

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.301 Solid State Circuits

Fall 2013
Problem Set 8 Solutions

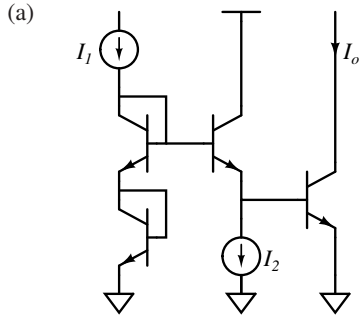
Issued : Nov 19, 2013
Due : Nov 26, 2013

Problem 1: Translinear Jungle Gym

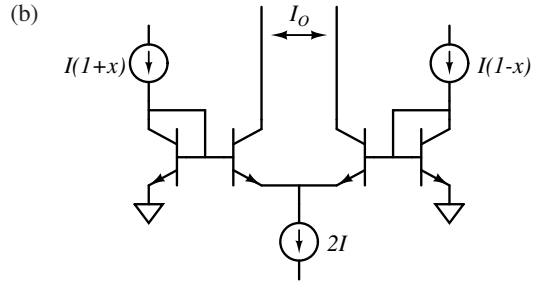
For each of the following circuits use the Gilbert Principle to determine I_o as a function of the other circuit variables. All of these circuits simplify to simple expressions.

A differential output is denoted by an I_o superimposed on an arrow, and double emitter arrows with $2A_E$ indicate that transistor has double the emitter area of the other transistors, thus its I_S is twice as large.

Finally, use the method of open circuit time constants to estimate the $-3dB$ frequency for the circuit in part (a) only.



$$I_1 I_1 = I_2 I_0 \rightarrow I_0 = \frac{I_1^2}{I_2}$$



Write I_0 as $I(1+y)$ and $I(1-y)$. Then,

$$I(1+x)I(1-y) = I(1+y)I(1-x)$$

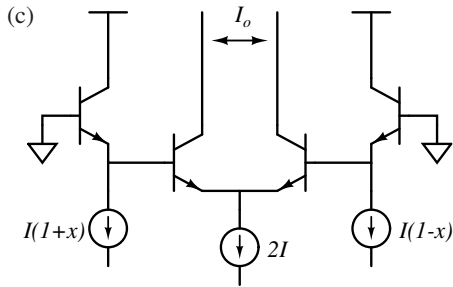
$$1-y+x-xy = 1+y-x-xy$$

$$x = y$$

The output currents are $I(1+x)$ and $I(1-x)$ so

$$I_0 = I(1+x) - I(1-x)$$

$$I_0 = 2x$$



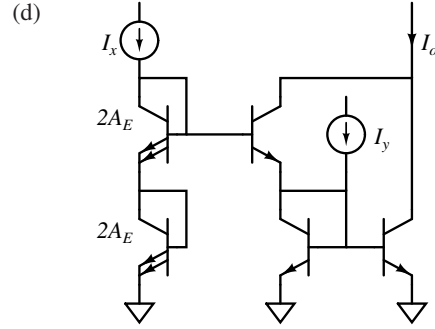
$$I(1+x)I(1+y) = I(1-y)I(1-x)$$

$$1+x+y+xy = 1-x-y+xy$$

$$2x = -2y$$

$$x = -y$$

$$I_0 = I(1-x) - I(1+x) = -2x$$



First find I_3 in terms of I_0 and I_y :

$$I_3 = I_0 - I_y, \text{ and}$$

$$I_3 = I_0 - I_3 - I_y$$

$$2I_3 = I_0 - I_y$$

$$I_3 = \frac{I_0 - I_y}{2}$$

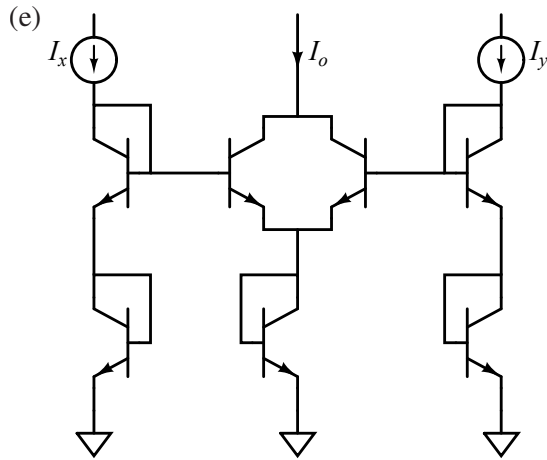
From the Gilbert loop in the left four transistors, we know that

