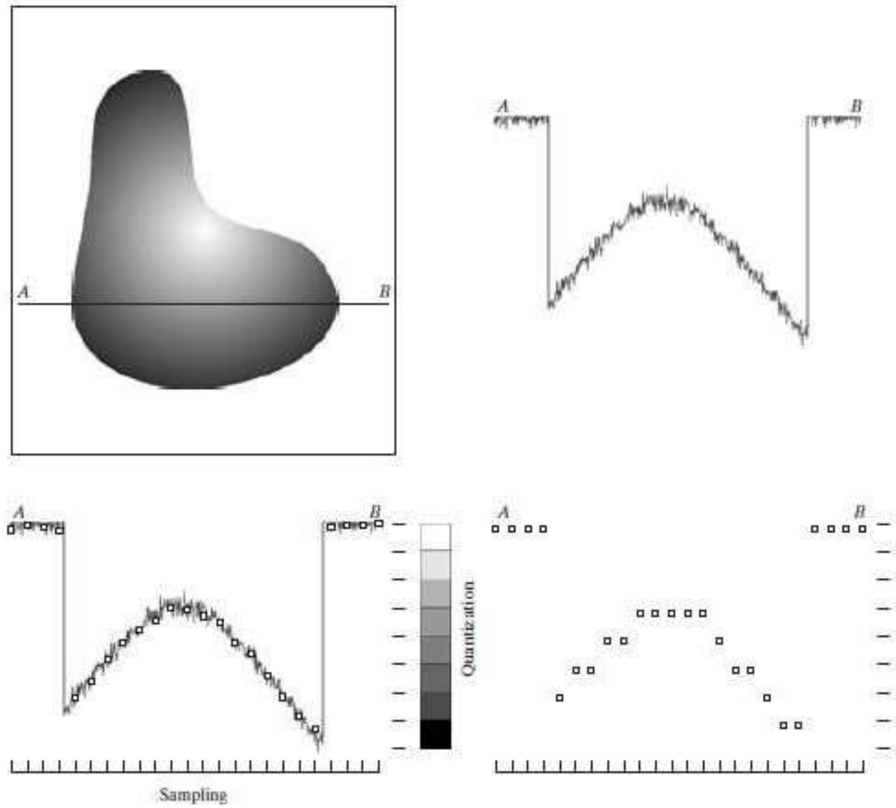
Digital Image Representations

Image Sampling and Quantization

- The output of most sensors is a continuous voltage waveform whose amplitude and spatial behavior are related to the physical phenomenon being sensed
- To create a digital image, we need to convert the continuous sensed data into digital form
- This involves two processes: *sampling* and *quantization*

Sampling and Quantization

- An image may be continuous with respect to the x- and y- coordinates, and also in amplitude
- To convert it to digital form, we have to sample the function in both coordinates and in amplitude
- Digitizing the coordinate values is called *sampling*
- Digitizing the amplitude values is called *quantization*



a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

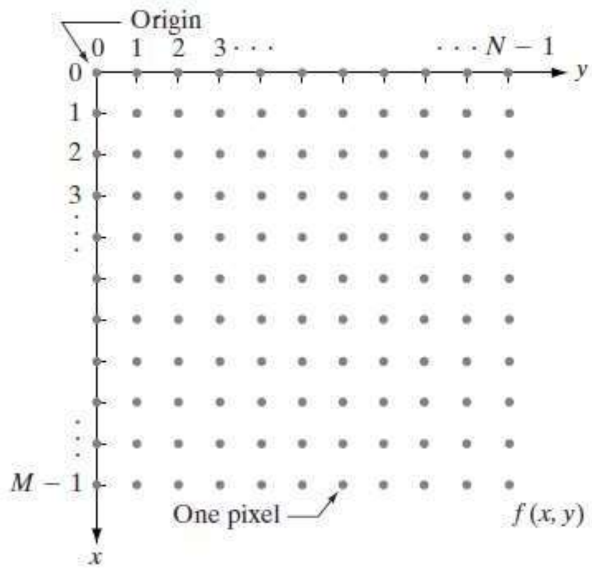
- The one-dimensional function shown in Fig. 2.16(b) is a plot of amplitude (gray level) values of the continuous image along the line segment AB in Fig. 2.16(a)
- The random variations are due to image noise
- To sample this function, we take equally spaced samples along line AB, as shown in Fig. 2.16(c)
- The location of each sample is given by a vertical tick mark in the bottom part of the figure
- The samples are shown as small white squares superimposed on the function.
- The set of these discrete locations gives the sampled function. However, the values of the samples still span (vertically) a continuous range of gray-level values

