

## **Time Series Modeling of Cash and Futures Commodity Prices**

*Joshua G. Maples\*, and B. Wade Brorsen*

---

\*Joshua G. Maples is an assistant professor at Mississippi State University and a former PhD student at Oklahoma State University and B. Wade Brorsen is a Regents Professor and A.J. and Susan Jacques Chair in the Department of Agricultural Economics, Oklahoma State University, Stillwater, Oklahoma; Maples receives financial support from a Sitlington Enriched Graduate Scholarship. Brorsen receives financial support from the A.J. and Susan Jacques Chair and the Oklahoma Agricultural Experiment Station and USDA National Institute of Food and Agriculture, Hatch Project number OKL02939.

***Abstract***

Commodity prices exhibit differing levels of mean reversion and unit root tests are a standard part of the analysis of commodity price series. Changing underlying means are inherent in commodity prices and can create biased estimates if not correctly specified when performing unit root tests. Prominent financial models include terms for both mean reversion and unit roots but assume that mean reversion occurs gradually over time. Other models such as the popular error correction models require the researcher to determine if prices are either mean-reverting or follow a unit root process. We discuss the models commonly used for commodity prices and how their assumptions align with how commodity spot and futures prices actually behave. We argue for using panel unit root tests for futures prices as they allow for differing underlying means across futures contracts. Cash prices are difficult as none of the currently available models captures their likely stochastic process. Current models, however, can still be useful as close approximations.

***Key Words:*** unit roots, mean reversion, commodity markets, time series

***JEL Classifications:*** Q02, G13

## Introduction

Time series models are a primary tool used to study cash and futures prices. A key point when choosing the best time series model for cash and futures prices is whether or not the two series are stationary. Commonly used tests for stationarity such as the Dickey Fuller test and the Phillips-Perron test are often used to determine whether prices follow a unit root process (i.e. prices can vary randomly between zero and infinity in the long run) or that the series are mean-reverting to an underlying mean (Dickey and Fuller 1981; Phillips and Perron 1988). A primary assumption of stationarity tests is whether the underlying means for spot and futures prices are constant or changing as these are the levels to which mean reversion is tested.

The discontinuity of futures prices creates a unique challenge in testing for unit roots. Since multiple contracts with varying expiration dates are traded within each year, it is difficult to align a continuous time series of futures prices. Generally this process involves using observations from the closest contract to expiration (nearby contract) and, thus combines prices from many different contracts into a single series. Ma, Mercer, and Walker (1992) argue that determining how and when which contracts are used is determined by the researcher can sometimes appear quite arbitrary (Ma, Mercer, and Walker 1992). Table 1 presents the rollover methods used by many previous studies to choose when to switch to the next available contract. We argue that selecting the rollover point is not what is important. The primary issue with the continuous contract approach with respect to stationarity tests is that the underlying mean is not constant. Figure 1 shows the five corn futures contracts traded in 2015 as well as a continuous series compiled from the nearby contract. Each of the expiring contracts clearly has a different mean and, therefore, the nearby series is compiled of observations with five different underlying

means. As we will show, a series of combined contracts with differing underlying means is not consistent with the assumptions of traditional unit root tests.

Price theory suggests that commodity spot prices should possess some level of mean reversion (Pindyck 2001; Lautier 2005; Wang and Tomek 2007). In particular, commodity spot prices are unlikely to sustain a level above the cost of production due to a resulting increase in production (Dixit and Pindyck 1994). A number of other studies also suggest mean reversion in commodity spot prices (e.g. Peterson, Ma and Ritchy 1992; Allen, Ma and Pace 1994; Walburger and Foster 1995; Irwin, Zulauf, and Jackson 1996; Schwartz 1997; Pindyck 2001; Casassus and Collin-Dufresne 2005; Tang 2012; Hart et al. 2015).

Contrary to the theory that commodity prices should be mean-reverting, much empirical evidence suggests that spot prices are more likely to follow some type of unit root (i.e. random walk) process (e.g. Ardeni 1989; Bessler and Covey 1991; Schroeder and Goodwin 1991; Beck 1994; Babula, Ruppel, and Bessler 1995; Foster, Havenner, and Walburger 1995; Zapata and Fortenbery 1996; Barkoulas et al. 1997; Goodwin and Holt 1999; McKenzie and Holt 2000; Harri, Nalley, and Hudson 2009; Franken, Parcell, and Tonsor 2011). Table 2 presents results from various studies that tested commodity prices for unit roots. While this is certainly not an exhaustive list, it shows that more often than not, tests fail to reject that commodity prices (cash and futures) follow a unit root process. While price theory supports mean reversion in the long run, it is hard to believe that prices are mean-reverting to a constant underlying long-run mean even in the longest available time series. So while price theory may suggest spot prices should revert back to the cost of production, the cost of production is not constant over time.

The changing cost of production can be seen in Figure 2 where the spot corn price in Omaha, NE and the cost of production for a bushel of corn in Iowa are compared. While this

figure does not account for transportation costs, it shows that the spot price generally moves in relation to the underlying cost of production. Perhaps more importantly, figure 2 documents the changing cost of production for corn. Others have also documented that commodity prices do not revert to a constant mean (e.g. Cuddington and Urzua 1987; Dempster et al. 2008). Tang (2012) further noted that if the long-run mean is stochastic and nonstationary, mean reversion tests for a constant mean should reject mean reversion.

Wang and Tomek (2007) argued that structural changes shift the underlying mean and discussed the important conundrum of theory suggesting mean-reversion while many empirical studies suggest unit root processes and stated that they found no theoretical evidence to believe that cash commodity prices possess unit roots. Wang and Tomek (2007) suggested that not considering structural breaks can explain why unit root tests find a series to be nonstationary. The problem with this argument is if structural breaks are frequent enough, how does it differ from a unit root process? Others have suggested methods to determine the frequency and magnitude of structural breaks and how this should impact commodity price models (e.g. Zivot and Andrews 1992; Lumsdaine and Papell 1997; Pesaran, Pettenuzzo, and Timmermann 2006; Lee and Brorsen 2016). While these studies identify where it can be statistically shown that structural changes occur, the underlying mean for agricultural commodities changes by at least a small amount each crop year as production practices and costs evolve.

Whether a series is mean-reverting or possesses a unit root is important for both theoretical and empirical models. Theoretically, the presence of unit roots suggests that short-run departures of prices from their underlying long-run equilibriums are persistent and irreversible which disagrees with most economic price theory (Campbell and Perron 1991). Empirically,

failing to account for nonstationary series in economic models can lead to spurious regression (Granger and Newbold 1974).

Most agricultural commodities are grown on an annual basis and this can have a key impact on when prices mean-revert as opposed to following a random walk process (though this impact is lessened due to the storability of most commodities). Corn and soybeans in the United States are predominantly harvested in the early fall while wheat harvest begins in late spring. Harvest affects prices as they will either represent new crop or old crop depending on whether the prices are prior to harvest or after harvest. Lence and Hayenga (2001) showed in a multi-year rollover framework that it is virtually impossible to secure a high price for the next crop year by hedging old crop futures prices. Thus, the mean reversion of spot prices may occur across crop years while prices act as random walks within crop years (Kim and Brorsen 2012).

Futures prices in efficient markets should not wander too far from the underlying spot value at rollover because this is where the two series must converge. If spot and futures prices drift apart in an efficient market, arbitrage should bring them back together (Schwartz and Szakmary 1994). Thus, we can reasonably expect that the equilibrium futures price is equal to the expected spot price.

The changing underlying means for commodity spot prices holds a curious place in the financial and commodity prices literature. Much attention has been paid to prices containing both mean-reverting and random components in univariate models (i.e. to explain the behavior of one price) of commodity prices. The seminal work by Schwartz (1997) introduced the short-term long-term model to allow for prices to be random (i.e. follow a Brownian motion process) in the long-run while allowing for mean reversion (i.e. Ornstein Uhlenbeck process) in the short-run. Variations of this model have become common in the commodity price literature (e.g. Schwartz

and Smith 2000; Lucia and Schwartz 2002; Paschke and Prokopczuk 2010; Tang 2012; Hart et al. 2015; etc.), though most extensively in univariate settings such as forecasting or real options models due to the added complications of estimation (Pindyck 2001). A key drawback of these models is that the factors are not actually observable and estimation techniques such as the Kalman Filter (or Bayesian methods) must be applied to infer non-observable variables from observable data (Lautier 2005). Further, these models typically assume linear mean reversion. The linear mean reversion is an issue for agricultural crops because when prices are high mean reversion is expected to primarily occur across crop years.

