Co-Design for Planetary Defense

1.144 Final Presentation

Niclas Scheuer
Ananth Venkatesh

Our planet is a lonely speck in the great enveloping cosmic dark... our responsibility to deal more kindly with one another, and to preserve and cherish the pale blue dot, the only home we've ever known.

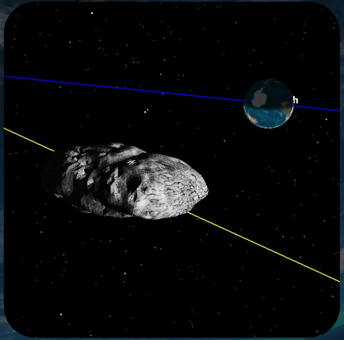
Carl Sagan

How to defend our planet from Near-Earth Objects

Categorical Approach for **transparency**, **tractability**, and **interoperability**

Multiple Stakeholders

Multiple Approaches



Complex Dynamics

NEO Defense Framework

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Motivation

Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.

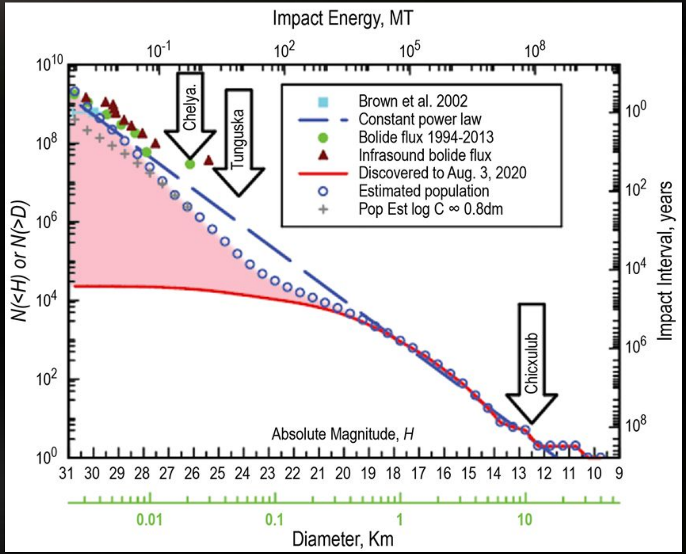
February 13, 2013

- 13-meter diameter meteor strikes Chelyabinsk, Russia
- > 1600 injured
- No prior warning

Motivation

Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.

Majority of NEOs of size 50-140m remain **undiscovered.**



"The number of undiscovered NEOs larger than 140m is on the order of 10,000"

- NASA Decadal Planetary Defense Study [3]

Motivation

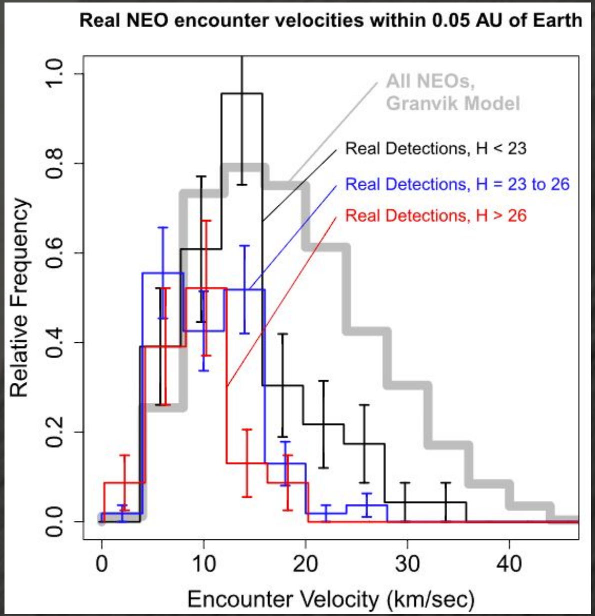
Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.

Majority of NEOs of size 50-140m remain **undiscovered.**

Current NEO warning systems give us limited warning times.

Warning Times [3]:
- 20m NEO: 1 day
- 100m NEO: 21 days

Current Interception Times [4]:
- 54 months



Size, Velocity, Albedo, Inclination, Semi-Major Axis, Eccentricity Data and Estimates Exist.

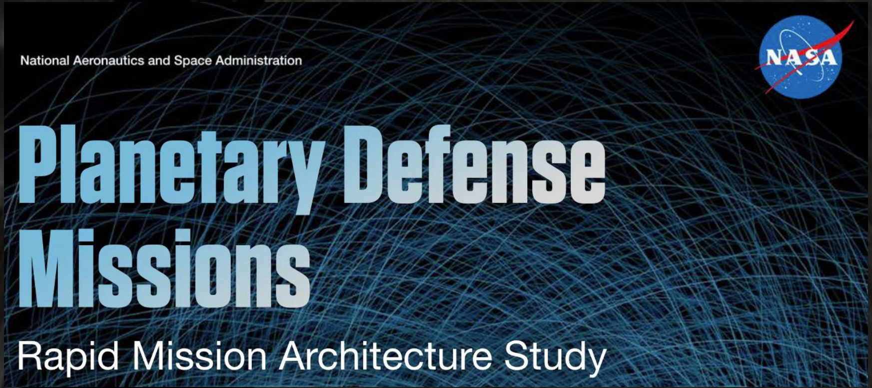
Modeling Threats

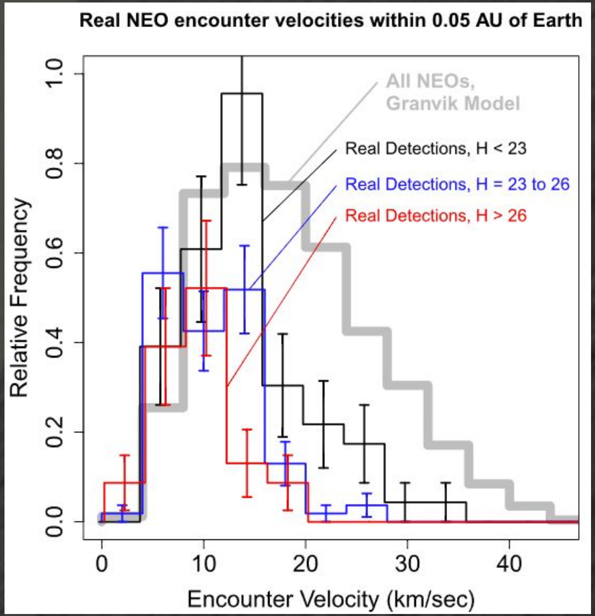
Models for NEO distributions exist and are piecewise validated by surveyors.

The population of near-earth asteroids revisited and updated

Alan W. Harris *, Paul W. Chodas

[a] *MoreData! Inc., La Cañada, CA 91011, USA*
[b] *Jet Propulsion Laboratory, Pasadena, CA 91109, USA*





Size, Velocity, Albedo, Inclination, Semi-Major Axis, Eccentricity Data and Estimates Exist.

