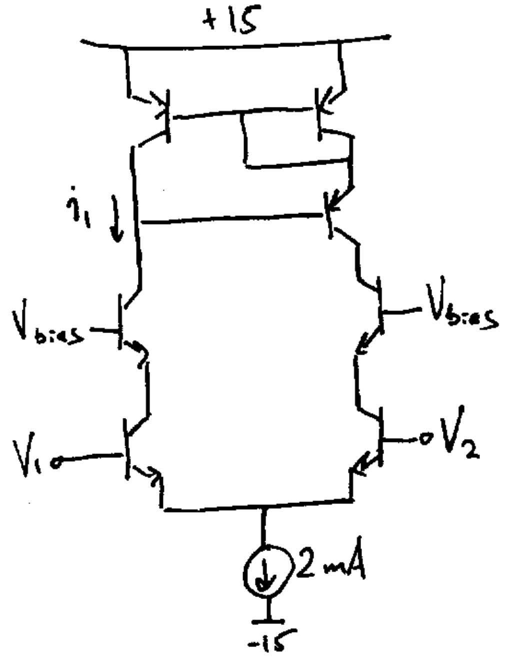
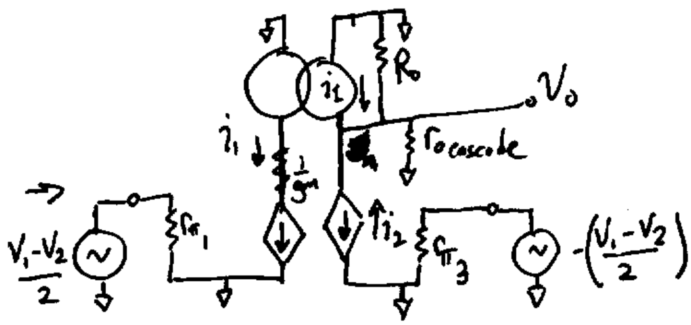
Pset 6 Solutions

1)



→



$$i_1 = g_m \frac{V_1 - V_2}{2}$$

$$i_2 = g_m \frac{V_1 - V_2}{2}$$

$$V_o = (i_1 + i_2)\, R_o \| r_{o\,cascode}$$

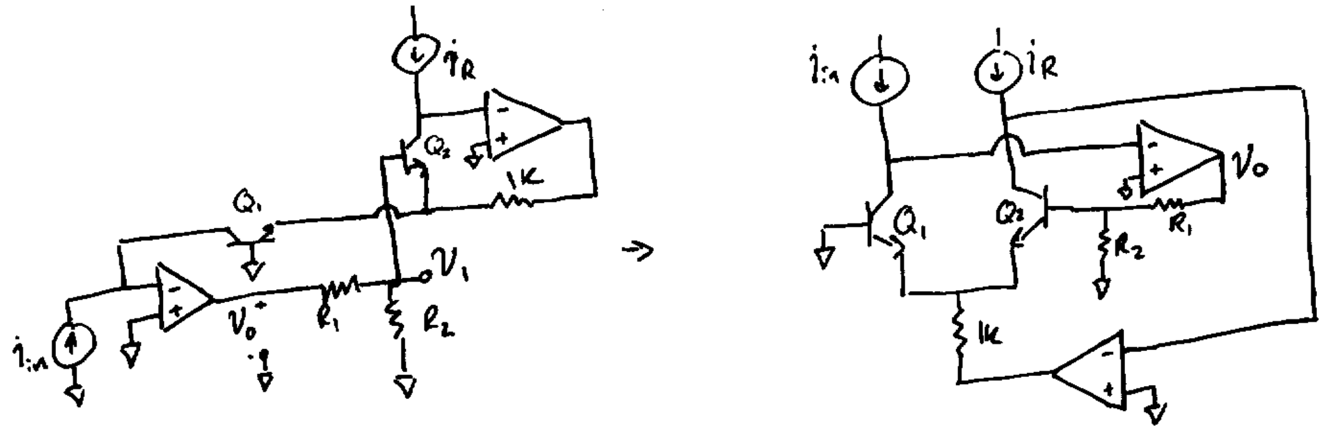
$$R_o = \frac{1}{2g_m} + \left(1 + \frac{\beta g_m}{2g_m}\right) r_{op} \approx \frac{\beta}{2} r_{op} = \frac{50}{2} \frac{V_{A\,pnp}}{I_C} = 25 \frac{50V}{1mA} = 1.25 M\Omega$$

$$r_{o\,cascode} = r_{oN} \| r_{\pi N} + r_{oN}(1 + g_m(r_{oN} \| r_{\pi N})) \approx \beta r_{oN}$$

$$r_{o\,cascode} = 200 \frac{V_{A\,NPN}}{I_C} = 200 \frac{100}{1mA} = 20 M\Omega$$

$$V_o = 2\frac{V_1 - V_2}{2} g_m (1.25 M\Omega \| 20 M\Omega) \rightarrow \frac{V_o}{V_1 - V_2} = \frac{0.04(1.25 M\Omega \| 20 M\Omega)}{\cancel{2}} \boxed{= 47000 \approx 50K}$$

