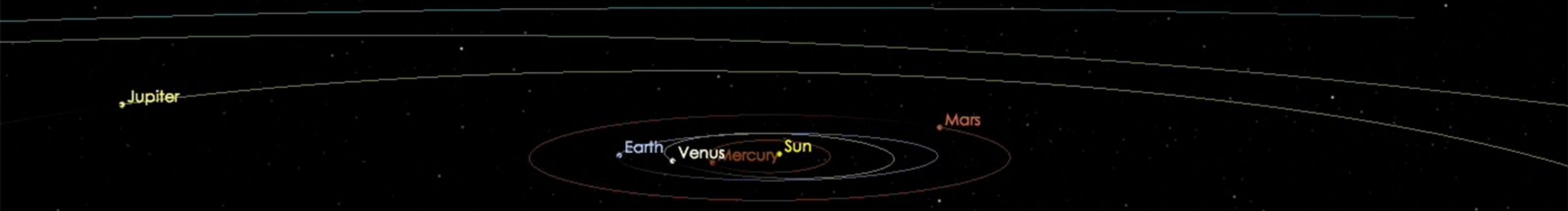


Co-Design for Planetary Defense

1.144 Final Presentation

Niclas Scheuer
Ananth Venkatesh

Our planet is a lonely speck in the great enveloping cosmic dark... our responsibility to deal more kindly with one another, and to preserve and cherish the pale blue dot, the only home we've ever known.

Carl Sagan

NEO Defense Framework

Table of Contents

Motivation

Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.

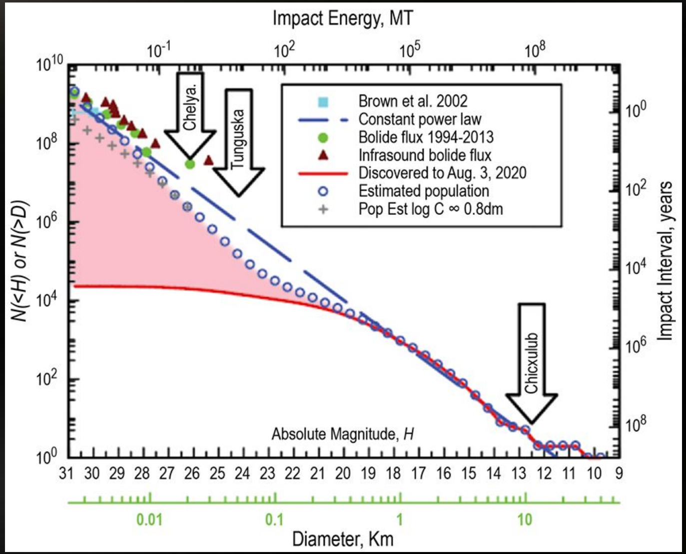
February 13, 2013

- 13-meter diameter meteor strikes Chelyabinsk, Russia
- > 1600 injured
- No prior warning

Motivation

Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.

Majority of NEOs of size 50-140m remain **undiscovered.**



"The number of undiscovered NEOs larger than 140m is on the order of 10,000"

- NASA Decadal Planetary Defense Study [3]

Motivation

Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.

Majority of NEOs of size 50-140m remain **undiscovered.**

Current NEO warning systems give us limited warning times.

Warning Times [3]:
- 20m NEO: 1 day
- 100m NEO: 21 days

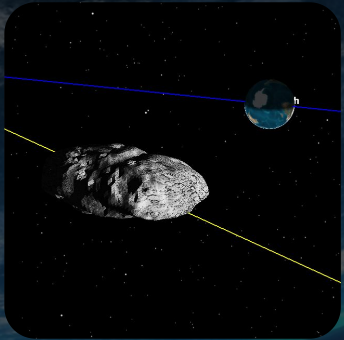
Current Interception Times [4]:
- 54 months

How to defend our planet from Near-Earth Objects

Categorical Approach for **transparency**, **tractability**, and **interoperability**

Multiple Stakeholders

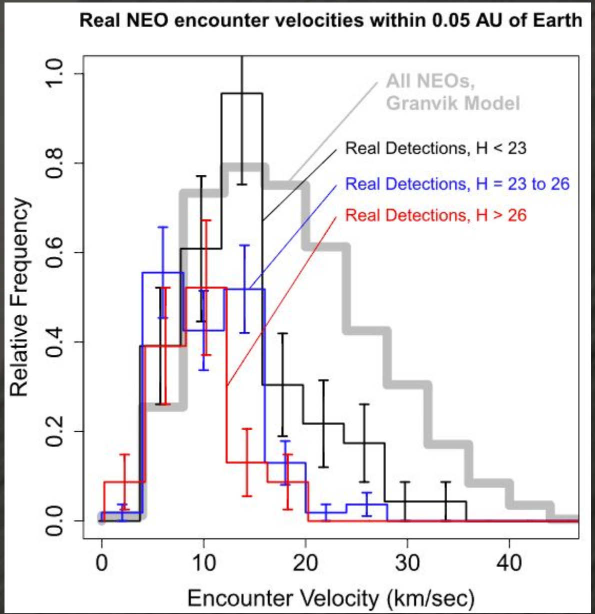
Multiple Approaches



Complex Dynamics

Modeling Threats

Models for NEO distributions exist and are piecewise validated by surveyors.

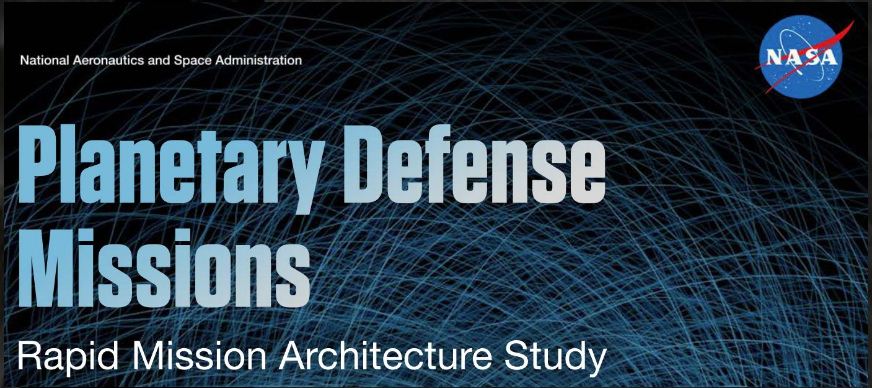


Size, Velocity, Albedo, Inclination, Semi-Major Axis, Eccentricity Data and Estimates Exist.

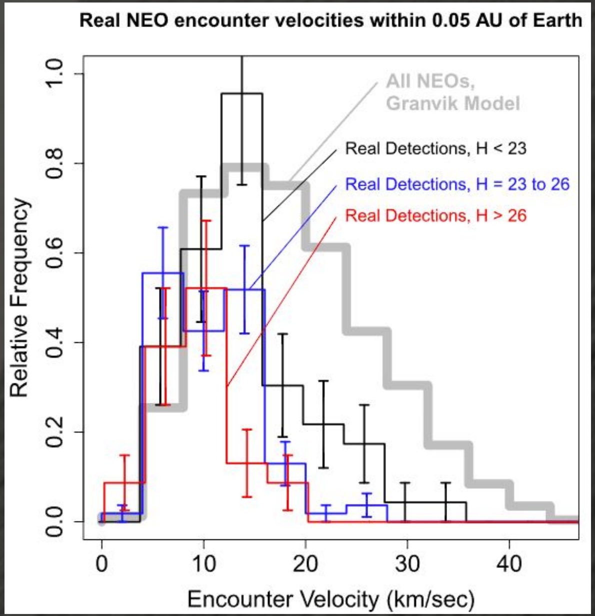
The population of near-earth asteroids revisited and updated

Alan W. Harris[a,*], Paul W. Chodas

[a] MoreData! Inc., La Cañada, CA 91011, USA
[b] Jet Propulsion Laboratory, Pasadena, CA 91109, USA



Modeling Threats



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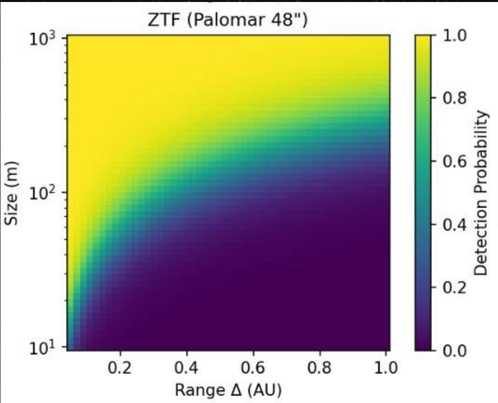
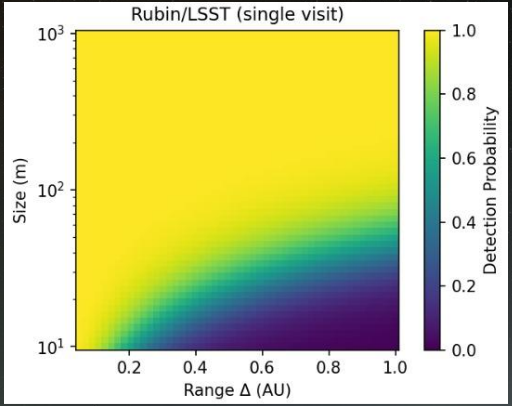
$$\text{SizeDistribution} : \text{Size} \to [0, 1]$$

$$\text{VelocityDistribution} : \text{Velocity} \to [0, 1]$$

$$\text{AlbedoDistribution} : \text{Albedo} \to [0, 1]$$

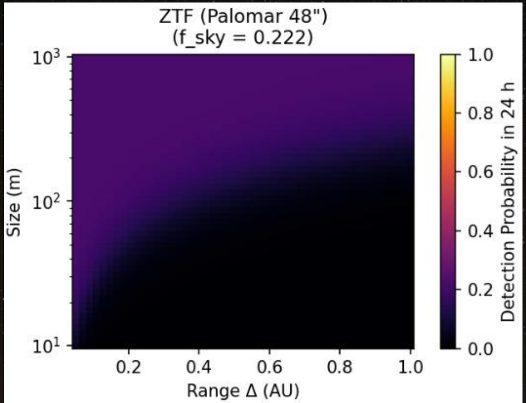


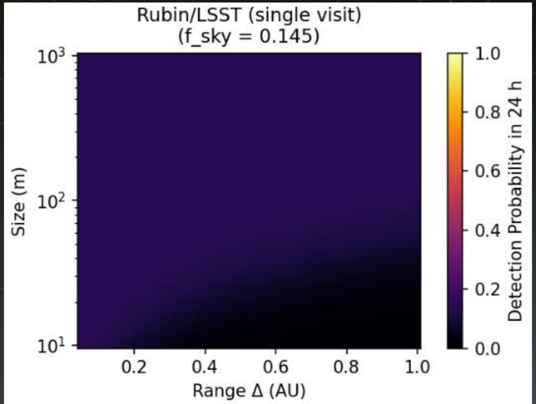


Modeling Sensors

Using publicly-available data on several ground stations, we characterize the <u>single-exposure detection probability</u> and <u>24-hour detection probability.</u>

