & e^{3t} \end{bmatrix}$$

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Step 8 of 9

Therefore,

$$e^{At} = \boxed{\begin{bmatrix} e^t & \frac{1}{2}e^{3t} - \frac{1}{2}e^t \\ 0 & e^{3t} \end{bmatrix}}$$

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Step 9 of 9

At $t = 0$:

$$e^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= I$$

Therefore, at $t = 0$, e^{At} is $\boxed{I}$.

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CH5.4 39P

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Step 1 of 3

Consider the following matrix:

$$A=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}$$

Compute A^2 and then fill the following blank:

$$e^{At}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}t & 3t\\ 0 & 0\end{bmatrix}+\frac{1}{2}\begin{bmatrix} & \\ & \end{bmatrix}$$
$$=\begin{bmatrix}e^t & \\ 0 & \end{bmatrix}$$

Provide feedback (0)

Step 2 of 3

Calculate the following:

$$A\cdot A=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}$$
$$A^2=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}$$

Therefore,

$$\boxed{A^2=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}}$$

Provide feedback (0)

Step 3 of 3

Provide feedback (0)

Step 2 of 3

Calculate the following:

$$A \cdot A = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

Therefore,

$$\boxed{A^2 = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}}$$

Provide feedback (0)

Step 3 of 3

Next put the value of A^2 in the expansion of e^{At}.

$$e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} t & 3t \\ 0 & 0 \end{bmatrix} + \frac{1}{2!}\begin{bmatrix} t^2 & 3t^2 \\ 0 & 0 \end{bmatrix} + \cdots$$

$$= \begin{bmatrix} e^t & 3(e^t - 1) \\ 0 & 1 \end{bmatrix}$$

Therefore,

$$\boxed{e^{At} = \begin{bmatrix} e^t & 3(e^t - 1) \\ 0 & 1 \end{bmatrix}}$$

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Step 1 of 3

Explain two reasons that why the exponential matrix e^{At} can never be singular.

(a) *Inverse*: A square matrix that is not invertible is called as singular matrix. However, determinant of e^{At} exists, so inverse will also exist. Inverse of e^{At} is given as follows:

Inverse: $\boxed{e^{-At}}$.

Determinant of the exponential is given as follows:

$$\begin{aligned}\det\left(e^{At}\right) &= e^{\lambda_1 t}\cdot e^{\lambda_2 t}\cdot\ldots\cdot e^{\lambda_n t}\\ &= e^{(\lambda_1+\lambda_2+\ldots+\lambda_n)t}\\ &= e^{\operatorname{trace}(At)}\end{aligned}$$

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Step 2 of 3

(b) Eigen values: If $Ax=\lambda x$ shows that λ is an Eigen values of A, then $e^{\lambda t}$ is an Eigen value matrix of e^{At}. That means $e^{\lambda t}$ can never be zero.

$$e^{At}x=\boxed{e^{\lambda t}x}.$$

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Step 3 of 3

Therefore, exponential matrix e^{At} can never be singular.

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Step 1 of 11

Consider the following system:

$$dx/dt = 0x - 4y$$
$$dy/dt = -2x + 2y$$

Find the solution for $x(t)$ and $y(t)$ that gets large as $t \to \infty$.

Provide feedback (0)

Step 2 of 11

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 0 & -4 \\ -2 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 11

Find the Eigen values and Eigen vectors and then find e^{At} from $Se^{\Lambda t}S^{-1}$. Check $u(t)$ at $t \to \infty$.

Provide feedback (0)

Step 4 of 11

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

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$u(t)$ at $t \to \infty$.

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Step 4 of 11

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A - \lambda I = \begin{bmatrix} 0-\lambda & -4 \\ -2 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(-\lambda)(2-\lambda) - 8 = 0$$

$$\lambda^2 - 2\lambda - 8 = 0$$

After solving following values are obtained:

$$\lambda_1 = 4$$
$$\lambda_2 = -2$$

Provide feedback (0)

Step 5 of 11

To calculate Eigen vectors do the following calculations:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 0-4 & -4 \\ -2 & 2-4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -4 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of x and y corresponding to $\lambda = 4$ are as follows:

$$x_1 = \begin{bmatrix} x \end{bmatrix}$$

Provide feedback (0)

Step 5 of 11

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 0-4 & -4 \\ -2 & 2-4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -4 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of x and y corresponding to $\lambda=4$ are as follows:

$$x_1=\begin{bmatrix} x \\ y \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda=-2$ is as follows:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 0+2 & -4 \\ -2 & 2+2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of x and y are as follows:

$$\begin{bmatrix} \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 11

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -2$ is as follows:

$$\begin{aligned} (A-\lambda I)x &= 0 \\ \begin{bmatrix} 0+2 & -4 \\ -2 & 2+2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 2 & -4 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

On solving values of x and y are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 7 of 11

