

# A Stochastic Method for the Generation of Optimized Building Layouts Respecting Urban Regulations

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# Outline

- Context, Motivation and Goal
- Proposed Method
- Case Studies

# Context: project e-PLU

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- Financed by European Regional Development Fund
- Aiming at transforming the paper urban planning documents into digital urban development services through a 3D simulation platform that estimates the legal building possibilities according to morphological and urban criteria



# Context: French urban planning rules

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- French local urban planning scheme: PLU (Plan Local d'Urbanisme)
  - Zoning plan;
  - Regulations for each zone type, including all or some of the 16 articles;
  - Each article describes a fixed theme, e.g.
    - Article 1: prohibited land use
    - Article 13: open space and plantation
    - Article 16: electronic communication infrastructures and network
- This work focuses on the PLU rules that regulate the spatial aspects for building development at the scale of a parcel:
  - Article 6: building position in relation to public roads,
  - Article 7: building position in relation to separative limits,
  - Article 8: building position in relation to other buildings,
  - Article 9: building footprint,
  - Article 10: building height,
  - Article 14: floor area ratio.

# Motivation and Goal

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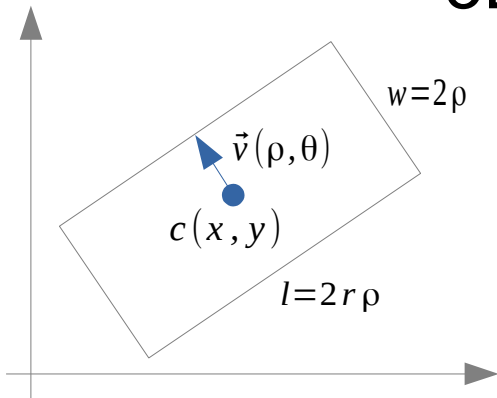
- Motivation:
  - To better understand urban planning rules (provided in text and/or with graphic illustration)
  - To assess the impact of the rules
  - To help preliminary design and presentation of new construction projects
- Goal:
  - To automatically generate 3D buildings on a parcel, complying with local urban planning rules, and of optimized urban indicators (e.g. floor area ratio)

# Outline

- Context, Motivation and Goal
- Proposed Method
  - Stochastic model
  - Optimization
  - Energetic modeling
- Case Studies

# Proposed Method: Stochastic Model (1)

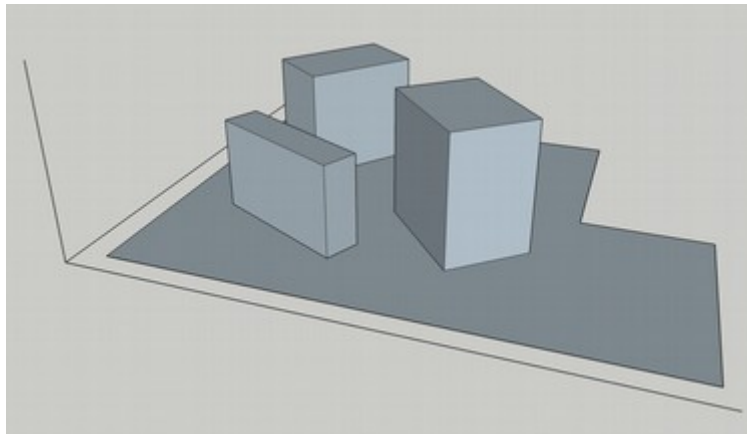
- Model the problem by a stochastic process: marked point process (MPP)
  - An object can be parameterized into a point (e.g its center) and its mark (all other geometric parameters)
  - In our problem, a building is represented by a 3D box with 6 parameters  $(x, y, \rho, \theta, r, h)$



- Footprint center  $c(x, y)$
- Footprint semi-minor axis vector  $\vec{v}(\rho, \theta)$
- Footprint aspect ratio (length/width)  $r = l/w$
- Height  $h$

# Proposed Method: Stochastic Model (2)

- Model the problem by a stochastic process: marked point process (MPP)



- A building layout is a configuration of an arbitrary number of buildings with different geometry.
- It can be considered as a realization of MPP of 3D box.

- A MPP is defined by a probabilized space  $(\Omega, \pi)$

$$\Omega = \bigcup_{n=0}^{\infty} K^n$$

- K: the set of possible values of a single object
- n: the number of objects

- For our problem

$$K = \underbrace{\overbrace{[0, 1] \times [0, 1]}^{\text{point}}}_{c'_i} \times \underbrace{\overbrace{[\rho_{min}, \rho_{max}] \times [0, \pi]}^{\text{mark}}}_{\vec{v}_i} \times \underbrace{\overbrace{1, [r_{max}]}_{r_i}} \times \underbrace{\overbrace{[h_{min}, h_{max}]}_{h_i}}.$$



## Proposed Method: Stochastic Model (3)

- Model the problem by a stochastic process: marked point process (MPP)
  - Probability distribution  $\pi$  of a MPP  $X$  can be defined by Gibbs measure:

$$\pi(X) = Z^{-1} e^{-E(X)}, \text{ where } Z = \int_X e^{-E(X)} d\mu(X)$$

- $Z$  is the normalization factor, with  $\mu(\cdot)$  as the distribution of the reference process (eg. Poisson process)
- $E(X)$  is the energy component which reflects the quality of a configuration: compliance with the PLU rules and optimization degree.
- Thus, the optimal building layout  $\tilde{X}$  can be found by maximizing the probability:  $\tilde{X} = \operatorname{argmax} \pi(\cdot)$

# Proposed Method: Optimization

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- Optimization approach: Simulated annealing
  - Problem: direct sampling from  $\pi$  is not possible!
  - Solution: coupling with RJMCMC (Reversible Jump Markov Chain Monte Carlo) sampler

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**Algorithm 1** Optimization: simulated annealing coupled with RJMCMC sampler.

