Classical Physics TFY4345
Computational exercises 2 and 3.

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1 Introduction

This project is an extension on the computational exercise 1, which consisted of looking at projectile motion in the xz-plane, with and without air density and drag correctional models. Here the model will be extended to three dimensions where the projectile moves in reference to the earth. Therefore the Coriolis effect will be an important aspect of the following model. This project considers the Paris Gun from WW1, where the shooting angle, azimuth, deflection angle and highest point of trajectory, will be found.

2 Paris Gun and the Coriolis effect

The Paris gun was used during WW1 to shoot long range missiles from Le mont de Joie near Crépy to Paris, which is a range of approximately 120km. The coordinates of Crépy is (49.605°N 3.514722°E), while the coordinates of Paris is (48.85667°N 2.350833°E). Here Crépy is set in the origin. Figure (1) shows the used coordinate system used for this project.

Other useful constants were used, such as the radius for the Earth $R = 6371\text{km}$, the rotational velocity of the Earth $\omega = 7.29 \cdot 10^{-5}\text{s}^{-1}$, initial speed of the cannon shell $v_0 = 1640\text{ms}^{-1}$ and projectile mass $m = 106\text{kg}$.

First the shooting direction (azimuth), in the local coordinate system (as in figure (1)), was found using the local coordinates given above. The shooting direction was found to be $\theta = -45.89°$. Secondly the shooting angle ϕ was found. This was found by plotting the shooting angle against distance travelled by the projectile d_L, and the distance between Crépy and Paris which is 118.6km corresponds to a shooting angle of $\phi = 12.25°$, which can be seen in figure (2). These calculations disregard the Coriolis effect. Then with the same shooting angle ϕ and shooting direction θ the Coriolis effect was "switched on", which was done to demonstrate the effect on the trajectory. This is shown in figure (3):

In figure (3) we can see that the projectile becomes slightly shifted to the left in its course, as it should be due to the location on the northern hemisphere and according to

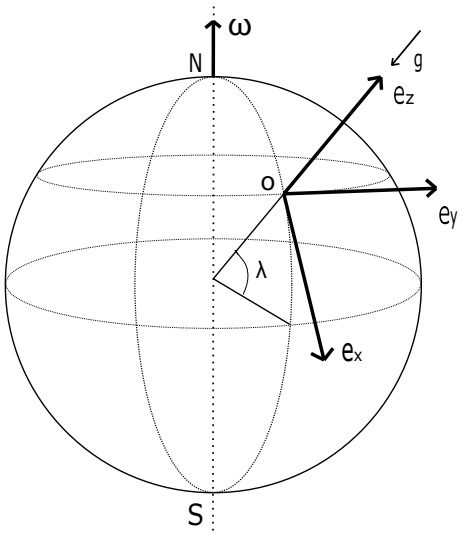

Figure 1: Coordinate system chosen for this project. Crépy is in the origin.

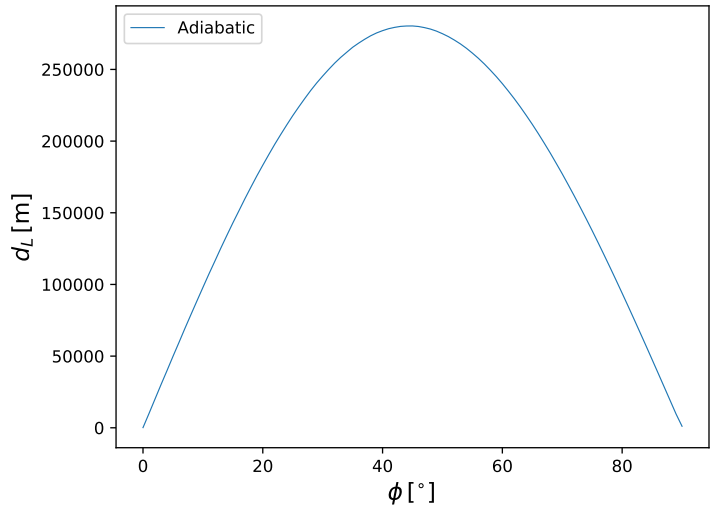

Figure 2: Graph of shooting angle against distance travelled by the projectile. $d_L = 118.6$km from Crépy to Paris, which corresponds to a shooting angle of $\phi = 12.25°$.

