CH5.1

Step 1 of 13

Def: suppose A is n by n matrix has n linearly independent eigen vectors. If these eigen vectors are the columns of a matrix S, then $S^{-1}AS$ is a diagonal matrix Λ.
The eigen values of A are on the diagonal of Λ.

Provide feedback (0)

Step 2 of 13

Given that $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

To find the eigen values and the respective eigen vectors of this matrix, we consider the characteristic equation of A to be $\det(A-\lambda I)=0$

i.e., $\begin{vmatrix}1-\lambda & 1\\ 1 & 1-\lambda\end{vmatrix}=0$

$\Rightarrow \lambda^2-2\lambda=0$

$\Rightarrow \lambda_1=0, \lambda_2=2$ are the eigen values of A.

Provide feedback (0)

Step 3 of 13

To get the respective eigen vectors, suppose x_1 is a vector such that it satisfies

$(A-\lambda_1 I)x_1=0$

$\Rightarrow \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}t_1\\ t_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$

Using the row operation $R_2 \to R_2-R_1$, we get $\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}t_1\\ t_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$

Writing the homogeneous equation from this, we get $t_1+t_2=0$

Putting $t_1=1$, we get $t_2=-1$ and thus, $x_1=\begin{pmatrix}t_1\\ t_2\end{pmatrix}=\begin{pmatrix}1\\ -1\end{pmatrix}$ is the eigen vector corresponding to $\lambda_1=0$

CH5.2 2P

$\Rightarrow \lambda_1 = 0, \lambda_2 = 2$ are the eigen values of A.

Provide feedback (0)

Step 3 of 13

To get the respective eigen vectors, suppose x_1 is a vector such that it satisfies

$(A - \lambda_1 I)x_1 = 0$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Using the row operation $R_2 \to R_2 - R_1$, we get $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Writing the homogeneous equation from this, we get $t_1 + t_2 = 0$

Putting $t_1 = 1$, we get $t_2 = -1$ and thus, $x_1 = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is the eigen vector corresponding to $\lambda_1 = 0$

Provide feedback (0)

Step 4 of 13

Similarly, suppose x_2 is the eigen vector satisfying $(A - \lambda_2 I)x_2 = 0$

i.e., $\Rightarrow \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Consequently, $x_2 = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is the eigen vector corresponding to $\lambda_2 = 2$

Since the eigen vectors of the distinct eigen vectors are linearly independent, the matrix whose columns are these eigen vectors will be non singular.

i.e., $S = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ is the non – singular matrix and $S^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

Provide feedback (0)

Step 5 of 13

Provide feedback (0)

Step 5 of 13

Now, $S^{-1}AS = \frac{1}{2}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix}0 & 0\\2 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}$$

$$= \begin{pmatrix}0 & 0\\0 & 2\end{pmatrix}$$

$$= \Lambda$$

This is the diagonal matrix whose diagonal entries are the eigen values 0 , 2 of the given matrix.

Provide feedback (0)

Step 6 of 13

Therefore, $A = S\Lambda S^{-1}$

$$= \begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}0 & 0\\0 & 2\end{pmatrix}\frac{1}{2}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$$

Provide feedback (0)

Step 7 of 13

$$A = \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix}$$

We consider the characteristic equation of A to be $\det(A - \lambda I) = 0$

i.e., $\begin{vmatrix}2-\lambda & 1\\0 & -\lambda\end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 2\lambda = 0$

$\Rightarrow \lambda_1 = 0, \lambda_2 = 2$ are the eigen values of A.

Provide feedback (0)

$$=\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & 2\end{pmatrix}\frac{1}{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$$

Provide feedback (0)

Step 7 of 13

$$A=\begin{bmatrix}2 & 1\\ 0 & 0\end{bmatrix}$$

We consider the characteristic equation of A to be $\det(A-\lambda I)=0$

i.e., $\begin{vmatrix}2-\lambda & 1\\ 0 & -\lambda\end{vmatrix}=0$

$\Rightarrow \lambda^2-2\lambda=0$

$\Rightarrow \lambda_1=0, \lambda_2=2$ are the eigen values of A.

Provide feedback (0)

Step 8 of 13

To get the respective eigen vectors, suppose x_1 is a vector such that it satisfies

$(A-\lambda_1 I)x_1=0$

$$\Rightarrow \begin{pmatrix}2 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}t_1\\ t_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$$

Writing the homogeneous equation from this, we get $2t_1+t_2=0$

Putting $t_1=1$, we get $t_2=-2$ and thus, $x_1=\begin{pmatrix}t_1\\ t_2\end{pmatrix}=\begin{pmatrix}1\\ -2\end{pmatrix}$ is the eigen vector corresponding to $\lambda_1=0$

Provide feedback (0)

Step 9 of 13

Similarly, suppose x_2 is the eigen vector satisfying $(A-\lambda_2 I)x_2=0$

to $\lambda_1 = 0$

Provide feedback (0)

Step 9 of 13

Similarly, suppose x_2 is the eigen vector satisfying $(A-\lambda_2 I)x_2 = 0$

Provide feedback (0)

Step 10 of 13

i.e., $\Rightarrow \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Using the row operation $R_2 \to R_2 + 2R_1$, we get $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Writing the homogeneous equations, we get $t_2 = 0$

But t_1 satisfy any scalar value and so, the solution is $t_2 = 0$ and $t_1 = k$

When $k = 1$, we get $x_2 = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is the eigen vector corresponding to $\lambda_2 = 2$

Provide feedback (0)

Step 11 of 13

Since the eigen vectors of the distinct eigen vectors are linearly independent, the matrix whose columns are these eigen vectors will be non singular.

i.e., $S = \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}$ is the non – singular matrix and $S^{-1} = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$

Provide feedback (0)

Step 12 of 13

Now $S^{-1}AS = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

i.e., $S = \begin{pmatrix} & \\ -2 & 0 \end{pmatrix}$ is the non – singular matrix and $S^{-1} = \frac{1}{2}\begin{pmatrix} & \\ 2 & 1 \end{pmatrix}$

Provide feedback (0)

Step 12 of 13

Now, $S^{-1}AS = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \Lambda$$

This is the diagonal matrix whose diagonal entries are the eigen values 0, 2 of the given matrix.

Provide feedback (0)

Step 13 of 13

Therefore, $A = S\Lambda S^{-1}$

$$= \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$$

Provide feedback (0)

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CH5.2 1P

Step 1 of 2

Def: suppose A is n by n matrix has n linearly independent eigen vectors. If these eigen vectors are the columns of a matrix S, then $S^{-1}AS$ is a diagonal matrix Λ.
The eigen values of A are on the diagonal of Λ.

Provide feedback (0)

Step 2 of 2

Given that A has eigen values are 1 and 4, and whose eigenvectors are $\begin{bmatrix}3\\1\end{bmatrix}$ and $\begin{bmatrix}2\\1\end{bmatrix}$.

In view of the above definition, we have

$$S=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix},$$

So, $S^{-1}=\dfrac{1}{3-2}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$

Also, the diagonal matrix is $\Lambda=\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}$

The required matrix is $A=S\Lambda S^{-1}$

$$\begin{aligned}&=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}\\&=\begin{bmatrix}3 & 8\\1 & 4\end{bmatrix}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}\\&=\begin{bmatrix}-5 & 18\\-3 & 10\end{bmatrix}\end{aligned}$$

Provide feedback (0)

CH5.2 3P

Algebra tutors who can help right now

CH5.2 2P

CH5.2 4P

Step 1 of 7

Given that $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

To find eigen values, we consider $|A-\lambda I|=0$

$$|A-\lambda I|=\begin{vmatrix}1-\lambda&1&1\\1&1-\lambda&1\\1&1&1-\lambda\end{vmatrix}=0$$

Applying the row operations upon this, we have

$$R_1\to R_1+R_2+R_3\Rightarrow\begin{vmatrix}3-\lambda&3-\lambda&3-\lambda\\1&1-\lambda&1\\1&1&1-\lambda\end{vmatrix}=0$$

$$\Rightarrow(3-\lambda)\begin{vmatrix}1&1&1\\1&1-\lambda&1\\1&1&1-\lambda\end{vmatrix}=0$$

$$R_2\to R_2-R_1,R_3\to R_3-R_1\Rightarrow(3-\lambda)\begin{vmatrix}1&1&1\\0&-\lambda&0\\0&0&-\lambda\end{vmatrix}=0$$

This is the determinant of the upper triangular matrix and so, the determinant is the product of diagonal entries.

So, $|A-\lambda I|=(3-\lambda)(1)(-\lambda)(-\lambda)=0$

$\Rightarrow\lambda_1=0,\lambda_2=0,\lambda_3=3$ are the eigen values of the given matrix.

Provide feedback (2)

Step 2 of 7

To find eigen vector for $\lambda_1=0$, we solve the homogeneous system $(A-\lambda I)x=0$ where x is any column vector of dimension 3.

$$\Rightarrow\begin{bmatrix}1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=0$$

Provide feedback (2)

Step 2 of 7

To find eigen vector for $\lambda_1 = 0$, we solve the homogeneous system $(A-\lambda I)x = 0$ where x is any column vector of dimension 3.

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

By applying the row operation on the coefficient matrix, the system reduces to $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

Provide feedback (0)

Step 3 of 7

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$\Rightarrow x_1 + x_2 + x_3 = 0$ ..... (1)

Supposing x_2, x_3 are free variables and letting $x_2 = k_1, x_3 = k_2$, we get $x_1 = -k_1 - k_2$

So, the solution set of $(A-\lambda I)x = 0$ is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -k_1 - k_2 \\ k_1 \\ k_2 \end{bmatrix}$

$$= k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So, the eigen vectors corresponding to the repeated eigen values $\lambda_1 = 0, \lambda_2 = 0$ are

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Provide feedback (0)

Step 4 of 7

Similarly, for $\lambda = 3$, we solve $(A-3I)x = 0$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Applying the row operations on the coefficient matrix,

$$R_2 \to 2R_2 + R_1, R_3 \to 2R_3 + R_1 \Rightarrow \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}$$

$$R_3 \to R_3 + R_2, R_2 / -3 \Rightarrow \begin{bmatrix} -2 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Provide feedback (0)

Step 5 of 7

This is the reduced matrix and so, we rewrite the homogeneous equations from this as

$$\left.\begin{aligned} -2x_1 + x_2 + x_3 &= 0 \\ x_2 - x_3 &= 0 \end{aligned}\right\} \qquad \ldots\ldots (2)$$

$\Rightarrow x_2 = x_3,$

$\Rightarrow x_1 = x_3$

Using $x_3 = 1$, we get the solution set $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to

$\lambda_3 = 3$

Provide feedback (0)

Provide feedback (0)

Step 6 of 7

While the number of eigen values is equal to the number of eigen vectors of A, we say that A is diagonalizable.
Using the eigen vectors as the columns of the matrix S, we see that the diagonalization of the given matrix is $A = S\Lambda S^{-1}$ where $S = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$,

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}, S^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 2 & -1 \\ -1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \qquad \ldots\ldots (3)$$

Provide feedback (0)

Step 7 of 7

On the other hand, we try for the other diagonalization.
Let us consider (1) and write $x_3 = -x_1 - x_2$

Then the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = m\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + n\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ where $x_1 = m, x_2 = n$ are the parameters.

So, the other possible eigen vectors corresponding to $\lambda_1 = 0, \lambda_2 = 0$ are $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Similarly, considering (2), the eigen vector corresponding to $\lambda_3 = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Using these eigen vectors as the columns of S, we get $A = S\Lambda S^{-1}$ where

$$S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } S^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix} \qquad \ldots\ldots (4)$$

On the other hand, we try for the other diagonalization.
Let us consider (1) and write $x_3 = -x_1 - x_2$

Then the solution set is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = m\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + n\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ where $x_1 = m, x_2 = n$ are the parameters.

So, the other possible eigen vectors corresponding to $\lambda_1 = 0, \lambda_2 = 0$ are $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

Similarly, considering (2), the eigen vector corresponding to $\lambda_3 = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Using these eigen vectors as the columns of S, we get $A = S\Lambda S^{-1}$ where

$$S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ and } S^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \quad \ldots\ldots (4)$$

Observe that (3) and (4) are the different diagonalizations for the same matrix.

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CH5.2 3P

Step 1 of 2

Suppose A is a 3×3 upper triangular matrix with the diagonal entries 1, 2, and7.

Then the characteristic equation is $|A-\lambda I| = \begin{vmatrix} 1-\lambda & b & c \\ 0 & 2-\lambda & d \\ 0 & 0 & 7-\lambda \end{vmatrix} = 0$ where b, c, d are any real numbers.

We use the first column to find the determinant of this matrix.

i.e., $(1-\lambda)\begin{vmatrix} 2-\lambda & d \\ 0 & 7-\lambda \end{vmatrix} - 0\begin{vmatrix} b & c \\ 0 & 7-\lambda \end{vmatrix} + 0\begin{vmatrix} b & c \\ 2-\lambda & d \end{vmatrix} = 0$

$= (1-\lambda)(2-\lambda)(7-\lambda) - 0 + 0$

So, the eigen values are $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 7$

Provide feedback (0)

CH5.2 5P

Step 2 of 2

We know that the eigen vectors of the distinct eigen values are linearly independent.

