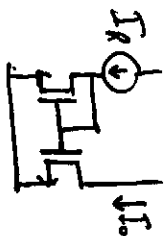
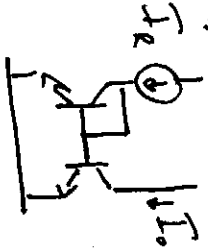


# Pset 5 solutions

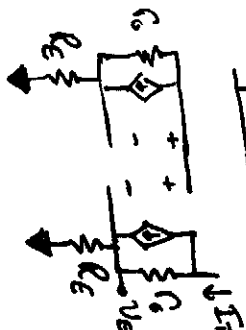
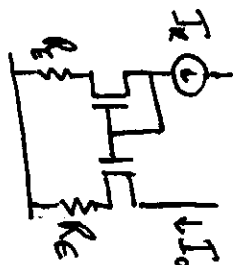
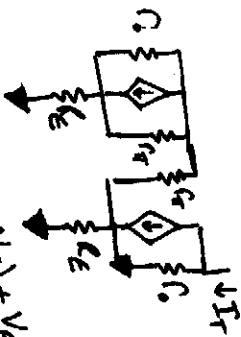
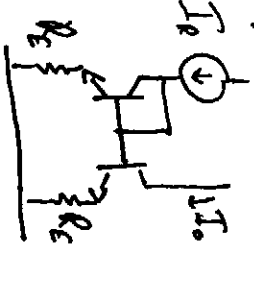
1) a)



$$R_o = r_o$$

$$R_o = r_o$$

b)



$$V_T = r_o(I_T - g_m V_E) + V_E$$

$$V_E = I_T R_E$$

$$V_T = V_E$$

$$V_T = r_o(1 + g_m R_E) + R_E = R_o$$

$$R_o = r_o + R_E + g_m R_E r_o$$

With bypass Cap

Nothing changes

$$R_o = r_o + R_E + g_m R_E r_o$$

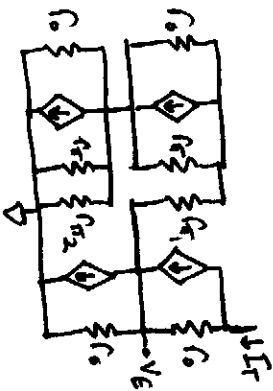
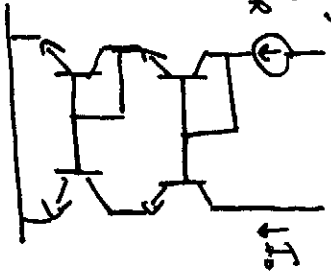
c) With bypass Cap

$$V_E = I_T (R_E \parallel r_\pi)$$

$$V_T = \frac{-V_E r_\pi}{r_\pi} = -V_E$$

$$R_o = r_o + \frac{\beta r_o R_E}{r_\pi + R_E} + R_E \parallel r_\pi$$

d)



$$V_T = V_E + (I_T - g_m V_{\pi_1}) r_o$$

$$V_E = (I_T - g_m V_{\pi_2}) (r_o || (r_{\pi_1} + \frac{1}{g_m} + \frac{1}{g_m}))$$

$$V_{\pi_2} = \frac{V_E \frac{1}{g_m}}{r_{\pi_1} + 2 \frac{1}{g_m}}$$

$$V_{\pi_1} = -\frac{V_E}{r_{\pi_1}}$$

$$V_E = (I_T - \frac{g_m V_E \frac{1}{g_m}}{r_{\pi_1} + 2 \frac{1}{g_m}}) (r_o || (r_{\pi_1} + \frac{2}{g_m}))$$

$$V_E + V_E \frac{r_o || (r_{\pi_1} + \frac{2}{g_m})}{r_{\pi_1} + 2 \frac{1}{g_m}} = I_T (r_o || (r_{\pi_1} + \frac{2}{g_m}))$$

