# Graphs for Decision-Making

EN 500.111 Week 8

## Presentation Work

- ➔ Meet with your groupmates for ~20 minutes to work on coordinating presentation
- ➔ I’ll be coming around to each group to check in and answer any questions
- ➔ https://github.com/mkirsche/TAG2020/blob/master/presentation/guidelines.md

## Outline

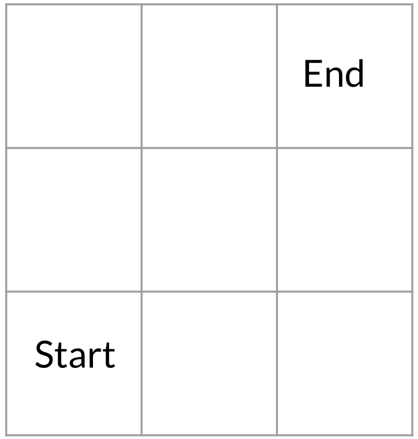
- ➔ Dynamic programming
- ➔ Machine Learning
    - ◆ Decision trees
    - ◆ Neural networks

## Counting Paths

Sometimes we're interested in knowing not only the shortest path, but how many routes there are from one node in a graph to another

- ➔ Finding traffic bottlenecks for road planning
- ➔ Checking whether people are in the same group within a social network

How many shortest paths are there from start to end in this grid (moving up/down/left/right)?

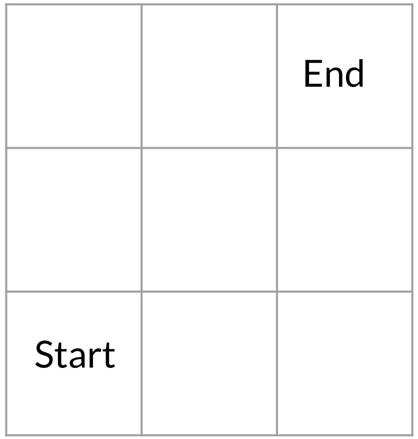


## Counting Paths

How many shortest paths are there from start to end in this graph?

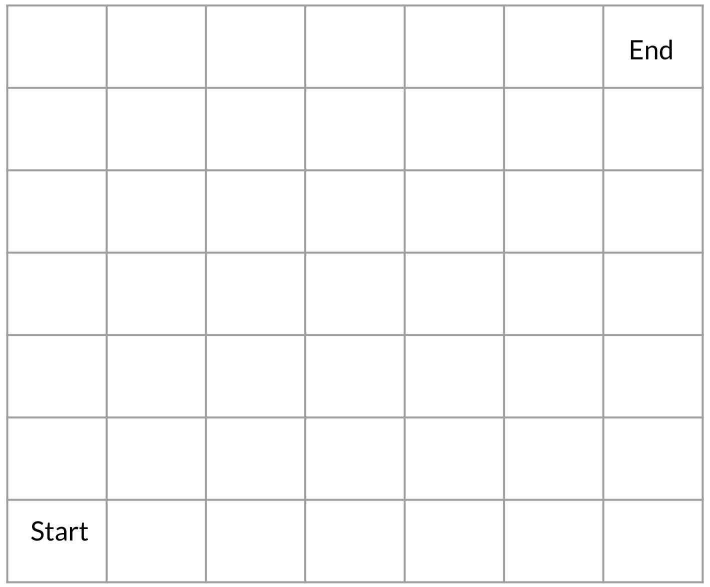
**6**

**RRUU**
**RURU**
**URUR**
**UURR**
**URRU**
**RUUR**



# Counting Paths

What about this graph?

