UK Macro History

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UK Macro Economic Relationships

We look at a subset of the full series between 1885 and 1985 with the following variables

- U_t: percent unemployment rate. Col ;
- P_t : PGDP: GDP deáator, 2013=100.
- Q_t: real UK GDP at market prices, geographically consistent estimate based on post 1922 borders. £ mn Chained Volume measure, 2013 prices. Col A1.B.
- RS_t: short interest rates, percent per annum, (Bank Rate).
- RL_t: long interest rates, percent per annum, (Consol/10 year debt) Col

and their subsequent transformations

- log GDP: LQ_t
- log GDP deflator: LPt
- inflation: $INFt = 100(LP_t - LP_{t-1})$;
- growth: $G_t = 100(LQ_t - LQ_{t-1})$

Expected relationships

We would expect to see evidence of economic cycles throughought the period. This would include cycles of growth, decreasing unemployment, potentially encouraged by easier monetary policy and lower short term interest rates. In due course this may subsequently lead to inflationary pressures as slack in the labour supply decreases and wages increase. Measures of control may increases in short term interest rates and subsequent cooling of economic growth. We would also expect inflationary pressures to lead to higher long term interest rates. We may see inflationary shocks or high periods of inflation leading to poor economic growth and higher unemployment. This timeframe encompasses significant socio-economic changes and wars which will lead to more extreme observations in the data.

Summary Statistics & Commentary

Table 1: Summary Statistics

Index	U	G	INF	RS	RL
Min. :1885	Min. : 0.2835	Min. :-10.2153	Min. :-14.7434	Min. : 2.000	Min. : 2.264
1st Qu.:1910	1st Qu.: 2.3972	1st Qu.: 0.9712	1st Qu.: -0.1722	1st Qu.: 3.000	1st Qu.: 2.904
Median :1935	Median : 4.3887	Median : 2.6406	Median : 2.3143	Median : 3.959	Median : 3.756
Mean :1935	Mean : 5.0910	Mean : 1.9688	Mean : 3.7673	Mean : 4.824	Mean : 5.061
3rd Qu.:1960	3rd Qu.: 6.8727	3rd Qu.: 3.9406	3rd Qu.: 6.6286	3rd Qu.: 5.496	3rd Qu.: 5.458
Max. :1985	Max. :15.3873	Max. : 9.4607	Max. : 23.1675	Max. :16.301	Max. :15.173

