§1.2 Row Reduction / Echelon Forms

A nonzero row or column in a matrix refers to a row or column that contains at least one nonzero entry.

A leading entry of a row refers to the leftmost nonzero entry (in a nonzero row).

A rectangular matrix is in echelon form (row echelon form) if

- All nonzero rows are above any rows of all zeros.
- Each leading entry of a row is in a column to the right of the leading entry of the row above it (stairstep down).
- All entries in a column below a leading entry are zeros.

Reduced echelon form (reduced row echelon form) satisfies the previous three conditions as well as

- The leading entry in each nonzero row is 1 (one).
- Each leading 1 (one) is the only nonzero entry in its column.

A pivot position in a matrix A is a location in A that corresponds to a leading 1 (one) in the reduced echelon form of A. A pivot column is a column of A that contains a pivot position.

A pivot is a nonzero number in a pivot position that is used as needed to create zeros via row operations.

Row Reduction Algorithm

1. Begin with the leftmost nonzero column. This is a pivot column. The pivot position is at the top.
2. Select a nonzero entry in the pivot column as a pivot. If necessary, interchange rows to move this entry into the pivot position.
3. Use row replacement operations to create zeros in all positions below the pivot.
4. Cover the row containing the pivot position and cover all rows, if any, above it. Apply steps 1-3 to the submatrix that remains. Repeat the process until there are no more nonzero rows to modify.
5. Beginning with the rightmost pivot and working upward and to the left, create zeros above each pivot. If a pivot is not 1 (one), make it 1 (one) by a scaling operation.

Forward Phase – Steps 1-4

Backward Phase – Step 5

The variables corresponding to pivot columns in the matrix are called basic variables.

Variables in all other columns are called free variables.

Solving a system amounts to finding a parametric description of the solution set or determining that the solution set is empty. The free variables act as parameters.