CH5.4

**Step 1** of 7

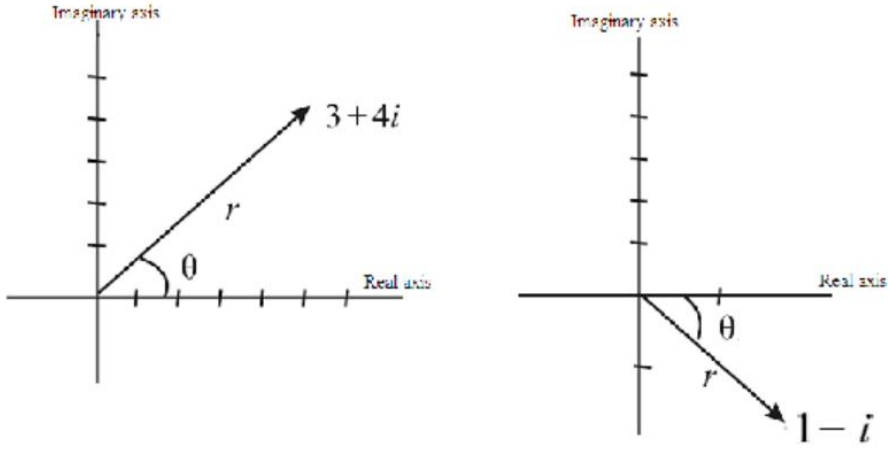
Given complex numbers are $3+4i$ and $1-i$.

a) We have to find the positions of the given complex numbers in the complex plane.
The positions of the given complex numbers are shown below.

Provide feedback (0)

CH5.5 2P

**Step 2** of 7

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**Step 3** of 7

b) We have to find the sum and product of the given complex numbers.
Now

$$\text{sum} = (3+4i)+(1-i)$$
$$= (3+1)+i(4-1)$$
$$= 4+3i$$

$$\text{Product} = (3+4i)(1-i)$$

Provide feedback (0)

**Step 3** of 7

b) We have to find the sum and product of the given complex numbers.

Now

$$\begin{aligned}\text{sum} &= (3+4i)+(1-i)\\ &= (3+1)+i(4-1)\\ &= 4+3i\end{aligned}$$

$$\begin{aligned}\text{Product} &= (3+4i)(1-i)\\ &= 3(1)+4i(1)-3(i)-4i^2\\ &= 3+4i-3i+4 \qquad (\text{since } i^2=-1)\\ &= 7+i\end{aligned}$$

Hence the sum of the given complex numbers is $\boxed{4+3i}$ and the product is $\boxed{7+i}$.

Provide feedback (0)

**Step 4** of 7

c) We have to find the conjugate and absolute values of the given complex numbers.

$$\begin{aligned}\text{Conjugate of } 3+4i &= \overline{3+4i}\\ &= 3-4i\end{aligned}$$

Therefore, the conjugate of $3+4i$ is $\boxed{3-4i}$.

Provide feedback (0)

**Step 5** of 7

$$\begin{aligned}\text{Conjugate of } 1-i &= \overline{1-i}\\ &= 1+i\end{aligned}$$

Therefore, the conjugate of $1-i$ is $\boxed{1+i}$.

Provide feedback (0)

$= 7+i$

Hence the sum of the given complex numbers is $\boxed{4+3i}$ and the product is $\boxed{7+i}$.

Provide feedback (0)

**Step 4** of 7

c) We have to find the conjugate and absolute values of the given complex numbers.

Conjugate of $3+4i = \overline{3+4i}$

$= 3-4i$

Therefore, the conjugate of $3+4i$ is $\boxed{3-4i}$.

Provide feedback (0)

**Step 5** of 7

Conjugate of $1-i = \overline{1-i}$

$= 1+i$

Therefore, the conjugate of $1-i$ is $\boxed{1+i}$.

Provide feedback (0)

**Step 6** of 7

Absolute value of $3+4i = |3+4i|$

$= \sqrt{3^2+4^2}$

$= \sqrt{9+16}$

$= 5$

Therefore, the absolute value of $3+4i$ is $\boxed{5}$.

Provide feedback (0)

**Step 7** of 7

Absolute value of $1-i = |1-i|$

Absolute value of $3+4i = |3+4i|$

$$= \sqrt{3^2+4^2}$$
$$= \sqrt{9+16}$$
$$= 5$$

Therefore, the absolute value of $3+4i$ is $\boxed{5}$.

Provide feedback (0)

**Step 7** of 7

Absolute value of $1-i = |1-i|$

$$= \sqrt{1^2+(-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2}$$

Therefore, the absolute value of $1-i$ is $\boxed{\sqrt{2}}$.

Since both absolute values are $5$ and $\sqrt{2}$ more than 1.
Hence both the numbers lie outside the unit circle.

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CH5.5 1P

**Step 1** of 4

a) We have to say what is the sum of a complex number and its conjugate.
Let $a+ib$ be a complex number.
Then the conjugate of $a+ib=\overline{(a+ib)}$
$=a-ib$
Now the sum of $a+ib$ and $a-ib$ is
$=(a+ib)+(a-ib)$
$=(a+a)+i(b-b)$
$=2a$
Therefore the sum of the complex number and its conjugate is $\boxed{\text{twice}}$ the real part of the given complex number.

Provide feedback (0)

**Step 2** of 4

b) We have to say what is the conjugate of a number on the unit circle.
Let $a+ib$ be a complex number on the unit circle.
Then $r=\sqrt{a^2+b^2}=1$
The conjugate of $a+ib=a-ib$
And the radius $r=\sqrt{a^2+b^2}=1$
Therefore, the conjugate of a complex number is also lie on the unit circle.

Provide feedback (0)

**Step 3** of 4

c) We have to find what is the product of two numbers on the unit circle.
Let $(a+ib),(c+id)$ are the complex numbers
Then the product
$(a+ib)(c+id)=ac+ibc+iad+i^2bd$
$=ac+ibc+iad-bd$
$=(ac-bd)+i(ad+bc)$

CH5.5 3P

Let $a+ib$ be a complex number on the unit circle.

Then $r=\sqrt{a^2+b^2}=1$

The conjugate of $a+ib=a-ib$

And the radius $r=\sqrt{a^2+b^2}=1$

Therefore, the conjugate of a complex number is also lie on the unit circle.

Provide feedback (0)

**Step 3** of 4

c) We have to find what is the product of two numbers on the unit circle.

Let $(a+ib),(c+id)$ are the complex numbers

Then the product

$$\begin{aligned}(a+ib)(c+id)&=ac+ibc+iad+i^2bd\\&=ac+ibc+iad-bd\\&=(ac-bd)+i(ad+bc)\end{aligned}$$

The radius $r=\sqrt{(ac-bd)^2+(ad+bc)^2}$

If radius $r<1$ the product lies in the unit circle.

If $r=1$ then the product lie on the unit circle.

Provide feedback (0)

**Step 4** of 4

d) We have to find what is the sum of two numbers on the unit circle.

Let $(a+ib),(c+id)$ are the complex numbers

Then the sum of two complex numbers

$(a+ib)+(c+id)=(a+c)+i(b+d)$

The radius $r=\sqrt{(a+c)^2+(b+d)^2}$

If $r\le 1$ then the sum lie on the unit circle.

Provide feedback (0)

CH5.5 2P

**Step 1** of 14

Given that $x = 2+i$ and $y = 1+3i$

We have to find $\bar{x}, x\bar{x}, xy, \frac{1}{x}$ and $\frac{x}{y}$.

Provide feedback (0)

**Step 2** of 14

Now

$$\bar{x} = \overline{2+i}$$
$$= 2-i$$

Therefore, $\boxed{\bar{x} = 2-i}$

Provide feedback (0)

**Step 3** of 14

Now

$$x.\bar{x} = (2+i)(2-i)$$
$$= 4-(i)^2 \quad \left(\text{Since } (a+b)(a-b) = a^2-b^2\right)$$
$$= 4-(-1)^2 \quad \left(\text{Since } i^2 = -2\right)$$
$$= 5$$

Therefore, $\boxed{x\bar{x} = 5}$

Provide feedback (0)

**Step 4** of 14

Now

$$xy = (2+i)(1+3i)$$
$$= 2+6i+i+3i^2$$

CH5.5 4P

Provide feedback (0)

**Step 4** of 14

Now

$$\begin{aligned} xy &= (2+i)(1+3i) \\ &= 2+6i+i+3i^2 \\ &= 2+7i-3 \qquad (\text{Since } i^2=-1) \\ &= -1+7i \end{aligned}$$

Therefore, $\boxed{xy=-1+7i}$

Provide feedback (0)

**Step 5** of 14

Now

$$\begin{aligned} \frac{1}{x} &= \frac{1}{2+i} \\ &= \frac{1}{2+i}\times\frac{2-i}{2-i} \qquad (\text{Rationalising the denominator}) \\ &= \frac{2-i}{2^2-i^2} \qquad (\text{Since } (a+b)(a-b)=a^2-b^2) \\ &= \frac{2-i}{4+1} \qquad (\text{Since } i^2=-1) \end{aligned}$$

Provide feedback (0)

**Step 6** of 14

Continuation to the above

$$\begin{aligned} &= \frac{2-i}{5} \\ &= \frac{1}{5}(2-i) \\ &= \frac{2}{5}-\frac{1}{5}i \end{aligned}$$

**Step 6** of 14

Continuation to the above

$$= \frac{2-i}{5}$$

$$= \frac{1}{5}(2-i)$$

$$= \frac{2}{5} - \frac{1}{5}i$$

Therefore, $\boxed{\frac{1}{x} = \frac{2}{5} - \frac{1}{5}i}$

Provide feedback (0)

**Step 7** of 14

Now

$$\frac{x}{y} = \frac{2+i}{1+3i}$$

$$= \frac{2+i}{1+3i} \times \frac{1-3i}{1-3i}$$

$$= \frac{(2+i)(1-3i)}{(1+3i)(1-3i)}$$

$$= \frac{2-6i+i-3i^2}{(1)^2-(3i)^2}$$

Provide feedback (0)

**Step 8** of 14

Continuation to the above

$$= \frac{(2+3)-5i}{1+9}$$

$$= \frac{5-5i}{10}$$

1 1

**Step 8** of 14

Continuation to the above

$$= \frac{(2+3)-5i}{1+9}$$

$$= \frac{5-5i}{10}$$

$$= \frac{1}{2} - \frac{1}{2}i$$

Therefore, $\boxed{\frac{x}{y} = \frac{1}{2} - \frac{1}{2}i}$

Provide feedback (0)

**Step 9** of 14

We have to verify that $|xy| = |x||y|$.

Now

Provide feedback (0)

**Step 10** of 14

$$|xy| = |(x+i)(1+3i)|$$

$$= |-1+7i|$$

$$= \sqrt{(-1)^2 + 7^2}$$

$$= \sqrt{1+49}$$

$$= \sqrt{50}$$

Provide feedback (0)

**Step 11** of 14

And

**Step 11** of 14

And

$$|x| = |2+i|$$
$$= \sqrt{2^2+1^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$

Provide feedback (0)

**Step 12** of 14

And

$$|y| = |1+3i|$$
$$= \sqrt{1^2+3^2}$$
$$= \sqrt{1+9}$$
$$= \sqrt{10}$$

Now

$$|x||y| = \sqrt{5}\cdot\sqrt{10}$$
$$= \sqrt{50}$$

Hence $\boxed{|xy| = |x|.|y|}$

Provide feedback (0)

**Step 13** of 14

We have to verify that $\left|\frac{1}{x}\right| = \frac{1}{|x|}$

Now

$$\left|\frac{1}{x}\right| = \left|\frac{1}{2+i}\right|$$
$$= \left|\frac{2}{5} - \frac{1}{5}i\right|$$

Provide feedback (0)

**Step 13** of 14

We have to verify that $\left|\frac{1}{x}\right| = \frac{1}{|x|}$

Now

$$\begin{aligned}\left|\frac{1}{x}\right| &= \left|\frac{1}{2+i}\right| \\ &= \left|\frac{2}{5} - \frac{1}{5}i\right| \\ &= \sqrt{\left(\frac{2}{5}\right)^2 + \left(-\frac{1}{5}\right)^2} \\ &= \sqrt{\frac{4}{25} + \frac{1}{25}} \\ &= \frac{1}{\sqrt{5}}\end{aligned}$$

Provide feedback (0)

**Step 14** of 14

And

$$\begin{aligned}\frac{1}{|x|} &= \frac{1}{|2+i|} \\ &= \frac{1}{\sqrt{4+1}} \\ &= \frac{1}{\sqrt{5}}\end{aligned}$$

Hence $\boxed{\left|\frac{1}{x}\right| = \frac{1}{|x|}}$

Provide feedback (0)

CH5.5 3P

**Step 1** of 7

Given complex number is $a+ib$.
We have to find the values of $a$ and $b$ on the unit circle at the angles $\theta = 30°, 60°, 90°$.

Provide feedback (0)

CH5.5 5P

**Step 2** of 7

We can write $a+ib$ as $r(\cos\theta + i\sin\theta)$
Since the complex number on the unit circle
So $r = 1$
Therefore the complex number becomes $a+ib = \cos\theta + i\sin\theta$.

Provide feedback (0)

**Step 3** of 7

Let $\theta = 30°$
Then complex number
$a+ib = \cos 30° + i\sin 30°$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}i \qquad \left(\text{Since } \cos 30° = \frac{\sqrt{3}}{2},\ \sin 30° = \frac{1}{2}\right)$$

Therefore, $\boxed{a = \frac{\sqrt{3}}{2},\ b = \frac{1}{2}}$ for $\theta = 30°$

Provide feedback (0)

**Step 4** of 7

Let $\theta = 60°$
Then complex number
$a+ib = \cos 60° + i\sin 60°$

Provide feedback (0)

**Step 4** of 7

Let $\theta = 60°$
Then complex number
$a + ib = \cos 60° + i \sin 60°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} i \qquad \left( \begin{array}{l} \text{Since } \sin 60° = \frac{\sqrt{3}}{2} \\ \cos 0° = \frac{1}{2} \end{array} \right)$$

Therefore, $\boxed{a = \frac{1}{2}, b = \frac{\sqrt{3}}{2}}$ for $\theta = 60°$

Provide feedback (0)

**Step 5** of 7

Let $\theta = 90°$
Then complex number
$a + ib = \cos 90° + i \sin 90°$

$$= 0 + 1i \qquad \left( \begin{array}{l} \text{Since } \sin 90° = 1 \\ \cos 90° = 0 \end{array} \right)$$

$$= i$$

Therefore, $\boxed{a = 0, b = 1}$ for $\theta = 90°$

Provide feedback (0)

**Step 6** of 7

Let the first, second and third complex numbers are $\frac{\sqrt{3}}{2} + \frac{1}{2} i, \frac{1}{2} + \frac{\sqrt{3}}{2} i, i$

Square of the first complex number is

$$\left( \frac{\sqrt{3}}{2} + \frac{1}{2} i \right)^2 = \left( \frac{\sqrt{3}}{2} + \frac{1}{2} i \right) \left( \frac{\sqrt{3}}{2} + \frac{1}{2} i \right)$$

**Step 6** of 7

Let the first, second and third complex numbers are $\frac{\sqrt{3}}{2}+\frac{1}{2}i, \frac{1}{2}+\frac{\sqrt{3}}{2}i, i$

Square of the first complex number is

$$\begin{aligned}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2 &= \left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\\ &= \frac{3}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{1}{4}i^2\\ &= \left(\frac{3}{4}-\frac{1}{4}\right)+\frac{2\sqrt{3}}{4}i\\ &= \frac{1}{2}+\frac{\sqrt{3}}{2}i\end{aligned}$$

Hence the square of the first complex number is equal to the second number.

Provide feedback (0)

**Step 7** of 7

Now we have to verify the cube of the first complex number is third number.
The cube of the first complex number is

$$\begin{aligned}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3 &= \left(\frac{\sqrt{3}}{2}\right)^3+3\left(\frac{3}{4}\right)\left(\frac{1}{2}i\right)+3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}i\right)^2+\left(\frac{1}{2}i\right)^3\\ &= \frac{3\sqrt{3}}{8}+\frac{9}{8}i-\frac{3\sqrt{3}}{8}+\frac{1}{8}i^3\\ &= \frac{9}{8}i-\frac{1}{8}i \qquad \left(\text{Since } i^2=-1\right)\\ &= \left(\frac{9-1}{8}\right)i\\ &= \left(\frac{8}{8}\right)i\\ &= i\end{aligned}$$

Hence the cube of the first complex number is third number.

CH5.5 4P

**Step 1** of 12

a) Given that $x = re^{i\theta}$

We have to find $x^2, x^{-1}$ and $\bar{x}$ in polar coordinates.

Provide feedback (0)

**Step 2** of 12

We have $x = re^{i\theta}$

Now

$$x^2 = \left(re^{i\theta}\right)^2$$
$$= r^2 e^{2i\theta}$$

Therefore, $\boxed{x^2 = r^2 e^{2i\theta}}$

Provide feedback (0)

**Step 3** of 12

Now

$$x^{-1} = \frac{1}{x}$$
$$= \frac{1}{re^{i\theta}}$$
$$= \frac{1}{r}.e^{-i\theta}$$

Therefore, $\boxed{x^{-1} = \frac{1}{r}.e^{-i\theta}}$

Provide feedback (0)

**Step 4** of 12

We can write $x = re^{i\theta} = r\left(\cos\theta + i\sin\theta\right)$

CH5.5 6P

**Step 4** of 12

We can write $x = re^{i\theta} = r(\cos\theta + i\sin\theta)$

Now

$$\bar{x} = \overline{\left(r(\cos\theta + i\sin\theta)\right)}$$
$$= r(\cos\theta - i\sin\theta)$$
$$= re^{-i\theta}$$

Therefore, $\boxed{\bar{x} = re^{-i\theta}}$

Provide feedback (0)

**Step 5** of 12

We have to find the points at which $x^{-1} = \bar{x}$

Let us take $x^{-1} = \bar{x}$

$$\Rightarrow \frac{1}{r}e^{-i\theta} = r.e^{-i\theta}$$
$$\Rightarrow \frac{1}{r} = r$$
$$\Rightarrow r^2 = 1$$
$$\Rightarrow |x|^2 = 1$$

Therefore, on the unit circle, we have $\boxed{x^{-1} = \bar{x}}$.

Provide feedback (0)

**Step 6** of 12

b) We have to sketch the path of the complex number $e^{-(1+i)t}$ at $t = 0$.

At $t = 0$, the complex number

$$e^{(-1+i)t} = e^0$$
$$= 1$$

Provide feedback (0)

Provide feedback (0)

**Step 7** of 12

$$\text{If } t=\frac{\pi}{2}$$

$$\Rightarrow e^{(-1+i)\frac{\pi}{2}}=e^{\frac{-\pi}{2}+i\frac{\pi}{2}}$$

$$=e^{\frac{-\pi}{2}}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$$

$$=e^{\frac{-\pi}{2}}(i)$$

Provide feedback (0)

**Step 8** of 12

$$\text{If } t=\pi$$

$$\Rightarrow e^{-\pi+i\pi}=e^{-\pi}(\cos\pi+i\sin\pi)$$

$$=e^{-\pi}(-1+0)$$

$$=-e^{-\pi}$$

Provide feedback (0)

**Step 9** of 12

$$\text{If } t=\frac{3\pi}{2}$$

Then

$$e^{(-1+i)\frac{3\pi}{2}}=e^{\frac{-3\pi}{2}}\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$$

$$=e^{\frac{-3\pi}{2}}i$$

Provide feedback (0)

Provide feedback (0)

**Step 10** of 12

If $t = 2\pi$

Then

$$e^{(1+i)2\pi} = e^{-2\pi}(\cos 2\pi + i \sin 2\pi)$$
$$= e^{-2\pi}$$

Provide feedback (0)

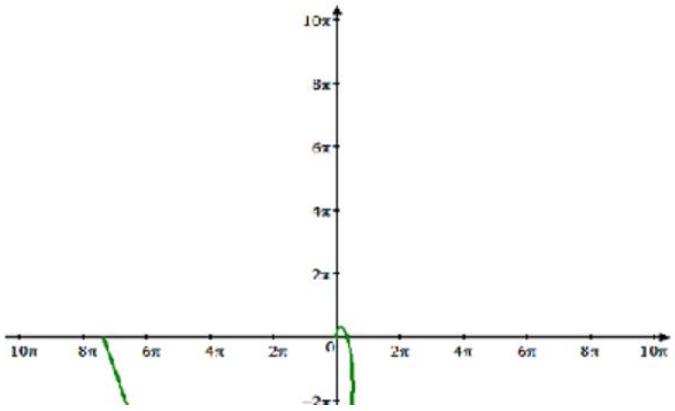
**Step 11** of 12

The path is a curve passing through the points $(1,0), \left(e^{\frac{-\pi}{2}}, 0\right), \left(-e^{-\pi}, 0\right), \left(e^{\frac{-3\pi}{2}}, 0\right), \left(e^{-2\pi}, 0\right)$.

The sketch of the path of the given number in the complex plane as $t$ increases from 0 to $2\pi$.

Provide feedback (0)

**Step 12** of 12

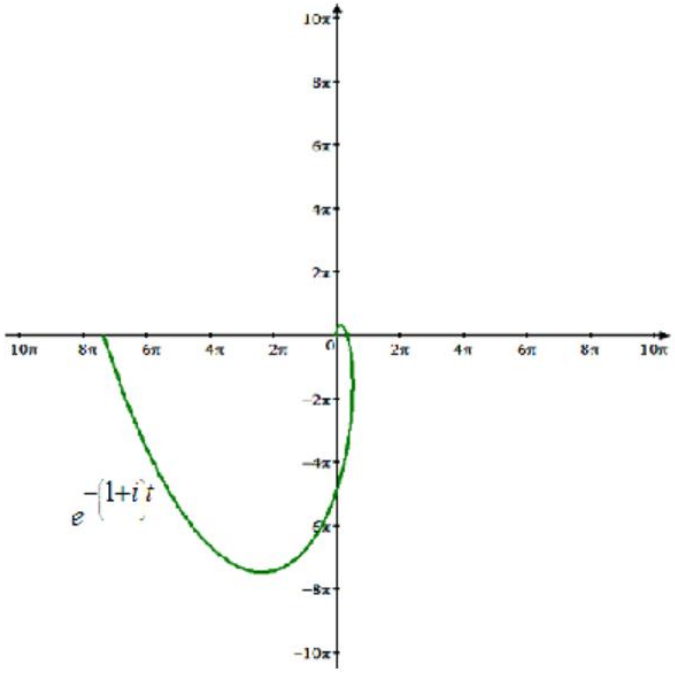


The path is a curve passing through the points $(1,0), \left(e^{\frac{-\pi}{2}}, 0\right), \left(-e^{-\pi}, 0\right), \left(e^{\frac{-3\pi}{2}}, 0\right), \left(e^{-2\pi}, 0\right)$.