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Construct an initial configuration  $X_0$

Define a temperature schedule  $S(\cdot)$

**repeat**

    Sample  $i \sim q(\cdot|X_t)$  //select a reversible kernel  $Q_i$

    Sample  $X'_t \sim Q_i(\cdot|X_t)$  //propose a new configuration

$R_\infty \leftarrow \frac{\mu(X'_t)Q_i(X_t|X'_t)}{\mu(X_t)Q_i(X'_t|X_t)}$  //compute Green ratio of the reference process

$R \leftarrow R_\infty e^{-\frac{\Delta E}{T}}$  //metropolis acceptance parameterized by delta energy and temperature

$a \leftarrow \min(1, R)$

    With probability  $\begin{cases} a & X_{t+1} \leftarrow X'_t \\ 1 - a & X_{t+1} \leftarrow X_t \end{cases}$

$T \leftarrow S(T)$  //Decrease T according to schedule

**until** EndTest=true

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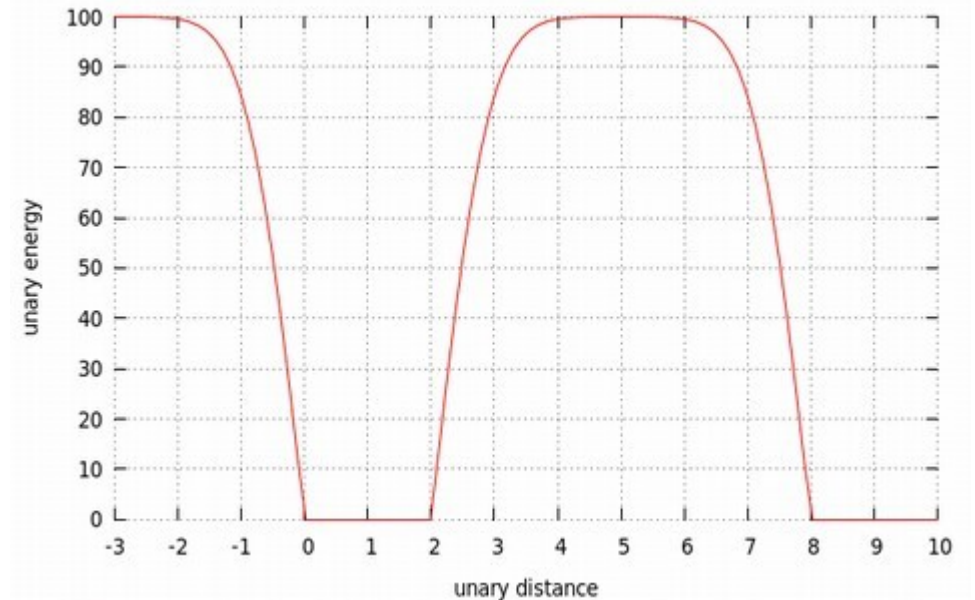
# Proposed Method: Energetic Modeling (1)

- Studied PLU articles
  - Article 6: building position (e.g. distance, angle) in relation to public roads (front borders)
  - Article 7: building position in relation to separative limits (side and back borders)
  - Article 8: building position in relation to other buildings (e.g. distance between buildings)
  - Article 9: building footprint (e.g. coverage ratio)
  - Article 10: building height (e.g. height limits, height difference between buildings)
  - Article 14: floor area ratio
- Grouped into 2 types of rules
  - Rule A: needs energetic modeling
  - Rule B: can be directly configured into geometric parameters
- Urban rules to energy terms or to parametric constraints
  - Rule A1: distance to parcel borders → unary distance energy
  - Rule A2: distance between buildings → binary distance energy
  - Rule A3: height difference between buildings → binary height energy
  - Rule A4: lot coverage ratio → global coverage energy
  - Rule A5: floor area ratio → global builtup energy
  - Rule B1: angle to parcel borders → constraint on parameter  $\rho$
  - Rule B2: building height limits → constraint on parameter  $h$

# Proposed Method: Energetic Modeling (2)

- The total energy is the sum of finite weighted energy terms, each of which is formed according to a specific rule.
- An example of energy formulation
  - If the distance from the  $i^{th}$  building to the  $j^{th}$  border  $d_{ij}$  should satisfy  $0 \leq d_{ij} \leq 2m \parallel d_{ij} \geq 8m$
  - The unary border energy of the  $i^{th}$  building to the  $j^{th}$  border  $E_{ij}^{u_d}$  can be defined by using Gaussian error function  $erf(.)$ :

$$E_{ij}^{u_d} = \begin{cases} -a * erf(d_{ij}) & d_{ij} \in [-\infty, 0) \\ 0 & d_{ij} \in [0, 2] \\ a * erf(d_{ij} - 2) & d_{ij} \in (2, 5] \\ -a * erf(d_{ij} - 8) & d_{ij} \in (5, 8) \\ 0 & d_{ij} \in [8, +\infty) \end{cases}$$



# Proposed Method: Implementation

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- Developed an open source tool:

BuildUP (Building generator for local Urban Planning)

<https://github.com/IGNF/BuildUP>

- Automatic generation of buildings (as 3D boxes) on a target parcel, complying with local urban planning rules, and of optimized urban indicators
- Automatic energetic modeling of common French PLU rules
- Optimization is realized using the open source c++ library `librjmc` <https://github.com/IGNF/librjmc> which provides a framework for stochastic optimization using RJMCMC sampler and simulated annealing

# Outline

- Context, Motivation and Goal
- Proposed Method
- Case Studies
  - Two case studies are conducted under the cooperation with our project partner  and evaluated by their urban planning experts

