Homework 7: Car Tracking

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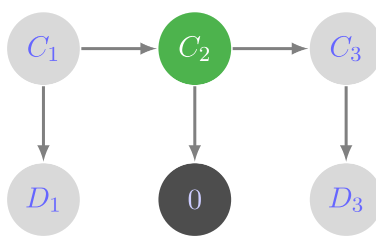
Setup:

- We want to drive our car from start to finish (green box).
- World is 2D grid with your car + K others. At each timestep t, you get noisy estimate of dist to other cars.
- Variables: Assume we are only concerned with one other car.
  - $C_t \in \mathbb{R}^2$: actual location of the other car. Unobserved.
  - $a_t \in \mathbb{R}^2$: your car's position. Observed and controlled by us.
  - $D_t \sim \mathcal{N}(||a_t - C_t||, \sigma^2)$
- Goal: Compute $P(C_t \mid D_1, \ldots, D_t)$.

Problem 1: Bayesian Network Basics

(a) *Suppose we have a sensor reading for the second timestep, $D_2 = 0$. Compute the posterior distribution $\mathbb{P}(C_2 = 1 \mid D_2 = 0)$.*

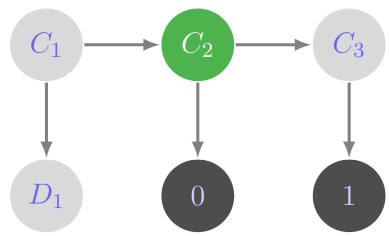
Below is the Bayesian network, where we've observed $D_2 = 0$:



$$
\begin{align}
\Pr\left[C_2 = 1 \mid D_2 = 0\right] &\propto \Pr\left[C_2 = 1, D_2 = 0\right] \tag{1}\\
&= \sum_{c_1} \Pr\left[C_2 = 1, D_2 = 0, c_1\right] \tag{2}\\
&= \sum_{c_1} \Pr\left[c_1\right] \Pr\left[C_2 = 1 \mid c_1\right] \Pr\left[D_2 = 0 \mid C_2 = 1\right] \tag{3}\\
&= 0.5 \sum_{c_1} \Pr\left[C_2 = 1 \mid c_1\right] \Pr\left[D_2 = 0 \mid C_2 = 1\right] \tag{4}\\
&= 0.5\eta \sum_{c_1} \Pr\left[C_2 = 1 \mid c_1\right] \tag{5}\\
&= 0.5\eta(\epsilon + (1 - \epsilon)) \tag{6}\\
&= 0.5\eta \tag{7}\\
\Pr\left[D_2 = 0\right] &= \sum_{c_2} \Pr\left[D_2 = 0, c_2\right] \tag{8}\\
&= \sum_{c_2} \Pr\left[D_2 = 0 \mid c_2\right] \sum_{c_1} \Pr\left[c_2 \mid c_1\right] \Pr\left[c_1\right] \tag{9}\\
&= (1 - \eta) \cdot (0.5(1 - \epsilon) + 0.5\epsilon) + \eta \cdot (0.5\epsilon + 0.5(1 - \epsilon)) \tag{10}\\
&= 1 \tag{11}\\
\therefore \Pr\left[C_2 = 1 \mid D_2 = 0\right] &= \frac{\Pr\left[C_2 = 1, D_2 = 0\right]}{\Pr\left[D_2 = 0\right]} = 0.5\eta \tag{12}
\end{align}
$$

(b) *Compute* $\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1)$

Now our Bayesian network looks like:



$$\Pr\left[C_2 = 1 \mid D_2 = 0, D_3 = 1\right] \propto \Pr\left[C_2 = 1, D_2 = 0, D_3 = 1\right] \tag{13}$$

$$= \sum_{c_1} \sum_{c_3} \Pr\left[D_3 = 1 \mid c_3\right] \Pr\left[c_3 \mid C_2 = 1\right] \Pr\left[C_2 = 1 \mid c_1\right] \Pr\left[c_1\right] \Pr\left[D_2 = 0 \mid C_2 = 1\right] \tag{14}$$