Supplementary Materials for CEMENT: Incomplete Multi-View Weak-Label Learning with Long-Tailed Labels

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Appendix

Algorithm Design

The optimization procedure in solving the optimization problem in Eq. (7) is outlined in Algorithm 1.

Algorithm 1 The Algorithm of CEMENT

Output: $\{\hat{\mathbf{X}}^v\}_{v=1}^m, \hat{\mathbf{Y}}$

1: while not converged do

Updating $\{\mathbf{P}^v\}_{v=1}^m$ according to Eq. (9). Updating \mathbf{P}^* according to Eq. (11). 2:

3:

Updating $\{\mathbf{U}^v\}_{v=1}^m$ according to Eq. (13). 4:

Updating W according to Eq. (15). 5:

Updating E according to Eq. (17). 6:

7: Updating α according to Eq. (19).

Updating β according to Eq. (22).

9: end while

10: Let $\hat{\mathbf{X}}^v = \mathbf{P}^v \mathbf{U}^v$, $v = 1, \dots, m$, and $\hat{\mathbf{Y}} = \mathbf{P}^* \mathbf{W} + \mathbf{E}$

More Details of Ablation Study

In this subsection, we provide the formulations of CEMENT-1, CEMENT-2 and CEMENT-3. CEMENT-1 only learns shared information from all feature views, and ignores individual information, which is defined by:

$$\min_{\substack{\boldsymbol{\alpha}, \mathbf{P}, \mathbf{P}^*, \\ \mathbf{U}^v, \mathbf{W}, \mathbf{E}}} - HSIC(\mathbf{P}, \mathbf{P}^*) + \sum_{v=1}^m \alpha_v ||\mathbf{O}_{\mathbf{X}}^v \odot (\mathbf{X}^v - \mathbf{P}\mathbf{U}^v)||_F^2 \\
+ \alpha_{m+1} ||\mathbf{O}_{\mathbf{Y}} \odot (\mathbf{Y} - \mathbf{P}^*\mathbf{W} - \mathbf{E})||_F^2 + \lambda ||\mathbf{E}||_1 \\
s.t. \ \alpha_v, \alpha_{m+1}, \mathbf{P}, \mathbf{P}^*, \mathbf{W}, \mathbf{U}^v \ge 0, v = 1, 2, \dots, m,$$
(A1)

where $\mathbf{P} \in \mathbb{R}^{n \times k}$ is the shared subspace, $k < \min\{d_1, d_2, \dots, d_m\}$. CEMENT-2 assumes that the label matrix \mathbf{Y} is low-rank by ignoring the tail label matrix \mathbf{E} ,

which is defined as:

$$\min_{\substack{\alpha,\beta,\mathbf{P}^{v},\mathbf{P}^{*},\\ \mathbf{U}^{v},\mathbf{W}}} - \sum_{v=1}^{m} \beta_{v} HSIC(\mathbf{P}^{v},\mathbf{P}^{*}) \\
+ \sum_{v=1}^{m} \alpha_{v} ||\mathbf{O}_{\mathbf{X}}^{v} \odot (\mathbf{X}^{v} - \mathbf{P}^{v} \mathbf{U}^{v})||_{F}^{2} \\
+ \alpha_{m+1} ||\mathbf{O}_{\mathbf{Y}} \odot (\mathbf{Y} - \mathbf{P}^{*} \mathbf{W})||_{F}^{2} \\
s.t. ||\beta||_{2} = 1, \alpha_{m+1}, \mathbf{P}^{*}, \mathbf{W} \ge 0, \\
\alpha_{v}, \beta_{v}, \mathbf{U}^{v}, \mathbf{P}^{v} > 0, v = 1, 2, \dots, m.$$
(A2)

CEMENT-3 only learns a shared subspace P^* among all views and labels, which does not need HSIC. It is defined

$$\min_{\substack{\boldsymbol{\alpha}, \mathbf{P}^*, \mathbf{U}^v \\ \mathbf{W}, \mathbf{E}}}, \sum_{v=1}^m \alpha_v || \mathbf{O}_{\mathbf{X}}^v \odot (\mathbf{X}^v - \mathbf{P}^* \mathbf{U}^v) ||_F^2 \\
+ \alpha_{m+1} || \mathbf{O}_{\mathbf{Y}} \odot (\mathbf{Y} - \mathbf{P}^* \mathbf{W} - \mathbf{E}) ||_F^2 + \lambda || \mathbf{E} ||_1$$

$$s.t. \ \alpha_v, \alpha_{m+1}, \mathbf{P}^*, \mathbf{W}, \mathbf{U}^v > 0, v = 1, 2, \dots, m.$$
(A3)

These three variants of CEMENT are introduced to investigate the effects of the components of CEMENT.