- In order to form a digital function, the gray-level values also must be converted (*quantized*) into discrete quantities
- The right side of Fig. 2.16(c) shows the gray-level scale divided into eight discrete levels, ranging from black to white
- The vertical tick marks indicate the specific value assigned to each of the eight gray levels
- The continuous gray levels are quantized simply by assigning one of the eight discrete gray levels to each sample
- The assignment is made depending on the vertical proximity of a sample to a vertical tick mark
- The digital samples resulting from both sampling and quantization are shown in Fig. 2.16(d)
- Starting at the top of the image and carrying out this procedure line by line produces a two-dimensional digital image

Representing digital images

- The result of sampling and quantization is a matrix of real numbers
- There are two different methods to represent digital images
- Assume that an image f(x, y) is sampled so that the resulting digital image has M rows and N columns
- The values of the coordinates (x, y) now become *discrete* quantities
- For notational clarity and convenience, we shall use integer values for these discrete coordinates

- Thus, the values of the coordinates at the origin are (x, y)=(0, 0)
- The next coordinate values along the first row of the image are represented as (x, y)=(0, 1)
- It is important to keep in mind that the notation (0, 1) is used to signify the second sample along the first row
- It does *not* mean that these are the actual values of physical coordinates when the image was sampled



- The complete M*N digital image can be written in the following compact matrix form

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}.$$

- The right side of this equation is by definition a digital image
- Each element of this matrix array is called an *image element*, *picture element*, *pixel*, or *pel*

- This digitization process requires decisions about values for M, N, and L, the number of discrete gray levels allowed for each pixel
- There are no requirements on *M* and *N*, other than that they have to be positive integers
- The number of gray levels typically is an integer power of 2, ie $L = 2^k$
- We assume that the discrete levels are equally spaced and that they are integers in the interval [0, L-1].
- The range of values spanned by the gray scale is called the *dynamic range* of an image
- An image whose gray levels span a significant portion of the gray scale as having a high dynamic range
- When an appreciable number of pixels exhibit this property, the image will have high contrast
- Conversely, an image with low dynamic range tends to have a dull, washed out gray look

- Given M,N and k ($k = \log_2 L$) , the number of bits required to store a digitized image is

  $$b=M*N*k$$

  - When M=N, this equation becomes

  $$b = N * N * k = N^2k$$

- When an image have 2^k gray levels, it is referred to as a “k-bit image”
- For example, an image with 256 possible gray-level values ie L = 256, then k = 8 and the image is called an 8-bit image

Relationship between pixels

- Here an image is denoted by $f(x, y)$ and pixel are represented using lowercase letters, such as p and q

1. Neighbors of a pixel

- A pixel p at coordinates (x, y) has four *horizontal* and *vertical* neighbors whose coordinates are given by

$$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$

- This set of pixels, called the 4-*neighbors* of p, is denoted by $N_4(p)$
- Each pixel is a unit distance from (x, y), and some of the neighbors of p lie outside the digital image if (x, y) is on the border of the image
- The four *diagonal* neighbors of p have coordinates (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

  and are denoted by $N_D(p)$
- These points, together with the 4-neighbors, are called the 8-*neighbors* of p, denoted by $N_8(p)$
- As before, some of the points in $N_D(p)$ and $N_8(p)$ fall outside the image if (x, y) is on the border of the image

Adjacency, Connectivity, Regions, and Boundaries

2. Connectivity

- To establish if two pixels are connected, it must be determined if they are neighbors and if their gray levels satisfy a specified criterion of similarity (say, if their gray levels are equal)
- For instance, in a binary image with values 0 and 1, two pixels may be 4-neighbors, but they are said to be connected only if they have the same value.

Two pixels are said to be connected if and only if

1. They are neighbors
2. Their gray levels must be equal

Adjacency of pixels

- Let *V* be the set of gray-level values used to define adjacency
- In a binary image,V={1} if we are referring to adjacency of pixels with value 1
- In a grayscale image, the idea is the same, but set *V* typically contains more elements
- For example, in the adjacency of pixels with a range of possible gray- level values 0 to 255, set *V* could be any subset of these 256 values
- We consider three types of adjacency:

(a) 4-*adjacency*

Two pixels p and q with values from *V* are 4-adjacent if q is in the set *N*4(p)

Conditions :

1. Value of p and q are in {V}
2. q is a 4-connected neighbor of p

(b) 8-*adjacency*

Two pixels p and q with values from *V* are 8-adjacent if q is in the set *N*8(p)

Conditions :

1. Value of p and q are in {V}
2. q is an 8-connected neighbor of p

(c) m-adjacency (mixed adjacency)

