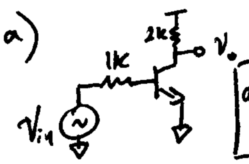
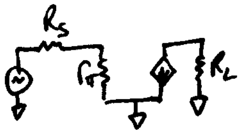
Pset 4 solutions

1) $\beta = 200,\ I_c = 2.5mA,\ C_\pi = 50pF,\ C_\mu = 2pF \quad r_b = 0,\ r_o = \infty \Big\} \to g_m = \frac{I_c}{V_t} = 0.1℧,\ r_\pi = \frac{\beta}{g_m} = 2K$

a)



$$\boxed{a_v = -\frac{r_\pi}{R_s + r_\pi} g_m R_L = -133.3}$$

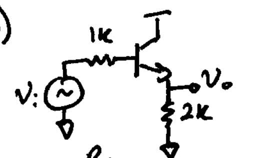


$r_{\pi_o} = r_\pi \| R_s = 666.6\Omega \to \tau_{\pi_o} = C_\pi r_{\pi_o} = 33.3ns$

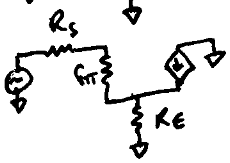
$r_{\mu_o} = r_\pi \| R_s + R_L + g_m R_L\, r_\pi \| R_s \approx 136K \to \tau_{\mu_o} = C_\mu r_{\mu_o} = 272ns$

$$\boxed{f_{-3dB} = \frac{1}{2\pi(\tau_{\mu_o} + \tau_{\pi_o})} = 521kHz}$$

b)



$$\boxed{a_v = \frac{(\beta+1)R_E}{(\beta+1)R_E + r_\pi + R_s} = 0.993}$$

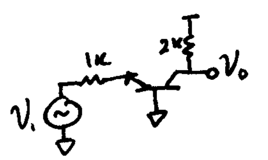


$r_{\pi_o} = r_\pi \| \frac{R_s + R_E}{1 + g_m R_E} = 14.8\Omega \to \tau_{\pi_o} = .74ns$

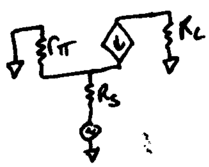
$r_{\mu_o} = R_s \| (r_\pi + (\beta+1)R_E) = 997\Omega \to \tau_{\mu_o} = 2ns$

$$\boxed{f_{-3dB} = \frac{1}{2\pi\, 2.74ns} = 58.1MHz}$$

c)



$$\boxed{a_v = \frac{\beta R_L}{(\beta+1)R_s + r_\pi} = 1.97}$$

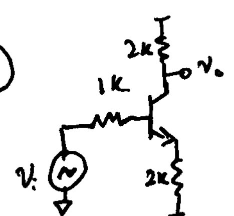


$r_{\pi_o} = r_\pi \| \frac{R_s}{1 + g_m R_s} = 9.9\Omega \to \tau_{\pi_o} = .493ns$

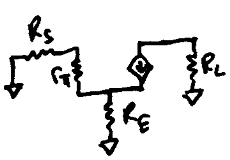
$r_{\mu_o} = R_L = 2K \to \tau_{\mu_o} = 4ns$

$$\boxed{f_{-3dB} = \frac{1}{2\pi \cdot 4.493ns} = 35.4MHz}$$

d)



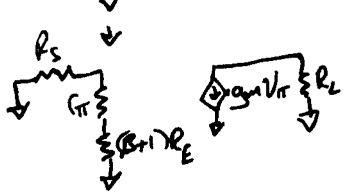
$$\boxed{a_v = \frac{-g_m R_L r_\pi}{r_\pi + R_s + (\beta+1)R_E} = -0.99}$$



$r_{\pi_o} = r_\pi \| \left(\frac{R_s + R_E}{1 + g_m R_E}\right) = 14.8\Omega \to \tau_{\pi_o} = .74ns$

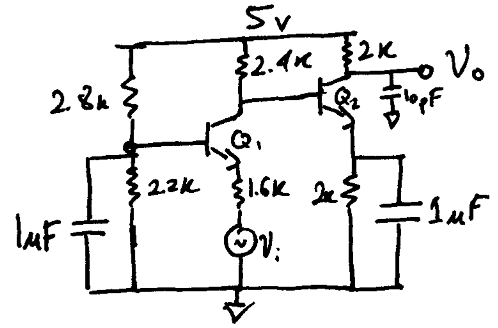
$r_{\mu_o} = R_s \| (r_\pi + (\beta+1)R_E) + R_L + \left(\frac{g_m r_\pi}{r_\pi + (\beta+1)R_E}\right)(R_s \| (r_\pi + (\beta+1)R_E))R_L = 3985\Omega$

