Supplementary Materials for CEMENT: Incomplete Multi-View Weak-Label Learning with Long-Tailed Labels

Zhiwei Li[1], **Lu Sun**[1*], **Mineichi Kudo**[2], **Kego Kimura**[2]

[1]ShanghaiTech University, [2]Hokkaido University

[1]{lizhw, sunlu1}@shanghaitech.edu.cn, [1]{mine, kimura5}@ist.hokudai.ac.jp

1 Appendix

1.1 Algorithm Design

The optimization procedure in solving the optimization problem in Eq. (7) is outlined in Algorithm 1.

Algorithm 1 The Algorithm of CEMENT

Input: $\{\mathbf{X}^v\}_{v=1}^m$, $\mathbf{Y}$, $\{\mathbf{O}_\mathbf{X}^v\}_{v=1}^m$, $\mathbf{O}_\mathbf{Y}$, k, λ, ϵ
Initialize: α, β, $\{\mathbf{U}^v\}_{v=1}^m$, $\mathbf{W}$, $\mathbf{E}$
Output: $\{\hat{\mathbf{X}}^v\}_{v=1}^m$, $\hat{\mathbf{Y}}$

while not converged **do**
    Updating $\{\mathbf{P}^v\}_{v=1}^m$ according to Eq. (9).
    Updating $\mathbf{P}^*$ according to Eq. (11).
    Updating $\{\mathbf{U}^v\}_{v=1}^m$ according to Eq. (13).
    Updating $\mathbf{W}$ according to Eq. (15).
    Updating $\mathbf{E}$ according to Eq. (17).
    Updating α according to Eq. (19).
    Updating β according to Eq. (22).
end while
Let $\hat{\mathbf{X}}^v = \mathbf{P}^v\mathbf{U}^v$, $v = 1, \ldots, m$, and $\hat{\mathbf{Y}} = \mathbf{P}^*\mathbf{W} + \mathbf{E}$

1.2 More Details of Ablation Study

In this subsection, we provide the formulations of CEMENT-1, CEMENT-2 and CEMENT-3. CEMENT-1 only learns shared information from all feature views, and ignores individual information, which is defined by:

$$\begin{aligned}\min_{\substack{\mathbf{P},\mathbf{P}^*,\\ \mathbf{U}^v,\mathbf{W},\mathbf{E}}} & -HSIC(\mathbf{P},\mathbf{P}^*) + \sum_{v=1}^m ||\mathbf{O}_\mathbf{X}^v \odot (\mathbf{X}^v - \mathbf{P}\mathbf{U}^v)||_F^2 \\ & + ||\mathbf{O}_\mathbf{Y} \odot (\mathbf{Y} - \mathbf{P}^*\mathbf{W} - \mathbf{E})||_F^2 + \lambda||\mathbf{E}||_1 \\ s.t.\ & \mathbf{P}, \mathbf{P}^*, \mathbf{W}, \mathbf{U}^v \geq 0, v = 1, 2, \ldots, m,\end{aligned} \tag{A1}$$

where $\mathbf{P} \in \mathbb{R}^{n\times k}$ is the shared subspace, $k < \min\{d_1, d_2, \ldots, d_m\}$. CEMENT-2 assumes that the label matrix $\mathbf{Y}$ is low-rank by ignoring the tail label matrix $\mathbf{E}$, which is defined as:

$$\begin{aligned}\min_{\substack{\alpha,\beta,\mathbf{P}^v,\mathbf{P}^*,\\ \mathbf{U}^v,\mathbf{W}}} & -\sum_{v=1}^m \beta_v HSIC(\mathbf{P}^v, \mathbf{P}^*) \\ & + \sum_{v=1}^m \alpha_v ||\mathbf{O}_\mathbf{X}^v \odot (\mathbf{X}^v - \mathbf{P}^v\mathbf{U}^v)||_F^2 \\ & + \alpha_{m+1}||\mathbf{O}_\mathbf{Y} \odot (\mathbf{Y} - \mathbf{P}^*\mathbf{W})||_F^2 \\ s.t.\ & ||\beta||_2 = 1, \alpha, \beta, \mathbf{P}^*, \mathbf{W} \geq 0, \\ & \mathbf{U}^v, \mathbf{P}^v \geq 0, v = 1, 2, \ldots, m.\end{aligned} \tag{A2}$$

