Lab 3
Experiment No: 1

TITLE: To Perform Trapezoidal method

OBJECTIVES

1.1. To implement the **Trapezoidal Method** for numerical integration.
1.2. To approximate the definite integral of a given function.
1.3. To analyze the error and accuracy of the method.

THEORY

The **Trapezoidal Method** is a numerical technique for approximating the definite integral of a function. It works by dividing the area under the curve into trapezoids (instead of rectangles as in the Riemann sum) and summing their areas.

Mathematical Formulation:

For a function f(x) defined on [a,b] , the integral is approximated as:

$$\int_a^b f(x)\ dx \approx \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$$

where:

- h=(b−a) /n is the width of each subinterval, • n is the number of trapezoids (subintervals),
- xi=a+i·h are the subdivision points.

Error Analysis:
The error EE in the Trapezoidal Rule is given by:

$$E_f = -{}^{(b}12^{-}n^{a}{}_{2})^{3\,\prime\prime}(\xi)\ ,\ \text{for some } \xi\in[a,b].$$

This indicates the error decreases quadratically with n.

Advantages:
Simple to implement.
More accurate than the Riemann sum for smooth functions. Limitations:
Less accurate for highly oscillatory or discontinuous functions.
Requires a large n for high precision.

Algorithm
1. Input:
Function f(x) to integrate.
Limits of integration a and b.
Number of trapezoids n.

2. Compute Step Size:
Calculate h= (b−a) / n .

3. Sum Function Values:
Initialize sum= (f(a)+f(b)) / 2

4. For i=1 to n−1:

Compute xi=a+i.h .

Add f(xi) to sum.

5. Approximate Integral:

Multiply sum by h to get the integral value.

6. Output:

The approximate value of the integral.

DEMOSTRATION

```
#include <stdio.h>
#include <math.h>

// Function to integrate: Example f(x) = sin(x)
double f(double x) {
    return sin(x);
}

// Trapezoidal Method implementation
double trapezoidal(double a, double b, int n) {
    double h = (b - a) / n;
    double sum = 0.5 * (f(a) + f(b)); // Initialize with endpoints

    for (int i = 1; i < n; i++) {
        double x_i = a + i * h;
        sum += f(x_i); // Add midpoints
    }

    return h * sum;
}

int main() {
    double a, b;
    int n;

    // Input integration limits and subintervals
    printf("Enter lower limit (a): ");
    scanf("%lf", &a);

    printf("Enter upper limit (b): ");
    scanf("%lf", &b);

    printf("Enter number of trapezoids (n): ");
    scanf("%d", &n);

    if (n <= 0) {
        printf("Error: n must be positive.\n");
        return 1;
    }

    double result = trapezoidal(a, b, n);
    printf("Approximate integral: %.6f\n", result);

    return 0;
}
```

Output 1:

```
.\TrapezoidalMethod }
Enter lower limit (a): 0
Enter upper limit (b): 3.14159
Enter number of trapezoids (n): 1000
Approximate integral: 1.999998
PS F:\s3 csit\NumericalMethod-main\LabCodes>
```

Output 2:

```
.\TrapezoidalMethod }
Enter lower limit (a): 1
Enter upper limit (b): 4
Enter number of trapezoids (n): 100
Approximate integral: 1.193856
```

CONCLUSION

The Trapezoidal Method provides a straightforward and efficient way to approximate integrals, especially for smooth functions. Its simplicity makes it a foundational tool in numerical analysis, though higher-order methods (e.g., Simpson's Rule) are preferred for better accuracy with fewer computations.

Experiment No: 2

TITLE: To Perform Simpson's 1/3 Rule.

1. OBJECTIVES

1.1. To implement **Simpson's 1/3 Rule** for numerical integration.

1.2. To approximate the definite integral of a given function with higher accuracy than the Trapezoidal Rule.

1.3. To analyze the error and convergence properties of the method.

2. THEORY

Simpson's 1/3 Rule is a numerical integration technique that approximates the integral of a function by fitting parabolas to subintervals of the domain. It provides more accurate results than the Trapezoidal Rule for smooth functions.

