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Practice Problems #5

Problem 1) Given any linear programming problem, you can change it into a different one that is familiar. In other words, any LP problem can be reduced to another via simple transformation. All you have to do to go from max to min is multiply the coefficients by -1.

Problem 2) max $x - y$

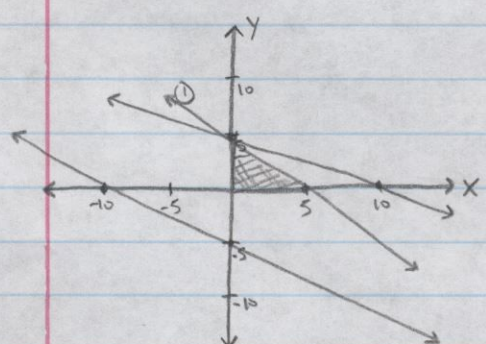
s.t. $x + y \leq 5$

$x + 2y \leq 10$

$-x - 2y \leq 10$

$x, y \geq 0$

This problem can be solved using the graphing method. The first step is plotting the equations as shown below.



① $x + y \leq 5$
when $x=0$, $y=5$
when $y=0$, $x=5$

② $x+2y \leq 10$
when $x=0$, $y=5$
when $y=0$, $x=10$

③ $-x-2y \leq 10$
when $x=0$, $y=-5$
when $y=0$, $x=-10$

Since x and y have to be greater than 0, we are only interested in the shaded region.

The vertices of the shaded region are (0,0), (0,5), and (5,0). To find the max, we plug these in to the original equation.

	(0,0)	(0,5)	(5,0)
x-y	0-0	0-5	5-0
value	0	-5	5

With the given constraints, the values that yield the max output for the given function are $\boxed{x=5 \text{ and } y=0.}$

Problem 3) $\min \max \{x, y\}$

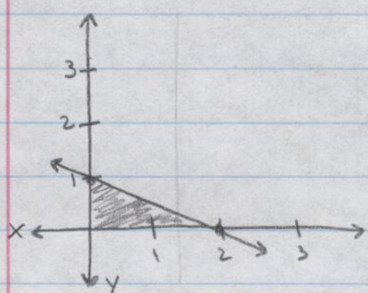
s.t. $x + 2y \geq 2$

which turns into $\min \max \{x, y\}$

$-x - 2y \leq -2$

$x, y \geq 0$

This problem can also be solved graphically, as the previous one.



① $-x - 2y \leq -2$

when $x=0$, $y=1$

when $y=0$, $x=2$

The intersects of this problem are $(0,0)$, $(0,1)$, $(2,0)$.

To solve the problem, we can plug the points into the original equation.

	$(0,0)$	$(0,1)$	$(2,0)$
$\max\{x, y\}$	0	1	2

The min value out of those is 0, which corresponds to the values $\boxed{x=0 \text{ and } y=0}$.

Problem 4) Given set of points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, find values m and b such that minimizes

$$\max_{1 \le i \le n} |y_i - mx_i - b|$$

Solution

Let z be the maximum absolute deviation, or $\max_i |y_i - mx_i - b|$

min z

s.t. $y_i - mx_i - b \le z$ for $i=1$ to n

$-y_i + mx_i + b \le z$ for $i=1$ to n

The only constraints this problem has are that the answer given at each point must be less than the maximum absolute deviation, which is a max of each calculation.

Given instance:

(1,3) (2,5) (3,7) (5,11) (7,14) (8,15) (10,19)

To solve this problem, you must turn plug in all the points for x and y, and constantly update the max absolute deviation as you iterate through them.

$3 = 1m + b$

$5 = 2m + b$

$7 = 3m + b$

$11 = 5m + b$

$14 = 7m + b$

$15 = 8m + b$

$19 = 10m + b$

After you find all of the absolute deviations, you can cycle through them to find what the one with the minimum absolute deviation is and base the line of best fit off that one.

You could also do the simplex method on this.

min z

s.t. $y_i - mx_i - b \leq z$

$-y_i + mx_i + b \leq z$

Which can be re-written as...

(with s values being slack values)

$$\begin{cases} z - s_1 - s_2 \\ y_i - mx_i - b + 1s_1 + 0s_2 \\ -y_i + mx_i + b + 0s_1 + 1s_2 \end{cases}$$

The simplex method can be applied to every point [(1,3), (2,5), etc.] and the minimun value can be determined out of the answers to give the slope and intercept for the line of best fit.

The only problem is that I'm not sure how that would be done with the constraints being compared to the function itself.