$$\frac{I_x^2}{4} = I_3(I_3 + I_y)$$

$$\frac{I_x^2}{4} = \frac{I_0 - I_y}{2} \cdot \frac{I_0 + I_y}{2}$$

$$\frac{I_x^2}{4} = \frac{I_0^2 - I_y^2}{4}$$

$$I_0^2 = I_x^2 + I_y^2$$



We know the current through Q5 (the transistor connected to the emitters of the diff pair) is I_o , but we don't know the currents through the diff pair transistors, I_6 and I_7 . From the left side loop:

$$I_x^2 = I_6 I_o$$

$$I_6 = \frac{I_x^2}{I_o}$$

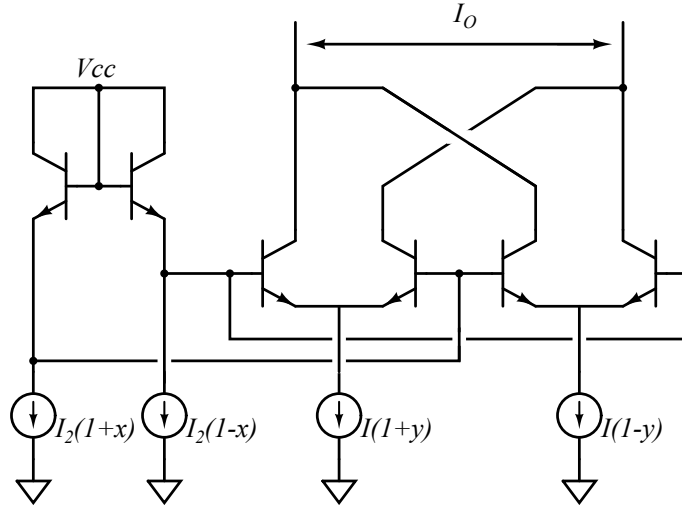
From the right side Gilbert loop we can similarly write that $I_7 = \frac{I_y^2}{I_o}$. Put them together:

$$I_o = I_6 + I_7$$

$$= \frac{I_x^2}{I_o} + \frac{I_y^2}{I_o}$$

$$I_o^2 = I_x^2 + I_y^2$$

(f)



Name the transistors Q1 through Q6 from left to right. Write the output currents as $I(1-z)$ and $I_O(1+z)$. Using the Gilbert loop formed by Q1, Q2, Q3, and Q4:

$$\begin{aligned}
 I_2(1+x)I_y &= I_2(1-x)I_3 \\
 I_2(1+x)(I(1+y) - I_3) &= I_2(1-x)I_3 \\
 I(1+y) + xI(1+y) - I_3 - xI_3 &= I_3 - xI_3 \\
 I(1+y+x+xy) &= 2I_3 \\
 I_3 &= \frac{1}{2}I(1+y)(1+x)
 \end{aligned}$$

Using the Gilbert loop from Q1, Q2, Q5, and Q6, we can similarly argue that

$$I_5 = \frac{1}{2}I(1-x)(1-y)$$

At the output,

$$\begin{aligned}
 I_O(1+z) &= I_3 + I_5 \\
 &= \frac{1}{2}I(1+x+y+xy) + \frac{1}{2}I(1-y-x+xy) \\
 I(1+z) &= I(1+xy) \\
 z &= xy
 \end{aligned}$$

For the -3dB frequency of the circuit in part (a), assume the output node has some load impedance $R_L < r_o$. This is reasonable because a current-source input load would look like $\frac{1}{gm}$ and a resistive load would likely be smaller than r_o . For the worst-case OCT's, R_π is $\frac{1}{gm}$ for all transistors. For the diode-connected transistors, R_μ is 0 since the base is shorted to the collector. The output transistor's $R_\mu = \frac{1}{gm} + 2R_o$. The middle transistor's $R_\mu = \frac{2}{gm}$.