The sketch of the path of the given number in the complex plane as $t$ increases from 0 to $2\pi$.

Provide feedback (0)

**Step 12** of 12




Provide feedback (0)

**Algebra** tutors who can help right now

CH5.5 5P

**Step 1** of 4

Given that $x=\begin{bmatrix}2-4i\\4i\end{bmatrix}, y=\begin{bmatrix}2+4i\\4i\end{bmatrix}$

We have to find the lengths and inner product of the given numbers.

Provide feedback (0)

**Step 2** of 4

Now the length of $x$ is

$$\begin{aligned}\|x\|^2 &= |2-4i|^2+|4i|^2\\ &= \sqrt{2^2+(-4)^2}+\sqrt{4^2}\\ &= (4+16)+16\\ &= 36\\ \Rightarrow \|x\| &= \sqrt{36}\\ &= 6\end{aligned}$$

Therefore, the length of $x$ is $\boxed{6}$.

Provide feedback (0)

**Step 3** of 4

Now the length of $y$ is

$$\begin{aligned}\|y\|^2 &= |2+4i|^2+|4i|^2\\ &= \sqrt{2^2+4^2}+\sqrt{4^2}\\ &= (4+16)+16\\ &= 36\\ \Rightarrow \|y\| &= \sqrt{36}\\ &= 6\end{aligned}$$

Therefore, the length of $y$ is $\boxed{6}$.

CH5.5 7P

$$= 6$$

Therefore, the length of $x$ is $\boxed{6}$.

Provide feedback (0)

**Step 3** of 4

Now the length of $y$ is

$$\begin{aligned}\|y\|^2 &= |2+4i|^2 + |4i|^2 \\ &= \sqrt{2^2+4^2} + \sqrt{4^2} \\ &= (4+16)+16 \\ &= 36 \\ \Rightarrow \|y\| &= \sqrt{36} \\ &= 6\end{aligned}$$

Therefore, the length of $y$ is $\boxed{6}$.

Provide feedback (0)

**Step 4** of 4

Now the inner product of $x$ and $y$ is

$$\begin{aligned}\bar{x}^T \cdot y &= \begin{bmatrix} 2+4i & -4i \end{bmatrix} \begin{bmatrix} 2+4i \\ 4i \end{bmatrix} \\ &= (2+4i)^2 + (-4i)(4i) \\ &= 4+16i^2+16i-16i^2 \\ &= 4-16+16i+16 \qquad (\text{Since } i^2=-1) \\ &= 4+16i\end{aligned}$$

Hence the inner product of $x$ and $y$ is $\boxed{4+16i}$.

Provide feedback (2)

CH5.5 6P

**Step 1** of 5

Given that $A=\begin{bmatrix}1 & i & 0\\ i & 0 & 1\end{bmatrix}$

We have to find $A^H$ and $C=A^H A$.

Provide feedback (0)

**Step 2** of 5

We know that the conjugate transpose of a matrix is called a Hermitian matrix.

Now the conjugate of $A$ is $\overline{A}=\begin{bmatrix}1 & i & 0\\ i & 0 & 1\end{bmatrix}$

And the transpose of conjugate of $A$ is $\overline{A}^T=\begin{bmatrix}1 & -i\\ -i & 0\\ 0 & 1\end{bmatrix}$

Therefore, $\boxed{A^H=\begin{bmatrix}1 & -i\\ -i & 0\\ 0 & 1\end{bmatrix}}$

Provide feedback (0)

**Step 3** of 5

Now

$$\begin{aligned}C&=A^H A\\ &=\begin{bmatrix}1 & -i\\ -i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & i & 0\\ i & 0 & 1\end{bmatrix}\\ &=\begin{bmatrix}1(1)-i(i) & 1(i)-i(0) & 1(0)-i(1)\\ -i(1)+0(i) & -i(i)+0(0) & -i(0)+0(1)\\ 0(1)+1(i) & 0(i)+1(0) & 0(0)+1(1)\end{bmatrix}\end{aligned}$$

CH5.5 8P

**Step 3** of 5

Now

$$C = A^H A$$

$$= \begin{bmatrix} 1 & -i \\ -i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & i & 0 \\ i & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)-i(i) & 1(i)-i(0) & 1(0)-i(1) \\ -i(1)+0(i) & -i(i)+0(0) & -i(0)+0(1) \\ 0(1)+1(i) & 0(i)+1(0) & 0(0)+1(1) \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 5

Continuation to the above

$$= \begin{bmatrix} 1-i^2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix} \qquad (\text{Since } i^2 = -1)$$

Therefore, $\boxed{C = \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}}$

Provide feedback (0)

**Step 5** of 5

Since $C = A^H A$

And

$C^H = (A^H A)^H$

$$= \begin{bmatrix} -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix} \qquad (\text{Since } i^2 = -1)$$

Therefore, $\boxed{C = \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}}$

Provide feedback (0)

**Step 5** of 5

Since $C = A^H A$
And

$$C^H = \left(A^H A\right)^H$$
$$= A^H \left(A^H\right)^H$$
$$= A^H A \qquad \left(\text{Since } \left(A^H\right)^H = A\right)$$
$$= C$$

Therefore, $\boxed{C^H = C}$ and $\boxed{C^H = \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}}$.

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CH5.5 7P

**Step 1** of 5

Let $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

We have

$$\begin{bmatrix} 1 & i & 0 \\ i & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For elimination, subtract $i$ times the first row from the second row. This gives:

$$\begin{bmatrix} 1 & i & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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CH5.5 9P

**Step 2** of 5

Thus, we get

$$\begin{aligned} x + iy &= 0 \\ y + z &= 0 \end{aligned}$$

This gives,

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**Step 2** of 5

Thus, we get

$$x + iy = 0$$
$$y + z = 0$$

This gives,

$$x = -iy$$
$$y = -z$$

So, let

$$z = 1$$
$$y = -1$$
$$x = i$$

Therefore, $\boxed{\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix}}$ is a solution to the system $Ax = 0$.

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**Step 3** of 5

(b) Thus, the null space of the matrix $A$ is $\{c(i,-1,1) \mid c \in \mathbf{C}\}$.

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**Step 3** of 5

(b) Thus, the null space of the matrix $A$ is $\{c(i,-1,1) \mid c \in \mathbf{C}\}$.

We need to show that this nullspace is orthogonal to $C(A^{\mathrm{H}})$ and is not orthogonal to $C(A^{\mathrm{T}})$.

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**Step 4** of 5

Now $A^{\mathrm{H}}$ is obtained by taking the conjugate transpose of the matrix $A$.

Thus, $A^{\mathrm{H}} = \begin{bmatrix} 1 & -i \\ -i & 0 \\ 0 & 1 \end{bmatrix}$.

Thus, we get

$$\begin{aligned}(1,i,0)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} &= i - i + 0 \\ &= 0\end{aligned}$$

$$\begin{aligned}(i,0,1)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} &= i^2 + 0 + 1 \\ &= 0\end{aligned}$$

Now $A^H$ is obtained by taking the conjugate transpose of the matrix $A$.

Thus, $A^H = \begin{bmatrix} 1 & -i \\ -i & 0 \\ 0 & 1 \end{bmatrix}$.

Thus, we get

$$\begin{aligned}(1,i,0)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} &= i - i + 0 \\ &= 0\end{aligned}$$

$$\begin{aligned}(i,0,1)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} &= i^2 + 0 + 1 \\ &= 0\end{aligned}$$

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**Step 5** of 5

Therefore, the nullspace is orthogonal to $C\left(A^H\right)$.

Now consider,

$$(1,-i,0)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} = i + i + 0$$

$$= 0$$

Provide feedback (0)

**Step 5** of 5

Therefore, the nullspace is orthogonal to $C\left(A^{\mathrm{H}}\right)$.

Now consider,

$$\begin{aligned}(1,-i,0)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} &= i+i+0 \\ &= 2i \\ (-i,0,1)\begin{bmatrix} i \\ -1 \\ 1 \end{bmatrix} &= -i^2+0+1 \\ &= 2\end{aligned}$$

This shows that show that the nullspace is not orthogonal to $C\left(A^{\mathrm{T}}\right)$.

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**Step 1** of 2

a) We have to find the relation between the determinant of $A^H$ and the determinant of $A$.

We know that $\det A^T = \det A$

But $\det A^H = \overline{(\det A)}$ $\quad$ (Since $A^H$ = Transpose of conjugate matrix)

Hence $\boxed{\det\left(A^H\right) = \overline{(\det A)}}$

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**Step 2** of 2

b) We have to prove that the determinant of a Hermitian matrix is real.

Let $A$ be any Hermitian matrix

Then $A^H = A$

Taking determinant on both sides, we get

$$\det\left(A^H\right) = \det A$$

$$= \left(\overline{\det A}\right) \qquad \left(\text{Since } \det A^H = \overline{(\det A)}\right)$$

$$= \text{real number} \qquad \begin{pmatrix} \text{Since} \\ a+ib = a-ib \\ \Rightarrow ib = -ib \\ \Rightarrow b = 0 \end{pmatrix}$$

Hence the determinant of a Hermitian matrix is real.

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CH5.5 9P

CH5.5 11P

**Step 1** of 7

(a) A real symmetric matrix is given by

$$A^{H} = A$$

Consider an *n* by *n* symmetric matrix, then there are *n* entries on the diagonal and $(n-1)+\cdots+1$ entries above the diagonal that can be chosen arbitrary.

Since, other $1+\cdots+(n-1)$ entries below the diagonal are determined by the symmetry of the matrix.

Therefore, there are only $n+(n-1)+\cdots+1=\frac{1}{2}n(n+1)$ degrees of freedom in the selection of the $n^2$ entries in a *n* by *n* real symmetric matrix.

Therefore, degrees of freedom of an *n* by *n* symmetric matrix is $\boxed{\frac{1}{2}n(n+1)}$.

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**Step 2** of 7

Consider an *n* by *n* diagonal matrix, then there are *n* entries on the diagonal and it can be chosen arbitrary.

Therefore, degrees of freedom of an *n* by *n* diagonal matrix is $\boxed{n}$.

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**Step 3** of 7

**Step 3** of 7

Consider the entries of *ith* and *jth* columns of an orthogonal matrix, as $a_{ki}$ and $a_{lj}$ respectively.

Then by using the condition of orthogonal we have

$(a_{ki}, a_{lj}) = 0$ , here $i \neq j$

This condition show a symmetric, so the degree of freedom is given by $\frac{n(n-1)}{2}$ .

And

$(a_{ki}, a_{li}) = 1$ , for $i = 1, \cdots, n$

This condition show the degree of freedom is given by *n*.

We know that an *n* by *n* matrix has $n^2$ degrees of freedom.

To find the total degrees of freedom of an *n* by orthogonal matrix, do the following calculation:

$$\begin{aligned} n^2 - \frac{1}{2}(n)(n-1) - n &= \frac{2n^2 - n^2 + n - 2n}{2} \\ &= \frac{n^2 - n}{2} \\ &= \frac{(n)(n-1)}{2} \end{aligned}$$

To find the total degrees of freedom of an *n* by orthogonal matrix, do the following calculation:

$$\begin{aligned} n^2 - \frac{1}{2}(n)(n-1) - n &= \frac{2n^2 - n^2 + n - 2n}{2} \\ &= \frac{n^2 - n}{2} \\ &= \frac{(n)(n-1)}{2} \end{aligned}$$

Provide feedback (0)

**Step 4** of 7

Therefore, degrees of freedom of an *n* by *n* orthogonal matrix is $\boxed{\frac{(n)(n-1)}{2}}$.

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**Step 5** of 7

(b) We know that Hermitian matrix *A* is equal to their conjugate transpose.

$$A^H = \overline{A}^T$$

Every entry in the transposed matrix is same as the complex conjugate of the corresponding entry in the original matrix.

$$\left(A^H\right)_{ij} = \overline{A_{ji}}.$$

(b) We know that Hermitian matrix $A$ is equal to their conjugate transpose.

$$A^H = \overline{A}^T$$

Every entry in the transposed matrix is same as the complex conjugate of the corresponding entry in the original matrix.

$$\left(A^H\right)_{ij} = \overline{A_{ji}} .$$

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**Step 6** of 7

Consider a 3 by 3 matrix $A$ as follows:

$$A = \begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Here $a,b,c,d,e,f,g,h,k$ all are real numbers.

Now to find the transpose matrix, change $A_{ij}$ by $A_{ji}$ .

$$A^T = \begin{bmatrix} a & b-ic & e-if \\ b+ic & d & h-ik \\ e+if & h+ik & g \end{bmatrix}$$

Taking the complex conjugate of entry, we get

$$A^T = \begin{bmatrix} a & b-ic & e-if \\ b+ic & d & h-ik \\ e+if & h+ik & g \end{bmatrix}$$

Taking the complex conjugate of entry, we get

$$\overline{A}^T = \begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Hence we get $A = \overline{A}^T$ .

Here nine real numbers determine the Hermitian matrices *A*, so the real degrees of freedom are 9.

Therefore, 3 by 3 Hermitian matrices *A* have $\boxed{9}$ real degrees of freedom.

Provide feedback (0)

**Step 7** of 7

We know that a complex *n* by *n* matrix has $2n^2$ degrees of freedom.

Since the unitary matrix *U* satisfied that

$$UU^H = I$$

So, we get $n^2$ terms in a unitary matrix.

Hence, *n* by *n* unitary matrix has $n^2$ real degrees of freedom.

$$\overline{A}^T = \begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Hence we get $A = \overline{A}^T$ .

Here nine real numbers determine the Hermitian matrices *A*, so the real degrees of freedom are 9.

Therefore, 3 by 3 Hermitian matrices *A* have $\boxed{9}$ real degrees of freedom.

Provide feedback (0)

**Step 7** of 7

We know that a complex *n* by *n* matrix has $2n^2$ degrees of freedom.

Since the unitary matrix *U* satisfied that

$$UU^H = I$$

So, we get $n^2$ terms in a unitary matrix.

Hence, *n* by *n* unitary matrix has $n^2$ real degrees of freedom.

Therefore, 3 by 3 unitary matrices *A* have $\boxed{9}$ real degrees of freedom.

Provide feedback (0)

CH5.5 10P

**Step 1** of 27

Given that $P=\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$, $Q=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $R=\begin{bmatrix}3 & 4\\ 4 & -3\end{bmatrix}$

We have to write $P$, $Q$ and $R$ in the form $\lambda_1 x_1 x_1^H+\lambda_2 x_2 x_2^H$.

Provide feedback (0)

**Step 2** of 27

We find the eigenvalues of $P$.

The characteristic equation of $P$ is $|P-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}\frac{1}{2}-\lambda & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}-\lambda\end{vmatrix}=0$$

$$\Rightarrow\left(\frac{1}{2}-\lambda\right)^2-\frac{1}{4}=0$$

$$\Rightarrow\lambda^2-\lambda=0$$

$$\Rightarrow\lambda(\lambda-1)=0$$

$$\Rightarrow\lambda=0,1$$

Hence the eigenvalues of $P$ are $\lambda_1=0,\lambda_2=1$.

Provide feedback (0)

**Step 3** of 27

We know that $x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ is an eigenvector of $A$ if and only if $x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ is the nonzero solution of $|P-\lambda I|x=0$

$\begin{bmatrix}1 & 1\end{bmatrix}$

CH5.5 12P

**Step 3** of 27

We know that $x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of $A$ if and only if $x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the nonzero solution of $|P-\lambda I|x=0$

That is $\begin{bmatrix} \frac{1}{2}-\lambda & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$ ..... (1)

Provide feedback (0)

**Step 4** of 27

For $\lambda=0$, (1) becomes

$(P-0I)x=0$

$$\Rightarrow \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$$

The Augmented matrix is

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 5** of 27

Add $(-1)$ times of row 1 to row 2, we get

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 5** of 27

Add $(-1)$ times of row 1 to row 2, we get

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}x_1 + \frac{1}{2}x_2 = 0$$

Here $x_1$ is free variable.

Provide feedback (0)

**Step 6** of 27

Let $x_1 = k$, where $k$ is a parameter.

$$\Rightarrow x_2 = -k$$

Therefore, $x = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Hence the eigenvector of $P$ corresponding to $\lambda = 0$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Therefore, $x_1$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Provide feedback (0)

**Step 7** of 27

For $\lambda_2 = 1$, (1) becomes

$$(P - 1I)x = 0$$

Step 6 of 27

Let $x_1 = k$, where $k$ is a parameter.

$\Rightarrow x_2 = -k$

Therefore, $x = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Hence the eigenvector of $P$ corresponding to $\lambda = 0$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Therefore, $x_1$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Provide feedback (0)

Step 7 of 27

For $\lambda_2 = 1$, (1) becomes

$(P - 1I)x = 0$

$$\Rightarrow \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The Augmented matrix is

$$\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 27

Add row 1 to row 2, we get

Provide feedback (0)

Step 9 of 27

$$\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 8** of 27

Add row 1 to row 2, we get

Provide feedback (0)

**Step 9** of 27

$$\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{-1}{2}x_1 + \frac{1}{2}x_2 = 0$$

Here $x_2$ is free variable.

Provide feedback (0)

**Step 10** of 27

Let $x_2 = k$, where $k$ is a parameter

$\Rightarrow x_1 = k$.

Therefore, $x = \begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence the eigenvector of $P$ corresponding to $\lambda = 1$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Therefore, $x_2$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

Let $x_2 = k$, where $k$ is a parameter

$\Rightarrow x_1 = k$.

Therefore, $x = \begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence the eigenvector of $P$ corresponding to $\lambda = 1$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Therefore, $x_2$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 11** of 27

Hence $P$ can be written as

$$P = \lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$$

$$= 0\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} + 1\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Therefore, $\boxed{P = 0\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} + 1\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}$

Provide feedback (0)

**Step 12** of 27

Now we have to write $Q$ as $\lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$

The characteristic equation of $Q$ is $|Q - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^2 - 1 = 0$

**Step 12** of 27

Now we have to write $Q$ as $\lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$

The characteristic equation of $Q$ is $|Q - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda = \pm 1$$

$$\Rightarrow \lambda_1 = 1 \text{ and } \lambda_2 = -1$$

Therefore, the eigenvalues of $Q$ are $\lambda_1 = 1$ and $\lambda_2 = -1$.

Provide feedback (0)

**Step 13** of 27

Now find the eigenvectors of $Q$.

We know that $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of $A$ if and only if $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the nonzero solution of $|Q - \lambda I| x = 0$

That is $\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ ..... (1)

For $\lambda_1 = 1$, (1) becomes

$$(Q - I) x = 0$$

$$\Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 14** of 27

The Augmented matrix is

$$\begin{bmatrix} -1 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 14** of 27

The Augmented matrix is

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

Add row 1 to row 2, we get

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow -x_1 + x_2 = 0$

Here $x_2$ is free variable

Provide feedback (0)

**Step 15** of 27

Let $x_2 = k$, where $k$ is a parameter

Then $x_1 = k$

Therefore, $x = \begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence the eigenvector of $Q$ corresponding to $\lambda = 1$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Therefore, $x_1$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 16** of 27

For $\lambda_2 = -1$, (1) becomes

$(Q + I)x = 0$

$\Rightarrow \begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \end{bmatrix} = 0$

The Augmented matrix is

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

Add row 1 to row 2, we get

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow -x_1 + x_2 = 0$

Here $x_2$ is free variable

Provide feedback (0)

**Step 15** of 27

Let $x_2 = k$, where $k$ is a parameter

Then $x_1 = k$

Therefore, $x = \begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence the eigenvector of $Q$ corresponding to $\lambda = 1$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Therefore, $x_1$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 16** of 27

For $\lambda_2 = -1$, (1) becomes

$(Q + I)x = 0$

$$\Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The augmented matrix is

Provide feedback (0)

Provide feedback (0)

**Step 16** of 27

For $\lambda_2 = -1$, (1) becomes

$(Q+I)x = 0$

$$\Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The augmented matrix is

Provide feedback (0)

**Step 17** of 27

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Add $(-1)$ times of row 1 to row 2, we get

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow x_1 + x_2 = 0$

Here $x_1$ is free variable.

Let $x_2 = k$, where $k$ is a parameter.

Then $x_2 = -k$

Provide feedback (0)

**Step 18** of 27

Therefore, $x = \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Hence the eigenvector of $Q$ corresponding to $\lambda = -1$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

**Step 18** of 27

Therefore, $x = \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Hence the eigenvector of $Q$ corresponding to $\lambda = -1$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Therefore, $x_2$ with length scaled to 1 is $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Provide feedback (0)

**Step 19** of 27

So we can write $Q$ as

$$Q = \lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$$

$$= 1\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} - 1\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Therefore, $\boxed{Q = 1\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} - 1\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}}$

Provide feedback (0)

**Step 20** of 27

We have to write $R$ as $\lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$

Given that $R = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

The characteristic equation of $R$ is $|R - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 4 \\ & \end{vmatrix} = 0$$

**Step 20** of 27

We have to write $R$ as $\lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$

Given that $R = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

The characteristic equation of $R$ is $|R - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(-3-\lambda) - 16 = 0$$

$$\Rightarrow \lambda^2 - 25 = 0$$

$$\Rightarrow \lambda = \pm 5$$

Therefore the eigenvalues of $R$ are $\boxed{\lambda_1 = 5 \text{ and } \lambda_2 = -5}$.

