Chapter: CH1.6 Problem: 1P

show all steps

CH1.5

CH1.6 2P

Step 1 of 3

Given that $A_1 = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$

We know that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\Rightarrow A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow A_1^{-1} = \frac{1}{0(0) - 3(2)}\begin{pmatrix} 0 & -2 \\ -3 & 0 \end{pmatrix}$$

$$\Rightarrow A_1^{-1} = \frac{1}{-6}\begin{pmatrix} 0 & -2 \\ -3 & 0 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{pmatrix}}$$

Provide feedback (0)

Step 2 of 3

Given that $A_2 = \begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$

$$A_2 = \begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$$

$$\Rightarrow A_2^{-1} = \frac{1}{2.2 - 4.0}\begin{pmatrix} 2 & 0 \\ -4 & 2 \end{pmatrix}$$

$$= \frac{1}{\ldots}\begin{pmatrix} 2 & 0 \end{pmatrix}$$

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Step 2 of 3

Given that $A_2 = \begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$

$$A_2 = \begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$$

$$\Rightarrow A_2^{-1} = \frac{1}{2.2-4.0}\begin{pmatrix} 2 & 0 \\ -4 & 2 \end{pmatrix}$$

$$\Rightarrow A_2^{-1} = \frac{1}{4}\begin{pmatrix} 2 & 0 \\ -4 & 2 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{2} \end{pmatrix}}$$

Provide feedback (0)

Step 3 of 3

Given that $A_3 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$A_3 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\Rightarrow A_3^{-1} = \frac{1}{\cos\theta.\cos\theta + \sin\theta.\sin\theta}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\Rightarrow A_3^{-1} = \boxed{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

Provide feedback (0)

CH1.6 1P

Step 1 of 5

(a) Given permutation matrix is $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

We have to find the inverse of the given permutation matrix.

Provide feedback (0)

Step 2 of 5

We have $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

We now find the inverse of the permutation matrix using Gauss-Jordan Elimination

Now $\begin{bmatrix} P & e_1 & e_2 & e_3 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & 0 & 1 & 0 \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 \end{bmatrix}$

Exchanging R_1 and $R_3 = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 \end{bmatrix}$

By the Gauss-Jordan elimination process the inverse of the given permutation matrix is $P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Provide feedback (0)

Step 3 of 5

CH1.6 3P

Provide feedback (0)

Step 3 of 5

Given matrix is $P=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

We have to find the inverse of the given permutation matrix.

Provide feedback (0)

Step 4 of 5

We have $P=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

Now $\begin{bmatrix}P & e_1 & e_2 & e_3\end{bmatrix}=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{1}&1&0&0\\\mathbf{1}&\mathbf{0}&\mathbf{0}&0&1&0\\\mathbf{0}&\mathbf{1}&\mathbf{0}&0&0&1\end{bmatrix}$

Exchanging row 1 and row 2 $=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{0}&0&1&0\\\mathbf{0}&\mathbf{0}&\mathbf{1}&1&0&0\\\mathbf{0}&\mathbf{1}&\mathbf{0}&0&0&1\end{bmatrix}$

Exchanging row 2 and row 3 $=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{0}&0&1&0\\\mathbf{0}&\mathbf{1}&\mathbf{0}&0&0&1\\\mathbf{0}&\mathbf{0}&\mathbf{1}&1&0&0\end{bmatrix}$

By the gauss-Jordan elimination process the inverse of the given permutation matrices is $P^{-1}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$

We have to find the inverse of the given permutation matrix.

Provide feedback (0)

Step 4 of 5

We have $P=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

Now $[P\ e_1\ e_2\ e_3]=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{1}&1&0&0\\\mathbf{1}&\mathbf{0}&\mathbf{0}&0&1&0\\\mathbf{0}&\mathbf{1}&\mathbf{0}&0&0&1\end{bmatrix}$

Exchanging row 1 and row 2 $=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{0}&0&1&0\\\mathbf{0}&\mathbf{0}&\mathbf{1}&1&0&0\\\mathbf{0}&\mathbf{1}&\mathbf{0}&0&0&1\end{bmatrix}$

Exchanging row 2 and row 3 $=\begin{bmatrix}\mathbf{1}&\mathbf{0}&\mathbf{0}&0&1&0\\\mathbf{0}&\mathbf{1}&\mathbf{0}&0&0&1\\\mathbf{0}&\mathbf{0}&\mathbf{1}&1&0&0\end{bmatrix}$

By the gauss-Jordan elimination process the inverse of the given permutation matrices is $P^{-1}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$

Provide feedback (0)

Step 5 of 5

(b) The inverse of permutation matrices is transpose of it self since permutation matrices is some row exchange of identity matrices.

Provide feedback (0)

CH1.6 2P

Step 1 of 2

Consider the matrix equation is $AB = C$

Find a formula for A^{-1}.

Multiplying $AB = C$ with A^{-1} both sides on left side gives

Again multiplying with C^{-1} on right sides gives

Hence the formula for A^{-1} is $\boxed{A^{-1} = BC^{-1}}$

Provide feedback (0)

Step 2 of 2

Now find a formula for A^{-1} from $PA = LU$

Multiplying right side of $PA = LU$ with $A^{-1}L^{-1}U^{-1}$ on both sides

CH1.6 4P

CH1.6 2P

Step 1 of 2

Consider the matrix equation is $AB = C$

Find a formula for A^{-1}.

Multiplying $AB = C$ with A^{-1} both sides on left side gives

$$\begin{aligned} A^{-1}(AB) &= A^{-1}C \\ (A^{-1}A)B &= A^{-1}C && \text{(By Associative property)} \\ IB &= A^{-1}C && (\text{Since } A^{-1}A = I) \\ B &= A^{-1}C && (\text{Since } IB = B) \end{aligned}$$

Again multiplying with C^{-1} on right sides gives

$$\begin{aligned} BC^{-1} &= (A^{-1}C)C^{-1} \\ BC^{-1} &= A^{-1}(CC^{-1}) && \text{(By associative property)} \\ BC^{-1} &= A^{-1}(I) && (\text{Since} CC^{-1} = I) \\ BC^{-1} &= A^{-1} \end{aligned}$$

Hence the formula for A^{-1} is $\boxed{A^{-1} = BC^{-1}}$

Provide feedback (0)

CH1.6 4P

Step 2 of 2

Now find a formula for A^{-1} from $PA = LU$

Multiplying right side of $PA = LU$ with $A^{-1}L^{-1}U^{-1}$ on both sides

$$\begin{aligned} PA(A^{-1}L^{-1}U^{-1}) &= LU(A^{-1}L^{-1}U^{-1}) \\ P(AA^{-1}L^{-1}U^{-1}) &= LU(L^{-1}U^{-1}A^{-1}) && \text{(By Commutative property)} \\ P(U^{-1}L^{-1}) &= (LUL^{-1}U^{-1})(A^{-1}) && \text{(By Commutative property)} \end{aligned}$$

Again multiplying with C^{-1} on right sides gives

$$BC^{-1} = \left(A^{-1}C\right)C^{-1}$$
$$BC^{-1} = A^{-1}\left(CC^{-1}\right) \qquad \text{(By associative property)}$$
$$BC^{-1} = A^{-1}(I) \qquad \left(\text{Since} CC^{-1} = I\right)$$
$$BC^{-1} = A^{-1}$$

Hence the formula for A^{-1} is $\boxed{A^{-1} = BC^{-1}}$

Provide feedback (0)

Step 2 of 2

Now find a formula for A^{-1} from $PA = LU$

Multiplying right side of $PA = LU$ with $A^{-1}L^{-1}U^{-1}$ on both sides

$$PA\left(A^{-1}L^{-1}U^{-1}\right) = LU\left(A^{-1}L^{-1}U^{-1}\right)$$
$$P\left(AA^{-1}L^{-1}U^{-1}\right) = LU\left(L^{-1}U^{-1}A^{-1}\right) \qquad \text{(By Commutative property)}$$
$$P\left(IL^{-1}U^{-1}\right) = \left(LUL^{-1}U^{-1}\right)\left(A^{-1}\right) \qquad \left(\begin{array}{l}\text{By Commutative property}\\ \text{and } AA^{-1} = I\end{array}\right)$$
$$P\left(L^{-1}U^{-1}\right) = \left(LL^{-1}UU^{-1}\right)\left(A^{-1}\right) \qquad \left(\begin{array}{l}\text{By Commutative property}\\ \text{And } IL^{-1}U^{-1} = L^{-1}U^{-1}\end{array}\right)$$
$$PL^{-1}U^{-1} = (I)\left(A^{-1}\right) \qquad \left(\begin{array}{l}LL^{-1} = I\\ UU^{-1} = I\\ IA^{-1} = A^{-1}\end{array}\right)$$
$$PL^{-1}U^{-1} = A^{-1}$$

Hence the formula for A^{-1} is $\boxed{A^{-1} = PL^{-1}U^{-1}}$

Provide feedback (0)

CH1.6 3P

Step 1 of 2

(a) Given that A is invertible $\Rightarrow A.A^{-1} = A^{-1}.A = I$

We have

$AB = AC$

Multiply with A^{-1} on both sides

$\Rightarrow A^{-1}.(AB) = A^{-1}.(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C$

$\Rightarrow I.B = I.C \quad (\text{since } A^{-1}A = I)$

$\Rightarrow B = C$

Provide feedback (0)

Step 2 of 2

(b) Given that $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Let $B = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix}$ (That is $B \neq C$)

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

Therefore $A.B = A.C$ but $B \neq C$

Provide feedback (1)

CH1.6 5P

CH1.6 4P

Step 1 of 1

Given that inverse of A^2 is B

Therefore we have $A^2B = BA^2 = I$

We have $A^2B = I$

$$\Rightarrow (AA)B = I$$

$$\Rightarrow A(AB) = I \ (\because \text{by associative law})$$

$\Rightarrow$ Inverse of A is AB.

Provide feedback (0)

CH1.6 6P

CH1.6 5P

Step 1 of 8

Given matrix is $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

We have to find the inverse of the given matrix using Gauss-Jordan method.

Provide feedback (0)

Step 2 of 8

Now $\quad [A_1 \ e_1 \ e_2 \ e_3] = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & 0 & 0 & 1 \end{bmatrix}$

Subtracting R_1 from R_2 $= \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & -1 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & 0 & 0 & 1 \end{bmatrix}$

Subtracting R_3 from R_2 $= \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -1 & 1 & -1 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & 0 & 0 & 1 \end{bmatrix}$

Hence the inverse of A_1 is $\boxed{A_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}$

Provide feedback (0)

Step 3 of 8

Given matrix is $A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \end{bmatrix}$

CH1.6 7P

Provide feedback (0)

Step 3 of 8

Given matrix is $A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

We have to find the inverse of the given matrix using Gauss-Jordan method.

Provide feedback (0)

Step 4 of 8

Now

$$\left[A_2 \;\; e_1 \;\; e_2 \;\; e_3\right] = \begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} & 1 & 0 & 0 \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{-1} & \mathbf{2} & 0 & 0 & 1 \end{bmatrix}$$

$$\textbf{pivot} = \mathbf{2} \rightarrow \begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} & 1 & 0 & 0 \\ \mathbf{0} & \frac{\mathbf{3}}{\mathbf{2}} & \mathbf{-1} & \frac{1}{2} & 1 & 0 \\ \mathbf{0} & \mathbf{-1} & \mathbf{2} & 0 & 0 & 1 \end{bmatrix}$$

$$\textbf{pivot} = \frac{\mathbf{3}}{\mathbf{2}} \rightarrow \begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} & 1 & 0 & 0 \\ \mathbf{0} & \frac{\mathbf{3}}{\mathbf{2}} & \mathbf{-1} & \frac{1}{2} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix} = \left[U \quad L^{-1}\right]$$

Provide feedback (1)

Step 5 of 8

Step 5 of 8

This completes the first half-forward elimination. The upper triangular U appears in the first three columns. The other three columns are the same as L^{-1}.
Now the second will go from U to I (multiplying by U^{-1}). That takes L^{-1} to $U^{-1}L^{-1}$ which is A_2^{-1}
Creating zeros above the pivots, we reach A_2^{-1}

$$\textbf{second half } \begin{bmatrix} U & L^{-1} \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 & \frac{3}{4} & \frac{3}{2} & \frac{9}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

Provide feedback (0)

Step 6 of 8

Continuation to the above

$$\textbf{zeors above pivots} \rightarrow \begin{bmatrix} 3 & 0 & 0 & \frac{9}{4} & \frac{3}{2} & \frac{3}{4} \\ 0 & \frac{3}{2} & 0 & \frac{3}{4} & \frac{3}{2} & \frac{9}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

$$\textbf{Divide by pivots} \rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

Hence the inverse of A_2 is $\boxed{A_2^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}}$

Provide feedback (2)

Step 7 of 8

Given matrix is $A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

We have to find the inverse of the given matrix using Gauss-Jordan method.

Provide feedback (0)

Step 8 of 8

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & 0 & 1 & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 1 \end{bmatrix}$$

Now

$$\mathbf{pivot} = \mathbf{1} \rightarrow \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & -1 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -1 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 \end{bmatrix}$$

Hence the inverse of A_3 is $\boxed{A_3^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$

CH1.6 6P

Step 1 of 7

We have to find three 2 by 2 matrices A, other than $A = I$ and $A = -I$, that are their own inverses $A^2 = I$.

Provide feedback (0)

CH1.6 8P

Step 2 of 7

Let the 2 by 2 matrices be $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$\begin{aligned} \text{Now } A^2 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0(0)+1(1) & 0(1)+1(0) \\ 1(0)+0(1) & 1(1)+0(0) \end{bmatrix} \\ &= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Provide feedback (0)

Step 3 of 7

Let the 2 by 2 matrices be $A = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

Now

Now

$$A^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\right) & \frac{\sqrt{3}}{2}\left(\frac{1}{2}\right)+\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) \\ \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) & \frac{1}{2}\left(\frac{1}{2}\right)+\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) \end{bmatrix}$$

Provide feedback (0)

Step 4 of 7

Continuation to the above

$$= \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{4}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Provide feedback (0)

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Provide feedback (0)

Step 5 of 7

Let the 2 by 2 matrices be $A = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Now

$$A^2 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) & \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) \\ \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) & \frac{1}{2}\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Continuation to the above

Provide feedback (0)

Provide feedback (0)

Step 6 of 7

Continuation to the above

Provide feedback (0)

Step 7 of 7

$$= \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{4}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Hence the required matrices are $\boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}}$.

