MeerFish

This is a (non-exhaustive) description of the analysis framework used in the MeerFish code. Please contact the author (steve.cunnington@port.ac.uk) if you feel something is missing/incorrect or needs further clarification.

We begin by defining the HI power spectrum as

$$P_{\mathrm{HI}}(k^{\mathrm{f}},\mu^{\mathrm{f}},z)=\alpha_{\parallel}^{-1}(z)\alpha_{\perp}^{-2}(z)\,\overline{T}_{\mathrm{HI}}^{2}(z)\left(b_{\mathrm{HI}}(z)+f(z)\mu^{2}+b_{\phi}^{\mathrm{HI}}(z)f_{\mathrm{NL}}\mathcal{M}(k,z)^{-1}\right)^{2} \times\, P_{\mathrm{m}}(k,z)\,\mathcal{B}_{\mathrm{beam}}^{2}(k,\mu,z) \tag{1}$$

where $\overline{T}_{\mathrm{HI}}$ is the background mean HI brightness temperature, b_{HI} is the linear bias for HI, and f is the growth rate of structure. We include the possibility for local primordial non-Gaussianity (PNG) to influence the power, which occurs by the bias acquiring a scale-dependent correction given by the $b_{\phi}f_{\mathrm{NL}}\mathcal{M}^{-1}$ term. b_{ϕ}^{HI} is the PNG bias parameter for HI. f_{NL} modulates the PNG contribution and $\mathcal{M}(k,z)=(2/3)k^{2}T_{\mathrm{m}}(k)D_{\mathrm{md}}(z)/(\Omega_{\mathrm{m}}H_{0}^{2})$, where T_{m} is the matter transfer function and D_{md} is the linear growth factor normalized to $1/(1+z)$. P_{m} is the matter power spectrum obtained from a Boltzmann solver, and the transverse fluctuations contributing to the power are affected by the telescope beam which acts as

$$\mathcal{B}_{\mathrm{beam}}(k,\mu,z)=\exp\left[-\frac{1}{2}k_{\perp}^{2}R_{\mathrm{b}}^{2}\right]=\exp\left[-\frac{k^{2}}{2}(1-\mu^{2})R_{\mathrm{b}}^{2}\right] \tag{2}$$

where $R_{\mathrm{b}}=\chi(z)\theta_{\mathrm{FWHM}}(z)/2\sqrt{2\ln 2}\;\rightarrow\;\alpha_{\parallel}\chi(z)\theta_{\mathrm{FWHM}}(z)/2\sqrt{2\ln 2}$. θ_{FWHM} is radio telescope specific but for a generic dish size with diameter D_{dish} and observations at a frequency ν, we can say $\theta_{\mathrm{FWHM}}=c/(\nu D_{\mathrm{dish}})$.

Lastly, in Equation 1, the Alcock-Paczynski dilation parameters for the transverse and radial directions are given by

$$\alpha_{\perp}(z)=\frac{D_{\mathrm{A}}(z)/r_{\mathrm{s}}}{(D_{\mathrm{A}}(z)/r_{\mathrm{s}})_{\mathrm{f}}}\,,\quad \alpha_{\parallel}(z)=\frac{(H(z)r_{\mathrm{s}})_{\mathrm{f}}}{H(z)r_{\mathrm{s}}}\,. \tag{3}$$

and the true wavenumbers are distorted by

$$k_{\perp}=k_{\perp}^{\mathrm{f}}/\alpha_{\perp}\,,\quad k_{\parallel}=k_{\parallel}^{\mathrm{f}}/\alpha_{\parallel}\,. \tag{4}$$

where the fiducial indices denote the fiducial (reference) cosmology assumed in the measured wavenumbers, and the wavenumber without an f index denotes the true wavenumbers for the true cosmology. These are conventionally transposed into k and μ using the factor $F_{\mathrm{AP}}=\alpha_{\parallel}/\alpha_{\perp}$, where

$$k=\frac{k_{\mathrm{f}}}{\alpha_{\perp}}\left[1+(\mu_{\mathrm{f}})^{2}\left(F_{\mathrm{AP}}^{-2}-1\right)\right]^{1/2} \tag{5}$$

$$\mu = \frac{\mu_\mathrm{f}}{F_\mathrm{AP}}\left[1+(\mu_\mathrm{f})^2\left(F_\mathrm{AP}^{-2}-1\right)\right]^{-1/2}. \tag{6}$$

Finally, to correct for any change in volume caused by the different cosmologies, the power spectrum model must be multiplied by $\alpha_\parallel^{-1}\alpha_\perp^{-2}$, which explains this factor at the front of Equation 1.