# Case Study A

## The PLU rules to obey:

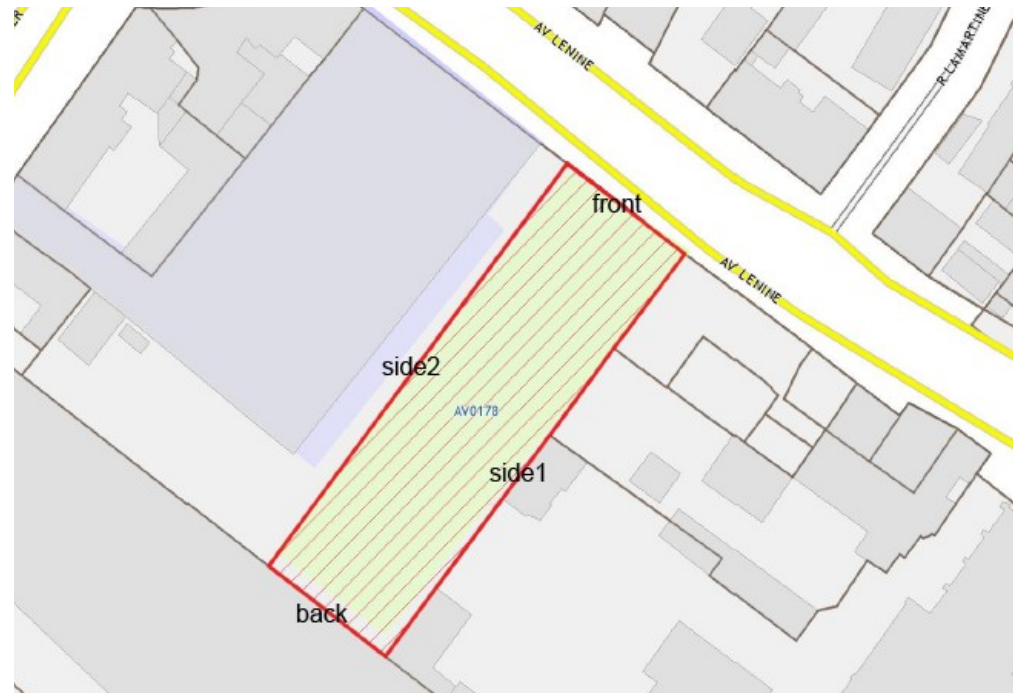
- Article 6(1):  $d_{Front} > 3m$   
6(2):  $\theta_{bldg} = \theta_{front}$
- Article 7:  
 $d_{Side1} = 0 \ \& \ d_{Side2} = 0 \parallel d_{Side1} = 0 \ \& \ d_{Side2} \geq \max(6, h) \parallel d_{Side2} = 0 \ \& \ d_{Side1} \geq \max(6, h)$   
 $d_{Back} > 4m$
- Article 8:  $d_{Pair} > 6m$
- Article 9: lot coverage ratio  $\leq 0.6$
- Article 10(1):  $6m \leq h \leq 18m$   
10(2):  $h_{Diff} \leq 6m$
- Article 14: floor area ratio  $\leq 4$

## Additional constraints:

- Width of each building fixed to 5m or 10m
- Height limit varies by width  
e.g.  $h \leq 6m$  if  $w = 5m$ ;  $h \leq 18m$  if  $w = 10m$

## Optimization goal:

Maximize floor area ratio (FAR)



12 avenue Lénine, La Courneuve, France

This case study is a bit different than the one presented in the paper  
(with additional constraints and modified rules)

# Case Study A

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Energy function:  $E = 30E^{u_d} + 20E^{b_d} + 20E^{b_h} + 20E^{g_c} + 50E^{g_f}$

Unary border energy  $E^{u_d}$

Binary distance energy  $E^{b_d}$

Global coverage energy  $E^{g_c}$

Binary height energy  $E^{b_h}$

Global buildup energy  $E^{g_f}$

- Test environment: 32 bit Linux (Ubuntu 14.04); HP workstation Z210 with 3.3GHz dual core CPU (Intel Core i5-2500) and 8 GB memory
- 100 experiments with 1.5 million iterations per experiment for simulated annealing
  - 32 optimal results (FAR $\geq$ 2.8 && no big violation of the rules)
  - 68 good results (2.8>FAR $\geq$ 2.0 && no big violation of the rules)
  - 0 bad results (FAR<2.0 || big violation of the rules)
  - ~ 56 seconds/experiment