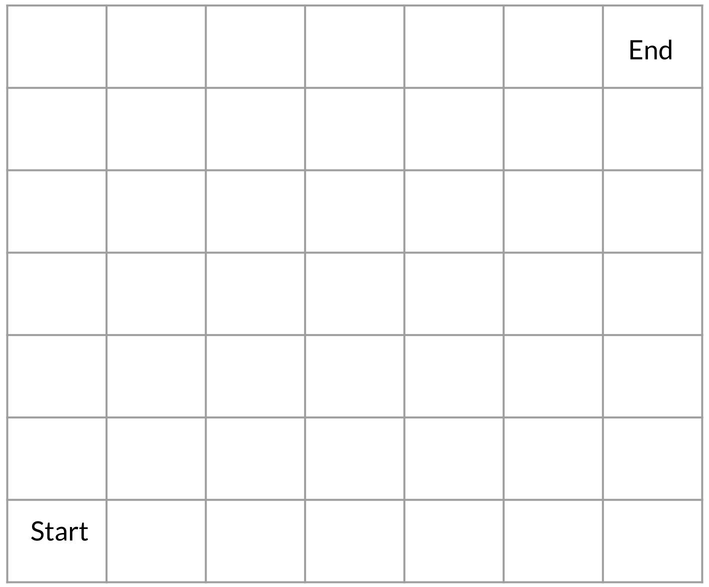


## Counting Paths

What about this graph?

A whole lot - any shortest path has 6 rightward movements and 6 upward movements, so it’s the number of ways to arrange those.

12 choose 6 = 924 paths



## Counting Paths

What about this graph?

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | End |
| | | X | | | | |
| | | | X | | X | |
| | X | | | | | |
| | | | | X | | |
| | | X | X | | | |
| Start | | | | | | |

## Counting Paths

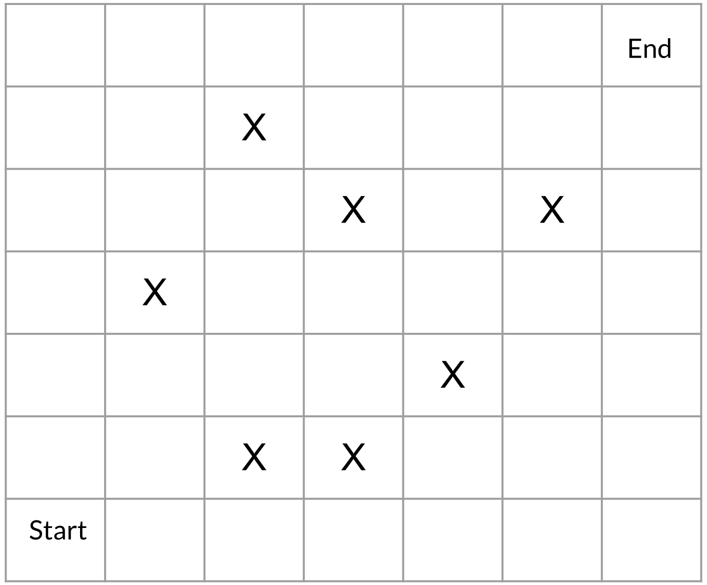
What are some things we know about the solution?

About movements

- 12 total moves
- 6 right moves and 6 up moves

About sequence of squares

- First square is start - (0, 0)
- Last square is end - (6, 6)
- Second square is (1, 0) or (0, 1)
- Second to last square is (5, 6) or (6, 5)