While 1, 2, and 7 are the distinct eigen values, the respective eigen vectors x_1, x_2, x_3 are linearly independent.

So, the matrix S whose columns are x_1, x_2, x_3 is a non singular matrix.

So, S^{-1} exists such that $A = S\Lambda S^{-1}$ where Λ is the diagonal matrix whose diagonal entries are respectively the eigen values of A.

So, $\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

Provide feedback (0)

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Matt L.

CH5.2 4P

Step 1 of 9

A matrix A is not diagonalizable if the sum of the algebraic multiplicities of the eigen values is not equal to the sum of the geometric multiplicities of the respective eigen vectors.

Given that $A_1 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(-2-\lambda)+4 = 0$$

$$\Rightarrow -4-2\lambda+2\lambda+\lambda^2+4 = 0$$

$$\Rightarrow \lambda^2 = 0$$

$\Rightarrow \lambda = 0, 0$ says that the eigen value 0 has algebraic multiplicity 2.

Provide feedback (0)

CH5.2 6P

Step 2 of 9

To find the eigen vector corresponding to this eigen value, we solve the homogeneous system $(A - \lambda I)x = 0$

i.e, $\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Applying the row operation upon the coefficient matrix, $R_2 \to R_2 - R_1,\ R_1/2$ we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$$x_1 - x_2 = 0$$

$$\Rightarrow x_1 = x_2$$

Putting $x_1 = 1$, we get the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ which is the eigen vector for $\lambda = 0, 0$.

We easily see that the number of eigen vectors = geometric multiplicity = 1 for $\lambda = 0, 0$

So, algebraic multiplicity is not equal to the geometric multiplicity and thus

CH5.2 4P

Step 1 of 9

A matrix A is not diagonalizable if the sum of the algebraic multiplicities of the eigen values is not equal to the sum of the geometric multiplicities of the respective eigen vectors.

Given that $A_1 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(-2-\lambda)+4 = 0$$

$$\Rightarrow -4-2\lambda+2\lambda+\lambda^2+4 = 0$$

$$\Rightarrow \lambda^2 = 0$$

$\Rightarrow \lambda = 0, 0$ says that the eigen value 0 has algebraic multiplicity 2.

Provide feedback (0)

CH5.2 6P

Step 2 of 9

To find the eigen vector corresponding to this eigen value, we solve the homogeneous system $(A - \lambda I)x = 0$

i.e, $\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Applying the row operation upon the coefficient matrix, $R_2 \to R_2 - R_1,\ R_1/2$ we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$$x_1 - x_2 = 0$$

$$\Rightarrow x_1 = x_2$$

Putting $x_1 = 1$, we get the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ which is the eigen vector for $\lambda = 0, 0$.

We easily see that the number of eigen vectors = geometric multiplicity = 1 for $\lambda = 0, 0$

So, algebraic multiplicity is not equal to the geometric multiplicity and thus

Step 2 of 9

To find the eigen vector corresponding to this eigen value, we solve the homogeneous system $(A-\lambda I)x=0$

i.e, $\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Applying the row operation upon the coefficient matrix, $R_2 \to R_2 - R_1,\ R_1/2$ we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2$

Putting $x_1 = 1$, we get the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ which is the eigen vector for $\lambda = 0, 0$.

We easily see that the number of eigen vectors = geometric multiplicity = 1 for $\lambda = 0, 0$

So, algebraic multiplicity is not equal to the geometric multiplicity and thus,

$A_1 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is not diagonalizable.

Provide feedback (0)

Step 3 of 9

$$A_2 = \begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix}$$

$$|A_2 - \lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 0 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(2+\lambda) = 0$$

$\Rightarrow \lambda = -2, 2$ are the eigen values of A_2

Provide feedback (0)

Step 4 of 9

$\Rightarrow (2-\lambda)(2+\lambda)=0$

$\Rightarrow \lambda=-2,2$ are the eigen values of A_2

Provide feedback (0)

Step 4 of 9

The eigen vector corresponding to $\lambda=-2$ is obtained by solving the homogeneous system $(A_2-\lambda I)x=0$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations $R_2 \to 2R_2-R_1,\ R_1/4$ on the coefficient matrix, it becomes

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous equations from this.

i.e., $x_1=0$

Replacing any real number in place of x_2, the system gets satisfied.

So, $x_1=0$, $x_2=k$ is the solution where k is the parameter.

Putting $k=1$, the solution is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ which is the eigen vector corresponding to $\lambda=-2$

Provide feedback (0)

Step 5 of 9

Similarly, for $\lambda=2$, we solve $(A_2-\lambda I)x=0$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2/2 \Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

This is the reduced matrix and so, the homogeneous equation is $x_1-2x_2=0$

In other words, $x_1=2x_2$

Provide feedback (0)

Step 5 of 9

Similarly, for $\lambda = 2$, we solve $(A_2 - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 / 2 \Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

This is the reduced matrix and so, the homogeneous equation is $x_1 - 2x_2 = 0$

In other words, $x_1 = 2x_2$

Putting $x_2 = 1$, we get $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ as the solution set of the system which is the eigen vector corresponding to $\lambda = 2$.

The sum of the algebraic multiplicities of the eigen values $= 2$

The sum of the geometric multiplicities of the eigen vectors $= 2$

While both are equal, we can say that $A_2 = \begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix}$ is diagonalizable.

Provide feedback (0)

Step 6 of 9

$$A_3 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 9

$$|A_3 - \lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 0 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(2-\lambda) = 0$$

$\Rightarrow \lambda = 2, 2$ are the eigen values of A_3

Observe that the algebraic multiplicity of the eigen value 2 is 2.

Step 7 of 9

$$|A_3 - \lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 0 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(2-\lambda) = 0$$

$\Rightarrow \lambda = 2, 2$ are the eigen values of A_3

Observe that the algebraic multiplicity of the eigen value 2 is 2.

Provide feedback (0)

Step 8 of 9

To get the eigen vector corresponding to this eigen value, we solve $(A_3 - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operation $R_1 \leftrightarrow R_2 / 2$ on the coefficient matrix, it becomes

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous equations from this as

$1 \cdot x_1 + 0 \cdot x_2 = 0$

So, $x_1 = 0, x_2 = k$ satisfies this equation where k is any real number.

So, putting $k = 1$, the solution set is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ which is the eigen vector corresponding to the eigen value 2.

Provide feedback (0)

Step 9 of 9

We easily see that the number of eigen vectors is 1 and so the geometric multiplicity is 1.
But the algebraic multiplicity is 2.

So, $A_3 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ is not diagonalizable.

So, putting $k = 1$, the solution set is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ which is the eigen vector corresponding to the eigen value 2.

Provide feedback (0)

Step 9 of 9

We easily see that the number of eigen vectors is 1 and so the geometric multiplicity is 1. But the algebraic multiplicity is 2.

So, $A_3 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ is not diagonalizable.

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CH5.2 5P

Step 1 of 3

a) Let λ be an eigen value of A and the respective eigen vector is x.
Then we have $Ax = \lambda x$
Multiplying with A on both sides

$$A^2x = A(\lambda x)$$
$$= \lambda(Ax) \text{ while } \lambda \text{ is the real number, it commutes with the matrix } A.$$
$$= \lambda(\lambda x)$$
$$= \lambda^2 x$$

But given that $A^2 = I$.
So, we get $Ix = \lambda^2 x$ and thus, $\lambda^2 = 1$
Thus, the eigen values of A are -1 and 1.

Provide feedback (1)

Step 2 of 3

b) If A is 2×2 matrix and not equal to - I or I, then the trace of A is the sum of the eigen values $= -1 + 1 = 0$
Also, its determinant is nothing but the product of the eigen values $= -1\times 1 = -1$

Provide feedback (1)

Step 3 of 3

c) The first row of $A = (3,-1)$

Let $A = \begin{bmatrix} 3 & -1 \\ x & y \end{bmatrix}$

The characteristic equation is $\begin{vmatrix} 3-\lambda & -1 \\ x & y-\lambda \end{vmatrix} = 0 = (\lambda+1)(\lambda-1)$ while - 1 and 1 are the eigen values.

$\Rightarrow \lambda^2 - (3+y)\lambda + (3y+x) = 0$

Sum of the roots is the trace of the matrix $= 0$ and product of the roots is the determinant of the matrix $= -1$ given in the above results

CH5.2 7P

Step 3 of 3

c) The first row of $A=(3,-1)$

Let $A=\begin{bmatrix}3 & -1\\ x & y\end{bmatrix}$

The characteristic equation is $\begin{vmatrix}3-\lambda & -1\\ x & y-\lambda\end{vmatrix}=0=(\lambda+1)(\lambda-1)$ while - 1 and 1 are the eigen values.

$\Rightarrow \lambda^2-(3+y)\lambda+(3y+x)=0$

Sum of the roots is the trace of the matrix $=0$ and product of the roots is the determinant of the matrix = -1 given in the above results.

i.e., $3+y=0, 3y+x=-1$

$\Rightarrow y=-3, x=8$ are the possible entries of A.

Using these, we get $A=\begin{bmatrix}3 & -1\\ 8 & -3\end{bmatrix}$ such that $A^2=I$ is satisfied.

Thus, the required matrix is $A=\begin{bmatrix}3 & -1\\ 8 & -3\end{bmatrix}$

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CH5.2 6P

Step 1 of 5

The number λ is an eigen value of A if and only if $A - \lambda I$ is singular
In other words, $\det(A - \lambda I) = 0$
This is the characteristic equation. Each λ is associated with eigen vector x such that $Ax = \lambda x$.

Further, the eigen vectors corresponding to the distinct eigen values are linearly independent.
Considering the linearly independent eigen vectors as the columns of a matrix S, we can see that $A = S\Lambda S^{-1}$ where Λ is the diagonal matrix whose diagonal entries are nothing but the eigen values of A.

Now, raising both sides to the power n, we get

$$\begin{aligned} A^n &= \left(S\Lambda S^{-1}\right)^n \\ &= \left(S\Lambda S^{-1}\right)\left(S\Lambda S^{-1}\right)\ldots\left(S\Lambda S^{-1}\right) \quad n \text{ times} \\ &= S\Lambda\left(S^{-1}S\right)\Lambda\left(S^{-1}S\right)\Lambda\ldots\Lambda S^{-1} \\ &= S\Lambda I\Lambda\ldots\Lambda S^{-1} \\ &= S\Lambda^n S^{-1} \end{aligned}$$

Provide feedback (0)

CH5.2 8P

Step 2 of 5

In view of the above discussion, we get A^{100} for the given matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ as

The characteristic equation of A is $\det(A - \lambda I) = 0$

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} 4-\lambda & 3 \\ 1 & 2-\lambda \end{vmatrix} = 0 \\ &= (4-\lambda)(2-\lambda) - 3 \\ &= 8 - 4\lambda - 2\lambda + \lambda^2 - 3 \\ &= \lambda^2 - 6\lambda + 5 \end{aligned}$$

$$\Rightarrow (\lambda - 5)(\lambda - 1) = 0$$

$$\Rightarrow \lambda_1 = 1, \lambda_2 = 5$$

CH5.2 6P

This is the characteristic equation. Each λ is associated with eigen vector x such that $Ax = \lambda x$.

Further, the eigen vectors corresponding to the distinct eigen values are linearly independent.
Considering the linearly independent eigen vectors as the columns of a matrix S, we can see that $A = S\Lambda S^{-1}$ where Λ is the diagonal matrix whose diagonal entries are nothing but the eigen values of A.

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$$\begin{aligned} A^n &= \left(S\Lambda S^{-1}\right)^n \\ &= \left(S\Lambda S^{-1}\right)\left(S\Lambda S^{-1}\right)\ldots\left(S\Lambda S^{-1}\right) \quad n \text{ times} \\ &= S\Lambda\left(S^{-1}S\right)\Lambda\left(S^{-1}S\right)\Lambda\ldots\Lambda S^{-1} \\ &= S\Lambda I\Lambda\ldots\Lambda S^{-1} \\ &= S\Lambda^n S^{-1} \end{aligned}$$

Provide feedback (0)

Step 2 of 5

In view of the above discussion, we get A^{100} for the given matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ as

The characteristic equation of A is det $(A - \lambda I) = 0$

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} 4-\lambda & 3 \\ 1 & 2-\lambda \end{vmatrix} = 0 \\ &= (4-\lambda)(2-\lambda) - 3 \\ &= 8 - 4\lambda - 2\lambda + \lambda^2 - 3 \\ &= \lambda^2 - 6\lambda + 5 \end{aligned}$$

$$\Rightarrow (\lambda - 5)(\lambda - 1) = 0$$

$$\Rightarrow \lambda_1 = 1, \lambda_2 = 5$$

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Step 3 of 5

CH5.2 8P

$$= \lambda^2 - 6\lambda + 5$$
$$\Rightarrow (\lambda - 5)(\lambda - 1) = 0$$
$$\Rightarrow \lambda_1 = 1, \lambda_2 = 5$$

Provide feedback (0)

Step 3 of 5

Let x_1 is the vector such that $(A - \lambda_1 I)x_1 = 0$

i.e., $\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Using the row operation $R_2 \to 3R_2 - R_1$, we get $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Rewriting the system from this, we get $t_1 + t_2 = 0$

Putting $t_1 = 1$, we get $t_2 = -1$ and thus, $x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_1 = 1$.