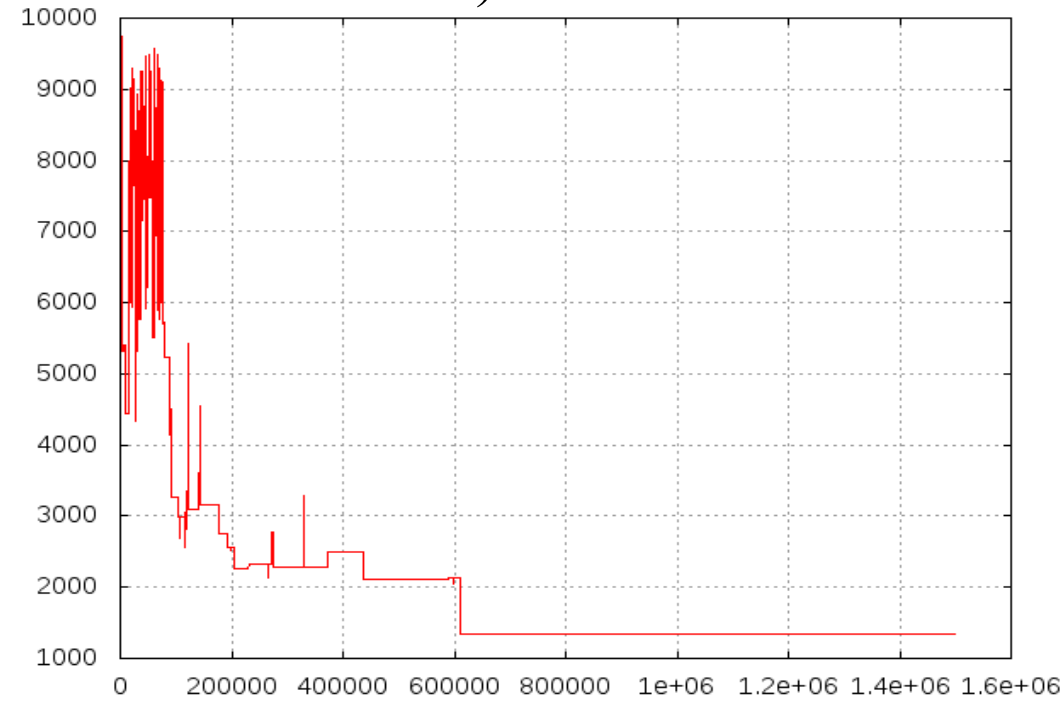
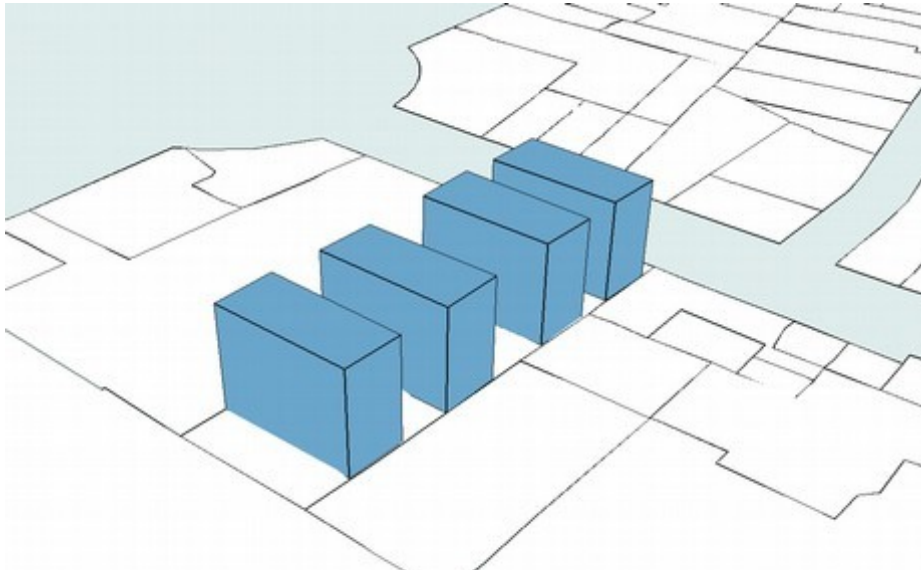
Statistics of 100 experiments

| floor area ratio |         |         | lot coverage ratio |          |         | CPU time (ms) |         |         |
|------------------|---------|---------|--------------------|----------|---------|---------------|---------|---------|
| min              | max     | average | min                | max      | average | min           | max     | average |
| 2.11511          | 3.02159 | 2.44782 | 0.352518           | 0.503598 | 0.40797 | 55892.4       | 58101.5 | 56413.4 |

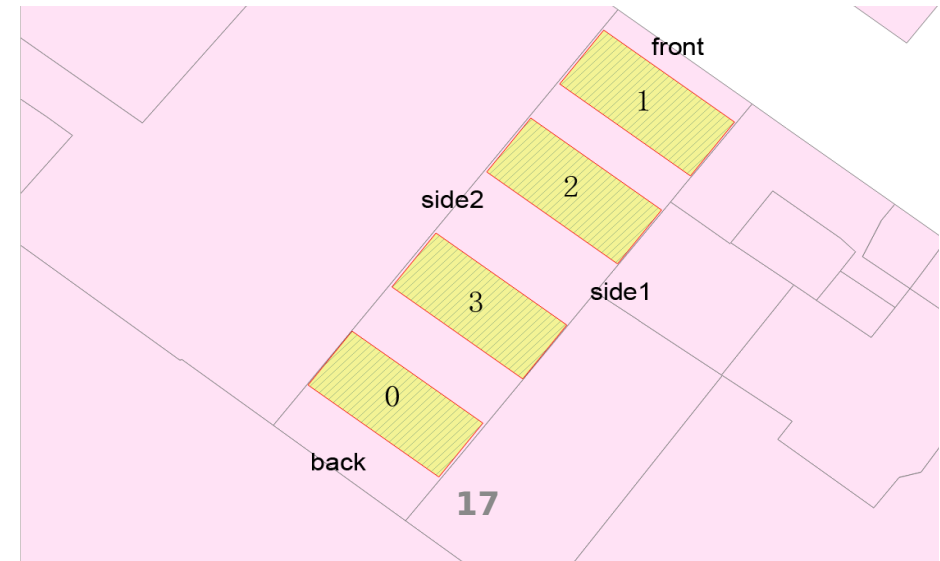


# Case Study A

Best optimization result (with highest floor area ratio)



|                               | dFront      | dSide1      | dSide2      | dBack       | height       |
|-------------------------------|-------------|-------------|-------------|-------------|--------------|
| bldg <sub>0</sub>             | 59.892      | 0.349779    | 0.0719926   | 7.62962     | 18           |
| bldg <sub>1</sub>             | 3.58819     | 0.17575     | 0.290706    | 63.9367     | 18           |
| bldg <sub>2</sub>             | 19.989      | 0.14313     | 0.0129      | 47.5358     | 18           |
| bldg <sub>3</sub>             | 41.3897     | 0.0846339   | 0.348109    | 26.1356     | 18           |
| Binary distance               |             |             |             |             |              |
| $d_{01}^b$                    | $d_{02}^b$  | $d_{03}^b$  | $d_{12}^b$  | $d_{13}^b$  | $d_{23}^b$   |
| 46.309                        | 29.798      | 8.569       | 6.360       | 27.802      | 11.435       |
| Lot coverage ratio = 0.503598 |             |             |             |             |              |
| Floor area ratio = 3.02159    |             |             |             |             |              |
| Energy                        |             |             |             |             |              |
| $30E^{u_d}$                   | $20E^{b_d}$ | $20E^{b_h}$ | $20E^{g_c}$ | $50E^{g_f}$ | total energy |
| 863.867                       | 0           | 0           | 0           | 478.647     | 1342.51      |