CEMENT-3 only learns a shared subspace $\mathbf{P}^*$ among all views and labels, which does not need HSIC. It is defined as:

$$\begin{aligned}\min_{\substack{\alpha,\mathbf{P}^*,\mathbf{U}^v,\\ \mathbf{W},\mathbf{E}}} & \sum_{v=1}^m \alpha_v ||\mathbf{O}_\mathbf{X}^v \odot (\mathbf{X}^v - \mathbf{P}^*\mathbf{U}^v)||_F^2 \\ & + \alpha_{m+1}||\mathbf{O}_\mathbf{Y} \odot (\mathbf{Y} - \mathbf{P}^*\mathbf{W} - \mathbf{E})||_F^2 + \lambda||\mathbf{E}||_1 \\ s.t.\ & \alpha, \mathbf{P}^*, \mathbf{W}, \mathbf{U}^v \geq 0, v = 1, 2, \ldots, m.\end{aligned} \tag{A3}$$

These three variants of CEMENT are introduced to investigate the effects of the components of CEMENT.

1.3 More Experimental Results

Settings	View-1	View-2	View-3	View-4
Rank	low	high	low	high
Related	y	y	n	n

Table A1: The descriptions of designed synthetic datasets. Here *low/high* means that the matrix is low/high-rank, and *y/n* means that the subspace of this view is related/unrelated to the subspace produced by the label matrix.

	View-1	View-2	View-3	View-4	Label
α	0.0541	0.0285	0.2165	0.0025	0.0389
β	0.6241	0.6436	0.2724	0.3495	\

Table A2: The learned values of α and β on the designed synthetic dataset shown in Table A1.

*Contact Author

Experiments On The Synthetic Dataset. This part aims to explore the effects of α and β. To this end, we carefully design a synthetic dataset with 4 views, each of which has 60, 60, 50, and 50 features. The dataset contains 100 samples. As shown in Table A1, the matrices of View-1 and View-3 are low-rank, while the matrix of View-2 and View-4 are high-rank. Meanwhile, View-1 and View-2 are related to the label matrix, while View-2 and View-4 are unrelated. As shown in Table A2, α assigns larger values to α_1 (View-1), α_3 (View-3) and α_5 (Label) than α_2 (View-2) and α_4 (View-4). It is because the feature matrices View-1, View-3 and the label matrix Label are successfully recovered due to the low-rankness. β assigns larger values to View-1 and View-2, which subspaces that correlated to the subspace produced by the label matrix, than the others. Thus, we can conclude that the variables α and β has the abilities to release the effects caused by the reconstruction errors and measure the importance of each view, respectively.

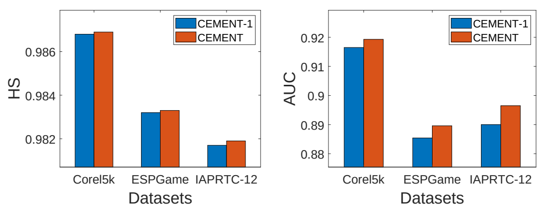

Figure A1: Case Study of CEMENT-1 and CEMENT for handling noisy views on the Corel5k, ESPGame and IAPRTC-12 datasets.

Case Study on Handling Noisy Views. To demonstrate the significance of CEMENT on investigating noisy views, we randomly add noises to half of views, and then evaluate the performances of CEMENT and CEMENT-1 on three image datasets (Corel5k, ESPGame and IAPRTC-12). The results are shown in Fig. A1. It can be seen that CEMENT outperforms CEMENT-1 in all the metrics on the three datasets, indicating that weighting embedding importance of noisy views is beneficial to recover the unobserved labels.

Parameter And Convergence Analysis. Fig. A2 analyzes the sensitivity of CEMENT w.r.t. λ and k on the five image datasets and the Emotions dataset. We plot the convergence curve of the optimization algorithm on the seven used datasets, as shown in Fig. A3. We terminate the optimization algorithm of CEMENT once the relative change of its objective value is below 10^{-4}. To show the convergence curve clearly, we omit the objective value of the first iteration in Fig. A3. We observe that the objective value monotonically decreases as the number of iterations increases, and it converges within 200 iterations on these datasets.

Experiments On The Real Datasets. To show the effectiveness of our method CEMENT, we conduct experiments under different combinations of $r\%$ and $s\%$. We first set $r\% = 10\%, 20\%, 30\%$, and 40% to evaluate the performances under incomplete views with 50% missing labels. The results are shown in Table A3, Table A4, Table A5, Table A6, respectively. Then we conduct the performance under complete views with 50% missing labels, and the results are shown in Table A7. It can be seen that our method CEMENT obtains the best performances on most of the cases.

