AE731

Theory of Elasticity

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upcoming schedule

- Sep 23 - Exam return, Traction vector
- Sep 25 - Stress Transformation
- Sep 30 - Equilibrium Equations
- Oct 2 - Material Characterization, HW3 Due

exam

average

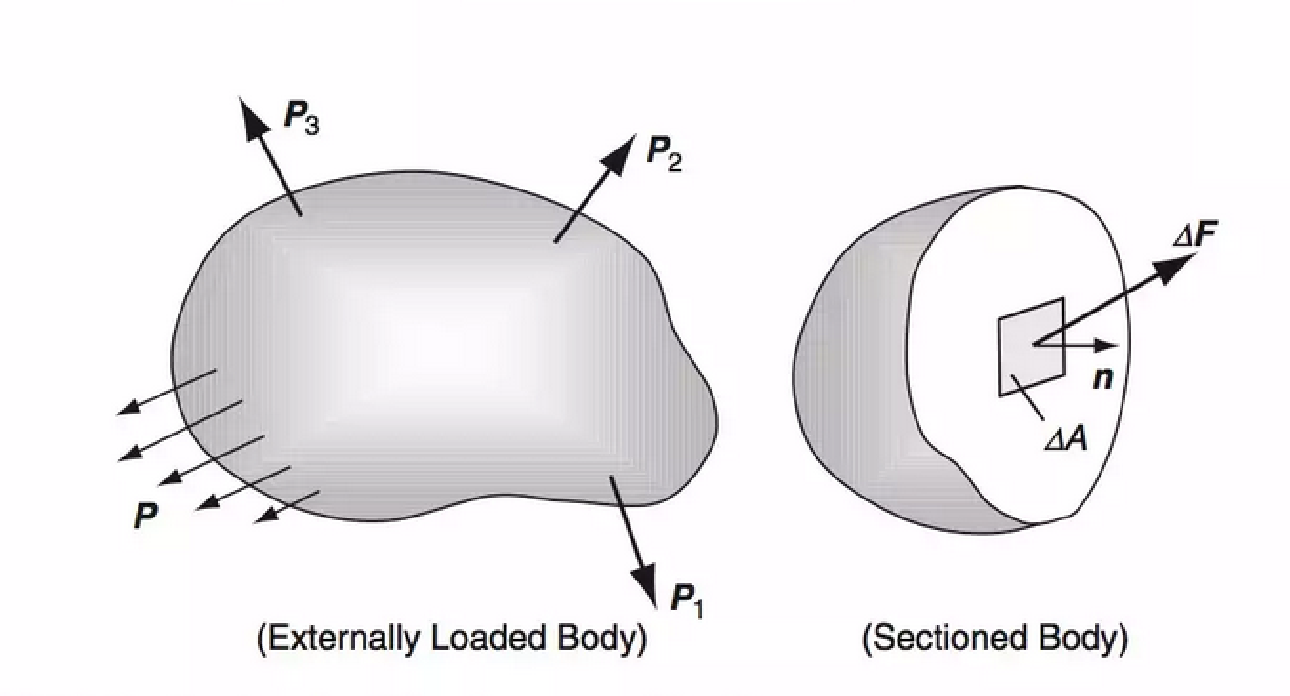
- Scores were very high on this exam (class average of 88%), so there is no curve
- Standard deviation of 13.4%

problems

- Problem 1 had lowest average, followed by Problem 4
- Problem 4 had the highest average, followed by Problem 3

traction vector and stress tensor

traction



P_3
P_2
P
P_1
ΔF
n
ΔA
(Externally Loaded Body)
(Sectioned Body)