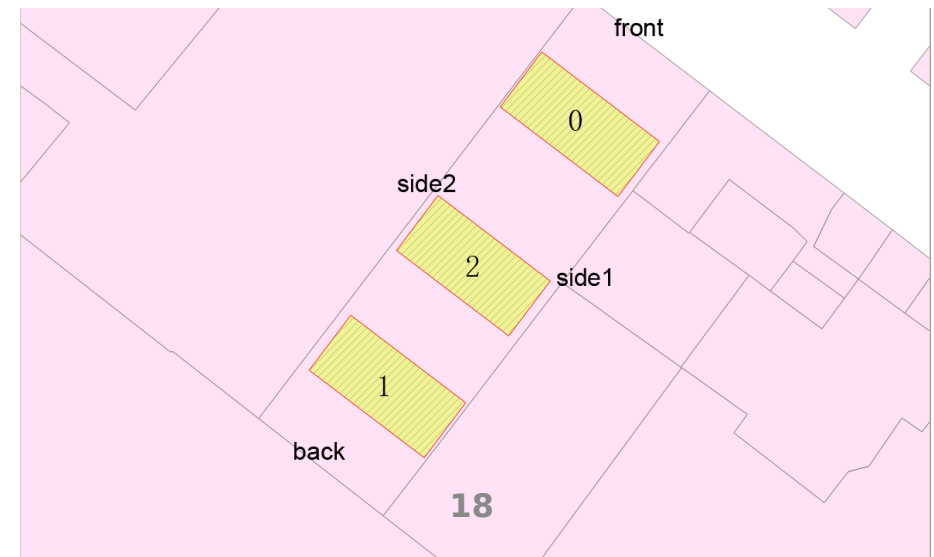
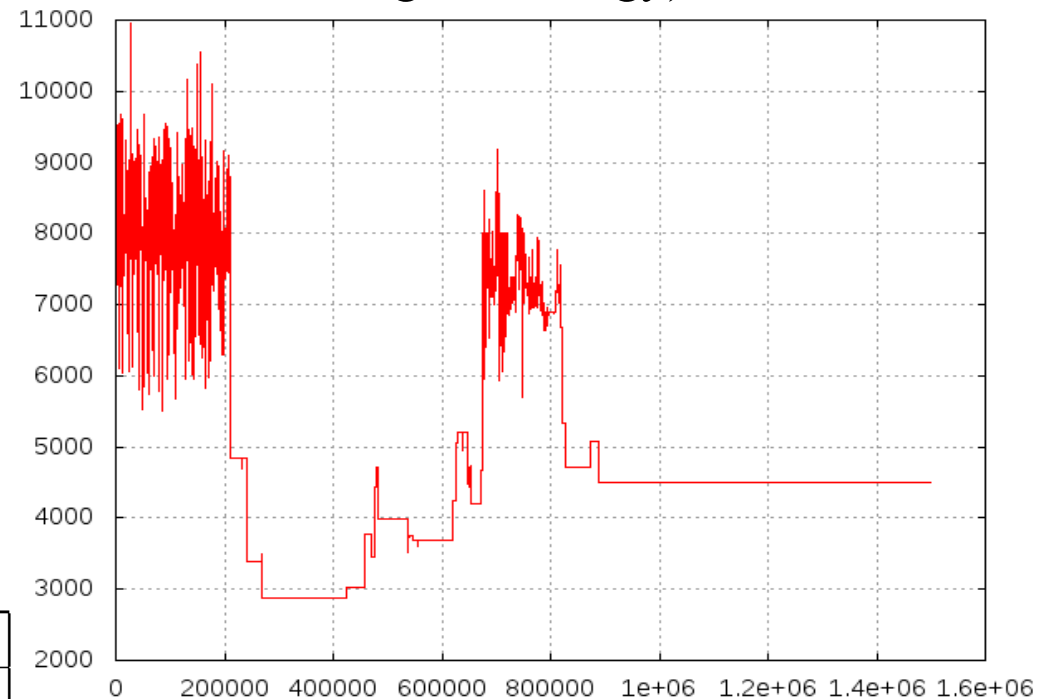


# Case Study A

Worst optimization result (with lowest floor area ratio and highest energy)



|                               | dFront      | dSide1      | dSide2      | dBack       | height       |
|-------------------------------|-------------|-------------|-------------|-------------|--------------|
| bldg <sub>0</sub>             | 10.2911     | 1.18468     | 0.271806    | 57.2335     | 18           |
| bldg <sub>1</sub>             | 57.4885     | 0.453683    | 1.49288     | 10.0512     | 18           |
| bldg <sub>2</sub>             | 35.9913     | 1.44529     | 0.982767    | 31.5421     | 18           |
| Binary distance               |             |             |             |             |              |
| $d_{01}^b$                    | $d_{02}^b$  | $d_{12}^b$  |             |             |              |
| 37.283                        | 15.627      | 11.524      |             |             |              |
| Lot coverage ratio = 0.352518 |             |             |             |             |              |
| Floor area ratio = 2.11511    |             |             |             |             |              |
| Energy                        |             |             |             |             |              |
| $30E^{u_d}$                   | $20E^{b_d}$ | $20E^{b_h}$ | $20E^{g_c}$ | $50E^{g_f}$ | total energy |
| 2720.69                       | 0           | 0           | 0           | 1776.41     | 4497.1       |



# Case Study B

## The PLU rules to obey:

- Article 6(1):  $(dFront1=0 \parallel dFront1>2.5m) \ \&\& \ (dFront2=0 \parallel dFront2>2.5m)$
- Article 6(2):  $\theta_{bldg} = \theta_{front}$
- Article 7:  
 $dSide1 \geq \max((h-3)/2, 3) \ \&\& \ dSide2 \geq \max((h-3)/2, 3)$   
 $dBack > 4m$
- Article 8:  $dPair > \max(hMax/2, (hMax-3), 8)$
- Article 9: no rule on lot coverage ratio
- Article 10(1):  $3.2m \leq h \leq 24m$
- Article 10(2):  $hDiff \leq 6m$
- Article 14: floor area ratio  $\leq 4$