Similarly to the HI power spectrum, we can model the galaxy power spectrum as

$$P_\mathrm{g}(k^\mathrm{f},\mu^\mathrm{f},z) = \alpha_\parallel^{-1}(z)\,\alpha_\perp^{-2}(z)\left(b_\mathrm{g}(z)+f(z)\mu^2+b_\phi^\mathrm{g}(z)f_\mathrm{NL}\mathcal{M}(k,z)^{-1}\right)^2 P_\mathrm{m}(k,z)\,. \tag{7}$$

Observed power spectra noise and errors: Neglecting shot-noise, which will be minimal for a HI intensity mapping survey, the observed power spectrum is given as

$$P_\mathrm{HI}^\mathrm{obs}(k^\mathrm{f},\mu^\mathrm{f},z) = P_\mathrm{HI}(k^\mathrm{f},\mu^\mathrm{f},z)+P_\mathrm{N}(z)\,, \tag{8}$$

where P_N is the noise power spectrum contribution, which for HI intensity mapping is given by the telescope's thermal noise, which is modelled as

$$P_\mathrm{N} = V_\mathrm{pix}\sigma_\mathrm{N}^2 \tag{9}$$

where the pixel volume is determined by the pixel area A_pix so that

$$V_\mathrm{pix} = \Omega_\mathrm{A}\int_{z_0}^{z_1}\mathrm{d}z\frac{\mathrm{d}V}{\mathrm{d}z\mathrm{d}\Omega} = \Omega_\mathrm{A}\int_{z_0}^{z_1}\mathrm{d}z\frac{c\,\chi^2(z)}{H(z)}\,, \tag{10}$$

where $\Omega_\mathrm{A} = A_\mathrm{pix}(\pi/180)^2$ and the redshift intervals are determined by the frequency of observation ν and the telescope's frequency resolution $\delta\nu$ so that $z_0 = \nu_\mathrm{21cm}/(\nu+\delta\nu)-1$ and $z_1 = \nu_\mathrm{21cm}/(\nu-\delta\nu)-1$, with $\nu_\mathrm{21cm} = 1420.4\,\mathrm{MHz}$, the frequency of the 21cm line. The RMS of the noise fluctuations is given by the radiometer equation;

$$\sigma_\mathrm{N}(\nu) = \frac{T_\mathrm{sys}(\nu)}{\sqrt{2\,\delta\nu\,t_\mathrm{pix}}}\,. \tag{11}$$

The system temperature is given by

$$T_\mathrm{sys}(\nu) = T_\mathrm{rx}(\nu)+T_\mathrm{spl}+T_\mathrm{CMB}+T_\mathrm{gal}(\nu) \tag{12}$$

where the contribution from spill-over is $T_\mathrm{spl} = 3\,\mathrm{K}$, the background contribution from the CMB is $T_\mathrm{CMB} = 2.73\,\mathrm{K}$ and the contribution from our own Galaxy is $T_\mathrm{gal} = 15\,\mathrm{K}(408\mathrm{MHz}/\nu)^{2.75}$, which is tuned to the match the average sky temperature excluding $|b| < 10^\circ$ in galactic latitude. Lastly, the receiver temperature is

$$T_\mathrm{rx}(\nu) = 7.5\,\mathrm{K}+10\,\mathrm{K}\left(\frac{\nu}{\mathrm{GHz}}-0.75\right)^2\,, \tag{13}$$

which has been tuned to match recent intensity mapping observations with the MeerKAT pilot survey. The time per pixel t_pix in the noise RMS (Equation 11), is given by the total observation time $t_{\rm obs}$ from the survey, shared among each pixel. Thus we define it as

$$t_{\rm pix} = N_{\rm dish} t_{\rm obs}\ \left(\theta_{\rm FWHM}/3\right)^2 / A_{\rm sur} \tag{14}$$

where we assume each pixel is 1/3 the size of the telescope's beam size (approximately correct for MeerKAT map-making). In single-dish mode, we get a set of observations per dish, hence observation time per pixel is multiplied by the number of dishes $N_{\rm dish}$.

The observed counterpart for the galaxies will contain shot-noise as the noise contribution defined as

$$P_{\rm g}^{\rm obs}(k^{\rm f}, \mu^{\rm f}, z) = P_{\rm g}(k^{\rm f}, \mu^{\rm f}, z) + P_{\rm SN}(z)\,, \tag{15}$$

where $P_{\rm SN} = 1\,/\,\bar{n}_{\rm g}$ is the galaxy shot noise, with $\bar{n}_{\rm g}$ as the galaxy number density for the survey.