2)



a) $I_{c_1} = i_{in} = I_s e^{\frac{V_{be_1}}{V_{th}}} \rightarrow V_{be_1} = \ln\left(\frac{i_{in}}{I_s}\right) V_{th}$

$I_{c_2} = i_R = I_s e^{\frac{V_{be_2}}{V_{th}}} \rightarrow V_{be_2} = \ln\left(\frac{i_R}{I_s}\right) V_{Th}$

$V_1 = -V_{be_1} + V_{be_2} = \left(-\ln\left(\frac{i_{in}}{I_s}\right) + \ln\left(\frac{i_R}{I_s}\right)\right) V_{Th} = V_{Th} \ln\left(\frac{i_R}{i_{in}}\right)$

$V_1 = V_o \frac{R_2}{R_1+R_2} \Rightarrow V_o = \frac{R_1+R_2}{R_2} V_1$; $R_1 = 15.7 R_2 \rightarrow 16.7\, V_{Th} \ln\left(\frac{i_R}{i_{in}}\right)$

In the form of $A \log_{10}(x) \rightarrow 16.7\, V_{Th} \ln\left(\frac{I_R}{I_{in}}\right) =$

$16.7\, V_{Th} \ln\left(\frac{I_R}{I_{in}}\right) = A \log_{10}(x) = A \frac{\ln(x)}{\ln(10)}$

$\ln(10) \times 16.7 V_{Th} \ln\left(\frac{I_R}{I_{in}}\right) = A \ln\left(\frac{I_R}{I_{in}}\right) \rightarrow \boxed{A = \ln(10) \times 16.7 V_{Th} \approx 1}$

$\boxed{x = \frac{I_R}{I_{in}}}$ or $\boxed{A = -1 \quad x = \frac{I_{in}}{I_R}}$

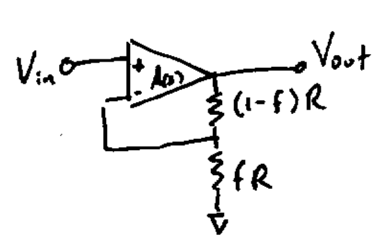
b) $A = 1 \rightarrow \log_2(x) = \frac{R_1+R_2}{R_2} V_{Th} \ln\left(\frac{I_R}{I_{in}}\right)$

$\frac{\cancel{\ln(x)}}{\ln(2)} = \frac{R_1+R_2}{R_2} V_{Th} \cancel{\ln(x)}$

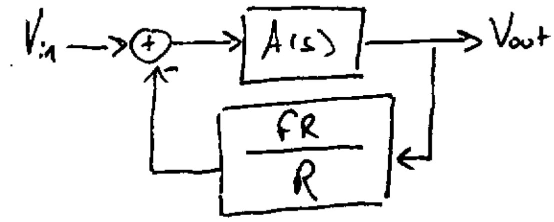
$\frac{1}{V_{Th} \ln(2)} = \frac{R_1}{R_2} + 1 \rightarrow \frac{R_1}{R_2} = \frac{1}{V_{Th} \ln(2)} - 1 = \boxed{54.7}$

$\boxed{R_1 = 54.7 R_2}$

3)



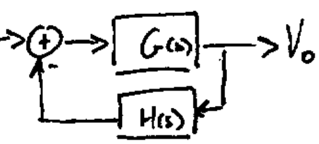
$A(s) = \dfrac{10^6}{10^{-6}s+1}$ $f = [1, 0.1, 0.01, 0.001]$



a) $\cancel{\frac{V_o}{V_i}}$

$V_{out} = A(s)(v_+ - v_-)$; $v_- = V_{out}\dfrac{fR}{fR+R-fR} = fV_{out}$

$v_+ = V_{in}$

Black's Formula: 

$$\frac{V_o}{V_i} = \frac{G(s)}{1+GH(s)} \rightarrow \frac{V_{out}}{V_{in}} = \frac{A(s)}{1+fA(s)} = \frac{\frac{a_o}{\tau s+1}}{1+\frac{fa_o}{\tau s+1}} = \frac{a_o}{1+a_of}\,\frac{1}{\frac{\tau}{1+a_of}s+1}$$

