Solutions/Scoring Guide
20 Points Automatic

Describe all solutions in parametric vector form of the system whose augmented matrix is

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -7 \\ 0 & 1 & 0 & 0 & -10 & 9 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -7 \\ 0 & 1 & 0 & 0 & -10 & 9 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & -10 & 9 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim$$

+4

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -28 & 27 \\ 0 & 1 & 0 & 0 & -10 & 9 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

+3

$$\mathbf{x} = \begin{bmatrix} 28x_5 + 27 \\ 10x_5 + 9 \\ x_3 \\ -2x_5 + 7 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 28 \\ 10 \\ 0 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 27 \\ 9 \\ 0 \\ 7 \\ 0 \end{bmatrix}$$

+1 +1 +1

Determine if **b** is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & 6 \\ 0 & 8 & 9 \\ -5 & 10 & -30 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 2 \\ 0 & 8 & 9 & -4 \\ -5 & 10 & -30 & -2 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 2 \\ 0 & 8 & 9 & -4 \\ 0 & 0 & 0 & 8 \end{array}\right]$$ +4

The system for this augmented matrix is inconsistent, so **b** is not a linear combination of the columns of A. +4

Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 6 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -4 \\ -16 \\ 8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 6 \\ 12 \\ h \end{bmatrix}$. For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

$$\left[\begin{array}{cc|c} 1 & -4 & 6 \\ 6 & -16 & 12 \\ -1 & 8 & h \end{array}\right] \sim$$ +1

$$\left[\begin{array}{cc|c} 1 & -4 & 6 \\ 0 & 8 & -24 \\ 0 & 4 & h+6 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{cc|c} 1 & -4 & 6 \\ 0 & 1 & -3 \\ 0 & 4 & h+6 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{cc|c} 1 & -4 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & h+18 \end{array}\right]$$ +2

The vector $\mathbf{b}$ is in $\text{span}\{\mathbf{a}_1, \mathbf{a}_2\}$ when $h + 18$ is zero, that is, when $h = -18$. +3

Determine if the columns of the matrix $A = \begin{bmatrix} 0 & -8 & 16 \\ 3 & 1 & -14 \\ -1 & 5 & -3 \\ 1 & -5 & -2 \end{bmatrix}$ form a linearly independent set.

$$\left[\begin{array}{ccc|c} 0 & -8 & 16 & 0 \\ 3 & 1 & -14 & 0 \\ -1 & 5 & -3 & 0 \\ 1 & -5 & -2 & 0 \end{array}\right] \sim$$ +1

$$\left[\begin{array}{ccc|c} 1 & -5 & -2 & 0 \\ 3 & 1 & -14 & 0 \\ -1 & 5 & -3 & 0 \\ 0 & -8 & 16 & 0 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|c} 1 & -5 & -2 & 0 \\ 0 & 16 & -8 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & -8 & 16 & 0 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|c} 1 & -5 & -2 & 0 \\ 0 & 8 & -4 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & -8 & -16 & 0 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|c} 1 & -5 & -2 & 0 \\ 0 & 8 & -4 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -20 & 0 \end{array}\right]$$ +2

There are no free variables. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution and so the columns of A are linearly independent. +1

Find the value(s) of h for which the vectors are linearly dependent.

$$\begin{bmatrix} 2 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -4 & -2 & 0 \\ -2 & 2 & 2 & 0 \\ -4 & 7 & h & 0 \end{array}\right] \sim$$ +1

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ -2 & 2 & 2 & 0 \\ -4 & 7 & h & 0 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & h-4 & 0 \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & h-4 & 0 \end{array}\right]$$ +2

The homogeneous system has a nontrivial solution if and only if $h - 4 = 0$ (making x_3 a free variable). Therefore the vectors are linearly dependent if and only if $h = 4$. +3

Find the inverse of $A = \begin{bmatrix} 5 & 5 \\ -9 & -5 \end{bmatrix}$.

$$\begin{bmatrix} 5 & 5 \\ -9 & -5 \end{bmatrix}^{-1} = \frac{1}{-25-(-45)} \begin{bmatrix} -5 & -5 \\ 9 & 5 \end{bmatrix}$$

+4 +4

$$= \frac{1}{20} \begin{bmatrix} -5 & -5 \\ 9 & 5 \end{bmatrix}$$ +1

$$= \begin{bmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{9}{20} & \frac{1}{4} \end{bmatrix}$$ +1

Find the inverse of $A = \begin{bmatrix} 1 & 0 & -2 \\ 6 & 1 & 3 \\ 2 & -7 & 3 \end{bmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 6 & 1 & 3 & 0 & 1 & 0 \\ 2 & -7 & 3 & 0 & 0 & 1 \end{array}\right] \sim$$ +1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 15 & -6 & 1 & 0 \\ 0 & -7 & 7 & -2 & 0 & 1 \end{array}\right] \sim$$ +1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 15 & -6 & 1 & 0 \\ 0 & 0 & 112 & -44 & 7 & 1 \end{array}\right] \sim$$ +1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 15 & -6 & 1 & 0 \\ 0 & 0 & 1 & -\frac{11}{28} & \frac{1}{16} & \frac{1}{112} \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{28} & \frac{1}{16} & -\frac{15}{112} \\ 0 & 0 & 1 & -\frac{11}{28} & \frac{1}{16} & \frac{1}{112} \end{array}\right] \sim$$ +2

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{14} & \frac{1}{8} & \frac{1}{56} \\ 0 & 1 & 0 & -\frac{3}{28} & \frac{1}{16} & -\frac{15}{112} \\ 0 & 0 & 1 & -\frac{11}{28} & \frac{1}{16} & \frac{1}{112} \end{array}\right]$$ +2

$$A^{-1} = \begin{bmatrix} \frac{3}{14} & \frac{1}{8} & \frac{1}{56} \\ -\frac{3}{28} & \frac{1}{16} & -\frac{15}{112} \\ -\frac{11}{28} & \frac{1}{16} & \frac{1}{112} \end{bmatrix}$$ +1

Find an LU factorization of $A = \begin{bmatrix} 2 & 3 & -3 & -4 \\ 6 & 12 & -7 & -11 \\ 10 & 24 & -6 & -15 \\ 4 & 15 & -9 & -10 \end{bmatrix}$.

$$\begin{bmatrix} 2 & 3 & -3 & -4 \\ 6 & 12 & -7 & -11 \\ 10 & 24 & -6 & -15 \\ 4 & 15 & -9 & -10 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & 3 & -3 & -4 \\ 0 & 3 & 2 & 1 \\ 0 & 9 & 9 & 5 \\ 0 & 9 & -3 & -2 \end{bmatrix} \sim$$ +2

$$\begin{bmatrix} 2 & 3 & -3 & -4 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & -9 & -5 \end{bmatrix} \sim$$ +2

$$\begin{bmatrix} 2 & 3 & -3 & -4 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U$$ +1

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 5 & 3 & 1 & 0 \\ 2 & 3 & -3 & 1 \end{bmatrix}$$ +5