1.

1. (1 punct) Valoarea lui λ astfel încât planele $2x - y + 2z - 2 = 0$, $x - 2y + z + 4 = 0$ și $x + \lambda y + z - 4 = 0$ să fie fețele laterale ale unei prisme triunghiulare este:

○ 3; ○ 0; ○ 1; ○ -3.

nu știu

2.

2. (1 punct) Se consideră paralelogramul construit pe vectorii $\mathbf{u}(2,4,-5)$ și $\mathbf{v} = (1,2,3)$. Atunci un vector de lungime 1, coliniar cu una dintre diagonalele paralelogramului este vectorul:

○ $\frac{1}{7}(3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k})$; ● $\frac{1}{7}(3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$; ○ $\frac{1}{\sqrt{69}}(-\mathbf{i} - 2\mathbf{j} + 8\mathbf{k})$; ○ $\frac{1}{\sqrt{69}}(\mathbf{i} + 2\mathbf{j} + 8\mathbf{k})$.

diagonale paralelogramului: $u + v = d_1$
$u - v = d_2$

$d_1 = (2,4,-5) + (1,2,3)$
$= (3,6,-2)$

$d_2 = (2,4,-5) - (1,2,3) = (1,2,-8)$

3. Normalizarea vectorilor (obținerea versorilor):

Un vector de lungime 1 (versor) se obține împărțind fiecare componentă la norma vectorului.

- **Pentru** $(3, 6, -2)$:

$$\text{Norma} = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7.$$

Versorul:

$$\left(\frac{3}{7}, \frac{6}{7}, -\frac{2}{7}\right) = \frac{1}{7}(3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}).$$

- **Pentru** $(1, 2, -8)$:

$$\text{Norma} = \sqrt{1^2 + 2^2 + (-8)^2} = \sqrt{1 + 4 + 64} = \sqrt{69}.$$

Versorul:

$$\left(\frac{1}{\sqrt{69}}, \frac{2}{\sqrt{69}}, \frac{-8}{\sqrt{69}}\right)$$

$$= \frac{1}{\sqrt{69}}(i + 2j - 8k)$$ nu e

$$= \frac{1}{\sqrt{69}}(i+2j-8k)$$ nu e în răspunsuri

3.

○ $\frac{1}{7}(3\mathbf{i}-6\mathbf{j}-2\mathbf{k})$; ○ $\frac{1}{7}(3\mathbf{i}+6\mathbf{j}-2\mathbf{k})$; ○ $\frac{1}{\sqrt{69}}(-\mathbf{i}-2\mathbf{j}+8\mathbf{k})$; ...

3. (1 punct) Care dintre următoarele ecuații reprezintă dreapta de intersecție a planelor $4x-4y-z+11=0$ și $x+2y-z-1=0$?
○ $\frac{x-2}{1}=\frac{y}{1}=\frac{z-3}{4}$; ○ $\frac{2x-13}{4}=\frac{4y-15}{4}=\frac{z}{4}$; ○ $\frac{x}{2}=\frac{y-2}{1}=\frac{z-3}{4}$; ○ $\frac{x-4}{-2}=\frac{y-4}{2}=\frac{z-11}{2}$.

... 20. Coordonatele vârfului A sunt $(-5,0)$, iar cele ale ... lui C pot fi:

$$\begin{cases} 4x-4y-z+11=0 \\ x+2y-z-1=0 \end{cases}$$

$$x=0 \Rightarrow \begin{cases} -4y-z=-11 \\ +2y-z=1 \end{cases} \quad +$$

$$-2y=-10$$

$$y=5 \qquad z=2y-1$$

$$z=9$$

$$\Rightarrow P(0,5,9)$$

$$\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} = \begin{vmatrix} -4 & -1 \\ 2 & -1 \end{vmatrix} = 4+2=6$$

$$\begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} = \begin{vmatrix} -1 & 4 \\ -1 & 1 \end{vmatrix} = -1+4=3 \qquad \vec{d}(6,3,12)$$

$$3\underbrace{(2,1,4)}$$

$$\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = \begin{vmatrix} 4 & -4 \\ 1 & 2 \end{vmatrix} = 8+4=12$$

$$\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = \begin{vmatrix} 4 & -4 \\ 1 & 2 \end{vmatrix} = 8+4 = 12$$

echivalent

$$\Delta: \frac{x-0}{2} = \frac{y-5}{1} = \frac{z-9}{4}$$

- $\frac{x-2}{1} = \frac{y}{1} = \frac{z-3}{4}$; ✗ vector director greșit
- $\frac{2x-13}{4} = \frac{4y-15}{4} = \frac{z}{4}$; ✗ vector director greșit
- $\frac{x}{2} = \frac{y-2}{1} = \frac{z-3}{4}$; ✓ același vector director

