CS M51A, Sec. 1, Class Exercises No. 2 - SOLUTIONS

Exercise 2.41

CSP equivalent:

$$
\begin{aligned}
E(x,y,z) &= x' + x(x'y + y'z)' \\
&= x'(y+y')(z+z') + x((x'y)'.(y'z)') \\
&= x'yz + x'yz' + x'y'z + x'y'z' + x(x+y').(y+z') \\
&= x'yz + x'yz' + x'y'z + x'y'z' + x(xy + xz' + y'y + y'z') \\
&= x'yz + x'yz' + x'y'z + x'y'z' + xy + xz' + xy'z' \\
&= x'yz + x'yz' + x'y'z + x'y'z' + xy(z+z') + xz'(y+y') + xy'z' \\
&= x'yz + x'yz' + x'y'z + x'y'z' + xyz + xyz' + xyz' + xy'z' + xy'z' \\
&= x'yz + x'yz' + x'y'z + x'y'z' + xyz + xyz' + xy'z' \\
&= \sum m(0,1,2,3,4,6,7)
\end{aligned}
$$

CPS equivalent:

$$
\begin{aligned}
E(x,y,z) &= x' + x(x'y + y'z)' \\
&= (x'+x).(x' + (x'y+y'z)') \\
&= x' + (x'y)'(y'z)' \\
&= (x' + (x'y)')(x' + (y'z)') \\
&= (x' + x + y')(x' + y + z') \\
&= 1.(x' + y + z') \\
&= \prod M(5)
\end{aligned}
$$

Exercise 2.49

(a)

- Inputs: x, y where $x, y \in \{0, 1, 2, 3\}$
- Output: $z \in \{0, 1, 2, 3, 4, 6, 9\}$
- Function: $z = x \cdot y$

(b) The table for the arithmetic function is

x, y	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	4	6
3	0	3	6	9

(c) considering binary representation for inputs and outputs we obtain the following table:

	x		y		z			
i	x_1	x_0	y_1	y_0	z_3	z_2	z_1	z_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0
2	0	0	1	0	0	0	0	0
3	0	0	1	1	0	0	0	0
4	0	1	0	0	0	0	0	0
5	0	1	0	1	0	0	0	1
6	0	1	1	0	0	0	1	0
7	0	1	1	1	0	0	1	1
8	1	0	0	0	0	0	0	0
9	1	0	0	1	0	0	1	0
10	1	0	1	0	0	1	0	0
11	1	0	1	1	0	1	1	0
12	1	1	0	0	0	0	0	0
13	1	1	0	1	0	0	1	1
14	1	1	1	0	0	1	1	0
15	1	1	1	1	1	0	0	1

one-set$(z_3) = \{15\}$
one-set$(z_2) = \{10, 11, 14\}$
one-set$(z_1) = \{6, 7, 9, 11, 13, 14\}$
one-set$(z_0) = \{5, 7, 13, 15\}$

Exercise 2.50 (a) A conditional expression is

$$L = \begin{cases} ON \text{ **if** an odd number of switches are ON} \\ OFF \text{ **otherwise**} \end{cases}$$

The state of the light cannot be considered as an input because a feedback loop is formed and the circuit generated with this loop will not be stable (it will oscillate for some combinations of the switches).
(b) Calling w, x, y, z the four switches, the table for the switching functions is:

$wxyz$	L
0000	0
0001	1
0010	1
0011	0
0100	1
0101	0
0110	0
0111	1
1000	1
1001	0
1010	0
1011	1
1100	0
1101	1
1110	1
1111	0

(c) The minimal sum of products expression for this function is

$$L = w'x'y'z + w'x'yz' + w'xy'z' + w'xyz + wx'y'z' + wx'yz + wxy'z + wxyz'$$