On the other hand, multivariate models using commodity prices, such as studies using the popular Engle and Granger (1987) error correction model are performed after determining if spot and futures price series are *either* mean-reverting *or* follow a unit root process (e.g. McKenzie and Holt 2000; Joseph, Garcia and Peterson 2013). Error correction models are an extremely valuable tool for estimating models that involve multiple prices that are cointegrated. However, the first prerequisite for two series to be cointegrated is that each follows the same order unit root process and, therefore, do not possess mean reversion.

An important motivation for this article is the apparent disconnect between how commodity prices are treated in univariate frameworks (e.g. forecasting, options valuation, etc.) and multivariate models of spot and futures prices (e.g. error correction models, etc.). By using extensions from the prominent Schwartz (1997) and Schwartz and Smith (2000) models, we argue that the current methods used to test for stationarity in multivariate analysis can be improved. We show and suggest that panel unit root tests more adequately test the way commodity processes actually behave. We argue that the panel unit root tests are more correctly specified for futures prices than the current continuous contract methods used. We show

theoretically that the current methods for aligning commodity futures prices as continuous contracts create a series with multiple underlying means. For cash prices, we test the hypothesis that price series should be divided into within crop-year and across crop-year segments. Our findings have implications for cointegration tests of cash and futures prices as a correctly specified cointegration test is subject to the specification of cash and futures prices. Next, the data are described and results are presented. Finally, conclusions are drawn and implications are considered.

## Conceptual Framework

To begin the discussion, consider a vector error correction model of all prices in the economy:

$$(1) \quad \Delta \mathbf{P}_t = \boldsymbol{\alpha}_t + \boldsymbol{\rho}_t \mathbf{u}_{t-1} + \mathbf{v}_t$$

where  $\mathbf{P}_t$  is a vector of prices,  $\mathbf{u}_{t-1}$  is a vector of the error correction terms which captures the cointegration dynamics of spot prices with all of the other prices in the economy,  $\boldsymbol{\alpha}_t$  is a vector of possibly time varying intercepts,  $\Delta$  is the first difference operator, and  $\mathbf{v}_t$  is a stationary error term.

This ECM includes every price. For example, there are a multitude of wheat prices as wheat prices vary across type, location, and quality. There can be a multitude of cointegrating vectors as well. The cointegration convergence rate in (1) is specified to vary across time. Considerable research has found thresholds in ECM models (Goodwin and Piggot 2001; Meyer and von Cramon-Taubadel 2004; Han, Chung, and Surathkal 2016). It is these thresholds that act together to keep prices from going to zero or to infinity. For example, if wheat prices fall enough



cattle feeders begin feeding wheat so wheat and corn prices become cointegrated. Further, many parameters change over time due to changes in technology, government policies and tastes and preferences. Thus, while such a model may describe the actual economy as a whole, a simplified model is needed to make it empirically tractable.

One such univariate model is the Ornstein-Uhlenbeck (OU) model and assumes that price follows a stochastic mean-reverting process. The OU process is shown as the solution of a stochastic differential equation as

$$(2) \quad dP_t = k(\bar{P} - P_t)dt + dz_t$$

where  $P_t$  is the commodity spot price,  $\bar{P}$  is the long-run mean spot price,  $k$  is the mean reversion speed, and  $dz_t$  are increments to a standard Brownian motion process. The OU process is the continuous time version of a stationary first order autoregressive process (AR(1)). The model assumes that the long-run mean  $\bar{P}$  is a constant.

The OU model in equation (2) can be extended by allowing the long-run equilibrium  $\bar{P}$  to possess a unit root and differing rates of adjustment  $k$  for the mean reverting and unit root processes. These extensions lead to one of the most prominent financial models as derived by Schwartz and Smith (2000) is built around this assumption. We use their two factor model to express the dynamics of commodity prices. This is not a novel concept. Autoregressive integrated moving average ARIMA forecasting models and other literature have long suggested both mean reversion and random components. This model for a commodity spot price allows for mean reversion and uncertainty in the equilibrium level to which prices revert and is written as

$$(3) \quad P_t = x_t + z_t$$

where  $P_t$  denotes the log of the spot price (to ensure positive prices) of a commodity in year  $t$  and is decomposed into two stochastic factors to represent short-term deviations ( $x_t$ ) as well as the stochastic long-run equilibrium ( $z_t$ ). Note that these two factors are not actually observable for most commodity prices. These factors must be inferred from available data such as long-maturity futures contracts to provide information about the long-run equilibrium and differences between long-term and short-term futures prices to provide information about short-run deviations (Lautier 2005).

Seasonality is also well-documented in commodity price series. Sørensen (2002) used a deterministic seasonal component to account for pronounced seasonal patterns in the prices of soybeans, corn and wheat. Jin et al. (2012) and Hart et al. (2015) used similar approaches to correct for seasonality in commodity prices while Paschke and Prokopczuk (2009) found seasonal effects in natural gas, heating oil, and gasoline. Others have also considered seasonality as playing an important role in specifying commodity prices (Manoliu and Tompaidis 2002; Lucia and Schwartz 2002; Brooks, Prokopczuk, and Wu 2013). Todorova (2004) and Mirantes et al. (2012) model seasonality as a stochastic factor. Kim and Brorsen (2012) used a seasonal mean reversion model to estimate when producers should sell stored grain. They found that, unless prices are extremely low, it is optimal for producers to sell stored grain before the mean reversion begins which they found to be between 38 weeks and 42 weeks after harvest for corn, soybeans, and wheat. Since our focus is on agricultural commodities, we also include seasonality in the spot price (e.g. Sørensen 2002, Schwartz and Smith 2000; Manoliu and Tompaidis 2002; and Todorova 2004). Thus, we amend equation (3) to become

$$(4) \quad P_t = s(t) + x_t + z_t$$

where  $s(t)$  is a seasonality term similar to Sørensen (2002) used to capture the seasonality effects inherent to most agricultural production practices. Similar to Schwartz and Smith (2000), the short-term (mean reversion) state variable  $x_t$  follows an Ornstein-Uhlenbeck process with mean reversion parameter  $k$  and volatility parameter  $\sigma_x$ , and the stochastic long-term state variable  $z_t$  follows a Brownian motion process with drift rate  $\mu$  and diffusion rate  $\sigma_z$  as shown in

$$(5) \quad dx_t = -kx_t dt + \sigma_x dW_{x_t}$$

$$(6) \quad dz_t = \mu dt + \sigma_z dW_{z_t}$$

where  $W_{1t}$  and  $W_{2t}$  are standard Weiner processes and are assumed to have a constant correlation coefficient  $\rho_{xz}$ . As in Tang (2012), the  $z_t$  factor can be thought of as accounting for permanent shocks while  $x_t$  is the mean reversion factor.

Note that the Schwarz-type models are approximations to the true stochastic process in (1). For a storable commodity, arbitrage should constrain expected price rises to not exceed the rate of storage cost and can occur gradually as assumed in the model. But, price decreases for a seasonal commodity such as corn should occur across crop years and are unconstrained in size. Further, cointegration thresholds should prevent prices from going as high or as low as is allowed by the Brownian motion assumption. In addition, as is widely recognized, there can be structural change in parameters. We continue by using equation (4), but acknowledge that it is an approximation.