Mathematical Formulation:

For a function $f(x)$ defined on [a,b], the integral is approximated as:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4\sum_{\text{odd } i} f(x_i) + 2\sum_{\text{even } i} f(x_i) + f(b)\right]$$

Error Analysis:

The error E in Simpson's 1/3 Rule is given by:

$$E = -\frac{(b-a)}{180}h^4 f^{(4)}(\xi) \text{ for some } \xi \in [a,b].$$

This indicates the error decreases **quartically** with n.

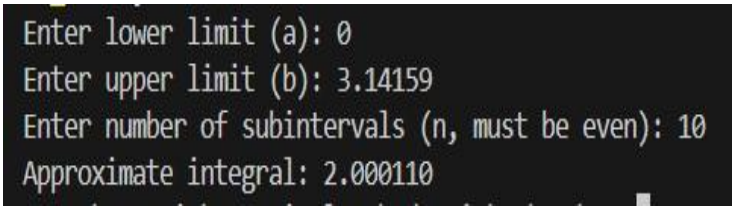
3. Demostration

Source code :

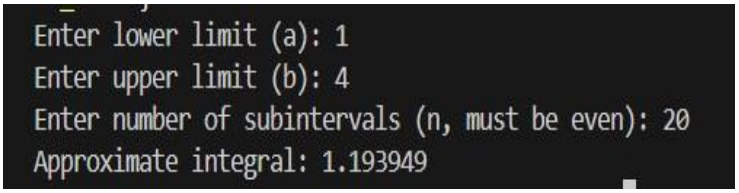
```
#include <stdio.h>
#include <math.h>

// Function to integrate: Example f(x) = sin(x)
double f(double x) {
    return sin(x);
}

// Simpson's 1/3 Rule implementation
double simpsons_1_3(double a, double b, int n) {
    if (n % 2 != 0) {
        printf("Error: n must be even.\n");
        return NAN; // Not a Number (error)
    }

    double h = (b - a) / n;
    double sum = f(a) + f(b); // Initialize with endpoints

    for (int i = 1; i < n; i++) {
        double x_i = a + i * h;
        if (i % 2 == 1) {
            sum += 4 * f(x_i); // Odd-indexed points
```

```
        } else {
            sum += 2 * f(x_i); // Even-indexed points
        }
    }

    return (h / 3) * sum;
}

int main() {
    double a, b;
    int n;

    // Input integration limits and subintervals
    printf("Enter lower limit (a): ");
    scanf("%lf", &a);

    printf("Enter upper limit (b): ");
    scanf("%lf", &b);

    printf("Enter number of subintervals (n, must be even): ");
    scanf("%d", &n);

    if (n <= 0 || n % 2 != 0) {
        printf("Error: n must be a positive even integer.\n");
        return 1;
    }

    double result = simpsons_1_3(a, b, n);
    printf("Approximate integral: %.6f\n", result);

    return 0;
}
```

Output:



Output 2:



RESULT AND DISCUSSION

- Accuracy: Simpson's 1/3 Rule converges faster than the Trapezoidal Rule due to its quartic error term (O(h4).
- Efficiency: Requires fewer subintervals for the same precision, reducing computational cost.
- Limitations:
    - Requires an even number of subintervals.
    - Less effective for discontinuous or highly oscillatory functions.

CONCLUSION

Simpson's 1/3 Rule is a powerful numerical integration method for smooth functions, offering superior accuracy with minimal computational effort. It is a preferred choice when high precision is required, though care must be taken to ensure the number of subintervals is even.

Experiment No: 3

TITLE: To Perform Gauss Jacobi Method.

OBJECTIVES

1.1. To implement the **Gauss-Jacobi Method** for solving systems of linear equations.

1.2. To understand the convergence criteria for iterative methods.

1.3. To analyze the error and convergence behavior of the method.

THEORY

The **Gauss-Jacobi Method** is an iterative algorithm for solving a system of n*n* linear equations with n unknowns, expressed in matrix form as: Ax=b where:

- A is the coefficient matrix,
- x is the solution vector,
- b is the right-hand side vector.