$$\tau = \frac{4C_\pi}{gm} + C_\mu \left(\frac{3}{gm} + 2R_o \right) \quad (1)$$

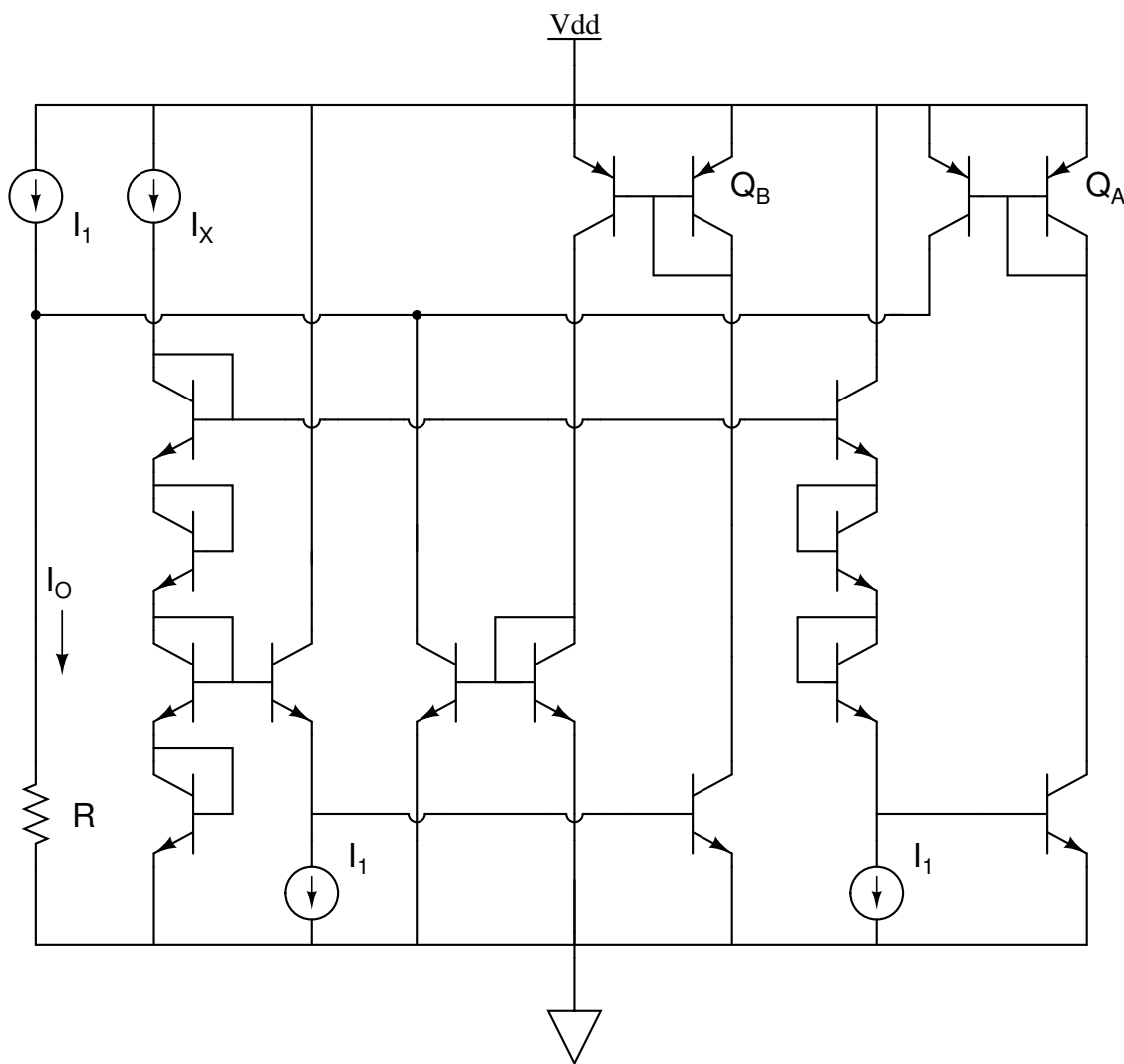
$$f_{-3dB} = \frac{gm}{2\pi(4C_\pi + (3 + 2gmR_o)C_\mu)} \quad (2)$$

This circuit is *fast*.

Problem 2: Translinear Approximator

Find $I_o = f(I_x)$, assuming well-matched transistors, negligible base currents, and $I_1 = 1\text{A}$. Also assume Q_A and Q_B have emitter areas $24A_E$ and $2A_E$, respectively, while all other transistors have emitter area A_E .

What famous function does I_o approximate for small I_x ?



