Exercise session 12, 4 December, 2019

# Numerical Methods in Informatics (L + E)

**Exercise 1**

Apply the Gauss–Newton method to the following problems.
**a.** Find the least–square approximation to $\theta$ to the system

$$\begin{cases} \theta^2 + 2 = 15 \\ -2\cos(\theta) = 1.5. \end{cases} \tag{1}$$

**b. Signal analysis**[1] The signal intensity is modeled as a sum of $m$ Gaussian functions (also called peaks)

$$f_k(t; a_k, \sigma_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-(t-a_k)^2/(2\sigma_k^2)\right), \tag{2}$$

given a set of *expected values* $a_k$ and of *variances* $\sigma_k$, we set

$$f(t, \underline{a}, \underline{\sigma}) = \sum_{k=1}^{m} f_k(t; a_k, \sigma_k). \tag{3}$$

Set $m = 5$ and set $\underline{a} = [2.3, 3.25, 4.82, 5.3, 6.6]$, and $\underline{\sigma} = [0.2, 0.34, 0.50, 0.23, 0.39]$. Generate some noisy signal adding to the function $f$ a random normal noise multiplied by 0.05 (`0.05*randn(size)`). Starting now from some initial values $\underline{a} = [2, 3, 4, 5, 6]$ and $\underline{\sigma} = [0.3, 0.3, 0.6, 0.3, 0.3]$ as initial guess, use the Gauss–Newton algorithm to recast the original signal.

**Exercise 2** ODE solvers

**a.** Implement the Euler forward and Euler backward methods for linear systems of equations.

**b.** Test them on a fine (1000 elements) and a coarse (8 elements) grid on the equation

$$\frac{du(t)}{dt} = cu(t), \quad t \in [0, 10] \tag{4}$$

where $u(0) = 1$ and $c = -1.5$. What do you observe?
**c.** Apply the methods to a linear system

$$\begin{cases} \frac{du_1(t)}{dt} = -0.2u_1(t) + u_2(t) \\ \frac{du_2(t)}{dt} = +0.2u_1(t) - u_2(t) \end{cases} \tag{5}$$

where $u0 = [0.1; 0.9]$ and $t \in [0, 10]$. This system describe the quantities of chemical particles in time. The total mass should be conserved and the quantities should stay positive. Test both methods with coarse and finer meshes. What do you observe?

**d.** Implement the nonlinear explicit Euler method.
**e.** Apply the method to the pendolum equation

$$y'' = -g\sin(y) \tag{6}$$

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that can be rewritten in a system

$$\begin{cases} \frac{dy_1(t)}{dt} = y_2(t), \\ \frac{dy_2(t)}{dt} = -g\sin(y_1(t)). \end{cases} \tag{7}$$

Use again $y0 = [0.1; 0.9]$ and $t \in [0, 10]$.