$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \to [0, 1]$$





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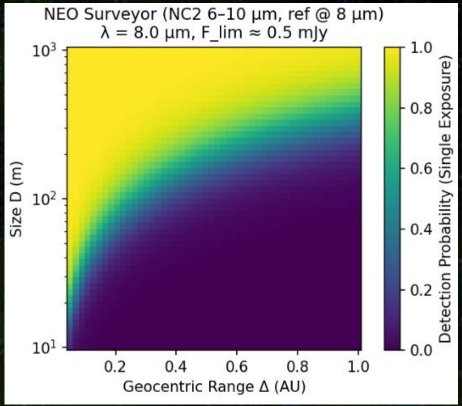
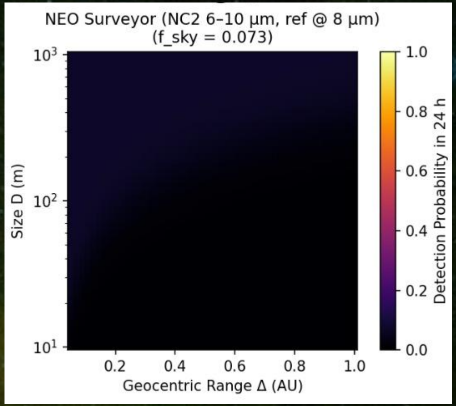
$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \to [0, 1]$$

Consider:

- Limited sensor cone of sensor
- 24h isotropic sky-coverage approximation

$$\mathbb{P}_{24\text{ hr}} : \text{Range} \times \text{Size} \to [0, 1]$$





Modeling Sensors

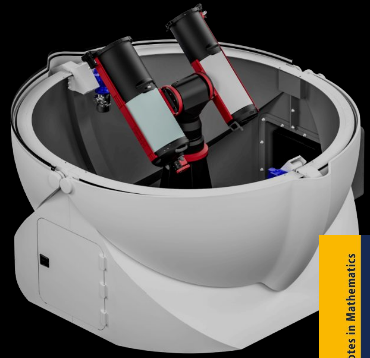
Similar calculation for space-based sensors, using infrared emissivity.

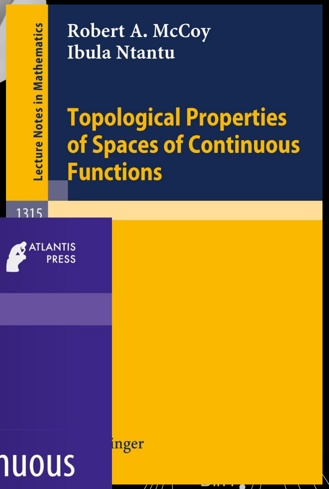
- 24h isotropic sky-coverage approximation
- Efficacy scaling based on cone angle

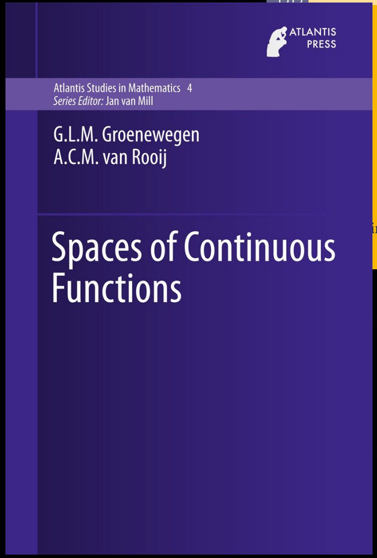
$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \to [0,1]$$
$$\mathbb{P}_{24\text{ hr}} : \text{Range} \times \text{Size} \to [0,1]$$

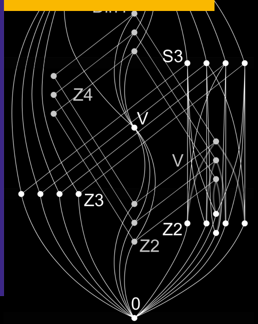
Sensor topology

Sensor detection functions are distributions over **range x size** invariant to constant multiples of the same **range–size** pair; parametrized by **range:size** ratio





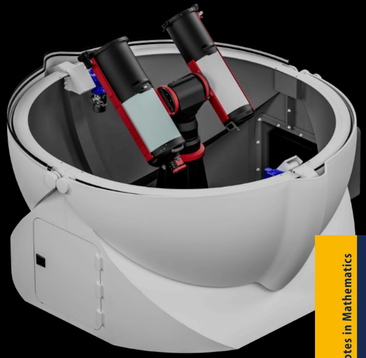




Sensor topology

Dimensionless poset of **range:size** ratios forms domain for sensor detection functions, which are **monotone maps** from the opposite of this poset to the **unit interval [0, 1]**.

Specifically, low range:size gives high probability of detection and high range:size gives low probability of detection. We then have an ordering of sensors based on the standard ordering of monotone maps.



Combining Sensors

Posets:

Range x Size

$$(r_1, s_1) \leq (r_2, s_2) \text{ if } r_1 \geq r_2, s_1 \leq s_2$$

Probabilities

$$[0,1], \leq$$

Monotone Maps:

$$\mathbb{P}_{24\text{hr}} : \text{NEOProperties} \to \text{Probability}$$
$$(r_1, s_1) \leq (r_2, s_2) \Rightarrow P_{24h}(r_1, s_1) \leq P_{24h}(r_2, s_2)$$

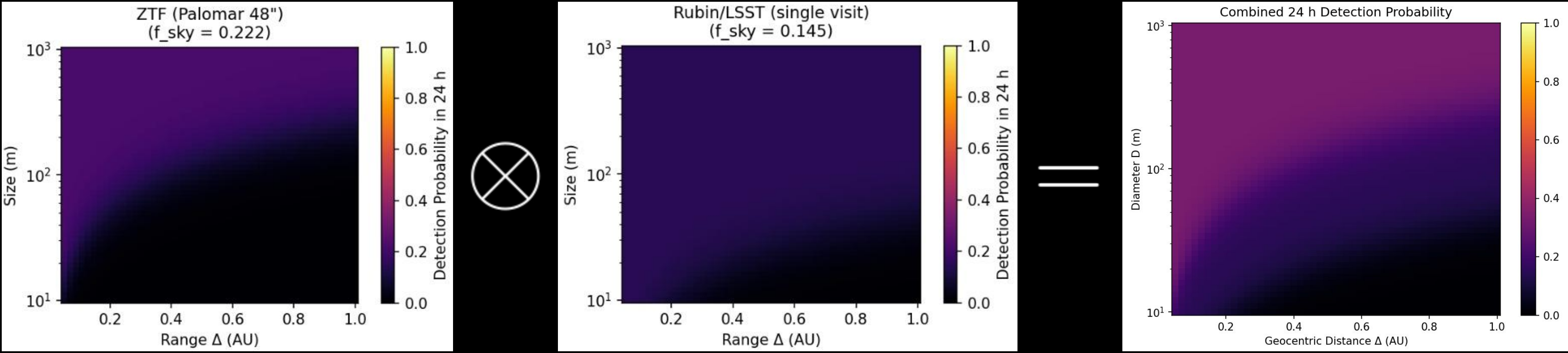
Monoidal probability fusion

$$p_1 \otimes p_2 = 1 - (1 - p_1)(1 - p_2)$$

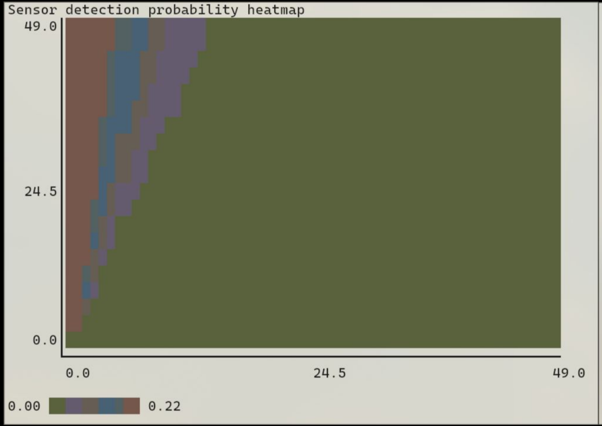
Commutative monoid on probabilities; gives a new detection function better than or the same as its composite detection functions

(This is **not** the probability *monad*)

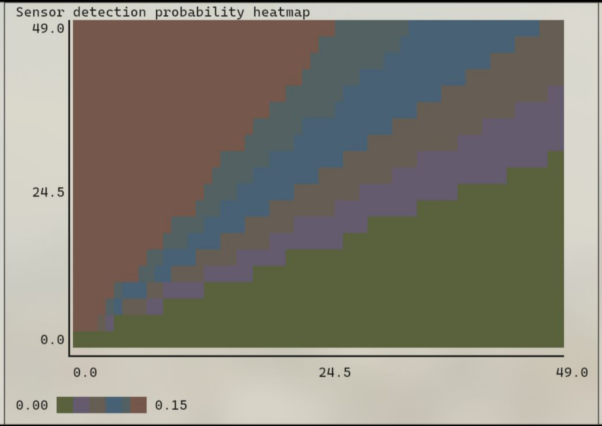
Combining Sensors



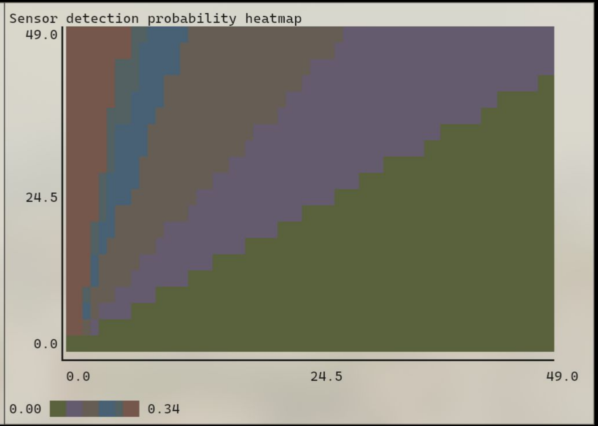
Discretized MCDP sensor combinations



ZTF (Palomar)
max detection probability:
22%



Rubin/LSST
max detection probability:
15%



Combined
max detection probability:
34%

Detection

Over the distributions

$$\text{SizeDistribution} : \text{Size} \to [0,1]$$
$$\text{VelocityDistribution} : \text{Velocity} \to [0,1]$$
$$\text{AlbedoDistribution} : \text{Albedo} \to [0,1]$$

what is the probability that a NEO stays undetected?

