8-1
1

Ex: Compute AB if $A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$.

A is 3×2.

B is 2×2.

Therefore AB is 3×2.

$$AB = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

2-1
2

Ex: How many rows does B have if BC is a 3×4 matrix?

The product BC exists if and only if the number of columns of B equals the number of rows of C. Since the product does exist, its size will be rows of B $\times$ columns of C. With BC being 3×4, we conclude that B has 3 rows.

2.1

3

Ex: Compute AB and AC if

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}.$$

$$AB = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

2-1
4

Ex: Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$. Construct a 2×2 matrix B such that AB is the zero matrix. Use two different nonzero columns for B.

Let $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

Then $AB = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$= \begin{bmatrix} 3b_{11} - 6b_{21} & 3b_{12} - 6b_{22} \\ -b_{11} + 2b_{21} & -b_{12} + 2b_{22} \end{bmatrix}$$

Since AB is the zero matrix, it follows that $b_{11} = 2b_{21}$ and $b_{12} = 2b_{22}$. By inspection,

set $B = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$.

Ex: Let $\vec{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

Compute $\vec{u}^T\vec{v}$, $\vec{v}^T\vec{u}$, $\vec{u}\vec{v}^T$, and $\vec{v}\vec{u}^T$.

The product $\vec{u}^T\vec{v}$ is a 1×1 matrix, which usually is identified with a real number and is written without the matrix brackets. The product $\vec{u}\vec{v}^T$ is an $n \times n$ matrix.

$$\vec{u}^T\vec{v} = \begin{bmatrix} -2 & 3 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -2a + 3b - 4c$$

$$\vec{v}^T\vec{u} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} = -2a + 3b - 4e$$

$$\vec{u}\vec{v}^T = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \\ -4a & -4b & -4c \end{bmatrix}$$

$$\vec{v}\vec{u}^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} -2 & 3 & -4 \end{bmatrix} = \begin{bmatrix} -2a & 3a & -4a \\ -2b & 3b & -4b \\ -2c & 3c & -4c \end{bmatrix}$$