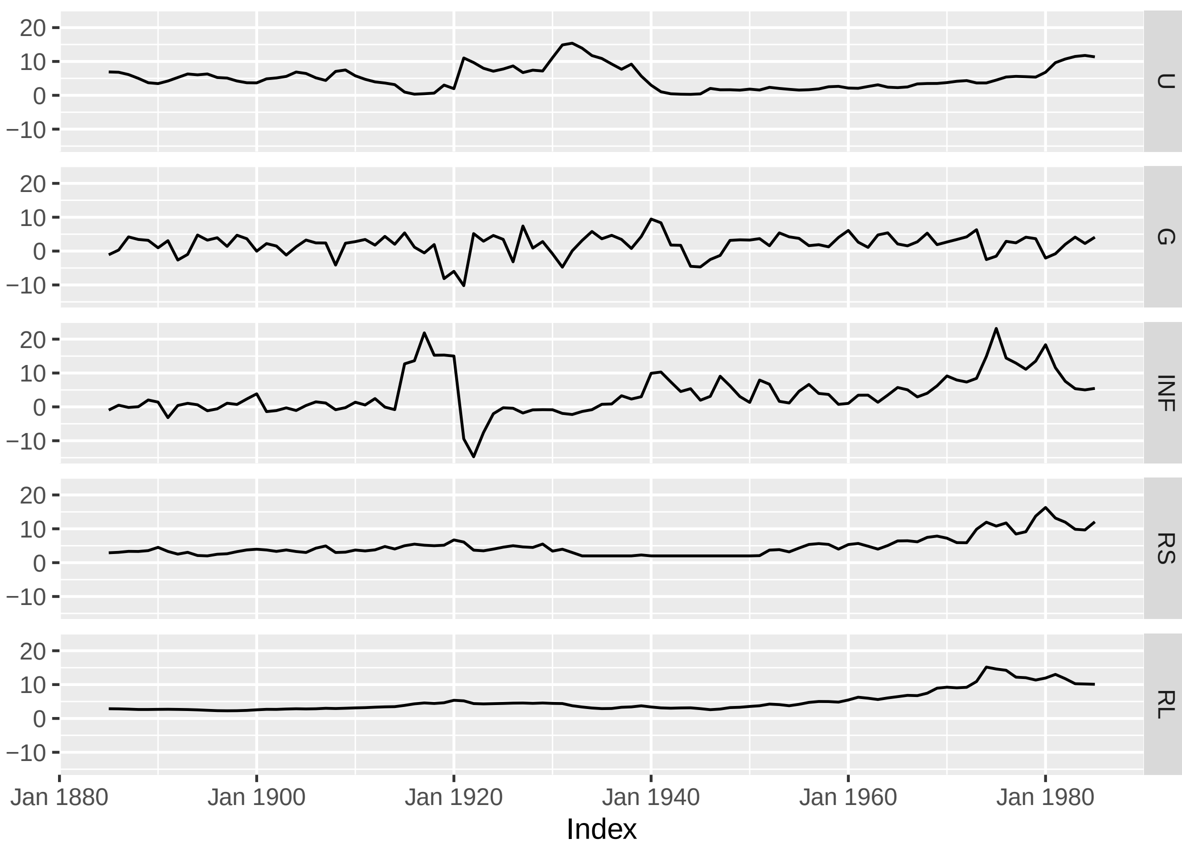


Figure 1: Plots of Economic Series between 1885 and 1985

Unemployment averages at 5% over the period with an average growth rate of 1.96%. The mean of inflation is 3.76% with the short and long term interest rates averaging at 4.8% and 5% respectively. We see peaks of inflation at 23% during the 1970s oil crisis, similar sustained periods of high inflation during WWI and unemployment at 15% following the great depression. Short term and long term interest rates show significant increases during the 1970s and onwards.

Table 2: Pearson Correlation Coefficients

	U	G	INF	RS	RL
U	1.0000000	-0.0883304	-0.4006868	0.1359800	0.1201737
G	-0.0883304	1.0000000	-0.1001583	-0.0861742	-0.0204763
INF	-0.4006868	-0.1001583	1.0000000	0.5439262	0.5781197
RS	0.1359800	-0.0861742	0.5439262	1.0000000	0.9114121
RL	0.1201737	-0.0204763	0.5781197	0.9114121	1.0000000

Unrestricted Model

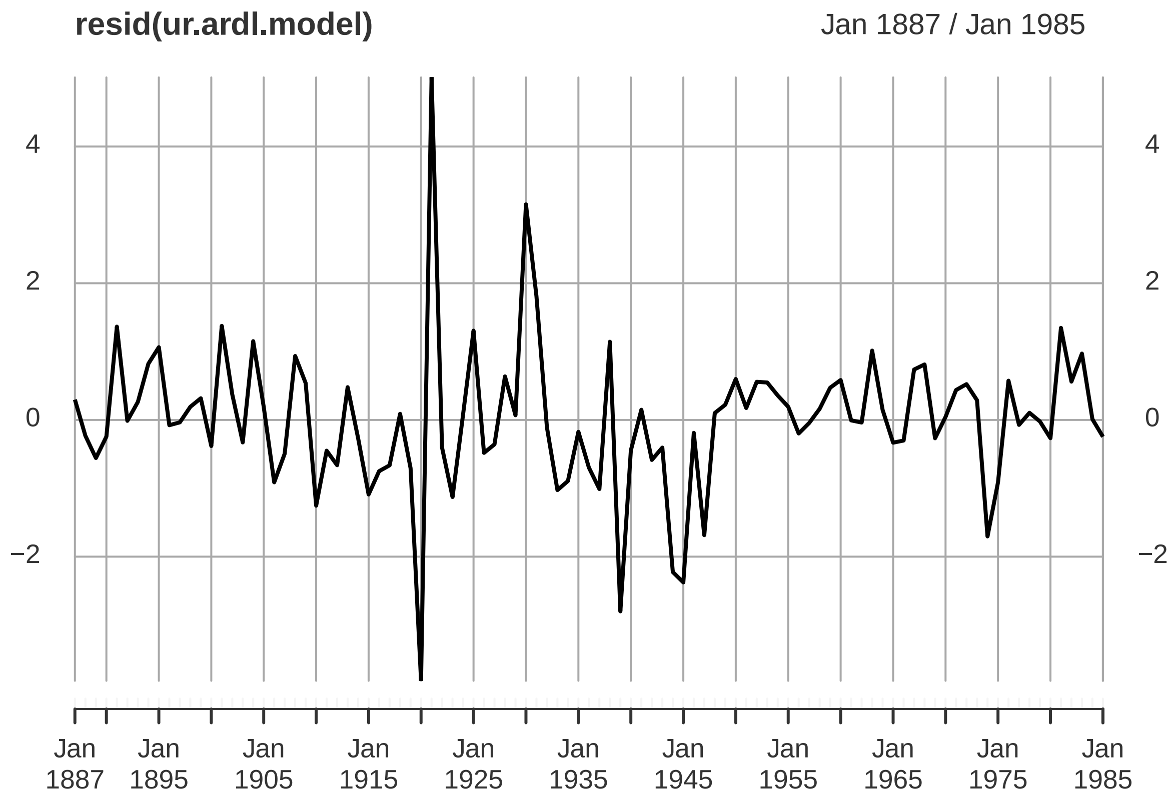
We consider the unrestricted model $U_t = \alpha_0 + \alpha_1 U_{t-1} + \alpha_2 U_{t-2} + \beta_0 LQ_t + \beta_1 LQ_{t-1} + \beta_2 LQ_{t-2} + \gamma t + \epsilon_{1t}$ and observe the following

