Name:

Homework 6

Due 13 Nov 2019

1. Use Hooke's Law to explicitly develop the strain energy relationships in terms of stress (U_σ) and strain (U_ϵ) only from the general relationship (Equation 1).

$$U = \frac{1}{2}\sigma_{ij}\epsilon_{ij} \tag{1}$$

2. Use the relationships in Equation 2 to prove the symmetry relationship in Hooke's Law, $C_{ijkl} = C_{klij}$.

$$\sigma_{ij} = \frac{\partial U_\epsilon}{\partial \epsilon}, \epsilon_{ij} = \frac{\partial U_\sigma}{\partial \sigma} \tag{2}$$

3. The stress field for a beam of length $2L$ (in the x-direction) and rectangular cross-section of depth $2c$ (in the y-direction) under applied bending moments M is

$$\sigma_x = -\frac{3M}{2c^3}y$$
$$\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$

   Find the strain energy density and the total strain energy for a unit thickness in the z-direction.

4. The stress field for a rod of circular cross-section is given by

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0$$
$$\tau_{xz} = -\mu\alpha y$$
$$\tau_{yz} = \mu\alpha x$$

   Find the strain energy density and total strain energy for a rod of radius R and length L, where α is a constant and the rod axis lies along the z-axis.

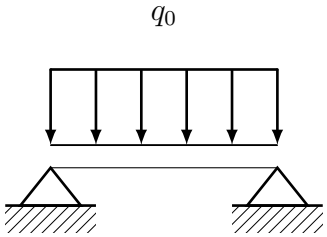

Figure 1: Beam for Problem 5

5. Use the Ritz method to approximate the solution for a simply supported Euler-Bernoulli beam of length L under a uniform load q_0. The approximation will take the form

$$w = w_0 + \sum_{j=1}^{N} c_j w_j \tag{3}$$

Compare two trial solutions, one as a polynomial

$$w_j = x^j(L - x) \tag{4}$$

And the other as a trigonometric function

$$w_j = \sin \frac{j\pi x}{L} \tag{5}$$

For each function, plot the beam deflection (w) for $N = 1$, $N = 2$ and $N = 3$. Compare to the exact solution

$$w = \frac{q_0 x}{24EI}(L^3 + x^3 - 2Lx^2) \tag{6}$$