

Ex: Compute the determinant of $\begin{bmatrix} 0 & 4 & 1 \\ 5 & -3 & 0 \\ 8 & 3 & 1 \end{bmatrix}$.

1st Row

$$0 \begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix} - 4 \begin{vmatrix} 5 & 0 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 5 & -3 \\ 2 & 3 \end{vmatrix}$$

$$= -4(5) + (21)$$

$$= 1$$

3-1

8

Ex: Compute the determinant of $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix}$.

1st Column

$$-2 \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 4 \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= -2(4) + (-6) - 4(-11)$$

$$= 30$$

3-1

3

Ex: Compute the determinant of $\begin{bmatrix} 5 & -2 & 2 \\ 0 & 3 & -3 \\ 8 & -4 & 7 \end{bmatrix}$.

3rd Row

$$8 \begin{vmatrix} -2 & 2 \\ 3 & -3 \end{vmatrix} - (-4) \begin{vmatrix} 5 & 2 \\ 0 & -3 \end{vmatrix} + 7 \begin{vmatrix} 5 & -2 \\ 0 & 3 \end{vmatrix}$$

$$= 8(0) + 4(-15) + 7(15)$$

$$= 45$$

3-1

4

Ex: Compute the determinant of $\begin{bmatrix} 4 & 1 & 2 \\ 4 & 0 & 3 \\ 3 & -2 & 5 \end{bmatrix}$.

1st Row

$$-4 \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} + 0 \begin{vmatrix} 4 & 2 \\ 3 & 5 \end{vmatrix} - 3 \begin{vmatrix} 4 & 1 \\ 3 & -2 \end{vmatrix}$$

$$= -4(9) - 3(-11)$$

$$= -3$$

3-1
5

Ex: Compute the determinant of $\begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{bmatrix}$.

2nd Row

$$-3 \begin{vmatrix} 1 & -2 & 2 \\ 2 & -4 & 5 \\ 2 & 0 & 5 \end{vmatrix}$$

3rd Row

$$-3 \left[2 \begin{vmatrix} -2 & 2 \\ -4 & 5 \end{vmatrix} + 5 \begin{vmatrix} 1 & -2 \\ 2 & -4 \end{vmatrix} \right]$$

$$= -3 [2(-2) + 5(0)]$$

$$= 12$$

3-1

6

Ex: Compute the determinant of $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 7 & -2 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 3 & -8 & 4 & -3 \end{bmatrix}$.

Matrix is triangular

Multiply along the main diagonal.

$$(3)(-2)(3)(-3)$$

$$= 54$$

3-1

7

Ex: Compute the determinant of

$$\begin{bmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{bmatrix}.$$

4th Row

$$-2 \begin{vmatrix} 3 & 2 & 4 & 0 \\ 0 & -4 & 1 & 0 \\ -5 & 6 & 7 & 1 \\ 2 & 3 & 2 & 0 \end{vmatrix}$$

4th Column

$$-2 \left[-1 \begin{vmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{vmatrix} \right]$$

2nd Row

$$-2 \left[-1 \left[-4 \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} - 1 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} \right] \right]$$

$$= -2 \left[-1 \left[-4(-2) - (5) \right] \right]$$

$$= 6$$