Sapienza Università di Roma

Department of Computer, Control and Management Engineering

FP1 Control of the Variable Length Pendulum

Underactuated Robotics

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1 Introduction

Ch. 8.1. Introduction [1].

2 Problem Formulation

Let's consider a variable length pendulum (VLP) with massless rod and without any friction. Let $\theta(t)$ be the angle between the pendulum and the y-axis (counterclockwise), let $l(t)$ be the length of the pendulum, let m be the mass of the pendulum and $f(t)$ be the force acting on the mass. Let (x_G, y_G) be the coordinates of the mass:

$$x_G = l(t)\sin\theta(t) \tag{1}$$

$$y_G = l(t)\cos\theta(t) \tag{2}$$

Let T be the kinetic energy of the VLP:

$$T = \frac{1}{2}m(\dot{x}_G^2 + \dot{y}_G^2) = \frac{1}{2}m(l(t)\dot{\theta}^2(t)) + \frac{1}{2}m(\dot{l}(t))^2 \tag{3}$$

Let P be the potential energy of the VLP:

$$P = mgy_G = -mgl(t)\cos\theta(t) \tag{4}$$

The Lagrangian of the VLP is $L = T - P$. Using Euler-Lagrange equation, it is possible to determine the motion equation of the VLP:

$$\ddot{\theta}(t) + \frac{2\dot{l}(t)\dot{\theta}(t)}{l(t)} + \frac{g\sin\theta(t)}{l(t)} = 0 \tag{5}$$

$$m\ddot{l}(t) - ml(t)\dot{\theta}^2(t) - mg\cos\theta(t) = f(t) \tag{6}$$

with $f(t)$ force applied to the mass.

Let's consider $u = \ddot{l}(t)$ as control input for the next sections. Since there is no control input controlling directly $\theta(t)$, the system is underactuated.

Let's assume $m = 1$. Let the total mechanical energy of the system E_T be:

$$E_T = T + P = \frac{1}{2}\dot{l}^2(t) + \frac{1}{2}(l(t)\dot{\theta}(t))^2 - gl(t)\cos\theta(t) \tag{7}$$

Let the desired trajectory of swing E_r be:

$$E_r = \frac{1}{2}(l_r\dot{\theta}(t))^2 - gl_r\cos\theta(t) \tag{8}$$

with E_r desired energy and l_r desired length of the pendulum. Moreover, being $\theta_{\max} \in (0, \pi]$ maximal angle of the desired swing:

$$E_r = -gl_r\cos\theta_{\max} \tag{9}$$

Given the above assumptions, let's consider the problem of trajectory tracking control, which consists in determining whether it is possible to design a control law u such that:

$$\lim_{t\to\infty} E_T = E_r \qquad \lim_{t\to\infty} \dot{l} = 0 \qquad \lim_{t\to\infty} l = l_r \tag{10}$$

3 Total Energy Shaping

Let's consider the following Lyapunov candidate:

$$V_c = \frac{1}{2}(E_T - E_r)^2 + \frac{1}{2}k_P(l - l_r)^2 + \frac{1}{2}k_D\dot{l}^2 \tag{11}$$

with k_D and k_P control parameters.

Proof here.

Controller (8.15):

$$u = \frac{(E_T - E_r)(l\dot{\theta}^2 + g\cos\theta) - k_P(l - l_r) - k_V\dot{l}}{E_T - E_r + k_D} \tag{12}$$

Lemma 2.2 not applicable here.

Difficulty of finding a constant k_D satisfying (8.17) for all $t \geq 0$.

3.1 Experiments

Ch. 8.5 (singular points in controller 8.18).

4 Partial Energy Shaping

Let's consider a partial kinetic energy E_P consisting only of the sum of the kinetic energy of rotation and potential energy of the VLP:

$$E_P = \frac{1}{2}(l(t)\dot{\theta}(t))^2 - gl(t)\cos(t) \tag{13}$$

Let's consider the following Lyapunov candidate:

$$V = \frac{1}{2}(E_P - E_r)^2 + \frac{1}{2}k_P(l - l_r)^2 + \frac{1}{2}k_D\dot{l}^2 \tag{14}$$

Stuff.

Let's take Controller (8.23):

$$u = \frac{(E_P - E_r)(l\dot{\theta}^2 + g\cos\theta) - k_P(l - l_r) + k_V\dot{l}}{k_D} \tag{15}$$

LaSalle's invariant principle.

Largest Invariant Set.

Lemma 8.1.

4.1 Motion Analysis

Ch. 8.4 (Convergence of Energy in 8.4.1, Closed-Loop Equilibrium Points in 8.4.2).

4.2 Experiments

Ch. 8.5 (controller 8.23 with initial state $(-\pi/6, 2, 0, 0)$ and $(-\pi/3, l_{de}, 0, 0)$).

5 Conclusion

Ch. 8.6.

References

[1] X. Xin and Y. Liu, *Control Design and Analysis for Underactuated Robotic Systems*. Springer Publishing Company, Incorporated, 2014.