Problem 2) a. $f(n) = 3n + 6$ $\quad g(n) = 10000n - 500$

- both linear functions, $\boxed{f = \Theta(g)}$

b. $f(n) = n^{1/2}$ $\quad g(n) = n^{2/3}$

- the growth rate of $n^{1/2}$ is slower than that of $n^{2/3}$, and since this is directly proportional to asymtotical growth rate, $\boxed{f = O(g)}$

c. $f(n) = \log(7n)$ $\quad g(n) = \log(n)$

- the constant does not affect it, therefore $\boxed{f = \Theta(g)}$

d. $f(n) = n^{1.5}$ $\quad g(n) = n \log n$

- $\boxed{f = \Omega(g)}$ because $n^{1.5}$ grows much faster

e. $f(n) = \sqrt{n} = n^{1/2}$ $\quad g(n) = (\log n)^3$

- $\boxed{f = \Theta(g)}$ because they grow at the same rate, which means their runtime will be asymtotically equal

f. $F(n) = n2^n$ $\quad g(n) = 3^n$

- both grow at the exact rate, so $\boxed{f = \Theta(g)}$

Problem 3) c is a positive real number, $g(n) = 1 + c + c^2 + \dots + c^n$

Show that...

a. $\Theta(1)$ if $c < 1$

If c is less than one, the function will look something like $g(n) = 1 + [>0]^n$, so runtime will remain $O(1)$

b. $O(n)$ if $c = 1$

If c is one, the function will only add 1 everytime an n is added, which is a linear increase ($\Theta(n)$).

For $c = 1$, $g(n) = 1 + 1 + 1 = n + 1$ which is $\Theta(n)$

c. $O(c^n)$ if $c > 1$

Since c is now greater than 1, it will increase exponentially as you progress through this function, such as $c=2$, $g(n) = 1 + 2 + 4 + 8 + 16 + \dots$, therefore the runtime will be $\Theta(c^2)$

Problem 4) Show that $\log(n!) = \Theta(n \log n)$

To show that $f = \Theta(g)$, we must prove that $\underline{f = O(g)}$ and also that $\underline{f = \Omega(g)}$.

<u>lower band</u> $\log(n!) \leq n\log(n)$

if $n=3\ldots$ $\log(1) + \log(2) + \log(3) \leq 3[\log(3) + \log(3) + \log(3)]$

so $\log(n!) = O(n\log(n))$

<u>upper band...</u>

$\log(n!) \geq n\log(n)$

$$\log(1) + \ldots + \log(n/2) + \ldots + \log(n) \geq \log(n/2) + \ldots + \log(n)$$
$$\geq \log(n/2) + \ldots + \log(n/2)$$
$$\geq n/2 \log(n/2)$$

By looking at the latter half of the equations, we can show the upper limit, therefore $\log(n!) = \Omega(n\log n)$

Since we can prove both the lower limit and the upper limit, the equations are asymtotically equal.

Problem 1) 2^{7n} fastest

7^{2n}

$1000(\log n)^3$

$n^{\log_3 7}$

$n^{1/2}$

$5^{\log_3 n}$

$n\log n$

$2^{\log_2 n}$

$(\log n + 1)^3$ slowest

The reason for this order from slowest to fastest increasing is based on the kind of function each one is. The exponential functions will increase faster than any of the other ones, so regardless of constants, they will be towards the top. $2^{\log_2 n}$ cancels out to simply equal n, giving that spot in the table an asymptotic run time of $\Theta(n)$. The next one up, $n\log n$, is linearithmic thus giving a longer asymptotic run time.