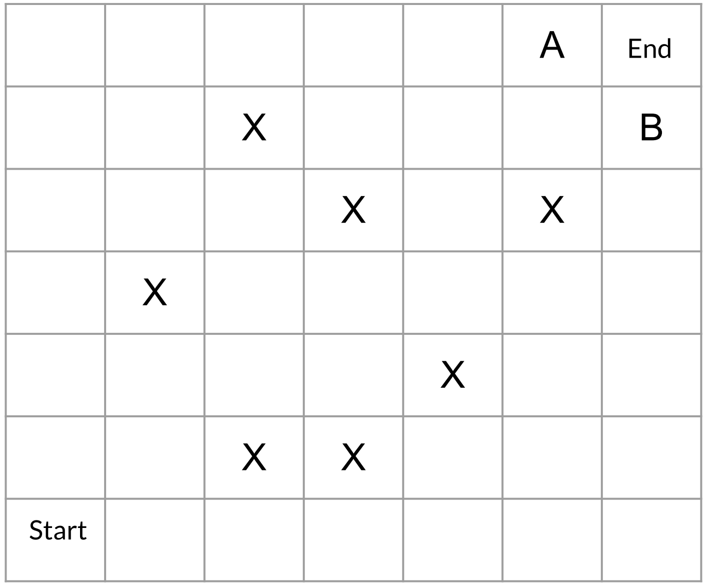


## Counting Paths

Any shortest path has to end with either (5, 6) -> (6, 6) or (6, 5) -> (6, 6). So if we count the shortest paths to A and to B, we can add those up to get the number of shortest paths to the end.

About sequence of squares

- First square is start - (0, 0)
- Last square is end - (6, 6)
- Second square is (1, 0) or (0, 1)
- Second to last square is (5, 6) or (6, 5)



## Counting Paths

So now we want to find the number of paths to A and B, which we can do the same way.

Paths to End = Paths to A + Paths to B
Paths to A = Paths to C + Paths to D
Paths to B = Paths to D + Paths to E

So we can use the number of shortest paths to closer destinations and combine them to eventually get the number of paths to the end.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | C | A | End |
| | | X | | | D | B |
| | | | X | | X | E |
| | X | | | | | |
| | | | | X | | |
| | | X | X | | | |
| | | | | | | |

## Counting Paths

We also know that for anything on the left side and the bottom, there's only one way to get there (all right or all up)

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | | | | C | A | End |
| 1 | | X | | | D | B |
| 1 | | | X | | X | E |
| 1 | X | | | | | |
| 1 | | | | X | | |
| 1 | | X | X | | | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

## Counting Paths

And we can start filling in the number of paths to other squares, starting with the ones closer to the start (as soon as we have the counts for the squares below and to the left of them).

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | | | | | | End |
| 1 | | X | | | | |
| 1 | | | X | | X | |
| 1 | X | | | | | |
| 1 | | | | X | | |
| 1 | 2 | X | X | | | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

## Counting Paths

Eventually we fill in the whole grid and have our answer of 34.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 9 | 15 | 34 |
| 1 | 2 | X | 0 | 6 | 6 | 19 |
| 1 | 1 | 4 | X | 6 | X | 13 |
| 1 | X | 3 | 6 | 6 | 8 | 13 |
| 1 | 3 | 3 | 3 | X | 2 | 5 |
| 1 | 2 | X | X | 1 | 2 | 3 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