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.452	12.12	-0.3672	0.7143
stats::lag(U, k = 1)	0.9582	0.1052	9.112	1.689e-14
stats::lag(U, k = 2)	0.03401	0.1097	0.3099	0.7574
LQ	-30.79	3.852	-7.995	3.692e-12
stats::lag(LQ, k = 1)	26.67	7.327	3.639	0.000451
stats::lag(LQ, k = 2)	4.535	5.253	0.8632	0.3902
trend	0.001319	0.02005	0.06581	0.9477

Table 4: Fitting linear model: dyn(U ~ stats::lag(U, k = 1) + stats::lag(U, k = 2) + LQ + stats::lag(LQ, k = 1) + stats::lag(LQ, k = 2) + trend)

Observations	Residual Std. Error	R^2	Adjusted R^2
99	1.1	0.9049	0.8987

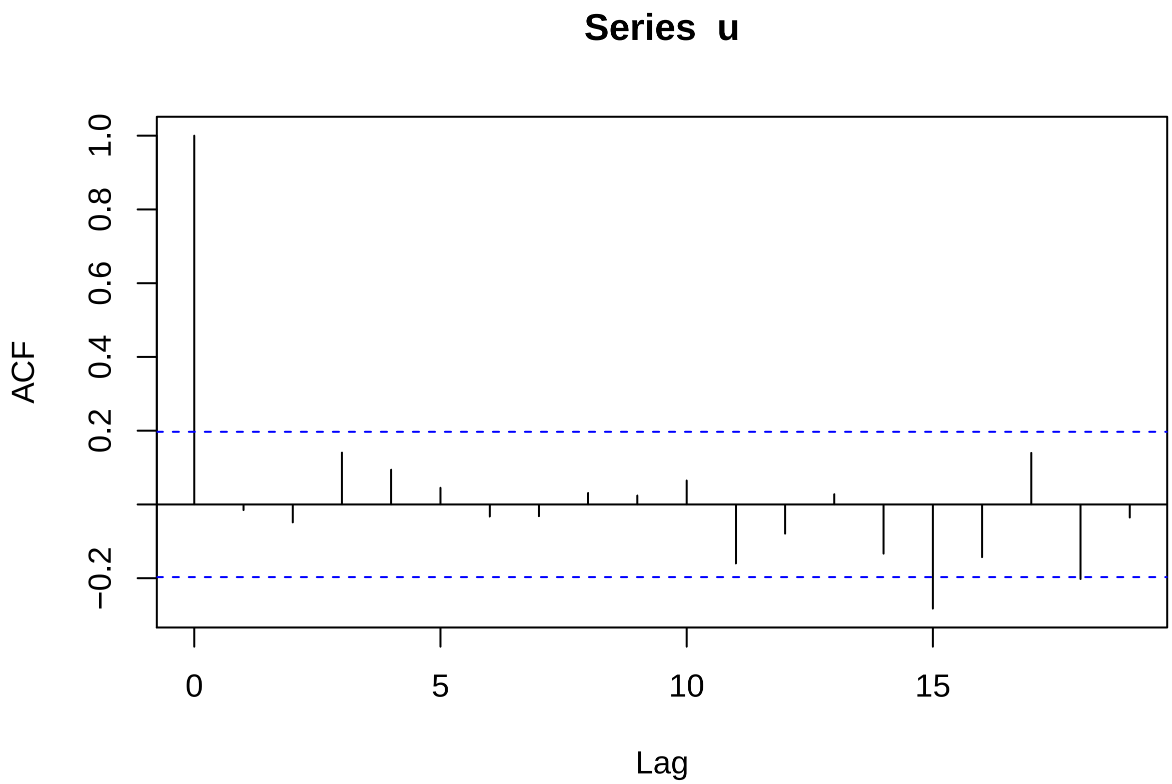
We note the coefficient of 1 period lagged unemployment, $\alpha_1 = 0.96$, as individually significant, suggesting a 1% increase in lagged unemployment will lead to a 0.96% increase in the following period. We also note that the log GDP and lagged log GDP LQ as significant, with a 1% increase in GDP proposing a -0.3% reduction in unemployment. However, we note that an increase in lagged GDP leading to a increase in unemployment, perhaps signalling some behaviour of the business cycle. The trend, t nor 2 period lagged unemplpoyment, U_{t-2} and 2 period lagged log GDP, LQ_{t-1} are deemed to be significant.