Provide feedback (0)

Step 4 of 5

Let x_2 is the vector such that $(A - \lambda_2 I)x_2 = 0$

i.e., $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Using the row operation $R_2 \to R_2 + R_1$ on this, we get $\begin{bmatrix} -1 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Rewriting the system, we get $-t_1 + 3t_2 = 0$

Putting $t_2 = 1$, we get $t_1 = 3$ and thus, $x_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_2 = 5$

Provide feedback (0)

Let x_2 is the vector such that $(A - \lambda_2 I)x_2 = 0$

i.e., $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Using the row operation $R_2 \to R_2 + R_1$ on this, we get $\begin{bmatrix} -1 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Rewriting the system, we get $-t_1 + 3t_2 = 0$

Putting $t_2 = 1$, we get $t_1 = 3$ and thus, $x_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_2 = 5$

Provide feedback (0)

Step 5 of 5

Considering the eigen vectors as the columns of S, we get $S = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

$S^{-1} = \frac{1}{4}\begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}$

So, $A = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}\frac{1}{4}\begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}$

The above notes provides $A^{100} = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}^{100}\frac{1}{4}\begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}$

$$= \frac{1}{4}\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 5^{100} \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}$$

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CH5.2 9P

Step 1 of 5

(a) Consider the following:

$$\begin{aligned} A &= uv^{\mathrm{T}} \\ Au &= uv^{\mathrm{T}}u \\ &= u\left(v^{\mathrm{T}}u\right) \\ &= \left(v^{\mathrm{T}}u\right)u \end{aligned}$$

Thus, we have shown that $Au = \left(v^{\mathrm{T}}u\right)u$.

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Step 2 of 5

When u is a column and v is a row, $v^{\mathrm{T}}u$ is simply a number.

Thus, we have shown that Au is equal to a number multiplied by u. Therefore, u is an eigenvector.

Since, $Au = \left(v^{\mathrm{T}}u\right)u$, the corresponding eigenvalue is $\boxed{\lambda = v^{\mathrm{T}}u}$.

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Step 3 of 5

(b) Since $A = uv^{\mathrm{T}}$ is a rank 1 matrix, it has only one independent row. Therefore, it has only one nonzero pivot, rest all are zero pivots.

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Step 3 of 5

(b) Since $A = uv^{\mathrm{T}}$ is a rank 1 matrix, it has only one independent row. Therefore, it has only one nonzero pivot, rest all are zero pivots.

Therefore, the other eigenvalues of A are zeros.

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Step 4 of 5

(c) Let u and v be as follows: $u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}}$ and $v = (v_1, v_2, \ldots, v_n)^{\mathrm{T}}$.

Therefore, we get

$$\begin{aligned} A &= uv^{\mathrm{T}} \\ &= \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} (v_1, v_2, \ldots, v_n)^{\mathrm{T}} \\ &= \begin{bmatrix} u_1v_1 & u_1v_2 & \cdots & u_1v_n \\ u_2v_1 & u_2v_2 & \cdots & u_2v_n \\ \cdots & \cdots & \cdots & \cdots \\ u_nv_1 & u_nv_2 & \cdots & u_nv_n \end{bmatrix} \end{aligned}$$

Provide feedback (0)

$$= \begin{bmatrix} \vdots \\ u_n \end{bmatrix} (v_1, v_2, \ldots, v_n)$$

$$= \begin{bmatrix} u_1v_1 & u_1v_2 & \ldots & u_1v_n \\ u_2v_1 & u_2v_2 & \ldots & u_2v_n \\ \ldots & \ldots & \ldots & \ldots \\ u_nv_1 & u_nv_2 & \ldots & u_nv_n \end{bmatrix}$$

Provide feedback (0)

Step 5 of 5

Therefore, the trace (A) = $u_1v_1 + u_2v_2 + \ldots + u_nv_n$. Now consider $v^{\mathrm{T}}u$.

$$v^{\mathrm{T}}u = (v_1, v_2, \ldots, v_n)^{\mathrm{T}} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$$= v_1u_1 + v_2u_2 + \ldots + v_nu_n$$

$$= u_1v_1 + u_2v_2 + \ldots + u_nv_n$$

This is equal to sum of the eigenvalues.

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Step 1 of 1

Given that $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}, B=\begin{bmatrix} q & r \\ s & t \end{bmatrix}$

$$AB=\begin{bmatrix} aq+bs & ar+bt \\ cq+ds & cr+dt \end{bmatrix}$$

Trace of $AB = aq+bs+cr+dt$ ..... (1)

$$BA=\begin{bmatrix} q & r \\ s & t \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$=\begin{bmatrix} aq+rc & ab+rd \\ sa+tc & sb+td \end{bmatrix}$$

Trace of $BA = aq+rc+sb+bd$

While addition of real numbers is abelian and product is commutative, we get

$aq+sb+rc+td = aq+bs+cr+dt$ ..... (2)

from (1) and (2), we get trace of AB = trace of BA

Trace of $(AB-BA)=0$

On the other hand, we know that the trace of I_2 is 2.

From these, we can write $AB-BA \neq I$

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Step 1 of 4

If A is a square matrix, λ is an eigen value and x is the eigen vector, then we have $Ax = \lambda x$

Multiplying A on both sides, we see that

$$\begin{aligned} A^2x &= A(\lambda x) \\ &= \lambda(Ax) \\ &= \lambda(\lambda x) \\ &= \lambda^2 x \end{aligned}$$

So, the eigen value of A^2 is λ^2

Provide feedback (0)

Step 2 of 4

In view of this observation, while the eigen values of A are 1, 2, and 4, the eigen values of A^2 are 1, 4, and 16.

We know that the trace of a matrix is nothing but the sum of the eigen values of that matrix.

So, the trace of A^2 is $1 + 4 + 16 = 21$

Provide feedback (0)

Step 3 of 4

Further, we can write $Ax = \lambda x$ as $\frac{1}{\lambda}x = A^{-1}x$

From this, the eigen value of A^{-1} is $\frac{1}{\lambda}$ ..... (1)

Also, we follow that $|A - \lambda I| = |A^T - \lambda I|$

From this, the eigen values of a matrix and its transpose are one and the same. ..... (2)

Putting (1) and (2) together, we confirm that the eigen values of $(A^{-1})^T$ are $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}$

Provide feedback (0)

CH5.2 11P

Provide feedback (0)

Step 3 of 4

Further, we can write $Ax = \lambda x$ as $\frac{1}{\lambda}x = A^{-1}x$

From this, the eigen value of A^{-1} is $\frac{1}{\lambda}$ ..... (1)

Also, we follow that $|A - \lambda I| = |A^T - \lambda I|$

From this, the eigen values of a matrix and its transpose are one and the same. ..... (2)

Putting (1) and (2) together, we confirm that the eigen values of $\left(A^{-1}\right)^T$ are $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}$

Provide feedback (0)

Step 4 of 4

We know that the determinant of the matrix is the product of its eigen values

So, the determinant of $\left(A^{-1}\right)^T$ is $\frac{1}{1} \times \frac{1}{2} \times \frac{1}{4} = \boxed{\frac{1}{8}}$

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Step 1 of 3

Given that the eigen values of A are 1, 1, and 2

a) Suppose A has to be a matrix of order 3×3 with these eigen values.
While the determinant of A is the product of eigen values, we see that the determinant is $2 \neq 0$
So, A is non singular and thus invertible.

Provide feedback (0)

Step 2 of 3

b) The statement is false in at least one case.

For, $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

While A is a triangular matrix, the eigen values λ are nothing but the diagonal entries 1, 1, and 2 which are as given in the question.
But the eigen value 1 is repeated while its eigen vector is obtained by solving

$$(A-I)x = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = k, x_2 = x_3 = 0$ where k is any real number satisfies the system.

Therefore, the eigen vector is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

While the algebraic multiplicity of the eigen value 1 is 2 and its geometric multiplicity is the number of eigen vectors is 1, the inequality of these confirms that the non singular matrix S does not exist such that $S^{-1}AS = \Lambda$
Therefore, A is not diagonalizable.

Provide feedback (0)

Step 3 of 3

CH5.2 12P

Therefore, the eigen vector is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

While the algebraic multiplicity of the eigen value 1 is 2 and its geometric multiplicity is the number of eigen vectors is 1, the inequality of these confirms that the non singular matrix S does not exist such that $S^{-1}AS = \Lambda$

Therefore, A is not diagonalizable.

Provide feedback (0)

Step 3 of 3

c) In certain cases, the statement is true and in some other cases, it is false.

So, we consider $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

The eigen values are 1, 1, and 2

It is a diagonal matrix already.

So, it can be written as $S^{-1}AS = \Lambda$ where S is the identity matrix and the diagonal matrix Λ is A itself.

This example shows that in every matrix with eigen values 1, 1, and 2 is diagonalizable also.

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Step 1 of 4

Let the eigen vectors of matrix A are multiples of the following:

$$x = (1,0,0)$$

Then to answer the following are true or false.

Provide feedback (0)

Step 2 of 4

(a) Matrix A is not invertible.

$\boxed{\text{False}}$, as eigen values are not known.

Provide feedback (0)

Step 3 of 4

(b) Matrix A has repeated eigen values.

$\boxed{\text{True}}$, as eigen vectors are multiples of vector x.

Provide feedback (1)

Step 4 of 4

(c) Matrix A is not diagonalizable.

$\boxed{\text{True}}$, as (a) is false.

Provide feedback (1)

CH5.2 13P

CH5.2 12P

Step 1 of 5

Given that $A=\begin{bmatrix}5 & 4\\ 4 & 5\end{bmatrix}$

To find eigen values, we consider the characteristic equation $|A-\lambda I|=0$.

$$\Rightarrow\begin{vmatrix}5-\lambda & 4\\ 4 & 5-\lambda\end{vmatrix}=0$$

$$\Rightarrow(5-\lambda)^2-16=0$$

$$\Rightarrow\lambda^2+25-10\lambda-16=0$$

$$\Rightarrow\lambda^2-10\lambda+9=0$$

$$\Rightarrow(\lambda-1)(\lambda-9)=0$$

$$\Rightarrow\lambda=1,9$$

The eigen values are $\lambda=1,9$

Provide feedback (0)

Step 2 of 5

To find the eigen vector for $\lambda=1$, we solve $(A-I)(x)=0$

$$\Rightarrow\begin{bmatrix}4 & 4\\ 4 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

Applying row operations $R_2\to R_2-R_1, R_1/4\text{R}$, we get $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

The coefficient matrix is the reduced matrix and so, we rewrite the homogeneous equations from this.

$x_1+x_2=0$

Putting $x_1=1$, we get $x_2=-1$ and thus, the solution set is $\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}1\\ -1\end{bmatrix}$ is the eigen vector corresponding to $\lambda=1$

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CH5.2 14P

Provide feedback (0)

Step 3 of 5

Similarly, for the eigen value $\lambda = 9$, we solve $(A-9I)(x)=0$

$$\Rightarrow \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations and reducing it to the echelon form, the homogeneous equation is $x_1 - x_2 = 0$

Putting $x_1 = 1$, the eigen vector is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ corresponding to $\lambda = 9$

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Step 4 of 5

Using the eigen vectors as the columns of the matrix, we get $S = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ and

$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ such that

$$\begin{aligned} S^{-1}AS &= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ -1 & 9 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 18 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix} \\ &= \Lambda \end{aligned}$$

Provide feedback (0)

$$= \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$$
$$= \Lambda$$

Provide feedback (0)

Step 5 of 5

Now, we can write this equation as $A = S\Lambda S^{-1}$
Applying the n powers on both sides, it becomes $A^n = S\Lambda^n S^{-1}$ and observe Λ^n is the nth powers of the diagonal entries.

Putting $n = \frac{1}{2}$, we get $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}^{\frac{1}{2}} = S\Lambda^{\frac{1}{2}}S^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1^{1/2} & 0 \\ 0 & 9^{1/2} \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$= \frac{1}{2} \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$= \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$
$$= R$$

While $1^{1/2} = \pm 1, 9^{1/2} = \pm 3$, we follow that there are three more square roots to the given matrix.

