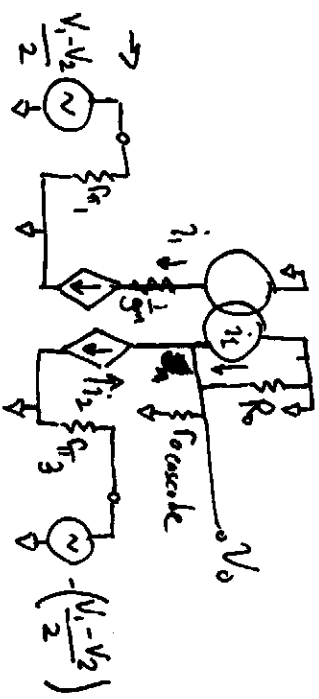
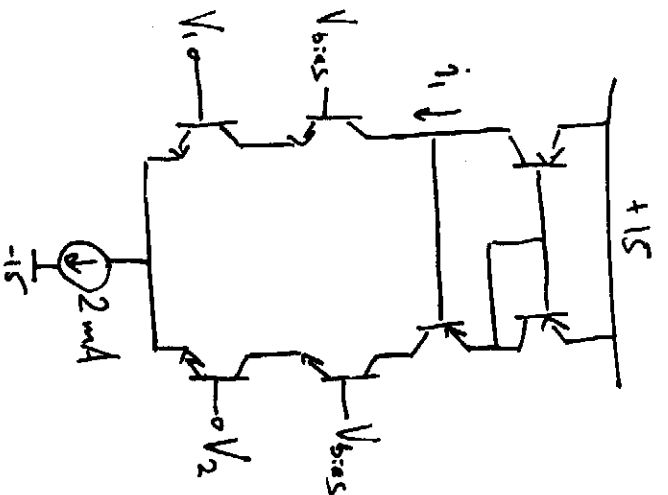


Set 6 Solutions



$$i_1 = g_m \frac{V_1 - V_2}{2}$$

$$i_2 = g_m \frac{V_1 - V_2}{2}$$

$$V_o = (i_1 + i_2) R_o \parallel r_{o, \text{cascode}}$$

$$R_o = \frac{1}{2g_m} + \left(1 + \frac{\beta g_m}{2g_m}\right) r_{o1} \approx \frac{\beta}{2} r_{o1} = \frac{50}{2} \frac{V_{A, PM}}{I_c} = 25 \frac{50V}{1\mu A} = 1.25M\Omega$$

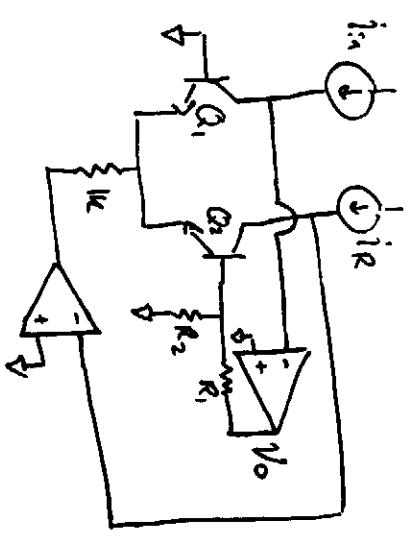
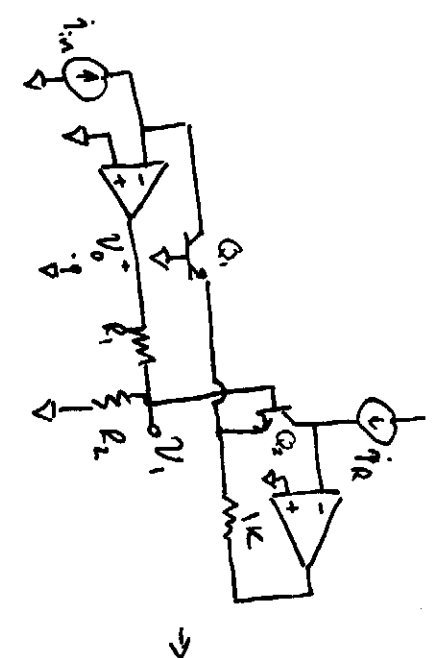
$$r_{o, \text{cascode}} = r_{oN} \parallel r_{pN} + r_{oN}(1 + g_m(r_{oN} \parallel r_{pN})) \approx \beta r_{oN}$$

$$r_{o, \text{cascode}} = 200 \frac{V_{A, NM}}{I_c} = 200 \frac{100}{1\mu A} = 20M\Omega$$

$$V_o = 2 \frac{V_1 - V_2}{2} g_m (1.25M\Omega \parallel 20M\Omega) \rightarrow \frac{V_o}{V_1 - V_2} = 0.04 (1.25M\Omega \parallel 20M\Omega)$$

$$= 47000 \approx 50k$$

2)



a) $I_{C1} = I_{in} = I_S e^{\frac{V_{be1}}{V_T}} \rightarrow V_{be1} = \left(\frac{I_{in}}{I_S} \right) V_T$

