§5.2 The Characteristic Equation

How do we compute the eigenvalues of a matrix that is not triangular?

We know that the eigenvalues of a square matrix A are the scalars λ such that $(A - \lambda I)\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

If $(A - \lambda I)\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then $A - \lambda I$ is not invertible.

$A - \lambda I$ fails to be invertible when its determinant is zero.

Recall if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $ad - bc$ is $\det A$.

A scalar λ is an eigenvalue of a square matrix A if and only if λ satisfies the equation $\det(A - \lambda I) = 0$.

Characteristic Equation – $\det(A - \lambda I) = 0$

Characteristic Polynomial – $\det(A - \lambda I)$

Multiplicity: The number of times a particular eigenvalue λ is repeated.