Provide feedback (0)

**Step 21** of 27

Now find the eigenvectors of $R$.

We know that $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of $A$ if and only if $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the nonzero solution of $|R - \lambda I| x = 0$

That is $\begin{bmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ ..... (1)

Provide feedback (0)

**Step 22** of 27

For $\lambda_1 = 5$, (1) becomes

$(R - 5I)x = 0$

$$\Rightarrow \begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The Augmented matrix is

$\begin{bmatrix} -2 & 4 & 0 \end{bmatrix}$

That is $\begin{bmatrix} 4 & -3-\lambda \end{bmatrix}\begin{bmatrix} x_2 \end{bmatrix} = 0$ ..... (1)

Provide feedback (0)

**Step 22** of 27

For $\lambda_1 = 5$, (1) becomes

$(R-5I)x = 0$

$\Rightarrow \begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

The Augmented matrix is

$\begin{bmatrix} -2 & 4 & 0 \\ 4 & -8 & 0 \end{bmatrix}$

$\Rightarrow -2x_1 + 4x_2 = 0$

Here $x_2$ is free variable.

Provide feedback (0)

**Step 23** of 27

Let $x_2 = k$, where $k$ is a parameter

$\Rightarrow x_1 = 2k$

Therefore, $x = \begin{bmatrix} 2k \\ k \end{bmatrix} = k\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Hence the eigenvector of $R$ corresponding to $\lambda = 5$ is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Therefore, $x_1$ with length scaled to 1 is $\frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 24** of 27

For $\lambda_2 = -5$, (1) becomes

Therefore, $x = \begin{bmatrix} \\ k \end{bmatrix} = k\begin{bmatrix} \\ 1 \end{bmatrix}$

Hence the eigenvector of $R$ corresponding to $\lambda = 5$ is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Therefore, $x_1$ with length scaled to 1 is $\frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 24** of 27

For $\lambda_2 = -5$, (1) becomes

$(R+5I)x = 0$

$\Rightarrow \begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

The Augmented matrix is

Provide feedback (0)

**Step 25** of 27

$\begin{bmatrix} 8 & 4 & 0 \\ 4 & 2 & 0 \end{bmatrix}$

Add $\left(\frac{-1}{2}\right)$ times of row 1 to row 2, we get

$\begin{bmatrix} 8 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow 8x_1 + 4x_2 = 0$

Here $x_1$ is free variable.

Provide feedback (0)

**Step 26** of 27

$\Rightarrow \begin{bmatrix} 4 & 2 \end{bmatrix} \begin{bmatrix} x_2 \end{bmatrix} = 0$

The Augmented matrix is

Provide feedback (0)

**Step 25** of 27

$$\begin{bmatrix} 8 & 4 & 0 \\ 4 & 2 & 0 \end{bmatrix}$$

Add $\left(\frac{-1}{2}\right)$ times of row 1 to row 2, we get

$$\begin{bmatrix} 8 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow 8x_1 + 4x_2 = 0$

Here $x_1$ is free variable.

Provide feedback (0)

**Step 26** of 27

Let $x_1 = k$, where $k$ is a parameter

$\Rightarrow x_2 = -2k$

Therefore, $x = \begin{bmatrix} k \\ -2k \end{bmatrix} = k \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Hence the eigenvector of $R$ corresponding to $\lambda = -5$ is $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Therefore, $x_2$ with length scaled to 1 is $\frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Provide feedback (0)

**Step 27** of 27

So we can write $R$ as

Here $x_1$ is free variable.

Provide feedback (0)

**Step 26** of 27

Let $x_1 = k$, where $k$ is a parameter

$\Rightarrow x_2 = -2k$

Therefore, $x = \begin{bmatrix} k \\ -2k \end{bmatrix} = k\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Hence the eigenvector of $R$ corresponding to $\lambda = -5$ is $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Therefore, $x_2$ with length scaled to 1 is $\frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Provide feedback (0)

**Step 27** of 27

So we can write $R$ as

$$R = \lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$$

$$= 5\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} - 5\begin{bmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{bmatrix}$$

Therefore, $\boxed{R = 5\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} - 5\begin{bmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{bmatrix}}$

Provide feedback (0)

CH5.5 11P

CH5.5 13P

Step 1 of 7

Determine true or false:

(a) If $A$ is Hermitian, then $A+iI$ is invertible.

If $A$ is Hermitian, then $A^H = A$. Consider the following matrix:

$$A = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$$

Here,

$$\begin{aligned} A^H &= \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} \\ &= A \end{aligned}$$

Therefore the matrix is Hermitian.

Provide feedback (2)

Step 2 of 7

Matrix $A+iI$ will be:

$$\begin{aligned} A+iI &= \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} + i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} + \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \\ &= \begin{bmatrix} 1+i & -i \\ i & 1+i \end{bmatrix} \end{aligned}$$

This matrix will be invertible if determinant is non-zero. So, calculate the determinant:

$$\det(A+iI) = (1+i)^2 + i^2$$

Provide feedback (2)

Step 2 of 7

Matrix $A+iI$ will be:

$$\begin{aligned} A+iI &= \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} + i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} + \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \\ &= \begin{bmatrix} 1+i & -i \\ i & 1+i \end{bmatrix} \end{aligned}$$

This matrix will be invertible if determinant is non-zero. So, calculate the determinant:

$$\begin{aligned} \det(A+iI) &= (1+i)^2 + i^2 \\ &= 1+i^2+2i+i^2 \\ &= -1+2i \\ &\neq 0 \end{aligned}$$

Therefore, if $A$ is Hermitian, then $A+iI$ is invertible is $\boxed{\text{true}}$.

Provide feedback (1)

Step 3 of 7

(b) If $Q$ is Orthogonal, then $Q+\frac{1}{2}I$ is invertible.

Consider the following matrix:

$$Q = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Orthogonal: If $OO^T = I$, then $O$ is said to be orthogonal matrix.

Step 3 of 7

(b) If $Q$ is Orthogonal, then $Q+\frac{1}{2}I$ is invertible.

Consider the following matrix:

$$Q=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$$

Orthogonal: If $QQ^T=I$, then $Q$ is said to be orthogonal matrix.

$$\begin{aligned}QQ^T&=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\cdot\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\\&=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\\&=I\end{aligned}$$

Therefore, matrix $Q$ is orthogonal matrix.

Provide feedback (0)

Step 4 of 7

Matrix $Q+\frac{1}{2}I$ will be:

$$Q+\frac{1}{2}I=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}+\frac{1}{2}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}1/2&0&0\\0&1/2&0\end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$
$$= I$$

Therefore, matrix $Q$ is orthogonal matrix.

Provide feedback (0)

**Step 4** of 7

Matrix $Q+\frac{1}{2}I$ will be:

$$\begin{aligned} Q+\frac{1}{2}I &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & 1 & 0 \\ 0 & 1/2 & 1 \\ 1 & 0 & 1/2 \end{bmatrix} \end{aligned}$$

This matrix will be invertible if determinant is non-zero. So, calculate the determinant:

$$\begin{aligned} \det\left(Q+\frac{1}{2}I\right) &= \frac{1}{2}\left(\frac{1}{4}\right)-(-1) \\ &= \frac{1}{8}+1 \\ &= \frac{9}{8} \\ &\neq 0 \end{aligned}$$

Provide feedback (0)

**Step 5** of 7

$$\begin{aligned} &8 \\ &= \frac{9}{8} \\ &\neq 0 \end{aligned}$$

Provide feedback (0)

**Step 5** of 7

Therefore, if $Q$ is Orthogonal, then $Q+1/2I$ is invertible is $\boxed{\text{true}}$.

Provide feedback (0)

**Step 6** of 7

(c) If $A$ is real then $A+iI$ is invertible.

Consider the following matrix:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

As all the elements of matrix $A$ is real elements, so $A$ is real.

Provide feedback (0)

**Step 7** of 7

Matrix $A+iI$ will be:

$$\begin{aligned} A+iI &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \\ &= \begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix} \end{aligned}$$

(c) If $A$ is real then $A+iI$ is invertible.

Consider the following matrix:

$$A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

As all the elements of matrix $A$ is real elements, so $A$ is real.

Provide feedback (0)

Step 7 of 7

Matrix $A+iI$ will be:

$$\begin{aligned}A+iI &= \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}+i\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}+\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}\\ &= \begin{bmatrix}i & 1\\ -1 & i\end{bmatrix}\end{aligned}$$

This matrix will be invertible if determinant is non-zero. So, calculate the determinant:

$$\begin{aligned}\det(A+iI) &= (i)^2+1\\ &= -1+1\\ &= 0\end{aligned}$$

Therefore, if $A$ is real then $A+iI$ will be invertible is $\boxed{\text{false}}$.

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CH5.5 12P

**Step 1** of 5

Given that $A$ is a symmetric 3 b y 3 matrix with eigenvalues $0, 1, 2$.

Since $A$ is symmetric $\Rightarrow A^T = A$

a) We have to explain the properties of the corresponding unit eigenvectors $u, v, w$.
Since $A$ is symmetric.
So the eigenvectors corresponding to different eigenvalues are orthogonal.
Therefore here $u, v, w$ are unit eigenvectors corresponding to eigenvalues $0, 1, 2$.
Hence $u, v, w$ are orthogonal to each other.

Provide feedback (0)

**Step 2** of 5

b) We have to describe the Nullspace, left Nullspace, row space and column space of $A$ in terms of $u, v, w$.
Since $u$ is the eigenvector corresponding to the eigenvalue $\lambda = 0$.
Therefore the null space is spanned by $u$.
The left null space is also spanned by $u$.
The remaining two eigenvectors $v, w$ corresponding to the eigenvalues $\lambda = 1, 2$ spans the row space as well as column space.

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**Step 3** of 5

c) We have to find a vector $x$ that satisfies $Ax = v + w$
We have the eigenvalue of $v$ is 1 and the eigenvalue of $w$ is 2.
Therefore, $Av = v, Aw = 2w$
So we take $x$ as $x = 1 \cdot v + 2 \cdot w$
This solution is not unique because we can add any multiple of $u$ to $x$.

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**Step 4** of 5

d) We have to find the conditions on $b$ such that the solution to $Ax = b$ have a

CH5.5 14P

**Step 4** of 5

d) We have to find the conditions on $b$ such that the solution to $Ax = b$ have a solution.
Since $A$ is symmetric
So $A = A^T$
Since $\lambda$ is an eigenvalue and $b$ is a solution.
So $\lambda$ and $b$ are real.
We have the eigenvectors are perpendicular.
Therefore, $xb^T \cdot u = 0$
Therefore if $b^T u = 0$ then $Ax = b$ has solution.

Provide feedback (0)

**Step 5** of 5

e) We have to find $S^{-1}$ and $S^{-1}AS$ if $u, v, w$ are the columns of $S$.
Since $A$ is real and symmetric.
So the eigenvectors are also real.
These orthonormal eigenvectors form an orthogonal matrix $S$ with
$S^T S = I$
$\Rightarrow S^T = S^{-1}$
And $S^{-1}AS = \wedge = diag(0,1,2)$
Therefore, $\boxed{S^T = S^{-1} \text{ and } S^{-1}AS = diag(0,1,2)}$

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CH5.5 15P

**Step 1** of 10

Consider the following matrices:

$$A=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$$

$$B=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$$

Let following are the classes of matrices:

Orthogonal, invertible, projection, permutation, Hermitian, rank-1, diagonalizable and Markov matrices. Determine $A$ and $B$ belongs to which classes of matrices. Also find the Eigen values of $A$ and $B$.

Provide feedback (0)

**Step 2** of 10

*Orthogonal*: If $PP^T=I$, then $P$ is said to be orthogonal matrix.

$$\begin{aligned}AA^T&=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\cdot\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\\&=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\\&=I\end{aligned}$$

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Step 2 of 10

*Orthogonal*: If $PP^T = I$, then $P$ is said to be orthogonal matrix.

$$\begin{aligned} AA^T &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

$$\begin{aligned} BB^T &= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ &= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ &\neq I \end{aligned}$$

Therefore, only matrix $\boxed{A \text{ is orthogonal}}$.

*Invertible*: Orthogonal matrices are always invertible. As $A^{-1} = A^T$. Therefore matrix $A$ is $\boxed{\text{invertible}}$, but matrix $\boxed{B \text{ is not invertible}}$.

Provide feedback (0)

Provide feedback (0)

Step 3 of 10

*Projection*: If $P^2 = P$, then $P$ is said to be projection matrix.

$$\begin{aligned} A \cdot A &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\ &\neq A \end{aligned}$$

$$\begin{aligned} B \cdot B &= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ &= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ &= B \end{aligned}$$

Therefore, only matrix $\boxed{B \text{ is a projection}}$ matrix.

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Step 4 of 10

*Permutation* matrix: Every row and column contains a single 1 with 0's everywhere else.

Therefore, only matrix $\boxed{A \text{ is a permutation}}$ matrix.

Provide feedback (0)

Step 4 of 10

*Permutation* matrix: Every row and column contains a single 1 with 0's everywhere else.

Therefore, only matrix $\boxed{A \text{ is a permutation}}$ matrix.

Provide feedback (0)

Step 5 of 10

*Rank*-1 matrix: One row is non-zero after reduction.

Matrix $A$ cannot be reduced in one non-zero row. Matrix $A$ has rank 4. But matrix $B$ can be reduced into one non-zero row.

Therefore, $\boxed{\text{matrix B is rank-1 matrix}}$.

Provide feedback (0)

Step 6 of 10

*Hermitian*: If $P^H = P$, then $P$ is said to be Hermitian matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A^H = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\neq A$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

Provide feedback (0)

Step 6 of 10

*Hermitian*: If $P^H = P$, then $P$ is said to be Hermitian matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A^H = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\neq A$$

$$B = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$B^H = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= B$$

Therefore, matrix $A$ is not Hermitian while matrix $B$ is Hermitian.

Provide feedback (0)

Step 7 of 10

*Diagonalizable*: Sufficient Eigen vectors make the matrix diagonalizable.

Matrix $B$ has only two Eigen values and corresponding to that it has two Eigen vectors,

Provide feedback (0)

Step 7 of 10

*Diagonalizable*: Sufficient Eigen vectors make the matrix diagonalizable.

Matrix $B$ has only two Eigen values and corresponding to that it has two Eigen vectors, which are not sufficient to diagonalize the matrix.

Therefore, matrix $\boxed{B \text{ is not diagonalizable}}$.

Whereas matrix $A$ has sufficient Eigen vectors corresponding to all Eigen values. Therefore, matrix $\boxed{A \text{ is diagonalizable.}}$

Provide feedback (1)

Step 8 of 10

*Markov* matrix: In Markov matrix every element of the matrix is positive and sum of the column elements equals to 1.

Every element of both matrices is positive and individual sum of the columns equal to 1. Therefore, both matrices $\boxed{A \text{ and } B \text{ are Markov}}$ matrices.

Provide feedback (0)

Step 9 of 10

*Eigen values*: To calculate the Eigen values do the following calculations:

$$A-\lambda I=\begin{bmatrix} 0-\lambda & 1 & 0 & 0 \\ 0 & 0-\lambda & 1 & 0 \\ 0 & 0 & 0-\lambda & 1 \\ 1 & 0 & 0 & 0-\lambda \end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(\lambda)^4-1=0$$

Step 9 of 10

*Eigen values*: To calculate the Eigen values do the following calculations:

$$A-\lambda I=\begin{bmatrix}0-\lambda & 1 & 0 & 0\\ 0 & 0-\lambda & 1 & 0\\ 0 & 0 & 0-\lambda & 1\\ 1 & 0 & 0 & 0-\lambda\end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$(\lambda)^4-1=0$$

After solving following Eigen values are obtained:

$$\boxed{\begin{aligned}\lambda_1&=1\\ \lambda_2&=-1\\ \lambda_3&=i\\ \lambda_4&=-i\end{aligned}}$$

Provide feedback (0)

Step 10 of 10

Similarly, Eigen values of matrix $B$ will be:

$$B-\lambda I=\frac{1}{4}\begin{bmatrix}1-\lambda & 1 & 1 & 1\\ 1 & 1-\lambda & 1 & 1\\ 1 & 1 & 1-\lambda & 1\\ 1 & 1 & 1 & 1-\lambda\end{bmatrix}$$

$$\det(B-\lambda I)=0$$

$$\lambda^4-\lambda^3=0$$

After solving following Eigen values are obtained:

$$\det(A-\lambda I)=0$$
$$(\lambda)^4-1=0$$

After solving following Eigen values are obtained:

$$\boxed{\begin{aligned}\lambda_1&=1\\ \lambda_2&=-1\\ \lambda_3&=i\\ \lambda_4&=-i\end{aligned}}$$

Provide feedback (0)

**Step 10** of 10

Similarly, Eigen values of matrix $B$ will be:

$$B-\lambda I=\frac{1}{4}\begin{bmatrix}1-\lambda & 1 & 1 & 1\\ 1 & 1-\lambda & 1 & 1\\ 1 & 1 & 1-\lambda & 1\\ 1 & 1 & 1 & 1-\lambda\end{bmatrix}$$
$$\det(B-\lambda I)=0$$
$$\lambda^4-\lambda^3=0$$

After solving following Eigen values are obtained:

$$\boxed{\begin{aligned}\lambda_1&=1\\ \lambda_2&=0\end{aligned}}$$

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CH5.5 14P

CH5.5 16P

**Step 1** of 2

A real symmetric matrix *A* is given by

$$A^H = A$$

We know that the entries of a real symmetric matrix are symmetric to the diagonal.

$$a_{ij} = a_{ji}$$

Consider an *n* by *n* symmetric matrix, then there are *n* entries on the diagonal and $(n-1)+\cdots+1$ entries above the diagonal that can be chosen arbitrary.

Since, other $1+\cdots+(n-1)$ entries below the diagonal are determined by the symmetry of the matrix.

So, there are only

$$n+(n-1)+\cdots+1=\frac{n(n+1)}{2}$$

degrees of freedom in the selection of the $n^2$ entries in an *n* by *n* real symmetric matrix *S*.

Therefore, dimension of an *n* by *n* symmetric matrix is $\boxed{\frac{n(n+1)}{2}}$.

Provide feedback (0)

Provide feedback (0)

**Step 2** of 2

If $A$ is a real symmetric matrix and $Q$ is an orthogonal matrix, then we can write:

$$\begin{aligned} A &= A^H \\ &= QAQ^H \\ &= \begin{bmatrix} q_1 & \cdots & q_1 \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} q_1^H \\ \vdots \\ q_n^H \end{bmatrix} \\ &= q_1 q_1^H \lambda_1 + \cdots + q_n q_n^H \lambda_n \end{aligned}$$

Here matrix $q_n q_n^H$ is the projection matrix onto $q_n$ , every symmetric matrix $A$ is a combination of $n$ projections matrices.

If matrix $A$ changes then the projections also changes.

Since the dimension exceeds $n$, there is no basis of $n$ fixed projection matrices, in the space $S$ of symmetric matrices.

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Step 1 of 1

One significant fact about the Eigen values of each of the following are as follows:

(a) A real symmetric matrix: Hermitian matrix or real symmetric matrix has all Eigen values real.

(b) A stable matrix: Stability is governed by the real parts of the Eigen values. All solutions of $du/dt = Au$ approaches to zero if and only if all Eigen values satisfy $\mathrm{Re}(\lambda) < 0$.

(c) An orthogonal matrix: Eigen values of orthogonal matrix has absolute value 1, $|\lambda| = 1$.

(d) A Markov matrix: One Eigen value of matrix $A$ is 1, $\lambda_1 = 1$ and other Eigen values satisfy $|\lambda_i| \le 1$.

(e) A defective matrix: Matrices that can not be diagonalize are called as defective matrix. These matrices have repeated Eigen values.

(f) A singular matrix: A singular matrix with zero determinant has one or more of its Eigen values equal to zero.

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**Step 1** of 2

Given that $U, V$ are unitary matrices

We have to show that $UV$ is also unitary.

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CH5.5 18P

**Step 2** of 2

Since $U$, $V$ are unitary matrices.

$\Rightarrow U^H U = I$ and $V^H V = I$ ..... (1)

Now

$$\begin{aligned}(UV)^H (UV) &= \left(V^H U^H\right)(UV) \\ &= V^H \left(U^H U\right) V \\ &= V^H (IV) \qquad \left(\text{Since by } (1)\right) \\ &= V^H V \\ &= I \qquad \left(\text{Since by } (1)\right)\end{aligned}$$

Therefore $\boxed{UV \text{ is also unitary}}$.

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**Step 1** of 4

Let $U$ be and unitary matrix.
We have to show that $|\det U| = 1$.

Provide feedback (0)

**Step 2** of 4

Since $U$ is unitary
Then $U^H U = I$
Let $\lambda$ be the eigenvalue of $U$.
Then $Ux = \lambda x$

$$\begin{aligned}
\text{But } \|Ux\| &= \|x\| \qquad (\text{Since } U \text{ is unitary}) \\
\Rightarrow \|x\| &= \|\lambda x\| \qquad (\text{Since } Ux = \lambda x) \\
&= |\lambda|\|x\| \\
\Rightarrow |\lambda| &= 1
\end{aligned}$$

Therefore, every eigenvalue of $U$ has absolute value $|\lambda| = 1$.

Provide feedback (0)

**Step 3** of 4

Now

$$\begin{aligned}
|\det U| &= |\text{ Product of eigen values}| \\
&= |\lambda| \\
&= 1
\end{aligned}$$

Hence $\boxed{|\det U| = 1}$ if $U$ is unitary.

Provide feedback (0)

**Step 4** of 4

CH5.5 19P

Since $U$ is unitary
Then $U^H U = I$
Let $\lambda$ be the eigenvalue of $U$.
Then $Ux = \lambda x$
But $\|Ux\| = \|x\|$ (Since $U$ is unitary)

$$\Rightarrow \|x\| = \|\lambda x\| \quad (\text{Since } Ux = \lambda x)$$
$$= |\lambda|\|x\|$$
$$\Rightarrow |\lambda| = 1$$

Therefore, every eigenvalue of $U$ has absolute value $|\lambda| = 1$.