## Additional constraints:

- Width of each building fixed to 5m or 10m
- Height limit varies by width  
e.g.  $h \leq 6m$  if  $w=5m$ ;  $h \leq 18m$  if  $w=10m$

## Optimization goal:

Maximize the floor area ratio



Quai de l'aéroplane chatelier, L'Île-Saint-Denis, France

# Case Study B

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Energy function:  $E = 30E^{u_d} + 20E^{b_d} + 20E^{b_h} + 20E^{g_c} + 50E^{g_f}$

Unary border energy  $E^{u_d}$

Binary distance energy  $E^{b_d}$       Binary height energy  $E^{b_h}$

Global coverage energy  $E^{g_c}$       Global buildup energy  $E^{g_f}$

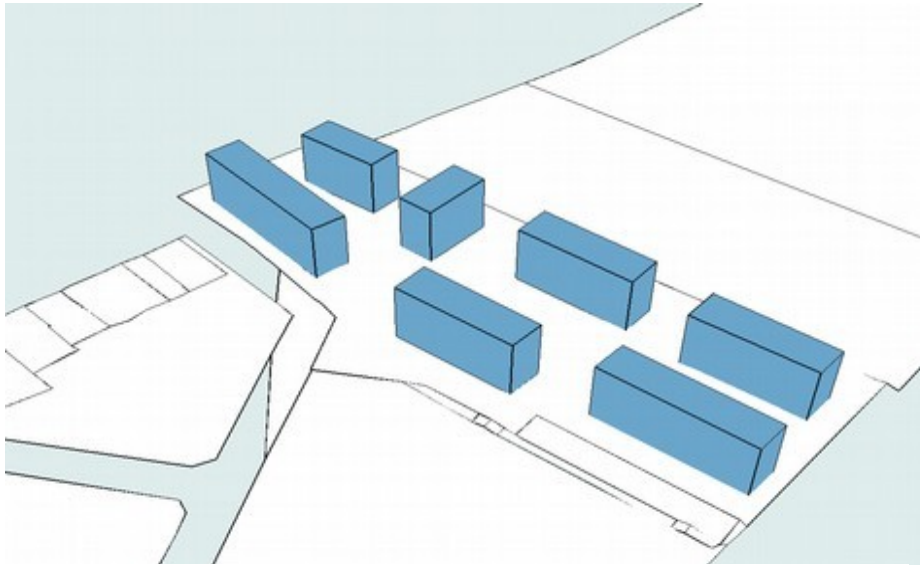
- Test environment: 32 bit Linux (Ubuntu 14.04); HP workstation Z210 with 3.3GHz dual core CPU (Intel Core i5-2500) and 8 GB memory
- 100 experiments with 1.5 million iterations per experiment for simulated annealing
  - 76 optimal results (FAR $\geq$ 1.0 && no big violation of the rules)
  - 24 good results (1.0>FAR $\geq$ 0.8 && no big violation of the rules)
  - 0 bad results (FAR<0.8 || big violation of the rules)
  - CPU time ~ 115 seconds

Statistics of 100 experiments

| floor area ratio |         |         | lot coverage ratio |         |          | CPU time (ms) |        |         |
|------------------|---------|---------|--------------------|---------|----------|---------------|--------|---------|
| min              | max     | average | min                | max     | average  | min           | max    | average |
| 0.816634         | 1.26306 | 1.05065 | 0.136106           | 0.21051 | 0.175109 | 113133        | 116516 | 115078  |

# Case Study B

Best optimization result (with highest floor area ratio)



|                   | dFront1 | dFront2 | dSide1  | dSide2  | height |
|-------------------|---------|---------|---------|---------|--------|
| bldg <sub>0</sub> | 9.85336 | 63.3457 | 83.449  | 47.9744 | 18     |
| bldg <sub>1</sub> | 36.4654 | 57.6205 | 82.763  | 11.4325 | 18     |
| bldg <sub>2</sub> | 7.60716 | 4.43858 | 133.293 | 42.9113 | 18     |
| bldg <sub>3</sub> | 29.4159 | 137.227 | 13.7346 | 12.1943 | 18     |
| bldg <sub>4</sub> | 36.8309 | 7.35529 | 131.968 | 13.6948 | 18     |
| bldg <sub>5</sub> | 4.59922 | 126.305 | 8.17203 | 34.6657 | 18     |
| bldg <sub>6</sub> | 29.9015 | 111.54  | 51.9139 | 8.68311 | 18     |

