

Jupiter

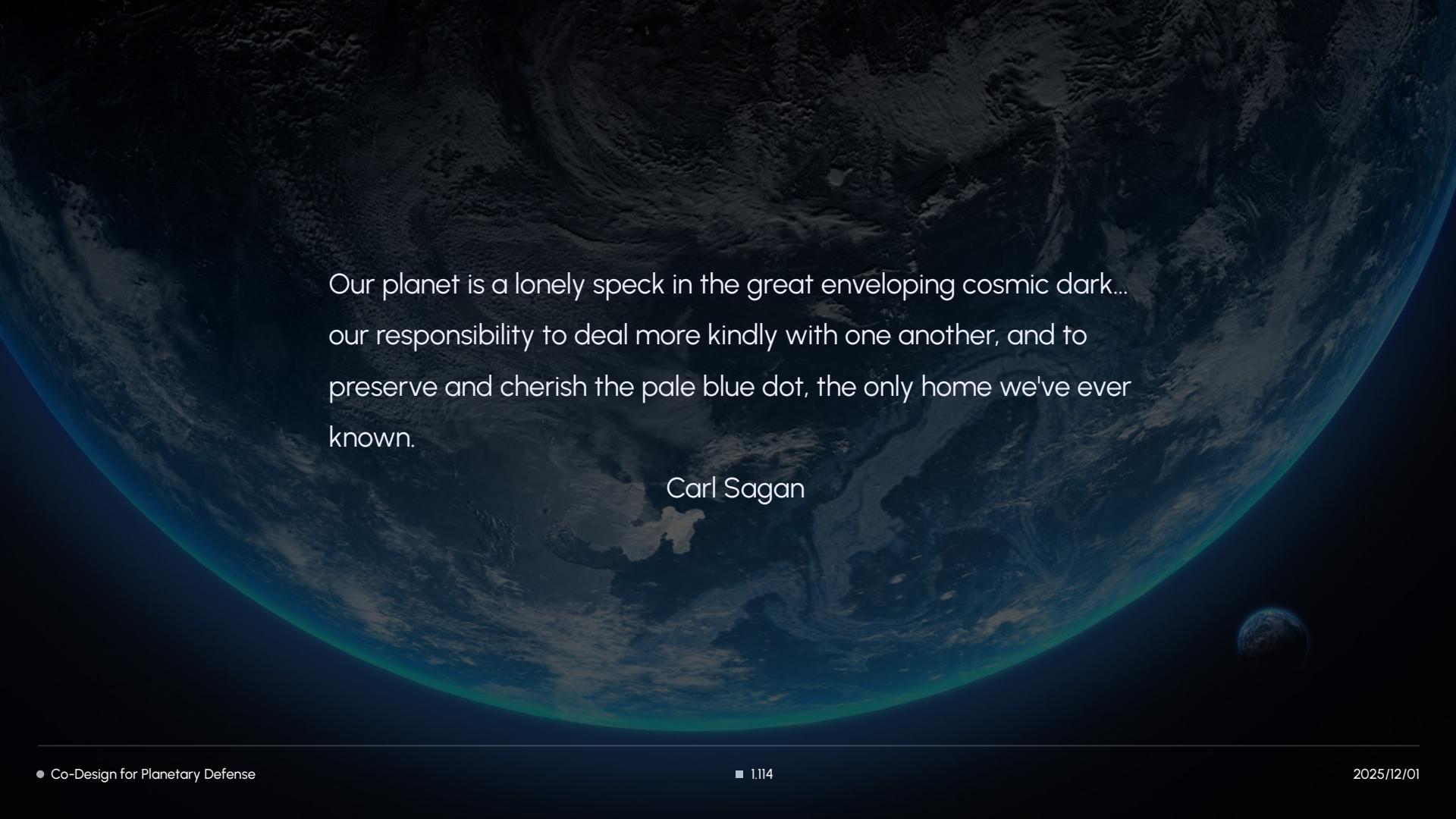
Earth Venus Mercury Sun

Mars

# Co-Design for Planetary Defense

1.144 Final Presentation

Niclas Scheuer  
Ananth Venkatesh



Our planet is a lonely speck in the great enveloping cosmic dark...  
our responsibility to deal more kindly with one another, and to  
preserve and cherish the pale blue dot, the only home we've ever  
known.

Carl Sagan

# NEO Defense Framework

## Table of Contents

- Motivation
- Modeling Threats
- Modeling Sensors
- Solving Detection Problem
- Results
- Compositionality and Nuclear Weapons
- Analysis & Discussion

# Motivation



Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.



**February 13, 2013**

- 13-meter diameter meteor strikes Chelyabinsk, Russia
- > 1600 injured
- No prior warning

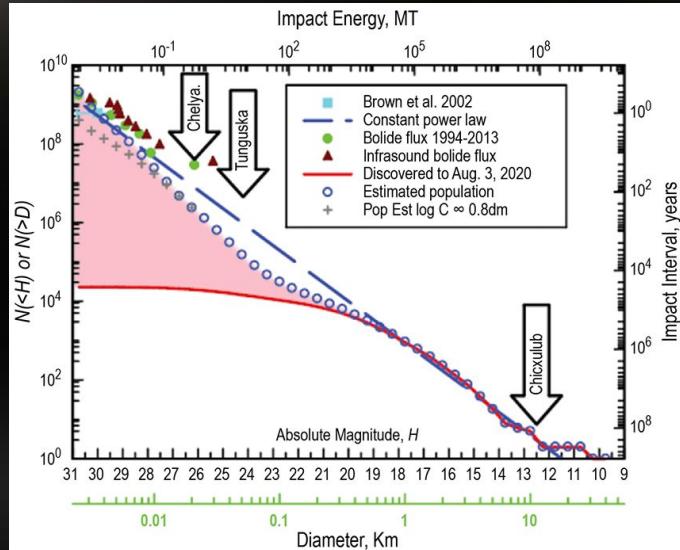
# Motivation



Near-Earth Objects (NEO) of size ( $>50\text{m}$ ) pose a significant risk to human life on Earth.



Majority of NEOs of size 50–140m remain **undiscovered**.



"The number of undiscovered NEOs larger than 140m is on the order of 10,000"

- NASA Decadal Planetary Defense Study [3]

# Motivation



Near-Earth Objects (NEO) of size (>50m) pose a significant risk to human life on Earth.



Majority of NEOs of size 50-140m remain **undiscovered**.



Current NEO warning systems give us limited warning times.



## Warning Times [3]:

- 20m NEO: 1 day
- 100m NEO: 21 days

## Current Interception Times [4]:

- 54 months

# How to defend our planet from Near-Earth Objects



Categorical Approach for **transparency**, **tractability**, and **interoperability**



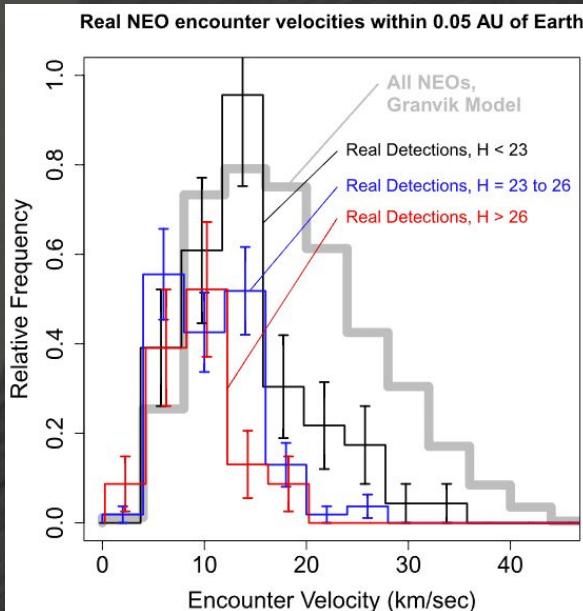
Multiple Stakeholders



Complex Dynamics



Multiple Approaches



Size, Velocity, Albedo, Inclination,  
Semi-Major Axis, Eccentricity Data and  
Estimates Exist.

# Modeling Threats

Models for NEO distributions exist and are piecewise validated by surveyors.

The population of near-earth asteroids revisited and updated

Alan W. Harris <sup>a</sup>, Paul W. Chodas

<sup>a</sup> MoreData! Inc., La Cañada, CA 91011, USA

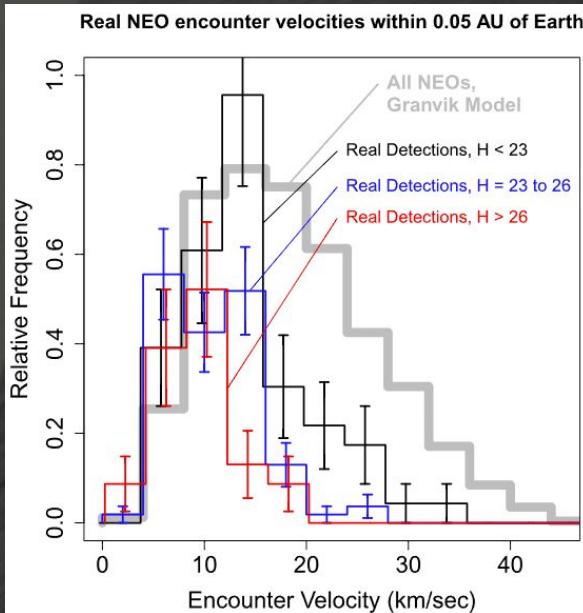
<sup>b</sup> Jet Propulsion Laboratory, Pasadena, CA 91109, USA

National Aeronautics and Space Administration



# Planetary Defense Missions

Rapid Mission Architecture Study



Size, Velocity, Albedo, Inclination,  
Semi-Major Axis, Eccentricity Data and  
Estimates Exist.