Provide feedback (0)

**Step 3** of 4

Now

$$|\det U| = |\text{ Product of eigen values}|$$
$$= |\lambda|$$
$$= 1$$

Hence $\boxed{|\det U| = 1}$ if $U$ is unitary.

Provide feedback (0)

**Step 4** of 4

We have to describe all the 2 b y 2 matrices that are unitary.
The columns, rows and eigenvalues of a unitary are orthogonal and orthonormal and every eigenvalue $\lambda$ have absolute value $|\lambda| = 1$.

Hence the $2\times 2$ unitary matrix is of the form $\boxed{U = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}}$ bring the eigenvalues $e^{it}$ and $e^{-it}$.

Provide feedback (0)

CH5.5 18P

**Step 1** of 7

Let unitary matrix $U$ is defined as follows:

$$U=\begin{bmatrix} 1/\sqrt{3} & i/\sqrt{2} & a \\ 1/\sqrt{3} & 0 & b \\ i/\sqrt{3} & 1/\sqrt{2} & c \end{bmatrix}$$

Find the values of $a, b$ and $c$. Also find the freedom in column 3.

Provide feedback (0)

**Step 2** of 7

Recall that Unitary matrix has orthonormal columns. So, each column of unitary matrix must be orthogonal to each other columns. Let $C_1, C_2$ and $C_3$ be the column of unitary matrix. Dot product of each column will be zero.

$$\begin{aligned} C_1^H \cdot C_2 &= \left(1/\sqrt{3}\right)\cdot\left(i/\sqrt{2}\right)+\left(1/\sqrt{3}\right)\cdot(0)+\left(-i/\sqrt{3}\right)\cdot\left(1/\sqrt{2}\right) \\ &= i/\sqrt{6}-i/\sqrt{6} \\ &= 0 \end{aligned}$$

$$\begin{aligned} C_2^H \cdot C_3 &= \left(-i/\sqrt{2}\right)\cdot a+(0)\cdot b+\left(1/\sqrt{2}\right)\cdot c \\ &= \frac{-ia+c}{\sqrt{2}} \end{aligned}$$

However, $C_2^H \cdot C_3 = 0$. So,

$$\begin{aligned} C_2^H \cdot C_3 &= 0 \\ \frac{-ia+c}{\sqrt{2}} &= 0 \\ -ia+c &= 0 \\ c &= ia \end{aligned}$$

CH5.5 20P

$$C_2^H \cdot C_3 = \left(-i/\sqrt{2}\right)\cdot a + (0)\cdot b + \left(1/\sqrt{2}\right)\cdot c$$
$$= \frac{-ia+c}{\sqrt{2}}$$

However, $C_2^H \cdot C_3 = 0$. So,

$$C_2^H \cdot C_3 = 0$$
$$\frac{-ia+c}{\sqrt{2}} = 0$$
$$-ia+c = 0$$
$$c = ia$$

This gives one equation in unknown values.

Provide feedback (0)

Step 3 of 7

Similarly, dot product of other two columns will also give an equation in unknown values.

$$C_1^H \cdot C_3 = \left(1/\sqrt{3}\right)\cdot(a) + \left(1/\sqrt{3}\right)\cdot(b) + \left(-i/\sqrt{3}\right)\cdot(c)$$
$$= \frac{a+b-ic}{\sqrt{3}}$$

However, $C_1^H \cdot C_3 = 0$. So,

$$C_1^H \cdot C_3 = 0$$
$$\frac{a+b-ic}{\sqrt{3}} = 0$$
$$a+b-ic = 0$$
$$a+b = ic$$

Or

Similarly, dot product of other two columns will also give an equation in unknown values.

$$\begin{aligned} C_1^H \cdot C_3 &= \left(1/\sqrt{3}\right)\cdot(a)+\left(1/\sqrt{3}\right)\cdot(b)+\left(-i/\sqrt{3}\right)\cdot(c) \\ &= \frac{a+b-ic}{\sqrt{3}} \end{aligned}$$

However, $C_1^H \cdot C_3 = 0$. So,

$$\begin{aligned} C_1^H \cdot C_3 &= 0 \\ \frac{a+b-ic}{\sqrt{3}} &= 0 \\ a+b-ic &= 0 \\ a+b &= ic \end{aligned}$$

Or

$$\begin{aligned} a+b &= i(ia) \\ b &= -a-a \\ &= -2a \end{aligned}$$

Provide feedback (0)

**Step 4** of 7

Every vector in unitary matrix is orthonormal. Therefore,

$$\begin{aligned} C_3 \cdot C_3 &= 1 \\ |a|^2+|b|^2+|c|^2 &= 1 \end{aligned}$$

Substitute the values of $b$ and $c$ from the above two equations and solve them to get the value of one unknown:

$$\begin{aligned} |a|^2+|-2a|^2+|ia|^2 &= 1 \\ 6|a|^2 &= 1 \end{aligned}$$

$$b = -a - a$$
$$= -2a$$

Provide feedback (0)

**Step 4** of 7

Every vector in unitary matrix is orthonormal. Therefore,

$$C_3 \cdot C_3 = 1$$
$$|a|^2 + |b|^2 + |c|^2 = 1$$

Substitute the values of $b$ and $c$ from the above two equations and solve them to get the value of one unknown:

$$|a|^2 + |-2a|^2 + |ia|^2 = 1$$
$$6|a|^2 = 1$$
$$a = \frac{1}{\sqrt{6}}$$

Provide feedback (0)

**Step 5** of 7

Therefore, values of other unknown are:

$$a = \boxed{\frac{1}{\sqrt{6}}}$$
$$b = -2a$$
$$= \boxed{\frac{-2}{\sqrt{6}}}$$
$$c = ia$$
$$= \boxed{\frac{i}{\sqrt{6}}}$$

Therefore, values of other unknown are:

$$a = \boxed{\frac{1}{\sqrt{6}}}$$

$$b = -2a$$

$$= \boxed{\frac{-2}{\sqrt{6}}}$$

$$c = ia$$

$$= \boxed{\frac{i}{\sqrt{6}}}$$

Provide feedback (0)

**Step 6** of 7

To find the freedom of a column 3 find the independent constraints.

First constraint: Dot product with column $C_1$ will be zero, $C_1 \cdot C_3 = 0$.
Second constraint: Dot product with column $C_2$ will be zero, $C_2 \cdot C_3 = 0$.
Third constraint: Unitary length $C_3 \cdot C_3 = 1$.

Provide feedback (0)

**Step 7** of 7

Total, 3 constraints are applicable. Therefore, degree of freedom of a column 3 is $\boxed{3.}$

Provide feedback (0)

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Scotty G.

Andrea M.

CH5.5 19P

Step 1 of 9

Consider the following skew-Hermitian matrix:

$$K = \begin{bmatrix} i & i \\ i & i \end{bmatrix}$$

Diagonalize matrix $K$ and compute $e^{Kt} = Se^{\Lambda t}S^{-1}$. Verify that $e^{Kt}$ is unitary. Also find the derivative of $e^{Kt}$ at $t = 0$.

Provide feedback (0)

Step 2 of 9

First step is to find the Eigen values of matrix $K$. To calculate the Eigen values do the following calculations:

$$K - \lambda I = \begin{bmatrix} i-\lambda & i \\ i & i-\lambda \end{bmatrix}$$

$$\det(K - \lambda I) = 0$$

$$(i-\lambda)^2 + 1 = 0$$

$$(\lambda^2 - 2\lambda i) = 0$$

After solving following values are obtained:

$$\lambda_1 = 2i$$

$$\lambda_2 = 0$$

Provide feedback (0)

Step 3 of 9

To calculate Eigen vectors do the following calculations:

$$(K - \lambda I)x = 0$$

CH5.5 21P

Provide feedback (0)

Step 3 of 9

To calculate Eigen vectors do the following calculations:

$$(K-\lambda I)x=0$$

$$\begin{bmatrix} i-\lambda & i \\ i & i-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -i & i \\ i & -i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of $y$ and $z$ corresponding to $\lambda=2i$ are as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 9

Similarly, Eigen vector corresponding to Eigen value $\lambda=0$ is as follows:

$$(K-\lambda I)x=0$$

$$\begin{bmatrix} i-\lambda & i \\ i & i-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & i \\ i & i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$\begin{bmatrix} y \end{bmatrix}$$

$$\begin{bmatrix}1\end{bmatrix}$$

Provide feedback (0)

**Step 4** of 9

Similarly, Eigen vector corresponding to Eigen value $\lambda = 0$ is as follows:

$$\begin{aligned}(K-\lambda I)x &= 0\\ \begin{bmatrix} i-\lambda & i \\ i & i-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}\\ \begin{bmatrix} i & i \\ i & i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}\end{aligned}$$

On solving values of $y$ and $z$ are as follows:

$$\begin{aligned}x_2 &= \begin{bmatrix} y \\ z \end{bmatrix}\\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}\end{aligned}$$

Provide feedback (0)

**Step 5** of 9

Therefore, diagonalisation of a matrix can be done as below:

$$\begin{aligned}K &= S\Lambda S^{-1}\\ &= \boxed{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2i & 0 \\ 0 & 0 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}\end{aligned}$$

Provide feedback (0)

**Step 6** of 9

$$= \boxed{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2i & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$$

Provide feedback (0)

**Step 6** of 9

Now, compute the following:

$$\begin{aligned} e^{Kt} &= Se^{\Lambda t}S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{2it} & 0 \\ 0 & e^{0t} \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{2it} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} e^{2it}+1 & e^{2it}-1 \\ e^{2it}-1 & e^{2it}+1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

**Step 7** of 9

Therefore,

$$\boxed{e^{Kt} = \frac{1}{2} \begin{bmatrix} e^{2it}+1 & e^{2it}-1 \\ e^{2it}-1 & e^{2it}+1 \end{bmatrix}}$$

Provide feedback (0)

**Step 8** of 9

Recall that a matrix $U$ is unitary if $U^H U = I$. To verify that $e^{Kt}$ is unitary, do the following calculations:

$$(e^{Kt})^H e^{Kt} = \frac{1}{2} \begin{bmatrix} e^{-2it}+1 & e^{-2it}-1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} e^{2it}+1 & e^{2it}-1 \end{bmatrix}$$

Provide feedback (0)

Step 8 of 9

Recall that a matrix $U$ is unitary if $U^H U = I$. To verify that $e^{Kt}$ is unitary, do the following calculations:

$$\begin{aligned}
\left(e^{Kt}\right)^H e^{Kt} &= \frac{1}{2}\begin{bmatrix} e^{-2it}+1 & e^{-2it}-1 \\ e^{-2it}-1 & e^{-2it}+1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} e^{2it}+1 & e^{2it}-1 \\ e^{2it}-1 & e^{2it}+1 \end{bmatrix} \\
&= \frac{1}{4}\begin{bmatrix} e^{-2it}+1 & e^{-2it}-1 \\ e^{-2it}-1 & e^{-2it}+1 \end{bmatrix}\begin{bmatrix} e^{2it}+1 & e^{2it}-1 \\ e^{2it}-1 & e^{2it}+1 \end{bmatrix} \\
&= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\
&= I
\end{aligned}$$

Therefore, $\boxed{e^{Kt} \text{ is unitary}}$.

Provide feedback (0)

Step 9 of 9

Derivative of $e^{Kt}$ at $t=0$ will be:

$$\begin{aligned}
e^{Kt} &= \frac{1}{2}\begin{bmatrix} e^{2it}+1 & e^{2it}-1 \\ e^{2it}-1 & e^{2it}+1 \end{bmatrix} \\
\frac{de^{Kt}}{dt} &= \frac{1}{2}\begin{bmatrix} 2ie^{2it} & 2ie^{2it} \\ 2ie^{2it} & 2ie^{2it} \end{bmatrix} \\
\left(\frac{de^{Kt}}{dt}\right)_{t=0} &= \begin{bmatrix} i & i \\ i & i \end{bmatrix} \\
&= K
\end{aligned}$$

Therefore, derivative of $e^{Kt}$ at $t=0$ is $\boxed{K \text{ matrix.}}$

Provide feedback (0)

CH5.5 20P

**Step 1** of 5

We have to describe all the 3 by 3 matrices that are simultaneously Hermitian, unitary and diagonal.

Provide feedback (0)

CH5.5 22P

**Step 2** of 5

Let $A$ be any $3\times3$ matrix.
Suppose $A$ is Hermitian, unitary and diagonal.
Since $A$ is diagonal.
So all the elements are zeros except the diagonal elements.

Provide feedback (0)

**Step 3** of 5

Since $A$ is Hermitian
$\Rightarrow A^H = A$
$\Rightarrow$ The diagonal elements must be real since any eigenvalue is real.
Since $A$ is unitary
So every eigenvalue of $A$ has absolute value $|\lambda| = \pm1$.
Since the absolute value of the eigenvalue is $|\lambda| = \pm1$
So this is possible for the matrices having diagonal elements $\pm1$ or $-1$ and the remaining elements are zeros.

Provide feedback (0)

**Step 4** of 5

The possible matrices with the above property are 8.
They are

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}0&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$$

So this is possible for the matrices having diagonal elements $\pm 1$ or $-1$ and the remaining elements are zeros.

Provide feedback (0)

**Step 4** of 5

The possible matrices with the above property are 8.
They are

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}0&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$$

$$\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$$

Provide feedback (0)

**Step 5** of 5

Hence the matrices those are simultaneously Hermitian, unitary and diagonal are

$$\boxed{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}0&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\\ \begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}}$$

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CH5.5 21P

**Step 1** of 11

Given that every matrix $Z$ can be split into a Hermitian part $A$ and a skew-Hermitian part $K$, that is $Z = A + K$.

Also given that the real part of $Z$ is half of $Z + Z^H$.

Given $Z = \begin{bmatrix} 3+i & 4+2i \\ 0 & 5 \end{bmatrix}$ and $z = \begin{bmatrix} i & i \\ -i & i \end{bmatrix}$

We have to find a formula for the imaginary part $K$ and split the given matrices into $Z = A + K$.

Provide feedback (0)

**Step 2** of 11

Here $A$ is Hermitian and $K$ is skew-Hermitian.

We have

$$A = \frac{1}{2}\left(Z + Z^H\right)$$

$$K = \frac{1}{2}\left(Z - Z^H\right)$$

Now we find the matrices $A$ and $K$.

We have $Z = \begin{bmatrix} 3+i & 4+2i \\ 0 & 5 \end{bmatrix}$

Then $Z^H = \begin{bmatrix} 3-i & 0 \\ 4-2i & 5 \end{bmatrix}$

Provide feedback (0)

**Step 3** of 11

Now

$$A = \frac{1}{2}\left(Z + Z^H\right)$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3+i & 4+2i \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 3-i & 0 \\ 4-2i & 5 \end{bmatrix}\right\}$$

CH5.5 23P

Then $Z^H = \begin{bmatrix} 3-i & 0 \\ 4-2i & 5 \end{bmatrix}$

Provide feedback (0)

**Step 3** of 11

Now

$$A = \frac{1}{2}\left(Z + Z^H\right)$$
$$= \frac{1}{2}\left\{\begin{bmatrix} 3+i & 4+2i \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 3-i & 0 \\ 4-2i & 5 \end{bmatrix}\right\}$$
$$= \frac{1}{2}\begin{bmatrix} 3+i+3-i & 4+2i+0 \\ 0+4-2i & 5+5 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 6 & 4+2i \\ 4-2i & 10 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 2+i \\ 2-i & 5 \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 11

Now $\overline{A} = \begin{bmatrix} 3 & 2-i \\ 2+i & 5 \end{bmatrix}$

And $A^H = \overline{A}^T = \begin{bmatrix} 3 & 2+i \\ 2-i & 5 \end{bmatrix}$

Therefore, $A^H = A$

Hence $A$ is Hermitian.

Provide feedback (0)

**Step 5** of 11

Now

**Step 5** of 11

Now

$$K=\frac{1}{2}\left(Z-Z^{H}\right)$$

$$=\frac{1}{2}\left\{\begin{bmatrix}3+i & 4+2i\\ 0 & 5\end{bmatrix}-\begin{bmatrix}3-i & 0\\ 4-2i & 5\end{bmatrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}3+i-3+i & 4+2i-0\\ 0-4+2i & 5-5\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}2i & 4+2i\\ -4+2i & 0\end{bmatrix}$$

$$=\begin{bmatrix}i & 2+i\\ -2+i & 0\end{bmatrix}$$

Provide feedback (0)

**Step 6** of 11

Now $\overline{K}=\begin{bmatrix}-i & 2-i\\ -2-i & 0\end{bmatrix}$

And $K^{H}=\overline{K}^{T}=\begin{bmatrix}-i & -2-i\\ 2-i & 0\end{bmatrix}$

Now

$$-K^{H}=-\begin{bmatrix}-i & -2-i\\ 2-i & 0\end{bmatrix}$$

$$=\begin{bmatrix}i & 2+i\\ -2+i & 0\end{bmatrix}$$

Since $K=-K^{H}$

So $K$ is a skew-symmetric matrix.

Therefore, $\boxed{Z=\begin{bmatrix}3+i & 4+2i\\ 0 & 5\end{bmatrix}=\begin{bmatrix}3 & 2+i\\ 2-i & 5\end{bmatrix}+\begin{bmatrix}i & 2+i\\ -2+i & 0\end{bmatrix}}$

Provide feedback (0)

Provide feedback (0)

**Step 7** of 11

Now we have to write $Z = \begin{bmatrix} i & i \\ -i & i \end{bmatrix}$ as a sum of Hermitian and skew-Hermitian matrices.

Given $Z = \begin{bmatrix} i & i \\ -i & i \end{bmatrix}$

$Z$ can be split into $Z = A + K$

Here A is Hermitian and K is a skew-Hermitian matrices.

Provide feedback (0)

**Step 8** of 11

Since $Z = \begin{bmatrix} i & i \\ -i & i \end{bmatrix}$

So $Z^H = \begin{bmatrix} -i & i \\ -i & -i \end{bmatrix}$

$$A = \frac{1}{2}\left\{Z + Z^H\right\}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} i & i \\ -i & i \end{bmatrix} + \begin{bmatrix} -i & i \\ -i & -i \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} i-i & i+i \\ -i-i & i-i \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 2i \\ -2i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$

Provide feedback (0)

Provide feedback (0)

**Step 9** of 11

Now $\overline{A} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

And $A^H = \overline{A}^T = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$

Therefore, $A^H = A$

Hence $A$ is Hermitian.

Provide feedback (0)

**Step 10** of 11

Now

$$\begin{aligned} K &= \frac{1}{2}\left\{Z - Z^H\right\} \\ &= \frac{1}{2}\left\{\begin{bmatrix} i & i \\ -i & i \end{bmatrix} - \begin{bmatrix} -i & i \\ -i & -i \end{bmatrix}\right\} \\ &= \frac{1}{2}\begin{bmatrix} i+i & i-i \\ -i+i & i+i \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 2i & 0 \\ 0 & 2i \end{bmatrix} \\ &= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \end{aligned}$$

Provide feedback (0)

**Step 11** of 11

Now $\overline{K} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

**Step 10** of 11

Now

$$K = \frac{1}{2}\left\{Z - Z^H\right\}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} i & i \\ -i & i \end{bmatrix} - \begin{bmatrix} -i & i \\ -i & -i \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} i+i & i-i \\ -i+i & i+i \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2i & 0 \\ 0 & 2i \end{bmatrix}$$

$$= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

Provide feedback (0)

**Step 11** of 11

Now $\overline{K} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

And $K^H = \overline{K}^T = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

Now

$$-K^H = -\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

Since $K = -K^H$

So $K$ is a skew-symmetric matrix.

Therefore, $\boxed{Z = \begin{bmatrix} i & i \\ -i & i \end{bmatrix} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} + \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}}$

Provide feedback (0)

CH5.5 22P

**Step 1** of 22

Given that $P=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$ and $U=\begin{bmatrix}1&1&1&1\\1&\omega&\omega^2&\omega^3\\1&\omega^2&\omega^3&\omega\\1&\omega^3&\omega&\omega^2\end{bmatrix}$

We have to show that the columns of the matrix $U$ are the eigenvectors of $P$.

Provide feedback (0)

**Step 2** of 22

We have to find the eigenvalues of $P$.

The characteristic equation of $P$ is

$$|P-\lambda I|=0$$

$$\Rightarrow\begin{vmatrix}-\lambda&1&0&0\\0&-\lambda&1&0\\0&0&-\lambda&1\\1&0&0&-\lambda\end{vmatrix}=0$$

$$\Rightarrow-\lambda\begin{vmatrix}-\lambda&1&0\\0&-\lambda&1\\0&0&-\lambda\end{vmatrix}-1\begin{vmatrix}1&1&0\\0&-\lambda&1\\1&0&-\lambda\end{vmatrix}=0$$

Provide feedback (0)

**Step 3** of 22

Continuation to the above

$$\Rightarrow-\lambda\left\{-\lambda\left(\lambda^2\right)\right\}-1\left\{-1(-1)\right\}=0$$

$$\Rightarrow\lambda^4-1=0$$

$$\Rightarrow\lambda^4=1$$

The $\lambda$ values are $4^{\text{th}}$ roots of unity they are $\lambda=1,\omega,\omega^2,\omega^3$

Hence the eigenvalues of $P$ are $\lambda=1,\omega,\omega^2,\omega^3$

CH5.5 24P

Provide feedback (0)

**Step 3** of 22

Continuation to the above

$$\Rightarrow -\lambda\left\{-\lambda\left(\lambda^2\right)\right\}-1\left\{-1(-1)\right\}=0$$

$$\Rightarrow \lambda^4-1=0$$

$$\Rightarrow \lambda^4=1$$

The $\lambda$ values are $4^{\text{th}}$ roots of unity they are $\lambda=1,\omega,\omega^2,\omega^3$

Hence the eigenvalues of $P$ are $\lambda=1,\omega,\omega^2,\omega^3$

Provide feedback (0)

**Step 4** of 22

We know that $x=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$ is an eigenvector of $P$ if and only if $x=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$ is the nonzero solution of the system $(P-\lambda I)x=0$.

