Rubik's Cube Group

Akil Marshall

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1 Introduction

It is my goal to apply group theory and search techniques from A.I. to study techniques to find solutions to the Rubik's cube.

Currently the wikipedia page and this article are my primary research leads.

Ultimately I would like to be able to build an environment that can easily facilitate the development of differnt solution seeking methods.

2 What is a Group

A group is a set of objects with method of combinding them called the group operation $\langle G, \cdot \rangle$. The set cannot be empty and the group operation must obey 4 conditions:

- For all $a, b \in G$,

  $$a \cdot b \in G$$

  The group operation is **closed** in G.

- For all $a, b, c \in G$,

  $$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$

  The group operation is **associative** in G.

- There exists $e \in G$ such that for any $a \in G$,

  $$a \cdot e = a = e \cdot a.$$

  There exists an **identity** element of the binary operation in G.

- For all $a \in G$ there exists $a^{-1} \in G$ such that

  $$a \cdot a^{-1} = e = a^{-1} \cdot a.$$

  There exists **inverse** elements of the binary operation in G.

3 The Rubik's Cube Group

It turns out that the Rubik's cube puzzle forms a group. The set consists of the physical manipulations you can do to the puzzle and the binary operation is the composition of those manipulations.[1] to to Put plainly if you turn a turn a Rubik's cube it results in a cube (closure), the "identity" is not manipulating the cube, and each turn and be undone (inverse). With these realizations we will build up a theory to describe manipulations (elements of the groups).

[1]The turns are also associative, suppose τ and σ are some turns of the cube you'll find that $(\sigma\tau\tau^{-1})\sigma^{-1} = \sigma(\tau\tau^{-1}\sigma^{-1})$.