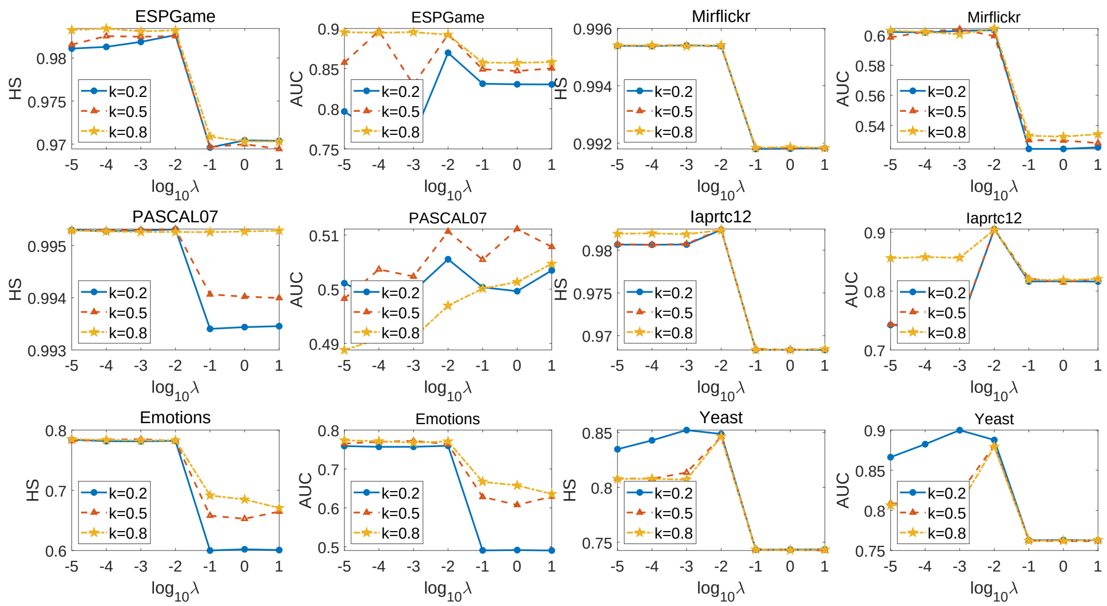

Figure A2: Hyperparameter sensitivity analysis of CEMENT under different combinations of λ and k on the five image datasets and the Emotions dataset.

