AE731

Theory of Elasticity

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upcoming schedule

- Sep 14 - Exam return, Traction vector
- Sep 16 - Stress Transformation
- Sep 17 - Homework 2 Self-grade Due
- Sep 21 - Equilibrium Equations
- Sep 23 - Material Characterization
- Sep 24 - Homework 3 Due

exam

average

problems

traction vector and stress tensor

traction

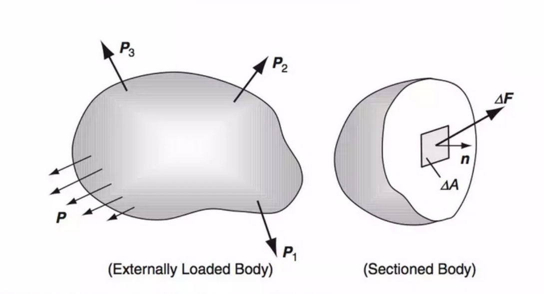


Figure 1: traction vector illustration

traction

- The traction vector is defined as

$$\hat{t}^n(x, \hat{n}) = \lim_{\Delta A \to 0} \frac{\Delta \hat{f}}{\Delta A}$$

- By Newton's third law (action-reaction principle)

$$\hat{t}^n(x, \hat{n}) = -\hat{t}^n(x, -\hat{n})$$

traction

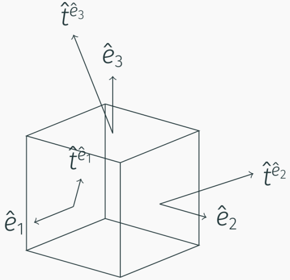


Figure 2: traction illustrated on a cube

- If we consider the special case where the normal vectors, $\hat{n}$, align with the coordinate system ($\hat{e}_1$,$\hat{e}_2$,$\hat{e}_3$)
- On the 1-face:

$$\hat{n} = \hat{e}_1 : \qquad \hat{t}^n = t_i^{(\hat{e}_1)}\hat{e}_i = t_1^{(\hat{e}_1)}\hat{e}_1 + t_2^{(\hat{e}_1)}\hat{e}_2 + t_3^{(\hat{e}_1)}\hat{e}_3$$

- On the 2-face:

$$\hat{n} = \hat{e}_2 : \qquad \hat{t}^n = t_i^{(\hat{e}_2)}\hat{e}_i = t_1^{(\hat{e}_2)}\hat{e}_1 + t_2^{(\hat{e}_2)}\hat{e}_2 + t_3^{(\hat{e}_2)}\hat{e}_3$$

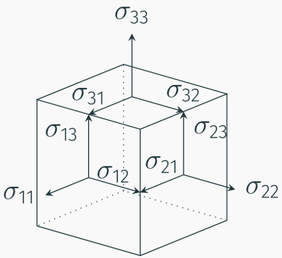
- And on the 3-face:

$$\hat{n} = \hat{e}_3 : \qquad \hat{t}^n = t_i^{(\hat{e}_3)}\hat{e}_i = t_1^{(\hat{e}_3)}\hat{e}_1 + t_2^{(\hat{e}_3)}\hat{e}_2 + t_3^{(\hat{e}_3)}\hat{e}_3$$

stress tensor

- To simplify the notation, we introduce the stress tensor

$$\sigma_{ij} = t_j^{(\hat{e}_i)}$$



traction

- We can find some interesting information about the traction vector by considering an arbitrary tetrahedron with some traction $\hat{t}^{(n)}$ applied to the surface

traction

- If we consider the balance of forces in the x_1-direction

$$t_1 dA - \sigma_{11} dA_1 - \sigma_{21} dA_2 - \sigma_{31} dA_3 + b_1 \rho dV = 0$$

- The area components are:

$$dA_1 = n_1 dA$$
$$dA_2 = n_2 dA$$
$$dA_3 = n_3 dA$$

- And $dV = \frac{1}{3} h dA$.

traction

$$t_1 dA - \sigma_{11} n_1 dA - \sigma_{21} n_2 dA - \sigma_{31} n_3 dA + b_1 \rho \frac{1}{3} h dA = 0$$

- If we let $h \to 0$ and divide by dA

$$t_1 = \sigma_{11} n_1 + \sigma_{21} n_2 + \sigma_{31} n_3$$

- We can write this in index notation as

$$t_1 = \sigma_{i1} n_i$$

traction

- We find, similarly

$$t_2 = \sigma_{i2} n_i$$
$$t_3 = \sigma_{i3} n_i$$

traction

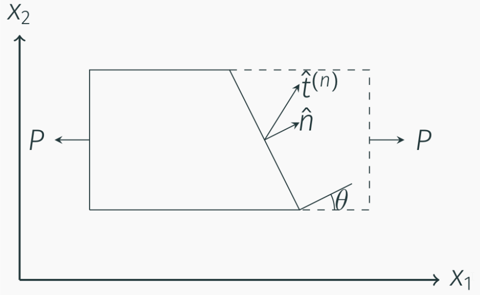
- We can further combine these results in index notation as

$$t_j = \sigma_{ij} n_i$$

- This means with knowledge of the nine components of σ_{ij}, we can find the traction vector at any point on any surface

example

- Consider a block of material with a uniformly distributed force acting on the 1-face. Find the tractions on an arbitrary interior plane



example

- First we consider a vertical cut on the interior 1-face ($n_i = \langle 1, 0, 0 \rangle$)
- Next we represent the force P as a vector, $p_i = \langle P, 0, 0 \rangle$
- Balancing forces yields

$$t_i A - p_i = 0$$

- We find $t_1 = \frac{P}{A} = \sigma_{11}$, $t_2 = 0 = \sigma_{12}$ and $t_3 = 0 = \sigma_{13}$

example

- No force is applied in the other directions, so it is trivial to find the rest of the stress tensor

$$\sigma_{ij} = \begin{bmatrix} P/A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

example

- We can now consider any arbitrary angle of interior cut.
- The normal for a cut as shown in the diagram will be $n_i = \langle cos\theta, \sin\theta, 0 \rangle$.
- We can again use $t_j = \sigma_{ij} n_i$ to find t_j for any angle

$$t_1 = \frac{P}{A}\cos\theta$$
$$t_2 = 0$$
$$t_3 = 0$$