MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.301: Solid-State Circuits — Fall 2012

PROBLEM SET 9 SOLUTION

Issued: November 17, 2012

Suggested Reading: Read as many of the following as you can. All of the recommended references are on reserve at Barker Library.

1. Lundberg sections 40-44.
2. Gray and Searle (in library) chapters 21 and 22.

Problem 1: In this problem, a transistor is controlled by supplying a base current drive as seen in Figure 1. Analyze the dynamics of the transistor and sketch q_F, q_S, i_C and i_B versus time. Each sketch should clearly indicate important slopes, final values, time constants, etc. Read this whole problem before starting to solve the first part.

1. Assume that the transistor remains in the forward active region. Determine the time constants and final values etc, and sketch the curves.

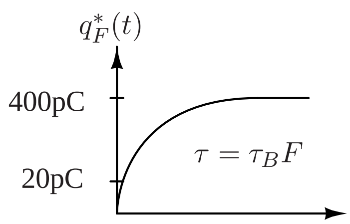
Assuming we stay in the forward active region:
The initial values are all zero, but

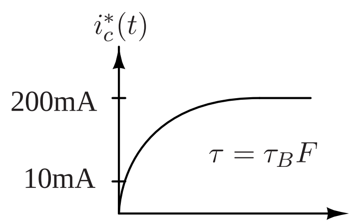
$$i_c^*(\infty) = \beta_F i_B = 200\text{mA} \qquad q_F^*(\infty) = i_C \tau_F = 400\text{pC}$$

The time constant for these transistors is $\tau_{BF} = \beta\tau_F = 400\text{ns}$, thus

$$q_F^* = 400\text{pC}(1 - e^{-t/\tau_{BF}}) \qquad i_c^* = 200\text{mA}(1 - e^{-t/\tau_B F})$$

sketches (not to scale):





2. Since $\beta_F i_B > i_{C(SAT)}$, the device will not remain in the forward active region for all time. Indicate on your graphs the point at which saturation occurs. Find q_{BO}, the final values (in saturation) for i_C and q_S and evaluate the time constant τ_S. Continue the sketches. How long does it take to transverse the active region?

Saturation occurs at

$$i_{c(\text{sat})} = \frac{V_{CC} - V_{CE(\text{sat})}}{R} = \frac{10\text{V}}{1k} = 10\text{mA}$$

$$t = -\tau_{BF} \ln \frac{i_c^*(\infty) - i_{c(\mathrm{sat})}}{i_C^*(\infty)} = 20.5\mathrm{ns} \qquad \text{(time to traverse FAR)}$$

$$q_{B0} = i_{c(\mathrm{sat})}\tau_F = 20\mathrm{pC}$$

It takes $\frac{i_{c(\mathrm{SAT})}}{\beta_F} = 50\mu\mathrm{A}$ of base drive to saturate the device. Everything else contributes to q_S:

$$\tau_s = \frac{\tau_{BF}(\beta_R + 1) + \tau_{BR}\beta_F}{\beta_R + \beta_F + 1} = 115\mathrm{ns}$$

$$q_s(\infty) = \tau_s(i_B - i_{B0}) = (115\mathrm{ns})(0.95\mathrm{mA}) = 109\mathrm{pC}$$

$$q_s(t') = 109\mathrm{pC}(1 - e^{-t'/\tau_s}), \qquad t' = t - 20.5\mathrm{ns}$$

We find q_F in the saturation region by using the charge-control equations:

$$i_{C(sat)} = \frac{q_F}{\tau_F} - q_R\left(\frac{1}{\tau_R} + \frac{1}{\tau_{BR}}\right) - \frac{dq_R}{dt} \qquad (1)$$

$$i_{B(sat)} = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt} + \frac{q_R}{\tau_{BR}} + \frac{dq_R}{dt} \qquad (2)$$

$$i_{B(sat)} - i_{B0} = \frac{q_s}{\tau_s} + \frac{dq_s}{dt} \qquad (3)$$

$$q_{B0} + q_s = q_F + q_R. \qquad (4)$$

Putting the equation 4 into the equation 3 gives us

$$\begin{aligned} i_{B(sat)} - i_{B0} &= \frac{q_F + q_R - q_{B0}}{\tau_s} + \frac{d(q_F + q_R - q_{B0})}{dt} && (5) \\ &= \frac{q_F + q_R - q_{B0}}{\tau_s} + \frac{dq_F}{dt} + \frac{dq_R}{dt}. && (6) \end{aligned}$$

