Quiz Problems (50 points total)

Problem 1 (15 points)

Minimize the number of states of the system that correspond to the table shown below. Show the final minimized table. The input is x and the output is z.

	Input	
PS	$x=0$	$x=1$
A	$K,0$	$C,1$
B	$G,0$	$E,1$
C	$B,0$	$F,0$
D	$A,1$	$G,0$
E	$I,0$	$D,1$
F	$A,0$	$J,1$
G	$B,0$	$E,1$
H	$A,1$	$B,0$
I	$E,0$	$D,1$
J	$G,0$	$A,0$
K	$A,0$	$K,1$
	NS,z	

Solution Looking at the output values, we can write
$P_1 = (A, B, E, F, G, I, K)$ (output 0/1), (C, J) (output 0/0), (D, H) (output 1/0)

From here we move to:

	group1							g2		g3	
	A	B	E	F	G	I	K	C	J	D	H
0	1	1	1	1	1	1	1	1	1	1	1
1	2	1	3	2	1	3	1	1	1	1	1

$P_2 = (A, F), (B, G, K), (E, I), (C, J), (D, H)$

	g1		g2			g3		g4		g5	
	A	F	B	G	K	E	I	C	J	D	H
0	2	1	2	2	1	3	3	2	2	1	1
1	4	4	3	3	2	5	5	1	1	2	2

$P_3 = (A), (F), (B, G), (K), (E, I), (C, J), (D, H)$

	g1	g2	g3		g4	g5		g6		g7	
	A	F	B	G	K	E	I	C	J	D	H
0			3	3		5	5	3	3	1	1
1			5	5		7	7	2	1	3	3

$P_4 = (A), (F), (B, G), (K), (E, I), (C), (J), (D, H)$

	g1	g2	g3		g4	g5		g6	g7	g8	
	A	F	B	G	K	E	I	C	J	D	H
0			3	3		5	5			1	1
1			5	5		8	8			3	3

Now $P_5 = P_4$ and no more reductions are possible. The minimal table is:

PS	$x = 0$	$x = 1$
$G1$	$G4$, 0	$G6$, 1
$G2$	$G1$, 0	$G7$, 1
$G3$	$G3$, 0	$G5$, 1
$G4$	$G1$, 0	$G4$, 1
$G5$	$G5$, 0	$G8$, 1
$G6$	$G3$, 0	$G2$, 0
$G7$	$G3$, 0	$G1$, 0
$G8$	$G1$, 1	$G3$, 0
	NS, z	

Problem 2 (20 points)

Design a binary string detector using the state vector approach to cover all possible states, and deriving the minimum number of states afterwards. The detector takes a stream of binary bits as input, one bit at each clock. The output is 1 when it detects a string of 001, 011 or 100. The input signal is $x(t)$, and the output signal is $z(t)$.

1. **(5 points)** Fill in the empty slots in the state transition table.

   Solution

PS	$x = 0$	$x = 1$
S_{init}	S_0, 0	S_1, 0
S_0	S_{00}, 0	S_{01}, 0
S_1	S_{10}, 0	S_{11}, 0
S_{00}	S_{00}, 0	S_{01}, 1
S_{01}	S_{10}, 0	S_{11}, 1
S_{10}	S_{00}, 1	S_{01}, 0
S_{11}	S_{10}, 0	S_{11}, 0
	NS, z	

2. **(15 points)** Minimize the number of states in the transition table, and show the final minimized table. Use $G1$ to $G7$ for the minimized group names.

   Solution Looking at the output of the previous table, we first get P_1 as shown:

   $P_1 = (S_{init}, S_0, S_1, S_{11})$ (output 0/0), (S_{00}, S_{01}) (output 0/1), and (S_{10}) (output is 1/0).

	group 1				group 2		g3
	S_{init}	S_0	S_1	S_{11}	S_{00}	S_{01}	S_{10}
0	1	2	3	3	2	3	
1	1	2	1	1	2	1	

$P_2 = (S_{init}), (S_0), (S_1, S_{11}), (S_{00}), (S_{01}), (S_{10})$

	g1	g2	g3		g4	g5	g6
	S_{init}	S_0	S_1	S_{11}	S_{00}	S_{01}	S_{10}
0			6	6			
1			3	3			

We can see that P_3 will be the same as P_2 and we stop here. By naming each group, we can write the following table.

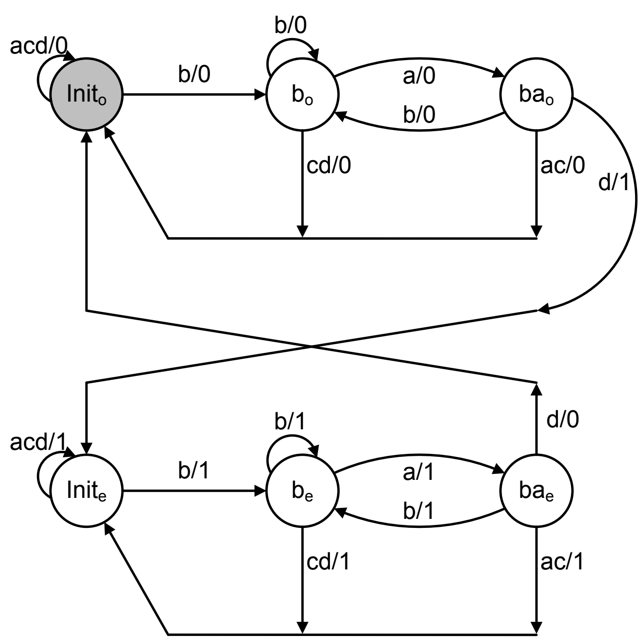
PS	$x = 0$	$x = 1$
$G1$	$G2$, 0	$G3$, 0
$G2$	$G4$, 0	$G5$, 0
$G3$	$G6$, 0	$G3$, 0
$G4$	$G4$, 0	$G5$, 1
$G5$	$G6$, 0	$G3$, 1
$G6$	$G4$, 1	$G5$, 0
	NS, z	

Problem 3 (15 points)

A sequential system has an input set $I = \{a, b, c, d\}$ and an output set $O = \{0, 1\}$. The system tracks the number of occurrences of the string *bad* in $x(0, t)$. It outputs 1 if the value is an odd number and 0 otherwise. Show the state diagram of the system. Use the minimum-number-of-states approach. (*Hint: You may need multiple instances of the same string detector.*)

For each state, clearly show the transition path for all variables in the input set I and the output values. Clearly identify the initial state.

Solution We need two separate instances of the string detector for *bad*, one for the even case and one for the odd case. Every time we encounter the string *bad*, we jump from one detector to the other as shown. The initial state is shaded in gray.



For a Moore machine, all states for the odd case output 1, all states for the even case output 0.