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Practice Problems #4

Problem 1) ⓐ $a=2 \quad b=3 \quad d=0$

$\log_3 2 > 0$ $\quad \boxed{\Theta(n^{\log_3 2})}$

ⓑ $a=5 \quad b=4 \quad d=1$

$\log_4 5 > 1$ $\quad \boxed{\Theta(n^{\log_4 5})}$

ⓒ $a=7 \quad b=7 \quad d=1$

$\log_7 7 = 1$ $\quad \boxed{\Theta(n \log n)}$

ⓓ $a=9 \quad b=3 \quad d=2$

$\log_3 9 = 2$ $\quad \boxed{\Theta(n^2 \log n)}$

ⓔ $a=8 \quad b=2 \quad d=3$

$\log_2 8 = 3$ $\quad \boxed{\Theta(n^3 \log n)}$

ⓕ $T(n) = T(n-1) + 2 \qquad T(n-1) = T(n-2) + 2$

$= T(n-2) + 2 + 2$

$= T(n-2) + 4 \qquad T(n-2) = T(n-3) + 2$

$= T(n-3) + 2 + 4$

$= T(n-3) + 6$

$= T(n-k) + 2k$

$T(n) = 2n - 2$ $\quad \boxed{\Theta(n)}$

ⓖ $T(n) = T(n-1) + c^n \qquad T(n-1) = T(n-2) + c^n$

$= T(n-2) + 2c^n$

$= T(n-k) + kc^n$ $\quad \boxed{\Theta(nc^n)}$

ⓗ $T(n) = 2T(n-1) + 1 \qquad T(n-1) = 2T(n-2) + 1$

$= 4T(n-2) + 2$

$= 2k\, T(n-k) + k$ $\quad \boxed{\Theta(n^2)}$

(i) $T(n) = 49\,T(n/25) + n^{3/2}\log n$

$a = 49 \qquad b = 25 \qquad d = ?$

$\log_{25} 49$

(j) $T(n) = T(n-1) + n^c \qquad T(n-1) = T(n-2) + (n-1)^c$

$= T(n-2) + (n-1)^c + n^c$

$= T(n-3) + (n-2)^c + (n-1)^c + n^c$

$= T(n-k) + (n-k-1)^c$

(k) $T(n) = T(\sqrt{n}) + 1$

$= T(\sqrt{n-1}) + 2$

$= T(\sqrt{n-2}) + 3$

$= T(\sqrt{n-k}) + (k+1)$

Problem 2) This proof is wrong when he makes the statement about the middle horses. If we have $n=2$, by his proof, there is no middle horse, making his logic fall apart.

Problem 3) This problem can be solved by checking the medians of the two arrays, and if they're not equal, keep dividing the two arrays until you are left with two members in each array, in which case you can determine the median.

get medians from a and b

if they're equal

  return either

if $median_a > median_b$

  median will be in either $a[1 \ldots n/2]$ or $b[n/2 \ldots n]$

if $median_a < median_b$

  median will be in either $a[n/2 \ldots n]$ or $b[1 \ldots n/2]$

if size of both arrays == 2

$$median = [\max(a[0], b[0]) + \min(a[1], b[1])] / 2$$

Problem 4) (a) The Stooge Sort works because it sorts the first and last element of the array, then it procceeds to sort the first $2/3^{rds}$, the last $2/3^{rds}$, and then the first $2/3^{rds}$ again, recursevely, until the whole array is sorted.

(b) Yes, Stooge Sort should still sort correctly if we replace k with m, because it should still go through the array the same way as before. First two-thirds, last two thirds, then the first again.

(c) By looking at the algorithm, we can see that the recurence relation is

$$T(n) = 3T(\tfrac{2}{3}n) + 1$$

(d) $T(n) = 3T(\frac{2}{3}n) + 1$

$a = 3 \quad b = 3 \quad d = 0$

$\log_3 3 > 0 \qquad \Theta(n^{\log_3 3})$