

# Forecasting Economic and Financial Time Series

## Week 1: Introduction

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MSc/PGCE Option, EMS Birkbeck

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# Module Aims and Objectives

- ▶ Examines how to make and evaluate forecasts of economic and financial time series for decision making.
- ▶ It contains a mixture of technical and practical material and a prior knowledge of time series econometrics at the level of the Autumn Econometrics module is assumed.
- ▶ Students who complete the course should be able to:
  - ▶ use a range of models to produce point and probability forecasts;
  - ▶ do economic and statistical evaluation of forecasts and understand the role of loss functions;
  - ▶ understand the limitations of point forecasts and be able to quantify forecast uncertainty through the use and evaluation of density forecasts.
  - ▶ interpret OBR and Bank of England forecasts

# Reading

- ▶ Key texts: *Elements of Forecasting*, by Diebold, and *Economic Forecasting*, Elliott and Timmerman, level of the course will be between the two.
- ▶ Ghysels & Marcellino *Applied Economic Forecasting* good on models.
- ▶ Some other useful readings on course outline and we will put material on Moodle.
- ▶ More popular books
  - ▶ Silver, N. (2012) *The signal and the noise: The Art and Science of Prediction*, Penguin.
  - ▶ Tetlock P.E. and D. Gardner (2015) *Superforecasting: The Art and Science of Prediction*, Random House
- ▶ Tetlock's superforecasters are primarily concerned with discrete defined events rather than continuous variables.

## Schedule by weeks (only a forecast)

1. Jan 15: Introduction,
2. Jan 22: Trends, Cycles and seasonality.
3. Jan 29: AR, MA, and ARIMA models.
4. Feb 5: Basic Bayes for forecasting, VARS
5. Feb 12: Forecast evaluation,
6. Feb 19: Reading Week No lecture
7. Feb 26: Factor Models and combining models.
8. March 5: Volatility modelling, ARCH and GARCH, Density forecasting and evaluating density forecasts.
9. March 12: *Applications*
10. March 19: *Applications*
11. March 26: Revision and round up: using economic models in forecasting and responses to forecast failure.

# Introduction

- ▶ Forecasting is common in any activity where decision making under uncertainty is important
- ▶ Some define
  - ▶ risk as where we know the event space, the set of possible events, and the probabilities attached to them;
  - ▶ uncertainty as where we know the event space but cannot attach probabilities to them; and
  - ▶ unawareness as where we do not know the set of possible events, where inconceivable events may occur, what Nassim Taleb labelled "Black Swans" or Donald Rumsfeld "unknown unknowns".
- ▶ We are primarily concerned with risk, which is more susceptible to statistical modelling.
- ▶ Lots of other definitions of risk, ISO31000: the effect of uncertainty on objectives. This can be positive as well as negative: death is certain, life is uncertain.

# Axiomatic Probability

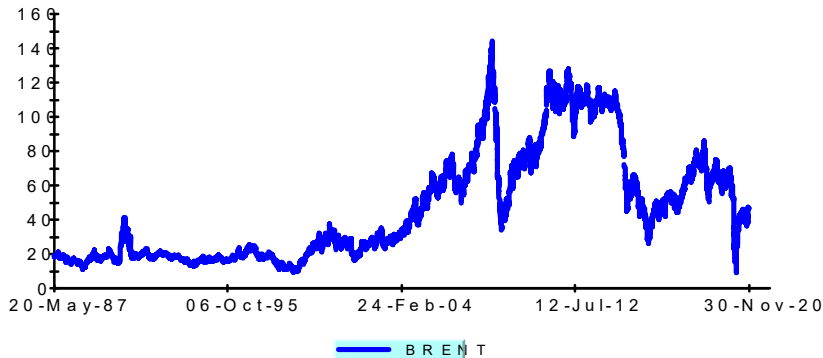
- ▶ A probability space  $(\Omega, \mathcal{F}, P)$  consists of
  - ▶ a sample space  $\Omega$ , the set of all possible outcomes,
  - ▶ an event space  $\mathcal{F}$  given by a  $\sigma$  field a set of subsets of  $\Omega$  and
  - ▶ a probability function  $P$ , assigning probabilities to events  $\mathcal{F}$  in such that  $P(.) : \mathcal{F} \rightarrow [0, 1]$ .
- ▶ A random variable maps  $\Omega$  onto the real line:  $X(.) : \Omega \rightarrow \mathbb{R}$
- ▶ How do we use these in practice. Use the oil price as an example.

# The oil price

- ▶ The event space?
- ▶ We do not know what things that may influence the oil price over different horizons. My January 2020 list was
  - ▶ Geopolitical factors: drone attacks on Saudi refineries; tensions in Saudi, Russia, Iran, Venezuela, Nigeria,.....
  - ▶ Geological factors: reserves/discoveries, costs of exploiting them, ...
  - ▶ Logistic factors: inventories in Cushing Oklahoma, restrictions on tankers, blocking Gulf of Hormuz, ..
  - ▶ Demand factors: growth in China, US, ...
  - ▶ Technological factors: alternative fuel sources, ...
- ▶ Left out the most important factor.
- ▶ Financial markets love risk but hate uncertainty. How do they turn uncertainty into risk? Make it a random variable: price defined over a space.

