§4.2 Null Space/Column Space/Row Space

The null space of an $m \times n$ matrix A, denoted Nul A, is the set of all solutions of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.

The column space of an $m \times n$ matrix A, denoted Col A, is the set of all linear combinations of the columns of A. If $A = [\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_n]$, then
Col $A = \text{span}\{\mathbf{a}_1, \dots, \mathbf{a}_n\}$.

The column space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^m$.

In order to find a spanning set for the null space of a particular matrix, solve the corresponding homogeneous system and write the basic variables in terms of the free variables. Decompose the vector giving the general solution into a linear combination of vectors where the weights are the free variables. The spanning set produced is automatically linearly independent because the free variables are the weights on the spanning vectors. When the null space contains nonzero vectors, the number of vectors in the spanning set for the null space equals the number of free variables in the corresponding homogeneous system.

The row space of an $m \times n$ matrix A, denoted Row A, is the set of all linear combinations of the rows of A.

The row space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.