§1.5 Solution Sets of Linear Systems

A system of linear equations is said to be homogeneous if it can be written in the form $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$. A homogeneous system always has at least one solution, namely, $\mathbf{x} = \mathbf{0}$ (the zero vector in $\mathbb{R}^n$). The zero solution is referred to as the trivial solution. A nontrivial solution to $A\mathbf{x} = \mathbf{0}$ is a nonzero vector $\mathbf{x}$ that satisfies $A\mathbf{x} = \mathbf{0}$.

The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution if and only if the equation has at least one free variable.

Writing a Solution Set (of a consistent system) in Parametric Vector Form

1. Row reduce the augmented matrix to reduced echelon form.
2. Express each basic variable in terms of any free variables appearing in an equation.
3. Write a typical solution $\mathbf{x}$ as a vector whose entries depend on the free variables, if any.

4. Decompose $\mathbf{x}$ into a linear combination of vectors (with numeric entries) using the free variables as parameters.