About the course

Organisation

- Theoretical lectures on Tuesday 8:00 9:45 in BIN 2.A.01
- Exercise classes on Wednesday 8:00 9:45 in BIN 0.B.06

Homework

- Every 2 weeks
- Published on Tuesday morning (8:00) of week n
- Deadline on Tuesday morning (8:00) of week n + 1

During exercise class

- Correct homework
- New exercises
- Hands on moments (easy ones)
- We solve together (difficult ones)

MATLAB crash course

- First 2 lessons

Material

- Slides of Programming in MATLAB course 2018
- Notes Felix Fontain notes 2013
- Help toolbox of MATLAB
- Google

Outline

Introduction

Flow control (if-statements, loops, ...

Arrays

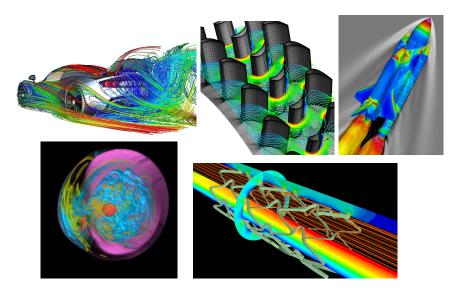
Visualisation

Exercises lesson 2

About MatLab

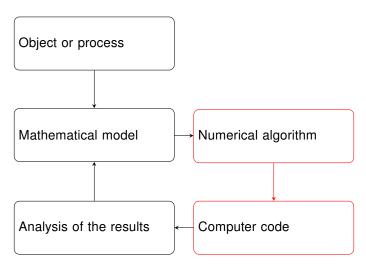
- MATrix LABoratory
- originally a FORTRAN program
- Data elaboration with floating points
- ready-to-use Toolboxes (e.g.: Image processing, financial tools, etc.)
- symbolic computations and exact arithmetic (Symbolic Math Toolbox) (but Mathematica is preferable for this..)
- equivalent software: Octave, Scilab

Numerical simulations



Mathematical modeling

The solution of a real industrial problem can be roughly represented by the following scheme.



Object

CAD model 3D mesh Navier-Stokes equations Numerical methods

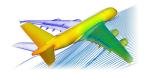
Software HPC simulation

Results Data visualization Analysis









Brief Design of Computers

Computer components:

- **CPU:** elaborates instructions in a machine code
- computer memory: stores data & instructions
- bus: communication system for data between components(CPU, memory, etc.)
- peripheral: everything connected to a computer (keyboard, usb, etc.)
- software, produced via Programming languages:
 - low-level
 - high-level

Python to Matlab

NumPy	Matlab	Notes
a and b	a&&b	-
a or b	a b	-
array([[1.,2.,3.],	[123;456]	2x3 matrix literal
[4.,5.,6.]])		
a[-1]	a(end)	last element a
a[1] or a[1,:]	a(2,:)	second row of a

Python to Matlab

NumPy	Matlab	Notes
a.transpose() or	a.'	transpose of a
a.T		
a.conj().transpose()	a'	conjugate trans-
or a.conj().T		pose of a
a.dot(b)	a * b	matrix multiply
a * b	a .* b	element-wise mul-
		tiply
a/b	a./b	element-wise
		divide
a**3	a.^ 3	element-wise
		exponentiation

Python to Matlab

Main differences:

- Indexing starts from 1
- Array/vector access is through ()
- Everything are arrays!
- Arrays have pass-by-value semantics as opposed to reference
- Possible to do Object Oriented, but widely not used

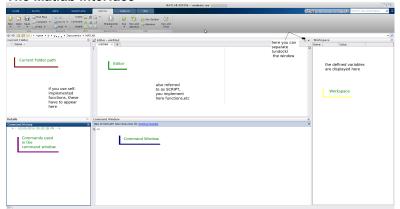
More info: https://docs.scipy.org/doc/numpy-dev/user/numpy-for-matlab-users.html

MATLAB

Install MATLAB on your machine with UZH license

- 1. Instructions on UZH website
- 2. Type Thinlinc password

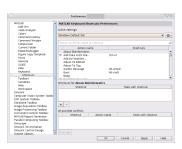
The Matlab interface



Watch out: shortcuts!