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CH5.2 13P

Step 1 of 2

A is diagonalizable and is $A = S\Lambda S^{-1}$

Then we follow that $\left(S\Lambda S^{-1}\right)^T = \left(S^{-1}\right)^T \left(S\Lambda\right)^T$ by the properties of transposition of matrices

$$= \left(S^T\right)^T \left(\Lambda^T S^T\right) \text{ because } S^{-1} = S^T$$
$$= S\Lambda S^T$$
$$= S\Lambda S^{-1}$$

Thus, we have shown that $\left(S\Lambda S^{-1}\right)^T = S\Lambda S^{-1}$

So, $S\Lambda S^{-1}$ is symmetric.

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Step 2 of 2

Further, we consider $S^T S =$

$$= S^{-1}S \text{ while } S^T = S^{-1}$$
$$= I$$

So, we follow that $s_i^T s_j = 0 \forall i \neq j$ and $s_i^T s_i = 1 \forall i$ where s_i, s_j are the rows of the matrix S.

This says that the rows of the matrix S are pair wise orthogonal.

Therefore, S is an orthogonal matrix.

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Step 1 of 8

Given that $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$

The characteristic equation of this matrix is $|A-\lambda I|=0$

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & 2\\ 0 & 3-\lambda\end{vmatrix}\\ &= (1-\lambda)(3-\lambda)\\ &= 3-\lambda-3\lambda+\lambda^2\\ &= \lambda^2-4\lambda+3\end{aligned}$$

$\Rightarrow (\lambda-1)(\lambda-3)=0$

$\Rightarrow \lambda_1=1, \lambda_2=3$ are the eigen values of A.

Provide feedback (0)

Step 2 of 8

For $\lambda_1=1$, suppose x_1 is the vector such that $(A-\lambda_1 I)x_1=0$

$$\Rightarrow \begin{bmatrix}0 & 2\\ 0 & 2\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$$

Using the operation $R_2 \to R_2 - R_1$, we get $\begin{bmatrix}0 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$

Rewriting the homogeneous equations, we get $2t_2=0$

So, $t_2=0$

But any value of t_1 satisfies the system.

So, putting $t_1=k$ a parameter and conveniently putting $k=1$, the eigen vector corresponding to $\lambda_1=1$ is $x_1=\begin{bmatrix}t_1\\ t_2\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$

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Provide feedback (0)

Step 3 of 8

For $\lambda_2 = 3$, suppose x_2 is the vector such that $(A - \lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$$

Using the operation $R_1 / -2$, we get $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$

Rewriting the homogeneous equations, we get $t_1 - t_2 = 0$

So, $t_2 = t_1$

Putting $t_1 = 1$, the eigen vector corresponding to $\lambda_2 = 3$ is $x_2 = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Provide feedback (0)

Step 4 of 8

Using the eigen vectors as the columns of S, we get $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and so, $S^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Consequently, $A = S\Lambda S^{-1}$

i.e., $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Provide feedback (0)

Step 5 of 8

Given that $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

The characteristic equation of this matrix is $|A - \lambda I| = 0$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix}$$

Step 5 of 8

Given that $A=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$

The characteristic equation of this matrix is $|A-\lambda I|=0$

$$|A-\lambda I|=\begin{vmatrix}1-\lambda & 1\\ 2 & 2-\lambda\end{vmatrix}$$

$\Rightarrow \lambda^2-3\lambda=0$

$\Rightarrow \lambda_1=0, \lambda_2=3$ are the eigen values of A.

Provide feedback (0)

Step 6 of 8

For $\lambda_1=0$, suppose x_1 is the vector such that $(A-\lambda_1 I)x_1=0$

$$\Rightarrow \begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$$

Using the operation $R_2 \to R_2-2R_1$, we get $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$

Rewriting the homogeneous equations, we get $t_1+t_2=0$

Putting $t_1=1$, the eigen vector corresponding to $\lambda_1=0$ is $x_1=\begin{bmatrix}t_1\\ t_2\end{bmatrix}=\begin{bmatrix}1\\ -1\end{bmatrix}$

Provide feedback (0)

Step 7 of 8

For $\lambda_2=3$, suppose x_2 is the vector such that $(A-\lambda_2 I)x_2=0$

$$\Rightarrow \begin{bmatrix}-2 & 1\\ 2 & -1\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$$

Using the operation $R_2 \to R_2+R_1$,, we get $\begin{bmatrix}-2 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$

Rewriting the homogeneous equations, we get $2t_1-t_2=0$

So $t_2=2t_1$

For $\lambda_2 = 3$, suppose x_2 is the vector such that $(A - \lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$$

Using the operation $R_2 \to R_2 + R_1$,, we get $\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$

Rewriting the homogeneous equations, we get $2t_1 - t_2 = 0$

So, $t_2 = 2t_1$

Putting $t_1 = 1$, the eigen vector corresponding to $\lambda_2 = 3$ is $x_2 = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Provide feedback (0)

Step 8 of 8

Using the eigen vectors as the columns of S, we get $S = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ and so,

$$S^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

Consequently, $A = S\Lambda S^{-1}$

i.e., $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

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Step 1 of 2

Given that $A = S\Lambda S^{-1}$

$$\begin{aligned} A^3 &= \left(S\Lambda S^{-1}\right)\left(S\Lambda S^{-1}\right)\left(S\Lambda S^{-1}\right) \\ &= S\Lambda\left(S^{-1}S\right)\Lambda\left(S^{-1}S\right)\Lambda S^{-1} \\ &= S\Lambda(I)\Lambda(I)\Lambda S^{-1} \\ &= S\Lambda^3 S^{-1} \end{aligned}$$

Observe that Λ^3 is nothing but the diagonal matrix whose diagonal entries are the cubes of the eigen values of A.

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Step 2 of 2

$$\begin{aligned} A^{-1} &= \left(S\Lambda S^{-1}\right)^{-1} \\ &= \left(S^{-1}\right)^{-1}(S\Lambda)^{-1} \\ &= S\left(\Lambda^{-1}S^{-1}\right) \qquad \text{since } \left(s^{-1}\right)^{-1} = s \\ &= S\Lambda^{-1}S^{-1} \end{aligned}$$

Observe that the diagonal entries of the diagonal matrix Λ^{-1} are the reciprocals of the eigen values of A.

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Step 1 of 1

A has $\lambda_1 = 2$ with eigen vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = 5$ with eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then $\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ is the diagonal matrix whose diagonal entries are the eigen values of A

and $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ whose columns are the eigen vectors in the order of the eigen values of A

By the process of diagonalizations, we know that $A = S\Lambda S^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & 5 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}}$$

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Step 1 of 3

Let $A = S\Lambda S^{-1}$. Then to determine eigen value matrix for $A+2I$, and eigen vector matrix. Also, fill in the following blanks:

$$A+2I = (\ )(\ )(\ )^{-1}$$

Provide feedback (0)

Step 2 of 3

To find the eigen value matrix do the following calculations:

$$A = S\Lambda S^{-1}$$
$$\Lambda = S^{-1}AS$$

Now, for $A+2I$ the eigen value matrix will be as follows:

$$\begin{aligned} S^{-1}(A+2I)S &= \left(S^{-1}A+2S^{-1}I\right)S \\ &= S^{-1}AS+2S^{-1}S \\ &= \Lambda+2I \end{aligned}$$

Therefore, eigen value matrix for $A+2I$ is: $\boxed{\Lambda+2I}$, and eigen vector matrix will be $\boxed{S}$

Provide feedback (1)

Step 3 of 3

Also, $A+2I$ can be written as follows:

$$\boxed{A+2I = (S)(\Lambda+2I)(S)^{-1}}$$

If the above relation is checked by substituting all the values, it is found to be true.

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Step 1 of 4

Given that n columns of S (eigen vectors of A) are independent.

a) We can see that the eigen vector corresponding to the eigen value $\lambda = 0$ is also non zero and can be linearly independent with the other eigen vectors of A.
So, the matrix S whose columns are eigen vectors of A may be invertible.
But due to the zero eigen value of A, we follow that the determinant of A is the product of eigen values of $A = 0$
So, A is not invertible.
Therefore, the linear independence of eigen vectors cannot confirm the invertibility of A.

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Step 2 of 4

b) If the eigen vectors of A are the linearly independent columns of the matrix S, then S is non singular and so invertible such that $S^{-1}AS = \Lambda$ the diagonal matrix whose diagonal entries are the eigen values of A.
Thus, A is diagonalizable.
So, the given statement is true.

Provide feedback (0)

Step 3 of 4

c) A is diagonalizable then the diagonal matrix Λ exists such that
$\Lambda = SAS^{-1}$
Hence S^{-1} is also exists.
Therefore S is invertible.

Provide feedback (0)

Step 4 of 4

d) If S is a matrix whose columns are linearly independent, then S is non singular.
This confirms that the eigen values of S are non zero.
But it is not necessary that the sum of the algebraic multiplicities of the eigen values is

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Therefore S is invertible.

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Step 4 of 4

d) If S is a matrix whose columns are linearly independent, then S is non singular.
This confirms that the eigen values of S are non zero.
But it is not necessary that the sum of the algebraic multiplicities of the eigen values is equal to the sum of the geometric multiplicities.
So, S is not necessarily invertible.

So, the statement is false.

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Step 1 of 1

Given that eigen vectors of A are the columns of I.

Therefore $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and S^{-1} exists such that $S^{-1}AS = \Lambda$ can be constructed.

So, A is diagonalizable.

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CH5.2 21P

CH5.2 20P

Step 1 of 7

Given that $A=\begin{bmatrix}4 & 0\\ 1 & 2\end{bmatrix}$

To find the eigen values of A, we consider $|A-\lambda I|=0$

$$\begin{vmatrix}4-\lambda & 0\\ 1 & 2-\lambda\end{vmatrix}=0$$

$$(4-\lambda)(2-\lambda)-0=0$$

$$\Rightarrow \lambda=2,4$$

The eigen values are $\lambda=2$ and 4

Provide feedback (0)

Step 2 of 7

To find the eigen vector corresponding to $\lambda=2$, we solve $(A-2I)x=0$

$$\Rightarrow \begin{bmatrix}2 & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

Using the row operation $R_2 \to 2R_2-R_1,\ R_1/2$ on the coefficient matrix, we get

$$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

This is the reduced matrix and so, rewriting the homogeneous equations from this, we get

$x_1=0$

So, any real number $k=x_2$ satisfies the system.

Therefore, putting $k=1$, the solution set $X_1=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 1\end{bmatrix}$ is the eigen vector of $\lambda=2$

Provide feedback (0)

Step 3 of 7

Similarly, for $\lambda=4$, we solve $(A-4I)x=0$

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Therefore, putting $\lambda = 1$, the solution set $X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the eigen vector of $\lambda = 2$

Provide feedback (0)

Step 3 of 7

Similarly, for $\lambda = 4$, we solve $(A - 4I)x = 0$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using $R_2 \leftrightarrow R_1$ on the coefficient matrix, we get $\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix and so, the homogeneous equation is $x_1 - 2x_2 = 0$

Putting $x_2 = 1$, we get $x_1 = 2$

So, $X_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 4$

Provide feedback (0)

Step 4 of 7

While the eigen vectors corresponding to distinct eigen values are linearly independent, and so, the matrix S whose columns are the eigen vectors of distinct eigen values is non singular and so invertible.

Writing $S = [X_1 X_2] = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$, we get $S^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}$ such that

$$\begin{aligned} S^{-1}AS &= \frac{1}{2}\begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} -2 & 4 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \\ &= \Lambda \text{ the diagonal matrix.} \qquad \ldots\ldots (1) \end{aligned}$$

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Step 5 of 7

Similarly, writing $S=\left[X_2 X_1\right]=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$, we get $S^{-1}AS=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}=\Lambda$ the diagonal matrix.

..... (2)

(1) and (2) are the two ways in which we can diagonalize the given matrix A.

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Step 6 of 7

We can write $A=S\Lambda S^{-1}$

Applying the nth powers on both sides, we get $A^n=S\Lambda^n S^{-1}$ where Λ^n is the diagonal matrix whose diagonal entries are the nth powers of the eigen values of A.

Putting $n=-1$, we get $A^{-1}=S\begin{bmatrix}\frac{1}{\lambda_1} & 0\\ 0 & \frac{1}{\lambda_2}\end{bmatrix}S^{-1}$

Provide feedback (0)

Step 7 of 7

Using $\lambda_1=2, \lambda_2=4$, we get $S^{-1}A^{-1}S=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{4}\end{bmatrix}=\Lambda$ and so,

$A^{-1}=\frac{1}{2}\begin{bmatrix}0 & 2\\ 1 & 1\end{bmatrix}\begin{bmatrix}1/2 & 0\\ 0 & 1/4\end{bmatrix}\begin{bmatrix}-1 & 2\\ 1 & 0\end{bmatrix}$ and using $\lambda_2=2, \lambda_1=4$, we get

$A^{-1}=\frac{1}{2}\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1/4 & 0\\ 0 & 1/2\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$ are the two ways possible.

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CH5.2 21P

Step 1 of 1

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be the matrix whose eigen vectors are $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Suppose λ_1 and λ_2 are the eigen values to which these are the respective eigen vectors.