### *Futures Prices*

We are next interested in deriving futures prices using the spot price. Forward prices have been shown to be essentially equal to the expected future spot price at maturity under the risk-

neutral framework assumption that interest rates are constant (Cox, Ingersoll, and Ross 1981; Duffie and Stanton 1992). Thus, we must adjust the spot price equations for risk neutrality in order to derive the model for futures prices. Following the risk adjustment procedure used by Schwartz and Smith (2000) and Lucia and Schwartz (2002), we develop the risk neutral model for spot price by introducing market price of risk parameters  $\lambda_x$  and  $\lambda_z$  into the risk neutral stochastic processes of  $dx_t$  and  $dz_t$  in equations (3) and (4), respectively, to specify assumed constant reductions in the drifts of each process. The processes then take the form

$$(7) \quad dx_t = (-kx_t - \lambda_x)dt + \sigma_x dW_{1t}$$

$$(8) \quad dz_t = (\mu_z - \lambda_z)dt + \sigma_z dW_{2t}$$

where the risk-neutral mean reversion process  $x_t$  now reverts to  $-\lambda_x\sigma_x/k$  instead of zero and the drift in the Brownian motion process  $z_t$  is now  $\mu_z - \lambda_z$ . Under the risk neutral framework of Lucia and Schwartz (2002), the futures price is the expected cash price at maturity:

$$(9) \quad F_{T,0} = E_0[P_T] = s(T) + e^{-kT}x_0 + z_0 + (1 - e^{-kT})\alpha + \mu T$$

where  $F_{T,0}$  denotes the futures price at time zero with  $T$  time until maturity,  $\alpha = -\lambda_x\sigma_x/k$  and  $\mu = \mu_z - \lambda_z\sigma_z$ .

Whereas spot prices are generally reported in a true continuous fashion, time series data for futures prices comes from multiple futures contracts. Since theory and arbitrage demand that spot prices and futures prices converge at maturity, the possibly gradual mean reversion in cash prices is expected to be reflected in discrete jumps in futures prices at rollover. We can show the behavior of futures price spreads at maturity using the model above. Using equations (9) and (4), the expected change at rollover can be calculated. To solve for this, we look at the calendar spread at the time of rollover:

$$(10) \quad F_{T+H,T} - F_{T,T} = s(T + H) - (1 - e^{-k(T+H)})x_t + (1 - e^{-k(T+H)})\alpha + \mu(T + H)$$

where  $H$  denotes the time to the next contract maturity. The only pieces left are seasonality  $s(T + H)$ , the mean reversion expected over the interval, and the latent factors  $\alpha$  and  $\mu$ . Thus, across contracts (at rollover), we can expect that futures prices will be mean reverting and the rollover point should be treated differently from the rest of the series.

Differencing a series of continuous contracts is complicated by the switching of contracts. Equation (9) shows that every futures contract has a different expected value and so combining multiple contract months results in differing means as contract maturity month changes. Differences taken between the last observation used of a contract and the first observation used of the next contract include the terms in (10), which can lead to outliers at the point where contracts are spliced together. Including a single dummy variable as in Franken, Parcell, and Tonsor (2011) only removes the mean of the spread terms in (10) and so the outliers remain. Including a different dummy variable for every rollover still leaves the outliers in the lagged values. Most past research has used differences taken before splicing, which eliminates the problem with outliers. However, splicing still combines differences from contracts with differing expected values.

### *Tests for Stationarity*

The Schwartz models suggest that commodity prices are comprised of both mean reverting and unit root processes. However, empirical estimation of these models is complicated by latent factors whose distribution must be inferred from observable data. The difficulty in

estimating the Schwartz model is a key reason that it is almost standard practice to instead assume a one-factor model where prices are either mean-reverting or follow a unit root.

One of the first tests performed in much commodity price analysis research is to test for a unit root. When working with both cash and futures data, generally, unit root tests are performed for each series and these results are used to determine if data transformations and cointegration tests are needed. Futures price series are often considered to have a unit root as they are thought to follow a random walk. The jury is less settled on unit roots in spot commodity prices as arguments such as Wang and Tomek (2007) and some empirical works do not support spot prices possessing unit roots. However, plenty of research has empirically found unit roots for spot price series.

The common practice of determining stationarity in commodity price series pits mean reversion and unit roots against each other and claims the winner to be the manner in which the series behaves. Thus, the reason so many empirical studies determine that prices must follow a unit root is because the random component outweighs the mean reversion component by enough to satisfy (or fail to reject) a statistical test such as Dickey and Fuller (1981) or Phillips and Perron (1988). It is certainly possible that the random component will dominate the mean reversion component in most available price series. Pindyck (1999) argued that even with series spanning more than a century, unit root tests are likely to be inconclusive due to the slow nature of mean reversion. He also suggested that the trend to which prices are reverting fluctuates over time. The storable nature of commodities also suggests that prices might mean-revert downward faster than they do upward since some participants may be in a position to wait-out low prices.

Perhaps the most common unit root test is the Dickey Fuller (DF) Test and its augmented version (ADF) (Dickey and Fuller 1981). We now consider the effects of applying the DF test to data generated from a Schwarz-type process. Consider a DF type regression

$$(11) \quad X_t = \beta_0 + \beta_1 X_{t-1} + \varepsilon_t$$

where  $X_t$  denotes a price series and  $\beta_1$  is the mean reversion parameter. Rejecting the null hypothesis of the DF test (and most univariate unit root tests) implies that the series mean-reverts in the long-run. If the intercept in (11) is zero then the convergence is to a constant price level. If the intercept is not zero, then convergence is to a linear trend. Failure to reject the null hypothesis of nonstationarity implies there is not enough statistical evidence to conclude against the process possessing a unit root. As specified by the conceptual model, an important weakness of the ADF test is that the underlying mean is changing. With futures, the mean differs with every change in contract maturity. With cash prices, the mean differs due to changes in the cost of production and the issue is how close these changes are to the linear trend assumed with a DF test. We now discuss the econometric problems from using the DF tests with futures and spot prices.

#### *Unit Root Tests for Futures Prices*

Using (13), the standard DF test for a commodity futures price series can be written as

$$(12) \quad F_t = \beta_0 + \beta_1 F_{t-1} + \varepsilon_t$$

where  $\beta_1$  denotes the mean reversion coefficient. The general practice is to test the null hypothesis of  $\beta_1 \geq 1$  which would imply the series possesses a unit root. However, also of interest is the underlying assumption of the alternative hypothesis. By adding and subtracting the

constant underlying mean  $\bar{F}$  from  $\beta_1 F_{t-1}$  in (12), it can be shown that the mean reversion coefficient  $\beta_1$  is estimating the rate of convergence to  $\bar{F}$  as in

$$(13) \quad F_t = (\beta_0 + \beta_1 \bar{F}) + \beta_1 (F_{t-1} - \bar{F}) + \varepsilon_t.$$

Therefore, alternative hypothesis of  $\beta_1 < 1$  is that the futures price reverts to a constant mean.

However, as was shown in (9) and (10), the expected values differ across futures contracts. Thus, the test shown in (13) would only apply to observations from a single contract month.

Performing this test on a continuous contracts series would fail to account for the different  $\bar{F}$  of each contract. Failing to account for a change in the underlying mean implies that the DF test is biased towards a false acceptance of a unit root (Wang and Tomek 2007). Perron (1990) showed that  $\beta_1$  approaches one as the magnitude of even a single change in the mean increases.

Much research has focused on how to accommodate a changing level of the underlying mean in unit root tests with respect to structural breaks. Perron (1989, 1990) and Perron and Vogelsang (1992) extended the ADF test to allow for structural breaks. Wang and Tomek (2007) modeled structural changes in spot milk prices and found that once these breaks were considered, some unit root tests switched from nonstationary to stationary. Arnade and Hoffman (2015) followed an approach similar to Zivot and Andrews (1992) to test for structural breaks in soybeans and soybean meal spot and futures prices. They identified three sub-periods with significantly different estimated coefficients on the lagged price of each unit root equation. Joseph, Garcia, and Peterson (2013) also used the Zivot and Andrews (1992) test and found a break in the mean level and trend for boxed beef prices around the 2008 financial crash. Lee and Brorsen (2016) specified a general model that included both mean-reverting and unit root processes as special cases and found that most shocks are permanent breaks for Oklahoma hard winter wheat, Illinois corn, and Illinois soybean basis which favors a unit root model.



The literature on structural breaks is similar to our scenario of a changing mean across futures contracts. However, most of the structural break methodology is suited for series with unknown periods of changing means. If breaks are infrequent and discrete, then the Zivot and Andrews (1992) approach should capture the necessary dynamics. In the case of commodity futures prices, we know the underlying mean changes for each futures contract. Therefore, each contract should be treated as its own cross-section. We suggest using a single contract for each crop year and we can amend (15) to allow for a different mean for each crop year's contract

$$(14) \quad F_{it} = (\beta_0 + \beta_1 \bar{F}_i) + \beta_1 (F_{t-1} - \bar{F}_i) + \varepsilon_{it}$$

where  $\bar{F}_i$  denotes the underlying mean for each crop year (contract)  $i$ . As a result, the univariate DF test is no longer applicable to the panel of futures contracts in (14). However, panel unit root tests can be used to get a joint test of the null hypothesis of a unit root.