traction

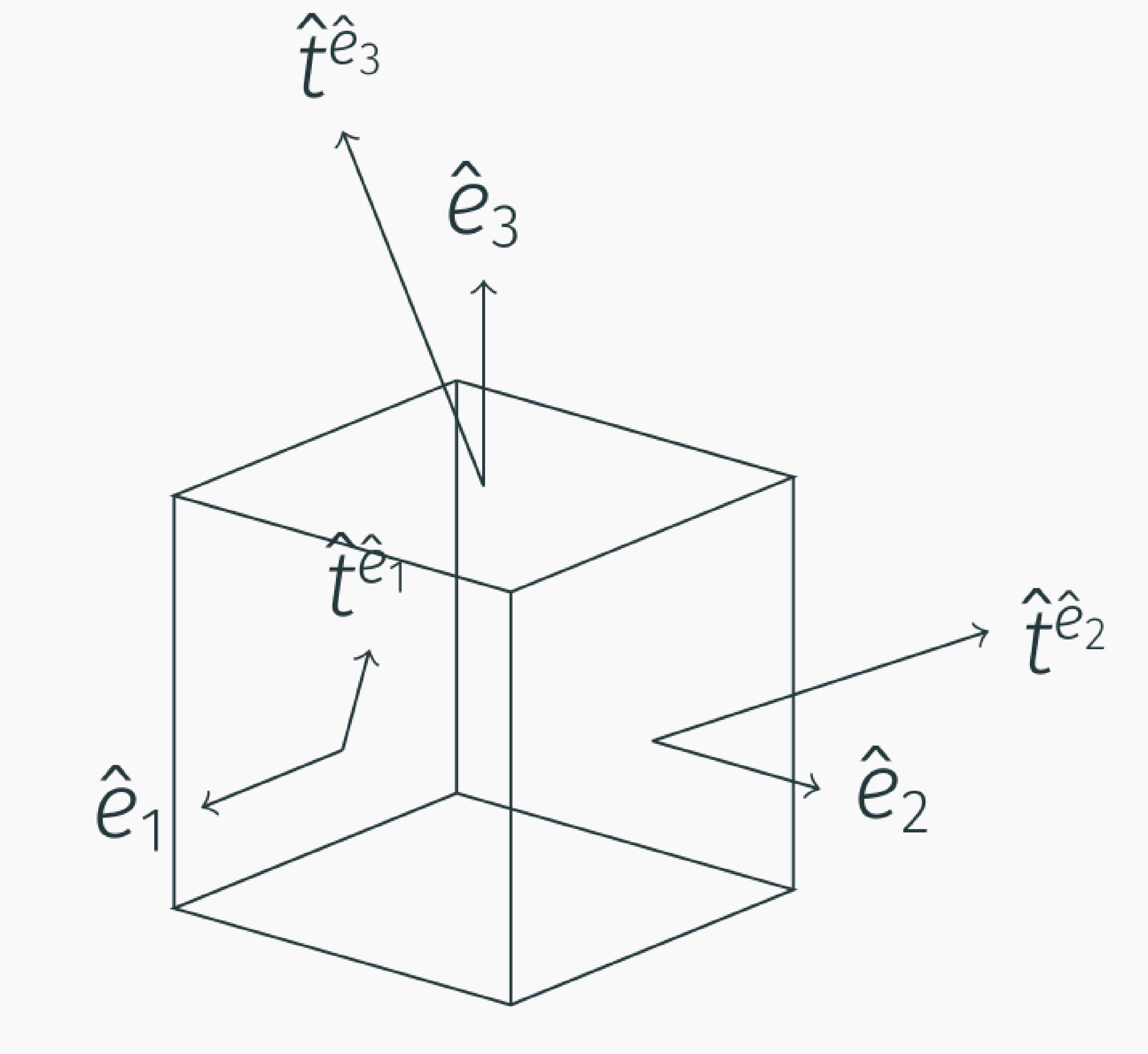
- The traction vector is defined as

$$\hat{t}^{\,n}(x,\hat{n}) = \lim_{\Delta A \to 0} \frac{\Delta \hat{f}}{\Delta A}$$

