Earthworks
Calculating earth volumes for terrain levelling

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Abstract

Terrain levelling analysis is a key element of cost estimation on construction projects at irregular ground areas. It determines the amount of earth that needs to be added (fill volume) and removed (cut volume) in order to bring all of the construction area to the same reference level (plateau). This study demonstrates how to obtain these volumes based on a data set of GPS points collected at a target construction site. The idea consists in building a triangular mesh from the points, determining the plane equation of each set of 3 points and applying a double integral to obtain the corresponding volume beneath it. The final volume is given by a simple summation. This operation is carried out for different reference levels of the surveyed terrain in order to obtain the optimal solution: the one at which the amount of earth that needs to be added equals the amount that needs to be removed. This solution is displayed in graphical form.

TAGS: #linearAlgebra #construction #calculus #python #terrainSurvey #topography

Assumptions

This document assumes the reader has previous knowledge of basic linear algebra (plane and line equations) and integral calculus.
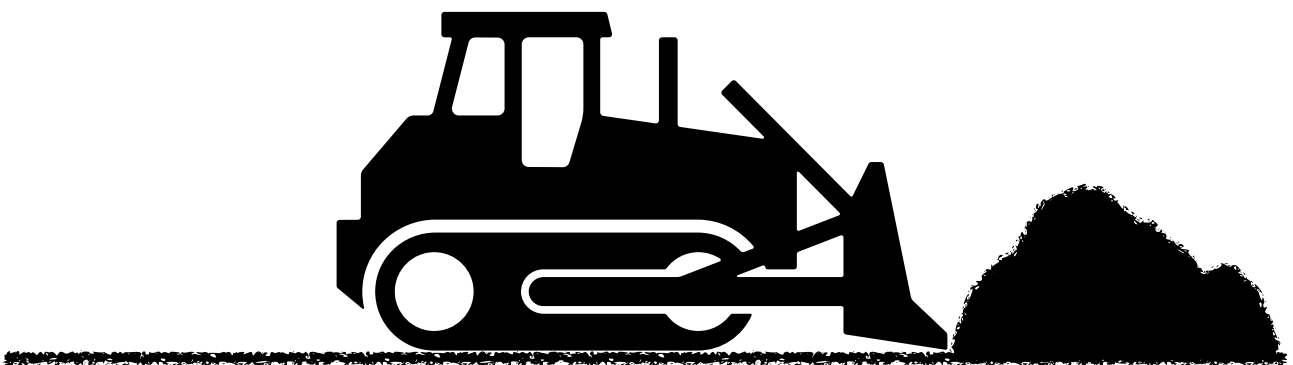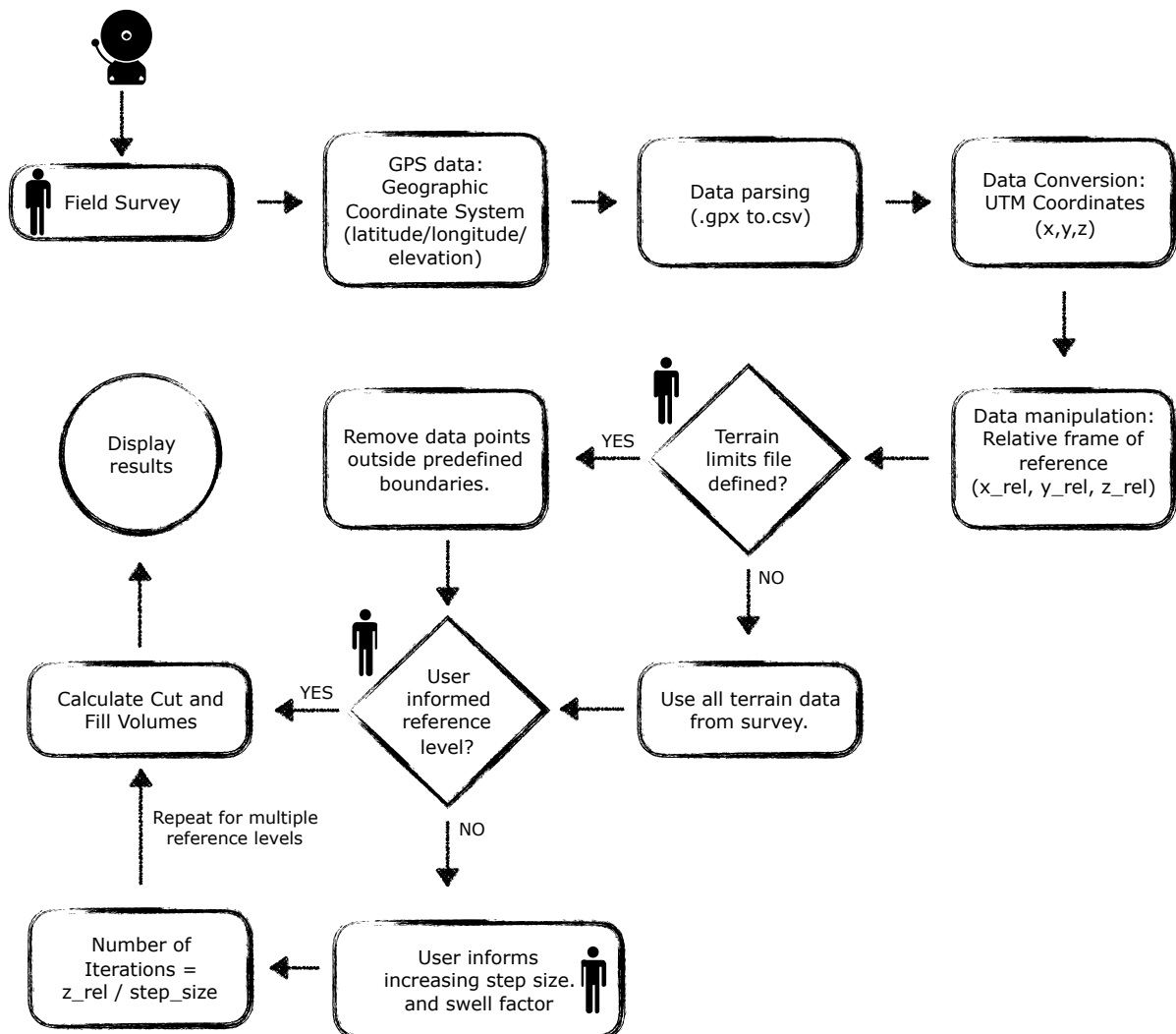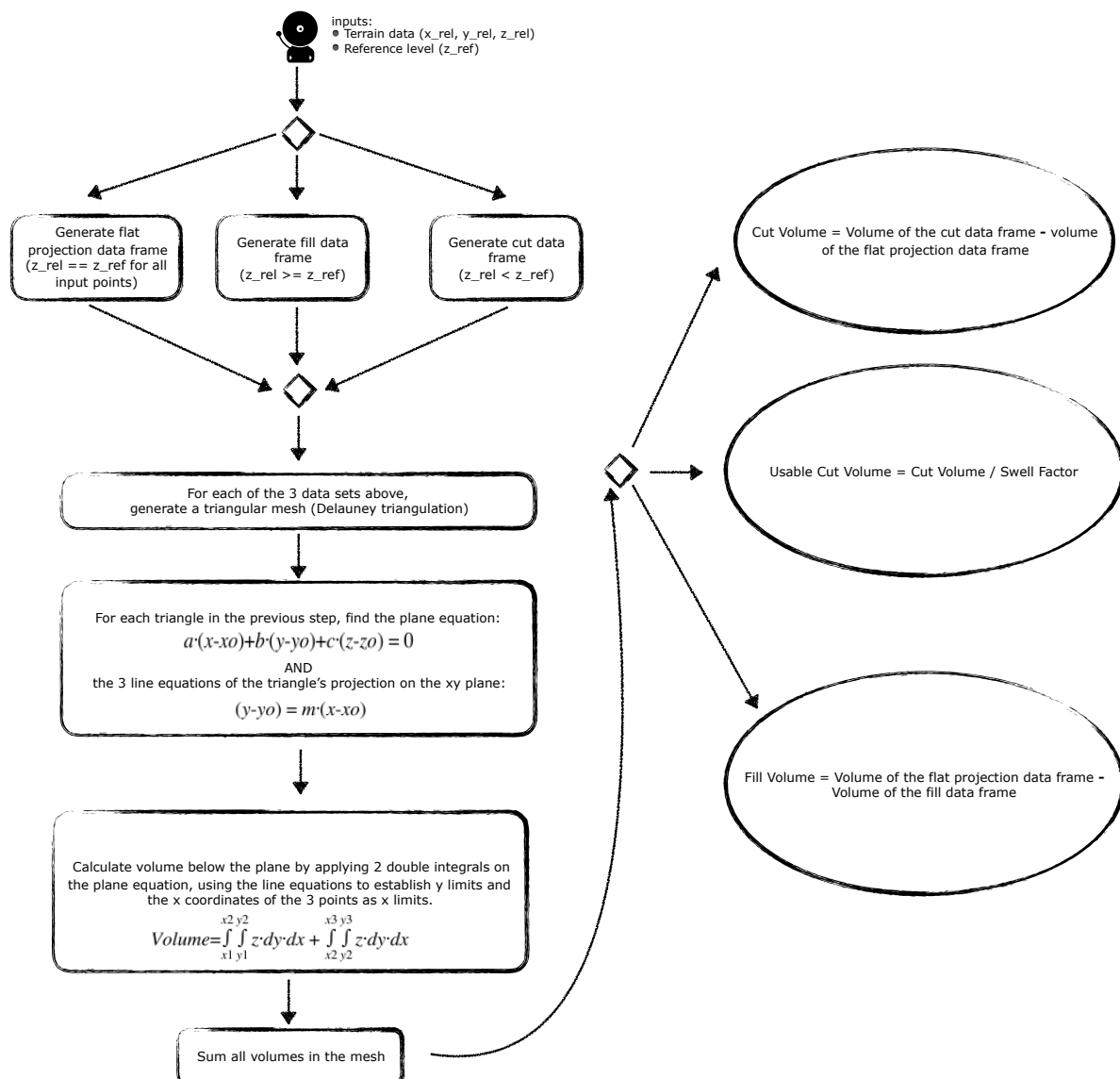
Motivation