#### *Unit Root Tests for Spot Prices*

The underlying mean of the spot price is expected to be related to the long-run marginal cost of production. If this mean is constant or follows a time trend, then the DF test in (12) would be correctly specified. However, if the cost of production changes stochastically each crop year as we hypothesize, then an adjustment is needed to allow for a different mean each year. Within the Schwartz model, this would imply the assumption that long-run equilibrium  $z_t$  changes each year. This changing mean can be accounted for by specifying spot price process in (4) as

$$(15) \quad P_{it} = s(t) + x_t + z_i$$

where  $i$  represents crop year. We must also consider the impact of our hypothesis that mean reversion occurs across crop years while prices follow a unit root within a crop year. This

requires the assumption that the mean reversion level  $k$  in (5) is a different level between crop years relative to within crop years. Specifically, we assume that  $k = 0$  within a crop year and  $k \neq 0$  between crop years as the new crop approaches. We can account for this assumption by creating separate panels for the “within crop year” period and “between crop year” period.

### *Panel Unit Root Tests*

We use panel unit root tests as they allow the underlying mean to vary across panels (e.g. crop years) as is specified in (14). We specify the following panel tests for futures prices, though they also apply for our specification of spot prices. However, spot prices are divided into separate panels for “within crop year” period and “between crop year” periods.

We begin with the popular Levin, Lin, and Chu (LLC) (2002) panel unit root test

$$(16) \quad \Delta F_{it} = \alpha_i + \rho F_{it-1} + \sum_{j=1}^k \beta_j \Delta F_{it-j} + \varepsilon_{it}$$

where  $i = 1, \dots, N$  denotes the number of panels and  $t$  represents the number of observations in each panel,  $\alpha_i$  represents any trend effects within each panel,  $\rho$  is the speed of adjustment (mean reversion parameter), and  $k$  is the lag length. For this test, the null hypothesis is that  $\rho = 0$  for all panels ( $i$ ) and the alternative is that  $\rho > 0$  for all panels. Thus, while the underlying means can be different across panels, the level of mean reversion is assumed to be constant across all panels which is similar to that of the DF and ADF tests.

Im, Pesaran, and Shin (2003) which we will denote as IPS, used a similar framework to LLC (2002), but relaxed the assumption of homogeneous rate of mean reversion for all panels.

While the rate of mean reversion is assumed to be constant for each panel in our model<sup>1</sup>, we include the IPS (2003) test to account for a possible change in the rate of mean reversion across crop years. Differing mean reversion rates would imply that prices mean-revert faster in some years than in others. The IPS (2003) panel unit root test is specified as

$$(17) \quad \Delta F_{it} = \alpha_i + \rho_i F_{it-1} + \sum_{j=1}^k \beta_j \Delta F_{it-j} + \varepsilon_{it}$$

where the speed of adjustment  $\rho_i$  is now allowed to be heterogeneous across panels and all other components are as defined in (16). The unit root test within each panel is based on the ADF  $t$  statistics and results are averaged across panels to provide a single test statistic for the data as shown by

$$(18) \quad \bar{t} = \frac{1}{N} \sum_{i=1}^N t_i$$

where  $t_i$  are the  $t$  statistics for each panel  $i$ . The null hypothesis for the IPS (2003) test is that all  $\rho_i = 0$  compared to the alternative hypothesis that at least one of the panels is stationary.

### *Cointegration*

The ideas on cointegration are not as fully developed as the rest of the paper, but we discuss cointegration anyway. A prerequisite for two series to be cointegrated is that they both follow a unit root process of the same order. If this condition holds, and there exists a linear combination of the two series that is stationary, then the variables can be defined as cointegrated

---

<sup>1</sup> Note the mean reversion level is hypothesized to be different for between crop year and within crop year segments for cash prices. However, we split cash prices in to separate panels for these two periods. Therefore each panel is assumed to have a constant mean reversion level.

(Engle and Granger 1987). By specifying the ECM equation from (1) with futures price as the dependent variable, we can use the solutions for spot (equation 4) and futures (equation 9) prices to show the cointegrating relationship between futures and spot prices to be

$$(19) \quad \Delta F_t = \alpha_t + \rho_t u_{t-1} + v_t$$

where  $\alpha_t$  includes the latent factors  $(1 - e^{kT})\alpha + \mu T$  from (9) and  $v_t$  captures any expectation differences between the futures price and the expected spot price. The error correction term  $u_{t-1}$  is the linear combination of  $F_t$  and  $P_t$  and is represented by  $u_{t-1} = P_t - F_t$  which, by substitution, can be specified as

$$(20) \quad u_{t-1} = (z_t - z_t) - (s(t) - s(T)) - (x_t - e^{-kT} x_0)$$

where the  $z_t$  terms are the unit root terms for futures and spot prices and are offset for any single time period  $t$ ,  $(s(t) - s(T))$  is the difference in seasonality, and  $x_t - e^{-kT} x_0$  is the difference between the mean reversion terms. A correctly specified cointegration test would account for the differences in mean reversion and seasonality. Alternatively, a plausible assumption can be made for each difference to be negligible at any single time period. Spot and futures prices are theoretically cointegrated under the assumptions of the Schwartz model. Empirical estimation of this cointegration model would require the use of latent factors that must be inferred from observable data. Though it is beyond the scope of this paper, it might be possible for these to be inferred from futures prices as in Hart et al. (2015). Panel unit root tests can help model differences in mean reversion when  $k \neq 0$ . Alternatively, for the case of  $k = 0$ , univariate unit root tests are appropriate.

By definition, cash and futures prices are cointegrated at maturity. To conduct a cointegration test using daily prices, the other factors such as cash price seasonality need to be

considered in the model. Most past empirical research has used daily prices and has not included variables to reflect cash price seasonality or cash market mean reversion. When empirical research has found cointegration in the past, it may mean that the seasonality and mean reversion factors are relatively small.

## Data

We perform the unit root tests on weekly cash and futures prices for corn, soybeans, wheat, and live cattle compiled by the Livestock Marketing Information Center (LMIC). All grain prices except soybean cash prices span from the beginning of the 1993/1994 crop year through the 2015/2016 crop year. Soybean cash prices were only available in this data set from the beginning of the 2004/2005 crop year through the end of the 2015/2016 crop year. The crop year for corn and soybeans begins on October 1<sup>st</sup> each year as suggested by Smith (2005). Wheat crop years start on June 1<sup>st</sup> each year. Since live cattle are produced throughout the year, calendar year panels are used. Summary statistics are provided in table 3. Corn cash prices are as reported in Chicago, IL and wheat cash prices are for hard red winter wheat #1 in Kansas City, MO and both are from the *National Weekly Feedstuff Wholesale Prices* reports (USDA, AMS 2016c). Soybean cash prices are as reported in central Illinois and are from the *Soybean Prices Compared with Value of Oil and Meal* reports (USDA, AMS 2016b). All grain prices are quoted in dollars per bushel (\$/bu). Live cattle cash prices are in dollars per hundred pounds (\$/cwt) and are reported by the *Five Area Weekly Weighted Average Direct Slaughter Cattle* reports (USDA, AMS 2016a).

Grain futures prices are weekly futures prices in \$/bu for Wednesday of each week. If Wednesday falls on a holiday, the Thursday price is used for that week. Live cattle futures prices

are reported in \$/cwt. We use a single contract for each crop year in order to avoid a combination of contracts with differing means. This panel approach avoids the combination of multiple contracts with differing underlying means as discussed in (14). For corn and soybeans, we use the May contract prices and thus our panel for each crop year spans from the first week in October through the second week in April. We avoid observations near maturity as is common in the commodity price literature as shown in table 2. For wheat, we use the March contract prices for each crop year and each panel spans from the first week in June through the last week in January. Live cattle futures prices are for the December contract and each panel is from the first week in January through the second week in November.

