

## **ComSci M51A**

**Lecture: Tuesdays, Thursdays 8:00 to 9:50**

**Discussion: Fridays 10:00 to 11:50**

- **Chapter 1-Introduction**

- **1.1-About Digital Systems**

- Digital system-a system in which signals have a finite number of discrete values
- Analog systems-signals have values from a continuous infinite set
- Systems with instants labeled at digital values labeled at discrete instants in which signals change due to the time being discretized are known as synchronous
- Those which changes may occur at any instant are called asynchronous
- Why are digital systems important?
  - Digital systems are used in information processing
    - Digital representation is well suited both numerical and nonnumerical information processing
    - Information processing can use a general-purpose system (a computer) which is programmed for a particular processing test
    - The finite number in digital signals can be represented with binary signals
      - Signals all represent just open and closed
    - Digital signals are quite insensitive to variations of component parameter values
      - Especially true for binary signals
    - Numerical digital systems can be made more accurate by adding the number of digits used in the representation
    - The advances of microelectronics technology in recent years have made possible the fabrication extremely complex digital signals, which are small, fast and cheap
      - Complex digital systems are built as integrated circuits composed of a large number of very simple devices
    - It is possible to select among different implementations of systems that trade-off speed and amount of hardware
  - When are digital systems used?
    - Analog and digital signals
      - Signals in there real world are analog, so it is necessary for us to convert digital signals to analog signals
      - Process of converting from analog to digital is called quantization or digitization
    - Combinational and sequential systems
      - Digital systems have two classes: combinational systems and sequential systems
        - Combination systems-the output at time t depends only on the input
          - These systems have no memory, because the system does not depend on previous systems
        - Sequential systems-the output at time t depends on the input at time t and possibly also depends on the input prior to t

- **1.2-Specification and Implementation, Analysis and Design**

- The specification of a system refers to a description of its function and of other characteristics required for its use
- Specification is related to what the system without reference to how it performs the operation
- Specs should be complete and as simple as possible
- Specifications of a systems must describe its function in a way that is adequate for two purposes:
  - To use the system as a component in more complex systems
  - To serve as the basis for the implementation of the system by a network of simpler components
- On the other hand, an implementation of a system refers to how the system is constructed from simpler components
  - With digital networks, this consists of the interconnection of digital modules
    - This network can be defined at many levels, ranging from complexity of the primitive models
      - I.e. gates to complex processors
- At the physical level, all digital systems are implemented by a complex interconnection of elementary elements such as transistors, resistors, and so on
- It is necessary to define intermediate levels of modules of increasing complexity, whose descriptions

- Modules are more than just conceptual entities used to simplify description of an implementation
  - Often built separately and then put together to finish the larger system
- The distinction between separation of specification of a system and its implementation is very important in complex systems because
  - It shields the description required to use the system from irrelevant implementation details
  - It allows choosing an implementation from different alternatives without influencing the description required for using the system
- Analysis of a system has the objective to determine its specification from implementation
- Design process consists of obtaining an implementation that satisfies the spec of a system
  - Top-down decomposes itself into subsystems in which are decomposed into smaller systems
    - Has the disadvantage that no systematic procedure exists to assure the decomposition at a particular level optimizes the final implementation
  - Bottom-up-Connects available modules to form subsystems and these subsystems are connected to the subsystems until the required functional specification is fulfilled
    - Disadvantage is similar to the top-down case
- Levels of an implementation
  - Module level consists of two registers and adder
  - Logical level consists of modules implemented with gates and flip-flops and signals are binary
  - Physical level are components that are realized in some technology
- **1.3-Computer-aided design tools**
  - CAD is used to help design of digital systems be efficient, timely, and economical
  - Description of digital systems is performed in a hierarchical manner
    - Description provides a logic diagram of the system at different levels, showing the modules and their interconnections
    - This process is called a schematic capture because the tool used to capture the schematic description of the digital system
    - Process is supported by libraries of standard components
    - An alternative method is using a hardware-description language
  - Synthesis and optimization tools help in obtaining an implementation from a given description in improving some characteristics such as the number of modules and network delays
  - Simulation tools are used to verify the operation of a system
    - These can be used to detect errors in a design and to determine characteristics
- **Chapter 2-Appendix A-Boolean Algebras**
- Boolean algebras is an important class of algebras
- Switching algebra is an instance of boolean algebras
  - **A.1-Boolean Algebra**
    - Boolean algebra is a tuple  $\{B, +, \cdot\}$ , where in
      - B is a set of elements
      - + and  $\cdot$  Are binary operations applied over the elements of B
    - Which satisfies the following postulates
      - P1: Commutative
        - $a+b=b+a$
        - $a \cdot b=b \cdot a$
      - P2: Associative
        - $a + (b \cdot c) = (a + b) \cdot (a + c)$
        - $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
      - P3: Identity
        - $0 + a = a + 0 = a$
        - $1 \cdot a = a \cdot 1 = a$
      - P4: Complement
        - $a+a'=1$
        - $a \cdot A'=0$
  - **A.2-Switching Algebra**
    - Switching algebra is a system used to describe switching functions by means of switching expressions
    - Switching algebra contains  $B=\{0,1\}$  and operations AND and OR