Two pixels p and q with values from *V* are m-adjacent if

(i) q is in *N*4(p), *or*

(ii) q is in *N*D(p) *and* the set has no pixels whose values are from *V*

- Mixed adjacency is a modification of 8-adjacency
- It is introduced to eliminate the ambiguities that often arise when 8- adjacency is used

Examples

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 54 & 10 & 100 & 5 \\ 81 & 150 & 2 & 34 \\ 20 & 200 & 3 & 45 \\ 71 & 70 & 147 & 56 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- Two image subsets *S*1 and *S*2 are adjacent if some pixel in *S*1 is adjacent to some pixel in *S*2
- A (*digital*) *path* (or *curve*) from pixel p with coordinates (x, y) to pixel q with coordinates (s, t) is a sequence of distinct pixels with coordinates (x0,y0),(x1,y1)……(xn,yn)

where (x0,y0) = (x,y) and (xn,yn) = (s,t) and pixels (xi,yi) and (xi-1, y i-1) are adjacent for 1<=i<=n

- In this case, n is the *length* of the path
- If (x0,y0) = (xn,yn), the path is a *closed* path
- We can define 4-, 8-, or m-paths depending on the type of adjacency specified

- Let *S* represent a subset of pixels in an image
- Two pixels p and q are said to be *connected* in *S* if there exists a path between them consisting entirely of pixels in *S*
- For any pixel p in *S*, the *set* of pixels that are connected to it in *S* is called a *connected component* of *S*
- If it only has one connected component, then set *S* is called a *connected set*

- Let *R* be a subset of pixels in an image
- We call *R* a *region* of the image if *R* is a connected set
- The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*
- If *R* happens to be an entire image (which we recall is a rectangular set of pixels), then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

Distance measures

For pixels p, q, and z, with coordinates (x, y), (s, t), and (v, w), respectively, D is a *distance function* or *metric* if

(a) $D(p, q) \geq 0$ $\left(D(p, q) = 0 \text{ iff } p = q\right)$,
(b) $D(p, q) = D(q, p)$, and
(c) $D(p, z) \leq D(p, q) + D(q, z)$.

The *Euclidean distance* between p and q is defined as

$$D_e(p, q) = \left[(x - s)^2 + (y - t)^2\right]^{\frac{1}{2}}. \tag{2.5-1}$$

For this distance measure, the pixels having a distance less than or equal to some value r from (x, y) are the points contained in a disk of radius r centered at (x, y).

The D_4 *distance* (also called *city-block distance*) between p and q is defined as

$$D_4(p, q) = |x - s| + |y - t|. \tag{2.5-2}$$

In this case, the pixels having a D_4 distance from (x, y) less than or equal to some value r form a diamond centered at (x, y). For example, the pixels with D_4 distance ≤ 2 from (x, y) (the center point) form the following contours of constant distance:

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

The pixels with $D_4 = 1$ are the 4-neighbors of (x, y).

The D_8 *distance* (also called *chessboard distance*) between p and q is defined as

$$D_8(p, q) = \max(|x - s|, |y - t|). \tag{2.5-3}$$

In this case, the pixels with D_8 distance from (x, y) less than or equal to some value r form a square centered at (x, y). For example, the pixels with D_8 distance ≤ 2 from (x, y) (the center point) form the following contours of constant distance:

$$\begin{matrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$

The pixels with $D_8 = 1$ are the 8-neighbors of (x, y).

- Note that the *D*4 and *D*8 distances between p and q are independent of any paths that might exist between the points because these distances involve only the coordinates of the points
- If we elect to consider m-adjacency, however, the *D*m distance between two points is defined as the shortest m-path between the points
- In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors
- For instance, consider the following arrangement of pixels and assume that p, p2 , and p4 have value 1 and that p1 and p3 can have a value of 0 or 1

$$\begin{array}{ccc} & p_3 & p_4 \\ p_1 & p_2 & \\ p & & \end{array}$$

- Suppose that we consider adjacency of pixels valued 1 (i.e.,V={1})
- If p1 and p3 are 0, the length of the shortest m-path (the *D*m distance) between p and p4 is 2
- If p1 is 1, then p2 and p will no longer be m-adjacent (see the definition of m-adjacency) and the length of the shortest m-path becomes 3 (the path goes through the points)
- Similar comments apply if p3 is 1 (and p1 is 0); in this case, the length of the shortest m-path also is 3
- Finally, if both p1 and p3 are 1 the length of the shortest m-path between p and p4 is 4
- In this case, the path goes through the sequence of points p p1 p2 p3 p4