1-5
1

Ex: Determine if the system has a nontrivial solution.

$$x_1 - 3x_2 + 7x_3 = 0$$
$$-2x_1 + x_2 - 4x_3 = 0$$
$$x_1 + 2x_2 + 9x_3 = 0$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ -2 & 1 & -4 & 0 \\ 1 & 2 & 9 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 5 & 2 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 0 & 12 & 0 \end{array}\right]$$

the system has only the trivial solution.

1-5
2

Ex: Determine if the system has a nontrivial solution.

$$-5x_1 + 7x_2 + 9x_3 = 0$$
$$x_1 - 2x_2 + 6x_3 = 0$$

$$\left[\begin{array}{ccc|c} -5 & 7 & 9 & 0 \\ 1 & -2 & 6 & 0 \end{array}\right]$$

More unknowns (columns) than equations (rows). The system will have a nontrivial solution.

1-5

3

Ex: Solve $x_1 + 3x_2 - 5x_3 = 0$

$x_1 + 4x_2 - 8x_3 = 0$

$-3x_1 - 7x_2 + 9x_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 3 & -5 & 0 \\ 1 & 4 & -8 & 0 \\ -3 & -7 & 9 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & -6 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = -4x_3$

$x_2 = 3x_3$

x_3 is free

In parametric form, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$

1-5
4

Ex: Solve $\left[\begin{array}{cccc|c} 1 & -2 & -9 & 5 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{array}\right]$

$$\left[\begin{array}{cccc|c} 1 & -2 & -9 & 5 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & -5 & -7 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{array}\right]$$

$x_1 = 5x_3 + 7x_4$

$x_2 = -2x_3 + 6x_4$

x_3 is free

x_4 is free

In parametric form, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5x_3 + 7x_4 \\ -2x_3 + 6x_4 \\ x_3 \\ x_4 \end{bmatrix}$

$$= x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

1-5
5

Ex: Solve $\left[\begin{array}{cccc|c} 1 & 3 & 0 & -4 & 0 \\ 2 & 6 & 0 & -8 & 0 \end{array}\right]$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -4 & 0 \\ 2 & 6 & 0 & -8 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 3 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = -3x_2 + 4x_4$

x_2 is free

x_3 is free

x_4 is free

In parametric form, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3x_2 + 4x_4 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

$$= x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

15
6

Ex: Solve $\left(\begin{array}{cccccc|c} 1 & 5 & 2 & -6 & 9 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$$\left[\begin{array}{cccccc|c} 1 & 5 & 2 & -6 & 9 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccccc|c} 1 & 5 & 2 & -6 & 9 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim$$

$$\left(\begin{array}{cccccc|c} 1 & 5 & 0 & 8 & 1 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = -5x_2 - 8x_4 - x_5$

x_2 is free

$x_3 = 7x_4 - 4x_5$

x_4 is free

x_5 is free

$x_6 = 0$

In parametric form $\vec{x} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ -4 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

15
7

$\vec{ex}$: Given $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, find one nontrivial solution of $A\vec{x} = 0$ (zero vector) by inspection.

Note that $\vec{a_2} = -\frac{3}{2}\vec{a_1}$ so that

$\frac{3}{2}\vec{a_1} + \vec{a_2} = 0$ (zero vector)

$\Rightarrow 3\vec{a_1} + 2\vec{a_2} = 0$ (zero vector)

thus $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ satisfies $A\vec{x} = 0$ (zero vector)