Provide feedback (0)

CH1.6 7P

Step 1 of 3

Given matrix is $A=\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}$

We have to show that A has no inverse by solving $Ax=0$.

Provide feedback (0)

Step 2 of 3

The matrix form of $Ax=0$ is

$$\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Subtracting 3 times row 1 from row 3 gives

$$\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

This shows that $x+y=0$ which has no solution

Hence the given matrix A has no inverse.

Provide feedback (0)

Step 3 of 3

Now we have to show that A has no inverse by solving $\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Given system is

$$\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By matrix multiplication

$$\begin{bmatrix}a+c & b+d\\ 3a+3c & 3b+3d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Equating the corresponding elements

$a+c=1$ $b+d=0$

CH1.6 9P

Step 2 of 3

The matrix form of $Ax = 0$ is

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Subtracting 3 times row 1 from row 3 gives

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This shows that $x + y = 0$ which has no solution

Hence the given matrix A has no inverse.

Provide feedback (0)

Step 3 of 3

Now we have to show that A has no inverse by solving $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given system is

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By matrix multiplication

$$\begin{bmatrix} a+c & b+d \\ 3a+3c & 3b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements

$a + c = 1 \qquad b + d = 0$

$3a + 3c = 0 \qquad 3b + 3d = 1$

But by try to solve $a + c = 1, 3a + 3b = 0$, it has no solution

Because they are parallel lines and leads to $0 = 1$ case.

Similarly, the other two equations also lead to $0 = 1$ condition.

Hence the given matrix A has no solution.

Provide feedback (0)

CH1.6 8P

Step 1 of 4

Given matrices $A = \begin{bmatrix} 2 & 1 & 4 & 6 \\ 0 & 3 & 8 & 5 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 9 \end{bmatrix}$

We have to show that A is not invertible.

Provide feedback (0)

Step 2 of 4

Subtracting $\frac{9}{7}$ times row 3 from row 4, we get

$$A = \begin{bmatrix} 2 & 1 & 4 & 6 \\ 0 & 3 & 8 & 5 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the last row consists all zeros.
Hence the matrix A has no inverse.

Provide feedback (0)

Step 3 of 4

Given that the third row of A^{-1}, multiplying A should give the third row $\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ of $A^{-1}A = I$

We have to explain why this is impossible.

Provide feedback (0)

Step 4 of 4

CH1.6 10P

Subtracting $\frac{9}{7}$ times row 3 from row 4, we get

$$A=\begin{bmatrix}2&1&4&6\\0&3&8&5\\0&0&0&7\\0&0&0&0\end{bmatrix}$$

Since the last row consists all zeros.
Hence the matrix A has no inverse.

Provide feedback (0)

Step 3 of 4

Given that the third row of A^{-1}, multiplying A should give the third row $\begin{bmatrix}0&0&1&0\end{bmatrix}$ of $A^{-1}A=I$
We have to explain why this is impossible.

Provide feedback (0)

Step 4 of 4

Let the third row of A^{-1} is $\begin{bmatrix}a&b&c&d\end{bmatrix}$
Then $A^{-1}A=I$

$$\begin{bmatrix}a&b&c&d\end{bmatrix}\begin{bmatrix}2&1&4&6\\0&3&8&5\\0&0&0&7\\0&0&0&9\end{bmatrix}=\begin{bmatrix}0&0&1&0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2a&a+3b&4a+8b&6a+5b+7c+9d\end{bmatrix}=\begin{bmatrix}0&0&1&0\end{bmatrix}$$

Equating the corresponding elements gives

$$2a=0,\ a+3b=0,\ 4a+8b=1,\ 6a+5b+7c+9d=0$$

This has no solution.
Hence A is not invertible.

CH1.6 9P

Step 1 of 11

Consider the matrix

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}$$

Use gauss-Jordan elimination process to find the inverse of the given matrix.

Provide feedback (0)

CH1.6 11P

Step 2 of 11

Assume the matrix of the form

$$\begin{bmatrix} A_1 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{4} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & 1 \end{bmatrix}$$

Divide 2nd row by 2 , 3rd row 3 and 4th row by 4.

That is, $R_2 \to R_2/2$, $R_3 \to R_3/3$ and $R_4 \to R_4/4$

Then the matrix can be changed to

$$\begin{bmatrix} A_1 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & 0 & \frac{1}{2} & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 0 & \frac{1}{3} & 0 \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

Step 3 of 11

Interchange 1st row and 4th row, 2nd and 3rd row.

That is, $R_1 \leftrightarrow R_4$ and $R_2 \leftrightarrow R_3$

Then the matrix can be changed to

$$\begin{bmatrix} A_1 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & \frac{1}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 0 & \frac{1}{3} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & 0 & \frac{1}{2} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \end{bmatrix}$$

Hence, inverse of the matrix A_1 is

$$\boxed{A_1^{-1} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}$$

Provide feedback (0)

Step 4 of 11

Consider the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \end{bmatrix}$$

Step 4 of 11

Consider the matrix

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{bmatrix}$$

Use gauss-Jordan elimination process to find the inverse of the given matrix.

Provide feedback (0)

Step 5 of 11

Assume the matrix of the form

$$\begin{bmatrix} A_2 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ -\frac{\mathbf{1}}{\mathbf{2}} & \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & -\frac{\mathbf{2}}{\mathbf{3}} & \mathbf{1} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & -\frac{\mathbf{3}}{\mathbf{4}} & \mathbf{1} & 0 & 0 & 0 & 1 \end{bmatrix}$$

Adding (½) times 1st row to 2nd row

That is, $R_2 \to R_2 + \left(\frac{1}{2}\right) R_1$

Then the matrix can be changed to

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & 1 & 1 & 0 & 0 \end{bmatrix}$$

Assume the matrix of the form

$$\begin{bmatrix} A_2 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Adding (½) times 1st row to 2nd row

That is, $R_2 \to R_2 + \left(\frac{1}{2}\right) R_1$

Then the matrix can be changed to

$$\begin{bmatrix} A_2 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 11

Adding $\left(\frac{2}{3}\right)$ times 2nd row to 3rd row.

That is, $R_3 \to R_3 + \left(\frac{2}{3}\right) R_2$

Then the matrix can be changed to

$$\begin{bmatrix} A_2 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & -\frac{\mathbf{3}}{\mathbf{4}} & \mathbf{1} & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 11

Adding $\left(\frac{3}{4}\right)$ times 3rd row to 4th row.

That is, $R_4 \to R_4 + \left(\frac{3}{4}\right) R_2$

Then the matrix can be changed to

$$\begin{bmatrix} A_2 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & 0 & \mathbf{1} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{bmatrix}$$

Hence, the inverse of A_2 is

$$A_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{} & \frac{2}{} & 1 & 0 \end{bmatrix}$$

Hence, the inverse of A_2 is

$$\boxed{A_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{bmatrix}}$$

Provide feedback (0)

Step 8 of 11

Consider the matrix

$$A_3 = \begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}$$

Use gauss-Jordan elimination process to find the inverse of the given matrix.

Provide feedback (0)

Step 9 of 11

Assume the matrix of the form

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} a & b & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ c & d & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & a & b & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & c & d & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} a & b & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ c & d & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & a & b & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & c & d & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{bmatrix}$$

Divide 1st row and 3rd row by a

That is, $R_1 \to R_1/a$ and $R_3 \to R_3/a$

Then the matrix can be changed to

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} 1 & \frac{b}{a} & \mathbf{0} & \mathbf{0} & \frac{1}{a} & 0 & 0 & 0 \\ c & d & \mathbf{0} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ \mathbf{0} & \mathbf{0} & c & d & 0 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting c times row 1 from row 2 and c times row 3 from row 4.

That is, $R_2 \to R_2 - c \cdot R_1$ and $R_4 \to R_4 - c \cdot R_3$

Then the matrix can be changed to

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} 1 & \frac{b}{a} & \mathbf{0} & \mathbf{0} & \frac{1}{a} & 0 & 0 & 0 \\ 0 & d-\frac{cb}{a} & \mathbf{0} & \mathbf{0} & \frac{c}{a} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ \mathbf{0} & \mathbf{0} & 0 & d-\frac{cb}{a} & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 11

Multiply 1st row and 4th row with $\frac{a}{ad-bc}$

That is, $R_1 \to \left(\frac{a}{ad-bc}\right)R_1$ and $R_4 \to \left(\frac{a}{ad-bc}\right)R_4$

Then the matrix can be changed to

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \begin{bmatrix} 1 & \frac{b}{a} & \mathbf{0} & \mathbf{0} & \frac{1}{a} & 0 & 0 & 0 \\ 0 & 1 & \mathbf{0} & \mathbf{0} & \frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ \mathbf{0} & \mathbf{0} & 0 & 1 & 0 & 0 & 0 & \frac{a}{ad-bc} \end{bmatrix}$$

Provide feedback (0)

Step 11 of 11

Subtracting $\left(\frac{b}{a}\right)$ times 2nd row from 1st row and $\left(\frac{b}{a}\right)$ times 3rd row from 4th row.

That is, $R_1 \to R_1 - \left(\frac{b}{a}\right)R_2$ and $R_4 \to R_4 - \left(\frac{b}{a}\right)R_3$

Then the matrix can be changed to

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \mathbf{0} & \mathbf{0} & \left(\frac{1}{a} - \frac{bc}{a(ad-bc)}\right) & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & \mathbf{0} & \mathbf{0} & \frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \end{bmatrix}$$

That is, $R_1 \to R_1 - \left(\frac{b}{a}\right)R_2$ and $R_4 \to R_4 - \left(\frac{b}{a}\right)R_3$

Then the matrix can be changed to

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \mathbf{0} & \mathbf{0} & \left(\frac{1}{a} - \frac{bc}{a(ad-bc)}\right) & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & \mathbf{0} & \mathbf{0} & \frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 & \left(\frac{1}{a} - \frac{bc}{a(ad-bc)}\right) & 0 \\ \mathbf{0} & \mathbf{0} & 0 & 1 & 0 & 0 & 0 & \frac{a}{ad-bc} \end{bmatrix}$$

Hence, the inverse of A_3 is

$$\boxed{A_3^{-1} = \begin{bmatrix} \frac{1}{a} - \frac{bc}{a(ad-bc)} & -\frac{b}{ad-bc} & 0 & 0 \\ \frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & \frac{1}{a} - \frac{bc}{a(ad-bc)} & 0 \\ 0 & 0 & 0 & \frac{a}{ad-bc} \end{bmatrix}}$$

Provide feedback (0)

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Scotty G.

Rahul K.

CH1.6 10P

CH1.6 12P

Step 1 of 4

(a) We have to give examples of A and B such that $A+B$ is not invertible although A and B are invertible.

Let $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ are two invertible matrices.

Then

$$\begin{aligned}A+B&=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\\ &=\begin{bmatrix}1-1 & 0+0\\ 0+0 & 1-1\end{bmatrix}\\ &=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\end{aligned}$$

Hence $A+B$ is not invertible although A and B are invertible.

Provide feedback (0)

Step 2 of 4

(b) We have to give examples of A and B such that $A+B$ is invertible although A and B are not invertible.

Let $A=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$ are two matrices which are not invertible because one of the rows of A and i is zero.

Now

$$\begin{aligned}A+B&=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\\ &=\begin{bmatrix}1+0 & 0+0\\ 0+0 & 0+1\end{bmatrix}\\ &=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\end{aligned}$$

Hence $A+B$ is invertible but A and B are not invertible matrices.

Step 3 of 4

(c) We have to give examples of A and B such that all of A, B and $A+B$ are invertible.

Let $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ are two invertible matrices

Then

$$A+B=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}=\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$$

$$\begin{aligned}A+B&=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\\ &=\begin{bmatrix}1+0 & 0+1\\ 0-1 & 1+0\end{bmatrix}\\ &=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\end{aligned}$$

Provide feedback (0)

Step 4 of 4

Now

$$\begin{aligned}\det(A+B)&=1(1)-(-1)1\\ &=1+1\\ &=2\end{aligned}$$

Since $\det(A+B)$ is nonzero.

Hence $A+B$ is also invertible.

Provide feedback (0)

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CH1.6 11P

Step 1 of 7

Let A be an invertible matrix.
We have to determine which of the properties of A remain true for A^{-1}.

Provide feedback (0)

CH1.6 13P

Step 2 of 7

(a) Let A be a triangular matrix.
We have to verify whether the inverse of triangular matrix is triangular or not.

Let $A=\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ be a triangular matrix.

Then

$$A^{-1}=\frac{1}{2(1)-0(3)}\begin{bmatrix}1 & -3\\ 0 & 2\end{bmatrix} \qquad \left(\text{Since } A=\begin{bmatrix}a & b\\ c & d\end{bmatrix} \text{ so } A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}1 & -3\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2} & -\frac{3}{2}\\ 0 & 1\end{bmatrix}$$

Hence A^{-1} is also a triangular matrix.

Provide feedback (0)

Step 3 of 7

(b) Let A be a symmetric matrix.
We have to verify whether the inverse of symmetric matrix is symmetric or not.
The inverse of symmetric matrices is also a symmetric.

Let $A=\begin{bmatrix}2 & 0\end{bmatrix}$, which a symmetric matrix since $A^T=A=\begin{bmatrix}2 & 0\end{bmatrix}$

(b) Let A be a symmetric matrix.
We have to verify whether the inverse of symmetric matrix is symmetric or not.
The inverse of symmetric matrices is also a symmetric.

Let $A=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$, which a symmetric matrix since $A^T=A=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$

Now

$$A^{-1}=\frac{1}{2(3)-0(0)}\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$$

$$=\frac{1}{6}\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{3}\end{bmatrix}$$

Now it is clear that $\left(A^{-1}\right)^T=A^{-1}$

Hence A^{-1} is also a symmetric matrix.

Provide feedback (0)

Step 4 of 7

(c) Let A be a tridiagonal matrix.
We have to verify whether the inverse of tridiagonal matrix is tridiagonal or not.

The inverse of tridiagonal matrices is also tridiagonal.

Let $A=\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}$ be a tridiagonal matrix.

Then

$$A^{-1}=\frac{1}{0(0)-2(3)}\begin{bmatrix}0 & -2\\ -3 & 0\end{bmatrix}$$

$$=\frac{1}{-6}\begin{bmatrix}0 & -2\\ -3 & 0\end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$$

Hence A^{-1} is also a tridiagonal matrix since its anti diagonal consists of nonzero elements.