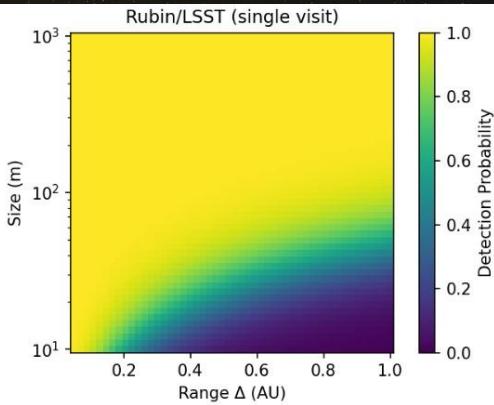
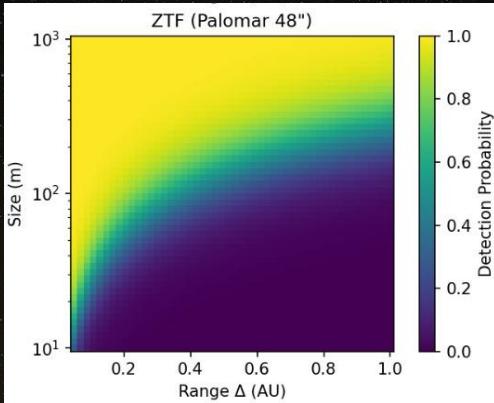
# Modeling Threats

Models for NEO distributions exist and are piecewise validated by surveyors.

SizeDistribution : Size  $\rightarrow [0, 1]$

VelocityDistribution : Velocity  $\rightarrow [0, 1]$

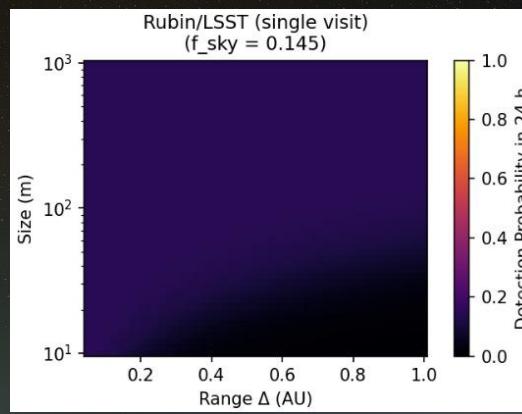
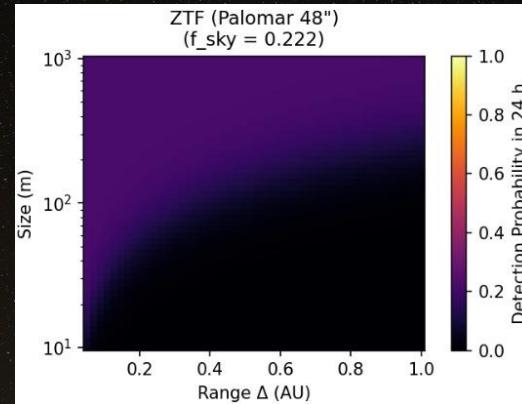
AlbedoDistribution : Albedo  $\rightarrow [0, 1]$



# Modeling Sensors

Using publicly-available data on several ground stations, we characterize the single-exposure detection probability and 24-hour detection probability.

$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \rightarrow [0, 1]$$



# Modeling Sensors

Using publicly-available data on several ground stations, we characterize the single-exposure detection probability and 24-hour detection probability.

$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \rightarrow [0, 1]$$

Consider:

- Limited sensor cone of sensor
- 24h isotropic sky-coverage approximation

$$\mathbb{P}_{24 \text{ hr}} : \text{Range} \times \text{Size} \rightarrow [0, 1]$$

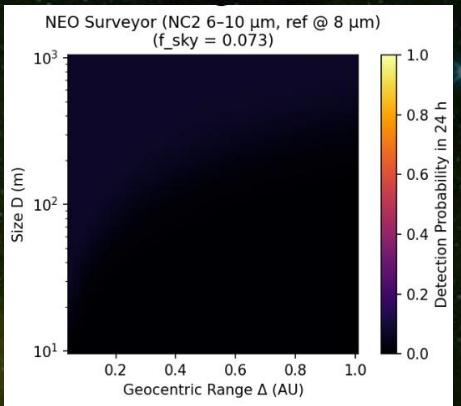
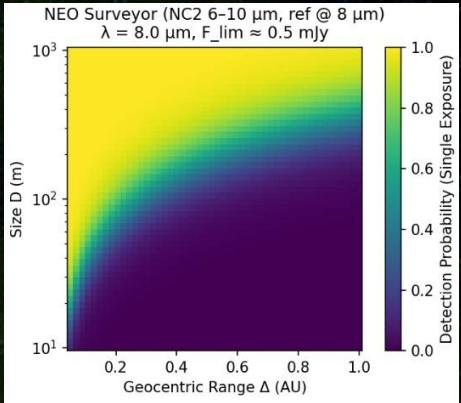
# Modeling Sensors

Similar calculation for space-based sensors, using infrared emissivity.

- 24h isotropic sky-coverage approximation
- Efficacy scaling based on cone angle

$$\mathbb{P}_{\text{single}} : \text{Range} \times \text{Size} \rightarrow [0, 1]$$

$$\mathbb{P}_{24 \text{ hr}} : \text{Range} \times \text{Size} \rightarrow [0, 1]$$



# Sensor topology

Sensor detection functions are distributions over **range x size** invariant to constant multiples of the same **range-size** pair; parametrized by **range:size** ratio



# Sensor topology

Dimensionless poset of **range:size** ratios forms domain for sensor detection functions, which are **monotone maps** from the opposite of this poset to the **unit interval  $[0, 1]$** .

Specifically, low range:size gives high probability of detection and high range:size gives low probability of detection. We then have an ordering of sensors based on the standard ordering of monotone maps.



# Combining Sensors

Posets:

Range x Size

$$(r_1, s_1) \leq (r_2, s_2) \text{ if } r_1 \geq r_2, s_1 \leq s_2$$

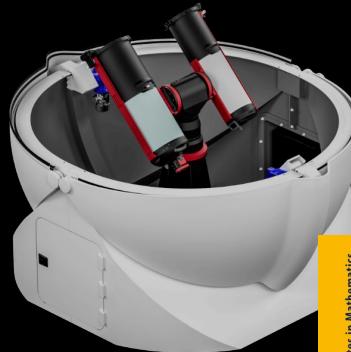
Probabilities

$$[0, 1], \leq$$

Monotone Maps:

$$\mathbb{P}_{24hr} : \text{NEOProperties} \rightarrow \text{Probability}$$

$$(r_1, s_1) \leq (r_2, s_2) \Rightarrow P_{24h}(r_1, s_1) \leq P_{24h}(r_2, s_2)$$



Robert A. McCoy  
Ibula Ntantu

**Topological Properties  
of Spaces of Continuous  
Functions**

1315

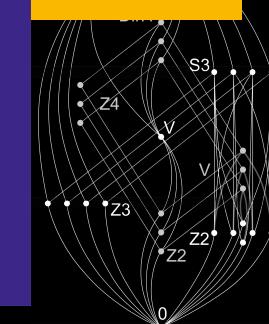


Atlantis Studies in Mathematics 4  
Series Editor: Jan van Mill

G.L.M. Groenewegen  
A.C.M. van Rooij

**Spaces of Continuous  
Functions**

Springer



# Monoidal probability fusion

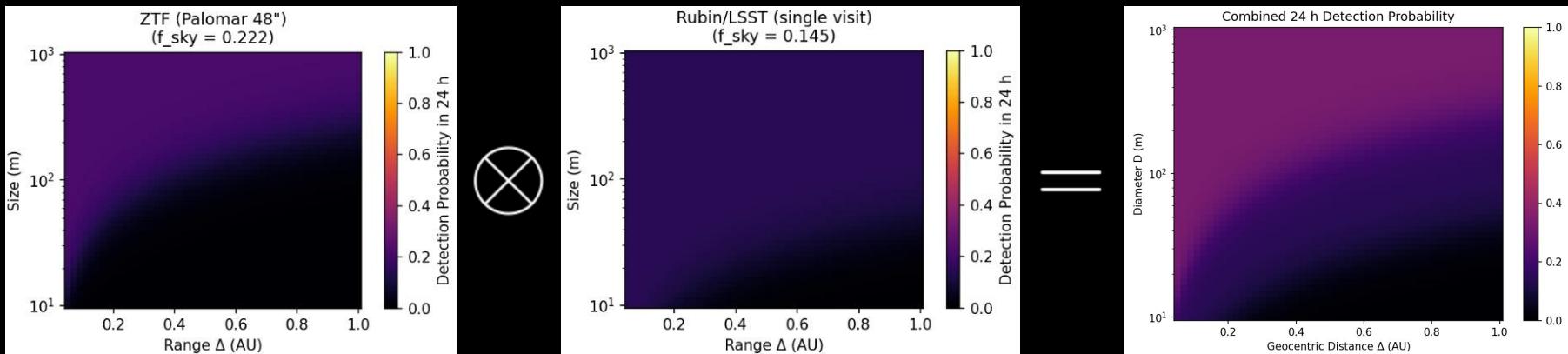
$$p_1 \otimes p_2 = 1 - (1 - p_1)(1 - p_2)$$

Commutative monoid on probabilities; gives a new detection function better than or the same as its composite detection functions

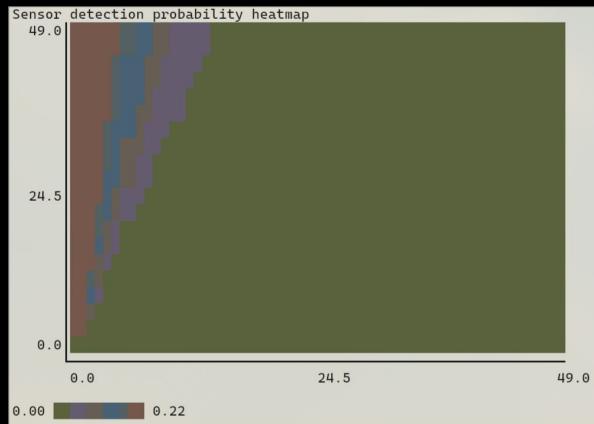
(This is **not** the probability monad)