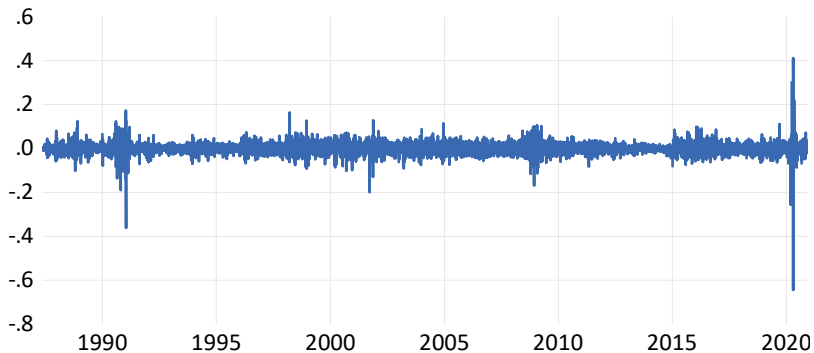
# Brent \$/barrel daily (breaks holidays) 1987-2020

Very close to log random walk  $\ln y_t = \alpha + \ln y_{t-1} + \varepsilon_t$





## Change in daily log Brent



# Frequency matters, descriptive stats for change in log Brent

	Daily	Monthly
Max	0.412	0.466
Min	-0.644	-0.511
SD	0.025	0.097
Skew	-1.93	-0.548
Kurtosis	75.20	6.857
N	8318	370

Daily sample 5/20/1987-11/30/2020,

Monthly sample 1990m1-2020m11

## Ron's advice

- ▶ Modelling and thinking about the future is essential to make decisions
- ▶ But do not forecast
  - ▶ Unless someone pays you large amounts to do it.
  - ▶ Since people pay, it is still worth learning how to forecast.
  - ▶ The forecasts will be wrong; so learn how to respond to forecast failure;
- ▶ Consider alternatives to forecasting: such as scenario planning, hedging/insurance, contingency plans. Try to be like bookmakers, make money whoever wins.
- ▶ Military Sayings
  - ▶ Failing to plan is planning to fail
  - ▶ No plan survives contact with the enemy
  - ▶ So you need plans B and C and D (unless plan B undermines credibility of plan A)
- ▶ Constructing scenarios helps you build contingency plans, you need a model to do it consistently

# Diebold's six questions for forecasters

1. **Decision environment and loss function. Why are we doing it?** A good forecast is one that leads to good decisions, judge this by some **Loss Function**.
2. **Forecast object. What are we forecasting?** A time series, such as sales, or an event such as a recession.
3. **Forecast Statement. How to we wish to state our forecast?** Point, interval or density forecast?
4. **Forecast Horizon. How far into the future do we wish to forecast?** This has implications for modelling strategy.
5. **Information set. What information is available at the time of forecasting?**, How much past history of the series? Other data? How big an estimation window?
6. **Method and complexity. Which method to use and how complex should the forecasting model be?** Parsimony (few estimated parameters) and shrinkage towards a prior may help.

# 1. Decision environment and loss function

- ▶ Forecasts are made in order to guide decisions and their value are thus judged on how they aid that decision in terms of minimising losses associated with a range of possible outcomes or states of the world.
- ▶ Consider a firm deciding how much inventory for next period. If you knew that demand would be high next period, then you would like to have a lot of inventory. If demand were low, then like to reduce your inventory.
- ▶ The problem then is how do you form a forecast or expectation of what demand will be next period?
- ▶ For the four possible combinations of inventory decisions and demand outcomes: in two we make the correct decision and in the other two, incorrect.
- ▶ Under symmetric loss, cost of unmet sales and unsold inventory the same.
- ▶ Under asymmetric loss, cost of unmet sales and unsold inventory are different.

## Types of Loss Function

- ▶ Denote forecast error  $e = y - \hat{y}$ , and assume loss only depends on the error.
- ▶ Quadratic Loss:  $L(e) = e^2$  : Mathematically convenient. Square makes it symmetric around the origin and increasing at an increasing rate on each side, so large errors are penalised much more than small ones. Forecast evaluation often uses root mean squared forecast error, RMSFE. Best forecast is the conditional mean (expected value). RMSFE not invariant to transformations, e.g. best model for forecasting level may not be best model for first differences.
- ▶ Absolute Loss:  $L(e) = |e|$  Symmetric with loss increasing linearly with the error. Forecast evaluation uses mean absolute error. Best forecast is the conditional median.
- ▶ Non symmetric Linex (linear exponential) loss function is

$$L(e) = 2(\exp(\alpha e) - \alpha e - 1)/\alpha^2$$

here  $\alpha$  controls the degree of asymmetry. Limit  $\alpha \rightarrow 0$  is quadratic. Underpredicting more costly when  $\alpha > 0$ . E&T