Provide feedback (1)

Step 5 of 7

(d) Let A be a matrix in which all the entries are whole numbers.
We have to verify whether all the entries of inverse of A are whole numbers are not.
The inverse of the matrix A in which all entries is whole numbers need not consists all whole numbers.
For instance let $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

Then

$$A^{-1} = \frac{1}{4(5)-2(3)} \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{20-6} \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{14} & -\frac{3}{14} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

consists all whole numbers.

For instance let $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$

Then

$$A^{-1}=\frac{1}{4(5)-2(3)}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}$$

$$=\frac{1}{20-6}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}$$

$$=\frac{1}{14}\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{5}{14} & -\frac{3}{14}\\ -\frac{1}{7} & \frac{2}{7}\end{bmatrix}$$

Provide feedback (0)

Step 6 of 7

Here the elements of A^{-1} are rational numbers.

Provide feedback (0)

Step 7 of 7

(e) Let A be a matrix in which all entries are fractions.
The inverse of the matrices A in which all entries of A are rational numbers consists of all rational numbers.
Since every whole number is also a rational number.
So it follows from the above example that A^{-1} is a matrix in which all entries are rational numbers.

Provide feedback (0)

CH1.6 12P

Step 1 of 3

Given that $A=\begin{pmatrix}3\\1\end{pmatrix}, B=\begin{pmatrix}2\\2\end{pmatrix}$

We have

$$A=\begin{pmatrix}3\\1\end{pmatrix}$$

$$\Rightarrow A^T=\begin{pmatrix}1 & 3\end{pmatrix}$$

$$B=\begin{pmatrix}2\\2\end{pmatrix}$$

$$\Rightarrow B^T=\begin{pmatrix}2 & 2\end{pmatrix}$$

Provide feedback (0)

Step 2 of 3

$$\begin{aligned}A^T.B&=\begin{pmatrix}1 & 3\end{pmatrix}_{1\times 2}.\begin{pmatrix}2\\2\end{pmatrix}_{2\times 1}\\&=(6+2)\\&=(8)_{1\times 1}\\&=\boxed{8}\end{aligned}$$

$$\begin{aligned}B^T.A&=\begin{pmatrix}2 & 2\end{pmatrix}_{1\times 2}.\begin{pmatrix}3\\1\end{pmatrix}_{2\times 1}\\&=(6+2)\\&=(8)_{1\times 1}\\&=\boxed{8}\end{aligned}$$

Provide feedback (1)

CH1.6 14P

$$A^T.B = \begin{pmatrix} 1 & 3 \end{pmatrix}_{1\times 2} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix}_{2\times 1}$$
$$= (6+2)$$
$$= (8)_{1\times 1}$$
$$= \boxed{8}$$

$$B^T.A = \begin{pmatrix} 2 & 2 \end{pmatrix}_{1\times 2} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}_{2\times 1}$$
$$= (6+2)$$
$$= (8)_{1\times 1}$$
$$= \boxed{8}$$

Provide feedback (1)

Step 3 of 3

$$AB^T = \begin{pmatrix} 3 \\ 1 \end{pmatrix}_{2\times 1} . \begin{pmatrix} 2 & 2 \end{pmatrix}_{1\times 2}$$
$$= \boxed{\begin{pmatrix} 6 & 6 \\ 2 & 2 \end{pmatrix}_{2\times 2}}$$

$$B.A^T = \begin{pmatrix} 2 \\ 2 \end{pmatrix}_{2\times 1} . \begin{pmatrix} 1 & 3 \end{pmatrix}_{1\times 2}$$
$$= \boxed{\begin{pmatrix} 6 & 6 \\ 2 & 2 \end{pmatrix}_{2\times 2}}$$

Provide feedback (1)

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CH1.6 13P

Step 1 of 5

Given that B is square matrix.

So let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2 by 2 square matrix.

Then $B^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Now

$$\begin{aligned} A &= B + B^T \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a & c \\ b & d \end{bmatrix} \\ &= \begin{bmatrix} a+a & b+c \\ c+b & d+d \end{bmatrix} \\ &= \begin{bmatrix} 2a & b+c \\ c+b & 2d \end{bmatrix} \end{aligned}$$

Now

$$A^T = \begin{bmatrix} 2a & b+c \\ c+b & 2d \end{bmatrix} = A$$

Hence $A = B + B^T$ is a symmetric matrix.

Provide feedback (0)

Step 2 of 5

Now

$$\begin{aligned} K &= B - B^T \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & c \\ b & d \end{bmatrix} \\ &= \begin{bmatrix} a-a & b-c \\ c-b & d-d \end{bmatrix} \\ &= \begin{bmatrix} 0 & b-c \end{bmatrix} \end{aligned}$$

CH1.6 15P

Provide feedback (0)

Step 2 of 5

Now

$$K = B - B^T$$

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$= \begin{bmatrix} a-a & b-c \\ c-b & d-d \end{bmatrix}$$

$$= \begin{bmatrix} 0 & b-c \\ c-b & 0 \end{bmatrix}$$

Now $K^T = \begin{bmatrix} 0 & c-b \\ b-c & 0 \end{bmatrix}$ and $-K = \begin{bmatrix} 0 & c-b \\ b-c & 0 \end{bmatrix}$

Here it is clear that $K^T = -K$

So $K = B - B^T$ is skew symmetric matrix.

By induction for all square matrices, it is true that $A = B + B^T$ is symmetric matrices and $K = B - B^T$ is skewing symmetric.

Provide feedback (0)

Step 3 of 5

Given that $B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

We have to write B as the sum of a symmetric matrix and a skew-symmetric matrix.

Provide feedback (0)

Step 4 of 5

We have $B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

We have $B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

So the transpose of B is $B^T = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

We can write $B = A + K$ where $A = B + B^T$ and $K = B - B^T$.

Now

$$\begin{aligned} A &= B + B^T \\ &= \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+1 & 3+1 \\ 1+3 & 1+1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \end{aligned}$$

The matrix A is a symmetric matrix since $A = A^T$.

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Step 5 of 5

And

$$\begin{aligned} K &= B - B^T \\ &= \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1-1 & 3-1 \\ 1-3 & 1-1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \end{aligned}$$

The matrix K is a skew-symmetric matrix since $K^T = -K$.

Hence $\boxed{B = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}}$

CH1.6 14P

Step 1 of 2

(a) In a symmetric matrix, the opposite elements to the main diagonal are equal.
If the order of the matrix is n then the number of entries to be chosen is $\frac{n(n+1)}{2}$ on and above diagonal.

Provide feedback (0)

Step 2 of 2

(b) In a skew symmetric matrix, the opposite elements to the main diagonal are additive inverse to each other
If the order of the matrices is n then the number of entries to be chosen is $\frac{n(n-1)}{2}$ on and above diagonal.

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CH1.6 16P

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CH1.6 15P

Step 1 of 3

(a) Let $A = LDU$ with 1s on the main diagonals of L and U.
We have to find the corresponding factorization of A^T.

Provide feedback (0)

CH1.6 17P

Step 2 of 3

Since $A = LDU$ where L, D are permutation matrices consists 1s on the diagonal
Now
$A = LDU$
Taking transpose on the both sides gives

$$(A)^T = (LDU)^T$$

$$A^T = U^T D^T L^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$$

$$A^T = U^T D L^T \qquad \left(\text{Since } D^T = D\right)$$

Hence $\boxed{A^T = U^T D L^T}$

Provide feedback (0)

Step 3 of 3

(b) We have to explain what triangular systems will give the solution to $A^T y = b$.
The upper triangular systems give the solution to the system $A^T y = b$ since the product becomes upper triangular matrices.

Provide feedback (0)

Algebra tutors who can help right now

(a) Let $L_1D_1U_1 = L_2D_2U_2$

Multiplying left sides with L_1^{-1} both sides gives

$L^{-1}{}_1L_1D_1U_1 = L_1^{-1}L_2D_2U_2$

$ID_1U_1 = L_1^{-1}L_2D_2U_2$

Since $L^{-1}{}_1L_1 = I, ID_1U_1 = D_1U_1$

$D_1U_1 = L_1^{-1}L_2D_2U_2$

Also, by multiplying right side with U_2^{-1}, we get

$D_1U_1 = L_1^{-1}L_2D_2U_2$

Provide feedback (0)

Step 2 of 3

$D_1U_1U_2^{-1} = L_1^{-1}L_2D_2U_2U_2^{-1}$

$D_1U_1U_2^{-1} = L_1^{-1}L_2D_2I$

Since $U^{-1}{}_1U_2^{-1} = I, ID_2 = D_2$

$D_1U_1U_2^{-1} = L_1^{-1}L_2D_2$

We know that the inverse of an upper triangular matrix is upper triangular and the inverse of the lower triangular matrix lower, product of lower triangular matrices is a lower triangular matrix and the product of upper triangular matrices is an upper triangular matrix.

Therefore $\boxed{L_1^{-1}L_2D = DU_1U_2^{-1}}$

Provide feedback (0)

Step 3 of 3

(b) $D_1U_1U_2^{-1} = L_1^{-1}L_2D_2$ is possible if and only if $L_1^{-1}L_2 = U_1U_2^{-1} = I$

Consequently, $D_1 = D_2$

Also, $L_1^{-1}L_2 = I \Rightarrow L_1 = L_2$ and $U_1U_2^{-1} = I \Rightarrow U_1 = U_2$

Therefore, the LDU factorization is unique for every invertible matrix A.

CH1.6 17P

Step 1 of 3

Given matrices are $A = \begin{bmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{bmatrix}$.

We have to find a condition on the entries so that A and B invertible.

Provide feedback (0)

Step 2 of 3

We have $A = \begin{bmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{bmatrix}$

The matrix A has inverse if $c \neq 0$ and $f \neq 0$

Because if $c = 0$ or $f = 0$ then the corresponding row entries become zeros and we know that if any row consists all zeros, the matrix does not possesses inverse.

Provide feedback (0)

Step 3 of 3

We have $B = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{bmatrix}$

The matrix B has inverse if $e \neq 0$.

Because if $e = 0$ then the corresponding row entries become zeros and we know that if any row consists all zeros the matrix does not possesses inverse.

Provide feedback (1)

CH1.6 19P

CH1.6 18P

Step 1 of 8

Given matrix is $A=\begin{bmatrix}1 & 3 & 5\\ 3 & 12 & 18\\ 5 & 18 & 30\end{bmatrix}$

We have to compute the symmetric LDL^T factorization of A.

Provide feedback (0)

Step 2 of 8

We have $A=\begin{bmatrix}1 & 3 & 5\\ 3 & 12 & 18\\ 5 & 18 & 30\end{bmatrix}$

Subtracting 3 times row 1 from row 2 and 5 times row 1 from row 3 gives

$$A\approx\begin{bmatrix}1 & 3 & 5\\ 0 & 3 & 3\\ 0 & 3 & 5\end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A\approx\begin{bmatrix}1 & 3 & 5\\ 0 & 3 & 3\\ 0 & 0 & 2\end{bmatrix}$$

Provide feedback (0)

Step 3 of 8

Here the pivot positions are $1, 3, 2$.

Applying the same row operations reversely on the identity matrix gives L.

Adding 3 times row 1 to row 2 and 5 times row 1 to row 3 gives $L=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 5 & 0 & 1\end{bmatrix}$

CH1.6 20P

Step 3 of 8

Here the pivot positions are $1, 3, 2$.

Applying the same row operations reversely on the identity matrix gives L.

Adding 3 times row 1 to row 2 and 5 times row 1 to row 3 gives $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$

Adding row 2 to row 3 gives $L \approx \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$

Now the transpose of L is $L^T = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Provide feedback (0)

Step 4 of 8

Since the pivots are 1, 3, and 2.

So the matrix D is $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Hence the LDL^T factorization of A is $\boxed{A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$.

Provide feedback (0)

Step 5 of 8

Given matrix is $A = \begin{bmatrix} a & b \\ & \end{bmatrix}$

Provide feedback (0)

Step 5 of 8

Given matrix is $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Subtracting $\left(\frac{c}{a}\right)$ times row 1 from row 2 gives $A \approx \begin{bmatrix} a & b \\ 0 & d-\frac{bc}{a} \end{bmatrix}$

Provide feedback (1)

Step 6 of 8

Applying the same row operations reversely on the identity matrices gives L.

Adding $\left(\frac{b}{a}\right)$ times row 1 to row 2 gives $L=\begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix}$

Provide feedback (0)

Step 7 of 8

The transpose of the matrix L is $L^T=\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}$

Provide feedback (0)

Step 8 of 8

The pivots are a and $d-\frac{bc}{}$

Subtracting $\left(\frac{-}{a}\right)$ times row 1 from row 2 gives $A \approx \begin{bmatrix} & \\ 0 & d-\frac{bc}{a} \end{bmatrix}$

Provide feedback (1)

Step 6 of 8

Applying the same row operations reversely on the identity matrices gives L.

Adding $\left(\frac{b}{a}\right)$ times row 1 to row 2 gives $L = \begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix}$

Provide feedback (0)

Step 7 of 8

The transpose of the matrix L is $L^T = \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}$

Provide feedback (0)

Step 8 of 8

The pivots are a and $d-\frac{bc}{a}$

So the matrix D is $D = \begin{bmatrix} a & 0 \\ 0 & d-\frac{bc}{a} \end{bmatrix}$

Hence the LDL^T factorization of A is $\boxed{A = \begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & d-\frac{cb}{a} \end{bmatrix}\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}}$

Provide feedback (0)

CH1.6 19P

Step 1 of 5

Given matrix is $A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$

We have to find the inverse of the given matrix.

Provide feedback (0)

Step 2 of 5

We use Gauss-Jordan elimination process to find the inverse of the given matrix.
Consider

$$\begin{bmatrix} A & e_1 & e_2 & e_3 & e_4 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 5

Subtracting $\frac{1}{4}$ times row 1 from row 2, $\frac{1}{3}$ times row 1 from row 3 and

Subtracting $\frac{1}{2}$ times row 1 from row 4 gives

CH1.6 21P

Step 3 of 5

Subtracting $\frac{1}{4}$ times row 1 from row 2, $\frac{1}{3}$ times row 1 from row 3 and

Subtracting $\frac{1}{2}$ times row 1 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 1 & 0 & -\frac{1}{3} & 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

Subtracting $\frac{1}{3}$ times row 2 from row 3 and $\frac{1}{2}$ times row 2 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -\frac{3}{8} & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 5

1

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & 1 & -\frac{3}{8} & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 5

Subtracting $\frac{1}{2}$ times row 3 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{3} & -\frac{1}{2} & 1 \end{bmatrix}$$

Hence the inverse of the given matrix is

$$\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ -\frac{1}{4} & -\frac{1}{3} & -\frac{1}{2} & 0 \end{bmatrix}}$$

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CH1.6 20P

Step 1 of 3

Suppose A and B are square matrices.
We have to show that $I - BA$ is invertible if $I - AB$ is invertible.