That is $\begin{bmatrix}-\lambda & 1 & 0 & 0\\0 & -\lambda & 1 & 0\\0 & 0 & -\lambda & 1\\1 & 0 & 0 & -\lambda\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=0$ ..... (1)

Provide feedback (0)

**Step 5** of 22

For $\lambda=1$, (1) becomes

$$(P-I)x=0$$

$$\begin{bmatrix}-1 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\end{bmatrix}$$

$(P-I)x=0$

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=0$$

The Augmented matrix

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \end{bmatrix}=0$$

Provide feedback (0)

**Step 6** of 22

Add row 1 to row 4

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Add row 2 to row 4.

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 7** of 22

Add row 3 to row 4

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 7** of 22

Add row 3 to row 4

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From this, we get the equations

$-x_1 + x_2 = 0$

$-x_2 + x_3 = 0$

$-x_3 + x_4 = 0$

Here $x_4$ is free variable.

Provide feedback (0)

**Step 8** of 22

Let $x_4 = k$, where $k$ is a parameter.

Then $x_3 = k, x_2 = k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} k \\ k \\ k \\ k \end{bmatrix}$$

$$= k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

**Step 8** of 22

Let $x_4 = k$, where $k$ is a parameter.

Then $x_3 = k, x_2 = k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} k \\ k \\ k \\ k \end{bmatrix}$$

$$= k\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence the eigenvector corresponding to the eigenvalue $\lambda = 1$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 9** of 22

For $\lambda = \varpi$, (1) becomes

$(P - \varpi I)x = 0$

$$\begin{bmatrix} -\varpi & 1 & 0 & 0 \\ 0 & -\varpi & 1 & 0 \\ 0 & 0 & -\varpi & 1 \\ 1 & 0 & 0 & -\varpi \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

The Augmented matrix

Provide feedback (0)

**Step 9** of 22

For $\lambda = \varpi$, (1) becomes

$(P - \varpi I)x = 0$

$$\begin{bmatrix} -\varpi & 1 & 0 & 0 \\ 0 & -\varpi & 1 & 0 \\ 0 & 0 & -\varpi & 1 \\ 1 & 0 & 0 & -\varpi \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

The Augmented matrix

$$\begin{bmatrix} -\varpi & 1 & 0 & 0 & 0 \\ 0 & -\varpi & 1 & 0 & 0 \\ 0 & 0 & -\varpi & 1 & 0 \\ 1 & 0 & 0 & -\varpi & 0 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 10** of 22

Add row 1 to $\varpi$ times of row 4

$$\begin{bmatrix} -\varpi & 1 & 0 & 0 & 0 \\ 0 & -\varpi & 1 & 0 & 0 \\ 0 & 0 & -\varpi & 1 & 0 \\ 0 & 1 & 0 & -\varpi^2 & 0 \end{bmatrix} = 0$$

Add row 2 to $\varpi$ times of row 4

$$\begin{bmatrix} -\varpi & 1 & 0 & 0 & 0 \\ 0 & -\varpi & 1 & 0 & 0 \\ 0 & 0 & -\varpi & 1 & 0 \\ 0 & 0 & 1 & -\varpi^3 & 0 \end{bmatrix} = 0$$

Provide feedback (0)

Provide feedback (0)

**Step 11** of 22

Add row 3 to $\varpi$ times of row 4

$$\begin{bmatrix} -\varpi & 1 & 0 & 0 & 0 \\ 0 & -\varpi & 1 & 0 & 0 \\ 0 & 0 & -\varpi & 1 & 0 \\ 0 & 0 & 0 & -\varpi^4+1 & 0 \end{bmatrix} = 0$$

From this, we get the equations

$$-\varpi x_1 + x_2 = 0$$
$$-\varpi x_2 + x_3 = 0$$
$$-\varpi x_3 + x_4 = 0$$
$$\left(-\varpi^4+1\right) x_4 = 0$$

Provide feedback (0)

**Step 12** of 22

Choose $x_4 = \varpi^3 k$, where $k$ is a parameter.

Then we get $x_3 = \varpi^2 k, x_2 = \varpi k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$
$$= \begin{bmatrix} k \\ \varpi k \\ \varpi^2 k \\ \varpi^3 k \end{bmatrix}$$
$$= k \begin{bmatrix} 1 \\ \varpi \\ \varpi^2 \end{bmatrix}$$

**Step 12** of 22

Choose $x_4 = \varpi^3 k$, where $k$ is a parameter.

Then we get $x_3 = \varpi^2 k, x_2 = \varpi k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} k \\ \varpi k \\ \varpi^2 k \\ \varpi^3 k \end{bmatrix}$$

$$= k \begin{bmatrix} 1 \\ \varpi \\ \varpi^2 \\ \varpi^3 \end{bmatrix}$$

Hence the eigenvector corresponding to the eigenvalue $\lambda = \varpi$ is $\begin{bmatrix} 1 \\ \varpi \\ \varpi^2 \\ \varpi^3 \end{bmatrix}$.

Provide feedback (0)

**Step 13** of 22

For $\lambda = \varpi^2$, (1) becomes

$(P - \varpi^2 I)x = 0$

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 \\ 0 & 0 & -\varpi^2 & 1 \\ 1 & 0 & 0 & -\varpi^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

The Augmented matrix

Provide feedback (0)

**Step 13** of 22

For $\lambda = \varpi^2$, (1) becomes

$$\left(P - \varpi^2 I\right)x = 0$$

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 \\ 0 & 0 & -\varpi^2 & 1 \\ 1 & 0 & 0 & -\varpi^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

The Augmented matrix

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^2 & 1 & 0 \\ 1 & 0 & 0 & -\varpi^2 & 0 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 14** of 22

Add row 1 to $\varpi^2$ times of row 4

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^2 & 1 & 0 \\ 0 & 1 & 0 & -\varpi^4 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^2 & 1 & 0 \\ 0 & 1 & 0 & -\varpi & 0 \end{bmatrix} = 0 \qquad \left(\text{Since } \varpi^3 = 1\right)$$

Provide feedback (0)

Provide feedback (0)

**Step 15** of 22

Add row 2 to $\varpi^2$ times of row 4

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^2 & 1 & 0 \\ 0 & 0 & 0 & -\varpi^3 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 16** of 22

Add row 3 to $\varpi^2$ times of row 4

$$\begin{bmatrix} -\varpi^2 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^2 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^2 & 1 & 0 \\ 0 & 0 & 0 & -\varpi^2+1 & 0 \end{bmatrix} = 0$$

From this, we get the equations

$$-\varpi^2 x_1 + x_2 = 0$$

$$-\varpi^2 x_2 + x_3 = 0$$

$$-\varpi^2 x_3 + x_4 = 0$$

$$\left(-\varpi^2 + 1\right) x_4 = 0$$

Provide feedback (0)

**Step 17** of 22

**Step 17** of 22

Choose $x_4 = \varpi k$, where $k$ is a parameter.

Then we get $x_3 = \varpi^3 k, x_2 = \varpi^2 k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} k \\ \varpi^2 k \\ \varpi^3 k \\ \varpi k \end{bmatrix}$$

$$= k \begin{bmatrix} 1 \\ \varpi^2 \\ \varpi^3 \\ \varpi \end{bmatrix}$$

Hence the eigenvector corresponding to the eigenvalue $\lambda = \varpi^2$ is $\begin{bmatrix} 1 \\ \varpi^2 \\ \varpi^3 \\ \varpi \end{bmatrix}$.

Provide feedback (0)

**Step 18** of 22

For $\lambda = \varpi^3$, (1) becomes

$\left(P - \varpi^3 I\right) x = 0$

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 \\ 0 & 0 & -\varpi^3 & 1 \\ 1 & 0 & 0 & -\varpi^3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 18** of 22

For $\lambda = \varpi^3$, (1) becomes

$$\left(P - \varpi^3 I\right)x = 0$$

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 \\ 0 & 0 & -\varpi^3 & 1 \\ 1 & 0 & 0 & -\varpi^3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

The Augmented matrix

Provide feedback (0)

**Step 19** of 22

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 1 & 0 & 0 & -\varpi^3 & 0 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 20** of 22

Add row 1 to $\varpi^3$ times of row 4

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 1 & 0 & -\varpi^6 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 1 & 0 & 0 & -\varpi^3 & 0 \end{bmatrix} = 0$$

Provide feedback (0)

**Step 20** of 22

Add row 1 to $\varpi^3$ times of row 4

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 1 & 0 & -\varpi^6 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} = 0 \qquad \left(\text{Since } \varpi^6 = \left(\varpi^3\right)^2 = 1\right)$$

Provide feedback (0)

**Step 21** of 22

Add row 2 to $\varpi^3$ times of row 4

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 0 & 1 & -\varpi^3 & 0 \end{bmatrix} = 0$$

Add row 3 to $\varpi^3$ times of row 4

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 0 & 0 & -\varpi^6+1 & 0 \end{bmatrix} = 0$$

Add row 2 to $\varpi^3$ times of row 4

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 0 & 1 & -\varpi^3 & 0 \end{bmatrix} = 0$$

Add row 3 to $\varpi^3$ times of row 4

$$\begin{bmatrix} -\varpi^3 & 1 & 0 & 0 & 0 \\ 0 & -\varpi^3 & 1 & 0 & 0 \\ 0 & 0 & -\varpi^3 & 1 & 0 \\ 0 & 0 & 0 & -\varpi^6+1 & 0 \end{bmatrix} = 0$$

From this, we get the equations

$$-\varpi^3 x_1 + x_2 = 0$$
$$-\varpi^3 x_2 + x_3 = 0$$
$$-\varpi^3 x_3 + x_4 = 0$$
$$\left(-\varpi^6 + 1\right) x_4 = 0$$

Provide feedback (0)

**Step 22** of 22

Choose $x_4 = \varpi^2 k$ , where $k$ is a parameter.

Then we get $x_3 = \varpi k, x_2 = \varpi^3 k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$
$$= \begin{bmatrix} k \\ \varpi^3 k \\ \varpi k \\ \varpi^2 k \end{bmatrix}$$

Provide feedback (0)

Please tell us how this step can be improved

**Step 22** of 22

Choose $x_4 = \varpi^2 k$ , where $k$ is a parameter.

Then we get $x_3 = \varpi k, x_2 = \varpi^3 k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} k \\ \varpi^3 k \\ \varpi k \\ \varpi^2 k \end{bmatrix}$$

$$= k \begin{bmatrix} 1 \\ \varpi^3 \\ \varpi \\ \varpi^2 \end{bmatrix}$$

Hence the eigenvector corresponding to the eigenvalue $\lambda = \varpi^3$ is $\begin{bmatrix} 1 \\ \varpi^3 \\ \varpi \\ \varpi^2 \end{bmatrix}$.

Hence the columns of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \varpi & \varpi^2 & \varpi^3 \\ 1 & \varpi^2 & \varpi^3 & \varpi \\ 1 & \varpi^3 & \varpi & \varpi^2 \end{bmatrix}$ are the eigenvectors of the given permutation matrix

Then we get $x_3 = \omega k, x_2 = \omega^2 k, x_1 = k$

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} k \\ \omega^3 k \\ \omega k \\ \omega^2 k \end{bmatrix}$$

$$= k \begin{bmatrix} 1 \\ \omega^3 \\ \omega \\ \omega^2 \end{bmatrix}$$

Hence the eigenvector corresponding to the eigenvalue $\lambda = \omega^3$ is $\begin{bmatrix} 1 \\ \omega^3 \\ \omega \\ \omega^2 \end{bmatrix}$.

Hence the columns of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^3 & \omega \\ 1 & \omega^3 & \omega & \omega^2 \end{bmatrix}$ are the eigenvectors of the given permutation matrix.

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**Step 1** of 4

Consider the following orthogonal matrix:

$$P=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$$

Find the circulant matrix $C$ that satisfies the following relation:

$$C=c_0I+c_1P+c_2P^2+c_3P^3$$

Also, find the four components of product of vector $x$ and circulant matrix $C$.

Provide feedback (0)

**Step 2** of 4

Circulant matrix $C$ can be calculated as follows:

$$\begin{aligned}C&=c_0I+c_1P+c_2P^2+c_3P^3\\&=c_0\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}+c_1\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}+c_2\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}+c_3\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\\&=\begin{bmatrix}c_0&0&0&0\\0&c_0&0&0\\0&0&c_0&0\\0&0&0&c_0\end{bmatrix}+\begin{bmatrix}0&c_1&0&0\\0&0&c_1&0\\0&0&0&c_1\\c_1&0&0&0\end{bmatrix}+\begin{bmatrix}0&0&c_2&0\\0&0&0&c_2\\c_2&0&0&0\\0&c_2&0&0\end{bmatrix}+\begin{bmatrix}0&0&0&c_3\\c_3&0&0&0\\0&c_3&0&0\\0&0&c_3&0\end{bmatrix}\\&=\begin{bmatrix}c_0&c_1&c_2&c_3\\c_3&c_0&c_1&c_2\\c_2&c_3&c_0&c_1\end{bmatrix}\end{aligned}$$

Also, find the four components of product of vector $x$ and circulant matrix $C$.

Provide feedback (0)

**Step 2** of 4

Circulant matrix $C$ can be calculated as follows:

$$
\begin{aligned}
C &= c_0 I + c_1 P + c_2 P^2 + c_3 P^3 \\
&= c_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + c_1 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} c_0 & 0 & 0 & 0 \\ 0 & c_0 & 0 & 0 \\ 0 & 0 & c_0 & 0 \\ 0 & 0 & 0 & c_0 \end{bmatrix} + \begin{bmatrix} 0 & c_1 & 0 & 0 \\ 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & c_1 \\ c_1 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & c_2 \\ c_2 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & c_3 \\ c_3 & 0 & 0 & 0 \\ 0 & c_3 & 0 & 0 \\ 0 & 0 & c_3 & 0 \end{bmatrix} \\
&= \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix}
\end{aligned}
$$

Therefore, circulant matrix is defined as follows:

$$
\boxed{C = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix}}
$$

Provide feedback (0)

**Step 3** of 4

Let $x = (x_0, x_1, x_2, x_3)$ be the vector. Product of circulant matrix and vector $x$ will be as

$$= \begin{bmatrix} c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix}$$

Therefore, circulant matrix is defined as follows:

$$\boxed{C = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix}}$$

Provide feedback (0)

**Step 3** of 4

Let $x = (x_0, x_1, x_2, x_3)$ be the vector. Product of circulant matrix and vector $x$ will be as follows:

$$Cx = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3 \\ c_3x_0 + c_0x_1 + c_1x_2 + c_2x_3 \\ c_2x_0 + c_3x_1 + c_0x_2 + c_1x_3 \\ c_1x_0 + c_2x_1 + c_3x_2 + c_0x_3 \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 4

Therefore, the four components of product of vector $x$ and circulant matrix $C$ is:

$$\boxed{(c_0x_0 + c_1x_1 + c_2x_2 + c_3x_3), (c_3x_0 + c_0x_1 + c_1x_2 + c_2x_3)}$$

$$\begin{bmatrix} c_1 & c_2 & c_3 & c_0 \end{bmatrix}$$

Provide feedback (0)

**Step 3** of 4

Let $x=(x_0,x_1,x_2,x_3)$ be the vector. Product of circulant matrix and vector $x$ will be as follows:

$$\begin{aligned} Cx &= \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \begin{bmatrix} c_0x_0+c_1x_1+c_2x_2+c_3x_3 \\ c_3x_0+c_0x_1+c_1x_2+c_2x_3 \\ c_2x_0+c_3x_1+c_0x_2+c_1x_3 \\ c_1x_0+c_2x_1+c_3x_2+c_0x_3 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

**Step 4** of 4

Therefore, the four components of product of vector $x$ and circulant matrix $C$ is:

$$\boxed{\begin{matrix} (c_0x_0+c_1x_1+c_2x_2+c_3x_3),(c_3x_0+c_0x_1+c_1x_2+c_2x_3) \\ (c_2x_0+c_3x_1+c_0x_2+c_1x_3),(c_1x_0+c_2x_1+c_3x_2+c_0x_3) \end{matrix}}$$

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Rohan M

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**Step 1** of 2

Thus, we get

$$Ax = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + \dots + a_{1n}x_n \\ a_{21}x_1 + \dots + a_{2n}x_n \\ \vdots \\ a_{n1}x_1 + \dots + a_{nn}x_n \end{bmatrix}$$

Thus, in the right hand side of $Ax$, we see an $n$ by 1 matrix, whose each entry has $n$ products.

Therefore, to carry out $Ax$, we have to do $n^2$ number of products.

Provide feedback (0)

**Step 2** of 2

Now consider a circulant matrix $C = F\Lambda F^{-1}$.

From the Fast Fourier Transform, we know that to multiply $x$ by $F$ and $F^{-1}$, we need only $n \log n$ multiplications. Also, for multiplying by $\Lambda$, we need only additional $n$ multiplications. The sum of all these number of multiplications is less than n2.

Therefore, to obtain $Cx$, it is easier to multiply first with $F^{-1}$, then $\Lambda$ and then by $F$ than multiplying directly by $C$.

$$\begin{bmatrix} a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_n \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + \ldots + a_{1n}x_n \\ a_{21}x_1 + \ldots + a_{2n}x_n \\ \vdots \\ a_{n1}x_1 + \ldots + a_{nn}x_n \end{bmatrix}$$

Thus, in the right hand side of $Ax$, we see an $n$ by 1 matrix, whose each entry has $n$ products.

Therefore, to carry out $Ax$, we have to do $n^2$ number of products.

Provide feedback (0)

**Step 2** of 2

Now consider a circulant matrix $C = F\Lambda F^{-1}$.

From the Fast Fourier Transform, we know that to multiply $x$ by $F$ and $F^{-1}$, we need only $n \log n$ multiplications. Also, for multiplying by $\Lambda$, we need only additional $n$ multiplications. The sum of all these number of multiplications is less than n2.

Therefore, to obtain $Cx$, it is easier to multiply first with $F^{-1}$, then $\Lambda$ and then by $F$ than multiplying directly by $C$.

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**Step 1** of 6

Given that $u=(1+i,1-i,1+2i)$ and $v=(i,i,i)$
We have to find the lengths of $u$ and $v$.

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**Step 2** of 6

We know that if $x=(x_1,\ldots,x_n)$ then the length of $x$ is $\|x\|=\sqrt{x_1^2+\ldots+x_n^2}$
Now the length of $u$ is

$$\begin{aligned}\|u\|^2 &= |1+i|^2+|1-i|^2+|1+2i|^2\\ &= (1^2+1^2)+(1^2+(-1)^2)+(1^2+2^2)\\ &= 2+2+5\\ &= 9\end{aligned}$$

$\Rightarrow \|u\|=\sqrt{9}=3$

Hence $\boxed{\|u\|=3}$

Provide feedback (0)

**Step 3** of 6

Now the length of $v$ is

$$\begin{aligned}\|v\|^2 &= |i|^2+|i|^2+|i|^2\\ &= 1^2+1^2+1^2\\ &= 3\end{aligned}$$

$\Rightarrow \|v\|=\sqrt{3}$

Hence $\boxed{\|v\|=\sqrt{3}}$

Provide feedback (0)

CH5.5 27P

Provide feedback (0)

**Step 4** of 6

Since $u=(1+i,1-i,1+2i)$

So $u=\begin{bmatrix}1+i\\1-i\\1+2i\end{bmatrix}$

Then $U^H=\begin{bmatrix}1-i & 1+i & 1-2i\end{bmatrix}$

And $v=(i,i,i)$

Then $v=\begin{bmatrix}i\\i\\i\end{bmatrix}$ and $v^H=\begin{bmatrix}-i & -i & -i\end{bmatrix}$

Provide feedback (0)

**Step 5** of 6

Now we have to find $u^H v$

Now

$$\begin{aligned}u^H v&=\begin{bmatrix}1-i & 1+i & 1-2i\end{bmatrix}\begin{bmatrix}i\\i\\i\end{bmatrix}\\&=\left[i(1-i)+i(1+i)+i(1-2i)\right]\\&=\left[i-i^2+i+i^2i-2i^2\right]\\&=\left[3i+2\right]\\&=\left[2+3i\right]\end{aligned}$$

Therefore, $\boxed{u^H v=\left[2+3i\right]}$

Provide feedback (0)

$$= [3i + 2]$$
$$= [2 + 3i]$$

Therefore, $\boxed{u^H v = [2 + 3i]}$

Provide feedback (0)

**Step 6** of 6

Now we have to find $v^H u$

Now

$$v^H u = \begin{bmatrix} -i & -i & -i \end{bmatrix} \begin{bmatrix} 1+i \\ 1-i \\ 1+2i \end{bmatrix}$$
$$= \left[-i(1+i) - i(1-i) - i(1+2i)\right]$$
$$= \left[-i - i^2 - i - i^2 - i - 2i^2\right]$$
$$= \left[-3i - 2i^2\right]$$
$$= [2 - 3i]$$

Therefore, $\boxed{v^H u = [2 - 3i]}$

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CH5.5 26P

**Step 1** of 6

Let $A$ be a matrix.
We have to prove that $A^H A$ is always Hermitian.

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**Step 2** of 6

We know that a matrix $A$ is Hermitian if $A^H = A$.
Let $P = A^H A$
Now

$$\begin{aligned} P^H &= \left(A^H A\right)^H \\ &= A^H \left(A^H\right)^H \\ &= A^H A \qquad \left(\text{Since } \left(A^H\right)^H = A\right) \\ &= P \end{aligned}$$

Since $P^H = P \Rightarrow P$ is a hermitatian matrix
Hence $A^H A$ is always Hermitian.

Provide feedback (0)

**Step 3** of 6

Given that $A = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$
We have to compute $A^H A$ and $AA^H$.