$I_{C2} = I_R = I_S e^{\frac{V_{be2}}{V_T}} \rightarrow V_{be2} = V_T \ln \left(\frac{I_R}{I_S} \right)$

$V_1 = -V_{be1} + V_{be2} = -\left(V_T \ln \left(\frac{I_{in}}{I_S} \right) \right) + V_T \ln \left(\frac{I_R}{I_S} \right)$

$V_1 = V_{T0} \frac{R_2}{R_1 + R_2} \Rightarrow V_2 = \frac{R_1 + R_2}{R_2} V_1 \Rightarrow V_2 = 15.7 R_2 \rightarrow 16.7 V_{T0} \ln \left(\frac{I_R}{I_{in}} \right)$

I_{in} the form of $A \log_{10}(x) \rightarrow 16.7 V_{T0} \ln \left(\frac{I_R}{I_{in}} \right) =$

$16.7 V_{T0} \ln \left(\frac{I_R}{I_{in}} \right) = A \log_{10}(x) = A \frac{\ln(x)}{\ln(10)}$

$\ln(10) \times 16.7 V_{T0} \ln \left(\frac{I_R}{I_{in}} \right) = A \ln \left(\frac{I_R}{I_{in}} \right) \rightarrow A = \ln(10) \times 16.7 V_{T0} \approx 1$

$x = \frac{I_R}{I_{in}}$ or $A = -1$
 $x = \frac{I_{in}}{I_R}$

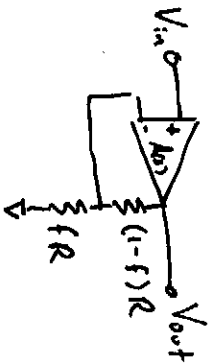
b) $A = 1 \rightarrow \log_2(x) = \frac{R_1 + R_2}{R_2} V_{T0} \ln \left(\frac{I_R}{I_{in}} \right)$

$\frac{\ln(x)}{\ln(2)} = \frac{R_1 + R_2}{R_2} V_{T0} \ln(x)$

$\frac{1}{V_{T0} \ln(2)} = \frac{R_1}{R_2} + 1 \rightarrow \frac{R_1}{R_2} = \frac{1}{V_{T0} \ln(2)} - 1 = 54.7$

$R_1 = 54.7 R_2$

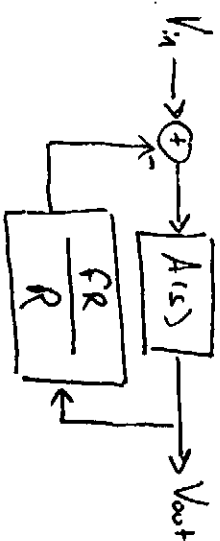
3)



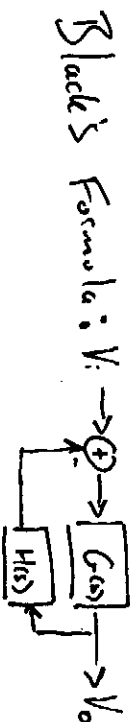
$$A(s) = \frac{10^6}{s^2 + 1}$$

$$f = [1, 0.1, 0.01, 0.001]$$

a) ~~Not~~



$$V_{out} = A(s)(V_{in} - V_{out}) \quad \text{for } V_{in} = V_{out} \quad \frac{fR}{fR + R - fR} = fV_{out}$$



