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<u>Ex:</u> Let $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$, $\vec{\varphi} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$.

Compute $\vec{w} \cdot \vec{w}$, $\vec{\varphi} \cdot \vec{w}$, $\dfrac{\vec{\varphi} \cdot \vec{w}}{\vec{w} \cdot \vec{w}}$.

$$\vec{w} \cdot \vec{w} = (3)(3) + (-1)(-1) + (-5)(-5) = 35$$

$$\vec{\varphi} \cdot \vec{w} = (6)(3) + (-2)(-1) + (3)(-5) = 5$$

$$\frac{\vec{\varphi} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} = \frac{5}{35} = \frac{1}{7}$$

Ex: Let $\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Compute $\dfrac{1}{\vec{u} \cdot \vec{u}} \vec{u}$.

$$\vec{u} \cdot \vec{u} = (-1)(-1) + (2)(2) = 5$$

$$\frac{1}{5} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} \\ \frac{2}{5} \end{bmatrix}$$

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Ex: Let $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$, $\vec{\varphi} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$.

Compute $\left(\dfrac{\vec{\varphi} \cdot \vec{w}}{\vec{\varphi} \cdot \vec{\varphi}} \right) \vec{\varphi}$.

$\vec{\varphi} \cdot \vec{w} = 5$

$\vec{\varphi} \cdot \vec{\varphi} = (6)(6) + (-2)(-2) + (3)(3) = 49$

$$\left(\frac{\vec{\varphi} \cdot \vec{w}}{\vec{\varphi} \cdot \vec{\varphi}} \right) \vec{\varphi} = \frac{5}{49} \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{30}{49} \\ -\frac{10}{49} \\ \frac{15}{49} \end{bmatrix}$$

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__Ex:__ Let $\vec{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$.

Compute $\|\vec{x}\|$.

$$\|\vec{x}\| = \sqrt{\vec{x} \cdot \vec{x}} = \sqrt{49} = 7$$

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Ex: Find a unit vector in the direction of the given vector.

$$\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$$

Let $\vec{v}$ represent the given vector.

$$\vec{v} \cdot \vec{v} = (-6)(-6) + (4)(4) + (-3)(-3) = 61$$

$$||\vec{v}|| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{61}$$

$$\frac{1}{||\vec{v}||}\vec{v} = \frac{1}{\sqrt{61}}\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -\frac{6}{\sqrt{61}} \\ \frac{4}{\sqrt{61}} \\ -\frac{3}{\sqrt{61}} \end{bmatrix}$$

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Ex: Find a unit vector in the direction of the given vector.

$$\begin{bmatrix} \frac{8}{3} \\ 2 \end{bmatrix}$$

Let $\vec{v}$ represent the given vector.

$$\vec{v} \cdot \vec{v} = \left(\frac{8}{3}\right)\left(\frac{8}{3}\right) + (2)(2) = \frac{100}{9}$$

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} = \frac{10}{3}$$

$$\frac{1}{\|\vec{v}\|}\vec{v} = \frac{3}{10}\begin{bmatrix} \frac{8}{3} \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5} \end{bmatrix}$$

Ex: Find the distance between $\vec{u} = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$ and $\vec{z} = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$.

$$\text{dist}(\vec{u}, \vec{z}) = \left[(-4-0)^2 + (-1-(-5))^2 + (8-2)^2\right]^{\frac{1}{2}}$$

$$= (16+16+36)^{\frac{1}{2}}$$

$$= \sqrt{68}$$

$$= 2\sqrt{17}$$

<u>Eg</u>: Determine if $\vec{u} = \begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$ are orthogonal.

$$\vec{u} \cdot \vec{v} = (12)(2) + (3)(-3) + (-5)(3) = 0$$

Thus $\vec{u}$ and $\vec{v}$ are orthogonal.

Ex: Determine if $\vec{y} = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}$ and $\vec{z} = \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$ are orthogonal.

$$\vec{y} \cdot \vec{z} = (-3)(1) + (7)(-8) + (4)(15) + (0)(-7) = 1$$

Therefore $\vec{y}$ and $\vec{z}$ are not orthogonal.