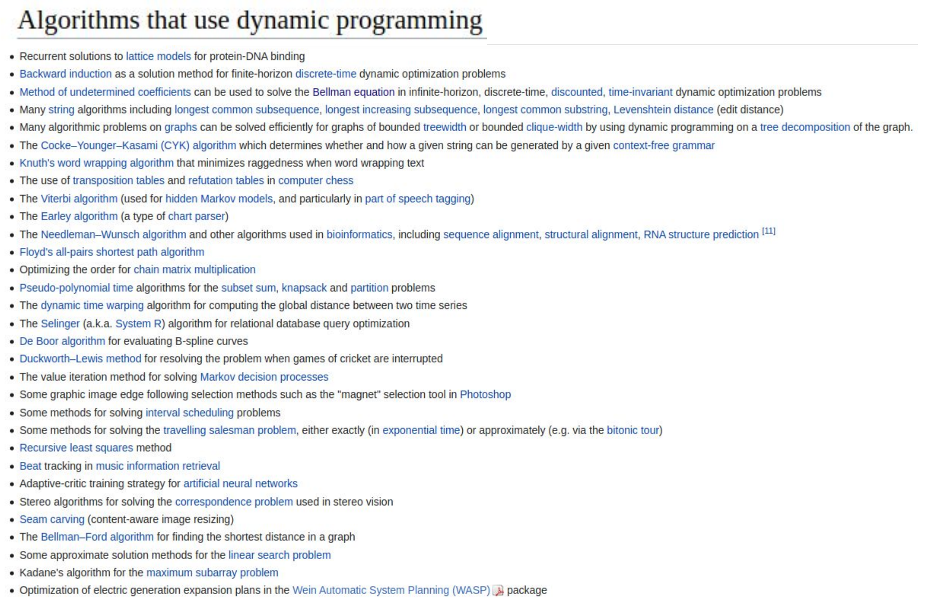
## Counting Paths

What if we want the total number of paths, including those that aren't shortest paths? Does this strategy still work?

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | End |
| | | X | | | | |
| | | | X | | X | |
| | X | | | | | |
| | | | | X | | |
| | | X | X | | | |
| Start | | | | | | |

# Dynamic Programming

- ➔ A technique introduced in the 1950s that involves breaking down a problem into smaller versions of the same problem
- ➔ Tons of applications in all different subject areas

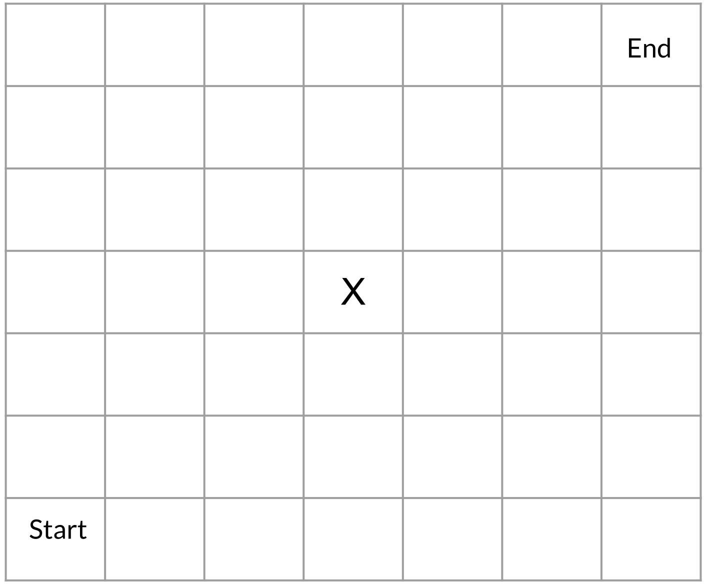


## Algorithms that use dynamic programming

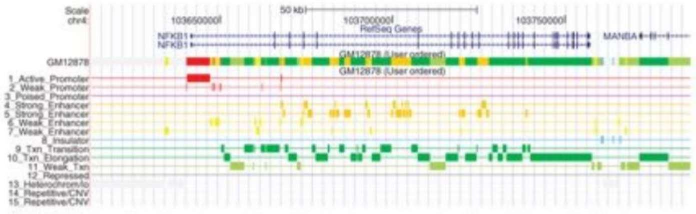
- Recurrent solutions to lattice models for protein-DNA binding
- Backward induction as a solution method for finite-horizon discrete-time dynamic optimization problems
- Method of undetermined coefficients can be used to solve the Bellman equation in infinite-horizon, discrete-time, discounted, time-invariant dynamic optimization problems
- Many string algorithms including longest common subsequence, longest increasing subsequence, longest common substring, Levenshtein distance (edit distance)
- Many algorithmic problems on graphs can be solved efficiently for graphs of bounded treewidth or bounded clique-width by using dynamic programming on a tree decomposition of the graph.
- The Cocke–Younger–Kasami (CYK) algorithm which determines whether and how a given string can be generated by a given context-free grammar
- Knuth's word wrapping algorithm that minimizes raggedness when word wrapping text
- The use of transposition tables and refutation tables in computer chess
- The Viterbi algorithm (used for hidden Markov models, and particularly in part of speech tagging)
- The Earley algorithm (a type of chart parser)
- The Needleman–Wunsch algorithm and other algorithms used in bioinformatics, including sequence alignment, structural alignment, RNA structure prediction [11]
- Floyd's all-pairs shortest path algorithm
- Optimizing the order for chain matrix multiplication
- Pseudo-polynomial time algorithms for the subset sum, knapsack and partition problems
- The dynamic time warping algorithm for computing the global distance between two time series
- The Selinger (a.k.a. System R) algorithm for relational database query optimization
- De Boor algorithm for evaluating B-spline curves
- Duckworth–Lewis method for resolving the problem when games of cricket are interrupted
- The value iteration method for solving Markov decision processes
- Some graphic image edge following selection methods such as the "magnet" selection tool in Photoshop
- Some methods for solving interval scheduling problems
- Some methods for solving the travelling salesman problem, either exactly (in exponential time) or approximately (e.g. via the bitonic tour)
- Recursive least squares method
- Beat tracking in music information retrieval
- Adaptive-critic training strategy for artificial neural networks
- Stereo algorithms for solving the correspondence problem used in stereo vision
- Seam carving (content-aware image resizing)
- The Bellman–Ford algorithm for finding the shortest distance in a graph
- Some approximate solution methods for the linear search problem
- Kadane's algorithm for the maximum subarray problem
- Optimization of electric generation expansion plans in the Wein Automatic System Planning (WASP) package

