AE731

Theory of Elasticity

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upcoming schedule

- Aug 28 - Tensor Calculus
- Sep 2 - Labor Day
- Sep 4 - Displacement and Strain, Homework 1 Due
- Sep 9 - Strain Transformation
- Sep 11 - Exam 1 Review

principal values

principal values

- In the 2D coordinate transformation example, we were able to eliminate one value from a vector using coordinate transformation
- For second-order tensors, we desire to find the “principal values” where all non-diagonal terms are zero

principal directions

- The direction determined by the unit vector, n_j, is said to be the *principal direction* or *eigenvector* of the symmetric second-order tensor, a_{ij} if there exists a parameter, λ, such that $a_{ij}n_j = \lambda n_i$
- Where λ is called the *principal value* or *eigenvalue* of the tensor

principal values

- We can re-write the equation $(a_{ij} - \lambda\delta_{ij})n_j = 0$
- This system of equations has a non-trivial solution if and only if $\det[a_{ij} - \lambda\delta_{ij}]=0$
- This equation is known as the characteristic equation, and we solve it to find the principal values of a tensor

example

- Find the principal values of the tensor

$$A_{ij} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

- From the characteristic equation, we know that det[$A_{ij} - \lambda\delta_{ij}$]=0, or

$$\begin{vmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

example

- Calculating the determinant gives $(1-\lambda)(4-\lambda) - 4 = 0$
- Multiplying out and simplifying, we find $\lambda^2 - 5\lambda = \lambda(\lambda - 5) = 0$
- This has the solution $\lambda = 0, 5$

invariants

invariants

- Every tensor has some invariants which do not change with coordinate transformation
- These are known as *fundamental invariants*
- The characteristic equation for a tensor in 3D can be written in terms of the invariants $\det[a_{ij} - \lambda\delta_{ij}] = -\lambda^3 + I_\alpha\lambda^2 - II_\alpha\lambda + III_\alpha = 0$

invariants

- The invariants can be found by the following equations

$$
\begin{aligned}
I_\alpha &= a_{ii} \\
II_\alpha &= \frac{1}{2}(a_{ii}a_{jj} - a_{ij}a_{ij}) \\
III_\alpha &= \det[a_{ij}]
\end{aligned}
$$

invariants

- In the principal direction, ${a_{ij}}'$ will be

$$a'_{ij} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

- Since invariants do not change with coordinate systems, we can also write the invariants as

$$\begin{aligned} I_\alpha &= \lambda_1 + \lambda_2 + \lambda_3 \\ II_\alpha &= \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 \\ III_\alpha &= \lambda_1\lambda_2\lambda_3 \end{aligned}$$