

Direct proof:

$$x_3[m] = \frac{1}{N} \sum_{k=0}^{N-1} x_1[k] x_2[k] W_N^{-km}$$

Circular shift:

$$x_2[((n-m))_N] \xleftrightarrow{\text{DFT}} x_2[k] W_N^{-km}$$

Hence:

$$x_2[k] W_N^{-km} = \sum_{n=0}^{N-1} x_2[((n-m))_N] W_N^{kn} \quad (\text{Analysis formula})$$

Substitute:

$$\begin{aligned} x_3[m] &= \frac{1}{N} \sum_{k=0}^{N-1} x_1[k] \left(\sum_{n=0}^{N-1} x_2[((n-m))_N] W_N^{kn} \right) \\ &= \sum_{n=0}^{N-1} x_2[((n-m))_N] \frac{1}{N} \sum_{k=0}^{N-1} x_1[k] W_N^{kn} \\ &= \sum_{n=0}^{N-1} x_1(n) x_2[((n-m))_N]. \end{aligned}$$