$$V_{in} = V_{out}$$

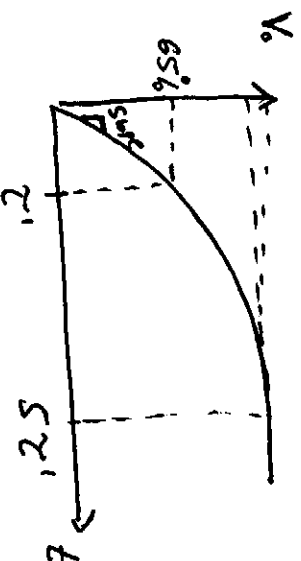
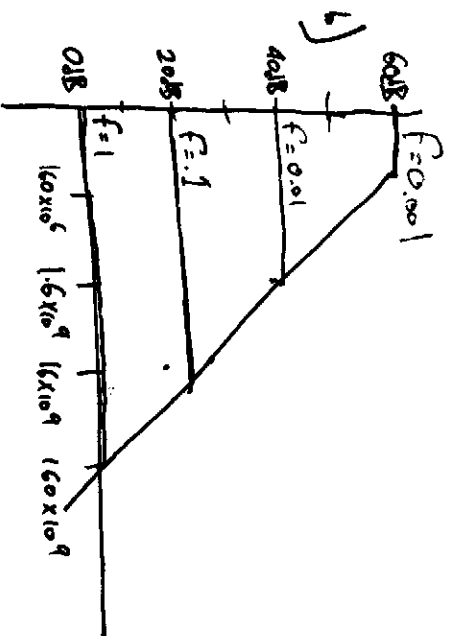
$$\frac{V_o}{V_i} = \frac{G(s)}{1 + GH(s)} \rightarrow \frac{V_{out}}{V_{in}} = \frac{A(s)}{1 + fA(s)} = \frac{\frac{10^6}{s^2 + 1}}{1 + \frac{10^6 f}{s^2 + 1}} = \frac{10^6}{s^2 + 1 + 10^6 f}$$

$$DC \text{ gain} = \frac{a_o}{1 + a_o f} \approx \frac{1}{f} = [1, 10, 100, 1000]$$

-3dB point for 1 pole - $\frac{1}{\tau}$

$$\frac{1}{\tau \frac{1}{1 + a_o f} s + 1} \rightarrow \frac{1}{\tau} = \frac{1 + a_o f}{2\pi \tau} \approx \frac{10^6 f}{2\pi \tau} = \frac{10^{12}}{2\pi \tau}$$

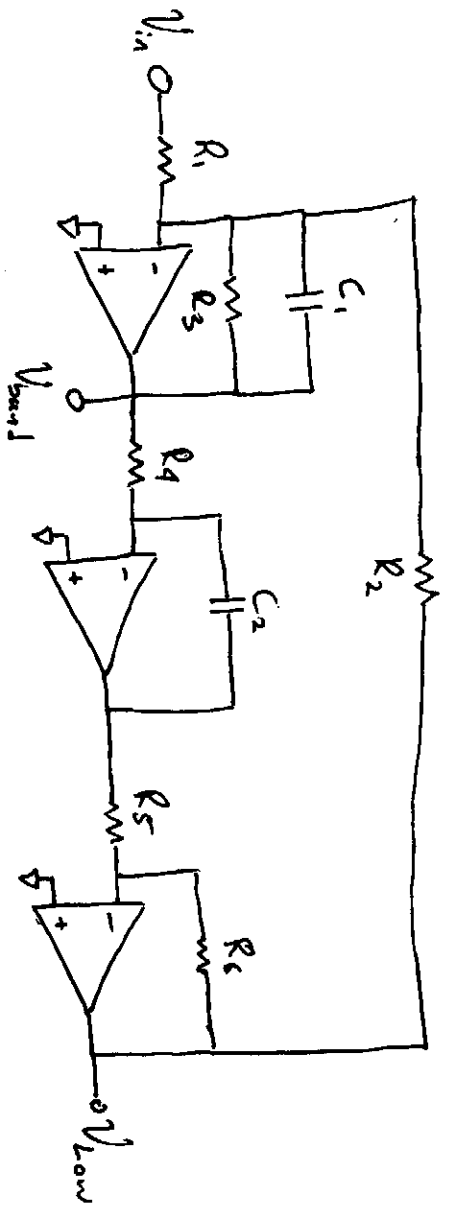
$$-3dB \text{ point} = [160 \text{ GHz}, 16 \text{ GHz}, 16 \text{ GHz}, 160 \text{ MHz}]$$



as f decreases, the step response slows, but the final value is larger.