$\hookrightarrow \tau_{\mu_o} = 7.97ns$



$$\boxed{f_{-3dB} = \frac{1}{2\pi(8.71ns)} = 18.27MHz}$$

2)



$V_{BE} = 0.6v$ $\beta = 200$ $C_\pi = 20pF$ $C_\mu = 2pF$ $r_b = 0$ $r_o = \infty$

a) Midband Voltage gain:

$V_{B_1} = 2.2v$

$V_{E_1} = 2.2v - 0.6v = 1.6v$

$I_{C_1} \approx \frac{1.6v}{1.6k} = 1mA \rightarrow g_m = 0.04 \mho, r_{\pi_1} = 5k\Omega$

$V_{B_2} = 2.6v$

$V_{E_2} = 2v$

$I_{C_2} = \frac{2v}{2k} = 1ma \rightarrow g_{m_2} = 0.04 \mho$

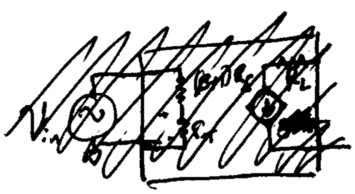
$r_{\pi_2} = 5k\Omega$

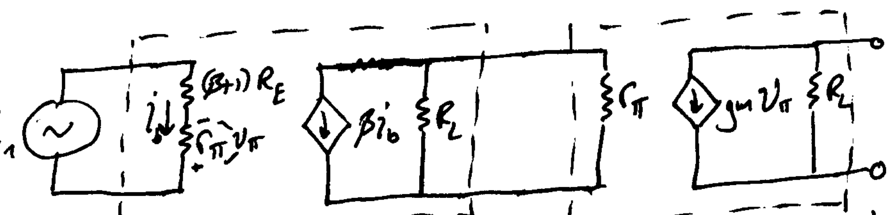
Stage 1 is a Common base Amplifier.

$a_v = \frac{\beta R_L}{(1+\beta)R_E + r_\pi}$

stage 2 is a Common Emitter amp.

$a_v = -g_m R_L$





CB:

$a_v = \frac{\beta R_L}{(1+\beta)R_E + r_\pi} \approx \frac{R_L'}{R_E}$

attenuation: $R_L' = R_L \| r_\pi$

CE:

$a_v = -g_m R_L$

$$\boxed{\text{Total gain: } -g_m R_L \times \frac{\beta (R_L \| r_\pi)}{(1+\beta)R_E + r_\pi} = -79.4}$$

b) $r_{\pi o1} = r_{\pi_1} \| \frac{R_{E_1}}{1+g_m R_{E_1}} = 24.5\Omega$

$r_{\mu o1} = R_{L_1} \| r_{\pi_2} = 1.62k\Omega$

$\tau_{\pi_1} = .49ns$

$\tau_{\mu_1} = 3.24ns$

$r_{\pi o2} = r_{\pi_2} \| R_{L_1} = 1.62k\Omega$

$r_{\mu o2} = r_{\pi_2} \| R_{L_1} + R_{L2} + g_m R_L (r_{\pi_2} \| R_{L_1}) = 133.22k\Omega$

$\tau_{\pi 2} = 32.4ns$ $\tau_{os} = (10pF)(2k\Omega) = 20ns$

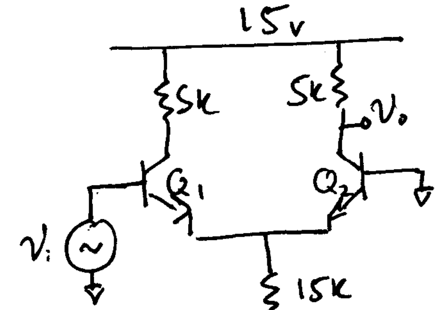
$\tau_{\mu 2} = 266.44ns$

$\sum \tau = 322.57ns \rightarrow \boxed{f_{3dB} = 494 kHz}$

3) Find a_v + f_{-3dB} for both amplifiers below

$V_{BE} = 0.6\text{v}$, $\beta = 200$, $C_\pi = 40\text{pF}$, ~~C~~ $C_\mu = 4\text{pF}$, $r_b = 0$, $r_o = \infty$

a)