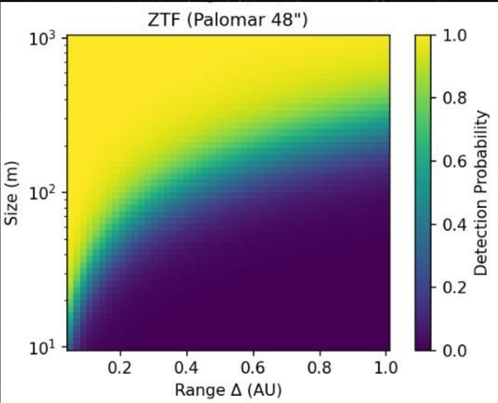
Modeling Threats

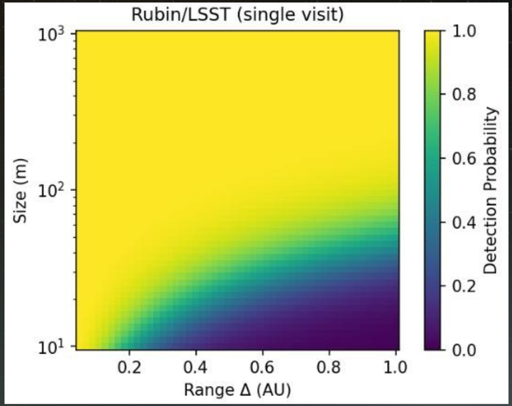
Models for NEO distributions exist and are piecewise validated by surveyors.

$$\text{SizeDistribution} : \text{Size} \to [0, 1]$$

$$\text{VelocityDistribution} : \text{Velocity} \to [0, 1]$$

$$\text{AlbedoDistribution} : \text{Albedo} \to [0, 1]$$

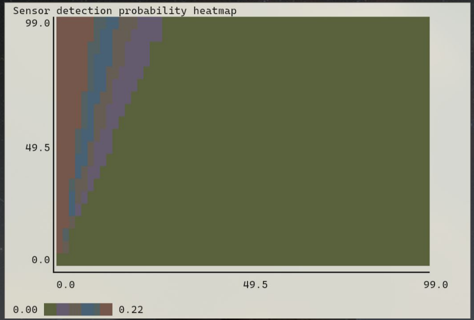


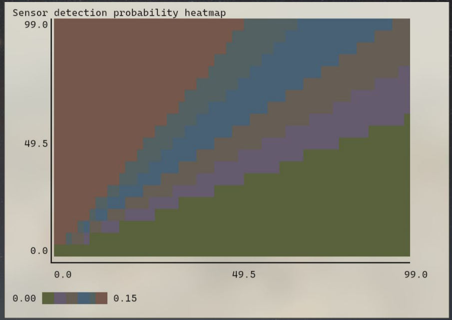


Modeling Sensors

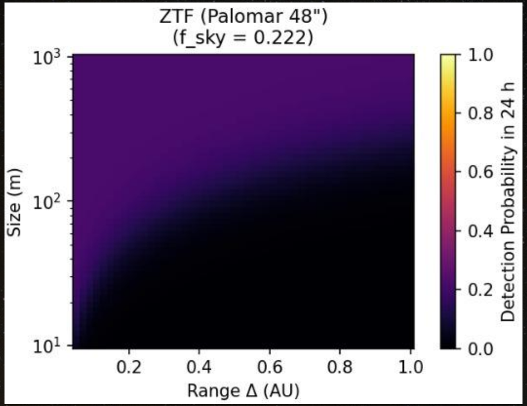
Using publicly-available data on several ground stations, we characterize the <u>single-exposure detection probability</u> and <u>24-hour detection probability.</u>

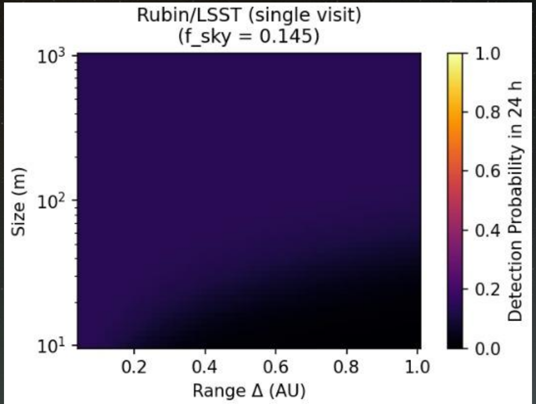
$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \to [0, 1]$$





Discretized and linearized Haskellian plots using ASCII magic





Modeling Sensors

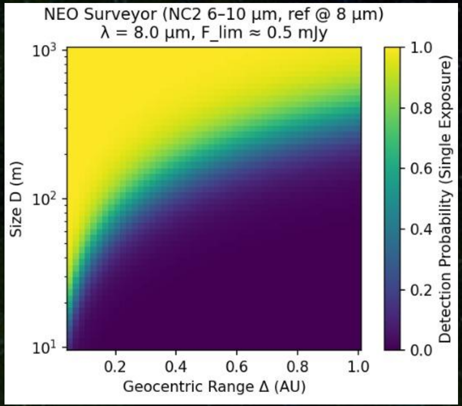
Using publicly-available data on several ground stations, we characterize the <u>single-exposure detection probability</u> and <u>24-hour detection probability.</u>

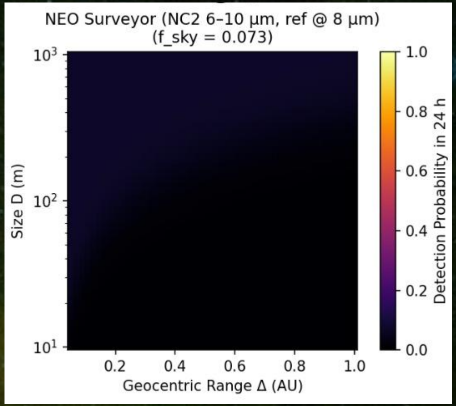
$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \to [0, 1]$$

Consider:

- Limited sensor cone of sensor
- 24h isotropic sky-coverage approximation

$$\mathbb{P}_{24\text{ hr}} : \text{Range} \times \text{Size} \to [0, 1]$$





Modeling Sensors

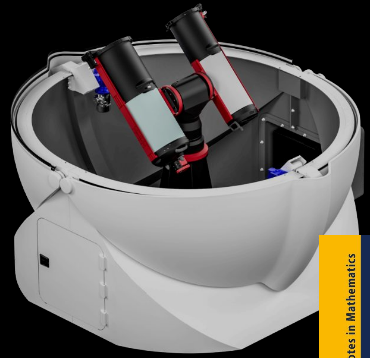
Similar calculation for space-based sensors, using infrared emissivity.

- 24h isotropic sky-coverage approximation
- Efficacy scaling based on cone angle

$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \to [0,1]$$
$$\mathbb{P}_{24\text{ hr}} : \text{Range} \times \text{Size} \to [0,1]$$

Sensor topology

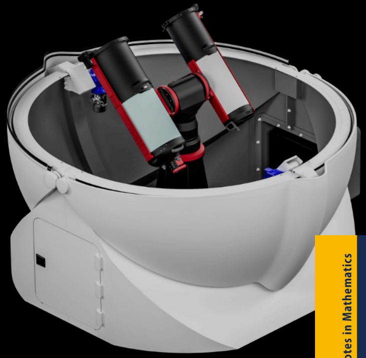
Sensor detection functions are distributions over **range x size** invariant to constant multiples of the same **range–size** pair; parametrized by **range:size** ratio



Sensor topology

Dimensionless poset of **range:size** ratios forms domain for sensor detection functions, which are **monotone maps** from the opposite of this poset to the **unit interval [0, 1]**.