The next step to get a first-order differential equation on q_F in the saturation region from the above equation is, to express q_R and $\frac{dq_R}{dt}$ in terms of q_F and $\frac{dq_F}{dt}$. From the equation 1 and equation 2, we get the results for this step as below:

$$\begin{aligned} q_R &= -\left(\frac{1}{\tau_{BF}\beta_R} + \frac{1}{\tau_F\beta_R} + \frac{1}{\tau_{BF}}\right) q_F - \left(1 + \frac{1}{\beta_R}\right)\frac{dq_F}{dt} + \left(1 + \frac{1}{\beta_R}\right) i_{B(sat)} + \frac{1}{\beta_R} i_{C(sat)}, \\ \frac{dq_R}{dt} &= \left(\frac{\tau_R}{\tau_{BF}} + \frac{\tau_R}{\tau_F}\right) q_F + \tau_R \frac{dq_F}{dt} - \tau_R\, i_{B(sat)} - \tau_R\, i_{C(sat)} \end{aligned}$$

Putting these two equations into the equation 6 results in the first-order equation on q_F. Note that we only need the time constant on q_F from this equation since the initial and final value to obtain the time-domain solution of q_F can be simply obtained, as will be explained later. The coefficient of the q_F term from the resulting equation is

$$c_1 = \frac{1}{\tau_s}\left(1 - \frac{1}{\tau_{BF}\beta_R} + \frac{1}{\tau_F\beta_R} + \frac{1}{\tau_{BF}}\right) + \frac{\tau_R}{\tau_{BF}} + \frac{\tau_R}{\tau_F}.$$

The coefficient of the $\frac{dq_F}{dt}$ term is

$$c_2 = \frac{1}{\tau_s} - \left(1 + \frac{1}{\beta_R}\right) + 1 + \tau_R.$$

Thus, we get the time constant of q_F in the saturation region as

$$\tau_{F(sat)} = \frac{c_2}{c_1} = 19.1\,\text{ns}.$$

The initial value of q_F in the saturation region is, by definition, q_{B0}:

$$q_{F(sat)}(0) = q_{B0} = 20\,\text{pC}$$

The final value of q_F in the saturation is obtained from the steady-state charge equations, which are obtained from the equation 1 and equation 2 by setting $\frac{dq_F}{dt} = \frac{dq_R}{dt} = 0$:

$$\begin{aligned} i_{C(sat)} &= \frac{q_{Fss}}{\tau_F} - q_{Rss}\left(\frac{1}{\tau_R} + \frac{1}{\tau_{BR}}\right) \\ i_{B(sat)} &= \frac{q_{Fss}}{\tau_{BF}} + \frac{q_{Rss}}{\tau_{BR}}. \end{aligned}$$

Because $i_{C(sat)} = 10\,\text{mA}$ and $i_{B(sat)} = 1\,\text{mA}$, solving the steady-state charge equations gives

$$\begin{aligned} q_{Fss} &= 39.8\,\text{pC}, \\ q_{Rss} &= 90.0\,\text{pC}. \end{aligned}$$

Note that the steady-state value q_{Fss} is the final value of q_F in the saturation region:

$$q_{F(sat)}(\infty) = q_{Fss} = 39.8\,\text{pC}.$$

Also note that, considering $q_{B0} = 20\,\text{pC}$ and $q_s = 109\,\text{pC}$, we see that the contribution of q_F to q_s is 39.8 pC $-$ 20 pC = 19.8 pC, which is much smaller than the 90 pC contribution of q_R to q_s.

With the $\tau_{F(sat)}$, $q_{F(sat)}(0)$, $q_{F(sat)}(\infty)$ that we have obtained so far, the expression on q_F in the saturation region is now determined as

$$\begin{aligned} q_{F(sat)}(t) &= q_{F(sat)}(\infty) + \left(q_{F(sat)}(0) - q_{F(sat)}(\infty)\right)\, e^{-t/\tau_{F(sat)}} \\ &= 39.8\,\text{pC} - 19.8\, e^{-t/19.1\,\text{ns}}\,\text{pC} \end{aligned}$$

(see plot summary at end of problem)

3. Now consider turning off the device, $i_B = 0$. Assume that the transistor remains saturated, that is $i_C = i_{C(SAT)}$ for all time. Determine the final values for q_S and sketch the curves.