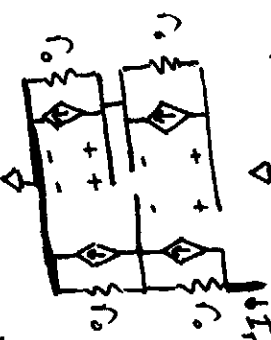
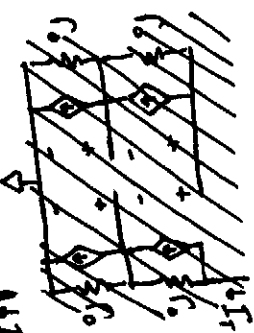
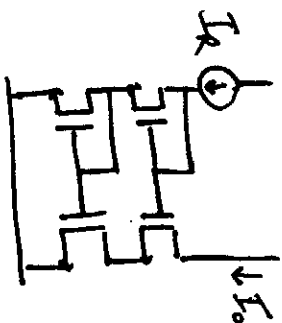
$$V_E (1 + \frac{r_o}{r_o + r_{\pi_1} + \frac{2}{g_m}}) = I_T (\frac{r_o (r_{\pi_1} + \frac{2}{g_m})}{r_o + r_{\pi_1} + \frac{2}{g_m}})$$

$$V_E = I_T \frac{1 + \frac{r_o}{r_o + r_{\pi_1} + \frac{2}{g_m}}}{\frac{r_o (r_{\pi_1} + \frac{2}{g_m})}{r_o + r_{\pi_1} + \frac{2}{g_m}}} = I_T \frac{r_o (r_{\pi_1} + \frac{2}{g_m})}{2 r_o + r_{\pi_1} + \frac{2}{g_m}}$$

$$V_{\pi_1} = -I_T \frac{r_{\pi_1} r_o (r_{\pi_1} + \frac{2}{g_m})}{(2 r_o + r_{\pi_1} + \frac{2}{g_m}) (r_{\pi_1} + \frac{2}{g_m})} = -I_T \frac{r_{\pi_1} r_o}{2 r_o + r_{\pi_1} + \frac{2}{g_m}}$$

$$V_T = I_T \left( \underbrace{\frac{r_o (r_{\pi_1} + \frac{2}{g_m})}{2 r_o + r_{\pi_1} + \frac{2}{g_m}}}_{< r_{\pi_1}} + \underbrace{r_o + \frac{2 r_o + r_{\pi_1} + \frac{2}{g_m}}{2}}_{\frac{\beta r_o}{2}} \right)$$

$$R_o \approx \frac{\beta r_o}{2}$$



$$V_T = V_E + (I_T - g_m V_{\pi_1}) r_o$$

$$V_E = (I_T - g_m V_{\pi_2}) r_o$$

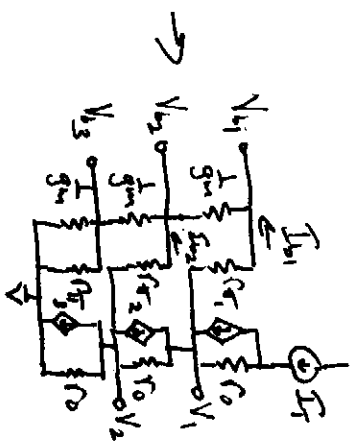
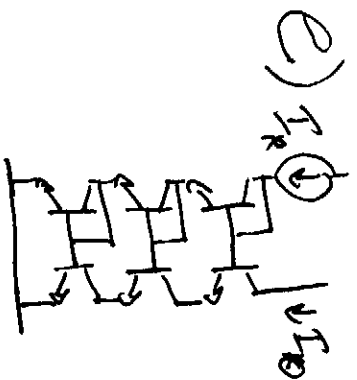
$$V_{\pi_1} = -V_E$$

$$V_{\pi_2} = 0$$

$$V_T = (I_T) r_o + (I_T - g_m I_T r_o) r_o$$

$$\frac{V_T}{I_T} = R_o = r_o + r_o + \beta r_o^2$$

$$R_o = 2 r_o + \beta r_o^2$$



$$V_{b3} = I_{b1} + I_{b2} \Rightarrow V_{b3} \approx \frac{I_{b1} + I_{b2}}{g_m}$$

$$V_{b2} = V_{b3} + \frac{I_{b1} + I_{b2}}{g_m} = 2 \frac{I_{b1} + I_{b2}}{g_m}$$

$$V_{b1} = V_{b2} + \frac{I_{b1}}{g_m} = 3 \frac{I_{b1}}{g_m} + 2 \frac{I_{b2}}{g_m}$$

