BE 1D growth rate

$$\left(G^{n+1} \cdot e^{ikp\Delta x} - G^{n} \cdot e^{ikp\Delta x}\right)/\Delta t$$

$$= \frac{\alpha}{\Delta x^2} G^{n+1}\left(e^{ik(p-1)\Delta x} + e^{ik(p+1)\Delta x} - 2e^{ikp\Delta x}\right)$$

$$G^{n+1} - G^{n} = G^{n+1} C\left(e^{-ik\Delta x} + e^{ik\Delta x} - 2\right)$$

$$G^{n+1}\left(1 - C(2\cos(k\Delta x) - 2)\right) = G^{n}$$

$$\frac{G^{n+1}}{G^{n}} = \frac{1}{1 - C(2\cos(k\Delta x) - 2)}$$

$\cos(k\Delta x)$ ranges from -1 to 1, so the rate can be from

$\frac{1}{1+4C}$ to 1. Since $C > 0$, $\boxed{\frac{1}{1+4C} < 1}$ and BE is unconditionally stable.