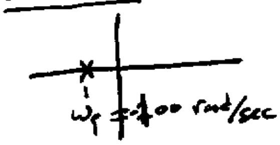
Pset 1 Solutions

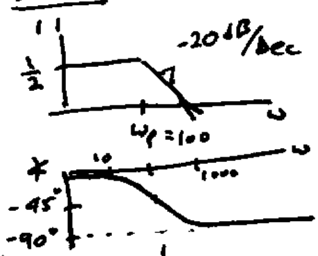
1) a) $\frac{V_2}{V_1}(s) = \frac{R \| \frac{1}{sC}}{R + R \| \frac{1}{sC}} = \frac{R}{R^2CS + 2R} = \frac{1}{RCS+2} = \frac{\frac{1}{2}}{\frac{RC}{2}s+1}$

Pole @ $\frac{2}{RC} = 100$ rad/sec

Pole-Zero



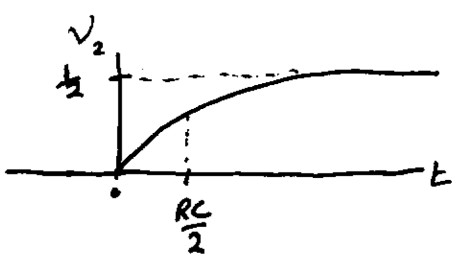
Bode



Step Response

$\mathcal{L}^{-1}\left(\frac{1}{s} \cdot \frac{\frac{1}{2}}{\frac{RC}{2}s+1}\right) = \mathcal{L}^{-1}\left(\frac{1}{s} \cdot \frac{\frac{1}{2} \cdot \frac{2}{RC}}{s + \frac{RC}{2}}\right)$, which is the form $\frac{A\alpha}{s(s+\alpha)}$

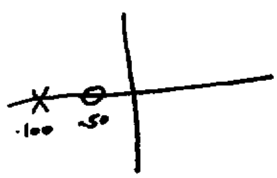
$= u(t)\frac{1}{2}(1-e^{-\alpha t}) = \frac{1}{2}(1-e^{-t/\tau}) = \frac{1}{2}(1-e^{-t/100})u(t)$



b) $\frac{V_3}{V_1}(s) = \frac{R}{R + \frac{R}{RCs+1}} = \frac{RCs+1}{RCs+2} = \frac{1}{2}\frac{RCs+1}{\frac{RC}{2}s+1} = \frac{1}{2}\frac{\frac{s}{50}+1}{\frac{s}{100}+1}$

$\omega_p = 100$ rad/sec $\omega_z = 50$ rad/sec

Pole-Zero



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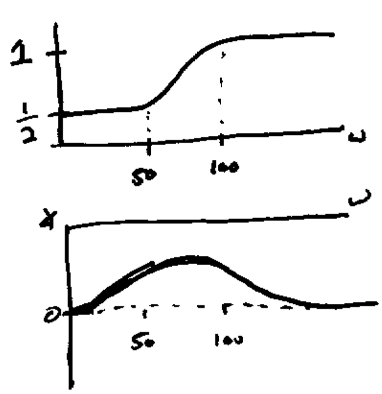


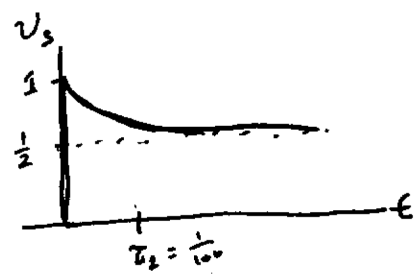
STEP Response

$\mathcal{L}^{-1}\left(\frac{A}{s} \cdot \frac{\tau_1 s+1}{\tau_2 s+1}\right) = \mathcal{L}^{-1}\left(\frac{A}{s} + \frac{A\left(\frac{\tau_1}{\tau_2}-1\right)}{s+\frac{1}{\tau_2}}\right)$

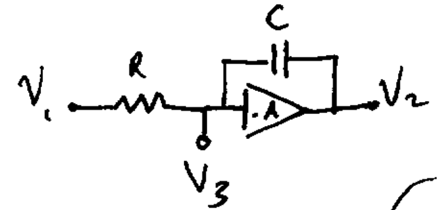
$= A\left(1 + \left(\frac{\tau_1 - \tau_2}{\tau_2}\right)e^{-t/\tau_2}\right)$

