§5.3 Diagonalization

The eigenvalue-eigenvector relationship for a particular square matrix A can provide a useful factorization of the form $A = PDP^{-1}$ where P is an invertible matrix and D is a diagonal matrix.

The factorization allows for an easy and quick computation of A^k for large values of k.

A square matrix A is called diagonalizable if $A = PDP^{-1}$ for some invertible P and diagonal D.

An $n \times n$ matrix A is diagonalizable if and only if A has n linearly independent eigenvectors. That is, $A = PDP^{-1}$ if and only if the columns of P are n linearly independent eigenvectors of A and the diagonal entries of D are the eigenvalues of A that correspond, respectively, to the eigenvectors in P.

A is diagonalizable if and only if there are enough eigenvectors to form a basis (eigenvector basis) of $\mathbb{R}^n$.

An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.

Steps to Diagonalize an $n \times n$ Square Matrix A

1. Determine the eigenvalues of A: $\lambda_1, \dots, \lambda_n$.
2. Determine n linearly independent eigenvectors: $\mathbf{v}_1, \dots, \mathbf{v}_n$.
3. Construct diagonal matrix D using $\lambda_1, \dots, \lambda_n$ along the main diagonal.
4. Construct matrix P using $\mathbf{v}_1, \dots, \mathbf{v}_n$.