The LLC (2002) test requires balanced panels and thus missing observations and varying numbers of weeks in each crop year create a problem for this test. The only missing observations occur in cash prices where two weekly prices were missing for corn and wheat and nine weekly prices were missing for soybeans. To fill these missing observations, we add the change in the futures price for the corresponding date and commodity to the most recent cash price. To account for a varying number of weekly observations per year, observations occurring on the first or last day of a crop year are omitted (i.e. September 30<sup>th</sup> or October 1<sup>st</sup> for corn and soybean prices). This results in each crop year possessing the balanced panels needed for the LLC (2002) test.

## **Empirical Results**

We begin by testing each series for stationarity using the traditional ADF test. This test assumes a constant underlying mean. Table 4 reports the results of the ADF test for each commodity price series in levels and in differences. As shown, the ADF test fails to reject the null hypothesis of a unit root for each price series in levels except for wheat futures prices and

wheat cash prices for February through May. The finding of mean reversion in wheat futures is surprising. The finding of mean reversion for cash prices is to be expected, especially in the between crop year span. All other variables are nonstationary according to the ADF in levels. After taking the first differences, the ADF tests reject the null hypothesis and appear stationary. Thus, we would conclude that each series, besides wheat, is  $I(1)$  from these results alone as is done in most of the studies listed in table 1 though this is certainly not an exhaustive list.

We next consider the LLC (2002) panel unit root test which assumes a constant rate of mean reversion across panels. Thus, for each commodity price series, the number of panels corresponds with the number of years available and the number of observations per year corresponds with the number of weeks in each year. Table 5 reports the results from this test. As shown, both series of corn and wheat cash prices are found to be mean-reverting in at least one panel. Likewise, wheat futures prices are found to be mean-reverting in at least one panel, an unexpected result. The LLC (2002) test fails to reject the null hypothesis of unit roots in all panels for the remaining variables.

Further exploration of wheat futures prices finds that upon deletion of observations after 2006, the null hypothesis of a unit root cannot be rejected. Thus, the finding of mean reversion in wheat futures prices is found only after 2006. This implies that the finding of mean reversion is driven by recent years when wheat prices have experienced dramatic changes. One possible explanation is the occurrence of a bubble in commodity prices around 2007-2008. A common concern is that increased buying pressure from financial index investors led to bubbles in commodity futures prices (e.g. Masters, 2008 2009). Other financial activity such as managed funds are also considered to have had an impact (Waggoner 2008). Empirical results are mixed on whether financial activity led to a bubble in wheat prices. Etienne, Irwin, and Garcia (2015)

provide an excellent discussion on the possibility of a bubble in wheat futures prices and found that any impact should be negligible. Alternatively, Gutierrez (2013) found support for a bubble in wheat prices, though he was unable to conclude that the bubble was caused specifically by financial activity. Our results are unable to offer specific insights on a bubble in wheat futures. However, we do find different wheat futures price behavior after 2006.

Next, we use the IPS (2003) panel unit root test to test each series for stationarity. This test allows for varying speed of adjustment as the panel is constructed across years to account for changing dynamics across crop years. As shown in table 6, the IPS (2003) panel unit root test results are similar to the LLC (2002) results. The IPS (2003) test fails to reject the null hypothesis of a unit root process in levels for all commodities except corn and wheat. The presence of a unit root process is rejected for corn cash prices from October to May at the one percent level and for corn futures prices at the ten percent level. Wheat cash prices are found to be stationary at the five percent level and wheat futures prices at the one percent level. Similar to the ADF test, after taking the first difference, every price series appears stationary.

Our hypothesis of mean reversion between crop years and a random walk within crop years is generally not supported. For each of the crops considered the LLC (2002) test results in a conclusion of mean reversion for both periods for corn and wheat cash prices and a conclusion of a unit root for both periods of soybean cash prices. Our results differ from traditionally used unit root tests shown in table 2 as we find mean reversion in corn and wheat cash prices. In particular, Wang and Tomek (2007) found that accounting for structural changes in the underlying mean can result in finding cash prices to be mean-reverting. Our results extend on this work by allowing the underlying means to change each crop year.



## Conclusions

Determining whether a series is mean-reverting or follows a unit root process is an integral part of time series analysis as it is often one of the first things tested. There seems to be a growing divide between how univariate analysis accounts for mean reversion as compared to multivariate analysis. In particular, the Schwartz-type (year) two factor models allow for some level of both mean reversion and unit roots whereas the popular error correction models require the researcher to determine if prices are either mean-reverting or follow a unit root process. Our application of panel unit root tests incorporates a key assumption from the Schwartz-type models (i.e. a changing underlying mean, though only across crop years). Thus, while we still ultimately find a series to be either mean-reverting or nonstationary, the results are from a test that allows for the changing underlying means.

We show that continuous contracts do not account for differing underlying means for each contract and for the mean reversion that occurs at rollover. Panel unit root tests are used by treating each crop year as a panel. The traditional unit root tests are unable to account for a possible shift in the underlying mean across crop years. It should not be expected that using panel unit root tests will drastically change results of previous studies. However, it is likely that results might be less sporadic due to a more uniform specification of the changing underlying mean.

Cash prices are difficult as none of the currently available models captures their likely stochastic process. The hypothesis that prices within a crop year and across a crop year should be treated as having differing mean reversion behavior is not well supported by our results. However, aligning cash prices as a single continuous series does not account for deviations from a linear trend for cost of production over time.

While the mean reversion in cash prices is easier to understand, the finding of mean reversion in wheat futures prices contrasts with the theory that futures prices should follow a unit root process. Further exploration using the LLC (2002) test found that this mean reversion was driven by price behavior in recent years. One possible explanation is trading by managed funds in recent years may have accentuated price swings.

The implications of our findings are important as they can influence modelling techniques. While the panel unit root tests are more appropriate for futures prices, the correct approach for cash prices is less clear-cut. Since cointegration tests rely on each price series, differing alignments of prices create difficulties in correctly specifying cointegration tests.

## References

- Allen, M. T., C. K. Ma, and R. D. Pace. 1994. "Over-Reactions in US Agricultural Commodity Prices." *Journal of Agricultural Economics* 45 (2): 240-251.
- Ardeni, P. G. 1989. "Does the Law of One Price Really Hold for Commodity Prices?" *American Journal of Agricultural Economics* 71 (3): 661-669.
- Arnade, C., and L. Hoffman. 2015. "The Impact of Price Variability on Cash/Futures Market Relationships: Implications for Market Efficiency and Price Discovery" *Journal of Agricultural and Applied Economics* 47 (4): 539-559.
- Babula, R. A., F. J. Ruppel, and D. A. Bessler. 1995. "US Corn Exports: The Role of the Exchange Rate." *Agricultural Economics* 13 (2): 75-88.
- Barkoulas, J., W. C. Labys, and J. Onochie. 1997. "Fractional Dynamics in International Commodity Prices." *Journal of Futures Markets* 17 (2): 161-189.
- Beck, S. E. 1994. "Cointegration and Market Efficiency in Commodities Futures Markets." *Applied Economics* 26: 249-257.
- Bessembinder, H. 1992. "Systematic Risk, Hedging Pressure, and Risk Premiums in Futures Markets." *Review of Financial Studies* 5 (4): 637-667.
- Bessler, D. A., and T. Covey. 1991. "Cointegration: Some Results on U.S. Cattle Prices." *The Journal of Futures Markets* (11): 461-474.
- Brooks, C., M. Prokopczuk, and Y. Wu. 2013. "Commodity Futures Prices: More Evidence on Forecast Power, Risk Premia and the Theory of Storage." *Quarterly Review of Economic Finance* 53 (1): 73-85.
- Campbell, J., and P. Perron. 1991. "Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots." *Macroeconomics Conference*, National Bureau of Economic Research, Cambridge, MA, February.
- Casassus, J., and P. Collin-Dufresne. 2005. "Stochastic Convenience Yield Implied from Commodity Futures and Interest Rates." *Journal of Finance* 60 (5): 2283-2331.
- Chen, S. L., J. D. Jackson, H. Kim, and P. Resiandini. 2014. "What Drives Commodity Prices?" *American Journal of Agricultural Economics* 96 (5): 1455-1468.
- Cox, J. C., J. E. Ingersoll, and S. A. Ross. 1981. "The Relation between Forward Prices and Futures Prices." *Journal of Financial Economics* 9 (4): 321-346.
- Cuddington, J., and M. Urzu. 1987. "Trends and Cycles in Primary Commodity Prices." Economics Department, Georgetown University, Washington, DC. Processed Paper.