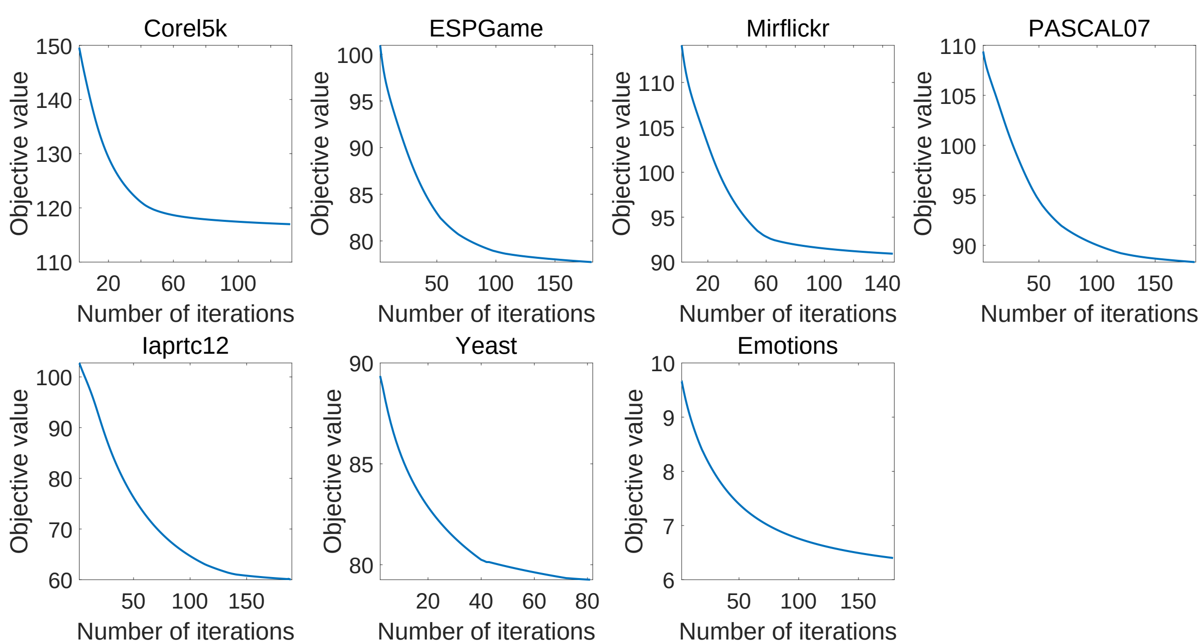

Figure A3: Convergence analysis of CEMENT on the five image datasets.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM^{3}L	CEMENT
Corel5k	HS	0.9273 ± 0.0030	0.9788 ± 0.0001	0.9789 ± 0.0003	0.9867 ± 0.0000	$\mathbf{0.9950 \pm 0.0001}$
	RS	0.8022 ± 0.0022	0.8917 ± 0.0031	0.9056 ± 0.0027	0.8775 ± 0.0020	$\mathbf{0.9851 \pm 0.0010}$
	AUC	0.8025 ± 0.0022	0.8919 ± 0.0029	0.9056 ± 0.0027	0.8775 ± 0.0020	$\mathbf{0.9823 \pm 0.0010}$
ESPGame	HS	0.9710 ± 0.0015	0.9708 ± 0.0001	0.9679 ± 0.0002	0.9804 ± 0.0000	$\mathbf{0.9918 \pm 0.0001}$
	RS	0.8041 ± 0.0005	0.8843 ± 0.0018	0.8459 ± 0.0029	0.8526 ± 0.0010	$\mathbf{0.9780 \pm 0.0007}$
	AUC	0.8041 ± 0.0005	0.8843 ± 0.0018	0.8459 ± 0.0029	0.8526 ± 0.0010	$\mathbf{0.9780 \pm 0.0007}$
Mirflickr	HS	0.8822 ± 0.0060	0.8731 ± 0.0000	0.8495 ± 0.0000	0.8815 ± 0.0001	$\mathbf{0.9330 \pm 0.0000}$
	RS	0.8168 ± 0.0008	0.8805 ± 0.0028	0.8378 ± 0.0044	0.8403 ± 0.0104	$\mathbf{0.9844 \pm 0.0025}$
	AUC	0.8035 ± 0.0003	0.8805 ± 0.0027	0.8378 ± 0.0044	0.8256 ± 0.0076	$\mathbf{0.9733 \pm 0.0018}$
Pascal07	HS	0.9326 ± 0.0013	0.8832 ± 0.0000	0.8936 ± 0.0001	0.9354 ± 0.0000	$\mathbf{0.9578 \pm 0.0000}$
	RS	0.7255 ± 0.0016	0.7192 ± 0.0059	0.7985 ± 0.0061	0.7350 ± 0.0072	$\mathbf{0.9735 \pm 0.0047}$
	AUC	0.7149 ± 0.0016	0.7192 ± 0.0059	0.7985 ± 0.0061	0.7118 ± 0.0072	$\mathbf{0.9735 \pm 0.0034}$
IAPRTC-12	HS	0.8950 ± 0.0024	0.9751 ± 0.0002	0.9734 ± 0.0001	0.9838 ± 0.0000	$\mathbf{0.9912 \pm 0.0002}$
	RS	0.7847 ± 0.0034	0.8977 ± 0.0010	0.8766 ± 0.0019	0.8842 ± 0.0009	$\mathbf{0.9797 \pm 0.0015}$
	AUC	0.7847 ± 0.0034	0.8977 ± 0.0010	0.8766 ± 0.0019	0.8835 ± 0.0010	$\mathbf{0.9797 \pm 0.0015}$
Yeast	HS	0.7054 ± 0.0003	0.8296 ± 0.0012	0.7416 ± 0.0062	0.7059 ± 0.0004	$\mathbf{0.8806 \pm 0.0006}$
	RS	0.7838 ± 0.0001	0.9124 ± 0.0010	0.8001 ± 0.0045	0.8133 ± 0.0009	$\mathbf{0.9786 \pm 0.0005}$
	AUC	0.7831 ± 0.0002	0.9124 ± 0.0010	0.8001 ± 0.0045	0.8126 ± 0.0009	$\mathbf{0.9786 \pm 0.0005}$
Emotions	HS	0.6493 ± 0.0008	0.6198 ± 0.0013	0.6819 ± 0.0139	0.6972 ± 0.0018	$\mathbf{0.8994 \pm 0.0039}$
	RS	0.6686 ± 0.0030	0.6199 ± 0.0079	0.7006 ± 0.0232	0.7281 ± 0.0038	$\mathbf{0.9821 \pm 0.0053}$
	AUC	0.6628 ± 0.0029	0.6199 ± 0.0079	0.7006 ± 0.0232	0.7211 ± 0.0052	$\mathbf{0.9821 \pm 0.0053}$