Provide feedback (0)

CH1.6 22P

Step 2 of 3

Take $B(I - AB) = (I - BA)B$

Multiplying both sides with B^{-1}, we get

$$B(I - AB)B^{-1} = (I - BA)BB^{-1}$$

$$\Rightarrow B(I - AB)B^{-1} = (I - BA)I \qquad (\text{Since } BB^{-1} = I)$$

$$\Rightarrow I - BA = B(I - AB)B^{-1}$$

Now

$$\begin{aligned}(I - BA)^{-1} &= \left(B(I - AB)B^{-1}\right)^{-1} \\ &= \left(B^{-1}\right)^{-1}(I - AB)^{-1}B^{-1} \qquad \left(\text{Since } (AB)^{-1} = B^{-1}A^{-1}\right) \\ &= B(I - AB)^{-1}B^{-1}\end{aligned}$$

Therefore, $I - BA$ is invertible if B and $I - AB$ is invertible.

Provide feedback (0)

Step 3 of 3

Suppose $I - BA$ is not invertible.
Then $BA = I$
And $BAx = x$, for some nonzero x has no solution.
Multiplying left side of $BAx = x$ with A gives

$$ABAx = Ax$$

$$\Rightarrow ABy = y \qquad (\text{Since let } Ax = y)$$

$$\Rightarrow (I - AB)y = 0$$

Therefore, $I - AB$ could not be invertible which is a contradiction to the hypothesis.
Hence $I - BA$ is invertible.

CH1.6 21P

Step 1 of 3

We know that $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\Rightarrow A^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A=\begin{pmatrix} 0 & 3 \\ 4 & 6 \end{pmatrix}$$

$$\Rightarrow A_1^{-1}=\frac{1}{0.6-3.4}\begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix}$$

$$\Rightarrow A_1^{-1}=\frac{1}{-12}\begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix}$$

$$=\boxed{\begin{pmatrix} \frac{-1}{2} & \frac{3}{4} \\ \frac{1}{3} & 0 \end{pmatrix}}$$

Provide feedback (2)

Step 2 of 3

$$B=\begin{pmatrix} a & b \\ b & 0 \end{pmatrix}$$

$$\Rightarrow B^{-1}=\frac{1}{0-b^2}\begin{pmatrix} 0 & -b \\ -b & a \end{pmatrix}$$

$$\Rightarrow B^{-1}=\frac{1}{b^2}\begin{pmatrix} 0 & -b \\ -b & a \end{pmatrix}$$

$$=\boxed{\begin{pmatrix} 0 & \frac{-1}{b} \\ -1 & a \end{pmatrix}}$$

CH1.6 23P

Step 2 of 3

$$B=\begin{pmatrix} a & b \\ b & 0 \end{pmatrix}$$

$$\Rightarrow B^{-1}=\frac{1}{0-b^2}\begin{pmatrix} 0 & -b \\ -b & a \end{pmatrix}$$

$$\Rightarrow B^{-1}=\frac{1}{b^2}\begin{pmatrix} 0 & -b \\ -b & a \end{pmatrix}$$

$$=\boxed{\begin{pmatrix} 0 & \frac{-1}{b} \\ \frac{-1}{b} & \frac{a}{b^2} \end{pmatrix}}$$

Provide feedback (2)

Step 3 of 3

$$C=\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$$

$$\Rightarrow C^{-1}=\frac{1}{3.7-4.5}\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

$$\Rightarrow C^{-1}=\frac{1}{1}\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

$$=\boxed{\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}}$$

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CH1.6 22P

Step 1 of 4

Given that $A^{-1} = \begin{bmatrix} x & t \\ y & z \end{bmatrix}$

We have to solve for the columns of A^{-1}.

Provide feedback (0)

Step 2 of 4

Given system is $\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Subtracting 2 times row 1 from row 2 gives

$$\begin{bmatrix} 10 & 20 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Subtracting 2 times row 2 from row 1 gives

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Dividing row 1 and row 2 by 10 gives

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$$

By row column multiplication of matrices, we get $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$

From this, we get $x = 0.5$, $y = -0.2$

Provide feedback (0)

CH1.6 24P

By row column multiplication of matrices, we get $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$

From this, we get $x = 0.5$, $y = -0.2$

Provide feedback (0)

Step 4 of 4

Given system is $\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Subtracting 2 times row 1 from row 2 gives

$$\begin{bmatrix} 10 & 20 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Subtracting 2 times row 2 from row 1 gives

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Dividing row 1 and row 2 by 10 gives

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$$

By row column multiplication of matrices, we get $\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$

From this, we get $t = -0.2$, $z = 0.1$

Hence the columns of A^{-1} are $\boxed{\begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix} \text{ and } \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}}$.

Provide feedback (0)

Algebra tutors who can help right now

Ian L.

Sonu M.

CH1.6 23P

Step 1 of 3

Given system is $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

We have to show that $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ has no inverse by trying to solve for the column (x, y) in the system $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x & t \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Provide feedback (0)

Step 2 of 3

We have the system $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

By row column multiplication of matrices, we have $\begin{bmatrix} x+2y \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

The second column shows that 0 = -3 case which is impossible

Therefore, $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ has no solution.

Provide feedback (0)

Step 3 of 3

Given system is $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

CH1.6 25P

Step 2 of 3

We have the system $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

By row column multiplication of matrices, we have $\begin{bmatrix} x+2y \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

The second column shows that 0 = -3 case which is impossible

Therefore, $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ has no solution.

Provide feedback (0)

Step 3 of 3

Given system is $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

By row column multiplication of matrices, we have $\begin{bmatrix} t+2z \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

The second column shows that 0 = 1 case which is impossible

Therefore, $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ has no solution.

Hence the given matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ has $\boxed{\text{no inverse}}$.

Provide feedback (0)

CH1.6 24P

Step 1 of 5

Let the matrix A be such that row 1 + row 2 = row 3.
That is

$$A=\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$$

We have to show that A has no inverse.

Provide feedback (0)

Step 2 of 5

We have $A=\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$

Subtracting row 1 from row 3 gives

$$A\approx\begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A\approx\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

The last row of A is zero.
Therefore, its determinant becomes zero.
So that the matrix A does not possesses inverse.

Provide feedback (0)

Step 3 of 5

CH1.6 26P

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

The last row of A is zero.
Therefore, its determinant becomes zero.
So that the matrix A does not possesses inverse.

Provide feedback (0)

Step 3 of 5

(a) The given system is $Ax = (1,0,0)$.
We have to show that the given system has no solution.
Now the system becomes

$$\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now performing (row 1 + row 2) –row3 gives

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

The last row shows that $0 = -1$ condition.
So the system $Ax = (1,0,0)$ has no solution.

Provide feedback (0)

Step 4 of 5

(b) We have to explain which right hand sides (b_1, b_2, b_3) might allow a solution to $Ax = b$.
The right hand sided must satisfy $b_1 + b_2 = b_3$
Then the system $Ax = b$ has a solution.

Step 4 of 5

(b) We have to explain which right hand sides (b_1, b_2, b_3) might allow a solution to $Ax = b$.

The right hand sided must satisfy $b_1 + b_2 = b_3$

Then the system $Ax = b$ has a solution.

Provide feedback (0)

Step 5 of 5

(c) We have to tell what happens to row 3 of A in elimination.

We have $A = \begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$

Subtracting row 1 from row 3 gives

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

Row 3 becomes a row of zeros and that has no third pivot.

Hence the row 3 becomes a zero row in elimination.

Provide feedback (0)

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CH1.6 25P

Step 1 of 5

Let the matrix A be such that column 1 + column 2 = column 3.

Let $A=\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}$

We have to show that A has no inverse.

Provide feedback (0)

Step 2 of 5

We have $A=\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}$

Subtracting column 1 from column 3, we get

$$A\approx\begin{bmatrix} a & d & d \\ b & e & e \\ c & f & f \end{bmatrix}$$

Subtracting column 2 from column 3, we get

$$A\approx\begin{bmatrix} a & d & 0 \\ b & e & 0 \\ c & f & 0 \end{bmatrix}$$

Since the last column of A is zero.
So its determinant becomes zero.
Hence the matrix A does not possesses inverse.

Provide feedback (0)

Step 3 of 5

(a) We have to find a nonzero solution x to $Ax=0$.

CH1.6 27P

Step 3 of 5

(a) We have to find a nonzero solution x to $Ax = 0$.
The given system $Ax = 0$ is

$$\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The augmented matrix is

$$\begin{bmatrix} a & d & a+d & 0 \\ b & e & b+e & 0 \\ c & f & c+f & 0 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 5

Subtracting column 1 from column 3, we get

$$A \approx \begin{bmatrix} a & d & d & 0 \\ b & e & e & 0 \\ c & f & f & 0 \end{bmatrix}$$

Subtracting column 2 from column 3, we get

$$A \approx \begin{bmatrix} a & d & 0 & 0 \\ b & e & 0 & 0 \\ c & f & 0 & 0 \end{bmatrix}$$

Therefore, the system $Ax = 0$ becomes

$$\begin{bmatrix} a & d & 0 & 0 \\ b & e & 0 & 0 \\ c & f & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This system has no solution.
Hence this system has no nonzero solution.

Provide feedback (0)

$$A \approx \begin{bmatrix} b & e & 0 & 0 \\ c & f & 0 & 0 \end{bmatrix}$$

Therefore, the system $Ax = 0$ becomes

$$\begin{bmatrix} a & d & 0 & 0 \\ b & e & 0 & 0 \\ c & f & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This system has no solution.
Hence this system has no nonzero solution.

Provide feedback (0)

Step 5 of 5

(b) We have to explain why there is no third pivot.

We have $A = \begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}$

Subtracting column 1 from column 3, we get

$$A \approx \begin{bmatrix} a & d & d \\ b & e & e \\ c & f & f \end{bmatrix}$$

Subtracting column 2 from column 3, we get

$$A \approx \begin{bmatrix} a & d & 0 \\ b & e & 0 \\ c & f & 0 \end{bmatrix}$$

From this we observe that column 3 becomes a column of zeros after elimination and that no third pivot.

Provide feedback (0)

CH1.6 26P

Step 1 of 3

Suppose A is an invertible matrix and exchange the first two rows of the matrix A.
Now we have to verify whether the new matrix is invertible or not.

Provide feedback (0)

CH1.6 28P

Step 2 of 3

Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Then A is invertible.
Now the exchange the first rows of A, we get a new matrix B as

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It is clear that B is also invertible since B is a permutation matrix and that have an inverse.

Provide feedback (0)

Step 3 of 3

Now we have to explain how we find B^{-1} from A^{-1}.
We got B by exchanging the first two rows of A.
So B^{-1} is obtained by exchanging the first two columns of A^{-1}.

Provide feedback (0)

Algebra tutors who can help right now

CH1.6 27P

Step 1 of 2

Suppose $M = ABC$ is an invertible matrix, where A, B and C are square matrices.
Then A, B and C are also invertible
We have to find a formula for B^{-1} that involves M^{-1} and A and C.

Provide feedback (0)

CH1.6 29P

Step 2 of 2

Since M, B are invertible.
So M^{-1}, B^{-1} exists.
Now multiplying both sides of $M = ABC$ with $B^{-1}M^{-1}$, we get

$$B^{-1}M^{-1}(M) = B^{-1}M^{-1}(ABC)$$

$$B^{-1}\left(M^{-1}M\right) = M^{-1}A\left(B^{-1}B\right)C \quad \text{(By Associative property)}$$

$$B^{-1}(I) = M^{-1}A(I)C \qquad \begin{pmatrix}\text{Since } M^{-1}M = I \\ \text{and } B^{-1}B = I\end{pmatrix}$$

$$B^{-1} = M^{-1}AC$$

Hence the formula for B^{-1} that involves M^{-1} and A and C is $\boxed{B^{-1} = M^{-1}AC}$.

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CH1.6 28P

Step 1 of 2

We have to prove that a matrix with a column of zeros cannot have an inverse.

Provide feedback (0)

CH1.6 30P

Step 2 of 2

Let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & 0 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & 0 \\ \vdots & \ddots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & 0 \end{bmatrix}$ be a matrix with a zero column.

We know that if a matrix A is invertible, then A has n pivot positions.
Since the given $n \times n$ matrix A has no pivot position in the last column.
So the matrix A is not invertible.
Hence a matrix with zero columns is not invertible.

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CH1.6 29P

Step 1 of 2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ times } \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} ad-bc & -ab+ab \\ cd-cd & -bc+ad \end{pmatrix}$$

$$= \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= (ad-bc)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\because\ ad-bc \neq 0)$$

Provide feedback (0)

Step 2 of 2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \boxed{\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}} \quad (\text{given that } ad-bc \neq 0)$$

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}^{-1} = \boxed{\frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \quad (\text{given that } ad-bc \neq 0)$$

Provide feedback (0)

CH1.6 31P

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CH1.6 30P

Step 1 of 6

(a) We have to find the matrix E that has the same effect as we subtract row 1 from row 2, subtract row 1 from row 3 and then subtract row 2 from row 3.

Provide feedback (0)

CH1.6 32P

Step 2 of 6

We have the elementary matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The matrix E that effects the three operations on the E is $E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$.

Since subtracting row 1 from row 2 in elementary matrix gives

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 6

Subtracting row 1 from row 3 in elementary matrices gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Subtracting row 2 from row 3 in elementary matrices gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Step 3 of 6

Subtracting row 1 from row 3 in elementary matrices gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Subtracting row 2 from row 3 in elementary matrices gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Now $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

$$= E$$

Hence the required matrix E is $\boxed{E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}}$.

Provide feedback (0)

Step 4 of 6

(b) We have to find the matrix L that has the same effect as these three reverse steps.
The steps are add row 2 to row 3, add row 1 to row 3 and then add row 1 to row 2.

Provide feedback (0)

Step 5 of 6

We have the elementary matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Provide feedback (0)

Step 5 of 6

We have the elementary matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Adding row 2 to row 3 in elementary matrix gives $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Adding row 1 to row 2 in elementary matrix gives $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Provide feedback (0)

Step 6 of 6

Adding row 1 to row 3 in elementary matrices gives $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Now $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

$= L$

After reversing the order of these 3 elementary matrices and changing -1 to 1,

Hence the required matrix L is $\boxed{L = E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}}$.