$=$ [illegible]

$= \frac{1}{2}\left(1 + \left(\frac{100}{50}-1\right)e^{-t/100}\right)$



1b)



$$\frac{V_2}{V_3} = -A \Rightarrow V_2 = -AV_3$$

$$\frac{V_2}{V_1}(s) = \frac{V_3}{V_1} \cdot \frac{V_2}{V_3}$$

$$\frac{V_3}{V_1}: \quad \frac{V_1 - V_3}{R} = \frac{V_3 - V_2}{\frac{1}{cs}} = \frac{V_3 + AV_3}{\frac{1}{cs}} = \frac{V_3(1+A)}{\frac{1}{cs}}$$

$$\frac{V_1}{R} = \frac{V_3(1+A)}{\frac{1}{cs}} + \frac{V_3}{R} = V_3\left(cs(1+A) + \frac{1}{R}\right)$$

$$\frac{V_3}{V_1} = \frac{1}{R\left(cs(1+A) + \frac{1}{R}\right)} = \frac{1}{RCs(1+A)+1}$$

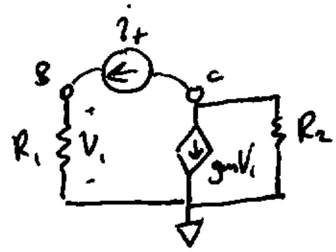
$$\boxed{\frac{V_2}{V_1} = \frac{-A}{RC(1+A)s+1}}$$

$$\boxed{\frac{V_3}{V_1} = \frac{1}{RC(1+A)s+1}}$$

$$Z_{in}(V_3) = \frac{V_3}{I_3} = \frac{V_3}{\frac{V_3 - V_2}{\frac{1}{cs}}}, \quad V_2 = -AV_3 \Rightarrow \frac{V_3}{\frac{V_3 + AV_3}{\frac{1}{cs}}} = \frac{V_3}{\frac{V_3(1+A)}{\frac{1}{cs}}} = \boxed{\frac{1}{(1+A)Cs}} = Z_{in}$$

The amplifier effectively multiplies the capacitance by its gain.
This is called miller capacitance, or the miller effect.

1c)



Use test current to establish V_1.

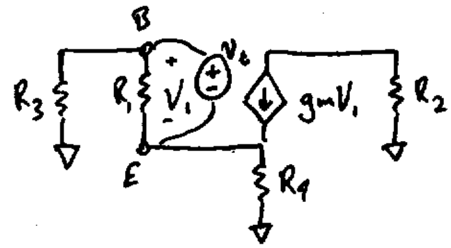
$$v_t = v_B - v_C$$

$$v_B = v_1 = i_t R_1$$

$$v_C = R_2(-g_m V_1 - i_t)$$

$$v_t = i_t R_1 + (i_t + g_m i_t R_1) R_2$$

$$\boxed{R_{BC} = \frac{v_t}{i_t} = R_1 + R_2 + g_m R_1 R_2}$$



Use test Voltage to establish V_1

$$i_t = \frac{V_e + V_t}{R_3} + \frac{V_t}{R_1}$$

$$g_m V_t - \frac{V_e}{R_4} + \frac{V_t}{R_1} - i_t = 0 \rightarrow V_e = R_4\left(\frac{V_t}{R_1} - i_t + v_t g_m\right)$$

$$\cancel{i_t = \frac{V_e}{R_4} + \frac{V_t}{R_1} + g_m V_t}$$

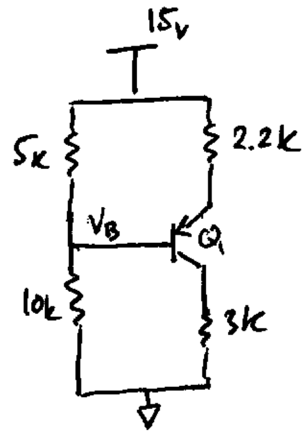
$$i_t = \frac{R_4\left(\frac{V_t}{R_1} - i_t + V_t g_m\right) + V_t}{R_3} + \frac{V_t}{R_1}$$