This work has started when I observed volume calculations for terrain levelling being carried out with a contour lines map and an iterative manual summation process for obtaining the cut and fill volumes for a fixed reference level. This project's main goal is to automate this process and also provide a graph that displays the effect that different reference elevation selections have in cut and fill volumes.

Block Diagrams

Illustration 1: Program workflow

Illustration 2: Algorithm for volume calculation

inputs:
- Terrain data (x_rel, y_rel, z_rel)
- Reference level (z_ref)

Generate flat projection data frame (z_rel == z_ref for all input points)

Generate fill data frame (z_rel >= z_ref)

Generate cut data frame (z_rel < z_ref)

For each of the 3 data sets above, generate a triangular mesh (Delauney triangulation)

For each triangle in the previous step, find the plane equation:

$$a \cdot (x-xo) + b \cdot (y-yo) + c \cdot (z-zo) = 0$$

AND

the 3 line equations of the triangle's projection on the xy plane:

$$(y-yo) = m \cdot (x-xo)$$

Calculate volume below the plane by applying 2 double integrals on the plane equation, using the line equations to establish y limits and the x coordinates of the 3 points as x limits.

$$Volume = \int_{x1}^{x2} \int_{y1}^{y2} z \cdot dy \cdot dx + \int_{x2}^{x3} \int_{y2}^{y3} z \cdot dy \cdot dx$$

Sum all volumes in the mesh

Cut Volume = Volume of the cut data frame - volume of the flat projection data frame

Usable Cut Volume = Cut Volume / Swell Factor

Fill Volume = Volume of the flat projection data frame - Volume of the fill data frame

Equipment and Software

The following were used during this study:

- GPS unit: Garmin Montana 680.

- Apple 13" Macbook Pro: 8GB RAM, 2,6GHz Intel Core i5 (Late 2013 model).

- Python version 3.6.1 making use of the following open source libraries: pandas, math, numpy, scipy, utm, plotly and bokeh.

This study should be replicable by anyone with access to a GPS and a computer running Python. results may differ mainly due to GPS precision and amount of points collected from the surveyed area.

Data collection and preliminary data analysis.

The GPS survey data used in this study (452 coordinate points) was obtained on Oct 27th 2017, at the vicinity of base coordinate 24°45'28.7"S 50°02'52.7"W, by

driving around the target area with the GPS device in a fixed position in track mode. The area in question was a potential construction site for a new electrical substation.

