

$$y_t \stackrel{iid}{\sim} \mathbb{E}[y_t] = \mu_0, \quad \text{var}(y_t) = \sigma_0^2$$

$t = 1, \dots, T$

estimator of  $\mu_0$ :

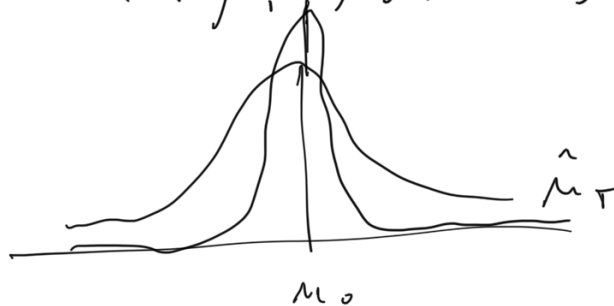
$$\hat{\mu}_T = \bar{y}_T = \frac{1}{T} \sum_t y_t$$

$$\begin{aligned} \mathbb{E}[\hat{\mu}_T] &= \mathbb{E}\left[\frac{1}{T} \sum_t y_t\right] = \frac{1}{T} \sum_t \underbrace{\mathbb{E}[y_t]}_{\mu_0} \\ &= \frac{1}{T} T \mu_0 = \mu_0 \end{aligned}$$

$$\begin{aligned} \text{var}(\hat{\mu}_T) &= \text{var}\left(\frac{1}{T} \sum_t y_t\right) \\ &= \frac{1}{T^2} \text{var}\left(\sum_t y_t\right) \\ &= \frac{1}{T^2} \sum_t \text{var}(y_t) \quad \text{b/c cov} = 0 \\ &= \frac{1}{T^2} T \sigma_0^2 \\ &= \frac{1}{T} \sigma_0^2 \rightarrow 0 \quad \text{as } T \rightarrow \infty \end{aligned}$$

$\Rightarrow \hat{\mu}_T$  is consistent:  $\varepsilon > 0$

$$\Pr(|\hat{\mu}_T - \mu_0| > \varepsilon) \leq \frac{\sigma_0^2}{T \varepsilon^2}$$



$$\hat{\mu}_T - \mu_0 \xrightarrow{P} 0 \quad \text{as } T \rightarrow \infty$$

$$\begin{aligned} \text{var}\left(\sqrt{T} (\hat{\mu}_T - \mu_0)\right) &= T \text{var}(\hat{\mu}_T - \mu_0) \\ &= T \text{var}(\hat{\mu}_T) \\ &= T \frac{\sigma_0^2}{T} \\ &= \sigma_0^2 \end{aligned}$$