We associate to $x[n]$ the periodic sequence $\tilde{x}[n] = x[((n))_N]$.

On a :

$$x[n] \overset{DFT}{\longleftrightarrow} X[k]$$

$$\tilde{x}[n] \overset{DFS}{\longleftrightarrow} \tilde{X}[k] \quad \text{où} \quad \tilde{X}[k] = X[((k))_N].$$

We know that : $\tilde{x}[n-m] \overset{DFS}{\longleftrightarrow} W_N^{mk} \tilde{X}[k]$.

Since, for $k \in [\![0, N-1]\!]$ $W_N^{mk} X[k] = W_N^{mk} \tilde{X}[k]$

$$\tilde{x}[((n-m))_N] \overset{DFT}{\longleftrightarrow} W_N^{mk} X[k].$$

Direct proof

$$\frac{1}{N} \sum_{k=0}^{N-1} W_N^{mk} X[k] W_N^{-kn} = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{k(m-n)}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{k((m-n))_N}$$

synthesis formula.

$$= x[((m-n))_N]$$