Brief Review of Camphor Boats

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January 15, 2015

1 Radial Extent of Camphor Spread

For a fixed c-boat, the flux of camphor from the c-boat onto the air-water interface and sublimation of camphor molecules from the air-water interface result in a steady state radial distribution of camphor around the c-boat. We measured the radial extent (i.e., distance out to which camphor molecules spread before sublimation) of camphor spread for different diameters of the c-boats. Figures **??**(a)-(d) show the radii of influence as a function of time for 1 mm, 2 mm, 3 mm and 4 mm diameter c-boats respectively. When there is *no* sublimation of the surface active molecules from the air-water interface, $r(t) \propto t^{\frac{3}{4}}$ [1]. In case of camphor one would expect a slower trend as the sublimation depletes camphor from the air-water interface. However, we fitted the measured $r(t)$ to $(1 - \mathrm{e}^{-t/\tau})$ as this function yielded the best fit results. We believe, this is *not* the correct functional form of the observed behavior as this yields $r(t) \propto t$ at small times which is faster than the expected behavior ($r(t) \propto t^{\frac{3}{4}}$) before sublimation dominates the flux of camphor onto the air-water interface.

1.1 Back-of-the-Envelope Thoughts

When a drop of surfactant is introduced on air-water interface, the surfactant molecules are drawn onto the interface to minimize the interfacial energy. The surfactant front spreads on air-water interface radially outwards with time as

$$r(t) \approx \left(\frac{\Delta\gamma^2}{\mu\rho}\right)^{\frac{1}{4}} \, t^{\frac{3}{4}} \tag{1}$$

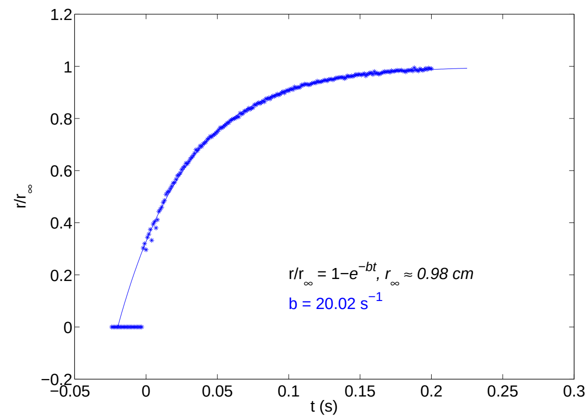
Substituting $\Delta\gamma = 20\left[\frac{\mathrm{mN}}{\mathrm{m}}\right]$, $\mu = 10^{-3}\left[\frac{\mathrm{N.s}}{\mathrm{m}^2}\right]$ and $\rho = 10^3\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$, we get

$$r(t) \approx \alpha \, t^{\frac{3}{4}}$$

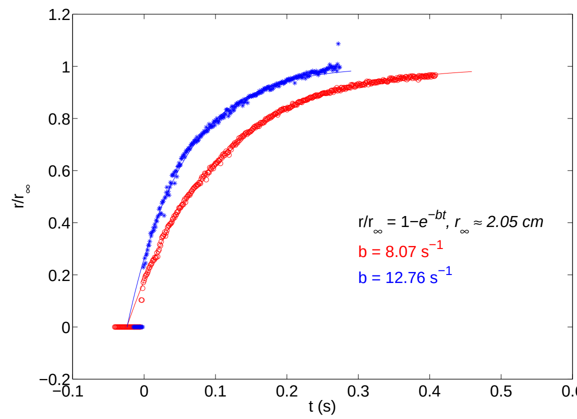
where, $\alpha = 0.14\,[\mathrm{m.s}^{-0.75}]$. The rate at which area increases is,

$$\frac{\mathrm{d}}{\mathrm{dt}}A(t) = \frac{3\pi}{2}\alpha^2 \, t^{\frac{1}{2}}$$

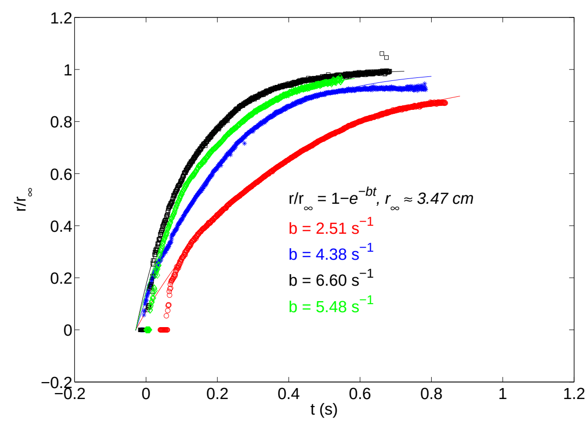
[1] M M Bandi, T Tallinen and L Mahadevan, Shock-driven jamming and periodic fracture of particulate rafts, *Europhys. Lett.*, 96, 36008, **2011**.



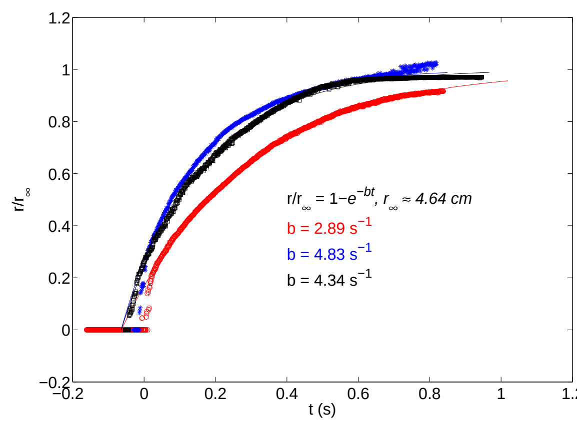
(a) Radius of Influence for 1 mm



(b) Radius of Influence for 2 mm



(c) Radius of Influence for 3 mm



(d) Radius of Influence for 4 mm

Figure 1: Radii of Influence for different diameters of c-boat

Effectively, one can think of sublimation as a process decreasing the surface area at a rate, $kA_{\text{eff}}(t)$ where k is the sublimation rate and $A_{\text{eff}}(t)$ area. Now one can write the rate equation for the effective area as,

$$\frac{\mathrm{d}}{\mathrm{dt}}A_{\text{eff}}(t) = \frac{\mathrm{d}}{\mathrm{dt}}A(t) - kA_{\text{eff}}(t)$$
$$\frac{\mathrm{d}}{\mathrm{dt}}\left[\mathrm{e}^{kt}A_{\text{eff}}(t)\right] = \frac{3\pi}{2}\alpha^2\ t^{\frac{1}{2}}\mathrm{e}^{kt}$$

Integrating we get,

$$A_{\text{eff}}(t) = \mathrm{e}^{-kt}\left[\pi\alpha^2\int t^{\frac{1}{2}}\mathrm{e}^{kt}\mathrm{dt} + \text{const.}\right]$$

Integrating we get,

$$A_{\text{eff}}(t) = \frac{\pi\alpha^2}{k}\left[t^{\frac{1}{2}} - \frac{1}{2kt^{\frac{1}{2}}}\left(1 + \frac{1}{2kt} + \frac{1\cdot 3}{(2kt)^2} + \frac{1\cdot 3\cdot 5}{(2kt)^3} + \cdots\right)\right] + \text{const}\cdot \mathrm{e}^{-kt} \tag{2}$$