§1.7 Linear Independence

A set of vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is said to be linearly independent if the vector equation $x_1\mathbf{v}_1 + \cdots + x_p\mathbf{v}_p = \mathbf{0}$ has only the trivial solution. The set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is said to be linearly dependent if there exist weights $c_1, \dots, c_p$, not all zero, such that $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$.

The columns of a matrix A are linearly independent if and only if the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

A set containing only one vector, $\mathbf{v}$, is linearly independent if and only if $\mathbf{v}$ is not the zero vector since the vector equation $x_1\mathbf{v} = \mathbf{0}$ has only the trivial solution when $\mathbf{v} \neq \mathbf{0}$. The zero vector is linearly dependent because $x_1\mathbf{0} = \mathbf{0}$ has many nontrivial solutions.

A set of two vectors is linearly dependent if at least one of the vectors is a multiple of the other. The set is linearly independent if and only if neither of the vectors is a multiple of the other.

A set of two or more vectors is linearly dependent if and only if at least one of the vectors in the set is a linear combination of the others.

If a set contains more vectors than there are entries in each vector, then the set is linearly dependent.

More unknowns than equations $\Longleftrightarrow$ More columns than rows $\Longleftrightarrow$ Must be a free variable and therefore a nontrivial solution

If a set of vectors contains the zero vector, then the set is linearly dependent.