- De Roon, F. A., T. E. Nijman, and C. Veld. 2000. "Hedging Pressure Effects in Futures Markets." *The Journal of Finance* 55 (3): 1437-1456.
- Dempster, M., E. Medova, and K. Tang. 2008. "Pricing and Hedging Long-Term Spread Options." *Journal of Banking Finance* 32: 2530-2540.
- Dickey, D. A., and W. A. Fuller. 1981. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." *Econometrica* 49: 1057-1072.
- Dixit, A. K., and R. S. Pindyck. 1994. "Investment Under Uncertainty." Princeton University Press, Princeton, NJ.
- Duffie, D., and R. Stanton. 1992. "Pricing Continuously Resettled Contingent Claims." *Journal of Economic Dynamics and Control* 16 (3): 561-573.
- Engle, R., and C. Granger. 1987. "Co-Integration and Error Correction: Representation, Estimation, and Testing." *Econometrica* 55: 251-276.
- Etienne, X. L. S. H. Irwin, and P. Garcia. 2015. "\$25 Spring Wheat was a Bubble, Right?" *Agricultural Finance Review* 75 (1): 114 – 132.
- Foster, K. A., A. M. Havenner, and A. M. Walburger. 1995. "System Theoretic Time-Series Forecasts of Weekly Live Cattle Prices." *American Journal of Agricultural Economics* 77 (4): 1012-1023.
- Franken, J. R., J. L. Parcell, and G. T. Tonsor. 2011. "Impact of Mandatory Price Reporting on Hog Market Integration." *Journal of Agricultural and Applied Economics* 43 (02): 229-241.
- Goodwin, B. K., and M. T. Holt. 1999. "Price Transmission and Asymmetric Adjustment in the US Beef Sector." *American Journal of Agricultural Economics* 81 (3): 630-637.
- Goodwin, B. K., and N. E. Piggott. 2001. "Spatial Market Integration in the Presence of Threshold Effects." *American Journal of Agricultural Economics* 83 (2): 302-317.
- Granger, C. W. J., and P. Newbold. 1974. "Spurious Regressions in Econometrics." *Journal of Econometrics* 2 (2): 111-120.
- Gutierrez, L. 2013. "Speculative Bubbles in Agricultural Commodity Markets." *European Review of Agricultural Economics* 40 (2): 217-238.
- Han, S., C. Chung, and P. Surathkal. 2016. "Impacts of Increased Corn Ethanol Production on Price Asymmetry and Market Linkages in Fed Cattle Markets." *Agribusiness* 1-25.
- Harri, A., L. Nalley, and D. Hudson. 2009. "The Relationship between Oil, Exchange Rates, and Commodity Prices." *Journal of Agricultural and Applied Economics* 41 (2): 501-510.

- Hart, C. E., S. H. Lence, D. J. Hayes, and N. Jin. 2016. "Price Mean Reversion, Seasonality, and Options Markets." *American Journal of Agricultural Economics* 98 (3): 707-725.
- Im, K., H. Pesaran, and K. Shin. 2003. "Testing for Unit Roots in Heterogeneous Panels." *Journal of Econometrics* 115 (1): 53-74.
- Irwin, S. H., C. R. Zulauf, and T. E. Jackson. 1996. "Monte Carlo Analysis of Mean Reversion in Commodity Futures Prices." *American Journal of Agricultural Economics* 78 (2): 387-399.
- Jin, N., S. Lence, C. Hart, and D. Hayes. 2012. "The Long-Term Structure of Commodity Futures." *American Journal of Agricultural Economics* 94 (3): 718-735.
- Joseph, K., P. Garcia, and P. E. Peterson. 2013. "Price Discovery in the U.S. Fed Cattle Market." In Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, St. Louis, MO.
- Kim, H. S., and B. W. Brorsen. 2012. "Can Real Option Value Explain Apparent Storage at a Loss?" *Applied Economics* 44 (16): 2081-2090.
- Lautier, D. 2005. "Term Structure Models of Commodity Prices: A Review." *The Journal of Alternative Investments* 8 (1): 42.
- Lee, Y., and B. W. Brorsen. 2016. "Permanent Shocks and Forecasting with Moving Averages." *Applied Economics* 1-13.
- Lence, S. H., and M. L. Hayenga. 2001. "On the Pitfalls of Multi-year Rollover Hedges: The Case of Hedge-to-Arrive Contracts." *American Journal of Agricultural Economics* 83 (1): 107-119.
- Levin, A., C. F. Lin, and C. S. J. Chu. 2002. "Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties." *Journal of Econometrics* 108 (1): 1-24.
- Lien, D., G. Lim, L. Yang, and C. Zhou. 2013. "Dynamic Dependence between Liquidity and the S&P 500 Index Futures-Cash Basis." *Journal of Futures Markets* 33 (4): 327-342.
- Lin, S. X., and M. N. Tamvakis. 2001. "Spillover Effects in Energy Futures Markets." *Energy Economics* 23 (1): 43-56.
- Lucia, J. J., and E. S. Schwartz. 2002. "Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange." *Review of Derivatives Research* 5 (1): 5-50.
- Lumsdaine, R. L., and D. H. Papell. 1997. "Multiple Trend Breaks and the Unit-Root Hypothesis." *Review of Economics and Statistics* 79 (2): 212-218.
- Ma, C. K., J. M. Mercer, and M. A. Walker. 1992. "Rolling Over Futures Contracts: A Note." *Journal of Futures Markets* 12 (2): 203-217.

- MacKinlay, A. C., and K. Ramaswamy. 1988. "Index-Futures Arbitrage and the Behavior of Stock Index Futures Prices." *Review of Financial Studies* 1 (2): 137-158.
- Manoliu, M., and S. Tompaidis. 2002. "Energy Futures Prices: Term Structure Models with Kalman Filter Estimation." *Applied Mathematical Finance* 9 (1): 21-43.
- Masters, M.W. 2008. Testimony before the Committee on Homeland Security and Governmental Affairs, United States Senate. May 2008.
- Masters, M.W. 2009. "Testimony before the Commodities Futures Trading Commission. August 5<sup>th</sup>, 2009.
- McKenzie, A. M., and M. T. Holt. 2000. "Market Efficiency in Agricultural Futures Markets." *Applied Economics* 34: 1519-1532.
- Meyer, J., and S. von Cramon-Taubadel. 2004. "Asymmetric Price Transmission: A Survey." *Journal of Agricultural Economics* 55 (3) : 581-611.
- Paschke, R., and M. Prokopczuk. 2009. "Integrating Multiple Commodities in a Model of Stochastic Price Dynamics." *Journal of Energy Markets* 2 (3): 47-82.
- Paschke, R., and M. Prokopczuk. 2010. "Commodity Derivatives Valuation with Autoregressive and Moving Average Components in the Price Dynamics." *Journal of Banking and Finance* 34 (11): 2742-2752.
- Perron, P. 1989. "The Great Crash, the Oil-Price Shock, and Unit-Root Hypothesis." *Econometrica* 57 (6): 1361-1401.
- Perron, P. 1990. "Testing for a Unit Root in a Time Series with a Changing Mean." *Journal of Business & Economic Statistics* 8 (2): 153-162.
- Perron, P, and T. J. Vogelsang. 1992. "Testing for a Unit Root in a Time Series with a Changing Mean: Corrections and Extensions." *Journal of Business & Economic Statistics* 10 (4): 467-470.
- Pesaran, M. H., D. Pettenuzzo, and A. Timmermann. 2006. "Forecasting Time Series Subject to Multiple Structural Breaks." *The Review of Economic Studies* 73 (4): 1057-1084.
- Peterson, R. L., C. K. Ma, and R. J. Ritchey. 1992. "Dependence in Commodity Prices." *Journal of Futures Markets* 12 (4): 429-446.
- Phillips, P. C., and P. Perron. 1988. "Testing for a Unit Root in Time Series Regression." *Biometrika* 75 (2): 335-346.
- Pindyck, R. S. 2001. "The Dynamics of Commodity Spot and Futures Markets: A Primer." *The Energy Journal* 22: 1-29.