Table A3: Experimental results on seven real-world datasets with $r\% = 10\%$, and $s\% = 10\%$. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM^{3}L	CEMENT
Corel5k	HS	0.8845 ± 0.0010	0.9781 ± 0.0002	0.9786 ± 0.0003	0.9868 ± 0.0000	$\mathbf{0.9929 \pm 0.0001}$
	RS	0.7985 ± 0.0017	0.8613 ± 0.0042	0.8934 ± 0.0055	0.8943 ± 0.0030	$\mathbf{0.9695 \pm 0.0011}$
	AUC	0.7996 ± 0.0016	0.8625 ± 0.0039	0.8934 ± 0.0055	0.8943 ± 0.0030	$\mathbf{0.9668 \pm 0.0011}$
ESPGame	HS	0.9732 ± 0.0065	0.9704 ± 0.0001	0.9681 ± 0.0002	0.9808 ± 0.0001	$\mathbf{0.9899 \pm 0.0001}$
	RS	0.8018 ± 0.0006	0.8668 ± 0.0022	0.8444 ± 0.0030	0.8520 ± 0.0014	$\mathbf{0.9554 \pm 0.0018}$
	AUC	0.8018 ± 0.0006	0.8668 ± 0.0022	0.8444 ± 0.0030	0.8520 ± 0.0014	$\mathbf{0.9554 \pm 0.0018}$
Mirflickr	HS	0.8763 ± 0.0029	0.8834 ± 0.0001	0.8562 ± 0.0000	0.8852 ± 0.0001	$\mathbf{0.9279 \pm 0.0000}$
	RS	0.8103 ± 0.0020	0.7225 ± 0.0041	0.8487 ± 0.0052	0.8379 ± 0.0066	$\mathbf{0.9760 \pm 0.0024}$
	AUC	0.7866 ± 0.0016	0.7225 ± 0.0041	0.8487 ± 0.0052	0.8159 ± 0.0055	$\mathbf{0.9650 \pm 0.0017}$
Pascal07	HS	0.9346 ± 0.0009	0.8691 ± 0.0001	0.8920 ± 0.0001	0.9350 ± 0.0001	$\mathbf{0.9488 \pm 0.0000}$
	RS	0.7248 ± 0.0008	0.8683 ± 0.0034	0.7941 ± 0.0076	0.7595 ± 0.0068	$\mathbf{0.9350 \pm 0.0056}$
	AUC	0.7006 ± 0.0006	0.8683 ± 0.0033	0.7941 ± 0.0076	0.7266 ± 0.0068	$\mathbf{0.9377 \pm 0.0038}$
IAPRTC-12	HS	0.8685 ± 0.0040	0.9746 ± 0.0001	0.9735 ± 0.0001	0.9840 ± 0.0001	$\mathbf{0.9892 \pm 0.0001}$
	RS	0.7828 ± 0.0002	0.8823 ± 0.0017	0.8757 ± 0.0020	0.8868 ± 0.0019	$\mathbf{0.9600 \pm 0.0022}$
	AUC	0.7808 ± 0.0005	0.8824 ± 0.0017	0.8757 ± 0.0020	0.8855 ± 0.0025	$\mathbf{0.9600 \pm 0.0022}$
Yeast	HS	0.7141 ± 0.0000	0.8245 ± 0.0108	0.7635 ± 0.0050	0.7141 ± 0.0007	$\mathbf{0.8654 \pm 0.0013}$
	RS	0.7821 ± 0.0007	0.8969 ± 0.0016	0.7941 ± 0.0027	0.8118 ± 0.0013	$\mathbf{0.9592 \pm 0.0014}$
	AUC	0.7803 ± 0.0002	0.8969 ± 0.0016	0.7941 ± 0.0027	0.8087 ± 0.0015	$\mathbf{0.9592 \pm 0.0014}$
Emotions	HS	0.7063 ± 0.0001	0.6244 ± 0.0048	0.6827 ± 0.0195	0.7053 ± 0.0020	$\mathbf{0.8846 \pm 0.0123}$
	RS	0.6576 ± 0.0008	0.6169 ± 0.0064	0.7015 ± 0.0290	0.7281 ± 0.0051	$\mathbf{0.9609 \pm 0.0140}$
	AUC	0.6419 ± 0.0023	0.6169 ± 0.0064	0.7015 ± 0.0290	0.7088 ± 0.0073	$\mathbf{0.9609 \pm 0.0140}$

