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Exercise session 8, 06 November 2019

Numerical Methods in Informatics (L + E)

Exercise 1

Compute the error of the trapezoidal quadrature formula.

Exercise 2

Apply the Richardson extrapolation for the trapezoidal formula (i.e., the Romberg integration) to the integral

$$I(f) = \int_0^2 \exp(-\frac{x^2}{2})\mathrm{d}x$$

with $H_1 = 1$ and $H_2 = 0.5$.
The exact value is given by $I(f,a,b) = \sqrt{\frac{\pi}{2}}\left(\mathrm{erf}(b/\sqrt{2}) - \mathrm{erf}(a/\sqrt{2})\right) = I(f,0,2) = \sqrt{\frac{\pi}{2}}\mathrm{erf}(\sqrt{2})$, where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$ is the error function (MATLAB command `erf`).
Compare the error for the simple trapezoidal formula with both integral and with the Romberg integration.

Exercise 3

The velocity of a falling parachutist can be estimated in terms of time by the following function:

$$v(t) = \frac{gm}{c_\mathrm{d}}(1 - \exp(-tc_\mathrm{d}/m))$$

where v is the velocity in (m/s), t is the time in s, g is the gravitation constant $g = 9.8$ (m/s^2), $m = 68.1$ kg is the mass of the parachutist, and $c_\mathrm{d} = 12.5$ kg/s is the drag coefficient. The distance covered in a certain time t can be computed by the integral of the velocity, i.e.

$$d(t) = \int_0^t v(s)\mathrm{d}s = \frac{gm}{c_\mathrm{d}}\int_0^t (1 - \exp(-sc_\mathrm{d}/m))\mathrm{d}s$$

(we can bring $\frac{gm}{c_\mathrm{d}}$ outside the integral as it is constant).

- In MATLAB, exploiting eventual existing routines, estimate the distance at final time $t_\mathrm{f} = 10\,\mathrm{s}$ by means of the composite trapezoidal rule, by using $M+1$ points.
- Apply the quadrature formula for the following values of M,
$$M = \{10^k : k = 1, \dots, 9\}.$$
- Given the exact value $d_\mathrm{ex} = 289.4351465112940\,\mathrm{m}$, for each value of M compute the percentage error $\varepsilon = 100 * (d_\mathrm{ex} - d)/d_\mathrm{ex}$. What can you say about the error behavior: is it progressively decreasing? If not, could you explain the possible cause?
Hint: a plot of the error may help you to understand its behavior.

Exercise 4

Adaptive Simpson quadrature (see slide).