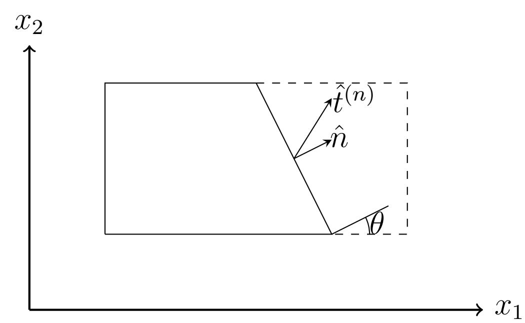
Name:

Homework 3

Due 2 Oct 2019

1. For the given states of stress, σ_{ij}, find the traction along an arbitrary interior plane as shown in the diagram.



(a)

$$\sigma_{ij} = \begin{bmatrix} k & \frac{k}{2} & 0 \\ \frac{k}{2} & 2k & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\sigma_{ij} = \begin{bmatrix} x_1 & \frac{1}{2}x_1x_3 & \frac{1}{2}x_1^2 \\ \frac{1}{2}x_1x_3 & x_3 & 0 \\ \frac{1}{2}x_1^2 & 0 & x_2 \end{bmatrix}$$

2. Find the principal stresses and maximum shear stress for a body in *plane stress*.

Note: In *plane stress*, $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$ which gives

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3. The plane stress solution for a semi-infinite elastic body subjected to a point load produces the following state of stress

$$\sigma_{ij} = \frac{-2P}{\pi(x^2+y^2)^2} \begin{bmatrix} x^2y & y^3 & 0 \\ y^3 & xy^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Calculate the maximum shear stress at any point in the body and plot contours of the result.

Hint: Let $x = A$ to find contours along the y-axis, then solve again for $y = B$ to find contours along the x-axis

4. Under what circumstances (if any) will the following stress fields be in static equilibrium?

   Note: When a stress component is not given, assume it is zero.

   (a)

$$\begin{aligned}
\sigma_{11} &= 3x_1 + g_2(x_2) \\
\sigma_{22} &= 4x_2 + g_1(x_1) \\
\sigma_{12} &= a + bx_1 + cx_1^2 + dx_2 + ex_2^2 + fx_1x_2 \\
\sigma_{21} &= \sigma_{12}
\end{aligned}$$

   (b)

$$\begin{aligned}
\sigma_{13} &= -Gx_2 \\
\sigma_{23} &= Gx_1
\end{aligned}$$

5. Use the equilibrium equations to find the stress in a bar under self-weight.

   Hint: Use body forces $F_x = \rho g$, $F_y = 0$, and $F_z = 0$ to find the stress as a function of the position along the bar.