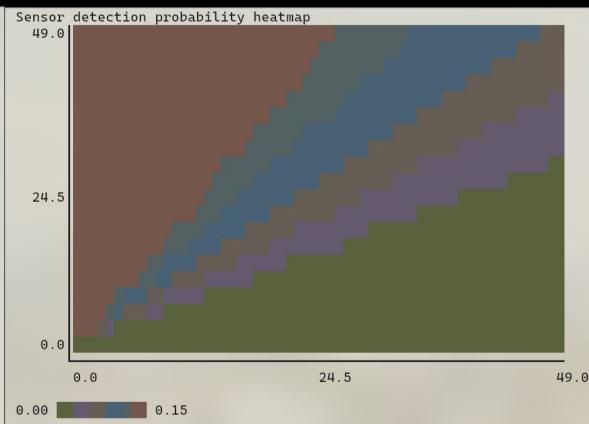
# Combining Sensors



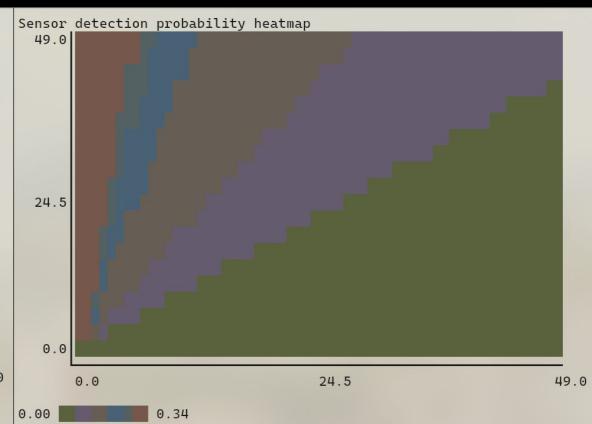
# Discretized MCDP sensor combinations



ZTF (Palomar)  
max detection probability:  
**22%**



Rubin/LSST  
max detection probability:  
**15%**



Combined  
max detection probability:  
**34%**

# Detection

Over the distributions

SizeDistribution : Size → [0, 1]

VelocityDistribution : Velocity → [0, 1]

AlbedoDistribution : Albedo → [0, 1]

what is the probability that a NEO stays undetected?



# Detection

Start survival function  
at  
 $S(\tau_k) = 1$



NEO begins  
undetected at a large  
time-to-impact TTI



# Detection

$$\lambda(\tau) \sim E[P_{24h}(R, S)]$$



Calculate the  
expectation of  
detecting the NEO in  
24h



# Detection

$$S(\tau_k) = S(\tau_{k+1})(1 - \lambda(\tau_k))$$



Update survival  
function



# Detection



As more time passes, we get  
more attempts at spotting the  
NEO.



> Slow NEOs easier to catch

# Detection



As the NEO gets closer, we get  
better at detecting!

$\mathbb{P}_{24\text{hr}}$  : NEOProperties  $\rightarrow$  Probability  
is a monotone map

# Detection

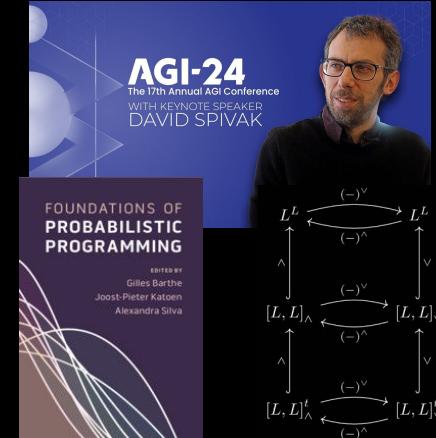


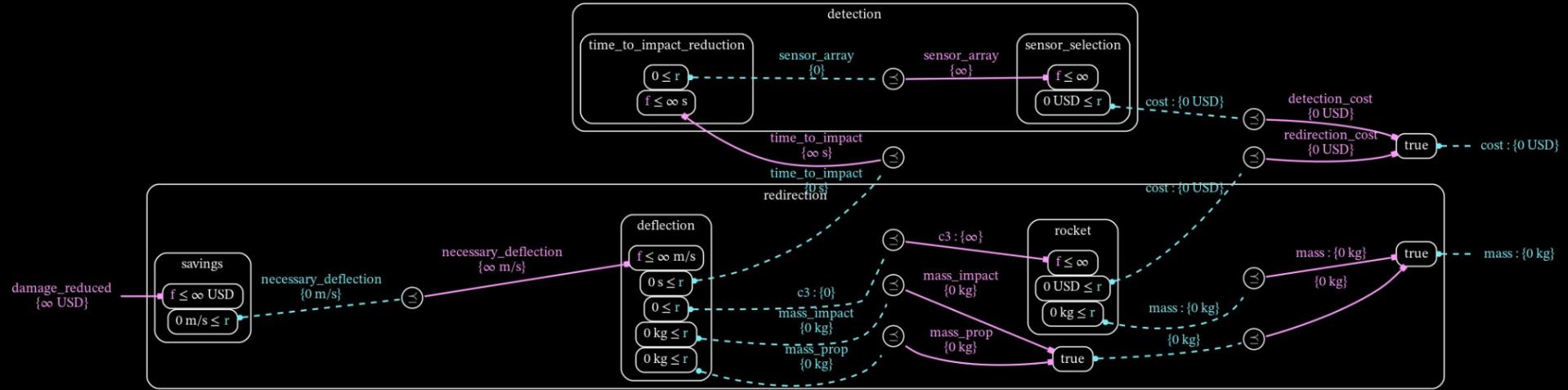
Survival function reaches 0



# The case for co-design

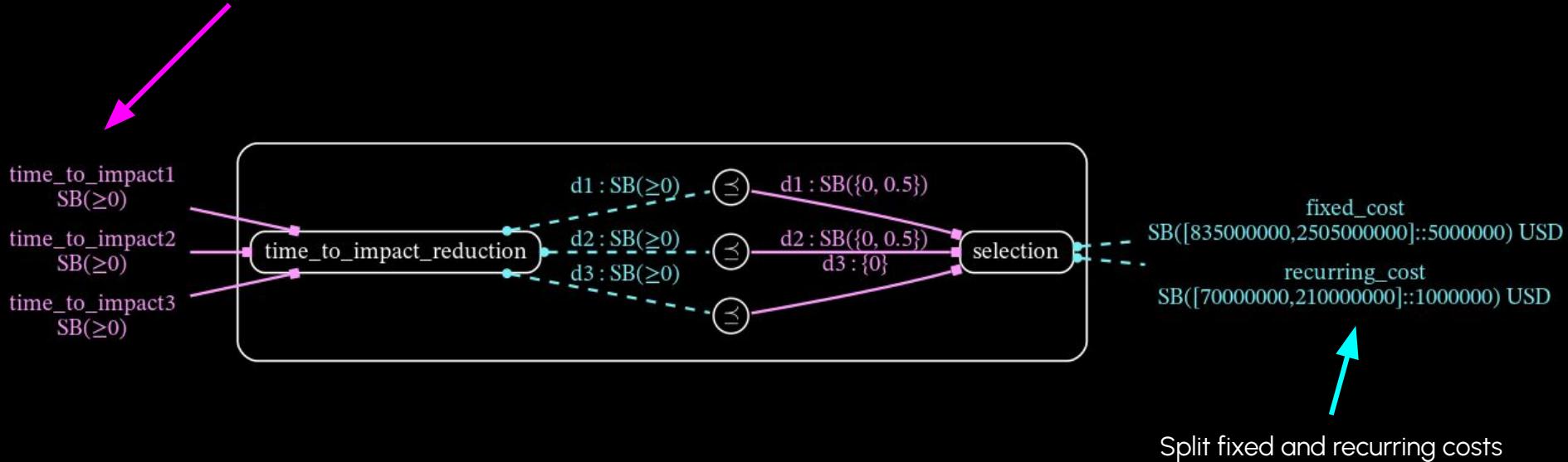
Categorical informatics,  
pareto fronts, probability  
monads, and the like





# System diagram

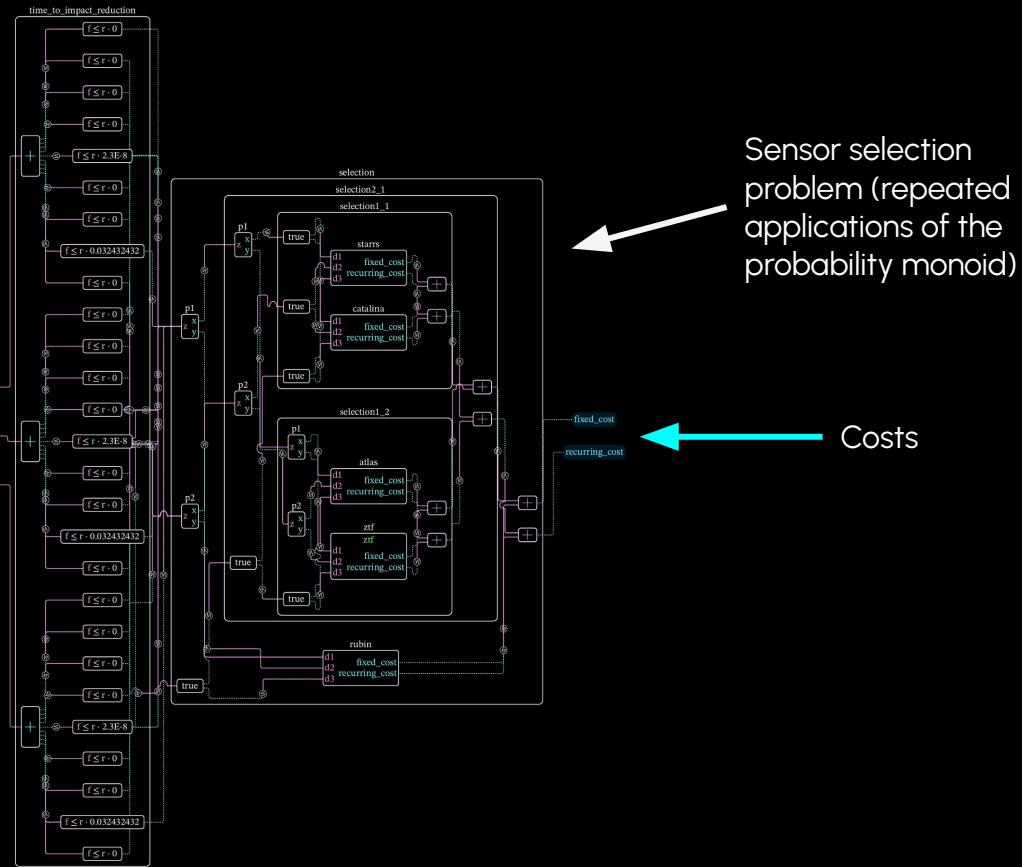
Instead of expected time to impact, compute the hazard function at representative times to get probabilities of detection



