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ELG 3106 : Electromagnetic Engineering

Design Study Impact of Broadband Antireflection Coating Design on Solar Power Production

Fall 2013

A good conceptual understanding of the reflection and transmission of an electromagnetic uniform plane wave normally incident on the boundary between two dielectric media is required. You must also be familiar with the MATLAB (see the "Introduction to MATLAB" presentation), and be able to implement matrix multiplication.

Introduction

The purpose of this design study is to demonstrate the impact design constraints have on system response. The system is a solar cell covered by an antireflection coating comprising either two or three layers, as shown in Figure 1, where choice of layers refractive indices and thicknesses are governed by the design approach. The wavelength distribution of light normally incident on the system is given by an idealized solar spectral irradiance (the so-called "blackbody" irradiance[1]),

$$I(\lambda) = \frac{6.16\times10^{15}}{\lambda^5\left(e^{2484/\lambda}-1\right)}, \qquad [1]$$

where I has units of power per unit area per unit wavelength (W/m^2/nm, and an area of 1 m^2 has been assumed) and λ is the wavelength in nm. The solar cell converts all light incident on it into electricity at 100% efficiency, regardless of the wavelength[2], but the antireflection coating has a transmissivity $T(\lambda)$, so the response is electrical power production given by

$$P = \int_{\lambda_1}^{\lambda_2} T(\lambda)\, I(\lambda)\, d\lambda\,, \qquad [2]$$

where the limits of integration are λ_1 = 200 nm and λ_2 = 2200 nm[3]. For T = 1, P = 1000 W/m^2.

[1] This neglects the filtering effects of the earth's atmosphere.
[2] This is not physically possible, but is used only to simplify calculations.
[3] This range delimits the bulk of the solar spectrum.

The object of this study is to demonstrate that the "obvious" approach to the design of broadband antireflection coatings does not necessarily maximize power production, and to explain why. The approach is to numerically integrate equation [2], using the transfer matrix method (TMM) to numerically simulate the wavelength-dependent transmissivity of the planar multilayer stack that comprises the antireflection coating on the solar cell. The theoretical description of the TMM presented below is completely general. It may be used to address any number of layers of arbitrary thickness.

The Design Approach

The focus is on double-layer and triple-layer antireflection coatings, an example of which is shown in Figure 1. We will assume that the sunshine incident on these systems may be represented by uniform plane waves of a given free-space wavelength at normal incidence. The challenge is to achieve a multilayer coating with as low a reflectivity (*i.e.*, as high a transmissivity) as possible across as broad a spectrum as possible. This depends on the number of layers used, their thicknesses and refractive indices. In general, the greater the number of layers used, for "optimal" choices of thickness and index, the greater the spectral range over which the reflectivity is suppressed. An example is shown in Figure 2. Note that without an antireflection coating, a reflectivity of about 30% is anticipated.

