

§3.3 Cramer's Rule

Let A be an invertible $n \times n$ matrix. For any \mathbf{b} in \mathbb{R}^n , the unique solution \mathbf{x} of $A\mathbf{x} = \mathbf{b}$ has entries given by

$$x_i = \frac{\det A_i(\mathbf{b})}{\det A}, \quad i = 1, 2, \dots, n$$

Let A be an invertible $n \times n$ matrix. Then

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

The matrix $\operatorname{adj} A$ is called the adjugate (or classical adjoint) of A and is given by

$$\begin{bmatrix} C_{11} & C_{21} & \cdots & C_{n1} \\ C_{12} & C_{22} & \cdots & C_{n2} \\ \vdots & \vdots & & \vdots \\ C_{1n} & C_{2n} & \cdots & C_{nn} \end{bmatrix}$$

The adjugate matrix is the transpose of the matrix of cofactors.