Zoom in on revised detection block

24 hour detection probabilities at representative times

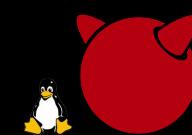
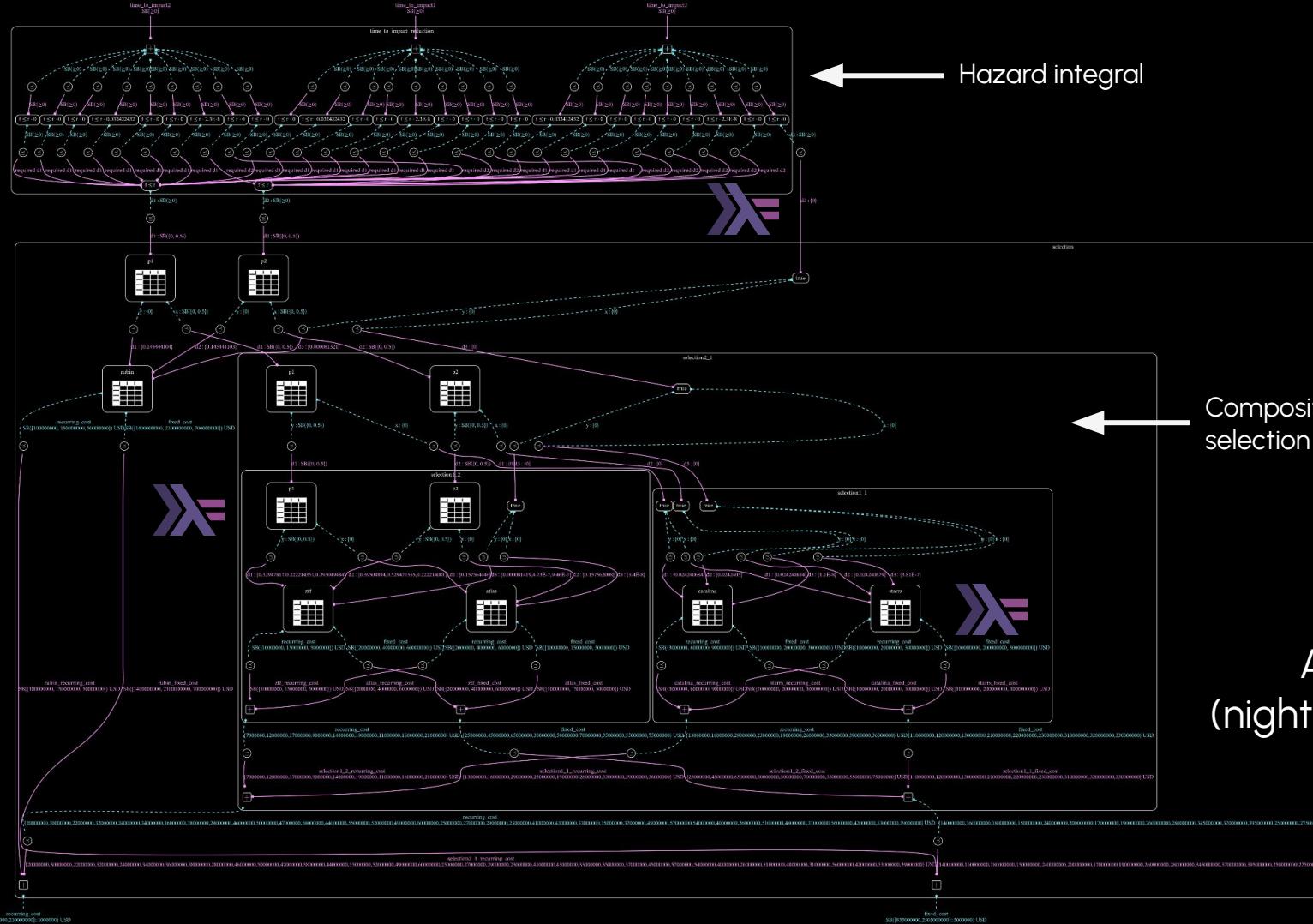
Hazard function  
(double integral approximated as Riemann sum)



Highly discretized (3 point approximation) DP representing sensor selection  
(full DP is orders of magnitude more cursed)

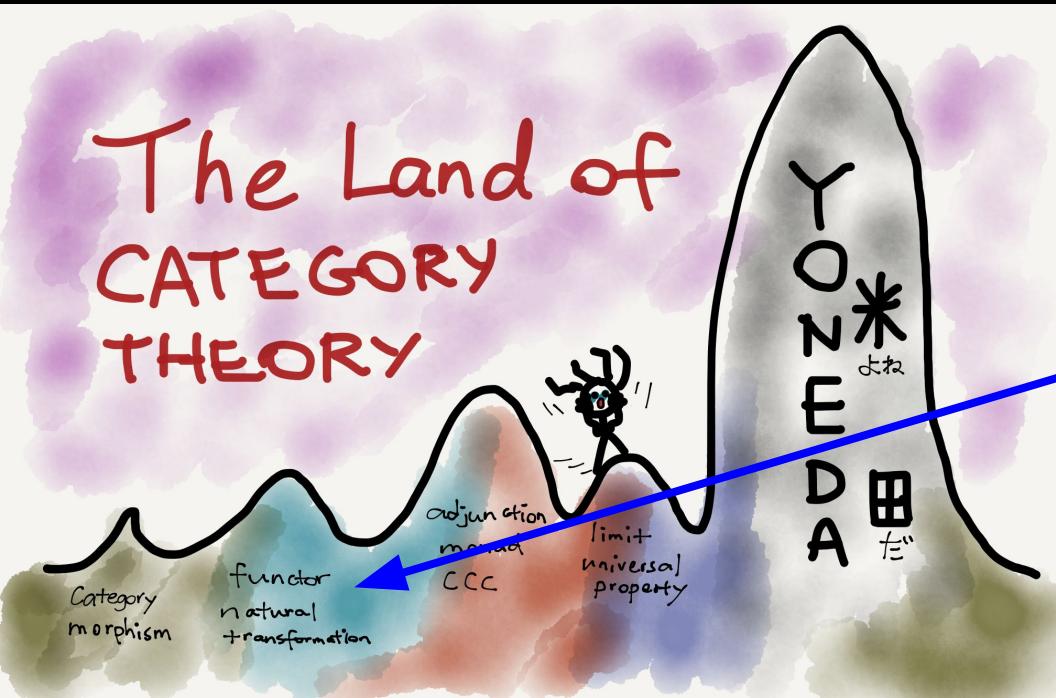
Sensor selection problem (repeated applications of the probability monoid)

Costs



Another view  
(nightmare design  
problem)

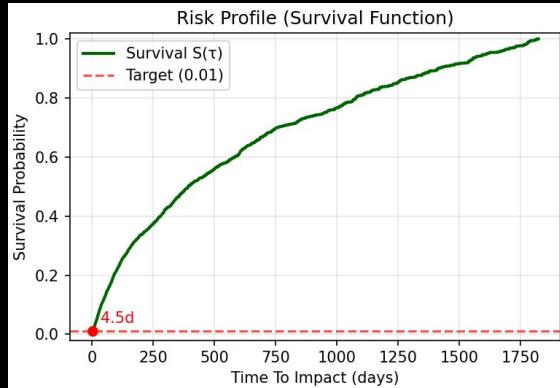
# The Land of CATEGORY THEORY



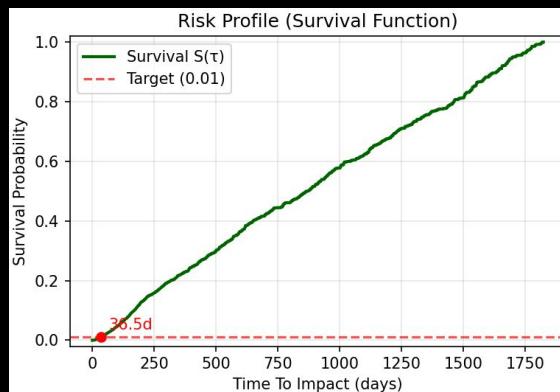
**Functionality** of the solution map means we can get away with very inefficient specifications, provided they're compositional

(we heavily abused this property in order to represent our problem in MCDP)

# Detection



- Survival decreases more rapidly close to Earth
- Adding more sensors increases warning time



# Detection DPI



Given a distribution of **NEO sizes** and **velocities**, what combination of investments in **detection** and **redirection** yields the greatest **lead time** for planetary defense?