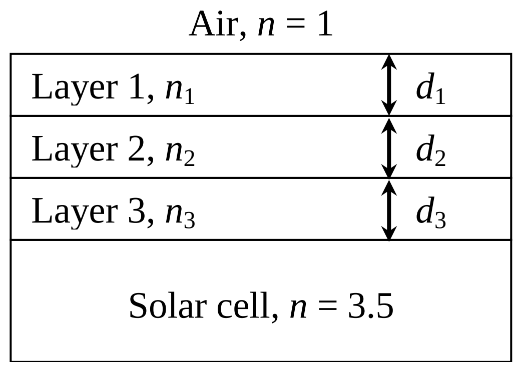

Figure 1: A triple-layer coating

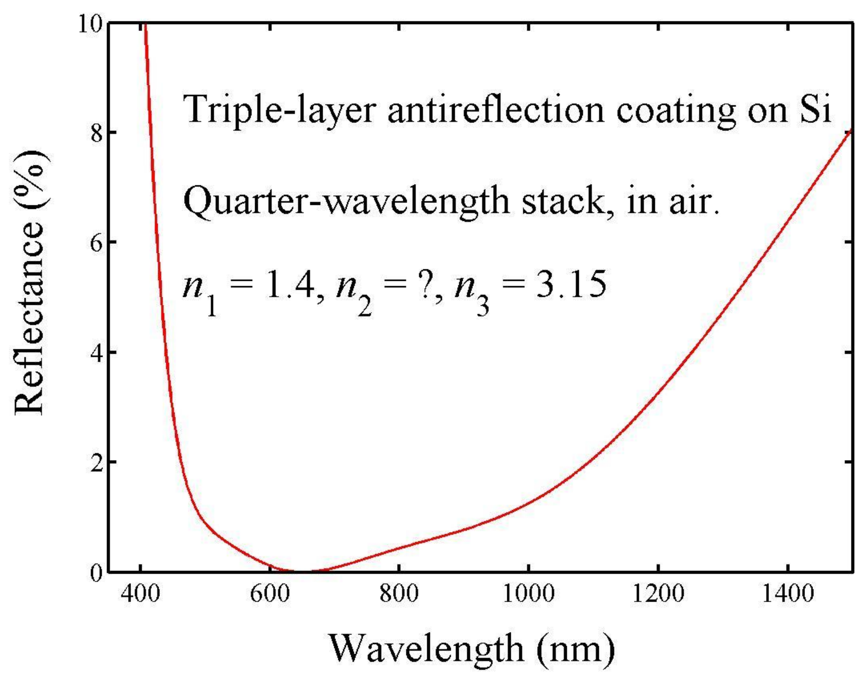

Figure 2: Impact of an antireflection coating upon the reflectivity.

In the analysis and simulations to be done, you are to presume that the thickness of each layer is equal to one-quarter the centre wavelength, λ_C, adjusted appropriately for the layer's refractive index. That is, the thickness should be one-quarter the material wavelength, not the in-air wavelength. Presume that the centre wavelength (in air) is $\lambda_C = 650$ nm, that the refractive index of air is unity, and that $n = 3.5$ is the refractive index of the solar cell.

The Double-Layer Coating

Apply the TMM theory described below to a double-layer antireflection coating. Determine the reflectivity at the centre wavelength analytically, and find the relationship between the four refractive indices that minimizes the reflectivity at the centre wavelength. If $n_1 = 1.4$, what is the optimal choice of n_2?

Now take your index and thickness values and provide a software description, implemented in MATLAB, that shows the full reflectivity spectrum as a function of free-space wavelength λ, ranging from 200 nm to 2200 nm. Calculate the power production, as per equation [1]. By choosing other index values and/or layer thicknesses, can you get better power production? If so, explain why, with illustrations.

The Triple-Layer Coating

Consider now the particular triple-layer antireflection coating whose reflectance is illustrated in Figure 2. Determine the reflectivity at the centre wavelength analytically, and find the relationship between the five refractive indices that minimizes the reflectance at the centre wavelength. Assuming $n_1 = 1.4$ and $n_3 = 3.15$, what value of n_2, to two significant figures, achieves this result? Modify your previous program to enable you to address triple-layer systems. With the same index assumptions, calculate the power production as a function of n_2, for values ranging from 1.6 to 3.0. What value of n_2 gives maximum power production? Does it agree with the value found using the analytic approach? Why or why not?

The Report

Your design study is to be structured as a formal technical report. Consult the document "Writing a Technical Report" and your *Fundamentals of Engineering Computation* notes for a review. You must show the application of the theory for both the double-layer and the triple-layer calculations, as described below.

You must completely specify each system with a diagram, fully labelling all parameters (cf. Figure 1). Report the full transfer matrix for each system, as a product of all the individual matrices (see TMM theory below), for arbitrary incident wavelength. Evaluate analytically each system at the central wavelength, present the formula for the reflectance at this wavelength, and determine the relationship between the index values that minimizes this reflectance. Provide the requested refractive index values.

You must construct an algorithm using the frequency-dependent TMM theory (use wavelength as a proxy for frequency). An algorithm is suggested at the end of this document. Code up the algorithm in MATLAB and numerically evaluate the power production as described above. Provide plots of the nominally optimal reflectivity (i.e., the reflectivity for the quarter-wave stack whose indices were analytically determined). Tabulate (or plot) the dependence of the power production on layer indices and thicknesses to ascertain if the design approach followed does indeed maximize power production. Discuss and suggest other optimization approaches.