Setting up the Keyboard shortcuts: Preferences > Matlab >

Keyboard > Shortcuts > Windows Default Set



About precision...

Matlab uses *floating-point numbers* \mathbb{F} - different from \mathbb{R} !

Example

>> 1/7 ans = 0.1429

Output formats:

>> format Type	Type	
· -	rat	1/7
	short	0.1429
	short g	0.14286
	short e	1.4286e-01
	long	0.142857142857143
	long g	0.142857142857143
	long e	1.428571428571428e-01

Storing of numbers

Representation of **floating-point numbers**¹:

$$x = (-1)^{s} \cdot (0.a_{1}a_{2} \dots a_{t}) \cdot \beta^{e} = (-1)^{s} \sum_{j=1}^{t} a_{j} \beta^{e-j} = (-1)^{s} \cdot m \cdot \beta^{e-t}$$

with

- $s \in \{0, 1\}$
- $a_1 \neq 0$
- $-\ eta \geq$ 2 is the basis adopted by the computer (usually 2)
- -m, the mantissa, is an integer whose length is the number of digits t
- $-a_i \in \{0, \dots, \beta 1\}$ are the stored digits
- *e* ∈ {*L*, . . . , *U*} is an integer number called *exponent*
- → floating-point numbers since decimal point not fixed
- \rightarrow digits $a_1 a_2 ... a_p$ are also called p first significant digits of x.

¹ Reference: Quarteroni, Saleri, Gervasio - Scientific Computing with Matlab and Octave, IVth ed. University of Zurich, I-Math 18.09.2019 Numerical Methods in Informatics

Exercises diy

Representation of **floating-point numbers**:

$$x = (-1)^{s} \cdot (0.a_{1}a_{2}...a_{t}) \cdot \beta^{e} = (-1)^{s} \sum_{j=1}^{t} a_{j}\beta^{e-j} = (-1)^{s} \cdot m \cdot \beta^{e-t}$$
$$a_{1} \neq 0$$

Use:

$$\beta = 2$$
 $t = 3$ $e \in \{-4, ..., 3\}$

- What is the smallest number that you can represent?
- What is the biggest number?
- What is the smallest absolute value of a number that you can represent?
- How many bits do you need to store one number?

Exercises diy

Representation of floating-point numbers:

$$x = (-1)^{s} \cdot (0.a_{1}a_{2}...a_{t}) \cdot \beta^{e} = (-1)^{s} \sum_{j=1}^{t} a_{j}\beta^{e-j} = (-1)^{s} \cdot m \cdot \beta^{e-t}$$
$$a_{1} \neq 0$$

Use:

$$\beta = 2$$
 $t = 3$ $e \in \{-4, ..., 3\}$

- What is the smallest number that you can represent? -7
- What is the biggest number? 7
- What is the smallest absolute value of a number that you can represent? $\frac{1}{32}$
- How many bits do you need to store one number? $s: 1, a_i: t-1=2, e: 3 \Rightarrow 6$ bits

Storing in MatLab¹

General: $\mathbb{F}(\beta,t,L,U)$ characterizes the set \mathbb{F} (namely the basis, # significant digits and range of e via (L,U). MatLab: $\mathbb{F}=\mathbb{F}(2,53,-1021,1024)$ are stored in 8 bytes (called double precision)

Smallest and largest positive values of \mathbb{F} :

$$x_{min} = \beta^{L-1}, \quad x_{max} = \beta^{u}(1 - \beta^{-t}).$$

MatLab:

>> realmin ans = 2.2250738585072e-308

>> realmax ans = 1.79769313486232e+308

¹ Reference: Quarteroni, Saleri, Gervasio - Scientific Computing with Matlab and Octave, IVth ed.

Exercises diy

Represent the following floating points in base 2, or explain why this is not possible to be done exactly.

- a) 15
- b) 0.5
- c) 0.1
- d) 1/3
- e) 1/256

Round-off error¹

Definition

It is the error generated when a real number is replaced with a floating-point number.

$$\frac{|x-fl(x)|}{|x|}\leq \frac{1}{2}\epsilon_M$$

with $\epsilon_M = \beta^{1-t}$ called the machine epsilon.