1.3 More Experimental Results

Settings	View-1	View-2	View-3	View-4
Rank	low	high	low	high
Related	у	у	n	n

Table A1: The descriptions of designed synthetic datasets. Here low/high means that the matrix is low/high-rank, and y/n means that the subspace of this view is related/unrelated to the subspace produced by the label matrix.

	View-1	View-2	View-3	View-4	Label
α	0.0541	0.0285	0.2165	0.0025	0.0389
β	0.6241	0.6436	0.2724	0.3495	\

Table A2: The learned values of α and β on the designed synthetic dataset shown in Table A1.

Experiments On The Synthetic Dataset. This part aims to explore the effects of α and β . To this end, we carefully design a synthetic dataset with 4 views, each of which has 60, 60, 50, and 50 features. The dataset contains 100 samples. As shown in Table A1, the matrices of View-1 and View-3 are low-rank, while the matrix of View-2 and View-4 are high-rank. Meanwhile, View-1 and View-2 are related to the label matrix, while View-2 and View-4 are unrelated. As shown in Table A2, α assigns larger values to α_1 (View-1), α_3 (View-3) and α_5 (Label) than α_2 (View-2) and α_4 (View-4). It is because the feature matrices View-1, View-3 and the label matrix Label are successfully recovered due to the low-rankness. β assigns larger values to View-1 and View-2, which subspaces that correlated to the subspace produced by the label matrix, than the others. Thus, we can conclude that the variables α and β has the abilities to release the effects caused by the reconstruction errors and measure the importance of each view, respectively.

Parameter And Convergence Analysis. Fig. A1 analyzes the sensitivity of CEMENT w.r.t. λ and k on the five image datasets and the Emotions dataset. We plot the convergence curve of the optimization algorithm on the five image datasets, as shown in Fig. A2. We observe that the objective value monotonically decreases as the number of iterations increases, and it converges within 200 iterations on these datasets.

Experiments On The Real Datasets. To show the effectiveness of our method CEMENT, we conduct experiments under different combinations of r% and s%. We first set r% = 10%, 20%, 30%, and 40% to evaluate the performances under incomplete views with 50% missing labels. The results are shown in Table A3, Table A4, Table A5, Table A6, respectively. Then we conduct the performance under complete views with 50% missing labels, and the results are shown in Table A7. It can be seen that our method CEMENT obtains the best performances on most of the cases.

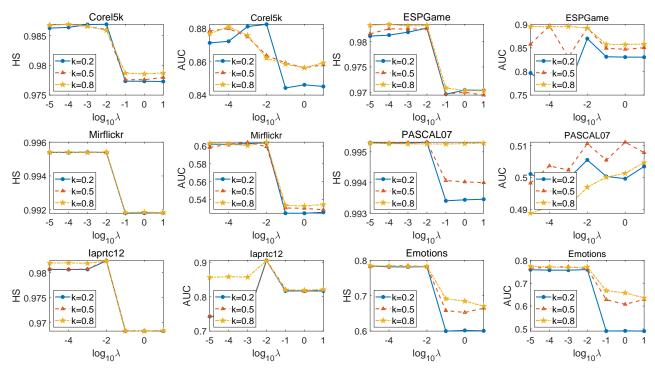


Figure A1: Hyperparameter sensitivity analysis of CEMENT under different combinations of λ and k on the five image datasets and the Emotions dataset.