~~Wird hier nicht benötigt~~

$$V_2 = (I_T - I_{b1} - I_{b2} - V_{b3} g_m) r_o = (I_T - 2I_{b1} - 2I_{b2}) r_o$$

$$I_{b2} = \frac{V_2 - V_{b2}}{r_{\pi}} = \frac{I_T r_o - 2I_{b1} r_o - 2I_{b2} r_o - \frac{2I_{b1} - 2I_{b2}}{g_m}}{r_{\pi}} \Rightarrow I_{b2} = \frac{I_T r_o - (2r_o + \frac{2}{g_m}) I_{b1}}{r_{\pi} + 2r_o + \frac{2}{g_m}}$$

$$V_1 = (I_T - I_{b1} + I_{b2} g_m) r_o + V_2 = (2I_T - 3I_{b1} + (\beta - 2) I_{b2}) r_o$$

$$I_{b1} = \frac{V_1 - V_{b1}}{r_{\pi}} = \frac{2I_T r_o - 3I_{b1} r_o + (\beta - 2) I_{b2} r_o (\frac{I_T r_o - (2r_o + \frac{2}{g_m}) I_{b1}}{r_{\pi} + 2r_o + \frac{2}{g_m}}) - 2 \frac{I_{b1}}{g_m} - \frac{2I_{b2} + I_{b1}}{g_m}}{r_{\pi}} = \frac{2I_T r_o - 3(\beta - \frac{\beta}{\beta - 2}) I_{b1} + ((\beta - 2) r_o - \frac{2}{\beta - 2}) (\frac{I_T r_o - (2r_o + \frac{2}{g_m}) I_{b1}}{r_{\pi} + 2r_o + \frac{2}{g_m}})}{r_{\pi}}$$

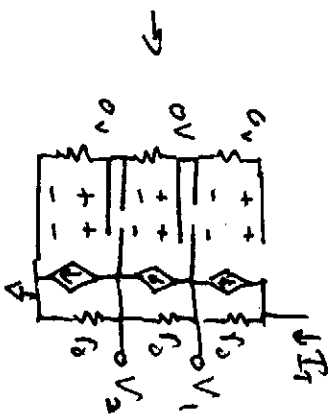
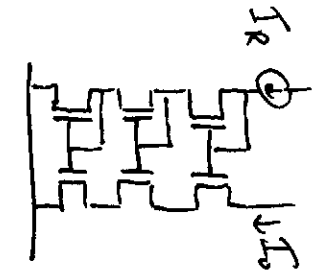
$$I_{b1} = I_T \left( \frac{2r_o + \frac{(\beta - 2) r_o^2}{r_{\pi}^2 + 2r_o r_{\pi}}}{(1 + \frac{3r_o}{r_{\pi}} + \frac{(\beta - 2) r_o^2}{r_{\pi}^2 + 2r_o r_{\pi}})} \right) \approx I_T$$

$$I_{b2} = \frac{I_T (-r_o - \frac{2}{\beta - 2})}{r_{\pi} + 2r_o} = \frac{-r_o}{r_{\pi} + 2r_o} I_T = -\frac{1}{2} I_T$$

$$V_T = V_1 + (I_T + g_m r_o I_{b1}) = (2I_T - 3I_T + (\beta - 2) \frac{I_T}{2}) r_o + I_T r_o + \beta I_T r_o = 3I_T r_o - 3I_T r_o + I_T r_o + \beta I_T r_o = (\frac{\beta}{2} + 1) I_T r_o$$

$$R_o = \frac{V_T}{I_T} = \left( \frac{\beta}{2} + 1 \right) r_o$$

c)



$$\begin{aligned} V_{\pi_3} &= 0 \\ V_{\pi_2} &= -V_2 \\ V_{\pi_1} &= -V_1 \end{aligned}$$

$$V_2 = (I_T - g_m V_{\pi_3}) r_o = I_T r_o$$

$$V_1 = V_2 + (I_T - g_m V_{\pi_2}) r_o = I_T (2 + g_m r_o) r_o$$

$$\begin{aligned} V_T &= V_1 + (I_T - g_m V_{\pi_1}) r_o = I_T (2 + g_m r_o) r_o + (1 + g_m (2 + g_m r_o) r_o) r_o \\ &= I_T (2 r_o + g_m r_o^2 + r_o + 2 g_m r_o^2 + g_m^2 r_o^3) \\ &= I_T (3 r_o + 3 g_m r_o^2 + g_m^2 r_o^3) \end{aligned}$$

$$R_i = \frac{V_T}{I_T} = 3 r_o + 3 g_m r_o^2 + g_m^2 r_o^3$$