We can observe potential heteroskedasticity in the residuals with greater variance in the early to mid 20th century and some notable outliers.

Diagnostic tests

```
## [1] "DW: 2.03"
```



We note that DW is close to 2 (2.03) so no serial correlation. We can also observe this visually in the included ACF chart. The inclusion of two lagged terms appears to take care of any serial correlation concerns.

We test for heteroskedasticity using $\hat{u}_t^2 = \alpha + b' z_t + v_t$ using the hypothesis that $b' = 0$. In this case we use $z_t = x_t$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	49.94	27.75	1.8	0.07515
stats::lag(U, k = 1)	-0.6911	0.2407	-2.871	0.005077
stats::lag(U, k = 2)	0.7352	0.2512	2.927	0.004318
LQ	-44.43	8.817	-5.039	2.327e-06
stats::lag(LQ, k = 1)	-4.211	16.77	-0.2511	0.8023
stats::lag(LQ, k = 2)	44.56	12.03	3.705	0.00036
trend	0.08493	0.04589	1.851	0.06745

Table 6: Fitting linear model: dyn(u2 ~ stats::lag(U, k = 1) + stats::lag(U, k = 2) + LQ + stats::lag(LQ, k = 1) + stats::lag(LQ, k = 2) + trend)

Observations	Residual Std. Error	R^2	Adjusted R^2
99	2.518	0.4145	0.3763

Table 7: F-Statistic for the test for heteroskedasticity

	x
value	10.85578
numdf	6.00000
dendf	92.00000

When we retrieve the F-statistic we see that this is 10.8 which means we can reject the null hypothesis that $b' = 0$ and of homoskedasticity and constant variance. Given what we observe visually in the residuals with differing regimes throughough the sample, a further variance ratio test (Goldfeld-Quandt) might be warranted

We also perform a RESET test of the form $\hat{u}_i^2 = \alpha + b\hat{y}_i^2 + \epsilon_i$. However in this case we can accept the null hypothesis that $b = 0$ and constant variance. However two of the three tests inform us of heteroskedasticity, perhaps giving us reason to investigate the result of this test

```
##
## Call:
## lm(formula = u2 ~ (yhat * yhat))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.2087 -1.0611 -0.7544 -0.3435 23.8840
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.59336    0.58951   1.007    0.317
## yhat         0.10500    0.09791   1.072    0.286
##
## Residual standard error: 3.186 on 97 degrees of freedom
## Multiple R-squared:  0.01172,    Adjusted R-squared:  0.001529
## F-statistic:  1.15 on 1 and 97 DF,  p-value: 0.2862
```

Similarly another BPG test using the lmtest package returns a p-value < 0.05 so we reject the null hypothesis of homoskedasticity. We can say our estimator is unbiased but not minimum variance and efficient.

```
##
##  studentized Breusch-Pagan test
##
## data:  ur.ardl.model
## BP = 41.037, df = 6, p-value = 2.847e-07
```

Testing for normality using the Jarque Bera test we see a χ^2 value of 380 and a p-value $<$ 2.2e-16, so we reject the null hypothesis of normality.

```
##
##  Jarque Bera Test
##
## data:  u
## X-squared = 150.86, df = 2, p-value < 2.2e-16
```

