

Exercise 1.15. The sine of an angle (specified in radians) can be computed by making use of the approximation $\sin x \approx x$ if x is sufficiently small, and the trigonometric identity

$$\sin x = 3 \sin \frac{x}{3} - 4 \sin^3 \frac{x}{3}$$

to reduce the size of the argument of \sin . (For purposes of this exercise an angle is considered “sufficiently small” if its magnitude is not greater than 0.1 radians.) These ideas are incorporated in the following procedures:

```
(define (cube x) (* x x x))
(define (p x) (- (* 3 x) (* 4 (cube x))))
  (if (not (> (abs angle) 0.1))
      angle
      (p (sine (/ angle 3.0)))))
```

- a. How many times is the procedure `p` applied when `(sine 12.15)` is evaluated?
- b. What is the order of growth in space and number of steps (as a function of a) used by the process generated by the `sine` procedure when `(sine a)` is evaluated?

Answer

a. 5 times.

b. The order of growth of the number of steps is $\Theta(\log a)$. Since the procedure is a recursive procedure, the order of growth in space grows linearly with the number of the recursive steps. So the order of growth in space is also $\Theta(\log a)$.