MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Electrical Engineering and Computer Science

6.301 Solid State Circuits

Fall 2013 | Issued : Nov 19, 2013

Problem Set 8 Solutions | Due : Nov 26, 2013

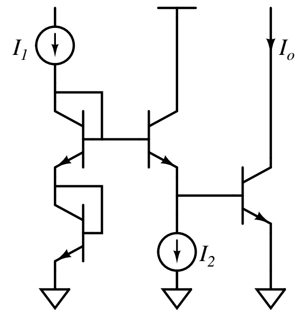
Problem 1: Translinear Jungle Gym

For each of the following circuits use the Gilbert Principle to determine I_o as a function of the other circuit variables. All of these circuits simplify to simple expressions.

A differential output is denoted by an I_o superimposed on an arrow, and double emitter arrows with $2A_E$ indicate that transistor has double the emitter area of the other transistors, thus its I_S is twice as large.

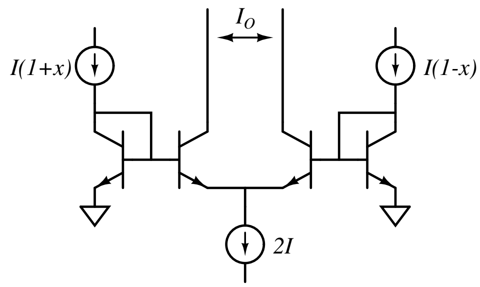
Finally, use the method of open circuit time constants to estimate the $-3dB$ frequency for the circuit in part (a) only.

(a)



$$I_1 I_1 = I_2 I_\mathsf{o} \longrightarrow I_\mathsf{o} = \frac{\mathsf{I}_1^2}{\mathsf{I}_2}$$

(b)



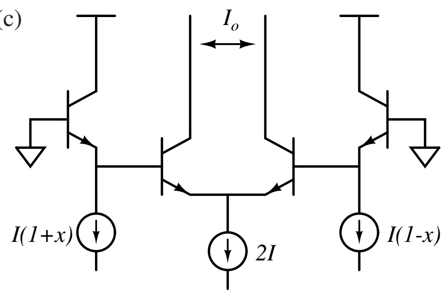
Write I_o as $I(1+y)$ and $I(1-y)$. Then,

$$I(1+x)I(1-y) = I(1+y)I(1-x)$$
$$1 - y + x - xy = 1 + y - x - xy$$
$$x = y$$

The output currents are $I(1+x)$ and $I(1-x)$ so

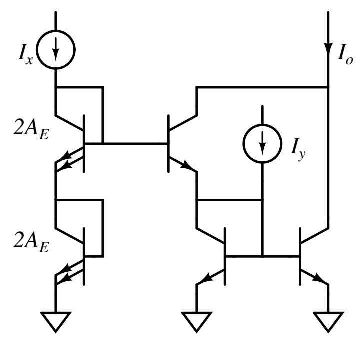
$$I_\mathsf{o} = I(1+x) - I(1-x)$$
$$I_\mathsf{o} = 2x$$

(c)



$$I(1+x)I(1+y) = I(1-y)I(1-x)$$
$$1 + x + y + xy = 1 - x - y + xy$$
$$2x = -2y$$
$$x = -y$$
$$I_\mathsf{o} = I(1-x) - I(1+x) = -2x$$

(d)