Then $S=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $S^{-1}=\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ and $\Lambda=\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ such that

$$\begin{aligned} A &= S\Lambda S^{-1} \\ &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1 & -\lambda_2 \\ \lambda_1 & \lambda_2 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} \lambda_1+\lambda_2 & \lambda_1-\lambda_2 \\ \lambda_1-\lambda_2 & \lambda_1+\lambda_2 \end{bmatrix} \end{aligned}$$

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Step 1 of 10

Given $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $A+B=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

Now $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{bmatrix}\end{aligned}$$

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CH5.2 24P

Step 2 of 10

Then

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{vmatrix}\\ &= (1-\lambda)^2\\ &= 1+\lambda^2-2\lambda\end{aligned}$$

Provide feedback (0)

Step 3 of 10

Becomes

$$\begin{aligned}\lambda^2-2\lambda+1&=0\\ \lambda^2-\lambda-\lambda+1&=0\\ \lambda(\lambda-1)-1(\lambda-1)&=0\\ (\lambda-1)(\lambda-1)&=0\end{aligned}$$

Step 3 of 10

Becomes

$$\lambda^2 - 2\lambda + 1 = 0$$
$$\lambda^2 - \lambda - \lambda + 1 = 0$$
$$\lambda(\lambda - 1) - 1(\lambda - 1) = 0$$
$$(\lambda - 1)(\lambda - 1) = 0$$
$$\lambda = 1, 1$$

That is $\lambda_1 = 1, \lambda_2 = 1$

Therefore the Eigen values of A is 1, 1

Provide feedback (0)

Step 4 of 10

Now $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$
$$= \begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}$$

Provide feedback (0)

Step 5 of 10

Then

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix}$$
$$= (1-\lambda)^2$$
$$= 1 + \lambda^2 - 2\lambda$$

Provide feedback (0)

Then

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix}$$
$$= (1-\lambda)^2$$
$$= 1+\lambda^2-2\lambda$$

Provide feedback (0)

Step 6 of 10

Becomes

$$\lambda^2 - 2\lambda + 1 = 0$$
$$\lambda^2 - \lambda - \lambda + 1 = 0$$
$$\lambda(\lambda-1) - 1(\lambda-1) = 0$$
$$(\lambda-1)(\lambda-1) = 0$$
$$\lambda = 1, 1$$

That is $\lambda_1 = 1, \lambda_2 = 1$

Therefore the Eigen values of B is 1, 1

Provide feedback (0)

Step 7 of 10

Now $A+B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$
$$= \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

Provide feedback (0)

Therefore the Eigen values of B is 1, 1

Provide feedback (0)

Step 7 of 10

Now $A+B=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{bmatrix}\end{aligned}$$

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Step 8 of 10

Then

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{vmatrix}\\ &= (2-\lambda)(2-\lambda)-1\\ &= 4-2\lambda-2\lambda+\lambda^2-1\\ &= \lambda^2-4\lambda+3\end{aligned}$$

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Step 9 of 10

$$\begin{aligned}\lambda^2-4\lambda+3&=0\\ \lambda^2-3\lambda-\lambda+3&=0\\ \lambda(\lambda-3)-1(\lambda-3)&=0\\ (\lambda-3)(\lambda-1)&=0\end{aligned}$$

That is, $\lambda_1=3$, $\lambda_2=1$

Step 9 of 10

$$\lambda^2 - 4\lambda + 3 = 0$$
$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$
$$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$$
$$(\lambda - 3)(\lambda - 1) = 0$$

That is $\lambda_1 = 3, \lambda_2 = 1$

Therefore the Eigen values of $A + B$ is 3, 1

Eigen values of $A + B$ are not equal to Eigen values of A plus Eigen values of B.

Because here the Eigen values of $A + B$ is 3, 1

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Step 10 of 10

Which are not equal to the Eigen values of A is 1, 1 plus the Eigen values of B is 1, 1

Here Eigen values of A is equal to the Eigen values of B

But Eigen values of $A + B$ are not equal to Eigen values of A plus Eigen values of B.

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Step 1 of 13

Given $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ $AB=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$ and $BA=\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$

Now $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{bmatrix}\end{aligned}$$

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Step 2 of 13

Then

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{vmatrix}\\ &= (1-\lambda)^2\\ &= 1+\lambda^2-2\lambda\end{aligned}$$

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Step 3 of 13

Becomes

$$\begin{aligned}\lambda^2-2\lambda+1&=0\\ \lambda^2-\lambda-\lambda+1&=0\\ \lambda(\lambda-1)-1(\lambda-1)&=0\\ (\lambda-1)(\lambda-1)&=0\end{aligned}$$

Step 3 of 13

Becomes

$$\lambda^2 - 2\lambda + 1 = 0$$
$$\lambda^2 - \lambda - \lambda + 1 = 0$$
$$\lambda(\lambda - 1) - 1(\lambda - 1) = 0$$
$$(\lambda - 1)(\lambda - 1) = 0$$
$$\lambda = 1, 1$$

Therefore the eigenvalues of A is 1, 1

Provide feedback (0)

Step 4 of 13

Now $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$
$$= \begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}$$

Provide feedback (0)

Step 5 of 13

Then

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix}$$
$$= (1-\lambda)^2$$
$$= 1 + \lambda^2 - 2\lambda$$

Provide feedback (0)

Provide feedback (0)

Step 6 of 13

Becomes

$$\begin{aligned}\lambda^2-2\lambda+1&=0\\ \lambda^2-\lambda-\lambda+1&=0\\ \lambda(\lambda-1)-1(\lambda-1)&=0\\ (\lambda-1)(\lambda-1)&=0\\ \lambda&=1,1\end{aligned}$$

Therefore the eigenvalues of B is 1, 1

Provide feedback (0)

Step 7 of 13

Now $AB=\begin{bmatrix}1&1\\1&2\end{bmatrix}$

$$\begin{aligned}A-\lambda I&=\begin{bmatrix}1&1\\1&2\end{bmatrix}-\lambda\begin{bmatrix}1&0\\0&1\end{bmatrix}\\ &=\begin{bmatrix}1&1\\1&2\end{bmatrix}-\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}\\ &=\begin{bmatrix}1-\lambda&1\\1&2-\lambda\end{bmatrix}\end{aligned}$$

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Step 8 of 13

Then

$$\begin{aligned}|A-\lambda I|&=\begin{vmatrix}1-\lambda&1\\1&2-\lambda\end{vmatrix}\\ &=(1-\lambda)(2-\lambda)-1\end{aligned}$$

Step 8 of 13

Then

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$
$$= (1-\lambda)(2-\lambda)-1$$
$$= 2-\lambda-2\lambda+\lambda^2-1$$
$$= \lambda^2-3\lambda+1$$

$\lambda^2-3\lambda+1=0$

This is in the form of $ax^2+bx+c=0$

Then

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Provide feedback (0)

Step 9 of 13

Now

$$\lambda = \frac{3\pm\sqrt{9-4}}{2}$$
$$= \frac{3\pm\sqrt{5}}{2}$$

Therefore the eigenvalues of AB is $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

Provide feedback (0)

Step 10 of 13

Now $BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 10 of 13

Now $BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} \end{aligned}$$

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Step 11 of 13

Now

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} \\ &= (2-\lambda)(1-\lambda) - 1 \\ &= 2 - \lambda - 2\lambda + \lambda^2 - 1 \\ &= \lambda^2 - 3\lambda + 1 \end{aligned}$$

$\lambda^2 - 3\lambda + 1 = 0$

Provide feedback (0)

Step 12 of 13

This is in the form of $ax^2 + bx + c = 0$
Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now

$3 + \sqrt{9-4}$

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix}$$
$$= (2-\lambda)(1-\lambda)-1$$
$$= 2-\lambda-2\lambda+\lambda^2-1$$
$$= \lambda^2-3\lambda+1$$
$$\lambda^2-3\lambda+1=0$$

Provide feedback (0)

Step 12 of 13

This is in the form of $ax^2+bx+c=0$
Then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Now

$$\lambda = \frac{3 \pm \sqrt{9-4}}{2}$$
$$= \frac{3 \pm \sqrt{5}}{2}$$

Therefore the eigenvalues of BA is $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

Provide feedback (0)

Step 13 of 13

Eigen values of AB are not equal to eigenvalues of A times Eigen values of B
Eigenvlaues of AB are equal to eigenvalues of BA

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Step 1 of 3

Given that the matrix A has eigen values 2, 2, and 5

a) The statement is true.

The det A = product of eigen values

$$= (2)(2)(5)$$

$$= 20 \neq 0$$

Therefore A is invertiable.

Provide feedback (0)

Step 2 of 3

b) We observe that the algebraic multiplicity of the eigen value 2 is 2 (number of times repeated)

But we don't know whether the corresponding eigen vectors are also two in number (geometric multiplicity).

So, A is diagonalizable if and only if the sum of the algebraic multiplicities is equal to the sum of the geometric multiplicities.

In our case, we have no confirmation about the geometric multiplicity of the eigen value 2. So, we cannot confirm that A is diagonalizable.

Therefore, the statement is false.

Provide feedback (0)

Step 3 of 3

c) If the homogeneous equation from $(A-\lambda I)x=0$ when the eigen value $\lambda=2$ is substituted is $x+y+z=0$, then the solution set is $x=-k-m$ where $y=k, z=m$ are the parameters.

So, it can be written as $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + m\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Putting $k=1, m=1$, the corresponding eigen vectors for $\lambda=2$ are $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

CH5.2 26P

parameters.

So, it can be written as $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + m\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Putting $k = 1, m = 1$, the corresponding eigen vectors for $\lambda = 2$ are $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

So, the geometric multiplicity of the eigen value $\lambda = 2$ is 2.
While the algebraic multiplicity = geometric multiplicity, we can see the given matrix A is diagonalizable.
So, the statement is false.

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Step 1 of 3

Given that the eigen values of A are 1 and 0.

Trace of A = sum of eigen values

$$= 1 + 0 = 1$$

$\det A =$ Product of eigen values

$$= 1(0)$$

A is singular matrix.

Therefore A is not invertiable.

Provide feedback (0)

Step 2 of 3

A has distinct eigen values. So, the eigen vectors corresponding to the distinct eigen values are linearly independent.

So, the matrix S whose columns are the eigen vectors of A is non singular and so,

S^{-1} exists such that $S^{-1}AS = \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ the diagonal matrix.

Therefore, A is diagonalizable.

Further, $A^n = S\Lambda^n S^{-1}$ and Λ^n is the diagonal matrix whose diagonal entries are the nth powers of the eigen values 1 and 0.

So, $A^n = S\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}S^{-1}$ for every positive integer n.

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Step 3 of 3

While the determinant of A is zero, it is not invertible.

So, $A^n = S\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}S^{-1}$ does not hold for other than positive integers.

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Step 1 of 8

We consider $A = \begin{bmatrix} 8 & x \\ y & 2 \end{bmatrix}$

Given $\det A = 25$

$\Rightarrow 16 - xy = 25$

$\Rightarrow xy = -9$ where x and y are integers.

The possible factors are $x = -1, y = 9$ or $x = -9, y = 1$, or $x = 1, y = -9$, or $x = -3, y = 3$ or $x = 3, y = -3$.

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Step 2 of 8

Given that the eigen value $\lambda = 5$ is repeated.

We easily see that $|A - \lambda I| = \begin{vmatrix} 8-\lambda & x \\ y & 2-\lambda \end{vmatrix} = (\lambda - 5)^2$ is satisfied by all the possible pairs of x and y obtained above.

(1) when $x = -1, y = 9$, we have $A = \begin{bmatrix} 2 & -1 \\ 9 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & -1 \\ 9 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & -1 \\ 9 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ respectively.

Provide feedback (0)

Step 3 of 8

(2) When $x = 1, y = -9$, we have $A = \begin{bmatrix} 2 & 1 \\ -9 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & 1 \\ -9 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

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Step 3 of 8

(2) When $x = 1, y = -9$, we have $A = \begin{bmatrix} 2 & 1 \\ -9 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & 1 \\ -9 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & 1 \\ -9 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ respectively.

Provide feedback (0)

Step 4 of 8

(3) When $x = 9, y = -1$, we have $A = \begin{bmatrix} 2 & 9 \\ -1 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & 9 \\ -1 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ respectively.

Provide feedback (0)

Step 5 of 8

(4) When $x = -9, y = 1$, we have $A = \begin{bmatrix} 2 & -9 \\ 1 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & -9 \\ 1 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & -9 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & -9 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems
$\begin{bmatrix} 3 \end{bmatrix}$ or $\begin{bmatrix} 3 \end{bmatrix}$

respectively.

Provide feedback (0)

Step 5 of 8

(4) When $x = -9, y = 1$, we have $A = \begin{bmatrix} 2 & -9 \\ 1 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & -9 \\ 1 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & -9 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & -9 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ respectively.

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Step 6 of 8

(5) When $x = -3, y = 3$, we have $A = \begin{bmatrix} 2 & -3 \\ 3 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & -3 \\ 3 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & -3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & -3 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ respectively.