Detection

Start survival function
at
$$S(\tau_k) = 1$$

NEO begins
undetected at a large
time-to-impact TTI

Detection

$$\lambda(\tau) \sim \mathrm{E}[P_{24h}(R, S)]$$

Calculate the expectation of detecting the NEO in 24h

Detection

$$S(\tau_k) = S(\tau_{k+1})(1 - \lambda(\tau_k))$$

Update survival
function

Detection

As more time passes, we get more attempts at spotting the NEO.

> Slow NEOs easier to catch

Detection

As the NEO gets closer, we get better at detecting!

$$\mathbb{P}_{24\text{hr}} : \text{NEOProperties} \rightarrow \text{Probability}$$

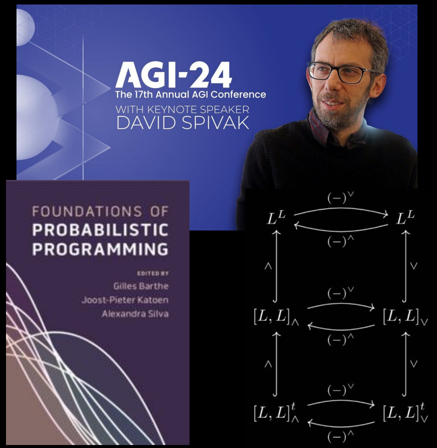
is a monotone map

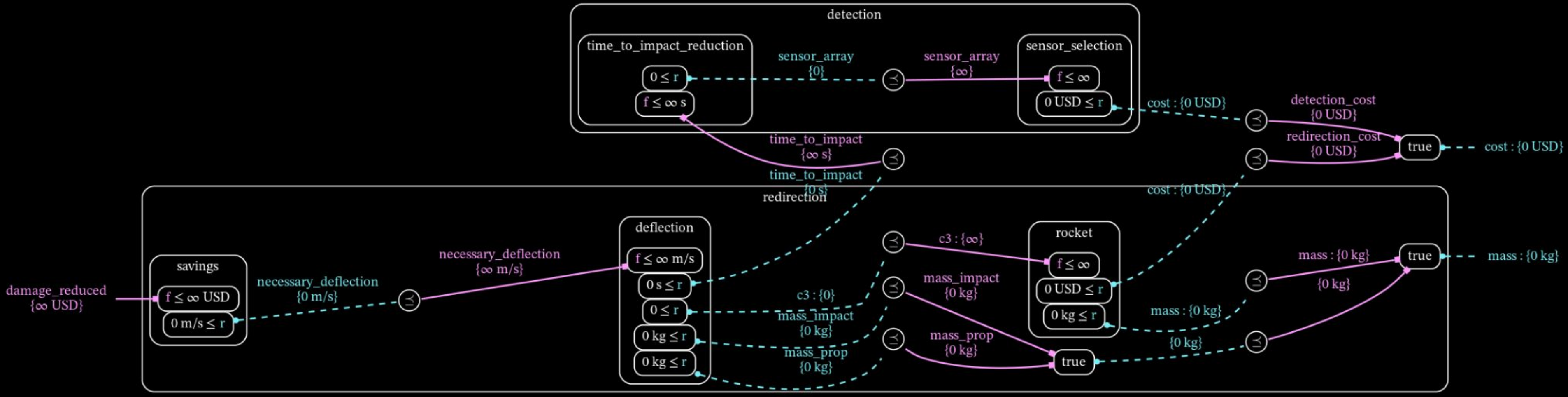
Detection

Survival function reaches 0

The case for co-design

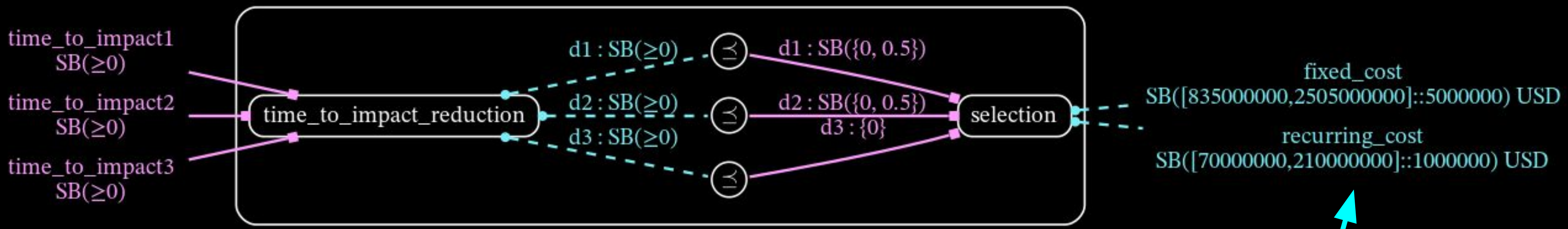
Categorical informatics, pareto fronts, probability monads, and the like





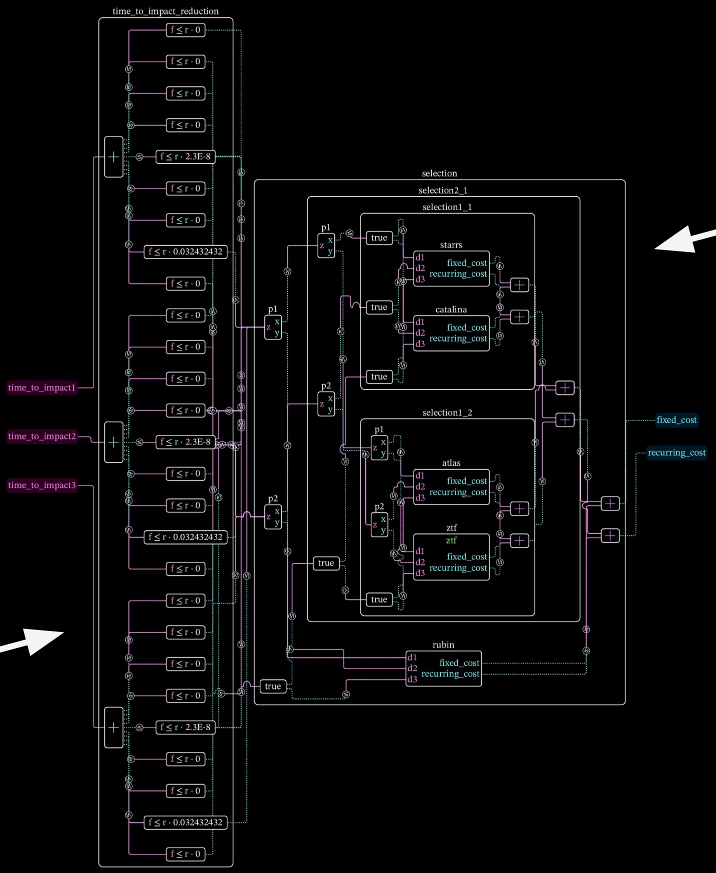
System diagram

Instead of expected time to impact, compute the hazard function at representative times to get probabilities of detection



Split fixed and recurring costs

Zoom in on revised detection block



Sensor selection problem (repeated applications of the probability monoid)

