

CN 2D growth rate

$$(G^{n+1} \cdot e^{ikp\Delta x} \cdot e^{ikq\Delta x} - G^n e^{ikp\Delta x} \cdot e^{ikq\Delta x}) / \Delta t$$

$$= \frac{\alpha}{2\Delta x^2} G^{n+1} \left[e^{ik(p-1)\Delta x} \cdot e^{ikq\Delta x} + e^{ik(p+1)\Delta x} \cdot e^{ikq\Delta x} + e^{ikp\Delta x} \cdot e^{ik(q-1)\Delta x} + e^{ikp\Delta x} \cdot e^{ik(q+1)\Delta x} - 4e^{ikp\Delta x} \cdot e^{ikq\Delta x} \right] + \frac{\alpha}{2\Delta x^2} G^n \left[\begin{matrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{matrix} \right]$$

$$G^{n+1} - G^n = \frac{\alpha \Delta t}{2\Delta x^2} G^{n+1} \left[e^{-ik\Delta x} + e^{ik\Delta x} + e^{-ik\Delta x} + e^{ik\Delta x} - 4 \right] + \frac{\alpha \Delta t}{2\Delta x^2} G^n \left[2e^{-ik\Delta x} + 2e^{ik\Delta x} - 4 \right]$$

$$G^{n+1} - G^n = \frac{C}{2} G^{n+1} (2(2\cos(k\Delta x) - 2)) + \frac{C}{2} G^n (2(2\cos(k\Delta x) - 2))$$

$$G^{n+1} (1 - 2C(\cos(k\Delta x) - 1)) = G^n (1 + 2C(\cos(k\Delta x) - 1))$$

$$\frac{G^{n+1}}{G^n} = \frac{1 + 2C(\cos(k\Delta x) - 1)}{1 - 2C(\cos(k\Delta x) - 1)} \quad \text{Again, cos term} = 1 \text{ means rate} = 1.$$

$$\text{cos term} = -1 \text{ means rate} = \boxed{\frac{1-4C}{1+4C}} \quad \text{This scales the graph of } C \text{ vs growth rate compared to 1D version}$$