CH1.6 32P

Step 1 of 3

We have to show that $A = 4*eye(4) - ones(4,4)$ is not invertible.

Provide feedback (0)

CH1.6 34P

Step 2 of 3

Given that $A = 4*eye(4) - ones(4,4)$

$$= 4*\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

Therefore, $A = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

Provide feedback (0)

Step 3 of 3

Now we have to multiply $A*ones(4,1)$

$$= \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

Therefore, $A = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

Provide feedback (0)

Step 3 of 3

Now we have to multiply $A*ones(4,1)$.

Now

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3(1)+(-1)1+(-1)1+(-1)1 \\ (-1)(1)+(3)1+(-1)1+(-1)1 \\ (-1)(1)+(-1)1+(3)1+(-1)1 \\ (-1)(1)+(-1)1+(-1)1+(3)1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1-1-1 \\ -1+3-1-1 \\ -1-1+3-1 \\ -1-1-1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, the system $Ax = 0$ has zero solution.
Hence A is not invertible.

Provide feedback (0)

CH1.6 33P

Step 1 of 7

Consider matrices of (2×2) with 1's and 0's. Determine how many are invertible.

Recall that a matrix is invertible if its determinant is non-zero.

Provide feedback (0)

Step 2 of 7

Determinants of all the matrices (2×2) containing 1's and 0's are as follows:

Consider the matrices with only one 1's.

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Provide feedback (0)

Step 3 of 7

Consider the matrices with two 1's.

CH1.6 35P

Provide feedback (0)

Step 3 of 7

Consider the matrices with two 1's.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Provide feedback (0)

Step 4 of 7

Consider the matrices with three 1's.

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each one of them are invertible.

Provide feedback (0)

Step 5 of 7

Consider the matrix with 1's positioned on the diagonal and anti-diagonal.

Consider the matrix with 1's positioned on the diagonal and anti-diagonal.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each one of them are invertible.

Provide feedback (0)

Step 6 of 7

Consider the matrix with all 1's and 0's.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Provide feedback (0)

Step 7 of 7

Therefore, from sixteen (2×2) matrices with 0's and 1's only 6 are invertible.

Provide feedback (0)

CH1.6 34P

Step 1 of 2

Given that

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$

$$= \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 - 3R_2$

$$= \begin{bmatrix} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

where $A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

Provide feedback (0)

Step 2 of 2

Given that

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 3R_1$

$$\sim \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 + \frac{4}{3}R_2$

$$\sim \begin{bmatrix} 1 & 0 & -3 & \frac{4}{3} \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

CH1.6 36P

$$= \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

where $A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

Provide feedback (0)

Step 2 of 2

Given that

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 3R_1$

$$\sim \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

Apply $R_1 \to R_1 + \frac{4}{3} R_2$

$$\sim \begin{bmatrix} 1 & 0 & -3 & \frac{4}{3} \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

Apply $R_2 \to \frac{1}{-3} R_2$

$$\sim \begin{bmatrix} 1 & 0 & -3 & \frac{4}{3} \\ 0 & 1 & 1 & \frac{1}{-3} \end{bmatrix}$$

$$\sim \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

where $A^{-1} = \begin{bmatrix} -3 & \frac{4}{3} \\ 1 & \frac{1}{-3} \end{bmatrix}$

Provide feedback (0)

CH1.6 35P

Step 1 of 4

Given that $[A \;\; I] = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$

We have to eliminate above and below the pivots to reduce $[A \;\; I]$ to $[I \;\; A^{-1}]$.

Provide feedback (0)

Step 2 of 4

Exchanging row 1 and row 2 gives

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting 2 times row 1 from row 2

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Subtracting 2 times row 3 from row 1 and adding 3 times row 3 to row 2

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 0 & 4 & 1 & -2 & 3 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Dividing row 2 by 4 gives

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

CH1.6 37P

Step 3 of 4

Subtracting 2 times row 3 from row 1 and adding 3 times row 3 to row 2

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-3} & 0 & 1 & -2 \\ \mathbf{0} & \mathbf{0} & 4 & 1 & -2 & 3 \\ \mathbf{0} & \mathbf{1} & \mathbf{2} & 0 & 0 & 1 \end{bmatrix}$$

Dividing row 2 by 4 gives

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-3} & 0 & 1 & -2 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

Adding 3 times row 2 to row 3, subtracting 2 times row 2 from row 3

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} & -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}$$

Exchanging row 2 and row 3

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -\frac{1}{2} & 1 & -\frac{1}{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Hence $\boxed{\begin{bmatrix} I & A^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}}$

$$\begin{bmatrix} 0 & 0 & 4 & 1 & -2 & 3 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Dividing row 2 by 4 gives

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

Adding 3 times row 2 to row 3, subtracting 2 times row 2 from row 3

$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ 0 & 1 & 2 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}$$

Exchanging row 2 and row 3

$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Hence $\boxed{\begin{bmatrix} I & A^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}}$

CH1.6 36P

Step 1 of 4

Given system is $\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We have to use Gauss- Jordan elimination on $\begin{bmatrix} A & I \end{bmatrix}$ to solve $AA^{-1} = I$.

Provide feedback (0)

Step 2 of 4

Given that $\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

Subtracting b times row 3 from row 1; subtracting c times row 3 from row 2

$$\begin{bmatrix} 1 & a & 0 & 1 & 0 & -b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

Subtracting a times row 2 from row 1

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -a & -b+ac \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Now $\begin{bmatrix} I & A^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & -a & -b+ac \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

CH1.6 38P

Subtracting a times row 2 from row 1

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -a & -b+ac \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Now $$\begin{bmatrix} I & A^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & -a & -b+ac \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 4

Now

$$AA^{-1} = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a & -b+ac \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+a(0)+b(0) & 1(-a)+a(1)+b(0) & 1(-b+ac)+a(-c)+b(1) \\ 0(1)+1(0)+c(0) & 0(-a)+1(1)+c(0) & 0(-b+ac)+1(-c)+c(1) \\ 0(1)+0(0)+1(0) & 0(-a)+0(1)+1(0) & 0(-b+ac)+0(-c)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & -a+a & -b+ac-ac+b \\ 0+0+0 & 0+1+0 & -c+c \\ 0+0+0 & 0+0+0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Hence $$\boxed{A^{-1} = \begin{bmatrix} 1 & -a & -b+ac \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}}$$

Provide feedback (0)

CH1.6 37P

CH1.6 39P

Step 1 of 3

Given that $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 3R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Therefore $A^{-1} = \boxed{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}}$

Provide feedback (0)

Step 2 of 3

Given that $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Step 2 of 3

Given that $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 3

Apply $R_1 \to R_1 - R_2, R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

Apply $R_2 \to R_2 - R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Therefore $A^{-1} = \boxed{\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}}$

CH1.6 38P

Step 1 of 2

Given that

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{bmatrix}$$

Apply $R_1 \leftrightarrow R_2$

$$\sim \begin{bmatrix} 2 & 2 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

Apply $R_1 \to R_1 - R_2$

$$\sim \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 2 of 2

Apply $R_1 \to \frac{R_1}{2}, R_2 \to \frac{R_2}{2}$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

where $A^{-1} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

Provide feedback (0)

CH1.6 40P

CH1.6 39P

Step 1 of 8

(a) Given statement is 'A 4 by 4 matrix with a row of zeros is not invertible'.
We have to decide whether the given statement is true or false.

Provide feedback (0)

Step 2 of 8

The given statement is **true**.
Because a 4 by 4 matrix with a row of zeros is not invertible since one of the row is full of zeros.
Therefore, there does not exist solution for the system $Ax = 0$.
Hence the given statement is **true.**

Provide feedback (0)

Step 3 of 8

(b) Given statement is 'A matrix with 1s down the main diagonal is invertible'.
We have to decide whether the given statement is true or false.

Provide feedback (0)

Step 4 of 8

The given statement is **true**.
Because a matrix with 1s down the main diagonal is invertible since all pivots in the main diagonal are non zero.
Therefore, the system has a solution.
Hence the given statement is **true**.

Provide feedback (0)

Step 5 of 8

CH1.6 41P

Step 5 of 8

(c) Given statement is 'If A is invertible then A^{-1} is also invertible'.
We have to decide whether the given statement is true or false.

Provide feedback (0)

Step 6 of 8

The given statement is **true**.

Since by inverse properties, we have $\left(A^{-1}\right)^{-1} = A$.

Hence the given statement is **true.**

Provide feedback (0)

Step 7 of 8

(d) Given statement is 'If A^T is invertible then A is also invertible'.
We have to decide whether the given statement is true or false.

Provide feedback (0)

Step 8 of 8

The given statement is **true**.

Because we know that inverse of A^T = transpose of A^{-1}

Hence the given statement is **true.**

Provide feedback (0)

Algebra tutors who can help right now

CH1.6 40P

CH1.6 42P

Step 1 of 3

Given that $A = \begin{pmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{pmatrix}$

Case (i):- If $c = 2$

Given matrix becomes $\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 8 & 7 & 2 \end{pmatrix}$

i.e. all the entries in row 1 and row 2 are same.
A is not invertible.

Provide feedback (0)

Step 2 of 3

Case (ii):- If $c = 7$

Given matrix becomes $\begin{pmatrix} 2 & 7 & 7 \\ 7 & 7 & 7 \\ 8 & 7 & 7 \end{pmatrix}$

That is all the entries in column 2 and columns 3 are same.
A is not invertible.

Provide feedback (0)

Step 3 of 3

Case (iii):- If $c = 0$

Given matrix becomes $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 8 & 7 & 0 \end{pmatrix}$

That is all entries in row 2 (or column 3) are zeros.
A is not invertible.

Provide feedback (0)

CH1.6 41P

Step 1 of 4

Given that $A=\begin{pmatrix} a & b & b \\ a & a & b \\ a & a & a \end{pmatrix}$

A is invertable if $|A|\neq 0$

$|A|\neq 0$

$$\Rightarrow \begin{vmatrix} a & b & b \\ a & a & b \\ a & a & a \end{vmatrix} \neq 0$$

$$\Rightarrow a(a^2-ab)-b(a^2-ab)+b(a^2-a^2)\neq 0$$

Provide feedback (0)

Step 2 of 4

$$\Rightarrow a(a^2-ab)-b(a^2-ab)\neq 0$$
$$\Rightarrow (a^2-ab)(a-b)\neq 0$$
$$\Rightarrow a(a-b)(a-b)\neq 0$$
$$\Rightarrow a\neq 0, a\neq b$$

Therefore A is invertible if $a\neq 0, a\neq b$

Provide feedback (0)

Step 3 of 4

Pivotes:-

Given that $A=\begin{pmatrix} a & b & b \\ a & a & b \\ a & a & a \end{pmatrix}$

$\begin{pmatrix} a & b & b \end{pmatrix}$

CH1.6 43P

$\Rightarrow a \neq 0, a \neq b$

Therefore A is invertible if $a \neq 0, a \neq b$

Provide feedback (0)

Step 3 of 4

Pivotes:-

Given that $A = \begin{pmatrix} a & b & b \\ a & a & b \\ a & a & a \end{pmatrix}$

$$A = \begin{pmatrix} a & b & b \\ a & a & b \\ a & a & a \end{pmatrix}$$

Apply $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\sim \begin{pmatrix} a & b & b \\ 0 & a-b & 0 \\ 0 & a-b & a-b \end{pmatrix}$$

Provide feedback (0)

Step 4 of 4

apply $R_3 \to R_3 - R_2$

$$\sim \begin{pmatrix} \boxed{a} & b & b \\ 0 & \boxed{a-b} & 0 \\ 0 & 0 & \boxed{a-b} \end{pmatrix}$$

which is upper triangular matrix

And the pivots are $\boxed{a, a-b, a-b}$.

Provide feedback (0)

CH1.6 42P

CH1.6 44P

Step 1 of 9

Given matrix is $A=\begin{bmatrix}1 & -1 & 1 & -1\\ 0 & 1 & -1 & 1\\ 0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1\end{bmatrix}$.

We have to find A^{-1} by elimination on $\begin{bmatrix}A & I\end{bmatrix}$.

Provide feedback (0)

Step 2 of 9

Now consider

$$\begin{bmatrix}A & I\end{bmatrix}=\begin{bmatrix}1 & -1 & 1 & -1 & 1 & 0 & 0 & 0\\ 0 & 1 & -1 & 1 & 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1\end{bmatrix}$$

Adding row 2 to row 1 gives

$$\begin{bmatrix}1 & 0 & 0 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & -1 & 1 & 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1\end{bmatrix}$$

Provide feedback (0)

Step 3 of 9

Adding row 3 to row 2 gives

$$\begin{bmatrix}1 & 0 & 0 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0\\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1\end{bmatrix}$$

Step 4 of 9

Adding row 4 to row 3 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the inverse of the given matrix A is $\boxed{A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$.

Provide feedback (0)

Step 5 of 9

Now we extend A to a 5 by 5 matrix by adding the row, column $(0,0,0,0,1)$.

Therefore,

$$B = \begin{bmatrix} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we find the inverse of the new matrix.

Provide feedback (0)

Step 6 of 9

Consider

$$[B \quad I] = \begin{bmatrix} 1 & -1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Step 6 of 9

Consider

$$\begin{bmatrix} B & I \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Adding row 2 to row 1 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

Adding row 3 to row 2 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 9

Adding row 4 to row 3 gives

Provide feedback (0)

Adding row 3 to row 2 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 9

Adding row 4 to row 3 gives

Provide feedback (0)

Step 9 of 9

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence the inverse of the new 5 by 5 matrix is $\boxed{B^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}$.

Provide feedback (0)

CH1.6 43P

Step 1 of 3

Suppose B has the columns of A in reverse order.
We have to solve $(A-B)x=0$ to show that $A-B$ is not invertible.

Provide feedback (0)

Step 2 of 3

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be any matrix.

Then by definition of the matrix B, we get $B=\begin{bmatrix} c & d \\ a & b \end{bmatrix}$.

Now

$$\begin{aligned} A-B &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} c & d \\ a & b \end{bmatrix} \\ &= \begin{bmatrix} a-c & b-d \\ c-a & d-b \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 3 of 3

Now the system $(A-B)x=0$ is $\begin{bmatrix} a-c & b-d \\ c-a & d-b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Subtracting row 1 from row 2 gives $\begin{bmatrix} a-c & b-d \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

In the above elimination, the last column becomes zero.
Therefore, the system has no solution.
Hence $A-B$ is not invertible.

Provide feedback (0)

CH1.6 45P

CH1.6 44P

Step 1 of 6

Given block matrices are $\begin{bmatrix} I & 0 \\ C & I \end{bmatrix}, \begin{bmatrix} A & 0 \\ C & D \end{bmatrix}, \begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$.