Specifically, low range:size gives high probability of detection and high range:size gives low probability of detection. We then have an ordering of sensors based on the standard ordering of monotone maps.



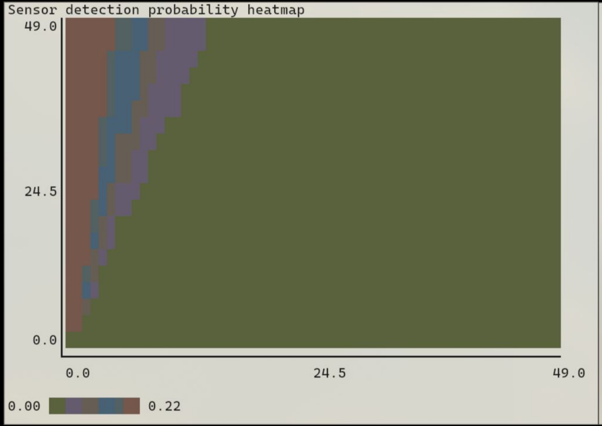
Monoidal probability fusion

$$p_1 \otimes p_2 = 1 - (1 - p_1)(1 - p_2)$$

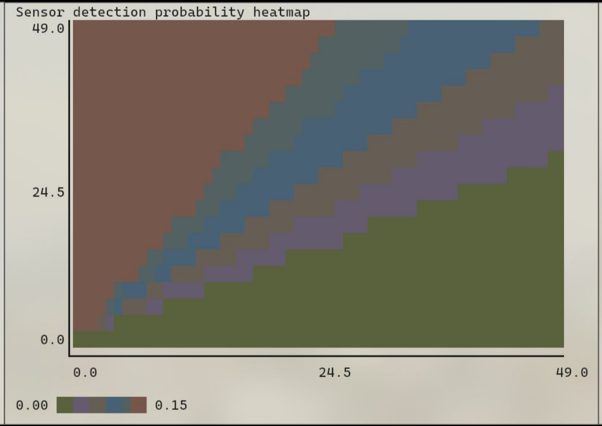
Commutative monoid on probabilities; gives a new detection function better than or the same as its composite detection functions

(This is **not** the probability *monad*)

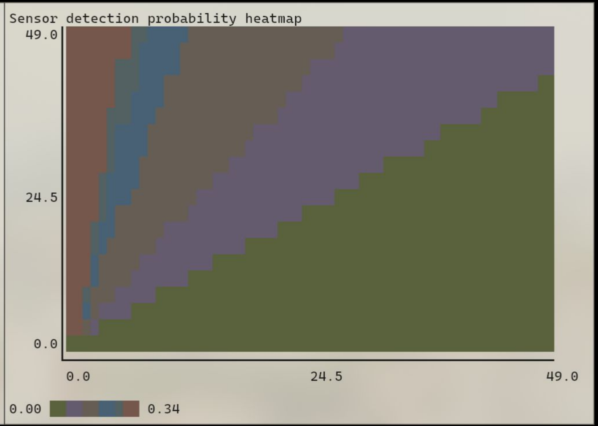
Monoidal probability fusion



ZTF (Palomar)
max detection probability:
22%



Rubin/LSST
max detection probability:
15%



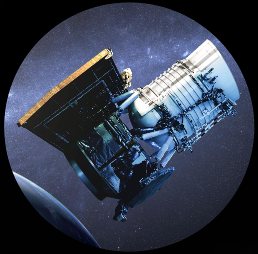
Combined
max detection probability:
34%

Detection Problem

Given a distribution of **NEO sizes** and **velocities**, what combination of investments in **detection** and **redirection** yields the greatest **lead time** for planetary defense?

Ground-based Observatories

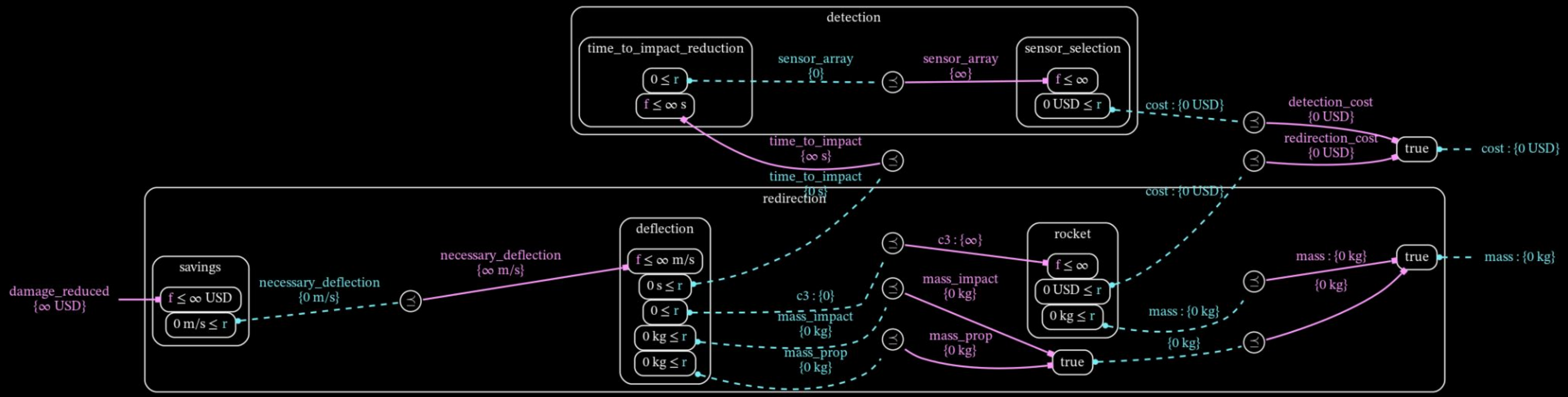
+



Space-based Observatories

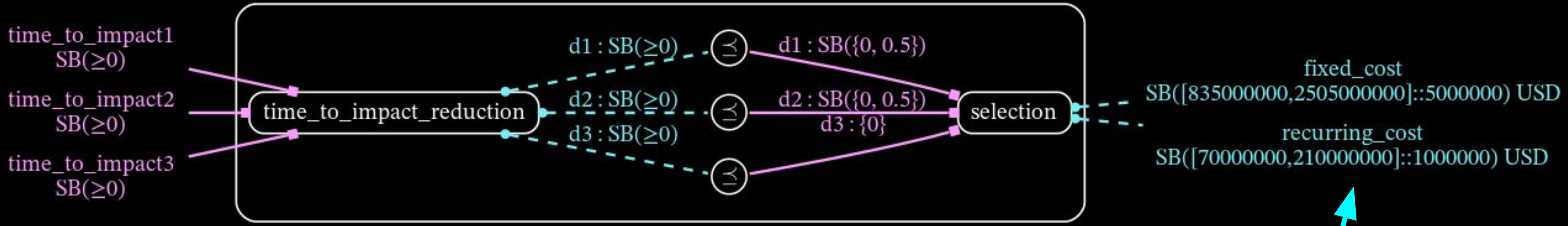
Detection Problem

Given a distribution of **NEO sizes** and **velocities**, what combination of investments in **detection** and **redirection** yields the greatest **lead time** for planetary defense?