Now $I_B = 0$, but we're still in saturation.

$$I_{OD} = I_B - I_{\text{BSAT}} = 0 - \frac{i_{\text{c(SAT)}}}{\beta_F} = -50\mu\text{A}$$

$$q_S(\infty) = I_{OD}\tau_S = -5.75\text{pC}$$

$$q_S(0) = 100\text{pC}$$

$$q_S(t) = [q_S(0) - q_S(\infty)][e^{-t/\tau_S}] + q_S(\infty) = -5.75\text{pC} + 115.8\text{pC}(\text{e}^{-\text{t}/\tau_\text{S}})$$

(see plot summary at end of problem)

4. Obviously, the transistor does not remain saturated but enters the forward active region when q_S equals zero. Determine the storage delay time, that is, the time during which the device remains saturated even though $i_B = 0$. Determine the time spent crossing the active region. Sketch the curves.

Solving for $t(q_S = 0) = t_{s0}$ gives:

$$t_{s0} = -\tau_S \ln\left(\frac{5.8\text{pC}}{115.8\text{pC}}\right) = 340\text{ns}$$

Once out of saturation, we have

$$q_F = q_{B0} e^{-t/\tau_{BF}}$$

$$i_c = \frac{q_F}{\tau_F} = \frac{q_{B0}}{\tau_F} e^{-t/\tau_{BF}}$$

We never get out of FAR, but after $5 \times \tau_{BF}$ we're within 1% of the final value:

$$t = 5(400\text{ns}) = 2\mu\text{s}$$

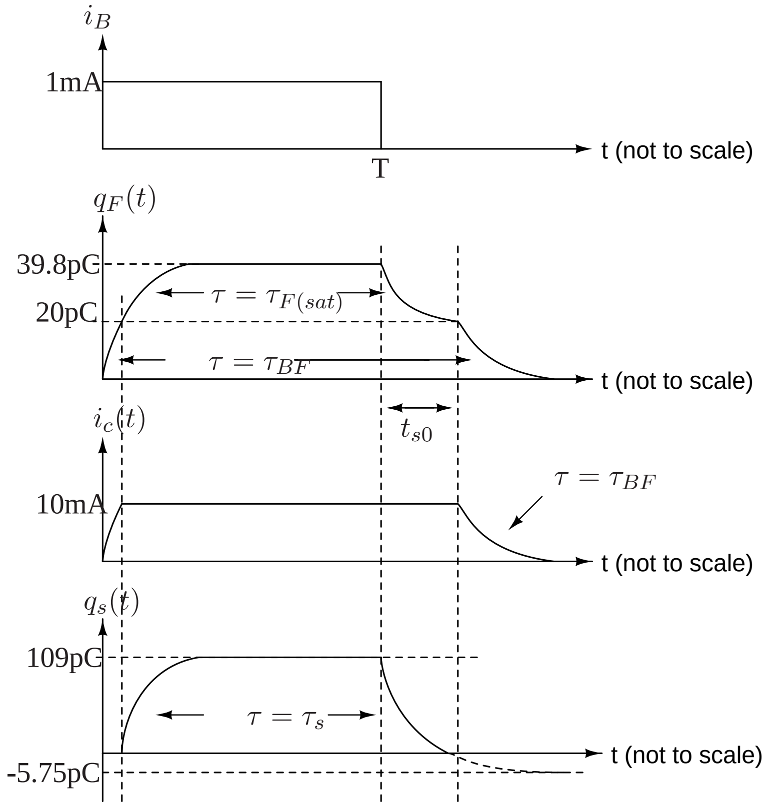
5. Compare turn-on times and turn-off times; explain the difference.

Turn on time $= 20.5$ns
Turn off time $= t_{s0} + t_{FAR} = 2.34\mu$s
During turn on we are actively injecting charge into the base ($I_B = 1$mA). During turn off we first have to remove stored base charge before the transistor even begins to turn off. Then we can reduce q_F. But $I_B = 0$ so we have to wait for recombination in the case to remove the charge for us.

6. It is observed that the storage delay time decreases if the input pulse duration is reduced. Explain.

When the pulse duration is reduced, we have less time to store excess charge (q_S) in the base. With less initial charge storage we can remove it faster.