$x=0,\ y=2,\ z=3 \in$ planelor?

$$\begin{cases} 4\cdot 0 - 4\cdot 2 - 3 + 11 = 0 \\ 0 + 2\cdot 2 - 3 - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} -8-3+11=0 \\ 4-4=0 \end{cases} (A)$$

- $\frac{x-4}{-2} = \frac{y-4}{2} = \frac{z-11}{2}$ ✗ vector director greșit

4

$\frac{y-4}{2} = \frac{z-11}{2}$.

4. (1 punct) Aria unui triunghi ABC este egală cu 20. Coordonatele vârfului A sunt $(-5,0)$, iar cele ale lui B sunt $(3,0)$. Vârful C se află pe dreapta $x = 1+t$, $y = -1+t$, $t \in \mathbb{R}$. Coordonatele lui C pot fi:
○ $(5,3)$; ● $(-3,-5)$; ○ $(-5,-7)$; ● $(7,5)$.

... trece prin punctul $M_0(8,6)$ și formează, împreună cu axele de

$A_{\Delta ABC} = 20$ $\quad A(-5,0)$

$B(3,\ )$

$$A_{\Delta ABC} = 20 \qquad A(-5,0)$$

$$B(3,0)$$

$C \in \Delta$

$$\Delta: \begin{cases} x = 1 + t \\ y = -1 + t \end{cases} \Rightarrow t = x - 1$$

$$\Rightarrow y = -1 + x - 1$$

$$\Rightarrow \underline{x - y - 2 = 0}$$

$$\Rightarrow x_C - y_C - 2 = 0$$

$$\Rightarrow x_C = y_C + 2$$

$$A_\Delta = \frac{1}{2} \cdot \left| \begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} \right|$$

$$= \frac{1}{2} \cdot \left| \begin{vmatrix} -5 & 0 & 1 \\ 3 & 0 & 1 \\ y_C+2 & y_C & 1 \end{vmatrix} \right|$$

$$= \frac{1}{2} \cdot \left| y_C \cdot (-1)^{3+2} \cdot \begin{vmatrix} -5 & 1 \\ 3 & 1 \end{vmatrix} \right|$$

$$= \frac{1}{2} \cdot \left| y_C \cdot (-5-3) \right|$$

$$= \frac{1}{2} \cdot \left| -8 y_C \right| = 4 |y_C| = 20$$

$$\Rightarrow y_C = \pm 5$$

$\Rightarrow y_C = \pm 5$

$y_C = 5 \Rightarrow x_C = 7$

$y_C = -5 \Rightarrow x_C = -3$

$\Rightarrow C(7, 5)$ sau $C(-3, -5)$ ✓

5

○ $(5,3)$; ○ $(-3,-5)$; ○ $(-5,-7)$; ○ $(7,5)$.

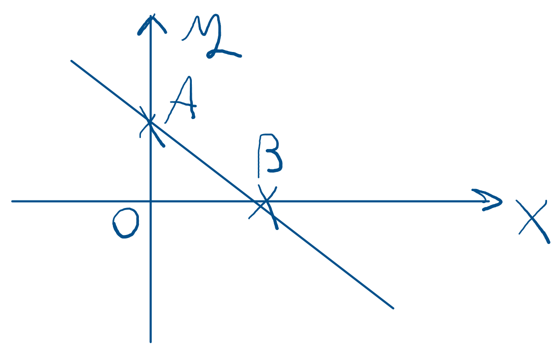
5. (1.5 puncte) Scrieţi ecuaţia dreptei care trece prin punctul $M_0(8,6)$ şi formează, împreună cu axele de coordonate, un triunghi de arie 12 $\frac{3}{2}x - y - 6 = 0$; $\frac{3}{8} \cdot x - y + 3 = 0$.

