Exam No: U112735 Student No: 13706806

Q2

Consider

$$y_t = \beta' x_t + u_t$$

a)

i) Serial correlation impacts the assumption $E[u_t u_{t-i}] = 0$ s.t. $E[u_t u_{t-i}] \neq 0$.

The estimator $\hat{\beta}$ is unbiased but not minimum variance.
The standard errors are biased

ii) generally, for serial correlation up to p we test the following

$$u_t = \sum_{i=1}^{p} e_i u_{t-i} + b' x_t + e_t$$

the null of $e_i = 0$ jointly using an F-test.

for $p = 4$

$$u_t = \sum_{i=1}^{4} e_i u_{t-i} + b' x_t + e_t$$

or

$$u_t = e_4 u_{t-4} + b' x_t + e_t$$

~~We~~

iii) We can attempt to account for serial correlation by adding more lags in the model, using robust standard errors or GLS

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b)

This is the presence of heteroskedasticity

i) The estimator $\hat{\beta}$ remains unbiased but again not minimum variance

ii) We can test for non constant variance using a BPG test of the following form

$$\hat{u}^2 = \alpha + b' z_t + \epsilon_t$$

for constant variance we need $b = 0$ so we test accordingly, null hypothesis of constant variance. using an F-test

Z_t can either be x_t (regressors) or their squares / cross product.
The fitted values $\hat{y}^2$ can also be used similar to the RESET test for functional form.

iii) We can try respecify the model with logs if possible, weight or scale the variables or estimating a GARCH.
Like before robust standard errors should be used if heteroskedasticity remains.

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c) Incorrect functional form

i) $\hat{\beta}$ is inconsistent & biased

ii) We can use a Reset test of the form

$$\hat{u}_t = \alpha + b\hat{y}_t^2 + \epsilon_t \qquad \hat{y} = \text{fitted value } \hat{\beta}X$$

testing for $b = 0$; if we reject the null then there may be an issue with functional form. (We are testing if $\hat{\beta}^2$ has a relationship to the residuals)

iii) We can estimate by non linear least squares or in the first instance try a new functional form

d)

i) Like heteroskedasticity above $\hat{\beta}$ is unbiased but not minimum variance

ii) We can perform a variance ratio or Goldfeld-Quandt test of the form

$$\frac{S_1^2}{S_2^2} = \frac{\hat{u}'\hat{u}.(T_1 - k)^{-1}}{\hat{u}_2'\hat{u}_2.(T_2 - k)^{-1}}$$

Note: put larger variance on top

where S_1^2 is sample variance of 1 period
S_2 " " " " 2 period

This is $F(T_1 - k, T_2 - k)$ distributed

iii) We can estimate the model over different periods or perhaps weight the variables according to the volatility

e) ~~Sh~~ Change in $\hat{\beta}$ over periods

i) We can say $\hat{\beta}$ is inconsistent & biased not reflective of β

ii) We can use a Chow test to test over different periods. The test statistic is

$$\frac{(\hat{u}'\hat{u} - \hat{u}_1'\hat{u}_1 - \hat{u}_2'\hat{u}_2) . k^{-1}}{(\hat{u}_1'\hat{u}_1 + \hat{u}_2'\hat{u}_2).(T_2 - k)^{-1}}$$

where $\hat{u}_1$ & $\hat{u}_2$ are the estimated residuals after the different periods T_1 & T_2 $(T - T_1)$ respectively.
$\hat{u}$ are the residuals for the whole period T
k : # of parameters.

Note that this test assumes constant variance.

iii) · As before estimate the two periods separately.

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Q3.

Consider

a $y_t = \alpha_t + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \epsilon_t$

a) A Covariance stationary process means that a process y_t has a stationary variance / covariance over a given period. This is also known as weakly stationary

The order of integration is the number of times a series must be differenced to become stationary

I(0): no difference, process is stationary

I(1): Integrated of order 1, process must be differenced once i.e $y_t - y_{t-1}$ is stationary

I(2): Integrated order two, process must be differenced twice i.e $y_t - y_{t-1} - y_{t-1} - y_{t-2}$ is stationary

b) $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \epsilon_t$

$y_t - y_{t-1} = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} - y_{t-1} + \epsilon_t$