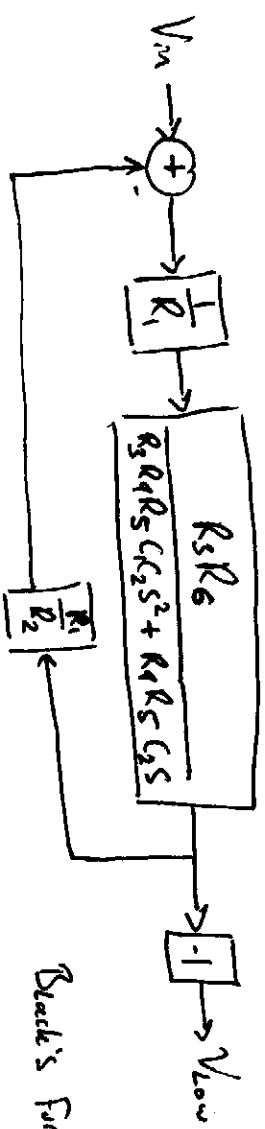
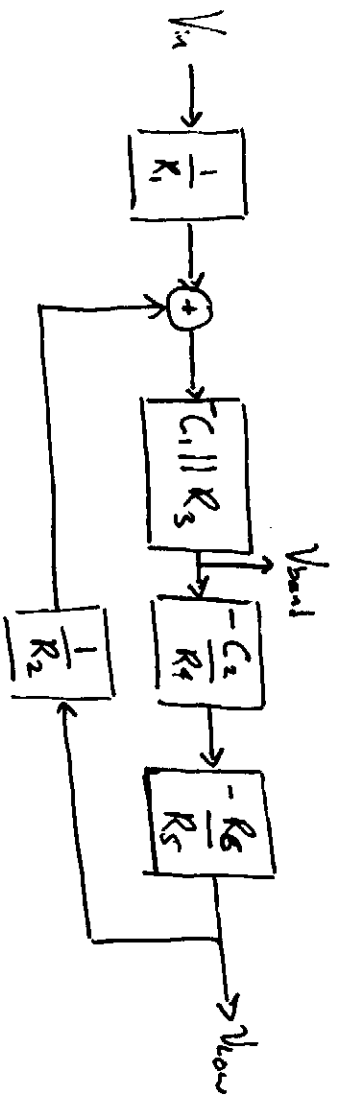
$$\tau = \frac{\tau}{1 + a_o f} \quad \text{Slope} = \frac{\text{final value}}{\tau}$$

$$\text{Final value} = DC \text{ gain} = \frac{1}{f}$$



$$C_1 \parallel R_3 = \frac{R_3}{R_3 C_1 s + 1}$$

$$\frac{C_2}{R_4} = \frac{1}{R_4 C_2 s}$$



Black's Formula: $\frac{1}{1 + F(s)G(s)}$



$$\frac{V_{low}}{V_{in}} = \frac{1}{1 + \frac{R_6}{R_3 R_4 R_5 C_1 C_2 s^2 + R_4 R_5 C_2 s}}$$

$$= \frac{\frac{1}{R_1} \left(\frac{R_3 R_6}{R_3 R_4 R_5 C_1 C_2 s^2 + R_4 R_5 C_2 s} \right)}{1 + \frac{R_6}{R_3 R_4 R_5 C_1 C_2 s^2 + R_4 R_5 C_2 s}} = \frac{\frac{1}{R_1} R_3 R_6}{R_3 R_4 R_5 C_1 C_2 s^2 + R_4 R_5 C_2 s + \frac{R_3 R_6}{R_1 R_2}}$$

Low pass!

$$\frac{V_{band}}{V_{in}} = \frac{V_{band}}{V_{low}} \frac{V_{low}}{V_{in}} = \left(\frac{R_6}{R_3} \frac{1}{R_4 C_2 s} \right) \frac{V_{low}}{V_{in}} = \frac{\frac{1}{R_1 C_1} s}{s^2 + \frac{1}{R_3 C_1} s + \frac{R_6}{R_2 R_4 R_5 C_1 C_2}}$$

Band Pass!

$$\omega_o = \sqrt{\frac{R_6}{R_2 R_4 R_5 C_1 C_2}}$$

$$Q = \frac{\frac{1}{R_3 C_1}}{\sqrt{\frac{R_6}{R_2 R_4 R_5 C_1 C_2}}} = \sqrt{\frac{R_2 R_4 R_5 C_1 C_2}{R_3^2 R_6}} = \sqrt{\frac{R_2 R_4 R_5 C_1 C_2}{R_3^2 R_6}}$$