CS M51A, Sec. 1, Class Exercises No. 1 - SOLUTIONS

Exercise 2.11

(a)

$$\begin{aligned} 34\,567 &= (0011\ 0100\ 0101\ 0110\ 0111)_{BCD} \\ &= (0110\ 0111\ 1000\ 1001\ 1010\ )_{Excess-3} \end{aligned}$$

(b) BCD does not have the complementary property, so an actual subtraction is needed:

$$99\,999 - 34\,567 = 65\,432 = (0110\ 0101\ 0100\ 0011\ 0010)_{BCD}$$

2421 code has the complementary property, such that the subtraction is done by complementing each bit:

$$\begin{aligned} 34\,567 &= (0011\ 0100\ 1011\ 1100\ 1101)_{2421\text{-code}} \\ 99\,999 - 34\,567 &= (1100\ 1011\ 0100\ 0011\ 0010)_{2421\text{-code}} \end{aligned}$$

Exercise 2.13

(a) $(1001010100011110)_2 = (1001\ 0101\ 0001\ 1110\ )_2 = (951E)_{16}$

(b) $(3456)_8 = (011\ 100\ 101\ 110)_2 = (011100101110)_2$

(c) To convert from radix-2 to radix-2^k we consider groups of k bits. The digits in radix-2^k are obtained converting each group (binary representation of the digit) into a single value in the new radix.

To convert from radix-2^k to radix-2, the digits in radix-2^k are converted to binary. The final vector, that corresponds to the concatenation of all digit representations in binary, is the radix-2 representation of the number.

Exercise 2.15

(a) Prove that $f_{\text{XOR}}(f_{\text{AND}}(x_1, x_0), f_{\text{AND}}(x_1, x_0)) = f_{\text{EQUIVALENCE}}(x_1, x_0)$

x_1	x_0	$f_{\text{AND}}(x_1, x_0)$	$f_{\text{XOR}}(f_{\text{AND}}(x_1, x_0), f_{\text{AND}}(x_1, x_0))$	$f_{\text{EQUIVALENCE}}(x_1, x_0)$
0	0	0	0	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1

The conclusion is:

$$f_{\text{XOR}}(f_{\text{AND}}(x_1, x_0), f_{\text{AND}}(x_1, x_0)) \neq f_{\text{EQUIVALENCE}}(x_1, x_0)$$

(b) Prove that $f_{\text{NAND}}(f_{\text{NAND}}(x_1, x_0), f_{\text{NAND}}(x_1, x_0)) = f_{\text{AND}}(x_1, x_0)$

x_1	x_0	$f_{\text{NAND}}(x_1, x_0)$	$f_{\text{NAND}}(f_{\text{NAND}}(x_1, x_0), f_{\text{NAND}}(x_1, x_0))$	$f_{\text{AND}}(x_1, x_0)$
0	0	1	0	0
0	1	1	0	0
1	0	1	0	0
1	1	0	1	1

The conclusion is:

$$f_{\text{NAND}}(f_{\text{NAND}}(x_1, x_0), f_{\text{NAND}}(x_1, x_0)) = f_{\text{AND}}(x_1, x_0)$$

Exercise 2.16

Each variable can have 2 values (0 or 1).
Total number of n-variable inputs: $\underbrace{2 \cdot 2 \cdot ... \cdot 2}_{n} = 2^n$

For each input, the output function can have 2 values.
Total number of functions: $\underbrace{2 \cdot 2 \cdot ... \cdot 2}_{2^n} = 2^{2^n}$