$V_E = 0 - 0.6\text{v} \rightarrow I_{RE} = \dfrac{14.4\text{v}}{15\text{k}\Omega} = .96\text{mA}$

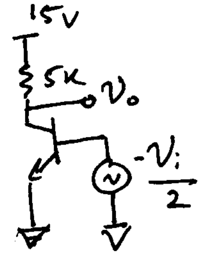
Current in both $Q_1 + Q_2$ are the same:

$I_{C_1} = I_{C_2} = 0.48\text{mA}$

Differential Gain: Half-circuit analysis:

$\underbrace{V_{in_{diff}} = V_1 - V_2 = V_i}$

We can model this single-ended input as a differential input with half the amplitude.



$g_m = 0.0192$

$V_\pi = -\dfrac{V_i}{2}$

$V_o = -g_m R_L V_\pi$

$\boxed{a_{v_{diff}} = 48}$

Common Mode Gain: Half-Circuit analysis: $V_{in_{common}} = \dfrac{V_1 + V_2}{2} = \dfrac{V_i}{2}$

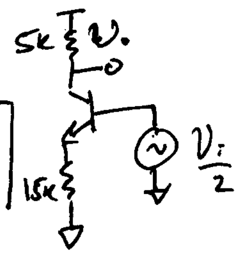
~~[illegible]~~

There is also a common-mode component to this input. We'll see if it makes a difference

$g_m = 0.0192$

$r_\pi = 10.4\text{k}\Omega$

$\boxed{a_{v_{comm}} = \dfrac{-g_m r_\pi R_L}{r_\pi + (\beta+1)R_E} \approx \dfrac{1}{6}}$



$\rightarrow$ Doesn't!

OCT'S:

$\boxed{r_E = \dfrac{r_\pi}{\beta+1} \| R_E = 51.5\Omega}$

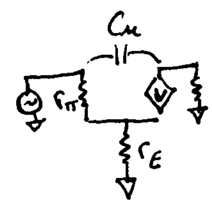
$r_{\pi o_1} = r_\pi \| \dfrac{R_S \overset{0}{\nearrow} + r_E}{1 + g_m r_E} = 25.8\Omega \rightarrow \tau_{\pi o_1} = 1.03\text{ns}$

$r_{\mu o_1} = R_L = 5\text{K} \longrightarrow \tau_{\mu o_1} = R_L C_\mu = 20\text{nS}$

$r_{\pi o 2} = r_\pi \| \dfrac{r_E}{1 + g_m r_E} = 25.8\Omega \longrightarrow \tau_{\pi o_2} = 1.03\text{ns}$

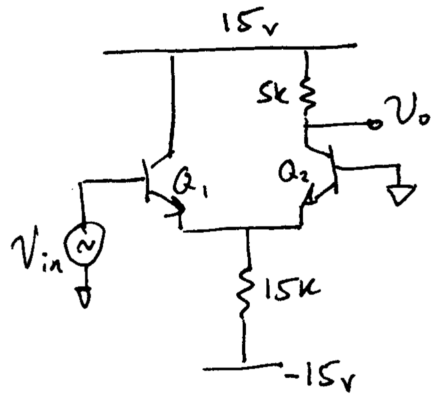
$r_{\mu o_2} = R_L = 5\text{K} \longrightarrow \tau_{\mu o_2} = 20\text{ns}$

$\Sigma = 42.06\text{ns} \rightarrow \boxed{f_{-3dB} = 3.78\text{MHz}}$



No source resistance, so $r_\mu = R_L$

b)



All bias parameters are identical.
Half-circuit analysis is identical. You could analyze it by thinking about an EF-CB cascade. Answer is the same.

$$\boxed{a_{v_{diff}} = 48}$$

$$\boxed{a_{v_{common}} \approx \frac{1}{6}}$$

OCT's:

Same → $r_{\pi o1} = r_\pi \| \frac{r_E}{1+g_m r_E} = 25.8\Omega \rightarrow \tau_{\pi o1} = \tau_{\pi o2} = 1.03ns$

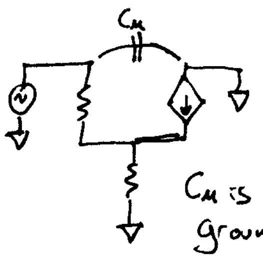
Same → $r_{\mu o2} = R_L = 5K \longrightarrow \tau_{\mu o2} = 20ns$

