

Gossamer Superconductivity

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An new superconducting hamiltonian is introduced for which the exact ground state is the Anderson resonating valence bond. It differs from the t-J and hubbard hamiltonians in possessing a powerful attractive force. Its superconducting state is characterized by a full and intact d-wave tunneling gap, quasiparticle photoemission intensities that are strongly suppressed, a suppressed superfluid density, and an incipient Mott-Hubbard gap. [Published as Phil. Mag. 86, 1165 (2006).]

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It has been known since the early work of Uemura¹ that the superfluid density and transition temperature of underdoped cuprate superconductors both vanish more-or-less linearly with doping and are proportional. The constant of proportionality is consistent with the transition being an order-parameter phase instability analogous to the Kosterlitz-Thouless transition in 2 dimensions³. This idea is supported by numerous other experiments, including the optical sum rule studies of Uchida², the giant proximity effect reported by Decca *et al*⁴, and the recent heat-transport measurements of Wang *et al*⁵ showing superconducting vortex-like effects above the transition temperature. The transport trends continue into the insulator, where Ando⁶ reports that high-temperature hall effect consistent with a carrier density roughly proportional to doping. At the same time, however, the d-wave gap in the quasiparticle spectrum grows monotonically as the doping decreases and saturates at a value of about 0.3 eV⁷⁻¹⁰. This has led to speculations that the tunneling pseudogap is the energy to make a pre-formed Cooper pair, which then condenses into a superfluid at a lower temperature. There is no direct evidence that the d-wave nodal structure survives into the insulator, but there is circumstantial evidence for this, notably the observation in La_{2-x}Sr_xCuO₄ by Yoshida *et al*¹¹ of dispersing quasiparticle bands near the d-wave node that become fainter as doping is reduced but do not shift or change their velocity scale. These bands are also detached from the lower Hubbard band, and simply materialize in mid-gap as the doping is increased from zero.

All of this behavior is consistent with the idea that the superconductivity persists deep into the “insulating” state, coexists with antiferromagnetism there, and fails to conduct only because its long-range order is disrupted, presumably on account of its low superfluid density¹². There are many ways the latter could occur, including impurity localization or crystallization of the order parameter, for such a *gossamer superconductor* is physically equivalent to a dilute gas of bosons and thus highly unstable.

The idea that the “insulator” might actually be a thin, ghostly superconductor is implicit in the mathematics of the Anderson resonating valence bond (RVB)¹³ worked out by various authors in the late 1980s¹⁴⁻¹⁶ and further extended recently by Paramahanti, Randeria, and

Trivedi¹⁷. Unfortunately, this idea has always run afoul of a basic premise of RVB theory that superconductivity should be a universal aspect of quantum antiferromagnetism. This premise is both confusing and fundamentally incorrect, as the conventional spin density wave ground state, which contains no superconductivity, is a perfectly good prototype for a quantum antiferromagnet. The real issue is not whether all antiferromagnets are superconductors but whether some of them are - *i.e.* whether there exists a second kind of antiferromagnetism distinguished from the first by a tiny background superfluid density. One would also like to know which hamiltonians favor this second kind of state over the first. It is not just the hamiltonians that stabilize antiferromagnetism, for these simply emphasize the aspects of the vacua that are the same and de-emphasize the aspects that are different. It has been known since the early work of Hsu¹⁸, for example, that these two kinds of vacuum have almost identical variational energies in the context of the t-J hamiltonian. This is consistent with the recent numerical-variational studies of Sorella *et al*¹⁹, who report that the t-J model superconducts in a region of its parameter space, even though previous numerical work on the same model reported antiferromagnetic stripe ordering²⁰. Becca *et al*²¹ have argued that the latter is an artifact lattice anisotropy. However the important point is that sensitivity to algorithmic detail and the inherent difficulty of determining the order, reflected in lack of agreement among different groups, demonstrate that models of this kind are highly conflicted and close to a quantum phase transition²². In other words, by exaggerating the magnetism these models confuse the issue rather than clarifying it. There is no persuasive evidence for superconductivity in the hubbard model²³⁻²⁶.

The purpose of this letter is to propose a new strategy for resolving the cuprate dilemma. Rather than struggle to diagonalize a conflicted hamiltonian, we shall modify the equations of motion to stabilize the gossamer superconductor. The easiest way to do this is by postulating a powerful attractive force between electrons, just as one would in any other superconductor. This solution is not unique, but it is experimentally falsifiable through the excitation spectrum of the state, which is model-dependent. Insofar as these properties match experiment, which has yet to be seen, it would suggest that coulomb interactions

are not sufficient to explain cuprate superconductivity.