Problem 1 plot summary:



Problem 2: Charge Control

1. Sketch q_F, q_S, i_C and i_B versus time for the circuit shown in Figure 2.

   We know how the circuit behaves with $C_B = 0$ from problem 1. We just need to know how much charge is injected for $C_B \neq 0$.

$$i_{CB} = C_B \frac{dv_{CB}}{dt}$$

$$i_{CB}dt = q_{CB} = C_B dV_{CB}; \qquad dV_{CB} = 10.6\delta(t)$$

$$q_{CB} = C_B(10.6\delta(t))$$

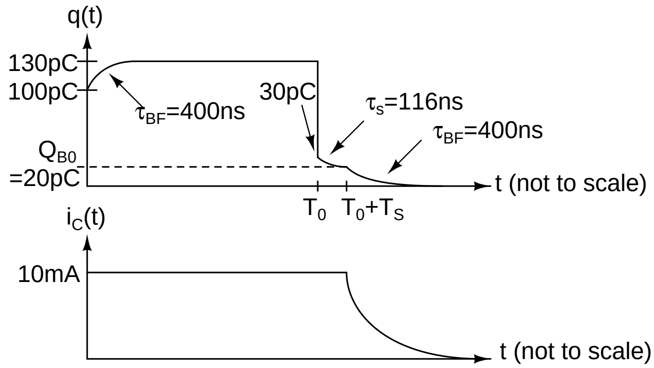
$$i_B = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt} = \underbrace{\frac{(10.6 - .6)\mathrm{V}}{10\mathrm{k}\Omega} u(t)}_{\text{resistor current}} + \underbrace{(10\mathrm{V})(10\mathrm{pF})\delta(\mathrm{t})}_{\text{capacitor current}} = I_B u(t) + Q_{CB}\delta(t)$$

   The solution (from the notes) is:

$$q_F(t) = I_B \tau_{BF}(1 - e^{-t/\tau_{BF}}) + Q_{CB} e^{-t/\tau_{BF}}$$

   During the storage time, $q_S(t) = -5.8\mathrm{pC} + 15.8\mathrm{pC}(\mathrm{e}^{-\mathrm{t}/\tau_\mathrm{S}}) \rightarrow \mathrm{T_S} = 116\mathrm{ns}$

   The total base charge q_B and i_C are sketched below. q_F and q_S could be obtained and sketched in the same manner as done with the Problem 1.



2. Now assume that you are free to choose the capacitor value. what value should be chosen so that final conditions for both the turn on and the turn off transient are established as quickly as possible?

   We want $Q_{CB} = 130\text{pC}$ to completely cancel $Q_{B0} + q_S(\infty)$.

$$C_B = \frac{Q_{CB}}{10\text{V}} = 13\text{pF}$$

Figure 1

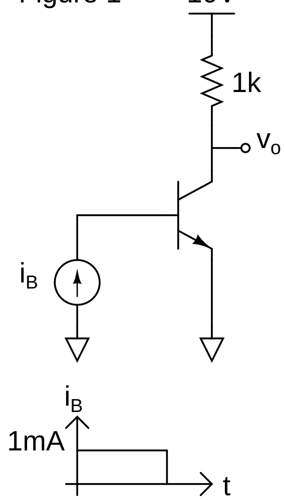
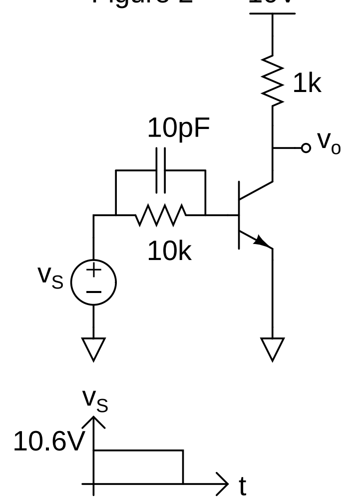


Figure 2



<u>For both circuits:</u> $\tau_F = 2ns$ $\quad\beta_F = 200$
$\tau_R = 10ns$ $\quad\beta_R = 10$
$v_{CE(SAT)} = 0V$ $\quad v_{BE} = 0.6V$ for $i_C > 0$
Ignore SCL's

Problem 3: Charge Control

1. Initially, the circuit shown in Figure 3 is assumed to be in equilibrium with the switch open. Calculate: q_F, q_S, i_C.

$$i_{c,max} = I_B \beta_F = 10\text{mA}$$
$$i_{c(\text{SAT})} = 36\text{V}/6\text{k}\Omega = 6\text{mA}$$
$$q_F = I_{c(\text{SAT})}\tau_F = 6\text{pC}$$

overdrive:

