

BE 3D growth rate

$$(G^{n+1} e^{ikp\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ikr\Delta x} - G^n \cdot e^{ikp\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ikr\Delta x}) / \Delta t$$

$$= \frac{\alpha}{\Delta x^2} G^{n+1} (e^{ik(p-1)\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ikr\Delta x} + e^{ik(p+1)\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ikr\Delta x} + e^{ikp\Delta x} \cdot e^{ik(q-1)\Delta x} \cdot e^{ikr\Delta x} + e^{ikp\Delta x} \cdot e^{ik(q+1)\Delta x} \cdot e^{ikr\Delta x} + e^{ikp\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ik(r-1)\Delta x} + e^{ikp\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ik(r+1)\Delta x} - 6 \cdot e^{ikp\Delta x} \cdot e^{ikq\Delta x} \cdot e^{ikr\Delta x})$$

$$G^{n+1} - G^n = \frac{\alpha \Delta t}{\Delta x^2} G^{n+1} (3e^{-ik\Delta x} + 3e^{ik\Delta x} - 6)$$

$$G^{n+1} (1 - 3C(e^{-ik\Delta x} + e^{ik\Delta x} - 2)) = G^n$$

$$\frac{G^{n+1}}{G^n} = \frac{1}{1 - 3 \cdot 2C(\cos(k\Delta x) - 1)}$$

cos goes from 1 to -1, so rate goes from 1 to

$$\boxed{\frac{1}{1 + 12C}}$$