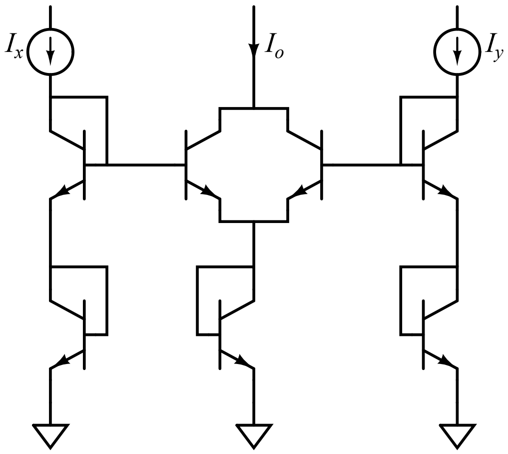
First find I_3 in terms of I_o and I_y:

$$I_3 = I_\mathsf{o} - I_\mathsf{y}, \text{ and}$$
$$I_3 = I_\mathsf{o} - I_3 - I_\mathsf{y}$$
$$2I_3 = I_\mathsf{o} - I_\mathsf{y}$$
$$I_3 = \frac{I_\mathsf{o} - I_\mathsf{y}}{2}$$

From the Gilbert loop in the left four transistors, we know that

$$\frac{I_\mathsf{x}^2}{4} = I_3(I_3 + I_\mathsf{y})$$
$$\frac{I_\mathsf{x}^2}{4} = \frac{I_\mathsf{o} - I_\mathsf{y}}{2} \quad \frac{I_\mathsf{o} + I_\mathsf{y}}{2}$$
$$\frac{I_\mathsf{x}^2}{4} = \frac{I_\mathsf{o}^2 - I_\mathsf{y}^2}{4}$$
$$I_\mathsf{o}^2 = I_\mathsf{x}^2 + I_\mathsf{y}^2$$

(e)



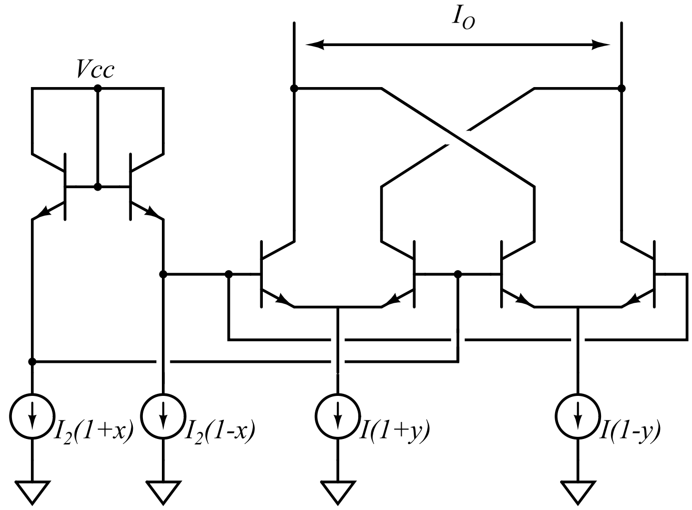
We know the current through Q5 (the transistor connected to the emitters of the diff pair) is I_o, but we don't know the currents through the diff pair transistors, I_6 and I_7. From the left side loop:

$$I_x^2 = I_6 I_o$$
$$I_6 = \frac{I_x^2}{I_o}$$

From the right side Gilbert loop we can similarly write that $I_7 = I_y^2 \triangleleft I_o$. Put them together:

$$\begin{aligned} I_o &= I_6 + I_7 \\ &= \frac{I_x^2}{I_o} + \frac{I_y^2}{I_o} \\ I_o^2 &= I_x^2 + I_y^2 \end{aligned}$$

(f)



Name the transistors Q1 throught Q6 from left to right. Write the output currents as $I(1-z)$ and $I_{\mathsf{o}}(1+z)$. Using the Gilbert loop formed by Q1, Q2, Q3, and Q4:

$$\begin{aligned}
I_2(1+x)I_{\mathsf{y}} &= I_2(1-x)I_3 \\
I_2(1+x)(I(1+y)-I_3) &= I_2(1-x)I_3 \\
I(1+y)+xI(1+y)-I_3-xI_3 &= I_3-xI_3 \\
I(1+y+x+xy) &= 2I_3 \\
I_3 &= \frac{1}{2}I(1+y)(1+x)
\end{aligned}$$