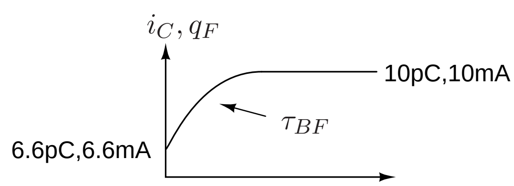
$$I_{OD} = I_B - i_{B0} = I_B - \frac{i_{c(\text{SAT})}}{\beta_F} = 0.1\text{mA} - 60\mu\text{A} = 40\mu\text{A}$$

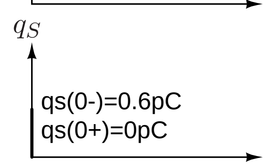
$$q_S(\infty) = I_{OD}\tau_S, \qquad \tau_S = \frac{\tau_{BF}(\beta_R + 1) + \tau_{BR}\beta_F}{\beta_F + \beta_R + 1} = 15.1\text{ns}$$
$$q_S(\infty) = I_{OD}\tau_S = 604\text{fC}$$

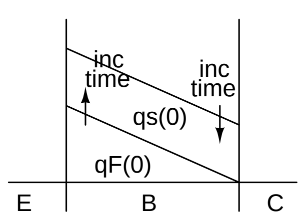
2. At $t = 0$, the switch is closed, assume instantaneous slosh. Also assume that, after the slosh transition, the circuit is quasistatic. Sketch and dimension graphs of $q_F(t)$, $q_S(t)$ and $i_C(t)$. Be sure to indicate relevant slopes, initial and final values, time constants, etc. Approximately sketch the distribution of charge in the base. Clearly illustrate the slosh transition and the equilibrium distribution.

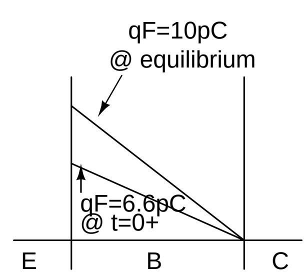
Slosh happens instantly when we come out of saturation... the total base charge is transferred to q_F

$$q_F(0^+) = q_F(0^-) + q_S(0^-) = 6.6\text{pC}$$









<u>Assume:</u> $\tau_{BF} = 100ns$ $\beta_F = 100$
$\tau_{BR} = 10ns$ $\beta_R = 5$
$v_{CE(SAT)} = 0V$
Ignore SCL's

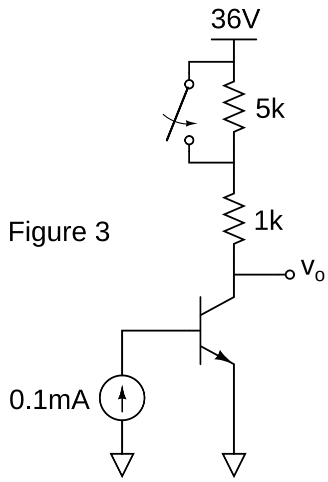
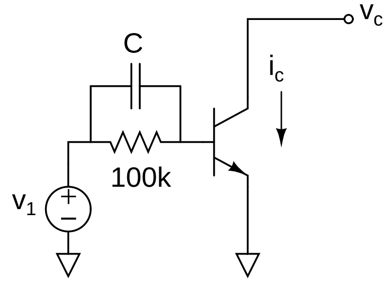


Figure 3

Problem 4: A transistor is connected in the circuit shown below.



The voltage v_1 has made a 0 to $10.6V$ step. The approximate collector current waveform that results is $i_C(t) = 1mA + 2mA(1 - e^{-t/100ns})$, $t > 0$. Determine the parameters τ_F, τ_{BF}, and β for the transistor. Also find the value of capacitor C. You may assume that $v_{BE} = 0.6V$ for any significant value of i_C.

Since i_c is proportional to q_F and q_F changes with a time constant of τ_{BF}, by inspection of

$$i_c = 1\text{mA} + 2\text{mA}(1 - e^{-t/100\text{ns}})$$

$$\tau_{BF} = 100\text{ns}$$

Furthermore,

$$\beta_F = \frac{i_c(\infty)}{i_B(\infty)} = \frac{3\text{mA}}{0.1\text{mA}} = 30$$

$$\tau_F = \tau_{BF}/\beta_F = 3.33\text{ns}$$

at $t = 0$, i_c "instantaneously" jumps to 1mA. This corresponds to a Δq_F of

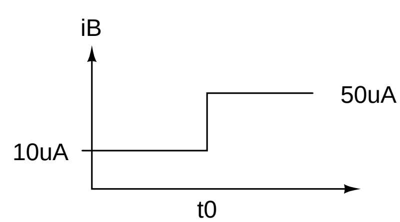
$$\Delta q_F = \Delta i_c \tau_F = (1\text{mA})(3.33\text{ns}) = 3.33\text{pC}$$

which is caused by the charge dump from $C\Delta V$

$$C = \Delta q_F/\Delta V = 3.33\text{pF}/10\text{V} = 0.33\text{pF}$$

Problem 5: The circuit shown below is driven by a voltage source, $v_s(t)$. You may assume that $v_{BE} = 0.6V$ for any I_C, when the transistor is in the forward active region. All space charge layer capacitances are zero.