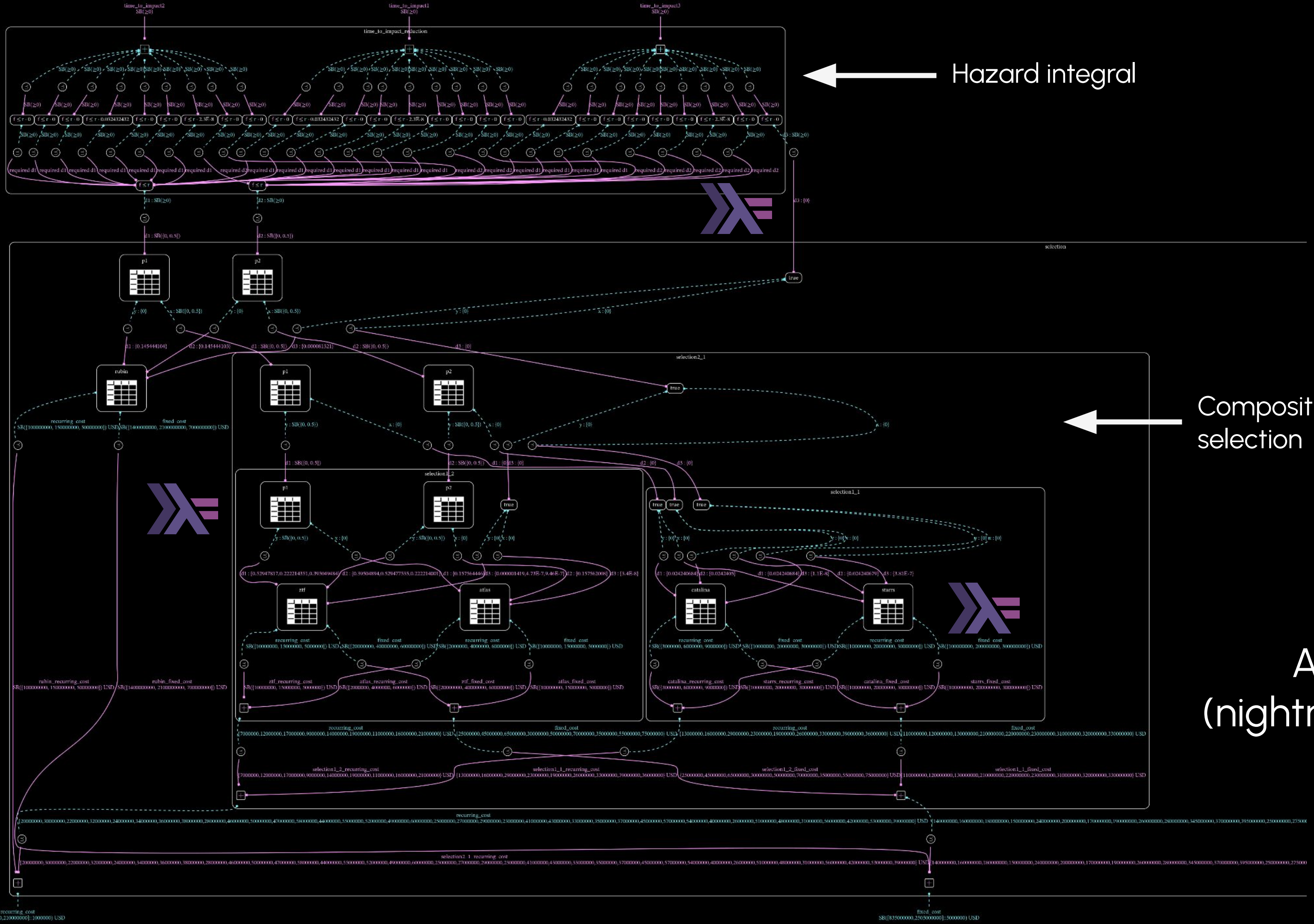
24 hour detection probabilities at representative times

Costs

Hazard function (double integral approximated as Riemann sum)

Highly discretized (3 point approximation) DP representing sensor selection

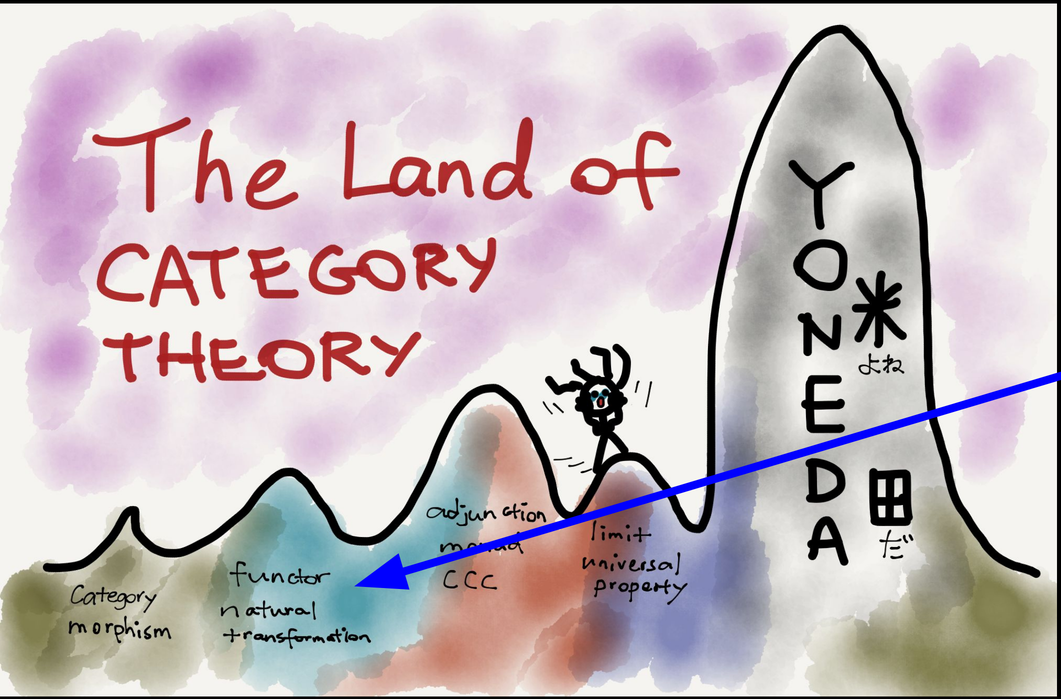
(full DP is orders of magnitude more cursed)



Hazard integral

Compositional sensor selection

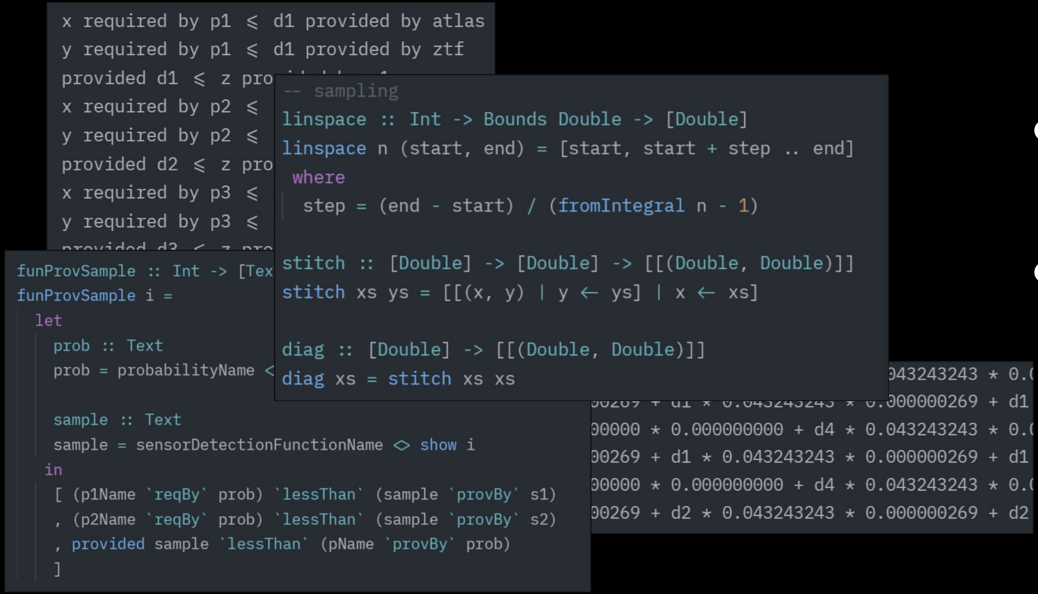
Another view (nightmare design problem)



Functionality of the solution map means we can get away with very inefficient specifications, provided they're compositional

(we heavily abused this property in order to represent our problem in MCDP)

MCDP

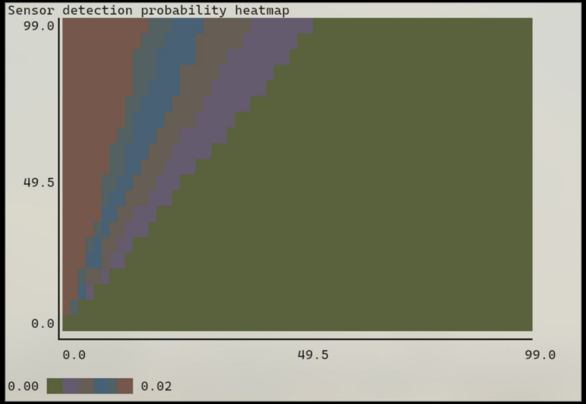


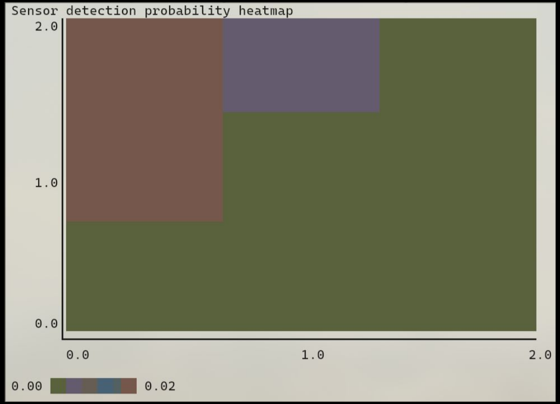
- Discretize everything
- Haskell-based templating metalanguage
- Dynamically set resolution and solve for antichains
- Describe sensor combination with compositions of the probability monoid

Results

(with MCDP)

- Crashed web editor multiple times
- Wasted tremendous amounts of compute
- Haskell code worked flawlessly
- MCDP killed itself several times likely due to (d)OOM issues





Discretization :(

2.58%

Maximum 24-hour detection probability with a $2.5 billion budget

(recurring cost bounded at 10% of total budget)