We consider a planar square lattice of sites j on which electrons may sit. The superconducting vacuum we wish to stabilize is $|\Psi\rangle = \Pi_\alpha |\Phi\rangle$, where

$$|\Phi\rangle = \prod_{\mathbf{k}}^N (u_{\mathbf{k}} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger) |0\rangle, \quad (1)$$

with $c_{\mathbf{k}\sigma} = N^{-1/2} \sum_j^N \exp(i\mathbf{k} \cdot \mathbf{r}_j) c_{j\sigma}$ as usual, and

$$\Pi_\alpha = \prod_j^N z_0^{(n_{j\uparrow} + n_{j\downarrow})/2} (1 - \alpha_0 n_{j\uparrow} n_{j\downarrow}) \quad (2)$$

The Bardeen-Cooper-Schrieffer (BCS) pairing amplitudes satisfy

$$\begin{bmatrix} \epsilon_{\mathbf{k}} - \mu & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}} & \mu - \epsilon_{\mathbf{k}} \end{bmatrix} \begin{bmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{bmatrix} = E_{\mathbf{k}} \begin{bmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{bmatrix} \quad (3)$$

and are normalized by $u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 = 1$. They are related to the hole doping δ by

$$\frac{1}{N} \sum_{\mathbf{k}} v_{\mathbf{k}}^2 = 1 - \frac{1}{N} \sum_{\mathbf{k}} u_{\mathbf{k}}^2 = \frac{1 - \delta}{2} \quad (4)$$

The important positive eigenvalue

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta_{\mathbf{k}}^2} \quad (5)$$

is the energy to make a quasiparticle (either an electron or a hole) when the parameter α_0 is zero. The parameter z_0 is a fugacity required to keep the electron density the same as one varies α_0 . Assuming the superconducting order parameter to be d-wave, so there is no on-site pairing amplitude, the charge states of a site are statistically independent and characterized by a fugacity z . The condition that the total charge on the site be $1 - \delta$, where δ is the doping, is $[2z + 2(1 - \alpha)z^2] / [1 + 2z + (1 - \alpha)z^2] = 1 - \delta$, where $1 - \alpha = (1 - \alpha_0)^2$, or

$$z = \frac{\sqrt{1 - \alpha(1 - \delta^2)} - \delta}{(1 - \alpha)(1 + \delta)} = \left(\frac{1 - \delta}{1 + \delta} \right) z_0 \quad (6)$$

The parameter z_0 is the factor by which z exceeds $(1 - \delta)/(1 + \delta)$, its value for $\alpha_0 = 0$.

The hamiltonian is constructed using the BCS annihilation operators

$$b_{\mathbf{k}\uparrow} = u_{\mathbf{k}} c_{\mathbf{k}\uparrow} + v_{\mathbf{k}} c_{-\mathbf{k}\downarrow}^\dagger \quad b_{\mathbf{k}\downarrow} = u_{\mathbf{k}} c_{\mathbf{k}\downarrow} - v_{\mathbf{k}} c_{-\mathbf{k}\uparrow}^\dagger \quad (7)$$

So long as $\alpha_0 \neq 1$, the partial projector Π_α has an inverse

$$\Pi_\alpha^{-1} = \prod_j^N z_0^{-(n_{j\uparrow} + n_{j\downarrow})/2} (1 + \beta_0 n_{j\uparrow} n_{j\downarrow}) \quad (8)$$

where $\beta_0 = \alpha_0/(1 - \alpha_0)$. This enables us to construct the modified annihilation operators

$$\tilde{b}_{\mathbf{k}\uparrow} = \Pi_\alpha b_{\mathbf{k}\uparrow} \Pi_\alpha^{-1} = \frac{1}{\sqrt{N}} \sum_j^N e^{i\mathbf{k} \cdot \mathbf{r}_j} \times \left[z_0^{-1/2} u_{\mathbf{k}} (1 + \beta_0 n_{j\downarrow}) c_{j\uparrow} + z_0^{1/2} v_{\mathbf{k}} (1 - \alpha_0 n_{j\uparrow}) c_{j\downarrow}^\dagger \right] \quad (9)$$

and likewise for $\tilde{b}_{\mathbf{k}\downarrow}$, for which $\tilde{b}_{\mathbf{k}\sigma} |\Psi\rangle = 0$. Thus $|\Psi\rangle$ is an eigenstate of the hamiltonian

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} \tilde{b}_{\mathbf{k}\sigma}^\dagger \tilde{b}_{\mathbf{k}\sigma} \quad (10)$$

with eigenvalue 0. However, this hamiltonian has only non-negative eigenvalues, since for any wavefunction $|\chi\rangle$

$$\langle \chi | \mathcal{H} | \chi \rangle = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} \langle \tilde{b}_{\mathbf{k}\sigma} \chi | \tilde{b}_{\mathbf{k}\sigma} \chi \rangle \geq 0 \quad (11)$$

Thus $|\Psi\rangle$ is a ground state of \mathcal{H} . However, it is also *the* ground state by virtue of adiabatic continuity. The state in question may be continuously deformed into a BCS state by taking α_0 slowly to zero. Since it does not cross a phase boundary in the process, the ground state and low-lying excitations must track in a one-to-one way.

