Intersection point of two lines (2 dimensions)

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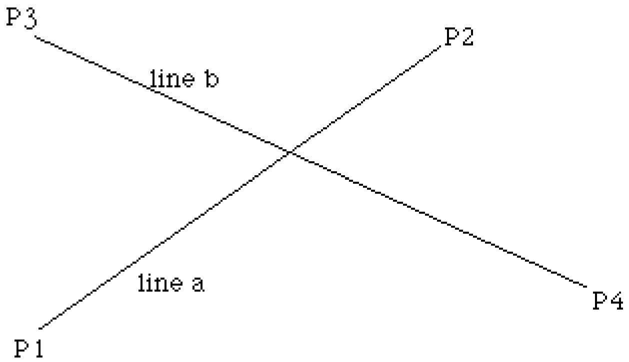
This note describes the technique and algorithm for determining the intersection point of two lines (or line segments) in 2 dimensions.



The equations of the lines are

$$\mathbf{P_a} = \mathbf{P1} + u_a\,(\mathbf{P2} - \mathbf{P1})$$

$$\mathbf{P_b} = \mathbf{P3} + u_b\,(\mathbf{P4} - \mathbf{P3})$$

Solving for the point where $\mathbf{P_a} = \mathbf{P_b}$ gives the following two equations in two unknowns (u_a and u_b)

$$x1 + u_a\,(x2 - x1) = x3 + u_b\,(x4 - x3)$$

and

$$y1 + u_a\,(y2 - y1) = y3 + u_b\,(y4 - y3)$$

Solving gives the following expressions for u_a and u_b

$$u_a = \frac{(x4 - x3)(y1 - y3) - (y4 - y3)(x1 - x3)}{(y4 - y3)(x2 - x1) - (x4 - x3)(y2 - y1)}$$

$$u_b = \frac{(x2 - x1)(y1 - y3) - (y2 - y1)(x1 - x3)}{(y4 - y3)(x2 - x1) - (x4 - x3)(y2 - y1)}$$

Substituting either of these into the corresponding equation for the line gives the intersection point. For example the intersection point (x,y) is

$$x = x1 + u_a\,(x2 - x1)$$

$$y = y1 + u_a\,(y2 - y1)$$

Notes:

- The denominators for the equations for u_a and u_b are the same.
- If the denominator for the equations for u_a and u_b is 0 then the two lines are parallel.
- If the denominator and numerator for the equations for u_a and u_b are 0 then the two lines are coincident.
- The equations apply to lines, if the intersection of line segments is required then it is only necessary to test if ua and ub lie between 0 and 1. Whichever one lies within that range then the corresponding line segment contains the intersection point. If both lie within the range of 0 to 1 then the intersection point is within both line segments.