BIRKBECK

(University of London)

MSc EXAMINATION FOR INTERNAL STUDENTS

Department of Economics, Mathematics, and Statistics

ECONOMETRICS - PART II

EMEC026S7

Monday, 04 June 2018, 2:30 - 4:40 pm (includes 10 minutes reading time)

Answer **ANY TWO** questions. All questions carry the same weight; the relative weight of sub-questions is indicated in square brackets Any results used in lectures can be used without proof, but needs to be properly stated.

---

1. Consider the structural equation

$$y_i = \beta x_i^2 + \epsilon_i,$$

and the reduced form equation

$$x_i = \gamma z_i + u_i,$$

where y_i, x_i and z_i are scalar random variables and where, with probability one, $\mathbb{E}\left[\epsilon_i | x_i\right] \neq 0$ and $\mathbb{E}[u_i|z_i] = \mathbb{E}[\epsilon_i|z_i] = 0$.

   (a) [25%] A researcher wants to estimate the structural parameter β and proceeds as follows. The researcher first regresses x_i on z_i using OLS and retrieves the fitted values $\hat{x}_i = \hat{\gamma}_N z_i$; the researcher then estimates β by means of an OLS regression of y_i onto $\hat{x}_i^2$. Provide an expression of the resulting estimator $\hat{\beta}_N$ of β as a function of the sample $\{(y_i, x_i, z_i), i = 1, \cdots, N\}$.

   (b) [25%] Derive the probability limit of $\hat{\beta}_N$ and show that, in general, this estimator is inconsistent for β.

   (c) [25%] Are there any further conditions that do not violate the assumptions made so far that would render $\hat{\beta}_N$ consistent for β?

   (d) [25%] Derive a consistent estimator for β. Be sure to demonstrate its consistency and to state any additional assumptions you may need.

2. Let Y_i denote an outcome measure for individual i, and let D_i be a binary treatment indicator that takes value 1 if i is receives the treatment and 0 otherwise. You are interested in estimating the average treatment effect. A vector of additional covariates $\mathbf{X}_i$ is also observed.

   (a) [25%] Suppose you maintain the assumption that Y_i and D_i are independent, conditional on $\mathbf{X}_i$. Show that the OLS estimator of δ in the regression

   $$Y_i = \alpha + \delta D_i + \mathbf{X}_i'\theta + \epsilon_i$$

   is a consistent estimator of the average treatment effect.

   (b) [25%] Suppose that the assumption in (a) is wrong and that instead Y_i and D_i are independent, conditional on $\mathbf{Z}_i' = [\mathbf{X}_i', W_i]$, i.e. the assumption requires to also condition on the additional covariate W_i. Suppose, however, that W_i is not observed. Show that under these circumstances the OLS estimator in (a) is inconsistent for the average treatment effect.

   (c) [25%] Consider the propensity score $p(\mathbf{X}_i) = \Pr(D_i = 1|\mathbf{X}_i)$ based on the covariates in (a). Under the assumptions about the true model in (b), show that Y_i and D_i are not independent, conditional on $p(\mathbf{X}_i)$.

   (d) [25%] Discuss how a consistent estimator of the average treatment effect can be obtained under the assumptions in (b). You are free to make additional assumptions, if needed.

3. In the analysis of labor mobility, researchers have employed models for the wage y_{it} of worker i in period t employed by firm j in that period of the following form:

$$y_{it} = \mathbf{x}'_{it}\beta + \alpha_i + \gamma_{j(i,t)} + \epsilon_{it},$$

where $\mathbf{x}_{it}$ are relevant time varying characteristics of the worker (such as experience), α_i is a consumer level time-invariant effect (such as innate ability), and $\gamma_{j(i,t)}$ is a firm-specific effect if worker i was employed by firm j in period t, where $j = 1, \cdots, J$ and J can be very large. The regression errors ϵ_{it} are assumed to be i.i.d. across i and t. The primary interest lies in estimating the vector β.

(a) [25%] Explain why standard fixed-effects panel methods are not applicable in this model.

(b) [25%] Stacking up the equations (3), the model can be written in matrix form as

$$\mathbf{y} = \mathbf{X}\beta + W\alpha + D\beta + \epsilon.$$

Let $\mathbf{Z} = [\mathbf{W} : \mathbf{D}]$, i.e. the matrix of $\mathbf{W}$ next to the matrix $\mathbf{D}$. Suppose, irrespective of the large number of columns in $\mathbf{Z}$, one can construct the matrix $P_Z = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$, and $M_Z = \mathbf{I} - P_z$. Provide an interpretation of $\hat{\beta} = \left(\mathbf{X}'M_z\mathbf{X}\right)^{-1}\mathbf{X}'M_z\mathbf{y}$.

(c) [25%] Suppose one considers the above model under the random effects assumption. Explain the pros and cons of this approach.

(d) [25%] The i.i.d. assumption on the regression errors ϵ_{ijt} may not hold. Someone suggests a test based on the statistic $\mathcal{T} = \frac{\hat{\epsilon}'P_Z\hat{\epsilon}}{\hat{\sigma}^2}$, where $\hat{\epsilon}$ is the vector of residuals of the regression in (b) and $\hat{\sigma}^2$ an estimate of their variance. Discuss this test statistic and its merits.