Let us now consider the quasiparticle excitations of this superconductor. The operators $\tilde{b}_{\mathbf{k}\sigma}$ no longer anticommute properly with their hermitian adjoints and thus cannot be used to create quasiparticles. The physical meaning of this is that the quasiparticles interact. Instead we will use the variational wavefunctions $|\mathbf{k}\sigma\rangle = \Pi_\alpha b_{\mathbf{k}\sigma}^\dagger |\Phi\rangle$ borrowed from the RVB literature. The expected energy is

$$\frac{\langle \mathbf{k}\sigma | \mathcal{H} | \mathbf{k}\sigma \rangle}{\langle \mathbf{k}\sigma | \mathbf{k}\sigma \rangle} = E_{\mathbf{k}} \frac{\langle \Psi | \Psi \rangle}{\langle \mathbf{k}\sigma | \mathbf{k}\sigma \rangle} \simeq E_{\mathbf{k}} \quad (12)$$

The last step requires evaluating the relevant norms, which can be done by hand only approximately^{17,18}. We repeat the arguments here for completeness. From

$$b_{\mathbf{k}\uparrow}^\dagger |\Phi\rangle = \frac{1}{u_{\mathbf{k}}} c_{\mathbf{k}\uparrow}^\dagger |\Phi\rangle = \frac{1}{v_{\mathbf{k}}} c_{-\mathbf{k}\downarrow} |\Phi\rangle \quad (13)$$

we find that

$$\frac{1}{N} \sum_{\mathbf{k}} u_{\mathbf{k}}^2 \frac{\langle \Phi | b_{\mathbf{k}\uparrow} \Pi_\alpha^2 b_{\mathbf{k}\uparrow}^\dagger | \Phi \rangle}{\langle \Phi | \Pi_\alpha^2 | \Phi \rangle} = \frac{\langle \Phi | c_{j\uparrow} \Pi_\alpha^2 c_{j\uparrow}^\dagger | \Phi \rangle}{\langle \Phi | \Pi_\alpha^2 | \Phi \rangle}$$

$$= z_0 \left[\frac{1 + (1 - \alpha)z}{1 + 2z + (1 - \alpha)z^2} \right] = \frac{1 + \delta}{2} \quad (14)$$

and

$$\begin{aligned} \frac{1}{N} \sum_{\mathbf{k}} v_{\mathbf{k}}^2 \frac{\langle \Phi | b_{\mathbf{k}\uparrow} \Pi_{\alpha}^2 b_{\mathbf{k}\uparrow}^{\dagger} | \Phi \rangle}{\langle \Phi | \Pi_{\alpha}^2 | \Phi \rangle} &= \frac{\langle \Phi | c_{j\uparrow}^{\dagger} \Pi_{\alpha}^2 c_{j\uparrow} | \Phi \rangle}{\langle \Phi | \Pi_{\alpha}^2 | \Phi \rangle} \\ &= \frac{1}{z_0} \left[\frac{z + z^2}{1 + 2z + (1 - \alpha)z^2} \right] = \frac{1 - \delta}{2} . \end{aligned} \quad (15)$$

For $j \neq j'$ we assume that the amplitude for a given configuration is weighted by the square root of its corresponding probability, and that these weights add equally. We then have

$$\begin{aligned} \frac{\langle \Phi | c_{j\uparrow} \Pi_{\alpha}^2 c_{j'\uparrow}^{\dagger} | \Phi \rangle}{\langle \Phi | \Pi_{\alpha}^2 | \Phi \rangle} \times \frac{\langle \Phi | \Phi \rangle}{\langle \Phi | c_{j\uparrow} c_{j'\uparrow}^{\dagger} | \Phi \rangle} \\ \simeq \frac{4}{1 - \delta^2} (z z_0) \left[\frac{1 + (1 - \alpha)z}{1 + 2z + (1 - \alpha)z^2} \right]^2 = 1 \end{aligned} \quad (16)$$

and

$$\begin{aligned} \frac{\langle \Phi | c_{j\uparrow}^{\dagger} \Pi_{\alpha}^2 c_{j'\uparrow} | \Phi \rangle}{\langle \Phi | \Pi_{\alpha}^2 | \Phi \rangle} \times \frac{\langle \Phi | \Phi \rangle}{\langle \Phi | c_{j\uparrow}^{\dagger} c_{j'\uparrow} | \Phi \rangle} \\ \simeq \frac{4}{1 - \delta^2} \left(\frac{z}{z_0} \right) \left[\frac{1 + z}{1 + 2z + (1 - \alpha)z^2} \right]^2 = 1 . \end{aligned} \quad (17)$$