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$\begin{aligned} e^{At} &= \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{4t} & 0 \\ 0 & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{3}\begin{bmatrix} e^{4t} & 2e^{-2t} \\ -e^{4t} & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{3}\begin{bmatrix} e^{4t}+2e^{-2t} & -2e^{4t}+2e^{-2t} \\ -e^{4t}+e^{-2t} & 2e^{4t}+e^{-2t} \end{bmatrix} \end{aligned}$$

Step 7 of 11

Recall that $e^{At} = Se^{\Lambda t}S^{-1}$. Therefore,

$$\begin{aligned} e^{At} &= \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{4t} & 0 \\ 0 & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{3}\begin{bmatrix} e^{4t} & 2e^{-2t} \\ -e^{4t} & e^{-2t} \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{3}\begin{bmatrix} e^{4t}+2e^{-2t} & -2e^{4t}+2e^{-2t} \\ -e^{4t}+e^{-2t} & 2e^{4t}+e^{-2t} \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 8 of 11

Recall that $u(t) = e^{At}u(0)$. To cancel the fraction lets consider initial values as $(1,-1)$.

$$\begin{aligned} u(t) &= e^{At}u(0) \\ &= \frac{1}{3}\begin{bmatrix} e^{4t}+2e^{-2t} & -2e^{4t}+2e^{-2t} \\ -e^{4t}+e^{-2t} & 2e^{4t}+e^{-2t} \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ &= \frac{1}{3}\begin{bmatrix} 3e^{4t} \\ -3e^{4t} \end{bmatrix} \\ &= \begin{bmatrix} e^{4t} \\ -e^{4t} \end{bmatrix} \end{aligned}$$

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Step 9 of 11

Therefore, the solution is as follows:

$$\boxed{x(t) = e^{4t}}$$

$$= \begin{bmatrix} e^{4t} \\ -e^{4t} \end{bmatrix}$$

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Step 9 of 11

Therefore, the solution is as follows:

$$\boxed{\begin{aligned} x(t) &= e^{4t} \\ y(t) &= -e^{4t} \end{aligned}}$$

As $t \to \infty$ the values of $x(t)$ and $y(t)$ becomes very large and the solution becomes unstable.

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Step 10 of 11

To avoid this instability the two equations of the system are exchanged as follows:

$$\begin{aligned} dy/dt &= -2x + 2y \\ dx/dt &= 0x - 4y \end{aligned}$$

Now the matrix becomes:

$$B = \begin{bmatrix} -2 & 2 \\ 0 & -4 \end{bmatrix}$$

Eigen values of changed system are less than zero $\lambda < 0$. As per the scientists the system is now stable.

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$$dx/dt = 0x - 4y$$

Now the matrix becomes:

$$B = \begin{bmatrix} -2 & 2 \\ 0 & -4 \end{bmatrix}$$

Eigen values of changed system are less than zero $\lambda < 0$. As per the scientists the system is now stable.

Provide feedback (0)

Step 11 of 11

Compare the two matrices A and B.

$$A = \begin{bmatrix} 0 & -4 \\ -2 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 2 \\ 0 & -4 \end{bmatrix}$$

It can be seen that values of x and y in matrix A are exchanged among themselves in matrix B. To change the values of x from values of y column values should be exchanged. Thus, correct matrix is as follows:

$$\boxed{u(y,x) = \begin{bmatrix} 2 & -2 \\ -4 & 0 \end{bmatrix}}$$

This matrix gives the same Eigen values as the original system matrix A.

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CH5.4 42P

Step 1 of 4

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

General solution is given as follows:

$$u(t) = c_1 e^{2t}\begin{bmatrix}2\\1\end{bmatrix} + c_2 e^{5t}\begin{bmatrix}1\\1\end{bmatrix}$$

Find the matrix A.

Provide feedback (0)

Step 2 of 4

General solution can be written as follows:

$$\begin{aligned} u(t) &= c_1 e^{2t}\begin{bmatrix}2\\1\end{bmatrix} + c_2 e^{5t}\begin{bmatrix}1\\1\end{bmatrix} \\ &= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}e^{2t} & 0\\0 & e^{5t}\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} \\ &= Se^{\Lambda t}c \end{aligned}$$

Provide feedback (0)

Step 3 of 4

Recall that $A = S\Lambda S^{-1}$. Therefore,

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\0 & 5\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix} \end{aligned}$$

CH5.5

$$\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}0 & e^{-t}\end{bmatrix}\begin{bmatrix}c_2\end{bmatrix}$$
$$= Se^{\Lambda t}c$$

Provide feedback (0)

Step 3 of 4

Recall that $A = S\Lambda S^{-1}$. Therefore,

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\0 & 5\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix} \\ &= \begin{bmatrix}4 & 5\\2 & 5\end{bmatrix}\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix} \\ &= \begin{bmatrix}-1 & 6\\-3 & 8\end{bmatrix} \end{aligned}$$

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Step 4 of 4

Therefore,

$$\boxed{A = \begin{bmatrix}-1 & 6\\-3 & 8\end{bmatrix}}$$

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