Binary distance: no violation

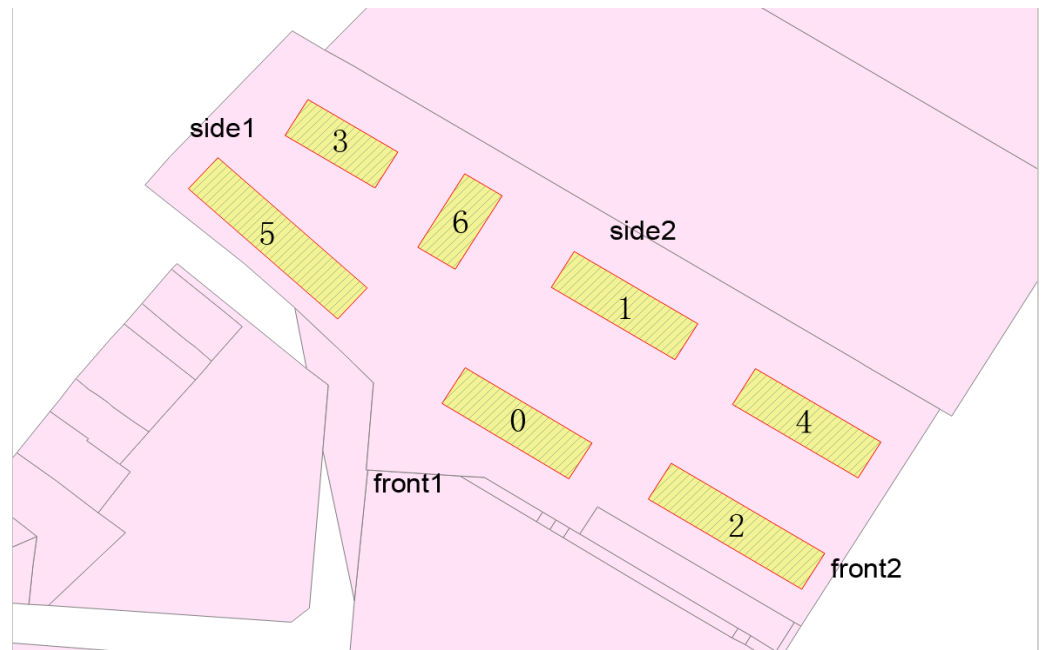
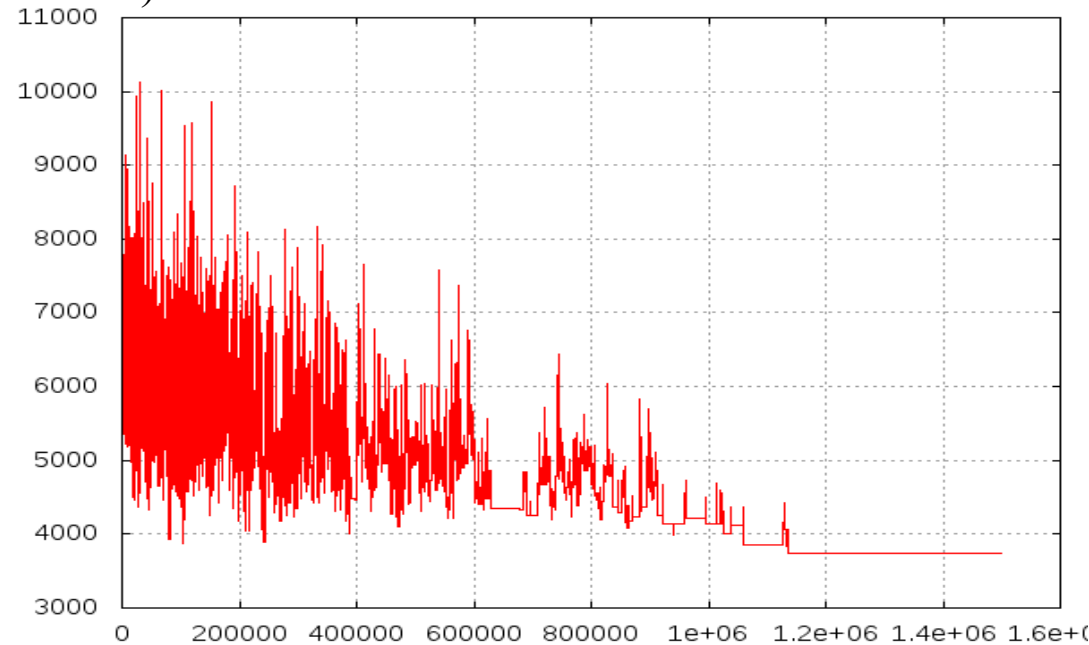
Binary height difference: no violation

Lot coverage ratio = 0.21051

Floor area ratio = 1.26306

Energy

| $30E^{u_d}$ | $20E^{b_d}$ | $20E^{b_h}$ | $20E^{g_c}$ | $50E^{g_f}$ | total energy |
|-------------|-------------|-------------|-------------|-------------|--------------|
| 0           | 0           | 0           | 0           | 3745.42     | 3745.42      |

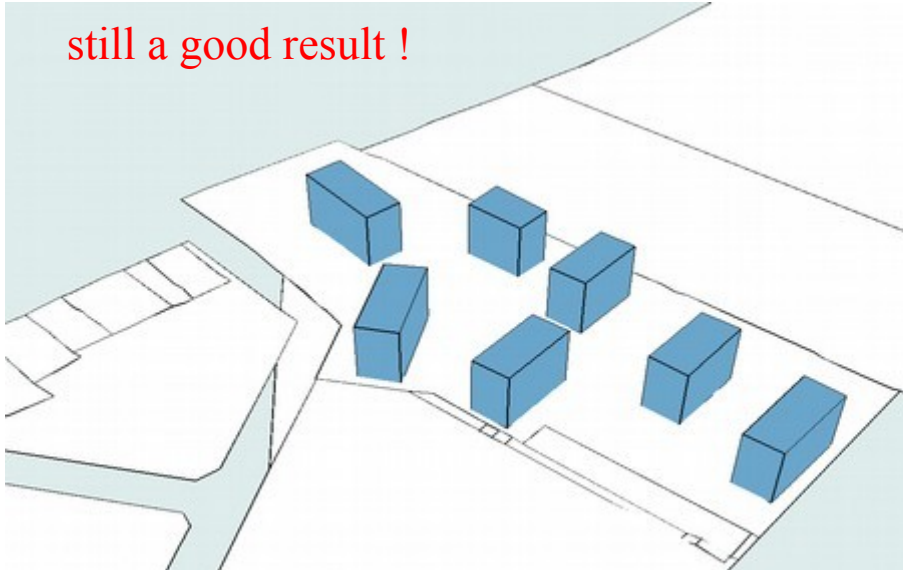




# Case Study B

Worst optimization result (with lowest floor area ratio)

still a good result !



|                   | dFront1 | dFront2 | dSide1  | dSide2  | height |
|-------------------|---------|---------|---------|---------|--------|
| bldg <sub>0</sub> | 9.18388 | 88.1194 | 73.6874 | 47.2649 | 18     |
| bldg <sub>1</sub> | 10.1186 | 3.05827 | 161.652 | 25.8945 | 18     |
| bldg <sub>2</sub> | 6.07522 | 60.0734 | 109.678 | 45.5575 | 18     |
| bldg <sub>3</sub> | 14.1849 | 68.2293 | 94.4153 | 9.07669 | 18     |
| bldg <sub>4</sub> | 19.0516 | 31.9352 | 132.365 | 20.9724 | 18     |
| bldg <sub>5</sub> | 18.5779 | 122.827 | 30.3905 | 25.1989 | 18     |
| bldg <sub>6</sub> | 41.7577 | 93.6583 | 64.31   | 9.94434 | 18     |