Let us now consider the low-energy spectroscopic properties of this model. Reasoning as above, we find the matrix element for photoemission to be

$$\frac{\langle -\mathbf{k} \downarrow | c_{\mathbf{k}\uparrow} | \Psi \rangle}{\sqrt{\langle -\mathbf{k} \downarrow | -\mathbf{k} \downarrow \rangle \langle \Psi | \Psi \rangle}} = g v_{\mathbf{k}} , \quad (18)$$

where

$$g^2 \simeq \frac{2\alpha_0}{\alpha} \left\{ 1 - \frac{\alpha_0}{\alpha} \left[\frac{1 - \sqrt{1 - \alpha(1 - \delta^2)}}{1 - \delta^2} \right] \right\} . \quad (19)$$

Inverse photoemission is the same except with $u_{\mathbf{k}}$ substituted for $v_{\mathbf{k}}$. The suppression of the photoemission intensity is matched by a similar suppression of the superfluid order parameter:

$$\frac{\langle \Psi | c_{j\uparrow} c_{j'\downarrow} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \simeq g^2 \frac{\langle \Phi | c_{j\uparrow} c_{j'\downarrow} | \Phi \rangle}{\langle \Phi | \Phi \rangle} . \quad (20)$$

We need not consider the case of $j = j'$ for a d-wave superconductor. Thus under strong projection near half-filling this model exhibits the pseudogap phenomenon:

The quasiparticle energies remain at their unperturbed values $E_{\mathbf{k}}$ as α_0 increases from 0 to 1, but the superfluid density decreases from 1 to $2|\delta|/(1 + |\delta|)$.

Let us now consider the formation of the Mott-Hubbard gap. Photoemission of a quasiparticle accounts for only a small fraction of the sum rule

$$\frac{\langle \Psi | c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \simeq g^2 v_{\mathbf{k}}^2 + (1 - g^2) \frac{1 - \delta}{2} . \quad (21)$$

The rest must occur at a higher energy scale, the value of which may be estimated by computing the expected energy of a hole. From the anticommutators

$$\{\tilde{b}_{\mathbf{k}\uparrow}, c_{j\uparrow}\} = \frac{1}{\sqrt{N}} e^{i\mathbf{k} \cdot \mathbf{r}_j} \left[-z_0^{1/2} \alpha_0 v_{\mathbf{k}} c_{j\uparrow} c_{j\downarrow}^{\dagger} \right] \quad (22)$$

$$\begin{aligned} \{\tilde{b}_{\mathbf{k}\downarrow}, c_{j\uparrow}\} &= \frac{1}{\sqrt{N}} e^{i\mathbf{k} \cdot \mathbf{r}_j} \left[-z_0^{-1/2} \beta_0 u_{\mathbf{k}} c_{j\uparrow} c_{j\downarrow} \right. \\ &\quad \left. + z_0^{1/2} v_{\mathbf{k}} (1 - \alpha_0 n_{j\downarrow}) \right] \end{aligned} \quad (23)$$

we obtain

$$\begin{aligned} \langle \Psi | c_{\mathbf{k}\sigma}^{\dagger} \mathcal{H} c_{\mathbf{k}\sigma} | \Psi \rangle &= z_0 \left[1 - \alpha_0 \frac{1 - \delta}{2} \right]^2 v_{\mathbf{k}}^2 E_{\mathbf{k}} \\ &+ \frac{1}{N} \sum_{\mathbf{q}} E_{\mathbf{k}+\mathbf{q}} \left[z_0 \alpha_0^2 A_{\mathbf{q}} v_{\mathbf{k}+\mathbf{q}}^2 + \frac{\beta_0^2}{\alpha_0} B_{\mathbf{q}} u_{\mathbf{k}+\mathbf{q}}^2 \right] , \end{aligned} \quad (24)$$