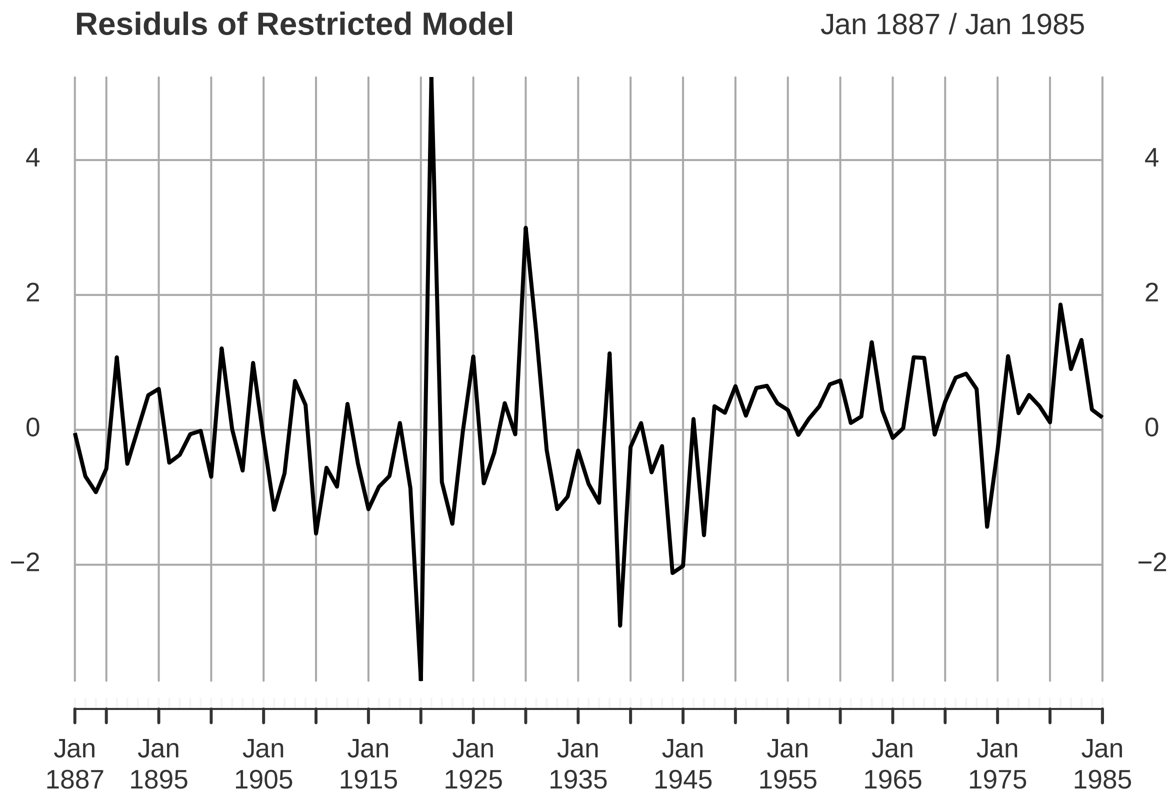
This implies our estimator is no longer the Maximum Likelihood estimator but is the minimum variance estimator in the class of linear unbiased estimators.

Performing a RESET test for functional form and non-linearity we see a p-value of > 0.05. We fail to reject the null hypothesis correct functional form and linearity. The RESET test takes the form of $y_t = \hat{\beta}_t x_t + \hat{u}_t$ then taking the residuals and $\hat{u}_t = b'x_t + c\hat{y}_t^2 + v_t$

```
##
##  RESET test
##
## data:  ur.ardl.model
## RESET = 0.16425, df1 = 2, df2 = 90, p-value = 0.8488
```

Restricted Model

We consider the model $\Delta U_t = \alpha_0 + \alpha_1 U_{t-1} + b_0 \Delta LQ_t + \epsilon_{2t}$




```
##
## Call:
## lm(formula = dyn(U ~ U.1 + LQ), data = r.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.7207 -0.6167  0.0248  0.5613  5.2258
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.67750    0.13222   5.124 1.54e-06 ***
## U.1           0.04115    0.07519   0.547    0.585
## LQ          -31.42355    3.49278  -8.997 2.13e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.11 on 96 degrees of freedom
##   (2 observations deleted due to missingness)
## Multiple R-squared:  0.4655, Adjusted R-squared:  0.4544
```

```
## F-statistic: 41.81 on 2 and 96 DF,  p-value: 8.707e-14
```

Fitting the restricted model we see that a level change in log GDP is significant at the 1% level, the change in unemployment is not significant. Plotting the residuals we see similar issues with the post-war period showing a different, possibly more constant variance. We see the same issues around the interwar years with large outliers and higher variance.

Again we no serial correlation with a DW statistic of approximately 2 (2.05).

```
durbinWatsonTest(as.vector(u))
```

```
## [1] 2.050358
```

Testing again for We test for heteroskedasticity using $\hat{u}_t^2 = \alpha + b'z_t + v_t$ using the hypothesis that $b' = 0$. In this case we use $z_t = x_t$

```
u2 <- u * u
summary(dyn$lm(u2 ~  U.1 + LQ, data = r.data))$fstatistic
```

```
##    value    numdf    dendf
## 15.92011  2.00000 96.00000
```

We see that the F-statstic is 15.9, suggesting the we fail to accept the null hypothesis of $b' = 0$ and of homoskedasticity. Similary a standalone BPG test shows we cannot accept the null hypothesis of constant variance

```
##
##  studentized Breusch-Pagan test
##
## data:  r.model
## BP = 24.657, df = 2, p-value = 4.423e-06
```

Testing for normality again we see that we do not accept the null hypothesis of normality using the Jarque Bera Test

```
##
##  Jarque Bera Test
##
## data:  u
## X-squared = 129.16, df = 2, p-value < 2.2e-16
```

Performing a RESET test to inform us on the functional form we see a RESET test statistic of 7.13 and a p-value of 0.001. We reject the null hyphothesis of non-linearity or correct functional form

```
##
##  RESET test
##
## data:  r.model
## RESET = 7.1321, df1 = 2, df2 = 94, p-value = 0.001307
```

Comparing the tests between the two models we see little difference bar the result of the RESET test which in the case of the restricted model fails for functional form. We observe heterskedasticity in both as well as non normality of the residuals. We do not observe any effects of serial correlation.