- Saghaian, S. H. 2010. "The Impact of the Oil Sector on Commodity Prices: Correlation or Causation?" *Journal of Agricultural and Applied Economics* 42 (3): 477-485.
- Schroeder, T. C., and B. K. Goodwin. 1991. "Price Discovery and Cointegration for Live Hogs." *The Journal of Futures Markets* 11 (6): 685-696.
- Schwartz, E. 1997. "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging." *Journal of Finance* 52 (3): 923-973.
- Schwartz, E, and J. Smith. 2000. "Short-Term Variations and Long-Term Dynamics in Commodity Prices." *Management Science* 46: 893-911.
- Schwarz, T. V., and A. C. Szakmary. 1994. "Price Discovery in Petroleum Markets: Arbitrage, Cointegration, and the Time Interval of Analysis." *Journal of Futures Markets* 14 (2): 147-167.
- Smith, A. 2005. "Partially Overlapping Time Series: A New Model for Volatility Dynamics in Commodity Futures." *Journal of Applied Econometrics* 20 (3): 405-422.
- Sørensen, C. 2002. "Modeling Seasonality in Agricultural Commodity Futures." *Journal of Futures Markets* 22 (5): 393-426.
- Tang, K. 2012. "Time-Varying Long-Run Mean of Commodity Prices and the Modeling of Futures Term Structures." *Quantitative Finance* 12 (5): 781-790.
- Todorova, M. I. 2004. "Modeling Energy Commodity Futures: Is Seasonality Part of It?" *The Journal of Alternative Investments* 7 (2): 10-32.
- Tomek, W.G. 1997. "Commodity Futures Prices as Forecasts." *Review of Agricultural Economics* 19 (1): 23-44.
- Trujillo-Barrera, A., M. Mallory, and P. Garcia. 2012. "Volatility Spillovers in US Crude Oil, Ethanol, and Corn Futures Markets." *Journal of Agricultural and Resource Economics* 37 (2): 247-262.
- U.S. Department of Agriculture, Agricultural Marketing Service (USDA, AMS). 2016a. Five Area Weekly Weighted Average Direct Slaughter Cattle. LM\_CT150, multiple reports.
- \_\_\_\_\_. 2016b. Soybean Prices Compared with Value of Oil and Meal. GX\_GR211, multiple reports.
- \_\_\_\_\_. 2016c. National Weekly Feedstuff Wholesale Prices. MS\_GR852, multiple reports.
- Walburger, A., and K. Foster. 1995. "Mean Reversion as a Test for Inefficient Price Discovery in U.S. Regional Cattle Markets." Published Abstract. *American Journal of Agricultural Economics* 77 (5): 1370.

- Waggoner, J. 2008. "Commodities Bubble Brews?" ABC News Online, May 23. Available online at <http://abcnews.go.com/Business/story?id=4906576&page=1> . Accessed November 20, 2016.
- Wang, D., and W. G. Tomek. 2007. "Commodity Prices and Unit Root Tests." *American Journal of Agricultural Economics* 89 (4): 873–889.
- Zapata, H. O., and T. R. Fortenbery. 1996. "Stochastic Interest Rates and Price Discovery in Selected Commodity Markets." *Review of Agricultural Economics* 18: 643-654.
- Zivot, E., and D. W. K. Andrews. 1992. "Further Evidence of the Great Crash, the Oil-Price Shock, and Unit-Root Hypothesis." *Journal of Business and Economic Statistics* 10 (3): 251-270.



**Table 1. Futures Contract Rollover Dates Used in Previous Research**

Authors	Rollover date	Futures contract(s)
Trujillo-Barrera, Mallory and Garcia (2012)	Third business day prior to the 25th of the month prior to expiration	Corn and ethanol
Ma, Mercer, and Walker (1992)	First notice day, first day of contract month, and delivery day	Soybeans, Gold, S&P 500, t-bond, and Japanese yen
Zapata and Fortenbery (1996)	First notice day	Corn and soybeans
Arnade and Hoffman (2015)	Last day of month prior to expiration	Soybeans and soymeal
Bessembinder (1992)	First day of delivery month	Foreign currency and agricultural futures
De Roon, Nijman, and Veld (2000)	First day of delivery month	20 futures markets, including agricultural commodities
Schroeder and Goodwin (1991)	The 15th of expiration month	Live hogs
Lien et al. (2013)	Both 6 and 10 working days before expiration	S&P 500 index
Franken, Parcell, and Tonsor (2011)	One week prior to expiration	Live hogs
Lin and Tamvakis (2001)	Five working days before expiration	Crude oil
Mackinlay and Ramaswamy (1988)	At expiration	S&P 500 Index
Bessler and Kling (1990)	At expiration	Live cattle
Bessler and Covey (1991)	At expiration	Live cattle

**Table 2. Unit Root Test Results for Agricultural Commodity Price Series from Previous Research**

Authors	Commodities	Series type	Series rejected unit root	Series failed to reject unit root
Beck (1994)	Cattle, cocoa, corn, hogs, soybeans	Cash	Hogs and soybeans	Cattle, cocoa, corn
Babula, Ruppel, and Bessler (1995)	Corn	Cash	None	All
Foster, Havenner, and Walburger (1995)	Live cattle	Cash	None	All
Goodwin and Holt (1999)	Producer, wholesale, and retail beef	Cash	None	All
Wang and Tomek (2007)	Barrows and gilts, corn, milk, and soybeans	Cash	Corn, soybeans, barrows and gilts	Milk
Saghaian (2010)	Corn, ethanol, soybeans, and wheat	Cash	None	All
Chen et al. (2014)	Barley, beef, cocoa, coffee, corn, cotton, hogs, rice, soybeans, and wheat	Cash	Barley	All else
Bessler and Covey (1991)	Live cattle	Cash and futures	None	All
Schroeder and Goodwin (1991)	Live hogs	Cash and futures	None	All
Zapata and Fortenbery (1996)	Corn and soybeans	Cash and futures	None	All
McKenzie and Holt (2002)	Corn, live cattle, hogs, and soybean meal	Cash and futures	None	All
Franken, Parcell, and Tonsor (2011)	Live hogs	Cash and futures	None	All
Arnade and Hoffman (2015)	Soybeans and soymeal	Cash and futures	None	All
Harri, Nalley, and Hudson (2009)	Corn, cotton, soybeans, soybean oil, and wheat	Futures	None	All
Trujillo-Barrera, Mallory, and Garcia (2012)	Corn and ethanol	Futures	None	All

Note: Some of these studies included more than just the agricultural prices listed. We omitted the non-agricultural commodities from this table.

**Table 3. Descriptive Statistics for Weekly Commodity Prices from 1993 to 2016**

Variable	Mean	SD	Min	Max
Corn cash price (\$/bu)	3.39	1.55	1.59	8.26
Corn futures price (\$/bu)	3.45	1.51	1.78	8.24
Soybean cash price (\$/bu) <sup>a</sup>	10.60	3.20	5.08	18.15
Soybean futures price (\$/bu)	8.27	3.18	4.13	17.50
Wheat cash price (\$/bu)	5.27	2.04	2.62	13.67
Wheat futures price (\$/bu)	4.86	1.90	2.66	12.50
Live cattle cash price (\$/cwt)	89.18	26.62	55.79	171.38
Live cattle futures price (\$/cwt)	89.44	25.96	56.72	172.09

<sup>a</sup> Soybean cash prices are only available starting at the beginning of the 2004/2005 crop year.

**Table 4. Augmented Dickey Fuller Tests for Commodity Prices**

Variable	Levels		Differences	
	Test Statistic	p-value	Test Statistic	p-value
Corn cash price Oct-May	-2.20	0.21	-29.89***	<0.01
Corn cash price June-Sept	-2.40	0.14	-23.21***	<0.01
Corn futures price	-1.96	0.31	-27.16***	<0.01
Soybean cash price Oct-May	-2.17	0.22	-21.16***	<0.01
Soybean cash price June-Sept	-1.83	0.37	-13.68***	<0.01
Soybean futures price	-1.93	0.32	-27.95***	<0.01
Wheat cash price June-Jan	-1.80	0.38	-28.83***	<0.01
Wheat cash price Feb-May	-2.71**	0.07	-21.38***	<0.01
Wheat futures price	-1.66	0.45	-27.42***	<0.01
Live cattle cash price	-1.04	0.74	-29.87***	<0.01
Live cattle futures price	-0.79	0.82	-34.38***	<0.01

Note: Single asterisk (\*), double asterisk (\*\*), and triple asterisk (\*\*\*), denote significance at the ten percent, five percent, and one percent level, respectively.

