BIRKBECK
(University of London)

MSc EXAMINATION FOR INTERNAL STUDENTS

Department of Economics, Mathematics and Statistics

ECONOMETRICS/ ECONOMETRICS for PG Certificate

EMEC026S7/BUEM007H7

Friday, 03 June 2016, 10.00 am - 12.10 pm (includes 10 minutes reading time)

Answer **ANY THREE** questions. All questions carry the same weight; the relative weight of sub-questions is indicated in square brackets.

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1. Consider the linear regression model

$$y_t = \mathbf{x}_t'\beta + u_t, \qquad t = 1, 2, \ldots, n,$$

where $\mathbf{x}_t$ is a $k \times 1$ vector of stochastic explanatory variables, β is a $k \times 1$ vector of unknown coefficients, and $\{u_t\}$ are unobservable random disturbances with zero mean. Suppose that some of the explanatory variables are endogenous so that $\mathsf{E}(\mathbf{x}_t u_t) \neq \mathbf{0}$. Assume that there exists an $m \times 1$ vector of instruments $\mathbf{w}_t$ such that $\mathsf{E}(\mathbf{w}_t u_t) = \mathbf{0}$.

(a) Explain how to estimate β by the generalised method of moments (GMM). Make sure to distinguish between the exactly identified (or just-identified) case and the over-identified case. [30%]

(b) Show that, in the exactly identified case, the GMM estimator of β is the same as the instrumental variables estimator

$$\widehat{\beta}_{\mathrm{IV}} = \left(\sum_{t=1}^{n} \mathbf{w}_t \mathbf{x}_t' \right)^{-1} \left(\sum_{t=1}^{n} \mathbf{w}_t y_t \right).$$

[15%]

(c) Outline a method for obtaining an asymptotically efficient GMM estimate of β in the over-identified case. [30%]

(d) Explain how a test for the validity of the instruments $\mathbf{w}_t$ may be carried out. Give the formula for the appropriate test statistic and state its null distribution. [25%]

2. Consider the linear simultaneous equations model

$$\begin{aligned} y_{1t} &= \beta_{12} y_{2t} + \gamma_{11} x_{1t} + u_{1t}, \\ y_{2t} &= \beta_{21} y_{1t} + \gamma_{22} x_{2t} + \gamma_{23} x_{3t} + u_{2t}, \end{aligned}$$

where (y_{1t}, y_{2t}) are endogenous variables, (x_{1t}, x_{2t}, x_{3t}) are exogenous variables, and (u_{1t}, u_{2t}) are homoskedastic and serially uncorrelated random disturbances with zero mean.

(a) Discuss the identifiability of each equation of the system in terms of the order and rank conditions for identification. [30%]

(b) Explain why the ordinary least squares estimator of $(\beta_{12}, \gamma_{11})$ is inconsistent. [20%]

(c) Describe, step by step, how to estimate the coefficients in the two equations using two-stage least squares. Explain how the two-stage least squares estimator is related to an appropriately constructed instrumental variables estimator. [30%]

(d) Would you estimate the coefficients in the two equations by indirect least squares? Explain. [20%]

3. Consider the vector autoregressive (VAR) model

$$\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \varepsilon_t,$$

where $\mathbf{y}_t$ is an $m \times 1$ random vector, $\mathbf{A}_1$ and $\mathbf{A}_2$ are $m \times m$ coefficient matrices, and $\{\varepsilon_t\}$ is an m-dimensional white noise process with zero mean and bounded, positive definite variance–covariance matrix $\mathbf{\Sigma}$. Let $\mathbf{I}_m$ denote the $m \times m$ identity matrix.