Comparing the AIC & BIC tests we see that AIC & BIC prefers the restricted model marginally.

Table 8: AIC & BIC comparisons for unrestricted (ur) and restricted (r) models

AIC.ur.ardl.model.	AIC.r.model.	BIC.ur.ardl.model.	BIC.r.model.
308.5393	306.5626	329.3003	316.9431

Looking at the likelihood ratio test $2(LL(\theta) - LL(\theta^*))$ we see

```
## [1] "LR Test: 6.02"
```

```
## [1] "Chi Sq CV: 9.49"
```

We have 7 parameters in the unrestricted model and 3 parameters in the restricted so we have imposed 4 restrictions. We have restricted the following coefficients $\alpha_2 = (\alpha_1 - 1); \beta_1 = -\beta_0; \beta_2 = 0; \gamma = 0$ with the remaining coefficients unchanged.

Using the likelihood ratio test (which is $\chi^2(4)$ distributed) we fail to reject the null hypothesis that the restrictions are binding/true.

ARIMA

We test the following variables, U_t, RL_t , G_t and INF_t for unit roots. Testing each for unit roots with and without trends we see the following

- U_t: We fail to reject the null hypothesis that unemployment has a unit root and is of at least I(1)
- RL_t: We fail to reject the null hypothesis that unemployment has a unit root and is of at least I(1)
- INF_t: Inflation does not have a unit root
- G_t: Growth does not have a unit root

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -21.6013  -0.7976   0.5862   2.1137  13.4810
##
## Coefficients:
##            Estimate Std. Error t value Pr(>|t|)
## z.lag.1    -0.18818    0.05798  -3.246  0.00161 **
## z.diff.lag  0.15017    0.10063   1.492  0.13885
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.96 on 97 degrees of freedom
## Multiple R-squared:  0.1007, Adjusted R-squared:  0.08213
## F-statistic: 5.429 on 2 and 97 DF,  p-value: 0.005821
##
##
```

```
## Value of test-statistic is: -3.2457
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -8.966 -0.755  1.275  2.632 10.640
##
## Coefficients:
##            Estimate Std. Error t value Pr(>|t|)
## z.lag.1    -0.41487    0.09906  -4.188  6.2e-05 ***
## z.diff.lag -0.12021    0.10133  -1.186    0.238
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.243 on 97 degrees of freedom
## Multiple R-squared:  0.2443, Adjusted R-squared:  0.2287
## F-statistic: 15.68 on 2 and 97 DF,  p-value: 1.261e-06
##
##
## Value of test-statistic is: -4.1881
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4967 -0.4548  0.0718  0.5684  9.2695
##
## Coefficients:
```

```
##            Estimate Std. Error t value Pr(>|t|)
## z.lag.1    -0.02529    0.02519  -1.004    0.318
## z.diff.lag  0.14093    0.10184   1.384    0.170
##
## Residual standard error: 1.492 on 97 degrees of freedom
## Multiple R-squared:  0.02527,    Adjusted R-squared:  0.005168
## F-statistic: 1.257 on 2 and 97 DF,  p-value: 0.2891
##
##
## Value of test-statistic is: -1.0038
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4097 -0.6137 -0.2345  0.4794  8.9620
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.377221   0.396819   0.951   0.3442
## z.lag.1     -0.099829   0.045615  -2.189   0.0311 *
## tt           0.003140   0.005237   0.600   0.5502
## z.diff.lag   0.175586   0.103506   1.696   0.0931 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.473 on 95 degrees of freedom
## Multiple R-squared:  0.06874,    Adjusted R-squared:  0.03933
## F-statistic: 2.337 on 3 and 95 DF,  p-value: 0.07853
##
##
## Value of test-statistic is: -2.1885 1.8541 2.7449
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.99 -3.43 -3.13
## phi2  6.22  4.75  4.07
## phi3  8.43  6.49  5.47

