Name:

Homework 7
Due 20 Nov 2019

1. Show which displacements will be functions of z in a full solution of the plane stress problem by integrating the strain-displacement relations
2. Identify all nonzero compatibility relations for a full solution of the plane stress problem. What form must ϵ_{33} take to satisfy compatibility?
3. Explicitly check the validity of the plane strain/plane stress transformation relations given in Table 1 by transforming:

Table 1: Conversion between plane strain and plane stress

	E	ν
Plane stress to plane strain	$\frac{E}{1-\nu^2}$	$\frac{v}{1-\nu}$
Plane strain to plane stress	$\frac{E(1+2\nu)}{(1+\nu)^2}$	$\frac{v}{1+\nu}$

(a) Equation 1 from Plane Strain to Plane Stress

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + F_x = 0 \tag{1}$$

(b) Equation 2 from Plane Stress to Plane Strain

$$\mu\nabla^2 v + \frac{E}{2(1-\nu)}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + F_y = 0 \tag{2}$$

(c) Equation 3 from Plane Stress to Plane Strain

$$\nabla^2(\sigma_{xx} + \sigma_{yy}) = -(1+\nu)\left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\right) \tag{3}$$

4. The plane stress solution for pure bending is given by

$$u = -\frac{Mxy}{EI}$$
$$v = -\frac{M}{2EI}(\nu y^2 + x^2 - L^2)$$

Where $-L \le x \le L$.

Transform this result to plane strain and plot a comparison of the y-displacement (v) for the two solutions along the x-axis for various Poisson's ratios.

5. The plane strain radial displacement solution for a hole of radius R under uniform far-field loading, T, is

$$u_r = \frac{T(1+\nu)}{E}\left[(1-2\nu)r + \frac{R^2}{r}\right]$$

Transform this result to plane stress and plot the displacement versus $\frac{r}{R}$ for both solutions. Also plot the displacement along the hole ($r = R$) for varying Poisson's ratio. Comment on the results.