

CN 1D growth rate

$$\frac{T_j^{n+1} - T_j^n}{\Delta t} = \frac{\alpha}{2\Delta x^2} \left[ T_{j-1}^{n+1} + T_{j+1}^{n+1} - 2T_j^{n+1} + T_{j-1}^n + T_{j+1}^n - 2T_j^n \right]$$

$$G^{n+1} \cdot e^{ikj\Delta x} - G^n \cdot e^{ikj\Delta x} = \frac{\alpha \Delta t}{2\Delta x^2} G^{n+1} \left[ e^{ik(j-1)\Delta x} + e^{ik(j+1)\Delta x} - 2e^{ikj\Delta x} \right] + \frac{\alpha \Delta t}{2\Delta x^2} G^n \left[ e^{ik(j-1)\Delta x} + e^{ik(j+1)\Delta x} - 2e^{ikj\Delta x} \right]$$

$$G^{n+1} - G^n = \frac{C}{2} G^{n+1} [e^{-ik\Delta x} + e^{ik\Delta x} - 2] + \frac{C}{2} G^n [e^{-ik\Delta x} + e^{ik\Delta x} - 2]$$

$$G^{n+1} \left( 1 - \frac{C}{2} (2\cos(k\Delta x) - 2) \right) = G^n \left( 1 + \frac{C}{2} (2\cos(k\Delta x) - 2) \right)$$

$$\frac{G^{n+1}}{G^n} = \frac{1 + C(\cos(k\Delta x) - 1)}{1 - C(\cos(k\Delta x) - 1)}$$

When the cos term is 1, the growth rate is 1. When the cos term is -1, the rate is  $\frac{1-2C}{1+2C}$  which is between -1

and 1, getting very close to 1 as  $C \rightarrow 0$ , and close to -1 as  $C \rightarrow \infty$ . At  $C = .5$ , the rate is 0. Does this mean we can eliminate the error in one step by choosing  $\Delta t$  and  $\Delta x$  carefully?