Theory of Elasticity

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upcoming schedule

- 23 Nov - Complex Methods
- 25 Nov - Thanksgiving (No Class)
- 30 Nov - Complex Methods
- 2 Dec - Final Exam Review
- 3 Dec - HW 8 Due, HW 7 Self-Grade Due
- 7 Dec - 5:40 - 7:30 Final Exam

outline

- group problems, review
- complex variable methods
- research and courses

group problems

group 1

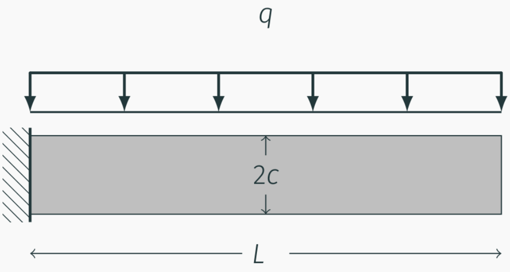


Figure 1: group 1 airy stress problem

group 2

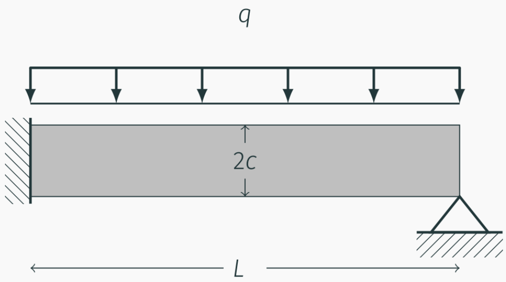


Figure 2: group 2 airy stress problem

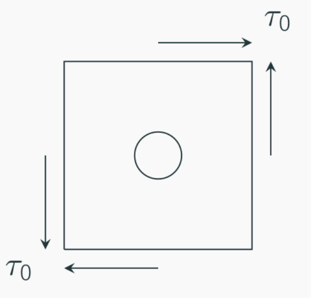


Figure 3: group 3 airy stress problem

complex variable methods

complex variables

- Complex variables are made up of a real portion and imaginary portion

$$z = x + iy$$

- Polar form is written as

$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

- We also define the complex conjugate, $\bar{z}$

$$\bar{z} = x - iy = re^{-i\theta}$$

complex variables

- A function of complex variables will also be made up of a real and imaginary portion

$$f(z) = f(x + iy) = u(x, y) + iv(x, y)$$

- We also define the complex conjugate of the complex function

$$f(\bar{z}) = u(x, y) - iv(x, y)$$

derivatives

- We can use complex conjugates to define derivatives of complex functions

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}$$
$$\frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right)$$

complex plane strain

$$\sigma_x = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial u}{\partial x}$$
$$\sigma_x = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial v}{\partial y}$$
$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Airy stress compatibility

$$\frac{\partial^4 \phi}{\partial z^2 \partial \bar{z}^2} = 0$$

complex Airy stress

$$\phi(z, \bar{z}) = \frac{1}{2}(z\gamma\bar{(}z) + \bar{z}\gamma z + \chi(z) + \chi\bar{(}z))$$

$$\phi(z, \bar{z}) = \Re(\bar{z}\gamma(z) + \chi(z))$$

complex potentials

- It is more convenient to write stresses directly in terms of complex potentials
- These are derived by combining complex Airy stress functions with Navier's equations

$$\sigma_x + \sigma_y = 2(\gamma'(z) + \gamma'\bar(z))$$
$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2(\bar{z}\gamma''(z) + \psi'(z))$$

uses for complex variables

- In Elasticity, complex variables are advantageous in many situations
- Conformal mappings - allows a solution for a simple shape to be mapped onto a more complicated shape
- With complex methods we can handle singularities, and quantify the order of a singularity

uses for complex variables

- Multivalued displacements (dislocations)
- Fracture mechanics
- Westergaard functions (crack analysis)

multiply connected domains

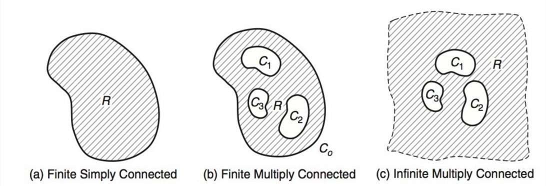
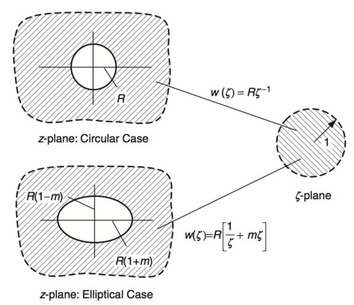


Figure 4: multiply connected domains

mapping

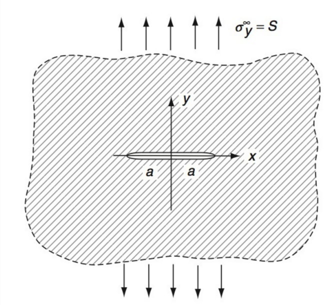


westergaard stress function

- The Westergaard stress function is convenient for many planar crack problems

$$
\begin{aligned}
\sigma_x &= Re[Z(z)] - yIm[Z'(z)] - A \\
\sigma_y &= Re[Z(z)] + yIm[Z'(z)] + A \\
\tau_{xy} &= -yRe[Z'(z)]
\end{aligned}
$$

crack example



crack example

- Consider the Westergaard stress function

$$Z(z) = \frac{Sz}{\sqrt{z^2 - a^2}} - \frac{S}{2}$$

research and courses

continuum mechanics

- AE 831, even years Fall
- A "bigger picture" version of 731
- Develop framework for large deformation
- Solids, fluids, and viscoelastic solids

continuum mechanics - research

- When carbon fiber composites are manufactured, there is always a time where both liquids and solids are present
- If the system is under any motion, the fluid influences the fibers and the fibers influence the fluid
- We can use continuum mechanics to model both together and predict where the fibers will be

micromechanics and multi-scale modeling

- AE 760AA, even years Spring
- Analytic and computational methods for multi-scale modeling
- Particularly applicable to various forms of composites (3D printed, molded composites, etc.)

fracture mechanics

- AE 737 (very applied class, AE 731 not pre-req), AE 837 (theoretical and numerical fracture mechanics methods, AE 731 is a pre-req)
- Research applications: characterize interlaminar fracture toughness, fatigue of aerospace structures, etc.