If f is a-periodic

$$f(y) = \sum_{k\in\mathbb{Z}} c_k(f)\, e^{+2i\pi \frac{k}{a} y} \qquad c_k(f) = \frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} f(y)\cdot e^{-i2\pi\frac{k}{a}y}\, dy.$$

We apply this result to $g_x(\xi) = e^{+2i\pi\xi x}$ $\quad \xi \in \left]-\frac{1}{2T}, +\frac{1}{2T}\right[$;

Then, $\quad g_x(\xi) = \sum_{k\in\mathbb{Z}} c_k(g_x)\, e^{+i2\pi\xi kT}$ $\quad$ (here, $a \leftarrow \frac{1}{T}$, $y \leftarrow \xi$)

and

$$c_k(g_x) = T\int_{-\frac{1}{2T}}^{+\frac{1}{2T}} g_x(\xi)\, e^{-i2\pi\xi kT}\, d\xi$$

$$= T\int_{-\frac{1}{2T}}^{+\frac{1}{2T}} e^{+i2\pi\xi x}\, e^{-i2\pi\xi kT}\, d\xi$$

$$= \frac{1}{i2\pi(x-kT)}\left[e^{+i2\pi\xi(x-kT)}\right]_{-\frac{1}{2T}}^{+\frac{1}{2T}} = \frac{\sin\left(\frac{\pi}{T}(x-kT)\right)}{\frac{\pi}{T}(x-kT)}$$