We have to find and check the inverses of these block matrices.

Provide feedback (0)

Step 2 of 6

Assuming that the inverses of I, A, C and D exists and they be I, A^{-1}, C^{-1} and D^{-1} respectively.

Let $M = \begin{bmatrix} I & 0 \\ C & I \end{bmatrix}$

For a 2 by 2 matrix, the inverse is $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ..... (1)

Provide feedback (0)

Step 3 of 6

By (1), the inverse of $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$ is

$$M^{-1} = \frac{1}{I.I - 0.C}\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$$

$$= \frac{1}{I^2}\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 \\ -C & I \end{bmatrix} \qquad (\text{Since } I^2 = I)$$

Therefore, the inverse of $\begin{bmatrix} I & 0 \\ C & I \end{bmatrix}$ is $\boxed{\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}}$.

CH1.6 46P

Step 4 of 6

Let $N=\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$

By (1), the inverse of $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ is

$$\begin{aligned} N^{-1} &= \frac{1}{A.D-0.C}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix} \\ &= \frac{1}{AD}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix} \\ &= (AD)^{-1}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix} && \left(\text{Since } \frac{1}{AD}=(AD)^{-1}\right) \\ &= D^{-1}A^{-1}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix} && \left(\text{Since } (AD)^{-1}=D^{-1}A^{-1}\right) \end{aligned}$$

Provide feedback (0)

Step 5 of 6

Continuation to the above

$$\begin{aligned} &= \begin{bmatrix} D^{-1}A^{-1}D & 0 \\ -D^{-1}A^{-1}C & D^{-1}A^{-1}A \end{bmatrix} \\ &= \begin{bmatrix} D^{-1}DA^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1}A^{-1}A \end{bmatrix} \\ &= \begin{bmatrix} A^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1} \end{bmatrix} && \begin{pmatrix} \text{Since } D^{-1}D=I \\ \text{and } A^{-1}A=I \end{pmatrix} \end{aligned}$$

Hence the inverse of the block matrix $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ is $\boxed{\begin{bmatrix} A^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1} \end{bmatrix}}$.

$$= \begin{bmatrix} D^{-1}A^{-1}D & 0 \\ -D^{-1}A^{-1}C & D^{-1}A^{-1}A \end{bmatrix}$$

$$= \begin{bmatrix} D^{-1}DA^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1}A^{-1}A \end{bmatrix}$$

$$= \begin{bmatrix} A^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1} \end{bmatrix} \qquad \begin{pmatrix} \text{Since } D^{-1}D = I \\ \text{and } A^{-1}A = I \end{pmatrix}$$

Hence the inverse of the block matrix $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ is $\boxed{\begin{bmatrix} A^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1} \end{bmatrix}}$.

Provide feedback (0)

Step 6 of 6

Let $P = \begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$

By (1), the inverse of $\begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$ is

$$P^{-1} = \frac{1}{0.D - I.I}\begin{bmatrix} D & -I \\ -I & 0 \end{bmatrix}$$

$$= \frac{1}{-I^2}\begin{bmatrix} D & -I \\ -I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -D & I \\ I & 0 \end{bmatrix}$$

Therefore, the inverse of $\begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$ is $\boxed{\begin{bmatrix} -D & I \\ I & 0 \end{bmatrix}}$.

Provide feedback (0)

CH1.6 46P

Step 1 of 2

For $Ax = b$ with $A = ones(4,4)$ is a singular matrix and so not invertible.
So, considering $b =$ rand (4, 1), to find the solution through MATLAB, it considers $x = A^{-1}b$ which does not exist.
So, obviously it displays that $Ax = b$ has no solution.

Provide feedback (0)

Step 2 of 2

By putting $b =$ ones (4, 1), the system reduces to one non zero row and all other rows zero in A/b.
So, it will pick x = (1, 0, 0, 0) and pinv (A) * b and the shortest solution
$x = (1,1,1,1)/4$

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CH1.6 48P

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Consider the following matrices equation:

$$M = I - UV$$
$$M^{-1} = I_n + U(I_m - VU)^{-1}V$$

Multiply M and M^{-1} to get /$\left(MM^{-1} = I\right)$. This M^{-1} shows the change in A^{-1} when a matrix is subtracted from A.

Provide feedback (0)

Step 2 of 3

Do the following calculations:

$$\begin{aligned} MM^{-1} &= (I_n - UV)\left(I_n + U(I_m - VU)^{-1}V\right) \\ &= I_n - UV + U(I_m - VU)^{-1}V - UVU(I_m - VU)^{-1}V \\ &= I_n - UV + U(I_m - VU)^{-1}V(I_m - VU) \\ &= I_n - UV + U(I_m - VU)^{-1}(I_m - VU)V \end{aligned}$$

$$\begin{aligned} &= I_n - UV + UV \\ MM^{-1} &= I_n \end{aligned}$$

Provide feedback (0)

Multiply M and M^{-1} to get $I\left(MM^{-1}=I\right)$. This M^{-1} shows the change in A^{-1} when a matrix is subtracted from A.

Provide feedback (0)

Step 2 of 3

Do the following calculations:

$$\begin{aligned}
MM^{-1} &= (I_n - UV)\left(I_n + U(I_m - VU)^{-1}V\right) \\
&= I_n - UV + U(I_m - VU)^{-1}V - UVU(I_m - VU)^{-1}V \\
&= I_n - UV + U(I_m - VU)^{-1}V(I_m - VU) \\
&= I_n - UV + U(I_m - VU)^{-1}(I_m - VU)V \\
&= I_n - UV + UV \\
MM^{-1} &= I_n
\end{aligned}$$

Provide feedback (0)

Step 3 of 3

Therefore, $\boxed{MM^{-1}=I_n}$.

CH1.6 48P

CH1.6 50P

Step 1 of 4

Given that $A=\begin{pmatrix}1 & 0\\ 9 & 3\end{pmatrix}$

We have $A=\begin{pmatrix}1 & 0\\ 9 & 3\end{pmatrix}$

$$\Rightarrow A^T=\begin{pmatrix}1 & 9\\ 0 & 3\end{pmatrix}$$

We know that $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

$$\Rightarrow A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$$

$$A=\begin{pmatrix}1 & 0\\ 9 & 3\end{pmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{3-0}\begin{pmatrix}3 & 0\\ -9 & 1\end{pmatrix}$$

$$\Rightarrow A^{-1}=\begin{pmatrix}1 & 0\\ -3 & \frac{1}{3}\end{pmatrix}$$

$$\Rightarrow \left(A^{-1}\right)^T=\boxed{\begin{pmatrix}1 & -3\\ 0 & \frac{1}{3}\end{pmatrix}}$$

Provide feedback (0)

Step 2 of 4

We have

$$A^T=\begin{pmatrix}1 & 9\\ 0 & 3\end{pmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1}=\frac{1}{}\begin{pmatrix}3 & -9\end{pmatrix}$$

Step 2 of 4

We have

$$A^T = \begin{pmatrix} 1 & 9 \\ 0 & 3 \end{pmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1} = \frac{1}{3-0}\begin{pmatrix} 3 & -9 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1} = \begin{pmatrix} 1 & -3 \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$= \left(A^{-1}\right)^T$$

Therefore $\boxed{\left(A^T\right)^{-1} = \left(A^{-1}\right)^T}$

Provide feedback (0)

Step 3 of 4

$$A = \begin{pmatrix} 1 & c \\ c & 0 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{0-c^2}\begin{pmatrix} 0 & -c \\ -c & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{c^2}\begin{pmatrix} 0 & c \\ c & -1 \end{pmatrix}$$

$$\Rightarrow \left(A^{-1}\right)^T = \boxed{\frac{1}{c^2}\begin{pmatrix} 0 & c \\ c & -1 \end{pmatrix}}$$

Provide feedback (0)

Step 4 of 4

Provide feedback (0)

Step 3 of 4

$$A=\begin{pmatrix}1 & c\\ c & 0\end{pmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{0-c^2}\begin{pmatrix}0 & -c\\ -c & 1\end{pmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{c^2}\begin{pmatrix}0 & c\\ c & -1\end{pmatrix}$$

$$\Rightarrow \left(A^{-1}\right)^T=\boxed{\frac{1}{c^2}\begin{pmatrix}0 & c\\ c & -1\end{pmatrix}}$$

Provide feedback (0)

Step 4 of 4

We have

$$A=\begin{pmatrix}1 & c\\ c & 0\end{pmatrix}$$

$$\Rightarrow A^T=\begin{pmatrix}1 & c\\ c & 0\end{pmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1}=\frac{1}{0-c^2}\begin{pmatrix}0 & -c\\ -c & 1\end{pmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1}=\frac{1}{c^2}\begin{pmatrix}0 & c\\ c & -1\end{pmatrix}$$

$$=\left(A^{-1}\right)^T$$

Therefore $\boxed{\left(A^T\right)^{-1}=\left(A^{-1}\right)^T}$

Provide feedback (0)

CH1.6 49P

Step 1 of 4

Given that $A=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$

We have

$$A=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$$

$$\Rightarrow A^T=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$

$$B=\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$$

$$\Rightarrow B^T=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}$$

Provide feedback (0)

Step 2 of 4

$$AB=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$$

$$AB=\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$$

$$\Rightarrow (AB)^T=\begin{pmatrix}1 & 2\\ 3 & 7\end{pmatrix}$$

Provide feedback (0)

Step 3 of 4

CH1.6 51P

$$\Rightarrow (A.B)^T = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

Provide feedback (0)

Step 3 of 4

$$B^T.A^T = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$
$$= (A.B)^T$$
$$A^T.B^T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$$
$$\neq (A.B)^T$$

Provide feedback (0)

Step 4 of 4

$$B^T.A^T = (A.B)^T$$
$$= (B.A)^T$$
$$= A^T.B^T \quad (\because AB = BA)$$

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CH1.6 50P

(a) Given that $\left((AB)^{-1}\right)^T$ comes from $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$.

We have to find the order.

In $\left((AB)^{-1}\right)^T$ the order of terms are $\left(A^{-1}\right)^T, \left(B^{-1}\right)^T$

$$\text{Since } \left((AB)^{-1}\right)^T = \left(B^{-1}A^{-1}\right)^T \qquad \left(\text{Since } (AB)^{-1} = B^{-1}A^{-1}\right)$$

$$= \left(A^{-1}\right)^T \left(B^{-1}\right)^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$$

Hence the $\left((AB)^{-1}\right)^T$ comes from $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$ in the order of $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$.

Provide feedback (0)

CH1.6 52P

Step 2 of 3

(b) Suppose U is an upper triangular matrix.

We have to find $\left(U^{-1}\right)^T$ is which triangular matrix.

Provide feedback (0)

Step 3 of 3

$$\text{Let } U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

$$\text{Then } U^{-1} = \begin{bmatrix} \frac{1}{a} & -\frac{b}{ad} & \frac{be-cd}{adf} \\ 0 & \frac{1}{d} & -\frac{e}{df} \\ 0 & 0 & \frac{1}{f} \end{bmatrix}$$

of (A^{-1}) and (B^{-1}).

Provide feedback (0)

Step 2 of 3

(b) Suppose U is an upper triangular matrix.

We have to find $\left(U^{-1}\right)^T$ is which triangular matrix.

Provide feedback (0)

Step 3 of 3

Let $U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

Then $U^{-1} = \begin{bmatrix} \frac{1}{a} & -\frac{b}{ad} & \frac{be-cd}{adf} \\ 0 & \frac{1}{d} & -\frac{e}{df} \\ 0 & 0 & \frac{1}{f} \end{bmatrix}$

Now $\left(U^{-1}\right)^T = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{b}{ad} & \frac{1}{d} & 0 \\ \frac{be-cd}{adf} & -\frac{e}{df} & \frac{1}{f} \end{bmatrix}$ which is a lower triangular matrix.

Therefore, if U is an upper triangular matrix then $\left(U^{-1}\right)^T$ is a $\boxed{\text{lower}}$ triangular matrix.

CH1.6 51P

We have to show that $A^2 = 0$ is possible but $A^T A = 0$ is not possible.

Provide feedback (0)

CH1.6 53P

Step 2 of 3

Let $A = \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$

Then

$$\begin{aligned} A^2 &= A.A \\ &= \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2(2)+2(-2) & 2(2)+2(-2) \\ (-2)(2)+(-2)(-2) & (-2)(2)+(-2)(-2) \end{bmatrix} \\ &= \begin{bmatrix} 4-4 & 4-4 \\ -4+4 & -4+4 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore, $A^2 = 0$

Provide feedback (0)

Step 3 of 3

Now $A^T = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

$$\begin{aligned} A^T.A &= \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 2(2)+(-2)(-2) & 2(2)+(-2)(-2) \\ (2)(2)+(-2)(-2) & (2)(2)+(-2)(-2) \end{bmatrix} \\ &\begin{bmatrix} 4+4 & 4+4 \end{bmatrix} \end{aligned}$$

Let $A = \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$

Then

$$A^2 = A.A$$

$$= \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+2(-2) & 2(2)+2(-2) \\ (-2)(2)+(-2)(-2) & (-2)(2)+(-2)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 4-4 \\ -4+4 & -4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, $A^2 = 0$

Provide feedback (0)

Step 3 of 3

Now $A^T = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

$$A^T.A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+(-2)(-2) & 2(2)+(-2)(-2) \\ (2)(2)+(-2)(-2) & (2)(2)+(-2)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 4+4 \\ 4+4 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

Hence it is clear that if $A \neq 0$ then $\boxed{A^2 = 0 \text{ but } A^T A \neq 0}$.

CH1.6 52P

Step 1 of 4

(a) Given that $x^T Ay = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

We have to verify that the row vector x^T times A times the column vector y produces what number.

Provide feedback (0)

Step 2 of 4

Now

$$\begin{aligned} x^T Ay &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1(0)+2(1)+3(0) \\ 4(0)+5(1)+6(0) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0+2+0 \\ 0+5+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 0(2)+1(5) \end{bmatrix} \\ &= \begin{bmatrix} 5 \end{bmatrix} \end{aligned}$$

Hence $\boxed{x^T Ay = \begin{bmatrix} 5 \end{bmatrix}}$

Provide feedback (0)

Step 3 of 4

CH1.6 54P

Hence $\boxed{x^T Ay = [5]}$

Provide feedback (0)

Step 3 of 4

(b) Now

$$x^T A = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+1(4) & 0(2)+1(5) & 0(3)+1(6) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

Therefore, $\boxed{\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}}$ is the row that is $x^T A$ times the column $y = (0,1,0)$.

