4-4

1

Ex: Find the vector $\vec{x}$ determined by the given coordinate vector $[\vec{x}]_{\mathcal{B}}$ and the given basis $\mathcal{B}$.

$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}$, $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$

$$\vec{x} = c_1 \vec{b}_1 + c_2 \vec{b}_2$$

$$= 8 \begin{bmatrix} 4 \\ 5 \end{bmatrix} + (-5) \begin{bmatrix} 6 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

4-4
2

ex: Find the vector $\vec{x}$ determined by the given coordinate vector $[\vec{x}]_\beta$ and the given basis β.

$$\beta = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}, \quad [\vec{x}]_\beta = \begin{bmatrix} -4 \\ 8 \\ -7 \end{bmatrix}$$

$$\vec{x} = c_1 \vec{b_1} + c_2 \vec{b_2} + c_3 \vec{b_3}$$

$$= -4 \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + 8 \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix} - 7 \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$$

4-4

3

Ex: Find the coordinate vector $[\vec{x}]_{\beta}$ of $\vec{x}$ relative to the given basis $\beta = \{\vec{b}_1, \ldots, \vec{b}_n\}$.

$\vec{b}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$.

$$\vec{x} = c_1\vec{b}_1 + c_2\vec{b}_2 \Rightarrow \begin{bmatrix} 4 \\ 0 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2\begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 5 & 4 \\ -2 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 5 & 4 \\ 0 & 4 & 8 \end{array}\right] \Rightarrow c_1 = -6,\ c_2 = 2$$

Thus $[\vec{x}]_{\beta} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$.

4-4

4

Ex: Find the coordinate vector $[\vec{x}]_{\beta}$ of $\vec{x}$ relative to the given basis $\beta = \{\vec{b}_1, \dots, \vec{b}_n\}$.

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \vec{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \quad \vec{b}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}.$$

$$\vec{x} = c_1\vec{b}_1 + c_2\vec{b}_2 + c_3\vec{b}_3 \Rightarrow \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 3 & 8 & 2 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Hence $[\vec{x}]_{\beta} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$.

4.4

5

<u>Ex</u>: Find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^n$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 7 \end{bmatrix} \right\}$$

The change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^3$ is

$$P_{\mathcal{B}} = \begin{bmatrix} 3 & 2 & 8 \\ -1 & 0 & -2 \\ 4 & -5 & 7 \end{bmatrix}.$$

Ex: Use an inverse matrix to find $[\vec{x}]_\beta$ for

$$\beta = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}, \quad \vec{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

$$\vec{x} = P_\beta [\vec{x}]_\beta \Rightarrow [\vec{x}]_\beta = P_\beta^{-1} \vec{x}$$

$$P_\beta = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix} \Rightarrow P^{-1}_\beta = \frac{1}{-2} \begin{bmatrix} 7 & -6 \\ -5 & 4 \end{bmatrix}$$

$$[\vec{x}]_\beta = P^{-1}_\beta \vec{x} = -\frac{1}{2} \begin{bmatrix} 7 & -6 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

Ex: The set $\beta = \{1-t^2,\ t-t^2,\ 2-2t+t^2\}$

is a basis for $\mathbb{P}_2$. Find the coordinate vector of

$\vec{p}(t) = 3 + t - 6t^2$ relative to β.

$$3+t-6t^2 = c_1(1-t^2) + c_2(t-t^2) + c_3(2-2t+t^2)$$

$$= (c_1+2c_3) + t(c_2-2c_3) + t^2(-c_1-c_2+c_3)$$

$c_1 + 2c_3 = 3 \qquad c_2 - 2c_3 = 1 \qquad -c_1 - c_2 + c_3 = -6$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ -1 & -1 & 1 & -6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Therefore $[\vec{p}]_\beta = \begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix}$.