$M_0 \in d$

$d: y - 6 = m(x - 8)$

$\Rightarrow y - 6 = mx - 8m$

$mx - y - 8m + 6 = 0$



axele de coord. $x = 0 \Rightarrow y = 6 - 8m$

$y = 0 \Rightarrow x = \frac{8m-6}{m}$

$$A_\Delta = \frac{1}{2} \cdot \left| \begin{vmatrix} 0 & 0 & 1 \\ 0 & 6-8m & 1 \\ \frac{8m-6}{m} & 0 & 1 \end{vmatrix} \right|$$

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 6-8m & 1 \\ \frac{8m-6}{m} & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 6-8m \\ \frac{8m-6}{m} & 0 \end{vmatrix}$$

$$= -1 \left(6-8m \cdot \frac{8m-6}{m} \right)$$

$$= +1 \frac{\left((8m-6)^2 \right)}{m} =$$

$$\Rightarrow \frac{1}{2} \left| \frac{(8m-6)^2}{m} \right| = 12$$

$$\Rightarrow (8m-6)^2 = 24|m|$$

$$[2(4m-3)]^2 = 24|m|$$

$$4(4m-3)^2 = 24|m|$$

$$(4m-3)^2 = 6|m|$$

I $m \geq 0 \Rightarrow 16m^2 - 24m + 9 = 6m$

1

$$16m^2 - 30m + 9 = 0$$

$$\Delta = 900 - 576$$

$$= 324 = 18^2$$

$$m_1 = \frac{30+18}{32} = \frac{48}{32} = \frac{24}{16} = \frac{12}{8} = \frac{3}{2} > 0$$

$$m_1 = \frac{30+18}{32} = \frac{48}{32} = \frac{24}{16} = \frac{12}{8} = \frac{3}{2} \geq 0$$

$$m_2 = \frac{30-18}{32} = \frac{12}{32} = \frac{3}{8} \geq 0$$

II $m < 0 \Rightarrow 16m^2 - 24m + 9 = -6m$

$$16m^2 - 18m + 9 = 0$$

$$\Delta = 324 - 576 < 0$$

$d: mx - y - 8m + 6 = 0$

$m = \frac{3}{2} \Rightarrow d: \frac{3}{2}x - y - 12 + 6 = 0$

$$\boxed{\frac{3}{2}x - y - 6 = 0}$$ ✓

$A(4, 0) \in d$

$B(0, -6) \in d$

$m = \frac{3}{8} \Rightarrow d: \frac{3}{8}x - y - 3 + 6 = 0$

$$\boxed{\frac{3}{8}x - y + 3 = 0}$$ ✓

$-20i + 7j - 11k$

6

coordonate, un triunghi de arie 12

6. (1.5 puncte) Calculaţi vectorul $\mathbf{b}\times(\mathbf{a}+\mathbf{b}-\mathbf{a}\times\mathbf{b})$, cu $\mathbf{a}(1,-1,1)$ şi $\mathbf{b} = 2\mathbf{i}+\mathbf{j}-3\mathbf{k}$

$\frac{x-1}{} = \frac{y+2}{} = \frac{z-5}{}$ şi $\Delta_2: \frac{x-7}{2} = \frac{y-2}{2} = \frac{z-1}{-2}$ sunt

$a(1,-1,1)$

$b(2,1,-3)$

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix} = i \cdot \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} - j \cdot \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + k \cdot \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= i(3-1) - j(-3-2) + k(1+2)$$

$$= 2i + 5j + 3k$$

$a + b = (3,0,-2)$

$$b \times (a + b - a \times b) = b \times ((3,0,-2) - (2,5,3))$$

$$= b \times (1,-5,-5)$$

$$= \begin{vmatrix} i & j & k \\ 2 & 1 & -3 \\ 1 & -5 & -5 \end{vmatrix} = i \begin{vmatrix} 1 & -3 \\ -5 & -5 \end{vmatrix} - j \begin{vmatrix} 2 & -3 \\ 1 & -5 \end{vmatrix} + k \begin{vmatrix} 2 & 1 \\ 1 & -5 \end{vmatrix}$$

$$= i(-5-15) - j(-10+3) + k(-10-1)$$

$$= \underline{-20i + 4j - 11k}$$ ✓

7.