Ground-based Observatories



Space-based Observatories

# Detection DPI

**Functionality Space F:**

$$f = \tau \in \mathbb{R}_{\geq 0}$$

**Requirement Space R:**

$$r = C \in \mathbb{R}_{\geq 0}$$

**Implementation Space I:**

Discrete multiset of sensors.

$$i = (n_1, n_2, \dots, n_k)$$

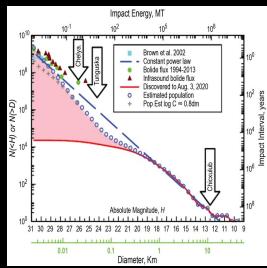
**Delivered Functionality:**

$$F(i) = \tau(i)$$

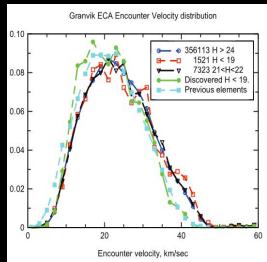
**Required Cost:**

$$C(i) = \sum_j n_j c_j$$

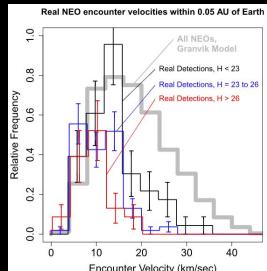




SizeDistribution : Size → [0, 1]

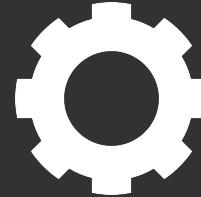


VelocityDistribution : Velocity → [0, 1]



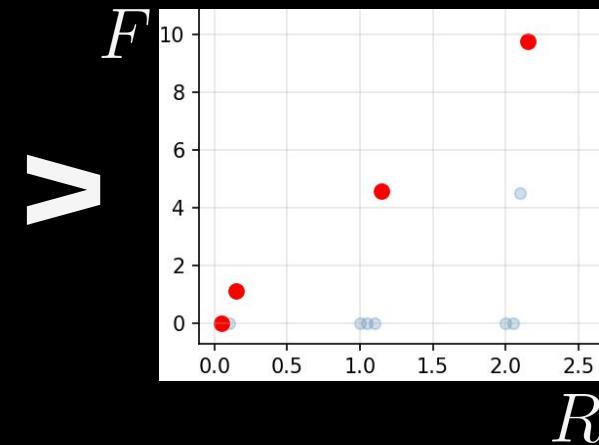
AlbedoDistribution : Albedo → [0, 1]

