$y_t \overset{iid}{\sim} \quad \mathbb{E}[y_t] = \mu_0 \quad , \quad var(y_t) = \sigma_0^2$

$t = 1, \dots, T$

estimator of μ_0:

$$\hat{\mu}_T = \bar{y}_T = \frac{1}{T} \sum_t y_t$$

$$\mathbb{E}[\hat{\mu}_T] = \mathbb{E}\left[\frac{1}{T}\sum_t y_t\right] = \frac{1}{T}\sum_t \underbrace{\mathbb{E}[y_t]}_{\mu_0}$$

$$= \frac{1}{T} T \mu_0 = \mu_0$$

$$var(\hat{\mu}_T) = var\left(\frac{1}{T}\sum_t y_t\right)$$

$$= \frac{1}{T^2} var\left(\sum_t y_t\right)$$

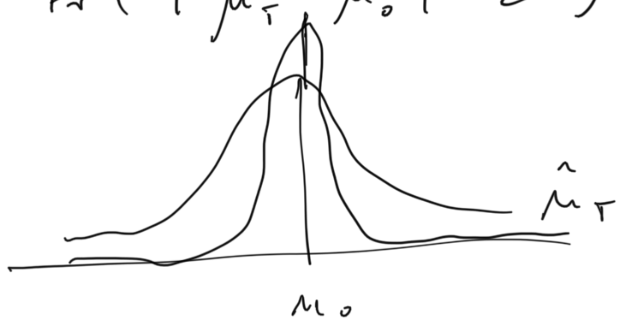
$$= \frac{1}{T^2} \sum_t var(y_t) \quad \text{b/c } cov = 0$$

$$= \frac{1}{T^2} T \sigma_0^2$$

$$= \frac{1}{T}\sigma_0^2 \to 0 \quad \text{as } T \to \infty$$

$\Rightarrow$ $\hat{\mu}_T$ is consistent: $\varepsilon > 0$

$$Pr(|\hat{\mu}_T - \mu_0| > \varepsilon) \leq \frac{\sigma_0^2}{T\varepsilon^2}$$



$$\hat{\mu}_T - \mu_0 \xrightarrow{P} 0 \quad \text{as } T \to \infty$$

$$var\left(\sqrt{T}(\hat{\mu}_T - \mu_0)\right) = T\, var(\hat{\mu}_T - \mu_0)$$

$$= T\, var(\hat{\mu}_T)$$

$$= T \frac{\sigma_0^2}{T}$$

$$= \sigma_0^2$$