A preliminary data output showed a terrain elevation difference of 28.72 meters. By observing a 3D scatter plot of the data, it was easy to identify an outlier, which was attributed to a temporary malfunction of the GPS device, so this point was discarded for the definitive analysis. The new terrain elevation difference after the outlier's removal was 12.36 meters, varying from 1,014.44 to 1,026.80 meters above sea level.
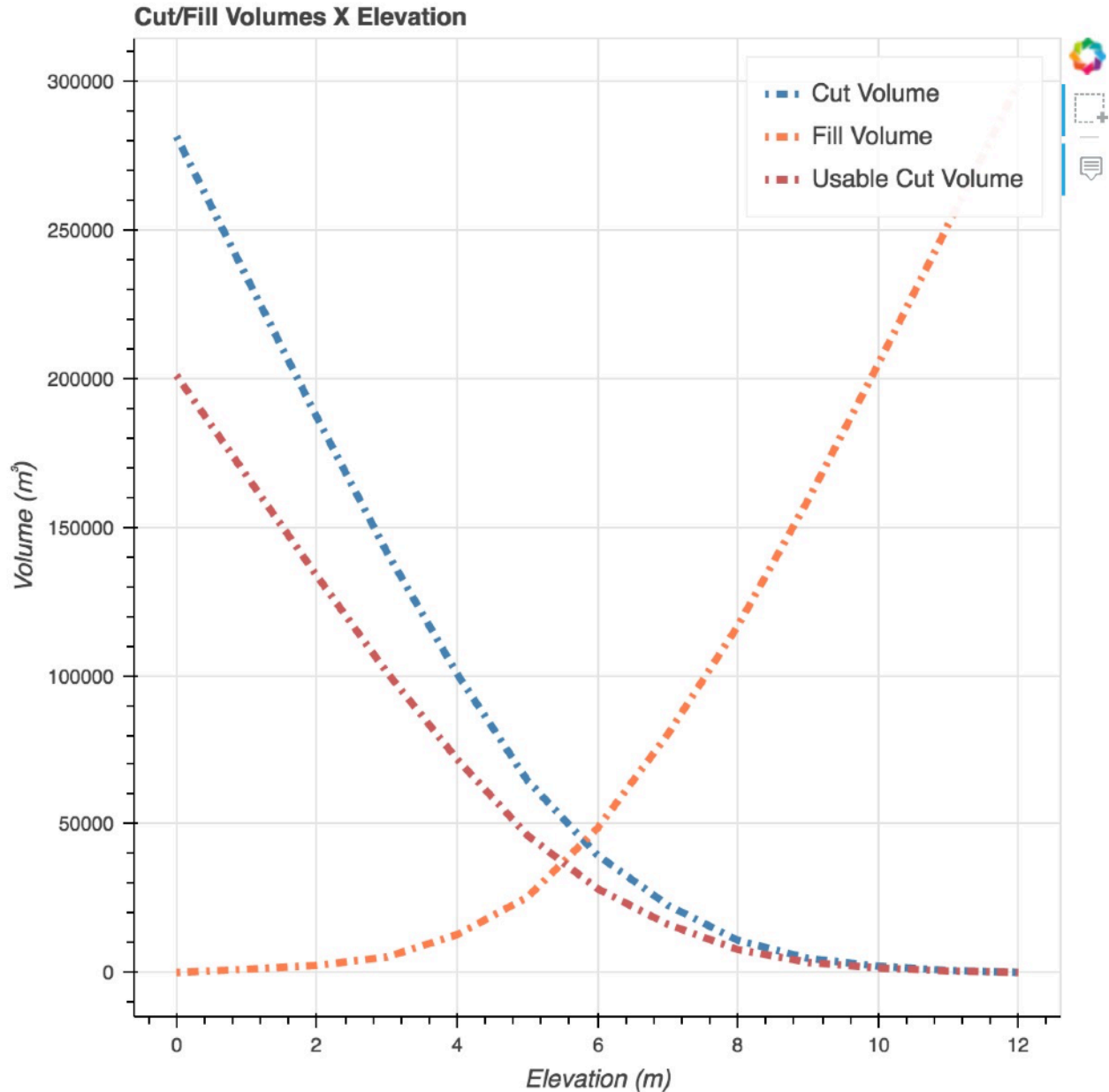
Illustration 3: 3D scatter plot of the survey data
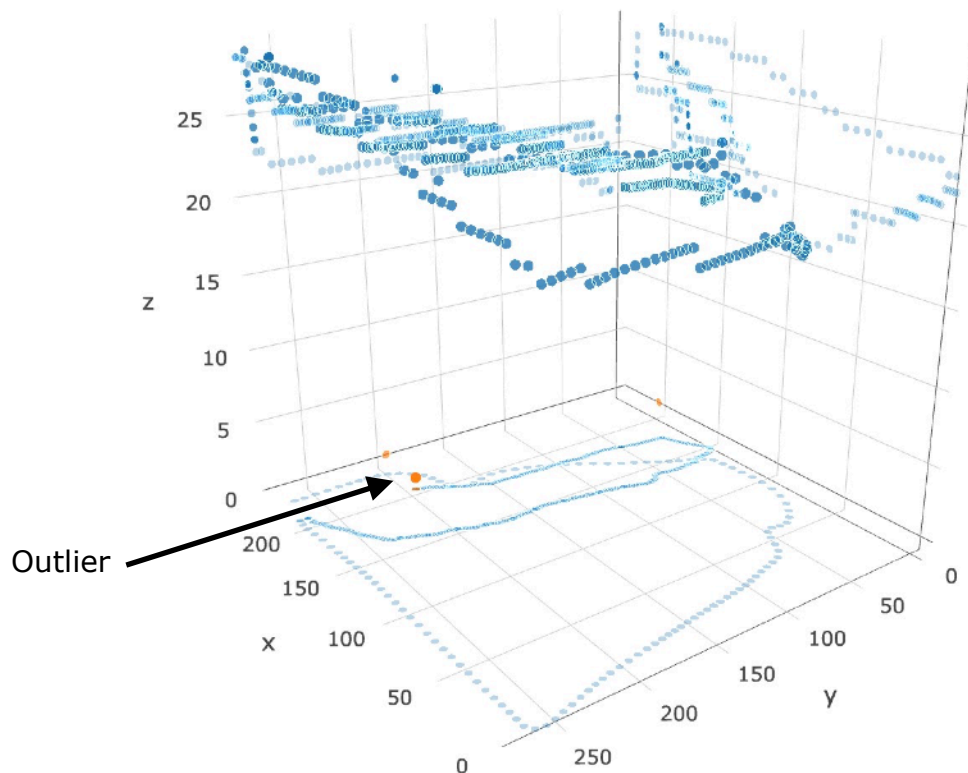
Volume calculation