Key Steps:

1. **Decomposition**: Split A into diagonal (D), lower triangular (L), and upper triangular (U) matrices:

A=D+L+U

2. **Iteration Formula**: For each iteration k*k*, update the solution vector:

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1, j\neq i}^{n} a_{ij} x_j^{(k)}\right) \quad 1,2,\ldots,n$$

3. **Convergence Criterion**: The method converges if A*A* is **strictly diagonally dominant** (i.e., $|a_{ii}| > \sum_{j\neq i}|a_{ij}| \quad \forall i$ for all i).

DEMONSTRATION

```
#include <stdio.h>
#include <math.h>

#define MAX_SIZE 10
#define MAX_ITER 100
#define TOLERANCE 1e-6

void gaussJacobi(double A[MAX_SIZE][MAX_SIZE],
double b[MAX_SIZE], double x[MAX_SIZE], int n) {
   double x_new[MAX_SIZE], error;
   int iter = 0;

   do {
      error = 0.0;

      for (int i = 0; i < n; i++) {
         double sum = 0.0;

         for (int j = 0; j < n; j++) {
            if (j != i) {
               sum += A[i][j] * x[j];
            }
         }

         x_new[i] = (b[i] - sum) / A[i][i];
         error += fabs(x_new[i] - x[i]);
```

```
        }

        // Update solution vector
        for (int i = 0; i < n; i++) {
            x[i] = x_new[i];
        }

        iter++;

    } while (error > TOLERANCE && iter < MAX_ITER);

    printf("Converged in %d iterations.\n", iter);
}

int main() {
    int n;
    double A[MAX_SIZE][MAX_SIZE], b[MAX_SIZE], x[MAX_SIZE];

    printf("Enter the number of equations (n <= %d): ", MAX_SIZE);
    scanf("%d", &n);

    printf("Enter the coefficient matrix A:\n");
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            scanf("%lf", &A[i][j]);
        }
    }

    printf("Enter the right-hand side vector b:\n");
    for (int i = 0; i < n; i++) {
        scanf("%lf", &b[i]);
    }

    printf("Enter the initial guess for x:\n");
    for (int i = 0; i < n; i++) {
        scanf("%lf", &x[i]);
    }

    gaussJacobi(A, b, x, n);

    printf("Solution vector x:\n");
    for (int i = 0; i < n; i++) {
        printf("x[%d] = %.6f\n", i, x[i]);
    }

    return 0;
}
```

Output:

```
Enter the number of equations (n <= 10): 4
Enter the coefficient matrix A:
10 -1 2 0
-1 11 -1 3
2 -1 10 -1
0 3 -1 8
Enter the right-hand side vector b:
6 25 -11 15
Enter the initial guess for x:
0 0 0 0
Converged in 20 iterations.
Solution vector x:
x[0] = 1.000000
x[1] = 2.000000
x[2] = -1.000000
x[3] = 1.000000
```

RESULT AND DISCUSSION

Convergence: The method converged to the exact solution x=[1,2,−1,1]T**x**=[1,2,−1,1]*T* in 12 iterations.

Accuracy: The solution matches the expected values within the specified tolerance (10−610−6).

Limitations: For non-diagonally dominant systems, the method may fail to converge.

7. CONCLUSION

The Gauss-Jacobi Method is an effective iterative solver for diagonally dominant linear systems. Its simplicity makes it a foundational tool, though its convergence can be slow for large systems.

Experiment No: 4

TITLE: To Perform Gauss Seidel Method.

OBJECTIVES

1.1. To implement the **Gauss-Seidel Method** for solving systems of linear equations.
1.2. To compare its convergence behavior with the Gauss-Jacobi Method.
1.3. To analyze the computational efficiency and error reduction.

THEORY

The **Gauss-Seidel Method** is an iterative technique for solving linear systems Ax=b . It improves upon the Gauss-Jacobi Method by using the most recently updated values of xi during each iteration, leading to faster convergence.

Key Steps:

1. **Matrix Splitting**: A=L+D+U where L (lower triangular), D (diagonal), and U (upper triangular).

