§2.2 The Inverse of a Matrix

An $n \times n$ matrix A is said to be invertible if there is an $n \times n$ matrix C such that $CA = I$ and $AC = I$ where $I = I_n$, the $n \times n$ identity matrix. We say that C is the inverse of A with $C = A^{-1}$.

Singular / not invertible

Nonsingular / invertible

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $ad - bc \neq 0$, then A is invertible and

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The quantity $ad - bc$ is called the determinant of A and we write

$$\det A = ad - bc$$

Thus a 2×2 matrix A is invertible if and only if $\det A \neq 0$.

Algorithm for finding A^{-1}

1. Row reduce the augmented matrix $[A \quad I]$.
2. If A is row equivalent to I, then $[A \quad I]$ is row equivalent to $[I \quad A^{-1}]$.
   Otherwise, A does not have an inverse.