Provide feedback (0)

**Step 4** of 6

We have $A = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$

CH5.5 28P

Provide feedback (0)

**Step 4** of 6

We have $A=\begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$

Then $\overline{A}=\begin{bmatrix} -i & 1 & -i \\ 1 & -i & -i \end{bmatrix}$

Therefore, $A^H=\overline{A}^T=\begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix}$

Provide feedback (0)

**Step 5** of 6

Now

$$A^H A=\begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix}\begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$$

$$=\begin{bmatrix} -i^2+1 & -i+i & -i^2+i \\ i-i & 1-i^2 & i-i^2 \\ -i^2-i & -i-i^2 & -i^2-i^2 \end{bmatrix}$$

$$=\begin{bmatrix} -(-1)+1 & -i+i & -(-1)+i \\ i-i & 1-(-1) & i-(-1) \\ -(-1)-i & -i-(-1) & -(-1)-(-1) \end{bmatrix} \qquad (\text{Since } i^2=-1)$$

$$=\begin{bmatrix} 2 & 0 & 1+i \\ 0 & 2 & 1+i \\ 1-i & 1-i & 2 \end{bmatrix}$$

Therefore, $\boxed{A^H A=\begin{bmatrix} 2 & 0 & 1+i \\ 0 & 2 & 1+i \end{bmatrix}}$

Provide feedback (0)

**Step 5** of 6

Now

$$A^H A = \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix} \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2+1 & -i+i & -i^2+i \\ i-i & 1-i^2 & i-i^2 \\ -i^2-i & -i-i^2 & -i^2-i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -(-1)+1 & -i+i & -(-1)+i \\ i-i & 1-(-1) & i-(-1) \\ -(-1)-i & -i-(-1) & -(-1)-(-1) \end{bmatrix} \qquad (\text{Since } i^2=-1)$$

$$= \begin{bmatrix} 2 & 0 & 1+i \\ 0 & 2 & 1+i \\ 1-i & 1-i & 2 \end{bmatrix}$$

Therefore, $\boxed{A^H A = \begin{bmatrix} 2 & 0 & 1+i \\ 0 & 2 & 1+i \\ 1-i & 1-i & 2 \end{bmatrix}}$

Provide feedback (0)

**Step 6** of 6

Now we compute $AA^H$

$$AA^H = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix} \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2+1-i^2 & i-i-i^2 \\ -i+i-i^2 & 1-i^2-i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -(-1)+1-(-1) & i-i-(-1) \end{bmatrix} \qquad (\text{Since } i^2=-1)$$

Provide feedback (0)

**Step 6** of 6

Now we compute $AA^H$

$$AA^H = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix} \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2+1-i^2 & i-i-i^2 \\ -i+i-i^2 & 1-i^2-i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -(-1)+1-(-1) & i-i-(-1) \\ -i+i-(-1) & 1-(-1)-(-1) \end{bmatrix} \qquad (\text{Since } i^2=-1)$$

$$= \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Hence $\boxed{AA^H = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}}$

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CH5.5 27P

**Step 1** of 2

Consider a matrix $A$ and vector $z$. If $Az = 0$, then $A^H Az = 0$. Multiply by $z^H$ to prove that $Az = 0$.

$$z^H A^H Az = 0$$
$$(Az)^H Az = 0$$
$$\|Az\|^2 = 0$$

This shows that length is zero. Or it can be said as $\boxed{Az = 0}$.

Provide feedback (0)

**Step 2** of 2

From the above calculations it can be said that the null space of $A$ and $A^H A$ are $\boxed{\text{same}}$. Matrix $A^H A$ will be an invertible Hermitian matrix when the null space of matrix $A$ contains only vector $\boxed{z = \{0\}}$.

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CH5.5 29P

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CH5.5 28P

**Step 1** of 5

Let $A$ be a Hermitian matrix

Then $A^H = A$

Let $c$ be any real number.

Now we have to verify that $cA$ is Hermitian or not.

Provide feedback (0)

**Step 2** of 5

We know that a matrix $A$ is Hermitian if $A^H = A$

Now

$$\begin{aligned}(cA)^H &= c\left(A^H\right) \\ &= cA \qquad \left(\text{since } A^H = A\right)\end{aligned}$$

Therefore, $(cA)^H = cA$

Hence $cA$ is a Hermitian matrix.

Provide feedback (0)

**Step 3** of 5

Let $c = i$

Now we have to show that $iA$ is skew-Hermitian.

Provide feedback (0)

**Step 4** of 5

We know that a matrix $A$ is skew-Hermitian if $A^H = -A$

Now

$$\begin{aligned}(iA)^H &= -i\left(A^H\right) \qquad \left(\text{Since } \bar{i} = -1\right) \\ &= -iA \qquad \left(\text{Since } A^H = A\right)\end{aligned}$$

Therefore, $(iA)^H = -iA$

CH5.5 30P

**Step 4** of 5

We know that a matrix $A$ is skew-Hermitian if $A^H = -A$

Now

$$(iA)^H = -i\left(A^H\right) \quad \left(\text{Since } \bar{i} = -1\right)$$

$$= -iA \quad \left(\text{Since } A^H = A\right)$$

Therefore, $(iA)^H = -iA$

Hence $iA$ is a skew-Hermitian matrix.

Provide feedback (0)

**Step 5** of 5

Let $A$ be a $3\times3$ Hermitian matrix.

Then by above, we have $cA$ is also Hermitian, for any real number $c$.

Therefore any scalar multiple of $A$ is also a Hermitian matrix.

Hence the $3\times3$ Hermitian matrices are a subspace.

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CH5.5 29P

CH5.5 31P

**Step 1** of 6

Consider a following matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Let following are the classes of matrices:

Orthogonal, Invertible, Hermitian, Unitary, factorizable into $LU$, factorizable into $QR$. Determine $P$ belongs to which classes of matrices.

Provide feedback (0)

**Step 2** of 6

Orthogonal: If $PP^T = I$, then $P$ is said to be orthogonal matrix.

$$\begin{aligned} PP^T &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

Therefore, $\boxed{\text{matrix } P \text{ is orthogonal matrix}}$.

Invertible: Orthogonal matrices are always invertible. As $P^{-1} = P^T$. Therefore matrix $P$ is also $\boxed{\text{invertible}}$.

Provide feedback (0)

Therefore, matrix $P$ is orthogonal matrix.

Invertible: Orthogonal matrices are always invertible. As $P^{-1} = P^T$. Therefore matrix $P$ is also invertible.

Provide feedback (0)

Step 3 of 6

Hermitian: If $P^H = P$, then $P$ is said to be Hermitian matrix.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P^H = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\neq P$$

Therefore, matrix $P$ is not Hermitian.

Provide feedback (0)

Step 4 of 6

Unitary: If columns of matrix are orthonormal, then matrix is Unitary.

Matrix $P$ has orthogonal columns with unitary length of each column vector. Therefore, matrix $P$ is unitary.

Provide feedback (0)

Step 5 of 6

Factorization into $LU$: Factorization of matrix $P$ into lower and upper triangular matrix requires no row exchanges. But to avoid zeros and to make the pivots of upper triangular matrix 1, row transformation is required.

$$P^H = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$
$$\neq P$$

Therefore, matrix $\boxed{P \text{ is not Hermitian}}$.

Provide feedback (0)

**Step 4** of 6

Unitary: If columns of matrix are orthonormal, then matrix is Unitary.

Matrix $P$ has orthogonal columns with unitary length of each column vector. Therefore, matrix $\boxed{P \text{ is unitary}}$.

Provide feedback (0)

**Step 5** of 6

Factorization into $LU$: Factorization of matrix $P$ into lower and upper triangular matrix requires no row exchanges. But to avoid zeros and to make the pivots of upper triangular matrix 1, row transformation is required.

Therefore, matrix $\boxed{P \text{ can not be factorised into } LU \text{ matrices}}$.

Provide feedback (0)

**Step 6** of 6

Factorization into $QR$: Matrix $P$ has independent columns which can be orthogonalised into matrix $Q$ by the Gram-Schmidt process.

Therefore, matrix $\boxed{P \text{ can be factorised into } QR \text{ matrices}}$.

Provide feedback (0)

CH5.5 30P

**Step 1** of 7

Given that $P=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$

We have to compute $P^2, P^3$ and $P^{100}$. And also we have to find the eigenvalues of $P$.

Provide feedback (0)

**Step 2** of 7

Now

$$P^2=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$$

$$=\begin{bmatrix}0(0)+1(0)+0(1) & 0(1)+1(0)+0(0) & 0(0)+1(1)+0(0)\\0(0)+0(0)+1(1) & 0(1)+0(0)+1(0) & 0(0)+0(1)+1(0)\\1(0)+0(0)+0(1) & 1(1)+0(0)+0(0) & 1(0)+0(1)+0(0)\end{bmatrix}$$

$$=\begin{bmatrix}0+0+1 & 0+0+0 & 0+1+0\\0+0+1 & 0+0+0 & 0+0+0\\0+0+0 & 1+0+0 & 0+0+0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

Therefore, $\boxed{P^2=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}}$

Provide feedback (0)

**Step 3** of 7

CH5.5 32P

**Step 3** of 7

Now

$$P^3 = P^2 P$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0(0)+0(0)+1(1) & 0(1)+0(0)+1(0) & 0(0)+0(1)+1(0) \\ 1(0)+0(0)+0(1) & 1(1)+0(0)+0(0) & 1(0)+0(1)+0(0) \\ 0(0)+1(0)+0(1) & 0(1)+1(0)+0(0) & 0(0)+1(1)+0(0) \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 7

Continuation to the above

$$= \begin{bmatrix} 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 1+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Therefore, $\boxed{P^3 = I}$

Provide feedback (0)

**Step 5** of 7

Now we have to compute $P^{100}$.

Now

$$P^{100} = P^{99} P$$

$$= (P^3)^{33} P$$

Provide feedback (0)

**Step 5** of 7

Now we have to compute $P^{100}$.
Now

$$\begin{aligned} P^{100} &= P^{99}P \\ &= \left(P^3\right)^{33} P \\ &= (I)^{33} P \qquad \left(\text{Since } P^3 = I\right) \\ &= IP \\ &= P \qquad \left(\text{Since } IP = PI = P\right) \end{aligned}$$

Hence $\boxed{P^{100} = P}$

Provide feedback (0)

**Step 6** of 7

Now we have to find the eigenvalues of $P$.
The characteristic equation of $P$ is

$$\begin{aligned} &|P - \lambda I| = 0 \\ &\Rightarrow \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0 \\ &\Rightarrow -\lambda\left(\lambda^2\right) - 1(-1) + 0 = 0 \\ &\Rightarrow -\lambda^3 + 1 = 0 \\ &\Rightarrow \lambda^3 = 1 \end{aligned}$$

Therefore the eigenvalues of $P$ are cube roots of unity $\boxed{\lambda = 1, \omega, \omega^2 = 1, e^{\frac{2\pi i}{3}}, e^{\frac{4\pi i}{3}}}$.

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**Step 7** of 7

Chapter: CH5.5 Problem: 31P

show all steps

**Step 7** of 7

CH5.5 31P

CH5.5 33P

**Step 1** of 9

Consider the following orthogonal matrix:

$$P=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$$

Find the unit Eigen vectors and unitary matrix. Determine the property of $P$ that makes these Eigen vectors orthogonal.

Provide feedback (0)

**Step 2** of 9

First step is to find the Eigen values of matrix $P$. To calculate the Eigen values do the following calculations:

$$P-\lambda I=\begin{bmatrix}0-\lambda & 1 & 0\\ 0 & 0-\lambda & 1\\ 1 & 0 & 0-\lambda\end{bmatrix}$$

$$\det(P-\lambda I)=0$$

$$(-\lambda)(\lambda^2)+1=0$$

$$(\lambda^3-1)=0$$

After solving following values are obtained:

$$\lambda_1=1$$

$$\lambda_2=e^{2\pi i/3}$$

$$\lambda_3=e^{4\pi i/3}$$

Provide feedback (0)

$$\lambda_1 = 1$$
$$\lambda_2 = e^{2\pi i/3}$$
$$\lambda_3 = e^{4\pi i/3}$$

Provide feedback (0)

Step 3 of 9

To calculate Eigen vectors do the following calculations:

$$(P - \lambda I)x = 0$$
$$\begin{bmatrix} 0-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ 1 & 0 & 0-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving, values of $x$, $y$ and $z$ corresponding to $\lambda = 1$ are as follows:

$$x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Provide feedback (0)

Step 4 of 9

Similarly, Eigen vector corresponding to Eigen value $\lambda = e^{2\pi i/3}$ is as follows:

$$(P - \lambda I)x = 0$$

Provide feedback (0)

Step 4 of 9

Similarly, Eigen vector corresponding to Eigen value $\lambda = e^{2\pi i/3}$ is as follows:

$$(P-\lambda I)x = 0$$

$$\begin{bmatrix} -e^{2\pi i/3} & 1 & 0 \\ 0 & -e^{2\pi i/3} & 1 \\ 1 & 0 & -e^{2\pi i/3} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On solving values of $x$, $y$ and $z$ are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ e^{2\pi i/3} \\ e^{4\pi i/3} \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 5 of 9

Similarly, Eigen vector corresponding to Eigen value $\lambda = e^{4\pi i/3}$ will be:

$$\begin{aligned} x_3 &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ e^{4\pi i/3} \\ e^{2\pi i/3} \end{bmatrix} \end{aligned}$$

Step 5 of 9

Similarly, Eigen vector corresponding to Eigen value $\lambda = e^{4\pi i/3}$ will be:

$$x_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ e^{4\pi i/3} \\ e^{2\pi i/3} \end{bmatrix}$$

Provide feedback (0)

Step 6 of 9

Therefore, Eigen vectors of matrix $P$ are as follows:

$$\boxed{(1,1,1),\left(1,e^{2\pi i/3},e^{4\pi i/3}\right),\left(1,e^{4\pi i/3},e^{2\pi i/3}\right)}$$

Provide feedback (0)

Step 7 of 9

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned}\|x\|^2 &= \left|(1)^2\right| + \left|\left(e^{2\pi i/3}\right)^2\right| + \left|\left(e^{4\pi i/3}\right)^2\right| \\ &= |1| + |1| + |1| \\ &= 3\end{aligned}$$

Let the length be $L$. So $L = \sqrt{3}$.

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To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned}\|x\|^2 &= \left|(1)^2\right| + \left|\left(e^{2\pi i/3}\right)^2\right| + \left|\left(e^{4\pi i/3}\right)^2\right| \\ &= |1| + |1| + |1| \\ &= 3\end{aligned}$$

Let the length be $L$. So $L = \sqrt{3}$.

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**Step 8** of 9

Therefore, unitary matrix will be as follows:

$$U = \boxed{\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{2\pi i/3} & e^{4\pi i/3} \\ 1 & e^{4\pi i/3} & e^{2\pi i/3} \end{bmatrix}}$$

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**Step 9** of 9

The property of $P$ that makes these Eigen vectors orthogonal is the orthogonality of matrix $P$ and Eigen value absolute value is $|\lambda| = 1$

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CH5.5 32P

CH5.5 34P

**Step 1** of 5

Consider the following orthogonal matrix:

$$P=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$$

Eigen vectors of $P$ are as follows:

$$(1,1,1),\left(1,e^{2\pi i/3},e^{4\pi i/3}\right)\left(1,e^{4\pi i/3},e^{8\pi i/3}\right)$$

Find the circulant matrix $C$ which has the same Eigen vectors as orthogonal matrix $P$ and satisfies the following relation:

$$C=2I+5P+4P^2$$

Also, find Eigen values of the circulant matrix $C$.

Provide feedback (0)

**Step 2** of 5

Circulant matrix $C$ can be calculated as follows:

$$\begin{aligned}C&=2I+5P+4P^2\\&=2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+5\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}+4\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\\&=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}+\begin{bmatrix}0&5&0\\0&0&5\\5&0&0\end{bmatrix}+\begin{bmatrix}0&0&4\\4&0&0\\0&4&0\end{bmatrix}\\&=\begin{bmatrix}2&5&4\\4&2&5\end{bmatrix}\end{aligned}$$

Circulant matrix C can be calculated as follows:

$$
\begin{aligned}
C &= 2I + 5P + 4P^2 \\
&= 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 5\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + 4\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 0 \\ 0 & 0 & 5 \\ 5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 4 \\ 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 5 & 4 \\ 4 & 2 & 5 \\ 5 & 4 & 2 \end{bmatrix}
\end{aligned}
$$

Therefore, circulant matrix is defined as follows:

$$
\boxed{C = \begin{bmatrix} 2 & 5 & 4 \\ 4 & 2 & 5 \\ 5 & 4 & 2 \end{bmatrix}}
$$

Provide feedback (0)

**Step 3** of 5

To calculate the Eigen values of circulant matrix, do the following calculations:

$$
\begin{aligned}
C - \lambda I &= \begin{bmatrix} 2-\lambda & 5 & 4 \\ 4 & 2-\lambda & 5 \\ 5 & 4 & 2-\lambda \end{bmatrix} \\
\det(C - \lambda I) &= 0 \\
(2-\lambda)^3\left((2-\lambda)^2 - 20\right) - 5(8 - 4\lambda - 25) + 4(6+\lambda) &= 0 \\
\left(-\lambda^3 + 6\lambda^2 + 48\lambda + 77\right) &= 0
\end{aligned}
$$

After solving following values are obtained:

Provide feedback (0)

**Step 3** of 5

To calculate the Eigen values of circulant matrix, do the following calculations:

$$C-\lambda I=\begin{bmatrix}2-\lambda & 5 & 4\\ 4 & 2-\lambda & 5\\ 5 & 4 & 2-\lambda\end{bmatrix}$$

$$\det(C-\lambda I)=0$$

$$(2-\lambda)^3\left((2-\lambda)^2-20\right)-5(8-4\lambda-25)+4(6+\lambda)=0$$

$$\left(-\lambda^3+6\lambda^2+48\lambda+77\right)=0$$

After solving following values are obtained:

$$\lambda_1=11$$

$$\lambda_2=-\frac{5}{2}+i\frac{\sqrt{3}}{2}$$

$$\lambda_3=-\frac{5}{2}-i\frac{\sqrt{3}}{2}$$

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**Step 4** of 5

These Eigen values can also be written as a combination of Eigen values of matrix $P$:

$$\begin{aligned}\lambda_1&=11\\&=2+5+4\\\lambda_2&=-\frac{5}{2}+i\frac{\sqrt{3}}{2}\\&=2\cdot 1+5e^{2\pi i/3}+4e^{4\pi i/3}\end{aligned}$$

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**Step 4** of 5

These Eigen values can also be written as a combination of Eigen values of matrix $P$:

$$\begin{aligned}
\lambda_1 &= 11 \\
&= 2+5+4 \\
\lambda_2 &= -\frac{5}{2}+i\frac{\sqrt{3}}{2} \\
&= 2\cdot 1+5e^{2\pi i/3}+4e^{4\pi i/3} \\
\lambda_3 &= -\frac{5}{2}-i\frac{\sqrt{3}}{2} \\
&= 2\cdot 1+5e^{4\pi i/3}+4e^{8\pi i/3}
\end{aligned}$$

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**Step 5** of 5

Therefore, following are the Eigen values of the circulant matrix:

$$\boxed{\begin{aligned}
\lambda_1 &= 11 \\
\lambda_2 &= -\frac{5}{2}+i\frac{\sqrt{3}}{2} \\
\lambda_3 &= -\frac{5}{2}-i\frac{\sqrt{3}}{2}
\end{aligned}}$$

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**Step 1** of 3

Suppose $U$ is unitary and $Q$ is a real orthogonal matrix.
We have to show that $U^{-1}$ and $UQ$ are unitary.

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CH5.5 35P

**Step 2** of 3

Since $U$ is unitary matrix.
So $U^H U = I$
Since $Q$ is real orthogonal matrix.
So $Q^T Q = I$
Now

$$\begin{aligned}
\left(U^{-1}\right)^H . U^{-1} &= \left(U^{-1}\right)^H U^H \qquad \left(\text{Since } U^H = U^{-1}\right) \\
&= \left(U^H\right)^H U^H \\
&= UU^H \qquad \left(\text{Since } \left(U^H\right)^H = U\right) \\
&= UU^{-1} \\
&= I
\end{aligned}$$

Since $\left(U^{-1}\right)^H U^{-1} = I$

Therefore $U^{-1}$ is also unitary.

Provide feedback (0)

**Step 3** of 3

Now

$$\begin{aligned}
(UQ)^H (UQ) &= \left(Q^H U^H\right)(UQ) \\
&= Q^H \left(U^H U\right) Q \\
&= Q^H (IQ) \\
&= Q^H Q
\end{aligned}$$

Since $Q$ is real orthogonal

Since $U$ is unitary matrix.
So $U^H U = I$
Since $Q$ is real orthogonal matrix.
So $Q^T Q = I$
Now

$$\begin{aligned}(U^{-1})^H . U^{-1} &= (U^{-1})^H U^H \quad (\text{Since } U^H = U^{-1}) \\ &= (U^H)^H U^H \\ &= UU^H \quad (\text{Since } (U^H)^H = U) \\ &= UU^{-1} \\ &= I\end{aligned}$$

Since $(U^{-1})^H U^{-1} = I$
Therefore $U^{-1}$ is also unitary.

Provide feedback (0)

**Step 3** of 3

Now

$$\begin{aligned}(UQ)^H (UQ) &= (Q^H U^H)(UQ) \\ &= Q^H (U^H U) Q \\ &= Q^H (IQ) \\ &= Q^H Q\end{aligned}$$

Since $Q$ is real orthogonal.

$$\begin{aligned}\text{So } Q^T &= Q^H \\ &= Q^T Q \\ &= I\end{aligned}$$

Therefore, $(UQ)^H (UQ) = I$
Hence $UQ$ is also unitary.

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CH5.5 34P

**Step 1** of 13

Consider the following matrices:

$$A=\begin{bmatrix} 0 & 1-i \\ i+1 & 1 \end{bmatrix}$$
$$K=\begin{bmatrix} 0 & -1+i \\ 1+i & i \end{bmatrix}$$

Diagonalize it to reach $A=U\Lambda U^H$ and $K=U\Lambda U^H$.