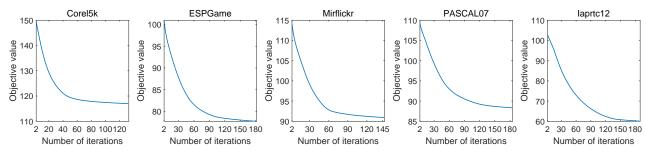


Figure A2: Convergence analysis of CEMENT on the five image datasets.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM ³ L	CEMENT
Corel5k	HS RS AUC	$\begin{array}{c} 0.9273 \pm 0.0030 \\ 0.8022 \pm 0.0022 \\ 0.8025 \pm 0.0022 \end{array}$	$\begin{array}{c} 0.9788 \pm 0.0001 \\ 0.8917 \pm 0.0031 \\ 0.8919 \pm 0.0029 \end{array}$	0.9789 ± 0.0003 0.9056 ± 0.0027 0.9056 ± 0.0027	$\begin{array}{c} 0.9867 \pm 0.0000 \\ 0.8775 \pm 0.0020 \\ 0.8775 \pm 0.0020 \end{array}$	$\begin{array}{c} 0.9950 \pm 0.0001 \\ 0.9851 \pm 0.0010 \\ 0.9823 \pm 0.0010 \end{array}$
ESPGame	HS RS AUC	$\begin{array}{c} 0.9710 \pm 0.0015 \\ 0.8041 \pm 0.0005 \\ 0.8041 \pm 0.0005 \end{array}$	$\begin{array}{c} 0.9708 \pm 0.0001 \\ 0.8843 \pm 0.0018 \\ 0.8843 \pm 0.0018 \end{array}$	0.9679 ± 0.0002 0.8459 ± 0.0029 0.8459 ± 0.0029	$\begin{array}{c} 0.9804 \pm 0.0000 \\ 0.8526 \pm 0.0010 \\ 0.8526 \pm 0.0010 \end{array}$	$\begin{array}{c} 0.9918 \pm 0.0001 \\ 0.9780 \pm 0.0007 \\ 0.9780 \pm 0.0007 \end{array}$
Mirflickr	HS RS AUC	$\begin{array}{c} 0.9787 \pm 0.0060 \\ 0.7292 \pm 0.0008 \\ 0.5065 \pm 0.0003 \end{array}$	$\begin{array}{c} 0.9873 \pm 0.0000 \\ 0.8090 \pm 0.0028 \\ 0.8091 \pm 0.0027 \end{array}$	0.9872 ± 0.0000 0.8384 ± 0.0044 0.8384 ± 0.0044	$\begin{array}{c} 0.9970 \pm 0.0001 \\ 0.5726 \pm 0.0104 \\ 0.3980 \pm 0.0076 \end{array}$	$\begin{array}{c} \textbf{0.9947} \pm \textbf{0.0000} \\ \textbf{0.9675} \pm \textbf{0.0025} \\ 0.7041 \pm 0.0018 \end{array}$
