HW: Section 9A #3, 4

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May 5, 2015

9A

#3

Suppose V is a real vector space and $v_1, \dots, v_m \in V$. Prove that $(v_1, \dots, v_m)$ is linearly independent in $V_{\mathbb{C}}$ if and only if $(v_1, \dots, v_m)$ is linearly independent in V.

"$\Longrightarrow$"

Suppose $\pi = (v_1, \dots, v_m)$ is linearly independent in $V_{\mathbb{C}}$. Then $\sum_{k=1}^m (a_k + ib_k)v_k = 0 \implies a_k = b_k = 0$ for $k = 1, \dots, m$. Then suppose $\sum_{k=1}^m a_k v_k = 0$. But $\sum_{k=1}^m a_k v_k = \sum_{k=1}^m (a_k + i(0))v_k$. Then by our assumption, $a_k + i(0) = 0$ for $k = 1, \dots, m$, which implies $a_k = 0$ for all $k = 1, \dots, m$. Thus π is linearly independent in V.

"$\Longleftarrow$"

Suppose π is linearly independent in V. Then $\sum_{k=1}^m a_k v_k = 0 \implies a_k = 0$ for all $k = 1, \dots, m$. Then suppose $\sum_{k=1}^m (a_k + ib_k)v_k = 0 + i(0)$. Then $\sum_{k=1}^m a_k v_k = \sum_{k=1}^m b_k v_k = 0$. Then by our assumption, $a_k = b_k = 0$ for $k = 1, \dots, m$. Then $(a_k + ib_k) = 0$ for $k = 1, \dots, m$. Thus π is linearly independent in $V_{\mathbb{C}}$.

Thus π is linearly independent in V if and only if π is linearly independent in $V_{\mathbb{C}}$. $\square$

#4

Suppose V is a real vector space and $v_1, \dots, v_m \in V$. Prove that $(v_1, \dots, v_m)$ spans $V_{\mathbb{C}}$ if and only if $(v_1, \dots, v_m)$ spans V.

"$\Longrightarrow$"

Suppose $\pi = (v_1, \dots, v_m)$ spans $V_{\mathbb{C}}$. Then $\forall v_{\mathbb{C}} \in V_{\mathbb{C}}$, $\exists (a_1 + ib_1), \dots, (a_m + ib_m) \in \mathbb{C}$ such that $v_{\mathbb{C}} = \sum_{k=1}^m (a_k + ib_k)v_k$. Then let $v \in V$, and consider $v + i(0) \in V_{\mathbb{C}}$. By our assumption, $\exists (a_1 + ib_1), \dots, (a_m + ib_m) \in \mathbb{C}$ such that $v + i(0) = \sum_{k=1}^m (a_k + ib_k)v_k$. Then $v = \sum_{k=1}^m a_k v_k$ (and $0 = \sum_{k=1}^m b_k v_k$). Thus π spans V.

"$\Longleftarrow$"

Suppose π spans V. Then $\forall v \in V$, $\exists a_1, \dots, a_m \in \mathbb{R}$ such that $v = \sum_{k=1}^m a_k v_k$. Then let $v_{\mathbb{C}} \in V_{\mathbb{C}}$, and note $v_{\mathbb{C}} = v_{\mathbb{R}} + iv_{\mathbb{I}}$ for some $v_{\mathbb{R}}, v_{\mathbb{I}} \in V$. By our assumption, $\exists a_1, \dots, a_m, b_1, \dots, b_m \in \mathbb{R}$

such that $v_{\mathbb{R}} = \sum_{k=1}^m a_k v_k$ and $v_{\mathbb{I}} = \sum_{k=1}^m b_k v_k$. Then $v_{\mathbb{C}} = \sum k = 1^m a_k v_k + i \sum k = 1^m b_k v_k = \sum_{k=1}^m (a_k + i b_k) v_k$. Thus π spans $V_{\mathbb{C}}$.

Thus π spans V if and only if π spans $V_{\mathbb{C}}$. $\square$