$$\underset{T\times T}{var(u)} := \mathbb{E}\big[\underbrace{(u-\mathbb{E}[u])}_{T\times 1}\underbrace{(u-\mathbb{E}[u])'}_{(T\times 1)'}\big]$$

$\mathbb{E}[u] = 0$ by A1

$$\begin{aligned} var(u) &= \mathbb{E}[(u-0)(u-0)'] \\ &= \mathbb{E}[uu'] \end{aligned}$$

in general,

$$var(u) = \mathbb{E}[uu'] - \mathbb{E}[u]\mathbb{E}[u]'$$

w, v random

$$\begin{aligned} var(w+v) &= \mathbb{E}[(w+v-\mathbb{E}[w+v])(w+v-\mathbb{E}[w+v])'] \\ &= \mathbb{E}[((w-\mathbb{E}[w])+(v-\mathbb{E}[v]))((w-\mathbb{E}[w])+(v-\mathbb{E}[v]))'] \\ &= \mathbb{E}[(w-\mathbb{E}[w])(w-\mathbb{E}[w])'] + \mathbb{E}[(v-\mathbb{E}[v])(v-\mathbb{E}[v])'] \\ &\quad + \mathbb{E}[(w-\mathbb{E}[w])(v-\mathbb{E}[v])'] + \mathbb{E}[(v-\mathbb{E}[v])(w-\mathbb{E}[w])'] \\ &= var(w) + var(v) \\ &\quad + cov(w,v) + cov(v,w) \end{aligned}$$

$\underset{T\times 1}{w}$ random, $\underset{K\times T}{A}$ fixed, non-stoch.

$$\underset{K\times 1}{v} = \underset{K\times T}{A}\,\underset{T\times 1}{w}$$

$$\begin{aligned} \underset{K\times K}{var(v)} &= \mathbb{E}[(v-\mathbb{E}[v])(v-\mathbb{E}[v])'] \\ &= \mathbb{E}[(Aw-\mathbb{E}[Aw])(Aw-\mathbb{E}[Aw])'] \\ &= \mathbb{E}[(Aw-A\mathbb{E}[w])(Aw-A\mathbb{E}[w])'] \\ &= \mathbb{E}[(A(w-\mathbb{E}[w]))(A(w-\mathbb{E}[w]))'] \\ &= A\,\mathbb{E}[(w-\mathbb{E}[w])(w-\mathbb{E}[w])']\,A' \\ &= A\,var(w)\,A' \end{aligned}$$

$\underset{T\times 1}{y} \in \mathbb{R}^T$, $\underset{T\times K}{X}$

$$Col(X) = \{w \in \mathbb{R}^T : w = X\alpha \text{ for some } \alpha \in \mathbb{R}^K\}$$

$$Col(X)^\perp = \{v \in \mathbb{R}^T : X'v = \underset{K\times 1}{0}\}$$

$$\mathbb{R}^T = Col(X) \oplus Col(X)^\perp$$

$$P_X = X(X'X)^{-1}X'$$

$w \in Col(X)$:
$$\begin{aligned} P_X w &= X(X'X)^{-1}X'w \\ &= X(X'X)^{-1}X'X\alpha \\ &= X\alpha \\ &= w \end{aligned}$$

$v \in Col(X)^\perp$:
$$\begin{aligned} P_X v &= X(X'X)^{-1}\underbrace{X'v}_{=0} \\ &= 0 \end{aligned}$$