Ex: Let W be the union of the first and third quadrants in the xy-plane. That is, let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, xy \geq 0 \right\}$.

- If $\vec{u}$ is in W and c is any scalar, is $c\vec{u}$ in W?

If $\vec{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ is in W, then the vector

$c\vec{u} = c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$ is in W because

$(cx)(cy) = c^2(xy) \geq 0$ since $xy \geq 0$.

- Find specific vectors $\vec{u}$ and $\vec{v}$ in W such that $\vec{u} + \vec{v}$ is not in W.

If $\vec{u} = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then $\vec{u}$ and $\vec{v}$ are in W but $\vec{u} + \vec{v}$ is not in W.

4-1
8

Ex: Determine if the given set is a subspace of $\mathbb{P}_n$ for an appropriate value of n.

All polynomials of the form $\vec{p}(t) = a + t^2$, where a is in $\mathbb{R}$.

No. The zero vector is not in the set.

4-1
3

Ex: Determine if the given set is a subspace of $\mathbb{P}_n$ for an appropriate value of n.

All polynomials in $\mathbb{P}_n$ such that $\vec{p}(0) = 0$.

Yes.

Let H represent the given set.

The zero vector is in the set H.

If $\vec{p}$ and $\vec{q}$ are in H, then $(\vec{p}+\vec{q})(0) = \vec{p}(0) + \vec{q}(0) = 0 + 0 = 0$,

so $\vec{p}+\vec{q}$ is in H.

For any scalar c, $(c\vec{p})(0) = c \cdot \vec{p}(0) = c \cdot 0 = 0$,

so $c\vec{p}$ is in H.

Thus H is a subspace.

4-1
4

ex! Let H be the set of all vectors of the form

$\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}$. Show that H is a subspace of $\mathbb{R}^3$.

Let $H = \text{span}\{\vec{v}\}$ where $\vec{v} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.

Thus H is a subspace of $\mathbb{R}^3$.

4.1

5

Ex: Let W be the set of all vectors of the form

$\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^4$.

Let $W = \text{span}\{\vec{u}, \vec{v}\}$ where $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix}$.

Thus W is a subspace of $\mathbb{R}^4$.

4-1
6

Ep: Define $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and let $\vec{w} = \begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}$.

Is $\vec{w}$ in the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ -1 & 3 & 6 & 7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 5 & 10 & 15 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

no solution

$\vec{w}$ is not in the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

Ex! Let W be the set of all vectors of the form

$$\begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \end{bmatrix},$$ where a, b, and c represent arbitrary

real numbers. Find a set S of vectors that spans W or give an example to show that W is not a vector space.

Since a vector in W may be written as

$$\vec{w} = a\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ -6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

we conclude that the zero vector is not in W and therefore W is not a vector space.

Ex: Let W be the set of all vectors of the form

$$\begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix},$$ where $a, b,$ and c represent arbitrary real numbers. Find a set S of vectors that spans W or give an example to show that W is not a vector space.

Since a vector in W may be written as

$$\vec{w} = a\begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix} + b\begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

we conclude that $S = \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

is a set that spans W.