System diagram

Instead of expected time to impact, compute the hazard function at representative times to get probabilities of detection

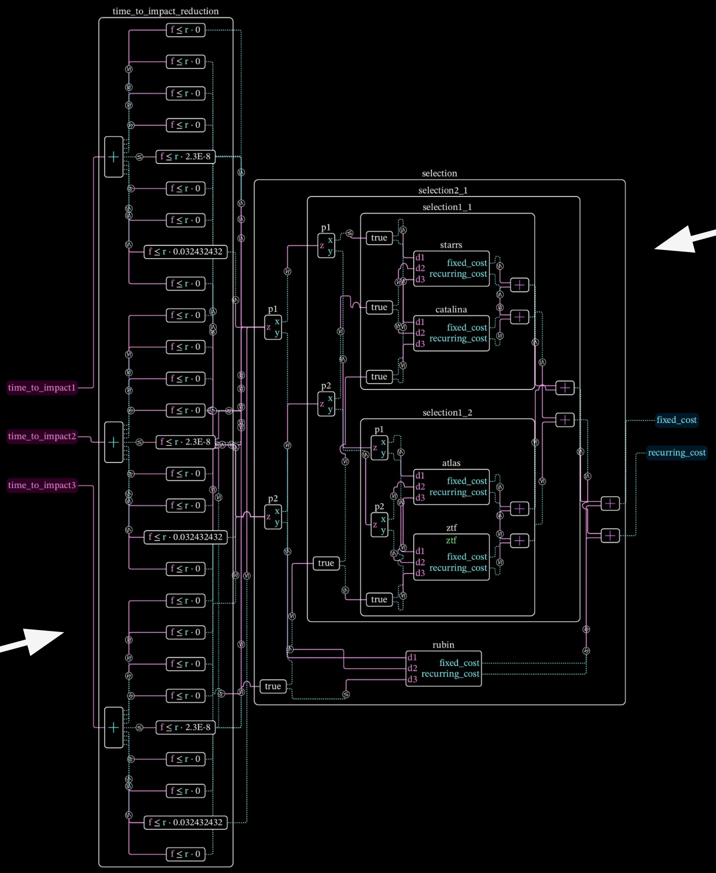


Split fixed and recurring costs

Zoom in on revised detection block

24 hour detection probabilities at representative times

Hazard function (double integral approximated as Riemann sum)

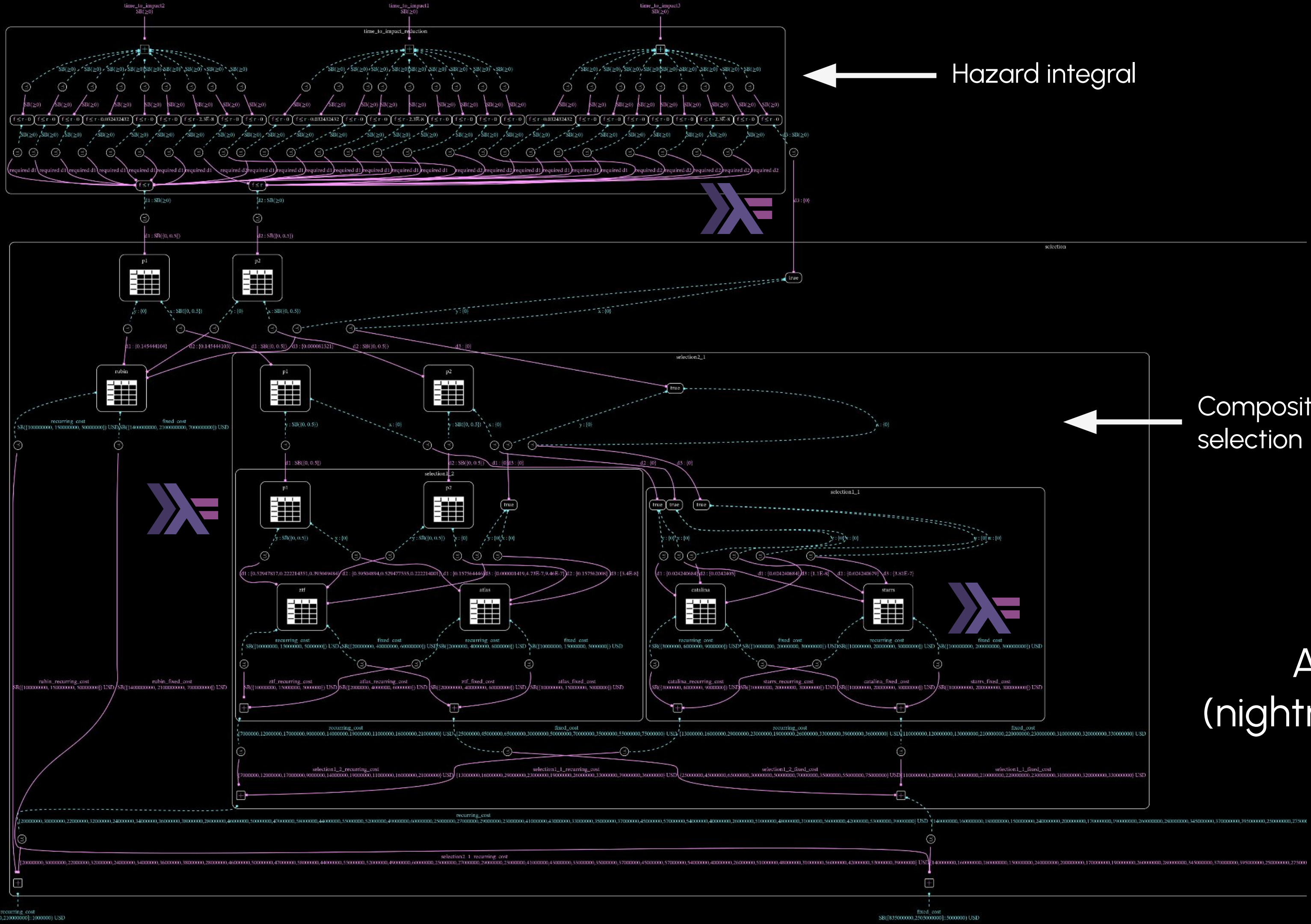


Sensor selection problem (repeated applications of the probability monoid)

Costs

Highly discretized (3 point approximation) DP representing sensor selection

(full DP is orders of magnitude more cursed)



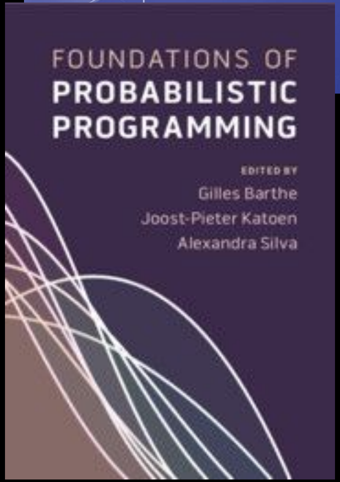
Hazard integral

Compositional sensor selection

Another view
(nightmare design
problem)