$\Delta y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{2-2} + \beta_3 y_{t-2} - \beta_3 y_{t-2} + \beta_3 y_{t-3} - y_{t-1} + \epsilon_t$

$= \alpha + \beta_1 y_{t-1} - y_{t-1} + y_{t-2}(\beta_2 + \beta_3) + \cancel{y_t \beta_3} - \beta_3 (y_{t-2} - y_{t-3})$

$= \cancel{\alpha + y_{t-1}(\beta_1 - 1) + y_{t-2}(\beta_2 + \beta_3) - \beta_3(y_{t-2} + y_{t-3})}$

$$\Delta y_t = \alpha + \rho_1 y_{t-1} - y_{t-1} + y_{t-2}(\rho_2+\rho_3) - \rho_3(y_{t-2} - y_{t-3})$$

$*(+ - (\rho_2+\rho) y_{t-1})$

$$= \alpha + \rho_1 y_{t-1} - y_{t-1} - y_{t-1}(\rho_2+\rho_3) + y_{t-1}(\rho_2+\rho_3) + y_{t-2}(\rho_2+\rho_3) - \rho_3(y_{t-2} - y_{t-3}) + \epsilon_t$$

$$= \alpha + \rho_1 y_{t-1} - y_{t-1} + y_{t-1}(\rho_2+\rho_3) - (\rho_2+\rho_3)(y_{t-1} - y_{t-2}) - \rho_3(y_{t-2} - y_{t-3}) + \epsilon_t$$

$$\Delta y_t = \alpha + y_{t-1}(\rho_1+\rho_2+\rho_3-1) - (\rho_2+\rho_3)(y_{t-1}-y_{t-2}) - \rho_3(y_{t-2}-y_{t-3})$$

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \epsilon_t$$

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t$$

We can test for $\beta = 0$ to test for unit root or stationarity (or non stationarity to be precise).

We have re parameterised the model but not imposed restrictions.

This is the augmented dicky fuller statistic.

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t$$

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c)

To test for I(2) against I(1) the test statistic is

$$\Delta^2 y_t = \alpha + \beta \Delta y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta^2 y_{t-i} + \epsilon_t$$

Again we are not imposing restrictions but simply re parameterising the initial y_t, then first differencing then second differencing

Explicitly we would rewrite

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + \epsilon_t$$

$$y_t - y_{t-1} - y_{t-1} - y_{t-2} = \Delta^2 y_t$$

$$= \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + y_{t-1} - y_{t-1} - y_{t-2}$$
$$+ \epsilon_t$$

⋮

d)

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + \epsilon_t + \gamma t$$

$$\Delta y_t = \cancel{\alpha + \rho_1 y_{t-1}} -$$

$$\Delta y_t = \alpha + (\rho_1 - 1)(y_{t-1} - gt) + \rho_2 y_{t-2} + \rho_3 y_{t-3} + \epsilon_t$$

$$\gamma = g(1-\rho)$$

as ~~before~~

we add / subtract $(\rho_2 + \rho_3) y_{t-1}$ / $\pm \rho_3 y_{t-2}$

as before

$\vdots$

$$\Delta y_t = \alpha + (\rho_1 + \rho_2 + \rho_3 - 1)(y_{t-1} - gt) \ldots\ldots$$

$$= \alpha + \beta(y_{t-1} - gt) + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t$$

the trend needs to be restricted when $\beta = 0$.

e) As we are dealing with small values $0 < \rho < 1$, it can be hard to distinguish between different values of ρ so it comes down to judgement.

- How intercept & trend are handled in the estimate is important and can cause variation again
- The sample of the data is important

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- can be 1(1) over T_1 but 1(0) over T.