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
```

```
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9742 -0.1174  0.0296  0.1992  3.7896
##
## Coefficients:
##            Estimate Std. Error t value Pr(>|t|)
## z.lag.1     0.00251    0.01077   0.233   0.8162
## z.diff.lag  0.26079    0.09952   2.620   0.0102 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6253 on 97 degrees of freedom
## Multiple R-squared:  0.07072,    Adjusted R-squared:  0.05156
## F-statistic: 3.691 on 2 and 97 DF,  p-value: 0.02851
##
##
## Value of test-statistic is: 0.2331
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.6500 -0.2288  0.0274  0.1283  3.8623
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.041577   0.128312   0.324  0.74663
## z.lag.1     -0.066426   0.030854  -2.153  0.03386 *
## tt           0.006769   0.003361   2.014  0.04682 *
## z.diff.lag   0.281447   0.098117   2.868  0.00508 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.6138 on 95 degrees of freedom
## Multiple R-squared:  0.1115, Adjusted R-squared:  0.08346
## F-statistic: 3.974 on 3 and 95 DF,  p-value: 0.01025
##
##
## Value of test-statistic is: -2.1529 1.9103 2.4806
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.99 -3.43 -3.13
## phi2  6.22  4.75  4.07
## phi3  8.43  6.49  5.47
```

We estimate a random walk (using OLS) for inflation of the following form $y_t = \alpha + y_{t-1} + \epsilon_t$

```
##
## Call:
## lm(formula = dyn(INF ~ stats::lag(INF, k = 1)), data = macro.series)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -21.9233  -1.6763  -0.3137   1.2183  12.4003
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)             0.93307    0.45890   2.033   0.0447 *
## stats::lag(INF, k = 1)  0.76829    0.06424  11.959   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.906 on 98 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.5934, Adjusted R-squared:  0.5893
## F-statistic:   143 on 1 and 98 DF,  p-value: < 2.2e-16
```

We estimate the random walk and the ARMA using Maximum Likelihood

```
##           Length Class  Mode
## coef        3    -none- numeric
## sigma2      1    -none- numeric
## var.coef    9    -none- numeric
## mask        3    -none- logical
## loglik      1    -none- numeric
## aic         1    -none- numeric
## arma        7    -none- numeric
## residuals 101    ts     numeric
## call        4    -none- call
## series      1    -none- character
## code        1    -none- numeric
## n.cond      1    -none- numeric
## nobs        1    -none- numeric
## model      10    -none- list
```

```
##           Length Class  Mode
## coef        2    -none- numeric
## sigma2      1    -none- numeric
```

```
## var.coef    4    -none- numeric
## mask        2    -none- logical
## loglik      1    -none- numeric
## aic         1    -none- numeric
## arma        7    -none- numeric
## residuals 101    ts     numeric
## call        4    -none- call
## series      1    -none- character
## code        1    -none- numeric
## n.cond      1    -none- numeric
## nobs        1    -none- numeric
## model      10    -none- list
```

Table 9: Restricted/Unrestricted Model comparison

ma.model.aic	rw.model.aic	X2....ma.model.loglik...rw.model.loglik.	qchisq.0.05..df...1..lower.tail...F.
563.557	566.3142	4.757261	3.841459

THe AIC for the ARMA and Random Walk is 563 & 566 respectively so using AIC we marginally choose the ARMA model over the random walk. Using BIC we see they are almost identical, 574 and 574.2. Using a likelihood ratio test of $2(LL(\theta) - LL(\hat{\theta}_r))$ we obtain a value of 4.75, which means we reject the null hypothesis at the 10% (cv = 3.84 $\chi^2(1)$) level but fail to reject at the 5% level (cv = 5.02 $\chi^2(1)$)

VAR

We test a 4 variable VAR with INF_t, RL_t, U_t and G_t. First we look at possible Granger Causality between the variables. We do not include all the GC tests in our results but we include those with the greatest significance. We can say reject the null hypothesis of no Granger Causality between

- inflation and unemployment, INF_t and U_t.
- growth and unemployment, G_t and U_t (and vice versa)
- interest rates and Unemployment
- inflation and growth, G_t is Granger causal of INF_t is the most significant finding (F-stat = 13.3)

```r
grangertest(U ~ INF, data=macro.subset[, c("U", "RL", "INF", "G")], order = 1)
```

```
## Granger causality test
##
## Model 1: U ~ Lags(U, 1:1) + Lags(INF, 1:1)
## Model 2: U ~ Lags(U, 1:1)
##   Res.Df Df      F   Pr(>F)
## 1     97
## 2     98 -1 7.3644 0.007874 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
grangertest(U ~ G, data=macro.subset[, c("U", "RL", "INF", "G")], order = 1)
```

```
## Granger causality test
##
## Model 1: U ~ Lags(U, 1:1) + Lags(G, 1:1)
## Model 2: U ~ Lags(U, 1:1)
##   Res.Df Df      F   Pr(>F)
## 1     97
## 2     98 -1 9.5584 0.002599 **
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
grangertest(G ~ U, data=macro.subset[, c("U", "RL", "INF", "G")], order = 1)
```