- By Newton's third law (action-reaction principle)

$$\hat{t}^{\,n}(x,\hat{n}) = -\hat{t}^{\,n}(x,-\hat{n})$$

traction



$\hat{t}^{\hat{e}_3}$
$\hat{e}_3$
$\hat{t}^{\hat{e}_1}$
$\hat{t}^{\hat{e}_2}$
$\hat{e}_1$
$\hat{e}_2$

traction

- If we consider the special case where the normal vectors, $\hat{n}$, align with the coordinate system ($\hat{e}_1$,$\hat{e}_2$,$\hat{e}_3$)
- On the 1-face:

$$\hat{n} = \hat{e}_1 : \qquad \hat{t}^{\,n} = t_i^{(\hat{e}_1)}\hat{e}_i = t_1^{(\hat{e}_1)}\hat{e}_1 + t_2^{(\hat{e}_1)}\hat{e}_2 + t_3^{(\hat{e}_1)}\hat{e}_3$$

- On the 2-face:

$$\hat{n} = \hat{e}_2 : \qquad \hat{t}^{\,n} = t_i^{(\hat{e}_2)}\hat{e}_i = t_1^{(\hat{e}_2)}\hat{e}_1 + t_2^{(\hat{e}_2)}\hat{e}_2 + t_3^{(\hat{e}_2)}\hat{e}_3$$

traction

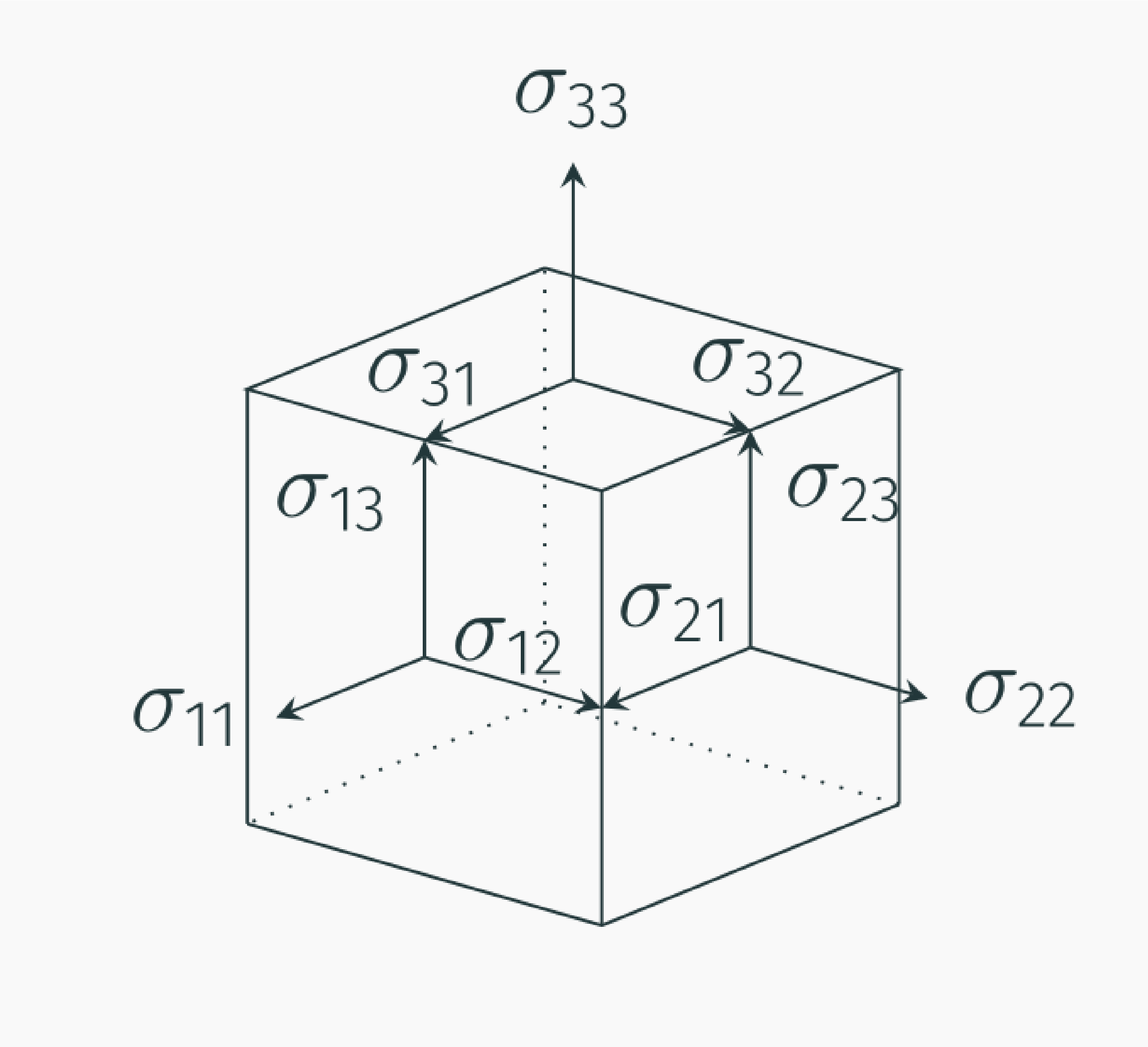
- And on the 3-face:

$$\hat{n} = \hat{e}_3 : \qquad \hat{t}^{\,n} = t_i^{(\hat{e}_3)} \hat{e}_i = t_1^{(\hat{e}_3)} \hat{e}_1 + t_2^{(\hat{e}_3)} \hat{e}_2 + t_3^{(\hat{e}_3)} \hat{e}_3$$

stress tensor

- To simplify the notation, we introduce the stress tensor

$$\sigma_{ij} = t_j^{(\hat{e}_i)}$$



σ_{33}
σ_{31}
σ_{32}
σ_{13}
σ_{23}
σ_{21}
σ_{12}
σ_{11}
σ_{22}

traction

- We can find some interesting information about the traction vector by considering an arbitrary tetrahedron with some traction $\hat{t}^{(n)}$ applied to the surface

traction

- If we consider the balance of forces in the x_1-direction

$$t_1 dA - \sigma_{11} dA_1 - \sigma_{21} dA_2 - \sigma_{31} dA_3 + b_1 \rho dV = 0$$

- The area components are:

$$\begin{aligned} dA_1 &= n_1 dA \\ dA_2 &= n_2 dA \\ dA_3 &= n_3 dA \end{aligned}$$

- And $dV = \frac{1}{3} h dA$.

traction

$$t_1 dA - \sigma_{11} n_1 dA - \sigma_{21} n_2 dA - \sigma_{31} n_3 dA + b_1 \rho \frac{1}{3} h dA = 0$$