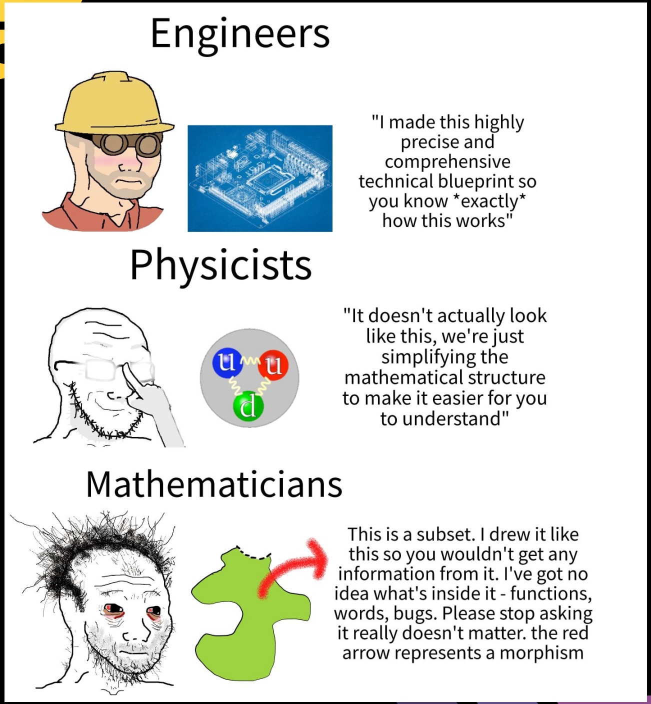
27 days

Average detection time with a $2.5 billion budget

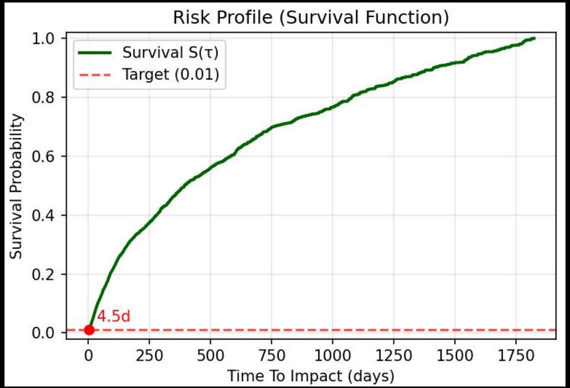
(recurring cost bounded at 10% of total budget)

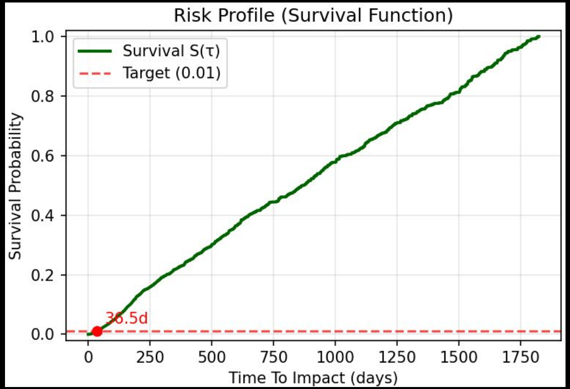
Reimplemented everything in Python for approximate high-fidelity solution

Haskell/MCDP pipeline gave provably optimal design solutions (for 3 data points)



Detection





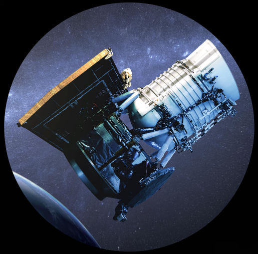
- Survival decreases more rapidly close to Earth
- Adding more sensors increases warning time

Detection DPI

Given a distribution of **NEO sizes** and **velocities**, what combination of investments in **detection** and **redirection** yields the greatest **lead time** for planetary defense?

Ground-based Observatories

+



Space-based Observatories

Detection DPI

Functionality Space F:

$$f = \tau \in \mathrm{R}_{\geq 0}$$

Requirement Space R:

$$r = C \in \mathrm{R}_{\geq 0}$$

Implementation Space I:

Discrete multiset of sensors.

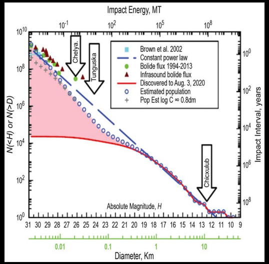
$$i = (n_1, n_2, \ldots, n_k)$$

Delivered Functionality:

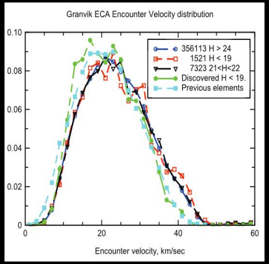
$$F(i) = \tau(i)$$

Required Cost:

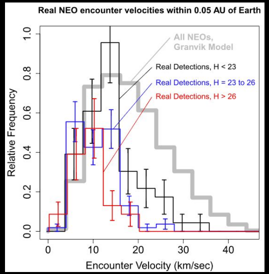
$$C(i) = \Sigma_j n_j c_j$$



$\text{SizeDistribution} : \text{Size} \to [0, 1]$



$\text{VelocityDistribution} : \text{Velocity} \to [0, 1]$



$\text{AlbedoDistribution} : \text{Albedo} \to [0, 1]$

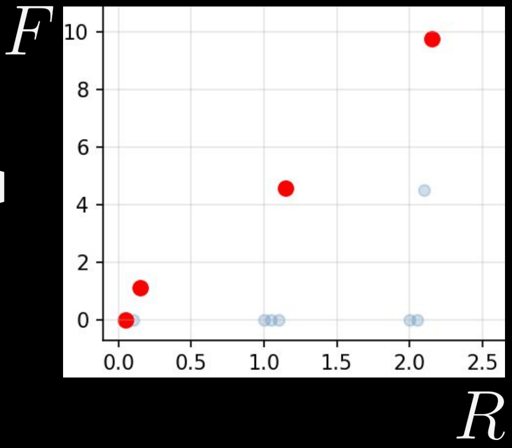
>

Monte Carlo Sampling of Design Problems

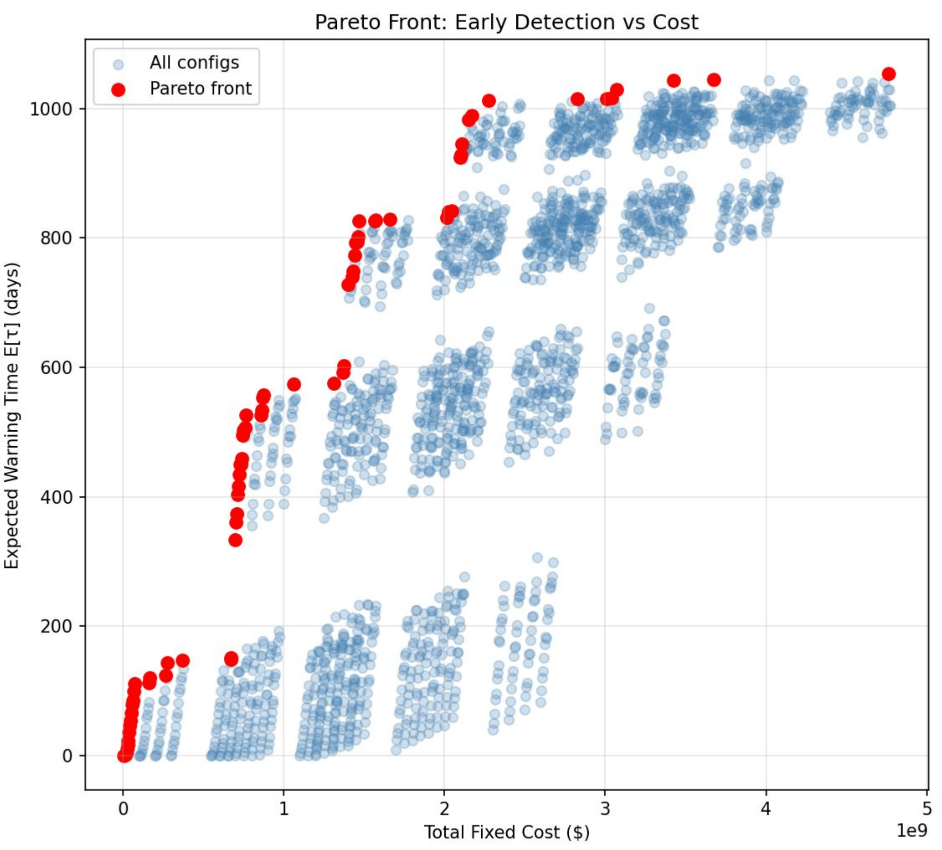
FixResMaxFun

FixFunMinRes

>

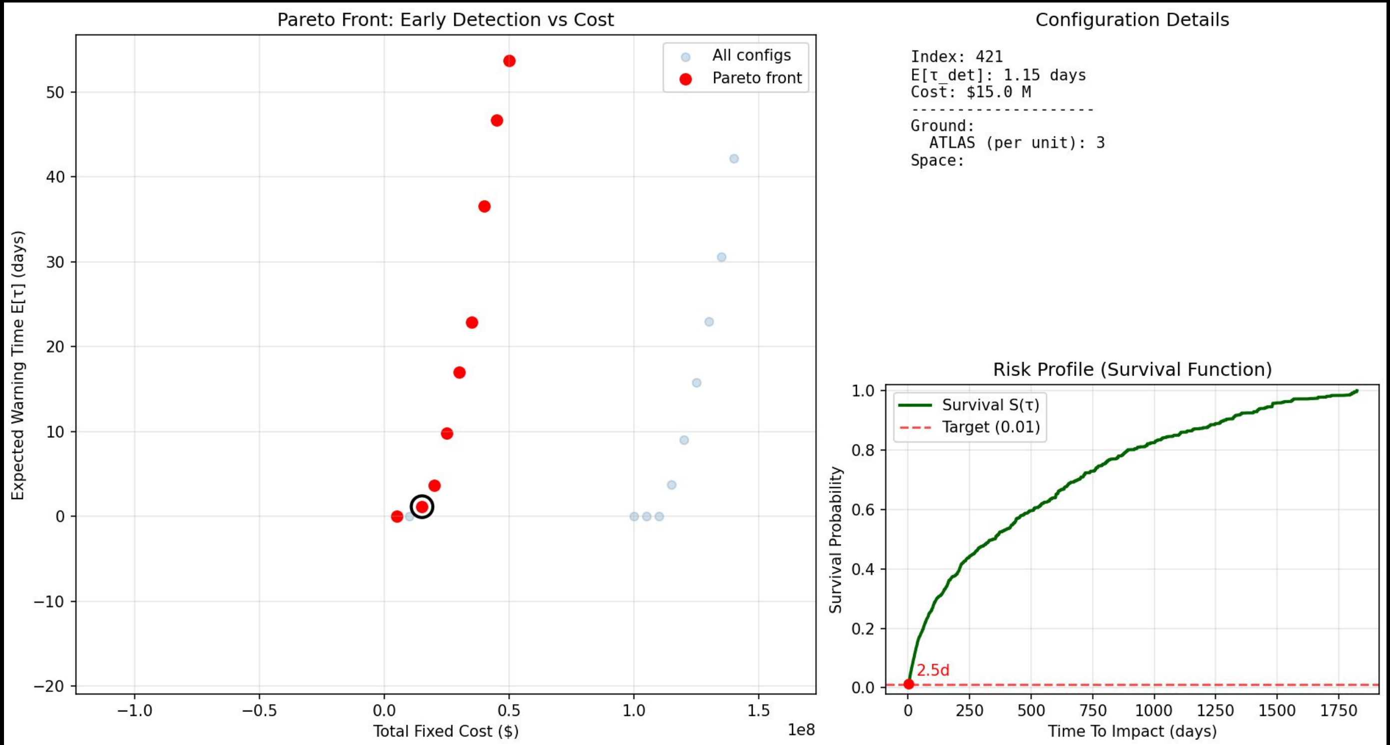


Results



Pareto Front: Early Detection vs Cost
All configs
Pareto front
Expected Warning Time E[τ] (days)
Total Fixed Cost ($)
1e9
0
200
400
600
800
1000
0
1
2
3
4
5

