

Notes on MPC Math in Kernel Module

Taran Lynn

August 7, 2018

1 Model

$l(t)$ denotes the RTT and $r(t)$ denotes the pacing rate at time t . Let N be the current time.

1.1 Basic Parameterized Model

$$\Delta l(n) = l(n) - \hat{l}(n)$$

$$\hat{l}(n) = \sum_{i=0}^p a_i l(n-i) + \sum_{j=0}^q b_j r(n-j)$$

$$a'_i = a_i + \frac{l(N-i)\Delta l}{\sum l^2 + \sum r^2}$$
$$b'_i = b_i + \frac{r(N-i)\Delta l}{\sum l^2 + \sum r^2}$$

$$\begin{aligned} \sum_{i=0}^p a'_i l(N-i) + \sum_{j=0}^q b'_j r(N-j) &= \sum_{i=0}^p a_i l(N-i) + \sum_{j=0}^q b_j r(N-j) \\ &\quad + \sum_{i=0}^p \frac{l(N-i)^2 \Delta l}{\sum l^2 + \sum r^2} + \sum_{j=0}^q \frac{r(N-j)^2 \Delta l}{\sum l^2 + \sum r^2} \\ &= \sum_{i=0}^p a_i l(N-i) + \sum_{j=0}^q b_j r(N-j) + \frac{(\sum l^2 + \sum r^2) \Delta l}{\sum l^2 + \sum r^2} \end{aligned}$$

$$\begin{aligned}
&= \hat{l}(N) + \Delta l(N) \\
&= l(N)
\end{aligned}$$

1.2 Square Root Parameterized Model

$$\Delta l(n) = l(n) - \hat{l}(n)$$

$$\hat{l}(n) = \sum_{i=0}^p a_i \sqrt{l(n-i)} + \sum_{j=0}^q b_j \sqrt{r(n-j)}$$

$$a'_i = a_i + \frac{\sqrt{l(N-i)}\Delta l}{\sum l + \sum r}$$

$$b'_i = b_i + \frac{\sqrt{r(N-i)}\Delta l}{\sum l + \sum r}$$

$$\begin{aligned} \sum_{i=0}^p a'_i \sqrt{l(N-i)} + \sum_{j=0}^q b'_j \sqrt{r(N-j)} &= \sum_{i=0}^p a_i \sqrt{l(N-i)} + \sum_{j=0}^q b_j \sqrt{r(N-j)} \\ &\quad + \sum_{i=0}^p \frac{l(N-i)\Delta l}{\sum l + \sum r} + \sum_{j=0}^q \frac{r(N-j)\Delta l}{\sum l + \sum r} \\ &= \sum_{i=0}^p a_i \sqrt{l(N-i)} + \sum_{j=0}^q b_j \sqrt{r(N-j)} + \frac{(\sum l + \sum r)\Delta l}{\sum l + \sum r} \\ &= \hat{l}(N) + \Delta l(N) \\ &= l \end{aligned}$$

Considering the control $r(N)$, we want to maximize

$$\begin{aligned} m(r(N)) &= \frac{\alpha r}{\mu_r(N-1)} - \frac{(1-\alpha)\hat{l}(N)}{\mu_l(N)} \\ &= \frac{(1-\alpha)r}{\mu_r(N-1)} - \frac{(1-\alpha)b_0\sqrt{r(N)}}{\mu_l(N)} - \frac{1-\alpha}{\mu_l(N)} [\hat{l}(N)]_{r(N)=0} \end{aligned}$$

$$\begin{aligned} \frac{d}{dr(N)} m(r(N)) &= 0 \\ \frac{\alpha}{\mu_r(N-1)} - \frac{(1-\alpha)b_0}{2\mu_l(N)\sqrt{r(N)}} &= 0 \end{aligned}$$

$$\sqrt{r(N)} = \frac{(1 - \alpha)b_0\mu_r(N - 1)}{2\alpha\mu_l(N)}$$

$$r(N) = \left(\frac{(1 - \alpha)b_0\mu_r(N - 1)}{2\alpha\mu_l(N)} \right)^2$$

α is a parameter that weighs which parameter we want to minimize more. $\alpha = 0$ only weighs the pacing rate ($r \rightarrow \infty$), and $\alpha = 1$ only weighs RTT ($r = 0$).