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Step 7 of 8

(6) When $x = 3, y = -3$, we have $A = \begin{bmatrix} 2 & 3 \\ -3 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & 3 \\ -3 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} -3 & -3 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} \\ -1 \end{bmatrix}$ or $\begin{bmatrix} \\ 1 \end{bmatrix}$ respectively.

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Step 7 of 8

(6) When $x = 3, y = -3$, we have $A = \begin{bmatrix} 2 & 3 \\ -3 & 8 \end{bmatrix}$ or $A = \begin{bmatrix} 8 & 3 \\ -3 & 2 \end{bmatrix}$

So, $Ax = \lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} -3 & -3 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ respectively.

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Step 8 of 8

In all the above cases, we see that for the eigen value 5, there is only one eigen vector available.
But the eigen value repeated two times say that the algebraic multiplicity not equal to the geometric multiplicity.
Therefore, A is not diagonalizable.

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Step 1 of 2

Given that $A=\begin{bmatrix}3 & 1\\ 0 & 3\end{bmatrix}$

$$A-3I=\begin{bmatrix}3 & 1\\ 0 & 3\end{bmatrix}-3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$

The determinant of this 2×2 matrix is zero.
So, rank $A-3I<2$
But we know that rank of the matrix with at least one non zero entry is greater than or equal to 1.
So, putting the above statements together, we get rank $A-3I=1$.

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Step 2 of 2

Changing any entry 0 in the given matrix, it can be diagonalizable.
For instance, $A=\begin{bmatrix}3 & 1\\ 16 & 3\end{bmatrix}$ has the eigen values -1 and 7 which are distinct and so, the corresponding eigen vectors will be linearly independent and so, it is diagonalizable.

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CH5.2 30P

Step 1 of 8

Given that $A=\begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}$

To find λ value take $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} .6-\lambda & .4 \\ .4 & .6-\lambda \end{vmatrix}=0$$

$$\Rightarrow (.6-\lambda)^2-.16=0$$

$$\Rightarrow \lambda^2-1.2\lambda+.36-.16=0$$

$$\Rightarrow \lambda^2-\frac{12}{10}\lambda+\frac{2}{10}=0$$

$$\Rightarrow 10\lambda^2-12\lambda+2=0$$

$$\Rightarrow 10\lambda^2-10\lambda-2\lambda+2=0$$

$$\Rightarrow 10\lambda(\lambda-1)-2(\lambda-1)=0$$

$$\Rightarrow (10\lambda-2)(\lambda-1)=0$$

$$\Rightarrow \lambda=\frac{1}{5},1$$

Provide feedback (0)

Step 2 of 8

To find eigen vector $\lambda=\frac{1}{5}$ take $\left(A-\frac{1}{5}I\right)x=0$

$$\Rightarrow \begin{bmatrix} \frac{6}{10}-\frac{1}{5} & \frac{4}{10} \\ \frac{4}{10} & \frac{6}{10}-\frac{1}{5} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient

Step 2 of 8

To find eigen vector $\lambda = \frac{1}{5}$ take $\left(A - \frac{1}{5}I\right)x = 0$

$$\Rightarrow \begin{bmatrix} \frac{6}{10} - \frac{1}{5} & \frac{4}{10} \\ \frac{4}{10} & \frac{6}{10} - \frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix, $R_2 \to R_2 - R_1, R_1 / 0.4 \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix, so, the homogeneous equation from this is $x_1 + x_2 = 0$

Putting $x_1 = 1$, the solution set is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to the eigen value $\lambda = \frac{1}{5}$

Provide feedback (0)

Step 3 of 8

Similarly, when $\lambda = 1$, $(A - \lambda I)x = 0$ is $\begin{bmatrix} -0.4 & 0.4 \\ 0.4 & -0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, proceeding as above, the eigen vector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

While the eigen values are distinct, the corresponding eigen vectors are linearly independent and so, matrix S whose columns are eigen vectors is non singular and thus

$$S^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$A = S\Lambda S^{-1}$ where $\Lambda = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$ whose diagonal entries are the eigen values of A.

So, $A^k = S\Lambda^k S^{-1}$

While the eigen values are distinct, the corresponding eigen vectors are linearly independent and so, matrix S whose columns are eigen vectors is non singular and thus

$$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$A = S\Lambda S^{-1}$ where $\Lambda = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$ whose diagonal entries are the eigen values of A.

So, $A^k = S\Lambda^k S^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0.2^k & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

As k approaches ∞, we get $A^k = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This shows that A^k does not approach 0 as k approaches ∞

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Step 4 of 8

$$B = \begin{bmatrix} .6 & .9 \\ .9 & .6 \end{bmatrix}$$

The characteristic equation is $(\lambda - 0.6)^2 - 0.09 = 0$

$\Rightarrow \lambda^2 - 1.2\lambda + 0.27 = 0$

$\Rightarrow \lambda^2 - 0.9\lambda - 0.3\lambda + 0.27 = 0$

$\Rightarrow (\lambda - 0.9)(\lambda - 0.3) = 0$

$\Rightarrow \lambda_1 = 0.3, \lambda_2 = 0.9$

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Step 5 of 8

Step 5 of 8

The eigen vector corresponding to $\lambda_1 = 0.3$ is obtained by solving $(B - \lambda_1 I)x = 0$

$$\Rightarrow \begin{bmatrix} 0.3 & 0.9 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

After the application of row operations on the coefficient matrix, it reduces to

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the eigen vector as above is $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ by putting $x_2 = -1$

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Step 6 of 8

Similarly, the eigen vector corresponding to $\lambda_2 = 0.9$ is obtained by solving $(B - \lambda_2 I)x = 0$

$$\Rightarrow \begin{bmatrix} -0.3 & 0.9 \\ 0.1 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the solution $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is the eigen vector corresponding.

Provide feedback (0)

Step 7 of 8

So, $B = S\Lambda S^{-1}$ where $S = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.9 \end{bmatrix}, S^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}$

Applying the kth power on both sides, it becomes $B^k = S\begin{bmatrix} (0.3)^k & 0 \\ 0 & (0.9)^k \end{bmatrix}S^{-1}$

Allowing k approach infinity, it becomes $= S\begin{bmatrix} 0 & 0 \end{bmatrix}S^{-1} = 0$

Provide feedback (0)

Step 7 of 8

So, $B = S\Lambda S^{-1}$ where $S = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.9 \end{bmatrix}, S^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}$

Applying the kth power on both sides, it becomes $B^k = S\begin{bmatrix} (0.3)^k & 0 \\ 0 & (0.9)^k \end{bmatrix}S^{-1}$

Allowing k approach infinity, it becomes $= S\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}S^{-1} = 0$

Provide feedback (0)

Step 8 of 8

We now take up the actual question in view of the above results, that $A^k = S\Lambda^k S^{-1}$ approaches zero as $k \to \infty$ if and only if every λ has absolute value less than $\boxed{1}$.

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Step 1 of 4

Given that $A=\begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}$

To find λ value take $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} .6-\lambda & .4 \\ .4 & .6-\lambda \end{vmatrix}=0$$

$$\Rightarrow (.6-\lambda)^2-.16=0$$

$$\Rightarrow \lambda^2-1.2\lambda+.36-.16=0$$

$$\Rightarrow \lambda^2-\frac{12}{10}\lambda+\frac{2}{10}=0$$

$$\Rightarrow 10\lambda^2-12\lambda+2=0$$

$$\Rightarrow 10\lambda^2-10\lambda-2\lambda+2=0$$

$$\Rightarrow 10\lambda(\lambda-1)-2(\lambda-1)=0$$

$$\Rightarrow (10\lambda-2)(\lambda-1)=0$$

$$\Rightarrow \lambda=\frac{1}{5},1$$

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Step 2 of 4

To find eigen vector $\lambda=\frac{1}{5}$ take $\left(A-\frac{1}{5}I\right)x=0$

$$\Rightarrow \begin{bmatrix} \frac{6}{10}-\frac{1}{5} & \frac{4}{10} \\ \frac{4}{10} & \frac{6}{10}-\frac{1}{5} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient

Step 2 of 4

To find eigen vector $\lambda = \frac{1}{5}$ take $\left(A - \frac{1}{5}I\right)x = 0$

$$\Rightarrow \begin{bmatrix} \frac{6}{10} - \frac{1}{5} & \frac{4}{10} \\ \frac{4}{10} & \frac{6}{10} - \frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix, $R_2 \to R_2 - R_1, R_1 / 0.4 \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix, so, the homogeneous equation from this is $x_1 + x_2 = 0$

Putting $x_1 = 1$, the solution set is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to the eigen value $\lambda = \frac{1}{5}$

Provide feedback (0)

Step 3 of 4

Similarly, when $\lambda = 1$, $(A - \lambda I)x = 0$ is $\begin{bmatrix} -0.4 & 0.4 \\ 0.4 & -0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, proceeding as above, the eigen vector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

While the eigen values are distinct, the corresponding eigen vectors are linearly independent and so, matrix S whose columns are eigen vectors is non singular and thus

$$S^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$A = S\Lambda S^{-1}$ where $\Lambda = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$ whose diagonal entries are the eigen values of A.

So, $A^k = S\Lambda^k S^{-1}$

Similarly, when $\lambda = 1$, $(A - \lambda I)x = 0$ is $\begin{bmatrix} -0.4 & 0.4 \\ 0.4 & -0.4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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$A = S\Lambda S^{-1}$ where $\Lambda = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$ whose diagonal entries are the eigen values of A.

So, $A^k = S\Lambda^k S^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0.2^k & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

As k approaches ∞, we get $A^k = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This shows that A^k does not approach 0 as k approaches ∞

Provide feedback (0)

Step 4 of 4

The limiting matrix $\Lambda^k = \begin{bmatrix} 0.2^k & 0 \\ 0 & 1^k \end{bmatrix}$, we see that the columns become 0 if the absolute value of the eigen value less than 1, one of the entries is 1 if the eigen value is 1 and one of the entries tend to infinity if the eigen value is greater than 1.

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Step 1 of 7

$$B=\begin{bmatrix}.6 & .9\\ .1 & .6\end{bmatrix}$$

The characteristic equation is $(\lambda-0.6)^2-0.09=0$

$$\Rightarrow \lambda^2-1.2\lambda+0.27=0$$
$$\Rightarrow \lambda^2-0.9\lambda-0.3\lambda+0.27=0$$
$$\Rightarrow (\lambda-0.9)(\lambda-0.3)=0$$
$$\Rightarrow \lambda_1=0.3, \lambda_2=0.9$$

Provide feedback (0)

Step 2 of 7

The eigen vector corresponding to $\lambda_1=0.3$ is obtained by solving $(B-\lambda_1 I)x=0$

$$\Rightarrow \begin{bmatrix}0.3 & 0.9\\ 0.1 & 0.3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

After the application of row operations on the coefficient matrix, it reduces to

$$\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

So, the eigen vector as above is $\begin{bmatrix}3\\ -1\end{bmatrix}$ by putting $x_2=-1$

Provide feedback (0)

Step 3 of 7

Similarly, the eigen vector corresponding to $\lambda_2=0.9$ is obtained by solving

$(B-\lambda_2 I)x=0$

$$\Rightarrow \begin{bmatrix}-0.3 & 0.9\\ 0.1 & -0.3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

CH5.2 32P

So, the eigen vector as above is $\begin{bmatrix} \\ -1 \end{bmatrix}$ by putting $x_2 = -1$

Provide feedback (0)

Step 3 of 7

Similarly, the eigen vector corresponding to $\lambda_2 = 0.9$ is obtained by solving

$(B - \lambda_2 I)x = 0$

$$\Rightarrow \begin{bmatrix} -0.3 & 0.9 \\ 0.1 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the solution $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is the eigen vector corresponding.