The case for co-design

Categorical informatics, probability monads, and the like

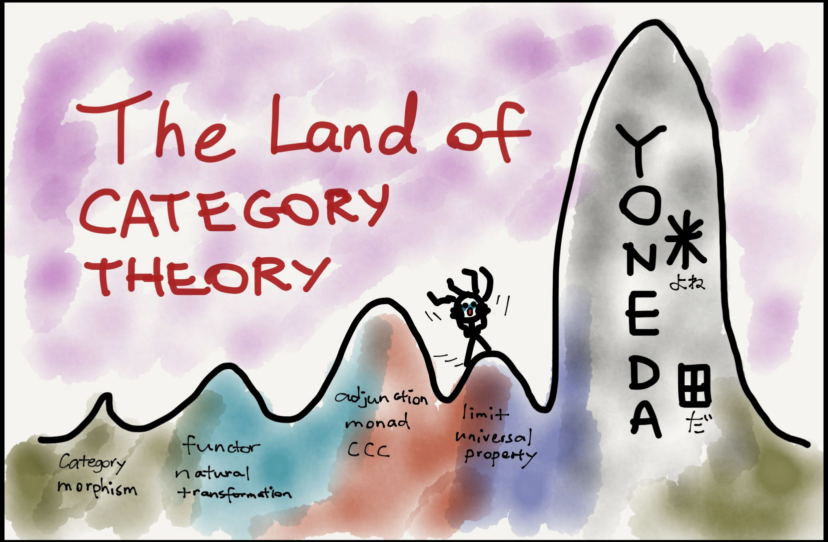


$$\begin{tikzcd}
L^L \arrow[r, bend left, "(-)^\vee"] & L^L \arrow[l, bend left, "(-)^\wedge"] \\
{[L,L]_\wedge} \arrow[u, hook, "\wedge"] \arrow[r, bend left, "(-)^\vee"] & {[L,L]_\vee} \arrow[u, hook, "\vee"'] \arrow[l, bend left, "(-)^\wedge"] \\
{[L,L]^t_\wedge} \arrow[u, hook, "\wedge"] \arrow[r, bend left, "(-)^\vee"] & {[L,L]^t_\vee} \arrow[u, hook, "\vee"'] \arrow[l, bend left, "(-)^\wedge"]
\end{tikzcd}$$

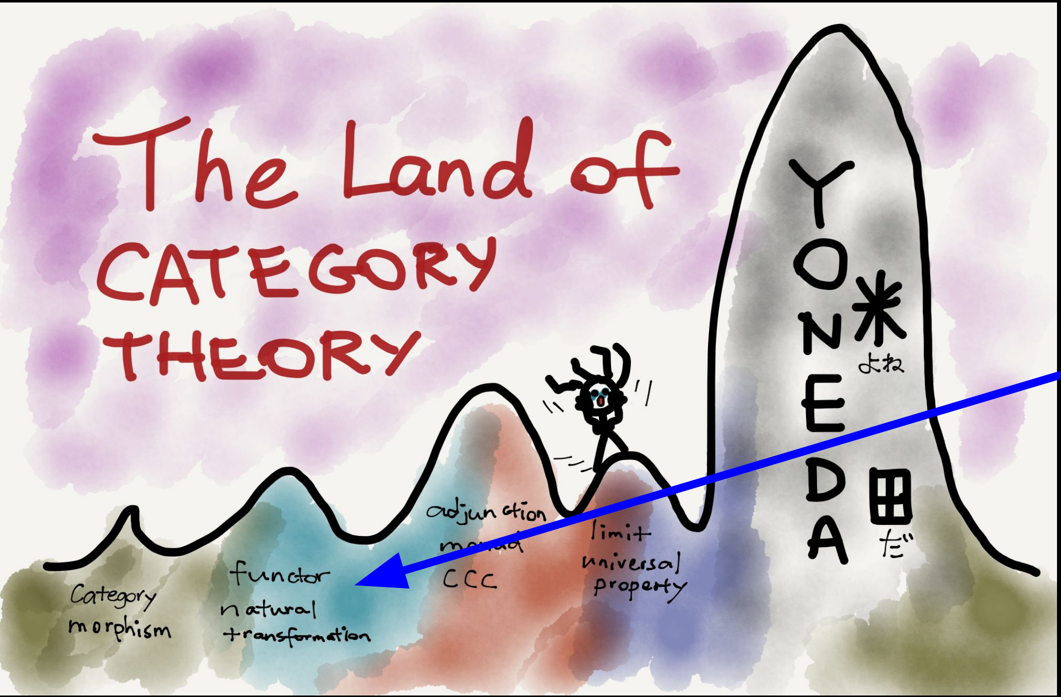
The Pareto Frontier

Given a fixed budget, what is the best we can do in terms of:

- Minimizing existential risk (i.e. detecting incoming asteroids early)
- Minimizing collateral damage (i.e. increasing our chances of preventing asteroid impact)
- What sensors do we invest in, and which interception mechanisms do we value?



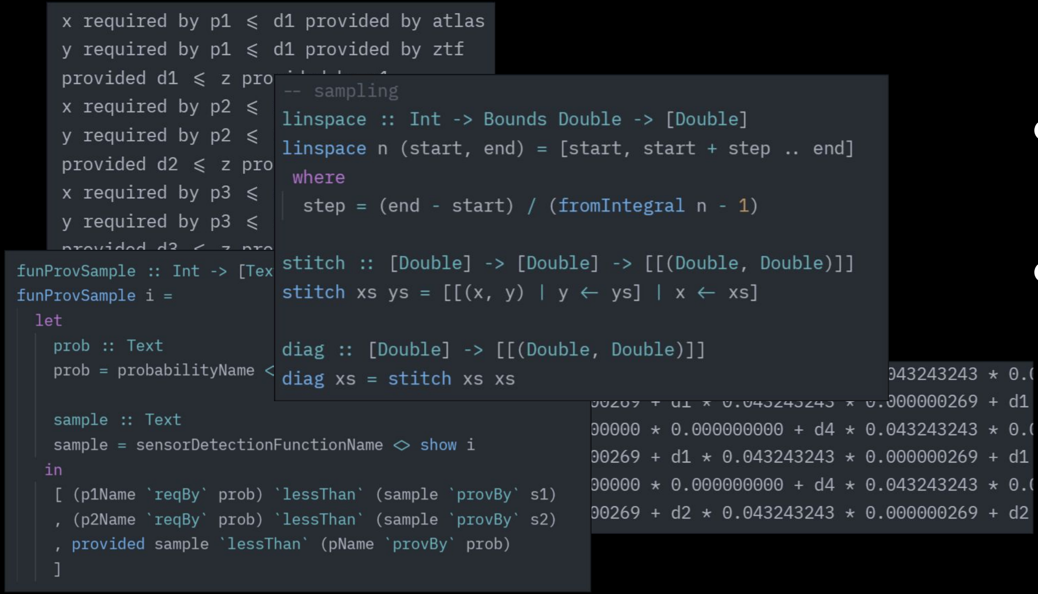
Categorically, we apply **FixResMaxFun** to obtain the Pareto frontier for detection probabilities given a fixed budget and **FixFunMinRes** to minimize costs required to achieve a given set of damage reduction targets



Functionality of the solution map means we can get away with very inefficient specifications, provided they're compositional

(we will heavily abuse this property in order to represent our problem in MCDP)

MCDP



- Discretize everything
- Haskell-based templating metalanguage
- Dynamically set resolution and solve for antichains
- Describe sensor combination with compositions of the probability monoid

Results

(with MCDP)

- Crashed web editor multiple times
- Wasted tremendous amounts of compute
- Haskell code worked flawlessly
- MCDP killed itself several times likely due to (d)OOM issues

