Name:

Homework 1

Due 4 September 2019

1. Given

$$S_{ij} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \qquad \text{and} \qquad a_i = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \tag{1}$$

   Find

   (a) S_{ii}

   (b) $S_{ij}S_{ij}$

   (c) $S_{ji}S_{ji}$

   (d) $S_{jk}S_{kj}$

   (e) $a_m a_m$

   (f) $S_{mn}a_m a_n$

2. Write the following equations in index notation

   (a) $s = A_1^2 + A_2^2 + A_3^2$

   (b) $\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} + \frac{\partial^2 \phi}{\partial x_3^2} = 0$

3. Let f be a scalar-valued function such that $f(x_i) = \sqrt{x_i x_i}$. Find $f_{,i}$

4. Show (by expansion) that:

$$(AB)_{,ii} = AB_{,ii} + 2A_{,i}B_{,i} + BA_{,ii}$$

   where A and B are scalars.

5. If S_{ij} is symmetric and A_{ij} is antisymmetric, show that $S_{ij}A_{ij} = 0$.

6. For an isotropic material, which is assumed to be linear and elastic, the stress-strain relationship is given by:

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\epsilon_{ij} + \frac{\nu}{1-2\nu}\epsilon_{kk}\delta_{ij}\right)$$

   Solve this equation for strain (ϵ_{ij}) in terms of stress (σ_{ij}). **Hint:** First find an expression for ϵ_{kk}, then use that to solve the full problem.

7. Find the second-order tensor, T_{ij} with respect to a coordinate system rotated $60°$ counter-clockwise about the x_2 axis (as shown in Figure 1)

$$T_{ij} = \begin{bmatrix} 6 & 9 & 8 \\ 5 & 3 & 4 \\ 1 & 2 & 7 \end{bmatrix}$$

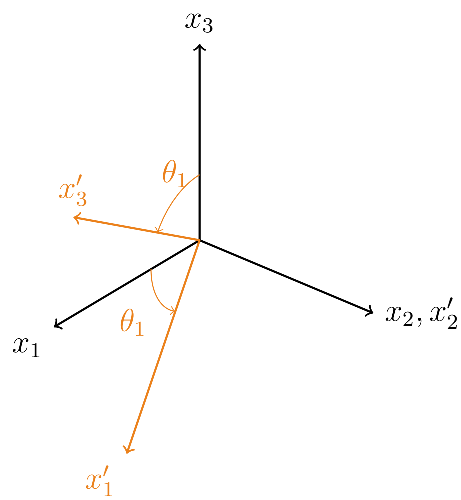

Figure 1: Axis description for Problem 7

8. For T_{ij} in Problem 7, find T''_{ij} in the coordinate system shown in Figure 2. The x'_i coordinate system is the same as shown in Problem 7, with a rotation of $\theta_1 = 60°$ about the x_2 axis. The x''_i coordinate system is obtained by rotating the x'_i coordinate system by $\theta_2 = 30°$ about the x'_3 axis.

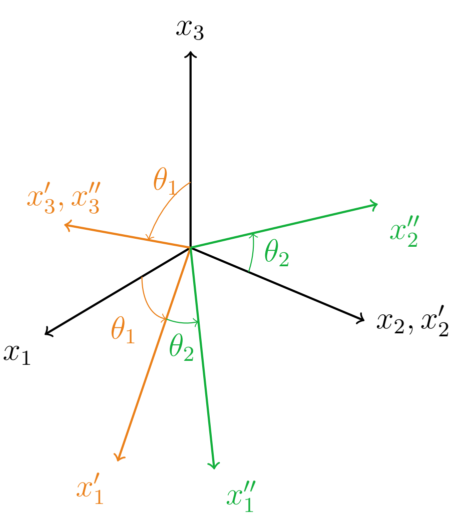

Figure 2: Axis description for Problem 8

9. For T_{ij} and T'_{ij} from Problem 7, and T''_{ij} from Problem 8, calculate the invariants, I_1, I_2, I_3. Comment on any findings.

10. For the stress tensor, σ_{ij}, find all principal values and their directions.

$$\sigma_{ij} = \begin{bmatrix} 4.750 & 2.165 & 1.500 \\ 2.165 & 2.250 & 0.866 \\ 1.5 & 0.866 & 4.000 \end{bmatrix}$$

    Hint: Round intermediate values to two decimal places, use a graphing calculator, MATLAB, or other computer method to solve cubic equations.