Using the Gilbert loop from Q1, Q2, Q5, and Q6, we can similarly argue that

$$I_5 = \frac{1}{2}I(1-x)(1-y)$$

At the output,

$$\begin{aligned}
I_{\mathsf{o}}(1+z) &= I_3+I_5 \\
&= \frac{1}{2}I(1+x+y+xy)+\frac{1}{2}I(1-y-x+xy) \\
I(1+z) &= I(1+xy) \\
z &= xy
\end{aligned}$$

For the -3dB frequency of the circuit in part (a), assume the output node has some load impedance $R_L < r_o$. This is reasonable because a current-source input load would look like $\frac{1}{gm}$ and a resistive load would likely be smaller than r_o. For the worst-case OCT's, R_π is $\frac{1}{gm}$ for all transistors. For the diode-connected transistors, R_μ is 0 since the base is shorted to the collector. The output transistor's $R_\mu = \frac{1}{gm} + 2R_o$. The middle transistor's $R_\mu = \frac{2}{gm}$.

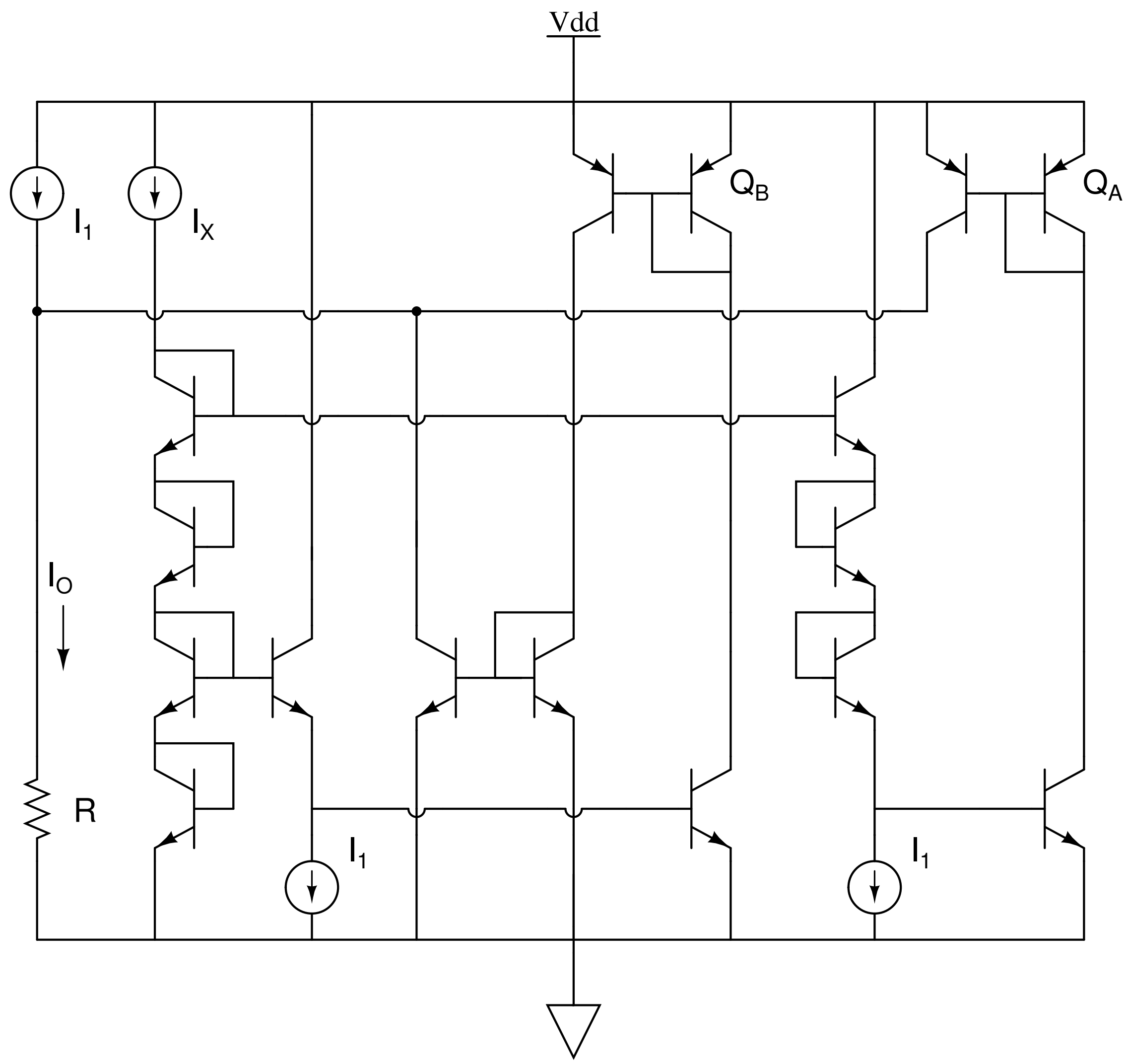
$$\tau = \frac{4C_\pi}{gm} + C_\mu(\frac{3}{gm} + 2R_o) \quad (1)$$