Provide feedback (0)

Step 4 of 4

(c) Now

$$Ay = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1(0)+2(1)+3(0) \\ 4(0)+5(1)+6(0) \end{bmatrix} \quad]$$

$$= \begin{bmatrix} 0+2+0 \\ 0+5+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Therefore, the row x^T times the column Ay is $\boxed{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}$.

CH1.6 53P

Step 1 of 3

Given block matrix is $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$.

We have to find M^T.

Provide feedback (0)

Step 2 of 3

Since $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

So $M^T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Hence the transpose of the given block matrix is $\boxed{M^T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}}$.

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Step 3 of 3

The given block matrix is symmetric if $C = B$.

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CH1.6 55P

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CH1.6 54P

Step 1 of 7

We have to explain why the inner product of x and y equals the inner product of Px and Py .

Provide feedback (0)

CH1.6 56P

Step 2 of 7

Let A be any matrix.
Now

$$\begin{aligned}(Px)^T A(Py) &= x^T P^T APy \\ &= x^T\left(P^T AP\right)y \\ &= x^T Ay \qquad \begin{pmatrix}\text{Since } P^T AP = A, \text{ for} \\ \text{any permutation matrix}\end{pmatrix}\end{aligned}$$

Hence the inner product of x and y equals the inner product of Px and Py .

Provide feedback (0)

Step 3 of 7

Given $x = (1,2,3)$ and $y = (1,4,2)$

We have to choose the permutation matrix P to show that $(Px)^T y$ is always not equal to $x^T\left(P^T y\right)$

Let the permutation matrix P be $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

Given $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

Step 3 of 7

Given $x=(1,2,3)$ and $y=(1,4,2)$

We have to choose the permutation matrix P to show that $(Px)^T y$ is always not equal to $x^T(P^T y)$

Let the permutation matrix P be $P=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$

Given $x=\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $y=\begin{bmatrix}1\\4\\2\end{bmatrix}$

Then $x^T=\begin{bmatrix}1&2&3\end{bmatrix}$ and $P^T=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

Provide feedback (0)

Step 4 of 7

Now

$$Px=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$=\begin{bmatrix}0(1)+1(2)+0(3)\\0(1)+0(2)+1(3)\\1(1)+0(2)+0(3)\end{bmatrix}$$

$$=\begin{bmatrix}2\\3\\1\end{bmatrix}$$

Therefore, $(Px)^T=\begin{bmatrix}2&3&1\end{bmatrix}$

Provide feedback (0)

Step 5 of 7

Now

$$(Px)^T y = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \left[2(1) + 3(4) + 1(2) \right]$$

$$= \left[2 + 12 + 2 \right]$$

$$= \left[16 \right]$$

Provide feedback (0)

Step 6 of 7

Now we find $x^T \left(P^T y \right)$

So

$$P^T y = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1) + 0(4) + 1(2) \\ 1(1) + 0(4) + 0(2) \\ 0(1) + 1(4) + 0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 7

Provide feedback (0)

Step 6 of 7

Now we find $x^T\left(P^T y\right)$

So

$$P^T y = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+0(4)+1(2) \\ 1(1)+0(4)+0(2) \\ 0(1)+1(4)+0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 7

Now

$$x^T\left(P^T y\right) = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$= \left[1(2)+2(1)+3(4)\right]$$

$$= \left[2+2+12\right]$$

$$= \left[16\right]$$

Hence thereexists no P such that $(Px)^T y \neq x^T\left(P^T y\right)$.

Provide feedback (0)

CH1.6 55P

CH1.6 57P

Step 1 of 4

Given that $A^T = A$ and $B^T = B$

a) We have to verify whether $A^2 - B^2$ is symmetric or not.

Now

$$\begin{aligned}
\left(A^2 - B^2\right)^T &= \left(A^2 + \left(-B^2\right)\right)^T \\
&= \left(A^2\right)^T + \left(-B^2\right)^T \quad \left(\text{Since } (A+B)^T = A^T + B^T\right) \\
&= \left(A^2\right)^T - \left(B^2\right)^T \\
&= \left(A^T\right)^2 - \left(B^T\right)^2 \quad \left(\text{Since } \left(A^m\right)^T = \left(A^T\right)^m, \text{ where } m \text{ is any scalar}\right) \\
&= A^2 - B^2 \quad \left(\text{Since } (A)^T = A\right)
\end{aligned}$$

Therefore, $\boxed{A^2 - B^2 \text{ is symmetric}}$.

Provide feedback (0)

Step 2 of 4

b) We have to verify whether $(A+B)(A-B)$ is symmetric or not.

Now

$$\begin{aligned}
\left[(A+B)(A-B)\right]^T &= \left[\left(A^2 - AB + BA - B^2\right)\right]^T \\
&= \left(A^2\right)^T - (AB)^T + (BA)^T - \left(B^2\right)^T \\
&= \left(A^T\right)^2 - B^T A^T + A^T B^T - \left(B^T\right)^2 \\
&= A^2 - BA + AB - B^2
\end{aligned}$$

Since $\left[(A+B)(A-B)\right]^T \neq (A+B)(A-B)$

Hence $\boxed{(A+B)(A-B) \text{ is not symmetric}}$.

Since $\lfloor (A+B)(A-B) \rfloor^T \neq (A+B)(A-B)$

Hence $\boxed{(A+B)(A-B) \text{ is not symmetric}}$.

Provide feedback (0)

Step 3 of 4

c) We have to verify whether ABA is symmetric or not.

Now

$$(ABA)^T = A^T B^T A^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$$

$$= ABA \qquad \left(\text{Since } A^T = A \text{ and } B^T = B\right)$$

Since $(ABA)^T = ABA$

Hence $\boxed{ABA \text{ is symmetric}}$.

Provide feedback (0)

Step 4 of 4

d) We have to verify whether $ABAB$ is symmetric or not.

Now

$$(ABAB)^T = B^T A^T B^T A^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$$

$$= BABA \qquad \left(\text{Since } A^T = A \text{ and } B^T = B\right)$$

Since $(ABAB)^T \neq ABAB$

Hence $\boxed{ABAB \text{ is not symmetric}}$.

Provide feedback (0)

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CH1.6 56P

Step 1 of 1

Given that $A = A^T$ needs a row exchange, then it also needs a column exchnage to stay symmetric.

In matrix language, PA loses the symmetry of A but $\boxed{PAP^T}$ recovers the symmetry since the rows in P are the columns in P^T.

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CH1.6 58P

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CH1.6 57P

Step 1 of 3

(a) Given that $A = A^T$, where A is a 5 by 5 matrix.
We have to find how many entries of A can be chosen independently.

Provide feedback (0)

CH1.6 59P

Step 2 of 3

We know that the number of entries that can be chosen independently in a symmetric matrix of order n are $\frac{n(n+1)}{2}$

Since A is a 5 by 5 matrix.
So $n = 5$
Therefore, the number of independent entries are

$$\begin{aligned}\frac{5(5+1)}{2} &= \frac{30}{2}\\ &= 15\end{aligned}$$

Hence the number of independent entries in the given matrix are $\boxed{15}$.

Provide feedback (0)

Step 3 of 3

(b) We have to explain how does L and D given the same number of choices in LDL^T.
The 5 by 5 matrices L and D gives the same number of choices in LDL^T because L and L^T have same elements in different order because they are symmetric matrices.

Provide feedback (0)

CH1.6 58P

Step 1 of 3

Given that R is a rectangular m by n matrix and A is m by m symmetric matrix.

(a) We have to show that $R^T AR$ is symmeric and find the shape of the matrix.

Now

$$\left(R^T AR\right)^T = R^T A\left(R^T\right)^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$$

$$= R^T AR \qquad \left(\text{Since } \left(R^T\right)^T = R\right)$$

Since $\left(R^T AR\right)^T = R^T AR$

So $\boxed{R^T AR \text{ is symmetric}}$.

Provide feedback (0)

Step 2 of 3

Now we have to fins the shape of the matrix.

The transpose of $R^T AR$ is $\left(R^T AR\right)^T = R^T A^T (R^T)^T = R^T A^T R =$ n by n order.

Since R is m by n

So R^T is n by m

Now R^T is n by m, A is m by m and A^T is m by m.

So R^T times A^T is of order n by m

And $R^T A^T$ times R is of order n by n.

Hence the shape of $R^T AR$ is $\boxed{n \text{ by } n}$.

Provide feedback (0)

Step 3 of 3

(b) We have to show why $R^T R$ has no negative numbers on its diagonal.

We have

$$\left(R^T R\right)_{ij} = (column\ j\ of\ R).(column\ j\ of\ R)$$

$$= square\ of\ column\ j$$

CH1.6 60P

CH1.6 58P

Now

$$\left(R^T AR\right)^T = R^T A\left(R^T\right)^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$$

$$= R^T AR \qquad \left(\text{Since } \left(R^T\right)^T = R\right)$$

Since $\left(R^T AR\right)^T = R^T AR$

So $\boxed{R^T AR \text{ is symmetric}}$.

Provide feedback (0)

Step 2 of 3

Now we have to fins the shape of the matrix.

The transpose of $R^T AR$ is $\left(R^T AR\right)^T = R^T A^T (R^T)^T = R^T A^T R =$ n by n order.

Since R is m by n

So R^T is n by m

Now R^T is n by m, A is m by m and A^T is m by m.

So R^T times A^T is of order n by m

And $R^T A^T$ times R is of order n by n.

Hence the shape of $R^T AR$ is $\boxed{n \text{ by } n}$.

Provide feedback (0)

Step 3 of 3

(b) We have to show why $R^T R$ has no negative numbers on its diagonal.

We have

$$\left(R^T R\right)_{ij} = (column\ j\ of\ R).(column\ j\ of\ R)$$

$$= square\ of\ column\ j$$

In R^T and R diagonal elements becomes squares of column elements and squares are always positive.

Hence the numbers on the diagonal of $R^T R$ are not negative.

Provide feedback (0)

CH1.6 60P

CH1.6 59P

Step 1 of 13

Given matrix is $A=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$

We have to factor the symmetric matrix A into $A=LDL^T$, where D is a diagonal matrix and L is a lower triangular matrix.

Provide feedback (0)

Step 2 of 13

We have $A=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$\rightarrow\begin{bmatrix}1 & 3\\ 0 & -7\end{bmatrix}$$

Provide feedback (0)

Step 3 of 13

Applying the same row operation reversely on the identity matrix gives L.

Adding 3 times row 1 to row 2 gives $\rightarrow\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

Therefore, $L=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

The transpose of the matrix L is $L^T=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$.

Provide feedback (0)

Step 4 of 13

CH1.6 61P

Step 4 of 13

Since the pivots are 1 and -7 .

So the matrix D is $D=\begin{bmatrix}1 & 0\\ 0 & -7\end{bmatrix}$

Hence the LDL^T factorization of the given matrix is $\boxed{\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -7\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}}$.

Provide feedback (0)

Step 5 of 13

Given matrix is $A=\begin{bmatrix}1 & b\\ b & c\end{bmatrix}$.

We have to factor the symmetric matrix A into $A=LDL^T$, where D is a diagonal matrix and L is a lower triangular matrix.

Provide feedback (0)

Step 6 of 13

Subtracting b times row 1 from row 2 gives

$$\rightarrow\begin{bmatrix}1 & b\\ 0 & c-b^2\end{bmatrix}$$

Provide feedback (0)

Step 7 of 13

Applying the same row operations reversely on the identity matrix gives L.

Adding b times row 1 to row 2 gives

$$\rightarrow\begin{bmatrix}1 & 0\\ b & 1\end{bmatrix}$$

Step 7 of 13

Applying the same row operations reversely on the identity matrix gives L.
Adding b times row 1 to row 2 gives

$$\rightarrow \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

Therefore, $L = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$

The transpose of the matrix L is $L^T = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$.

Provide feedback (0)

Step 8 of 13

Since the pivots are 1and $c - b^2$.

So the matrix D is $D = \begin{bmatrix} 1 & b \\ 0 & c-b^2 \end{bmatrix}$

Hence the LDL^T factorization of the given matrix is

$$\boxed{\begin{bmatrix} 1 & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & c-b^2 \end{bmatrix}\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}}$$

Provide feedback (0)

Step 9 of 13

Given matrix is $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

We have to factor the symmetric matrix A into $A = LDL^T$, where D is a diagonal matrix and L is a lower triangular matrix.

Provide feedback (0)

Step 10 of 13

Adding $\frac{1}{2}$ times row 1 to row 2 gives

$$\rightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Adding row 2 to $\frac{3}{2}$ times row 3 gives

$$\rightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 11 of 13

Applying the same row operations reversely on the identity matrices gives L.

Subtracting $\frac{1}{2}$ times row 1 to row 2 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Subtracting row 2 from $\frac{3}{2}$ times row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 1 & & 3 \end{bmatrix}$$

Step 11 of 13

Applying the same row operations reversely on the identity matrices gives L.

Subtracting $\frac{1}{2}$ times row 1 to row 2 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Subtracting row 2 from $\frac{3}{2}$ times row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$$

Provide feedback (0)

Step 12 of 13

Therefore, $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$

The transpose of L is $L^T = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$

Step 12 of 13

Therefore, $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$

The transpose of L is $L^T = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$

Provide feedback (0)

Step 13 of 13

Since the pivots are 2, $\frac{3}{2}$, and 2.

So the matrix D is $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Hence the LDL^T factorization of the given matrix is

$$\boxed{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}}$$

Provide feedback (0)

CH1.6 60P

CH1.6 62P

Step 1 of 3

Given that the voltages of the cities are x_B, x_C, x_S.

With unit resistances between the cities, the three currents are in y given as

$$y = Ax \text{ as } \begin{bmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_B \\ x_C \\ x_S \end{bmatrix}$$

Here the matrix A is $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

The transpose of A is $A^T = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$

Provide feedback (0)

Step 2 of 3

(a) We have to find the total currents $A^T y$ out of the three cities.

The total currents out of the three cities is $A^T y$.

Therefore,

$$A^T y = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{bmatrix}$$

$$= \begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}$$

Hence the total currents in three cities is $\boxed{\begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}}$.

Therefore,

$$A^T y = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{bmatrix}$$

$$= \begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}$$

Hence the total currents in three cities is $\boxed{\begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}}$.

Provide feedback (0)

Step 3 of 3

(b) We have to verify that $(Ax)^T y$ agrees with $x^T (A^T y)$.

