THIRD ORDER POINT-TO-POINT MOTION -PROFILE

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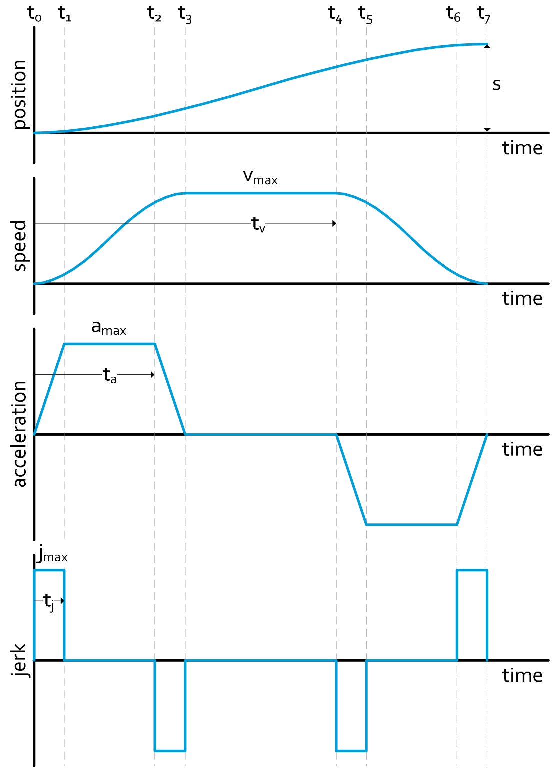
Introduction

In many mechatronic applications where a movement from A to B needs to be performed, a third order point to point motion profile is used.
To enable early insight in the relevant parameters of a motion profile it is useful to calculate and visualize the relevant parameters (position, speed, acceleration and jerk).

This sheet provides analytical formulas to calculate the quickest motion between point A to point B based on given maximum levels speed, acceleration and jerk.



Approach

The typical time plot of the parameters of a third order profile is depicted besides.
The difficulty in defining the motion trajectory is that the shape is not always the same. For example, there are cases where the maximum speed or acceleration level is not achieved, because there is not enough time to build up to the maximum before slowing down again.

These different cases (I … VI) are captured by the following criteria table.

	Trajectory instance						
	I	II	III	IV	V	VI	
v_{max}	<	>	<	<	>	>	$v_a = \frac{{a_{max}}^2}{j_{max}}$
s	>	<	<	<	>	>	$s_a = \frac{2 \cdot {a_{max}}^3}{{j_{max}}^2}$
s			>	<	>	<	$s_v = v_{max}\left[M\left(2\sqrt{\frac{v_{max}}{j_{max}}}\right) + N\left(\frac{v_{max}}{a_{max}} + \frac{a_{max}}{j_{max}}\right)\right]$ $M = 1, N = 0 \; if \; v_{max} j_{max} < {a_{max}}^2$ $M = 0, N = 1 \; if \; v_{max} j_{max} \geq {a_{max}}^2$
t_j	$\sqrt{\frac{v_{max}}{j_{max}}}$	$\sqrt[3]{\frac{s}{2 \cdot j_{max}}}$	$\sqrt{\frac{v_{max}}{j_{max}}}$	$\sqrt[3]{\frac{s}{2 \cdot j_{max}}}$	$\frac{a_{max}}{j_{max}}$	$\frac{a_{max}}{j_{max}}$	
t_a	t_j	t_j	t_j	t_j	$\frac{v_{max}}{a_{max}}$	$\frac{1}{2}\left(\sqrt{\frac{4 \cdot s \cdot {j_{max}}^2 + {a_{max}}^3}{a_{max} \cdot {j_{max}}^2}} - \frac{a_{max}}{j_{max}}\right)$	
t_v	$\frac{s}{v_{max}}$	$2 \cdot t_j$	$\frac{s}{v_{max}}$	$2 \cdot t_j$	$\frac{s}{v_{max}}$	$t_a + t_j$	

Then the following formula's define the motion parameters over time:

	Motion parameters			
	jerk	acceleration	velocity	position
$t_0 \ldots t_1$	j_{max}	$j_{max} \cdot (t - t_0)$	$\frac{1}{2} \cdot j_{max} \cdot (t - t_0)^2$	$\frac{1}{6} \cdot j_{max} \cdot (t - t_0)^3$
$t_1 \ldots t_2$	0	$a_1 = a_2$	$v_1 + a_1 \cdot (t - t_1)$	$p_1 + v_1 \cdot (t - t_1) + \frac{1}{2} \cdot a_1 \cdot (t - t_1)^2$
$t_2 \ldots t_3$	$-j_{max}$	$a_2 - j_{max} \cdot (t - t_2)$	$v_2 + a_2 \cdot (t - t_2) + \frac{1}{2} \cdot -j_{max} \cdot (t - t_2)^2$	$p_2 + v_2 \cdot (t - t_2) + \frac{1}{2} \cdot a_2 \cdot (t - t_2)^2 + \frac{1}{6} \cdot -j_{max} \cdot (t - t_2)^3$
$t_3 \ldots t_4$	0	0	$v_3 = v_4$	$p_3 + v_3 \cdot (t - t_3)$
$t_4 \ldots t_5$	$-j_{max}$	$-j_{max} \cdot (t - t_4)$	$v_4 + \frac{1}{2} \cdot -j_{max} \cdot (t - t_4)^2$	$p_4 + v_4 \cdot (t - t_4) + \frac{1}{6} \cdot -j_{max} \cdot (t - t_4)^3$
$t_5 \ldots t_6$	0	$a_5 = a_6$	$v_5 - a_{max} \cdot (t - t_5)$	$p_5 + v_5 \cdot (t - t_5) + \frac{1}{2} \cdot a_5 \cdot (t - t_5)^2$
$t_6 \ldots t_7$	j_{max}	$a_6 + j_{max} \cdot (t - t_6)$	$v_6 + a_6 \cdot (t - t_6) + \frac{1}{2} \cdot j_{max} \cdot (t - t_6)^2$	$p_6 + v_6 \cdot (t - t_6) + \frac{1}{2} \cdot a_6 \cdot (t - t_6)^2 + \frac{1}{6} \cdot j_{max} \cdot (t - t_6)^3$

$t_1 = t_j$	$t_2 = t_a$	$t_3 = t_a + t_j$	$t_4 = t_v$	$t_5 = t_v + t_j$	$t_6 = t_v + t_a$	$t_7 = t_v + t_j + t_a$

Source:
- Haihua MU et al. Third-order trajectory planning for high accuracy point-to-point motion