Name:

Exam 1

1. (10 pts.) Identify the dummy and free indexes in each of the following expressions. Indicate the tensor order of the expression. If index notation is used incorrectly, identify why it is used incorrectly and propose a correction.

   (a) $a_i b_j c_k + d_{ijk}$

   (b) $a_{ii} b_k + c_{kk} d_j$

   (c) $C_{ijkl} \epsilon_{kl}$

   (d) $f_{,ii} + b$

   (e) $g_{i,j} + h_{ij} f_{ik}$

2. (15 pts.) For an isotropic material, using Lamé constant notation, the strain can be found in terms of stress as

$$\epsilon_{ij} = \frac{1}{2\mu} \left(\sigma_{ij} - \frac{\lambda}{3\lambda + 2\mu} \sigma_{kk} \delta_{ij} \right)$$

Solve the equation for σ_{ij} (in terms of ϵ, λ, and μ).

Hint: First find an expression for σ_{kk}, then use that to solve the full problem.

3. (15 pts.) Find the vector, u_i with respect to a coordinate system rotated $\theta = 53.13°$ counter-clockwise about the x_1 axis (as shown in Figure 1).

$$u_i = \langle 5, 15, 10 \rangle$$

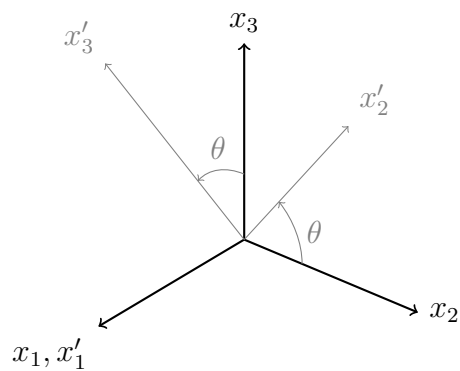Figure 1: Axis description for Problem 3

4. (20 pts.) The analytic strain field predicted for a certain experiment is proportional to two constants, A and B,

$$\epsilon_{ij} = \begin{bmatrix} 34A & -12A & 0 \\ -12A & 41A & 0 \\ 0 & 0 & 50B \end{bmatrix}$$
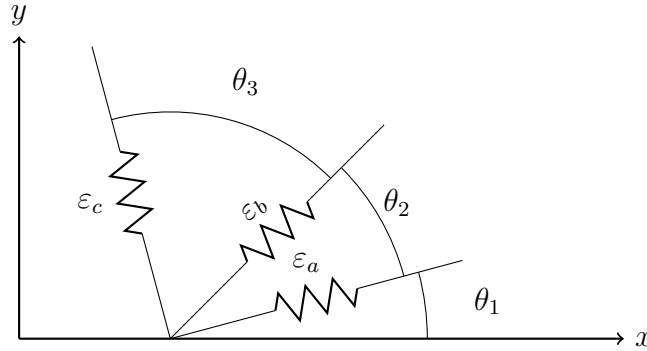
(a) Find the principal strains in terms of A and B

(b) Find the principal directions in terms of A and B

(c) What physical significance do principal strains and directions have?

5. (20 pts.) To study some particular material behaviors, we desire to create the following state of strain, with no rotation

$$\epsilon_{ij} = \begin{bmatrix} y^2z & 2xyz & xy^2 \\ 2xyz & x^2z & x^2y \\ xy^2 & x^2y & 0 \end{bmatrix}$$

Find the displacements to create this state of strain.

6. (20 pts.) Digital image correlation (DIC) can be used in stead of strain gages to measure strain during a tensile test. Often, strain gages are attached to the back side of a specimen to confirm the DIC measurements. After allowing another student to conduct a test for you, you realize they attached three separate gages at unknown angles instead of using a standard rosette. Assuming the DIC measurements are correct, find the angles for each of the strain gages.

DIC measurements:

$$\left\{ \begin{array}{c} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{array} \right\} = \left\{ \begin{array}{c} 0.006 \\ -0.002 \\ 0.000 \end{array} \right\}$$

Strain gage measurements:

$$\left\{ \begin{array}{c} \varepsilon_a \\ \varepsilon_b \\ \varepsilon_c \end{array} \right\} = \left\{ \begin{array}{c} 0.004 \\ 0.001 \\ -0.003 \end{array} \right\}$$

Strain transformation equations:

$$e'_x = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2} \cos 2\theta + e_{xy} \sin 2\theta$$
$$e'_y = \frac{e_x + e_y}{2} - \frac{e_x - e_y}{2} \cos 2\theta - e_{xy} \sin 2\theta$$
$$e'_{xy} = \frac{e_y - e_x}{2} \sin 2\theta + e_{xy} \cos 2\theta$$