Department of Computer Science and engg.- NITK Surathkal

CS-253 Design and Analysis of Algorithms

Assignment sheet- 1.A. Due Date: 15th January 2025

Asymptotic Analysis

1. An algorithm takes 0.5 seconds to run on an input of size 100. How long will it take to run on an input of size 1000 if the algorithm has a running time that is linear? quadratic? Cubic?

2. An algorithm is to be implemented and run on a processor that can execute a single instruction in an average of 10^{-9} seconds. What is the largest problem size that can be solved in one hour by the algorithm on this processor if the number of steps needed to execute the algorithm is n? n^2 ?, n^3 ? 1.3^n ? Assume n is the input size.

3. Suppose that the Insertion Sort sorting algorithm has a running time of $T(n) = 8n^2$, while the Counting Sort algorithm has a running time of $T(n) = 64n$. Find the largest positive input size for which Insertion Sort runs at least as fast as Counting Sort.

4. If you were given a full week to run your algorithm, which has running time $T(n) = 5 \cdot 10^{-9}(n^3)$ seconds, what would be the largest input size that could be used, and for which your algorithm would terminate after one week? Explain and show work.

5. Use big-O notation to state the growth of $f(n) = n + n \log n^2$. Defend your answer.

6. Which function grows faster: $f(n) = n \log^2 n$ or $g(n) = n^{0.3} \log^{36} n$. Defend your answer

7. Prove that $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.

8. Prove or disprove $f(n) + g(n) = \Theta(\min(f(n), g(n)))$

9. If $g(n) = o(f(n))$, then prove that $f(n) + g(n) = \Theta(f(n))$.

10. Use L'Hospital's rule to prove that $a^n = \omega(n^k)$, for every real $a > 1$ and integer $k \geq 1$.
    Hint: take k derivatives of the ratio a^n / n^k .

11. Show that $\log(n!) = \Theta(n \log n)$.

12. Prove that $n! = \omega(2^n)$.

13. Use mathematical induction to prove that $1 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$

14. Use mathematical induction to prove that, for all integers $k \geq 1$ and some $\varepsilon > 0$, $\log^k n = o(n^\varepsilon)$.

15. Show that for any real constant a and b, where $b>0$, $(n + a)^b = \Theta(n^b)$

16. Let f(n) and g(n) be asymptotically non negetive functions. Using the basic definition of Θ – notation , prove that $\max(f(n),g(n)) = \Theta(f(n) + g(n))$.