Pascal07	HS RS AUC	$\begin{array}{c} 0.9417 \pm 0.0013 \\ 0.6743 \pm 0.0016 \\ 0.6749 \pm 0.0016 \end{array}$	$\begin{array}{c} 0.9875 \pm 0.0000 \\ 0.8815 \pm 0.0059 \\ 0.8817 \pm 0.0059 \end{array}$	0.9875 ± 0.0001 0.8885 ± 0.0061 0.8885 ± 0.0061	$\begin{array}{c} 0.9954 \pm 0.0000 \\ 0.6862 \pm 0.0072 \\ 0.6862 \pm 0.0072 \end{array}$	$\begin{array}{c} \textbf{0.9963} \pm \textbf{0.0000} \\ \textbf{0.9651} \pm \textbf{0.0047} \\ 0.6058 \pm 0.0034 \end{array}$
IAPRTC-12	HS RS AUC	$\begin{array}{c} 0.8950 \pm 0.0024 \\ 0.7847 \pm 0.0034 \\ 0.7847 \pm 0.0034 \end{array}$	$\begin{array}{c} 0.9751 \pm 0.0002 \\ 0.8977 \pm 0.0010 \\ 0.8977 \pm 0.0010 \end{array}$	$\begin{array}{c} 0.9734 \pm 0.0001 \\ 0.8766 \pm 0.0019 \\ 0.8766 \pm 0.0019 \end{array}$	$\begin{array}{c} 0.9838 \pm 0.0000 \\ 0.8842 \pm 0.0009 \\ 0.8835 \pm 0.0010 \end{array}$	$\begin{array}{c} 0.9912 \pm 0.0002 \\ 0.9797 \pm 0.0015 \\ 0.9797 \pm 0.0015 \end{array}$
Yeast	HS RS AUC	$\begin{array}{c} 0.7054 \pm 0.0003 \\ 0.7838 \pm 0.0001 \\ 0.7831 \pm 0.0002 \end{array}$	$\begin{array}{c} 0.8296 \pm 0.0012 \\ 0.9124 \pm 0.0010 \\ 0.9124 \pm 0.0010 \end{array}$	$\begin{array}{c} 0.7416 \pm 0.0062 \\ 0.8001 \pm 0.0045 \\ 0.8001 \pm 0.0045 \end{array}$	$\begin{array}{c} 0.7059 \pm 0.0004 \\ 0.8133 \pm 0.0009 \\ 0.8126 \pm 0.0009 \end{array}$	$\begin{array}{c} 0.8806 \pm 0.0006 \\ 0.9786 \pm 0.0005 \\ 0.9786 \pm 0.0005 \end{array}$
Emotions	HS RS AUC	$\begin{array}{c} 0.6493 \pm 0.0008 \\ 0.6686 \pm 0.0030 \\ 0.6628 \pm 0.0029 \end{array}$	$\begin{array}{c} 0.6198 \pm 0.0013 \\ 0.6199 \pm 0.0079 \\ 0.6199 \pm 0.0079 \end{array}$	0.6819 ± 0.0139 0.7006 ± 0.0232 0.7006 ± 0.0232	$\begin{array}{c} 0.6972 \pm 0.0018 \\ 0.7281 \pm 0.0038 \\ 0.7211 \pm 0.0052 \end{array}$	$\begin{array}{c} 0.8994 \pm 0.0039 \\ 0.9821 \pm 0.0053 \\ 0.9821 \pm 0.0053 \end{array}$

