Homework 8: From Language to Logic

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Problem 4 Logical Inference

(a) *Some inferences that might look like they're outside the scope of Modus ponens are actually within reach. Suppose the knowledge base contains the following two formulas:*

$$KB = \{(A \vee B) \implies C, A\}$$

First, convert the knowledge base into conjunctive normal form (CNF). Then apply Modus ponens to derive C. Please show how your knowledge base changes as you apply derivation rules.

First we convert to CNF:

$$(A \vee B) \implies C \iff \neg(A \vee B) \vee C \tag{1}$$
$$\neg(A \vee B) \iff \neg A \wedge \neg B \tag{2}$$
$$(\neg A \wedge \neg B) \vee C \iff (\neg A \vee C) \wedge (\neg B \vee C) \tag{3}$$
$$\tag{4}$$

with the final formula in CNF: $(\neg A \vee C) \wedge (\neg B \vee C)$. This means that $\neg A \vee C$ and $\neg B \vee C$ are now in our knowledge base. We can apply Modus ponens as follows:

$$\neg A \vee C \iff A \implies C \tag{5}$$
$$\therefore \frac{A, A \implies C}{C} \tag{6}$$

(b) *Recall that Modus ponens is not complete, meaning that we can't use it to derive everything that's true. Suppose the knowledge base contains the following formulas:*

$$KB = \{A \vee B, B \rightarrow C, (A \vee C) \rightarrow D\}$$

In this example, Modus ponens cannot be used to derive D, even though D is entailed by the knowledge base. However, recall that the resolution rule is complete.

Your task: Convert the knowledge base into CNF and apply the resolution rule repeatedly to derive D.

We first convert each formula in the KB to CNF:

- $A \vee B$ is already in CNF.
- $B \implies C$ can be written as $\neg B \vee C$ which is in CNF.
- $(A \vee C) \implies D$ can be written as $\neg(A \vee C) \vee D \iff (\neg A \wedge \neg C) \vee D \iff (\neg A \vee D) \wedge (\neg C \vee D)$, with the last in CNF.

We now repeatedly apply the resolution rule as follows:

$$\frac{A \vee B, \neg A \vee D}{B \vee D} \tag{7}$$

$$\frac{\neg B \vee C, B \vee D}{C \vee D} \tag{8}$$

$$\frac{\neg C \vee D, C \vee D}{D} \tag{9}$$