Results

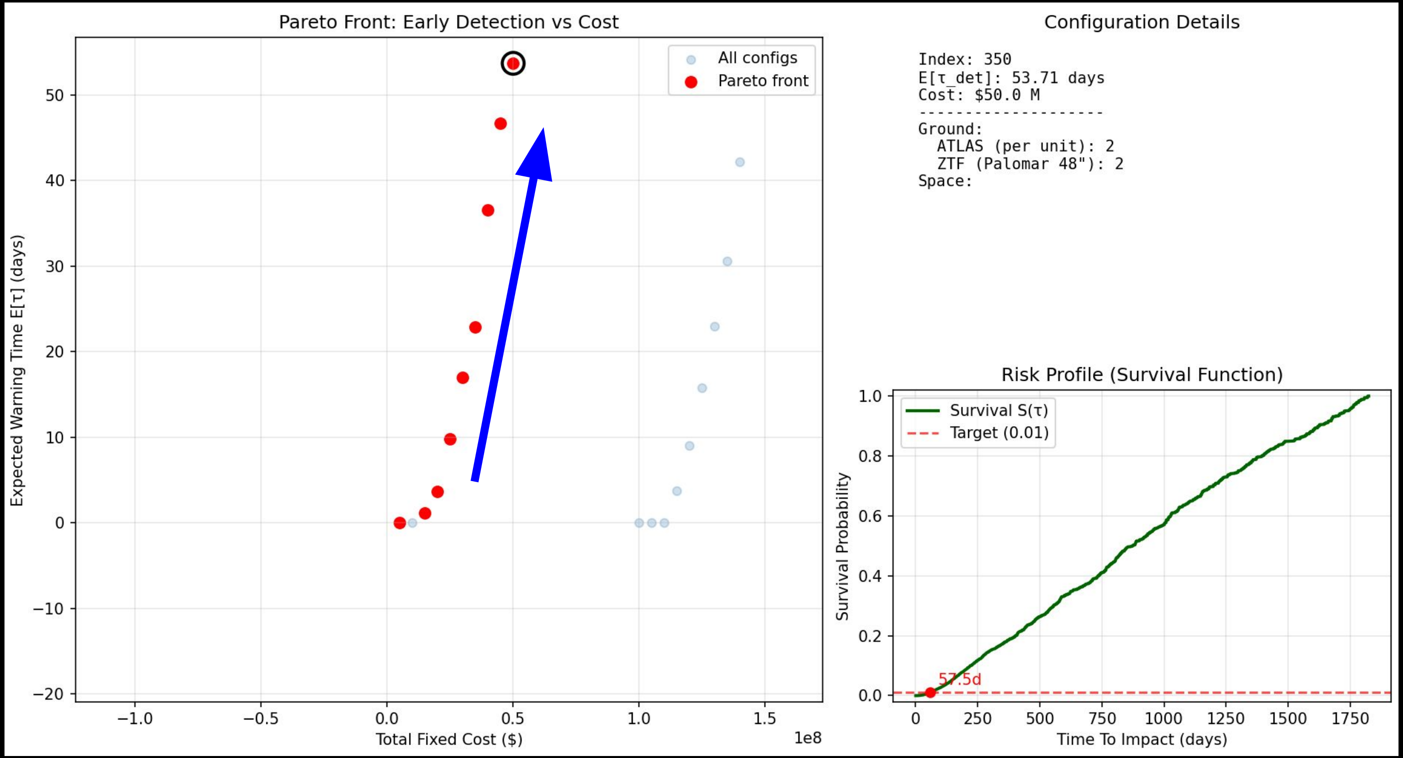


x3

3x **ATLAS** Observatory (Hawaii) provides warning times as available in literature.

~ $5,000,000 each

Results



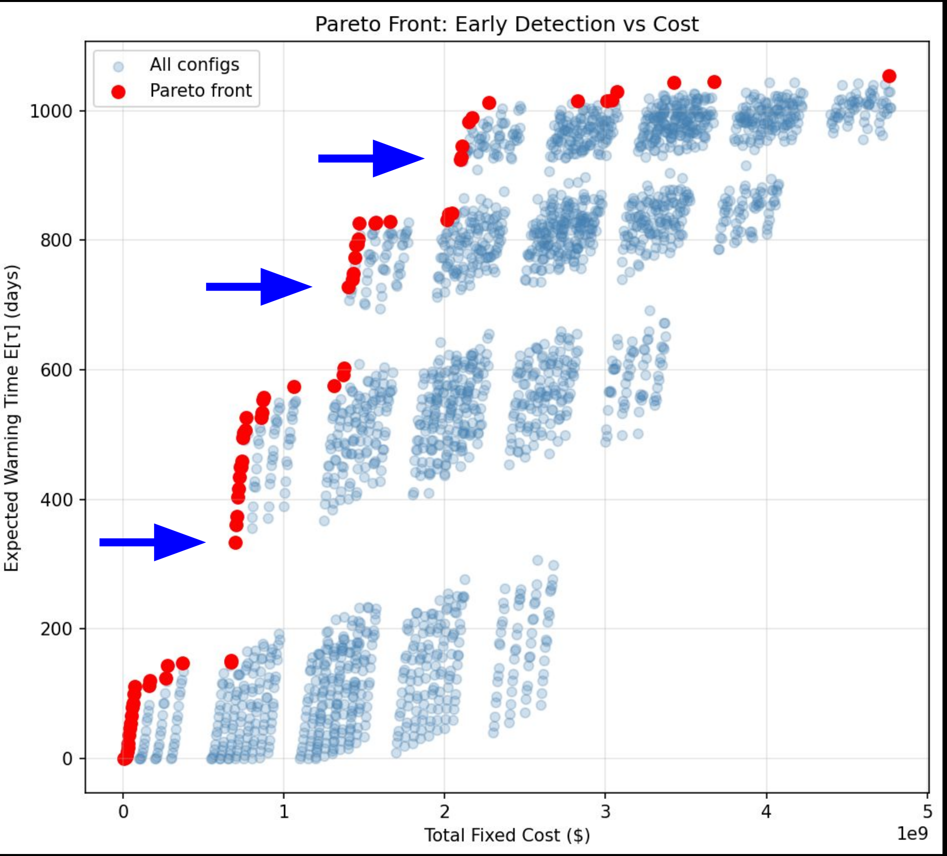
x2

x2

Combination of **ATLAS** & **ZTF** Observatories provide functionality with little cost increase

~ $5,000,000 (ATLAS)
~ $20,000,000 (ZTF)

Results



x3

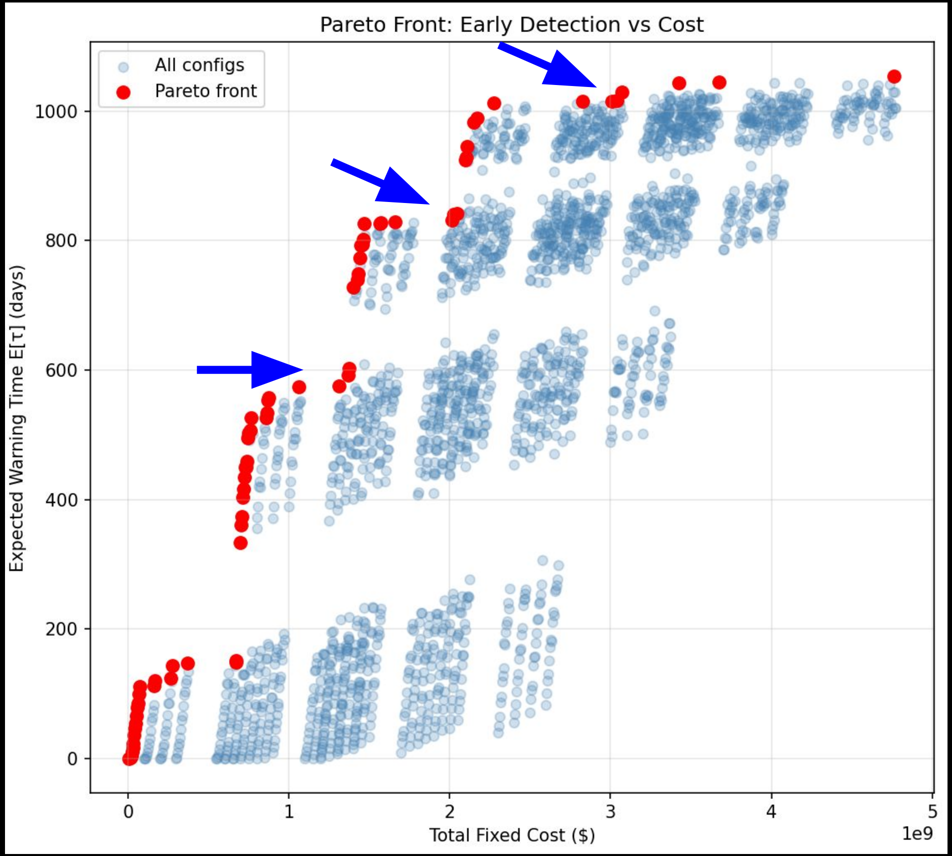
x2

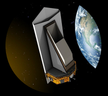
x1

RUBIN Observatory (Chile) provides the largest jumps, but is quite expensive.