Table A3: Experimental results on seven real-world datasets with r%=10%, and s%=10%. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	$NAIM^3L$	CEMENT
Corel5k	HS RS AUC	$\begin{array}{c} 0.8845 \pm 0.0010 \\ 0.7985 \pm 0.0017 \\ 0.7996 \pm 0.0016 \end{array}$	$\begin{array}{c} 0.9781 \pm 0.0002 \\ 0.8613 \pm 0.0042 \\ 0.8625 \pm 0.0039 \end{array}$	$\begin{array}{c} 0.9786 \pm 0.0003 \\ 0.8934 \pm 0.0055 \\ 0.8934 \pm 0.0055 \end{array}$	0.9868 ± 0.0000 0.8943 ± 0.0030 0.8943 ± 0.0030	$\begin{array}{c} 0.9929 \pm 0.0001 \\ 0.9695 \pm 0.0011 \\ 0.9668 \pm 0.0011 \end{array}$
ESPGame	HS RS AUC	$\begin{array}{c} 0.9732 \pm 0.0065 \\ 0.8018 \pm 0.0006 \\ 0.8018 \pm 0.0006 \end{array}$	$\begin{array}{c} 0.9704 \pm 0.0001 \\ 0.8668 \pm 0.0022 \\ 0.8668 \pm 0.0022 \end{array}$	0.9681 ± 0.0002 0.8444 ± 0.0030 0.8444 ± 0.0030	0.9808 ± 0.0001 0.8520 ± 0.0014 0.8520 ± 0.0014	$0.9899 \pm 0.0001 \ 0.9554 \pm 0.0018 \ 0.9554 \pm 0.0018$
Mirflickr	HS RS AUC	$\begin{array}{c} 0.9669 \pm 0.0029 \\ 0.7241 \pm 0.0020 \\ 0.5016 \pm 0.0016 \end{array}$	$\begin{array}{c} 0.9872 \pm 0.0001 \\ 0.7745 \pm 0.0041 \\ 0.7751 \pm 0.0041 \end{array}$	0.9872 ± 0.0000 0.8310 ± 0.0052 0.8310 ± 0.0052	$\begin{array}{c} \textbf{0.9968} \pm \textbf{0.0001} \\ 0.6079 \pm 0.0066 \\ 0.4212 \pm 0.0055 \end{array}$	0.9943 ± 0.0000 0.9335 ± 0.0024 0.6793 ± 0.0017
Pascal07	HS RS AUC	$\begin{array}{c} 0.8897 \pm 0.0009 \\ 0.6745 \pm 0.0008 \\ 0.6771 \pm 0.0006 \end{array}$	$\begin{array}{c} 0.9873 \pm 0.0001 \\ 0.8349 \pm 0.0034 \\ 0.8358 \pm 0.0033 \end{array}$	0.9874 ± 0.0001 0.8547 ± 0.0076 0.8547 ± 0.0076	$\begin{array}{c} 0.9951 \pm 0.0001 \\ 0.7165 \pm 0.0068 \\ 0.7165 \pm 0.0068 \end{array}$	$egin{array}{l} 0.9962 \pm 0.0000 \\ 0.9396 \pm 0.0056 \\ 0.5899 \pm 0.0038 \end{array}$
IAPRTC-12	HS RS AUC	$\begin{array}{c} 0.8685 \pm 0.0040 \\ 0.7828 \pm 0.0002 \\ 0.7808 \pm 0.0005 \end{array}$	$\begin{array}{c} 0.9746 \pm 0.0001 \\ 0.8823 \pm 0.0017 \\ 0.8824 \pm 0.0017 \end{array}$	0.9735 ± 0.0001 0.8757 ± 0.0020 0.8757 ± 0.0020	$\begin{array}{c} 0.9840 \pm 0.0001 \\ 0.8868 \pm 0.0019 \\ 0.8855 \pm 0.0025 \end{array}$	$\begin{array}{c} 0.9892 \pm 0.0001 \\ 0.9600 \pm 0.0022 \\ 0.9600 \pm 0.0022 \end{array}$
Yeast	HS RS AUC	$\begin{array}{c} 0.7141 \pm 0.0000 \\ 0.7821 \pm 0.0007 \\ 0.7803 \pm 0.0002 \end{array}$	$\begin{array}{c} 0.8245 \pm 0.0108 \\ 0.8969 \pm 0.0016 \\ 0.8969 \pm 0.0016 \end{array}$	$\begin{array}{c} 0.7635 \pm 0.0050 \\ 0.7941 \pm 0.0027 \\ 0.7941 \pm 0.0027 \end{array}$	$\begin{array}{c} 0.7141 \pm 0.0007 \\ 0.8118 \pm 0.0013 \\ 0.8087 \pm 0.0015 \end{array}$	$\begin{array}{c} 0.8654 \pm 0.0013 \\ 0.9592 \pm 0.0014 \\ 0.9592 \pm 0.0014 \end{array}$
Emotions	HS RS AUC	$\begin{array}{c} 0.7063 \pm 0.0001 \\ 0.6576 \pm 0.0008 \\ 0.6419 \pm 0.0023 \end{array}$	$\begin{array}{c} 0.6244 \pm 0.0048 \\ 0.6169 \pm 0.0064 \\ 0.6169 \pm 0.0064 \end{array}$	$\begin{array}{c} 0.6827 \pm 0.0195 \\ 0.7015 \pm 0.0290 \\ 0.7015 \pm 0.0290 \end{array}$	$\begin{array}{c} 0.7053 \pm 0.0020 \\ 0.7281 \pm 0.0051 \\ 0.7088 \pm 0.0073 \end{array}$	$\begin{array}{c} 0.8846 \pm 0.0123 \\ 0.9609 \pm 0.0140 \\ 0.9609 \pm 0.0140 \end{array}$

