§5.1 Eigenvectors and Eigenvalues

Let A be an $n \times n$ matrix. A nonzero vector $\mathbf{x}$ is an eigenvector if $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ.

A scalar λ is an eigenvalue if $A\mathbf{x} = \lambda\mathbf{x}$ has a nontrivial solution. That is, the equation $A\mathbf{x} = \lambda\mathbf{x}$ has a free variable.

$$A\mathbf{x} = \lambda\mathbf{x} \Rightarrow A\mathbf{x} - \lambda\mathbf{x} = \mathbf{0} \Rightarrow (A - \lambda I)\mathbf{x} = \mathbf{0}$$

A scalar λ is an eigenvalue if $(A - \lambda I)\mathbf{x} = \mathbf{0}$ has a nontrivial solution. That is, the equation $(A - \lambda I)\mathbf{x} = \mathbf{0}$ has a free variable.

The set of all solutions of $(A - \lambda I)\mathbf{x} = \mathbf{0}$ is the null space of the matrix $A - \lambda I$.

The null space of $A - \lambda I$ is a subspace of $\mathbb{R}^n$ and is called the eigenspace of A corresponding to λ.

The eigenspace consists of the zero vector and all eigenvectors corresponding to λ.

The eigenvalues of a triangular matrix are the main diagonal entries.