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**Step 2** of 13

First step is to find the Eigen values and Eigen vectors of matrix $A$. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix} 0-\lambda & 1-i \\ i+1 & 1-\lambda \end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(1-\lambda)(-\lambda)-(1-i^2)=0$$
$$(\lambda^2-\lambda-2)=0$$

After solving following values are obtained:

$$\lambda_1=2$$
$$\lambda_2=-1$$

Provide feedback (0)

**Step 3** of 13

To calculate Eigen vectors do the following calculations:

CH5.5 36P

$$\lambda_1 = 2$$
$$\lambda_2 = -1$$

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**Step 3** of 13

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x = 0$$
$$\begin{bmatrix} 0-2 & 1-i \\ i+1 & 1-2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} -2 & 1-i \\ i+1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of $y$ and $z$ corresponding to $\lambda = 2$ are as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 13

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -1$ is as follows:

$$(A-\lambda I)x = 0$$
$$\begin{bmatrix} 0+1 & 1-i \\ i+1 & 1+1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1-i \\ i+1 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$= \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 13

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -1$ is as follows:

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 0+1 & 1-i \\ i+1 & 1+1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1-i \\ i+1 & 2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -1+i \\ 1 \end{bmatrix}$$

Provide feedback (0)

**Step 5** of 13

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\|x\|^2 = \left|(1)^2\right| + \left|(1+i)^2\right|$$

$$= |1| + |(2i)|$$

$$= 3$$

Let the length be $L$. So $L = \sqrt{3}$.

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Step 5 of 13

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned}\|x\|^2 &= \left|(1)^2\right| + \left|(1+i)^2\right| \\ &= |1| + |(2i)| \\ &= 3\end{aligned}$$

Let the length be $L$. So $L = \sqrt{3}$.

Provide feedback (0)

Step 6 of 13

Now, the diagonalization of the matrix can be written as follows:

$$\begin{aligned}A &= U\Lambda U^H \\ &= \frac{1}{L}\begin{bmatrix} 1 & -1+i \\ 1+i & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\frac{1}{L}\begin{bmatrix} 1 & 1-i \\ -1-i & 1 \end{bmatrix}\end{aligned}$$

Here, $L = \sqrt{3}$

Provide feedback (0)

Step 7 of 13

Therefore, matrix $A$ is diagonalize to reach $\boxed{A = U\Lambda U^H}$.

Provide feedback (0)

Step 8 of 13

To diagonalize matrix $K$, do the following calculations. First step is to find the Eigen

Therefore, matrix $A$ is diagonalize to reach $\boxed{A = U\Lambda U^H}$.

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Step 8 of 13

To diagonalize matrix $K$, do the following calculations. First step is to find the Eigen values and Eigen vectors of matrix $K$. To calculate the Eigen values do the following calculations;

$$K - \lambda I = \begin{bmatrix} 0-\lambda & -1+i \\ 1+i & i-\lambda \end{bmatrix}$$

$$\det(K - \lambda I) = 0$$

$$(i-\lambda)(-\lambda) - (-1+i^2) = 0$$

$$(\lambda^2 - i\lambda + 2) = 0$$

After solving following values are obtained:

$$\lambda_1 = 2i$$
$$\lambda_2 = -i$$

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Step 9 of 13

To calculate Eigen vectors do the following calculations:

$$(K - \lambda I)x = 0$$

$$\begin{bmatrix} 0-2i & -1+i \\ 1+i & i-2i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2i & -1+i \\ 1+i & -i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Step 9 of 13

To calculate Eigen vectors do the following calculations:

$$(K-\lambda I)x=0$$

$$\begin{bmatrix} 0-2i & -1+i \\ 1+i & i-2i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2i & -1+i \\ 1+i & -i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of $y$ and $z$ corresponding to $\lambda=2i$ are as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 1-i \end{bmatrix}$$

Provide feedback (0)

Step 10 of 13

Similarly, Eigen vectors corresponding to Eigen value $\lambda=-i$ is as follows:

$$(K-\lambda I)x=0$$

$$\begin{bmatrix} 0+i & -1+i \\ 1+i & i+i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & -1+i \\ 1+i & 2i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$x_2=\begin{bmatrix} y \\ z \end{bmatrix}$$

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**Step 10** of 13

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -i$ is as follows:

$$(K - \lambda I)x = 0$$

$$\begin{bmatrix} 0+i & -1+i \\ 1+i & i+i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & -1+i \\ 1+i & 2i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} -1-i \\ 1 \end{bmatrix} \end{aligned}$$

Provide feedback (0)

**Step 11** of 13

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned} \|x\|^2 &= \left|(1)^2\right| + \left|(1-i)^2\right| \\ &= |1| + |(-2i)| \\ &= 3 \end{aligned}$$

Let the length be $L$. So $L = \sqrt{3}$.

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To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned}\|x\|^2 &= \left|(1)^2\right| + \left|(1-i)^2\right| \\ &= |1| + |(-2i)| \\ &= 3\end{aligned}$$

Let the length be $L$. So $L = \sqrt{3}$.

Provide feedback (0)

**Step 12** of 13

Now, the diagonalization of the matrix can be written as follows:

$$\begin{aligned}K &= U\Lambda U^H \\ &= \frac{1}{L}\begin{bmatrix} 1 & -1-i \\ 1-i & 1 \end{bmatrix}\begin{bmatrix} 2i & 0 \\ 0 & -i \end{bmatrix}\frac{1}{L}\begin{bmatrix} 1 & 1+i \\ -1+i & 1 \end{bmatrix}\end{aligned}$$

Here, $L = \sqrt{3}$

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**Step 13** of 13

Therefore, matrix $K$ is diagonalize to reach $\boxed{K = U\Lambda U^H}$.

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**Step 1** of 7

Consider the following orthogonal matrix:

$$Q=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Diagonalize it to reach $Q=U\Lambda U^{H}$.

Provide feedback (0)

**Step 2** of 7

First step is to find the Eigen values and Eigen vectors of matrix $Q$. To calculate the Eigen values do the following calculations;

$$\begin{aligned}Q-\lambda I&=\begin{bmatrix}\cos\theta-\lambda & -\sin\theta\\ \sin\theta & \cos\theta-\lambda\end{bmatrix}\\ \det(Q-\lambda I)&=0\\ (\cos\theta-\lambda)(\cos\theta-\lambda)+\sin^2\theta&=0\\ (\lambda^2-2\lambda\cos\theta+1)&=0\end{aligned}$$

After solving following values are obtained:

$$\begin{aligned}\lambda_1&=\cos\theta+i\sin\theta\\ \lambda_2&=\cos\theta-i\sin\theta\end{aligned}$$

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**Step 3** of 7

To calculate Eigen vectors do the following calculations:

$$(Q-\lambda I)x=0$$

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Step 3 of 7

To calculate Eigen vectors do the following calculations:

$$(Q-\lambda I)x=0$$

$$\begin{bmatrix} \cos\theta-\cos\theta-i\sin\theta & -\sin\theta \\ \sin\theta & \cos\theta-\cos\theta-i\sin\theta \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -i\sin\theta & -\sin\theta \\ \sin\theta & -i\sin\theta \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of $y$ and $z$ corresponding to $\lambda=\cos\theta+i\sin\theta$ are as follows:

$$\begin{aligned} x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -i \end{bmatrix} \end{aligned}$$

Provide feedback (0)

Step 4 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda=\cos\theta-i\sin\theta$ is as follows:

$$(Q-\lambda I)x=0$$

$$\begin{bmatrix} \cos\theta-\cos\theta+i\sin\theta & -\sin\theta \\ \sin\theta & \cos\theta-\cos\theta+i\sin\theta \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i\sin\theta & -\sin\theta \\ \sin\theta & i\sin\theta \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$x_2=\begin{bmatrix} y \end{bmatrix}$$

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Step 4 of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda = \cos\theta - i\sin\theta$ is as follows:

$$(Q-\lambda I)x = 0$$

$$\begin{bmatrix} \cos\theta - \cos\theta + i\sin\theta & -\sin\theta \\ \sin\theta & \cos\theta - \cos\theta + i\sin\theta \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i\sin\theta & -\sin\theta \\ \sin\theta & i\sin\theta \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} -i \\ 1 \end{bmatrix} \end{aligned}$$

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Step 5 of 7

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned} \|x\|^2 &= \left|(1)^2\right| + \left|(-i)^2\right| \\ &= |1| + |1| \\ &= 2 \end{aligned}$$

Let the length be $L$. So $L = \sqrt{2}$.

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$$= |1| + |1|$$
$$= 2$$

Let the length be $L$. So $L = \sqrt{2}$.

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**Step 6** of 7

Now, the diagonalization of the matrix can be written as follows:

$$\begin{aligned} Q &= U\Lambda U^H \\ &= \frac{1}{L}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}\begin{bmatrix} \cos\theta + i\sin\theta & 0 \\ 0 & \cos\theta - i\sin\theta \end{bmatrix}\frac{1}{L}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \end{aligned}$$

Here, $L = \sqrt{2}$

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**Step 7** of 7

Therefore, orthogonal matrix $Q$ is diagonalize to reach $\boxed{Q = U\Lambda U^H}$. All Eigen values of unitary matrix is $\boxed{|\lambda| = 1}$.

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**Step 1** of 7

Consider the following unitary matrix:

$$V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$$

Diagonalize it to reach $V = U\Lambda U^H$.

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**Step 2** of 7

First step is to find the Eigen values and Eigen vectors of matrix $V$. To calculate the Eigen values do the following calculations;

$$V - \lambda I = \frac{1}{\sqrt{3}}\begin{bmatrix} 1-\lambda & 1-i \\ 1+i & -1-\lambda \end{bmatrix}$$

$$\det(V - \lambda I) = 0$$

$$\frac{1}{\sqrt{3}}\left((-1-\lambda)(1-\lambda)-(1+i)(1-i)\right) = 0$$

$$\frac{1}{\sqrt{3}}\left(\lambda^2 - 3\right) = 0$$

After solving following values are obtained:

$$\lambda_1 = 1$$
$$\lambda_2 = -1$$

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**Step 3** of 7

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$$\det(V-\lambda I)=0$$

$$\frac{1}{\sqrt{3}}\left((-1-\lambda)(1-\lambda)-(1+i)(1-i)\right)=0$$

$$\frac{1}{\sqrt{3}}\left(\lambda^2-3\right)=0$$

After solving following values are obtained:

$$\lambda_1=1$$
$$\lambda_2=-1$$

Provide feedback (0)

Step 3 of 7

To calculate Eigen vectors do the following calculations:

$$(V-\lambda I)x=0$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}}-1 & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}}-1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1-\sqrt{3}}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{-1-\sqrt{3}}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of $y$ and $z$ corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix} y \\ z \end{bmatrix}$$
$$=\begin{bmatrix} \left(1+\sqrt{3}\right) \\ 1+i \end{bmatrix}$$

$$\begin{bmatrix} \frac{}{\sqrt{3}} & \frac{}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{-1-\sqrt{3}}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of $y$ and $z$ corresponding to $\lambda = 1$ are as follows:

$$\begin{aligned} x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} \left(1+\sqrt{3}\right) \\ 1+i \end{bmatrix} \end{aligned}$$

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**Step 4** of 7

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -1$ is as follows:

$$(V - \lambda I)x = 0$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}}+1 & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}}+1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1+\sqrt{3}}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{-1+\sqrt{3}}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} -1+i \\ \left(1+\sqrt{3}\right) \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}}+1 & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}}+1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1+\sqrt{3}}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{-1+\sqrt{3}}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of $y$ and $z$ are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} -1+i \\ \left(1+\sqrt{3}\right) \end{bmatrix} \end{aligned}$$

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Step 5 of 7

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned} \|x\|^2 &= \left|\left(1+\sqrt{3}\right)^2\right| + \left|(1+i)^2\right| \\ &= \left|1+3+2\sqrt{3}\right| + \left|1+i^2+2i\right| \\ &= \left|4+2\sqrt{3}\right| + \left|2i\right| \\ &= 6+2\sqrt{3} \end{aligned}$$

Let the length be $L$. So $L^2 = 6+2\sqrt{3}$.

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**Step 6** of 7

Now, the diagonalization of the matrix can be written as follows:

$$\begin{aligned} V &= U\Lambda U^H \\ &= \frac{1}{L}\begin{bmatrix} 1+\sqrt{3} & -1+i \\ 1+i & 1+\sqrt{3} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\frac{1}{L}\begin{bmatrix} 1+\sqrt{3} & 1-i \\ -1-i & 1+\sqrt{3} \end{bmatrix} \end{aligned}$$

Here, $L^2 = 6+2\sqrt{3}$

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**Step 7** of 7

Therefore, unitary matrix $V$ is diagonalize to reach $\boxed{V = U\Lambda U^H}$. All Eigen values of unitary matrix is $|\lambda| = 1$.

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**Step 1** of 3

Let $\mathbf{C}^n$ be the complex vector space. It contains $n$ independent unit coordinate vectors. Let $v_1, v_2, \ldots, v_n$ be an orthonormal basis for $\mathbf{C}^n$. If a matrix is formed by substituting orthonormal basis as column vectors then this matrix is called as unitary matrix.

Show that any vector $z$ equals to the following:

$$\left(v_1^H z\right)v_1 + \cdots + \left(v_n^H z\right)v_n.$$

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**Step 2** of 3

Recall that if $U$ is a unitary matrix then following is true:

$$UU^H = I$$

Columns of the unitary matrix are formed by orthonormal vectors.

Any vector $z$ can be written as follows:

$$\begin{aligned} z &= Iz \\ &= UU^H z \\ &= v_1\left(v_1^H z\right) + v_2\left(v_2^H z\right) + \cdots + v_n\left(v_n^H z\right) \\ &= \left(v_1^H z\right)v_1 + \left(v_2^H z\right)v_2 + \cdots + \left(v_n^H z\right)v_n \end{aligned}$$

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**Step 3** of 3

Therefore, vector $z$ can be written as $\boxed{\left(v_1^H z\right)v_1 + \cdots + \left(v_n^H z\right)v_n}$.

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**Step 1** of 3

Complex inner product: $\mathbf{u}^H\mathbf{v} = \bar{u}_1 v_1 + \cdots + \bar{u}_n v_n$.

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**Step 2** of 3

Consider the following function on the interval $0 \le x \le 2\pi$.

$$e^{-ix}, e^{ix}$$

These functions will be orthogonal if their complex inner product will be zero. For this first calculate the complex conjugate of first function and then integrate the product within the limits of variable $x$.

$$\begin{aligned}
\int_0^{2\pi} e^{ix} \cdot e^{ix} dx &= \int_0^{2\pi} e^{2ix} dx \\
&= \left[\frac{e^{2ix}}{2i}\right]_0^{2\pi} \\
&= \frac{1}{2i}\left[e^{4\pi i} - e^0\right] \\
&= \frac{1}{2i}\left[(-1)^4 - 1\right]
\end{aligned}$$

$$\int_0^{2\pi} e^{ix} \cdot e^{ix} dx = 0$$

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**Step 3** of 3

Therefore, complex inner product is zero, this shows that the functions are orthogonal.

$$\boxed{\int^{2\pi} e^{ix} \cdot e^{ix} dx = 0}$$

CH5.5 39P

Step 1 of 6

Complex inner product: $u^H v = \bar{u}_1 v_1 + \cdots + \bar{u}_n v_n$.

Orthogonality: $u^H v = 0$

Provide feedback (0)

Step 2 of 6

Consider the following vectors:

$$v = (1, i, 1)$$
$$w = (i, 1, 0)$$
$$z = (a, b, c)$$

These vectors form an orthogonal basis. Find vector $z$.

Provide feedback (0)

Step 3 of 6

If these vectors form an orthogonal basis then their inner product will be zero. Calculate the inner product of these vectors:

$$\begin{aligned} v \cdot w &= 1 \cdot i + (-i) \cdot 1 + 1 \cdot 0 \\ &= i - i + 0 \\ &= 0 \end{aligned}$$

Provide feedback (0)

Step 4 of 6

Similarly, inner product of other two vectors must also be zero.

CH5.5 41P

$$= 0$$

Provide feedback (0)

Step 4 of 6

Similarly, inner product of other two vectors must also be zero.

$$w \cdot z = 0$$
$$v \cdot z = 0$$

Calculate the following:

$$\begin{aligned} w \cdot z &= (-i) \cdot a + 1 \cdot b + 0 \cdot c \\ &= -ia + b \\ v \cdot z &= 1 \cdot a + (-i) \cdot b + 1 \cdot c \\ &= a - ib + c \end{aligned}$$

Provide feedback (0)

Step 5 of 6

On solving following results are obtained:

$$\begin{aligned} w \cdot z &= 0 \\ -ia + b &= 0 \\ b &= ia \end{aligned}$$

Similarly,

$$\begin{aligned} v \cdot z &= 0 \\ a - ib + c &= 0 \\ a + a + c &= 0 \\ 2a + c &= 0 \end{aligned}$$

Put $a = 1$ in the above results. Following values are obtained:

$$v \cdot z = 1 \cdot a + (-i) \cdot b + 1 \cdot c$$
$$= a - ib + c$$

Provide feedback (0)

**Step 5** of 6

On solving following results are obtained:

$$w \cdot z = 0$$
$$-ia + b = 0$$
$$b = ia$$

Similarly,

$$v \cdot z = 0$$
$$a - ib + c = 0$$
$$a + a + c = 0$$
$$2a + c = 0$$

Put $a = 1$ in the above results. Following values are obtained:

$$b = i$$
$$c = -2$$

Provide feedback (0)

**Step 6** of 6

Therefore, vector is $\boxed{z = (1, i, -2)}$. All these three vectors $v$, $w$, and $z$ are an orthogonal basis for $\boxed{\mathbf{C}^3}$.

Provide feedback (0)

CH5.5 40P

**Step 1** of 2

Given that $A = R + iS$ is a Hermitian matrix.
We have to verify that the real matrices $R$ and $S$ are symmetric or not.

Provide feedback (0)

CH5.5 42P

**Step 2** of 2

Now

$$A^H = A$$

$$\Rightarrow (R + iS) = (R + iS)^H$$

$$= R^H - iS^H$$

Since $R$, $S$ are real matrices.

So $R^H = R^T$ and $S^H = S^T$

$\Rightarrow R = R^T$ and $S = -S^T$

Hence $R$ is symmetric and $S$ is skew-symmetric.

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CH5.5 41P

**Step 1** of 2

Let $\mathbf{C}^n$ be the complex vector space. It contains $n$ independent unit coordinate vectors. Therefore, dimension of complex vector space $\mathbf{C}^n$ is $\boxed{n}$.

Simple basis of $\mathbf{C}^n$ will be $n$ independent unit coordinate vectors. These vectors are not real.

$$(i,0,0,\ldots,0),(0,i,0,\ldots,0),\ldots(0,0,0,\ldots,i)$$

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**Step 2** of 2

Therefore, nonreal basis for $\mathbf{C}^n$ are: $\boxed{(i,0,0,\ldots,0),(0,i,0,\ldots,0),\ldots(0,0,0,\ldots,i)}$

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CH5.5 42P

**Step 1** of 5

We have to find all the 1 by 1 matrices that are Hermitian and also unitary.

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CH5.5 44P

**Step 2** of 5

We know that a matrix $A$ is Hermitian if $A^H = A$ and matrix $B$ is unitary if $B^H B = I$.

Let us consider the 1 by 1 matrices $[1]$ and $[-1]$

Let $U = [1]$

Then

$$\begin{aligned} U^H U &= [1][1] \\ &= [1] \\ &= I \end{aligned}$$

Therefore $U = [1]$ is unitary.

And $U^H = [1] = U$

Hence $U$ is Hermitian matrix.

Therefore, $U = [1]$ is both Hermitian and unitary.

Provide feedback (0)

**Step 3** of 5

Let us consider $U = [-1]$

Now $U^H = [-1] = U$

Therefore, $U$ is Hermitian.

Now

$$\begin{aligned} U^H.U &= [-1][-1] \\ &= [1] \\ &= I \end{aligned}$$

Therefore, $U$ is unitary.

Hence $U$ is Hermitian and unitary matrix.

Therefore, $U$ is Hermitian.
Now

$$\begin{aligned} U^H.U &= [-1][-1] \\ &= [1] \\ &= I \end{aligned}$$

Therefore, $U$ is unitary.
Hence $U$ is Hermitian and unitary matrix.

Provide feedback (0)

**Step 4** of 5

Now we have to find all the 2 by 2 matrices that are both Hermitian and unitary.
Let us consider $2\times 2$ matrix

$$U = \begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix} \text{ with } a^2+b^2+c^2=1$$

Then

$$\begin{aligned} U^H &= \begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix} \\ &= U \end{aligned}$$

Since $U^H = U$
So $U$ is Hermitian.

Provide feedback (0)

**Step 5** of 5

Now

$$\begin{aligned} U^H U &= \begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix}\begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix} \\ &= \begin{bmatrix} a(a)+(b+ic)(b-ic) & a(b+ic)-(b+ic)a \\ (b-ic)(a)-a(b-ic) & +(b+ic)(b-ic)+a(a) \end{bmatrix} \\ &= \begin{bmatrix} a^2+b^2+c^2 & ab+iac-ab-aic \\ ab-iac-ab+aic & b^2+c^2+a^2 \end{bmatrix} \\ &\begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

$$U=\begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix} \text{ with } a^2+b^2+c^2=1$$

Then

$$U^H=\begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix}$$
$$=U$$

Since $U^H=U$

So $U$ is Hermitian.

Provide feedback (0)

**Step 5** of 5

Now

$$U^HU=\begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix}\begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix}$$
$$=\begin{bmatrix} a(a)+(b+ic)(b-ic) & a(b+ic)-(b+ic)a \\ (b-ic)(a)-a(b-ic) & +(b+ic)(b-ic)+a(a) \end{bmatrix}$$
$$=\begin{bmatrix} a^2+b^2+c^2 & ab+iac-ab-aic \\ ab-iac-ab+aic & b^2+c^2+a^2 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$=I$$

Since $U^HU=I$

So $U$ is unitary.

