Name:

Exam 2 Equation Sheet

Cauchy's Stress Theorem

$$t_i = \sigma_{ij} n_j \tag{1}$$

Strain-Displacement

Cartesian

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

Cylindrical

$$\begin{aligned} \epsilon_r = \frac{\partial u_r}{\partial r}, \epsilon_\theta = \frac{u_r}{r}, \epsilon_z = \frac{\partial u_z}{\partial z} \\ \epsilon_{r\theta} = \epsilon_{\theta z} = \epsilon_{zr} = 0 \end{aligned} \tag{3}$$

Spherical

$$\begin{aligned} \epsilon_r = \frac{\partial u_r}{\partial r}, \epsilon_\theta = \epsilon_\phi = \frac{u_r}{r} \\ \epsilon_{r\theta} = \epsilon_{r\phi} = \epsilon_{\theta\phi} = 0 \end{aligned} \tag{4}$$

Equilibrium

Cartesian

$$\sigma_{ij,j} + F_i = 0 \tag{5}$$

Navier

$$\mu \nabla^2 u_{i,kk} + (\lambda + \mu) u_{k,ki} + F_i = 0 \tag{6}$$

Constitutive

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij} - (3\lambda + 2\mu)\alpha \Delta T \delta_{ij} \tag{7}$$

$$\epsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} + \alpha \Delta T \delta_{ij} \tag{8}$$

	$\lambda =$	$\mu = G =$	$E =$	$\nu =$	$K =$
λ, μ			$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$	$\frac{3\lambda+2\mu}{3}$
G, E	$\frac{G(2G-E)}{E-3G}$			$\frac{E-2G}{2G}$	$\frac{GE}{3(3G-E)}$
G, ν	$\frac{2G\nu}{1-2\nu}$		$2G(1+\nu)$		$\frac{2G(1+\nu)}{3(1-2\nu)}$
E, ν	$\frac{\nu E}{(1+\nu)(1-2\nu)}$	$\frac{E}{2(1+\nu)}$			$\frac{E}{3(1-2\nu)}$
K, E	$\frac{3K(3K-E)}{9K-E}$	$\frac{3EK}{9K-E}$		$\frac{3K-E}{6K}$	
ν, K	$\frac{3K\nu}{1+\nu}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	$3K(1-2\nu)$		

Material Constants

Compatibility

Strain

$$\begin{aligned}
\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} &= 2\frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \\
\frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} &= 2\frac{\partial^2 \epsilon_{yz}}{\partial y \partial z} \\
\frac{\partial^2 \epsilon_z}{\partial x^2} + \frac{\partial^2 \epsilon_x}{\partial z^2} &= 2\frac{\partial^2 \epsilon_{zx}}{\partial z \partial x} \\
\frac{\partial^2 \epsilon_x}{\partial y \partial z} &= \frac{\partial}{\partial x}\left(-\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{zx}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z}\right) \\
\frac{\partial^2 \epsilon_y}{\partial z \partial x} &= \frac{\partial}{\partial y}\left(-\frac{\partial \epsilon_{zx}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x}\right) \\
\frac{\partial^2 \epsilon_z}{\partial x \partial y} &= \frac{\partial}{\partial z}\left(-\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{zx}}{\partial y}\right)
\end{aligned} \tag{9}$$

Beltrami-Michell

$$\begin{aligned}
(1+\nu)\nabla^2\sigma_x + \frac{\partial^2}{\partial x^2}(\sigma_x + \sigma_y + \sigma_z) &= 0 \\
(1+\nu)\nabla^2\sigma_y + \frac{\partial^2}{\partial y^2}(\sigma_x + \sigma_y + \sigma_z) &= 0 \\
(1+\nu)\nabla^2\sigma_z + \frac{\partial^2}{\partial z^2}(\sigma_x + \sigma_y + \sigma_z) &= 0 \\
(1+\nu)\nabla^2\tau_{xy} + \frac{\partial^2}{\partial x \partial y}(\sigma_x + \sigma_y + \sigma_z) &= 0 \\
(1+\nu)\nabla^2\tau_{yz} + \frac{\partial^2}{\partial y \partial z}(\sigma_x + \sigma_y + \sigma_z) &= 0 \\
(1+\nu)\nabla^2\tau_{zx} + \frac{\partial^2}{\partial z \partial x}(\sigma_x + \sigma_y + \sigma_z) &= 0
\end{aligned} \tag{10}$$

Plane Stress

$$\nabla^2(\sigma_{11} + \sigma_{22}) = -(1+\nu)\left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\right) \tag{11}$$

Plane Stress in Polar Coordinates

$$\nabla^2(\sigma_{rr} + \sigma_{\theta\theta}) = -4\rho(1+\nu)\left(\frac{\partial F_r}{\partial r} + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta}\right) \tag{12}$$

Where, in polar coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{13}$$

Stress transformation

$$\begin{aligned}
\sigma_x' &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\
\sigma_y' &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \\
\tau_{xy}' &= \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta
\end{aligned} \tag{14}$$