Matlab:
$$\epsilon_M = 2^{-52} \sim 2.22 \cdot 10^{-16}$$
:

ans =

2.22044604925031e-16

¹ Reference: Quarteroni, Saleri, Gervasio - Scientific Computing with Matlab and Octave, IVth ed.

Good to know: Sources of computational errors

Computational errors can arise due to

- 1. Incompleteness of mathematical model.
- 2. Errors in the initial (input) data (e.g. from experimental measurements).
- 3. Approximate methods to solve the equations of the mathematical model.
- Fixed-size data storage format on computers, round-off errors.
- 5. ...

Exercises... Get started: Scalar assignment

1. Type in the command line:

Note: >> is called prompt.

Exercises... Get started: Scalar assignment

Typing in the command line:

```
>> a = 2.45

>> A = 3.1

>> A = 7.2

>> a= 1.2,

>> a= 1.7, b=2.45

>> a= 1.7; b=2.45

>> a= 1.7, b=2.45

>> a= 1.7; b=2.45

>> a= 1.7; b=2.45
```

Get started: Cleaning of memory

- delete variable A from workspace:
 - >> clear A
- delete all variables from workspace:
 - >> clear
 - >> clear all
- clean up the visualization window:
 - >> clc
 - >> home

<u>Good habits:</u> Clean your workspace and visualization always before starting a new exercise.

Get started: Discover new instructions

- to see how to use an instruction, ex. called instruction:

>> help instruction

Example:

>> help format

Mathworks website

Expressions

Arithmetic operators (in decreasing precedence order)

brackets power/multiplic./division addition/subtraction == ~= | equivalent to/unequal to... < <= > >= | less/greater than... and/or/not

operations precedence arithmetical operations arithmetical operations equivalence operators equivalence operators Logic symbols

To know more:

>> doc 'operator precedence'

Expressions

Numerical expressions:

```
>> 1
>> 0.23, % or
>> .23
>> 23e-2 % or
>> 23*10^(-2)
>> 5+4i % or
>> 5+4j
```

Functions:

```
call with functionName(parameter1, parameter2, ...)
sin sqrt log floor
cos abs log2 ceil
tan exp log10 round
```

Predefined Variables

ans	results of last computation	
рi	$\pi = 3.14$	
eps	machine precision	
i,j	$\sqrt{-1}$	
nan	not a number (ex. as result of 0/0)	
inf	infinity (ex. as result of 1/0)	
realmin, realmax	smallest/biggest fluctuation-point number	

Attention:

don't overwrite these variables!!

To check the complete word list:

» iskeyword

Scripts

many commands ⇒ **Scripts** convenient

- click "new script"
- write the following:

```
>> z=23;
>> hallo=12*z
```

- save the script with the name "FirstScript.m".
- run the script by clicking on the run button
- run the script from command line via FirstScript
- by typing the inital letters of FirstScript and TAB: autofill
- workspace shows z and hallo variables

Comments

ightarrow Good habit: comment whatever you write

Comment types

```
% single line comment
%{
multi line
comment
%}
```

%% describes a codeblock, splits into different sections

Why comments?

- ease in reading codes
- easy to maintain
- documentation: describe in the beginning of the script a comment with
 - what the script does
 - what variables it takes as input or output
 - further useful info
- these comments are displayed when typing

```
>> help FirstTest
```

or

>> FirstTest + F1

Functions

- none, one or many inputs (arguments) and outputs
- go to New> Function
- change it to:

```
function y = power8(x)
% Compute y=x^8 (This is a comment, displayed in the help) y=x^8;
end
```

- save it to power8.m
- type in command line:

```
>> power8(2)
```

Note: The function name must be the same of the .m file!

Functions: multiple arguments

- in case of multiple inputs or outputs

```
function [y1, y2] = Multi_inout_function(x1,x2,x3)
  % example function with more than one input
  y1=x1;
  y2=x2+x3;
```

– call from command line:

```
>> [a,b] = Multi_inout_function(1,2,3)
```

– if not interested in some outputs:

```
>> a = L103_function(1,2,3)
>> [~,b]=L103_function(1,2,3)
```

 Variable inputs/outputs possible: see varargin, varargout, nargin, nargout.