Provide feedback (0)

Step 4 of 7

So, $B = S\Lambda S^{-1}$ where $S = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.9 \end{bmatrix}, S^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}$

Applying the kth power on both sides, it becomes $B^k = S\begin{bmatrix} (0.3)^k & 0 \\ 0 & (0.9)^k \end{bmatrix}S^{-1}$

Putting $k = 10$, we get $B^{10} = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} (0.3)^{10} & 0 \\ 0 & (0.9)^{10} \end{bmatrix}\frac{1}{6}\begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}$

$$= \frac{1}{6}\begin{bmatrix} 3\times 0.3^{10} & 3\times 0.9^{10} \\ -1\times 0.3^{10} & 0.9^{10} \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 3(0.3^{10} + 0.9^{10}) & 9(0.9^{10} - 0.3^{10}) \\ 0.9^{10} - 0.3^{10} & 3(0.9^{10} + 0.3^{10}) \end{bmatrix}$$

$$= \frac{3^{10}}{6}\begin{bmatrix} 3 & -9 \\ -1 & 3 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$$

Provide feedback (0)

Provide feedback (0)

Step 5 of 7

Using B^{10}, we find the products $B^{10}u_0$

(1) $u_0 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, then $B^{10}u_0$

$$= \left\{ \frac{3^{10}}{6}\begin{bmatrix} 3 & -9 \\ -1 & 3 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} \right\}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{3^{10}}{6}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

$$= 0.9^{10}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

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Step 6 of 7

(2) $u_0 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, then $B^{10}u_0$

$$= \left\{ \frac{3^{10}}{6}\begin{bmatrix} 3 & -9 \\ -1 & 3 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} \right\}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$= \frac{3^{10}}{6}\begin{bmatrix} 18 \\ -6 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= 0.3^{10}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 7

$$= 0.3^{10}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Provide feedback (0)

Step 7 of 7

(3) $u_0 = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$, then $B^{10}u_0$

$$= \left\{\frac{3^{10}}{6}\begin{bmatrix} 3 & -9 \\ -1 & 3 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}\right\}\begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$= \frac{3^{10}}{6}\begin{bmatrix} 18 \\ -6 \end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

$$= 0.3^{10}\begin{bmatrix} 3 \\ -1 \end{bmatrix} + 0.9^{10}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

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Step 1 of 3

Let $A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

The characteristic equation of A is $|A-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{vmatrix}=0$$

$$(2-\lambda)^2-1=0$$

$$\lambda^2-4\lambda+3=0$$

$$(\lambda-1)(\lambda-3)=0$$

$$\lambda=1,3$$

The eigen values are $\lambda_1=1,\lambda_2=3$

Provide feedback (0)

Step 2 of 3

The eigen vector corresponding to $\lambda_1=1$ is obtained by solving $(A-\lambda_1 I)x=0$

$$\Rightarrow\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

After reducing the coefficient matrix, we rewrite the homogeneous equation $x_1+x_2=0$

So, the solution set $X_1=\begin{bmatrix}1\\ -1\end{bmatrix}$ is the eigen vector corresponding to $\lambda_1=1$.

Similarly, solving $(A-\lambda_2 I)x=0$, we get $X_2=\begin{bmatrix}1\\ 1\end{bmatrix}$, the eigen vector corresponding to $\lambda_2=3$

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Step 3 of 3

The eigen vectors corresponding to the distinct eigen values are linearly independent and

CH5.2 33P

After reducing the coefficient matrix, we rewrite the homogeneous equation $x_1 + x_2 = 0$

So, the solution set $X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_1 = 1$.

Similarly, solving $(A - \lambda_2 I)x = 0$, we get $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, the eigen vector corresponding to $\lambda_2 = 3$

Provide feedback (0)

Step 3 of 3

The eigen vectors corresponding to the distinct eigen values are linearly independent and so, the matrix S whose columns are these eigen vectors is non singular.

Therefore, $S = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ such that $A = S\Lambda S^{-1}$

Further, $A^k = S\Lambda^k S^{-1}$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3^k \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 3^k \\ -1 & -3^k \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1+3^k & 3^k-1 \\ -1-3^k & 1-3^k \end{bmatrix}$$

By interchanging X_1 and X_2 in S, we get the stipulated form given.

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Step 1 of 3

Given that $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

The characteristic equation is $|B - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(2-\lambda) = 0$$

$$\Rightarrow \lambda_1 = 2, \lambda_2 = 3$$

Provide feedback (0)

Step 2 of 3

To get the eigen vector corresponding to $\lambda_1 = 2$, we solve $(\mathrm{B} - \lambda_1 \mathrm{I})x = 0$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the solution set of $x_1 + x_2 = 0$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector required.

Similarly, we solve $(\mathrm{B} - \lambda_2 \mathrm{I})x = 0$ to get $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The corresponding eigen vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 3 of 3

Using the eigen vectors, $S = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $S^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ such that

$B = S\Lambda S^{-1}$

Multiplying the respective sides with themselves for k times, we get $B^k = S\Lambda^k S^{-1}$

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The corresponding eigen vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Provide feedback (0)

Step 3 of 3

Using the eigen vectors, $S = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $S^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ such that

$B = S\Lambda S^{-1}$

Multiplying the respective sides with themselves for k times, we get $B^k = S\Lambda^k S^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2^k & 0 \\ 0 & 3^k \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2^k & 3^k \\ -2^k & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$$

$$= 3^k \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 2^k \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$$

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Step 1 of 1

By the properties of determinants, we have $|XY| = |X||Y|$ for every square matrices X and Y of size $n \times n$.

Using this, we have $A = S\Lambda S^{-1}$ where all the matrices are of same size and Λ is the diagonal matrix whose diagonal entries are nothing but the eigen values of A.

Applying determinant throughout, we get $|A| = |S\Lambda S^{-1}|$

By the above property, we get $|A| = |S||\Lambda||S^{-1}|$

We observe that the determinant is a scalar quantity and so, product of determinants is commutative.

So, the above equation can be written as $|A| = |S||S^{-1}||\Lambda|$

$$|A| = |SS^{-1}||\Lambda|$$

$$\Rightarrow |A| = |I||\Lambda|$$

$$\Rightarrow |A| = |\Lambda|$$

We know that the determinant of the diagonal matrix is nothing but the product of the diagonal entries.

Further, Λ has the diagonal entries nothing but the eigen values of A.

Therefore, $|A|$ is the product of eigen values of A.

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Step 1 of 1

Which is the sum of the λ's.

Explanation: Trace $AB = \left((aq + bs) + (cr + dt)\right)$

$= (qa + rc) + (sb + td)$ trace BA.

Proof for diagonalizable case: The trace of $S\Lambda S^{-1}$ is the trace of $(\Lambda S^{-1})S = \Lambda$.

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Step 1 of 4

Let $A = S\Lambda S^{-1}$. Matrix B is given as follows:

$$B = \begin{bmatrix} A & 0 \\ 0 & 2A \end{bmatrix}$$

Then to diagonalize the block matrix and to determine its Eigen values and Eigen vector matrices.

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Step 2 of 4

To diagonalize the block matrix B means to write the matrix in the following form:

$$B = S_B \Lambda_B S_B^{-1}$$

Here, S_B, and Λ_B represents Eigen vector matrices and Eigen value matrices.

Provide feedback (0)

Step 3 of 4

Let $A = S\Lambda S^{-1}$, then:

$$2A = S(2\Lambda)S^{-1}$$

Substitute the values in matrix B and do the following calculations:

$$\begin{aligned} B &= \begin{bmatrix} S\Lambda S^{-1} & 0 \\ 0 & S(2\Lambda)S^{-1} \end{bmatrix} \\ &= \begin{bmatrix} S & 0 \\ 0 & S^{-1} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & 2\Lambda \end{bmatrix} \begin{bmatrix} S^{-1} & 0 \\ 0 & S^{-1} \end{bmatrix} \\ &= S_B \Lambda_B S_B^{-1} \end{aligned}$$

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Let $A = S\Lambda S^{-1}$, then:

$$2A = S(2\Lambda)S^{-1}$$

Substitute the values in matrix B and do the following calculations:

$$\begin{aligned} B &= \begin{bmatrix} S\Lambda S^{-1} & 0 \\ 0 & S(2\Lambda)S^{-1} \end{bmatrix} \\ &= \begin{bmatrix} S & 0 \\ 0 & S^{-1} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & 2\Lambda \end{bmatrix} \begin{bmatrix} S^{-1} & 0 \\ 0 & S^{-1} \end{bmatrix} \\ &= S_B \Lambda_B S_B^{-1} \end{aligned}$$

Provide feedback (0)

Step 4 of 4

Therefore, $\boxed{\Lambda, 2\Lambda}$ are Eigen values and $\boxed{S_B}$ is Eigen vector matrices.

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Step 1 of 3

A is set of 4×4 matrix diagonalized by the eigen vector matrix S.

$\Rightarrow A_i = S\Lambda S^{-1}$ for every A_i matrix in A.

Let $A_1, A_2 \in A$

Then we have $A_1 = S\Lambda_1 S^{-1}$ and $A_2 = S\Lambda_2 S^{-1}$

$$A_1 + A_2 = S\Lambda_1 S^{-1} + S\Lambda_2 S^{-1}$$
$$= S(\Lambda_1 + \Lambda_2)S^{-1}$$

While the sum of diagonal matrices is diagonal, we have $\Lambda_1 + \Lambda_2$ is a diagonal matrix and so, $A_1 + A_2$ is diagonalized by S.

This confirms that $A_1 + A_2$ is in A. …… (1)

Provide feedback (0)

Step 2 of 3

Suppose a is any scalar and A_i is any member of A.

Then we have $a\,A_i = a\,S\Lambda S^{-1}$

We know that the scalar a commutes with the product of matrices and so, this equation can be written as $aA_i = S(a\Lambda)S^{-1}$

$a\Lambda$ is the product of a with the diagonal entries and allows the resultant matrix is also a diagonal matrix.

So, S diagonalizes aA_i.

In other words, aA_i is also a member of A. …… (2)

(1), (2) confirms that A is a subspace of all 2×2 matrices.

Provide feedback (0)

Step 3 of 3

If I diagonalizes A_i, then we write $A_i = IA_iI^{-1} = IA_iI$

But we know that every matrix A_i can be written like this regardless of whether A_i is diagonalizable or not.

diagonal matrix.
So, S diagonalizes aA_1.
In other words, aA_1 is also a member of A. ..... (2)
(1), (2) confirms that A is a subspace of all 2×2 matrices.

Provide feedback (0)

Step 3 of 3

If I diagonalizes A_1, then we write $A_1 = IA_1I^{-1} = IA_1I$
But we know that every matrix A_1 can be written like this regardless of whether A_1 is diagonalizable or not.
In other words, I cannot diagonalize any matrix.
Or, S cannot be replaced by I.
In other words, A_1 is a matrix spanned by all the four standard basis matrices.
Thus, the dimension of A is 4.

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Step 1 of 3

Let A be a square matrix such that $A^2 = A$.

Such a matrix is called Idempotent Matrix.

Suppose A has full set of eigenvectors and thus, A can be diagonalized. Therefore, the rank of the matrix A is n, where A is n by n matrix.

Provide feedback (0)

Step 2 of 3

Let us consider x to be an eigenvector with respect to the eigenvalue $\lambda = 1$.

Thus, we have $Ax = x$.

But, $A^2 = A$.

Thus, we get

$$\begin{aligned} A(Ax) &= Ax \\ A^2x &= x \end{aligned}$$

This clearly indicates that the vector x lies in the row space of the matrix A.

Provide feedback (2)

Step 3 of 3

CH5.2 39P

Thus, we have $Ax = x$.

But, $A^2 = A$.

Thus, we get

$$\begin{aligned} A(Ax) &= Ax \\ A^2x &= x \end{aligned}$$

This clearly indicates that the vector x lies in the row space of the matrix A.

Provide feedback (2)

Step 3 of 3

Let us consider y to be an eigenvector with respect to the eigenvalue $\lambda = 1$.

Thus, we have

$$\begin{aligned} Ay &= 0y \\ &= 0 \end{aligned}$$

This indicates that y must be in the nullspace of the matrix A.

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Step 1 of 2

Given that $Ax = \lambda x$
If $\lambda = 0$ then x is in the null space.
If $\lambda \neq 0$ then x is in the column space.
Also given that the dimension of the null space is $n - r$ and the dimension of the column space is r.
So the dimension of the given space is n.
We have to explain why every square matrix has no n linearly independent eigenvectors.

Provide feedback (0)

Step 2 of 2

Because there are two problems.
The null space and column space can overlap.
So x called be in both.
Second problem is if there may not be r independent eigenvectors in the column space.
If suppose λ have repeated eigenvalues then there may not be r independent eigenvectors.

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Step 1 of 3

The matrix A contains n eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Since, the matrix A is not a zero matrix, at least one of the eigenvalues of A must be nonzero.

Without loss of generality, let λ_n be a non zero eigenvalue.

Thus, we have $Ax = \lambda_n x$, where $\lambda_n \neq 0$ and $x \neq 0$.

Provide feedback (0)

Step 2 of 3

Let us apply the product $(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)$ to the vector x. Thus, we get

$$\begin{aligned}
((A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I))x &= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(A-\lambda_n I)x \\
&= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(Ax-\lambda_n Ix) \\
&= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(\lambda_n x-\lambda_n x) \\
&= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(0) \\
((A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I))x &= 0
\end{aligned}$$

Provide feedback (0)

Step 3 of 3

Thus, we have shown that $\boxed{(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)=0}$.

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$$\begin{aligned}((A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I))x &= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(A-\lambda_n I)x \\ &= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(Ax-\lambda_n Ix) \\ &= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(\lambda_n x-\lambda_n x) \\ &= ((A-\lambda_1 I)(A-\lambda_2 I)\cdots)(0)\end{aligned}$$

$$((A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I))x = 0$$

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Step 3 of 3

Thus, we have shown that $\boxed{(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)=0}$.

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CH5.2 40P

Step 1 of 4

Given Fibonacci's matrix is $A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$.

We have to verify the Cayley-Hamilton Theorem on the given Fibonacci's matrix.

