

## §2.5 The LU Factorization

Let  $A$  be an  $m \times n$  matrix that can be row reduced to echelon form. Then  $A$  can be written in the form  $A = LU$ , where  $L$  is an  $m \times m$  lower triangular matrix with 1's on the diagonal and  $U$  is an  $m \times n$  echelon form of  $A$ . Such a factorization is called an  $LU$  factorization of  $A$ . Furthermore, the matrix  $L$  is invertible and is called a unit lower triangular matrix.

An  $m \times n$  upper triangular matrix is one whose entries below the main diagonal are 0's.

An  $m \times n$  lower triangular matrix is one whose entries above the main diagonal are 0's.

Unit - 1's along the main diagonal

## Algorithm for an $LU$ Factorization

1. Reduce  $A$  to an echelon form  $U$  by a sequence of row replacement operations, if possible.
2. Place entries in  $L$  such that the same sequence of row operations reduces  $L$  to  $I$ .