Homework 3
Image Compression

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- ## Goal:

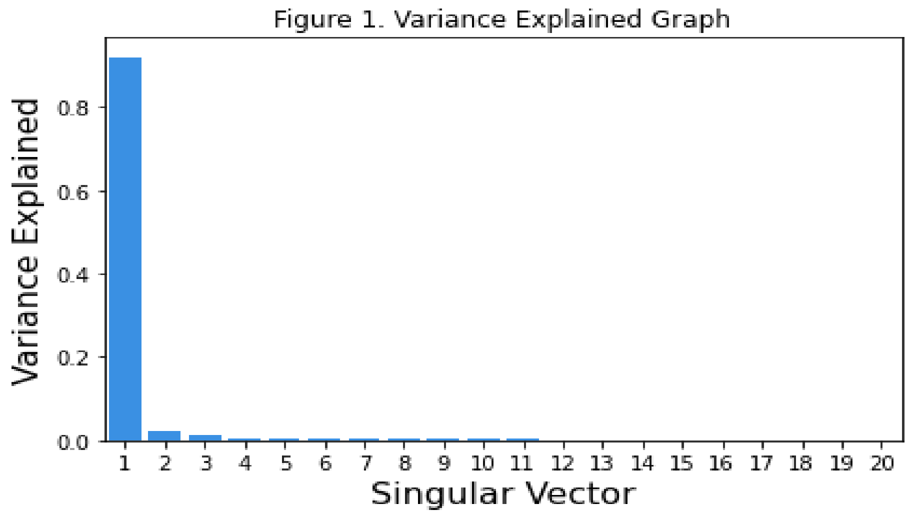
  The aim of this exercise is to see how the singular value decomposition (SVD) can be used to "compress" a graphical figure. The idea is to represent the figure as a matrix and then use the singular value decomposition to find the closest matrix of lower rank to the original matrix.

- ## SVD Recap:

  Singular Value Decomposition aka SVD is one of many matrix decomposition techniques that decomposes a matrix into 3 sub-matrices namely U, S, V where U is the left eigenvector, S is a diagonal matrix of singular values and V is called the right eigenvector. We can reconstruct SVD of an image by using "linalg.svd()" method of NumPy module.

- ## Steps:

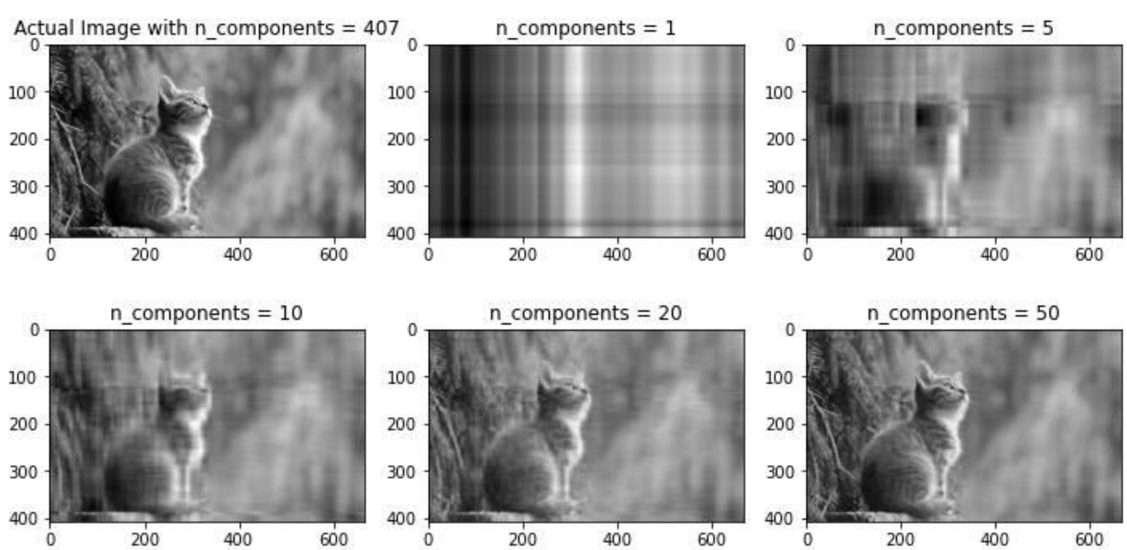
  We first read the image which using the "imread" method in "matplotlib" library in python which returns an array of height, width, and channel (RGB) For faster computations, the image is converted into grayscale by using the luminance formula (0.3 * R) + (0.59 * G) + (0.11 * B) and then applying this formula on the image array we get the grayscale image, then we apply SVD to the array which returns U, V, and S with 407 singular values.
  In the Figure 1 below we observe the variance of the image used over a singular vector.



The variance explained graph above clearly shows that about 92% of the information is explained by the first eigenvector and its corresponding eigenvalue themselves. Therefore, we can conclude that to reconstruct the image with just the top few eigenvectors is advisable.

- Results:

Reconstruction was done on the image using different number of components as we see in the figure below.



Though the 1st eigenvector contains 92% of the information, reconstructing the image solely from it does not give a clear picture as we see in the second picture compared to the first which is preserving the actual image components. Using the first 20 components was enough to show most of the details of the original picture with little blur starting from 407 which is a huge decrease and using 50 we get an image that we almost cannot notice the difference from the original. So, in conclusion we observe that using SVD we can get a pretty good compression of an image with great decrease in computation.

Finally, we evaluate these approximations by look at the relative spectral norm in Figure 2 below which has the following formula:

$$\frac{||A - \tilde{A}_k||_2}{||A||_2} = \frac{\sigma_{k+1}}{\sigma_1}$$

And by looking at the graph we can see how the choice of k will affect our approximation.

Figure 2. Relative Spectral Norm