Submission of the report may be done online. Softcopy submissions of the MATLAB mfiles are required and should include

The Transfer Matrix Method (TMM)

An arbitrary multilayer is shown in Figure 3 below, annotated by the electric field components at each interface. Note that, for convenience, this figure is oriented such that the incident wave travels to the right (in Figure 1, the incident wave travels downward). The subscripts indicate the layer, the + and – signs distinguish between forward and backward waves, respectively, and the prime is used to distinguish waves on the right hand side of an interface from those on the left.

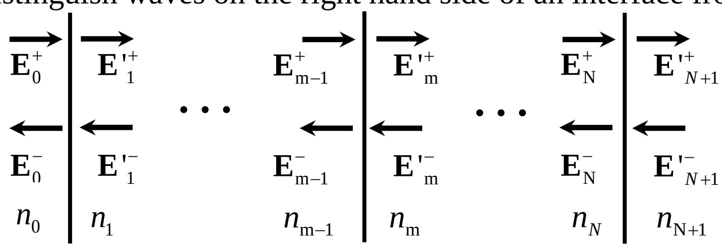

Figure 3: An arbitrary multilayer, used to illustrate the TMM theory.

The multilayer structure is composed of N layers, not including the unbounded media to the left (usually air, $n_0 = 1$) and to the right (denoted the substrate – silicon in our case), and N+1 interfaces. The layers have the refractive indices, n, as noted. The boundary conditions on the electric field vectors **E** on either side of an interface permit simple description by means of a 2×2 matrix. for the m^{th} interface, The relationship between the field components is

$$\begin{pmatrix} \mathbf{E}_{m-1}^+ \\ \mathbf{E}_{m-1}^- \end{pmatrix} = Q_{m-1,m} \begin{pmatrix} \mathbf{E'}_m^+ \\ \mathbf{E'}_m^- \end{pmatrix},$$

where

$$Q_{m-1,m} = \frac{1}{\tau_{m-1,m}} \begin{bmatrix} 1 & \Gamma_{m-1,m} \\ \Gamma_{m-1,m} & 1 \end{bmatrix}$$

is the dynamical matrix, which is defined in terms of the usual reflection and transmission coefficients

$$\Gamma_{m-1,m} = \frac{n_{m-1} - n_m}{n_{m-1} + n_m} \quad \text{and} \quad \tau_{m-1,m} = \frac{2n_{m-1}}{n_{m-1} + n_m}$$

respectively. The field components on the left and right hand sides of the m^{th} layer are related by the propagation matrix

$$\begin{pmatrix} \mathbf{E}'^{+}_{m} \\ \mathbf{E}'^{-}_{m} \end{pmatrix} = P_m \begin{pmatrix} \mathbf{E}^{+}_{m} \\ \mathbf{E}^{-}_{m} \end{pmatrix},$$

where

$$P_m = \begin{bmatrix} \exp(j\delta_m) & 0 \\ 0 & \exp(-j\delta_m) \end{bmatrix}$$

and $\delta_m = \frac{2\pi}{\lambda} n_m d_m$ is the phase thickness of the m^{th} layer, whose physical thickness is d_m; here λ is the wavelength in free-space (air). The repeated application of the above transformations for the N layers and the N+1 interfaces leads to a product of (N+1) 2×2 matrices that then relates the total field in the left hand unbounded medium to that of the total field in the right hand unbounded medium:

$$\begin{pmatrix} \mathbf{E}^{+}_{0} \\ \mathbf{E}^{-}_{0} \end{pmatrix} = \mathbf{T} \begin{pmatrix} \mathbf{E}'^{+}_{\mathrm{N+1}} \\ \mathbf{E}'^{-}_{\mathrm{N+1}} \end{pmatrix}$$

where **T** is the system transfer matrix

$$\mathbf{T} = \begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix} = Q_{0,1} \prod_{\mathrm{m=1}}^{\mathrm{N}} P_{\mathrm{m}} Q_{\mathrm{m,m+1}} \,.$$