Table A4: Experimental results on seven real-world datasets with r%=20%, and s%=20%. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM ³ L	CEMENT
Corel5k	HS RS AUC	$\begin{array}{c} 0.9094 \pm 0.0090 \\ 0.7815 \pm 0.0004 \\ 0.7877 \pm 0.0000 \end{array}$	$\begin{array}{c} 0.9773 \pm 0.0002 \\ 0.8334 \pm 0.0049 \\ 0.8362 \pm 0.0045 \end{array}$	0.9783 ± 0.0003 0.8673 ± 0.0049 0.8673 ± 0.0049	0.9870 ± 0.0001 0.9103 ± 0.0057 0.9103 ± 0.0057	$\begin{array}{c} 0.9908 \pm 0.0002 \\ 0.9537 \pm 0.0024 \\ 0.9511 \pm 0.0024 \end{array}$
ESPGame	HS RS AUC	$\begin{array}{c} 0.9346 \pm 0.0081 \\ 0.7940 \pm 0.0010 \\ 0.7930 \pm 0.0007 \end{array}$	$\begin{array}{c} 0.9699 \pm 0.0001 \\ 0.8510 \pm 0.0025 \\ 0.8511 \pm 0.0025 \end{array}$	0.9710 ± 0.0002 0.8414 ± 0.0032 0.8414 ± 0.0032	$\begin{array}{c} 0.9812 \pm 0.0001 \\ 0.8577 \pm 0.0022 \\ 0.8572 \pm 0.0022 \end{array}$	$\begin{array}{c} 0.9878 \pm 0.0001 \\ 0.9315 \pm 0.0014 \\ 0.9315 \pm 0.0014 \end{array}$
Mirflickr	HS RS AUC	$\begin{array}{c} 0.9008 \pm 0.0026 \\ 0.7105 \pm 0.0014 \\ 0.6913 \pm 0.0003 \end{array}$	$\begin{array}{c} 0.9872 \pm 0.0000 \\ 0.7415 \pm 0.0041 \\ 0.7435 \pm 0.0038 \end{array}$	0.9874 ± 0.0001 0.8083 ± 0.0054 0.8083 ± 0.0054	$\begin{array}{c} \textbf{0.9967} \pm \textbf{0.0000} \\ 0.6372 \pm 0.0101 \\ 0.4407 \pm 0.0058 \end{array}$	0.9938 ± 0.0000 0.8982 ± 0.0038 0.6536 ± 0.0028
Pascal07	HS RS AUC	$\begin{array}{c} 0.9300 \pm 0.0062 \\ 0.6400 \pm 0.0029 \\ 0.6585 \pm 0.0016 \end{array}$	$\begin{array}{c} 0.9871 \pm 0.0001 \\ 0.7856 \pm 0.0088 \\ 0.7881 \pm 0.0087 \end{array}$	0.9875 ± 0.0001 0.8193 ± 0.0114 0.8193 ± 0.0114	0.9951 ± 0.0001 0.7230 ± 0.0065 0.7230 ± 0.0065	$egin{array}{l} 0.9959 \pm 0.0000 \\ 0.8843 \pm 0.0233 \\ 0.5677 \pm 0.0044 \end{array}$
IAPRTC-12	HS RS AUC	$\begin{array}{c} 0.9325 \pm 0.0098 \\ 0.7616 \pm 0.0060 \\ 0.7585 \pm 0.0053 \end{array}$	$\begin{array}{c} 0.9742 \pm 0.0001 \\ 0.8694 \pm 0.0030 \\ 0.8697 \pm 0.0029 \end{array}$	$\begin{array}{c} 0.9735 \pm 0.0001 \\ 0.8723 \pm 0.0021 \\ 0.8723 \pm 0.0021 \end{array}$	$\begin{array}{c} 0.9842 \pm 0.0001 \\ 0.8987 \pm 0.0015 \\ 0.8958 \pm 0.0020 \end{array}$	$egin{array}{l} 0.9868 \pm 0.0002 \ 0.9373 \pm 0.0026 \ 0.9373 \pm 0.0026 \ \end{array}$
Yeast	HS RS AUC	$\begin{array}{c} 0.7285 \pm 0.0003 \\ 0.7811 \pm 0.0002 \\ 0.7665 \pm 0.0008 \end{array}$	$\begin{array}{c} 0.8280 \pm 0.0014 \\ 0.8810 \pm 0.0017 \\ 0.8810 \pm 0.0017 \end{array}$	0.7514 ± 0.0056 0.7885 ± 0.0029 0.7885 ± 0.0029	0.7217 ± 0.0011 0.8092 ± 0.0016 0.8007 ± 0.0019	$\begin{array}{c} 0.8507 \pm 0.0014 \\ 0.9403 \pm 0.0015 \\ 0.9403 \pm 0.0015 \end{array}$
Emotions	HS RS AUC	$\begin{array}{c} 0.6197 \pm 0.0325 \\ 0.6596 \pm 0.0367 \\ 0.6302 \pm 0.0351 \end{array}$	$\begin{array}{c} 0.6213 \pm 0.0150 \\ 0.6183 \pm 0.0097 \\ 0.6183 \pm 0.0097 \end{array}$	$\begin{array}{c} 0.6784 \pm 0.0099 \\ 0.6851 \pm 0.0163 \\ 0.6851 \pm 0.0163 \end{array}$	$\begin{array}{c} 0.7118 \pm 0.0041 \\ 0.7315 \pm 0.0070 \\ 0.7012 \pm 0.0095 \end{array}$	$\begin{array}{c} 0.8568 \pm 0.0189 \\ 0.9272 \pm 0.0245 \\ 0.9272 \pm 0.0245 \end{array}$