Multiple functions in a file

– multiple functions in a single file are possible:

```
function y=power8(x)
y=mypower8(x);
end

function y=mypower8(x)
y=x^8;
end
```

Attention: only the function with the same name as the file can be called by the command line or other scripts!!

Local variables in functions

interface between function and the workspace:

only via input output arguments

```
function y=power8_local(x)
% none; -> this appears in the function description
y=x^8
% the next line does not generate any
% variable in the workspace
myVar = 5;
end
```

This means: everything which does not appear in the first line of a function stays *local*

→ useful if variables are called the same in different functions

Anonymous Functions

- Functions (e.g. equations) written without .m:

Example

>>
$$f = @(x,y) x^y$$

>> $f(2,3)$

- the right hand side is an anonymous function.
- f is a function_handle.

Condition statement

Definition

ightarrow commands in the if-statement are **only considered** if statement fulfilled.

```
if (condition statement)
  (matLab commands)
  end
Example
  if x == 5 % is carried through, if x=5
   y=1;
  elseif x==6
   z=2;
  else
   y=3;
  end
```

for-loops

Definition

 \rightarrow carries out same commands for several times (e.g. for the terms in a vector)

```
for index=start:end
  ( commands )
end
```

Example

```
for k=1:10
  disp(k^2)
end
```

- continue: jumps to the loop starting
- break: interrupts the loop

while-loops

→ Execute loops, until a condition is specified.

```
a=0;
while a<5
  a=a+1;
end
```

Example (bad example - Ctrl+C interrupts script)

```
a=6;
while a>5
  a=a+1;
end
```

Exercises... Condition statements & Loops

- 1.home a) Write a script which contains:
 - if
 - for
 - while
 - continue
 - break
 - a call to a self implemented function
 - b) Check the execution through the debugger.
 - 2.live Compute the sum of the square of the first 1000 integer numbers, i.e., $\sum_{k=1}^{1000} k^2$ using a for loop
 - Compute the product of the first 100 integers $\prod_{k=1}^{100} k = 100!$ using a while loop
 - Compute the sums of the even integers < 100 and odd integers
 100 in one loop.

Arrays

Very important part of Matlab. Two types of arrays:

- 1. scalar (1 \times 1), vector ($n \times$ 1 or 1 \times n) and matrix ($n \times m$)
- 2. cell array of objects

Exercise... Arrays (live)

a) Try the following in the command line:

- b) Check the workspace and look at the information about a, b, c.
- c) What do: space or, or; do?
- d) Compute
 - >> a+b
 - >> a+c

Matrices

Example

```
>> A=[1 2 3; 4 5 6]
>> A=[1,2,3; 4, 5, 6]
```

Matrices in Matlab

- − definition via 「 ¹
- space or , divides columns
- ; divides lines
- to access values:

– change of single values:

$$>> A(2,3)=7$$

Size of matrices

Check the dimension of the array

- check the workspace
- call the variable through the command line without;
- use Matlab-inbuilt function size
 - >> size(a)
 - >> size(c)
 - >> size(A)
 - >> size(A,1)
 - >> size(A,2)
- to check the length of vectors:
 - >> length(a)
 - >> length(c)
 - >> length(A)
- go to last element of vector a:
 - >> a(end)

Built-in structures of matrices

A few examples of the matrix built-in layouts:

 $-n \times n$ identity matrix

- 3 × 8 matrix of ones

- 4 × 6 matrix of zeros

matrix with random values

equispaced points from a to b

- magic square

- empty matrix

– & many more..

