HC
1

Ex: Solve $A\vec{x} = \vec{b}$ if $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & -2 & -2 \end{array}\right]$$

$-2x_3 = -2 \Rightarrow x_3 = 1$

$5x_2 + 5x_3 = 1 \Rightarrow x_2 = -\frac{4}{5}$

$x_1 + 2x_2 + x_3 = 0 \Rightarrow x_1 = \frac{3}{5}$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$

1-4
2

Ex: Let $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$.

Is $\vec{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A?

$$\left[\begin{array}{ccc|c} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 5 & 8 & 7 & 2 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -7 & 7 & -8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{array}\right]$$

no solution

$\vec{u}$ is not in the subset spanned by the columns of A.

Ex: Let $\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$.

Does $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ span $\mathbb{R}^3$?

$$\begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -1 \\ -2 & 8 & -5 \end{bmatrix} \sim \begin{bmatrix} -2 & 8 & -5 \\ 0 & -3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

The matrix $[\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$ has a pivot in each row, so the columns of the matrix span $\mathbb{R}^3$, that is, $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ spans $\mathbb{R}^3$.

M4
4

Ex: Let $\vec{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$.

It can be shown that $3\vec{u} - 5\vec{v} - \vec{w} = 0$ (zero vector)

Find x_1 and x_2 that satisfy the equation

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}.$$

The equation in x_1 and x_2 involves the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$, and it may be viewed as $[\vec{u}\ \vec{v}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{w}$. By the definition of a matrix-vector product, $x_1\vec{u} + x_2\vec{v} = \vec{w}$. The stated fact that $3\vec{u} - 5\vec{v} - \vec{w} = 0$ can be rewritten as $3\vec{u} - 5\vec{v} = \vec{w}$. Therefore a solution is $x_1 = 3$ and $x_2 = -5$.