Table A4: Experimental results on seven real-world datasets with $r\% = 20\%$, and $s\% = 20\%$. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM^{3}L	CEMENT
Corel5k	HS	0.9094 ± 0.0090	0.9773 ± 0.0002	0.9783 ± 0.0003	0.9870 ± 0.0001	$\mathbf{0.9908 \pm 0.0002}$
	RS	0.7815 ± 0.0004	0.8334 ± 0.0049	0.8673 ± 0.0049	0.9103 ± 0.0057	$\mathbf{0.9537 \pm 0.0024}$
	AUC	0.7877 ± 0.0000	0.8362 ± 0.0045	0.8673 ± 0.0049	0.9103 ± 0.0057	$\mathbf{0.9511 \pm 0.0024}$
ESPGame	HS	0.9346 ± 0.0081	0.9699 ± 0.0001	0.9710 ± 0.0002	0.9812 ± 0.0001	$\mathbf{0.9878 \pm 0.0001}$
	RS	0.7940 ± 0.0010	0.8510 ± 0.0025	0.8414 ± 0.0032	0.8577 ± 0.0022	$\mathbf{0.9315 \pm 0.0014}$
	AUC	0.7930 ± 0.0007	0.8511 ± 0.0025	0.8414 ± 0.0032	0.8572 ± 0.0022	$\mathbf{0.9315 \pm 0.0014}$
Mirflickr	HS	0.8516 ± 0.0026	0.8656 ± 0.0000	0.8570 ± 0.0001	0.8872 ± 0.0000	$\mathbf{0.9214 \pm 0.0000}$
	RS	0.7845 ± 0.0014	0.8572 ± 0.0041	0.8449 ± 0.0054	0.8336 ± 0.0101	$\mathbf{0.9571 \pm 0.0038}$
	AUC	0.7602 ± 0.0003	0.8572 ± 0.0038	0.8449 ± 0.0054	0.8093 ± 0.0058	$\mathbf{0.9463 \pm 0.0028}$
Pascal07	HS	0.9315 ± 0.0062	0.8831 ± 0.0001	0.8927 ± 0.0001	0.9357 ± 0.0001	$\mathbf{0.9402 \pm 0.0000}$
	RS	0.7172 ± 0.0029	0.7171 ± 0.0088	0.8001 ± 0.0114	0.7773 ± 0.0065	$\mathbf{0.8909 \pm 0.0233}$
	AUC	0.6783 ± 0.0016	0.7171 ± 0.0087	0.8001 ± 0.0114	0.7281 ± 0.0065	$\mathbf{0.8909 \pm 0.0044}$
IAPRTC-12	HS	0.9325 ± 0.0098	0.9742 ± 0.0001	0.9735 ± 0.0001	0.9842 ± 0.0001	$\mathbf{0.9868 \pm 0.0002}$
	RS	0.7616 ± 0.0060	0.8694 ± 0.0030	0.8723 ± 0.0021	0.8987 ± 0.0015	$\mathbf{0.9373 \pm 0.0026}$
	AUC	0.7585 ± 0.0053	0.8697 ± 0.0029	0.8723 ± 0.0021	0.8958 ± 0.0020	$\mathbf{0.9373 \pm 0.0026}$
Yeast	HS	0.7285 ± 0.0003	0.8280 ± 0.0014	0.7514 ± 0.0056	0.7217 ± 0.0011	$\mathbf{0.8507 \pm 0.0014}$
	RS	0.7811 ± 0.0002	0.8810 ± 0.0017	0.7885 ± 0.0029	0.8092 ± 0.0016	$\mathbf{0.9403 \pm 0.0015}$
	AUC	0.7665 ± 0.0008	0.8810 ± 0.0017	0.7885 ± 0.0029	0.8007 ± 0.0019	$\mathbf{0.9403 \pm 0.0015}$
Emotions	HS	0.6197 ± 0.0325	0.6213 ± 0.0150	0.6784 ± 0.0099	0.7118 ± 0.0041	$\mathbf{0.8568 \pm 0.0189}$
	RS	0.6596 ± 0.0367	0.6183 ± 0.0097	0.6851 ± 0.0163	0.7315 ± 0.0070	$\mathbf{0.9272 \pm 0.0245}$
	AUC	0.6302 ± 0.0351	0.6183 ± 0.0097	0.6851 ± 0.0163	0.7012 ± 0.0095	$\mathbf{0.9272 \pm 0.0245}$

