

**Abstract**

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## CONTENTS

# MATHEMATICAL MODELING.

For this chapter, we'll:

- Get our framework from an algorithmic framework of thinking to algebraic framework
- Represent a problem with algebraic expressions.
- Make proper implementations of mathematical models.

## 1.1. INTRODUCTION TO MATHEMATICAL MODELING

Before we can begin to think about models, we must understand what they are. And, for the purposes of this book, we can define a mathematical model as a representation of a real life problem with an abstract representation of the phenomena we intend to study.

For an immediately relevant example, a lot of problems in graph theory can be explained in algebraic terms. Although these problems can be a bit more mathematical in nature, then we can use them to explain real-life problems. Such as:

- Shortest path in a graph, for computer networking and pathfinding.
- The traveling salesman problem, for pathfinding, DNA sequencing and microchip manufacturing
- The vehicular routing problem, for finding multiple instances of a TSP-esque solution on a same graph

As a practical example of such a graph-based problem, let's analyze the following:

### 1.1.1. EXAMPLE 1: THE TRANSPORT PROBLEM

Let a finite number of sources and a finite number of destinations, we can determine the number of elements we can route to every destination with a minimal cost for every source. For example, if we said such sources are factories and the destinations are warehouses, we could see a 3x3 graph as it follows:

### 1.1.2. EXAMPLE 2: COVER PROBLEMS

More than a specific problem, this is a subdivision of problems meant to cover a certain group of demands or coverage criteria according to a specific defined problem.

#### 1.1.2.1. THE HOSPITAL PROBLEM

For example, say we got a group of neighborhoods that are of Euclidean behavior, and we intend to put the minimum amount of hospitals that can cover the entire region we're considering. A hospital covers the neighborhood if located on and those who share a boundary with it. A boundary is a non-zero

For an example, imagine the following instance of this problem:

#### 1.1.2.2. THE FOUR COLOR THEOREM

The four color theorem is a very famous application of a cover problem.

### 1.1.3. EXAMPLE 3: MINIMUM SPANNING TREE

If we imagine a tree data structure such as a graph with no cycles, and a graph as an interlocked set of nodes, a minimum spanning tree would be the minimum cost path that can connect all nodes as a tree.

This problem has a set of requirements for it to be considered correct:

- There must be no cycles or subcycles in the solution
- the number of connections must be the number of nodes minus one

## 1.2. CLASSIFICATION OF MATHEMATICAL MODELS

These models can be, as most things in mathematics, categorized and calificated in

### 1.2.1. DETERMINISTIC MATHEMATICAL MODELS

These are models that can be predicted in a way that is certain.

### 1.2.2. STOCHASTIC MATHEMATICAM MODELS

A stochastic model is one that involves a certain amount of randomness. For example,

### 1.2.3. STATIC MATHEMATICAL MODELS

A static model is one that does not depend on time, implying that whatever time passes, the result of such a system should stay constant. Generally this will happen for models where we don't even consider time as a relevant variable.

### 1.2.4. DYNAMIC MATHEMATICAL MODELS

#### 1.2.4.1. CONTINUOUS TIME MODELS

A continuous time model is one where we assume time as a continuous function, that meaning, it is flowing in somewhat real time. Although the way computation has it should bring it to be a finite level of statuses, this should be a big enough sample such as we can assume the variable can be statistically generalized into a continuous system

#### 1.2.4.2. DISCRETE TIME MODELS

A fairly interesting application of discrete time models comes from finance, where there exists capitalization in a set time interval. For example, we could imagine the free cash flow of an economic project as a mathematical problem to be optimized, such as this:

#### 1.2.4.3. CONTINUOUS STATUS MODELS

In a continuous status model, the dependant variable can assume any value in the range of a specific interval.

#### 1.2.4.4. DISCRETE STATUS MODELS

In these models, the difference is analogous to the way a discrete time model varies from a continuous time model. Values for the dependant variable shouls only assume specific values in an interval. For example, a model where every value the dependant variable can take is in the range of natural numbers ( $\mathbb{N}$ ) would be a discrete status.

#### 1.2.4.5. EXAMPLES OF MODEL CLASSIFICATION

**1.2.4.5.1. NUMBER OF ATTENDANTS IN A BANK EVERY HOUR** This example would be a dynamic mathematical model where every

**1.2.4.5.2. TEMPERATURE OF A CLASSROOM EVERY HOUR** For this,

## 1.2.5. OPEN MATHEMATICAL MODELS

An open mathematical model is one where the input that is recievied will be external to this model and independent from itself. For example a translator, such as Google Translate or DeepL will model natural language such as the input recievied is not eventually retroactively giving feedback to the model. Although it is possible that in ML models there are eventually expected use cases introduced into the model, for out purposes, they will be considered open as long as at runtime the previous example does not feedback into further answers.

## 1.3.



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