## Direction -of -change, DoC, forecast loss function

- ▶ Often profit is the basis of the loss function.
- ▶ Financial asset returns, often focus on direction of change forecasts: go long on the asset going up in price, short on the asset going down.
- ▶ So a DoC forecast takes one of two values - up or down (include no change in one). Therefore the loss function might be:

$$\begin{aligned} L(y, \hat{y}) &= 0 & \text{if } \text{sign}\Delta y = \text{sign}\Delta \hat{y} \\ L(y, \hat{y}) &= 1 & \text{if } \text{sign}\Delta y \neq \text{sign}\Delta \hat{y} \end{aligned}$$

- ▶ If you predict the direction of change correctly, you incur no loss; but if your prediction is wrong, you are penalised.
- ▶ Profit can be very different from RMSFE, e.g. choice bonds and equities.

# Optimal Forecasts

- ▶ "Claims of forecast optimality are typically limited to restrictive classes of models and so can be very weak - optimality often does not extend too far in that there may be better forecast models available given the same data" E&T p39.
- ▶ Least Squares, gives the best (minimum variance), linear, unbiased, predictor, BLUP, conditional on the given  $X$ .
- ▶ But the loss function may not be quadratic (mean square error, minimum variance) and there may be better non-linear, biased predictors, or ones using a different  $X$  matrix.
- ▶ Econometrics often involves a trade-off between bias and variance but for asymmetric loss functions bias is a good thing.



## Questions about objectives

- ▶ **Question.** The US government weather service makes unbiased forecasts of the probability of rain the next day. But the US TV Weather Channel makes biased forecasts? Why? What direction do you think the bias takes?
- ▶ **Question:** What is the Decision Problem and Loss Function for (a) a professional economic forecaster (b) the Monetary Policy Committee of the Bank of England (c) The Office of Budget Responsibility.
- ▶ **Question:** Professional economic forecasts often fail rational expectations tests because they do not respond to new information quickly enough and are too smooth. Why is this rational?
- ▶ **Question:** Why doesn't the Bank of England publish interest rate forecasts?
- ▶ **Question:** Before OBR, why was public expenditure one of the most error prone elements of the Treasury forecast?

## 2. Forecast Object

Examples:

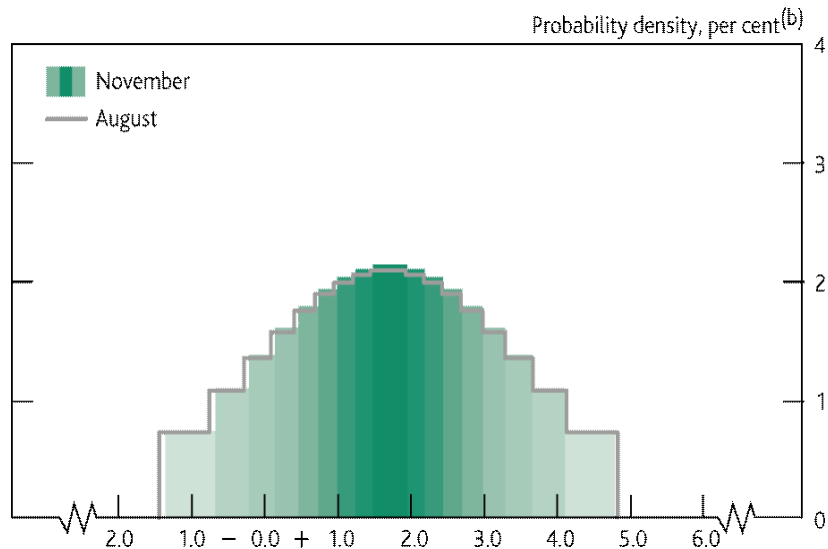
1. Event Outcome forecasts: certain to take place at a given time but the outcome is uncertain e.g. elections, sporting event. Usually discrete
2. Event Timing forecasts: when an event is certain to take place, the outcome is known, but the timing is uncertain, e.g. the next business cycle turning points: peaks and troughs. So for example, if the economy is currently in an expansion then the next turning point will be a peak, but there is substantial uncertainty as to its timing. Usually continuous, a date.
3. Time Series forecasts: future value of a time series is of interest and must be projected. Typically forecast is based on the history of the series and related series, plus subjective judgemental adjustments. Most economic data is in this form and usually continuous.