- Operations are used to evaluate switching expressions
- Theorem 1
  - The switching algebra is a Boolean Algebra
- P1: Commutativity of  $(+)$ ,  $(\cdot)$ 
  - Shown by inspection of the operation tables
    - Holds if operation is symmetric about the main diagonal
- P2: Distributivity of  $(+)$ ,  $(\cdot)$ 
  - Shown by perfect induction, by considering all possible values of the elements  $a, b, c$
- P3: Existence of additive and multiplicative identity element
  - $0 + 1 = 1 + 0 = 1$
  - $0 \cdot 1 = 1 \cdot 0 = 0$
- P4: Existence of the complement
  - 1 is the complement of 0 and 0 is the complement of 1
- **A.3-Important Theorems in Boolean Algebra**
  - Theorem 2: Principle of duality
    - Every algebraic identity deducible from the postulates of a Boolean algebra remains valid if
      - The operations  $+$  and  $\cdot$  are interchanged throughout
      - The identity elements 0 and 1 are also interchanged throughout
  - Theorem 3
    - Every element in B has a unique complement
  - Theorem 4
    - For any  $a$  in B  $a + a' = 1$ ,  $a \cdot a' = 0$
  - Theorem 5
    - The complement of the element 1 is 0 and vice versa
  - Theorem 6 Idempotent Law
    - For every  $a$  in B
      - $a + a = a$
      - $a \cdot a = a$
  - Theorem 7 Involution Law
    - For every  $a$  in B,
      - $(a')' = a$
  - Theorem 8 Absorption Law
    - For every pair of element  $a, b$  in B
      - $a + a \cdot b = a$
      - $a \cdot (a + b) = a$
  - Theorem 9
    - For every pair of elements  $a, b$  in B
      - $a + a' = a + b$
      - $a(a' + b) = ab$
  - Theorem 10
    - In a Boolean algebra, each of the binary operators,  $(+)$  and  $(\cdot)$  is associative. That is, for every  $a, b, c$  in B
      - $a + (b + c) = (a + b) + c$
      - $a(bc) = (ab)c$
  - Corollary 1
    - The order in applying the  $+$  and  $\cdot$  operator among  $n$  elements does not matter
  - Theorem 11 DeMorgan's Laws
    - For every pair of elements  $a, b$  in B:
      - $(a + b)' = a'b'$
      - $(ab)' = a' + b'$
  - Theorem 12 Generalized DeMorgan's Laws
    - Let  $\{a, b, \dots, c, d\}$  be a set elements in a Boolean algebra. Then, the following identities hold:
      - $(a + b + \dots + c + d)' = [(a + b + \dots + c) + d]'$
      - $= (a + b + \dots + c)'d'$

- Other examples of Boolean Algebras
  - Algebra of sets
  - Algebra of Logic (Propositional Calculus)
- **Chapter 3-Combinatorial Integrated Circuits: Characteristics and Capabilities**
  - Logic gates-and, or, nor, etc
  - Circuit level-Physical representation of the gates
  - PMOS-Pull-up
  - NMOS-Pull-down
  - The not gate is formed by simply two gates inverting the input value to allow for output
  - AND and OR gates must be created by adding a NOT gate to invert the value of the circuit
  - Transmission gates are often used to represent XOR
  - We cannot get instantaneous change, so we will have propagation delay
  - Total load is the sum of all load factors
    - Assume load factor is 1
- **Chapter 4-Description and Analysis of Gate Networks**
  - Gate network-Interconnections of data
  - Connections all carry signals
  - Description of a gate network contains
    - Graphical representation (Logic diagram)
    - Tabular representation (a net list)
    - Representation based on a hardware description language (a set of language statements)
  - Specification of a gate network includes
    - Functional specification
    - The input load factors of the network inputs
    - The fanout factors of the network outputs
    - The propagation delays through the network
  - Universal sets
    - {AND, OR, NOT}
    - {AND, NOT}
    - {OR, NOT}
    - {NAND}
    - {NOR}
- **Chapter 5-Design of Combinational Systems: Two-Level Gate Networks**
  - Two level networks are networks with two layers of gates
  - Karnaugh maps are a method of determining a minimal expression via a visual representation
  - Implicant is a value 1 in the karnaugh map
  - Prime implicant is the largest  $2^n$  value of grouping 1s
  - Essential Prime implicants contain at least one 1 which is not covered by any other primes
- **Chapter 7-Specification of Sequential Systems**
  - Sequential system
  - Synchronous and asynchronous
  - State diagrams
  - Initial case
  - States influence the output
  - Note input also influence output
  - Two types of state diagrams
- **Chapter 8-Sequential Networks**
  - Sequential Network is a collection of combinational and sequential networks
  - Canonical Implementation
    - Huffman-Moore implementation is based directly on the state description of a system
    - State register stores the state
    - Combinational network implements the transition and output functions
    - Synchronizing signal determines time instants at which the next state is loaded into the state register
    - Clock pulses
    - Initialize is the input to initialize the state

- Flip-Flop
  - JK
  - D
  - SR
  - T
- Only flips the output value
- We only really care about how everything initiates wrt the clock
- Clock will always run at any given time
- **Chapter 9-Standard Combinational Modules**
  - n-input binary decoder-combinational system that has n binary outputs
  - Coincident decoding and tree decoding
  - $2^n$  output binary-decoder
  - Additional I/O can be used with module enable E
  - $2^n$  input binary-multiplexer
  - $2^n$  output binary-demultiplexer
  - Multiplexer trees
  - Simple Shifter
  - p-shifter
  - Unidirectional shifter
- **Chapter 10**
  - Adders
    - Full
      - With carry in and carry out
    - Half
      - Without carry in and carry out
  - Carry ripple adder