

## CS M51A, Sec. 1, Class Exercises No. 4 - SOLUTIONS

### Exercise 4.7:

The operation specified by

$$g(x, y, z) = x'yz + xy' + y'z$$

is universal because  $g(x, 1, 1) = x'$  realizes the NOT operation, and  $g(x, 0, z) = x + z$  realizes the OR operation. Since the set  $\{OR, NOT\}$  is universal, the  $g$  operation is also universal.

**Exercise 4.14** (a) The switching expression for output  $z$  is

$$z = E(I_0s'_2s'_1s'_0 + I_1s'_2s'_1s_0 + I_2s'_2s_1s'_0 + I_3s'_2s_1s_0 + I_4s_2s'_1s'_0 + I_5s_2s'_1s_0 + I_6s_2s_1s'_0 + I_7s_2s_1s_0)$$

or

$$z = E \sum_{i=0}^7 I_i m_i(s_2, s_1, s_0)$$

(b) A high-level description is

$$z = \begin{cases} I_s & \text{if } E=1 \\ 0 & \text{otherwise} \end{cases}$$

where  $s = 4s_2 + 2s_1 + s_0$ .

This function corresponds to an 8-input multiplexer (selector), that is discussed in Chapter 9 of the textbook.

(c) The load factor of each input is 1, since each input is connected to only one gate input in the circuit.

The load applied to each gate is:

gate	Load
NOT connected to $E'$	8
NOT gates connected to $s_i$	5
NOT gates to restore $s_i$	4
AND gates	1
OR gate	$L_1 + 1$
NOT used to generate $z'$	$L_2$

where,  $L_1$  and  $L_2$  are the Load imposed to the outputs  $z$  and  $z'$  respectively.

The delay of this network cannot be calculated based on the table given because there are no expressions for an AND gate with 5 inputs and an OR gate with 8 inputs. Calling these delays:

$$\begin{aligned} t_{pLH}(\text{AND-5}) &= T_1 \\ t_{pHL}(\text{AND-5}) &= T_2 \\ t_{pLH}(\text{OR-8}) &= T_3 \\ t_{pHL}(\text{OR-8}) &= T_4 \end{aligned}$$

the critical path delay is:

$$\begin{aligned} t_{pLH}(s_2, z') &= t_{pLH}(\text{NOT}) + t_{pHL}(\text{NOT}) + t_{pHL}(\text{AND-5}) + t_{pHL}(\text{OR-8}) + t_{pLH}(\text{NOT}) \\ &= 0.02 + 0.038 \times 5 + 0.05 + 0.017 \times 4 + T_2 + T_4 + 0.02 + 0.038 \times L_2 \\ &= 0.35 + T_2 + T_4 + 0.038 \times L_2 \\ t_{pHL}(s_2, z') &= t_{pHL}(\text{NOT}) + t_{pLH}(\text{NOT}) + t_{pLH}(\text{AND-5}) + t_{pLH}(\text{OR-8}) + t_{pHL}(\text{NOT}) \\ &= 0.05 + 0.017 \times 5 + 0.02 + 0.038 \times 4 + T_1 + T_3 + 0.05 + 0.017 \times L_2 \\ &= 0.36 + T_1 + T_3 + 0.017 \times L_2 \end{aligned}$$

The values of  $T_1, T_2, T_3$  and  $T_4$  are obtained by decomposition of the AND-5 and OR-8 gates into the gates listed in Table 3.1 of the book. This decomposition is:

$$\text{AND}(x_4, x_3, x_2, x_1, x_0) = \text{AND}(\text{AND}(x_4, x_3, x_2), \text{AND}(x_1, x_0))$$

and

$$\text{OR}(x_7, x_6, \dots, x_1, x_0) = \text{OR}(\text{OR}(x_7, x_6, x_5, x_4), \text{OR}(x_3, x_2, x_1, x_0))$$

The resulting delays are:

$$\begin{aligned} T_1 &= t_{pLH}(\text{AND-3}) + t_{pLH}(\text{AND-2}) = 0.2 + 0.038 + 0.15 + 0.037 = 0.43 \\ T_2 &= t_{pHL}(\text{AND-3}) + t_{pHL}(\text{AND-2}) = 0.18 + 0.018 + 0.16 + 0.017 = 0.38 \\ T_3 &= t_{pLH}(\text{OR-4}) + t_{pLH}(\text{OR-2}) = 0.13 + 0.038 + 0.12 + 0.037(L_1 + 1) = 0.33 + 0.037L_1 \\ T_4 &= t_{pHL}(\text{OR-4}) + t_{pHL}(\text{OR-2}) = 0.45 + 0.025 + 0.2 + 0.019(L_1 + 1) = 0.69 + 0.019L_1 \end{aligned}$$

(d) timing diagram presented in figure 4.7.

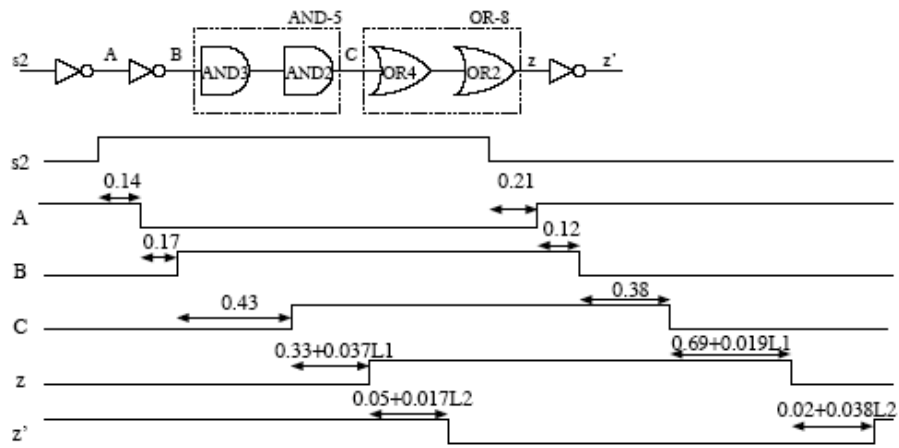


Figure 4.7: Timing diagram of Exercise 4.14