Discretization :(

2.58%

Maximum 24-hour detection probability with a $2.5 billion budget

(recurring cost bounded at 10% of total budget)

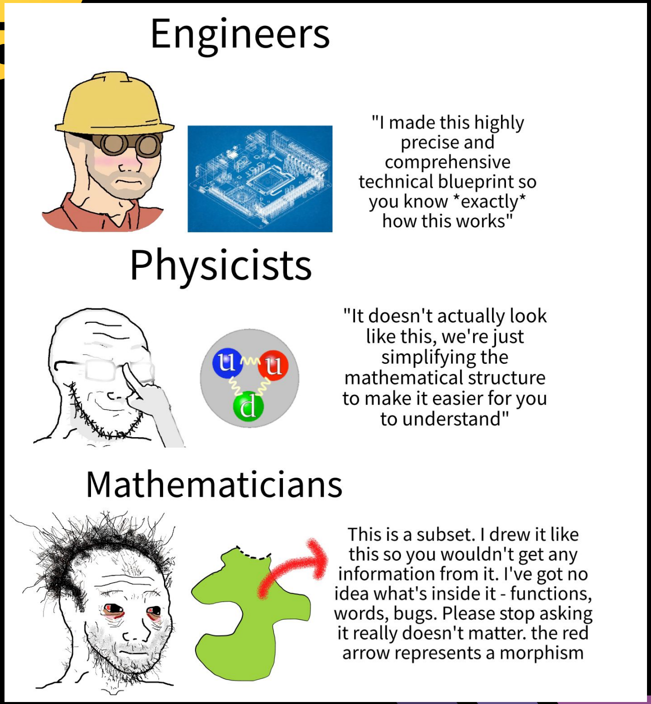
27 days

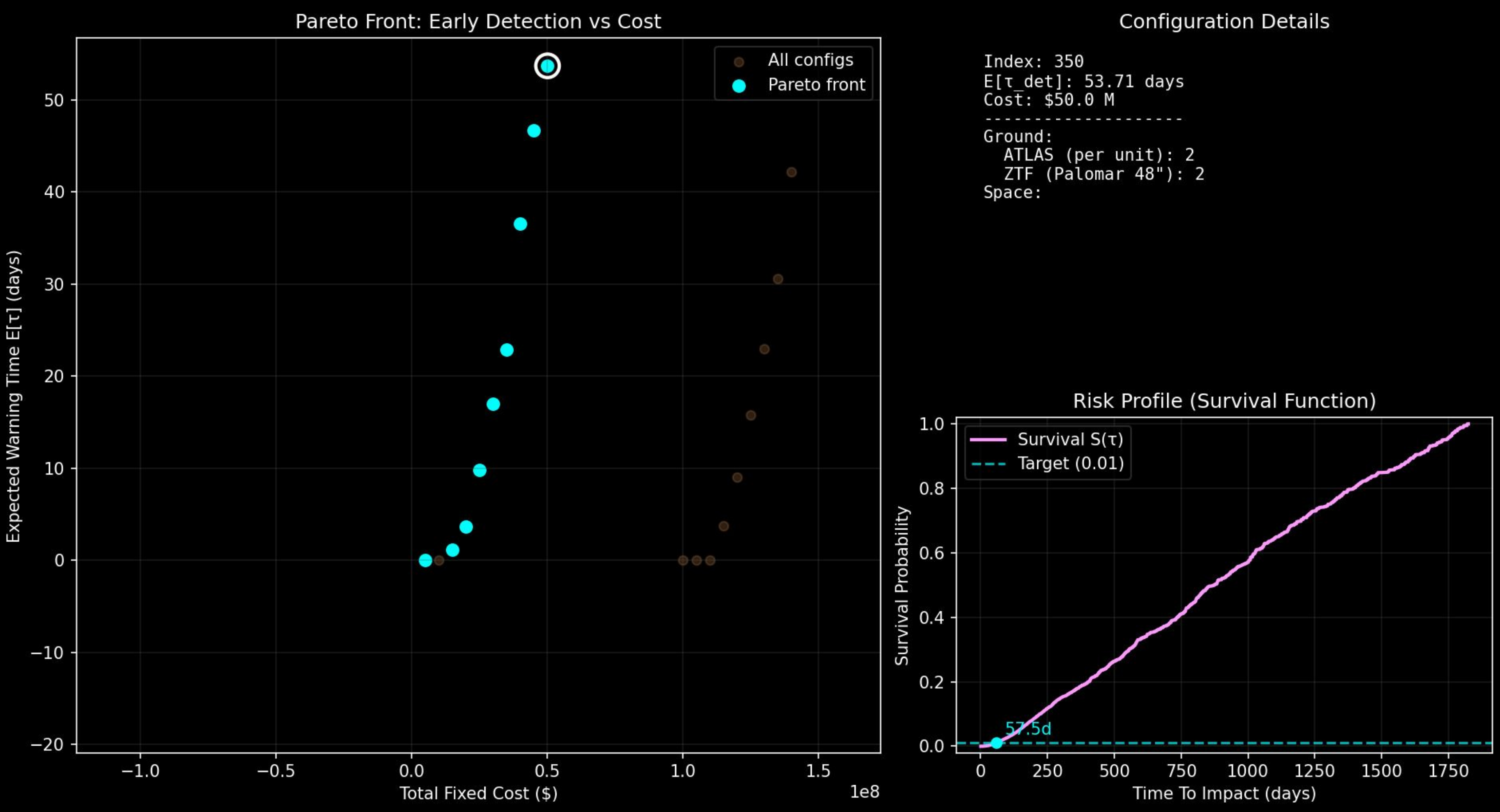
Average detection time with a $2.5 billion budget

(recurring cost bounded at 10% of total budget)

Reimplemented everything in Python for approximate high-fidelity solution

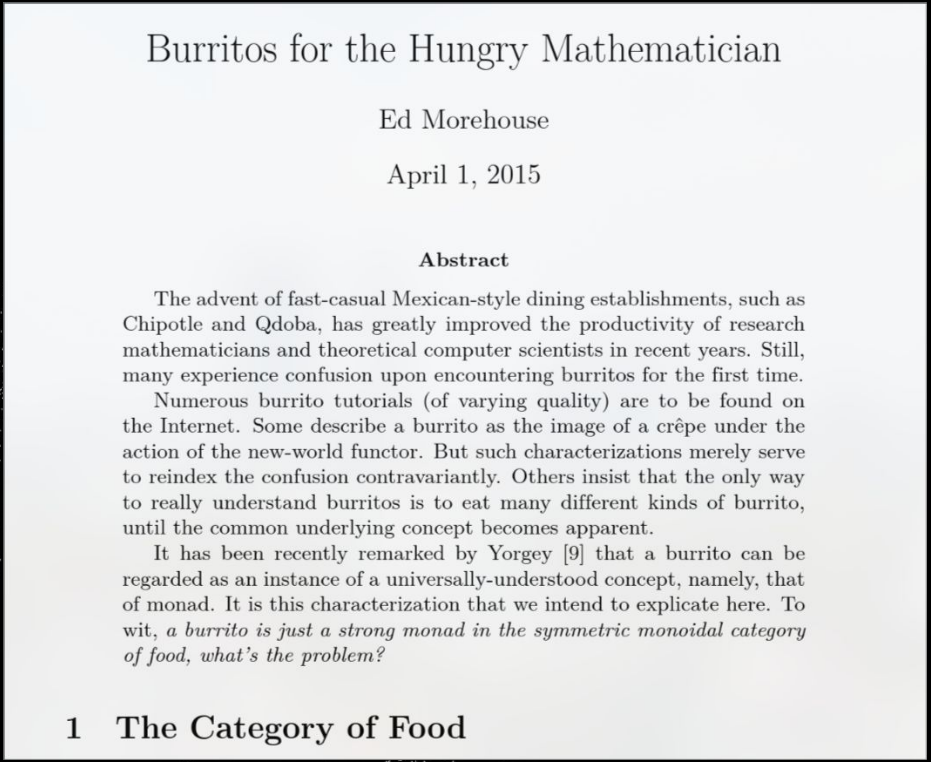
Haskell/MCDP pipeline gave provably optimal design solutions (for 3 data points)




