Name:

Homework 4

Due 9 Oct 2019

1. In one-dimensional problems, we can say that $\sigma = E\epsilon$. Explain in your own words, using Hooke's Law for an isotropic material, that $\sigma_{ij} \neq E\epsilon_{ij}$.

2. Given the following displacements

$$
\begin{aligned}
u &= -\frac{M(1-\nu^2)}{EI}xy \\
v &= \frac{M(1+\nu)\nu}{2EI}y^2 + \frac{M(1+\nu^2)}{2EI}\left(x^2 - \frac{l^2}{4}\right) \\
w &= 0
\end{aligned}
$$

   Find the corresponding strain and stress fields. Assume the material is isotropic, and that M, E, I, and l are constants.

3. For an isotropic material, the elastic modulus, E, shear modulus, G, and bulk modulus, K, must always be positive.

   (a) What limits must be imposed on the Poisson's ratio, ν?

   (b) What is the physical meaning of a negative Poisson's ratio?

Table 1: Selected material properties

	E (Gpa)	ν	μ, G (Gpa)	$\alpha(10^-6/°C)$
Aluminum	68.9	0.34	25.7	25.5
Nylon	28.3	0.4	10.1	102
Steel	207	0.29	80.2	13.5

4. Your good friend, who is getting a degree in "Humanities," thinks he has found some kind of super-material in his basement. What he shows you, however, looks a lot like a 20x20x5 (mm) block of anodized aluminum. To humor him, while simultaneously satisfying an obscure requirement for your Elasticity class, you set up an experiment to load it to 50 MPa in pure shear. Assuming it is aluminum, what deformation will you need to apply to obtain this state of stress?

5. Somewhat humbled to learn about his not-so-super material, your determined friend decides to use some of the material he has in his basement (Aluminum, Nylon, and Steel) to make bi-material rings. A revolved schematic drawing for his ring design is shown in Figure 1.

Figure 1: Revolved schematic of your friend's ring design

Your friend has the most steel, so he prefers to use steel as the main ring (white fill in schematic), with either aluminum or nylon as the "highlight." Using your knowledge of thermoelasticity, compare the maximum operating temperature of a ring with aluminum to a ring with nylon. Assume that the inside ring radius is 10 mm, the outside radius is 13 mm, and that the highlight has an inside radius of 12 mm. The ring has a height of 9 mm and the highlight has a height of 3 mm.

Note: The textbook summarizes field equations in cylindrical coordinates in Appendix A

6. Consider a bar with a square cross-section, constrained in the x-direction, but free in the y and z directions. Assuming it is isotropic, find the stress and strain for some change in temperature, ΔT.