### 3. Types of Forecast Statement

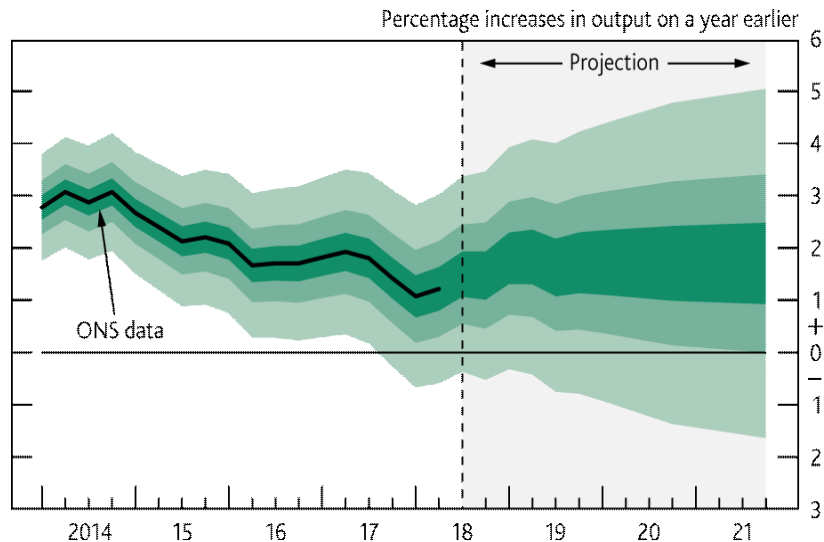
- ▶ Discrete: It will rain tomorrow:  $E_t(x_{t+1}) = \{1, 0\}$ , for event  $x_{t+1}$ .
- ▶ Probability: there is a 95% chance it will rain tomorrow.  $P_t(x_{t+1} = 1)$ . (Unfalsifiable on any particular day, like the typical English weather forecast: sunny with showers).
- ▶ Point: 10 mm of rain will fall tomorrow.  $E_t(y_{t+1})$ . For random variable  $y_{t+1}$ . Single numbers are clear and simple, but we need to know the degree of confidence we have in a number- how much uncertainty?
- ▶ Interval:  $P_t(a < y_{t+1} < b) = \alpha$ . There is a 95% chance that between 0 and 20 mm of rain will fall tomorrow: range of values in which we expect the realised value of series to fall with some (prespecified) probability
- ▶ Density:  $f_t(y_{t+1})$ . Probabilities of different amounts of rainfall tomorrow.

<i>mm</i>	0 – 5	5 – 10	10 – 15	15 – 20	> 20
Pr	0.1	0.3	0.35	0.2	0.05

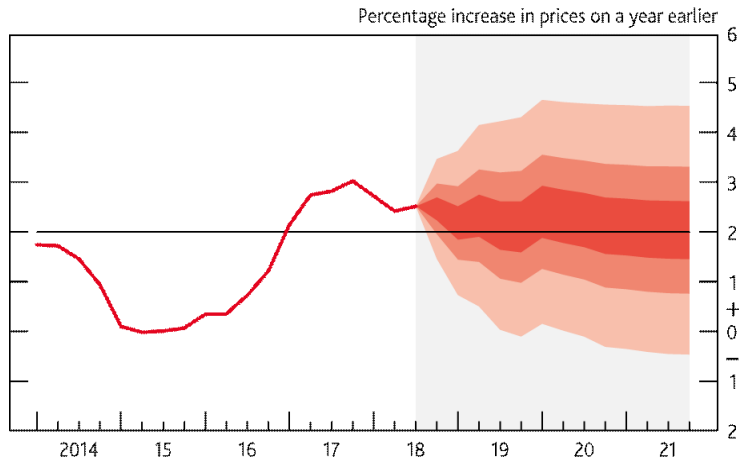
# Density, BoE Nov 2018 forecast GDP growth 2020Q4 on a year earlier, central 90% of distribution



## BofE Nov 2018 GDP Fan chart presents densities at different horizons



# CPI inflation projection based on market interest rate expectations



## 4. Forecast Horizon $h$

- ▶ Number of periods (months, quarters, depending on frequency) between when you make the forecast and the date of the forecast: talk in terms of  $h$ -step-ahead forecast
- ▶ The appropriate length of horizon depends on the decision problem involved.
- ▶ The forecast changes with the forecast horizon and the forecasting model may also differ. The short term forecasting model may differ from the long term model, for the same variable.
- ▶ Usually instead of the horizon being fixed e.g. every month we make a 4-month ahead forecast, the horizon includes all steps from 1-step-ahead to  $h$ -step-ahead.
- ▶ **Question:** Why do the Bank of England and the Office of Budget Responsibility's horizons differ?

## 5 Information set & notation

- ▶ For horizon  $h$  the mathematical expectation for period  $T + h$  based on information at  $T$  is written

$$E(y_{T+h} \mid \mathfrak{I}_T) \text{ or } E_T(y_{T+h})$$

- ▶ So in describing a forecast, you need to specify the period from which you are making the forecast,  $T$ , and the period for which you are forecasting  $T + h$ , e.g

$$y_{T+h/T}^f; \text{ or } y_{T+h/T}; \text{ or } {}_T y_{T+h}^f; \text{ or } {}_T y_{T+h}$$

- ▶ For the same information set distinguish between indirect/iterated and direct (linear projection) methods.