Pareto Front: Early Detection vs Cost
All configs
Pareto front
Expected Warning Time E[τ] (days)
Total Fixed Cost ($)
1e8
Configuration Details
Index: 350
E[τ_det]: 53.71 days
Cost: $50.0 M

Ground:
ATLAS (per unit): 2
ZTF (Palomar 48"): 2
Space:
Risk Profile (Survival Function)
Survival S(τ)
Target (0.01)
57.5d
Survival Probability
Time To Impact (days)

Modeling



Burritos for the Hungry Mathematician

Ed Morehouse

April 1, 2015

Abstract

The advent of fast-casual Mexican-style dining establishments, such as Chipotle and Qdoba, has greatly improved the productivity of research mathematicians and theoretical computer scientists in recent years. Still, many experience confusion upon encountering burritos for the first time.

Numerous burrito tutorials (of varying quality) are to be found on the Internet. Some describe a burrito as the image of a crêpe under the action of the new-world functor. But such characterizations merely serve to reindex the confusion contravariantly. Others insist that the only way to really understand burritos is to eat many different kinds of burrito, until the common underlying concept becomes apparent.

It has been recently remarked by Yorgey [9] that a burrito can be regarded as an instance of a universally-understood concept, namely, that of monad. It is this characterization that we intend to explicate here. To wit, *a burrito is just a strong monad in the symmetric monoidal category of food, what's the problem?*

1 The Category of Food

Modeling uncertainty has a natural monadic interpretation, and probability monads can be put in one-to-one correspondence with the different types of tortilla endofunctors. The proof is trivial given the generalization of the burrito presented by [Morehouse (2015)](#).

Modeling
Sensors

A sensor is characterized by its **angular resolution**, which relates range and diameter of target.

$$\mathrm{Sensor} : \mathrm{Range} \times \mathrm{Diameter} \to [0, 1]$$

$$P_i(r, d)$$

We combine this with the angular cadence, i.e. percent of sky covered in 24 hours.

$$\mathrm{DailySensor} : \mathrm{Range} \times \mathrm{Size} \times \mathrm{St} \to [0, 1]$$

$$\bar{\alpha}_i \approx \frac{c_i}{4\pi} \qquad \bar{P}_i(r, s) = \bar{\alpha}_i P_i(r, d)$$

Finally, we define combined detection probability:

$$F(r, d) = 1 - \Pi_i(1 - \bar{P}_{ground,i}(r, d))\Pi_j(1 - \bar{P}_{space,j}(r, d))$$

Modeling
Sensors

NEO absolute magnitude:

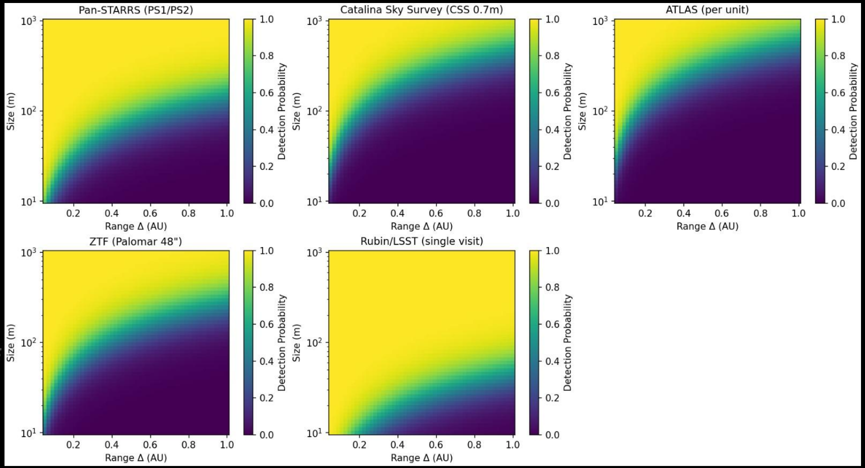
$$H = 5 \cdot \log_{10}\left(\frac{1329}{D_{\mathrm{km}\sqrt{p_v}}}\right)$$

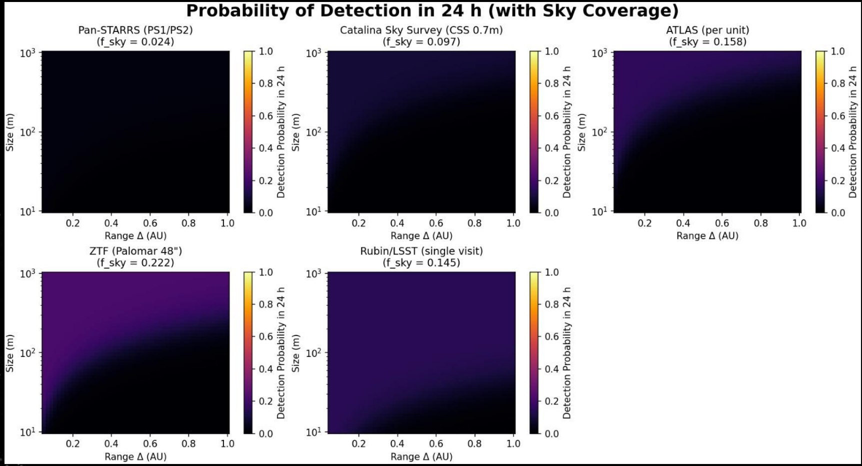
NEO apparent magnitude:

$$m \approx H + 5 \cdot \log_{10}(r_{\odot}\Delta)$$

Single-Exposure Detection Probability

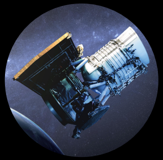
$$P_{\mathrm{det}} = \frac{1}{1 + \exp\left(\frac{m(r,d) - m_{\mathrm{lim}}}{k}\right)}$$





Modeling

Sensors



Equilibrium Temperature:

$$T(r) = 278\mathrm{K}\left(\frac{1-A}{\epsilon\mu}\right)^{\frac{1}{4}} r^{-\frac{1}{2}}$$

Planck Radiation

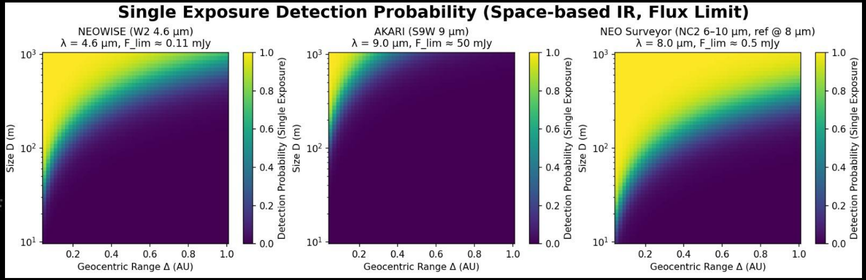
$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}$$