$$i_t + \frac{R_4}{R_3} i_t = \frac{R_4}{R_3}\left(\frac{V_t}{R_1} + V_t g_m\right) + \frac{V_t}{R_3} + \frac{V_t}{R_1} = V_t\left(\frac{R_4}{R_3}\left(\frac{1}{R_1} + g_m\right) + \frac{1}{R_3} + \frac{1}{R_1}\right)$$

$$\frac{V_t}{i_t} = \frac{1 + \frac{R_4}{R_3}}{\frac{R_4}{R_3}\left(\frac{1}{R_1} + g_m\right) + \frac{1}{R_3} + \frac{1}{R_1}} = \frac{R_3 + R_4}{\frac{R_4}{R_1} + R_4 g_m + 1 + \frac{R_3}{R_1}} = \frac{R_1(R_3 + R_4)}{R_4 + R_1 R_4 g_m + R_1 + R_3}$$

$$R_{Th} = \frac{R_1\left(\frac{R_3 + R_4}{1 + g_m R_4}\right)}{R_1 + \frac{R_3 + R_4}{1 + g_m R_4}} = \boxed{R_1 \,\|\, \left(\frac{R_3 + R_4}{1 + g_m R_4}\right)}$$

2)



$V_B = 10\,V$

$V_{BE} = -0.6\,V$

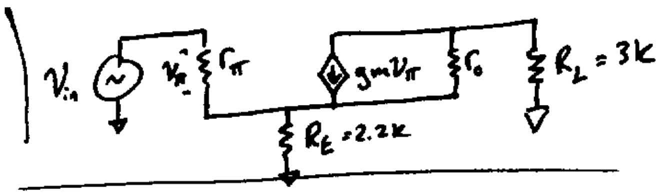
$V_E = 10.6\,V$

$V_{R_E} = 4.4\,V$

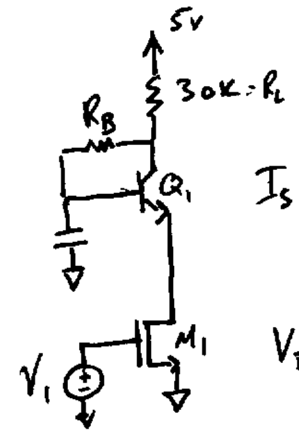
$I_{R_E} = 2.2\,mA = I_E \approx I_C$ or $I_C = (1+\frac{1}{\beta})I_E = 2.21\,mA$

$I_C = I_S e^{\frac{V_{BE}}{V_{th}}} \Rightarrow I_S = \frac{I_C}{e^{V_{EB}/V_{th}}} = \boxed{8.3\times10^{-14}\,A \text{ or } 8.31\times10^{-14}\,A}$

$$\boxed{\begin{aligned} g_m &= \frac{I_C}{V_{th}} = \frac{2.2\,mA}{.025\,V} = 0.088\,\mho \\ r_\pi &= \frac{\beta}{g_m} = \frac{200}{0.088\,\mho} = 2272\,\Omega \\ r_o &= \frac{V_A}{I_C} = \frac{80\,V}{2.2\,mA} = 36.3\,k\Omega \end{aligned}}$$



3)



$I_S = 10^{-15}A$, $V_A = \infty$, $V_{CEsat} = 0.2V$, $\beta = 200$

$V_T = 0.6V$, $\mu_n C_{ox} \frac{W}{L} = 200 \frac{\mu A}{V^2}$, $\lambda = 0$

a) $I_{D_1} \approx I_{C_1} = 60\mu A$

In Saturation,

$I_{D_1} = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)^2 = \frac{1}{2}\, 200\mu A/V^2 \,(V_1 - 0.6V)^2 = 60\mu A$

$\boxed{V_1 = 1.374V}$; or, with β considered : $\boxed{V_1 = 1.375}$

no real difference.

b) $V_C = V_o = V_{CC} - (I_{Cq} + I_{B2})R_L = \boxed{3.2V \text{ or } 3.19V}$

c) Q_1 is always in the FAR.

For M_1, $V_{DS} \geq V_{GS} - V_T \geq 1.37V - 0.6V \geq 0.77V$

$V_{DS_1} = V_o - R_B I_{B_1} - V_{BE_1} = 3.2V - R_B \frac{60\mu A}{\beta} - 0.6V \geq 0.77$

$\frac{3.2V - 0.77V - 0.6V}{.3\mu A} > R_B \rightarrow \boxed{R_B < 6.1 M\Omega}$