$5-\lambda$

Ex: Find the characteristic polynomial and the eigenvalues of

$$A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (5-\lambda)^2 - 9 = \lambda^2 - 10\lambda + 16 = (\lambda-2)(\lambda-8)$$

$$\det(A-\lambda I) = 0 \Rightarrow \lambda = 2, \lambda = 8.$$

$5-8$

λ

Ex: Find the characteristic polynomial and the eigenvalues of

$$A = \begin{bmatrix} 5 & -3 \\ -4 & 3 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & -3 \\ -4 & 3-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (5-\lambda)(3-\lambda) - 12 = \lambda^2 - 8\lambda + 3$$

$$\det(A-\lambda I) = 0 \Rightarrow \lambda = 4 \pm \sqrt{13}.$$

<u>Ex</u>: Find the characteristic polynomial and the eigenvalues of

$$A = \begin{bmatrix} 3 & -4 \\ 4 & 8 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & -4 \\ 4 & 8-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (3-\lambda)(8-\lambda) + 16 = \lambda^2 - 11\lambda + 40$$

$$\det(A - \lambda I) = 0 \Rightarrow \lambda = \frac{-11 \pm \sqrt{-39}}{2}$$

these values are complex numbers, so A has no real eigenvalues. There is no nonzero vector $\vec{x}$ in $\mathbb{R}^2$ such that $A\vec{x} = \lambda\vec{x}$, because a real vector $A\vec{x}$ cannot equal a complex multiple of $\vec{x}$.

5-2
4

Ex: Find the characteristic polynomial and the eigenvalues of

$$A = \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} 7-\lambda & -2 \\ 2 & 3-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (7-\lambda)(3-\lambda) + 4 = \lambda^2 - 10\lambda + 25 = (\lambda-5)^2$$

$$\det(A-\lambda I) = 0 \Rightarrow \lambda = 5 \text{ (multiplicity 2)}$$

$5-2$
5

Ex! Find the characteristic polynomial of

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} -\lambda & 3 & 1 \\ 3 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = -\lambda \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} - 3 \begin{vmatrix} 3 & 2 \\ 1 & -\lambda \end{vmatrix} + \begin{vmatrix} 3 & -\lambda \\ 1 & 2 \end{vmatrix}$$

1st Row

$$= -\lambda(\lambda^2 - 4) - 3(-3\lambda - 2) + (6 + \lambda)$$

$$= -\lambda^3 + 14\lambda + 12$$

5-8

6

Ex: Find the characteristic polynomial of

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} -1-\lambda & 0 & 1 \\ -3 & 4-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (2-\lambda) \begin{vmatrix} -1-\lambda & 0 \\ -3 & 4-\lambda \end{vmatrix}$$

3rd Row

$$= (2-\lambda)(-1-\lambda)(4-\lambda)$$

$$= (2-\lambda)(\lambda^2 - 3\lambda - 4)$$

$$= -\lambda^3 + 3\lambda^2 + 4\lambda + 2\lambda^2 - 6\lambda - 8$$

$$= -\lambda^3 + 5\lambda^2 - 2\lambda - 8$$

$5-\lambda$

7

<u>Eg</u>: Find the characteristic polynomial of

$$A = \begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}.$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & -2 & 3 \\ 0 & 1-\lambda & 0 \\ 6 & 7 & -2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda) \begin{vmatrix} 5-\lambda & 3 \\ 6 & -2-\lambda \end{vmatrix}$$

2nd Row

$$= (1-\lambda)\left[(5-\lambda)(-2-\lambda) - 18\right]$$

$$= (1-\lambda)(\lambda^2 - 3\lambda - 28)$$

$$= -\lambda^3 + 3\lambda^2 + 28\lambda + \lambda^2 - 3\lambda - 28$$

$$= -\lambda^3 + 4\lambda^2 + 25\lambda - 28$$

5-8

8

Ex! List the eigenvalues of

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 8 & -4 & 0 & 0 \\ 0 & 7 & 1 & 0 \\ 1 & -5 & 8 & 1 \end{bmatrix}.$$

Since A is triangular, its eigenvalues are the entries along the main diagonal.

Hence $\lambda = 5$

$\lambda = -4$

$\lambda = 1$ (multiplicity 2)