Operations with matrices

+	-		element-by-element sum/division
,	. ,		conjugate transposed (ex. A'), transposed (ex. A.')
*			multiplication (ex. B*A, 3*B)
^			Power for square matrix
.*	./	.\	element-by-element multiplication/division
. ^			element-by-element power
	&	~	element-by-element logic operations
==	~=	< > <= >=	element-by-element comparing operators
inv			inverse of matrix
\			$A \setminus b$ solves the system of equations $A * x = b$
diag			diagonal matrix (ex. B=diag([1,2]) or extract diagonal elements ex. diag(magic(5)) Note: diag(A,k) gives the k-th diagonal diag([3,2],k) builds a matrix with k-th diagonal

Broadcasting operations

Attention: new versions of MATLAB (from 2016b) are broadcasting the operations! You can add a scalar to a matrix

```
>> A = [1, 2; 3, 4];
>> A + 1
ans =
2 3
4 5
```

or vector to a matrix

```
>> b = [3, 2];
>> A+b
ans =
4 4
```

Broadcasting operations

Attention: new versions of MATLAB (from 2016b) is broadcasting the operations! You can also add a vector to a vector

```
>> b = [3, 2];
>> a = [1; 4];
>> a+b
ans =
4 3
7 6
```

Useful commands when dealing with matrices

- assembling matrices

change shape (columns are filled first)

- most functions react on matrices element-wise

submatrices:

```
>> A3(1:3, 2:4)

>> A3(1:3, [1 4 3])

>> A3(1:3, 1:2:6)

>> A3(1:5)

>> A3(1, 1:end)

>> A3(1, 1:end-1)

>> A3(:,1:3)

>> A3(:)
```

- the expression

a:b:c

Generates (row-)vector with values: a to c with step b. If b is omitted it defaults to 1.

Example

```
>> 1:3:10
>> .1:.2:5
>> plot(sin(0:.01:2*pi))
```

Some tips when dealing with matrices

- clear columns or lines:

 array dimension is changed automatically when new elements added:

Useful functions

max, min: find the maximum/ minimum of a vector.
 If applied to matrices: column-wise

```
>> max(A(:))
>> max(A)
```

- similarly: sum, mean, median, std
- to get the dimension along which the function operates:

```
\gg mean(A,2)
```

1.home Construct in Matlab the following matrices in an efficient way (use help diag):

$$C = egin{pmatrix} 5 & 1 & 0 & 0 & 0 & 5 & 5 & 6 & 7 & 8 \ 6 & 0 & 2 & 0 & 0 & 6 & 1 & 0 & 3 & 0 \ 7 & 0 & 0 & 3 & 0 & 7 & 0 & 2 & 0 & 4 \ 8 & 0 & 0 & 0 & 4 & 8 & 5 & 6 & 7 & 8 \end{pmatrix}$$

Exercises... Matrices (home except last)

2.home Generate a 10×10 random matrix.

Find the values and indexes of the biggest element of each row.

Possible hints: sort, max, find

3.home Do the inverse of a random vector (entry–wise).

4.home Write a function, which shifts cyclically a vector or matrix by a certain number of rows down and columns to the right (do not use circshift).

5.home Checkboard:

- a) Generate a random (rand) matrix A with size n × n and a second matrix with the same size as A and which takes some elements of A in the black fields, while zeros everywhere else. The structure of the second matrix should be similar to a chessboard.
- b) Make sure that your code works for the cases when n is even and odd.To check if you succeeded try spy.

- 6.home What in-built functions of MatLab can you use to compute, given a matrix of type $m \times n$:
 - average,
 - median,
 - mode.
- 7.home Check what the following functions do:
 - rank,
 - null,
 - rref,
 - orth.

8.home Consider A to be a square matrix, as

Check the following expressions:

9.home Consider A=rand(10) and check

- 10.diy Generate a table with N values of the function f(x) on the interval [a, b], s.t. you get an $N \times 2$ matrix having on each row x and f(x). Consider the following functions:
 - a) $f(x) = x^5 4x + 1$
 - b) $f(x) = exp(i\frac{x}{10})$

Displaying results

Example (Plot of a vector)

```
>> x=linspace(0,2*pi,200);
  >> y1=sin(x);
  >> v2=cos(x);
  >> plot([y1',y2'])
  >> plot(y1)
  >> plot(y2)
\rightarrow We are interested also in x:
  >> plot(x,[y1', y2'])
  >> plot(x,v1,x,v2)
Try:
  >> plot(y1, y2)
  >> plot([v1; v2])
```

what changes?