$r_{\mu o1} = 0 \longrightarrow \tau_{\mu o1} = 0ns$

$\Sigma = 22.06ns$

↓

$\boxed{f_{-3dB} = 7.22\,MHz}$



C_μ is incrementally grounded on both ends → $r_{\mu o1} = 0$

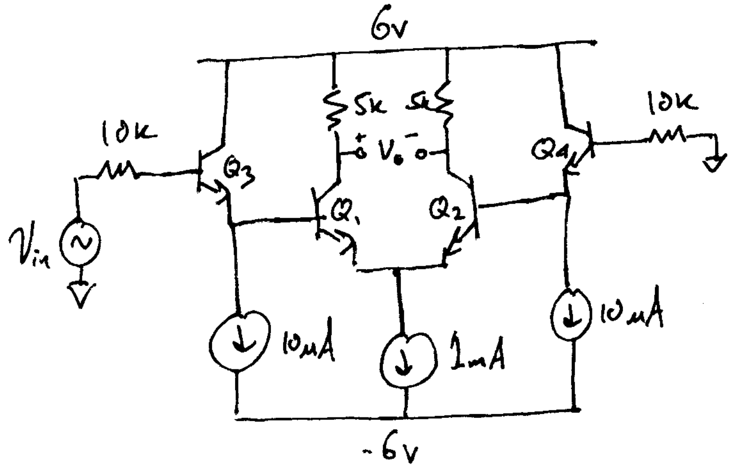
According to OCT's, this amplifier will operate twice as fast as the previous amplifier, due to incrementally grounding the collector of Q_1.

* In Practice:

Since we are using Q_1 to steer current in Q_2, it turns out that we don't care about the voltage gain in Q_1. While the voltage gain is attenuated in Q_1 at $f_{-3dB_1} = 3.78\,MHz$, the current gain goes unaffected (C_μ begins to source i_c). Thus $\tau_{\mu o1}$ should actually be ignored when calculating the ~~OCT's~~ -3dB bandwidth.

4) $I_S = 0.5fA$ $\beta = 200$ $C_{\mu o} = 0.5pF$ $C_{je} = 4pF$ $f_T = 500MHz$ @ $I_C = 1mA$, $V_{CB} = 2.5V$

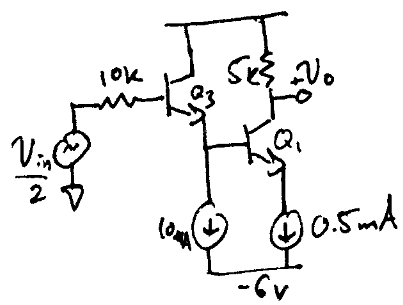
$m = 0.5$ & $\Psi_o = 0.7V$ $r_b = 0$ $r_o = \infty$



a) midband gain:

Common mode: Since the 1mA current source is ideal, the common mode gain ($\sim \frac{R_L}{R_E}$, $R_E \to \infty$) is Zero.

differential mode: Half-circuit analysis $\rightarrow V_{diff} = V_1 - V_2 = V_{in}$



$$a_{V_{Q3}} = \frac{(\beta+1)R_E}{(\beta+1)R_E + r_\pi + R_s} ; R_E \to \infty \rightarrow a_{V_{Q3}} = 1$$

$$a_{V_{Q1}} = -\frac{r_\pi}{R_{s_1} + r_\pi} g_m R_L \tfrac{1}{2} ; R_{s_1} = \frac{R_{s_3} + r_\pi}{\beta + 1} = 2.5k \rightarrow a_{V_{Q1}} = -\frac{80}{2}$$

$$\boxed{a_{V_{diff}} = 2 \times a_{V_{Q1}} = -80}$$

b)

@ $I_C = 1mA$, $f_T = 500MHz = \frac{g_m}{(C_\pi + C_\mu)2\pi} \rightarrow C_\pi = \frac{g_m}{(2\pi)500MHz} - C_\mu$

@ $V_{CB} = 2.5V$ $C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{\Psi_C}\right)^m} = \frac{0.5pF}{\left(1 + \frac{2.5}{0.7}\right)^{0.5}} = 234fF$

$$C_\pi = \frac{0.04}{1000\pi \times 10^6} - 234fF = 12.5pF$$

$\rightarrow$ find τ_F for simulation: $\tau_F = \frac{C_\pi - C_{jef}}{g_m} = \frac{12.5pF - C_{je} \times 2}{0.04} = \boxed{112.5pS = \tau_F}$ for later.