# Monte Carlo Sampling of Design Problems

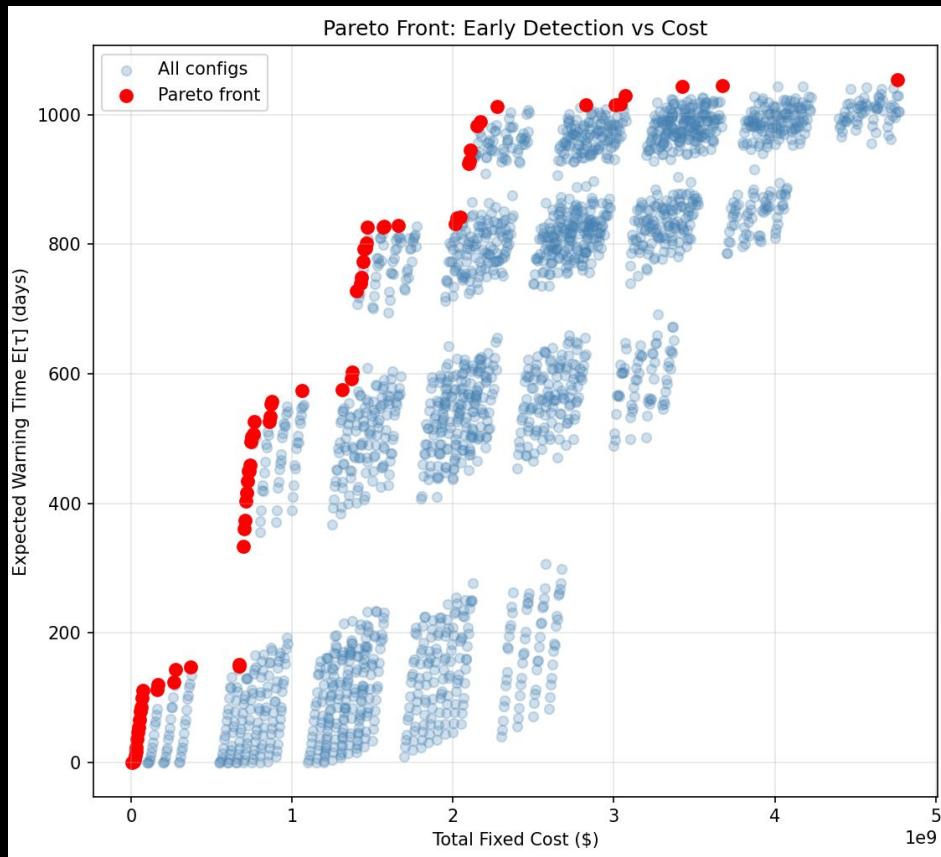


**FixResMaxFun**

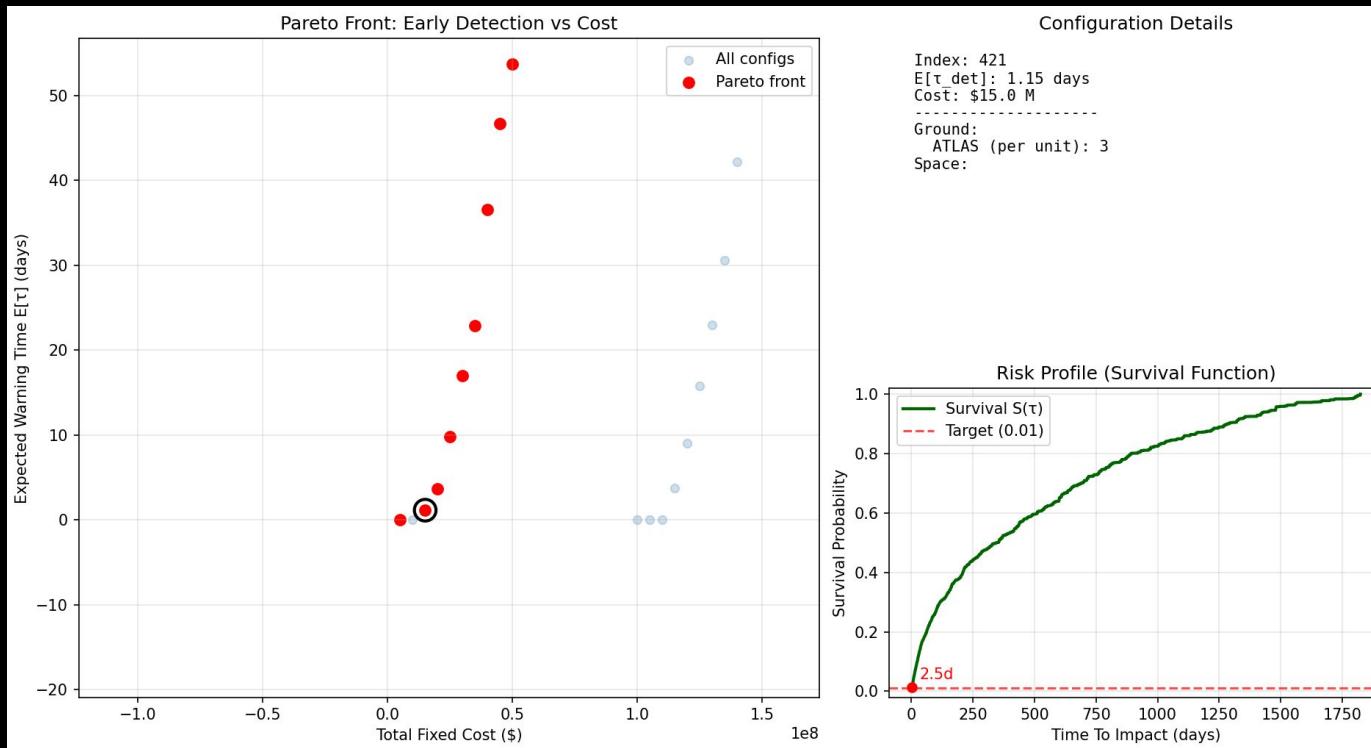
**FixFunMinRes**



# Results



# Results

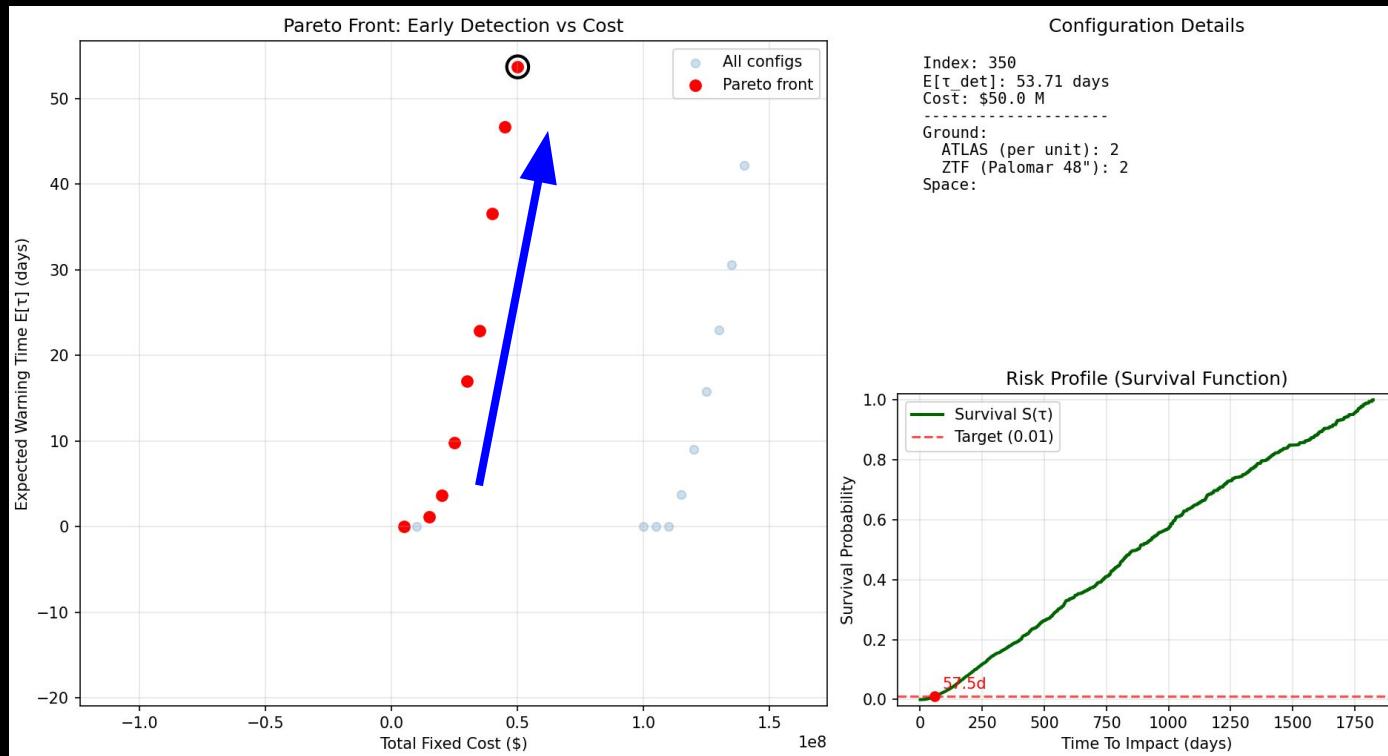


x3

3x ATLAS Observatory (Hawaii) provides warning times as available in literature.

~ \$5,000,000 each

# Results



x2

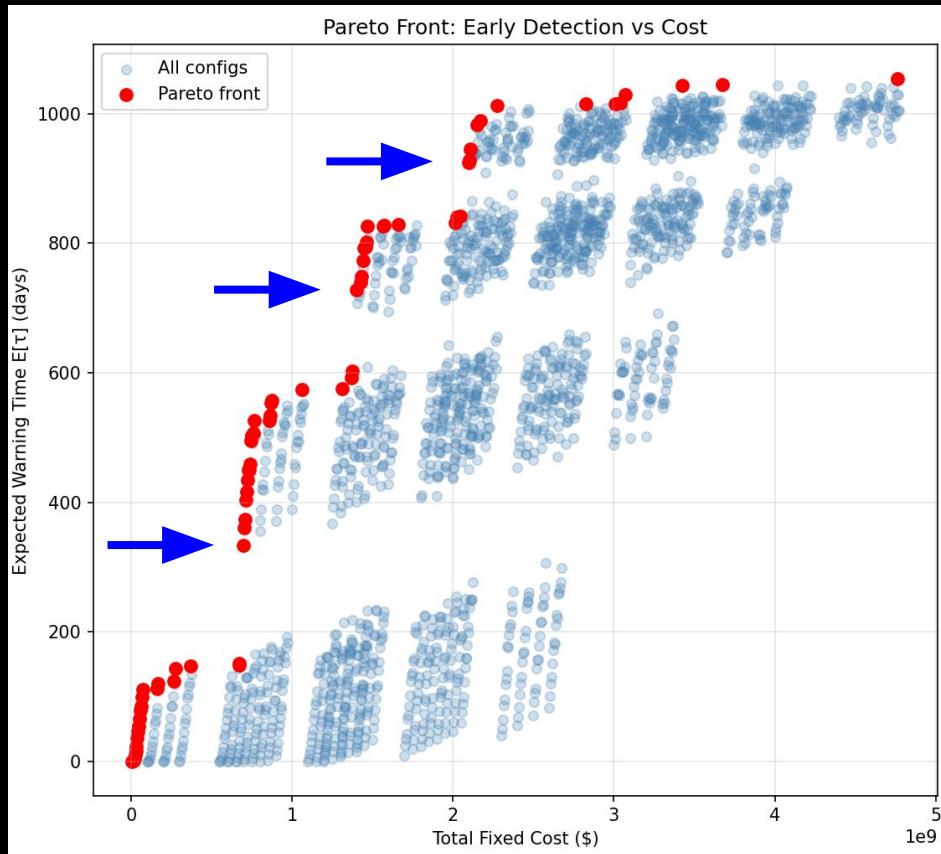
x2

Combination of  
**ATLAS & ZTF**  
Observatories provide  
functionality with little  
cost increase

~ \$5,000,000  
(ATLAS)

~ \$20,000,000 (ZTF)

# Results



x3



x2

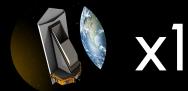
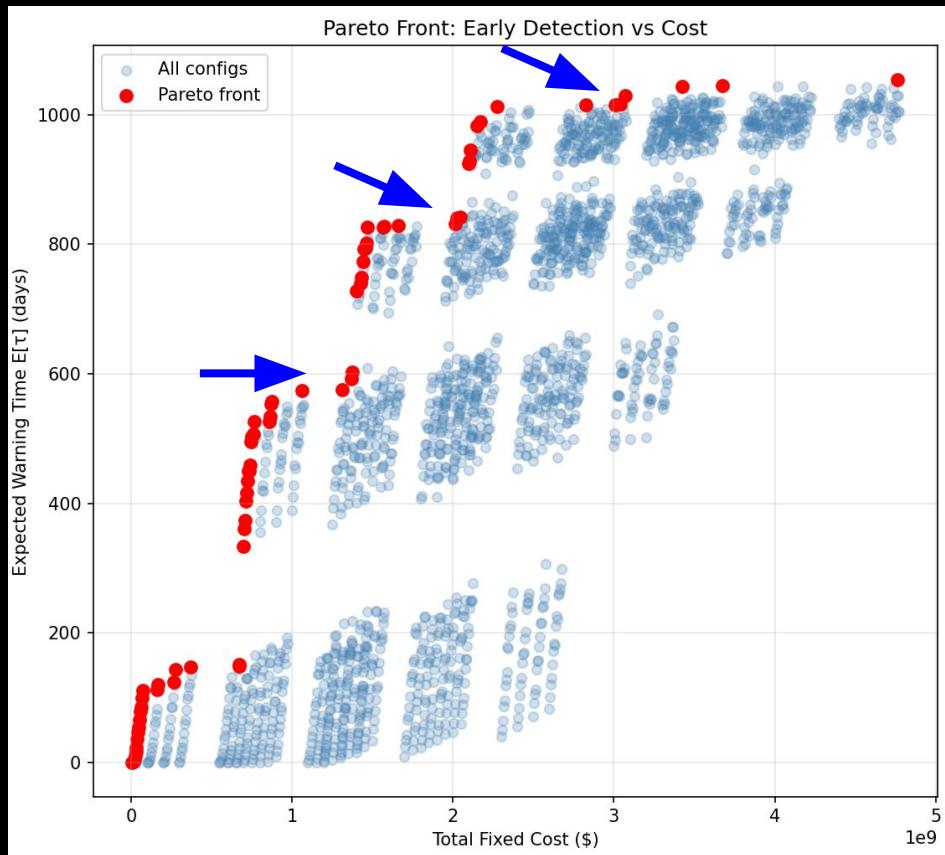


x1

**RUBIN** Observatory (Chile)  
provides the largest jumps,  
but is quite expensive.

~ \$700,000,000 each

# Results

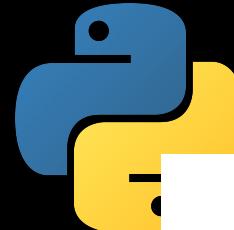


x1

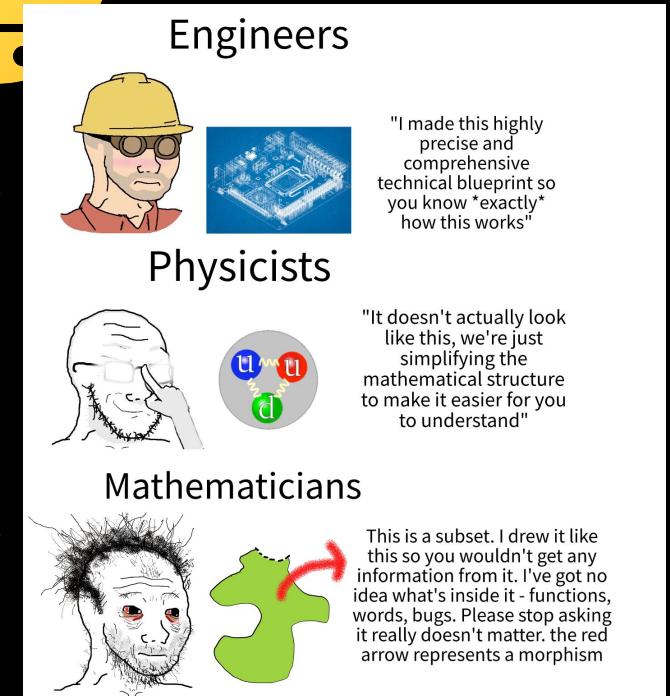
Orbital Surveyors like  
**NEOCAM** become valuable in  
the extremes.