Pictures complete of information

For **complete** and **useful** pictures, some features have to be added:

 Legend - when more items displayed, makes it clear what is what

```
>> legend('sin', 'cos')
```

Title

Colors and line tipology (if not specified: automatic colors)

axis labels

- change x-axis ranges to $[0,2\pi]$ and y-axis to [1.5,1.5]

How to save pictures?

To save your plots you can

- in your command window

```
>> fig = figure()
>> % call your plot()
>> savefig('my filename0')
>> savefig(fig,'my filename1')
>> saveas(fig, 'my filename2')
```

 on the figure window go to File > Save As and choose the format

Hint: to change the output format saveas(fig, 'my filename', FORMAT), where you can choose eps, png, etc.

Multiple information on same plot

- when for example in loops, an alternative to

```
>> plot(x, y1, x, y2)
```

is to apply hold on:

- >> plot(x, y1)
- >> hold on
- >> plot(x, y2)
- >> hold off

Multiple plots

if visualization desired in different figures: either

```
>> figure()
>> plot(x,y1)
or specifying
>> figure(1) % Choose figure 1
>> plot(x,y1)
>> figure(2) % Choose figure 2
>> plot(x,y2)
```

Theory..Sub-figures

For multiple sub-figures within a figure:

subplot(a,b,n) with n the place where the plot is collocated,
 while a,b the dimension of the fictive matrix where it is collocated.

Example

```
>> subplot(1,2,1)
>> plot(x,y1)
>> subplot(1,2,2)
>> plot(x,y2)
```

Good to know:

clf to clear active figures

close or close all to close the last or all figures.

Different scale plots

Plot the functions $f(x) = e^{3x}$, $g(x) = x^x$, $h(x) = \log(x)$ and j(x) = 3x on [0.01, 100].

- Use plot
- Use semilogx
- Use semilogy
- Use loglog.

Which plot is better for each function?

This is usefull when we are interested in logarithmic and exponential functions. (Very common in error plots)

Exercises... Plots

- 1.live Plot in one figure the functions $f(x) = e^{x/10} \sin(2\pi x)$ and $g(x) = log(3+x)\cos(4\pi x)$ on the interval [0, 1]. The plot be such that:
 - f is plotted in red colour and dashed lines,
 - $-\ g$ is in blue and it is alternating dots and dashes,
 - it should contain a title "Cute functions"
 - the x axis is delimited by 0 and 1 and it has a label "Time"
 - the y axis is delimited by -2 and 2 and it has a label "Money"
 - there is a legend: for f is "Marc" and for g is "John".

Then, save the plot as "my_cute_functions.fig" and close the figure (using the functions we learned).

Exercises... Plots

- 2.home Check help plot. Create with the obtained information the plots for:
 - a) a function $f = x^2 0.5$ on [-1, 1] with red and dashed lines;
 - b) the functions f = sin(s * x) with s = 1, 2, 3, 4 with different colors and a legend;
 - c) f = sin(2 * pi * x). Here the function values have to be displayed as little black circles at the points x = n/10.

1.live a) Write a function that approximates the derivative of an anonymous function f in a point x_0 using the Taylor expansion

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$
 (1)

- b) Apply the function on $f(x) = \sin(x)$ in $x_0 = 1.2$.
- Test the algorithm with different hs and plot the error for different hs.

2.home

- a) Write a function fibo(n) that computes the members of the Fibonacci sequence 1, 1, 2, 3, 5, 8, ...
 - once without loops as a recursive function

$$f(n) = f(n-1) + f(n-2) \quad \forall n > 2, \quad f(1) = f(2) = 1.$$

- once with loops
- b) Write a documentation text for the implemented function

3.live Check numerically:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

4.diy Bisection method

- a) Write a function, that takes a continuous function f and two values (boundary) a, b, such that $f(a) \cdot f(b) < 0$. The function will return the x for which f(x) = 0. This is done through the bisection method on the interval [a, b].
- b) Check the function on sin to compute pi.

5.home Newton method

a) Write a function, that takes as input a function f, its derivative f' and an initial value x_0 . This function will give as output a zero of f found with the Newton method, that reads

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Be careful, $f'(x) \neq 0$, so, choose properly x_0 .

 b) Check your implemented function using the function sin to compute pi.