

BE 2D growth rate

$$(G^{n+1} \cdot e^{ikp\Delta x} \cdot e^{ikq\Delta x} - G^n \cdot e^{ikp\Delta x} \cdot e^{ikq\Delta x}) / \Delta t$$

$$= \frac{\Delta t}{\Delta x^2} G^{n+1} (e^{ik(p-1)\Delta x} \cdot e^{ik(q-1)\Delta x} + e^{ik(p+1)\Delta x} \cdot e^{ikq\Delta x} + e^{ikp\Delta x} \cdot e^{ik(q-1)\Delta x} + e^{ikp\Delta x} \cdot e^{ik(q+1)\Delta x} - 4 \cdot e^{ikp\Delta x} \cdot e^{ikq\Delta x})$$

$$G^{n+1} - G^n = \frac{\Delta t}{\Delta x^2} G^{n+1} (e^{-ik\Delta x} + e^{ik\Delta x} + e^{-ik\Delta x} + e^{ik\Delta x} - 4)$$

$$G^{n+1} (1 - C(4\cos(k\Delta x) - 4)) = G^n$$

$$\frac{G^{n+1}}{G^n} = \frac{1}{1 - 4C(\cos(k\Delta x) - 1)}, \text{ but } \cos(k\Delta x) \text{ goes from } -1 \text{ to } 1,$$

so rate is from  $\boxed{\frac{1}{1+8C}}$  to 1, unconditionally stable.