Continued...

b)

for Q_3 + Q_4, $V_{BE} = V_T \ln\left(\frac{10uA}{0.5fA}\right) = 0.593 \approx 0.6v$

$V_{CB} = 6V \rightarrow C_\mu = \frac{0.5pF}{\sqrt{1+\frac{6v}{0.7v}}} = 161.6fF$

$C_\pi = g_m \tau_F + C_{jef} = 8.045pF$

$r_{\pi o_3} = r_{\pi_3} \| \frac{10k + r_{\pi_1}}{1+g_m r_{\pi_1}} = 4k\Omega \rightarrow \tau_{\pi o_3} = 32.2nS = \tau_{\pi o 4} = 32.2nS$

$r_{\mu o_3} = \cancel{r_{\pi_1} \| \frac{r_{\pi_3}}{\beta+1} = 199\Omega \rightarrow \tau_{\mu o_3} =}$

$= \cancel{r_{\pi_1} \| \frac{r_{\pi_3}}{\beta+1} + R_L + g_m R_L \, r_{\pi_4} \| \frac{r_{\pi_3}}{\beta+1} = 206.1k\Omega \rightarrow \tau_{\mu o_3} =}$

$= 10k \| (r_{\pi_3} + (\beta+1) r_{\pi_1}) = 9960 \rightarrow \tau_{\mu o_3} = \tau_{\mu o_4} = 1.61nS$

for Q_1 + Q_2, $V_{CB} = 4.1v \rightarrow C_\mu = 190.9fF$ $\quad R_S = \frac{10k + r_{\pi 3}}{\beta+1} = 2.5k$

$C_\pi = 10.25pF$ $\quad R_C = 5k$

$r_{\pi o_1} = r_{\pi_1} \| R_S = 2k \rightarrow \tau_{\pi o_{1,2}} = 20.5nS$ $\quad R_E = 0$

$r_{\mu o_1} = r_{\pi_1} \| R_S + R_L + g_m R_L \, r_{\pi_1} \| R_S = 207k \rightarrow \tau_{\mu o_{1,2}} = 39.98nS$

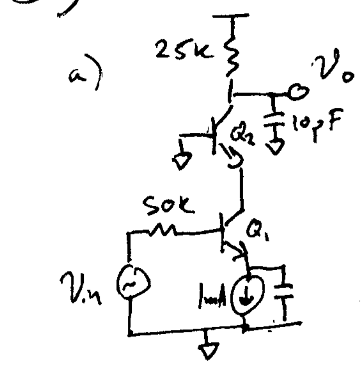
$\sum = 188.7nS \rightarrow f_{-3dB} = 843.5kHz$

5)

$\beta = 200, \; C_\mu = 2pF, \; C_{je} = 5pF, \; \tau_F = 250ps$

$C_\pi = C_b + C_{jeF} \; ; \; C_b = g_m \tau_F \rightarrow \frac{C_b}{I_c} = \frac{g_m \tau_F}{I_c} = \frac{q}{kT}\tau_F = \frac{10pF}{mA}$

a)



$C_\pi = 10pF + C_{je} \times 2 = 20pF \qquad r_\pi = 5K \quad g_m = 0.04$

$r_E = \frac{r_\pi}{\beta+1} = 25\Omega$

$$\boxed{a_v = -\frac{\beta}{\beta+1}\left(\frac{r_\pi g_m R_L}{R_S + r_\pi}\right) = -90}$$

$\tau_{\pi_1} = C_{\pi_1}(R_S \| r_{\pi_1}) = 90nS$

$\tau_{\mu_1} = C_{\mu_1}(R_S\|r_{\pi_1} + r_E + g_m r_E R_S\|r_{\pi_1}) = 18.2nS$