1.3 Step Model

$$\begin{aligned}
y_0 &= \bar{l} \\
\Delta l(i) &= l(i) - l(i-1) \\
\Delta r(i) &= r(i) - r(i-1)
\end{aligned}$$

$$\begin{aligned}
\delta l(N) &= l(N) - \hat{l}(N) \\
\hat{l}(N) &= y_0 + \sum_{i=0}^p a_i \Delta l(N-i) + \sum_{j=0}^q b_j \Delta r(N-j)
\end{aligned}$$

$$\begin{aligned}
a'_i &= a_i + \frac{\Delta l(N-i) \delta l(N)}{\sum (\Delta l)^2 + \sum (\Delta r)^2} \\
b'_i &= b_i + \frac{\Delta r(N-i) \delta l(N)}{\sum (\Delta l)^2 + \sum (\Delta r)^2}
\end{aligned}$$

$$\begin{aligned}
y_0 + \sum_{i=1}^p a'_i \Delta l(N-i) + \sum_{j=1}^q b'_j \Delta r(N-j) &= y_0 + \sum_{i=1}^p a_i \Delta l(i) + \sum_{j=1}^q b_j \Delta r(j) \\
&\quad + \sum_{i=1}^p \frac{(\Delta l(N-i))^2 \delta l(N)}{\sum (\Delta l)^2 + \sum (\Delta r)^2} + \sum_{j=1}^q \frac{(\Delta r(N-j))^2 \delta l(N)}{\sum (\Delta l)^2 + \sum (\Delta r)^2} \\
&= y_0 + (\hat{l}(N) - y_0) + \frac{(\sum (\Delta l)^2 + \sum (\Delta r)^2) \delta l(N)}{\sum (\Delta l)^2 + \sum (\Delta r)^2} \\
&= \hat{l}(N) + \delta l(N) \\
&= l(N)
\end{aligned}$$

1.3.1 Rate Prediction

Choose $r(N)$ such that $m(r(N))$ is minimized.

$$\begin{aligned}
m(r(N)) &= \frac{\left[\hat{l}(N) \right]_{\Delta r(N)=r(N)-r(N-1)}}{\mu_l(N)} - \frac{r(N)}{\mu_r(N-1)} \\
&= \frac{b_0 \Delta r(N)}{\mu_l(N)} - \frac{r(N)}{\mu_r(N-1)} + \left[\hat{l}(N) \right]_{\Delta r(N)=0}
\end{aligned}$$

$$= \frac{b_0(r(N) - r(N-1))}{\mu_l(N)} - \frac{r(N)}{\mu_r(N-1)} + [\hat{l}(N)]_{\Delta r(N)=0}$$

$$\begin{aligned} \frac{d}{dr(N)} m(r(N)) &= 0 \\ \frac{b_0}{\mu_l(N)} - \frac{1}{\mu_r(N-1)} &= 0 \\ b_0 &= \frac{\mu_l(N)}{\mu_r(N-1)} \end{aligned}$$

We want to move b_0 towards this value, so consider b'_0 .

$$\begin{aligned} b'_0 &= \frac{\mu_l(N)}{\mu_r(N-1)} \\ b_0 + \frac{\Delta r(N)\delta l(N)}{\sum(\Delta l)^2 + \sum(\Delta r)^2} &= \frac{\mu_l(N)}{\mu_r(N-1)} \\ b_0 + \frac{\Delta r(N)\delta l(N)}{\Delta r(N)^2 + \sum(\Delta l)^2 + \sum_{j=1}^N(\Delta r(N-j))^2} &= \frac{\mu_l(N)}{\mu_r(N-1)} \end{aligned}$$