6-4

Ex: The given set is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

$$\vec{e}_1 = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$$

Set $\vec{v}_1 = \vec{e}_1$.

$$\vec{v}_2 = \vec{e}_2 - \frac{\vec{e}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1$$

$$= \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix} - \frac{10}{20} \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix} \right\}$ is an orthogonal basis for W.

6.4
2

Ex: The given set is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

$$\vec{x}_1 = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix}$$

Set $\vec{v}_1 = \vec{x}_1$.

$$\vec{v}_2 = \vec{x}_2 - \frac{\vec{x}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1$$

$$= \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix} - \frac{(-100)}{50} \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} \right\}$ is an orthogonal basis for W.

6.4
3

Ex: The given set is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

$$\vec{x}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix}$$

Set $\vec{v}_1 = \vec{x}_1$.

$$\vec{v}_2 = \vec{x}_2 - \frac{\vec{x}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1$$

$$= \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix} - \frac{(-45)}{15} \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 6 \\ -3 \\ 0 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -3 \\ 0 \end{bmatrix} \right\}$ is an orthogonal basis for W.

6-4
4

Ex: Find an orthonormal basis of the subspace spanned by the vectors $\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix}$.

An orthogonal basis is $\left\{ \underset{\vec{v}_1}{\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}}, \underset{\vec{v}_2}{\begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}} \right\}$.

$\|\vec{v}_1\| = \sqrt{50} = 5\sqrt{2}$

$\|\vec{v}_2\| = \sqrt{54} = 3\sqrt{6}$

So $\left\{ \begin{bmatrix} \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \right\}$ is an orthonormal basis for W.

6-4
5

Ex: Find an orthogonal basis for the column space of

$$\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}.$$

Let $\vec{x}_1, \vec{x}_2, \vec{x}_3$ represent the columns of the matrix and

set $\vec{v}_1 = \vec{x}_1$.

$$\vec{v}_2 = \vec{x}_2 - \frac{\vec{x}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\vec{v}_1 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} - \frac{(-36)}{12}\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{v}_3 = \vec{x}_3 - \frac{\vec{x}_3 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\vec{v}_1 - \frac{\vec{x}_3 \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2}\vec{v}_2 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \frac{6}{12}\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} - \frac{30}{12}\begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$ is an orthogonal basis for the column space.

Ex: Find an orthogonal basis for the column space of

$$\begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}.$$

Let $\vec{x}_1, \vec{x}_2, \vec{x}_3$ represent the columns of the matrix and set $\vec{v}_1 = \vec{x}_1$.

$$\vec{v}_2 = \vec{x}_2 - \frac{\vec{x}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\vec{v}_1 = \begin{bmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{bmatrix} - \frac{16}{4}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{v}_3 = \vec{x}_3 - \frac{\vec{x}_3 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\vec{v}_1 - \frac{\vec{x}_3 \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2}\vec{v}_2 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{bmatrix} - \frac{14}{4}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{12}{8}\begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3 \\ 0 \\ -3 \\ 3 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ -3 \\ 3 \end{bmatrix} \right\}$ is an orthogonal basis for the column space.