```
## Granger causality test
##
## Model 1: G ~ Lags(G, 1:1) + Lags(U, 1:1)
## Model 2: G ~ Lags(G, 1:1)
##   Res.Df Df      F   Pr(>F)
## 1     97
## 2     98 -1 7.4898 0.007381 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
grangertest(RL ~ U, data=macro.subset[, c("U", "RL", "INF", "G")], order = 1)
```

```
## Granger causality test
##
## Model 1: RL ~ Lags(RL, 1:1) + Lags(U, 1:1)
## Model 2: RL ~ Lags(RL, 1:1)
##   Res.Df Df      F   Pr(>F)
## 1     97
## 2     98 -1 6.9314 0.009856 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
grangertest(INF ~ G, data=macro.subset[, c("U", "RL", "INF", "G")], order = 1)
```

```
## Granger causality test
##
## Model 1: INF ~ Lags(INF, 1:1) + Lags(G, 1:1)
## Model 2: INF ~ Lags(INF, 1:1)
##   Res.Df Df     F    Pr(>F)
## 1     97
## 2     98 -1 13.31 0.0004272 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Looking for cointegrating vectors using the Johansen test (trace) we see the following

```r
jo.test <- ca.jo(na.omit(macro.subset[, c("U", "G", "INF", "RL")]),type="trace", K=2, ecdet="none")
summ.johansen <- summary(jo.test)
print(summ.johansen)
```

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: trace statistic , with linear trend
##
## Eigenvalues (lambda):
## [1] 0.42454481 0.28539264 0.08230043 0.01916604
##
## Values of teststatistic and critical values of test:
##
```

```
##           test 10pct  5pct  1pct
## r <= 3 |  1.92  6.50  8.18 11.65
## r <= 2 | 10.42 15.66 17.95 23.52
## r <= 1 | 43.68 28.71 31.52 37.22
## r = 0  | 98.39 45.23 48.28 55.43
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##             U.l2       G.l2      INF.l2     RL.l2
## U.l2    1.000000  1.0000000  1.00000000 1.0000000
## G.l2    8.583983 -0.3739642  0.02561089 0.7724564
## INF.l2  1.761029  2.0177664  0.19540319 0.2803455
## RL.l2  -3.994050 -2.3254088 -0.58025596 2.2511118
##
## Weights W:
## (This is the loading matrix)
##
##               U.l2        G.l2      INF.l2        RL.l2
## U.d   -0.021063660  0.04472537 -0.10609981  0.008960492
## G.d   -0.070189366 -0.03659693  0.20215403 -0.023625683
## INF.d  0.048256797 -0.25812881  0.04750720  0.007889202
## RL.d   0.009049231 -0.01031197 -0.03693521 -0.006189780
```

```r
jo.test <- ca.jo(na.omit(macro.subset[, c("U", "G", "INF", "RL")]),type="eigen", K=2, ecdet="none")
summ.johansen <- summary(jo.test)
print(summ.johansen)
```

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: maximal eigenvalue statistic (lambda max) , with linear trend
##
## Eigenvalues (lambda):
## [1] 0.42454481 0.28539264 0.08230043 0.01916604
##
## Values of teststatistic and critical values of test:
##
##           test 10pct  5pct  1pct
## r <= 3 |  1.92  6.50  8.18 11.65
## r <= 2 |  8.50 12.91 14.90 19.19
## r <= 1 | 33.27 18.90 21.07 25.75
## r = 0  | 54.71 24.78 27.14 32.14
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##             U.l2       G.l2      INF.l2     RL.l2
## U.l2    1.000000  1.0000000  1.00000000 1.0000000
## G.l2    8.583983 -0.3739642  0.02561089 0.7724564
## INF.l2  1.761029  2.0177664  0.19540319 0.2803455
## RL.l2  -3.994050 -2.3254088 -0.58025596 2.2511118
##
```

```
## Weights W:
## (This is the loading matrix)
##
##                U.l2        G.l2      INF.l2        RL.l2
## U.d   -0.021063660  0.04472537 -0.10609981  0.008960492
## G.d   -0.070189366 -0.03659693  0.20215403 -0.023625683
## INF.d  0.048256797 -0.25812881  0.04750720  0.007889202
## RL.d   0.009049231 -0.01031197 -0.03693521 -0.006189780
```

We reject the hypothesis of 0, and 1 co-integrating vectors. However, we can say there are at least 2 co-integrating vectors, maybe 3 co-integrating vectors as we fail to reject the test for r<=3 co-integrating vectors.