**Table 5. Levin, Lin, and Chu (2002) Panel Unit Root Tests of Commodity Prices**

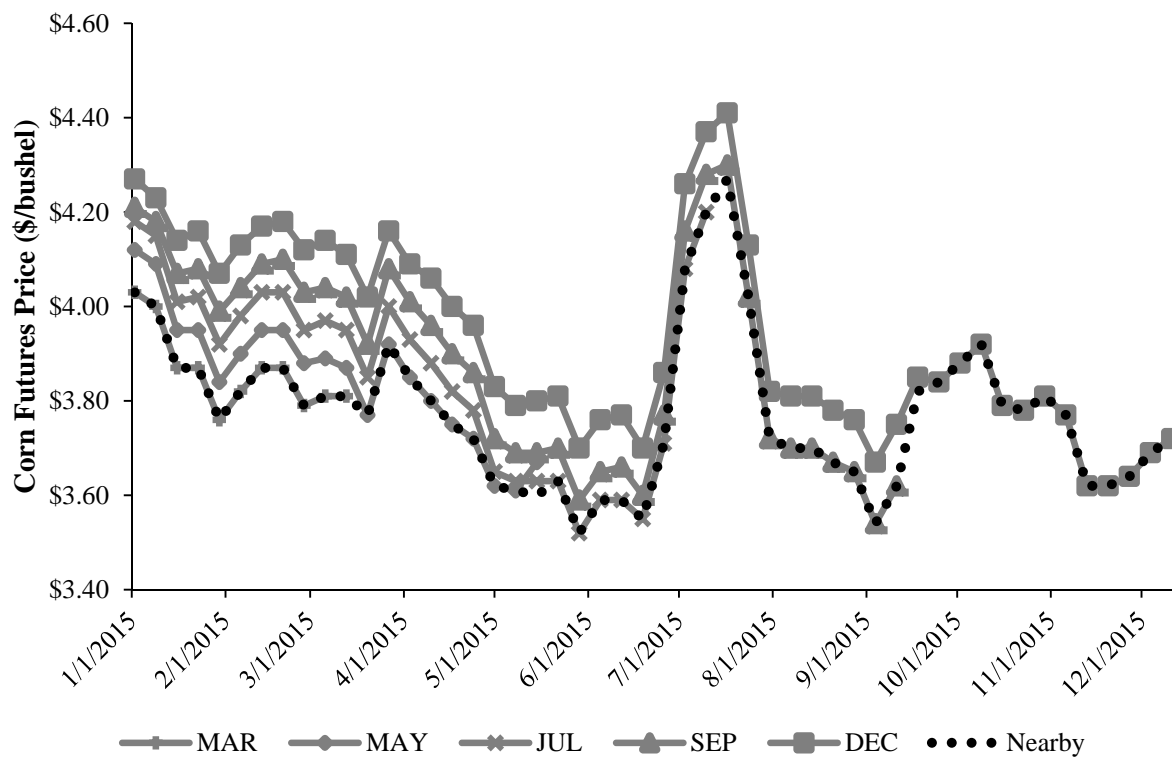
Variable	Test Statistic	p-value
Corn cash price Oct-May	-2.46***	0.01
Corn cash price June-Sept	-4.76***	<0.01
Corn futures price	-0.32	0.37
Soybean cash price Oct-May	-0.76	0.22
Soybean cash price June-Sept	0.20	0.58
Soybean futures price	-0.23	0.41
Wheat cash price June-Jan	-2.83***	<0.01
Wheat cash price Feb-May	-3.52***	<0.01
Wheat futures price	-3.12***	<0.01
Live cattle cash price	-1.13	0.13
Live cattle futures price	0.35	0.64

Note: Single asterisk (\*), double asterisk (\*\*), and triple asterisk (\*\*\*), denote significance at the ten percent, five percent, and one percent level, respectively.

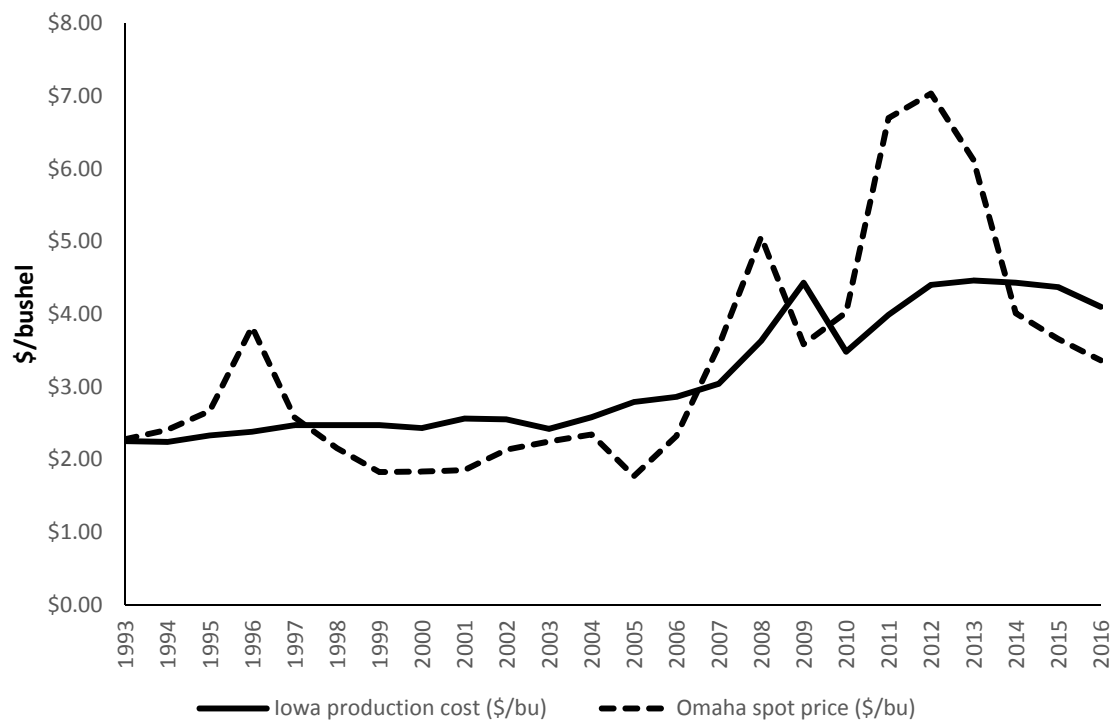
**Table 6. Im, Pearson, Shin (2003) Panel Unit Root Tests of Commodity Prices**

Variable	Levels		Differences	
	Test Statistic	p-value	Test Statistic	p-value
Corn cash price Oct-May	-2.66***	<0.01	-20.33***	<0.01
Corn cash price June-Sept	-1.00	0.16	-11.11***	<0.01
Corn futures price	0.40	0.65	-17.24***	<0.01
Soybean cash price Oct-May	-0.49	0.48	-14.14***	<0.01
Soybean cash price June-Sept	3.71	0.99	-7.71***	<0.01
Soybean futures price	0.57	0.72	-17.62***	<0.01
Wheat cash price June-Jan	-1.94**	0.03	-17.01***	<0.01
Wheat cash price Feb-May	-1.88**	0.03	-12.11***	<0.01
Wheat futures price	-1.54*	0.06	-16.56***	<0.01
Live cattle cash price	-0.04	0.48	-19.06***	<0.01
Live cattle futures price	0.39	0.65	-22.78***	<0.01

Note: Single asterisk (\*), double asterisk (\*\*), and triple asterisk (\*\*\*), denote significance at the ten percent, five percent, and one percent level, respectively.



**Figure 1.** 2015 corn futures prices for contracts ending in March, May, July, September, and December and the nearby (continuous) contract.



**Figure 2.** Corn production cost (\$/bu) in Iowa and annual average corn spot price (\$/bu) in Omaha, NE from 1993 to 2016.