~ $700,000,000 each

Results





x1

Orbital Surveyors like **NEOCAM** become valuable in the extremes.

~ $600,000,000 each

Compositionality and Nuclear Weapons

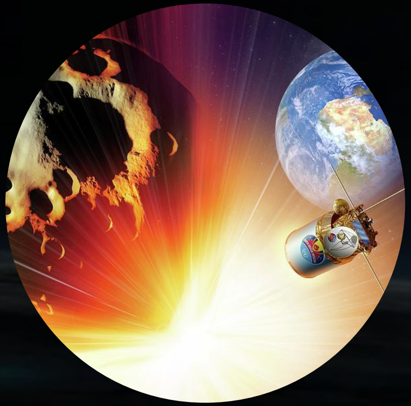
Redirecting asteroids close to Earth is **harder** than redirecting asteroids far from Earth.

We find that **warning time** → **intercept cost** is anti-monotone.

$$f : \tau \rightarrow \mathrm{R}_{\geq 0}$$

Total cost:

$$C_{total} = C_{sensors} + f(\tau)$$



Nuclear redirection offers the most performant protection on the shortest time scale of all redirection methods.

Compositionality and Nuclear Weapons

Redirecting asteroids close to Earth is **harder** than redirecting asteroids far from Earth.

We find that **warning time -> intercept cost** is anti-monotone.

$$f : \tau \to \mathrm{R}_{\geq 0}$$

Total cost:

$$C_{total} = C_{sensors} + f(\tau)$$

THEOREM

Solving the *Sensor DPI* automatically solves the *Sensor+Interception DPI*.

PROOF

The integrated design problem is:

$$\min_i \left(C_i + f(\tau_i)\right)$$

Per absurdum suppose solution A solves *Sensor+Interception DPI* optimally without solving *Sensor DPI* optimally.

Then there exists sensor B that dominates sensor A in *Sensor DPI*:

$$\tau_A < \tau_B$$
$$C_A \geq C_B$$

Because f is anti-monotone:

$$\tau_A < \tau_B \Rightarrow f(\tau_A) \geq f(\tau_B)$$

$$C_A + f(\tau_A) \geq C_B + f(\tau_B)$$

Analysis

Capable methods exist to identify asteroid threats early and to act on this information.

Our framework enables:

- Tractable and informed decision making
- Easy updating of sensor architectures
- Adjustment of risk parameters
- Parallelized algorithm that exploits dominance & anti-chains for speedup

Future Work:

- Interceptor Cost Modelling
- Further optimization for large state-spaces

Blue_Dome Public

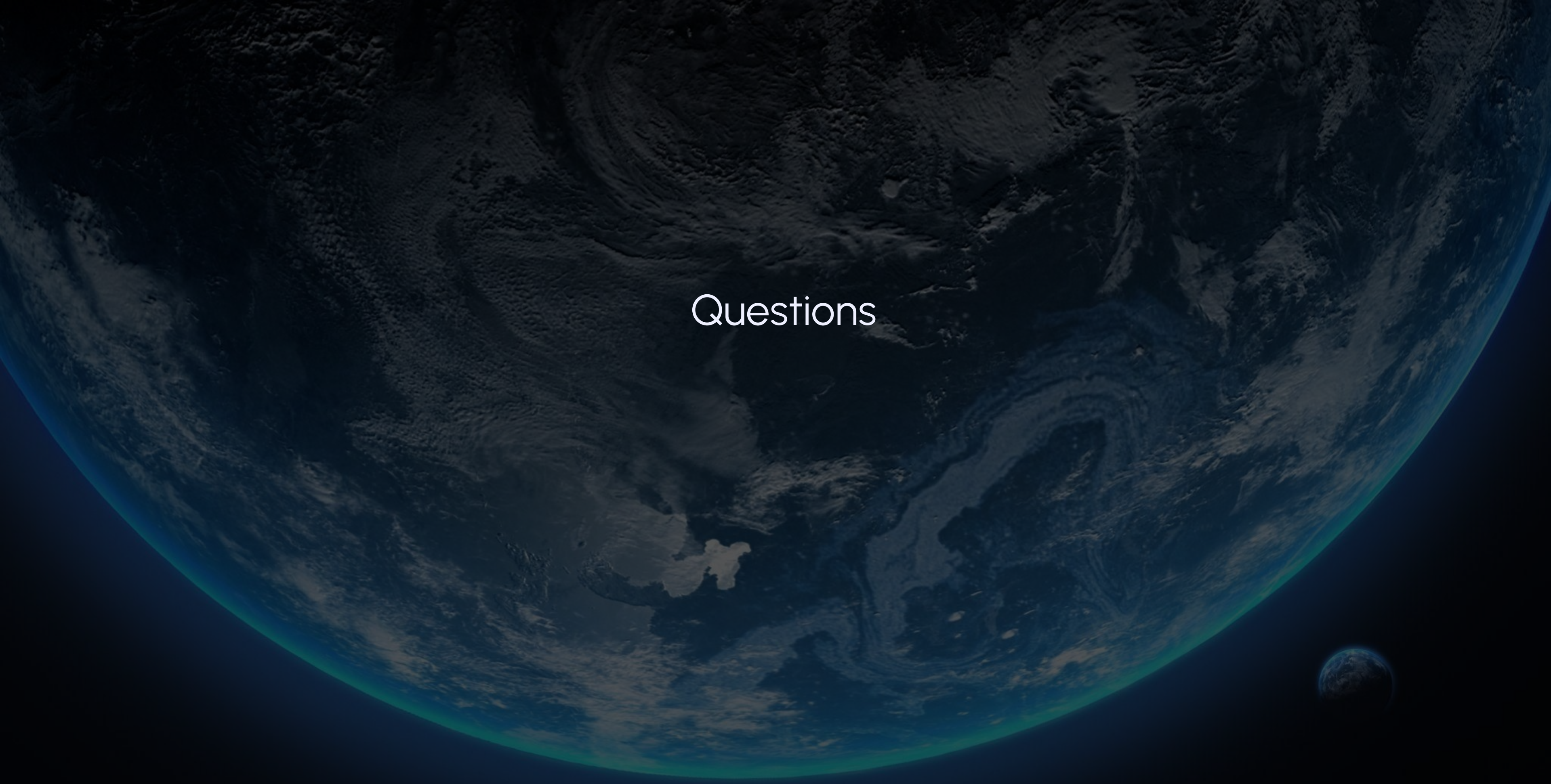
MIT 1.114 Project on using applied category theory for orbital defense. Simulations, code, and presentations for optimal sensor and interception strategies of asteroids.

E

https://github.com/nscheuer/Blue_Dome

2001_WN5 ICR Axes

25 Jun 2028 23:57:00.000 Time Step: 60.00 sec



Questions

Appendix: Ground Sensors

NEO absolute magnitude:

$$H = 5 \cdot \log_{10}(\frac{1329}{D_{\mathrm{km}\sqrt{p_v}}})$$

NEO apparent magnitude:

$$m \approx H + 5 \cdot \log_{10}(r_{\odot}\Delta)$$

Single-Exposure Detection Probability

$$P_{\mathrm{det}} = \frac{1}{1 + \exp(\frac{m(r,d) - m_{\mathrm{lim}}}{k})}$$

24h Isotropic Exposure Detection Probability

$$P_{24h} = \frac{c_i}{41253} P_{det}$$

Publicly-available information on ground-based sensors:

- 5-sigma sensitivity: m_{lim}
- Deg2 covered per 24h: c_i
- Fixed Cost
- Annual Operating Costs

Appendix: Ground Sensors

RUBIN (Chile)

ZTF (California)

ATLAS (Hawaii)

CSS (Arizona)

Pan-STARRS (Hawaii)

Appendix: Space Sensors

Equilibrium Temperature:

$$T(r) = 278\text{K}\left(\frac{1-A}{\epsilon\mu}\right)^{\frac{1}{4}} r^{-\frac{1}{2}}$$

Planck Radiation

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}$$

Flux at Earth

$$F_v(r,d) = \frac{\epsilon\pi B_\lambda(T)d^2}{4\Delta^2}\frac{\lambda^2}{c}$$

Single Exposure

$$P_{\text{det}} = \frac{1}{1+\exp\left(\frac{\log_{10}(F_{v,\text{lim}})-\log_{10}(F_v(r,d))}{k}\right)}$$

Publicly-available information on ground-based sensors:

- Target waveband:
- 5-sigma flux density limit:
- Deg2 covered per 24h
- Fixed Cost
- Annual Operating Costs

Modeling
Sensors

A sensor is characterized by its **angular resolution**, which relates range and diameter of target.

$$\mathrm{Sensor} : \mathrm{Range} \times \mathrm{Diameter} \to [0, 1]$$

$$P_i(r, d)$$

We combine this with the angular cadence, i.e. percent of sky covered in 24 hours.

$$\mathrm{DailySensor} : \mathrm{Range} \times \mathrm{Size} \times \mathrm{St} \to [0, 1]$$

$$\bar{\alpha}_i \approx \frac{c_i}{4\pi} \qquad \bar{P}_i(r, s) = \bar{\alpha}_i P_i(r, d)$$

Finally, we define combined detection probability:

$$F(r, d) = 1 - \Pi_i(1 - \bar{P}_{ground,i}(r, d))\Pi_j(1 - \bar{P}_{space,j}(r, d))$$

Modeling

Sensors

NEO absolute magnitude:

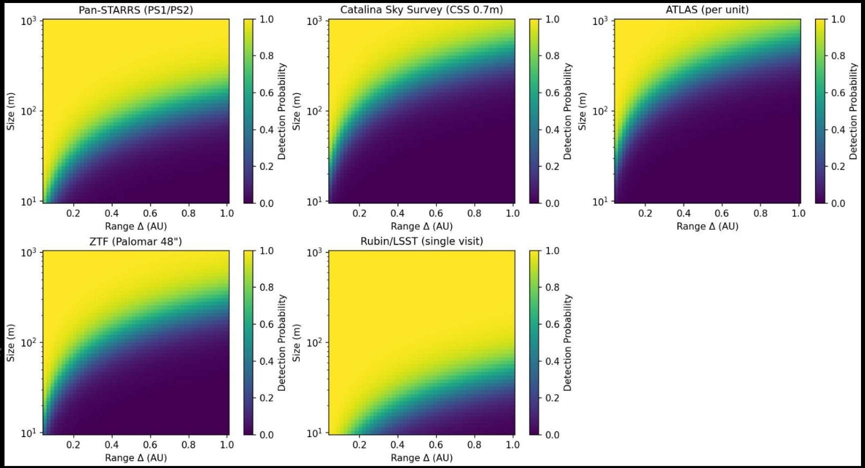
$$H = 5 \cdot \log_{10}\left(\frac{1329}{D_{\mathrm{km}\sqrt{p_v}}}\right)$$

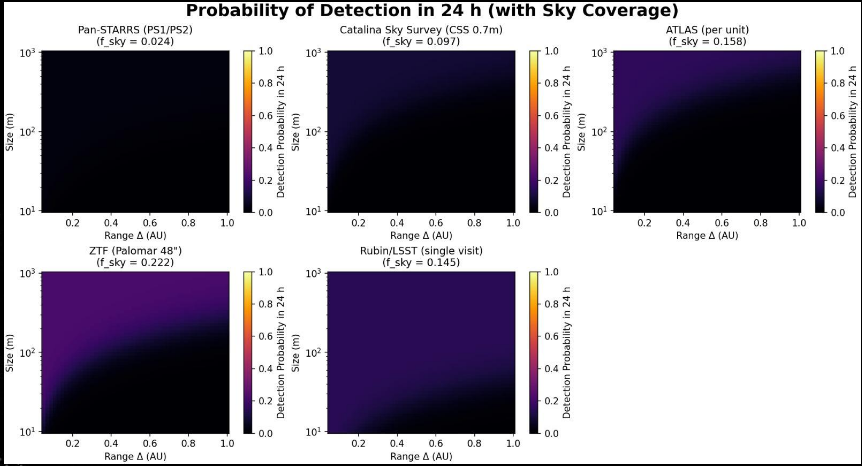
NEO apparent magnitude:

$$m \approx H + 5 \cdot \log_{10}(r_{\odot}\Delta)$$

Single-Exposure Detection Probability

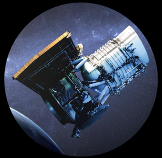
$$P_{\mathrm{det}} = \frac{1}{1 + \exp\left(\frac{m(r,d) - m_{\mathrm{lim}}}{k}\right)}$$





Modeling

Sensors



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Single Exposure

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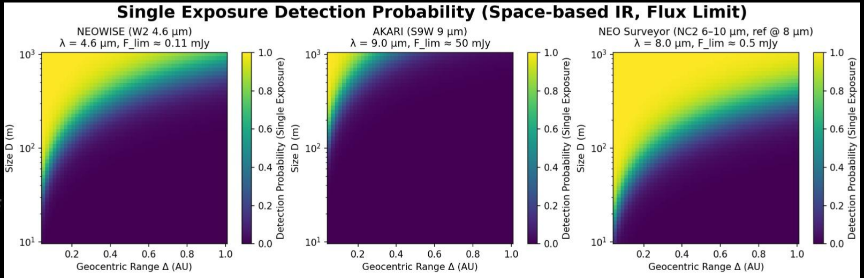
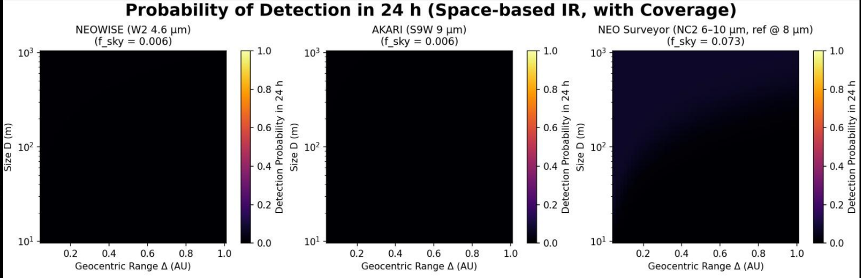
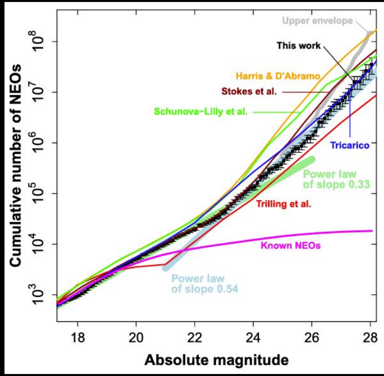
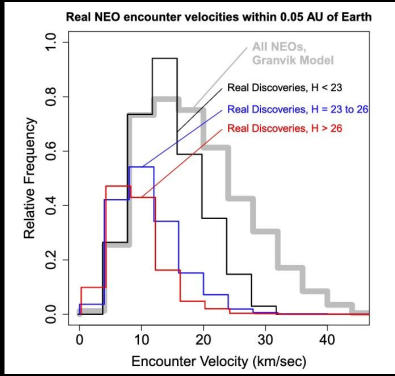




Modeling
Time to Impact

Provided the distributions:
- Size: $p_D(d)$
- Velocity: $p_V(v)$
- Orbit Geometry: $p_\Phi(\phi)$





Hazard Function:

$$\lambda(\tau) = \int_d \int_v \int_0^{\frac{\pi}{2}} F(v\tau cos(\phi), d) p_D(d) p_V(v) p_\Phi(\phi) d\phi dv ds$$

Survival:

$$S(\tau) = exp\left(-\int_\tau^{\tau_{max}} \lambda(u) du\right)$$

Probability Distribution Function:

$$p_\tau(t) = \frac{\lambda(t)S(t)}{1 - S(0)}$$

Modeling
Delta V

Provided the distributions:

- Size: $p_D(d)$
- Velocity: $p_V(v)$
- Time to Impact: $p_T(t)$

Tangential Delta-V that must be transferred for a 2-Earth Radii Miss [4][8]:

$$\Delta v = \frac{2R_\oplus}{t_{TTL}}$$

Interception angle plays an important role, difficult to model without simulation.

Modeling

Kinetic Interceptor

$$m_{\text{NEO}} = \rho \frac{2000}{6} D^3$$

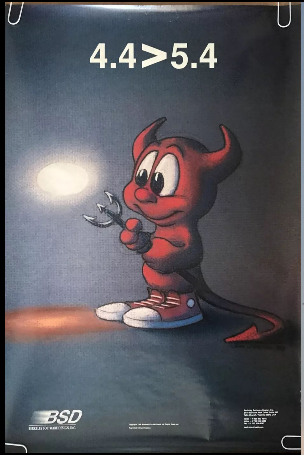
$$Rocket : Mass \times C_3 \to Bool$$

$$m_{sat} = m_{prop} + m_{impactor}$$

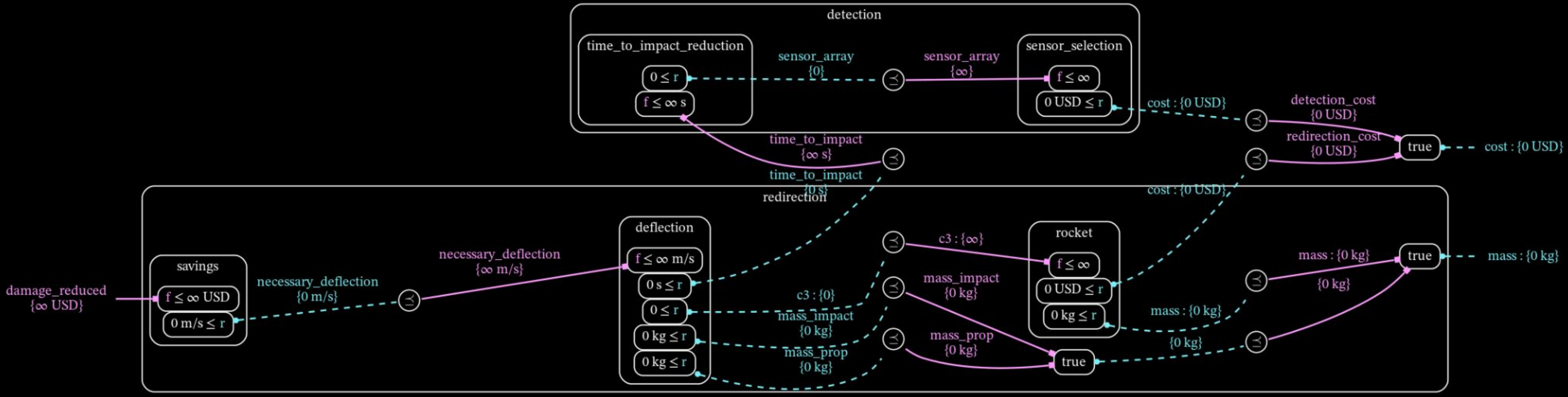
$$v_{impact} = v_e \ln\left(\frac{m_{sat}}{m_{impactor}}\right) + \sqrt{C_3}$$

$$t_{flight} = \frac{R}{\sqrt{v_{ast}^2 + v_{impact}^2}}$$

$$\Delta v_{req} = \frac{2R_{Earth}}{t_{TTL} - t_{flight}} \leq \frac{m_{impactor} v_{impact} \beta}{m_{ast}}$$



"The most important application of Toquos Theory is combating the devastating effects of Subgroup Psychosis stemming from several somnial attacks from the Number Devil, as he has the ability to invade dreams, especially those related to mathematics." —[Emma O'Neil]



Proprietary MCDPL-generated design schematics; for more information see:
https://code.functor.systems/q9i/blue-dome