Therefore $\boxed{\begin{bmatrix} a & b+ic \\ b-ic & -a \end{bmatrix}, a^2+b^2+c^2=1}$ is Hermitian and unitary matrix.

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CH5.5 43P

CH5.5 45P

**Step 1** of 2

Let $A^H$ be the complex conjugate of the square complex matrix $A$. Find the Eigen values of $A^H$, related to Eigen values of the square complex matrix $A$.

Eigen values of the square complex matrix $A$ is calculated as below:

$$\det(A-\lambda I)=0$$

Here, $\lambda$ is an Eigen value of matrix $A$.

If complex conjugate is considered then:

$$\det(A^H-\bar{\lambda} I)=0$$

Here, $\bar{\lambda}$ is an Eigen value of $A^H$.

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**Step 2** of 2

Therefore, Eigen values of $A^H$ are complex conjugate $\boxed{\bar{\lambda}}$ of Eigen value of matrix $A$.

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CH5.5 44P

**Step 1** of 5

A matrix is Hermitian matrix if $A = A^H$.

If $U$ is a unitary matrix then following is true:

$$UU^H = I$$

Columns of the unitary matrix are formed by orthonormal vectors.

Provide feedback (0)

**Step 2** of 5

If $u^H u = 1$ then to show that following is Hermitian and unitary:

$$I - 2uu^H$$

Also, find that rank-1 matrix $uu^H$ is the projection onto what line in $\mathbf{C}^n$.

Provide feedback (0)

**Step 3** of 5

Do the following calculations:

$$\begin{aligned}\left(uu^H\right)^H &= \left(u^H\right)^H u^H \\ &= uu^H\end{aligned}$$

Now, check for the $I - 2uu^H$ matrix:

$$\begin{aligned}\left(I - 2uu^H\right)^H &= \left(I - 2\left(uu^H\right)^H\right) \\ &= I - 2uu^H\end{aligned}$$

Above calculations shows that $I - 2uu^H$ is Hermitian matrix.

CH5.5 46P

Now, check for the $I-2uu^H$ matrix:

$$\begin{aligned}\left(I-2uu^H\right)^H &= \left(I-2\left(uu^H\right)^H\right)\\ &= I-2uu^H\end{aligned}$$

Above calculations shows that $I-2uu^H$ is Hermitian matrix.

Provide feedback (0)

**Step 4** of 5

Matrix $I-2uu^H$ will be unitary if:

$$\left(I-2uu^H\right)^H\cdot\left(I-2uu^H\right)=I.$$

Do the following calculations:

$$\begin{aligned}\left(I-2uu^H\right)^H\cdot\left(I-2uu^H\right) &= \left(I-2uu^H\right)\cdot\left(I-2uu^H\right)\\ &= \left(I-2uu^H\right)^2\\ &= I+4\left(uu^H\right)^2-4\left(I\cdot uu^H\right)\\ &= I-4\left(uu^H\right)+4\left(uu^H\cdot uu^H\right)\\ &= I-4\left(uu^H\right)+4\left(u\cdot 1\cdot u^H\right)\\ &= I-4\left(uu^H\right)+4\left(uu^H\right)\\ \left(I-2uu^H\right)^H\cdot\left(I-2uu^H\right) &= I\end{aligned}$$

Above calculations shows that $I-2uu^H$ is unitary matrix.

Provide feedback (0)

**Step 5** of 5

$$\left(I-2uu^H\right)^H\cdot\left(I-2uu^H\right)=I.$$

Do the following calculations:

$$\begin{aligned}\left(I-2uu^H\right)^H\cdot\left(I-2uu^H\right)&=\left(I-2uu^H\right)\cdot\left(I-2uu^H\right)\\&=\left(I-2uu^H\right)^2\\&=I+4\left(uu^H\right)^2-4\left(I\cdot uu^H\right)\\&=I-4\left(uu^H\right)+4\left(uu^H\cdot uu^H\right)\\&=I-4\left(uu^H\right)+4\left(u\cdot 1\cdot u^H\right)\\&=I-4\left(uu^H\right)+4\left(uu^H\right)\end{aligned}$$

$$\left(I-2uu^H\right)^H\cdot\left(I-2uu^H\right)=I$$

Above calculations shows that $I-2uu^H$ is unitary matrix.

Provide feedback (0)

**Step 5** of 5

Therefore, matrix $\boxed{I-2uu^H}$ is Hermitian and unitary. Rank-1 matrix $uu^H$ is the projection onto the line in $\mathbf{C}^n$ through $u$.

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CH5.5 45P

**Step 1** of 4

Suppose $A+iB$ is a unitary matrix, where $A$, $B$ are real.

We have to show that $Q=\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$ is orthogonal.

Provide feedback (0)

**Step 2** of 4

Since $A+iB$ is unitary

So $(A+iB)^H(A+iB)=I$

Now $(A+iB)^H(A+iB)=I$

$$\Rightarrow (A+iB)^H=(A+iB)^{-1}$$

$$\Rightarrow \left(A^H-iB^H\right)=\frac{1}{A+iB}$$

$$\Rightarrow A^T-iB^T=\frac{1}{A^2+B^2}[A-iB] \qquad \begin{pmatrix}\text{Since } A \text{ and } B \text{ are real} \\ \text{So } A^H=A^T \text{ and } B^H=B^T\end{pmatrix}$$

Provide feedback (0)

**Step 3** of 4

Comparing the real and imaginary parts on both sides, we get

$$A^T=\frac{A}{A^2+B^2}, B^T=\frac{B}{A^2+B^2}$$

Now we have $Q=\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$

Therefore,

$$Q^TQ=\begin{bmatrix} A^T & B^T \\ -B^T & A^T \end{bmatrix}\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$$

$$=\begin{bmatrix} A^TA+B^TB & -A^TB+B^TA \\ -B^TA+BA^T & BB^T+A^TA \end{bmatrix}$$

CH5.5 47P

$$\Rightarrow A^T - iB^T = \frac{1}{A^2+B^2}[A - iB] \qquad \left(\text{So } A^H = A^T \text{ and } B^H = B^T\right)$$

Provide feedback (0)

**Step 3** of 4

Comparing the real and imaginary parts on both sides, we get

$$A^T = \frac{A}{A^2+B^2}, B^T = \frac{B}{A^2+B^2}$$

Now we have $Q = \begin{bmatrix} A & -B \\ B & A \end{bmatrix}$

Therefore,

$$Q^T Q = \begin{bmatrix} A^T & B^T \\ -B^T & A^T \end{bmatrix}\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$$

$$= \begin{bmatrix} A^T A + B^T B & -A^T B + B^T A \\ -B^T A + BA^T & BB^T + A^T A \end{bmatrix}$$

Provide feedback (0)

**Step 4** of 4

Continuation to the above

$$= \begin{bmatrix} \frac{A}{A^2+B^2} + \frac{B}{A^2+B^2} & \frac{-AB}{A^2+B^2} + \frac{AB}{A^2+B^2} \\ \frac{-AB}{A^2+B^2} + \frac{AB}{A^2+B^2} & \frac{B^2}{A^2+B^2} + \frac{A^2}{A^2+B^2} \end{bmatrix} \qquad (\text{Since by } (1))$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Since $Q^T Q = I$

Therefore $Q$ is an orthogonal matrix.

Provide feedback (0)

CH5.5 46P

**Step 1** of 3

Suppose $A+iB$ is a Hermitian matrix, where $A$, $B$ are real.

We have to show that $Q=\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$ is symmetric.

Provide feedback (0)

**Step 2** of 3

Since the matrices $A$, $B$ are real.

So $A^H = A^T$ and $B^H = B^T$

Since $A+iB$ is a Hermitian matrix.

$$\Rightarrow (A+iB)^H = A+iB$$

$$\Rightarrow A^H - iB^H = A+iB$$

$$\Rightarrow A^T - iB^T = A+iB \qquad \left(\text{since } A^H = A^T, B^H = B^T\right)$$

Provide feedback (0)

**Step 3** of 3

Comparing the real and imaginary parts on both sides, we get

$A^T = A$ and $B^T = -B$

Now we have $Q=\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$

Therefore,

$$Q^T = \begin{bmatrix} A^T & B^T \\ -B^T & A^T \end{bmatrix}$$

$$= \begin{bmatrix} A & -B \\ B & A \end{bmatrix}$$

$$= Q$$

Since $Q^T = Q$

So $Q=\begin{bmatrix} A & -B \\ B & A \end{bmatrix}$ is symmetric matrix

CH5.5 48P

$$\Rightarrow (A+iB)^{H} = A+iB$$
$$\Rightarrow A^{H} - iB^{H} = A+iB$$
$$\Rightarrow A^{T} - iB^{T} = A+iB \qquad \left(\text{since } A^{H} = A^{T}, B^{H} = B^{T}\right)$$

Provide feedback (0)

**Step 3** of 3

Comparing the real and imaginary parts on both sides, we get

$A^{T} = A$ and $B^{T} = -B$

Now we have $Q = \begin{bmatrix} A & -B \\ B & A \end{bmatrix}$

Therefore,

$$Q^{T} = \begin{bmatrix} A^{T} & B^{T} \\ -B^{T} & A^{T} \end{bmatrix}$$
$$= \begin{bmatrix} A & -B \\ B & A \end{bmatrix}$$
$$= Q$$

Since $Q^{T} = Q$

So $Q = \begin{bmatrix} A & -B \\ B & A \end{bmatrix}$ is symmetric matrix.

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CH5.5 47P

**Step 1** of 2

We have to prove that the inverse of a Hermitian matrix is also Hermitian.

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CH5.5 49P

**Step 2** of 2

Let $A$ be a Hermitian matrix.
Then $A^H = A$
Let $A^{-1}$ be the inverse of $A$.
Then $AA^{-1} = I$
Now

$$\left(AA^{-1}\right)^H = (I)^H$$

$$\Rightarrow \left(A^{-1}\right)^H A^H = I \quad \left(\text{since } I^H = I\right)$$

$$\Rightarrow \left(A^{-1}\right)^H A = I \quad \left(\text{since } A^H = A\right)$$

Therefore $\left(A^{-1}\right)^H$ is the inverse of $A$.

Hence $A^{-1}$ is also Hermitian matrix.
Hence the inverse of a Hermitian matrix is again a Hermitian matrix.

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CH5.5 48P

**Step 1** of 17

Given that $A=\begin{bmatrix} 2 & 1-i \\ 1+i & 3 \end{bmatrix}$

We have to diagonalize the given matrix by constructing its eigenvalue matrix $\Lambda$ and its eigenvector matrix $S$.

Provide feedback (0)

**Step 2** of 17

First we find the eigenvalues of $A$.
The characteristic equation of $A$ is

$$|A-\lambda I|=0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1-i \\ 1+i & 3-\lambda \end{vmatrix}=0$$

$$\Rightarrow (2-\lambda)(3-\lambda)-(1-i^2)=0$$

Provide feedback (0)

**Step 3** of 17

Continuation to the above

$$\Rightarrow \lambda^2-5\lambda+6-2=0$$

$$\Rightarrow \lambda^2-9\lambda+4=0$$

$$\Rightarrow (\lambda-1)(\lambda-4)=0$$

$$\Rightarrow \lambda=1,4$$

Hence the eigenvalues of $A$ are 1, 4

Therefore, the eigenvalue matrix is $\Lambda=\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$.

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CH5.5 50P

Continuation to the above

$\Rightarrow \lambda^2 - 5\lambda + 6 - 2 = 0$

$\Rightarrow \lambda^2 - 9\lambda + 4 = 0$

$\Rightarrow (\lambda - 1)(\lambda - 4) = 0$

$\Rightarrow \lambda = 1, 4$

Hence the eigenvalues of $A$ are 1, 4

Therefore, the eigenvalue matrix is $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$.

Provide feedback (0)

**Step 4** of 17

Now we find the eigenvectors of the given matrix.

We know that $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is an eigenvector of $A$ if and only if $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the nonzero solution of the system $(A - \lambda I)x = 0$

That is $\begin{bmatrix} 2-\lambda & 1-i \\ 1+i & 3-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$ ..... (1)

Provide feedback (0)

**Step 5** of 17

For $\lambda = 1$, (1) becomes

$$\begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The Augmented matrix

$$\begin{bmatrix} 1 & 1-i & 0 \\ 1+i & 2 & 0 \end{bmatrix}$$

Add $(1-i)$ times of row 1 to row 2.

$$\begin{bmatrix} 1 & 1-i & 0 \\ 2 & 2-2i & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+i & 3-\lambda \end{bmatrix}\begin{bmatrix} x_2 \end{bmatrix}$$

Provide feedback (0)

**Step 5** of 17

For $\lambda = 1$, (1) becomes

$$\begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The Augmented matrix

$$\begin{bmatrix} 1 & 1-i & 0 \\ 1+i & 2 & 0 \end{bmatrix}$$

Add $(1-i)$ times of row 1 to row 2.

$$\begin{bmatrix} 1 & 1-i & 0 \\ 2 & 2-2i & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 6** of 17

Add $(-2)$ times of row 1 to row 2.

$$\begin{bmatrix} 1 & 1-i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From this, we get

$x_1 + (1-i)x_2 = 0$

Here $x_2$ is free variable.

Let $x_2 = k$, where $k$ is a free variable.

Then $x_1 = -(1-i)k$

Provide feedback (0)

**Step 7** of 17

Therefore,

Then $x_1 = -(1-i)k$

Provide feedback (0)

**Step 7** of 17

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= \begin{bmatrix} -(1-i)k \\ k \end{bmatrix}$$
$$= -k\begin{bmatrix} 1-i \\ 1 \end{bmatrix}$$

Therefore, the eigenvector corresponding to the eigenvalue $\lambda = 1$ is $\begin{bmatrix} 1-i \\ 1 \end{bmatrix}$.

Provide feedback (0)

**Step 8** of 17

For $\lambda = 4$, (1) becomes

$$\begin{bmatrix} -2 & 1-i \\ 1+i & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

The Augmented matrix is

$$\begin{bmatrix} -2 & 1-i \\ 1+i & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Add $(1(1-i))$ times of row 1 to 2 times of row 2

$$\begin{bmatrix} -2 & 1-i & 0 \\ 4 & -2+2i & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 9** of 17

The Augmented matrix is

$$\begin{bmatrix} -2 & 1-i \\ 1+i & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Add $\left(1(1-i)\right)$ times of row 1 to 2 times of row 2

$$\begin{bmatrix} -2 & 1-i & 0 \\ 4 & -2+2i & 0 \end{bmatrix}$$

Provide feedback (0)

**Step 9** of 17

Add 2 times of row 1 to row 2

Provide feedback (0)

**Step 10** of 17

$$\begin{bmatrix} -2 & 1-i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From this, we get

$-2x_1 + (1-i)x_2 = 0$

Here $x_2$ is free variable.

Let $x_2 = k$, where $k$ is a free variable.

Then $x_1 = \left(\frac{1-i}{2}\right)k$

Provide feedback (0)

**Step 11** of 17

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Provide feedback (0)

**Step 11** of 17

Therefore,

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{(1-i)}{2}k \\ k \end{bmatrix}$$
$$= \frac{k}{2}\begin{bmatrix} 1-i \\ 2 \end{bmatrix}$$

Therefore, the eigenvector corresponding to the eigenvalue $\lambda = 4$ is $\begin{bmatrix} 1-i \\ 2 \end{bmatrix}$.

Provide feedback (0)

**Step 12** of 17

Let $S = \begin{bmatrix} 1-i & 1-i \\ -1 & 2 \end{bmatrix}$

Then

$$S^{-1} = \frac{1}{2(1-i)+1(1-i)}\begin{bmatrix} 2 & -1+i \\ 1 & 1-i \end{bmatrix}$$
$$= \frac{1}{2-2i+1-i}\begin{bmatrix} 2 & -1+i \\ 1 & 1-i \end{bmatrix}$$
$$= \frac{1}{2-2i+1-i}\begin{bmatrix} 2 & -1+i \\ 1 & 1-i \end{bmatrix}$$

Provide feedback (0)

**Step 13** of 17

Continuation to the above

$$= \frac{}{2-2i+1-i}\begin{bmatrix} 1 & 1-i \end{bmatrix}$$

Provide feedback (0)

**Step 13** of 17

Continuation to the above

$$= \frac{1}{3-3i}\begin{bmatrix} 2 & -1+i \\ 1 & 1-i \end{bmatrix}$$

$$= \frac{3+3i}{18}\begin{bmatrix} 2 & -1+i \\ 1 & 1-i \end{bmatrix}$$

$$= \frac{1+i}{6}\begin{bmatrix} 2 & -1+i \\ 1 & 1-i \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 2+2i & -2 \\ 1+i & 2 \end{bmatrix}$$

Therefore, $S^{-1} = \frac{1}{6}\begin{bmatrix} 2+2i & -2 \\ 1+i & 2 \end{bmatrix}$

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**Step 14** of 17

Now

$$S\Lambda S^{-1} = \begin{bmatrix} 1-i & 1-i \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\frac{1}{6}\begin{bmatrix} 2+2i & -2 \\ 1+i & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-i & 1-i \\ -1 & 2 \end{bmatrix}\begin{bmatrix} \frac{1}{6}(2+2i)+0(1+i) & \frac{1}{6}(-2)+0(2) \\ 0(2+2i)+\frac{4}{6}(1+i) & 0(-2)+\frac{4}{6}(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1-i & 1-i \\ -1 & 2 \end{bmatrix}\begin{bmatrix} \frac{1+i}{3} & -\frac{1}{3} \\ \frac{2}{3}(1+i) & \frac{4}{3} \end{bmatrix}$$

Now

$$SAS^{-1}=\begin{bmatrix}1-i & 1-i\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 4\end{bmatrix}\frac{1}{6}\begin{bmatrix}2+2i & -2\\ 1+i & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-i & 1-i\\ -1 & 2\end{bmatrix}\begin{bmatrix}\frac{1}{6}(2+2i)+0(1+i) & \frac{1}{6}(-2)+0(2)\\ 0(2+2i)+\frac{4}{6}(1+i) & 0(-2)+\frac{4}{6}(2)\end{bmatrix}$$

$$=\begin{bmatrix}1-i & 1-i\\ -1 & 2\end{bmatrix}\begin{bmatrix}\frac{1+i}{3} & -\frac{1}{3}\\ \frac{2}{3}(1+i) & \frac{4}{3}\end{bmatrix}$$

Provide feedback (0)

**Step 15** of 17

Continuation to the above

$$=\begin{bmatrix}(1-i)\left(\frac{1+i}{3}\right)+(1-i)\left(\frac{2}{3}(1+i)\right) & (1-i)\left(-\frac{1}{3}\right)+(1-i)\frac{4}{3}\\ -1\left(\frac{1+i}{3}\right)+2\left(\frac{2}{3}(1+i)\right) & (-1)\left(-\frac{1}{3}\right)+(2)\frac{4}{3}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{3}+\frac{4}{3} & -(1-i)\frac{1}{3}+(1-i)\frac{4}{3}\\ -\frac{1}{2}(1+i)+\frac{4}{3}(1+i) & \frac{1}{3}+\frac{8}{3}\end{bmatrix}$$

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**Step 16** of 17

Continuation to the above

$$=\begin{bmatrix}\frac{6}{3} & (1-i)\frac{3}{3}\\ \frac{3}{3}(1+i) & \frac{9}{3}\end{bmatrix}$$

$$= \begin{bmatrix} -1\left(\frac{1+i}{3}\right)+2\left(\frac{2}{3}(1+i)\right) & (-1)\left(-\frac{1}{3}\right)+(2)\frac{4}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3}+\frac{4}{3} & -(1-i)\frac{1}{3}+(1-i)\frac{4}{3} \\ -\frac{1}{2}(1+i)+\frac{4}{3}(1+i) & \frac{1}{3}+\frac{8}{3} \end{bmatrix}$$

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**Step 16** of 17

Continuation to the above

$$= \begin{bmatrix} \frac{6}{3} & (1-i)\frac{3}{3} \\ \frac{3}{3}(1+i) & \frac{9}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1-i \\ 1+i & 3 \end{bmatrix}$$

Therefore, $A = S\Lambda S^{-1}$

Hence the matrix that diagonalizes the given matrix $A$ is $\boxed{S = \begin{bmatrix} 1-i & 1-i \\ -1 & 2 \end{bmatrix}}$.

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**Step 17** of 17

CH5.5 49P

**Step 1** of 4

Complex inner product: $\mathbf{u}^H\mathbf{v} = \bar{u}_1 v_1 + \cdots + \bar{u}_n v_n$.

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CH5.6

**Step 2** of 4

Consider the following orthonormal Eigen vectors form of matrix $A$.

$$\begin{aligned} A &= U\Lambda U^{-1} \\ &= U\Lambda U^H \end{aligned}$$

Prove the following:

$$AA^H = A^H A$$

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**Step 3** of 4

Calculate the following:

$$\begin{aligned} AA^H &= \left(U\Lambda U^H\right)\left(U\Lambda U^H\right)^H \\ &= \left(U\Lambda U^H\right)\left(\left(U^H\right)^H \Lambda U^H\right) \\ &= U\Lambda U^H U\Lambda U^H \\ &= \left(\left(U^H\right)^H \Lambda U^H\right)\left(U\Lambda U^H\right) \\ AA^H &= A^H A \end{aligned}$$

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**Step 4** of 4

Prove the following:

$$AA^H = A^H A$$

Provide feedback (0)

**Step 3** of 4

Calculate the following:

$$\begin{aligned} AA^H &= (U\Lambda U^H)(U\Lambda U^H)^H \\ &= (U\Lambda U^H)\left((U^H)^H \Lambda U^H\right) \\ &= U\Lambda U^H U\Lambda U^H \\ &= \left((U^H)^H \Lambda U^H\right)(U\Lambda U^H) \\ AA^H &= A^H A \end{aligned}$$

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**Step 4** of 4

Therefore, $\boxed{AA^H = A^H A}$. These are exactly the normal matrices

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