## How is it different from brute force?

- ➔ Consider some point in the middle of the grid
- ➔ There are tons of ways to get there and tons of ways to get to the end from there
- ➔ Brute force explores every pair of incoming path and outgoing path since it corresponds to a valid start -> end path
- ➔ The dynamic programming algorithm doesn’t care how incoming paths got to the X square and just adds them all to its tally before considering outgoing paths



# Another example



*ChromHMM*

Suppose we are trying to identify important regions of a genome. There are methods which establish different types of annotations (histone modifications, types of gene expression regulators, etc.) and assign each basepair a score for how likely each annotation is to apply to it. This is based on sequence context, similar sequence in already-annotated genomes, etc. We want to identify contiguous segments which contain a lot of bases with a similar annotation.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | G | C | G | C | T | A | C | A | C | G | T |
| 10 | -5 | -4 | 15 | 10 | 5 | 0 | 20 | -100 | 20 | 10 | -1 |

## Subarray sum problem

Given a list of numbers, find the contiguous block with the highest sum.

| A | G | C | G | C | T | A | C | A | C | G | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | -5 | -4 | 15 | 10 | 5 | 0 | 20 | -100 | 20 | 10 | -1 |

## Subarray sum problem

Solution:

Find the sum of the maximum subarray that ends at each position:

Max sum ending here:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | | | | | | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A<br>10 | G<br>-5 | C<br>-4 | G<br>15 | C<br>10 | T<br>5 | A<br>0 | C<br>20 | A<br>-100 | C<br>20 | G<br>10 | T<br>-1 |

## Subarray sum problem

Solution:

Find the sum of the maximum subarray that ends at each position.
For each position, we can either start a new subarray or take the sum from the previous position

Max sum ending here:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 5 | 1 | | | | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A<br>10 | G<br>-5 | C<br>-4 | G<br>15 | C<br>10 | T<br>5 | A<br>0 | C<br>20 | A<br>-100 | C<br>20 | G<br>10 | T<br>-1 |

## Subarray sum problem

Solution:

We can use this strategy to fill the entire list, and the answer is the biggest sum we get.