$$f_{-3dB} = \frac{gm}{2\pi(4C_\pi + (3 + 2gmR_o)C_\mu)} \quad (2)$$

This circuit is *fast*.

Problem 2: Translinear Approximator

Find $I_o = f(I_x)$, assuming well-matched transistors, negligible base currents, and $I_1 = 1A$. Also assume Q_A and Q_B have emitter areas $24A_E$ and $2A_E$, respectively, while all other transistors have emitter area A_E.
What famous function does I_o approximate for small I_x?



Solution:

Call the current through Q_A and Q_B I_A and I_B, respectively. The output current is:

$$I_0 = I_1 - \frac{I_A}{24} - \frac{I_B}{2} \tag{3}$$

We can find I_A from the Gilbert loop:

$$I_x^4 = I_1^3 I_A$$
$$I_A = \frac{I_x^4}{I_1^3}$$

and I_B:

$$I_x^2 = I_i I_B$$
$$I_B = \frac{I_x^2}{I_1}$$

Substituting into (3)

$$I_o = I_1 - \frac{I_x^2}{2I_1} + \frac{I_x^4}{24 I_1^3} \tag{4}$$

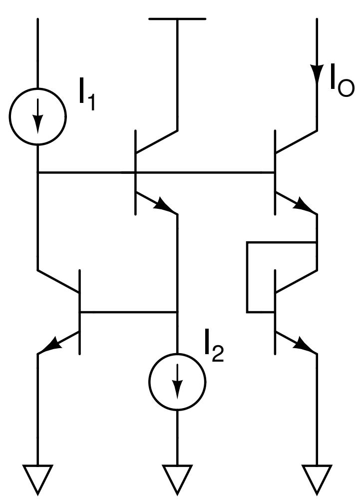
When $I_1 = 1$, (4) becomes:

$$I_o = 1 - \frac{I_x^2}{2} + \frac{I_x^4}{24}$$

Which is the first two terms of the Taylor series expansion of cosine.

Problem 3: Base Current Error

In the following circuit, assume $I_2 = 1mA$ and $\beta = 100$.



(a) Express I_o in terms of I_1 and I_2.
This is a simple Gilbert loop.

$$I_{c3}I_{c4} = I_{c1}I_{c2}$$
$$I_oI_o = I_1I_2$$
$$I_{o,ideal} = sqrtI_1I_2$$

(b) Assume we can tolerate a maximum I_o error due to β of 50%. For what range of I_1 is this circuit valid? With finite β we need to consider the effects of base current.

$$I_{o,real} = sqrtI_{c1}I_{c2}$$

$I_{o,real}$ should never exceed $I_{o,ideal}$, so $\frac{\sqrt{I_{c1}I_{c2}}}{\sqrt{I_1I_2}} = \frac{1}{2}$ should have at least two solutions which will provide the range of I_1.