Binary distance: no violation

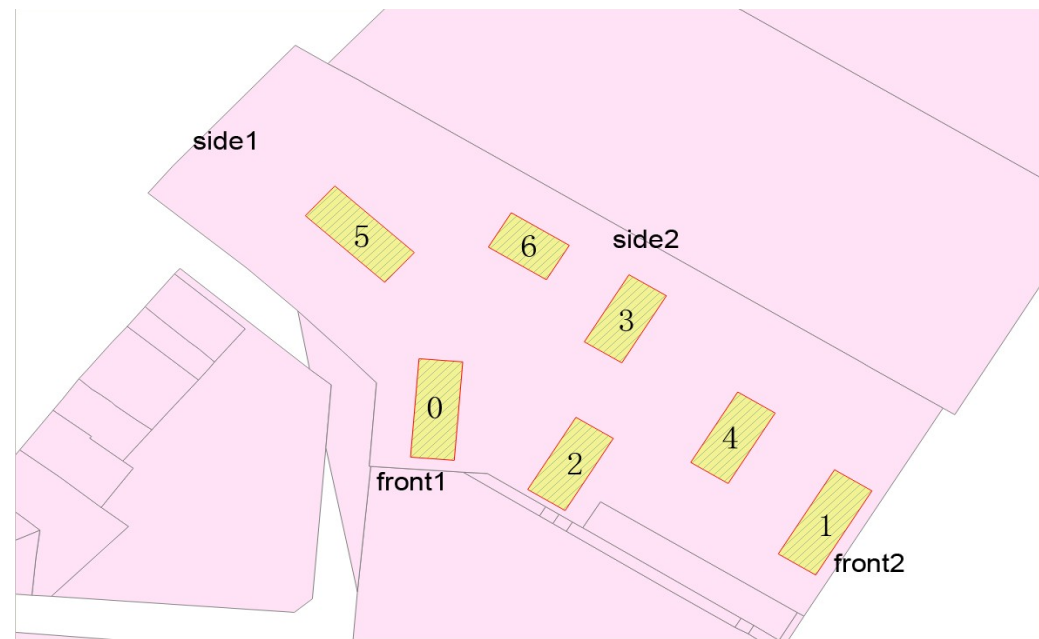
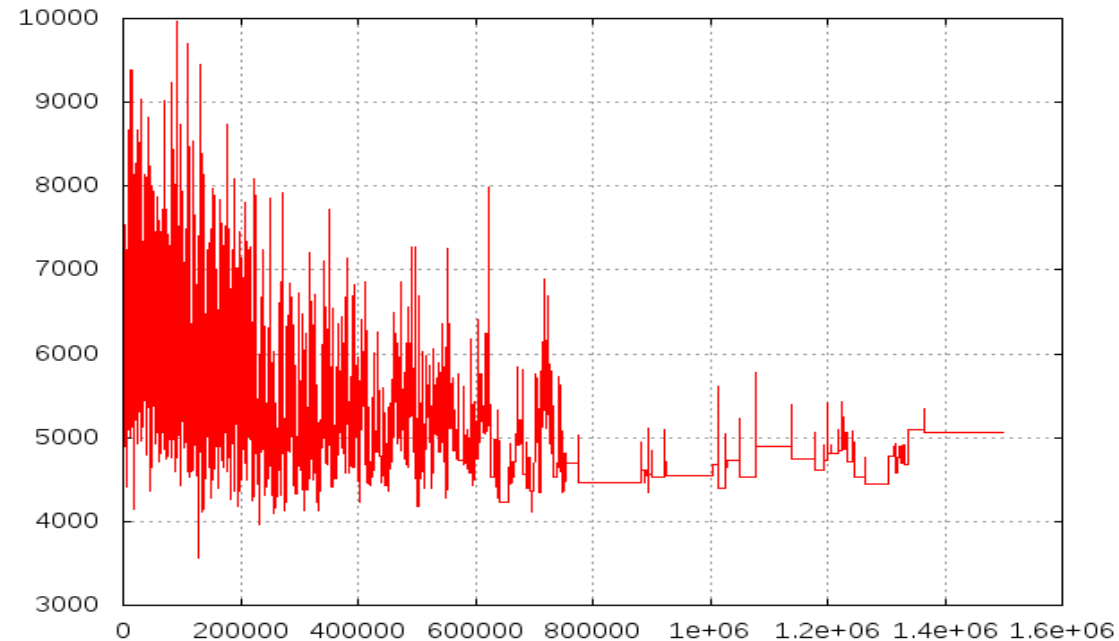
Binary height difference: no violation

Lot coverage ratio = 0.136106

Floor area ratio = 0.816634

Energy

| $30E^{u_d}$ | $20E^{b_d}$ | $20E^{b_h}$ | $20E^{g_c}$ | $50E^{g_f}$ | total energy |
|-------------|-------------|-------------|-------------|-------------|--------------|
| 0           | 0           | 0           | 0           | 5066.91     | 5066.91      |



# Conclusion and Future Work

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- Preliminary case study and test results are satisfying
  - Optimal/good solution can always be obtained
  - Fast computation
    - ~ 1 minute for a 1787.14 m<sup>2</sup> parcel and ~ 2 minutes for a 11020.9 m<sup>2</sup> parcel (time is not linear to the area due to the difference of applied urban rules)
- Future work
  - Try to increase the probability of generating optimal solutions, not just good ones
  - Conduct more case studies to evaluate the robustness and generality of our approach
  - Study more complex rules (e.g. concerning windows and roofs)
  - How to handle the diversity of urban rules?
  - How many rules can it support?
  - ...

# A Stochastic Method for the Generation of Optimized Building Layouts Respecting Urban Regulations

Thank You for Your Attention

**Shuang He, Julien Perret, Michaël Brasebin**

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