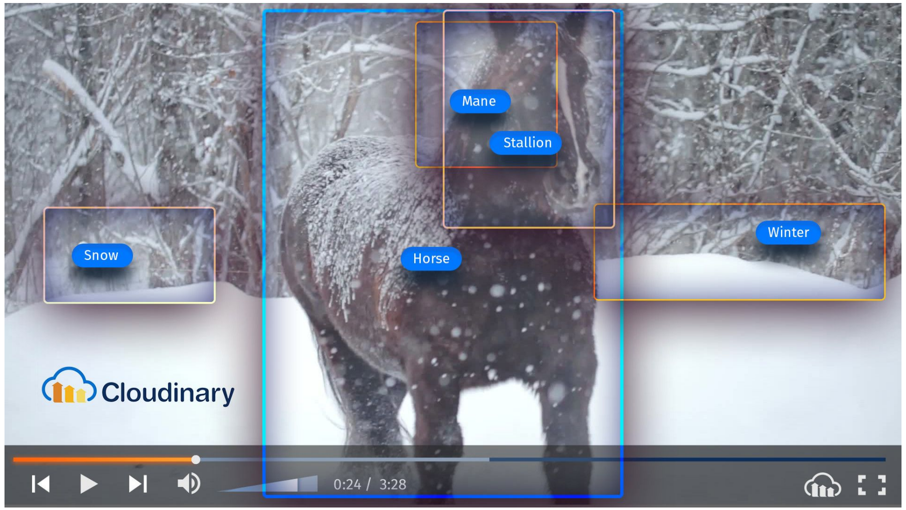
Max sum ending here:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 5 | 1 | 16 | 26 | 31 | 31 | 51 | -49 | 20 | 30 | 29 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | G | C | G | C | T | A | C | A | C | G | T |
| 10 | -5 | -4 | 15 | 10 | 5 | 0 | 20 | -100 | 20 | 10 | -1 |

## Classification Tasks

➔ We commonly want to use algorithms to perform classification, or figuring out what category an object belongs to:
  - Automated image/video tagging
  - Determining what organism a genomic read came from
  - Filtering spam in an email inbox
  - Triaging patients in a hospital
  - Determining whether someone has good enough credit to receive a loan
  - ...and much more!



https://cloudinary.com/blog/new_google_powered_add_on_for_automatic_video_categorization_and_tagging

# Decision Trees

- ➔ A tree where each leaf node is a category and each intermediate node is a question/feature which splits the data
- ➔ Uses training data (category already known) to decide on the best features to split on and to label leaf nodes, and then predicts category of new objects

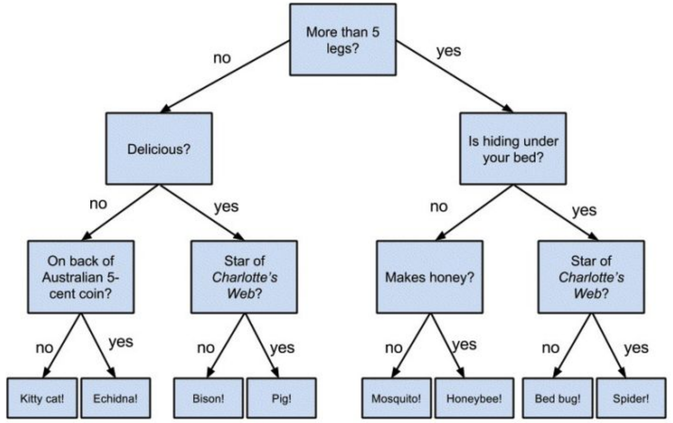
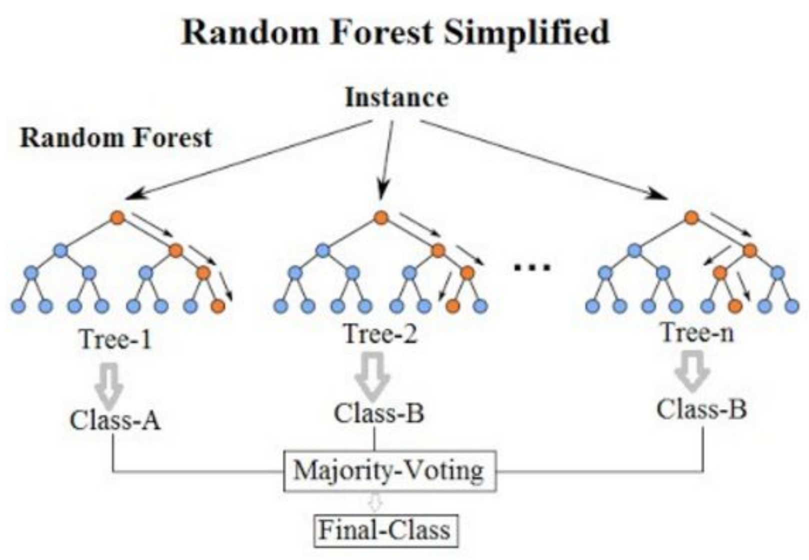


