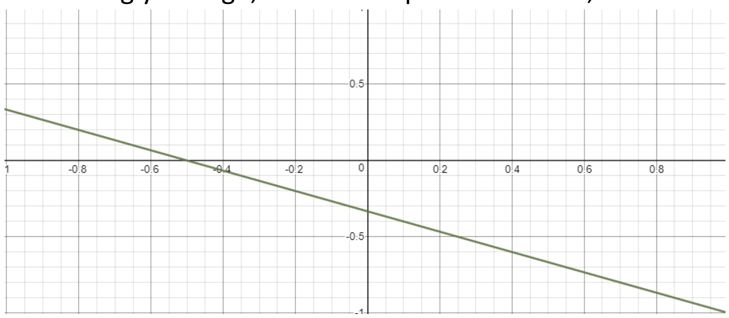
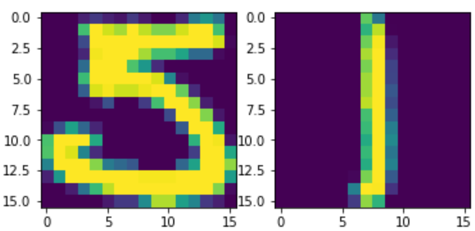
Nicholas Charchenko CSCE 489 Machine Learning HW1 Write-Up Due 2.1.2019

1. $w_0 + w_1x_1 + w_2x_2 = \mathbf{w}^T\mathbf{x}$ and we get the line $0 = w_0 + w_1x_1 + w_2x_2$. In slope-intercept form, this is $x_2 = -\frac{w_1}{w_2}x_1 - \frac{w_0}{w_2}$. In this case, $a = -\frac{w_1}{w_2}$ and $b = -\frac{w_0}{w_2}$.
For $w = [1,2,3]^T$, we get $x_2 = -\frac{2}{3}x_1 - \frac{1}{3}$ and for $w = -[1,2,3]^T$, we get $x_2 = -\left(\frac{-2}{-3}\right)x_1 - (\frac{-1}{-3})$.
Interestingly enough, these end up the same line, but we see that the separator is a line:



2.



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Data visualization done.
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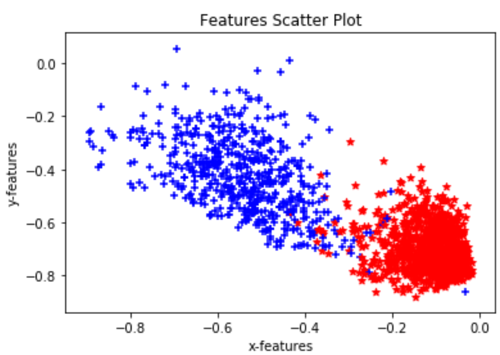


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Features visualization done.
Case 1: max iteration:10  train accuracy:0.980782  test accuracy: 0.959906.
Case 2: max iteration:30  train accuracy:0.976297  test accuracy: 0.948113.
Case 3: max iteration:50  train accuracy:0.969250  test accuracy: 0.938679.
Case 4: max iteration:100  train accuracy:0.971813  test accuracy: 0.941038.
Case 5: max iteration:200  train accuracy:0.977578  test accuracy: 0.948113.
Accuracy testing done.
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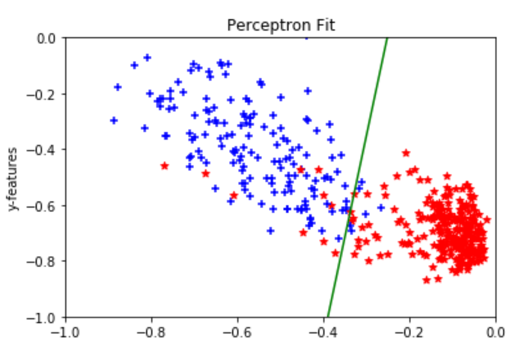


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Result visualization done.
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3. The matrix xy^T will look like $\begin{bmatrix} x_1y_1 & \cdots & x_1y_n \\ \vdots & \ddots & \vdots \\ x_my_1 & \cdots & x_my_n \end{bmatrix}$. We can clearly see that this matrix will only have rank one because all rows are a linear combination of the vector $[y_1, \dots, y_n]^T$, resulting in only one linearly independent vector.
4. $XY = x_1y_1^T + x_2y_2^T + \cdots + x_ny_n^T = \sum_{i=1}^{n} x_iy_i^T$ by the definition of matrix multiplication.
5. X^TX is symmetric if $X^TX = (X^TX)^T$. $(AB)^T = B^TA^T$. Therefore, $(X^TX)^T = X^TX$, proving symmetry. For X^TX to be positive semi-definite, then for any $x \in \mathbb{R}^n, x^TX^TXx \geq 0$. We can rewrite this as $(Xx)^TXx$, which is the square of a vector norm, which must be non-negative. Therefore, X^TX is positive semi-definite. To be positive semi-definite, this product must be strictly positive.
6. $\frac{\partial g}{\partial y} = 2ye^{y^2} + 3e^{3xy}x$
7. The characteristic polynomial is $-\lambda^3 + 8\lambda^2 - 3\lambda = -\lambda(\lambda^2 - 8\lambda + 3)$. We get eigenvalue 0. To get the remaining eigenvalues, we complete the square for $\lambda^2 - 8\lambda + 3 = 0$. Using the quadratic formula: $\lambda = \frac{8\pm\sqrt{64-12}}{2} = \frac{8\pm\sqrt{52}}{2} = 4 \pm \sqrt{13}$. The eigenvectors are $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} \sqrt{13}-3 \\ -\sqrt{13}+4 \\ 1 \end{pmatrix}, \begin{pmatrix} -3-\sqrt{13} \\ \sqrt{13}+4 \\ 1 \end{pmatrix}$.

   The eigen-decomposition is

   $$\begin{bmatrix} -1 & \sqrt{13}-3 & -3-\sqrt{13} \\ -1 & -\sqrt{13}+4 & \sqrt{13}+4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4+\sqrt{13} & 0 \\ 0 & 0 & 4-\sqrt{13} \end{bmatrix} \begin{bmatrix} -1/3 & -1/3 & 1/3 \\ \frac{5\sqrt{13}+13}{78} & \frac{13+2\sqrt{13}}{78} & \frac{7\sqrt{13}+26}{78} \\ -\frac{5\sqrt{13}-13}{78} & -\frac{2\sqrt{13}-13}{78} & \frac{26-7\sqrt{13}}{78} \end{bmatrix}$$

   The matrix is rank two because there are two non-zero rows in the reduced row echelon form of A: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

   This matrix is NOT positive definite, but it IS positive semi-definite because all eigenvalues are non-zero.

   The matrix determinant is equal to $2 * det\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} - det\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} + 3 * det\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} = 2 * 1 - 1 * (-1) + 3(-1) = 0$. Because we have a zero determinant, the matrix A **is singular**.