The errors on the power spectrum can then be estimated as

$$\sigma^2(k,\mu) = \frac{P^2_{\rm obs}(k,\mu)}{N_{\rm modes}(k,\mu)}\,, \tag{16}$$

where the number of *unique* modes is given by

$$N_{\rm modes}\ (k,\mu) = \frac{k^2 \Delta k \Delta \mu}{8\pi^2} V_{\rm sur}\,. \tag{17}$$

where the survey volume is determined by the minimum and maximum redshift ($z_{\rm min}$, $z_{\rm max}$) of the survey and its area $A_{\rm sur}$;

$$V_{\rm sur} = \Omega_{\rm sur} \int_{z_{\rm min}}^{z_{\rm max}} {\rm d}z \frac{c\,\chi^2(z)}{H(z)}\,, \tag{18}$$

where $\Omega_{\rm sur} = A_{\rm sur}(\pi/180)^2$.

Multipole expansion: It is convenient to adopt the multipole expansion of the anisotropic power spectrum in Equation 1. This is achieved using

$$P(k,\mu) = \sum_\ell P_\ell(k) \mathcal{L}_\ell(\mu)\,, \tag{19}$$

$$P_\ell(k,z) = (2\ell+1) \int_0^1 {\rm d}\mu\, P(k,\mu,z) \mathcal{L}_\ell(\mu)\,, \tag{20}$$

where

$$\mathcal{L}_0 = 1, \quad \mathcal{L}_2 = \frac{3\mu^2-1}{2}, \quad \mathcal{L}_4 = \frac{35\mu^4 - 30\mu^2 + 3}{8}\,. \tag{21}$$

and the errors on these multipoles is given by

$$\sigma_{P_\ell}(k) = \sqrt{(2\ell+1)^2 \int_0^1 {\rm d}\mu\, \sigma^2(k,\mu) \mathcal{L}_\ell^2(\mu)}\,. \tag{22}$$

Fisher forecasting: For the multipole expansion formalism, we define the Fisher matrix following Taruya+11

$$F_{ij}|_{\ell_{\max}} = \frac{V_{\rm sur}}{4\pi^2}\sum_{\ell,\ell'}^{\ell_{\max}}\int_{k_{\min}}^{k_{\max}} k^2 {\rm d}k \frac{\partial P_\ell(k)}{\partial\theta_i}\left[\widetilde{C}^{\ell\ell'}(k)\right]^{-1}\frac{\partial P_{\ell'}(k)}{\partial\theta_j}\,, \tag{23}$$

where the *reduced* covariance is given by

$$\widetilde{C}^{\ell\ell'}(k) = (2\ell+1)(2\ell'+1)\int_0^1 {\rm d}\mu \mathcal{L}_\ell(\mu)\mathcal{L}_{\ell'}(\mu)P_{\rm obs}(k,\mu)^2\,, \tag{24}$$

For computing the derivatives for the Fisher matrix, given Equation 20 we can say

$$\frac{\partial P_\ell(k)}{\partial\theta_i} = (2\ell+1)\int_0^1 {\rm d}\mu \frac{\partial}{\partial\theta_i}\left[P(k,\mu)\mathcal{L}_\ell(\mu)\right]\,, \tag{25}$$

since the integration limits do not depend on the parameters, and hence the derivative can be taken inside the integral. We then compute all derivatives numerically using a 5-point stencil, generically defined for a function $\mathcal{F}$ as

$$\frac{\partial\mathcal{F}}{\partial\theta} = \frac{-\mathcal{F}(\theta+2\delta\theta)+8\mathcal{F}(\theta+\delta\theta)-8\mathcal{F}(\theta-\delta\theta)+\mathcal{F}(\theta-2\delta\theta)}{12\,\delta\theta}\,, \tag{26}$$

where $\delta\theta$ is chosen such that it gives converged derivatives. Generally, we find that $\delta\theta\sim 10^{-2}$ is an appropriate choice for all parameters.