Solution:

Call the current through Q_A and Q_B I_A and I_B , respectively. The output current is:

$$I_o = I_1 - \frac{I_A}{24} - \frac{I_B}{2} \quad (3)$$

We can find I_A from the Gilbert loop:

$$I_x^4 = I_1^3 I_A$$

$$I_A = \frac{I_x^4}{I_1^3}$$

and I_B :

$$I_x^2 = I_i I_B$$

$$I_B = \frac{I_x^2}{I_1}$$

Substituting into (3)

$$I_o = I_1 - \frac{I_x^2}{2I_1} + \frac{I_x^4}{24I_1^3} \quad (4)$$

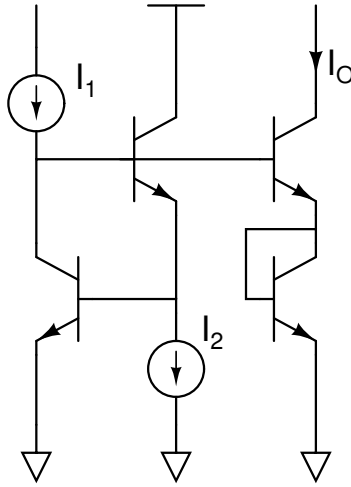
When $I_1 = 1$, (4) becomes:

$$I_o = 1 - \frac{I_x^2}{2} + \frac{I_x^4}{24}$$

Which is the first two terms of the Taylor series expansion of cosine.

Problem 3: Base Current Error

In the following circuit, assume $I_2 = 1mA$ and $\beta = 100$.



- (a) Express I_o in terms of I_1 and I_2 .
This is a simple Gilbert loop.

$$\begin{aligned} I_{c3}I_{c4} &= I_{c1}I_{c2} \\ I_oI_o &= I_1I_2 \\ I_{o,ideal} &= \sqrt{I_1I_2} \end{aligned}$$

- (b) Assume we can tolerate a maximum I_o error due to β of 50%. For what range of I_1 is this circuit valid? With finite β we need to consider the effects of base current.

$$I_{o,real} = \sqrt{I_{c1}I_{c2}}$$

$I_{o,real}$ should never exceed $I_{o,ideal}$, so $\frac{\sqrt{I_{c1}I_{c2}}}{\sqrt{I_1I_2}} = \frac{1}{2}$ should have at least two solutions which will provide the range of I_1 .

$$I_{c1} + \frac{I_{c2} + I_o}{\beta} = I_1 \rightarrow I_{c1} = I_1 - \frac{I_{c2} + \sqrt{I_{c1}I_{c2}}}{\beta} \quad (5)$$

$$I_{c2} = (I_2 - \frac{I_{c1}}{\beta}) \frac{\beta}{\beta + 1} \quad (6)$$

Substituting equation (1) in to equation (2) gives:

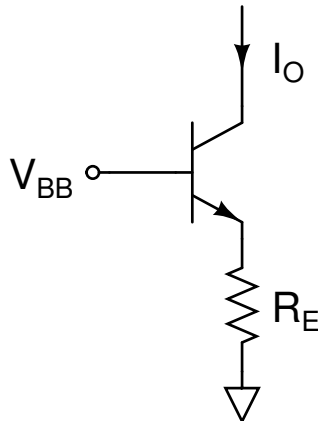
$$I_{c1} = I_1 - \left(\frac{I_2}{\beta + 1} - \frac{I_{c1}}{\beta(\beta + 1)} \right) - \frac{\frac{\beta}{\beta + 1}I_2I_{c1} - \frac{I_{c1}^2}{\beta + 1}}{\beta}$$

Solve for I_{c1} in terms of I_1 and $I_2 = 1\text{mA}$:

$$I_{c1} = \frac{5050(\sqrt{-101(I_1^2 - 0.1I_1 + 987E - 9)} + 10099(I_1 - 9.9E - 6))}{50994951}$$

Problem 4: Temperature Dependence and Compensation

When we design a circuit, we prefer that it operate over a wide range of temperature. Below is a voltage-biased current source with a temperature dependence heavily based on R_E and V_{be} . In the following circuit, assume that $\frac{1}{R} \frac{dR}{dT} = 600\text{ppm}/^\circ\text{C}$ and $\frac{dV_{be}}{dT} = -2\text{mV}/^\circ\text{C}$.



- (a) Find $\frac{dI_o}{dT}$.
- (b) Find the value of R_E in terms of I_o that minimizes $\frac{dI_o}{dT}$.

Problem 5: Your Own Personal Bandgap

Referencing the simplified schematic of the LM109 5V Bandgap Reference from lecture, design a modified version that produces an output voltage of 7.5V.

Explain the operation of your reference.