(a) State condition(s) under which $\{\mathbf{y}_t\}$ is covariance stationary. [10%]

(b) Show that the first three coefficient matrices in the moving average representation $\mathbf{y}_t = \sum_{k=0}^{\infty} \mathbf{\Psi}_k \varepsilon_{t-k}$ of the VAR are $\mathbf{\Psi}_0 = \mathbf{I}_m$, $\mathbf{\Psi}_1 = \mathbf{A}_1$ and $\mathbf{\Psi}_2 = \mathbf{A}_1^2 + \mathbf{A}_2$. Is your answer to (a) relevant for this calculation? Explain what the elements of $\mathbf{\Psi}_1$ and $\mathbf{\Psi}_2$ represent. [40%]

(c) Explain what the 'orthogonalised impulse responses' of the VAR are. Explain how to compute the orthogonalised impulse responses and why orthogonalisation is important. [40%]

(d) Suppose $\mathbf{A}_1 + \mathbf{A}_2 = \mathbf{I}_m$. What does this imply about the integration properties of $\{\mathbf{y}_t\}$? Explain. [10%]

4. Consider the linear regression model

$$y_t = \beta x_t + \varepsilon_t, \qquad t = 1, 2, \ldots, n,$$

where x_t is a non-stochastic explanatory variable. The disturbances $\{\varepsilon_t\}$ satisfy

$$\varepsilon_t = z_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2},$$

where $\alpha_0 > 0$, $\alpha_1 \geqslant 0$, and $\{z_t\}$ are independent and identically distributed random variables having the standard normal distribution (z_t is independent of ε_{t-i} for $i \geqslant 1$).

(a) Show that $\mathsf{E}(\varepsilon_t) = 0$, $\mathrm{Var}(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1}$, and $\mathrm{Cov}(\varepsilon_t, \varepsilon_{t-k}) = 0$ for all $k \geqslant 1$. What assumption(s) do you need to make to ensure that $\{\varepsilon_t\}$ is covariance stationary? [30%]

(b) What statistical properties does the ordinary least squares estimator of β have? Explain how to obtain an asymptotically efficient estimate of $(\beta, \alpha_0, \alpha_1)$. [25%]

(c) Explain how to test the null hypothesis that $\alpha_1 = 0$ using the Lagrange multiplier principle. [20%]

(d) Explain why the autocorrelation function and partial autocorrelation function of $\{\varepsilon_t^2\}$ may be useful in assessing the validity of the first-order ARCH assumption about $\{\varepsilon_t\}$. [25%]

5. Consider a univariate time series $\{y_t\}$ satisfying

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t, \qquad t = 1, 2, \ldots, n, \\ \mu_t &= \mu_{t-1} + u_t, \end{aligned}$$

where $\{\varepsilon_t\}$ and $\{u_t\}$ are independent and identically distributed random variables with $\mathsf{E}(\varepsilon_t) = \mathsf{E}(u_t) = 0$, $\text{Var}(\varepsilon_t) = \sigma_\varepsilon^2$ and $\text{Var}(u_t) = \sigma_u^2$. The disturbances $\{\varepsilon_t\}$ and $\{u_t\}$ are mutually independent as well as independent of the initial value μ_0. The trend component μ_t of the series and the disturbances (ε_t, u_t) are unobservable but y_t is observed.

(a) Show that, when $\sigma_\varepsilon^2 > 0$, y_t satisfies an ARIMA$(0, 1, 1)$ model. What is the ARIMA representation of y_t under the restriction $\sigma_\varepsilon^2 = 0$? [45%]

(b) What does the restriction $\sigma_u^2 = 0$ imply about the behaviour of the trend μ_t? Explain. [15%]

(c) Let $Y_{t-1} = \{y_1, \ldots, y_{t-1}\}$ be the information set available at time $t-1$. Assume that the conditional distribution of y_t given Y_{t-1} is normal with mean $\hat{\mu}_{t|t-1}$ and variance f_t, where $\hat{\mu}_{t|t-1}$ is the forecast of μ_t based on Y_{t-1} and f_t is the variance of the forecast error $\nu_t = y_t - \hat{\mu}_{t|t-1}$ computed by the Kalman filter. Explain how you may estimate the parameters $(\sigma_\varepsilon^2, \sigma_u^2)$. (Note that you are not required to describe how $\hat{\mu}_{t|t-1}$ and f_t are computed.) [40%]

End of Examination Paper