- If we let $h \to 0$ and divide by dA

$$t_1 = \sigma_{11} n_1 + \sigma_{21} n_2 + \sigma_{31} n_3$$

- We can write this in index notation as

$$t_1 = \sigma_{i1} n_i$$

traction

- We find, similarly

$$t_2 = \sigma_{i2} n_i$$
$$t_3 = \sigma_{i3} n_i$$

traction

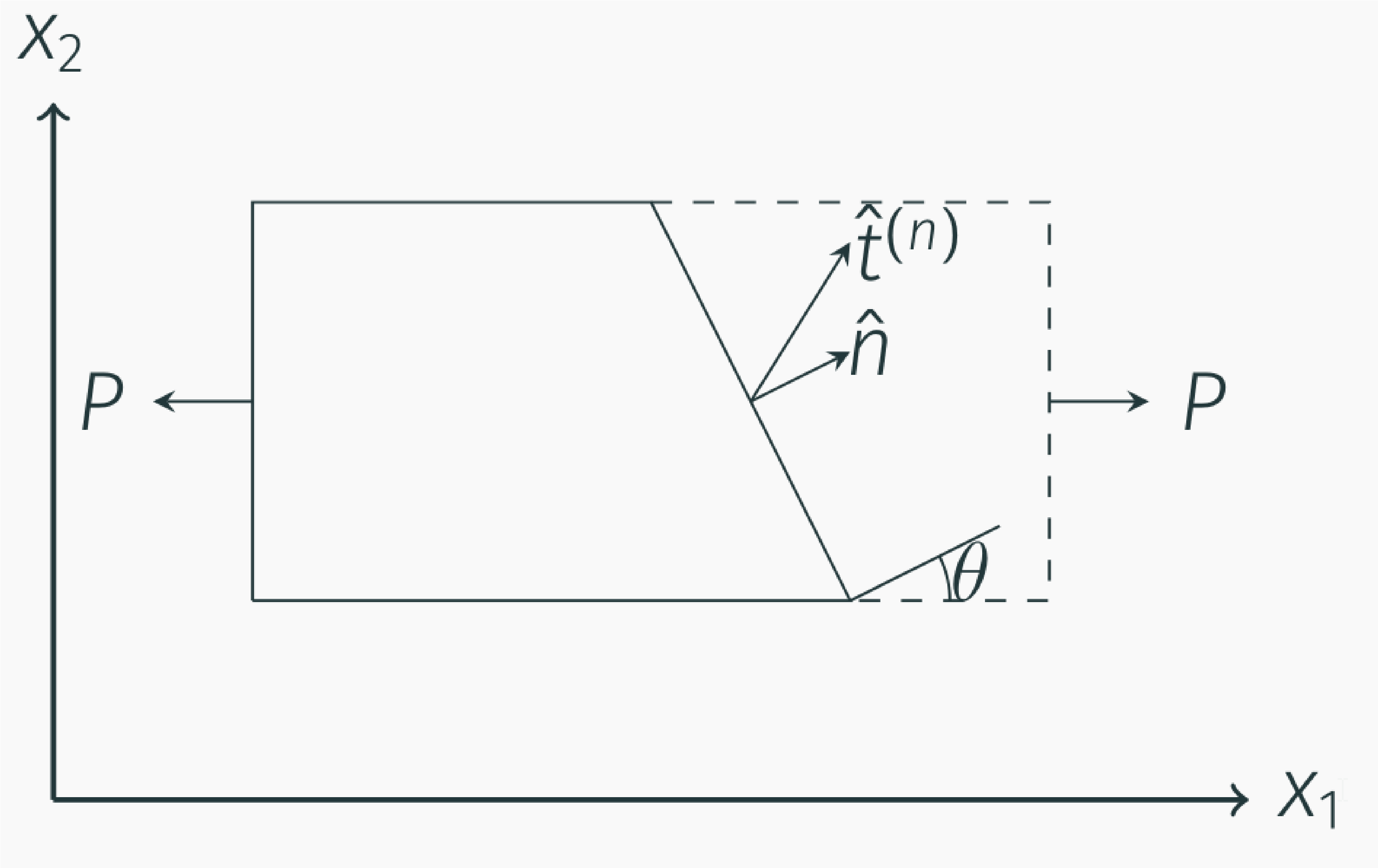
- We can further combine these results in index notation as

$$t_j = \sigma_{ij} n_i$$

- This means with knowledge of the nine components of σ_{ij}, we can find the traction vector at any point on any surface

example

- Consider a block of material with a uniformly distributed force acting on the 1-face. Find the tractions on an arbitrary interior plane


x_2
$\hat{t}^{(n)}$
$\hat{n}$
P
P
θ
x_1

example

- First we consider a vertical cut on the interior 1-face ($n_i = \langle 1, 0, 0 \rangle$)
- Next we represent the force P as a vector, $p_i = \langle P, 0, 0 \rangle$
- Balancing forces yields

$$t_i A - p_i = 0$$

- We find $t_1 = \frac{P}{A} = \sigma_{11}$, $t_2 = 0 = \sigma_{12}$ and $t_3 = 0 = \sigma_{13}$

example

- No force is applied in the other directions, so it is trivial to find the rest of the stress tensor

$$\sigma_{ij} = \begin{bmatrix} P/A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

example

- We can now consider any arbitrary angle of interior cut.
- The normal for a cut as shown in the diagram will be $n_i = \langle \cos\theta, \sin\theta, 0 \rangle$.
- We can again use $t_j = \sigma_{ij} n_i$ to find t_j for any angle θ.

$$t_1 = \frac{P}{A} \cos\theta$$
$$t_2 = 0$$
$$t_3 = 0$$