Ex: Determine if $\vec{b}$ is a linear combination of $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$.

$$\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 2 & 5 & 8 & -7 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

no solution

not a linear combination

13
2

Ex: Determine if $\vec{b}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{array}\right]$$

System has a solution

$\vec{b}$ is a linear combination

1-3
3

Ex: Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$, and $\vec{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$.

For what value(s) of h is $\vec{y}$ in the plane generated by $\vec{v}_1$ and $\vec{v}_2$?

$$\left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & 2h-3 \end{array}\right] \sim$$

$$\left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & 2h+7 \end{array}\right]$$

The vector $\vec{y}$ is in $\text{span}\{\vec{v}_1, \vec{v}_2\}$ when $0 = 2h+7$, that is, when $h = -\frac{7}{2}$.

1-3
4

Ex: Give a geometric description of span $\{\vec{v}_1, \vec{v}_2\}$ for the vectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$.

span $\{\vec{v}_1, \vec{v}_2\}$ is a plane in $\mathbb{R}^3$ through the origin, because neither vector is a multiple of the other. Every vector in the set has 0 as its second entry and so lies the xz-plane in ordinary 3-space. So span $\{\vec{v}_1, \vec{v}_2\}$ is the xz-plane.

13
5

Ex: Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$, and

let W be the set of all linear combinations of the columns of A.

— Is $\vec{b}$ in W?

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ -1 & 8 & 5 & 3 \\ 2 & 0 & 6 & 10 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 6 & 6 & 6 \\ 0 & 4 & 4 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

System has a solution, $\vec{b}$ is in W.

— Show that the third column of A is in W.

The third column of A is in W since

$$\vec{a}_3 = 0 \cdot \vec{a}_1 + 0 \cdot \vec{a}_2 + 1 \cdot \vec{a}_3.$$