Cross-correlation and multi-tracer: A cross-correlation power spectrum between the HI and galaxies is given by (dropping redshift dependence for brevity)

$$\begin{aligned}P_{\rm HI,g}(k^{\rm f},\mu^{\rm f}) = \alpha_\parallel^{-1}\alpha_\perp^{-2}\, r_{\rm HI,g}\overline{T}_{\rm HI}\left(b_{\rm HI}+f\mu^2+b_\phi^{\rm HI}f_{\rm NL}\mathcal{M}(k)^{-1}\right)\\ \times\left(b_{\rm g}+f\mu^2+b_\phi^{\rm g}f_{\rm NL}\mathcal{M}(k)^{-1}\right)P_{\rm m}(k)\,\mathcal{B}_{\rm beam}(k,\mu)\,,\end{aligned} \tag{27}$$

where $r_{\rm HI,g}(z)$ is the cross-correlation coefficient between the two tracers to modulate stochastic suppression in the cross-correlation. Similarly to the single tracer, the *multi-tracer* Fisher matrix will be given by

$$F_{ij}|_{\ell_{\max}} = \frac{V_{\rm bin}}{4\pi^2}\sum_{\ell,\ell'}^{\ell_{\max}}\int_{k_{\min}}^{k_{\max}} k^2 {\rm d}k \frac{\partial \boldsymbol{P}_{\tau,\ell}(k)}{\partial\theta_i}\left[\mathbf{C}^{\ell\ell'}_{\tau\tau'}(k)\right]^{-1}\frac{\partial \boldsymbol{P}_{\tau',\ell'}(k)}{\partial\theta_j} \tag{28}$$

but we are now implicitly integrating over the contributions within $\tau=\{\alpha,\beta,{\rm X}\}$ i.e. the different different generic tracers α and β, plus their cross-correlation X. For the combination of HI intensity mapping and a galaxy survey, we would have $\alpha\equiv$ HI and $\beta\equiv$ g. The data vector $\boldsymbol{P}_{\tau,\ell}$ represents the "non-observed" power spectra (i.e. without noise) containing each tracer combination and multipole i.e.

$$\boldsymbol{P}_{\tau,\ell}(k) = \{\boldsymbol{P}_{\alpha,\ell}(k),\boldsymbol{P}_{\beta,\ell}(k),\boldsymbol{P}_{{\rm X},\ell}(k)\} \tag{29}$$

with each element its own $n_\ell \times n_k$ vector. For the multipoles analysis, the covariance is given by

$$\mathbf{C}^{\ell\ell'}_{\tau\tau'}(k) = \begin{pmatrix} C^{\ell\ell'}_{\alpha\alpha} & C^{\ell\ell'}_{\alpha\beta} & C^{\ell\ell'}_{\alpha\mathrm{X}} \\ & C^{\ell\ell'}_{\beta\beta} & C^{\ell\ell'}_{\beta\mathrm{X}} \\ & & C^{\ell\ell'}_{\mathrm{XX}} \end{pmatrix}, \tag{30}$$

where each element represents the reduced covariance as before, given by the below;

$$C^{\ell\ell'}_{\alpha\alpha}(k) = (2\ell+1)(2\ell'+1)\int_0^1 \mathrm{d}\mu \mathcal{L}_\ell(\mu)\mathcal{L}_{\ell'}(\mu)P^{\mathrm{obs}}_\alpha(k,\mu)^2\,, \tag{31}$$

$$C^{\ell\ell'}_{\alpha\beta}(k) = (2\ell+1)(2\ell'+1)\int_0^1 \mathrm{d}\mu \mathcal{L}_\ell(\mu)\mathcal{L}_{\ell'}(\mu)P_\alpha(k,\mu)P_\beta(k,\mu)\,, \tag{32}$$

$$C^{\ell\ell'}_{\alpha\mathrm{X}}(k) = (2\ell+1)(2\ell'+1)\int_0^1 \mathrm{d}\mu \mathcal{L}_\ell(\mu)\mathcal{L}_{\ell'}(\mu)P^{\mathrm{obs}}_\alpha(k,\mu)P_\mathrm{X}(k,\mu)\,, \tag{33}$$

$$C^{\ell\ell'}_{\mathrm{XX}}(k) = \frac{(2\ell+1)(2\ell'+1)}{2}\int_0^1 \mathrm{d}\mu \mathcal{L}_\ell(\mu)\mathcal{L}_{\ell'}(\mu)\left[P^2_\mathrm{X}(k,\mu) + P^{\mathrm{obs}}_\alpha(k,\mu)P^{\mathrm{obs}}_\beta(k,\mu)\right]\,. \tag{34}$$

Each element is its own sub-matrix of size $n_\ell \times n_k$. We assume no mode coupling between k, thus the only non-zero off-diagonal components in the matrix come from multipole $(\ell\ell')$ and tracer $(\tau\tau')$ covariance.