

Introduction:

With given keys, find the shortest path to open all the lockers that contain tennis balls. There are m keys given initially, to open n lockers that contain t tennis balls. Each locker contains the keys to the adjacent lockers. Meaning locker i would contain the keys for lockers $i-1$ and $i+1$.

Algorithm 1 - Enumeration (Brute Force):

Pseudocode:

```
For i=0 to number of lockers
    If x is less than number of tennis balls
        If balls array at location x-1 equals i
            Then lockers array location i equals one
        Increment x
For i=1 to number of keys plus one
    Loop through the keys array after its been sorted
    Set the counter to zero
    Set the segments equal to the list function
    If the iterator is equal to one
        Set the total equal to the last ball location minus the first +1
        If there is a ball but no key
            Set total equal to balls array minus key array
        Else if there is a key for the last ball location
            Set the total equal to key array minus balls array
    at last location
        Set the counter equal to total
    Else
        Set the counter to i
        For j=0 to the length of the keyset array minus one
            Append the segments at j and j+1 in the keyset array
        For every pair found in these segments
            Increment count by either pair or lockers depending on
evalSegment
            Increase count based on the result of countLeft with arrays set to zero
            Increase count based on the result of countRight with arrays set to zero
        Decide which is less by doing the minimum between minLockers and count
Return minLockers
```

Runtime Analysis:

- The run time of the algorithm is $O(N \times 2^M)$. This is because the algorithm goes through every locker, as well as every combination possible for each key. It starts out by doing the possible combinations for key 1, then moves on to key 2, and so on. After getting this information, it will evaluate two numbers, the new cost and previous cost, and if the new cost is less than the previous, it will replace it as the preferred route.

Solutions:

- **Set #1:** 11
- **Set #2:** 14
- **Set #3:** 7
- **Set #4:** 14
- **Set #5:** 19
- **Set #6:** 1
- **Set #7:** 15
- **Set #8:** 8

Algorithm 2 - Dynamic Programming:

Pseudocode:

```
If the key is the first key then
    The total number of balls is checked to the left
    If the first key is less than the first ball
        Then add to the total balls
    Else if the first key is greater than the last ball
        Then add to the total balls
If there is a ball to the left of the first key
    Add to the total number of balls
While there are keys to the left of the key
    Go to the next position
    Set the left key
While loop
    Check if all balls are collected
        If so return
    Check if any balls left in the segment
        Check left
        Check right
    If the number from right is less than from the left
        Go from the left
    Count the number of balls left in the segment
        Add to the counter to track them
    Return them
```

Add the collected balls to the collected list
If not last segment
 Compare if close to get from left or right key
 Increment counter
If it's the last segment check for balls after it
 You will only be coming from the left in this case
 Add the left counter till find the balls
If no balls left in segment go to the next

Runtime Analysis:

- The pseudo code and actual code splits the keys in to segments, from the first locker to the first key from the first key to the second, from the I key to the I+1 key and from the last key to the N locker. From there it determines the shortest path from either the left or right key. The program will run in $O(N \times M^2)$ time because for every key it has to run left and right and check every locker on its way. There for each key gets run twice and each locker in its segment will get run for each time that segment is ran.

Solutions:

- **Set #1:** 99
- **Set #2:** 22
- **Set #3:** 68
- **Set #4:** 31
- **Set #5:** 103
- **Set #6:** 30
- **Set #7:** 87
- **Set #8:** 82