Name:

## Exam 1

1. (15 pts.) Any coordinate transformation involving any combination of rotations can be expressed in terms of three successive rotations. For some third-order tensor,  $T_{ijk}$ , write the coordinate transformation equations for T', T'' and T''' and use these to find one effective transformation,  $Q^e$ , which captures all three rotations,  $Q^1$ ,  $Q^2$ , and  $Q^3$ .

 $2.\ (15\ \mathrm{pts.})$  For the deformation shown, find the displacement, deformation gradient, strain, and rotation.

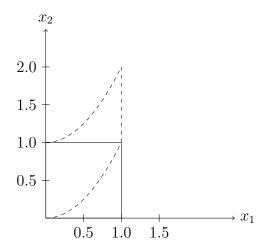


Figure 1: Deformation for Problem 2.

3. (20 pts.) Find the displacements that will place a material in a state of pure shear strain such that  $\epsilon_{12} = \epsilon_{13} = \epsilon_{23} = C$  with all other  $\epsilon_{ij} = 0$ . Are any additional assumptions needed to solve this problem? Is your solution unique? Why or why not?

4. (25 pts.) Solve the equation

$$\mu\{\delta_{kj}a_ia_i + \frac{1}{1-2\nu}a_ka_j\}U_k = P_j$$

for the vector  $U_i$ .

**Hint:** First multiply both sides by the vector  $a_i$ , choose an index for  $a_i$  which will result in a scalar equation.

5. (25 pts.) The analytic strain field predicted for a certain experiment is proportional to two constants, A and B,

$$\epsilon_{ij} = \begin{bmatrix} 2A & 0 & A \\ 0 & 2B & 0 \\ A & 0 & 2A \end{bmatrix}$$

- (a) Find the principal strains in terms of A and B
- (b) Find the principal directions in terms of A and B
- (c) What physical significance do principal strains and directions have?