~ \$600,000,000 each

Python for approximate  
high-fidelity solution



Haskell/MCDP pipeline gave  
provably optimal design solutions  
(for 3 data points)



# MCDP

```
x required by p1 ≤ d1 provided by atlas
y required by p1 ≤ d1 provided by ztf
provided d1 ≤ z pro
  ...
  -- sampling
x required by p2 ≤
y required by p2 ≤
provided d2 ≤ z pro
x required by p3 ≤
y required by p3 ≤
provided d3 ≤ z pro
funProvSample :: Int -> [Tex
funProvSample i =
let
  prob :: Text
  prob = probabilityName <
  sample :: Text
  sample = sensorDetectionFunctionName >> show i
in
  [(p1Name `reqBy` prob) `lessThan` (sample `provBy` s1),
   (p2Name `reqBy` prob) `lessThan` (sample `provBy` s2),
   provided sample `lessThan` (pName `provBy` prob)
]
```

```
linspace :: Int -> Bounds Double -> [Double]
linspace n (start, end) = [start, start + step .. end]
  where
    step = (end - start) / (fromIntegral n - 1)

stitch :: [Double] -> [Double] -> [[(Double, Double)]]
stitch xs ys = [[(x, y) | y <- ys] | x <- xs]

diag :: [Double] -> [[(Double, Double)]]
diag xs = stitch xs xs
```

043243243 \* 0.043243243 \* 0.00000269 + d1  
00000 \* 0.00000000 + d4 \* 0.043243243 \* 0.00000269 + d1  
00269 + d1 \* 0.043243243 \* 0.00000269 + d1  
00000 \* 0.00000000 + d4 \* 0.043243243 \* 0.00000269 + d2  
00269 + d2 \* 0.043243243 \* 0.00000269 + d2

- Discretize everything
- Haskell-based templating metalanguage
- Dynamically set resolution and solve for antichains
- Describe sensor combination with compositions of the probability monoid



tomie ✅ @tomieinlove · 4d

I wonder how much money OpenAI has lost in electricity costs from people saying “please” and “thank you” to their models.

1.2K 5.8K 181K 5.7M ⌂ ⌂



Sam Altman ✅

@sama

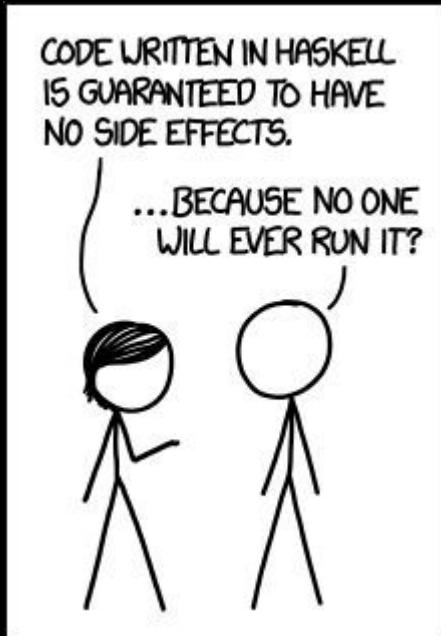
Replies to @tomieinlove

tens of millions of dollars well spent--you never know

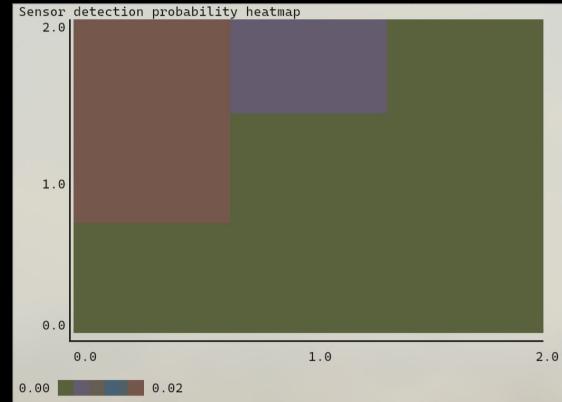
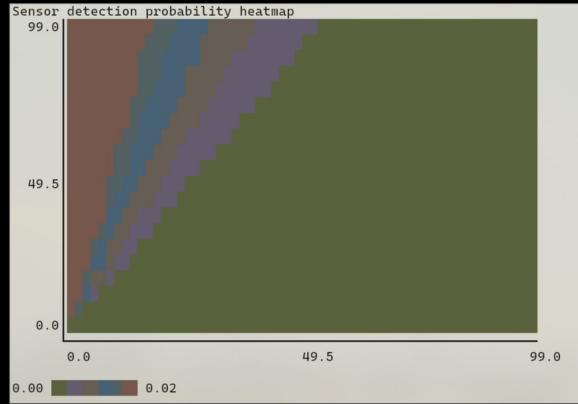
4:45 · 17 Apr 25 · 441K Views

# Results

(with MCDP)



- Crashed web editor multiple times
- Wasted tremendous amounts of compute
- Haskell code worked flawlessly
- MCDP killed itself several times likely due to (d)OOM issues



# Discretization :(

# 2.58%

Maximum 24-hour detection probability with a \$2.5 billion budget  
(recurring cost bounded at 10% of total budget)

# Compositionality and Nuclear Weapons

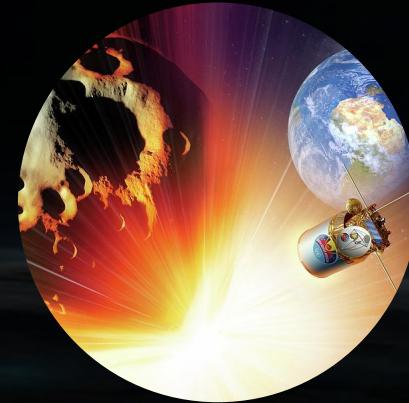
Redirecting asteroids close to Earth is **harder** than redirecting asteroids far from Earth.

We find that **warning time  $\rightarrow$  intercept cost** is anti-monotone.

$$f : \tau \rightarrow \mathbb{R}_{\geq 0}$$

Total cost:

$$C_{total} = C_{sensors} + f(\tau)$$



Nuclear redirection offers the most performant protection on the shortest time scale of all redirection methods.

# Compositionality and Nuclear Weapons

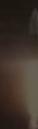
Redirecting asteroids close to Earth is **harder** than redirecting asteroids far from Earth.

We find that **warning time -> intercept cost** is **anti-monotone**.

$$f : \tau \rightarrow \mathbb{R}_{\geq 0}$$

Total cost:

$$C_{total} = C_{sensors} + f(\tau)$$



## THEOREM

Solving the *Sensor DPI* automatically solves the *Sensor+Interception DPI*.

## PROOF

The integrated design problem is:

$$\min_i (C_i + f(\tau_i))$$

Per absurdum suppose solution A solves *Sensor+Interception DPI* optimally without solving *Sensor DPI* optimally.

Then there exists sensor B that dominates sensor A in *Sensor DPI*:

$$\tau_A < \tau_B$$

$$C_A \geq C_B$$

Because  $f$  is anti-monotone:

$$\tau_A < \tau_B \Rightarrow f(\tau_A) \geq f(\tau_B)$$

$$C_A + f(\tau_A) \geq C_B + f(\tau_B)$$

# Analysis

Capable methods exist to identify asteroid threats early and to act on this information.

Our framework enables:

- Tractable and informed decision making
- Easy updating of sensor architectures
- Adjustment of risk parameters
- Parallelized algorithm that exploits dominance & anti-chains for speedup

Future Work:

- Interceptor Cost Modelling
- Further optimization for large state-spaces



# Questions

# Appendix: Ground Sensors

NEO absolute magnitude:

$$H = 5 \cdot \log_{10}\left(\frac{1329}{D_{\text{km}}\sqrt{p_v}}\right)$$

NEO apparent magnitude:

$$m \approx H + 5 \cdot \log_{10}(r_{\odot}\Delta)$$

Single-Exposure Detection Probability

$$P_{\text{det}} = \frac{1}{1 + \exp\left(\frac{m(r,d) - m_{\text{lim}}}{k}\right)}$$

24h Isotropic Exposure Detection Probability

$$P_{24h} = \frac{c_i}{41253} P_{\text{det}}$$

Publicly-available information on ground-based sensors:

- 5-sigma sensitivity:  $m_{\text{lim}}$
- Deg2 covered per 24h:  $c_i$
- Fixed Cost
- Annual Operating Costs

# Appendix: Ground Sensors



**RUBIN** (Chile)



**ZTF** (California)



**ATLAS** (Hawaii)



**CSS** (Arizona)



**Pan-STARRS** (Hawaii)

# Appendix: Space Sensors

Equilibrium Temperature:

$$T(r) = 278\text{K} \left( \frac{1 - A}{\epsilon\mu} \right)^{\frac{1}{4}} r^{-\frac{1}{2}}$$

Planck Radiation

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Flux at Earth

$$F_v(r, d) = \frac{\epsilon\pi B_\lambda(T)d^2}{4\Delta^2} \frac{\lambda^2}{c}$$

Single Exposure

$$P_{\text{det}} = \frac{1}{1 + \exp \left( \frac{\log_{10}(F_{v,\text{lim}}) - \log_{10}(F_v(r, d))}{k} \right)}$$

Publicly-available information on ground-based sensors:

- Target waveband:
- 5-sigma flux density limit:
- Deg2 covered per 24h
- Fixed Cost
- Annual Operating Costs

# Modeling Sensors

A sensor is characterized by its **angular resolution**, which relates range and diameter of target.