Table A5: Experimental results on seven real-world datasets with $r\% = 30\%$, and $s\% = 30\%$. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM^{3}L	CEMENT
Corel5k	HS	0.9094 ± 0.0090	0.9771 ± 0.0004	0.9783 ± 0.0002	0.9873 ± 0.0002	$\mathbf{0.9889 \pm 0.0001}$
	RS	0.7815 ± 0.0004	0.8084 ± 0.0073	0.8691 ± 0.0048	0.9012 ± 0.0080	$\mathbf{0.9385 \pm 0.0009}$
	AUC	0.7877 ± 0.0000	0.8139 ± 0.0063	0.8691 ± 0.0041	0.9012 ± 0.0080	$\mathbf{0.9361 \pm 0.0009}$
ESPGame	HS	0.9346 ± 0.0081	0.9721 ± 0.0002	0.9711 ± 0.0001	0.9817 ± 0.0001	$\mathbf{0.9856 \pm 0.0001}$
	RS	0.7940 ± 0.0010	0.8355 ± 0.0022	0.8422 ± 0.0041	0.8733 ± 0.0023	$\mathbf{0.9118 \pm 0.0021}$
	AUC	0.7930 ± 0.0007	0.8357 ± 0.0021	0.8422 ± 0.0041	0.8717 ± 0.0024	$\mathbf{0.9118 \pm 0.0021}$
Mirflickr	HS	0.8626 ± 0.0026	0.8616 ± 0.0001	0.8572 ± 0.0001	0.8897 ± 0.0001	$\mathbf{0.9107 \pm 0.0000}$
	RS	0.7978 ± 0.0014	0.8479 ± 0.0049	0.8423 ± 0.0091	0.8340 ± 0.0102	$\mathbf{0.9285 \pm 0.0021}$
	AUC	0.7621 ± 0.0003	0.8479 ± 0.0050	0.8423 ± 0.0091	0.8015 ± 0.0077	$\mathbf{0.9184 \pm 0.0015}$
Pascal07	HS	0.9366 ± 0.0062	0.8828 ± 0.0001	0.8861 ± 0.0001	$\mathbf{0.9427 \pm 0.0001}$	0.9343 ± 0.0000
	RS	0.7248 ± 0.0029	0.7199 ± 0.0135	0.7763 ± 0.0110	0.7823 ± 0.0058	$\mathbf{0.8807 \pm 0.0091}$
	AUC	0.6614 ± 0.0016	0.7199 ± 0.0128	0.7763 ± 0.0110	0.7117 ± 0.0058	$\mathbf{0.8807 \pm 0.0060}$
IAPRTC-12	HS	0.9325 ± 0.0098	0.9737 ± 0.0001	0.9735 ± 0.0002	$\mathbf{0.9844 \pm 0.0001}$	0.9842 ± 0.0001
	RS	0.7616 ± 0.0060	0.8515 ± 0.0043	0.8686 ± 0.0036	0.9078 ± 0.0018	$\mathbf{0.9152 \pm 0.0026}$
	AUC	0.7585 ± 0.0053	0.8522 ± 0.0041	0.8686 ± 0.0036	0.9028 ± 0.0033	$\mathbf{0.9152 \pm 0.0026}$
Yeast	HS	0.7285 ± 0.0003	0.8163 ± 0.0020	0.7488 ± 0.0046	0.7294 ± 0.0010	$\mathbf{0.8370 \pm 0.0009}$
	RS	0.7811 ± 0.0002	0.8645 ± 0.0029	0.7846 ± 0.0028	0.8071 ± 0.0019	$\mathbf{0.9230 \pm 0.0009}$
	AUC	0.7665 ± 0.0008	0.8645 ± 0.0029	0.7846 ± 0.0028	0.7911 ± 0.0028	$\mathbf{0.9230 \pm 0.0009}$
Emotions	HS	0.6197 ± 0.0325	0.6177 ± 0.0182	0.6682 ± 0.0099	0.7200 ± 0.0017	$\mathbf{0.8370 \pm 0.0213}$
	RS	0.6596 ± 0.0367	0.6202 ± 0.0136	0.6763 ± 0.0143	0.7358 ± 0.0132	$\mathbf{0.9028 \pm 0.0333}$
	AUC	0.6302 ± 0.0351	0.6202 ± 0.0136	0.6763 ± 0.0143	0.6906 ± 0.0106	$\mathbf{0.9028 \pm 0.0333}$

Table A6: Experimental results on seven real-world datasets with $r\% = 40\%$, and $s\% = 40\%$. The best results are highlighted in boldface.

