Name:

Homework 8

Due 4 Dec 2019

1. Find the polynomial form of an Airy stress function which can solve the cantilever beam shown. Carefully note which boundary conditions are exact and which have been replaced with a statically equivalent boundary using Saint Venant's principle. **Note:** You do not need to solve for the constants.

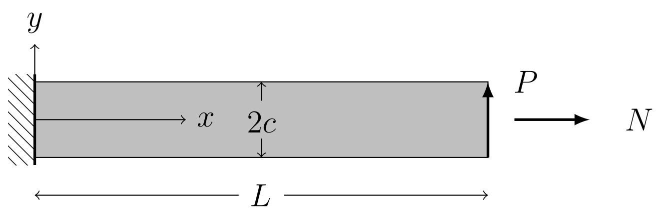


Figure 1: Cantilever beam for Problem 1

2. Use the given polynomial Airy stress function to find the stress field for a cantilever beam with a distributed load.

$$\phi = Ax^2 + By^3 + Cx^2y + Dx^2y^3 + Ey^5 \tag{1}$$

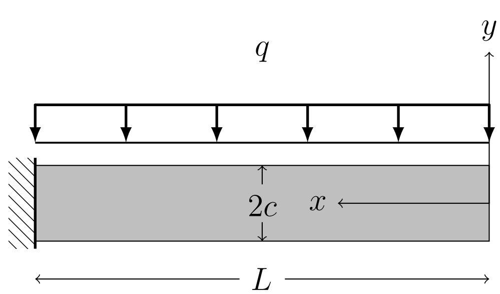
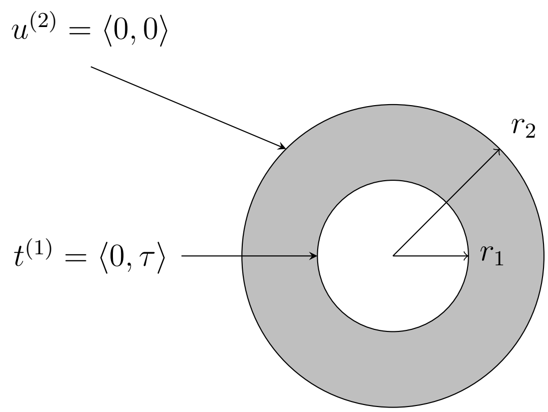


Figure 2: Cantilever beam for Problem 2

3. Consider the axisymmetric problem of an annular disk with a fixed outer radius under uniform shear loading at the inner radius. Find the stress and displacement solution.



Hint: Instead of our usual axisymmetric Airy stress function, which produces no shear stress, try to find a form for the Airy stress function which will have shear stress (thus θ in the Airy stress function), but will NOT have any θ dependence in the stress terms.

4. Show that the curved beam with end loadings can be solved with a superposition of the Airy stress function

$$\phi = \left[Ar^3 + \frac{B}{r} + Cr + Dr\log r\right]\cos\theta \tag{2}$$

with the pure bending solution

$$\phi = a_1 \log r + a_2 r^2 + a_3 r^2 \log r \tag{3}$$

where

$$\begin{aligned}
a_1 &= -\frac{4M}{N}a^2b^2\log\left(\frac{b}{a}\right) \\
a_2 &= \frac{M}{N}\left[b^2 - a^2 + 2(b^2\log b - a^2\log a)\right] \\
a_3 &= -\frac{2M}{N}(b^2 - a^2)
\end{aligned}$$

and

$$N = (b^2 - a^2)^2 - 4a^2b^2\left[\log\left(\frac{b}{a}\right)\right]^2$$

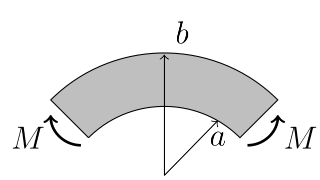


Figure 3: Pure bending problem

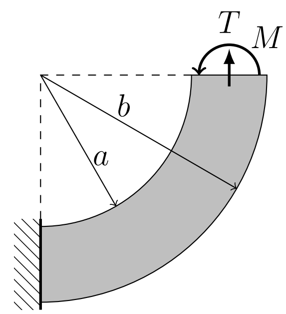
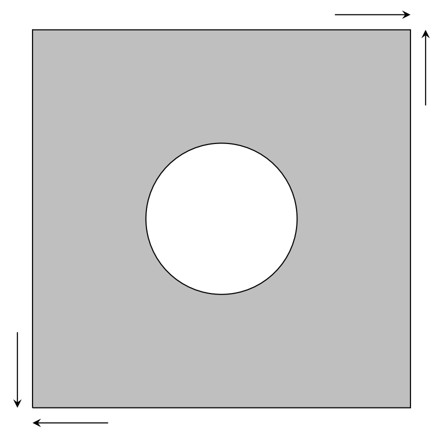


Figure 4: Curved beam with end loads

5. Use the Airy stress method to solve the problem of an infinite rectangular plate subjected to a pure shear stress (σ_{xy}) at infinity. Plot σ_θ around the hole.



6. An elastic disk is perfectly bonded to a rigid ring. This composite disk rotates at a constant angular velocity of Ω. Find the stress and displacement fields.

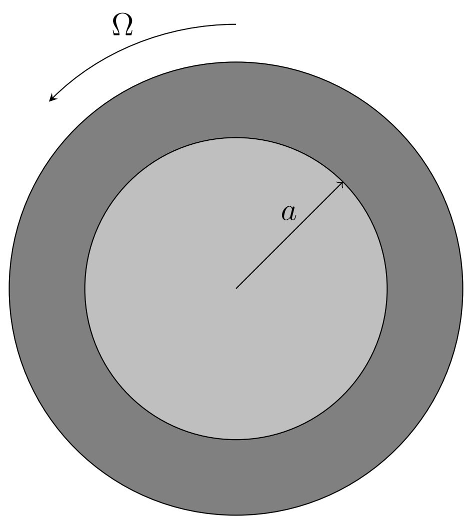


Figure 5: Diagram for an elastic disk bonded to a rigid ring. The light gray is the elastic disk, while the dark grey is the rigid ring.

Hint: Show that the particular solution,

$$\phi_P = \frac{\kappa - 1}{16(\kappa + 1)} \rho \Omega^2 r^4$$

satisfies compatibility, then use it in the solution.