BIRKBECK
(University of London)

MSc EXAMINATION FOR INTERNAL STUDENTS

Department of Economics, Mathematics and Statistics

Econometrics / Econometrics for Postgraduate Certificate

EMEC026S7 / BUEM007H7

Thursday, 06 June 2019, 9.30 am - 11.40 pm (includes 10 minutes reading time)

2 hours 10 minutes

This paper has five questions. Answer **ANY THREE** questions. All questions carry the same weight; the relative weight of sub-questions is indicated in square brackets.

Please turn over

1. Consider the simultaneous equations model

$$
\begin{aligned}
y_{1t} &= \beta_1 y_{2t} + u_{1t}, \\
y_{2t} &= \beta_2 y_{1t} + \gamma_1 x_{1t} + \gamma_2 x_{2t} + u_{2t},
\end{aligned}
$$

where (y_{1t}, y_{2t}) are endogenous variables, (x_{1t}, x_{2t}) are strictly exogenous variables, and (u_{1t}, u_{2t}) are homoskedastic and serially uncorrelated random disturbances with zero mean.

(a) Discuss the identifiability of the coefficients in each of the two equations in terms of the order and rank conditions for identification. [25%]

(b) Explain why the ordinary least squares estimator of β_1 is inconsistent. [20%]

(c) Describe, step by step, how to estimate any identifiable coefficients using two-stage least squares. Explain how the two-stage least squares estimator is related to an appropriately constructed instrumental variables estimator. [30%]

(d) Explain how to test for endogeneity of y_{2t} in the first equation using a Wu–Durbin–Hausman test. State clearly what the null and alternative hypotheses for the test are. [25%]

2. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size $n \geqslant 4$ from the Laplace distribution with location parameter μ $(-\infty < \mu < \infty)$ and scale parameter $\beta > 0$. Let $\theta = (\mu, \beta)'$.

   (a) Explain how to estimate θ by the generalized method of moments (GMM) using the fact that $\mathsf{E}(X_1) = \mu$ and $\mathsf{E}[(X_1 - \mu)^2] = 2\beta^2$. (Note that you are not required to obtain a closed-form expression for the GMM estimator of θ). [25%]

   (b) Explain how your answer to (a) would change if you used the additional information that $\mathsf{E}[(X_1 - \mu)^4] = 24\beta^4$. [25%]

   (c) Explain how to obtain an asymptotically efficient GMM estimator of θ using the moment conditions of part (b). [25%]

   (d) Explain how to test the validity of the moment conditions used to compute the GMM estimator of θ in part (c). Give the formula for the appropriate test statistic and state its asymptotic null distribution. [25%]

3. Consider the structural vector autoregressive (VAR) model

$$\mathbf{B}\mathbf{y}_t = \mathbf{\Phi}\mathbf{y}_{t-1} + \mathbf{u}_t,$$

where $\mathbf{y}_t$ is an $m \times 1$ vector of observable variables, $\mathbf{B}$ and $\mathbf{\Phi}$ are $m \times m$ matrices of coefficients, with $\det \mathbf{B} \neq 0$, and $\{\mathbf{u}_t\}$ is an unobservable m-dimensional white noise process with $\mathsf{E}(\mathbf{u}_t) = \mathbf{0}$ and $\mathsf{E}(\mathbf{u}_t\mathbf{u}_t') = \mathbf{I}_m$, where $\mathbf{I}_m$ is the $m \times m$ identity matrix. The model is structural in the sense that the elements of $\mathbf{u}_t$ have clear interpretations in terms of an underlying economic model. Assume that $\{\mathbf{y}_t\}$ is covariance stationary.

(a) Show that the model can be represented in reduced form as

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \varepsilon_t,$$

where $\mathbf{A} = \mathbf{B}^{-1}\mathbf{\Phi}$ and $\varepsilon_t = \mathbf{B}^{-1}\mathbf{u}_t$, with $\mathsf{E}(\varepsilon_t) = \mathbf{0}$ and $\mathsf{E}(\varepsilon_t\varepsilon_t') = \mathbf{B}^{-1}(\mathbf{B}^{-1})'$. [15%]

(b) Show that the first three coefficient matrices in the moving-average representation $\mathbf{y}_t = \sum_{k=0}^{\infty} \mathbf{\Psi}_k\varepsilon_{t-k}$ of the reduced-form VAR are $\mathbf{\Psi}_0 = \mathbf{I}_m$, $\mathbf{\Psi}_1 = \mathbf{A}$ and $\mathbf{\Psi}_2 = \mathbf{A}^2$. Explain what the elements of $\mathbf{\Psi}_1$ and $\mathbf{\Psi}_2$ represent. [35%]