$\tau_{\pi_2} = C_{\pi_2}\left(\frac{r_\pi}{\beta+1}\right) = 0.5nS$

$\tau_{\mu_2} = C_\mu R_L = 50nS$

$\tau_{Lo} = C_L R_L = 250nS$

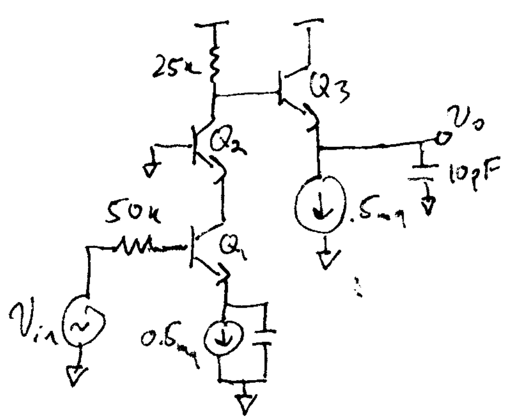
$\sum = 408.7nS$

$$\boxed{f_{-3dB} = 389.6 \, KHz}$$

b) $C_\pi = 5pF + 10pF = 15pF \qquad r_\pi = 10k \quad g_m = 0.02 \quad r_E = \frac{10k}{200} = 50$

$a_{v_3} = 1$, since current source $R_E = \infty$. $r_{inEF} \sim \infty$

$$\boxed{a_v = \frac{-\beta}{\beta+1}\left(\frac{r_\pi g_m R_L}{R_S + r_\pi}\right) = -82.9}$$



$\tau_{\pi_1} = 125nS$

$\tau_{\mu_1} = 33.4nS$

$\tau_{\pi_2} = 0.74nS$

same ↑

$\tau_{\mu_2} = 50nS$

$\tau_{\pi_3} = \frac{C_\pi}{g_m} = 0.75nS$

$\tau_{\mu_3} = C_\mu R_L = 50nS$

$\tau_{Lo} = C_L\left(\frac{R_L + r_\pi}{\beta+1}\right) = 1.78nS$

$\sum = 261 nS$

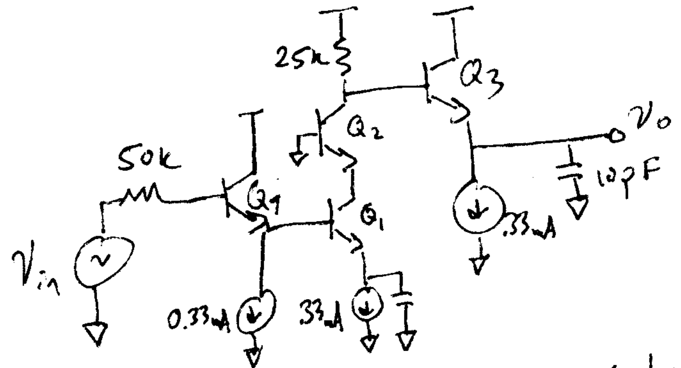
$f_{-3dB} = 608.7 \, KHz$

$r_E = \frac{15k}{200} = 75\Omega$

c)

$I_C = \frac{1}{3}\,mA \rightarrow g_m = 0.013 \rightarrow r_\pi = 15\cancel{000}\,k\Omega$

$C_\pi = 13.33\ pF$



$$\boxed{a_v = -\frac{(\beta+1)\, r_\pi\, g_m R_L}{R_s + r_\pi + (\beta+1) r_\pi} = 318}$$

$R_s' = \frac{R_s + r_\pi}{\beta+1}$ ← Source resistance into Cascode.

$= 323.4\Omega$

$$\left.\begin{aligned}
\tau_{\pi_1} &= C_\pi (R_s' \| r_\pi) = 4.2\ ns \\
\tau_{\mu_1} &= C_\mu (R_s' \| r_\pi + r_E + g_m r_E R_s' \| r_\pi) = 1.4\ ns \\
\tau_{\pi_2} &= C_\pi \left(\frac{r_\pi}{\beta+1}\right) = 0.99\ ns \\
\tau_{\mu_2} &= C_\mu R_L = 50\ ns \\
\tau_{\pi_3} &= \frac{C_\pi}{g_m} = 1\ ns \\
\tau_{\mu_3} &= C_\mu R_L = 50\ ns \\
\tau_{\pi_4} &= C_\pi \left(\frac{R_s + r_\pi}{\beta+1}\right) = 4.3\ ns \\
\tau_{\mu_4} &= C_\mu R_s = 100\ ns \\
\tau_{Lo} &= C_L \left(\frac{R_L + r_\pi}{\beta+1}\right) = 1.99\ ns
\end{aligned}\right\}$$

$\Sigma = 213.89$

$$\boxed{f_{-3dB} = 744.4\ kHz}$$