Now

$$(Ax)^T y = (x^T A^T) y \qquad (\text{Since } (AB)^T = B^T A^T)$$

$$= x^T (A^T y) \qquad (\text{By Associative property})$$

Therefore, $(Ax)^T y$ agrees with $x^T (A^T y)$.

Now

$$x^T (A^T y) = [x_B \quad x_C \quad x_S] \begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}$$

$$= [x_B y_{BC} + x_B y_{BS} - x_C y_{BC} + x_C y_{CS} - x_A y_{CS} - x_A y_{BS}]$$

Therefore, $x^T (A^T y) = [x_B y_{BC} + x_B y_{BS} - x_C y_{BC} + x_C y_{CS} - x_A y_{CS} - x_A y_{BS}]$

Provide feedback (0)

CH1.6 61P

CH1.6 63P

Step 1 of 4

Given that producing x_1 trucks and x_2 planes requires $x_1 + 50x_2$ tons of steel, $40x_1 + 1000x_2$ pounds of rubber, and $2x_1 + 50x_2$ months of labor.
Suppose the unit costs y_1, y_2, y_3 are \$700 per ton, \$3 perpound and \$3000 per month.
We have to find the value of one truck and one plane.

Provide feedback (0)

Step 2 of 4

Given that the truck and the plane are the components of $A^T y$.
From the given data, we get

$$Ax = \begin{bmatrix} x_1 + 50x_2 \\ 40x_1 + 1000x_2 \\ 2x_1 + 50x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Provide feedback (0)

Step 3 of 4

And $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{pmatrix} 700 \\ 3 \\ 3000 \end{pmatrix}$

We have the matrix A is $A = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix}$

The transpose of A is $A^T = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix}$.

Provide feedback (0)

Step 3 of 4

And $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{pmatrix} 700 \\ 3 \\ 3000 \end{pmatrix}$

We have the matrix A is $A = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix}$

The transpose of A is $A^T = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix}$.

Provide feedback (0)

Step 4 of 4

The values of one truck and one plane is $A^T y$.
Therefore,

$$A^T y = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix} \begin{bmatrix} 700 \\ 3 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 1(700)+40(3)+2(3000) \\ 50(700)+1000(3)+50(3000) \end{bmatrix}$$

$$= \begin{bmatrix} 700+120+6000 \\ 35000+3000+150000 \end{bmatrix}$$

$$= \begin{bmatrix} 6820 \\ 188000 \end{bmatrix}$$

Hence the value od one truck is $\boxed{\$6,820}$ and the value of one plane is $\boxed{\$188,000}$.

CH1.6 62P

Step 1 of 3

Given that producing x_1 trucks and x_2 planes requires $x_1 + 50x_2$ tons of steel, $40x_1 + 1000x_2$ pounds of rubber, and $2x_1 + 50x_2$ months of labor.

Suppose the unit costs y_1, y_2, y_3 are \$700 per ton, \$3 perpound and \$3000 per month.

Given that Ax gives the amounts of steel, rubber, and labor to produce x.

We have to find A.

Provide feedback (0)

Step 2 of 3

From the given data, we get

$$Ax = \begin{bmatrix} x_1 + 50x_2 \\ 40x_1 + 1000x_2 \\ 2x_1 + 50x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Therfore, $\boxed{A = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix}}$

Provide feedback (0)

Step 3 of 3

Here $(Ax)^T y$ is the $\boxed{\text{cost}}$ of inputs while $x^T (A^T y)$ is the $\boxed{\text{value}}$ of outputs.

Provide feedback (0)

CH1.6 64P

CH1.6 63P

CH1.6 65P

Step 1 of 3

Factorization of *A* into triangular times symmetric:

Let *A* be factorized into *LDU*.

$$A = LDU$$

Then matrix *A* can be written as $L(U^T)^{-1}$ times $U^T DU$. Here, $L(U^T)^{-1}$ is a matrix with diagonal all 1's.

Explain why $L(U^T)^{-1}$ is triangular and why $U^T DU$ is symmetric.

Provide feedback (0)

Step 2 of 3

Recall that if matrix *A* is factored into *LDU* with no row exchanges then matrix *U* is exactly L^T.

Now consider the product of lower triangular matrix and inverse transpose of upper triangular matrix:

$$\begin{aligned} L(U^T)^{-1} &= L\left((L^T)^T\right)^{-1} \\ &= L(L)^{-1} \\ &= I \end{aligned}$$

In other way it can also be said that product of two triangular matrices results into a triangular matrix. As here *L* and *U* are both triangular matrices.

$$\begin{aligned} L\left(U^T\right)^{-1} &= L\left(\left(L^T\right)^T\right)^{-1} \\ &= L(L)^{-1} \\ &= I \end{aligned}$$

In other way it can also be said that product of two triangular matrices results into a triangular matrix. As here L and U are both triangular matrices.

Therefore, $L\left(U^T\right)^{-1}$ is triangular.

Provide feedback (0)

Step 3 of 3

Recall that $A = A^T$ then matrix A is symmetric. Now, do the following calculations:

$$\begin{aligned} \left(U^T D U\right)^T &= U^T D^T \left(U^T\right)^T \\ &= U^T D^T U \\ &= U^T D U \end{aligned}$$

Here, $D = D^T$ is taken as it is a pivot matrix. Therefore, above calculations shows that $U^T D U$ is symmetric.

Provide feedback (0)

CH1.6 64P

CH1.6 66P

Step 1 of 6

Suppose $G_1 = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1} & - & l_{m(m-1)} & 1 \end{bmatrix} \right\}$ the set of lower triangular matrices of square order m having 1's on the diagonal.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1} & - & l_{m(m-1)} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ k_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ k_{m1} & - & k_{m(m-1)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21}+k_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1}+l_{m2}k_{21}+\ldots+k_{m1} & - & l_{m(m-1)}+k_{m(m-1)} & 1 \end{bmatrix}$$

This shows that the product of lower triangular matrices of order m is a lower triangular matrix of order m.

Inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1} & - & l_{m(m-1)} & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ (-1)^{m-1} l_{21} l_{32} \ldots l_{m(m-1)} + \ldots & - & -l_{m(m-1)} & 1 \end{bmatrix}$ which is also a lower triangular matrix having 1's on the main diagonal.

Further, the identity matrix is also a lower triangular matrix.

Associativity of product of lower triangular matrices is lower triangular.

Using all these properties, we see that the set of all square lower triangular matrices of order m form a group.

Provide feedback (0)

Step 2 of 6

We observe that G_2 : the diagonal invertible matrices have no diagonal entry 0

So, the product of such matrices is also a diagonal matrix having no diagonal entry 0.

Therefore, closure law holds under multiplication of matrices

Associativity of matrix multiplication holds for any matrices.

Already it is given that G_2 is invertible and the inverse matrix is also a diagonal matrix

Step 2 of 6

We observe that G_2 : the diagonal invertible matrices have no diagonal entry 0
So, the product of such matrices is also a diagonal matrix having no diagonal entry 0.
Therefore, closure law holds under multiplication of matrices
Associativity of matrix multiplication holds for any matrices.
Already it is given that G_2 is invertible and the inverse matrix is also a diagonal matrix having no entry zero on the main diagonal.
We easily see that the identity matrix of order m is also a diagonal matrix having no entry 0 on the main diagonal.
Therefore, G_2 is a group under multiplication of matrices.

Provide feedback (0)

Step 3 of 6

Symmetric matrices is not a Group

Let us consider the symmetric matrices $A=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}; B=\begin{pmatrix}4 & 2\\ 2 & 0\end{pmatrix}$

$$AB=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}\begin{pmatrix}4 & 2\\ 2 & 0\end{pmatrix}$$
$$=\begin{pmatrix}10 & 4\\ 10 & 2\end{pmatrix}$$ is not a symmetry matrix.

That is the product of symmetric matrices does not obey closure law.

Provide feedback (0)

Step 4 of 6

A positive matrix is a square matrix with all eigen values positive.
Their product is also a positive matrix, inverse of a positive matrix is positive and identity matrix is a positive matrix and associativity under multiplication holds.
Therefore, the set of positive matrices form a group.

Provide feedback (0)

Let us consider the symmetric matrices $A=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}; B=\begin{pmatrix}4 & 2\\ 2 & 0\end{pmatrix}$

$$AB=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}\begin{pmatrix}4 & 2\\ 2 & 0\end{pmatrix}$$

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Provide feedback (0)

Step 4 of 6

A positive matrix is a square matrix with all eigen values positive.
Their product is also a positive matrix, inverse of a positive matrix is positive and identity matrix is a positive matrix and associativity under multiplication holds.
Therefore, the set of positive matrices form a group.

Provide feedback (0)

Step 5 of 6

The product of permutation matrix is a square matrix with each row has only one entry 1 and other entries zero, similarly with columns.
The product of permutation matrices is a permutation matrix and the inverse of a permutation matrix is a permutation matrix.
Therefore, the permutation matrices form a group

Provide feedback (0)

Step 6 of 6

Further, the set of matrices of same order, non singular form a group under multiplication of matrices.

Provide feedback (0)

CH1.6 65P

Step 1 of 1

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 0 & 3 & 1 \\ 1 & 0 & 3 & 2 \\ 3 & 2 & 0 & 1 \end{bmatrix}$$

We easily see that every row contains the numbers 0, 1, 2, and 3
It is not symmetric while $1,2^{th}$ entry 1 and $2,1^{th}$ entry is 2 such that both are not equal.

The determinant of this matrix is $-1\begin{vmatrix} 2 & 3 & 1 \\ 1 & 3 & 2 \\ 3 & 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 0 & 2 & 3 \\ 1 & 3 & 1 \\ 2 & 3 & 2 \end{vmatrix}$

$$= (-1)(3)\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} + (-1)(1)\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} + 2(-1)\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} + 2(2)\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= -3(6-3) - 1(6-3) - 2(4-9) + 4(2-9)$$

$$= -9 - 3 + 10 - 28$$

$$= -29$$

$$\neq 0$$

So, the matrix is invertible and its inverse is

$$\begin{bmatrix} -1 & -\frac{7}{6} & \frac{11}{6} & \frac{1}{2} \\ 2 & \frac{17}{6} & -\frac{25}{6} & -\frac{1}{2} \\ 1 & \frac{11}{6} & -\frac{13}{6} & -\frac{1}{2} \\ -1 & -\frac{13}{6} & \frac{17}{6} & \frac{1}{2} \end{bmatrix}$$

CH1.6 67P

Provide feedback (0)

CH1.6 66P

Step 1 of 2

Suppose a typical matrix is $A = \begin{bmatrix} a_{11} & a_{12} & - & a_{1n} \\ a_{21} & a_{ij} & - & a_{2n} \\ - & - & - & - \\ a_{n1} & a_{n2} & - & a_{nn} \end{bmatrix}$ is an $n \times n$ matrix.

We know by definition that A^T is an $n \times n$ matrix such that ij^{th} entry b_{ij} of A^T is nothing but the ji^{th} entry a_{ji} of A.

$$A^T = \begin{bmatrix} a_{11} & a_{21} & - & a_{n1} \\ a_{12} & a_{ji} & - & a_{n2} \\ - & - & - & - \\ a_{1n} & a_{2n} & - & a_{nn} \end{bmatrix}$$

Provide feedback (0)

CH1.6 68P

Step 2 of 2

Now, to reorder the rows of A, we use the interchange of rows R_i and R_j

Then we follow that except the ij^{th} entry of ith row and j^{th} column, every other entry will be different from the entries of the same row or same column.

So, this reordering of A left the matrix different from A^T.

Similarly, any reordering of the rows and columns of A leave the other entries in the respective positions different.

Thus, the reordering of A and the transposition of A are different matrices.

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CH1.6 67P

Step 1 of 4

By partitioning a square matrix as two sub square matrices $A_{r\times r}$, $C_{n-r\times n-r}$ on the north west and south east corners, $D_{r\times n-r}, E_{n-r\times r}$ matrices along the anti – diagonal are such that $d_{ij} = e_{ji}$ for every i^{th} row and j^{th} column.

In the given case, we have $C_{n-r\times n-r}$ is a zero matrix.

So, the given matrix looks like $B = \begin{bmatrix} A & D \\ E & C \end{bmatrix}$

We follow that $B^T = \begin{bmatrix} A^T & E^T \\ D^T & C^T \end{bmatrix}$

While C is a zero matrix, we get C^T is also a zero matrix.

Further, we have noted that $D^T = E, E^T = D$

Therefore, B^T is a North West matrix.

Provide feedback (0)

Step 2 of 4

$$B^2 = \begin{bmatrix} A^2 + DE & AD + DC \\ EA + CE & ED + C^2 \end{bmatrix}$$

We observe that the South West matrix in this square is $ED + C^2$ not necessarily zero while E and D are not known to be the zero matrices.

Therefore, B^2 is not a North West matrix

Provide feedback (0)

Step 3 of 4

$$B^{-1} = \frac{1}{|AC| - |DE|}\begin{bmatrix} C & -D \\ -E & A \end{bmatrix}$$

We follow that the North west corner is the zero matrix and the South East is a non zero matrix.

Therefore, the inverse of a North West matrix is a South East matrix.

CH1.6 69P

$$B^2 = \begin{bmatrix} EA+CE & ED+C^2 \end{bmatrix}$$

We observe that the South West matrix in this square is $ED+C^2$ not necessarily zero while E and D are not known to be the zero matrices.
Therefore, B^2 is not a North West matrix

Provide feedback (0)

Step 3 of 4

$$B^{-1} = \frac{1}{|AC|-|DE|}\begin{bmatrix} C & -D \\ -E & A \end{bmatrix}$$

We follow that the North west corner is the zero matrix and the South East is a non zero matrix.
Therefore, the inverse of a North West matrix is a South East matrix.

Provide feedback (0)

Step 4 of 4

Suppose $B = \begin{bmatrix} X & Y \\ Z & W \end{bmatrix}$ is a North West matrix.

Then by definition, we have W is a zero square sub matrix of B.

Suppose $C = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$ is a South East matrix.

Then P is a zero matrix and the respective sizes of the sub matrices in both B and C are multipliable.

$$BC = \begin{bmatrix} XP+YR & XQ+YS \\ ZP+WR & ZQ+WS \end{bmatrix}$$

Using $W=0$ and $P=0$ in this, we get $BC = \begin{bmatrix} YR & XQ+YS \\ 0 & ZQ \end{bmatrix}$

The product of matrix is neither North West nor South East.

Provide feedback (0)

CH1.6 68P

Step 1 of 1

Random A is invertible.
Since all the Random matrices are almost surely invertible.

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CH1.6 70P

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CH1.6 70P

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CH1.7

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