Sensor : Range  $\times$  Diameter  $\rightarrow [0, 1]$

$$P_i(r, d)$$

We combine this with the angular cadence, i.e. percent of sky covered in 24 hours

DailySensor : Range  $\times$  Size  $\times$  St  $\rightarrow [0, 1]$

$$\bar{\alpha}_i \approx \frac{c_i}{4\pi} \quad \bar{P}_i(r, s) = \bar{\alpha}_i P_i(r, d)$$

Finally, we define combined detection probability:

$$F(r, d) = 1 - \Pi_i(1 - \bar{P}_{ground,i}(r, d)) \Pi_j(1 - \bar{P}_{space,j}(r, d))$$

# Modeling Sensors



NEO absolute magnitude:

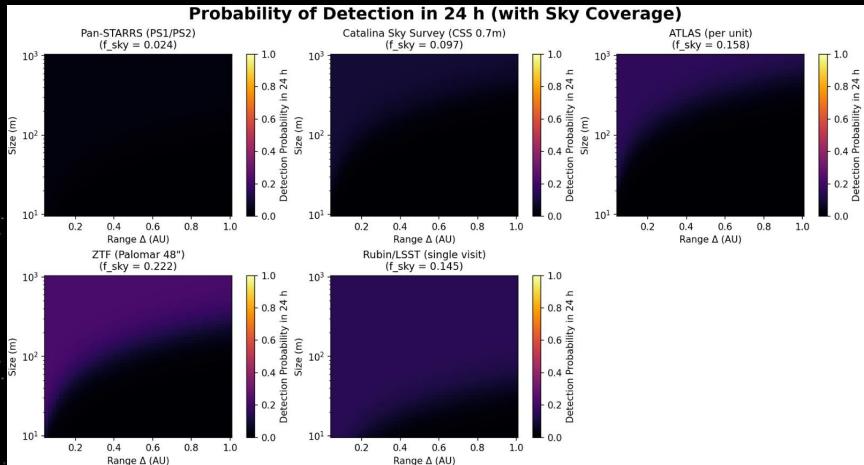
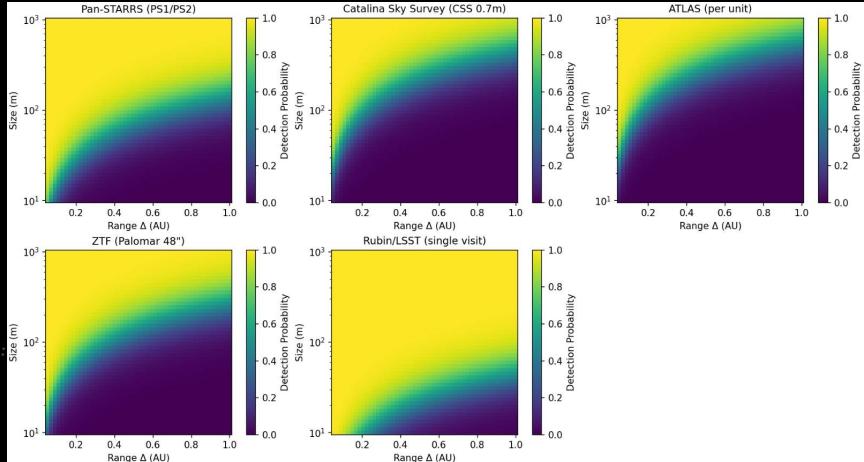
$$H = 5 \cdot \log_{10}\left(\frac{1329}{D_{\text{km}} \sqrt{p_v}}\right)$$

NEO apparent magnitude:

$$m \approx H + 5 \cdot \log_{10}(r_{\odot} \Delta)$$

Single-Exposure Detection Probability

$$P_{\text{det}} = \frac{1}{1 + \exp\left(\frac{m(r, d) - m_{\text{lim}}}{k}\right)}$$



# Modeling Sensors



Equilibrium Temperature:

$$T(r) = 278\text{K} \left( \frac{1 - A}{\epsilon\mu} \right)^{\frac{1}{4}} r^{-\frac{1}{2}}$$

Planck Radiation

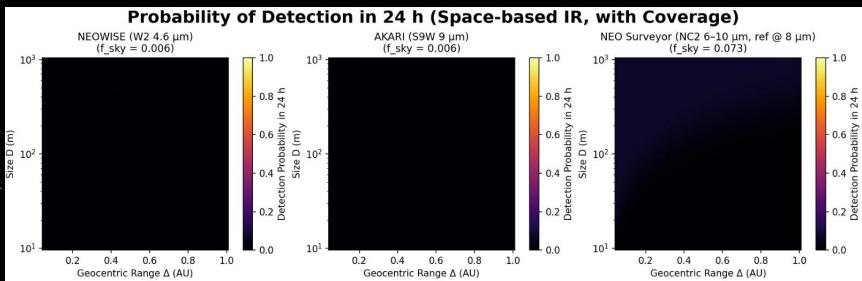
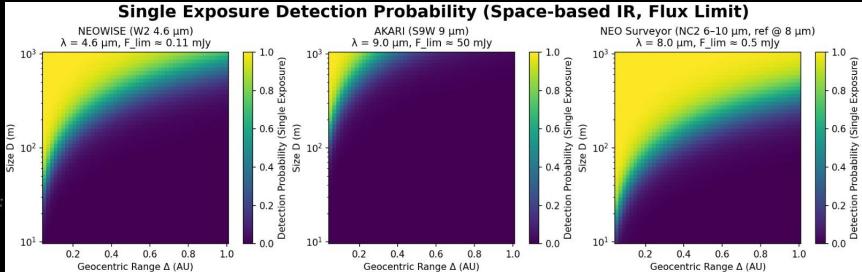
$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Flux at Earth

$$F_v(r, d) = \frac{\epsilon\pi B_\lambda(T)d^2}{4\Delta^2} \frac{\lambda^2}{c}$$

Single Exposure

$$P_{\text{det}} = \frac{1}{1 + \exp \left( \frac{\log_{10}(F_{v,\text{lim}}) - \log_{10}(F_v(r,d))}{k} \right)}$$



# Modeling Time to Impact

Provided the distributions:

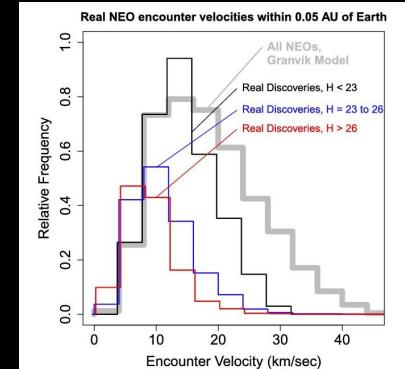
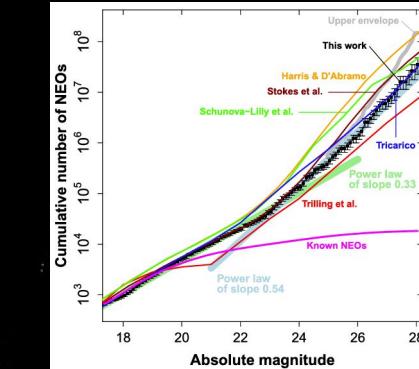
- Size:  $p_D(d)$
- Velocity:  $p_V(v)$
- Orbit Geometry:  $p_\Phi(\phi)$

Hazard Function:

$$\lambda(\tau) = \int_d \int_v \int_0^{\frac{\pi}{2}} F(v\tau \cos(\phi), d) p_D(d) p_V(v) p_\Phi(\phi) d\phi dv ds$$

Survival:

$$S(\tau) = \exp\left(- \int_{\tau}^{\tau_{max}} \lambda(u) du\right)$$



Probability Distribution Function:

$$p_\tau(t) = \frac{\lambda(t)S(t)}{1 - S(0)}$$

# Modeling Delta V

Provided the distributions:

- Size:  $p_D(d)$
- Velocity:  $p_V(v)$
- Time to Impact:  $p_T(t)$

Tangential Delta-V that must be transferred for  
a 2-Earth Radii Miss [4][8]

$$\Delta v = \frac{2R_{\oplus}}{t_{TTL}}$$

Interception angle plays an important role,  
difficult to model without simulation.

# Modeling Kinetic Interceptor

$$m_{\text{NEO}} = \rho \frac{2000}{6} D^3$$

*Rocket : Mass × C<sub>3</sub> → Bool*

$$m_{\text{sat}} = m_{\text{prop}} + m_{\text{impactor}}$$

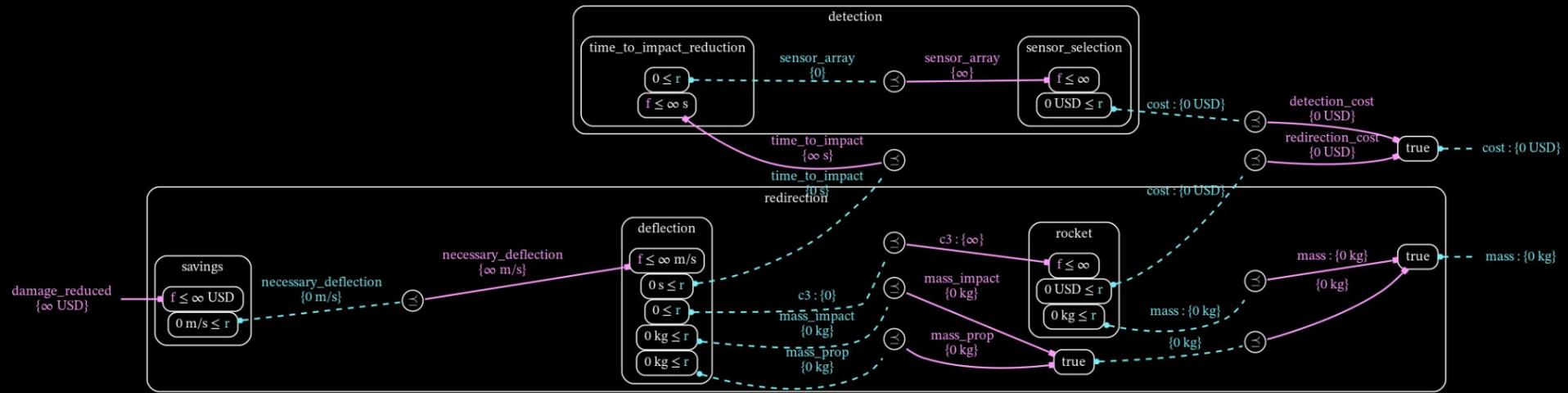
$$v_{\text{impact}} = v_e \ln \left( \frac{m_{\text{sat}}}{m_{\text{impactor}}} \right) + \sqrt{C_3}$$

$$t_{\text{flight}} = \frac{R}{\sqrt{v_{\text{ast}}^2 + v_{\text{impact}}^2}}$$

$$\Delta v_{\text{req}} = \frac{2R_{\text{Earth}}}{t_{\text{TTL}} - t_{\text{flight}}} \leq \frac{m_{\text{impactor}} v_{\text{impact}} \beta}{m_{\text{ast}}}$$



*"The most important application of Toquos Theory is combating the devastating effects of Subgroup Psychosis stemming from several somnial attacks from the Number Devil, as he has the ability to invade dreams, especially those related to mathematics." —Emma O'Neil*



Proprietary MCDPL-generated design schematics; for more information see:  
<https://code.functor.systems/q9i/blue-dome>