1. Assume that $C = 0$. Sketch and dimension $v_o(t)$ and $q_B(t)$. Note that $q_b(t)$ is the excess minority charge in the base.



$$i_{c,max} = \beta_F i_{Bmax} = (100)(50\mu\text{A}) = 5\text{mA} \qquad \rightarrow \qquad \text{No Saturation}$$

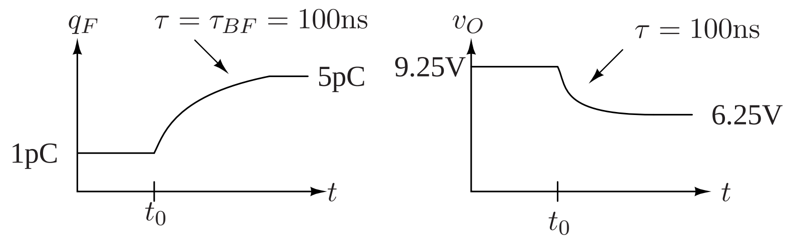
$$i_B = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt}$$

$$\begin{aligned} q_F(\infty) &= i_B \tau_{BF} \\ &= i_B \beta_F \tau_F \\ &= (50\mu\text{A})(100)(1\text{ns}) \\ &= 5\text{pC} \end{aligned}$$

$$i_C = \frac{q_F}{\tau_F}$$

$$i_C(0) = 1\text{mA}$$

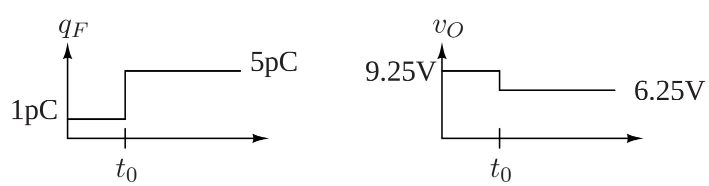
$$i_C(\infty) = 5\text{mA}$$



2. If $C = 1pF$, sketch and dimension $v_o(t)$ and $q_B(t)$. You may assume that the quasistatic approximation is valid.

$$q_{\text{dump}} = CdV = 1\text{pF} \times 4\text{V} = 4\text{pC}$$

This is exactly what we need to charge the base!



3. Suppose that when you assembled the circuit, you inadvertently inserted the transistor upside down. The emitter and collector leads are now reversed. Recalculate parts 1 and 2 for $v_o(t)$ and $q_R(t)$.

Still not saturated: $i_{c,\max} = \beta_R i_{B,\max} = (5)(50\mu\text{A}) = 250\mu\text{A}$

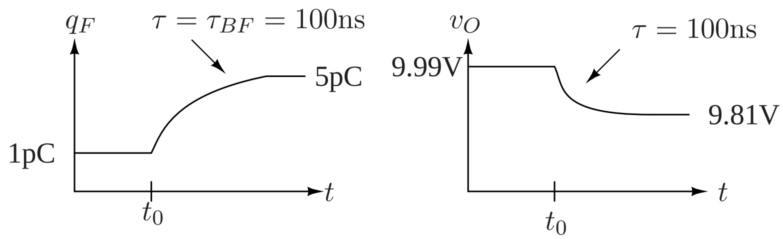
$$i_B = \frac{q_R}{\tau_{BR}} + \frac{dq_R}{dt}$$

$$\begin{aligned} q_R(\infty) &= i_B \tau_{BR} = i_B \beta_R \tau_R \\ &= (50\mu\text{A})(5)(20\text{ns}) \\ &= 5\text{pC} \end{aligned}$$

(this is the same)

$$i_c(0) = 50\mu\text{A}$$
$$i_c(\infty) = 250\mu\text{A}$$

These are smaller!



<u>Assume:</u> $\tau_F = 1ns \quad \beta_F = 100$
$\tau_R = 20ns \quad \beta_R = 5$