## Iterated forecast for $h=1,2,3$ by AR2

Iterated refers to multi-step ahead time series forecasts made using a one-period model, iterated forward for the desired number of periods.

Estimate a single equation using data up to  $T$

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$$

and iterate model forward from time  $T$ , substituting forecasts for unknown future values

$${}_T y_{T+1}^i = \alpha + \rho_1 y_T + \rho_2 y_{T-1}$$

$${}_T y_{T+2}^i = \alpha + \rho_1 {}_T y_{T+1}^i + \rho_2 y_T$$

$${}_T y_{T+3}^i = \alpha + \rho_1 {}_T y_{T+2}^i + \rho_2 {}_T y_{T+1}^i$$

## Linear Projection of order 2 for $h=1,2,3$

The direct method uses a horizon-specific estimated model, where the dependent variable is the multi-period ahead variable being forecast. Estimate 3 equations corresponding to  $h=1,2,3$  periods ahead (first is the same as the iterated equation)

$$y_t = \alpha^1 + \rho_1^1 y_{t-1} + \rho_2^1 y_{t-2} + \varepsilon_t^1$$

$$y_t = \alpha^2 + \rho_1^2 y_{t-2} + \rho_2^2 y_{t-3} + \varepsilon_t^2$$

$$y_t = \alpha^3 + \rho_1^3 y_{t-3} + \rho_2^3 y_{t-4} + \varepsilon_t^3$$

Use each set of estimates to forecast

$${}_T y_{T+1}^d = \alpha^1 + \rho_1^1 y_T + \rho_2^1 y_{T-1}$$

$${}_T y_{T+2}^d = \alpha^2 + \rho_1^2 y_T + \rho_2^2 y_{T-1}$$

$${}_T y_{T+3}^d = \alpha^3 + \rho_1^3 y_T + \rho_2^3 y_{T-1}$$

Implicitly the linear projection is allowing for longer lags. Could be matched by a VAR with long lags.

## 6. Method and complexity

Simple model often forecast well and can be difficult to beat.  
Examples below use

- ▶ no estimated parameters:
  - ▶ Tomorrow is the same as today:  $y_{T+1/T}^f = y_T$ ; works for random walk  $y_t = y_{t-1} + \varepsilon_t$
  - ▶ Tomorrow is the same as today plus change from yesterday:  $y_{T+1/T}^f = y_T + (y_T - y_{T-1})$ ; works for random walk with time varying drift  $y_t = \alpha_t + y_{t-1} + \varepsilon_t$  or for  $\Delta^2 y_t = \varepsilon_t$
  - ▶ Rather than today use same day last week for periodic variables:  $y_{T+7/T}^f = y_T$ .
- ▶ just historic mean:
  - ▶  $y_{T+1/T}^f = \bar{y}$  for  $y_t = \alpha + \varepsilon_t$ , where  $\varepsilon_t$  is white noise
  - ▶ Same as today plus mean change  $y_{T+1/T}^f = y_T + \overline{\Delta y}$  for random walk with drift  $y_t = \alpha + y_{t-1} + \varepsilon_t$

# Why can simple models be better than complicated ones.

- ▶ Can estimate the parameters of simple models more precisely, more degrees of freedom: errors in estimating parameters is a source of forecast errors. Trade-off bias and variance.
- ▶ Overfitting is a major problem in forecasting models. So only include a variable in a forecasting model if its t ratio is **much** larger than 2.
- ▶ Simple models reduce the scope for data mining i.e. tailoring a model to maximise its fit to the idiosyncrasies historical data, which have no relationship with the unrealised future.
- ▶ Random walk processes are common, so forecasting that tomorrow will be the same as today often works well (but may not be much use for policy-making).
- ▶ Simple models are more easily interpreted, understood, and scrutinised. Anomalies are more easily identified.
- ▶ Easier to communicate an intuitive feel for the behaviour of simple models, which makes them more useful in decision-making process.

# Forecasting Techniques

- ▶ Judgemental (guess)
- ▶ Surveys (ask someone else to guess)
- ▶ Technical (look at charts)
- ▶ Time series models:
  - ▶ univariate: trend extrapolation; exponential smoothing; ARIMA; structural unobserved component time-varying-parameter (Kalman Filter) models
  - ▶ Multivariate, VAR (not VaR), transfer functions
- ▶ Machine learning procedures like artificial neural networks .
- ▶ Econometric Models: more theory than time series and conditional on exogenous variables, e.g. policy assumptions.
- ▶ Will emphasise time-series and econometric methods.