We set the incident, reflected and transmitted field components as $\mathbf{E}_{\mathrm{i}} = \mathbf{E}^{+}_{0}$, $\mathbf{E}_{\mathrm{r}} = \mathbf{E}^{-}_{0}$ and $\mathbf{E}_{\mathrm{t}} = \mathbf{E}^{+}_{\mathrm{N+1}}$, respectively. In terms of the transfer matrix components, Γ and τ may be written as

$$\Gamma = \frac{\mathbf{E}_{\mathrm{r}}}{\mathbf{E}_{\mathrm{i}}} = \frac{T_{2,1}}{T_{1,1}} \quad \text{and} \quad \tau = \frac{\mathbf{E}_{\mathrm{t}}}{\mathbf{E}_{\mathrm{i}}} = \frac{1}{T_{1,1}} \,.$$

Note that these quantities are in general complex. The reflectance and transmittance are determined below.

Assuming $\mathbf{E}^{-}_{0} = \mathbf{E}'^{-}_{N+1} = 0$, conservation of energy yields

$$\left|\mathbf{S}^{+}_{0}\right| = \left|\mathbf{S}^{-}_{0}\right| + \left|\mathbf{S}^{+}_{N+1}\right| \quad \Rightarrow \quad \frac{\left|\mathbf{E}^{+}_{0}\right|^2}{2\eta^{*}_{0}} = \frac{\left|\mathbf{E}^{-}_{0}\right|^2}{2\eta^{*}_{0}} + \frac{\left|\mathbf{E}^{+}_{N+1}\right|^2}{2\eta^{*}_{N+1}},$$

where $\eta = \sqrt{\frac{\mu}{\varepsilon}}$. Since

$$\Gamma \equiv \frac{\mathbf{E}_{\mathrm{r}}}{\mathbf{E}_{\mathrm{i}}} \equiv \frac{\mathbf{E}^{-}_{0}}{\mathbf{E}^{+}_{0}} = \frac{T_{2,1}}{T_{1,1}} \quad \text{and} \quad \tau \equiv \frac{\mathbf{E}_{\mathrm{t}}}{\mathbf{E}_{\mathrm{i}}} \equiv \frac{\mathbf{E}'^{+}_{N+1}}{\mathbf{E}^{+}_{0}} = \frac{1}{T_{1,1}},$$

then

$$\frac{\left|\mathbf{E}^{+}_{0}\right|^2}{2\eta^{*}_{0}} = \frac{\left|\Gamma\right|^2\left|\mathbf{E}^{+}_{0}\right|^2}{2\eta^{*}_{0}} + \frac{\left|\tau\right|^2\left|\mathbf{E}^{+}_{0}\right|^2}{2\eta^{*}_{N+1}} \quad \Rightarrow \quad \left|\Gamma\right|^2 + \left|\tau\right|^2\left(\frac{\eta^{*}_{0}}{\eta^{*}_{N+1}}\right) = 1 \,.$$

For a lossless, nonmagnetic, medium, we then have

$$|\Gamma|^2 + |\tau|^2 \left(\frac{n_{N+1}}{n_0} \right) = 1,$$

and hence may define

$$R = |\Gamma|^2 \quad \text{and} \quad T = |\tau|^2 \left(\frac{n_{N+1}}{n_0} \right).$$

Suggested Functional Algorithm

(1) Set material parameters

> Refractive indices
> Reflection and transmission coefficients for each interface
> Elements of the dynamical matrix – define as matrix

(2) Set design parameters

> Centre wavelength
> Layer thickness

(3) Calculate transfer matrix

> In a loop over the wavelength (ranging from 400 to 1400 nm), do...
>
> > Define the propagator matrices
> > Do matrix multiplication for that wavelength
> > Extract reflectivity and transmissivity for that wavelength

(4) Plot reflectivity vs wavelength, calculate power production.

Note that this algorithm requires you to manually update the refractive indices. Alternatively, you could place steps (1) – (4) within loops over the relevant refractive index and/or layer thicknesses, modifying step (4) to test for maximum power production.