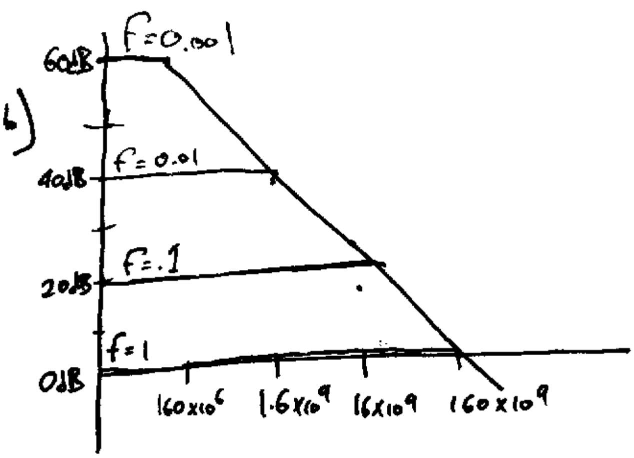
$$\boxed{\text{DC gain} = \frac{a_o}{1+a_of} \approx \frac{1}{f} = [1, 10, 100, 1000]}$$

-3dB point for 1 pole = $\frac{1}{\tau}$ $\rightarrow$ $\dfrac{1}{\underbrace{\frac{\tau}{1+a_of}}_{\tau}s+1}$

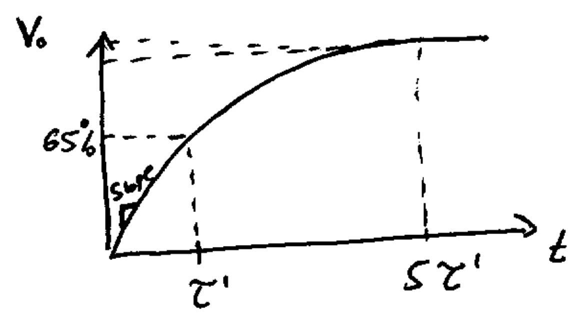
$$\tau \rightarrow \frac{1}{2\pi\tau} = \frac{1+a_of}{2\pi\tau} \approx \frac{10^6 f}{2\pi 10^{-6}} = \frac{10^{12}}{2\pi}f$$

$$\boxed{-3dB_{point} = [160\text{ GHz}, 16\text{ GHz}, 1.6\text{ GHz}, 160\text{ MHz}]}$$

b)



c)