Table A5: Experimental results on seven real-world datasets with r%=30%, and s%=30%. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM ³ L	CEMENT
Corel5k	HS RS AUC	$\begin{array}{c} 0.9094 \pm 0.0090 \\ 0.7815 \pm 0.0004 \\ 0.7877 \pm 0.0000 \end{array}$	0.9771 ± 0.0004 0.8084 ± 0.0073 0.8139 ± 0.0063	$\begin{array}{c} 0.9783 \pm 0.0002 \\ 0.8691 \pm 0.0048 \\ 0.8691 \pm 0.0041 \end{array}$	$\begin{array}{c} 0.9873 \pm 0.0002 \\ 0.9012 \pm 0.0080 \\ 0.9012 \pm 0.0080 \end{array}$	$\begin{array}{c} 0.9889 \pm 0.0001 \\ 0.9385 \pm 0.0009 \\ 0.9361 \pm 0.0009 \end{array}$
ESPGame	HS RS AUC	$\begin{array}{c} 0.9346 \pm 0.0081 \\ 0.7940 \pm 0.0010 \\ 0.7930 \pm 0.0007 \end{array}$	$\begin{array}{c} 0.9721 \pm 0.0002 \\ 0.8355 \pm 0.0022 \\ 0.8357 \pm 0.0021 \end{array}$	$\begin{array}{c} 0.9711 \pm 0.0001 \\ 0.8422 \pm 0.0041 \\ 0.8422 \pm 0.0041 \end{array}$	$\begin{array}{c} 0.9817 \pm 0.0001 \\ 0.8733 \pm 0.0023 \\ 0.8717 \pm 0.0024 \end{array}$	$\begin{array}{c} 0.9856 \pm 0.0001 \\ 0.9118 \pm 0.0021 \\ 0.9118 \pm 0.0021 \end{array}$
Mirflickr	HS RS AUC	$\begin{array}{c} 0.9008 \pm 0.0026 \\ 0.7105 \pm 0.0014 \\ 0.6913 \pm 0.0003 \end{array}$	$\begin{array}{c} 0.9871 \pm 0.0001 \\ 0.7193 \pm 0.0049 \\ 0.7244 \pm 0.0050 \end{array}$	$\begin{array}{c} 0.9874 \pm 0.0001 \\ 0.8091 \pm 0.0091 \\ \textbf{0.8091} \pm \textbf{0.0091} \end{array}$	$\begin{array}{c} \textbf{0.9967} \pm \textbf{0.0001} \\ 0.6436 \pm 0.0102 \\ 0.4439 \pm 0.0077 \end{array}$	0.9933 ± 0.0000 $\mathbf{0.8667 \pm 0.0021}$ 0.6311 ± 0.0015
Pascal07	HS RS AUC	$\begin{array}{c} 0.9300 \pm 0.0062 \\ 0.6400 \pm 0.0029 \\ 0.6585 \pm 0.0016 \end{array}$	$\begin{array}{c} 0.9869 \pm 0.0001 \\ 0.7182 \pm 0.0135 \\ 0.7235 \pm 0.0128 \end{array}$	$\begin{array}{c} 0.9875 \pm 0.0001 \\ 0.7955 \pm 0.0110 \\ \textbf{0.7955} \pm \textbf{0.0110} \end{array}$	$\begin{array}{c} 0.9952 \pm 0.0001 \\ 0.7094 \pm 0.0058 \\ 0.7094 \pm 0.0058 \end{array}$	$\begin{array}{c} \textbf{0.9957} \pm \textbf{0.0000} \\ \textbf{0.8712} \pm \textbf{0.0091} \\ 0.5486 \pm 0.0060 \end{array}$
IAPRTC-12	HS RS AUC	$\begin{array}{c} 0.9325 \pm 0.0098 \\ 0.7616 \pm 0.0060 \\ 0.7585 \pm 0.0053 \end{array}$	$\begin{array}{c} 0.9737 \pm 0.0001 \\ 0.8515 \pm 0.0043 \\ 0.8522 \pm 0.0041 \end{array}$	$\begin{array}{c} 0.9735 \pm 0.0002 \\ 0.8686 \pm 0.0036 \\ 0.8686 \pm 0.0036 \end{array}$	$\begin{array}{c} \textbf{0.9844} \pm \textbf{0.0001} \\ 0.9078 \pm 0.0018 \\ 0.9028 \pm 0.0033 \end{array}$	$0.9842 \pm 0.0001 \\ 0.9152 \pm 0.0026 \\ 0.9152 \pm 0.0026$
Yeast	HS RS AUC	$\begin{array}{c} 0.7285 \pm 0.0003 \\ 0.7811 \pm 0.0002 \\ 0.7665 \pm 0.0008 \end{array}$	$\begin{array}{c} 0.8163 \pm 0.0020 \\ 0.8645 \pm 0.0029 \\ 0.8645 \pm 0.0029 \end{array}$	$\begin{array}{c} 0.7488 \pm 0.0046 \\ 0.7846 \pm 0.0028 \\ 0.7846 \pm 0.0028 \end{array}$	$\begin{array}{c} 0.7294 \pm 0.0010 \\ 0.8071 \pm 0.0019 \\ 0.7911 \pm 0.0028 \end{array}$	$\begin{array}{c} 0.8370 \pm 0.0009 \\ 0.9230 \pm 0.0009 \\ 0.9230 \pm 0.0009 \end{array}$
Emotions	HS RS AUC	$\begin{array}{c} 0.6197 \pm 0.0325 \\ 0.6596 \pm 0.0367 \\ 0.6302 \pm 0.0351 \end{array}$	$\begin{array}{c} 0.6177 \pm 0.0182 \\ 0.6202 \pm 0.0136 \\ 0.6202 \pm 0.0136 \end{array}$	$\begin{array}{c} 0.6682 \pm 0.0099 \\ 0.6763 \pm 0.0143 \\ 0.6763 \pm 0.0143 \end{array}$	$\begin{array}{c} 0.7200 \pm 0.0017 \\ 0.7358 \pm 0.0132 \\ 0.6906 \pm 0.0106 \end{array}$	$\begin{array}{c} 0.8370 \pm 0.0213 \\ 0.9028 \pm 0.0333 \\ 0.9028 \pm 0.0333 \end{array}$