Dataset	Metric	lrMMC	McWL	iMVWL	NAIM^{3}L	CEMENT
Corel5k	HS	0.9053 ± 0.0060	0.9780 ± 0.0002	0.9784 ± 0.0003	$\mathbf{0.9873} \pm \mathbf{0.0001}$	0.9869 ± 0.0001
	RS	0.7740 ± 0.0000	0.8182 ± 0.0050	0.8545 ± 0.0068	0.8729 ± 0.0051	$\mathbf{0.9213} \pm \mathbf{0.0022}$
	AUC	0.7835 ± 0.0006	0.8280 ± 0.0038	0.8545 ± 0.0068	0.8729 ± 0.0051	$\mathbf{0.9190} \pm \mathbf{0.0021}$
ESPGame	HS	0.9516 ± 0.0041	0.9730 ± 0.0002	0.9713 ± 0.0001	0.9821 ± 0.0001	$\mathbf{0.9833} \pm \mathbf{0.0002}$
	RS	0.8022 ± 0.0001	0.8442 ± 0.0024	0.8415 ± 0.0027	0.8534 ± 0.0022	$\mathbf{0.8899} \pm \mathbf{0.0020}$
	AUC	0.7961 ± 0.0009	0.8447 ± 0.0025	0.8415 ± 0.0027	0.8498 ± 0.0034	$\mathbf{0.8899} \pm \mathbf{0.0020}$
Mirflickr	HS	0.8930 ± 0.0024	0.8668 ± 0.0000	0.8588 ± 0.0001	0.8933 ± 0.0000	$\mathbf{0.9032} \pm \mathbf{0.0001}$
	RS	0.8183 ± 0.0036	0.8604 ± 0.0045	0.8495 ± 0.0080	0.8342 ± 0.0056	$\mathbf{0.9108} \pm \mathbf{0.0025}$
	AUC	0.7778 ± 0.0018	0.8604 ± 0.0036	0.8495 ± 0.0080	0.7915 ± 0.0045	$\mathbf{0.9010} \pm \mathbf{0.0020}$
Pascal07	HS	0.9403 ± 0.0067	0.8841 ± 0.0000	0.8931 ± 0.0000	$\mathbf{0.9417} \pm \mathbf{0.0000}$	0.9239 ± 0.0001
	RS	0.7304 ± 0.0011	0.7142 ± 0.0046	0.7983 ± 0.0101	0.7579 ± 0.0080	$\mathbf{0.8281} \pm \mathbf{0.0049}$
	AUC	0.6543 ± 0.0017	0.7142 ± 0.0046	0.7983 ± 0.0101	0.6821 ± 0.0080	$\mathbf{0.8364} \pm \mathbf{0.0039}$
IAPRTC-12	HS	0.9492 ± 0.0028	0.9745 ± 0.0002	0.9737 ± 0.0002	0.9846 ± 0.0001	$\mathbf{0.9816} \pm \mathbf{0.0002}$
	RS	0.7868 ± 0.0006	0.8627 ± 0.0038	0.8679 ± 0.0024	0.8865 ± 0.0019	$\mathbf{0.8909} \pm \mathbf{0.0030}$
	AUC	0.7770 ± 0.0004	0.8641 ± 0.0036	0.8679 ± 0.0024	0.8787 ± 0.0025	$\mathbf{0.8909} \pm \mathbf{0.0030}$
Yeast	HS	0.7354 ± 0.0002	0.8322 ± 0.0013	0.7635 ± 0.0050	0.7355 ± 0.0005	$\mathbf{0.8234} \pm \mathbf{0.0011}$
	RS	0.7788 ± 0.0006	0.8835 ± 0.0020	0.7941 ± 0.0027	0.8045 ± 0.0012	$\mathbf{0.9062} \pm \mathbf{0.0011}$
	AUC	0.7521 ± 0.0004	0.8835 ± 0.0020	0.7941 ± 0.0027	0.7749 ± 0.0018	$\mathbf{0.9062} \pm \mathbf{0.0011}$
Emotions	HS	0.6444 ± 0.0006	0.6369 ± 0.0052	0.6999 ± 0.0123	0.7270 ± 0.0019	$\mathbf{0.7914} \pm \mathbf{0.0242}$
	RS	0.6617 ± 0.0016	0.6133 ± 0.0197	0.7222 ± 0.0190	0.7219 ± 0.0075	$\mathbf{0.8280} \pm \mathbf{0.0189}$
	AUC	0.6137 ± 0.0009	0.6133 ± 0.0197	0.7222 ± 0.0190	0.6611 ± 0.0084	$\mathbf{0.8280} \pm \mathbf{0.0189}$

Table A7: Experimental results on seven real-world datasets with $r\% = 0\%$, and $s\% = 50\%$. The best results are highlighted in boldface.