*Figure 17-1. A "guess the animal" decision tree*

https://medium.com/analytics-vidhya/a-guide-to-machine-learning-in-r-for-beginners-decision-trees-c24dfd490abb

## Random Forest

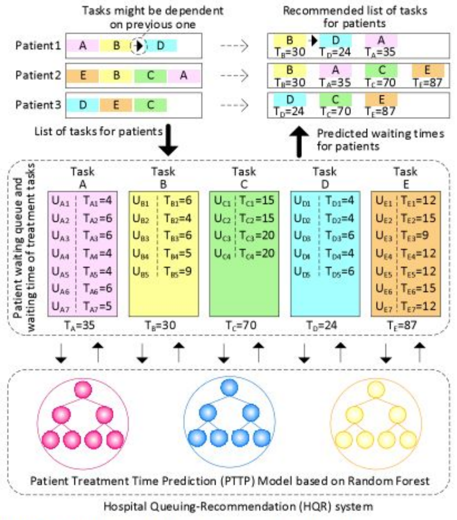
- ➔ A collection of decision trees where each one is split on different features and uses different training data
- ➔ To perform classification, they vote on the category a given object falls into



(Wikipedia)

## Hospital Queue Time Minimization

- ➔ One place random forests have been applied is to predict hospital queue times based on limited resources
- ➔ This can be used to recommend the order in which treatments/tests/etc. Can be done for different patients



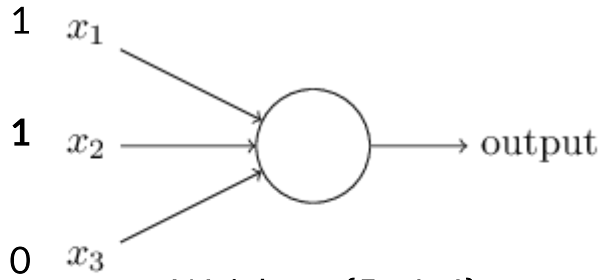
Tasks might be dependent on previous one
Recommended list of tasks for patients
Patient 1
Patient 2
Patient 3
List of tasks for patients
Predicted waiting times for patients
Patient waiting queue and waiting time of treatment tasks
Task A
Task B
Task C
Task D
Task E
$T_A=35$
$T_B=30$
$T_C=70$
$T_D=24$
$T_E=87$
Patient Treatment Time Prediction (PTTP) Model based on Random Forest
Hospital Queuing-Recommendation (HQR) system

**FIGURE 1.** **Workflow of patient treatment and wait model.**

"A Parallel Patient Treatment Time Prediction Algorithm and Its Applications in Hospital Queuing-Recommendation in a Big Data Environment"

## Another Classification Method - Perceptrons

- Takes a series of binary (0 or 1) inputs and classifies as 0 (no) or 1 (yes)
- The perceptron stores a series of weights and a threshold value
- It classifies by adding up $\text{weight}_i * x_i$ for each of the inputs and checking if it's greater than (1) or less than (0) the threshold