Table A6: Experimental results on seven real-world datasets with r%=40%, and s%=40%. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM ³ L	CEMENT
Corel5k	HS RS AUC	$\begin{array}{c} 0.9053 \pm 0.0060 \\ 0.7740 \pm 0.0000 \\ 0.7835 \pm 0.0006 \end{array}$	$\begin{array}{c} 0.9780 \pm 0.0002 \\ 0.8182 \pm 0.0050 \\ 0.8280 \pm 0.0038 \end{array}$	$\begin{array}{c} 0.9784 \pm 0.0003 \\ 0.8545 \pm 0.0068 \\ 0.8545 \pm 0.0068 \end{array}$	$egin{array}{l} 0.9873 \pm 0.0001 \\ 0.8729 \pm 0.0051 \\ 0.8729 \pm 0.0051 \end{array}$	0.9869 ± 0.0001 0.9213 ± 0.0022 0.9190 ± 0.0021
ESPGame	HS RS AUC	$\begin{array}{c} 0.9516 \pm 0.0041 \\ 0.8022 \pm 0.0001 \\ 0.7961 \pm 0.0009 \end{array}$	$\begin{array}{c} 0.9730 \pm 0.0002 \\ 0.8442 \pm 0.0024 \\ 0.8447 \pm 0.0025 \end{array}$	$\begin{array}{c} 0.9713 \pm 0.0001 \\ 0.8415 \pm 0.0027 \\ 0.8415 \pm 0.0027 \end{array}$	$\begin{array}{c} 0.9821 \pm 0.0001 \\ 0.8534 \pm 0.0022 \\ 0.8498 \pm 0.0034 \end{array}$	$\begin{array}{c} 0.9833 \pm 0.0002 \\ 0.8899 \pm 0.0020 \\ 0.8899 \pm 0.0020 \end{array}$
Mirflickr	HS RS AUC	$\begin{array}{c} 0.9013 \pm 0.0024 \\ 0.7089 \pm 0.0036 \\ 0.6889 \pm 0.0018 \end{array}$	$\begin{array}{c} 0.9873 \pm 0.0000 \\ 0.7167 \pm 0.0045 \\ 0.7268 \pm 0.0036 \end{array}$	0.9875 ± 0.0001 0.7956 ± 0.0080 0.7956 ± 0.0080	$\begin{array}{c} \textbf{0.9969} \pm \textbf{0.0000} \\ 0.6196 \pm 0.0056 \\ 0.4258 \pm 0.0045 \end{array}$	0.9928 ± 0.0001 0.8354 ± 0.0025 0.6087 ± 0.0020
Pascal07	HS RS AUC	$\begin{array}{c} 0.9713 \pm 0.0067 \\ 0.6196 \pm 0.0011 \\ 0.6500 \pm 0.0017 \end{array}$	$\begin{array}{c} 0.9871 \pm 0.0000 \\ 0.7346 \pm 0.0046 \\ 0.7449 \pm 0.0046 \end{array}$	$0.9874 \pm 0.0000 \\ 0.7573 \pm 0.0101 \\ 0.7573 \pm 0.0101$	$\begin{array}{c} \textbf{0.9954} \pm \textbf{0.0000} \\ 0.6822 \pm 0.0080 \\ 0.6822 \pm 0.0080 \end{array}$	0.9952 ± 0.0001 $\mathbf{0.8294 \pm 0.0049}$ 0.5256 ± 0.0039
IAPRTC-12	HS RS AUC	$\begin{array}{c} 0.9492 \pm 0.0028 \\ 0.7868 \pm 0.0006 \\ 0.7770 \pm 0.0004 \end{array}$	$\begin{array}{c} 0.9745 \pm 0.0002 \\ 0.8627 \pm 0.0038 \\ 0.8641 \pm 0.0036 \end{array}$	0.9737 ± 0.0002 0.8679 ± 0.0024 0.8679 ± 0.0024	$\begin{array}{c} \textbf{0.9846} \pm \textbf{0.0001} \\ 0.8865 \pm 0.0019 \\ 0.8787 \pm 0.0025 \end{array}$	0.9816 ± 0.0002 0.8909 ± 0.0030 0.8909 ± 0.0030
Yeast	HS RS AUC	$\begin{array}{c} 0.7354 \pm 0.0002 \\ 0.7788 \pm 0.0006 \\ 0.7521 \pm 0.0004 \end{array}$	$\begin{array}{c} 0.8322 \pm 0.0013 \\ 0.8835 \pm 0.0020 \\ 0.8835 \pm 0.0020 \end{array}$	$\begin{array}{c} 0.7635 \pm 0.0050 \\ 0.7941 \pm 0.0027 \\ 0.7941 \pm 0.0027 \end{array}$	$\begin{array}{c} 0.7355 \pm 0.0005 \\ 0.8045 \pm 0.0012 \\ 0.7749 \pm 0.0018 \end{array}$	$\begin{array}{c} 0.8234 \pm 0.0011 \\ 0.9062 \pm 0.0011 \\ 0.9062 \pm 0.0011 \end{array}$
Emotions	HS RS AUC	$\begin{array}{c} 0.6444 \pm 0.0006 \\ 0.6617 \pm 0.0016 \\ 0.6137 \pm 0.0009 \end{array}$	$\begin{array}{c} 0.6369 \pm 0.0052 \\ 0.6133 \pm 0.0197 \\ 0.6133 \pm 0.0197 \end{array}$	$\begin{array}{c} 0.6999 \pm 0.0123 \\ 0.7222 \pm 0.0190 \\ 0.7222 \pm 0.0190 \end{array}$	$\begin{array}{c} 0.7270 \pm 0.0019 \\ 0.7219 \pm 0.0075 \\ 0.6611 \pm 0.0084 \end{array}$	$\begin{array}{c} 0.7914 \pm 0.0242 \\ 0.8280 \pm 0.0189 \\ 0.8280 \pm 0.0189 \end{array}$

Table A7: Experimental results on seven real-world datasets with r%=0%, and s%=50%. The best results are highlighted in boldface.