(1.5 puncte) Calculaţi vectorul $\mathbf{b} \times (\mathbf{a} + \mathbf{b} - \mathbf{a} \times \mathbf{b})$, cu a(...

(2 puncte) Verificaţi dacă dreptele $\Delta_1: \frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-5}{4}$ şi $\Delta_2: \frac{x-7}{3} = \frac{y-2}{2} = \frac{z-1}{-2}$ sunt coplanare şi, în caz afirmativ, scrieţi ecuaţia planului determinat de ele $2x-16y-13z+31=0$.

$$\Delta_1: \frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-5}{4} = t$$

$$\Delta_2: \frac{x-7}{3} = \frac{y-2}{2} = \frac{z-1}{-2} = t$$

$$\vec{d_1}(2,-3,4)$$

$$\vec{d_2}(3,2,-2)$$

$$P_1 \in d_1 \Rightarrow P_1(1,-2,5)$$

$$P_2 \in d_2 \Rightarrow P_2(7,2,1)$$

$$\overrightarrow{P_1P_2}(6,4,-4)$$

$$\begin{vmatrix} 2 & -3 & 4 \\ 3 & 2 & -2 \\ 6 & 4 & -4 \end{vmatrix} = 2 \cdot \begin{vmatrix} 2 & -3 & 4 \\ 3 & 2 & -2 \\ 3 & 2 & -2 \end{vmatrix} = 0$$

$\Rightarrow$ dr. coplanare

$\Rightarrow$ dr. coplanare

$\frac{2}{3} \neq \frac{-3}{3} \neq \frac{4}{-2} \Rightarrow$ nu sunt paralele

$\Rightarrow$ se $\cap$ într-un punct

$\Rightarrow$ drepte concurente

Ecuaţia planului determinat de două drepte concurente

Dreptele concurente

$$\Delta_1): \frac{x-x_0}{l_1} = \frac{y-y_0}{m_1} = \frac{z-z_0}{n_1}$$

şi

$$(\Delta_2): \frac{x-x_0}{l_2} = \frac{y-y_0}{m_2} = \frac{z-z_0}{n_2},$$

care trec prin punctul $M_0(x_0, y_0, z_0)$, determină un plan de ecuaţie

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0.$$

găsim pct. de intersecție

$$\Delta_1: \begin{cases} x = 2t+1 \\ y = -3t-2 \\ z = 4t+5 \end{cases} \qquad \Delta_2: \begin{cases} x = 3s+7 \\ y = 2s+2 \\ z = -2s+1 \end{cases}$$

$$\begin{cases} 2t+1 = 3s+7 & (1) \\ -3t-2 = 2s+2 & (2) \\ 4t+5 = -2s+1 & (3) \end{cases}$$

$(2)+(3) \Rightarrow t+3 = 3$

$\Rightarrow t = 0$

$$\Rightarrow \begin{cases} 3t+7=1 \Rightarrow t=-2 \\ 2t+2=-2 \Rightarrow t=-2 \\ -2t+1=5 \Rightarrow t=-2 \end{cases}$$

$$\Rightarrow P(1,-2,5)$$

$$\Rightarrow \begin{vmatrix} x-1 & y+2 & z-5 \\ 2 & -3 & 4 \\ 3 & 2 & -2 \end{vmatrix} = 0$$

$$= (x-1)\begin{vmatrix} -3 & 4 \\ 2 & -2 \end{vmatrix} + (y+2)\begin{vmatrix} 4 & 2 \\ -2 & 3 \end{vmatrix} + (z-5)\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix}$$

$$= (x-1)(6-8) + (y+2)(12+4) + (z-5)(4+9)$$

$$= -2(x-1) + 16(y+2) + 13(z-5)$$

$$= -2x + 16y + 13z + 2 + 32 - 65$$

$$\begin{array}{r} 65- \\ 34 \\ \hline 31 \end{array}$$

$$= -2x + 16y + 13z - 31$$

$$\Rightarrow \boxed{2x - 16y - 13z + 31 = 0}$$

$$\boxed{2x - 16y - 13z + 31 = 0}$$