- A

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Q6

u_t : Unemployment

q_t : log real GDP.

$$u_t = \alpha_0 + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2} + ct + \epsilon_{1t} \quad (2)$$

$$\Delta u_t = a_0 + a_1 u_{t-1} + a_2 \Delta u_{t-1} + b_0 \Delta q_t + b_1 q_{t-1} + b_2 \Delta q_{t-1} + ct \quad (3)$$

$$(2) \quad u_t - u_{t-1} = \alpha_0 + \alpha_1 u_{t-1} - u_{t-1} + \alpha_2 u_{t-2} + \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2}$$

$$\left[\pm a_2 u_{t-1} \quad \pm \beta_2 q_{t-1} \right]$$

$$= \alpha_0 + \alpha_1 u_{t-1} - u_{t-1} + \alpha_2 u_{t-2} - a_2 u_{t-1} + \alpha_2 u_{t-1} + \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-1} - \beta_2 q_{t-1} + \beta_2 q_{t-2}$$

$$= \alpha_0 + \alpha_1 u_{t-1} - u_{t-1} + \alpha_2 u_{t-1} \cancel{- \alpha_2 u_{t-1}} - \alpha_2 (u_{t-1} - u_{t-2}) + \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-1} - \beta_2 (q_{t-1} - q_{t-2}) + ct + \epsilon_t$$

$$= a_0 + u_{t-1} (\alpha_1 - 1 + \alpha_2) - \alpha_2 \Delta u_{t-1} + \beta_2 q_{t-1} - \beta_2 (\Delta q_{t-1}) + \beta_0 q_t + \beta_1 q_{t-1} + ct + \epsilon_t$$

$\beta_1 = -\beta_0$

$$= a_0 + u_{t-1} (\alpha_1 - 1 + \alpha_2) - \alpha_2 \Delta u_{t-1} + \beta_2 q_{t-1} - \beta_2 \Delta q_{t-1} + \beta_0 \Delta q_t + ct + \epsilon_t$$

so

$a_0 = \alpha_0$

$a_1 = (\alpha_1 + \alpha_2 - 1)$

$a_2 = -\alpha_2$

$b_0 = \beta_0$

$b_1 = \beta_2$

$b_2 = -\beta_2$

b) Long run effect (3)

$$\Delta u_t = \alpha_0 + a_1 u_{t-1} + a_2 \Delta u_{t-1} + b_0 \Delta q_t + b_1 q_{t-1} + \cancel{b} + b_2 \Delta q_{t-1} + ct + \epsilon_t$$

$u_t = u_{t-1} = u_{t-2}$

$q_t = q_{t-1} = q_{t-2}$

$$0 = \alpha_0 + a_1 u_t + a_2 0 + b_0 0 + b_1 q_t + b_2 0 + ct + \epsilon$$

$a_1 u_t = -\alpha_0 - b_1 q_t - ct$

$u_t = \frac{-\alpha_0}{\alpha_1} - \frac{b_1}{a_1} q_t - ct$

$-\frac{b_1}{a_1} = \theta_q$ = long run coeff

$= \frac{-2}{-0.03} = 66 \ldots$

This is not a reasonable estimate, the estimate for unemployment is causing this to blow up / not necessarily significant

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c)

(4) $\Delta u_t = \alpha + a_1 u_{t-1} - \overset{b_1}{\cancel{2b}}\Delta q_t + \varepsilon_t$

~~ps~~ in ~~a~~ we imposed restriction $\beta_1 = -\beta_0$ to go from 2 to (3)

model (2) $\Delta u_t = \alpha_0 + a_1 u_{t-1} + \Delta u_{t-1} a_2 + b_0 \Delta q_t + b_1 q_{t-1} + b_2 \Delta q_{t-1} + \varepsilon_t$

so imposing the following

$a_2 = 0$, $b_1 = 0$, $b_2 = 0$

gives us model 4

which going back to our parameters

$a_2 = -\alpha_2$; ~~or~~ $\alpha_2 = 0$

$b_1 = \beta_2 = 0 = b_2$

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d) if the change in growth is 0 then only previous unemployment impacts unemployment

the t-stat gives $\frac{-24}{2.81} = -8.5$

which means we reject the null of growth ~~bee~~

e)

~~Att~~	MLL	AIC	BIC	k
	-30.58	-37.58	-45.30	(1)
	-30.58	-37.58	-45.3	(2)
	-34	-38.34	-42.75	(3)
	-41.15	-44.15	-47.46	(4)

We choose model 1 & 2 by AIC as they are the same

~~The~~ We choose model 3 by BIC

Given similar SER etc. between three models, I would choose model 3, less parameter to estimate ~~and it seems~~ ~~the~~ and it seems the parameter estimate errors are less