With the implementation of the algorithm displayed on illustration 2, it was possible to obtain the Cut Vs. Fill volume for varying terrain reference elevation (volume curves) which is displayed in the graph below.

Illustration 4: Volume curves

At the intersection points between the blue and orange curves of the above graph, the cut and fill volumes are equal, which in theory would be the optimal scenario as it would not require any material to be moved from/to outside of the construction area. That elevation would be approximately 5.8m and would require the movement of about 44,640m^3 of material (cut and fill volumes).

Now accounting for the fact that the material cut from the construction site itself cannot be 100% used for filling a volume of equal amount due to its swelling factor, it is possible to find how much of the cut volume is actually usable for filling.

For the sake of this example let us consider to be the one of clay, which is about 1.4. This means that in an area covered by clay, 1.4m^3 of the terrain cut can be used to fill a volume of 1m^3. By applying that factor to the calculated cut volume, we obtain the usable cut volume (red curve), which translates to the amount of volume that can be filled using the corresponding cut volume. Finally we obtain the optimal terrain elevation to be at 5.5m resulting in a cut volume of 52,780m^3 and a fill volume of 37,130m^3.

Conclusion and further development

As this study shows that an acceptable level of precision can be achieved with a simple GPS unit and a bit of math. So the next step to make it more useable is to create an mobile app that can survey a terrain using the device's internal GPS and its CPU/GPU to perform all required calculations to obtain the graphics and visualize data on the go.

Calculated volume values are within the same order of magnitude of other known conventional methods of calculation, which indicates the program is working as expected. To further validate this, a point cloud of regular geometry having a previously known volume was generated and submitted to the program for volume evaluation.

A future version of this application can also be enhanced to take a drop factor into account. A drop factor of 1.5%, for example, means that if you walk the terrain for 100 meters, the difference in elevation will be of 1.5 meters. That is regular practice on construction sites in order to build a natural drainage system so that there's no accumulation of puddles of water from the rain and should also result in smaller volumes of materials being moved.

References

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