(c) Explain what the 'orthogonalized impulse responses' of the reduced-form VAR are. Explain how to compute the orthogonalized impulse responses using the Cholesky factorization of the matrix $\mathbf{\Sigma} = \mathsf{E}(\varepsilon_t\varepsilon_t')$ and why orthogonalization of ε_t is important. [35%]

(d) Are there any advantages to using $\mathbf{B}$ instead of the the matrix associated with the Cholesky factorization of $\mathbf{\Sigma}$ when orthogonalizing ε_t? Explain. [15%]

4. Consider the linear regression model

$$y_t = \beta x_t + \varepsilon_t, \qquad t = 1, 2, \ldots, n,$$

where x_t is a non-stochastic explanatory variable. The disturbances $\{\varepsilon_t\}$ satisfy

$$\varepsilon_t = z_t \left(\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2\right)^{1/2},$$

where $\alpha_0 > 0$, $\alpha_1 \geqslant 0$, $\alpha_2 \geqslant 0$, and $\{z_t\}$ are independent and identically distributed random variables having the $\mathsf{N}(0, 1)$ distribution (z_t is independent of ε_{t-i} for $i \geqslant 1$).

(a) Show that $\mathsf{E}(\varepsilon_t) = 0$, $\mathsf{E}(\varepsilon_t^2) = \alpha_0/(1 - \alpha_1 - \alpha_2)$, and $\mathsf{E}(\varepsilon_t \varepsilon_{t-k}) = 0$ for all $k \geqslant 1$. What assumption(s) do you need to make to ensure that $\{\varepsilon_t\}$ is covariance stationary? [30%]

(b) What statistical properties does the ordinary least squares estimator of β have? Explain how to obtain an asymptotically efficient estimate of $(\beta, \alpha_0, \alpha_1, \alpha_2)$. [25%]

(c) Explain how to test the null hypothesis that $\alpha_1 = \alpha_2 = 0$ using the Lagrange multiplier principle. [20%]

(d) Explain why the autocorrelation function and partial autocorrelation function of $\{\varepsilon_t^2\}$ may be useful in assessing the validity of the second-order ARCH assumption about $\{\varepsilon_t\}$. [25%]

5. Consider a univariate time series $\{y_t\}$ satisfying the local level model

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t, \qquad t = 1, 2, \ldots, n, \\ \mu_t &= \mu_{t-1} + u_t, \end{aligned}$$

where $\{\varepsilon_t\}$ and $\{u_t\}$ are independent and identically distributed random variables with $\mathsf{E}(\varepsilon_t) = \mathsf{E}(u_t) = 0$, $\mathsf{E}(\varepsilon_t^2) = \sigma_\varepsilon^2$ and $\mathsf{E}(u_t^2) = \sigma_u^2$. The disturbances $\{\varepsilon_t\}$ and $\{u_t\}$ are mutually independent as well as independent of the initial value μ_0. The trend component μ_t of the time series and the disturbances (ε_t, u_t) are unobservable but y_t is observable.

(a) Show that, if $\sigma_\varepsilon^2 > 0$, then y_t satisfies an ARIMA$(0, 1, 1)$ model. What is the ARIMA representation of y_t when $\sigma_\varepsilon^2 = 0$? [40%]

(b) What does the restriction $\sigma_u^2 = 0$ imply about the behaviour of the trend component μ_t? Explain. [15%]

(c) Let $Y_{t-1} = \{y_1, \ldots, y_{t-1}\}$ be the information set available at time $t-1$. Assume that the conditional distribution of y_t given Y_{t-1} is $\mathsf{N}(\hat{\mu}_{t|t-1}, f_t)$, where $\hat{\mu}_{t|t-1}$ is the optimal forecast of μ_t based on Y_{t-1} and f_t is the variance of the forecast error $\nu_t = y_t - \hat{\mu}_{t|t-1}$ computed by the Kalman filter. Explain how you may estimate the parameters $(\sigma_\varepsilon^2, \sigma_u^2)$ and state what the statistical properties of the proposed estimator will be. (Note that you are not required to describe how $\hat{\mu}_{t|t-1}$ and f_t are computed.) [30%]

(d) Explain what the difference between a filtered estimate and a smoothed estimate of μ_t is. Is there any advantage to using a smoothed estimate instead of a filtered estimate? Explain. [15%]

End of Examination Paper