where

$$\begin{aligned} A_{\mathbf{q}} &= \sum_j \left[\frac{\langle \Psi | \mathbf{S}_0 \cdot \mathbf{S}_j | \Psi \rangle}{\langle \Psi | \Psi \rangle} \right. \\ &\quad \left. + \frac{1}{4} \frac{\langle \Psi | n_0 n_j | \Psi \rangle}{\langle \Psi | \Psi \rangle} - \frac{(1 - \delta)^2}{4} \right] e^{i\mathbf{q} \cdot \mathbf{r}_j} \end{aligned} \quad (25)$$

$$B_{\mathbf{q}} = \sum_j \frac{\langle \Psi | c_{0\uparrow}^{\dagger} c_{0\downarrow}^{\dagger} c_{j\downarrow} c_{j\uparrow} | \Psi \rangle}{\langle \Psi | \Psi \rangle} e^{i\mathbf{q} \cdot \mathbf{r}_j} . \quad (26)$$

Proceeding similarly with the state $c_{\mathbf{k}\uparrow}^{\dagger} | \Psi \rangle$, we find that the energy to inject an electron is the exact particle-hole conjugate of this expression, produced from it by interchanging $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$, negating δ , and substituting of $1 - n_j$ for n_j . Thus at half-filling the electron spectral function is symmetric. The density and superfluid correlations become suppressed at half-filling in the $\alpha_0 \rightarrow 1$ limit, while the magnetic correlations become enhanced. Approximating the latter by the correlation function of the Néel vacuum, we obtain at $\delta = 0$

$$\lim_{\alpha_0 \rightarrow 1} \frac{\langle \Psi | c_{\mathbf{k}\sigma}^\dagger \mathcal{H} c_{\mathbf{k}\sigma} | \Psi \rangle}{\langle \Psi | c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} | \Psi \rangle} = \lim_{\alpha_0 \rightarrow 1} \frac{\langle \Psi | c_{\mathbf{k}\sigma} \mathcal{H} c_{\mathbf{k}\sigma}^\dagger | \Psi \rangle}{\langle \Psi | c_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}^\dagger | \Psi \rangle}$$

$$\simeq \frac{1}{1 - \alpha_0} \left[\frac{1}{N} \sum_{\mathbf{q}} E_{\mathbf{q}} + \frac{1}{2} E_{\mathbf{k}} \right]. \quad (27)$$

Thus the spectral function consists of Mott-Hubbard “lobes” at high energies with a faint band of states at mid-gap associated with the gossamer quasiparticles. The chemical potential does not jump in this model when δ is tuned from negative to positive, as it does in the Hubbard model²⁶.

Let us now consider antiferromagnetism. At half-filling the largest term in Eq. (10) is an on-site coulomb repulsion of magnitude $2 \sum_{\mathbf{k}} E_{\mathbf{k}} / N(1 - \alpha_0)$. If this repulsion is made slightly larger, the system becomes unstable to spin density wave formation on top of the superconductivity at the nesting wavevector of the d-wave nodes. The case of half-filling has been worked out in detail by Hsu¹⁸ and we only quote the result here. The quasiparticle dispersion relation Eq. (5) becomes modified to $E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta_{\mathbf{k}}^2 + \Delta_0^2}$, where the energy gap Δ_0 is related to the site magnetization $\langle \Psi | S_j^z | \Psi \rangle / \langle \Psi | \Psi \rangle = \pm m$ by

$$m \simeq \frac{m_0}{1 - \alpha_0(1 - 4m_0^2)} \quad m_0 = \Delta_0 / \frac{2}{N} \sum_{\mathbf{k}} E_{\mathbf{k}}. \quad (28)$$

It was observed by Hsu¹⁸ that strong projection enhances the magnetization by roughly a factor of 2 over its unprojected value, enabling the Heisenberg model value of $m = 0.307$ to be achieved with a rather modest value of Δ_0 , about 1/3 the zone average of $E_{\mathbf{k}}$. In contrast to the case worked out by him, however, the gap here has physical meaning and can be detected in a tunneling or photoemission experiment. This small quasiparticle gap is a key characteristic of the gossamer superconductor.

Phase fluctuations have been left out of Eq. (10) on the grounds that they are irrelevant to the fermi spectrum, which is characterized by an energy scale much higher than the superconducting T_c . However, they are essential for accounting for both the Uemura plot and strange-metal transport above T_c . The transport of a dilute gas of bosons formed when the order parameter dephases would not exhibit any traditional metallic behavior and indeed would tend to “short out” conduction by the fermions. Lattice-mediated crystallization of a gossamer superconductor would also provide a ready explanation for why the static stripes²⁷ are observed at $\delta = 1/8$ in some cuprates and not others, why the stripe commensuration wavevector is so peculiar, why stripes can be destroyed by moderate pressure²⁸, and why the materials insulate when subjected to strong magnetic fields²⁹.

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