Econometrics II - Class Exercises

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Week 1: No class.

Week 2: Review: Linear and Partitioned Regression; Law of Iterated Expectations; Kronecker Products.

Week 3: Consider the simultaneous equations model:

$$\begin{aligned} y_{1t} &= \gamma_{10} + \beta_{12} y_{2t} + u_{1t}, \\ y_{2t} &= \gamma_{20} + \beta_{21} y_{1t} + \gamma_{21} x_{1t} + u_{2t}, \end{aligned}$$

where y_{1t} and y_{2t} are endogenous variables, x_{1t} is an exogenous variable, and u_{1t} and u_{2t} are serially uncorrelated and homoscedastic disturbances with zero mean.

(a) Assess the identifiability of the parameters that appear as coefficients in the model in terms of order and rank conditions.

(b) Derive the reduced-form equations for y_{1t} and y_{2t}.

(c) Show that the indirect least-squares and two-stage least-squares estimators of β_{12} are equivalent.

Week 4: Consider the following recursive system:

$$\begin{aligned} y_{1t} + \gamma_{11} x_{1t} + \gamma_{12} x_{2t} &= u_{1t}, \\ \beta_{21} y_{1t} + y_{2t} + \gamma_{23} x_{3t} &= u_{2t}, \end{aligned}$$

where y_{1t} and y_{2t} are endogenous variables, x_{1t} , x_{2t} and x_{3t} are exogenous variables, and u_{1t} and u_{2t} are serially uncorrelated and homoscedastic zero-mean disturbances such that $\mathrm{E}(u_{1t}u_{2t}) = 0$. (A linear system $\mathbf{B}\mathbf{y}_t + \mathbf{\Gamma}\mathbf{x}_t = \mathbf{u}_t$ is called recursive if $\mathbf{B}$ is triangular and the variance–covariance matrix of $\mathbf{u}_t$ is diagonal).

(a) Assess the identifiability of the two equations of the system in terms of the order and rank conditions for identification.

(b) Derive the reduced-form equations for y_{1t} and y_{2t}.

(c) Show that the ordinary least squares estimates of the coefficients of each equation of the system are consistent.

Week 5: Exam 2011, question 5.

Week 6: Spring Term Reading Week.

Week 7: 1. Consider the difference-in-means estimator $\bar{d} = \bar{Y}_1 - \bar{Y}_0$, where $\bar{y}_j = \frac{\sum_i Y_i 1_{\{D_i=j\}}}{\sum_i 1_{\{D_i=j\}}}$, $j \in \{0, 1\}$.

(a) Show that $\bar{d}$ is a biased estimator of the average treatment effect of the treated (ATT), and derive an expression for the bias.

(b) Suppose $D_i = 1$ ($D_i = 0$) if individual i participates (does not participate) in a job training program. Let Y_{0i} denote an individual's potential earnings in the absence of the job training. In this setting, would one expect $\mathbb{E}[Y_{i0}|D_i = 1)] > \mathbb{E}[Y_{0i}|D_i = 0]$, or the opposite inequality? Explain your reasoning.

(c) Explain the direction of the bias of $\bar{d}$ for the ATT in the setting of (b).

2. Suppose Y_i is student i's test score; $D_i = 1$ if student i attended a Catholic high school, and zero otherwise; and $Z_i = 1$ if student i is catholic, $i = 1, \cdots, N$. The objective is to estimate the effect on test score of attending a Catholic high school for students who are Catholic and therefore chose to study at a Catholic high school.

(a) What constitutes the compliant group in this case? Who are the defiers?

(b) Under what condition is the local average treatment effect (LATE) for the compliant group identified?

(c) Derive a consistent estimator for the LATE.

(d) Show that the estimator in (c) is identical to the IV estimator in the model

$$Y_i = \alpha + \beta D_i + \epsilon_i,$$

where D_i is instrumented by Z_i, $i = 1, \cdots, N$.

Week 8: Exam 2011, question 6.

Week 9: Let $\{X_1, \ldots, X_n\}$ be a random sample from the $N(\mu, \sigma^2)$ distribution.

(a) Write down the moment conditions for the first four moments of the sample.

(b) Explain how you would estimate μ and σ^2 from the moment conditions in (a).

(c) Construct a chi-squared specification test for these moment conditions. How many degrees of freedom does the test have? What do you conclude if the test rejects?

Week 10:

1. Consider the following panel data settings.

   (a) Suppose that panel data $\{y_{it}, x_{it}, i = 1, \ldots, N, t = 1, 2\}$ are observed; note that each individual i is observed in only two periods $t = 1, 2$, and that y_{it} and x_{it} are both scalar random variables. Show that in the model

   $$y_{it} = \alpha + \beta x_{it} + a_i + \epsilon_{it}$$

   the OLS estimator of β using first-differenced data is identical to the 'within' estimator.

   (b) Suppose that, instead of the previous data and model, panel data for more than 3 periods are observed, and the model considered is

   $$y_{it} = \alpha + \beta x_{it} + a_i + \delta_i t + \epsilon_{it},$$

   i.e. it includes a time trend which may affect different people in different ways. Discuss problems using OLS with first-differenced data and suggest alternatives.

2. Consider panel data $\{(y_{it}, x'_{it}), i = 1, \ldots, N, t_i \in \mathcal{T}_i\}$, where $\mathcal{T}_i$, $i = 1, \ldots, N$, are the individual specific time periods for which data on individual i are available. Suppose the sets $\mathcal{T}_i$ differ across i. Adopt the convention that $\mathcal{T}_i = \{1, \ldots, T_i\}$. Then this is an instance of unbalanced panel data. Suppose you are interested in the coefficient β_0 in the standard individual-specific effects model

   $$y_{it} = \alpha_i + \mathbf{x}'_{it}\beta_0 + \epsilon_{it}, \text{ for } i = 1, \ldots, N \text{ and } t \in \mathcal{T}_i.$$

(a) Describe the fixed-effects (least squares dummy variable) estimator for such cases.

(b) Describe the random-effects (GLS) estimator for such cases, assuming the variance-covariance parameters $\text{var}(\mathbf{y}_i|\mathbf{x}_i) = \sigma^2_{0\alpha}\iota_{T_i}\iota'_{T_i} + \sigma^2_{0\epsilon}\mathbf{I}$ (with no correlation across i) are known; here, $\iota_{T_i} = (1, \ldots, 1)'$ is a $T_i \times 1$ vector, $\mathbf{I}$ is a $T_i \times T_i$ identity matrix, $\mathbf{y}_i = (y_{i1}, \ldots, y_{iT_i})'$ and $\mathbf{x}_i = [\mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT_i}]'$.

(c) Describe a feasible random-effects estimator for this case.

Week 11: Review, Q&A