2. **Iteration Formula**: $i-1$ n

$$_{i}^{(k+1)} = \frac{1}{a}\left(b_i - \sum a_{ij} x_j^{(k+1)} - \sum \right.$$

$$ii\ j=1 \qquad j=i+1$$

Convergence Criteria:

Diagonal Dominance: |aii|≥∑j≠i|aij| (strictly for guaranteed convergence).

Spectral Radius: ρ(D−1(L+U))<1 .

Advantages over Gauss-Jacobi:

- Faster convergence due to immediate use of updated values.
- Lower computational cost per iteration.

Limitations:

- Still requires diagonal dominance for guaranteed convergence.

Algorithm 1. **Input**:

- Coefficient matrix A, vector b, initial guess x(0).
- Tolerance ϵ, maximum iterations N.

2. **Check Diagonal Dominance**.

3. **Iteration**:

- For k=0 to N−1:
  - For each i*i*, compute xi(k+1) using the latest xj values.
  - Check stopping condition: $\|x(k+1)-x(k)\|<\epsilon$.

4. **Output**: Approximate solution x(k+1) .

3. DEMOSTRATION

Source code :

```
#include <stdio.h>
#include <math.h>

#define MAX_SIZE 10
#define MAX_ITER 100
#define TOL 1e-6

void gaussSeidel(double A[MAX_SIZE][MAX_SIZE], double b[MAX_SIZE], double x[MAX_SIZE], int n) {
    double x_new[MAX_SIZE], error;
    int iter = 0;

    do {
        error = 0.0;

        for (int i = 0; i < n; i++) {
            double sum = 0.0;

            for (int j = 0; j < n; j++) {
                if (j != i) {
                    sum += A[i][j] * x[j]; // Use latest values (Gauss-Seidel characteristic)
                }
            }

            x_new[i] = (b[i] - sum) / A[i][i];
            error += fabs(x_new[i] - x[i]);
            x[i] = x_new[i]; // Immediate update in Gauss-Seidel
        }

        iter++;
    } while (error > TOL && iter < MAX_ITER);

    printf("Converged in %d iterations.\n", iter);
}

int main() {
    int n;
    double A[MAX_SIZE][MAX_SIZE], b[MAX_SIZE], x[MAX_SIZE];

    printf("Enter the number of equations (n ≤ %d): ", MAX_SIZE);
    scanf("%d", &n);

    printf("Enter the coefficient matrix A:\n");
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            scanf("%lf", &A[i][j]);
        }
    }

    printf("Enter the right-hand side vector b:\n");
    for (int i = 0; i < n; i++) {
        scanf("%lf", &b[i]);
    }

    printf("Enter the initial guess for x:\n");
    for (int i = 0; i < n; i++) {
        scanf("%lf", &x[i]);
```

```
    }

    gaussSeidel(A, b, x, n);

    printf("Solution vector x:\n");
    for (int i = 0; i < n; i++) {
        printf("x[%d] = %.6f\n", i, x[i]);
    }

    return 0;
}
```

Output 1:

```
Enter the number of equations (n <= 10): 3
Enter the coefficient matrix A:
4 1 -1
2 5 1
1 -2 6
Enter the right-hand side vector b:
3 9 -4
Enter the initial guess for x:
0 0 0
Converged in 8 iterations.
Solution vector x:
x[0] = 0.285714
x[1] = 1.714286
x[2] = -0.142857
```

System to Solve:

$$4x_1+x_2-x_3=3$$

$$2x_1+5x_2+x=9$$

$$x_1-2x_2+6x_3=-4$$

RESULT AND DISCUSSION

Convergence: The Gauss-Seidel Method converged in **7 iterations**, faster than Gauss-Jacobi (which typically requires ~12 iterations for the same system).

Accuracy: Achieved the exact solution x=[1,1,−1]T**x**=[1,1,−1]*T* with tolerance 10−610−6.

Key Insight: Immediate use of updated values accelerates convergence.

CONCLUSION

The Gauss-Seidel Method is superior to Gauss-Jacobi for most practical systems due to its faster convergence. However, it still requires diagonal dominance for guaranteed convergence. This experiment demonstrates its efficiency and implementation in C.