# Models are useful

- ▶ Provide a Reproducible framework for systematic thought
- ▶ Act as a library for data and relationships
- ▶ Help you find out where you are now
- ▶ Impose consistency, make sure things add up
- ▶ Allow you to follow through complicated relationships
- ▶ Help you use judgement and other information in a coherent way
- ▶ Help you to ask clear questions, explain the answers, and provide a story
- ▶ Provide confidence intervals, densities and probabilities
- ▶ Allow you to learn from forecast evaluation
- ▶ Do it all quickly on a computer.

But you cannot rely on their forecasts: Why?

# Forecasting is difficult, particularly about the future

- ▶ Efficient markets (prices are unpredictable version, not price is right version). This near unpredictability of many variables is one of the few empirical regularities in economics.
- ▶ Tails of the unexpected: Black Swans arrive in flocks, e.g. pandemic.
- ▶ Structural Breaks: past may not be a guide to the future. Goodhart's Law: every well established econometric relationship breaks down as soon as it is used for policy.
- ▶ Economic forecasts are often conditional on assumed assumed values for exogenous variables, e.g. government policy, which may change
- ▶ Feedbacks: system responds to forecasts
- ▶ Data very bad and out of date
- ▶ Sensitivity to initial conditions. The fact that we do not know where we are now can produce chaotic outcomes.

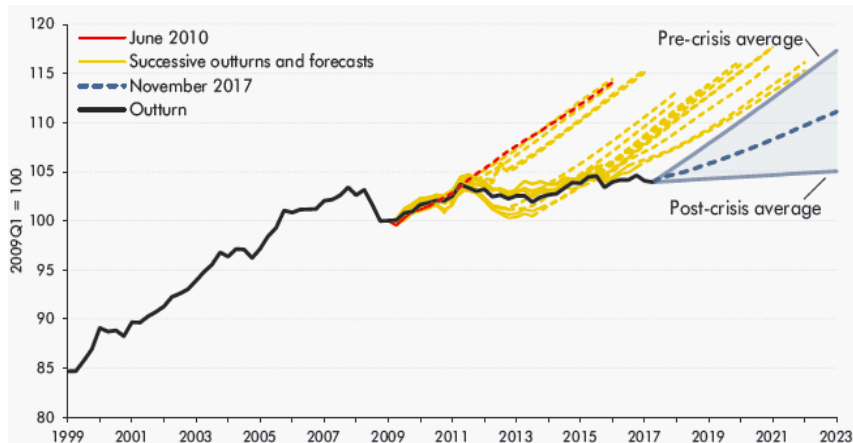
# Approach

- ▶ We will emphasise forecasting using statistical models, but there will always be a judgmental component.
- ▶ Before we make a forecast, we need to know why we are doing it: decision/loss involved?
- ▶ The decision problem determines the type of model built and how its parameters are estimated from historical data
- ▶ The estimated model seeks to summarise/approximate dynamic patterns in the data for a particular purpose
- ▶ The estimated model provides a statistical characterisation of what we expect in the present, conditional on the past, from which we infer what to expect in the future (conditional on the present and past)
- ▶ Since the forecasting model extrapolates from observed historical data, we hope there is no structural change.
- ▶ Ex post we evaluate the forecasts to try to improve them..



# Features 1: trends and structural breaks: OBR

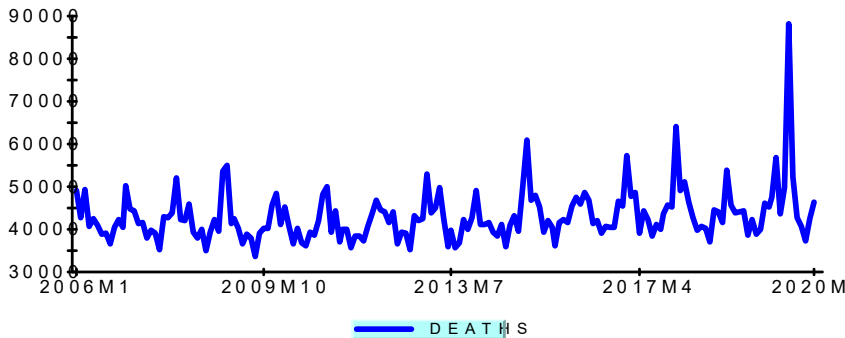
## Productivity forecasts



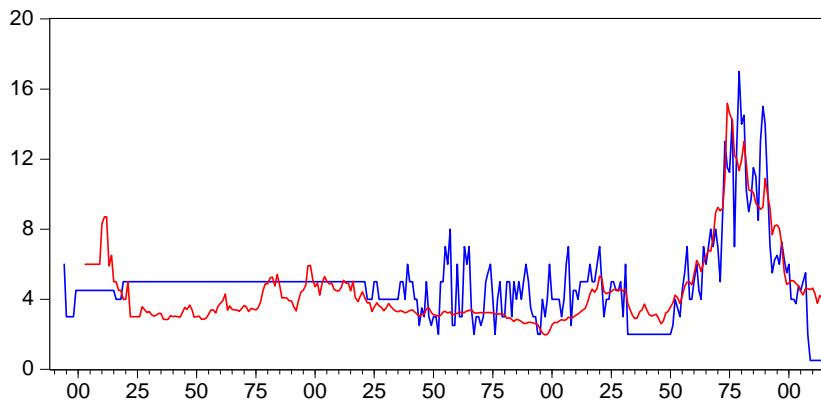
Note: Solid lines represent the outturn data that underpinned the forecast.