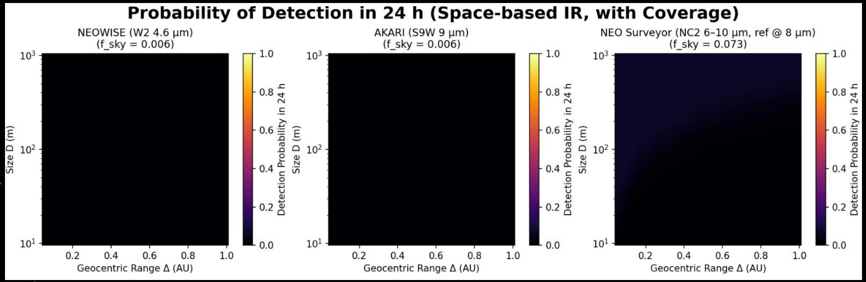
Flux at Earth

$$F_v(r,d) = \frac{\epsilon\pi B_\lambda(T) d^2}{4\Delta^2}\frac{\lambda^2}{c}$$

Single Exposure

$$P_{\mathrm{det}} = \frac{1}{1+\exp\left(\frac{\log_{10}(F_{v,\mathrm{lim}})-\log_{10}(F_v(r,d))}{k}\right)}$$



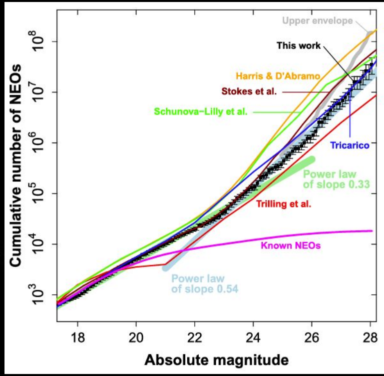


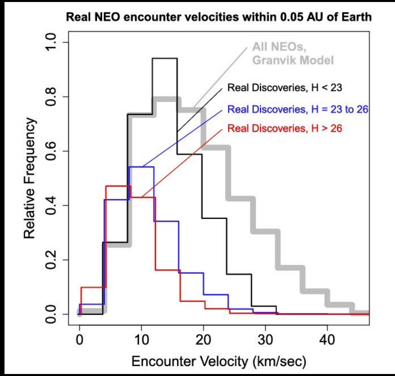
Modeling

Time to Impact

Provided the distributions:

- Size: $p_D(d)$
- Velocity: $p_V(v)$
- Orbit Geometry: $p_\Phi(\phi)$





Hazard Function:

$$\lambda(\tau) = \int_d \int_v \int_0^{\frac{\pi}{2}} F(v\tau cos(\phi), d) p_D(d) p_V(v) p_\Phi(\phi) d\phi dv ds$$

Survival:

$$S(\tau) = exp\left(-\int_\tau^{\tau_{max}} \lambda(u) du \right)$$

Probability Distribution Function:

$$p_\tau(t) = \frac{\lambda(t)S(t)}{1 - S(0)}$$

Modeling
Delta V

Provided the distributions:

- Size: $p_D(d)$
- Velocity: $p_V(v)$
- Time to Impact: $p_T(t)$

Tangential Delta-V that must be transferred for a 2-Earth Radii Miss [4][8]:

$$\Delta v = \frac{2R_{\oplus}}{t_{TTL}}$$

Interception angle plays an important role, difficult to model without simulation.

Modeling

Kinetic Interceptor

$$m_{\mathrm{NEO}} = \rho \frac{2000}{6} D^3$$

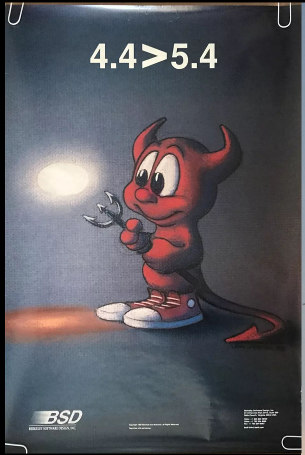
$$Rocket : Mass \times C_3 \rightarrow Bool$$

$$m_{sat} = m_{prop} + m_{impactor}$$

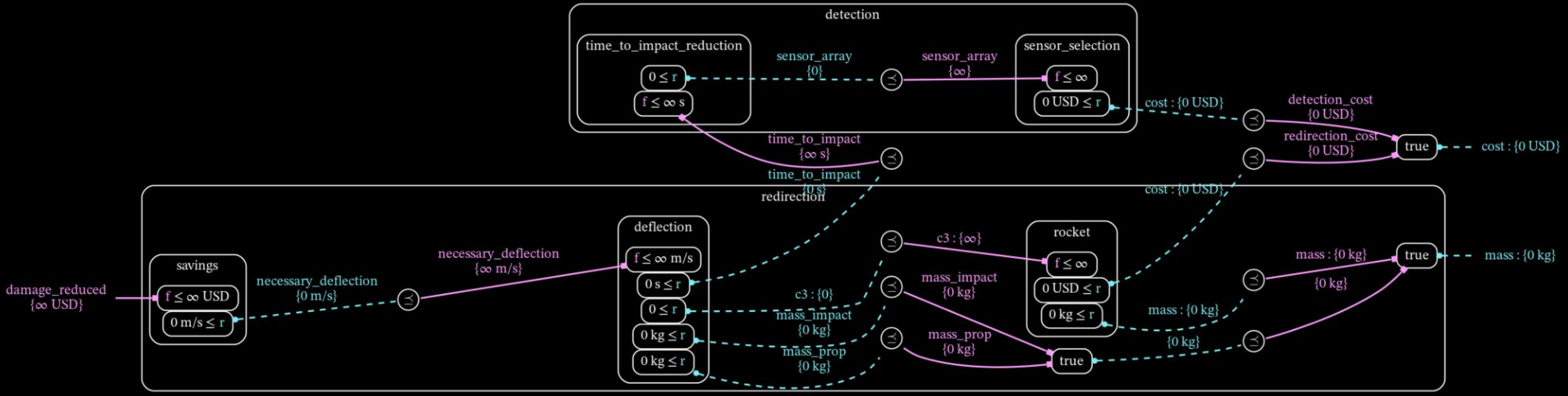
$$v_{impact} = v_e \ln\left(\frac{m_{sat}}{m_{impactor}}\right) + \sqrt{C_3}$$

$$t_{flight} = \frac{R}{\sqrt{v_{ast}^2 + v_{impact}^2}}$$

$$\Delta v_{req} = \frac{2R_{Earth}}{t_{TTL} - t_{flight}} \leq \frac{m_{impactor} v_{impact} \beta}{m_{ast}}$$



"The most important application of Toquos Theory is combating the devastating effects of Subgroup Psychosis stemming from several somnial attacks from the Number Devil, as he has the ability to invade dreams, especially those related to mathematics." —Emma O'Neil



Proprietary MCDPL-generated design schematics; for more information see:
https://code.functor.systems/q9i/blue-dome