$$I_{c1} + \frac{I_{c2} + I_o}{\beta} = I_1 \rightarrow I_{c1} = I_1 - \frac{I_{c2} + \sqrt{I_{c1}I_{c2}}}{\beta} \tag{5}$$

$$I_{c2} = (I_2 - \frac{I_{c1}}{\beta})\frac{\beta}{\beta+1} \tag{6}$$

Substituting equation (1) in to equation (2) gives:

$$I_{c1} = I_1 - (\frac{I_2}{\beta+1} - \frac{I_{c1}}{\beta(\beta+1)}) - \frac{\frac{\beta}{\beta+1}I_2I_{c1} - \frac{I_{c1}^2}{\beta+1}}{\beta}$$

Solve for I_{c1} in terms of I_1 and $I_2 = 1\text{mA}$:

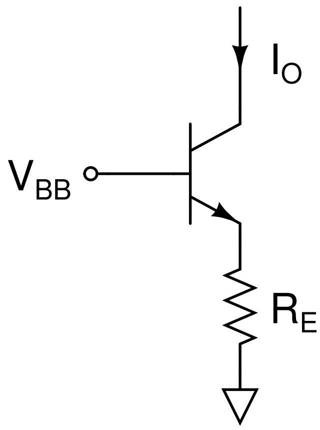
$$I_{c1} = \frac{5050(\sqrt{-101(I_1^2 - 0.1I_1 + 987 \times 10^{-9})} + 10099(I_1 - 9.9 \times 10^{-6}))}{50994951}$$

Solving $\frac{\sqrt{I_{c1}I_{c2}}}{\sqrt{I_1I_2}} = \frac{1}{2}$ for I_1:

$$12.5\mu A < I_1 < 74.4mA$$

Problem 4: Temperature Dependence and Compensation

When we design a circuit, we prefer that it operate over a wide range of temperature. Below is a voltage-biased current source with a temperature dependence heavily based on R_E and V_{be}. In the following circuit, assume that $\frac{1}{R}\frac{dR}{dT} = 600ppm/^\circ C$ and $\frac{dV_{be}}{dT} = -2mV/^\circ C$.



(a) Find $\frac{dI_o}{dT}$.

Assume $I_B = 0$. Then $I_o = I_E = \frac{V_{BB}-V_{BE}}{R_E}$.

$$
\begin{aligned}
\frac{dI_o}{dT} &= \frac{d}{dT}(\frac{V_{BB}}{R_E}) - \frac{d}{dT}(\frac{V_{BE}}{R_E}) \\
&= \frac{-V_{BB}}{R_E^2}\frac{dR_E}{dT} - \frac{R_E\frac{dV_{BE}}{dT} - V_{BE}\frac{dR_E}{dT}}{R_E^2} \\
&= -\frac{V_{BB}}{R_E}(\frac{1}{R_E}\frac{dR_E}{dT}) + \frac{V_{BE}}{R_E}(\frac{1}{R_E}\frac{dR_E}{dT}) - \frac{1}{R_E}(\frac{dV_{BE}}{dT}) \\
&= -I_o(\frac{1}{R_E}\frac{dR_E}{dT}) - \frac{1}{R_E}(\frac{dV_{BE}}{dT})
\end{aligned}
$$

(b) Find the value of R_E in terms of I_o that minimizes $\frac{dI_o}{dT}$.

$$
\begin{aligned}
\frac{dI_o}{dT} &= -I_o(\frac{1}{R_E}\frac{dR_E}{dT}) - \frac{1}{R_E}(\frac{dV_{BE}}{dT}) \\
&= -I_o(600 \times 10^{-6}/^\circ C) - \frac{1}{R_E}(-2 \times 10^{-3}V/^\circ C) \\
&\rightarrow R_{E_{min}} = \frac{3.33}{I_o}
\end{aligned}
$$