CSCE 489: Machine Learning (Spring 2019)

Homework #1
Due 2/1/2019

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1. You need to submit a report in hard-copy before lecture and your code to eCampus. Your hard-copy report should include (1) answers to the non-programming part, and (2) analysis and results of the programming part. Your submission to eCampus should be your code files ONLY. Please put all your code files into a compressed file named "HW#_FirstName_LastName.zip"
2. Hard-copy is due in class before lecture, and code files are due 9:10AM on eCampus on the due date.
3. Unlimited number of submissions are allowed on eCampus and the latest one will be graded.
4. LFD refers to the textbook "Learning from Data".
5. Please read and follow submission instructions. No exception will be made to accommodate incorrectly submitted files/reports.
6. All students are highly encouraged to typeset their reports using Word or latex. In case you decide to hand-write, please make sure your answers are clearly readable.

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1. (10 points) Problem 1.2 in LFD (page 33).
2. (25 points) **Perceptron for Handwritten Digits Recognition**: The handwritten digits files are in the "data" folder: train.txt and test.txt. The starting code is in the "code" folder. In the data file, each row is a data example. The first entry is the digit label ("1" or "5"), and the next 256 are grayscale values between -1 and 1. The 256 pixels correspond to a 16×16 image. You are expected to implement your solution based on the given codes. The only file you need to modify is the "solution.py" file. You can test your solution by running "main.py" file. Note that code is provided to compute a two-dimensional feature (symmetry and average intensity) from each digit image; that is, each digit image is represented by a two-dimensional vector before being augmented with a "1" to form a three-dimensional vector as discussed in class. These features along with the corresponding labels should serve as inputs to your Perceptron algorithm.

   (a) (5 points) Familiarize yourself with the data by completing the *show_images* function. Include the images you plotted in your report.

   (b) (5 points) In this assignment, we have already extracted two features, symmetry and average intensity, to distinguish between 1 and 5. Familiarize yourself with the features by completing the *show_features* function and include the 2-D scatter plot into your report. For each sample, plot the two features with a red $*$ if the label is 1 and a blue $+$ if the label is 5.

   (c) (10 points) Complete the *Perceptron* class. You can test your accuracy results using the "test_accuracy" function in "main.py".

   (d) (5 points) Complete the *show_result* function to plot the test data with the separators. Include the images you plotted into your report.

Deliverable: You should submit (1) a hard-copy report (along with your write-up for other questions) that summarizes your results and (2) the "solution.py" file to the Blackboard.

Note: Please read the "Readme.txt" file carefully before you start this assignment. Please do NOT change anything in the "main.py" and "helper.py" files when you program.

3. (10 points) Given $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, show that the rank of matrix xy^T is one.

4. (10 points) Given

$$X = [x_1, x_2, \cdots, x_n] \in \mathbb{R}^{m \times n}$$

where $x_i \in \mathbb{R}^m$ for all i, and

$$Y = \begin{bmatrix} y_1^T \\ y_2^T \\ \vdots \\ y_n^T \end{bmatrix} \in \mathbb{R}^{n \times p}$$

where $y_i \in \mathbb{R}^p$ for all i. Show that

$$XY = \sum_{i=1}^{n} x_i y_i^T.$$

5. (10 points) Given $X \in \mathbb{R}^{m \times n}$, show that the matrix $X^T X$ is symmetric and positive semi-definite. When is it positive definite?

6. (10 points) Given $g(x, y) = e^x + e^{y^2} + e^{3xy}$, compute $\frac{\partial g}{\partial y}$.

7. (25 points) Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix},$$

   (a) Compute the eigenvalues and corresponding eigenvectors of A. You are allowed to use Matlab to compute the eigenvectors (but not the eigenvalues).

   (b) What is the eigen-decomposition of A?

   (c) What is the rank of A?

   (d) Is A positive definite? Is A positive semi-definite?

   (e) Is A singular?