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Step 2 of 4

The characteristic equation of A is

$$|A-\lambda I|=0$$

$$\Rightarrow\begin{vmatrix}1-\lambda & 1\\ 1 & -\lambda\end{vmatrix}=0$$

$$\Rightarrow -\lambda(1-\lambda)-1=0$$

$$\Rightarrow \lambda^2-\lambda-1=0$$

Therefore $\det(A-\lambda I)=\lambda^2-\lambda-1$

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Step 3 of 4

By Cayley – Hamilton theorem, every square matrix satisfies its characteristic equation.

Therefore, $A^2-A-I=0$

To prove this $A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1(1)+1(1) & 1(1)+1(0)\\ 1(1)+0(1) & 1(1)+0(0)\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$$

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Step 3 of 4

By Cayley – Hamilton theorem, every square matrix satisfies its characteristic equation.
Therefore, $A^2 - A - I = 0$

To prove this $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+1(1) & 1(1)+1(0) \\ 1(1)+0(1) & 1(1)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

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Step 4 of 4

Therefore,

$$A^2 - A - I = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2-1 & 1-1-0 \\ 1-1-0 & 1-0-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence the given Fibonacci matrix satisfies the Cayley-Hamilton theorem.

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CH5.2 41P

Step 1 of 5

Given that $A=\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

Also given that $\det(A-\lambda I)=(\lambda-a)(\lambda-b)$

We have to check that given matrix satisfies the Cayley-Hamilton theorem.

Provide feedback (0)

Step 2 of 5

By Cayley-Hamilton theorem, every square matrix satisfies its characteristic polynomial.

Since $\det(A-\lambda I)=(\lambda-a)(\lambda-b)$

So the characteristic polynomial of A is $(\lambda-a)(\lambda-b)$.

Therefore, by Cayley-Hamilton theorem, we have to verify that $(A-aI)(A-dI)=0$

Provide feedback (0)

Step 3 of 5

Now

$$\begin{aligned} A-aI &= \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} - a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} - \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \\ &= \begin{bmatrix} a-a & b-0 \\ 0-0 & d-a \end{bmatrix} \\ &= \begin{bmatrix} 0 & b \\ 0 & d-a \end{bmatrix} \end{aligned}$$

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Provide feedback (0)

Step 4 of 5

And

$$A-dI=\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}-d\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$=\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}-\begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}$$
$$=\begin{bmatrix} a-d & b-0 \\ 0-0 & d-d \end{bmatrix}$$
$$=\begin{bmatrix} a-d & b \\ 0 & 0 \end{bmatrix}$$

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Step 5 of 5

Therefore,

$$(A-aI)(A-dI)=\begin{bmatrix} 0 & b \\ 0 & d-a \end{bmatrix}\begin{bmatrix} a-d & b \\ 0 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 0(a-d)+b(0) & 0(b)+b(0) \\ 0(a-d)+(d-a)(0) & 0(b)+(d-a)(0) \end{bmatrix}$$
$$=\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$
$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$=0$$

Therefore, the given matrix A satisfies its characteristic polynomial $(A-aI)(A-dI)$.
Hence the given matrix satisfies Cayley–Hamilton theorem.

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CH5.2 42P

Step 1 of 5

Given that $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $AB = BA$.

Also given that $B = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix}$

We have to show that B is also diagonal.

Provide feedback (0)

Step 2 of 5

Since $AB = BA$

So

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1(\mathbf{a})+0(\mathbf{c}) & 1(\mathbf{b})+0(\mathbf{d}) \\ 0(\mathbf{a})+2(\mathbf{c}) & 0(\mathbf{b})+2(\mathbf{d}) \end{bmatrix} = \begin{bmatrix} \mathbf{a}(1)+\mathbf{b}(0) & \mathbf{a}(0)+\mathbf{b}(2) \\ \mathbf{c}(1)+\mathbf{d}(0) & \mathbf{c}(0)+\mathbf{d}(2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ 2\mathbf{c} & 2\mathbf{d} \end{bmatrix} = \begin{bmatrix} \mathbf{a} & 2\mathbf{b} \\ \mathbf{c} & 2\mathbf{d} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} a & 2b \\ c & 2d \end{bmatrix}$$

Provide feedback (0)

Step 3 of 5

From this, we get $\mathbf{b} = \mathbf{2b}$ and $\mathbf{2c} = \mathbf{c}$

$\Rightarrow \mathbf{b} = 0$, $\mathbf{c} = 0$

Hence $B = \begin{bmatrix} \mathbf{a} & \mathbf{0} \\ \mathbf{0} & \mathbf{2d} \end{bmatrix}$

Hence B is also diagonal.

CH5.2 44P

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From this, we get $\mathbf{b} = 2\mathbf{b}$ and $2\mathbf{c} = \mathbf{c}$

$\Rightarrow \mathbf{b} = 0$, $\mathbf{c} = 0$

Hence $B = \begin{bmatrix} \mathbf{a} & \mathbf{0} \\ \mathbf{0} & \mathbf{2d} \end{bmatrix}$

Hence B is also diagonal.

Provide feedback (0)

Step 4 of 5

Let λ be the eigenvalue of A.

Then $Ax = \lambda x$

Multiply both sides with B, we get

$BAx = B\lambda x$

$\Rightarrow ABx = B\lambda x \qquad (\text{Since } AB = BA)$

$= \lambda Bx$

Thus x and Bx are eigenvectors of A with same λ.

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Step 5 of 5

But the eigenvectors of A are distinct.

$\Rightarrow Bx$ must be multiple of x

$\Rightarrow x$ is an eigenvector of B as well as x is an eigen vector of A.

Therefore A and B have same eigenvectors but different eigenvalues.

Here we are getting the value of B.

$\Rightarrow \mathbf{b} = 0$, $\mathbf{c} = 0$

Hence B forms a two dimensional subspace of matrix space.

Since B has two nonzero columns.

So the rank of B is $\boxed{2}$.

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CH5.2 43P

Step 1 of 3

If $B = A$, then $AB = BA$.

If $B = 0$, then $AB = BA$.

If *A* is invertible and if $B = A^{-1}$, then $AB = BA$.

If *B* is *I*, that is identity matrix, then also $AB = BA$.

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Step 2 of 3

Note that, in each of these cases, $AB - BA = 0$, that is, a zero matrix.

Let *A* be a 5 by 5 matrix. Suppose $AB - BA = 0$.

Since, no other information is mentioned about *A*, it may not be invertible. In such case the equation $B = A^{-1}$ is meaningless. Also, *B* should not be the zero matrix. Further, if B is to be different from *A*, the equation $B = A$ is also meaningless.

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Step 3 of 3

Therefore, the required solution is $\boxed{B = I}$.

Note that, $B \neq 0$, $B \neq A$ and $AB - BA = 0$.

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Let A be a 5 by 5 matrix. Suppose $AB - BA = 0$.

Since, no other information is mentioned about A, it may not be invertible. In such case the equation $B = A^{-1}$ is meaningless. Also, B should not be the zero matrix. Further, if B is to be different from A, the equation $B = A$ is also meaningless.

Provide feedback (0)

Step 3 of 3

Therefore, the required solution is $\boxed{B = I}$.

Note that, $B \neq 0$, $B \neq A$ and $AB - BA = 0$.

Therefore, for this matrix, the 25 by 25 matrix is singular.

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CH5.2 44P

Step 1 of 12

Given that $A=\begin{bmatrix}0.6 & 0.3\\ 0.4 & 0.8\end{bmatrix}$ and $A^{\infty}=\begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{2}{3} & \frac{1}{3}\end{bmatrix}$

We have to find the eigenvalues and eigenvectors for both the given Markov matrices.

Provide feedback (0)

Step 2 of 12

Now we find the eigenvalues of A.

The characteristic equation of A is $|A-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}0.6-\lambda & 0.2\\ 0.4 & 0.8-\lambda\end{vmatrix}=0$$

$$\Rightarrow(0.6-\lambda)(0.8-\lambda)-0.08=0$$

$$\Rightarrow\lambda^{-2}1.4\lambda+0.4=0$$

$$\Rightarrow\lambda^2-\lambda-0.4\lambda+0.4=0$$

$$\Rightarrow\lambda(\lambda-1)-0.4(\lambda-1)=0$$

$$\Rightarrow\lambda=0.4,1$$

Therefore, eigenvalues are $\boxed{\lambda=0.4 \text{ and } 1}$.

Provide feedback (0)

Step 3 of 12

We know that x is the eigenvector of A if and only if x is the nonzero solution of $(A-\lambda I)x=0$

That is $\begin{bmatrix}0.6-\lambda & 0.2\\ 0.4 & 0.8-\lambda\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$ ..... (1)

For $\lambda=0.4$, (1) becomes

$\begin{bmatrix}0.2 & 0.2\end{bmatrix}\begin{bmatrix}x_1\end{bmatrix}$

CH5.3

Step 3 of 12

We know that x is the eigenvector of A if and only if x is the nonzero solution of $(A-\lambda I)x=0$

That is $\begin{bmatrix} 0.6-\lambda & 0.2 \\ 0.4 & 0.8-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$ ..... (1)

For $\lambda=0.4$, (1) becomes

$$\begin{bmatrix} 0.2 & 0.2 \\ 0.4 & 0.4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$$

Provide feedback (0)

Step 4 of 12

The augmented matrix is

$$\begin{bmatrix} 0.2 & 0.2 & 0 \\ 0.4 & 0.4 & 0 \end{bmatrix}$$

Add (-2) times of row 1 to row 2, we get

$$\begin{bmatrix} 0.2 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From this, we get $0.2x_1+0.2x_2=0$

Here x_2 is free variable.

Provide feedback (0)

Step 5 of 12

Let $x_2=k$, where k is a parameter

Then $x_1=-k$

Therefore,

$$x=\begin{bmatrix} -k \\ k \end{bmatrix}$$

$$=k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} -1 \end{bmatrix}$

Provide feedback (0)

Step 5 of 12

Let $x_2 = k$, where k is a parameter

Then $x_1 = -k$

Therefore,

$$x = \begin{bmatrix} -k \\ k \end{bmatrix}$$

$$= k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Therefore, the eigenvector corresponding to $\lambda = 0.4$ is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Step 6 of 12

For $\lambda = 1$, (1) becomes

$$\begin{bmatrix} -0.4 & 0.2 \\ 0.4 & -0.2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Provide feedback (0)

Step 7 of 12

The augmented matrix is

$$\begin{bmatrix} -0.4 & 0.2 & 0 \\ 0.4 & -0.2 & 0 \end{bmatrix}$$

Add row 1 to row 2, we get

$$\begin{bmatrix} -0.4 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From this, we get $-0.4x_1 + 0.2x_2 = 0$

Here x_1 is free variable.

$$\begin{bmatrix} -0.4 & 0.2 \\ 0.4 & -0.2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Provide feedback (0)

Step 7 of 12

The augmented matrix is

$$\begin{bmatrix} -0.4 & 0.2 & 0 \\ 0.4 & -0.2 & 0 \end{bmatrix}$$

Add row 1 to row 2, we get

$$\begin{bmatrix} -0.4 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From this, we get $-0.4x_1 + 0.2x_2 = 0$

Here x_1 is free variable.

Provide feedback (0)

Step 8 of 12

Let $x_1 = k$, where k is a free variable.

Then $-0.4x_1 = -0.2x_2$

$\Rightarrow x_2 = 2k$

Therefore,

$$x = \begin{bmatrix} k \\ 2k \end{bmatrix}$$

$$= k\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Therefore, the eigenvector corresponding to $\lambda = 1$ is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Provide feedback (0)

Provide feedback (0)

Step 9 of 12

Let $S = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$ and

Then

$$A = S\Lambda S^{-1}$$
$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

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Step 10 of 12

Now

$$A^{\infty} = S\Lambda^{\infty}S^{-1}$$
$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} (.4) & 0 \\ 0 & 1 \end{bmatrix}^{\infty} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} (.4)^{\infty} & 0 \\ 0 & 1^{\infty} \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Provide feedback (0)

Step 11 of 12

Since $|\lambda| < 1$

So as $n \to \infty$, $\lambda^n \to 0$

Hence the eigenvalues of A^{∞} are $\boxed{0 \text{ and } 1}$

And the eigenvectors of A^{∞} are $\boxed{\begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$.

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Provide feedback (0)

Step 11 of 12

Since $|\lambda| < 1$

So as $n \to \infty$, $\lambda^n \to 0$

Hence the eigenvalues of A^∞ are $\boxed{0 \text{ and } 1}$

And the eigenvectors of A^∞ are $\boxed{\begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$.

Provide feedback (0)

Step 12 of 12

We have to explain why A^{100} is close to A^∞.

Now

$$A^{100} = S\Lambda^{100}S^{-1}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.4 & 0 \\ 0 & 1 \end{bmatrix}^{100} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} (0.4)^{100} & 0 \\ 0 & (1)^{100} \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Since $|\lambda| < 1$

So $n \to 100$, $\lambda^{100} \to 0$

Therefore A^{100} close to A^∞.

Provide feedback (0)