the right hand rule. Without the Coriolis effect the distance travelled by the projectile was $d_{L1} = 118.6$km, with the Coriolis effect the distance was $d_{L2} = 118.1$km, which gives a difference of $\Delta d_L = d_{L1} - d_{L2} = 506$m. Without the Coriolis effect the maximum height for the trajectory was $h_1 = 6435.7$m, the maximum height with the Coriolis effect was found to be $h_2 = 6408.2$m, which in turn gives a difference of $\Delta h = h_1 - h_2 = 27.5$m. The deflection angle $\Delta\theta$ was found to be $\Delta\theta = 0.302°$.The total deflection was found to be $d = 803.1$m.

Next the code was modified such that one could shoot in any direction from Crépy, where the user is provided to choose the coordinates, within the shooting range. The firing angle and azimuth is then automatically corrected for the Coriolis deflection. This is demonstrated in figures (4), (5) and (6).

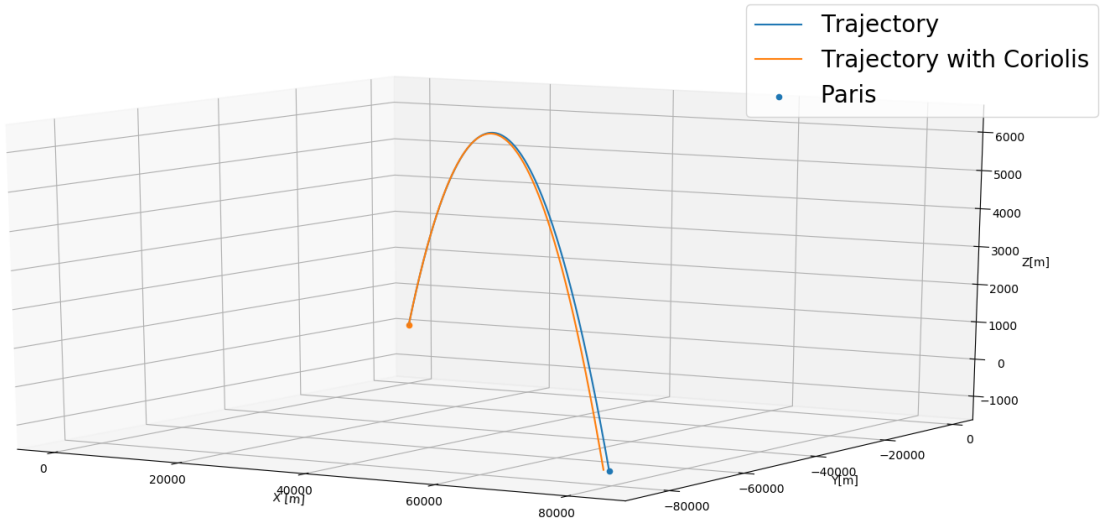

Figure 3: Trajectory from Crépy to Paris, with and without the Coriolis effect.