Source: ONS, OBR

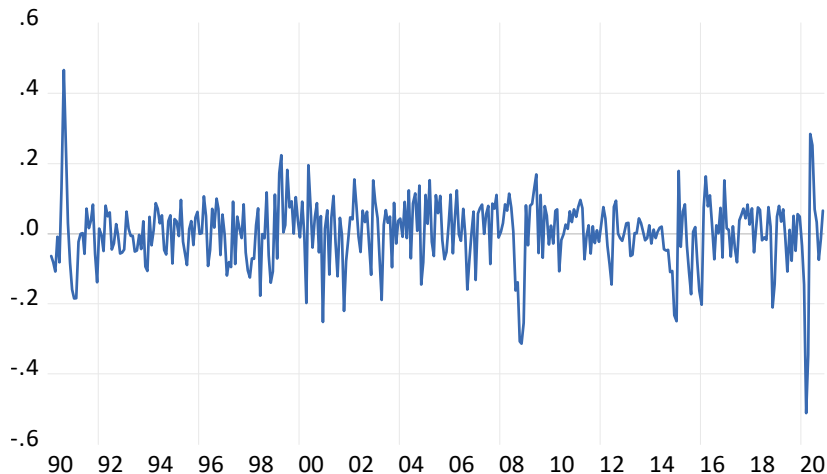
## Features 2: Seasonality and outliers: monthly death rates England and Wales



## Features 3: non-stationarity and cointegration, UK Bank rate and Long Rate 1688 2014



## Features 4? Volatility clusters and excess kurtosis: change in log monthly Brent



# GARCH allows for volatility clustering and excess kurtosis

Dependent Variable: DLBRENT

Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)

Date: 12/09/20 Time: 17:23

Sample (adjusted): 1990M03 2020M11

Included observations: 369 after adjustments

Convergence achieved after 43 iterations

Coefficient covariance computed using outer product of gradients

Presample variance: backcast (parameter = 0.7)

GARCH = C(3) + C(4)\*RESID(-1)^2 + C(5)\*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.009684	0.004151	2.332968	0.0196
DLBRENT(-1)	0.167722	0.052248	3.210146	0.0013
Variance Equation				
C	0.005521	0.001023	5.396651	0.0000
RESID(-1)^2	0.478356	0.118499	4.036793	0.0001
GARCH(-1)	-0.108827	0.094449	-1.152226	0.2492
T-DIST. DOF	9.386235	4.058259	2.312872	0.0207
R-squared	0.063741	Mean dependent var		0.002098
Adjusted R-squared	0.061190	S.D. dependent var		0.096607
S.E. of regression	0.093605	Akaike info criterion		-2.076977
Sum squared resid	3.215618	Schwarz criterion		-2.013387
Log likelihood	389.2022	Hannan-Quinn criter.		-2.051716
Durbin-Watson stat	1.694700			

# Questions

- ▶ Are you interested in a single variable? Univariate models like ARIMA.
- ▶ Are you interested in multiple variables?
  - ▶ Single endogenous variable conditional on other exogenous variables? Static regression, ARDL/ECM dynamic regressions.
  - ▶ Multiple endogenous variables, no exogenous variables? VAR/VECM.
  - ▶ Multiple endogenous variables, some exogenous variables? Simultaneous equations.
- ▶ You need to be able to forecast the exogenous variables or you may want to condition on a particular variable: what would happen to the exchange rate if interest rates followed a particular path.
- ▶ What is exogenous?

# Regression Analysis from a Forecasting Perspective 1

- ▶ We have a dependent variable,  $y_t$  and a  $k \times 1$  vector of predictors  $\mathbf{x}_t$ , and we wish to find the linear function of  $\mathbf{x}_t$  that gives us the best forecast of  $y_t$ , where "best forecast" means minimising the sum of squared forecast errors, for the sample of data considered
- ▶ Under the Gauss Markov assumptions, **without exogeneity**, OLS gives us the BLUP, Best (minimum variance) linear unbiased predictor in

$$y_t = \boldsymbol{\beta}' \mathbf{x}_t + \varepsilon_t;$$
$$\varepsilon_t \sim iid(0, \sigma^2).$$

- ▶ In many ways prediction is an easier problem than estimation.