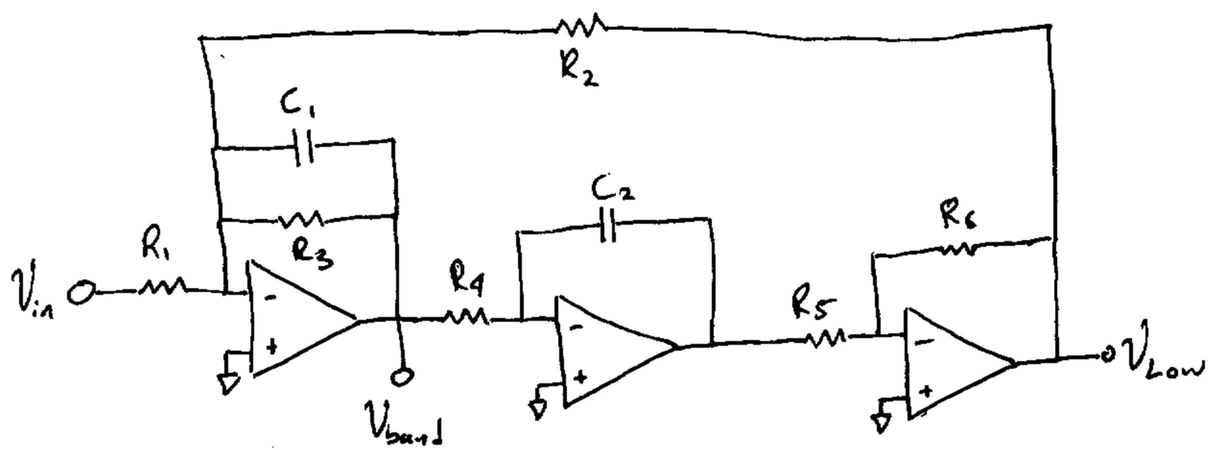


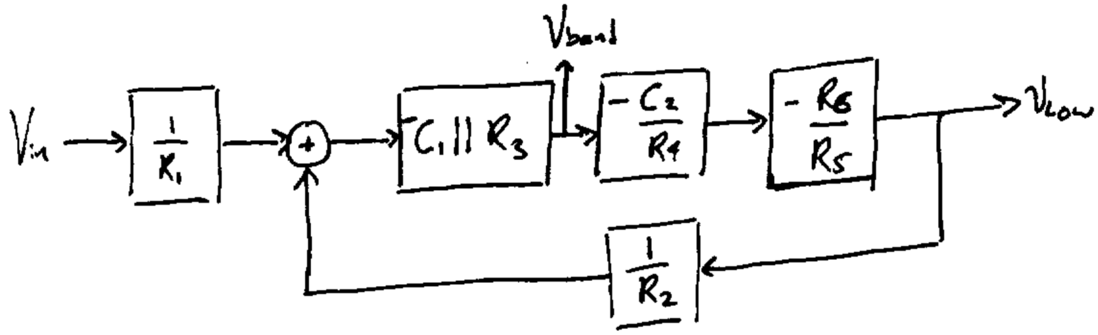
as f decreases, the step response slows, but the final value is larger.

$\tau' = \dfrac{\tau}{1+a_of}$ Slope $= \dfrac{\text{final value}}{\tau'}$

Final value = DC Gain. = $\frac{1}{f}$

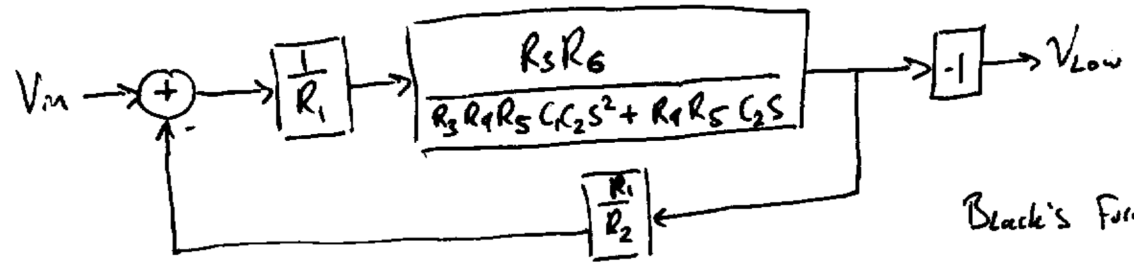
4

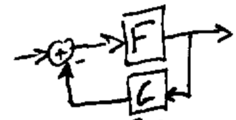




$$C_1 \| R_3 = \frac{R_3}{R_3C_1s+1}$$

$$\frac{C_2}{R_4} = \frac{1}{R_4C_2s}$$





Black's Formula: $\dfrac{F(s)}{1+F(s)G(s)}$

$$\frac{V_{Low}}{V_{in}} = \frac{\frac{1}{R_1}\left(\frac{R_3R_6}{R_3R_4R_5C_1C_2s^2+R_4R_5C_2s}\right)}{1+\frac{R_1}{R_1R_2}\left(\frac{R_3R_6}{R_3R_4R_5C_1C_2s^2+R_4R_5C_2s}\right)} = \frac{\frac{1}{R_1}R_3R_6}{R_3R_4R_5C_1C_2s^2+R_4R_5C_2s+\frac{R_3R_6}{R_2}} = \frac{\frac{R_6}{R_1R_5}\frac{1}{R_4C_1C_2}}{s^2+\frac{1}{R_3C_1}s+\frac{R_6}{R_2R_4R_5C_1C_2}}$$

Low pass!

$$\boxed{\frac{V_{band}}{V_{in}} = \frac{V_{band}}{V_{Low}}\frac{V_{Low}}{V_{in}} = \frac{1}{\left(\frac{R_6}{R_5}\frac{1}{R_4C_2s}\right)}\frac{V_{Low}}{V_{in}} = \frac{\frac{1}{R_1C_1}s}{s^2+\frac{1}{R_3C_1}s+\frac{R_6}{R_2R_4R_5C_1C_2}}}$$

Band pass!

$$\omega_0 = \sqrt{\frac{R_6}{R_2R_4R_5C_1C_2}}$$

$$\zeta = \frac{\frac{1}{R_3C_1}}{\sqrt{\frac{R_6}{R_2R_4R_5C_1C_2}}} = \frac{\sqrt{R_2R_4R_5C_1C_2}}{\sqrt{R_3^2C_1^2R_6}} = \sqrt{\frac{R_2R_4R_5C_2}{R_3^2R_6C_1}}$$