1-7

Ex: Determine if the vectors are linearly independent.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

the homogeneous system has only the trivial solution. the vector are linearly independent.

1-7

2

Ex: Determine if the vectors are linearly independent.

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$$

Neither vector is a multiple of the other.

The vectors are linearly independent.

Ex: Determine if the columns of the matrix form a linearly independent set.

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 5 & 4 & 6 & 0 \\ -4 & -3 & 0 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 4 & -9 & 0 \\ 0 & -3 & 12 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 4 & -9 & 0 \\ 0 & -3 & 12 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system has no free variables and only the trivial solution. The columns of the matrix are linearly independent.

1-7
4

Ex: Determine if the columns of the matrix form a linearly independent set.

$$\begin{bmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{bmatrix}$$

The corresponding homogeneous system would have 3 equations and 4 unknowns. There are more variables than equations, so there must be a free variable. Therefore the homogeneous system has a nontrivial solution, and the columns of the given matrix are linearly dependent.

1-7

5

Ex: For what values of h is $\vec{v}_3$ in span$\{\vec{v}_1, \vec{v}_2\}$ and for what values of h is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ linearly dependent.

$\vec{v}_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & h+6 \end{array}\right]$$

contradiction, $\vec{v}_3$ is in span$\{\vec{v}_1, \vec{v}_2\}$ for no value of h.

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ -5 & 10 & -9 & 0 \\ -3 & 6 & h & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & h+6 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

For all h, x_2 is a free variable so the homogeneous equation has a nontrivial solution.

Thus $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent.

1-7

6

Ex: Find the value(s) of h for which the vectors are linearly dependent.

$$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 4 & 0 \\ -4 & 7 & h & 0 \\ 2 & -6 & 8 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 0 \\ 0 & -5 & h+16 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

For all h, x_3 is a free variable so the homogeneous equation has a nontrivial solution.

Therefore the vectors are linearly dependent.

1-7

7

Ex: Find the value(s) of h for which the vectors

are linearly dependent.

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ -1 & 7 & 1 & 0 \\ 3 & 8 & h & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 23 & h-3 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 23 & h-3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & h-26 & 0 \end{array}\right]$$

The homogeneous system has a nontrivial solution if and only if $h-26=0$ (making x_3 a free variable).

Therefore the vectors are linearly dependent if and only if $h=26$.

1-7

8

<u>Ex</u>: Determine whether the vectors are linearly independent.

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

linearly dependent since $\vec{v}_2 = \frac{3}{2}\vec{v}_1$.

1-7

9

Ex: Determine whether the vectors are linearly independent.

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

linearly dependent since each of the four vectors are in $\mathbb{R}^2$.

1-7

10

Ex: Determine whether the vectors are linearly independent.

$$\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

linearly dependent since the set of vectors contains the zero vector.

1-7

11

Ex: Given $A = \begin{pmatrix} 4 & 1 & 6 \\ -7 & 5 & 3 \\ 9 & -3 & 3 \end{pmatrix}$, observe that the

first column plus twice the second column equals the third column. Find a nontrivial solution of $A\vec{x} = 0$ (zero vector)

$$\vec{a}_1 + 2\vec{a}_2 = \vec{a}_3 \Rightarrow \vec{a}_1 + 2\vec{a}_2 - \vec{a}_3 = 0 \text{ (zero vector)}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$