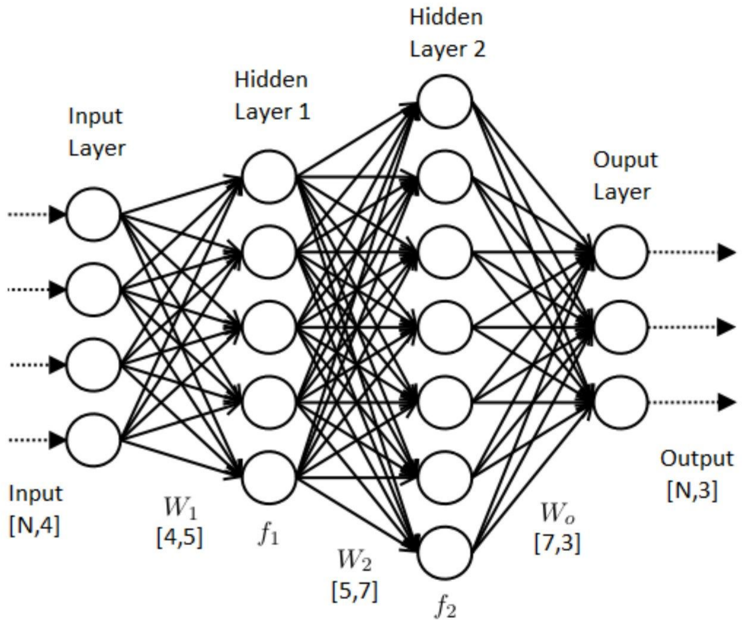


Weights = {5, -1, 4}
Threshold = 4.5

Sum = 1*5 + 1*-1 + 0*4 = 4
Sum < Threshold, so output 0.

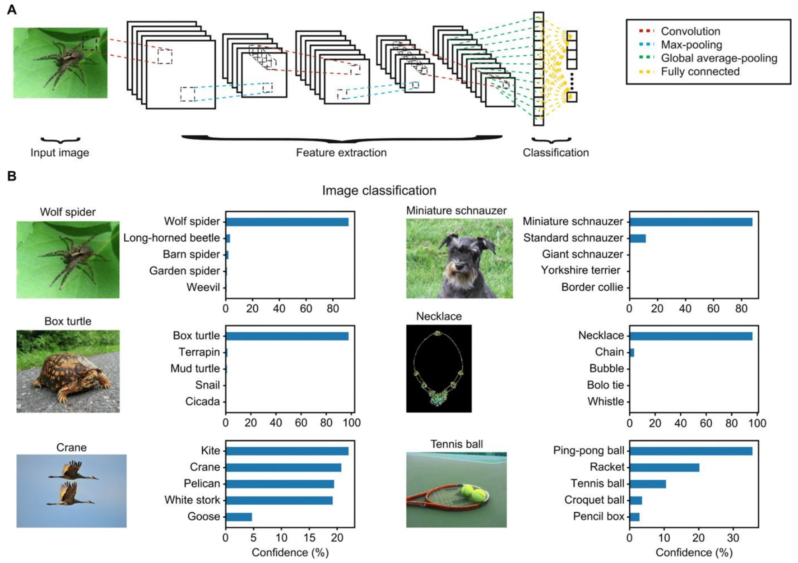
## Artificial Neural Networks

- ➔ (Very) loosely based on neurons in the human brain
- ➔ The idea is to perform classification tasks by combining a large number of perception-like nodes
- ➔ Nodes are arranged in layers, with edges between nodes in consecutive layers
- ➔ Weights of edges are learned from training data and reflect how much an input from one layer affects the outputs of the next layer



## Image Classification

- ➔ Neural networks commonly used to classify images
- ➔ Takes images as a series of pixel intensities and is able to infer even very complex patterns
- ➔ Lots of ongoing research for how neural networks should be arranged and what types of functions should be used



## Conclusions

- ➔ Dynamic programming gives us faster algorithms than brute force when we can divide a task into smaller versions of the same task
- ➔ Graphs are used for the underlying structure of many “machine learning” classification techniques, including random forests and neural networks