## Regression Analysis from a Forecasting Perspective 2

- ▶ If the regression model holds, then the expected value of  $y_{T+h}$  conditional on  $\mathbf{x}$  taking on a particular value  $\mathbf{x}_{T+h}$ , is:

$$E_t(y_t \mid \mathbf{x}_{T+h}) = \boldsymbol{\beta}' \mathbf{x}_{T+h}$$

- ▶ The regression function is the conditional expectation of  $y$ . The expectation of future  $y$  conditional on current information is a good forecast under quadratic loss.
- ▶ Estimate  $\boldsymbol{\beta}$  and  $\sigma^2$  e.g. by OLS to get  $\hat{\boldsymbol{\beta}}$  and  $\hat{\sigma}^2$ , forecast is then

$$\hat{y}_{T+h} = \hat{\boldsymbol{\beta}}' \mathbf{x}_{T+h}$$

- ▶ If  $k = 1$ , and  $x_{1t} = 1$ , then the best forecast is  $\hat{\beta}_1$  the mean, which is a baseline forecast for a stationary process.
- ▶ For ex post forecasts  $\mathbf{x}_{T+h}$  and  $y_{T+h}$  are known and forecasts can be evaluated. For ex ante (true) forecasts they are not known.



# Forecast errors 1

- ▶ Forecast conditional on known  $\mathbf{x}_{T+h}$  (e.g. time trend)

$$\hat{y}_{T+h} = \hat{\beta}' \mathbf{x}_{T+h}$$

- ▶ "Truth"

$$y_{T+h} = \beta' \mathbf{x}_{T+h} + \varepsilon_{T+h}$$

- ▶ Forecast error

$$y_{T+h} - \hat{y}_{T+h} = (\beta - \hat{\beta})' \mathbf{x}_{T+h} + \varepsilon_{T+h}$$

- ▶ Two sources of forecast error (estimation error,  $(\beta - \hat{\beta})$  and disturbances  $\varepsilon_{T+h}$ ), may also have to forecast  $\mathbf{x}_{T+h}$ .

## Forecast errors 2

- ▶ Estimate on data  $t = 1, 2, \dots, T$  :  $y_t = \hat{\alpha} + \hat{\beta}x_t + \hat{\gamma}y_{t-1} + \hat{u}_t$
- ▶ Forecast next period using forecast of exogenous variable,  $x_{T+1}^f$ , preliminary data for  $T$ ,  $y_T^P$ , and judgemental adjustment,  $u_{T+1}^f$  :

$$y_{T+1/T}^f = \hat{\alpha} + \hat{\beta}x_{T+1/T}^f + \hat{\gamma}y_T^P + u_{T+1/T}^f$$

- ▶ "Truth" ignoring estimation error  
 $y_{T+1} = \hat{\alpha} + \hat{\beta}x_{T+1} + \hat{\gamma}y_T + \hat{u}_{T+1}$
- ▶ Decompose error into wrong: forecast of exogenous variables; starting data; and judgemental adjustment:

$$\begin{aligned} y_{T+1} - y_{T+1/T}^f &= \hat{\beta}(x_{T+1} - x_{T+1/T}^f) + \hat{\gamma}(y_T - y_T^P) \\ &\quad + (\hat{u}_{T+1} - u_{T+1/T}^f) \end{aligned}$$

- ▶ Could be wrong model. Use real time data not revised.

# Econometric Modelling in practice

- ▶ Assemble data (usually badly measured and out of date).
- ▶ Estimate equations, e.g. making endogenous variables a function of their lags and exogenous (e.g. policy) variables
- ▶ Get some software to run the model, hope it works
- ▶ Try to nowcast (guess) where you are now
- ▶ Starting from last forecast, update beliefs on the basis of your previous forecast errors.
- ▶ Make assumptions about future values of exogenous variables, e.g. from market expectations of interest rates
- ▶ Run model forward to get initial forecasts
- ▶ Use judgement to add or subtract a bit to get forecast that you like and can tell a story about
- ▶ If a policy maker, experiment with alternative policy setting to determine least bad forecast outcome.

# Choices

- ▶ Since they convey more information interval or density forecasts should be preferred, but point forecasts remain the most popular. Many hate the BoE fan charts
- ▶ Construction of density forecast requires either (1) additional, and possibly incorrect, distributional assumptions relative to those required of a point forecast (2) or computer intensive simulations like bootstrap.
- ▶ Point forecasts are often easier to understand and to act on than interval or density forecasts - hence more information which is costly to process may not be helpful.
- ▶ Probability forecasts convey uncertainties surrounding forecasts and are straightforward to use in decision theoretic contexts.
- ▶ May want joint probabilities, e.g. of recession and inflation over 3%.

# Six questions for forecasters

1. Decision environment and loss function. Why are we doing it what is the loss function?
2. Forecast object. What are we forecasting?
3. Forecast Statement. How to we wish to state our forecast? Point, Probability, Density?
4. Forecast Horizon. How far into the future do we wish to forecast?
5. Information set. What information is available at the time of forecasting? Estimation window?
6. Method and complexity. Which method is best suited to the problem and how complex should the forecasting model be?