References

[1] TFY4345 2018 – Compulsory Exercise 2. 2018

[2] TFY4345 2018 – Compulsory Exercise 3. 2018

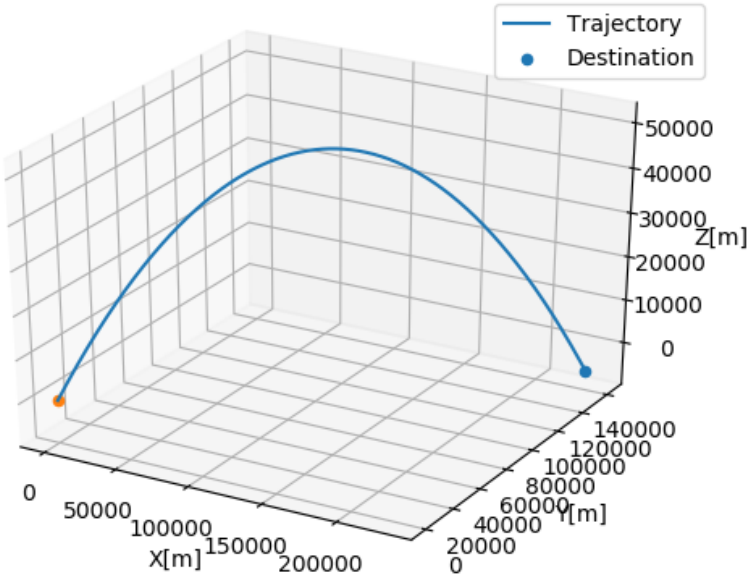

Figure 4: Projectile fired from Crépy to a destination of Latitude 47.5° and Longitude 5.4°. The projectile was shoot to the east of Crépy.

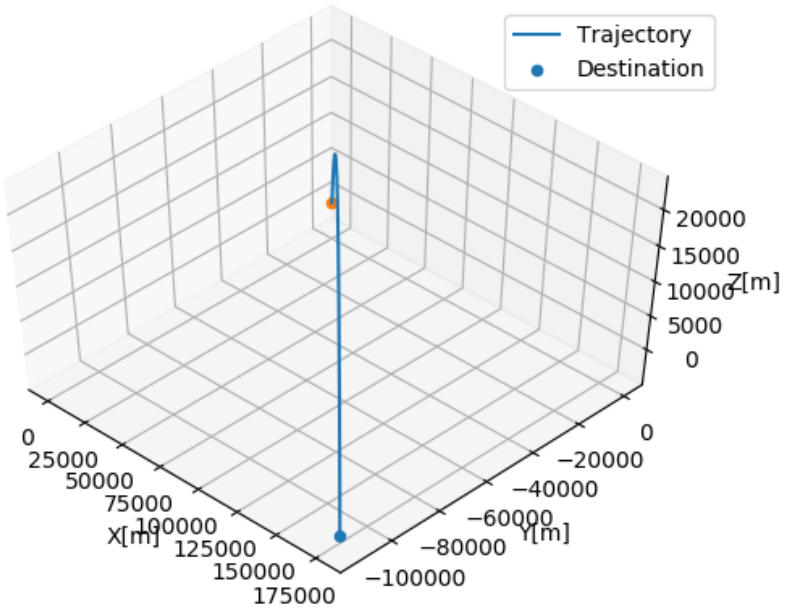

Figure 5: Projectile fired from Crépy to a destination of Latitude 48.0° and Longitude 2.0°. The projectile was shot in approximately the same direction as Paris, but further.

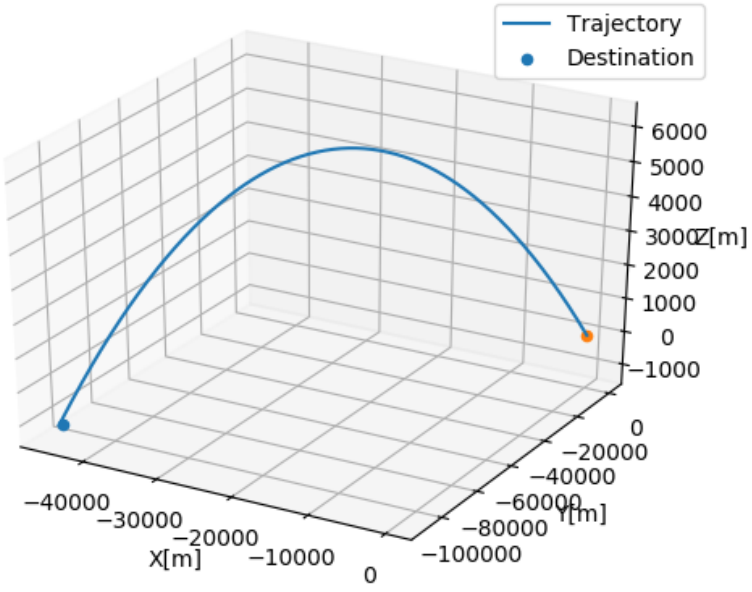

Figure 6: Projectile fired from Crépy to a destination of Latitude 50.0° and Longitude 2.0°. The projectile was shot to the south west of Crépy.