Ex: Solve using Cramer's Rule.

$$4x_1 + x_2 = 6$$
$$3x_1 + 2x_2 = 7$$

$$\left[\begin{array}{cc|c} 4 & 1 & 6 \\ 3 & 2 & 7 \end{array}\right]$$

$$x_1 = \frac{\begin{vmatrix} 6 & 1 \\ 7 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}} = \frac{5}{5} = 1$$

$$x_2 = \frac{\begin{vmatrix} 4 & 6 \\ 3 & 7 \end{vmatrix}}{\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}} = \frac{10}{5} = 2$$

3-3
2

Ex: Solve using Cramer's Rule.

$$-5x_1 + 2x_2 = 9$$
$$3x_1 - x_2 = -4$$

$$\left[\begin{array}{cc|c} -5 & 2 & 9 \\ 3 & -1 & -4 \end{array}\right]$$

$$x_1 = \frac{\begin{vmatrix} 9 & 2 \\ -4 & -1 \end{vmatrix}}{\begin{vmatrix} -5 & 2 \\ 3 & -1 \end{vmatrix}} = \frac{-1}{-1} = 1$$

$$x_2 = \frac{\begin{vmatrix} -5 & 9 \\ 3 & -4 \end{vmatrix}}{\begin{vmatrix} -5 & 2 \\ 3 & -1 \end{vmatrix}} = \frac{-7}{-1} = 7$$

Ex: Solve using Cramer's Rule.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ -x_1 \quad + 2x_3 &= 2 \\ 3x_1 + x_2 \quad &= 2 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ -1 & 0 & 2 & 2 \\ 3 & 1 & 0 & 2 \end{array}\right]$$

$$x_1 = \frac{\begin{vmatrix} 4 & 3 & 1 \\ 2 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 0 \end{vmatrix}} = \frac{1\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}}{1\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}} = \frac{2-2(-2)}{-1-2(-8)} = \frac{2}{5}$$

$$x_2 = \frac{\begin{vmatrix} 1 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 0 \end{vmatrix}} = \frac{1\begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}}{15} = \frac{-8-2(-10)}{15} = \frac{4}{5}$$

$$x_3 = \frac{\begin{vmatrix} 1 & 3 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 0 \end{vmatrix}} = \frac{-3\begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ -1 & 2 \end{vmatrix}}{15} = \frac{-3(8)-(6)}{15} = \frac{6}{5}$$

Ex: Solve using Cramer's Rule.

$$\begin{aligned} x_1 + x_2 + x_3 &= 5 \\ x_1 - 2x_2 - 3x_3 &= -1 \\ 2x_1 + x_2 - x_3 &= 3 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & -2 & -3 & -1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$$x_1 = \frac{\begin{vmatrix} 5 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}} = \frac{5\begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ 3 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix}}{1\begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}} = 4$$

$$x_2 = \frac{\begin{vmatrix} 1 & 5 & 1 \\ 1 & -1 & -3 \\ 2 & 3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}} = \frac{1\begin{vmatrix} -1 & -3 \\ 3 & -1 \end{vmatrix} - 5\begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}}{5} = -2$$

$$x_3 = \frac{\begin{vmatrix} 1 & 1 & 5 \\ 1 & -2 & -1 \\ 2 & 1 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}} = \frac{1\begin{vmatrix} -2 & -1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + 5\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}}{5} = 3$$

3-1

5

Ex: Compute the adjugate and use it to find the inverse of the matrix.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \qquad C_{12} = -\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} = 2 \qquad C_{13} = \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} = -2$$

$$C_{21} = -\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 2 \qquad C_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1 \qquad C_{23} = -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1$$

$$C_{31} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5 \qquad C_{32} = -\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = -7 \qquad C_{33} = \begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} = 4$$

$$\operatorname{adj} A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & 1 & -7 \\ -2 & -1 & 4 \end{bmatrix} \qquad \det A = -1\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} = -3$$

$$A^{-1} = \frac{1}{\det A}\operatorname{adj} A = -\frac{1}{3}\begin{bmatrix} 1 & 2 & -5 \\ 2 & 1 & -7 \\ -2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} & \frac{5}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{7}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{4}{3} \end{bmatrix}$$

3-3

6

Ex: Compute the adjugate and use it to find the inverse of the matrix.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 4 \end{bmatrix}$$

$$C_{11} = \begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} = 8 \qquad C_{12} = -\begin{vmatrix} 0 & 1 \\ 2 & 4 \end{vmatrix} = 2 \qquad C_{13} = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

$$C_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix} = 4 \qquad C_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0 \qquad C_{23} = -\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -2$$

$$C_{31} = \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -5 \qquad C_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = -1 \qquad C_{33} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2$$

$$\operatorname{adj} A = \begin{bmatrix} 8 & 4 & -5 \\ 2 & 0 & -1 \\ -4 & -2 & 2 \end{bmatrix} \qquad \det A = 1\begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} + 2\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -2$$

$$A^{-1} = \frac{1}{\det A}\operatorname{adj} A = -\frac{1}{2}\begin{bmatrix} 8 & 4 & -5 \\ 2 & 0 & -1 \\ -4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -2 & \frac{5}{2} \\ -1 & 0 & \frac{1}{2} \\ 2 & 1 & -1 \end{bmatrix}$$

Ex: Compute the adjugate and use it to find the inverse of the matrix.

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

$$C_{11} = \begin{vmatrix} -3 & 1 \\ 0 & -2 \end{vmatrix} = 6 \qquad C_{12} = -\begin{vmatrix} 0 & 1 \\ 0 & -2 \end{vmatrix} = 0 \qquad C_{13} = \begin{vmatrix} 0 & -3 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{21} = -\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = 4 \qquad C_{22} = \begin{vmatrix} 1 & 4 \\ 0 & -2 \end{vmatrix} = -2 \qquad C_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{31} = \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = 14 \qquad C_{32} = -\begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = -1 \qquad C_{33} = \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -3$$

$$\operatorname{adj} A = \begin{pmatrix} 6 & 4 & 14 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{pmatrix} \qquad \det A = 6$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A = \frac{1}{6} \begin{pmatrix} 6 & 4 & 14 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{pmatrix} = \begin{pmatrix} 1 & \frac{2}{3} & \frac{7}{3} \\ 0 & -\frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$$